Many-body Characterization of Particle-Conserving Topological Superfluids

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INT-15-1 - March 2015

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Mostly: PRL 113, 267002 (2014)
arXiv:1407.3793

Motivation for this work

- Most of the work on Topological Quantum Matter is based on the Mean-Field quasi-particle picture

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Reasons:

- Mathematically Simple (Lie algebraic with poly-complexity)
- Intuitive after Landau's work on quasiparticles
- Topological invariants easy to derive and compute
  ((full or partial) Chern #, Berry phases, Winding #, Bott index, Hopf index, ... → a Zoo of Numbers)

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Bulk-Boundary Correspondence

One-to-one correspondence between topological vacuum and symmetry-protected boundary (or defect) modes:

Boundary (defect) modes {
- Topo-Insulators: Fermions
- Topo-Superfluids: Majorana (or Fermions)

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  - Open QS

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- Do we have a Bulk-Boundary Correspondence ?
- What is the experimental signature ?
- What is a localized Many-body mode ?
- What is the meaning/fate of Majorana modes ?
- How much of the Mean-field picture survives ?

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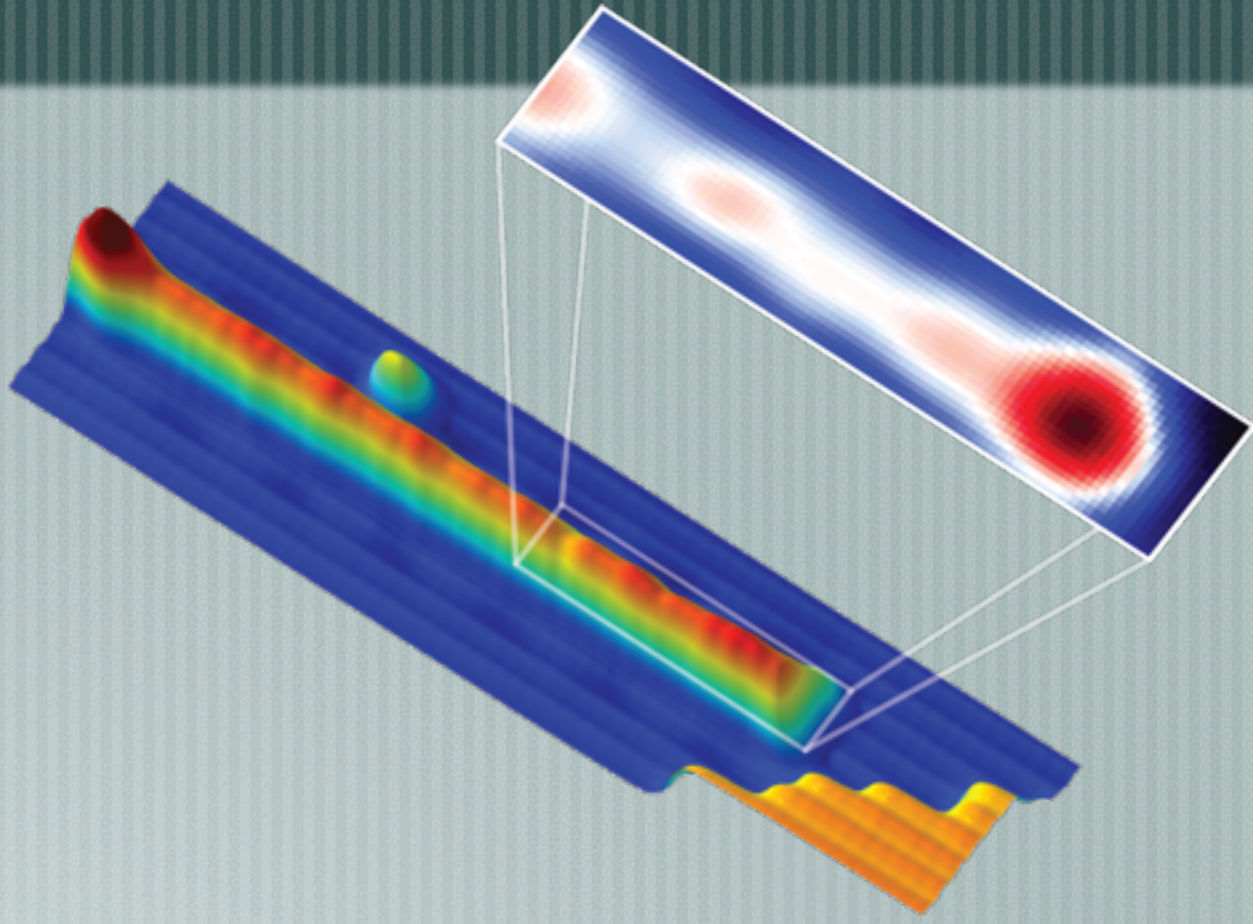
- Why do we care ?



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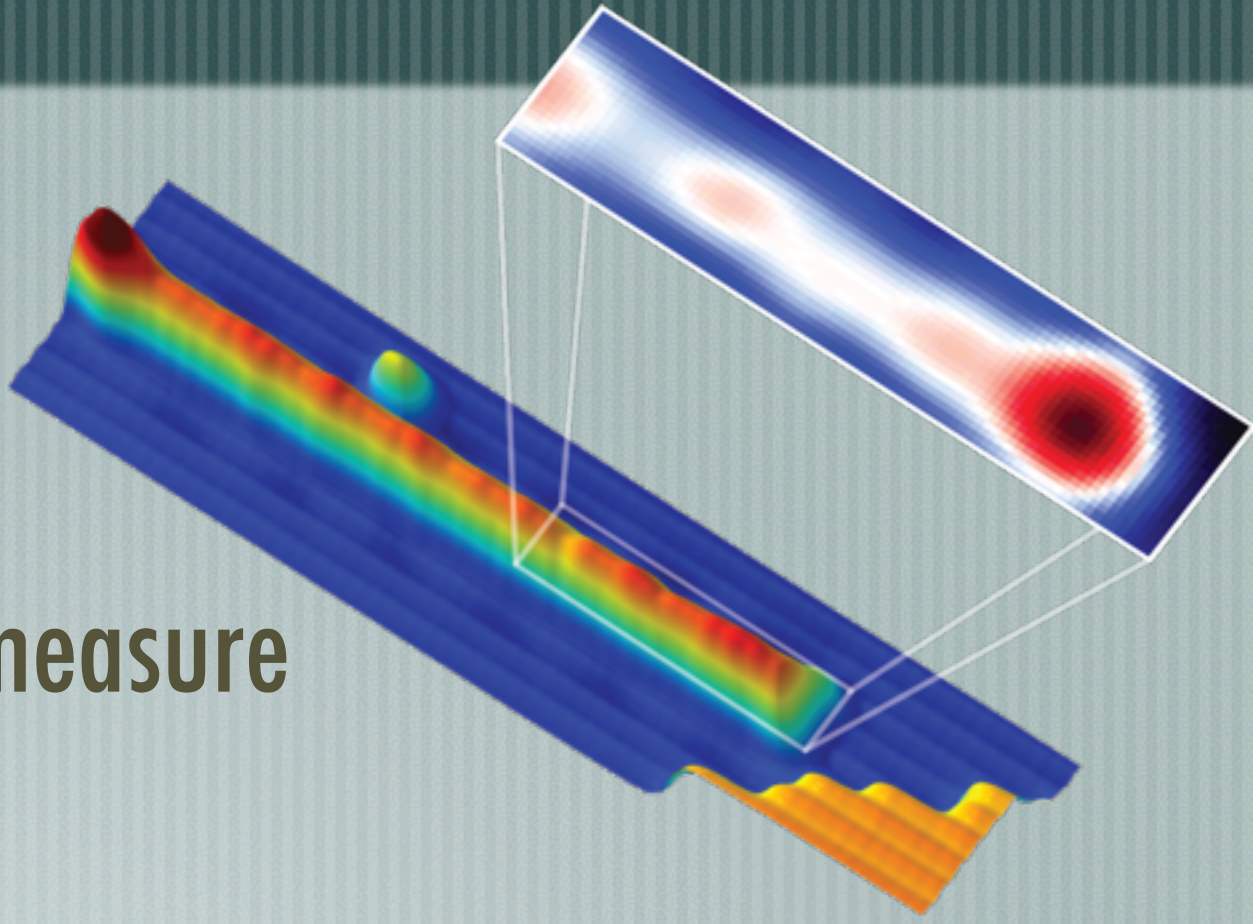

Why do we care ?

- We need to understand what to measure

- Majorana fermions are key components of many information processing devices (Topo Comp) because of its supposed protection (against decoherence) and non-Abelian braiding properties

Main Messages

- Introduce a number-conserving, interacting fermion superfluid model: The Richardson-Gaudin-Kitaev wire
- Characterization of Topological Superfluidity in generic interacting many-body systems: Fermion Parity Switches
- Meaning to emergent many-body Majorana zero-energy modes

Majorana Fermions in the Mean-Field Framework

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Majoranas are part of the hardware

Bogoliubov-de Gennes: No Conservation particle number

Conservation of fermion parity $\mathbb{Z}_2$

$$H_{\mathsf{mf}} = \sum_{i,j}^{L} A_{ij}\, c_i^\dagger c_j + \frac{1}{2}\sum_{i,j}^{L}(B_{ij}\, c_i^\dagger c_j^\dagger + B_{ij}^*\, c_j c_i)$$

$$\boxed{H_{\mathsf{mf}} = \frac{1}{2}(\Psi^\dagger H_{\mathsf{BdG}}\Psi + \mathrm{Tr}A)}$$

with: $H_{\mathsf{BdG}} = \begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix}$

$$A = A_R + \mathrm{i}A_I,\ B = B_R + \mathrm{i}B_I \qquad ,\ A = A^\dagger,\ B^T = -B$$

$$A_R = A_R^T,\ A_I = -A_I^T,\ B_R = -B_R^T,\ B_I = -B_I^T$$

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or:

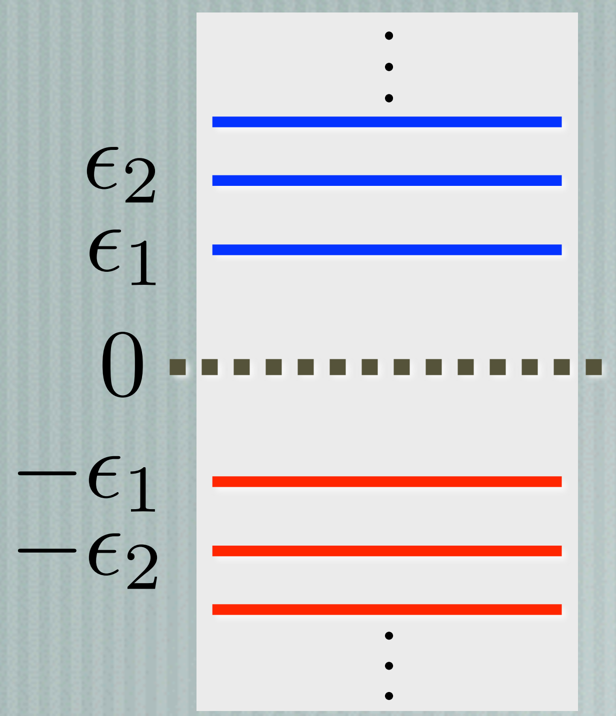
$$H_{\mathsf{BdG}} = \mathrm{i}\mathbb{1}_2 \otimes A_I + \mathrm{i}\sigma^x \otimes B_I + \mathrm{i}\sigma^y \otimes B_R + \sigma^z \otimes A_R$$

Single-particle spectrum: The anti-unitary operator

$$\mathcal{C} = K\sigma^x \otimes \mathbb{1}_L$$

anti-commutes with the BdG Hamiltonian: $\{H_{\mathsf{BdG}}, \mathcal{C}\} = 0$

This, in turn, implies that the single-particle spectrum is of the form:



$$H_{\mathsf{BdG}}\,\phi_n = \epsilon_n \phi_n \qquad , \; \phi_n = \begin{pmatrix} X^n \\ Y^n \end{pmatrix}$$

with quasi-particle bogoliubons:

$$b_n^\dagger = \sum_i^L (X_i^n c_i^\dagger + Y_i^n c_i)$$, in terms of which:

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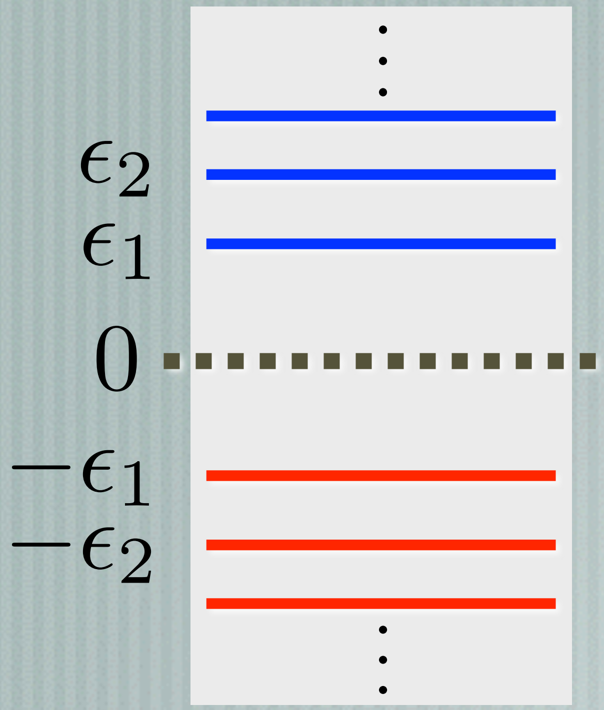

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Many-body ground-state energy $\mathcal{E}_0$

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$$\epsilon_1 = 0 \qquad \text{(doubly degenerate)}$$

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Zero modes are Majorana fermions by default

Example: Beyond Kitaev's paradigm ⇨ Power-law Majoranas

$$H_{\mathsf{mf}} = -t_1 \sum_{i=1}^{L} (c_i^\dagger c_{i+1} + \mathsf{h.c}) - t_2 \sum_{i=1}^{L} (c_i^\dagger c_{i+2} + \mathsf{h.c}) - \mu \sum_{i=1}^{L} n_i$$

$$-\Delta \sum_{i>j}^{L} \eta(i-j) c_j^\dagger c_i^\dagger - \Delta^* \sum_{i>j}^{L} \eta(i-j) c_i c_j$$

with: $\eta(i-j) = \frac{4}{L} \sum_k \sin(k(i-j))\, \eta_k$

$$\eta_k = \sin\left(\frac{k}{2}\right) \sqrt{t_1 + 4t_2 \cos^2\left(\frac{k}{2}\right)}$$

1) $t_1 = 0,\ t_2 \neq 0 :\ \ \eta(m) = \sqrt{t_2}\, \delta_{m,1}$

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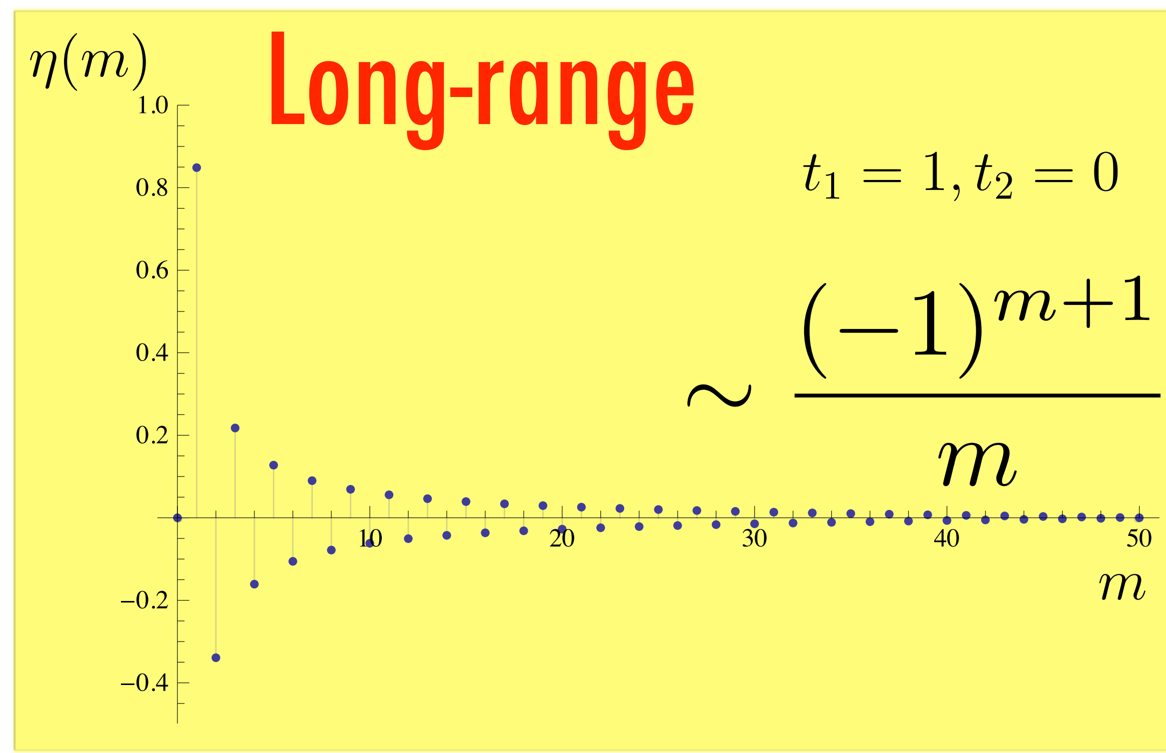

It is trivial to show that :

- There exists a Topologically non-trivial quantum phase

- There exists Power-law Majorana fermions
  (there is no symmetry-sweet-spot with deconfined Majoranas)

- There exists 4π-periodic Josephson effect

The Richardson-Gaudin-Kitaev wire
(A Number conserving Topological Superfluid)

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What is a topological superfluid ?

