On ‚Higgs‘ modes and the optical conductivity in O(2) models in condensed matter physics

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in collaboration with:

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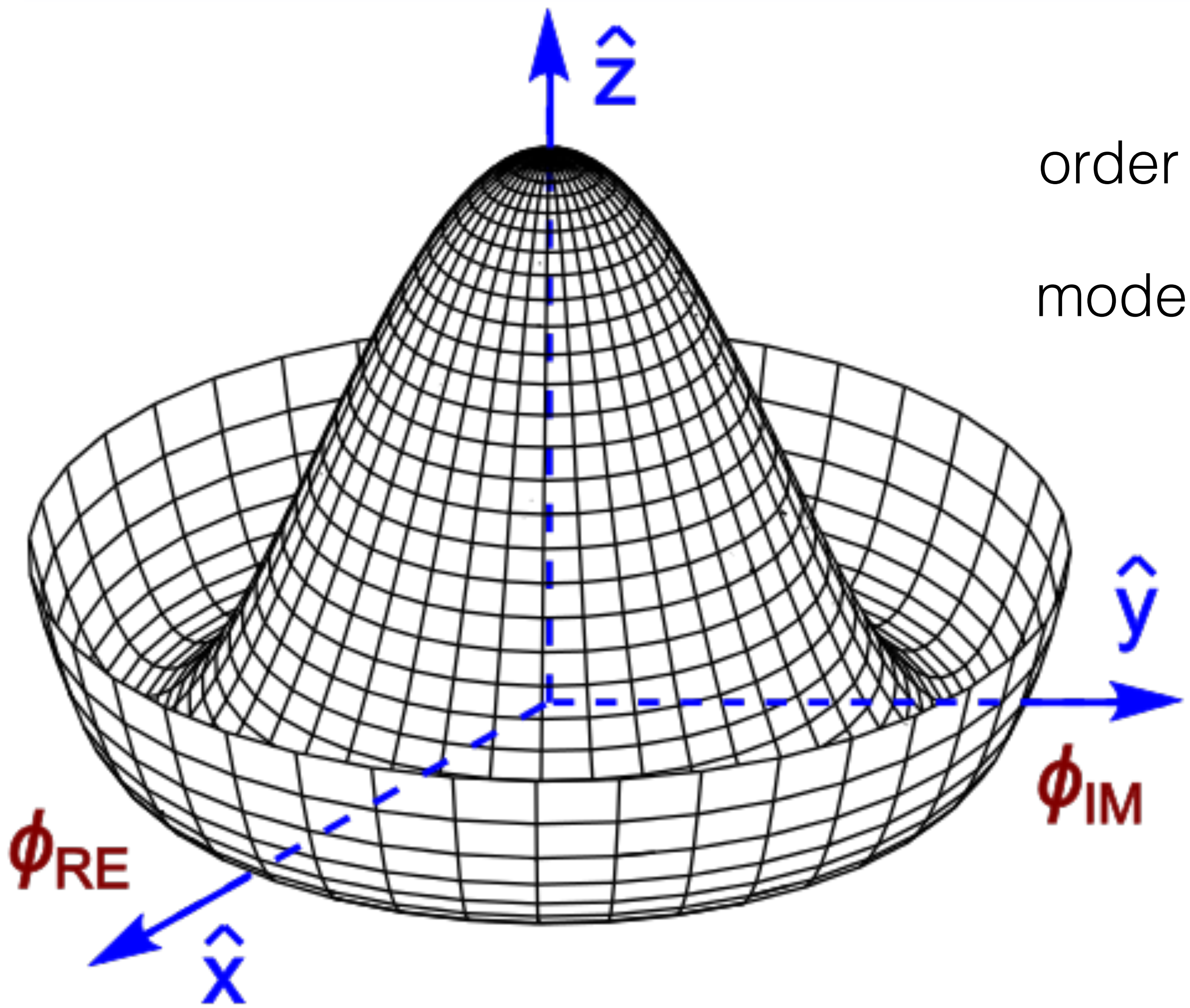
Ref: PRL 2012, PRL 2013, PRL 2014

Mexican hat potential



order parameter: $\Psi(r,t) = |\Psi(r,t)|\, e^{i\phi(r,t)}$

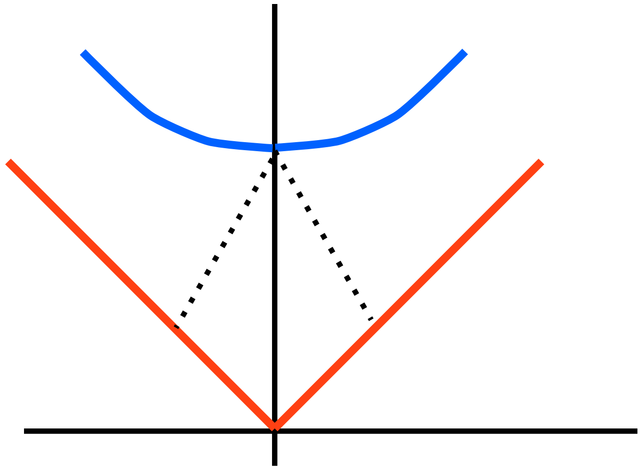
models: O(N)

O(2) : superfluids
O(3) : antiferromagnets

fluctuations of the modulus of order parameter = scalar

hence amplitude mode is hard to couple to

necessary condition: explicit/ emergent Lorentz invariance



decomposition of fluctuations of order parameter into:
- longitudinal & transverse
- radial & tangential

this help understanding behavior of different correlation functions

global gauge invariance

Consider a relativistic quantum field theory with mass m, and a complex scalar field

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \frac{1}{2}\lambda(\phi^*\phi)^2$$

or, for negative mass,

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + m^2 \phi^* \phi - \frac{1}{2}\lambda(\phi^*\phi)^2$$

The Lagrangian has a global U(1) symmetry

$$\phi(x) \rightarrow \phi(x) e^{i\theta}$$

In terms of the Mexican hat potential,

$$V(\phi) = -\frac{1}{2}\lambda\nu\phi^*\phi + \frac{1}{2}\lambda(\phi^*\phi)^2 \qquad \nu = -\frac{-2m^2}{\lambda}$$

the minimum occurs for

$$|\phi|^2 = \frac{\nu^2}{2}$$

courtesy of I. Bloch

global gauge invariance

We pick one of the minima and expand around it,

$$\phi = \frac{1}{\sqrt{2}}(\nu + \varphi_1 + i\varphi_2)$$

The low-energy Lagrangian is then

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu \varphi_1)^2 + (\partial_\mu \varphi_2)^2\right] - \frac{1}{2}\lambda\nu^2\varphi_1^2 + \ldots$$

where we see a massless Goldstone mode and a massive Higgs mode.

local gauge invariance

Consider now the case of coupling to a gauge field and local gauge invariance,

$$\theta \to \theta(x)$$

$$A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \theta(x)$$

$$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi$$

$$\mathcal{L} = D_\mu \phi^* D^\mu \phi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} - V(\phi)$$

Breaking the symmetry now leads to

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu \varphi_1)^2 + (\partial_\mu \varphi_2 + e\nu A_\mu)^2\right] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\lambda\nu^2\varphi_1^2 + \ldots$$

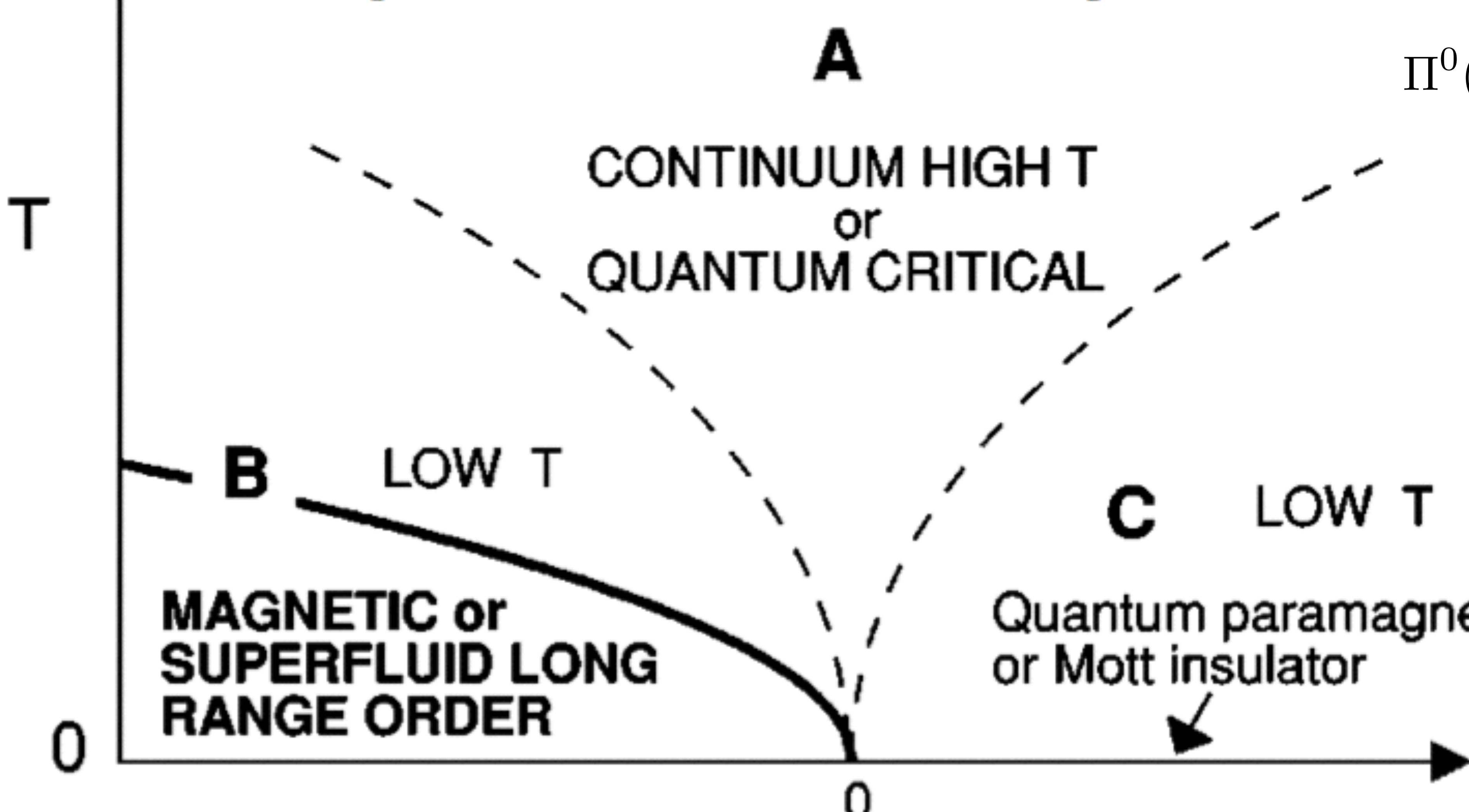
plasmon

exactly same terms as for global gauge invariance

O(N) field theories

$$\mathcal{Z}_Q = \int \mathcal{D}\phi_\alpha(x,\tau) \exp\left(- \int d^d x \int_0^{1/T} d\tau \mathcal{L}_Q \right)$$

