Spin-1 Spin-Orbit Coupled Bose gas

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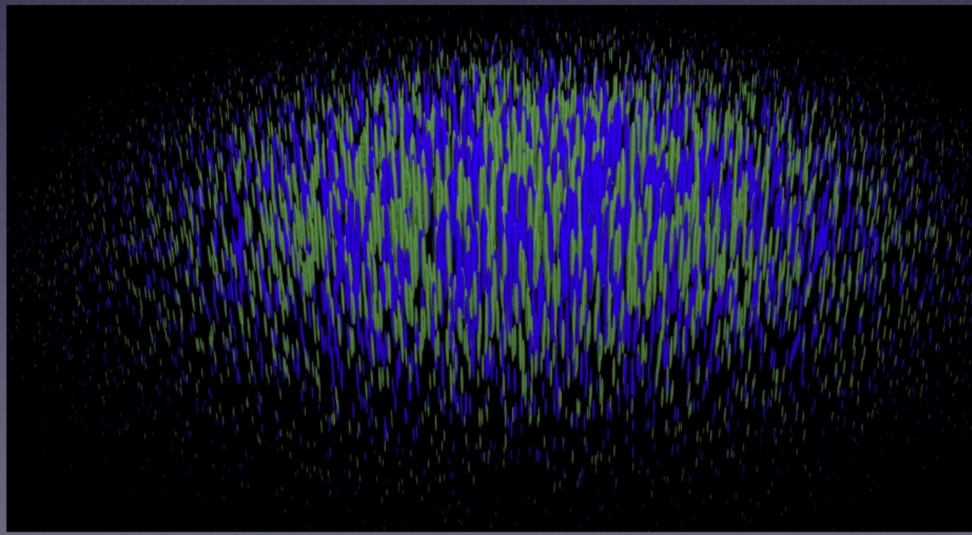

Collaborators

Theory

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Erich Mueller
Cornell

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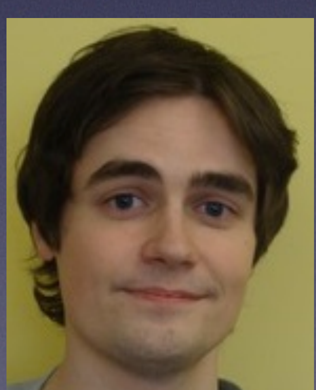

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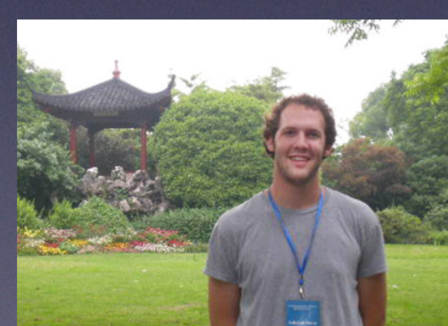

Jed Pixley
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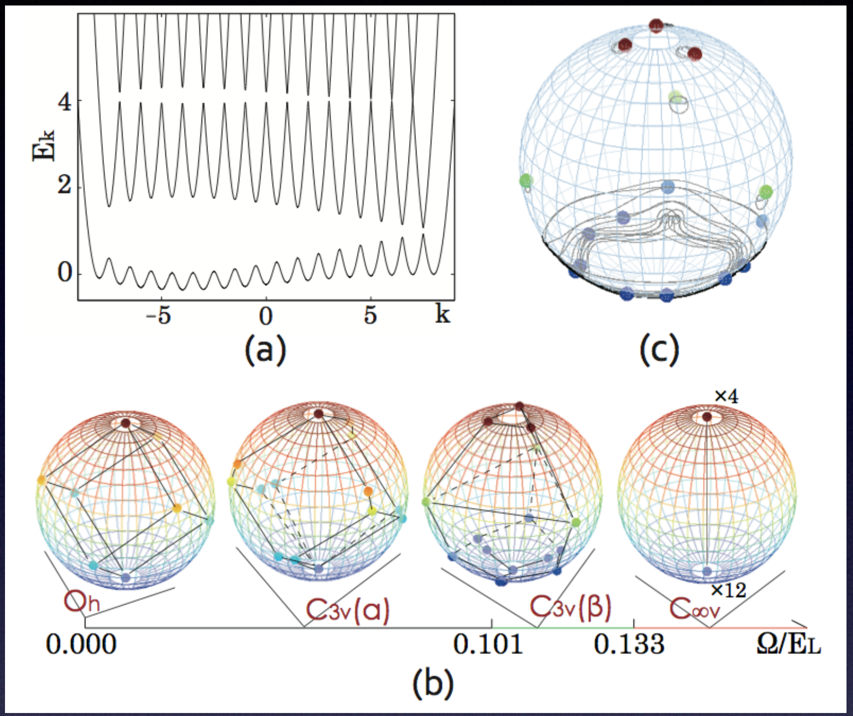
Experiments

Ben Lev, *Stanford*

Ian Spielman,*NIST/JQI*

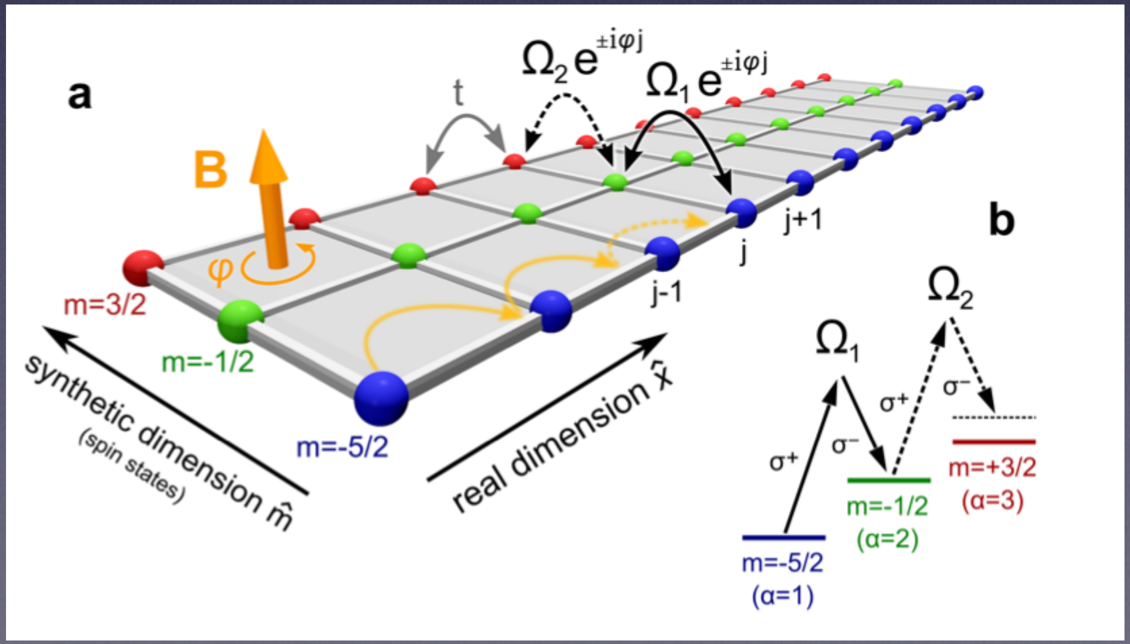
Large spin/Synthetic B Fields

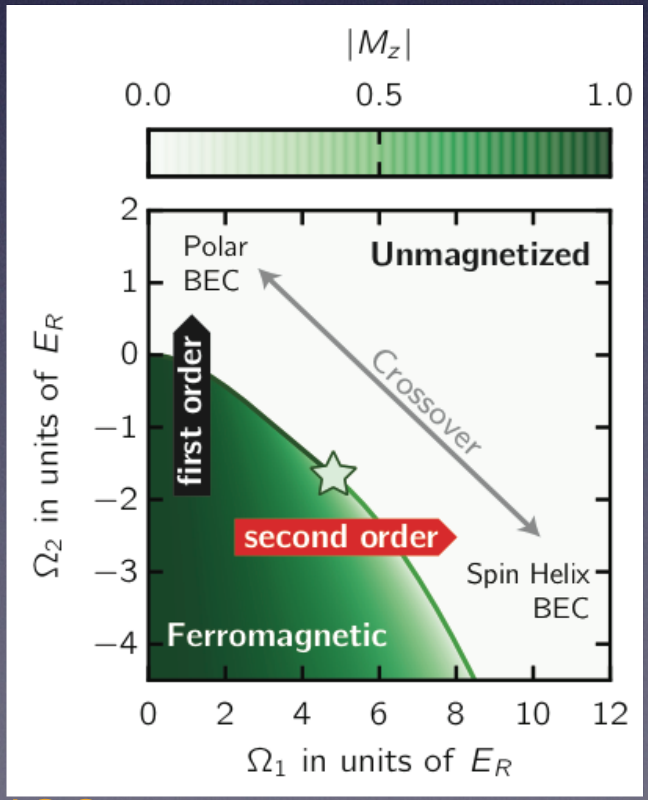


Dy/Erbium in Synthetic Gauge Fields

Cui et al. PRA 88 (2013)

First Experimental Realization of spin-S SOC gas





Campbell et al. arXiv:1501.05984

Mancini et al. 1502.02495, Celi et al.1502.02496

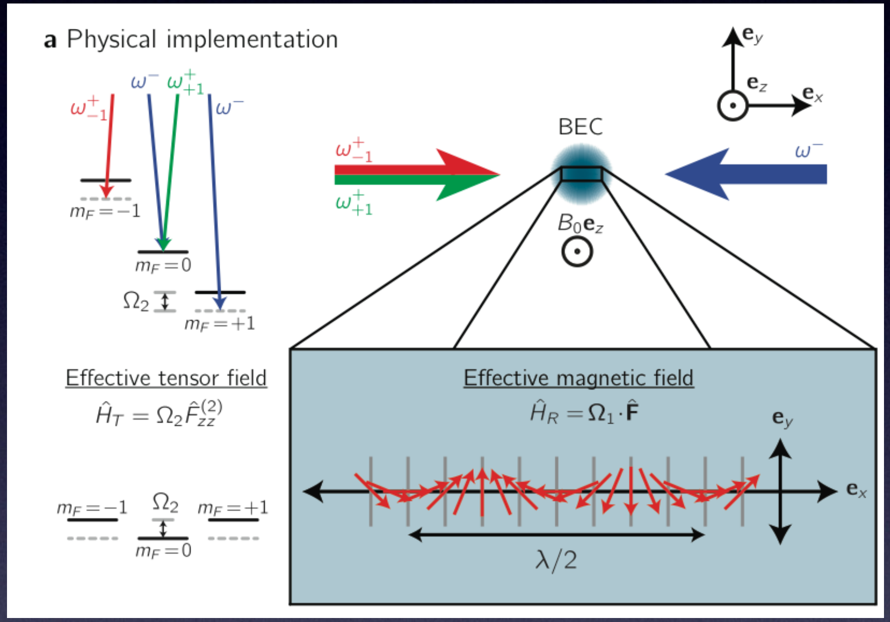
New Physics with Large Spin systems?

- *Interplay between interactions, SOC (gauge fields) and large spin?*
- *What kinds of ordered states can we expect?*
- *How do we understand these theoretically?*

This talk...

