Phases of strongly-interacting bosons on a two-leg ladder

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My collaborators

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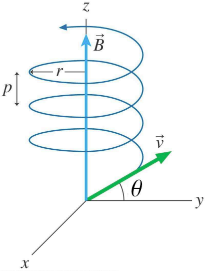
S. Greschner

T. Vekua

Electron in a magnetic field

Cyclotron orbit



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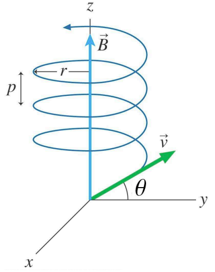
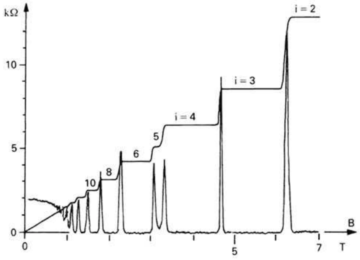


Landau levels (highly degenerate)

$$E_n = \hbar\omega_c(n + 1/2)$$

Quantum Hall Effect : quantization of conductance



K. von Klitzing, RMP (1986)

Electron in a magnetic field

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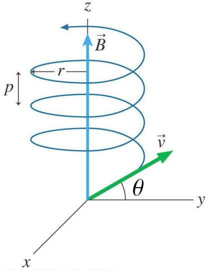
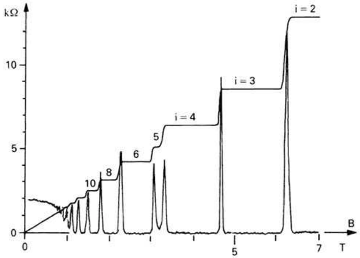


Quantum Hall Effect :
quantization of conductance



K. von Klitzing, RMP (1986)

Landau levels (highly degenerate)

$$E_n = \hbar\omega_\mathrm{c}(n + 1/2)$$

Modifies transport properties & creates Topological transitions

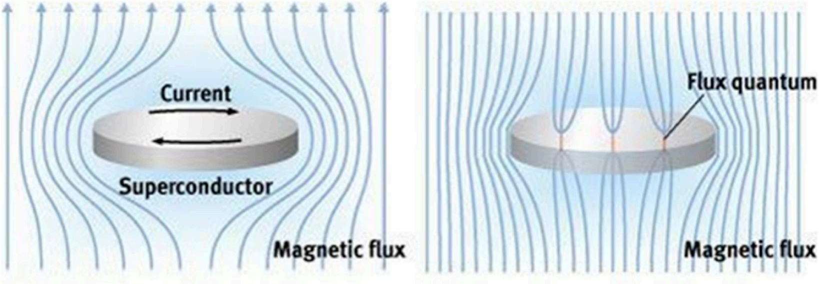
With interactions : Fractional QHE

$\Rightarrow$ Unusual excitations, fractional charge and statistics

Type-II Superconductor in a magnetic field

Meissner effect W. Meissner & R. Ochsenfeld, Naturwissenschaften (1933)

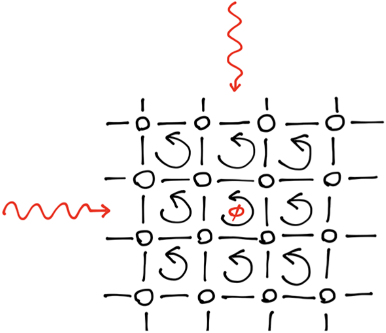
Meissner state $H < H_{c1}$ Vortex lattice $H_{c1} < H < H_{c2}$



Source : Quantumlevitation.com

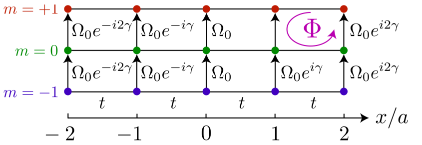
Ultra-cold atoms in synthetic gauge fields

Engineer effective $\vec{B}$ field or SO coupling on lattices :



C. Chin & E.J. Mueller, PRL Viewpoint (2013)

In 'Synthetic dimensions'



A. Celi *et al.*, PRL (2014)

Realizations :

M. Mancini *et al.*, arXiv1502 :02495

B. Stuhl *et al.*, arXiv1502 :02496

- Interplay interactions/ gauge fields/ dimensionality
  $\Rightarrow$ Promising candidate to search for new phases of matter.