$$\mathcal{L}_Q = \frac{1}{2}\left[\frac{1}{c^2}(\partial_\tau \phi_\alpha)^2 + (\nabla_x \phi_\alpha)^2 + (r_c + r)\phi_\alpha^2 \right] + \frac{u}{4!}(\phi_\alpha^2)^2.$$



d>3 : u is irrelevant (Gaussian free field theory)

$$\chi^0(q) = \frac{u}{q^2 + (r_c + r)}$$

mean-field pole at amplitude mass

$$\Pi^0(q) \sim \int \frac{1}{k^2(k+q)^2} \frac{d^{d+1}k}{(2\pi)^{d+1}}$$

IR divergence d=2, n=1,2

fig from S. Sachdev

more on field theory

usually the low effective field theory is of the form

$$Z = \int \mathcal{D}\Psi^*(x,\tau)\mathcal{D}\Psi(x,\tau)\exp\left(-\int d^dx \int d\tau \mathcal{L}[\Psi^*,\Psi]\right)$$

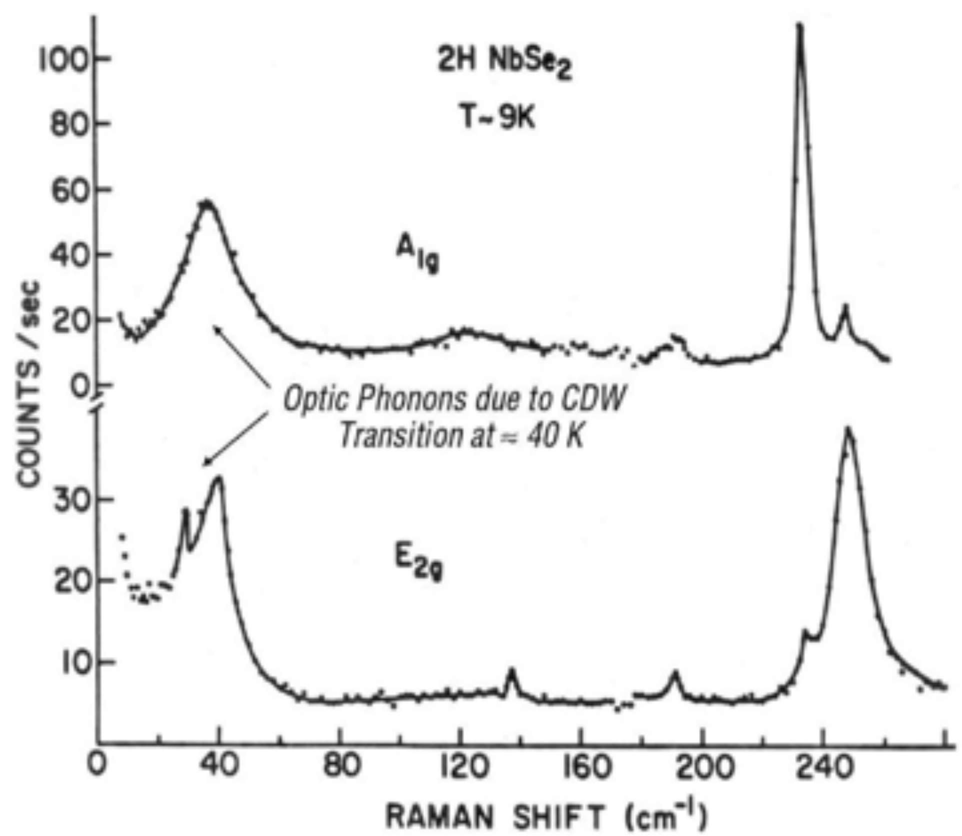
$$\mathcal{L}[\Psi^*,\Psi] = -\frac{\partial r}{\partial \mu}\Psi^*\frac{\partial \Psi}{\partial \tau} + |\partial_\tau \Psi|^2 + c^2|\nabla\Psi|^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4 + \cdots$$

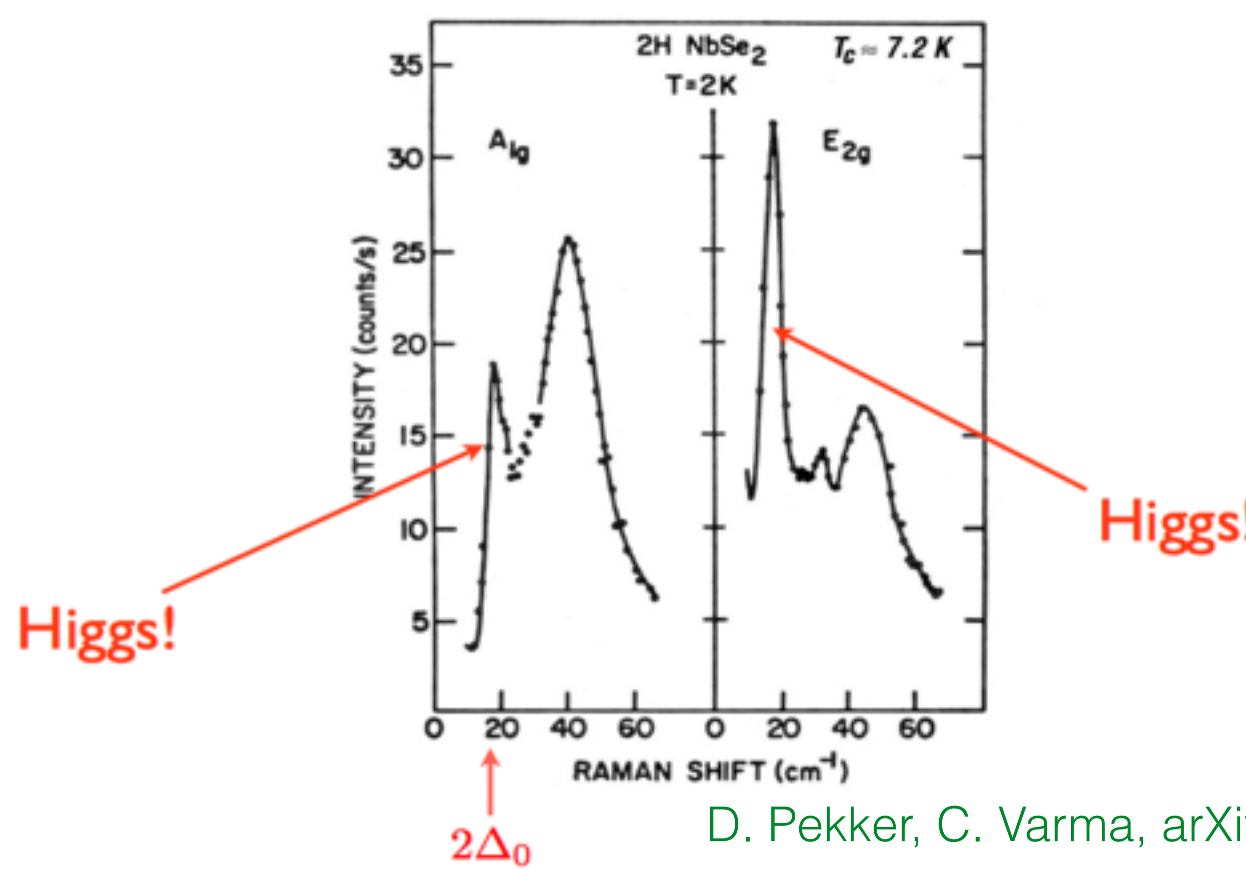
S. Sachdev, Quantum Phase Transitions, 1999

ph - symmetry needed for it to vanish (superconductors, not metals nor superfluids)

It is hard to couple to the Higgs mode:

Raman spectra of NbSe2





D. Pekker, C. Varma, arXiv (2014)

PRL 100, 205701 (2008) PHYSICAL REVIEW LETTERS week ending 23 MAY 2008

Quantum Magnets under Pressure: Controlling Elementary Excitations in $TlCuCl_3$

Ch. Rüegg,[1] B. Normand,[2,3] M. Matsumoto,[4] A. Furrer,[5] D. F. McMorrow,[1] K. W. Krämer,[6] H.-U. Güdel,[6] S. N. Gvasaliya,[5] H. Mutka,[7] and M. Boehm[7]