Concrete example: Spin-1 Bosons

- *Spin-1 Phenomenology*
- *Spin-1/2 SOC*
- *Spin-1 spin-orbit coupled system*
- *Finite temperature physics*



Campbell et al. arXiv:1501.05984

Simplest Case: Spin-1 Bosons

$F = 1, m_F = 1, 0, -1$

$I = 3/2$ alkalis ($J = 0, S = 1/2$), ^{23}Na, ^{87}Rb, ^{39}K

S-wave interactions rotationally invariant in real and spin space

Symmetry allowed interaction channels: $F = 0, 2$

$$\mathcal{V}_{int} = V_0 \mathcal{P}_0 + V_2 \mathcal{P}_2$$

$$\mathcal{V}_{int} = V_i + V_d \mathbf{S}_1 \cdot \mathbf{S}_2$$

$V_i \propto a_0 + 2\, a_2$, $V_d \propto a_2 - a_0$

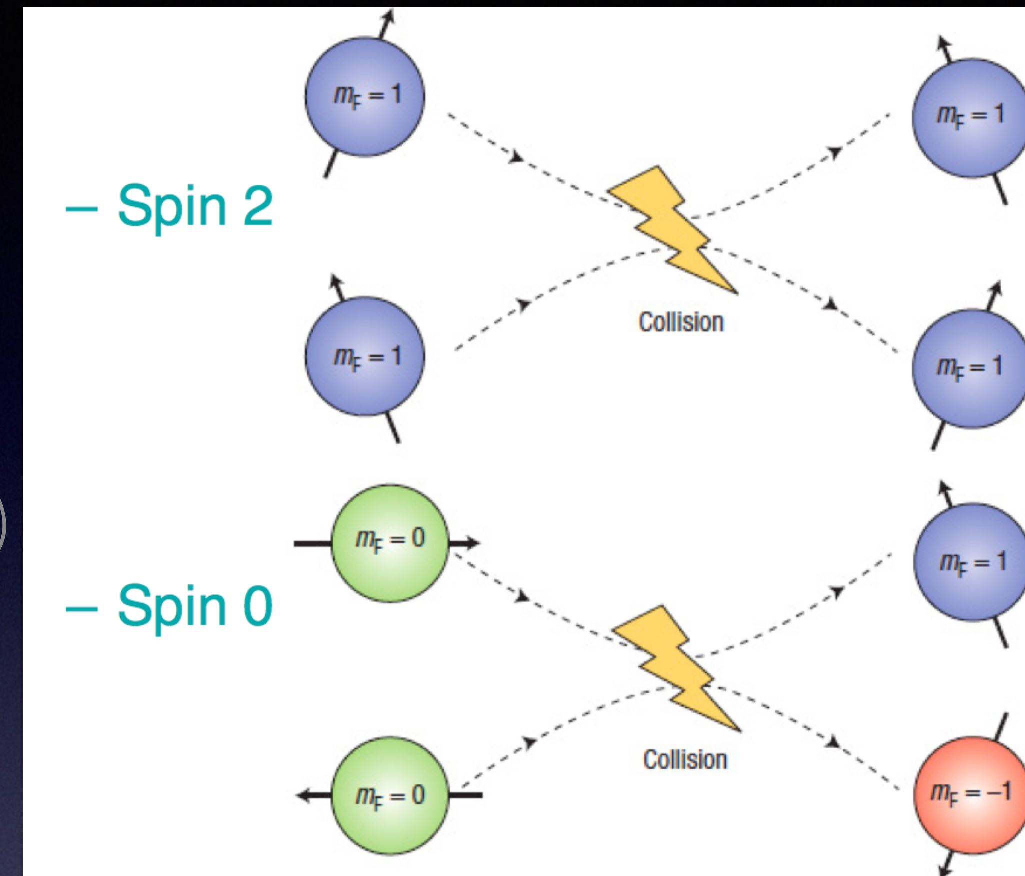
Ho PRL (1998)
Machida and Ohmi, JPSJ (1998)

Ground States

Interaction Hamiltonian ($\sim n^2$)

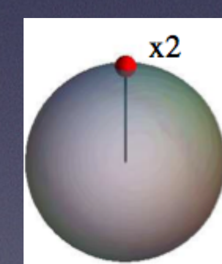
spin-independent part $c_0 n^2$ ($c_0 \propto a_0 + 2a_2$)

spin-dependent part $\rightarrow c_2 S_1 \cdot S_2$ ($c_2 \propto a_2 - a_0$)



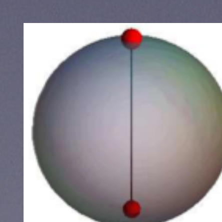
$c_2 < 0$: Ferromagnetic state

$$\zeta = e^{i\theta}\mathcal{U}\begin{pmatrix}1\\0\\0\end{pmatrix}$$



$c_2 > 0$: Polar state

$$\zeta = e^{i\theta}\mathcal{U}\begin{pmatrix}0\\1\\0\end{pmatrix}$$



Alternate view: Orders

Hermitian operator $Q_{ab} = \frac{\psi_a^* \psi_b}{\sum_c \psi_c^* \psi_c}$

$$Q_{ab} = Q^{(0)}\delta_{ab}/3 + i\epsilon_{abc}Q_c^{(1)}/2 + Q_{ab}^{(2)}$$

Magnetic Order (vector) $\mathbf{Q}^{(1)} = \mathbf{s} = \langle \mathbf{S} \rangle / \rho$

Nematic Order (tensor) $Q_{ab}^{(2)} = (2/3)\delta_{ab} - (\langle S_a S_b \rangle + \langle S_b S_a \rangle)/2\rho^2$

Uniaxial/Biaxial nematic

magnetic and nematic order generally do not coexist

$$\begin{aligned} 0 &= \mathrm{Tr}(Q^2) - (\mathrm{Tr}Q) \\ &= -2/3 + \mathbf{Q}^{(1)} \cdot \mathbf{Q}^{(1)}/2 + \mathrm{Tr}((Q^{(2)})^2) \end{aligned}$$

Mueller, PRA (2004)

Ground States

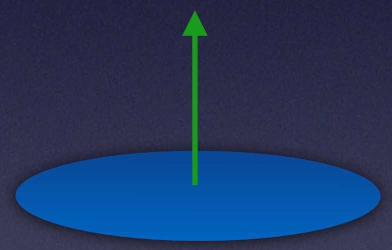
Interaction Hamiltonian ($\sim n^2$)

spin-independent part $c_0 n^2$ ($c_0 \propto a_0 + 2\, a_2$)

spin-dependent part $\rightarrow c_2 S_1 \cdot S_2$ ($c_2 \propto a_2 - a_0$)

$c_2 < 0$: Ferromagnetic state

$$\zeta = e^{i\theta}\mathcal{U}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

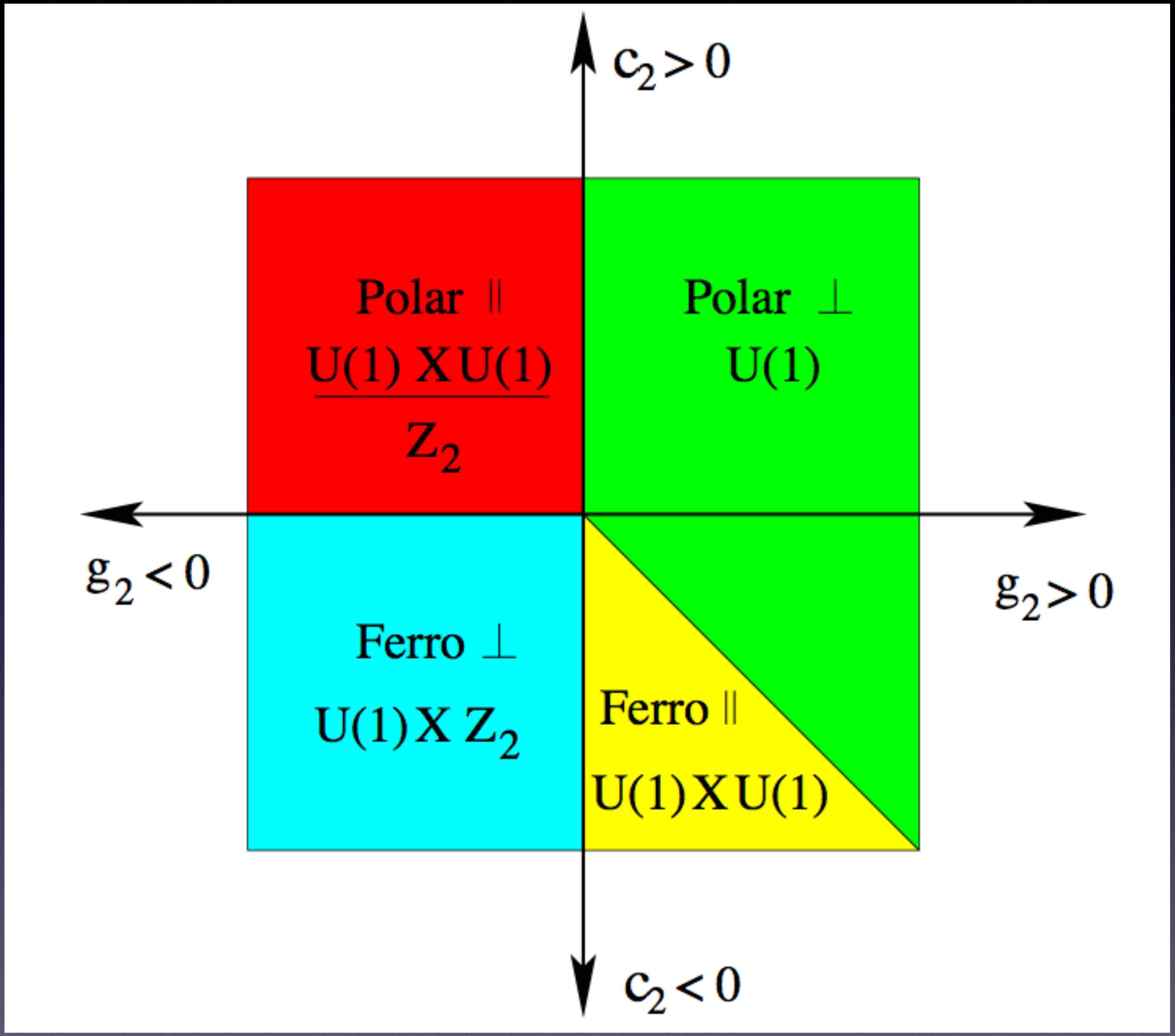


$c_2 > 0$: Polar state

$$\zeta = e^{i\theta}\mathcal{U}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

Unlike spin-1/2, spin-1 doesn't necessarily point anywhere!

Spin-1 Phase Diagram



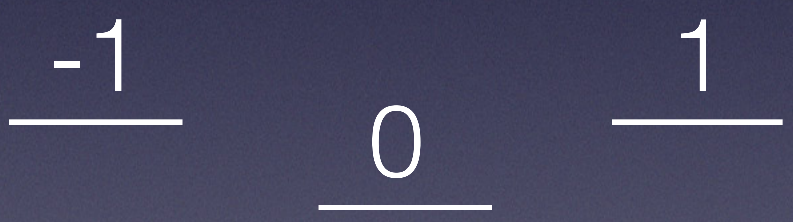
Mukherjee, Xu and Moore, PRB (2007)

Spin-spin interactions

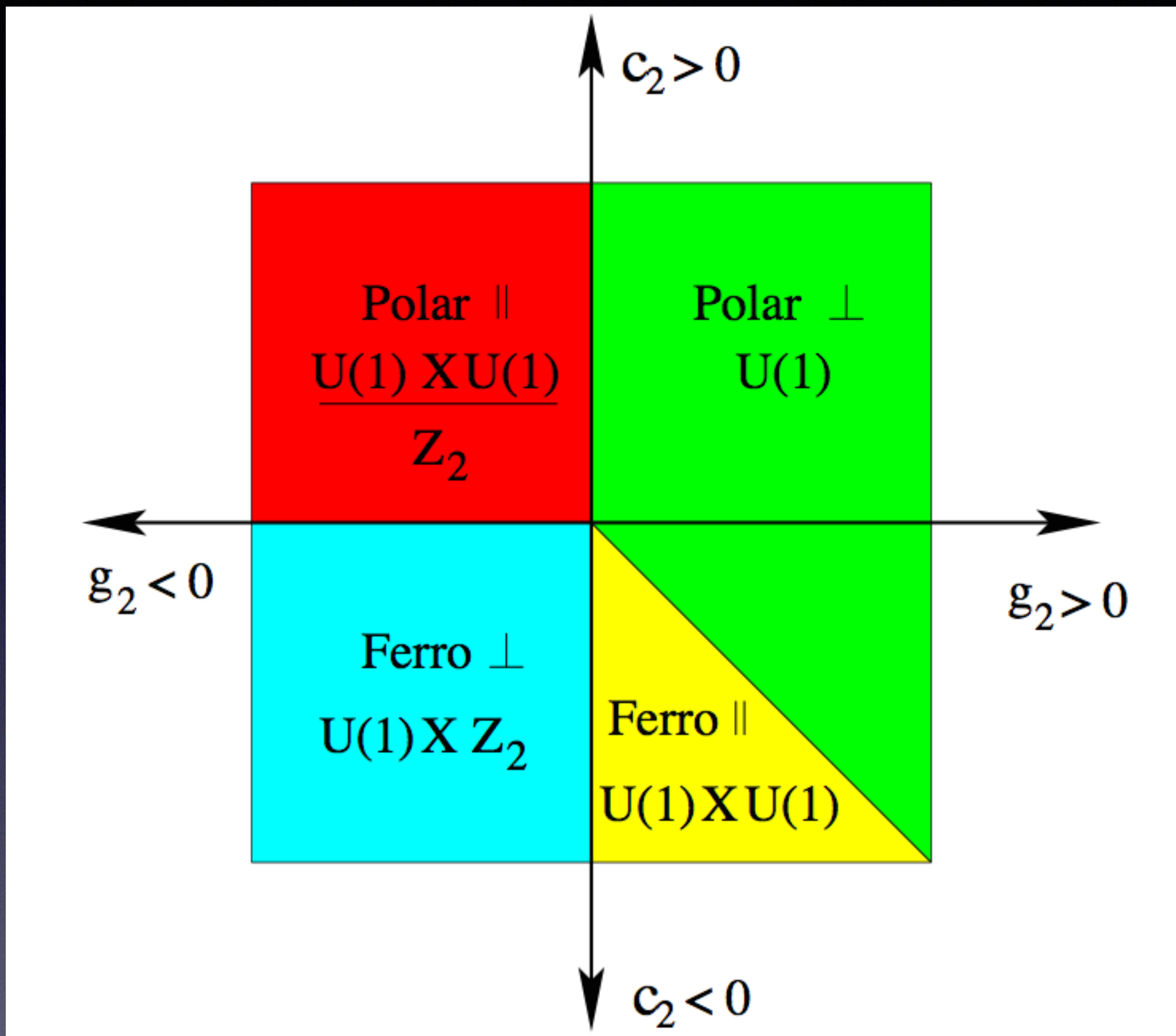
$$E = c_2 \langle \vec{\mathbf{S}} \rangle^2 + g_2 \langle S_z^2 \rangle$$