  J. Dalibard & F. Gerbier, RMP (2011)
- UCA permit to study **bosons** in magnetic fields
- Ladders = simplest "2D" system

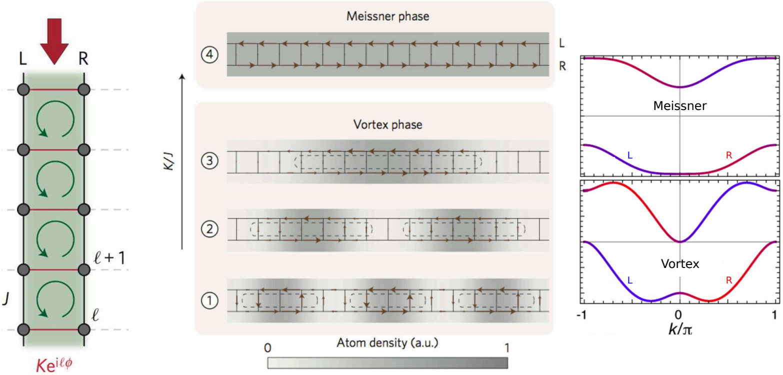
Weakly-interacting bosons on ladders

Vortex/ Meissner Phase Transition

(ladders with uniform $\pi/2$ magnetic flux)

Theo : E. Orignac & T. Giamarchi, PRB (2001) , D. Hügel & B. Paredes, PRA (2014)

Exp : M. Atala *et al.*, Nat. Phys. (2014)



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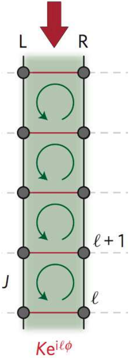

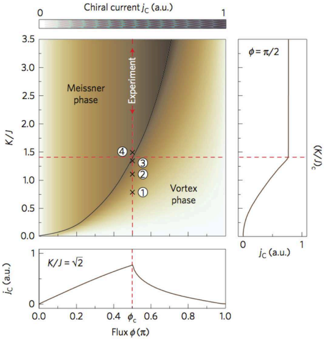

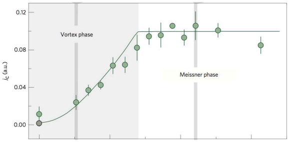

Interacting bosons on ladders in uniform flux

A few theoretical results

- Field-theoretical studies — E. Orignac & T. Giamarchi, PRB (2001)
  A. Tokuno & A. Georges, NJP (2014)
- Mean-field approach — R. Wei & E. J. Mueller, PRA (2014)
- Chiral Mott Insulator — A. Dhar *et al.*, PRA (2012) ; PRB (2013)
- Mott Insulator with Meissner Currents
  A. Petrescu & K. Le Hur, PRL (2013)
- Study of the momentum distribution
  M.-C. Cha & J.-G. Shin, PRA (2011)
- Laughlin-1/2 states
  A. Petrescu & K. Le Hur, PRB (2015)

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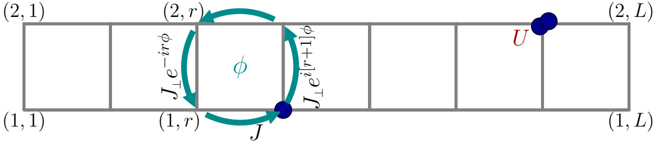
  A. Petrescu & K. Le Hur, PRB (2015)

$\Rightarrow$ Explore the phase diagram numerically

Interacting bosons on a two-leg ladder in a uniform flux : Model

$$\mathcal{H}_{\mathsf{K}} = -J \sum_{\ell=1,2;r=1}^{L} \left(a^\dagger_{\ell,r+1} a_{\ell,r} + \mathrm{H.c.}\right) - J_\perp \sum_{r=1}^{L} \left(e^{-ir\phi} a^\dagger_{1,r} a_{2,r} + \mathrm{H.c.}\right)$$

$$\mathcal{H}_{\mathsf{U}} = \frac{U}{2} \sum_{\ell=1,2;r=1}^{L} n_{\ell,r}(n_{\ell,r} - 1)$$