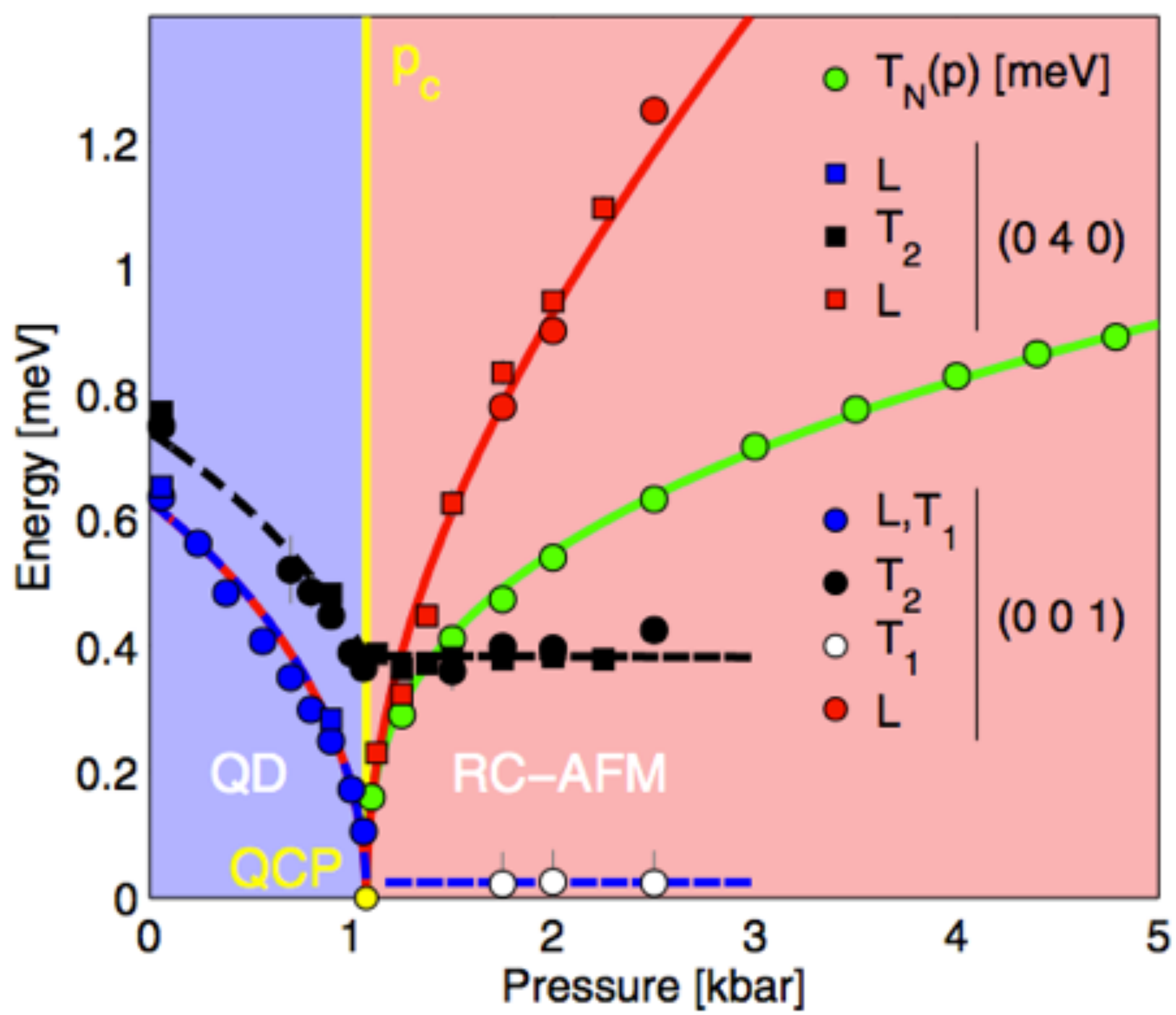


FIG. 3 (color online). Summary of INS results for the gaps of all three triplet excitations as functions of pressure at $T = 1.85$ K. Data for $T_N(p)$ from Ref. [5]. Modes L and T_1 are degenerate within experimental resolution at $p < p_c$. Red symbols show the longitudinal mode L at $p > p_c$. Solid and dashed lines are theoretical fits.

(3d quantum antiferromagnet)

other systems:

- pump-probe experiments Matsunaga et al. PRL (2013)
- He3 (p-wave)
- Raman spectrum of LCO? Muschler et al (2010); Weidinger & Zwerger (2015)

two dimensions

PHYSICAL REVIEW B VOLUME 49, NUMBER 17 1 MAY 1994-I

Theory of two-dimensional quantum Heisenberg antiferromagnets with a nearly critical ground state

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Subir Sachdev and Jinwu Ye
Department of Physics and Applied Physics, P.O. Box 208120, Yale University, New Haven, Connecticut 06520-8120
(Received 21 April 1993; revised manuscript received 6 January 1994)

PHYSICAL REVIEW B VOLUME 59, NUMBER 21 1 JUNE 1999-I

Universal relaxational dynamics near two-dimensional quantum critical points

Subir Sachdev
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(Received 4 November 1998)

$$\chi_{\perp}(k,\omega)=\frac{N_0^2}{\rho_s(0)[k^2-(\omega/c)^2]},$$

$$\chi_{\parallel}(k,\omega)=\frac{N_0^2}{\rho_s(0)}\frac{1}{\sqrt{k^2-(\omega/c)^2}[\sqrt{k^2-(\omega/c)^2}+16\rho_s(0)/cn]}$$

longitudinal susceptibility has branch cut

no pole-like structure at a frequency of order $\rho_s(0)$

two dimensions

VOLUME 92, NUMBER 2 PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2004

Anomalous Fluctuations in Phases with a Broken Continuous Symmetry

W. Zwerger
Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria
(Received 7 April 2003; published 16 January 2004)

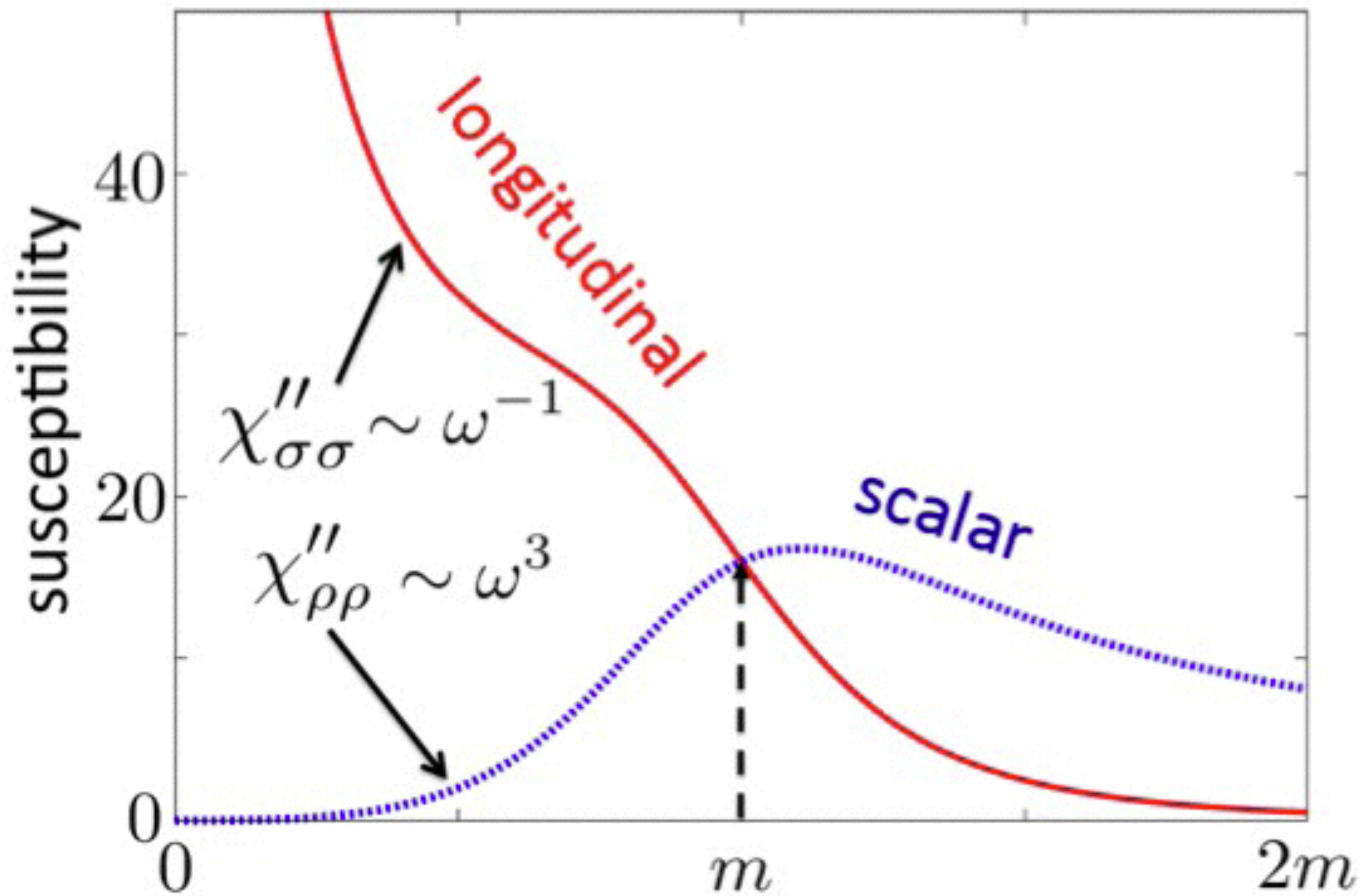
derived same formula's, and used them in the dynamic structure factor:

$$S(q,\omega) = 2m_s^2\xi_J \frac{N-1}{N}\left[\frac{\pi}{2q}\delta(\omega - cq) + \frac{\xi_J}{16}\frac{\theta(\omega - cq)}{\sqrt{\omega^2 - c^2q^2}}\right]$$

``The longitudinal fluctuations of the Neel order thus lead to a critical continuum above the spin wave pole at w~ cq, which decays only algebraically. The continuum results from the decay of a normally massive amplitude mode with momentum p into a pair of spin waves with momenta q and p-q, which is possible for any w > cq, with a singular cross section because of the large phase space. The amplitude mode is thus completely overdamped in two dimensions."