Quadratic Zeeman

$g_2 > 0$



Spin-1 Phase Diagram



Mukherjee, Xu and Moore, PRB (2007)

Spin-spin interactions

$$E = c_2 \langle \vec{\mathbf{S}} \rangle^2 + g_2 \langle S_z^2 \rangle$$

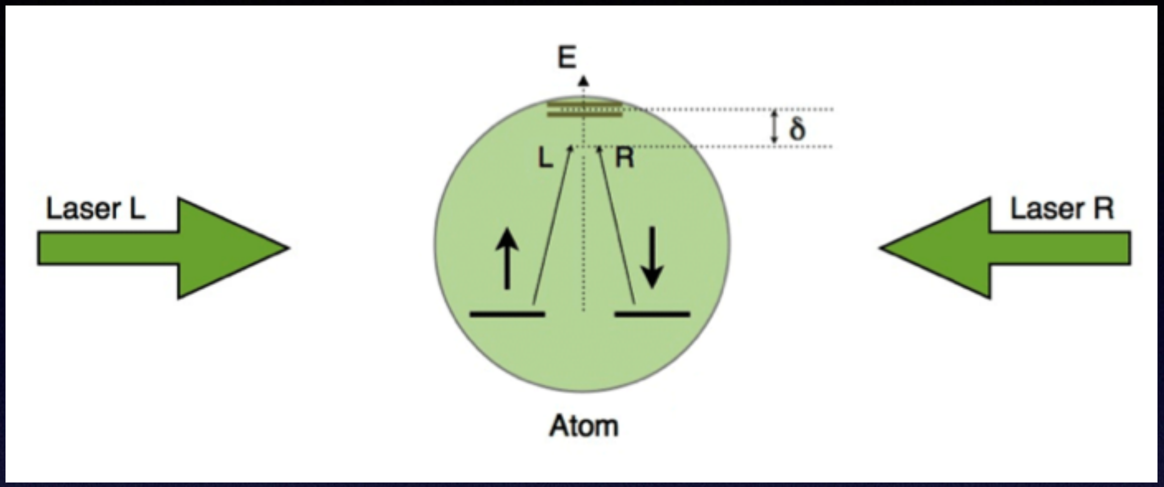
Quadratic Zeeman

$g_2 < 0$

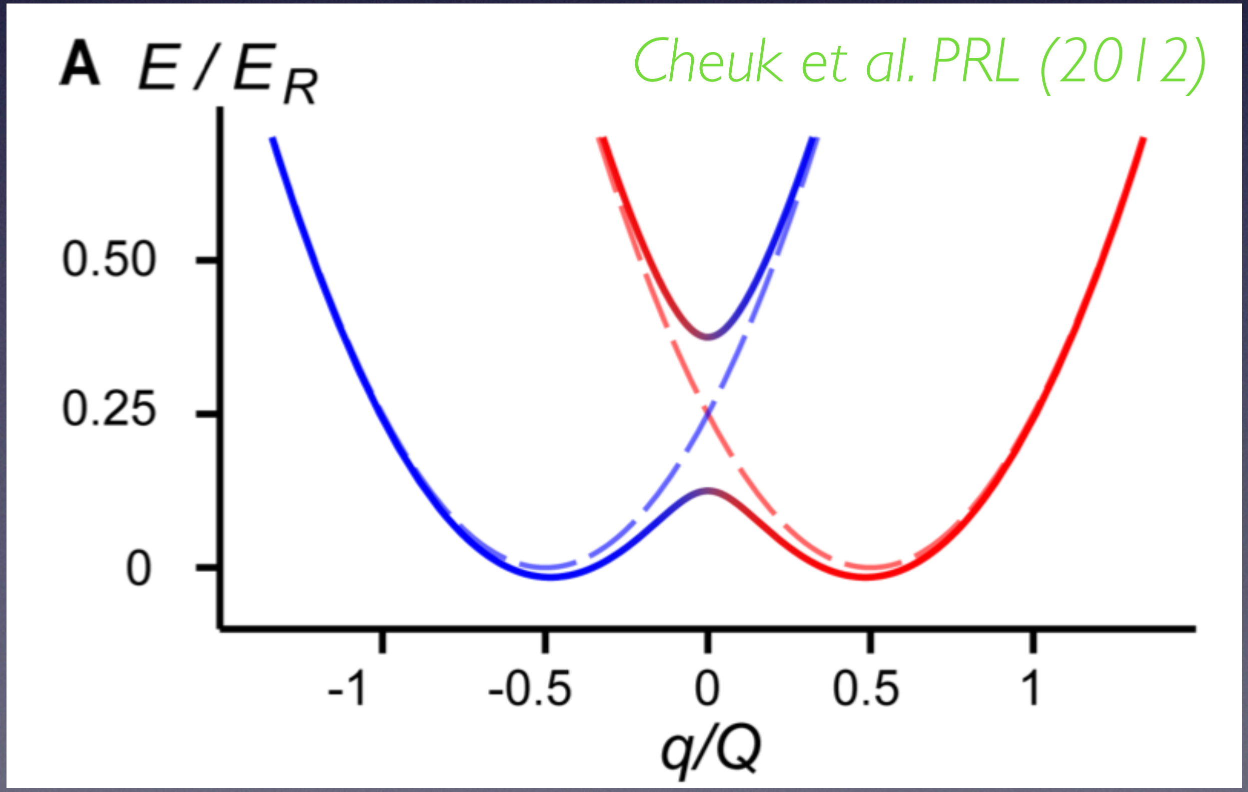
-1 0 1

In condensate: density-density interaction ($c_0 n^2$) not important

Spin-Orbit Coupling



Mueller, Physics Viewpoint (2012)



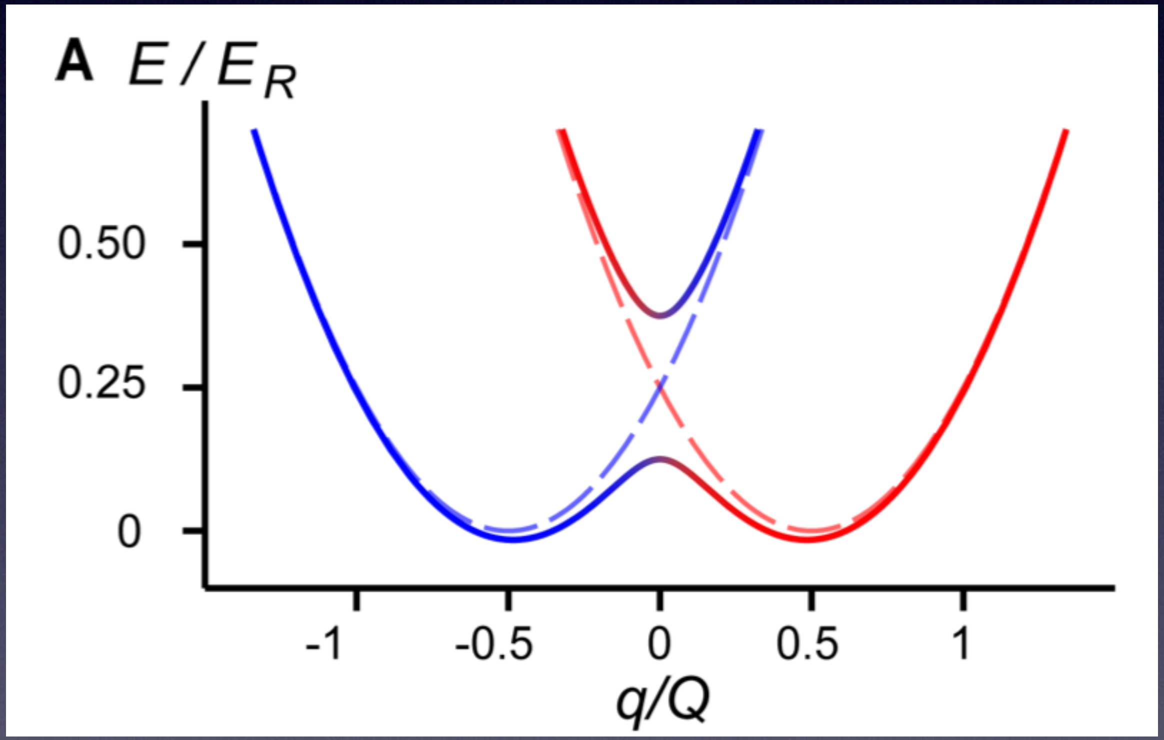
1D SOC

$$h_0 = \frac{1}{2}\left[\left(p_x - k_0\sigma_z\right)^2 + p_\perp^2\right] + \frac{\Omega}{2}\sigma_x + \frac{\delta}{2}\sigma_z$$

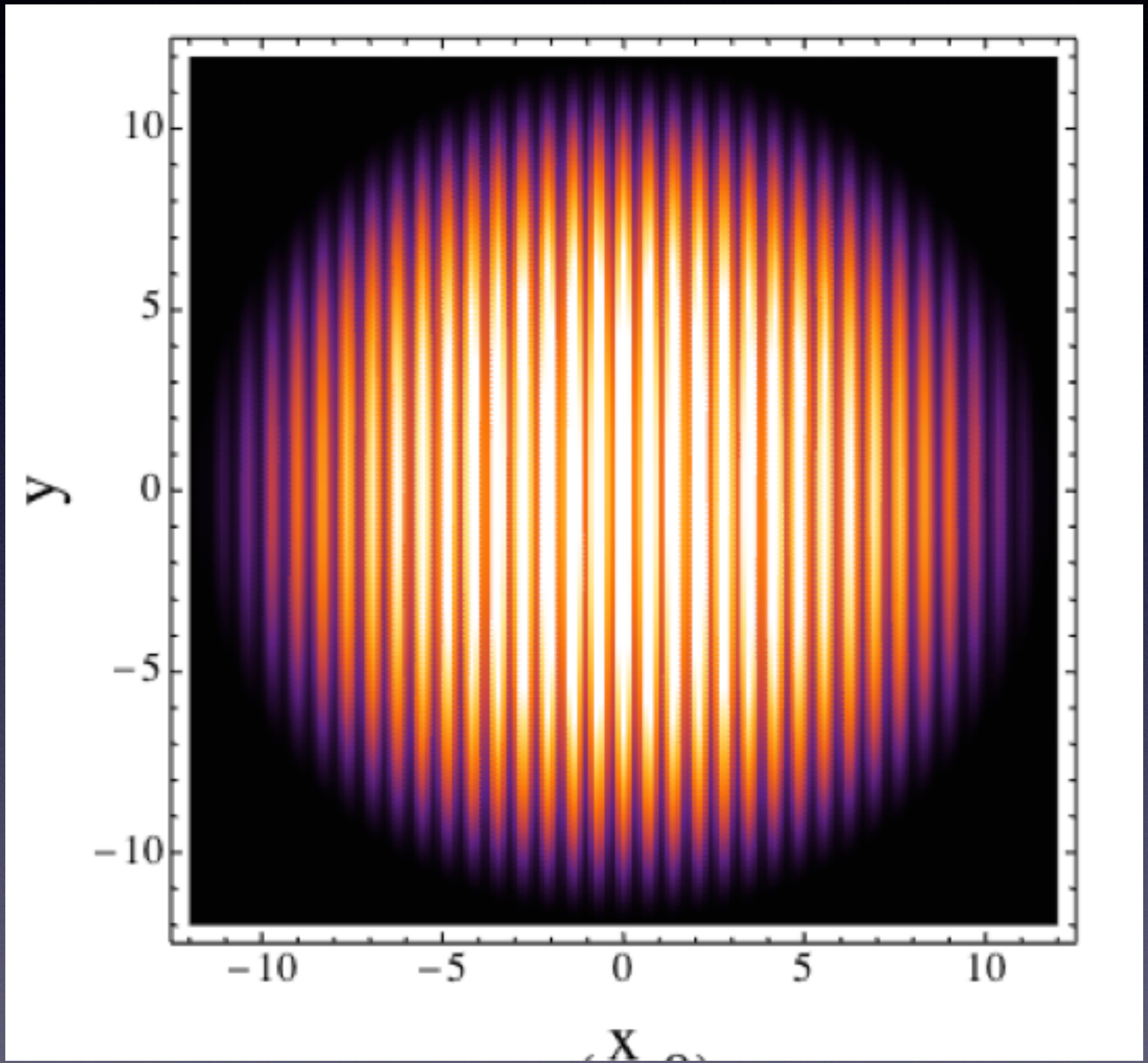
Lin et al. Nature (2011)
Wang et al. PRL (2012)

SOC makes Stripes!