A number-conserving fermionic superfluid: $(G > 0)$

$$H_{\mathsf{RGK}} = \sum_{k \in \mathcal{S}_k^{\phi}} \varepsilon_k \, \hat{c}_k^{\dagger} \hat{c}_k - 8G \sum_{k,k' \in \mathcal{S}_{k+}^{\phi}} \eta_k \eta_{k'} \hat{c}_k^{\dagger} \hat{c}_{-k}^{\dagger} \hat{c}_{-k'} \hat{c}_{k'}$$

Free-fermion band:

$$\varepsilon_k = -2t_1 \cos k - 2t_2 \cos 2k$$

Interaction potential:

$(\eta_k = -\eta_{-k})$

$$\eta_k = \sin\left(\frac{k}{2}\right) \sqrt{t_1 + 4t_2 \cos^2\left(\frac{k}{2}\right)}$$

Boundary conditions:

Flux= $\Phi = \dfrac{\phi}{2\pi} \Phi_0$ $\quad (\Phi_0 = h/2e)$

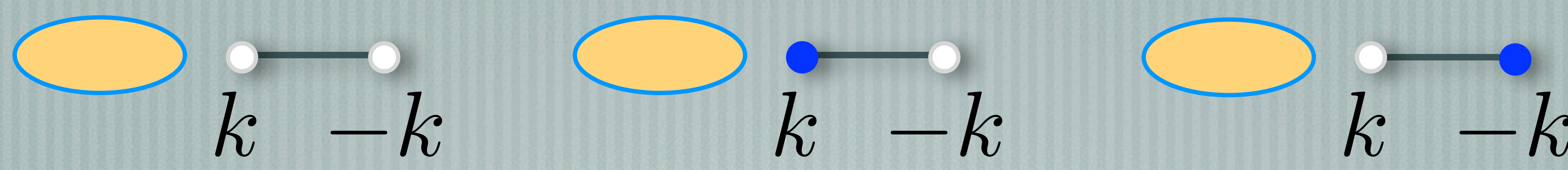
Periodic: $(\phi = 0)$ $\quad \mathcal{S}_k^0 = \mathcal{S}_{k+}^0 \oplus \mathcal{S}_{k-}^0 \oplus \{0, -\pi\}$

Anti-Periodic: $(\phi = 2\pi)$ $\quad \mathcal{S}_k^{2\pi} = \mathcal{S}_{k+}^{2\pi} \oplus \mathcal{S}_{k-}^{2\pi}$

The Richardson-Gaudin-Kitaev wire is integrable (Bethe ansatz)

$$N = 2M + N_\nu + N_{\text{inactive}}$$ (L orbitals)

Paired | **Unpaired** | **Inactive levels**



$\nu_k = 0$ | $\nu_k = +1$ | $\nu_k = -1$

seniority

$$N_\nu = \sum_k |\nu_k|$$

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Eigenspectrum:

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with Gaudin (Bethe) equations:

$$\sum_{k \in \mathcal{S}_{k+}^{\phi}} \frac{s_k}{\eta_k^2 - E_\alpha} - \sum_{\beta(\neq\alpha)} \frac{1}{E_\beta - E_\alpha} = \frac{Q_\phi}{E_\alpha}$$

$$Q_\phi = 1/2G - \sum_{k \in \mathcal{S}_{k+}^{\phi}} s_k + M - 1 \quad , \quad s_k = \frac{1}{2}(1 - |\nu_k|) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$$

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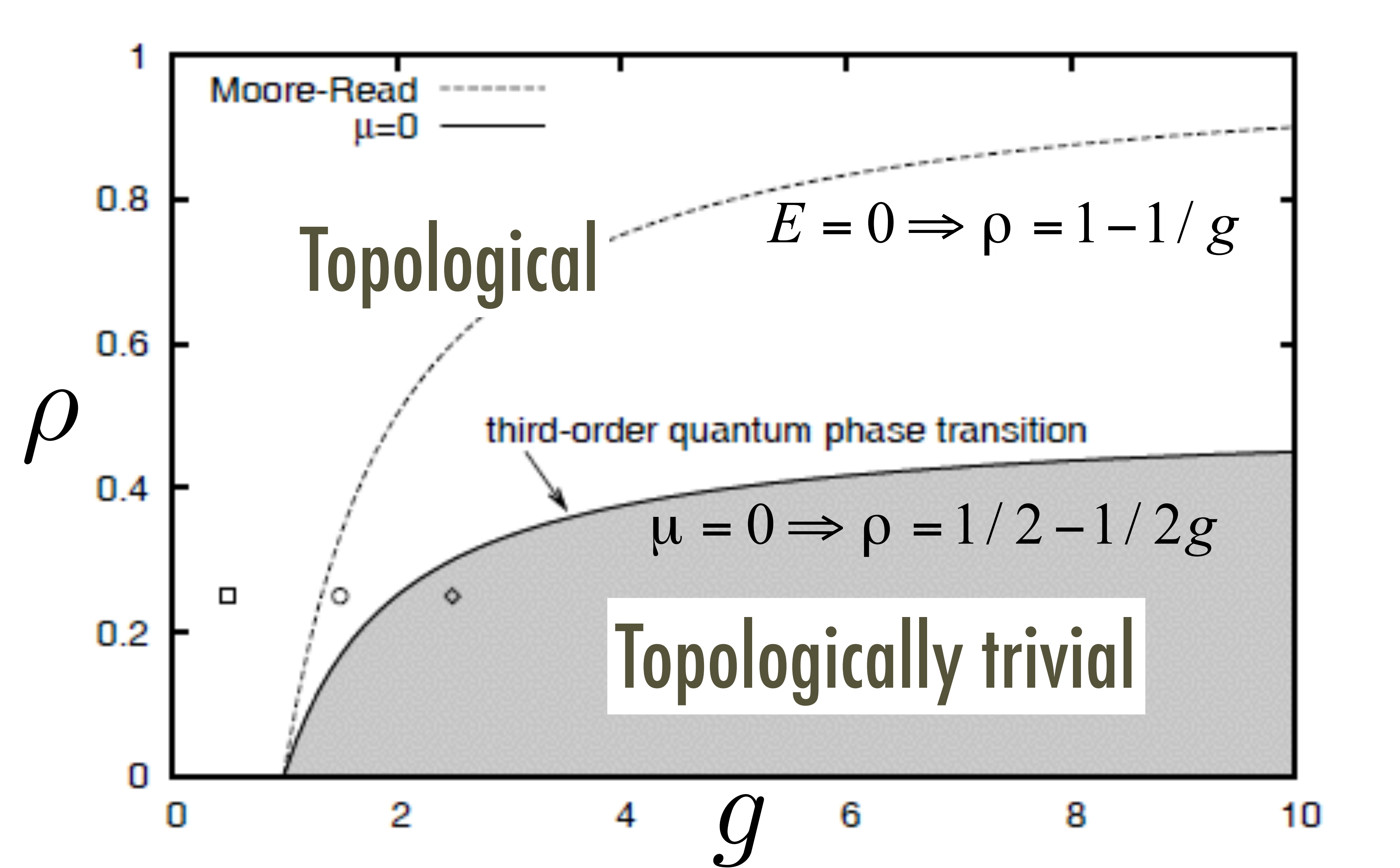
Quantum Phase Diagram

The phase diagram can be parametrized in terms of the density $\rho = N/L$ and the rescaled coupling $g = GL/2$



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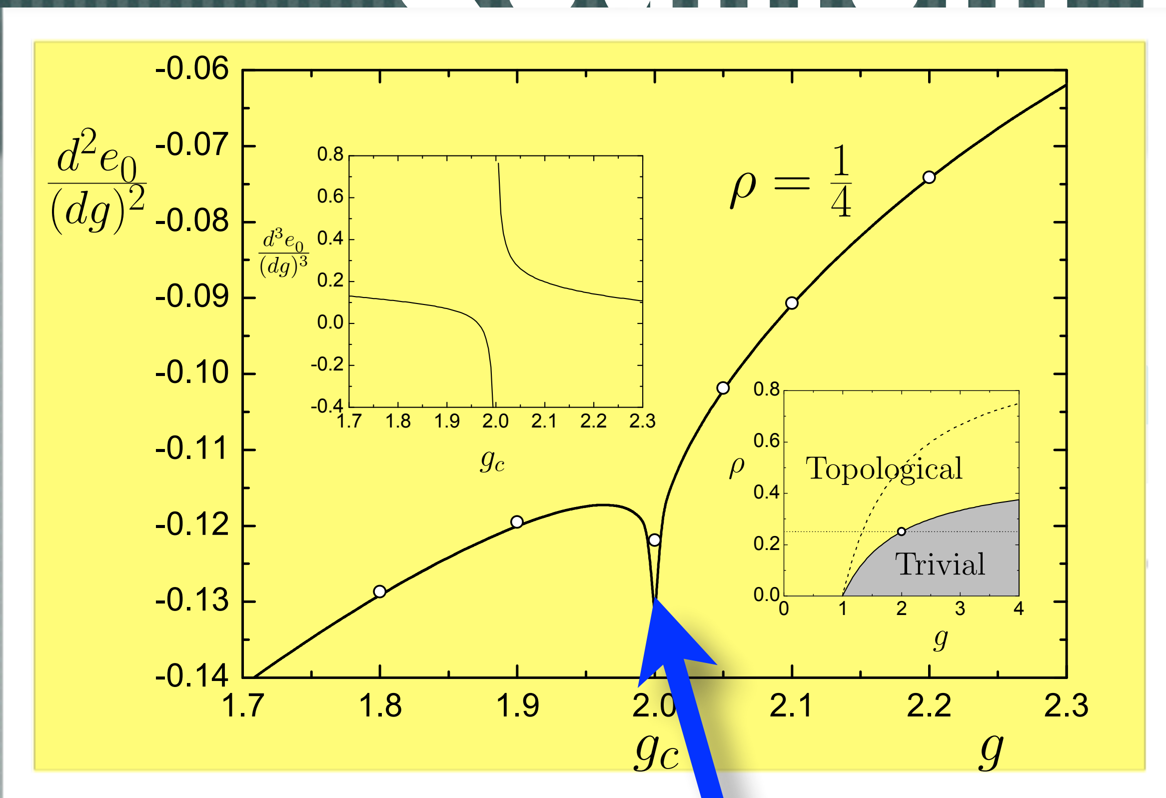