Scalar and longitudinal susceptibility

Chubukov, Sachdev, Ye '93
Podolsky, Auerbach, Arovas '11
S. Huber, G. Blatter, E. Altman



Universal scaling predictions

$$\chi''_{\rho\rho}(\omega) \propto \Delta^{3-2/\nu} F(\omega/\Delta)$$

$$\Delta \propto (U_C - U)^{\nu}, \quad \nu = 0.6717$$

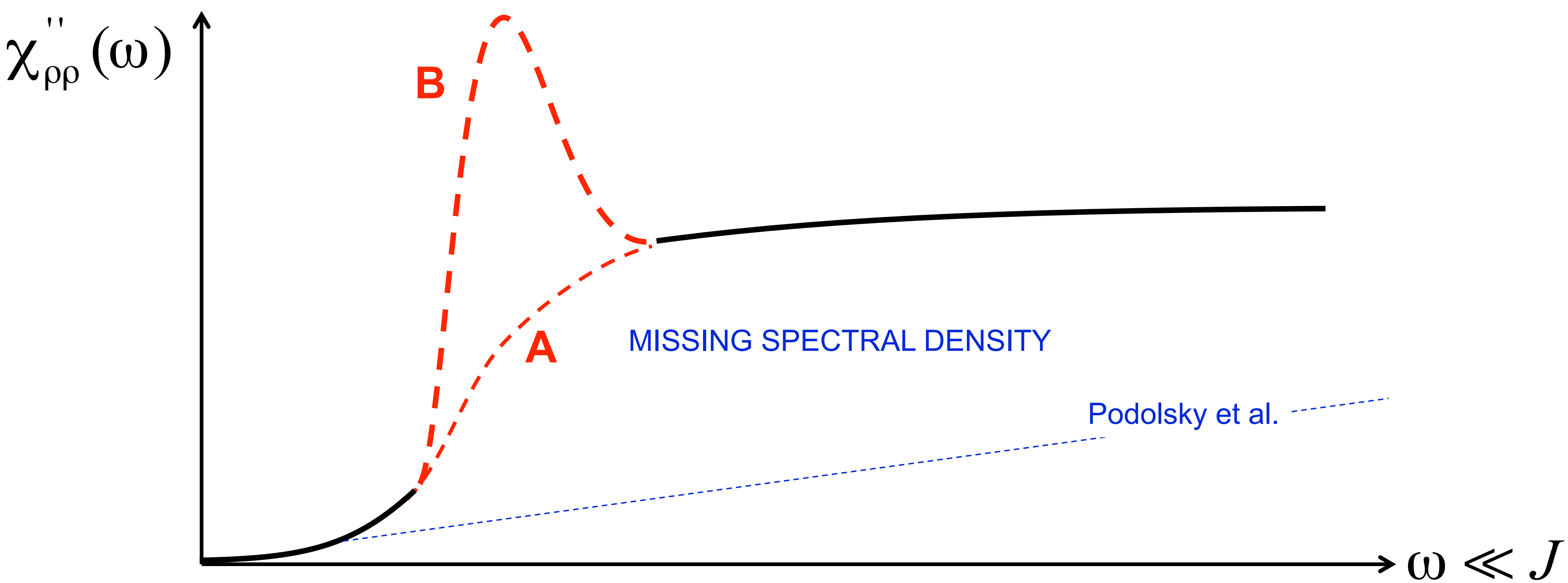
Chubukov, Sachdev, Ye '93

Sachdev '99

$\omega \ll \Delta$ $\qquad$ $\omega \gg \Delta$

$$\chi''_{\rho\rho}(\omega) \propto \omega^3 / (1 - U/U_C)^2$$

$$\chi''_{\rho\rho}(\omega) \propto \omega^{3-2/\nu} = \omega^{0.0225}$$

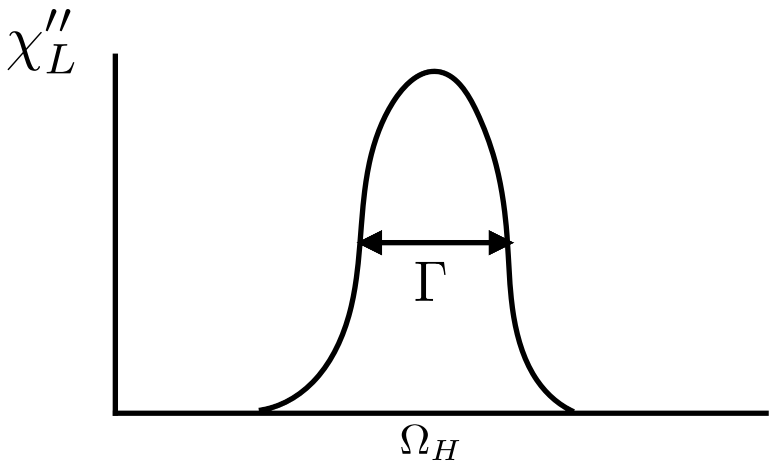


Two has more than three

„The model I came up with in 1964 is just the invention of a rather strange sort of medium that looks the same in all directions and produces a kind of refraction that is a little bit more complicated than that of light in glass or water“ — P. Higgs

d=3+1

Longitudinal response: finite width peak



Gaussian fixed point

Higgs peak is critically well defined

$$\frac{\Gamma}{\omega_H} \sim \frac{1}{\ln|g-g_c|}$$

Energy ratio: $$\frac{\omega_H}{\Delta} = \sqrt{2}$$

Affleck & Wellman, PRB 92

d=2+1

Longitudinal response IR divergent

universal scaling function

Strongly coupled fixed point

Higgs peak is marginally defined

$$\frac{\Gamma}{\omega_H} \to \text{const}$$

Energy ratio: $$\frac{\omega_H}{\Delta} \neq \sqrt{2}$$

D. Podolsky

Physics of Bose-Hubbard in a nutshell

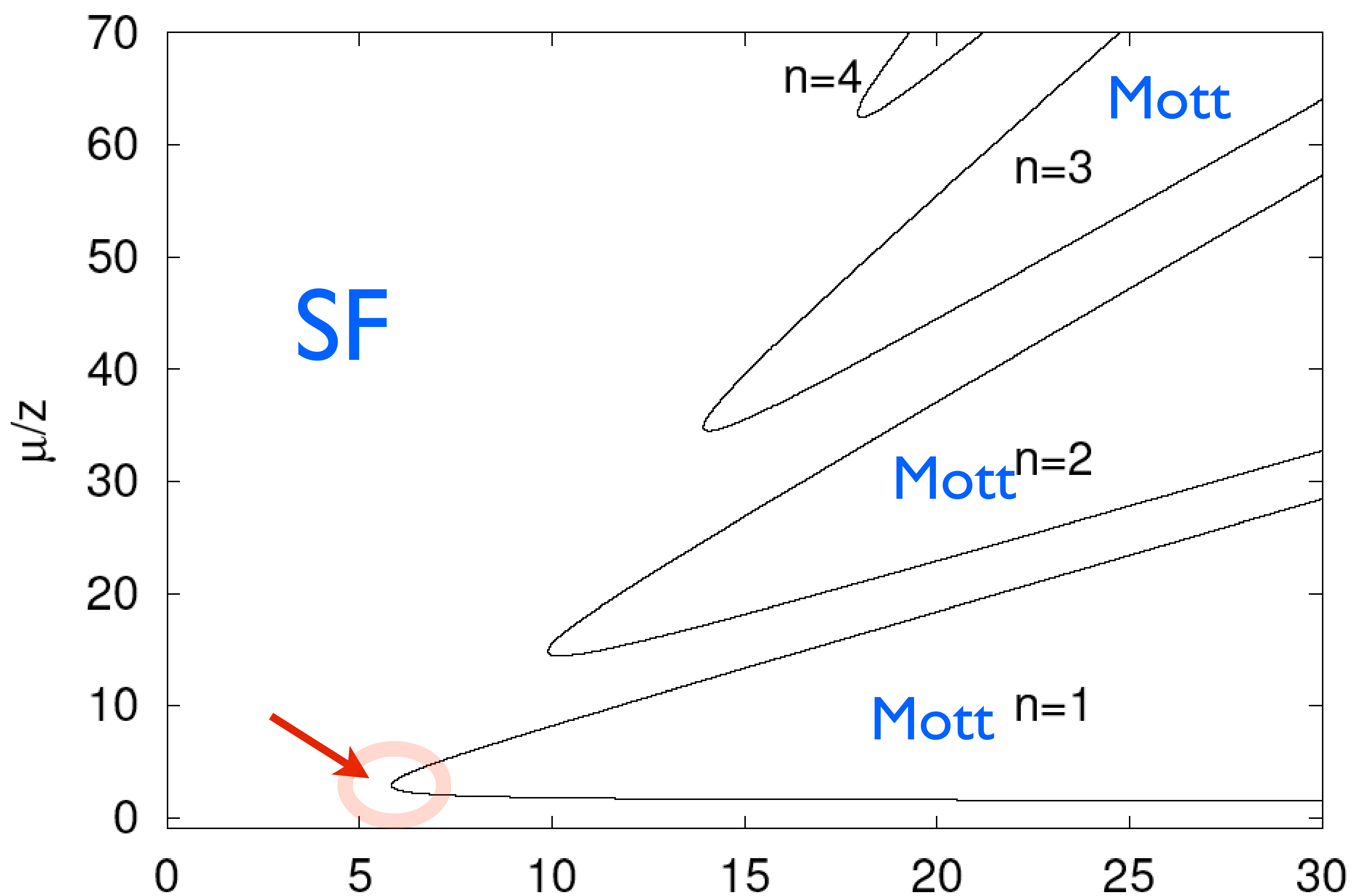
$$H = -t \sum_{<ij>} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) - \sum_i \mu_i n_i$$

M. P.A. Fisher et al, PRB 1989

U(1) symmetry global $b_i \to b_i e^{i\phi}$
decoupling approximation (mean-field)

$$b_i^\dagger b_j = \psi(b_i^\dagger + b_j) - \psi^2$$
$$\psi = \langle b_i \rangle = \langle b_i^\dagger \rangle$$