Single-particle term favors stripe formation!



$$h_0 = \frac{1}{2}\left[(p_x - k_0\sigma_z)^2 + p_\perp^2\right] + \frac{\Omega}{2}\sigma_x + \frac{\delta}{2}\sigma_z$$



Ho and Zhang PRL (2011)

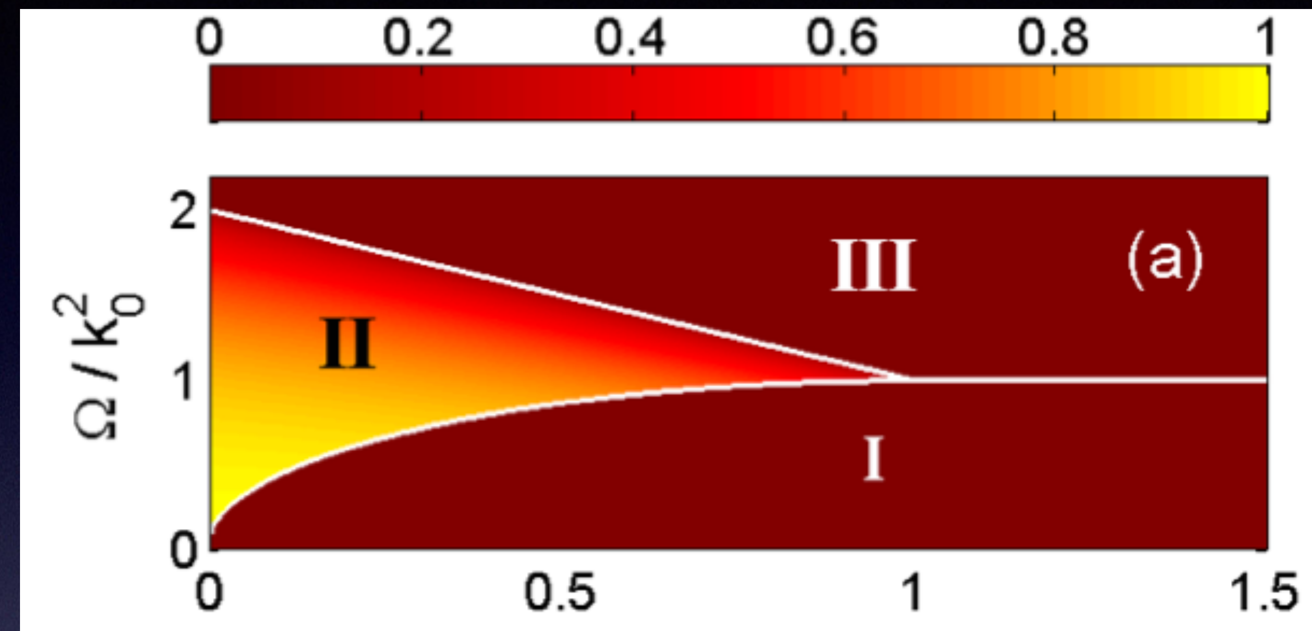
Li, Pitaevskii and Stringari, PRL (2012)

Density penalty: Contrast tiny in most experimental conditions

Spin-1/2 MF Phase diagram

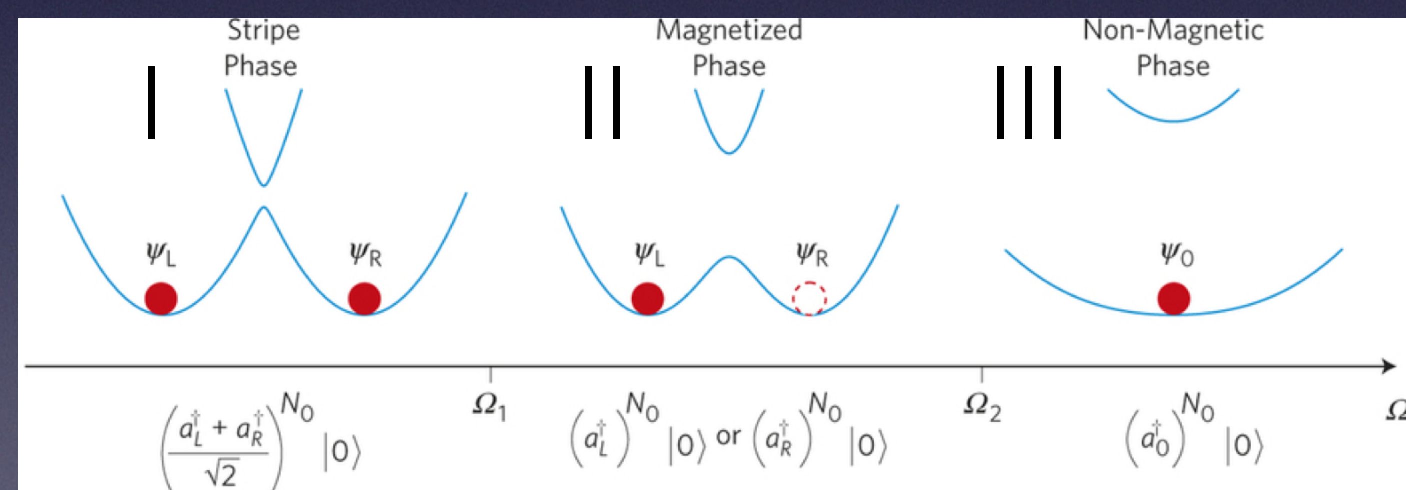
$$h_0 = \frac{1}{2}\left[(p_x - k_0\sigma_z)^2 + p_\perp^2\right] + \frac{\Omega}{2}\sigma_x + \frac{\delta}{2}\sigma_z$$

$$n(x) = n\left[1 + 2|C_1C_2|\frac{\sqrt{k_0^2 - k_1^2}}{k_0}\cos(2k_1x + \phi)\right]$$



Li, Pitaevskii and Stringari, PRL (2012)

Ji et al. Nature Physics (2014)



g $(n_{up}+n_{down})$^2

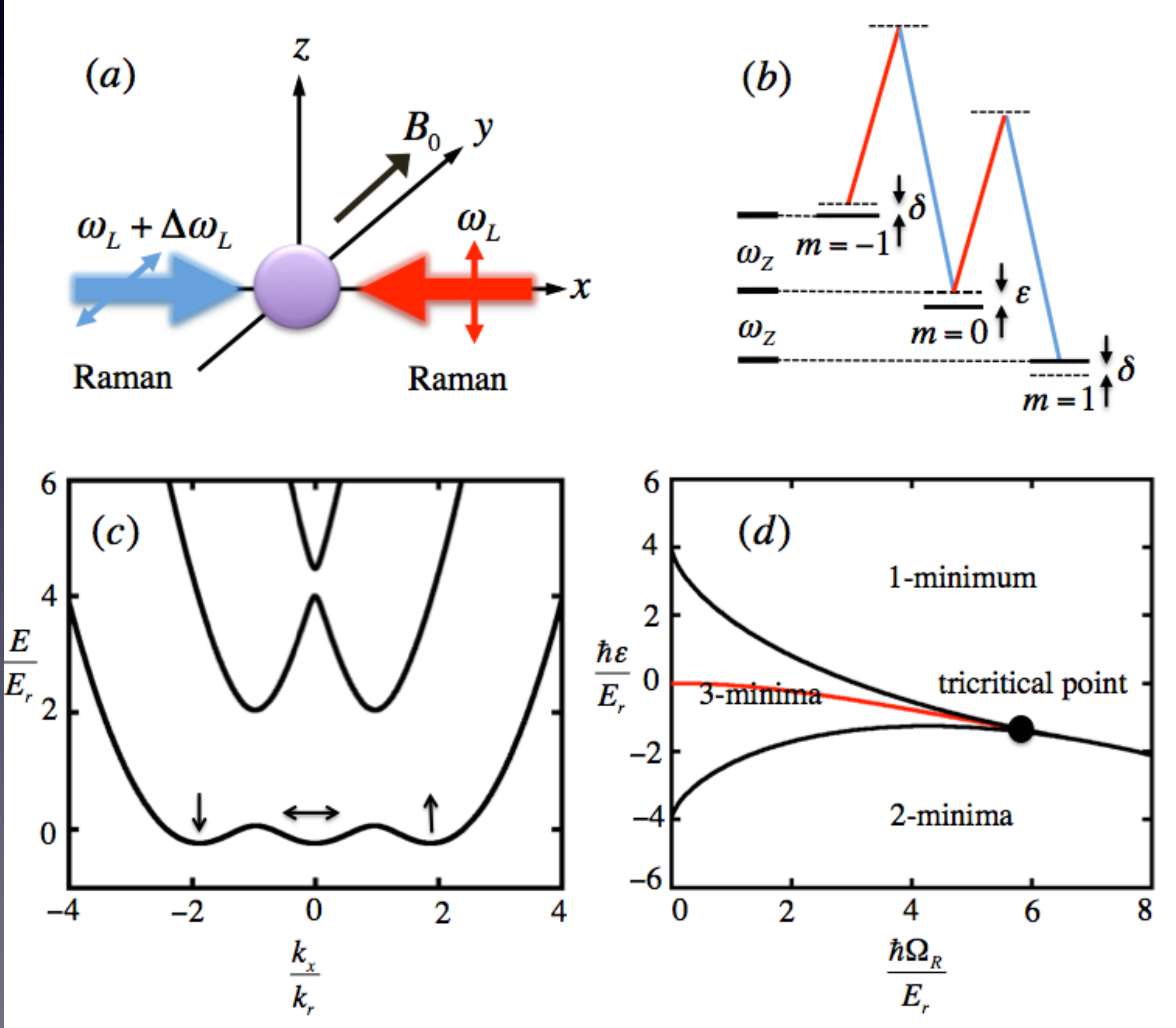
Density penalty: Contrast tiny in most experimental situations!

Can interactions offset this density energy cost?

Single-particle physics

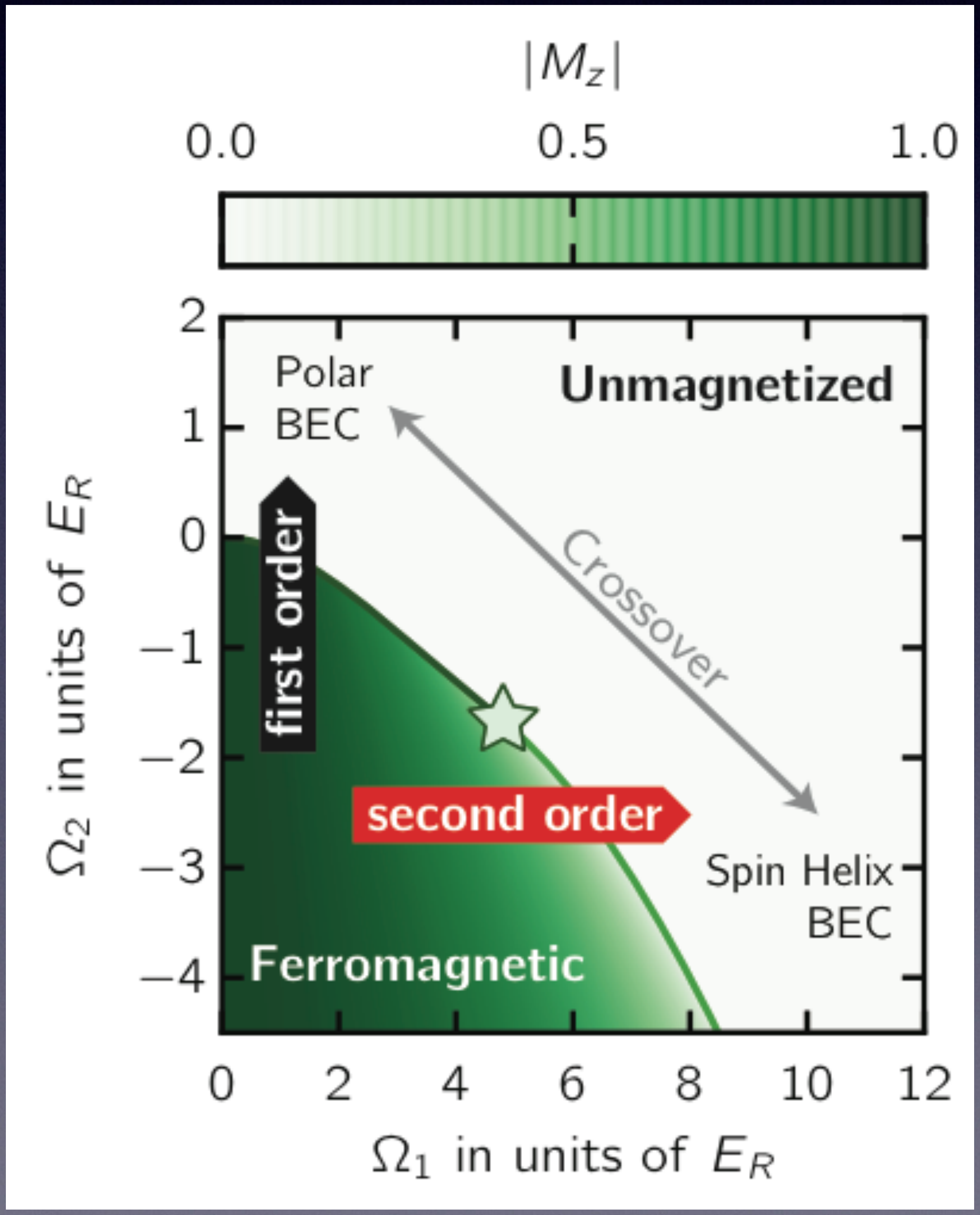
$$\mathcal{H}_{soc} = \frac{\hbar^2(k_x - k_0 S_z)^2}{2m} + \frac{\hbar^2 k_\perp^2}{2m} + \frac{\Omega}{2}S_x + \frac{\delta}{2}S_z + \frac{q}{2}S_z^2$$