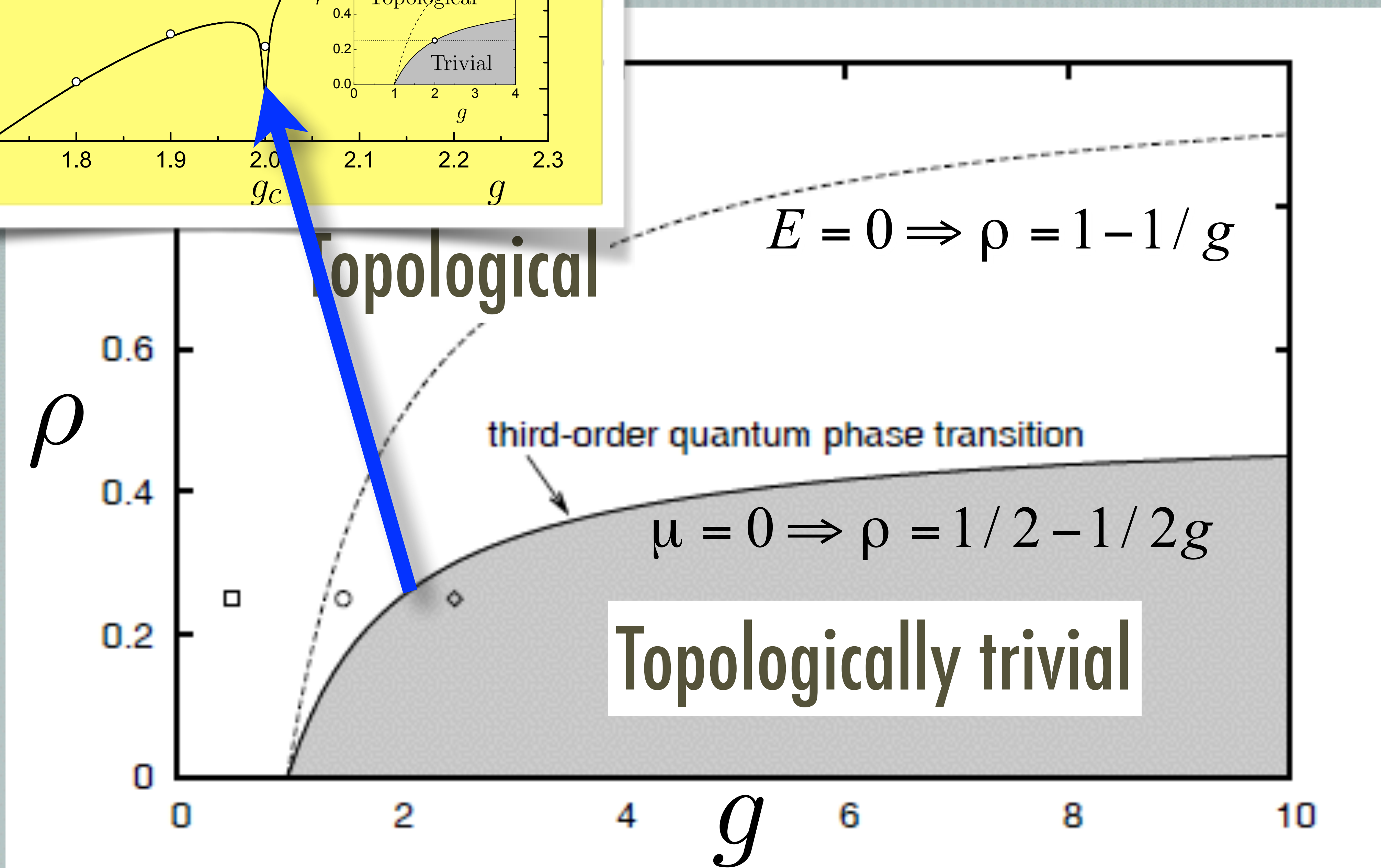

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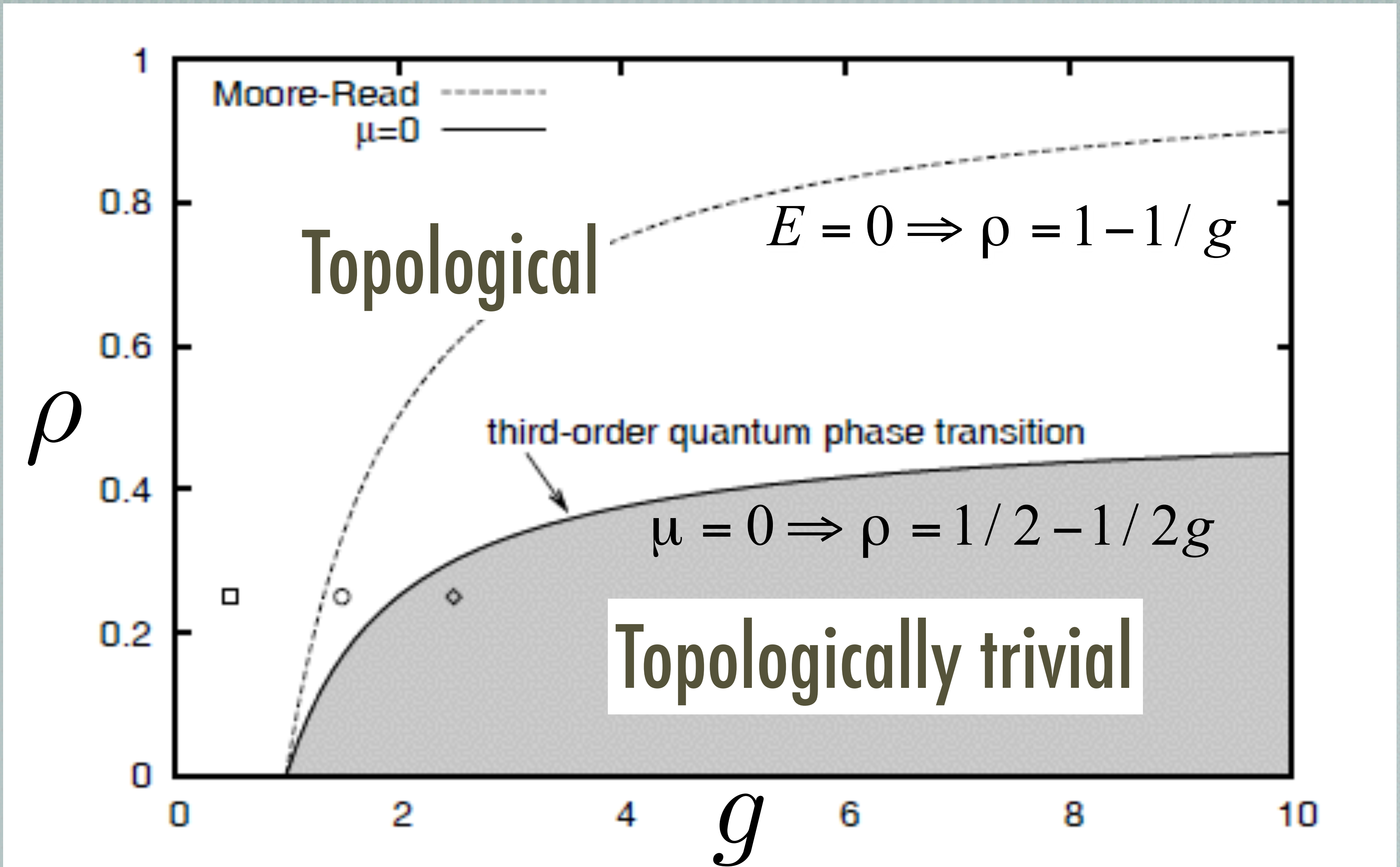

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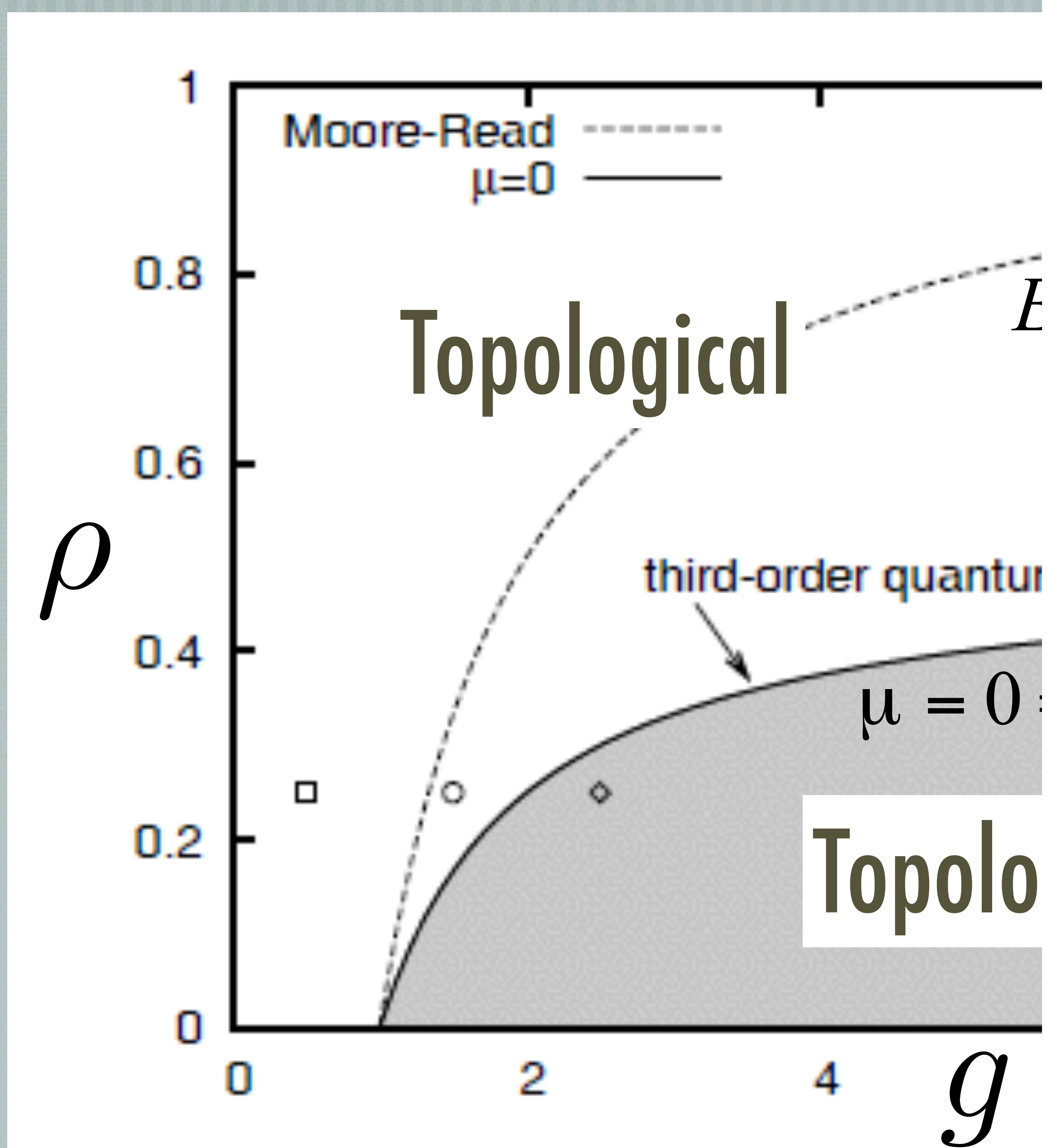

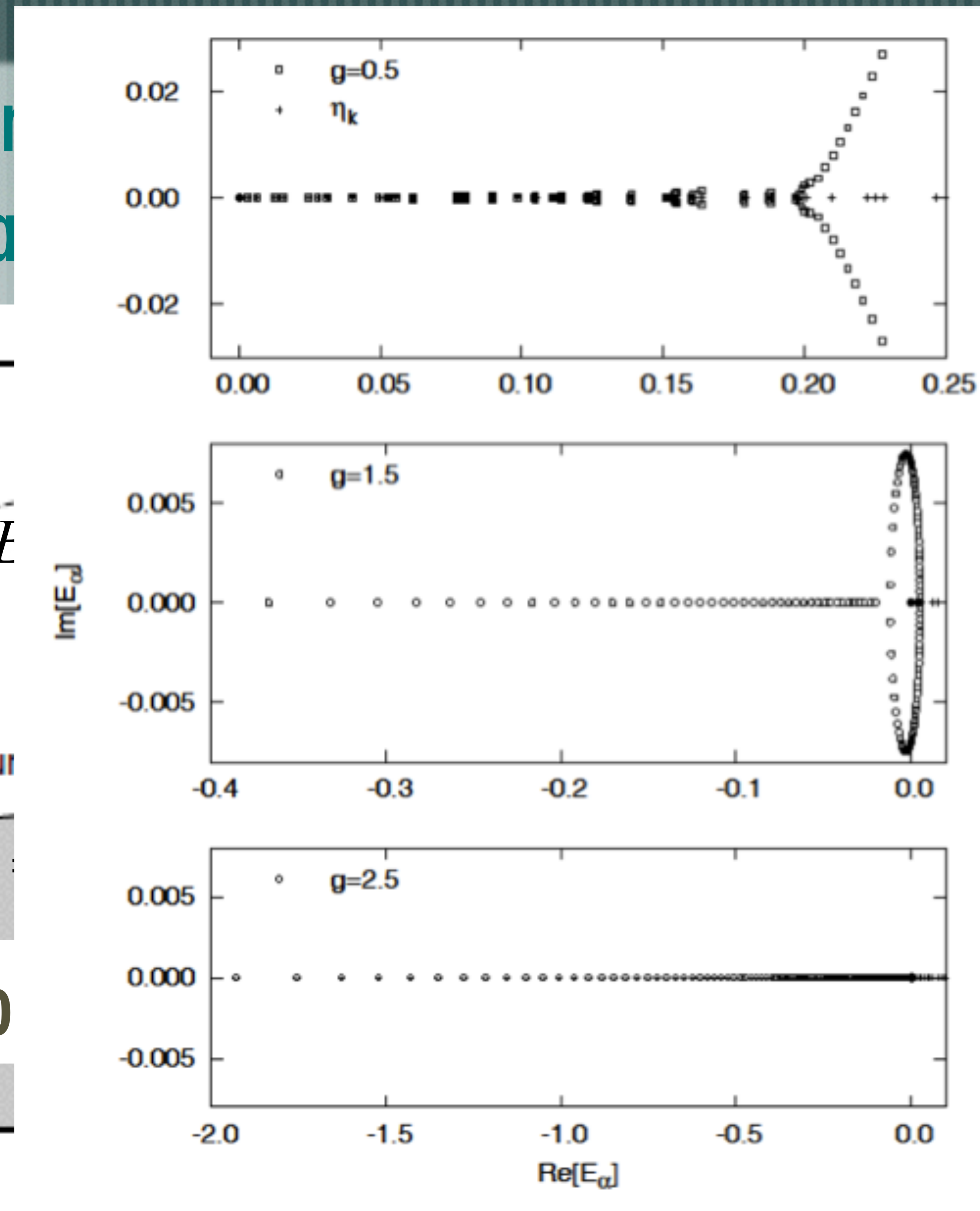

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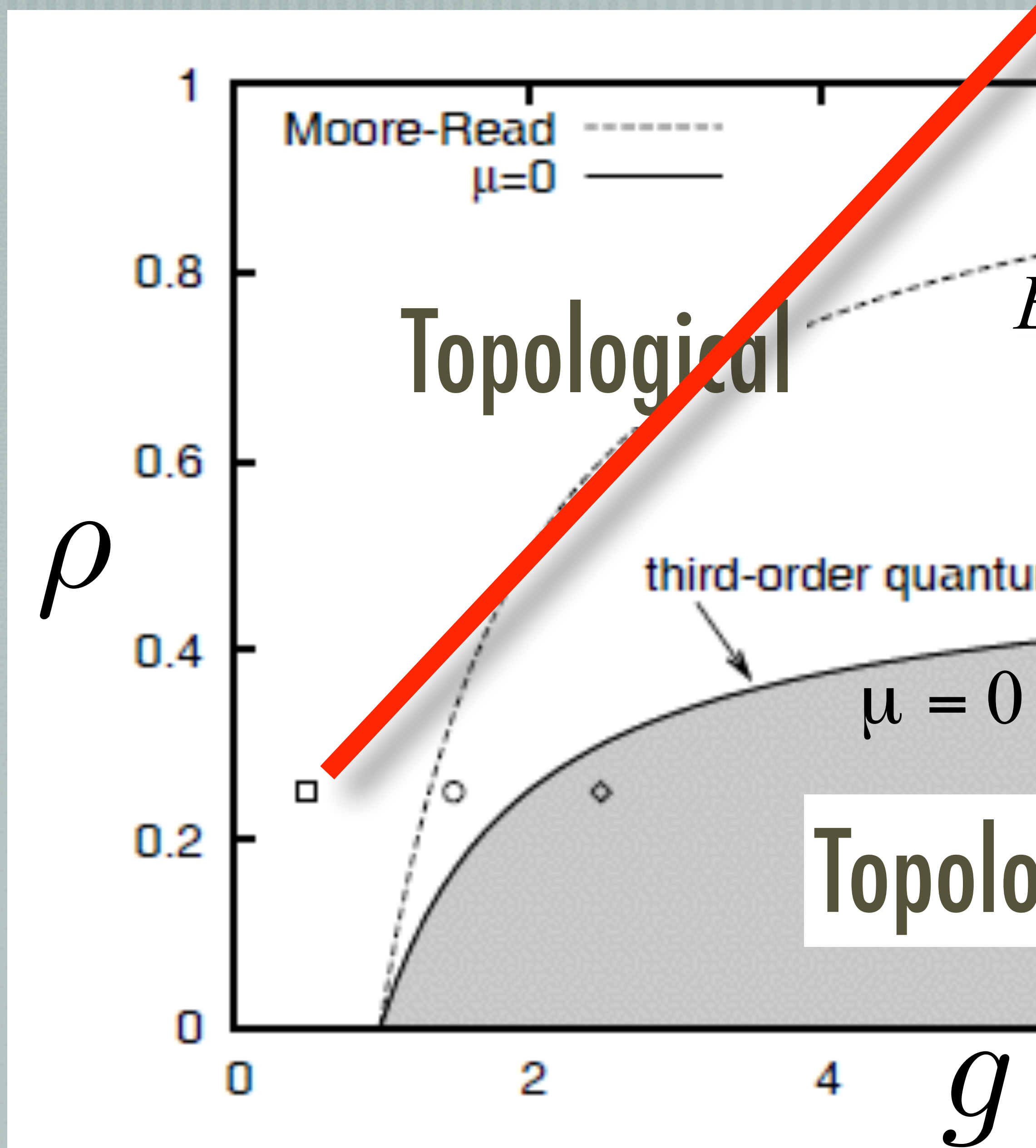