Mott phase:
- Integer density
- zero compressibility
- gap
- insulating

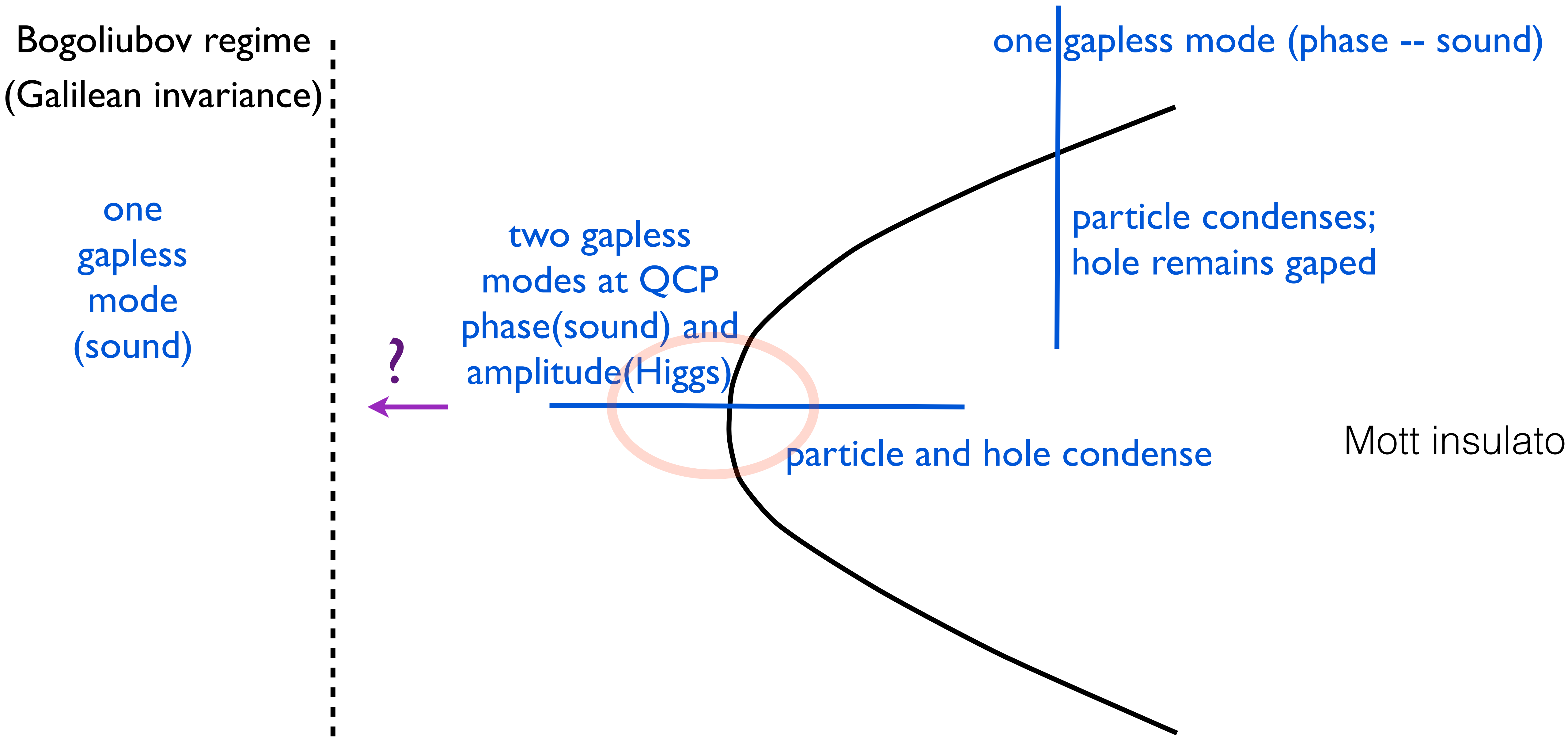


$$Z = \int \mathcal{D}\Psi^*(x,\tau)\mathcal{D}\Psi(x,\tau) \exp\left(-\int d^dx \int d\tau \mathcal{L}\left[\Psi^*, \Psi\right]\right)$$
$$\mathcal{L}\left[\Psi^*, \Psi\right] = -\frac{\partial r}{\partial \mu}\Psi^* \frac{\partial \Psi}{\partial \tau} + |\partial_\tau \Psi|^2 + c^2 |\nabla \Psi|^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4 + \cdots$$

S. Sachdev, Quantum Phase Transitions, 1999

must vanish!

sketch



$$Z = \int \mathcal{D}\Psi^*(x,\tau)\mathcal{D}\Psi(x,\tau)\exp\left(-\int d^d x \int d\tau \mathcal{L}\left[\Psi^*,\Psi\right]\right)$$

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must vanish!