Density Matrix Renormalization Group

Ground state and first excited states of 1D many-body systems

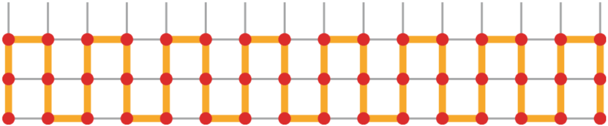
S. R. White, Phys. Rev. Lett. **69**, 2863 (1992)

U. Schollwöck, Rev. Mod. Phys. **77**, 259 (2005)

Successfully applied to (quasi-)2D systems

S. Yan *et al.*, Science (2011)

S. Depenbrock *et al.*, PRL (2012)



E. M. Stoudenmire *et al.*, Annu. Rev. Cond. Mat. Phys. (2012)

Outline

- Hard-Core Bosons

  M. P., F. Heidrich-Meisner, I. P. McCulloch, S. Greschner, T. Vekua and U. Schollwöck, PRB **91** 140406(R) (2015)

  - Phase Diagram : MI and SF, Meissner and vortex phases
  - A few observables : chiral current and vortex density

- Finite interactions

  S. Greschner, M. P., F. Heidrich-Meisner, I. P. McCulloch, U. Schollwöck and T. Vekua, *submitted* (2015)

  - Phase Diagram : vortex lattice phases
  - Chiral current : spontaneous reversal !

Outline

- **Hard-Core Bosons**

  M. P., F. Heidrich-Meisner, I. P. McCulloch, S. Greschner, T. Vekua and U. Schollwöck, PRB **91** 140406(R) (2015)

  - **Phase Diagram**
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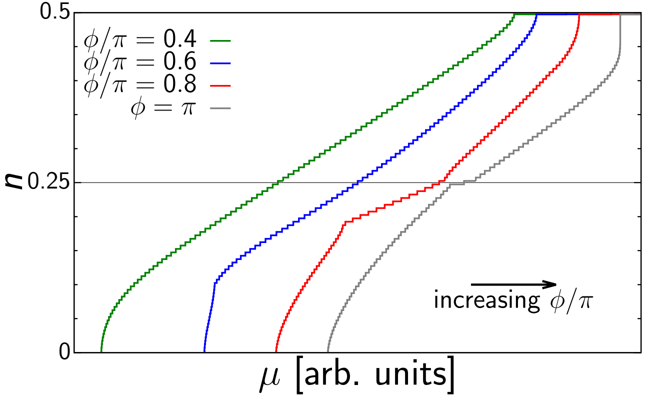
  S. Greschner, M. P., F. Heidrich-Meisner, I. P. McCulloch, U. Schollwöck and T. Vekua, *submitted* (2015)

  - Phase Diagram
  - Chiral current

Equation of state

Filling $n = N/2L$ vs. chemical potential μ



Plateau (at $n = 0.5$, and at $n = 0.25$ for $\phi = \pi$)

$\Rightarrow$ incompressible state

Kink $\Rightarrow$ change in the number of gapless excitations (+ use S_{vN})

Fractionally filled Mott insulator

Hard-core bosons, no flux

- Strong rungs $(J_\perp \gg J)$: $\mathcal{H}_\kappa \sim -J_\perp \sum_r \left(a^\dagger_{1,r} a_{2,r} + \mathrm{H.c.} \right)$

$s_z = 0$ triplet	$(\vert 1,0\rangle_r + \vert 0,1\rangle_r)/\sqrt{2}$	$E = -J_\perp$
$s_z = \pm 1$ triplets	$\vert 1,1\rangle_r$ and $\vert 0,0\rangle_r$	$E = 0$
singlet	$(\vert 1,0\rangle_r - \vert 0,1\rangle_r)/\sqrt{2}$	$E = J_\perp$

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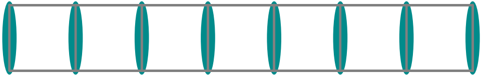
singlet	$(\|1,0\rangle_r - \|0,1\rangle_r)/\sqrt{2}$	$E = J_\perp$