Lan, Ohberg PRA (2014)
Natu, Li, Cole PRA (2015)



Experiments

Model SINGLE-PARTICLE transitions from FM to spin Helix



Ω_1: Rabi coupling

Ω_2: Quadratic Z

At spin helix transition,
number of minima changes to 3->2

At F-polar transition,
minima goes from 3 -> 1

Campbell et al. arXiv:1501.05984

Variational Ansatz

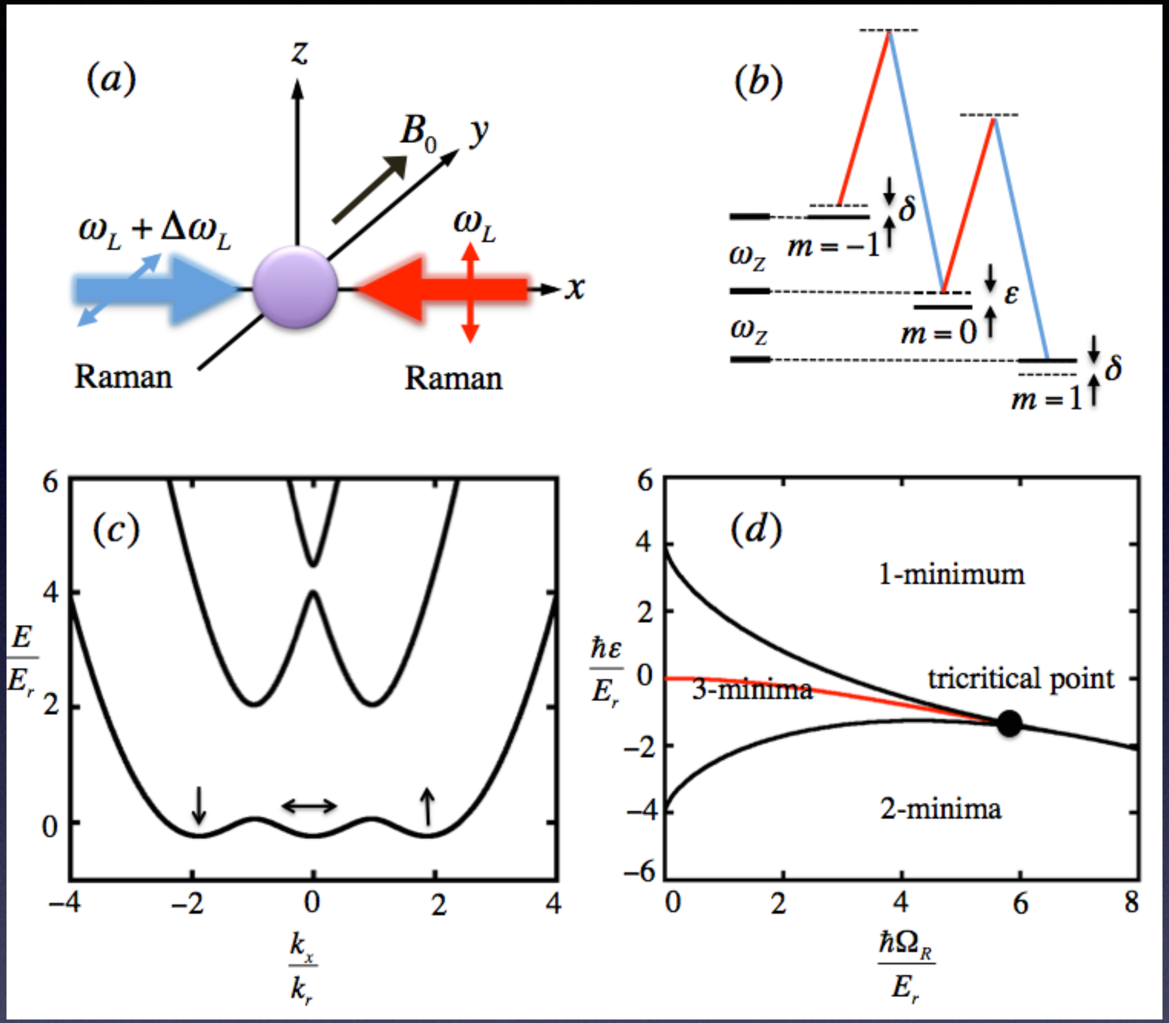
complex number

$$\psi = \sqrt{\frac{N}{V}}\left(\chi_+ e^{ik_1 x}\phi_+ + \chi_0\phi_0 + \chi_- e^{-ik_1 x}\phi_-\right)$$

3-component spinor

Minimize

$$\mathcal{H}_{int} = \frac{1}{2}\int d^3\mathbf{r}\ \psi^\dagger_\alpha \psi^\dagger_\beta \psi_\gamma \psi_\delta (c_0 \delta_{\alpha\delta}\delta_{\beta\gamma} + c_2 \mathbf{S}_{\alpha\delta}\cdot \mathbf{S}_{\beta\gamma})$$



Lan, Ohberg PRA (2014)

Choose Raman coupling to be in 3 minimum regime.
Vary interactions and q.

Possible Orders

TABLE I: Orders in spin-orbit coupled spin-1 gas.

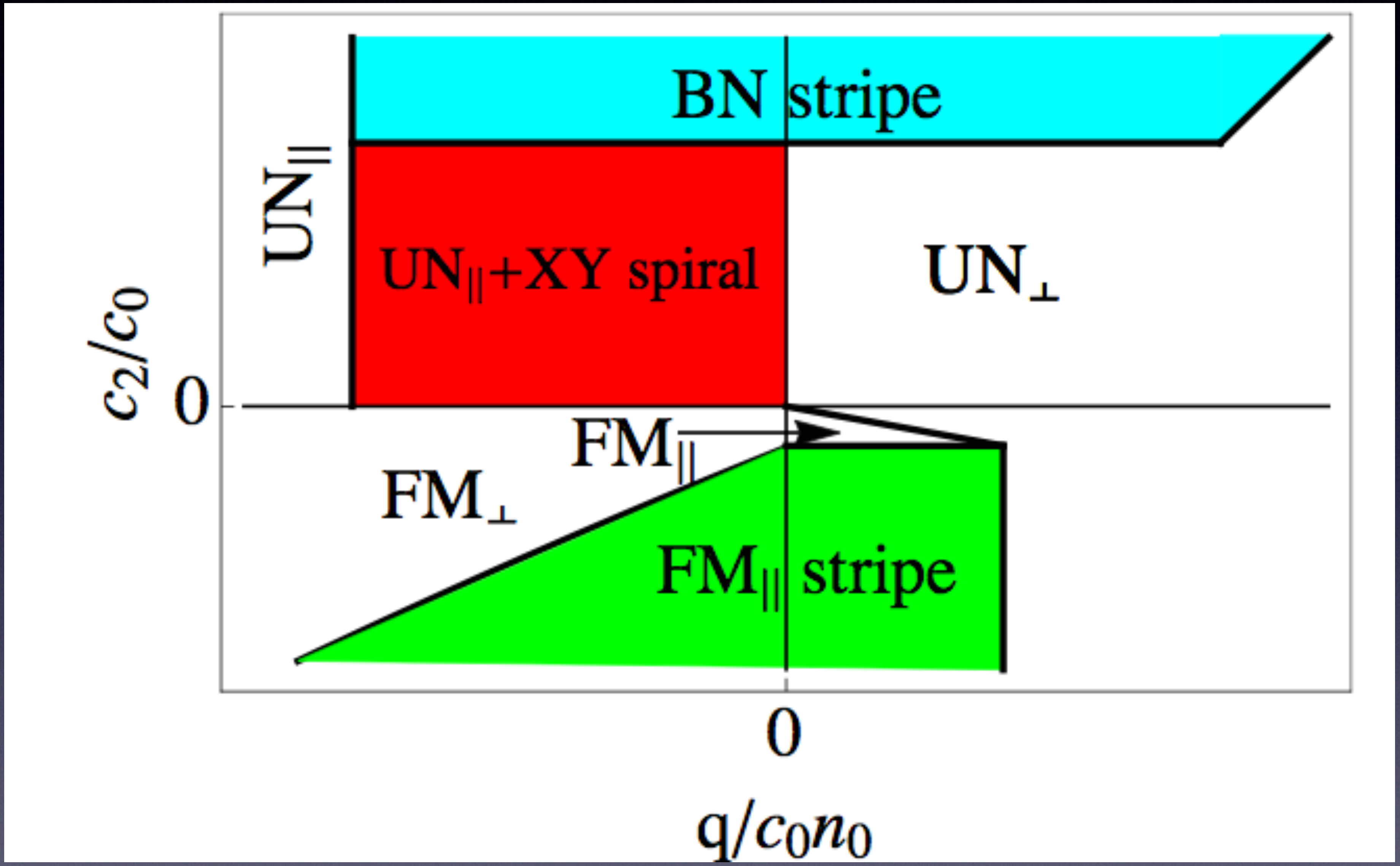
Order	Symbol	Order Parameter
ferromagnetic	$FM_{\|\|/\perp}$	$\langle S^i \rangle \neq 0$
Uniaxial nematic	$UN_{\|\|/\perp}$	$\lambda_1 \neq 0, \lambda_2 = \lambda_3 = 0$
Biaxial nematic	BN	$\lambda_1 < \lambda_2 < \lambda_3$
Translation	stripe, XY spiral	$\langle S^i(\mathbf{r}) \rangle \sim \cos(k_1 r)$
		$n(r) \sim \cos(k_2 r)$

Magnetic Order (vector) $\mathbf{Q}^{(1)} = \mathbf{s} = \langle \mathbf{S} \rangle / \rho$

Nematic Order (tensor) $Q^{(2)}_{ab} = (2/3)\delta_{ab} - (\langle S_a S_b \rangle + \langle S_b S_a \rangle)/2\rho^2$

Uniaxial/Biaxial nematic

Phase Diagram



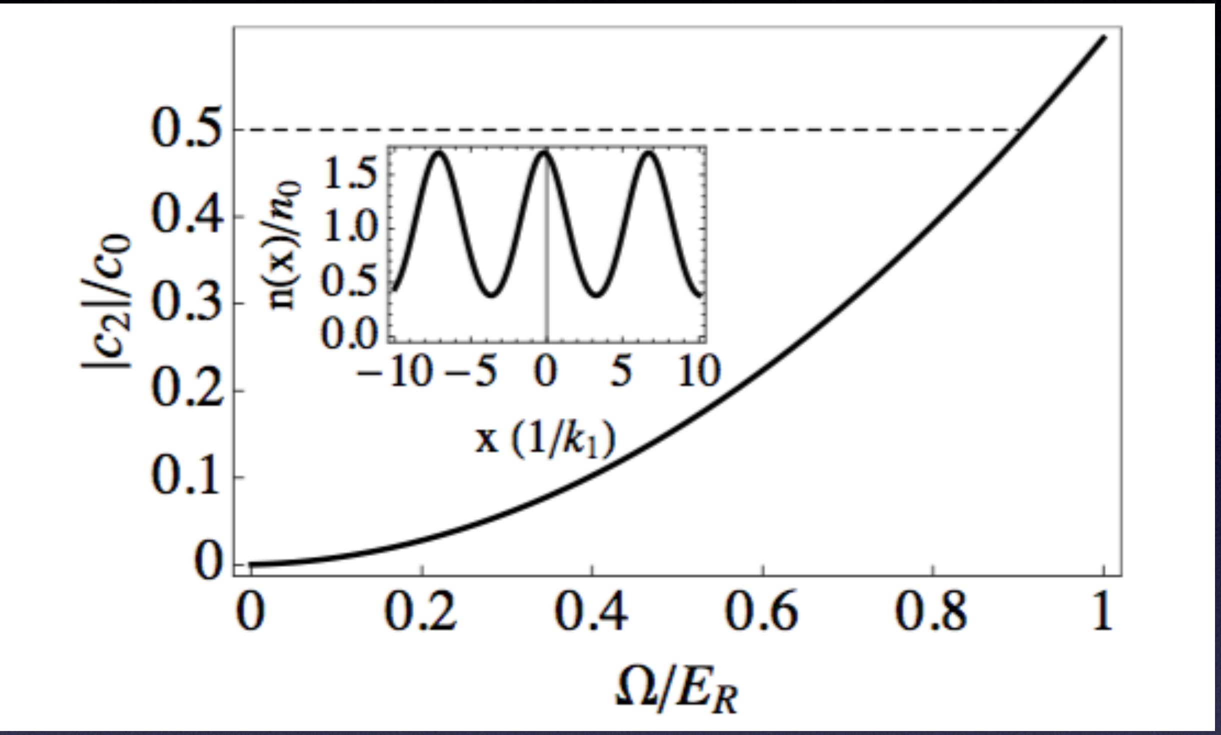
Stripe Phase driven by interactions!