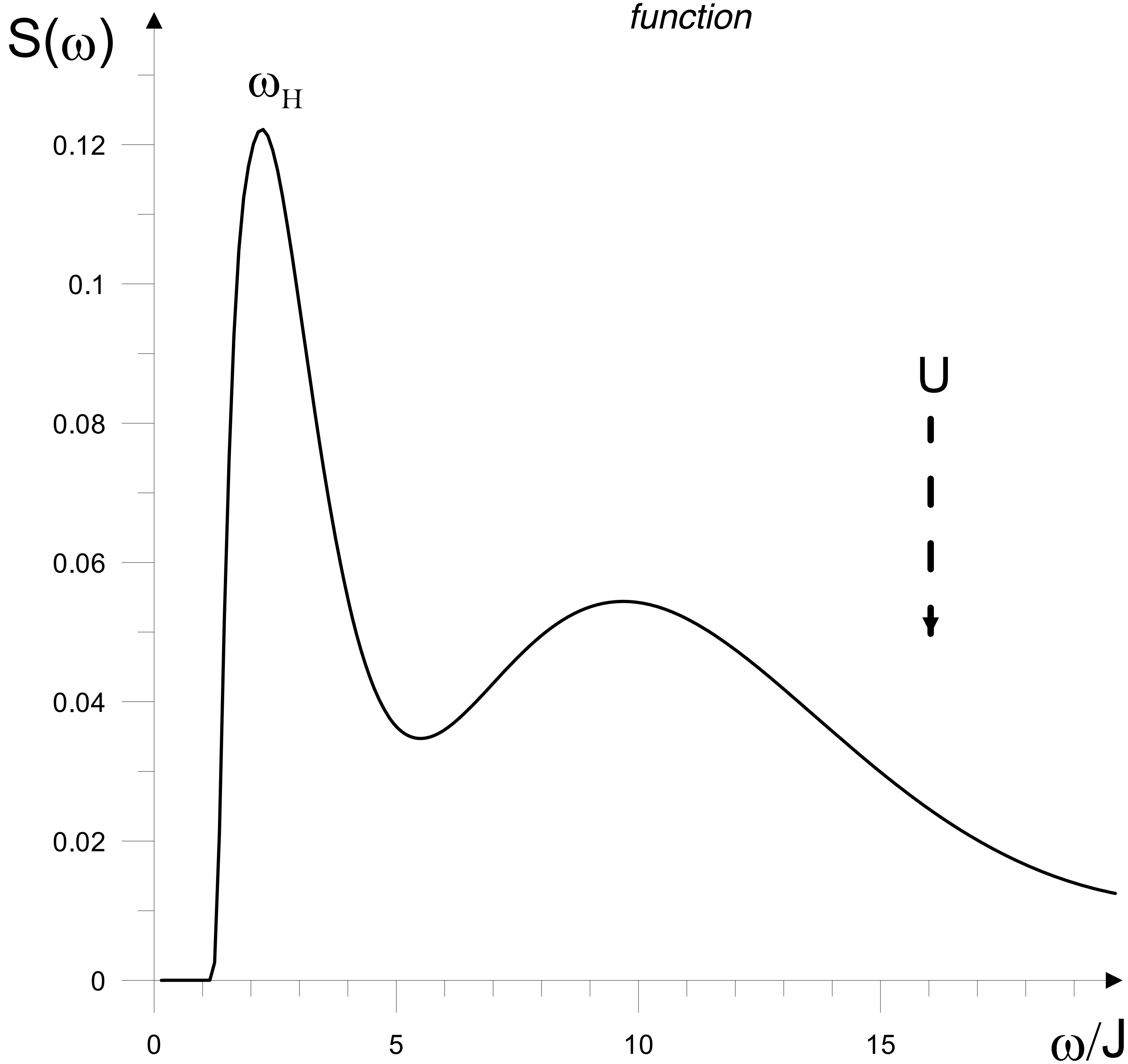
M. P.A. Fisher et al, PRB 1989

S. Sachdev, Quantum Phase Transitions, 1999

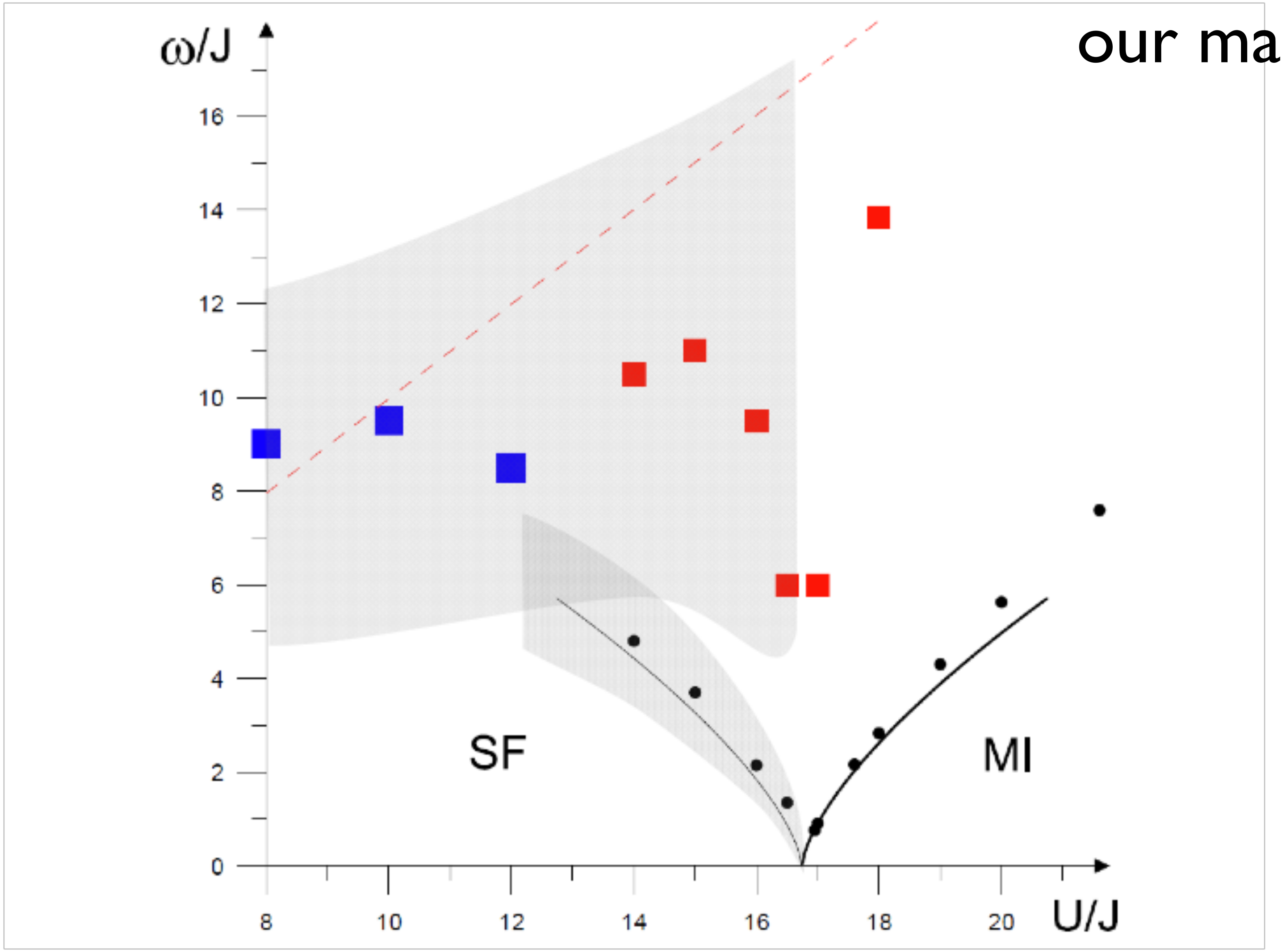
Our response

This involves analytic continuation of the *kinetic energy correlation function*

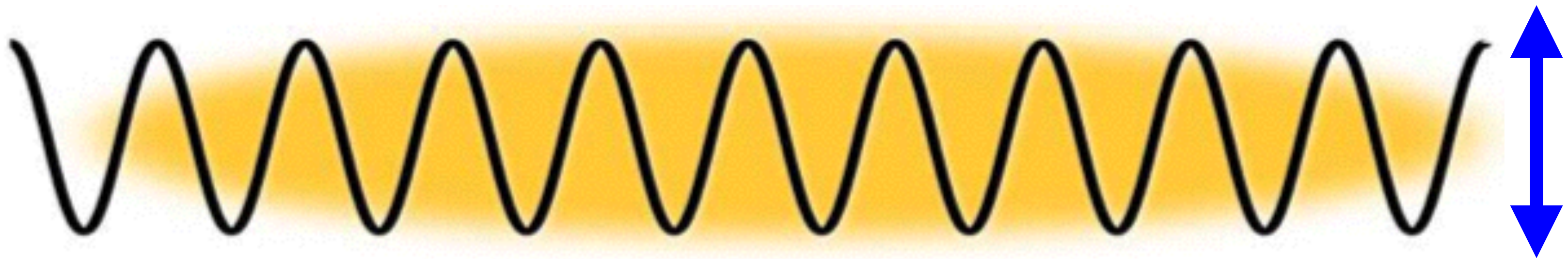
U=16



Long Monte Carlo simulations (LMC)



our main result



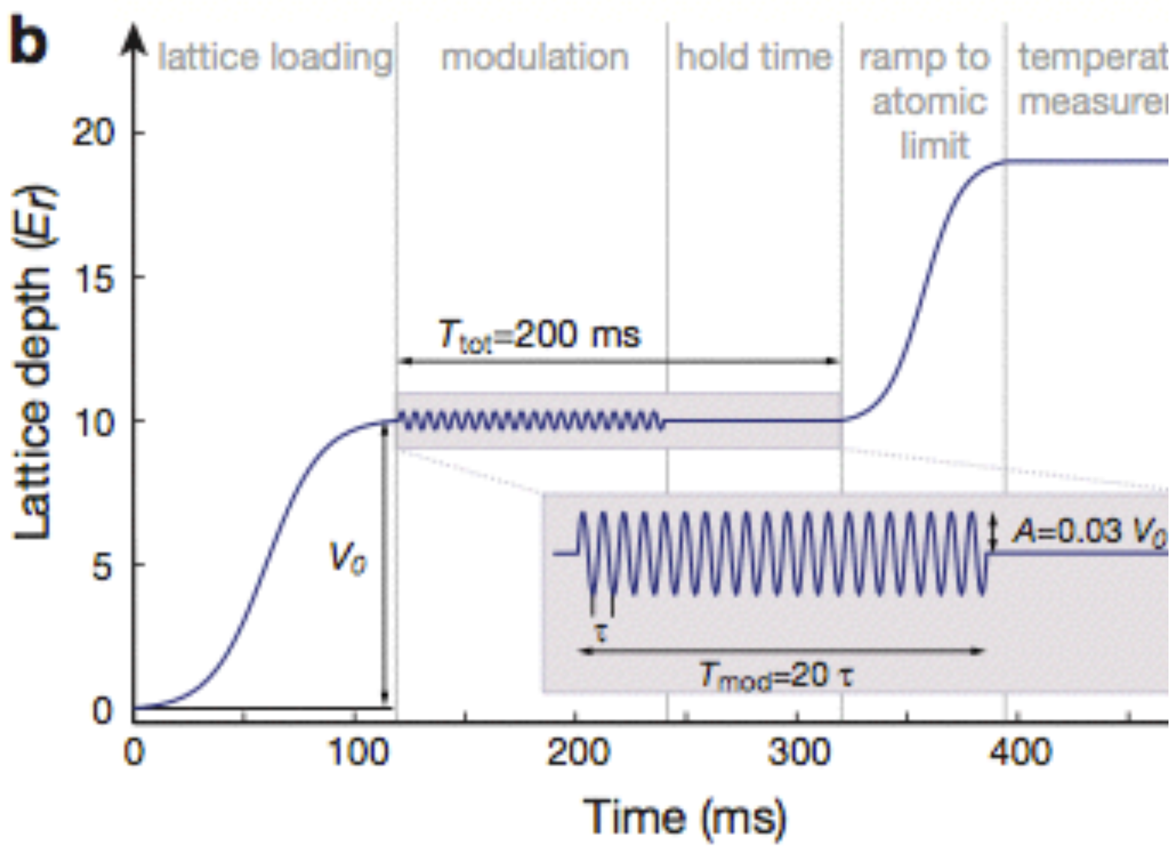
amplitude modulation

Technique pioneered in Zurich (Stoeferle et al);
see also Kollath et al, etc

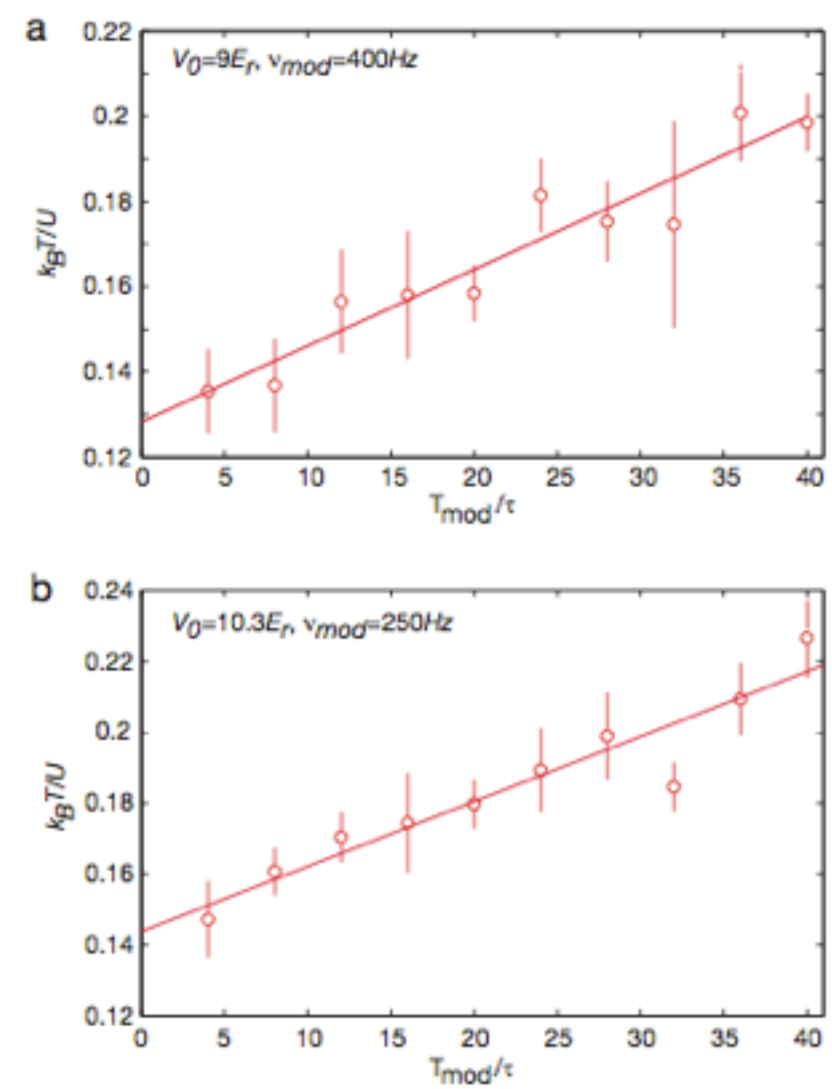
The 'Higgs' Amplitude Mode at the Two-Dimensional Superfluid-Mott Insulator Transition

Manuel Endres[1], Takeshi Fukuhara[1], David Pekker[2], Marc Cheneau[1], Peter Schauß[1], Christian Gross[1], Eugene Demler[3], Stefan Kuhr[1,4], and Immanuel Bloch[1,5]