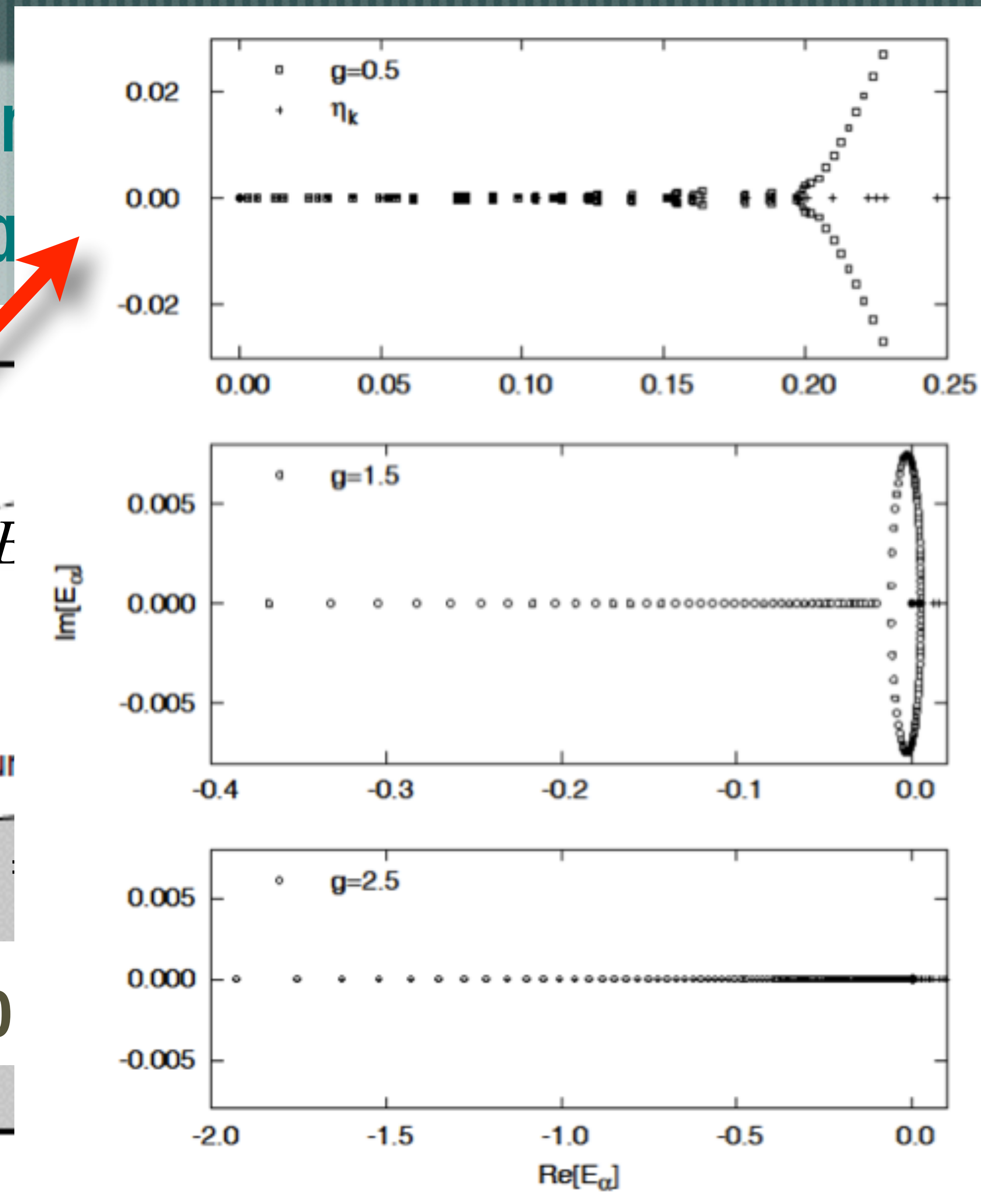

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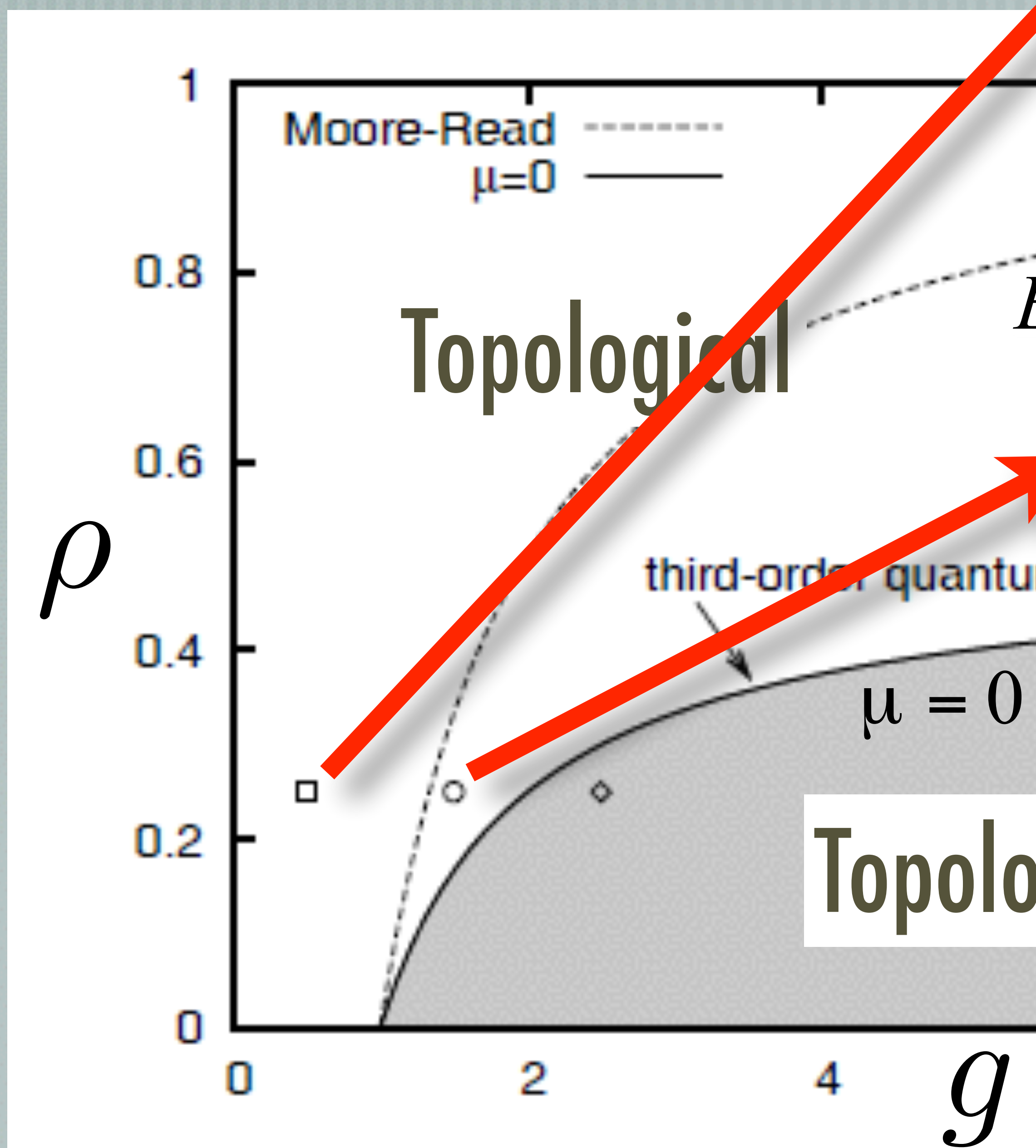