1 boson per rung $\Rightarrow$ 'rung-triplet' phase

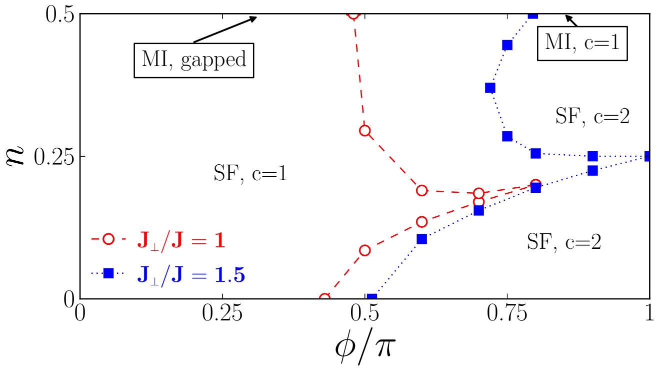


$\Rightarrow$ **Mott insulator extends down to $J_\perp/J = 0$ for HCBs**

T. Vekua *et al.*, PRB (2003) ; F. Crépin *et al.*, PRB (2011).

Phase diagram for Hard-Core Bosons

Parameter space : rung-tunneling $J_\perp/J$, flux ϕ, filling $n = N/2L$.

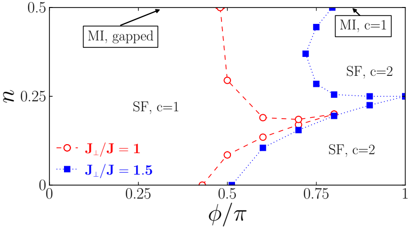


Upper half $0.5 < n < 1$ related by **particle-hole symmetry**.
Phase diagram **consistent with field-theory**.

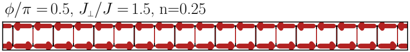
Current patterns

Local currents

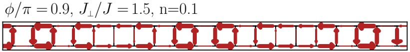
$$j^{\parallel}_{\ell,r} = iJ\left(a^\dagger_{\ell+1,r}a_{\ell,r} - \text{H.c.}\right) \quad \text{and} \quad j^{\perp}_{r} = iJ_\perp\left(e^{-ir\phi}a^\dagger_{1,r}a_{2,r} - \text{H.c.}\right)$$



Meissner-SF



Vortex-SF



cf E. Orignac & T. Giamarchi, PRB (2001) , A. Tokuno & A. Georges, NJP (2014) , M. Atala *et al.*, Nat. Phys. (2014)

Current patterns

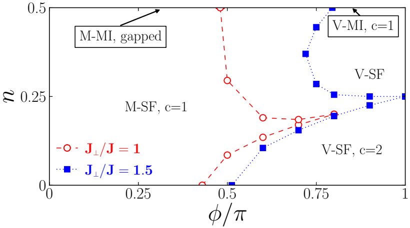
Meissner-MI

$\phi/\pi = 0.5,\ J_\perp/J = 2,\ n = 0.5$

Vortex-MI

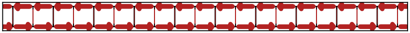
$\phi/\pi = 0.5,\ J_\perp/J = 0.5,\ n = 0.5$

cf A. Petrescu & K. Le Hur, PRL (2013)



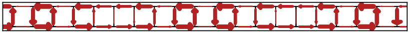
Meissner-SF

$\phi/\pi = 0.5,\ J_\perp/J = 1.5$, n=0.25



Vortex-SF

$\phi/\pi = 0.9,\ J_\perp/J = 1.5$, n=0.1



Tractable limit : around $\phi \sim \pi$

For strong rung-tunneling $J_\perp \gg J$ and $n < 0.5$: effective spin $\mathcal{H}$

Pseudo-spin-1/2 on each rung r :
$(|1,0\rangle_r + e^{ir\phi}|0,1\rangle_r)/\sqrt{2} \rightarrow |{\downarrow}\rangle_r$ and $|0,0\rangle_r \rightarrow |{\uparrow}\rangle_r$.

$$\mathcal{H}_{\mathbf{eff}}(\phi=\pi) = \frac{J^2}{2|J_\perp|}\sum_r \left(2S_r^z S_{r+1}^z - \left[S_r^+(\frac{1}{2} - S_{r+1}^z)S_{r+2}^- + \mathrm{H.c.}\right]\right)$$

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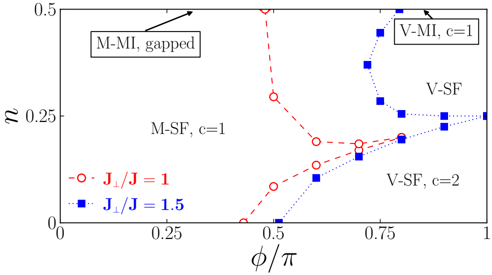
- $(n = 0.25, \phi = \pi)$ **becomes gapped**
  $\Rightarrow$ V-SF splitted in 2 lobes (for $J_\perp \gtrsim 1.3J$)