Nature 2012



190(36) particles

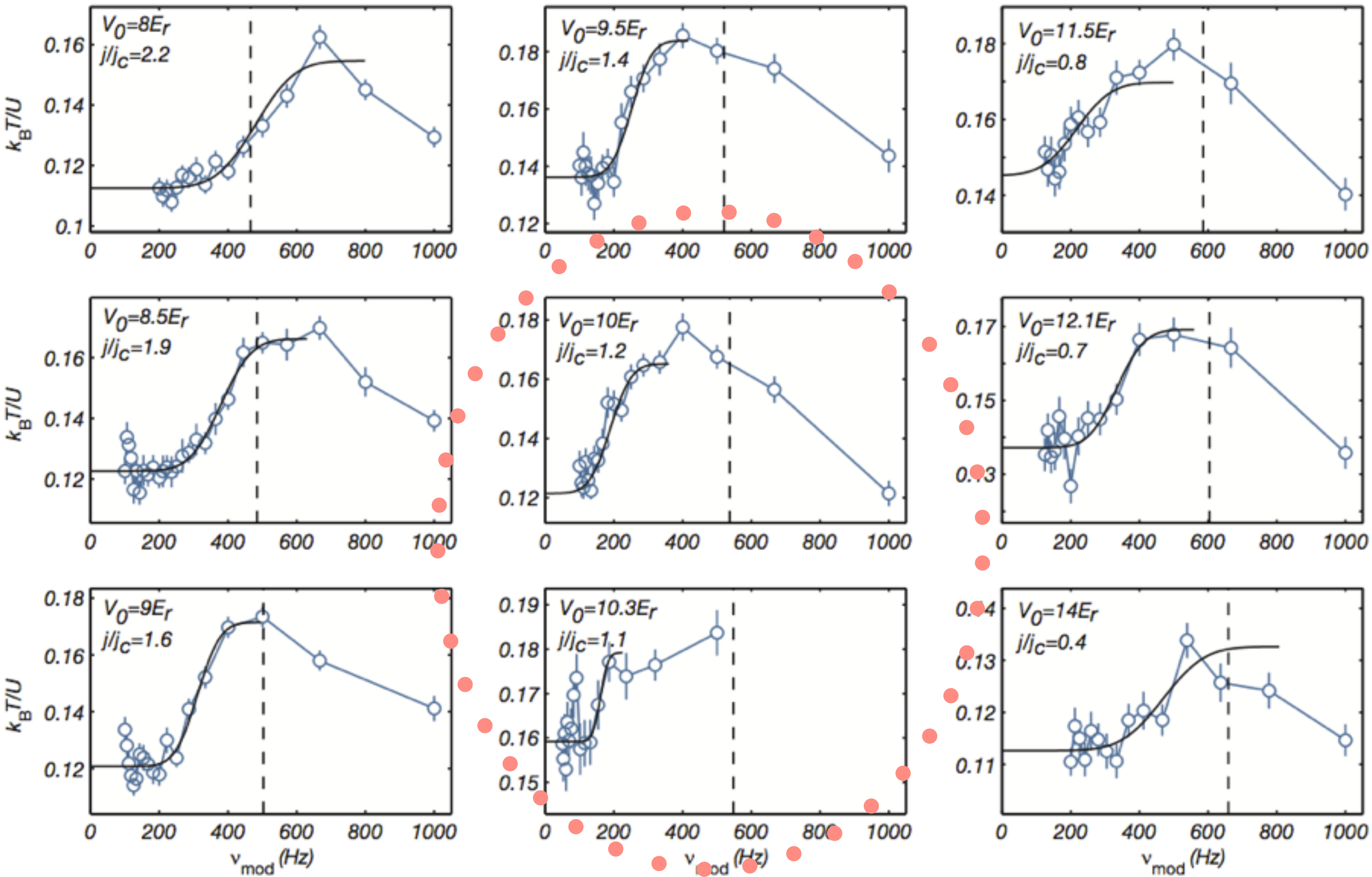


$$\chi_\omega = \langle K(\tau)K(0)\rangle_\omega + \langle K \rangle$$

Energy dissipation rate $\omega \, \mathrm{Im}\, \chi_\omega$

Total energy absorbed: $\Delta E = \omega \, \mathrm{Im}\, \chi_\omega \left(2\pi M / \omega\right) \propto \mathrm{Im}\, \chi_\omega$

The experimental results



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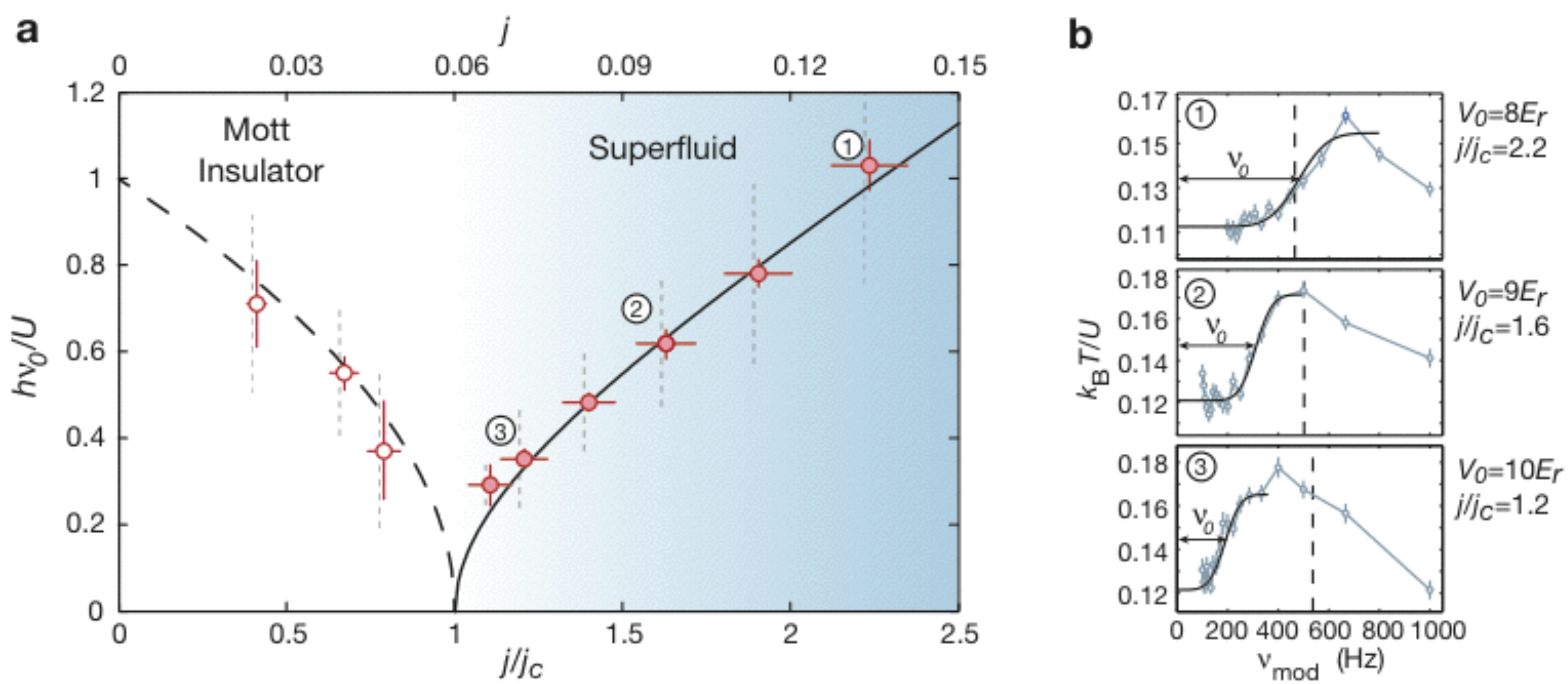
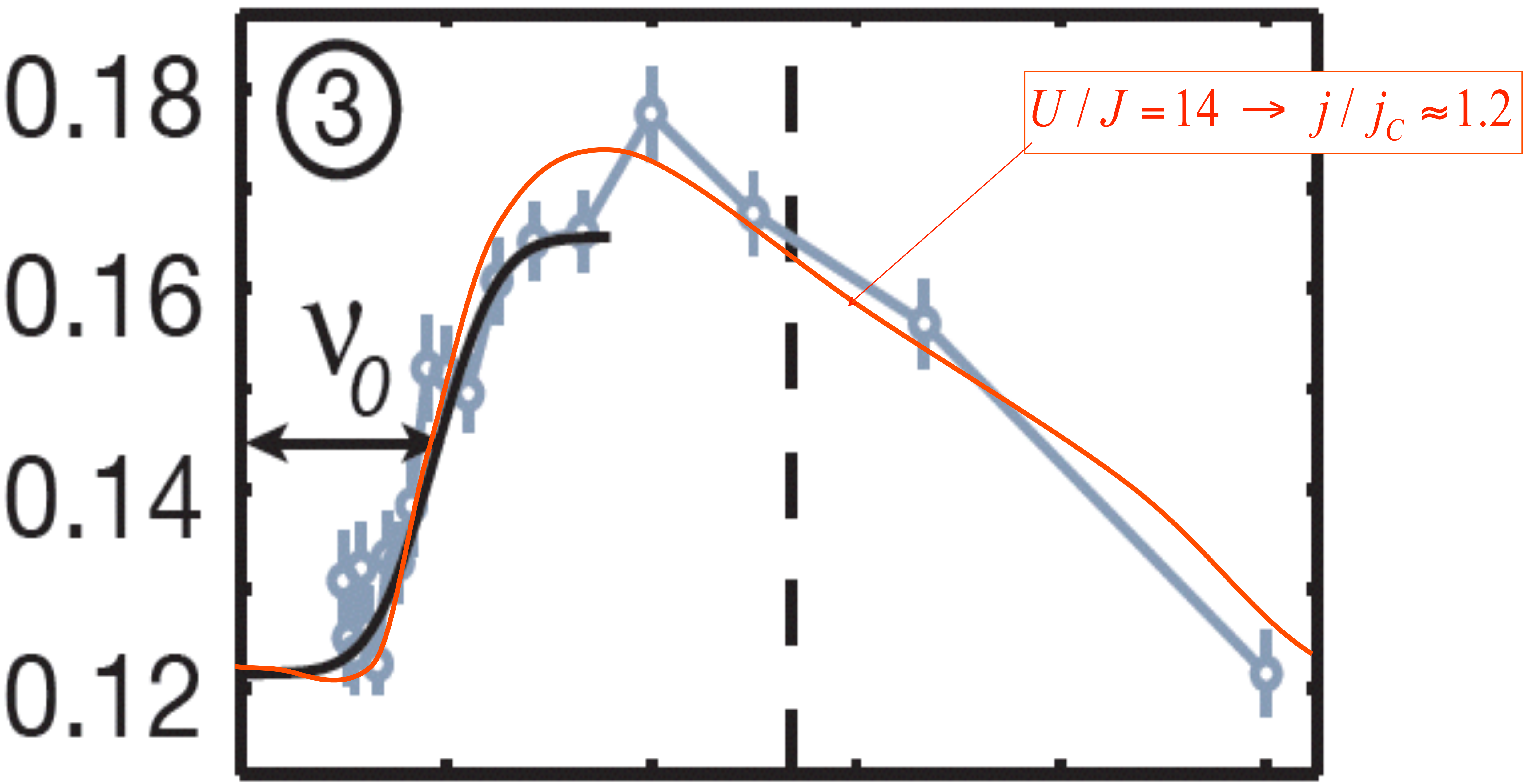