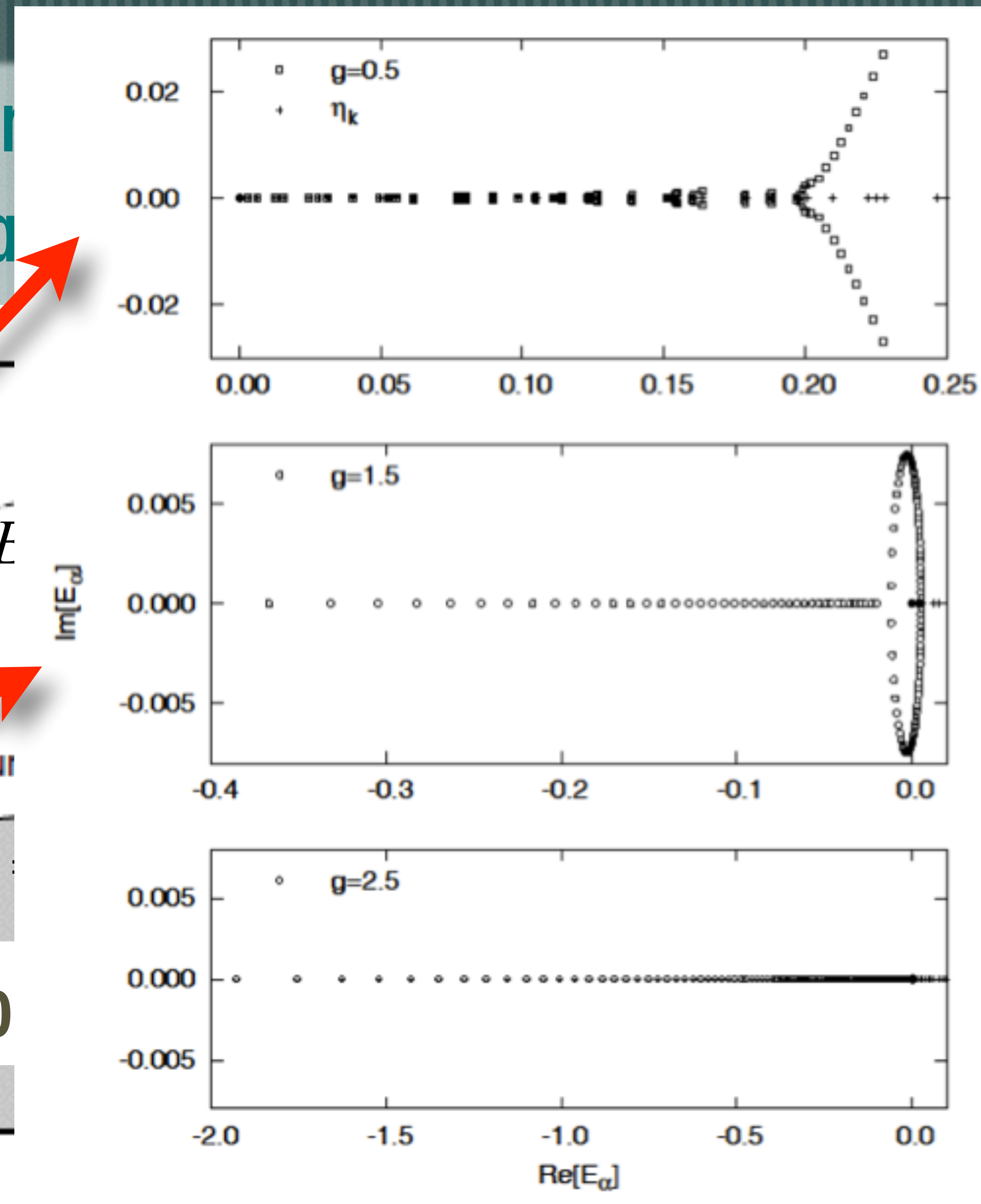

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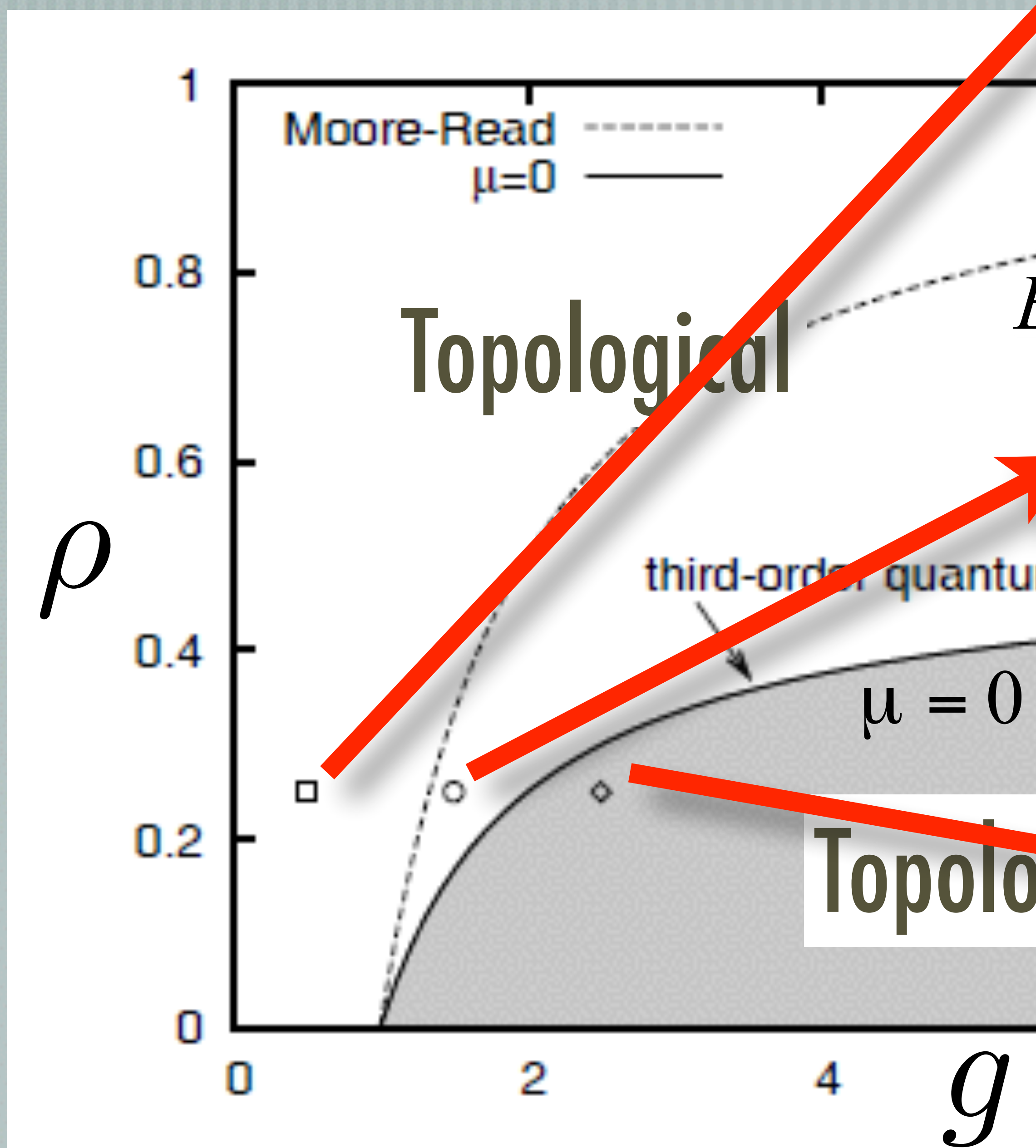
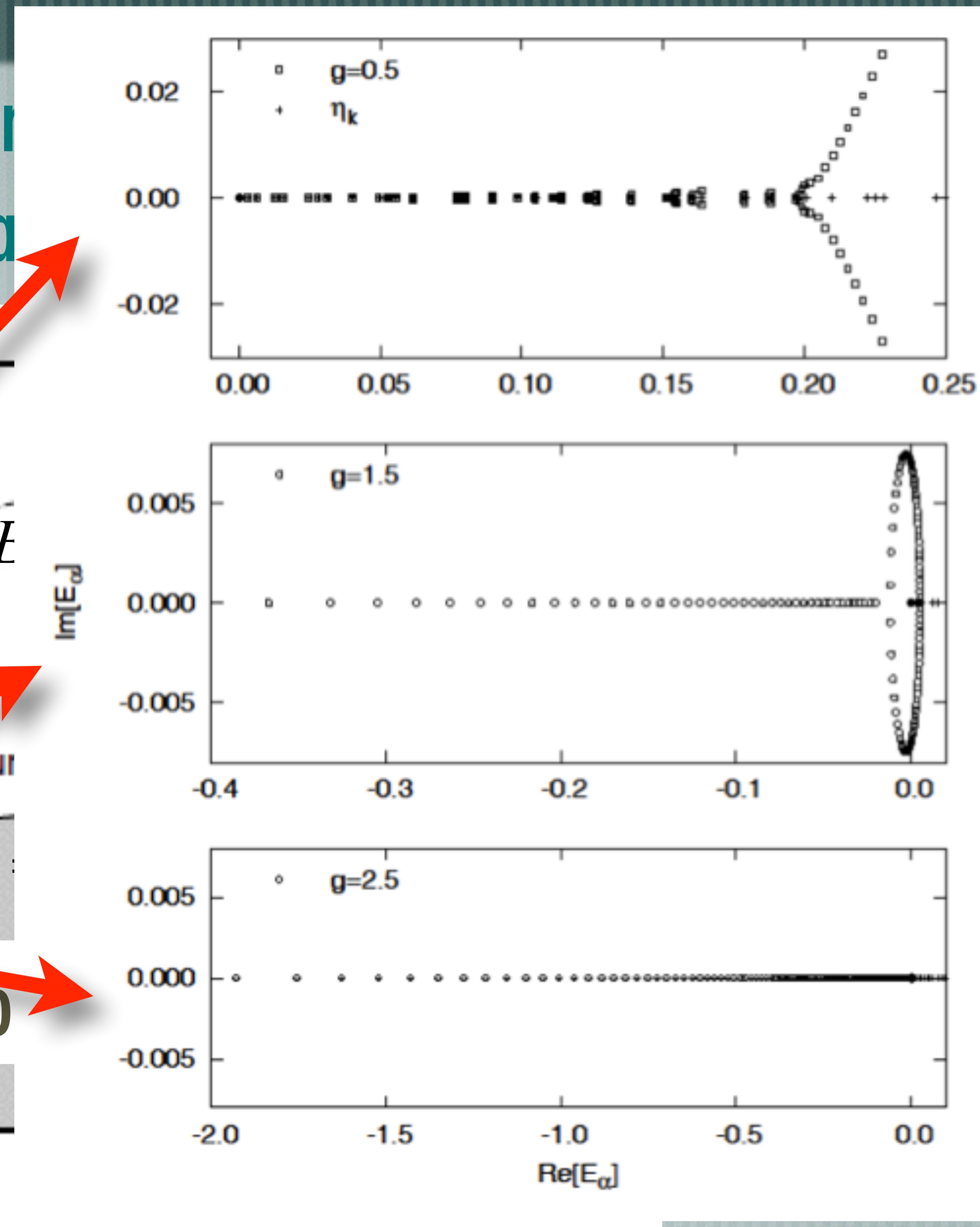
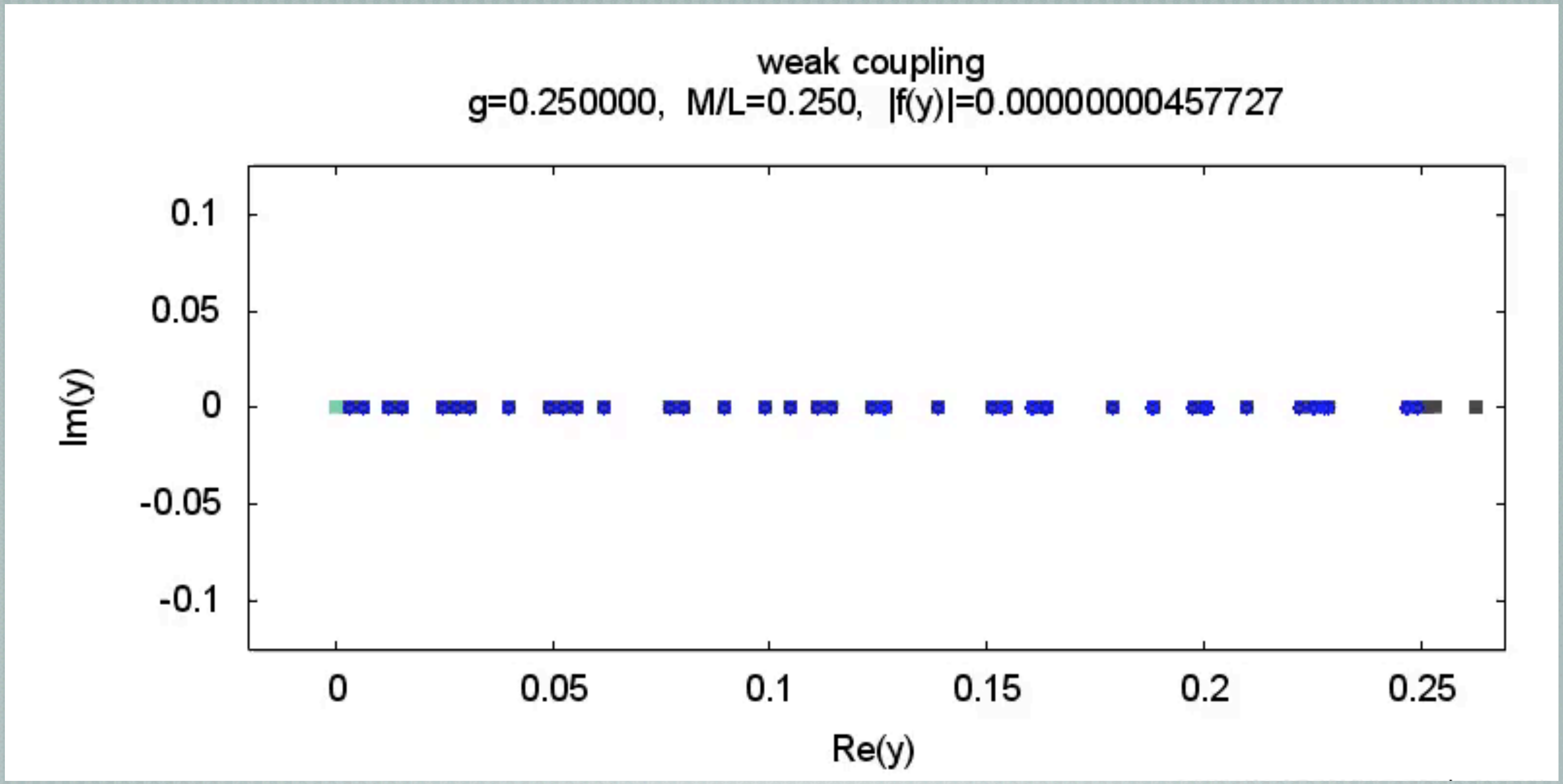




Behavior of the Pairons: The Movie



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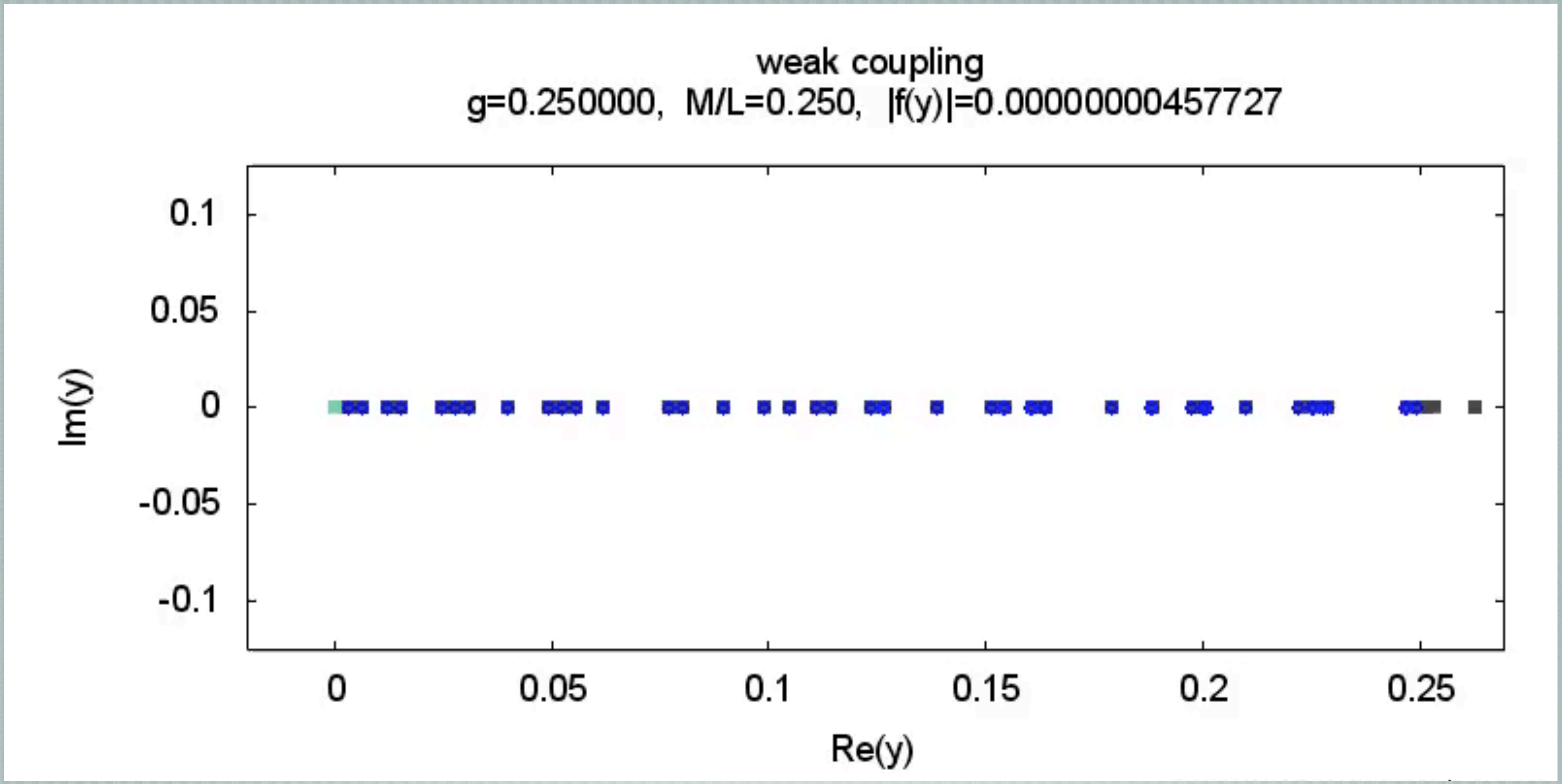
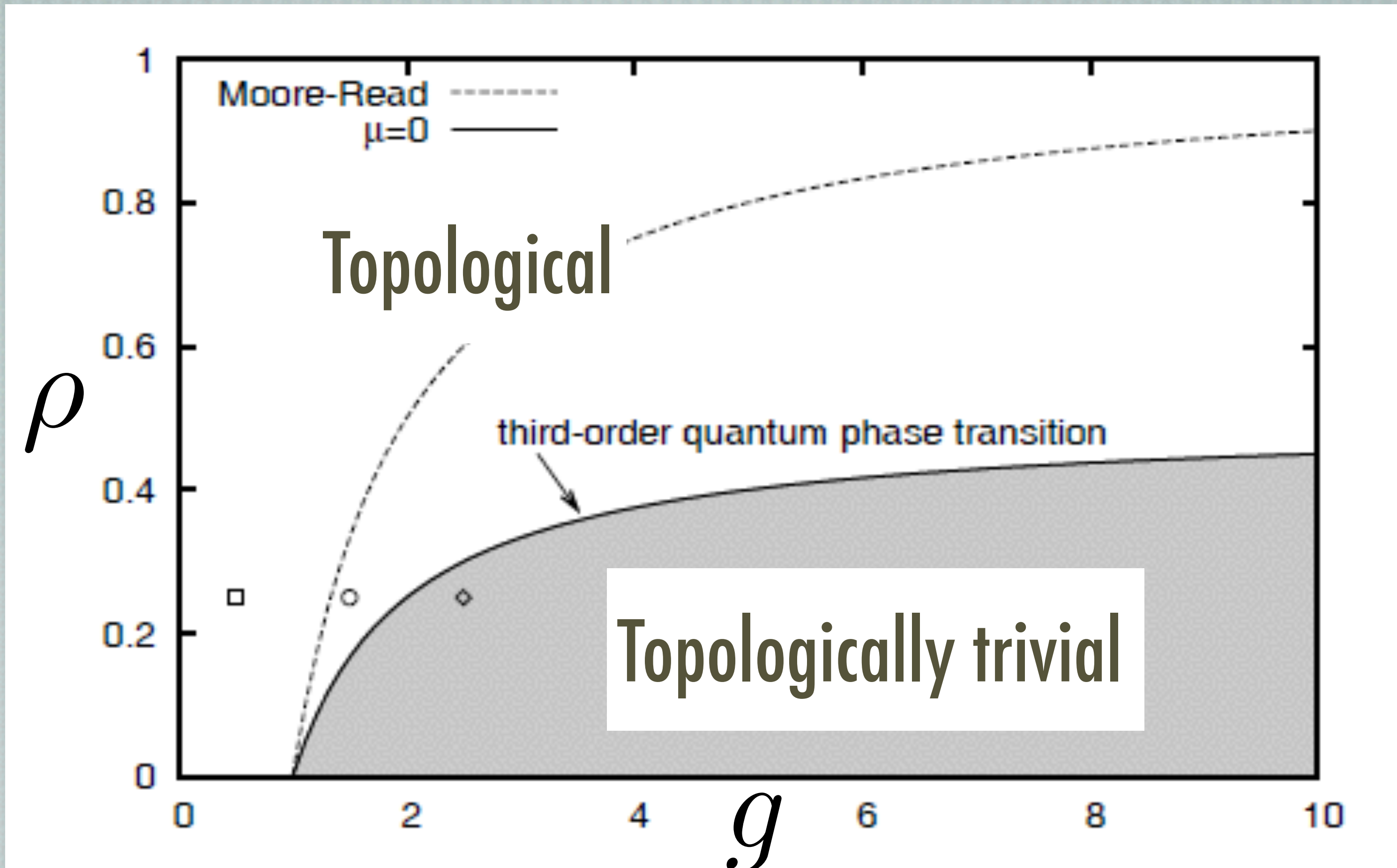


Topological Invariant

It turns out that for the RGK wire there is a topological invariant related to a Winding number: Occupation number

$$\mathcal{N}_k = \tfrac{1}{2} - s_k - 4 s_k \gamma^2 \sum_{\alpha=1}^{M} \frac{\eta_k^2}{(\eta_k^2 - E_\alpha)^2} \frac{\partial E_\alpha}{\partial \gamma}$$