Tractable limit : around $\phi \sim \pi$

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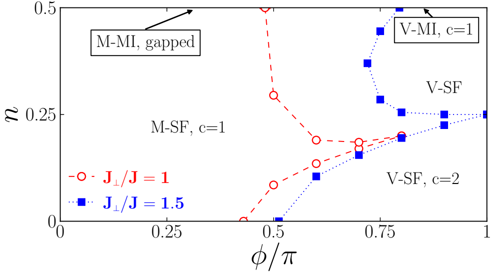
$$\mathcal{H}_{\text{eff}}(\phi = \pi) = \frac{J^2}{2|J_\perp|} \sum_r \left(2S^z_r S^z_{r+1} - \left[S^+_r (\frac{1}{2} - S^z_{r+1}) S^-_{r+2} + \text{H.c.} \right] \right)$$

- $(n = 0.25, \phi = \pi)$ **becomes gapped**
  $\Rightarrow$ V-SF splitted in 2 lobes (for $J_\perp \gtrsim 1.3J$)
- **V-MI and upper lobe of V-SF disappears** (for $J_\perp \gtrsim 1.7J$)

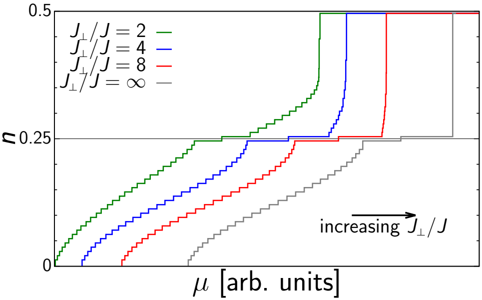


Tractable limit : around $\phi \sim \pi$

For strong rung-tunneling $J_\perp \gg J$ and $n < 0.5$: effective spin $\mathcal{H}$

$$\mathcal{H}_{\text{eff}}(\phi=\pi) = \frac{J^2}{2|J_\perp|}\sum_r \left(2S_r^z S_{r+1}^z - \left[S_r^+(\frac{1}{2} - S_{r+1}^z)S_{r+2}^- + \mathrm{H.c.}\right]\right)$$

- **Meta-magnetic transition at $\phi = \pi$:**
  jump in density from $n > 0.25$ to $n = 0.5$ (for $J_\perp \gtrsim 1.5J$)



Outline

- **Hard-Core Bosons**

  M. P., F. Heidrich-Meisner, I. P. McCulloch, S. Greschner, T. Vekua and U. Schollwöck, PRB **91** 140406(R) (2015)

  - Phase Diagram
  - **A few observables**

- Finite interactions

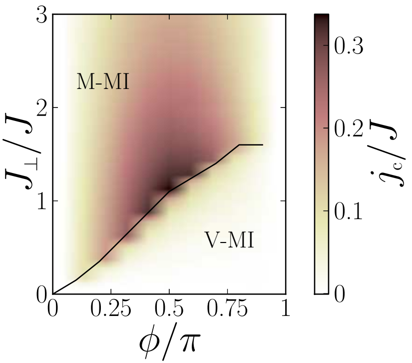
  S. Greschner, M. P., F. Heidrich-Meisner, I. P. McCulloch, U. Schollwöck and T. Vekua, *submitted* (2015)

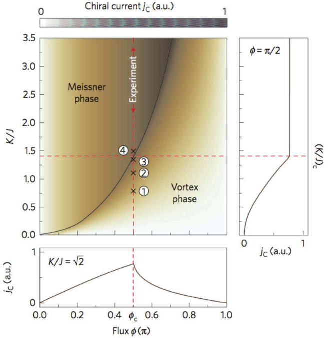
  - Phase Diagram
  - Chiral current

Chiral current, HCB at $n = 0.5$

$$j_c = \frac{1}{2L} \left| \sum_r \langle j^{\parallel}_{1,r} - j^{\parallel}_{2,r} \rangle \right|$$

cf non-interacting phases :