Natu, Li, Cole PRA (2015)

Critical interaction for Stripes

$$\mathcal{H}_{soc} = \frac{\hbar^2(k_x - k_0 S_z)^2}{2m} + \frac{\hbar^2 k_\perp^2}{2m} + \frac{\Omega}{2} S_x + \frac{\delta}{2} S_z + \frac{q}{2} S_z^2$$

$$\psi = \sqrt{\frac{N}{V}} (\chi_+ e^{ik_1 x}\phi_+ + \chi_0 \phi_0 + \chi_- e^{-ik_1 x}\phi_-)$$



Wave-function of any component has stripes

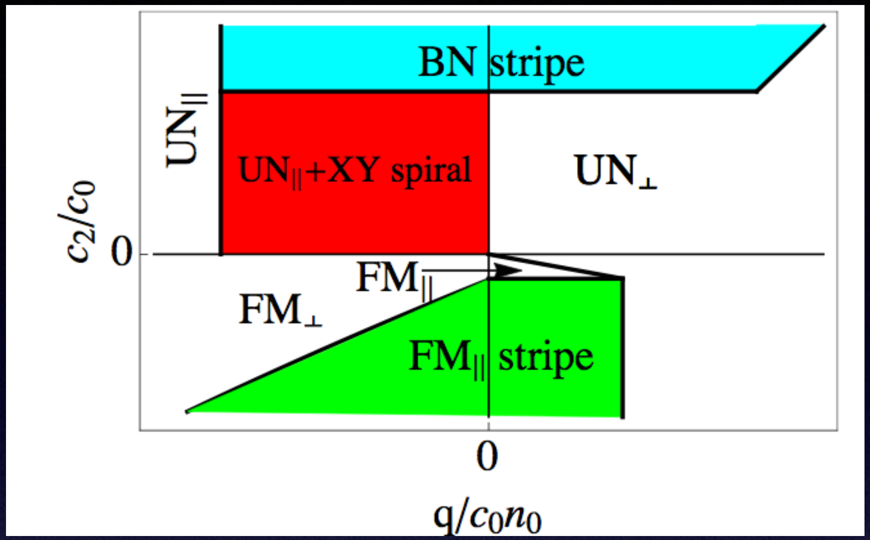
But forming stripes costs density energy

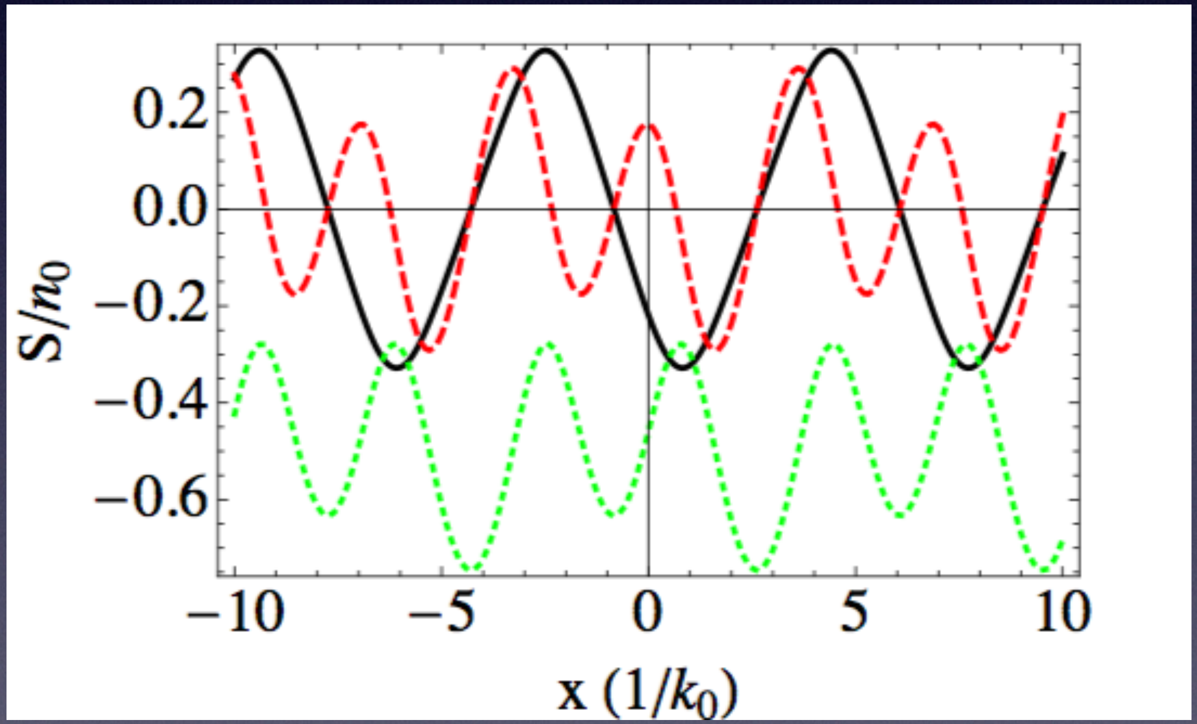
In spin-1 gas, this can be offset by c_2

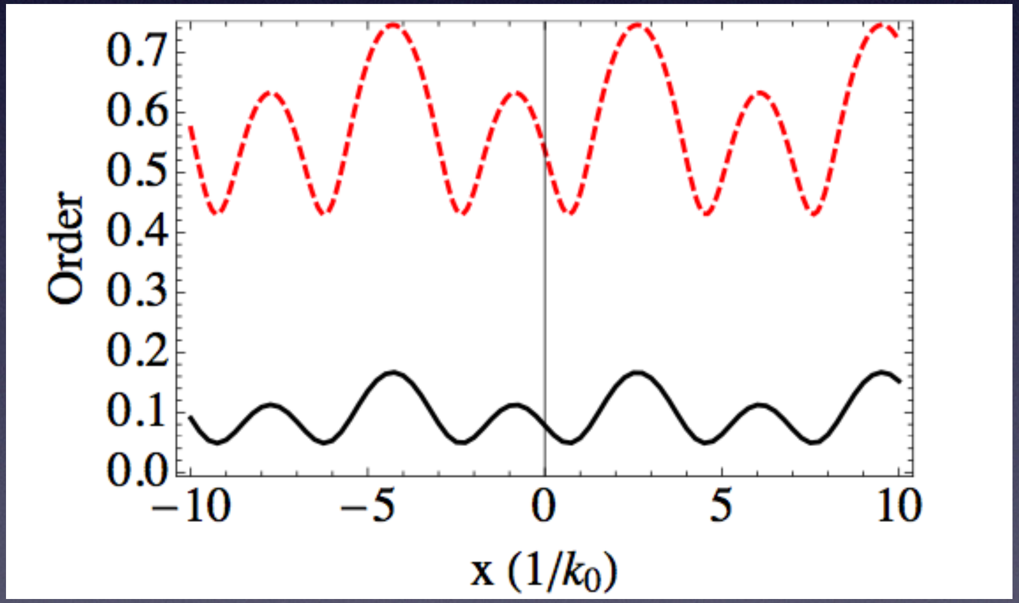
Stripe amplitude can be enhanced by enhancing Ω by going to larger c_2

Ferronematic

In presence of Raman field,
Nematicity and Spiral spin order coexist
with density wave order







Spatially oscillating Biaxial nematic phase coexisting with FM
in presence of SOC!

Natu, Li, Cole PRA (2015)

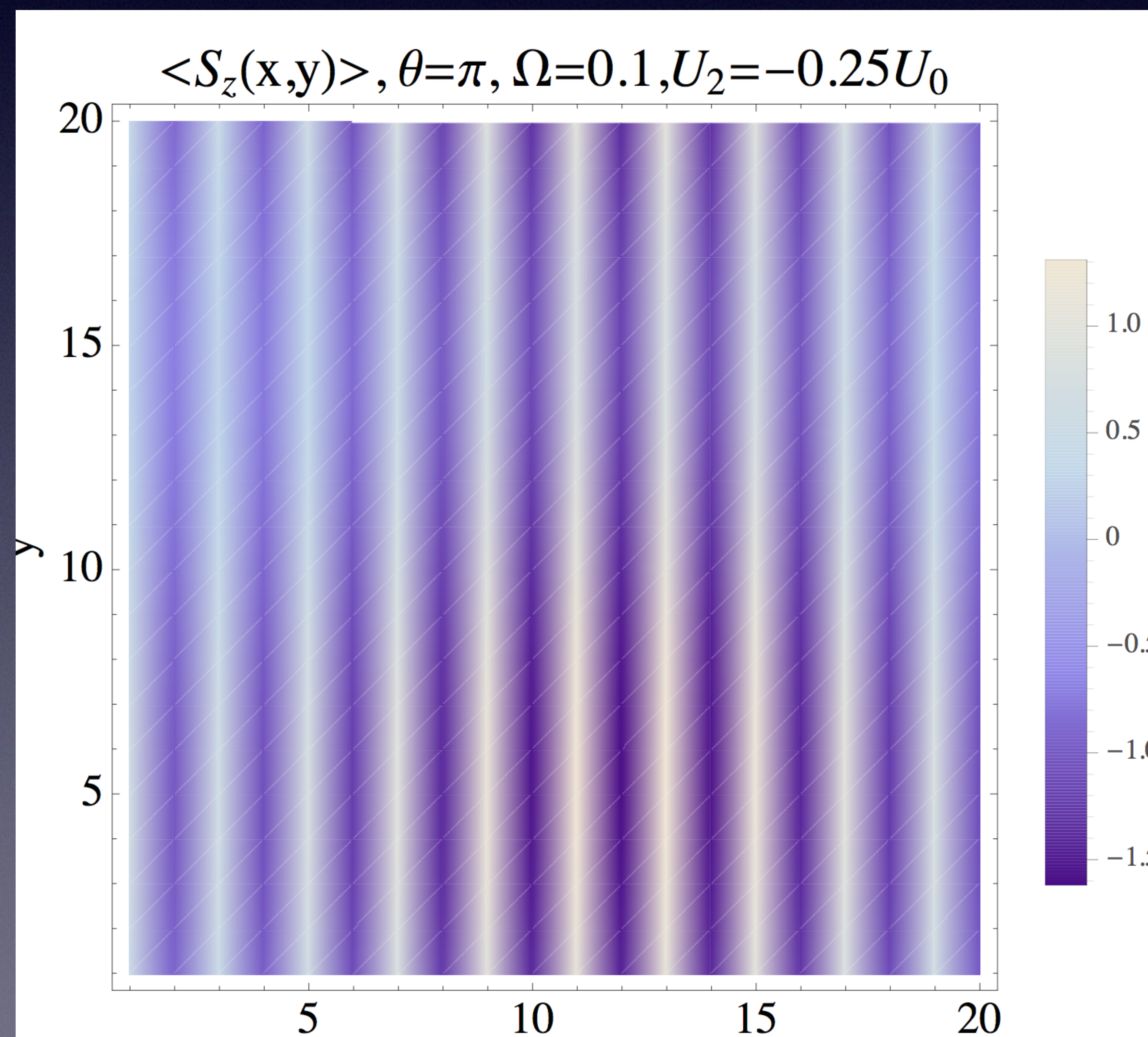
SdW in a lattice

$$H - \mu \hat{N} = -t \sum_{\langle i,j \rangle} \left(a_{i\alpha}^{\dagger} R_{ij}^{\alpha\beta} a_{j\beta} + \mathrm{H.c} \right) + \frac{U_0}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) + \frac{U_2}{2} \sum_i \left(\mathbf{S}^2 - 2\hat{n}_i \right) - \mu \sum_i \hat{n}_i + \Omega_R \sum_i S_i^z$$

$$\begin{aligned} R_{ii+\hat{x}}^{\alpha\beta} &= e^{i\theta T_y} \\ R_{ii+\hat{y}}^{\alpha\beta} &= \mathbf{1} \\ R_{ii+\hat{z}}^{\alpha\beta} &= \mathbf{1} \end{aligned}$$