Topological Invariant

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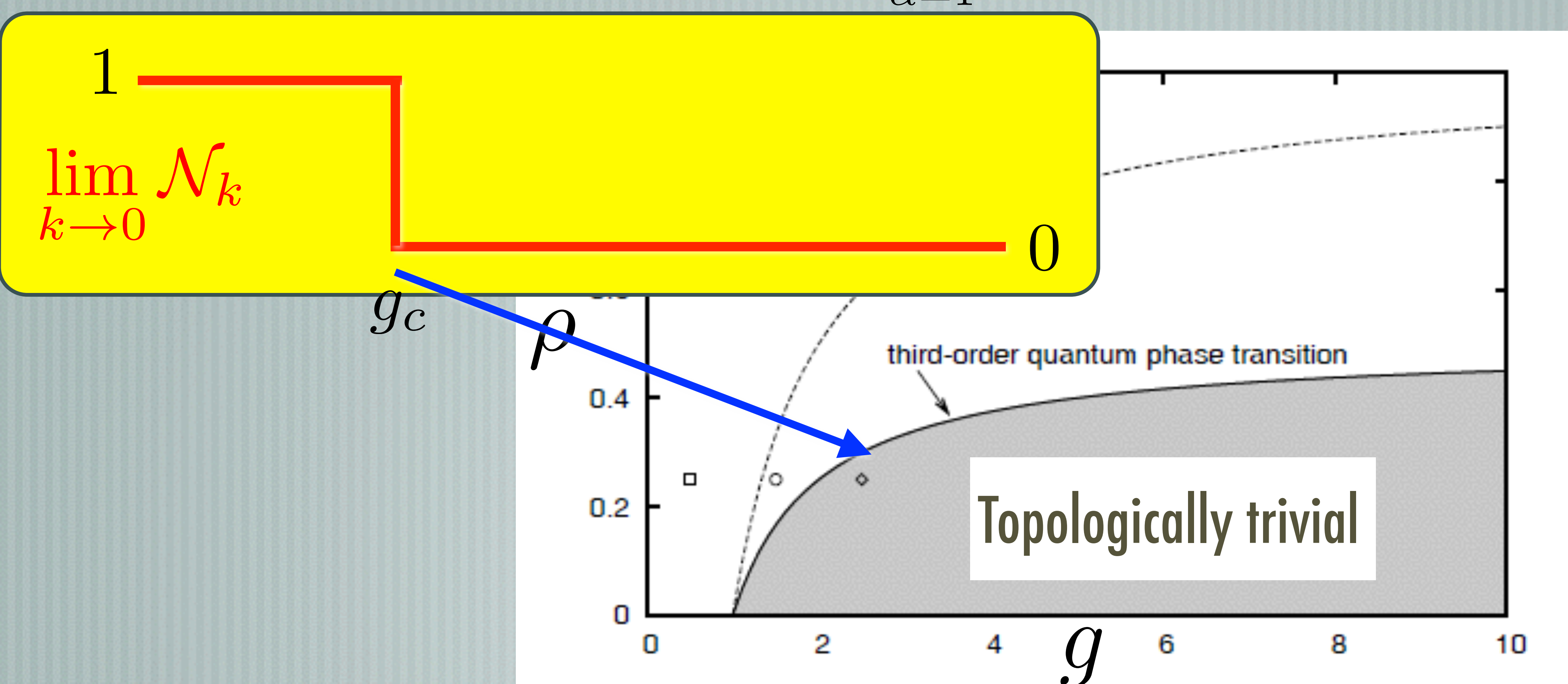


Many-Body Characterization of Topological Superfluids

How does one distinguish Topo from Trivial?

Many-Body Characterization of Topological Superfluids

How does one distinguish Topo from Trivial?

Many-Body Characterization of Topological Superfluids

Fermion Parity Switches

Fermion parity Switches:

A quantitative criterion to establish Topological Superfluidity that exploits the behavior of the ground state energy of a system with N, $N+1$, and $N-1$ particles, for both periodic $(\Phi = 0)$ and anti-periodic $(\Phi = \Phi_0 = \frac{h}{2e})$ BC

To identify the parity Switches: $N \in$ even

Even sector

$$\mathcal{E}_0^{\text{even}}(\Phi) = \mathcal{E}_0^{\Phi}(N)$$

Odd sector

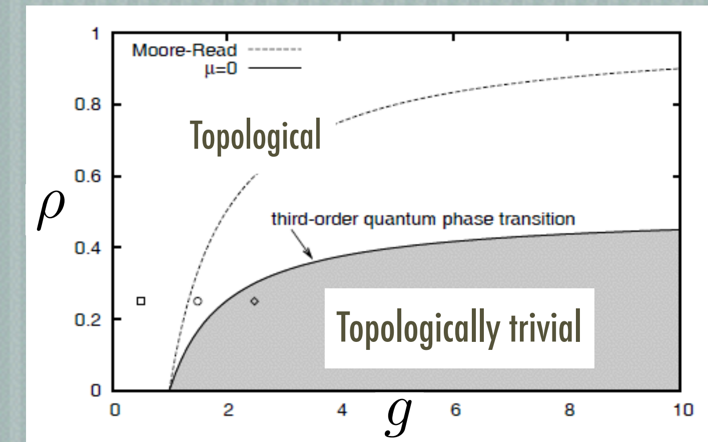
$$\mathcal{E}_0^{\text{odd}}(\Phi) = \frac{1}{2}(\mathcal{E}_0^{\Phi}(N+1) + \mathcal{E}_0^{\Phi}(N-1))$$

Define: $\chi(\Phi) = \mathcal{E}_0^{\text{odd}}(\Phi) - \mathcal{E}_0^{\text{even}}(\Phi)$ = Inverse compressibility

Parity Switch:

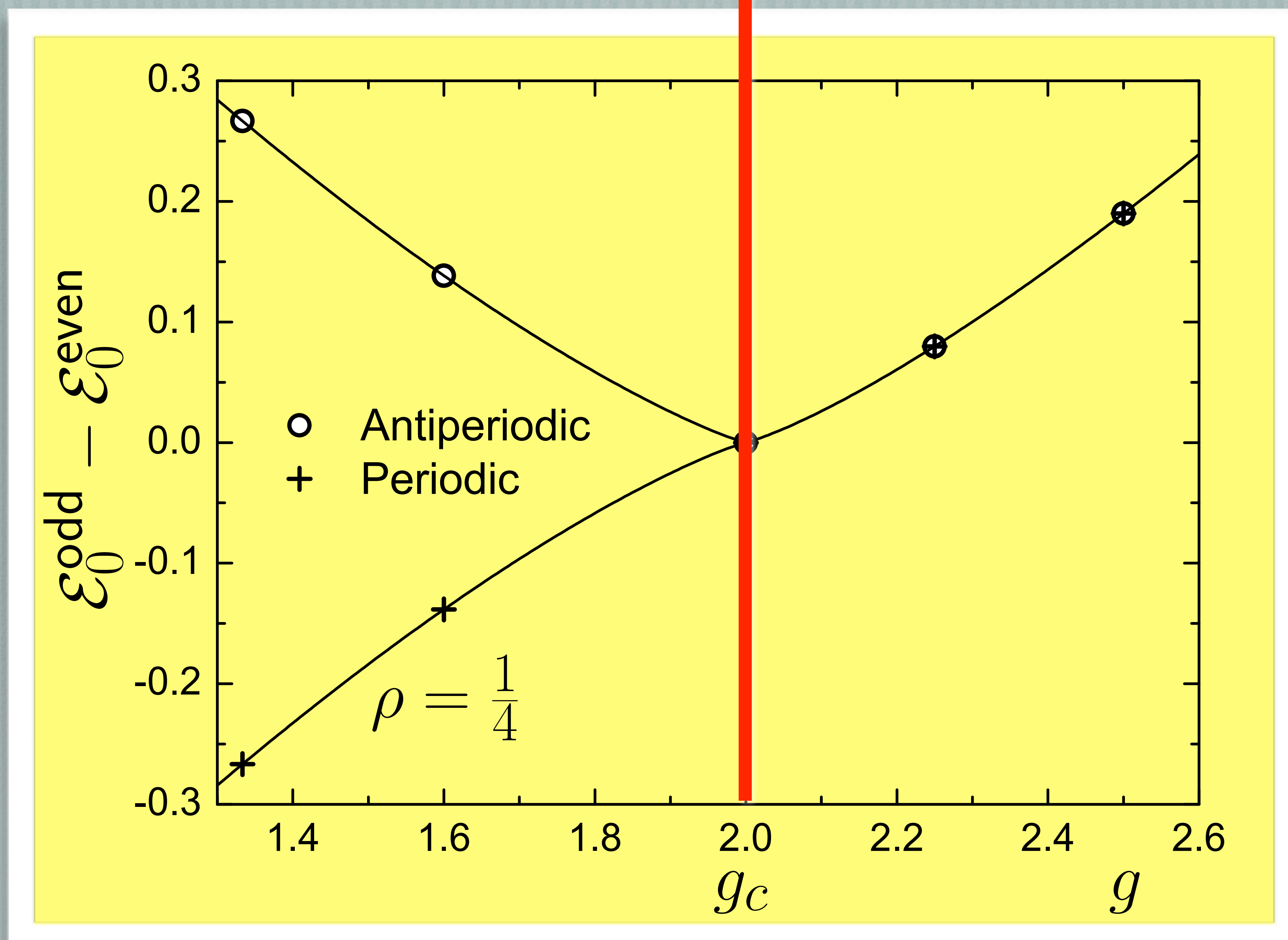
$$\mathcal{P}_N(\Phi) = \text{sign}[\chi(\Phi)]$$

In our Richardson-Gaudin-Kitaev wire:

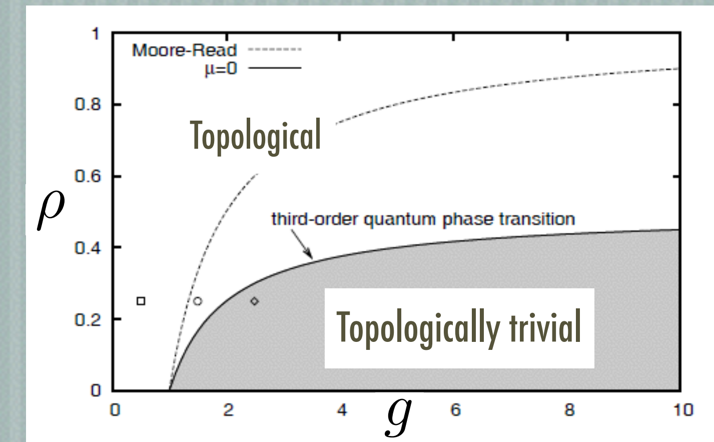


Topological

Topologically Trivial



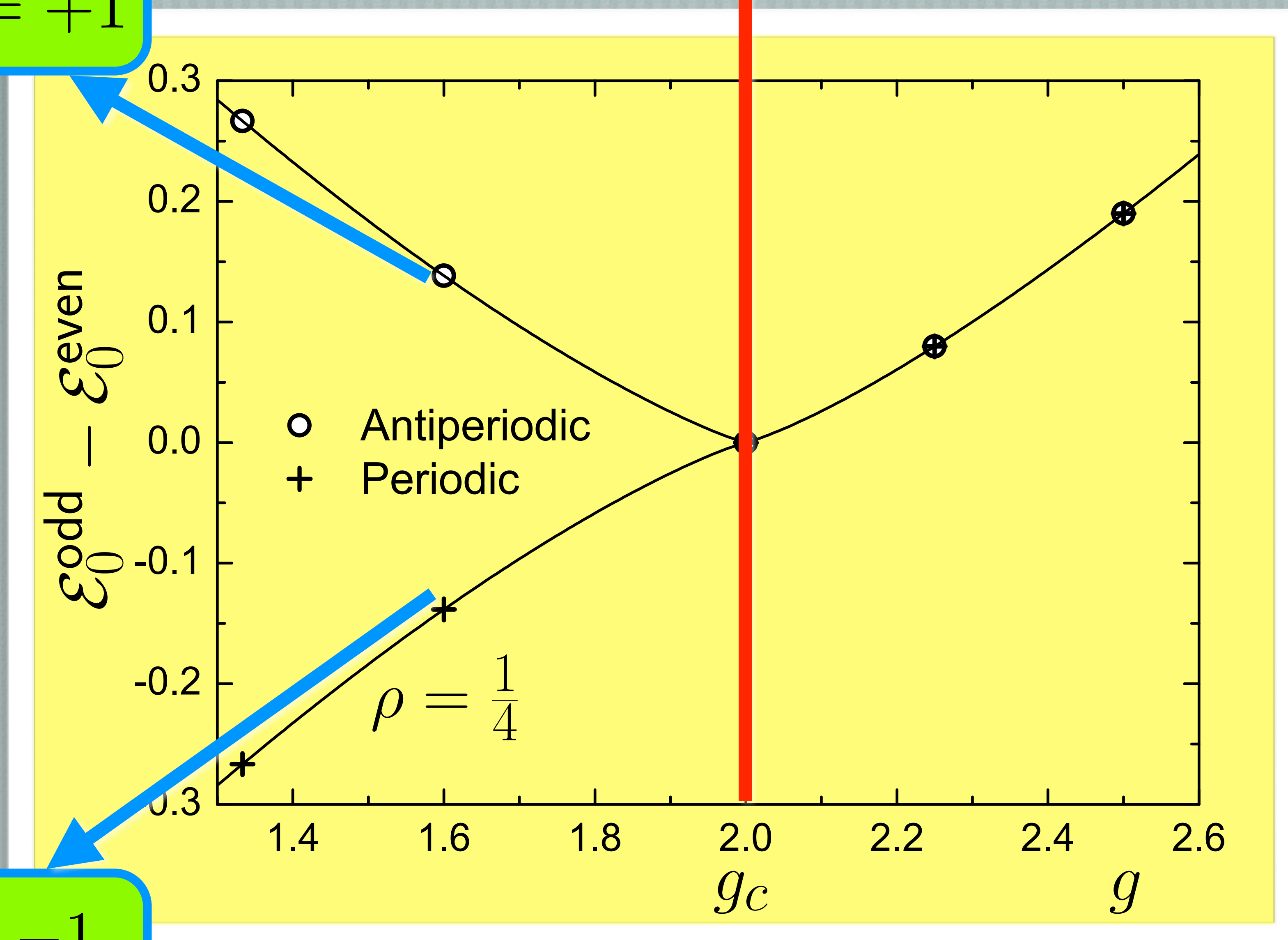
In our Richardson-Gaudin-Kitaev wire:



Topological

Topologically Trivial

$\mathcal{P}_N(\Phi = \Phi_0) = +1$



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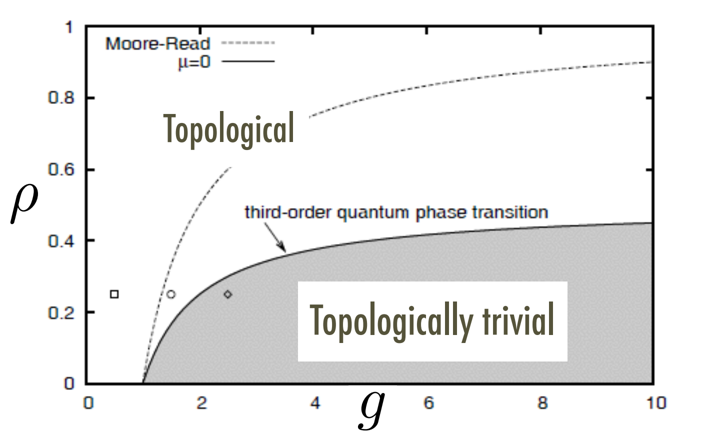
In our Richardson-Gaudin-Kitaev wire:



Topological | **Topologically Trivial**

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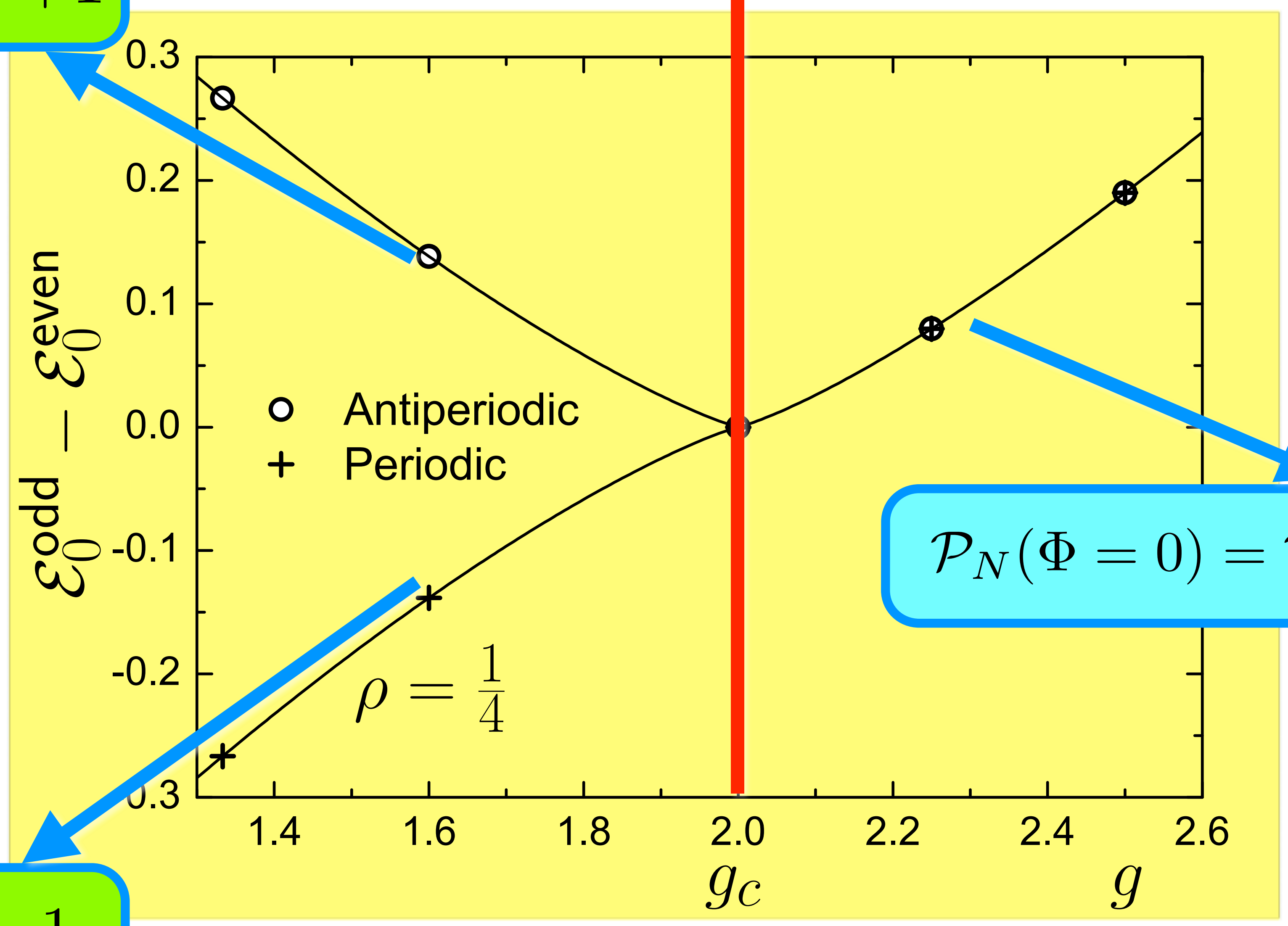

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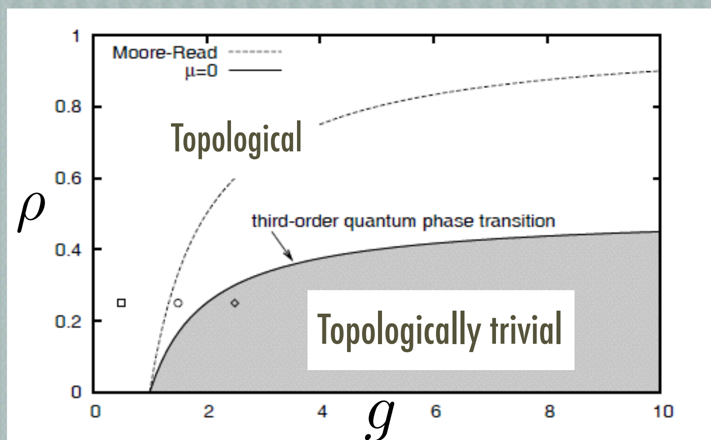
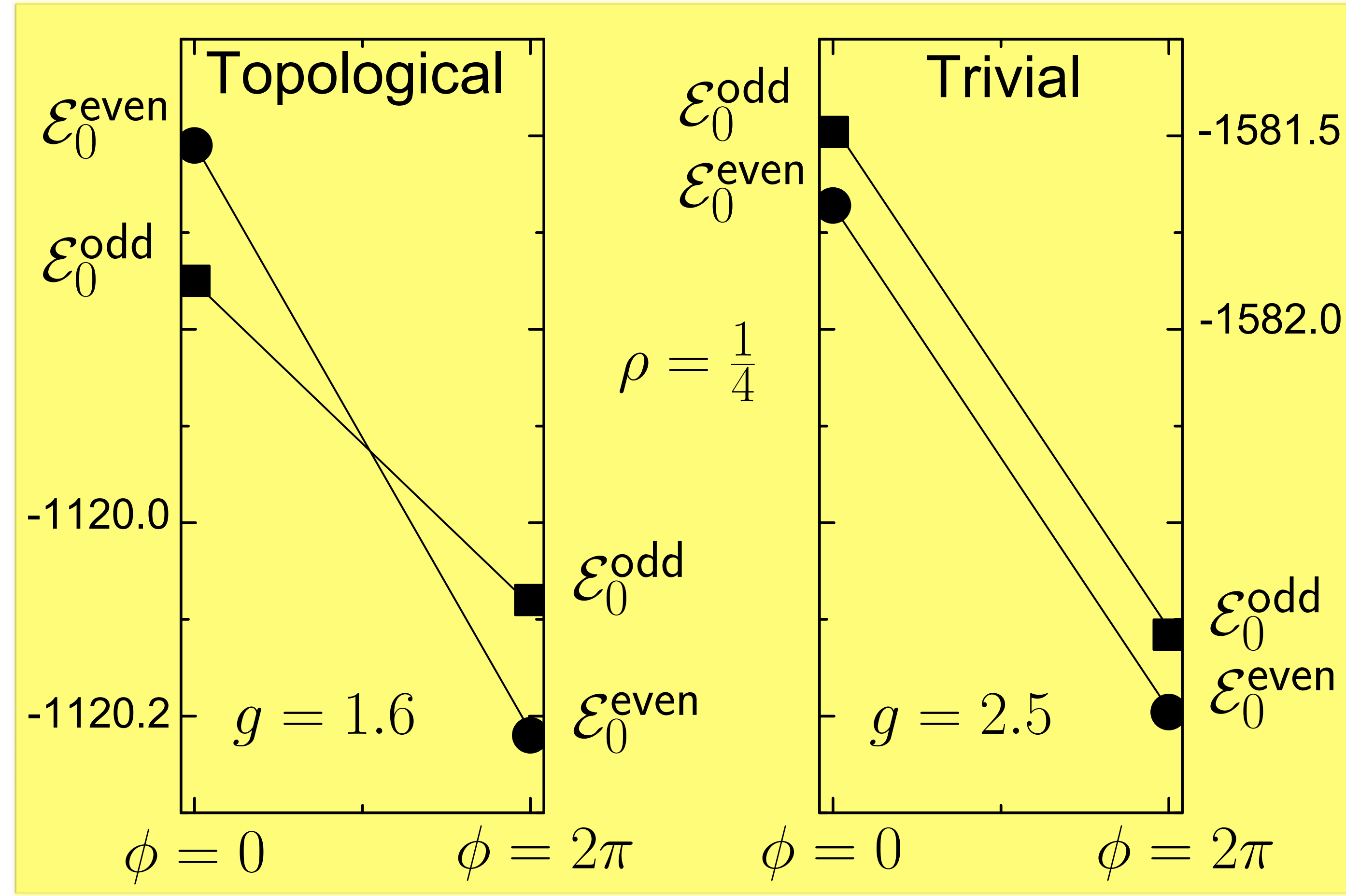
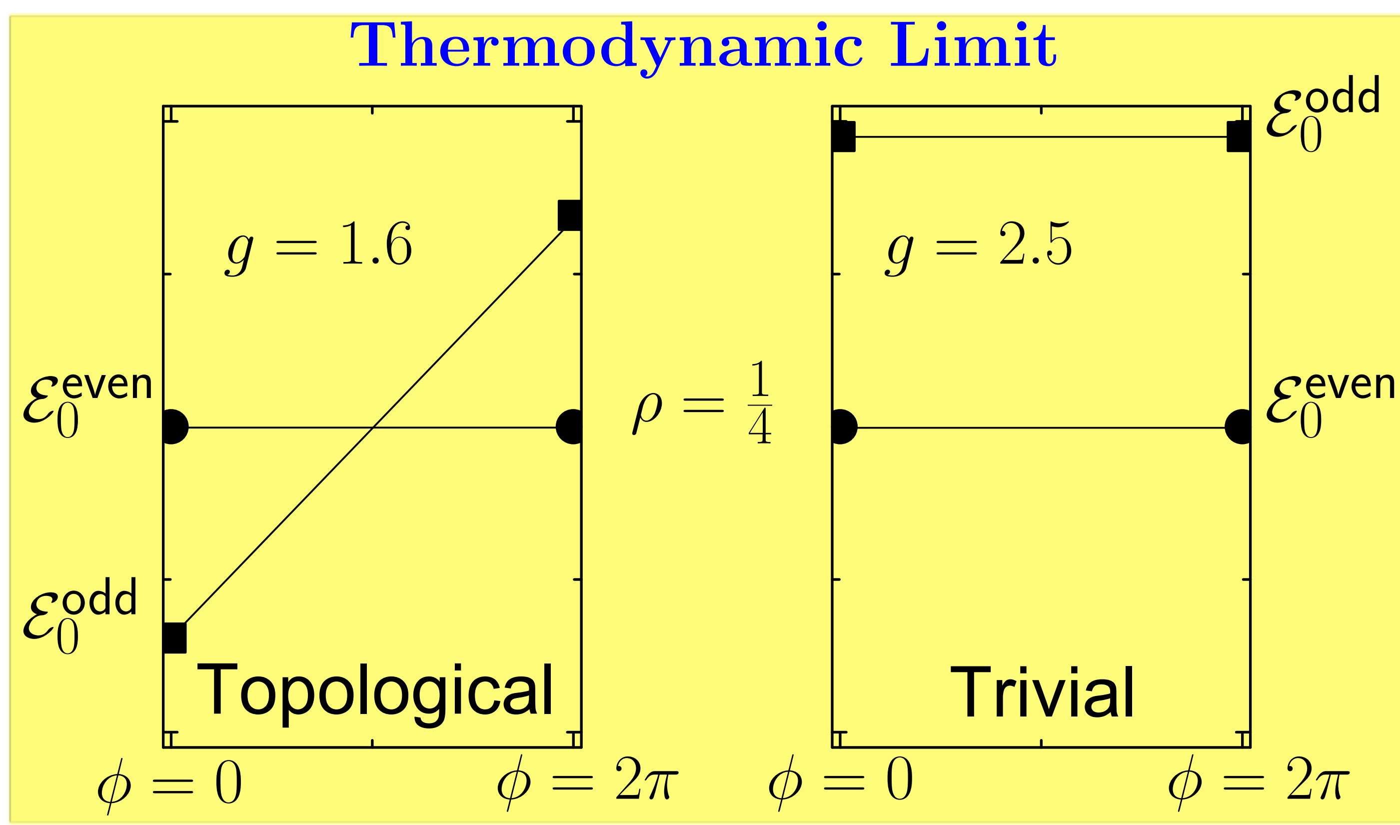
Another way of understanding the meaning of parity switches:







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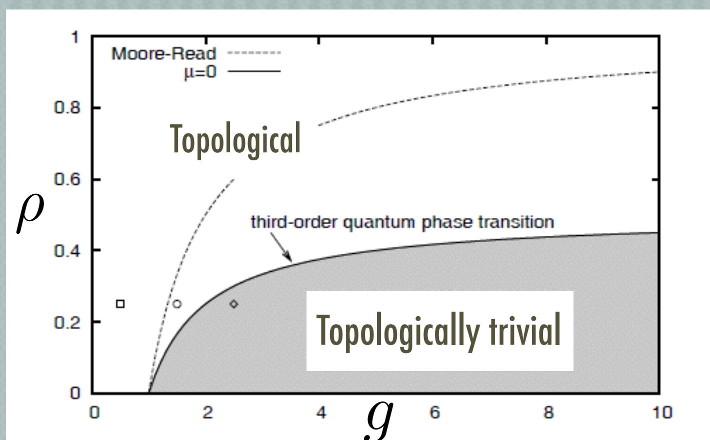
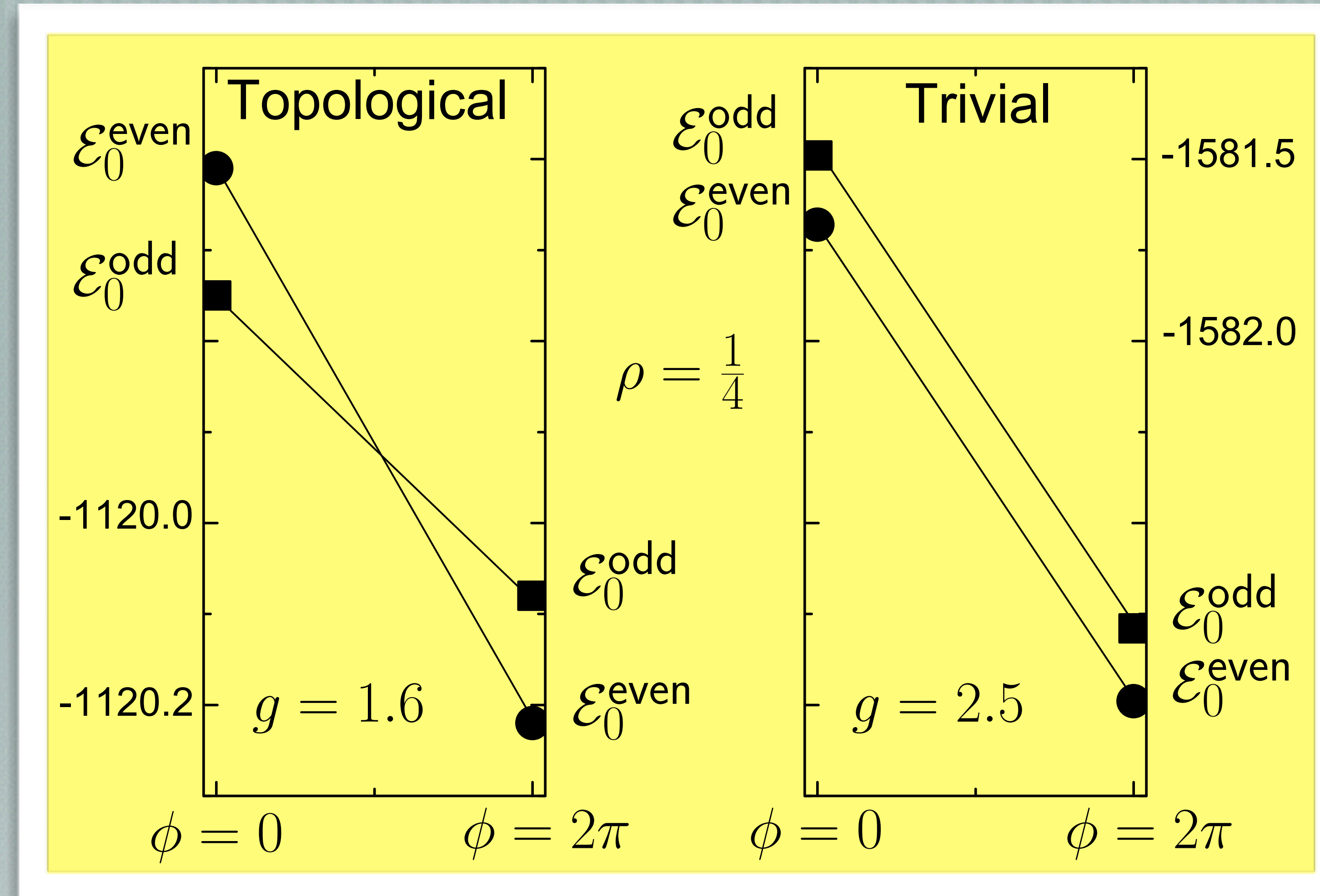
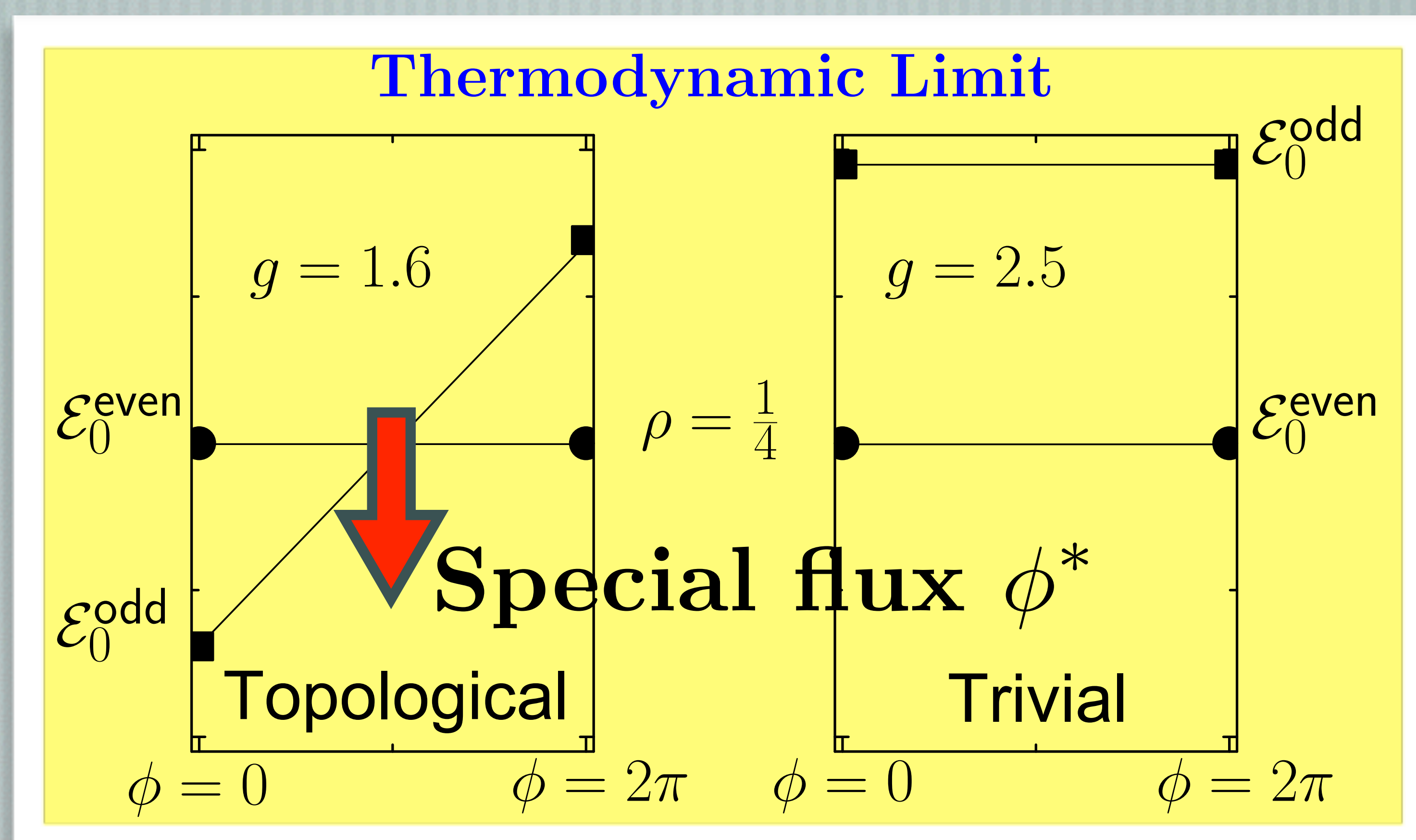
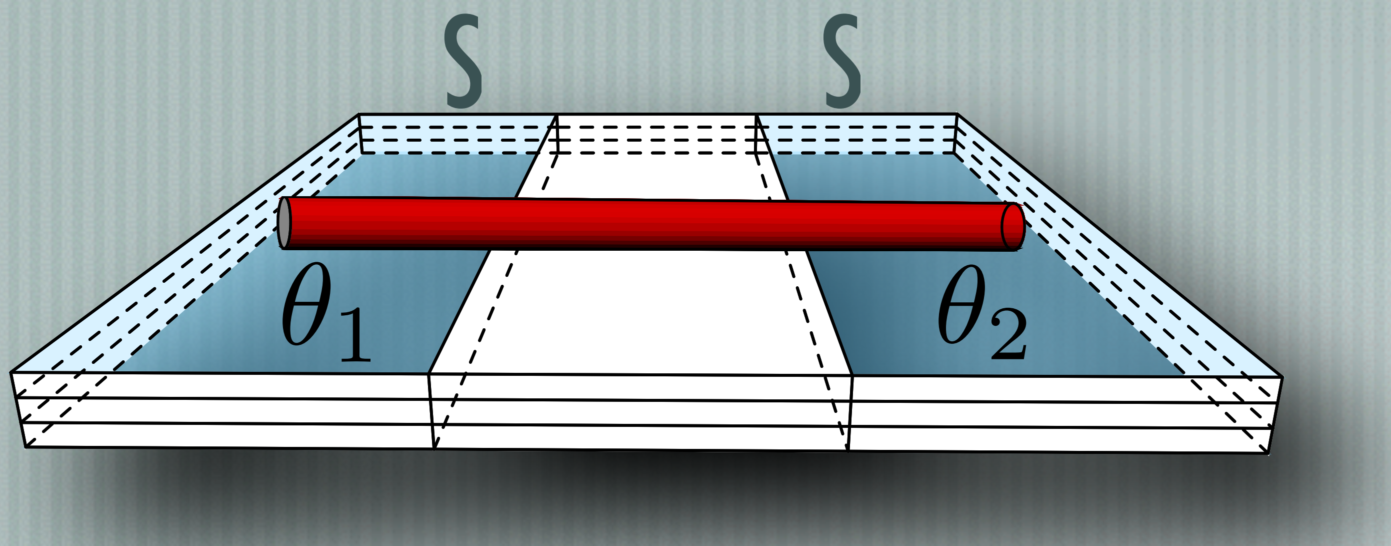


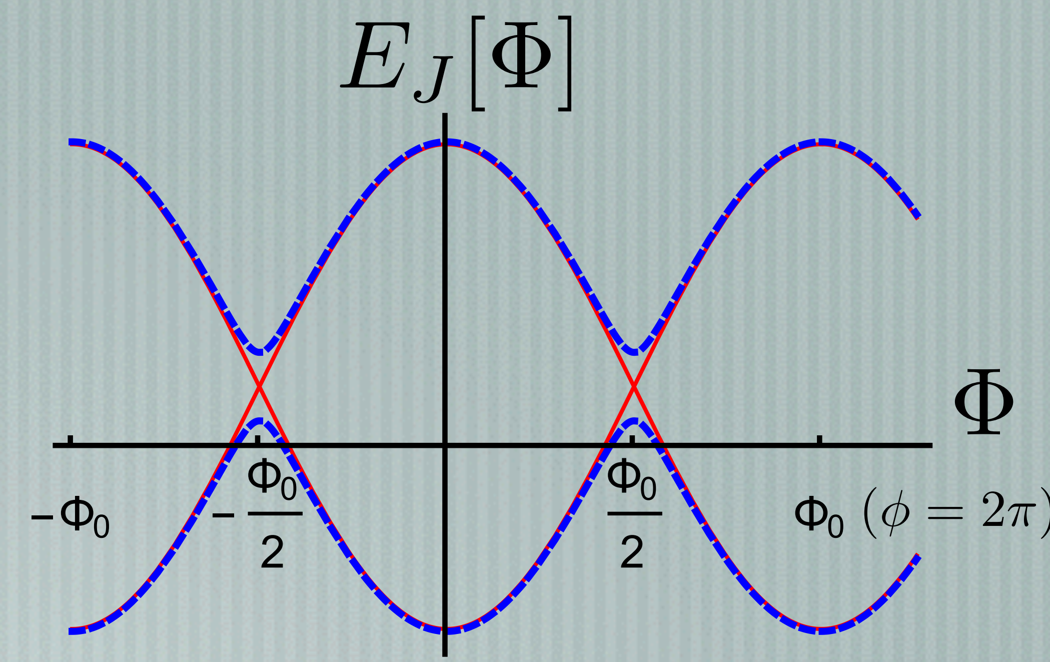




Fermion parity Switches = Fractional Josephson effect



$$\Phi = \theta_1 - \theta_2$$



Despite number-conservation the Fractional Josephson effect remains a physical experimental signature of Topological Superfluidity

Majoranas ?
Many-Body Zero Modes

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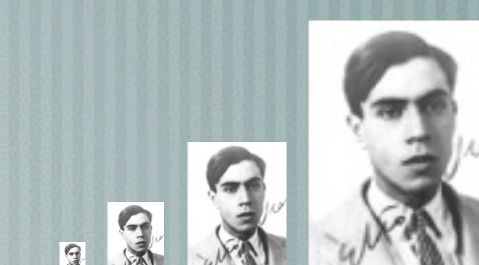
Many-Body Zero Modes



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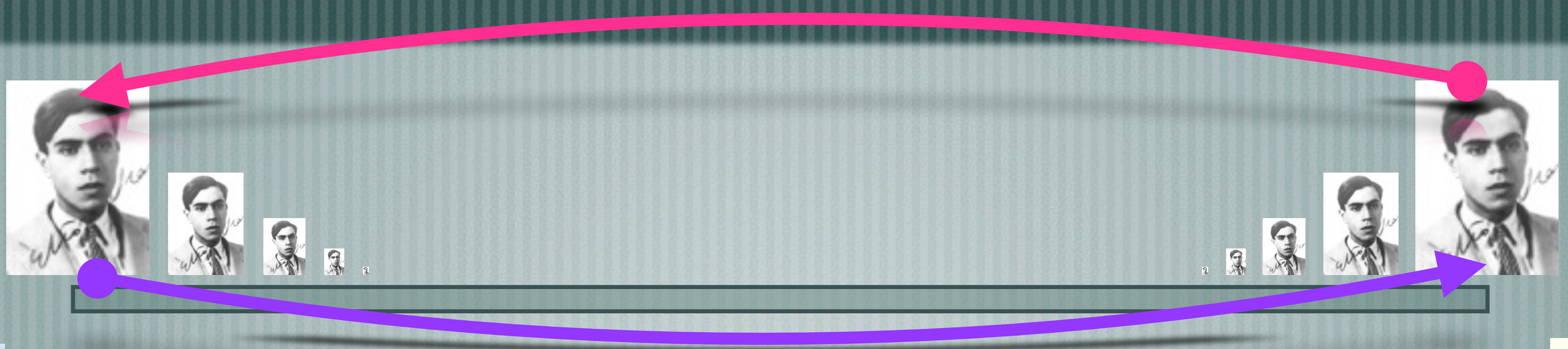

What is the meaning/fate of Majorana Modes?

Majoranas ?

Many-Body Zero Modes

For a given flux ϕ : $N \in$ even

$$\lvert\Psi_0^{\mathsf{odd}}\rangle = \frac{\lvert\Psi_0^{N+1}\rangle + e^{\mathrm{i}\varphi}\lvert\Psi_0^{N-1}\rangle}{\sqrt{2}},$$	$$\lvert\Psi_0^{\mathsf{even}}\rangle = \lvert\Psi_0^{N}\rangle$$
$$H_{\mathsf{RGK}}\lvert\Psi_0^{\mathsf{odd}}\rangle = \mathcal{E}_0^{\mathsf{odd}}(\phi)\lvert\Psi_0^{\mathsf{odd}}\rangle + \delta\mathcal{E}_0^{\mathsf{odd}}(\phi)\lvert\widetilde{\Psi}_0^{\mathsf{odd}}\rangle,$$	$$H_{\mathsf{RGK}}\lvert\Psi_0^{\mathsf{even}}\rangle = \mathcal{E}_0^{\mathsf{even}}(\phi)\lvert\Psi_0^{\mathsf{even}}\rangle$$
$$\langle\Psi_0^{\mathsf{odd}}\rvert H_{\mathsf{RGK}}\lvert\Psi_0^{\mathsf{odd}}\rangle = \mathcal{E}_0^{\mathsf{odd}}(\phi)$$	$$\langle\Psi_0^{\mathsf{even}}\rvert H_{\mathsf{RGK}}\lvert\Psi_0^{\mathsf{even}}\rangle = \mathcal{E}_0^{\mathsf{even}}(\phi)$$

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$$\lvert\widetilde{\Psi}_0^{\mathsf{odd}}\rangle = \frac{\lvert\Psi_0^{N+1}\rangle - e^{\mathrm{i}\varphi}\lvert\Psi_0^{N-1}\rangle}{\sqrt{2}}, \qquad \langle\Psi_0^{\mathsf{odd}}\lvert\widetilde{\Psi}_0^{\mathsf{odd}}\rangle = 0 \quad \text{(Orthonormality)}$$

At the particular flux ϕ^*

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$$\langle\Psi_0^{\text{even}}|H_{\text{RGK}}|\Psi_0^{\text{even}}\rangle = \mathcal{E}_0^{\text{even}}(\phi^*) = \mathcal{E}_0^{\text{odd}}(\phi^*) = \langle\Psi_0^{\text{odd}}|H_{\text{RGK}}|\Psi_0^{\text{odd}}\rangle$$

At this particular flux ϕ^* one can define Emergent zero modes

In terms of transition operators:

$$\hat{T} = |\Psi_0^{\text{even}}\rangle\langle\Psi_0^{\text{odd}}|, \quad \hat{T}^2 = 0, \quad \{\hat{T}, \hat{T}^\dagger\} = \hat{P}_0, \quad \text{and } \hat{P}_0 = |\Psi_0^{\text{even}}\rangle\langle\Psi_0^{\text{even}}| + |\Psi_0^{\text{odd}}\rangle\langle\Psi_0^{\text{odd}}|$$

define:

$$\Gamma_1 = \hat{T} + \hat{T}^\dagger \ , \quad \text{and } \mathrm{i}\Gamma_2 = \hat{T} - \hat{T}^\dagger$$

that satisfy Majorana's algebra:

$$\Gamma_1^2 = \hat{P}_0 = \Gamma_2^2 \ , \quad \text{and} \quad \{\Gamma_1, \Gamma_2\} = 0$$

and Parity relation:

$$\mathrm{i}\Gamma_2\Gamma_1 = |\Psi_0^{\text{even}}\rangle\langle\Psi_0^{\text{even}}| - |\Psi_0^{\text{odd}}\rangle\langle\Psi_0^{\text{odd}}|$$

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Are these Majorana Zero-Energy Modes?

Consider the commutators:

$$[H_{\mathsf{RGK}}, \Gamma_1] = \ \delta\mathcal{E}_0^{\mathsf{odd}}(\phi^*)(|\widetilde{\Psi}_0^{\mathsf{odd}}\rangle\langle\Psi_0^{\mathsf{even}}| - |\Psi_0^{\mathsf{even}}\rangle\langle\widetilde{\Psi}_0^{\mathsf{odd}}|),$$

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Γ_1 and Γ_2 are Emergent Majorana zero-Energy Modes

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Moreover, they connect even and odd parity sectors:

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Violation of charge-superselection rule?

Conclusions

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What is the meaning of Many-body Majorana localization ?