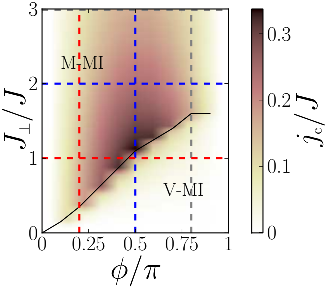




M. Atala *et al.*, Nat. Phys. (2014)

Vortex phase is disfavored

Chiral current, HCB at $n = 0.5$



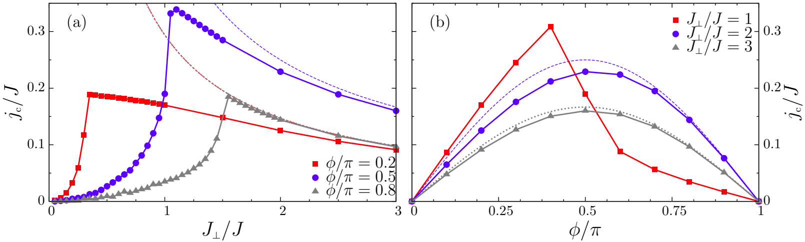
At $J_\perp \ll J$ and small ϕ :

$$j_c \sim (J_\perp/\phi)^2$$

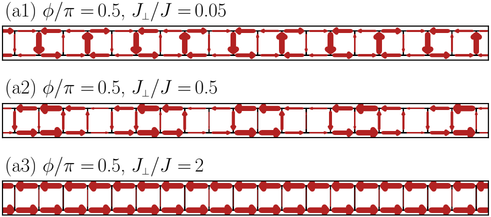
At $J_\perp \gg J$:

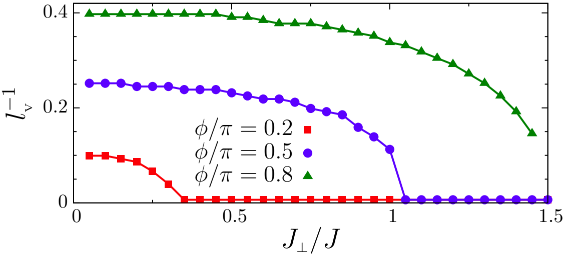
$$j_c = \frac{J^2}{J_\perp} \sin(\phi)$$

no saturation for HCB in gapped phases



Vortex density, HCB at $n = 0.5$





Inverse spatial extension of the vortices : $\rho_\mathrm{V} = l_\mathrm{V}^{-1}$ (obtained from the Fourier transform of the rung current $\langle j_r^\perp \rangle$) .

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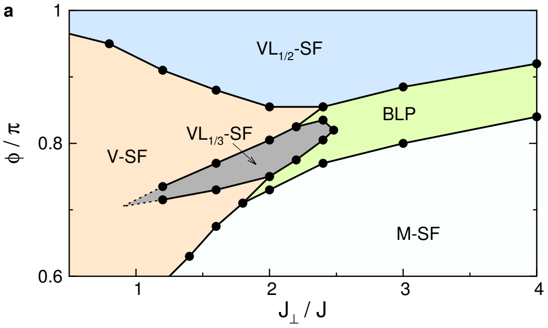
  - Phase Diagram
  - A few observables

- **Finite interactions**

  S. Greschner, M. P., F. Heidrich-Meisner, I. P. McCulloch, U. Schollwöck and T. Vekua, *submitted* (2015)

  - **Phase Diagram**
  - Chiral current

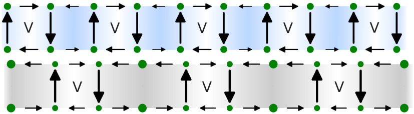
Phase Diagram, at $U = 2J$ and $n = 0.8$



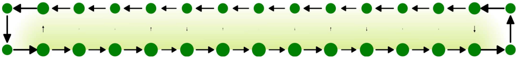
Vortex Lattices E. Orignac & T. Giamarchi, PRB (2001)

at $\rho_{\rm V} = 1/2$

and $\rho_{\rm V} = 1/3$



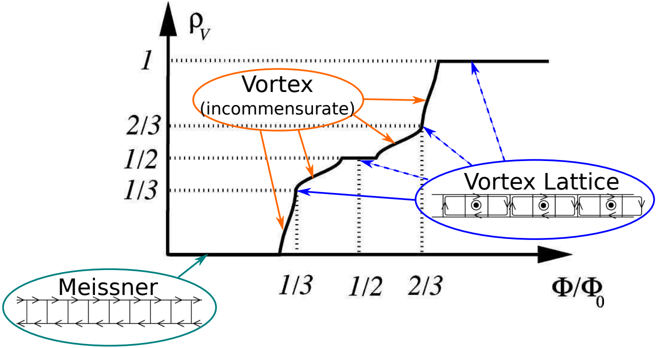
Biased Leg Phase R. Wei & E. J. Mueller, PRA (2014)