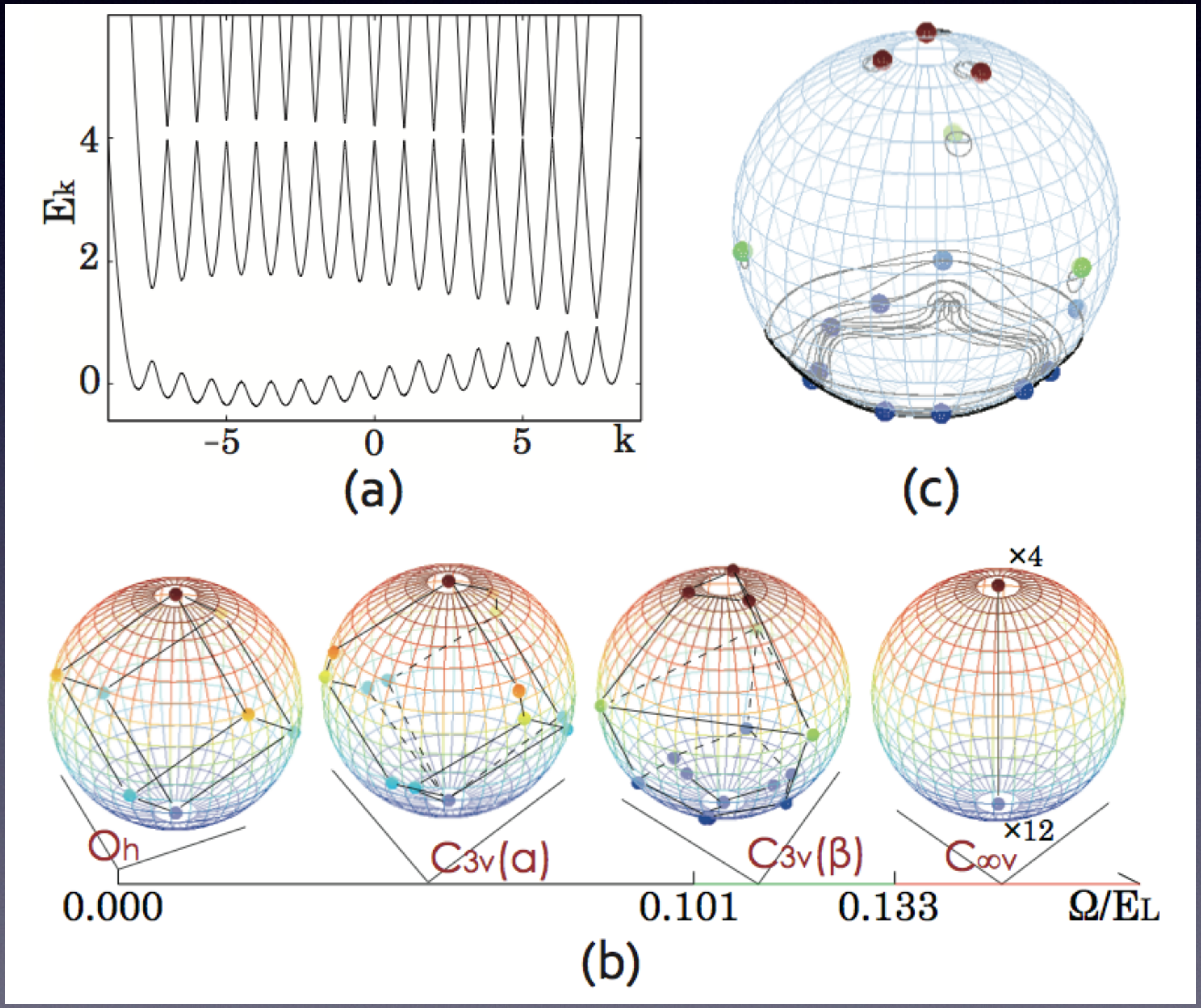
Preliminary!

- Spin-1 SOC Bose Hubbard model
- Gutzwiller phase diagram



Mo' Spin… Mo' Orders

Spin-S atoms host generalizations of spin and nematic order!



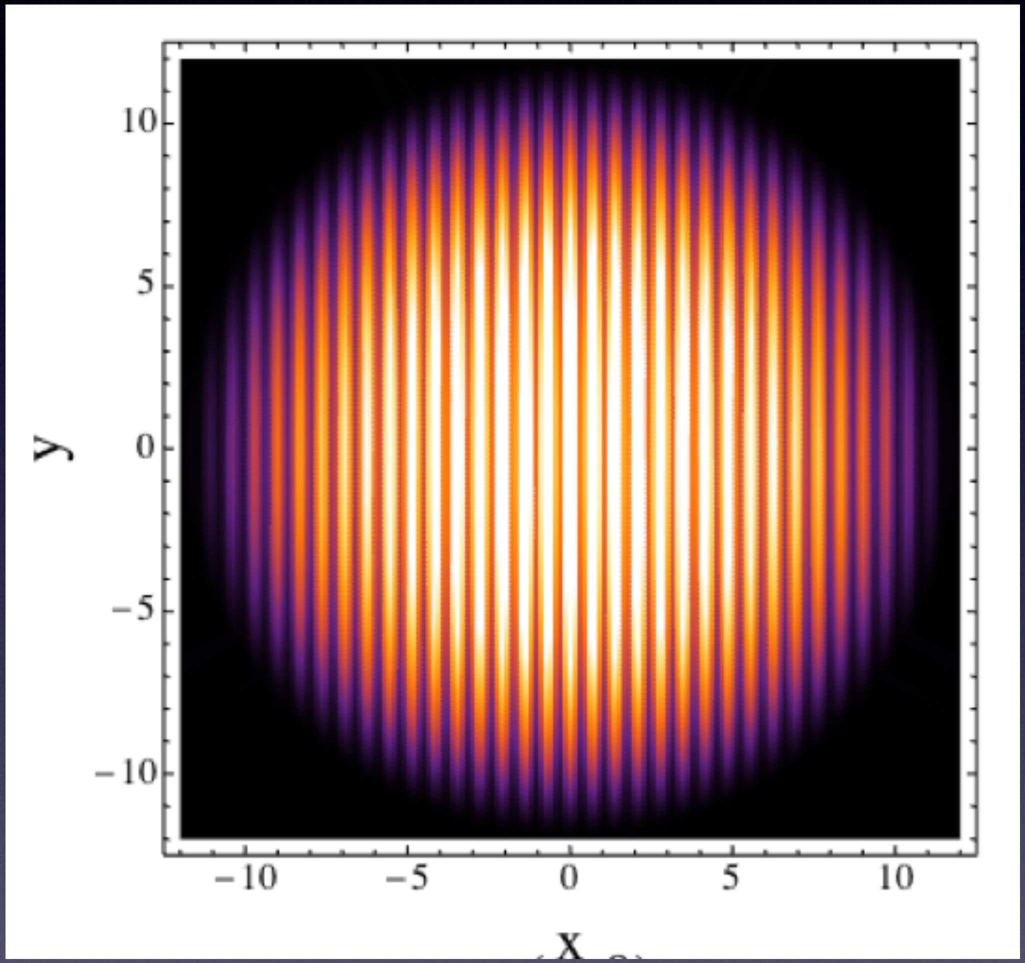
Spin-S viewed as 2S spin 1/2 on a Bloch sphere

Generalized platonic solids

Cui et al. PRA (R) (2013)

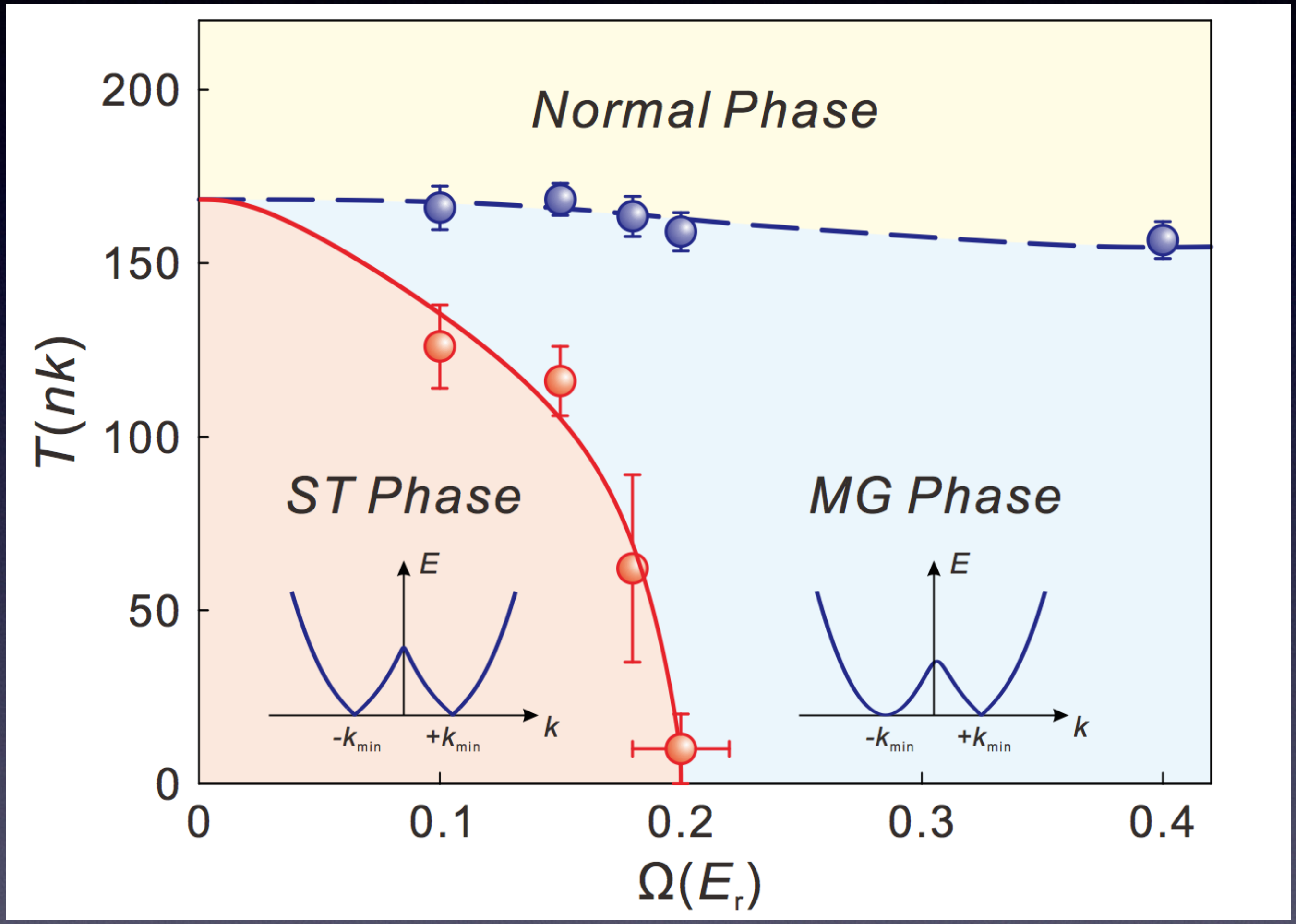
Finite Temperature Physics of Interacting Bose Gases

T = 0 Theory



Ho and Zhang, PRL (2011)

Finite T Experiment



Ji et al. Nature Physics (2014)
Z. Q. Yu. PRA (2014)

Density wave melts before condensation transition

Our Approach

Compute instabilities of Normal phase to **stripe** ordering

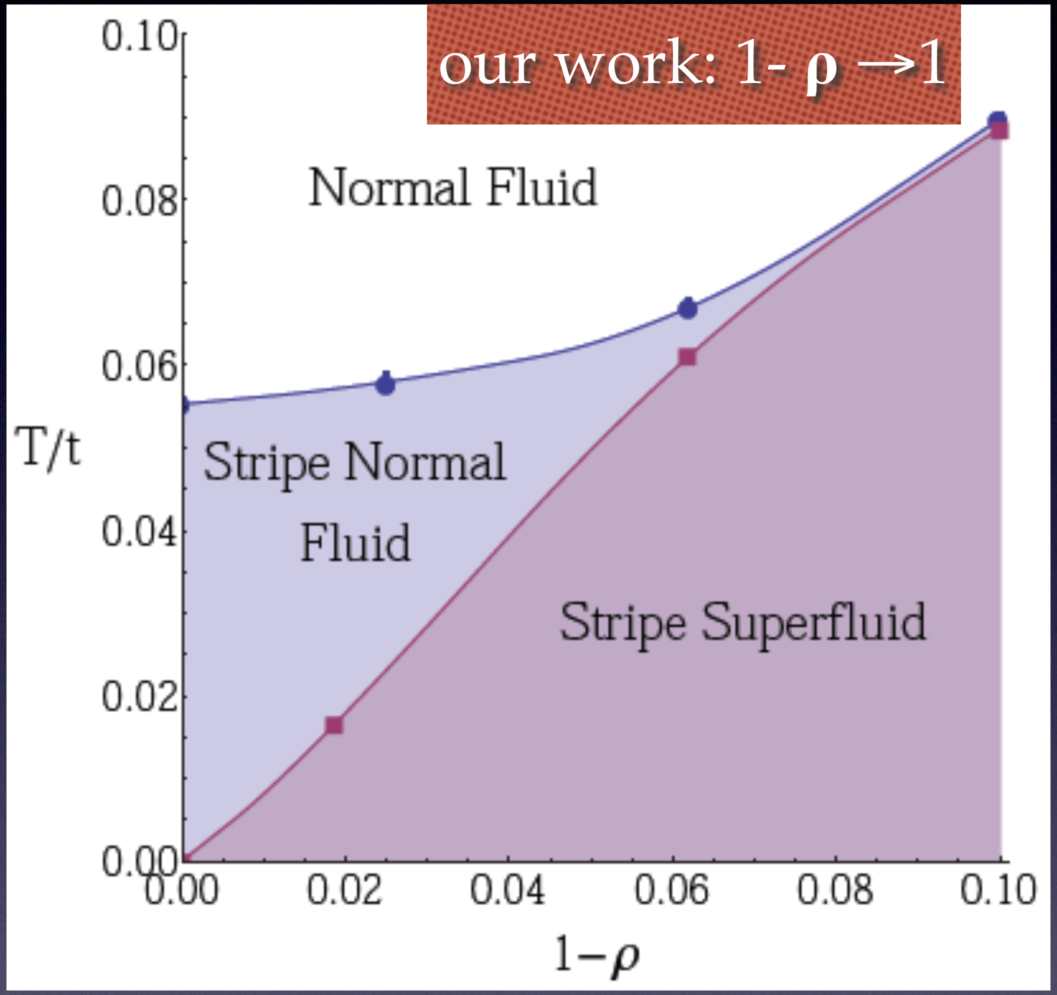
OTHER WORK

Within our *continuum* calculations we find no evidence for stripe formation above T_{BEC}