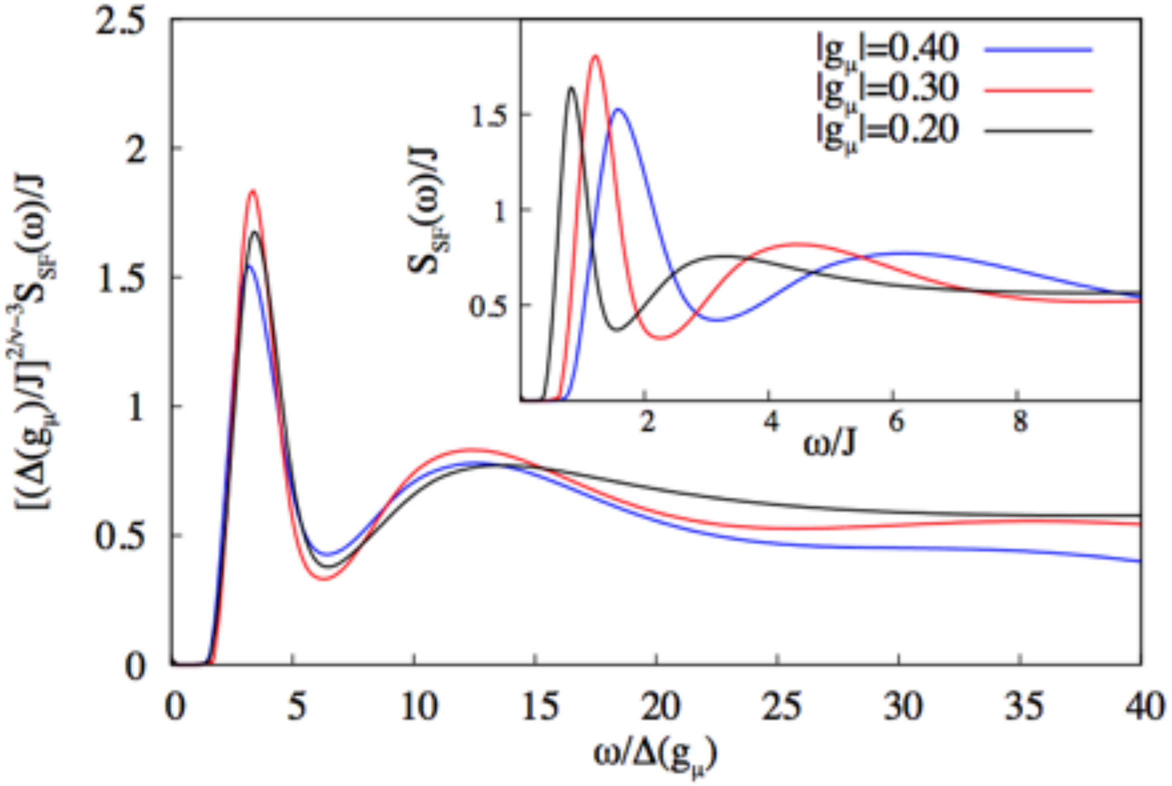
softening of onset of spectral weight on approach to the critical point

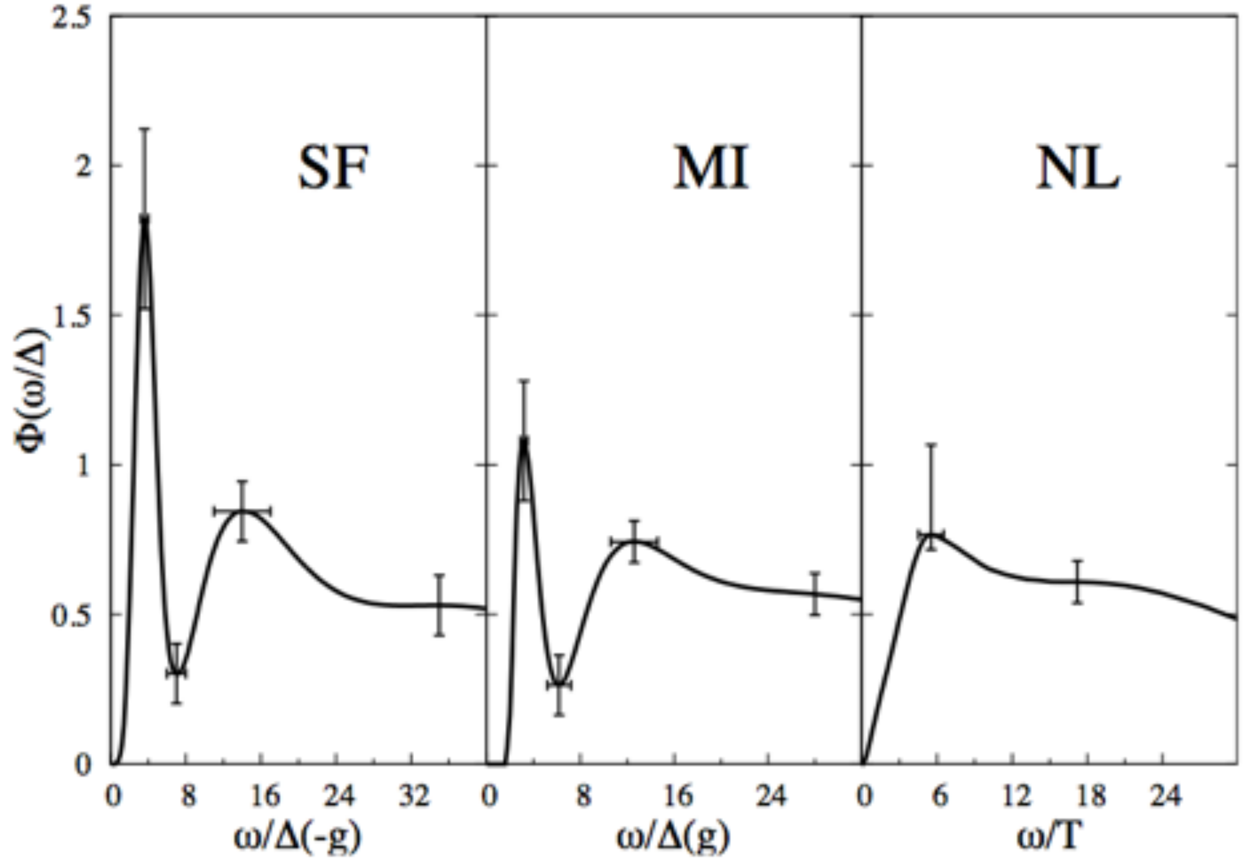
Attempt to compare signals (amplitude adjusted)

Take a realistic temperature and trapping parameters into account



universal scaling function





results by Podolsky et al

$$S_E = \frac{1}{g}\left[-\sum_{\langle i,j\rangle}\vec{\phi}_i\cdot\vec{\phi}_j - \mu\sum_i|\vec{\phi}_i|^2 + \sum_i(|\vec{\phi}_i|^2)^2\right]$$

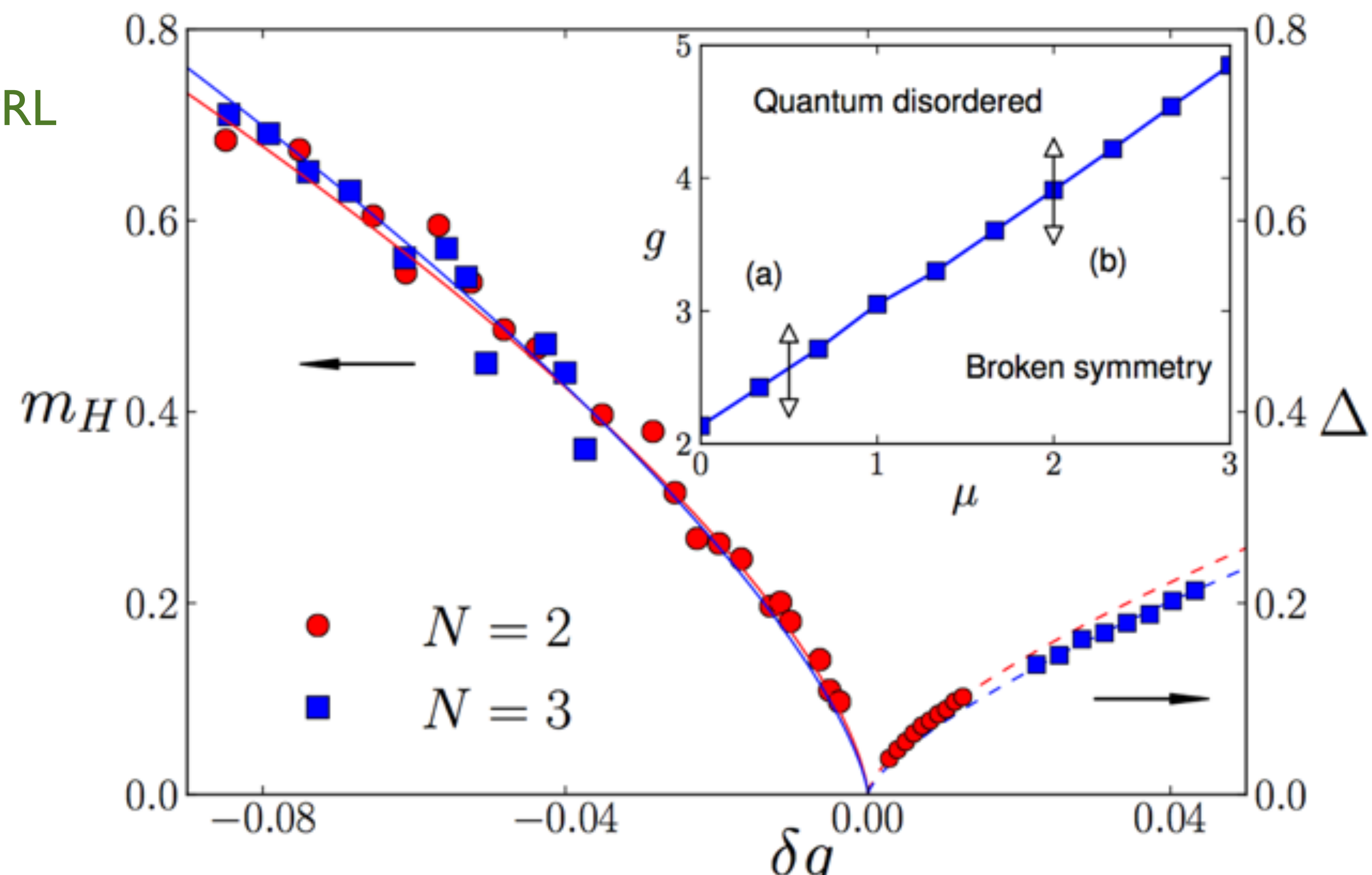
S. Gazit et al, arXiv:1212.3759, PRL

on SF side:

$\omega_H = 2.1(3)\Delta$

compare to ours:

$\omega_H = 3.2(8)\Delta$



conclusion and future work

- conditions under which amplitude/Higgs mode can be seen as a sharp and universal peak in correlation functions
- strongly interacting fixed point in 2d; also conductivity accurately computed (cf AdS/CFT correspondence)
- further experiments would be welcome though challenging
- universal scaling function determined; explicit demonstration of Lorentz symmetry under way
- what about (artificial) graphene (Gross-Neveu criticality)? what about 1d?

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Kun Chen, Longxiang Liu, Youjin Deng, Nikolay Prokof'ev
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W. Zwerger, D. Manske, M. Dressel