Vortex Lattices

Commensurate/ incommensurate transitions

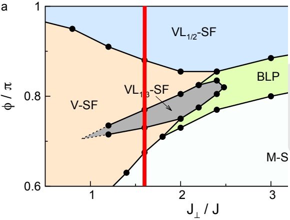
E. Orignac & T. Giamarchi, PRB (2001)

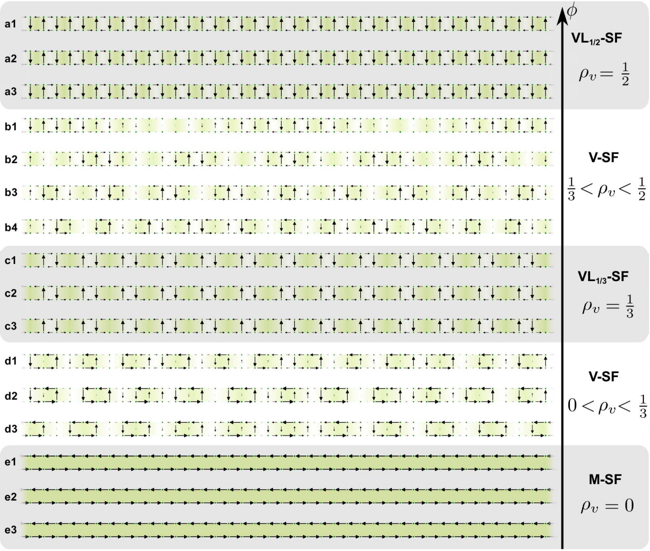


The size of the plateaux reduces with interactions

Phase Diagram, at $U = 2J$ and $n = 0.8$

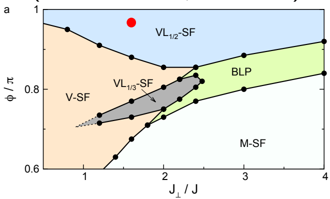
cut at $J_\perp = 1.6J$



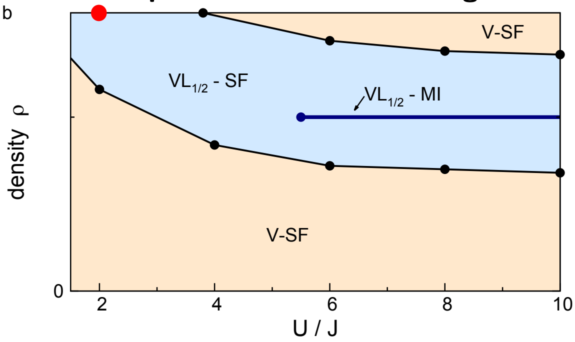


Phase Diagram : Mott Insulating phases

$(J_\perp = 1.6J,\ \phi = 0.9\pi)$



MI phase at fractional filling



Outline

- Hard-Core Bosons

  M. P., F. Heidrich-Meisner, I. P. McCulloch, S. Greschner, T. Vekua and U. Schollwöck, PRB **91** 140406(R) (2015)

  - Phase Diagram
  - A few observables

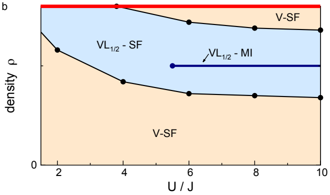
- **Finite interactions**

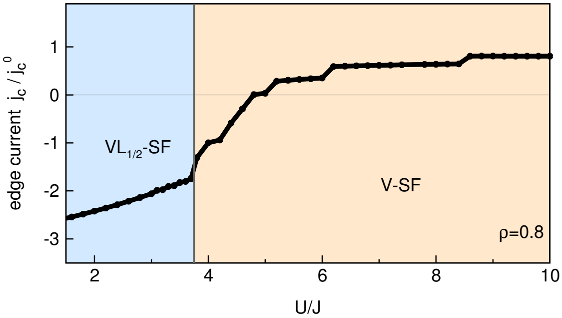
  S. Greschner, M. P., F. Heidrich-Meisner, I. P. McCulloch, U. Schollwöck and T. Vekua, *submitted* (2015)

  - Phase Diagram
  - **Chiral current**

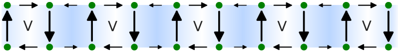
Chiral current : Swimming against the tide !

$(J_\perp = 1.6J,\ \phi = 0.9\pi)$
cut at $n = 0.8$



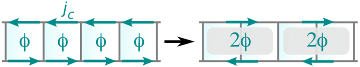


In and near the $\mathrm{VL}_{1/2}$, the current circulates in the clock-wise direction, on average :

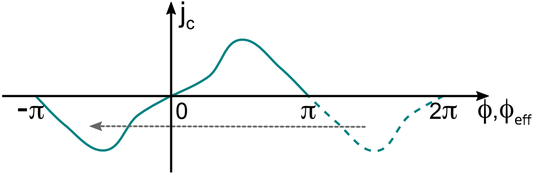


Chiral current : Swimming against the tide !

- In the $\mathrm{VL}_{1/2}$ phase : spontaneous symmetry breaking
  $\Rightarrow$ **doubling of the unit cell**



- j_c is odd and 2π-periodic with the flux



- If we have $\odot\,\mathcal{B}$ on $\square$ with $\phi \in [\pi/2, \pi]$;
  the effective flux on $\square\square$ (2ϕ) corresponds to $\bigotimes\mathcal{B}$.
  $\Rightarrow$ **spontaneous current reversal**

Chiral current : Swimming against the tide !

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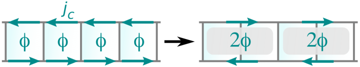

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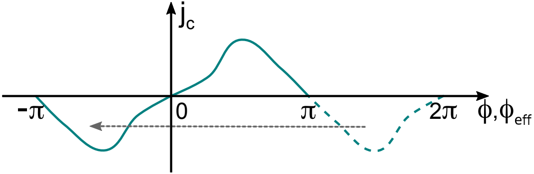

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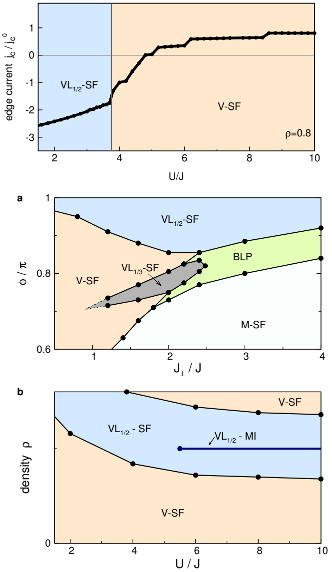
$\Rightarrow$ Genuine many-body effect

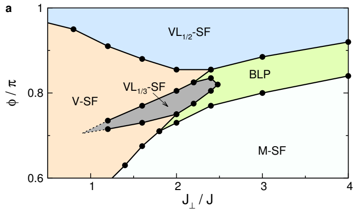
Current reversal : Remarks



- The amplitude of the reversal is $\sim 3j_c^0$
- The VL$_{1/3}$ does not lead to a current reversal (because $\phi > 2\pi/3$)
- The current reversal is robust against the presence of a mass gap
- Study at $n \gg 1$: the current reversal occurs at arbitrary small U/J
- It is stable against a finite number of defects, and at finite T

Conclusion

Strongly-interacting bosons on a ladder with uniform flux :
Phase diagram and observables (DMRG and field theory)



Plethora of phases :

- Mott insulating and superfluid, Meissner and Vortex phases
- Vortex lattices ($\rho_{\mathrm{V}} = 1/2$ and $1/3$)
- Biased Leg Phase

Vortex lattice phases,
Spontaneous current reversal

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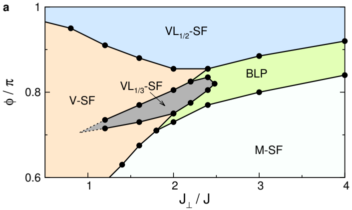

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Thank you for your attention !