Lattice calculation:

Gutzwiller mean-field theory

C. Hickey and A. Paramekanti, PRL (2015)

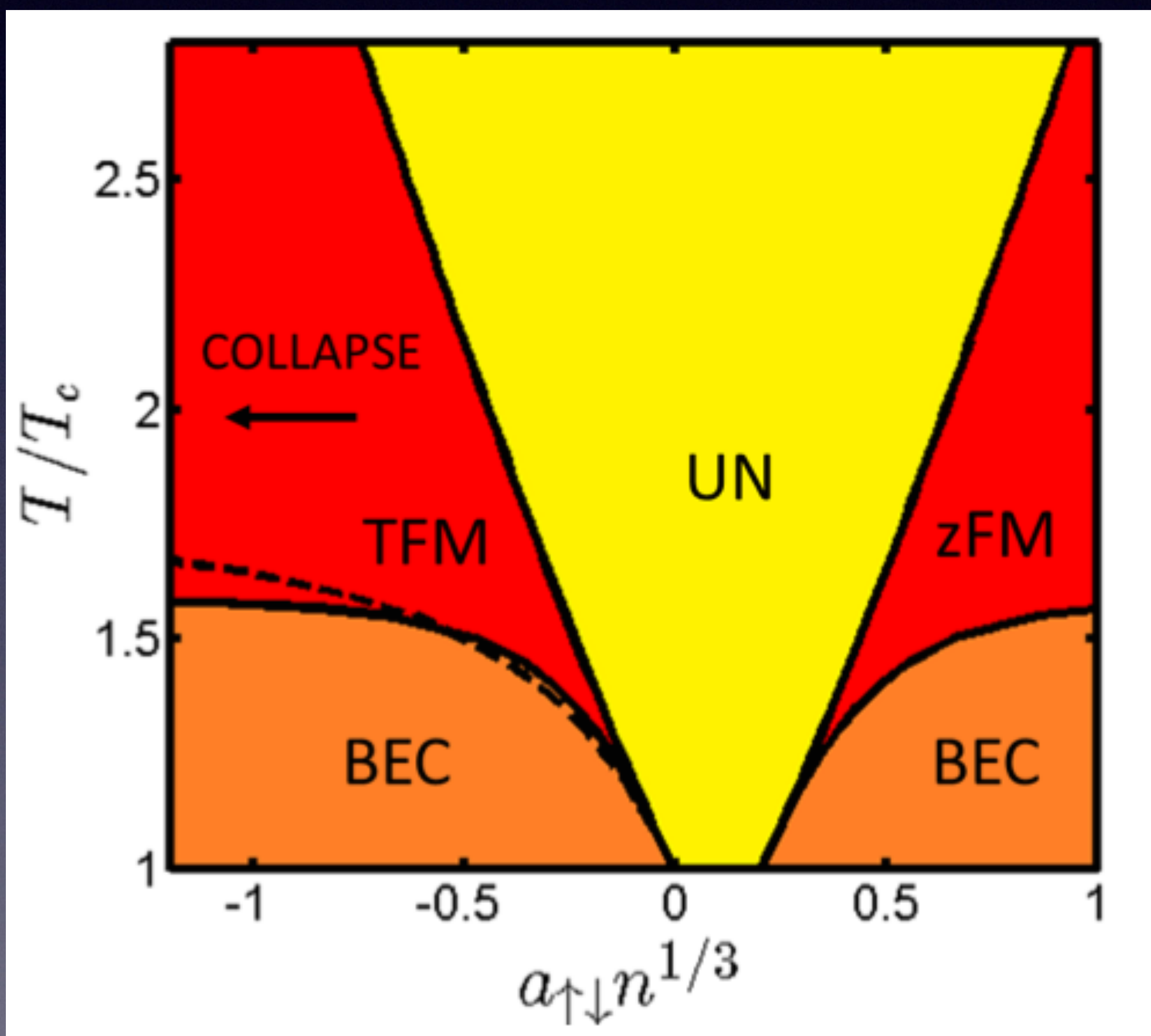


Consistent with work of Hickey and Paramekanti

Consistent with experiments in the continuum

Spin-1/2 Bosons without SOC

Itinerant pseudo-spin 1/2 bosons with short range interactions



$$\hat{H}_{\rm kin} = \int d\boldsymbol{r} \sum_{\sigma=\uparrow,\downarrow} \hat{\psi}^\dagger_\sigma(\boldsymbol{r}) \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu\right) \hat{\psi}_\sigma(\boldsymbol{r})$$

$$\hat{H}_{\rm int} = \int d\boldsymbol{r} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \frac{g_{\sigma,\sigma'}}{2} \hat{\psi}^\dagger_\sigma(\boldsymbol{r}) \hat{\psi}^\dagger_{\sigma'}(\boldsymbol{r}) \hat{\psi}_{\sigma'}(\boldsymbol{r}) \hat{\psi}_\sigma(\boldsymbol{r})$$

$$g_{\sigma,\sigma'} = 4\pi\hbar^2 a_{\sigma,\sigma'}/m$$

$$g_{\uparrow\uparrow} = g_{\downarrow\downarrow} = g > 0$$

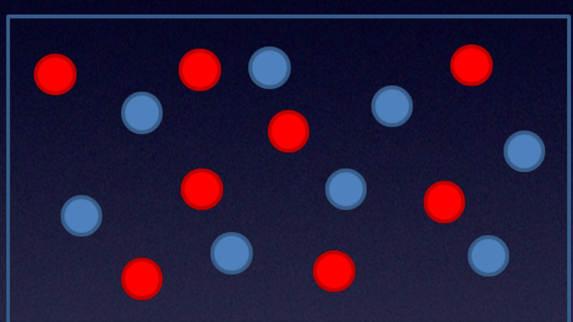
Ising FM: repulsive interactions
XY FM: attractive interactions

Radic, Natu, Galitski, PRL 113 185302 (2014)

STONER FERROMAGNETISM

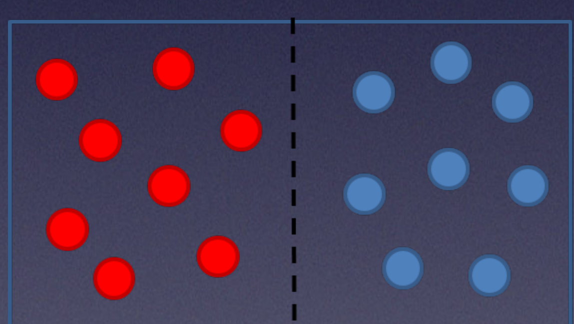
Textbook model for **itinerant** magnetism

Spin up and down electrons with repulsive, ***short-range*** density interactions



$$\hat{H}_0 = \frac{\mathbf{p}^2}{2m} + g \int d\boldsymbol{r} \check{n}_\uparrow(\boldsymbol{r}) \check{n}_\downarrow(\boldsymbol{r})$$

$$E_F 2Vn[\tfrac{3}{10}\{(1+\eta)^{5/3} + (1-\eta)^{5/3}\} + \tfrac{2}{3\pi} k_F a (1+\eta)(1-\eta)]$$



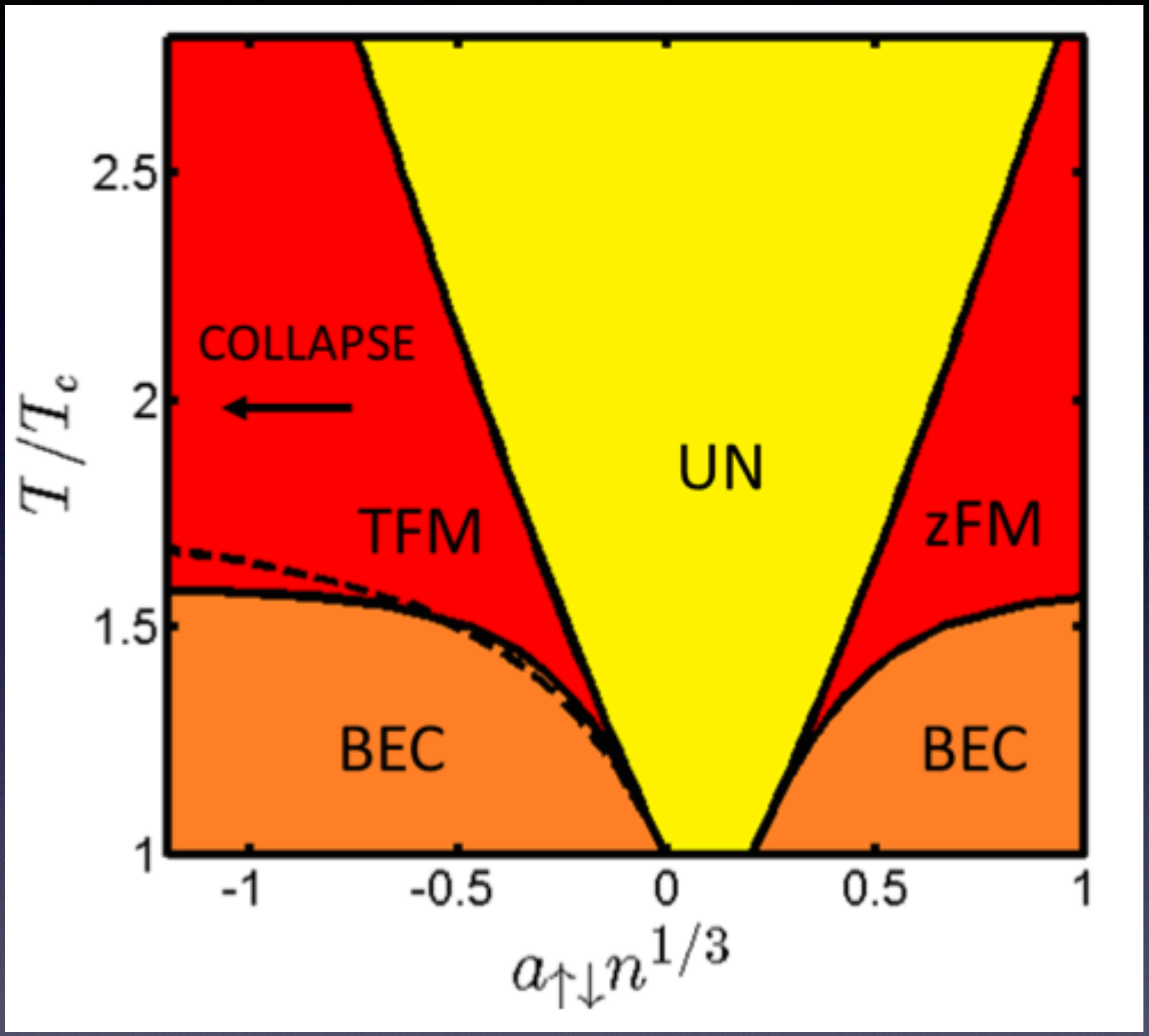
- lower interaction energy

- higher kinetic energy

$$\eta = \Delta n / n = (n_1 - n_2)/(n_1 + n_2)$$

For $k_F a > 1$, energy is minimized by setting $\eta > 0$!

MAGNETISM OR SUPERFLUIDITY?



Order parameters $n_{\sigma,\sigma'} = \frac{1}{V}\sum_k \langle \hat{a}^\dagger_{\boldsymbol{k},\sigma}\hat{a}_{\boldsymbol{k},\sigma'}\rangle$

Ising magnetism: $n_{\uparrow\uparrow}-n_{\downarrow\downarrow}$

XY magnetism: $n_{\uparrow\downarrow}$

Condensation: $\langle \hat{a}_{k=0}\rangle \neq 0$

$$\hat{H}_{\mathrm{HF}} = \sum_{\boldsymbol{k},\sigma,\sigma'} \hat{a}^\dagger_{\boldsymbol{k},\sigma}\mathcal{H}_{\sigma,\sigma'}(\boldsymbol{k})\hat{a}_{\boldsymbol{k},\sigma'} - E_0$$

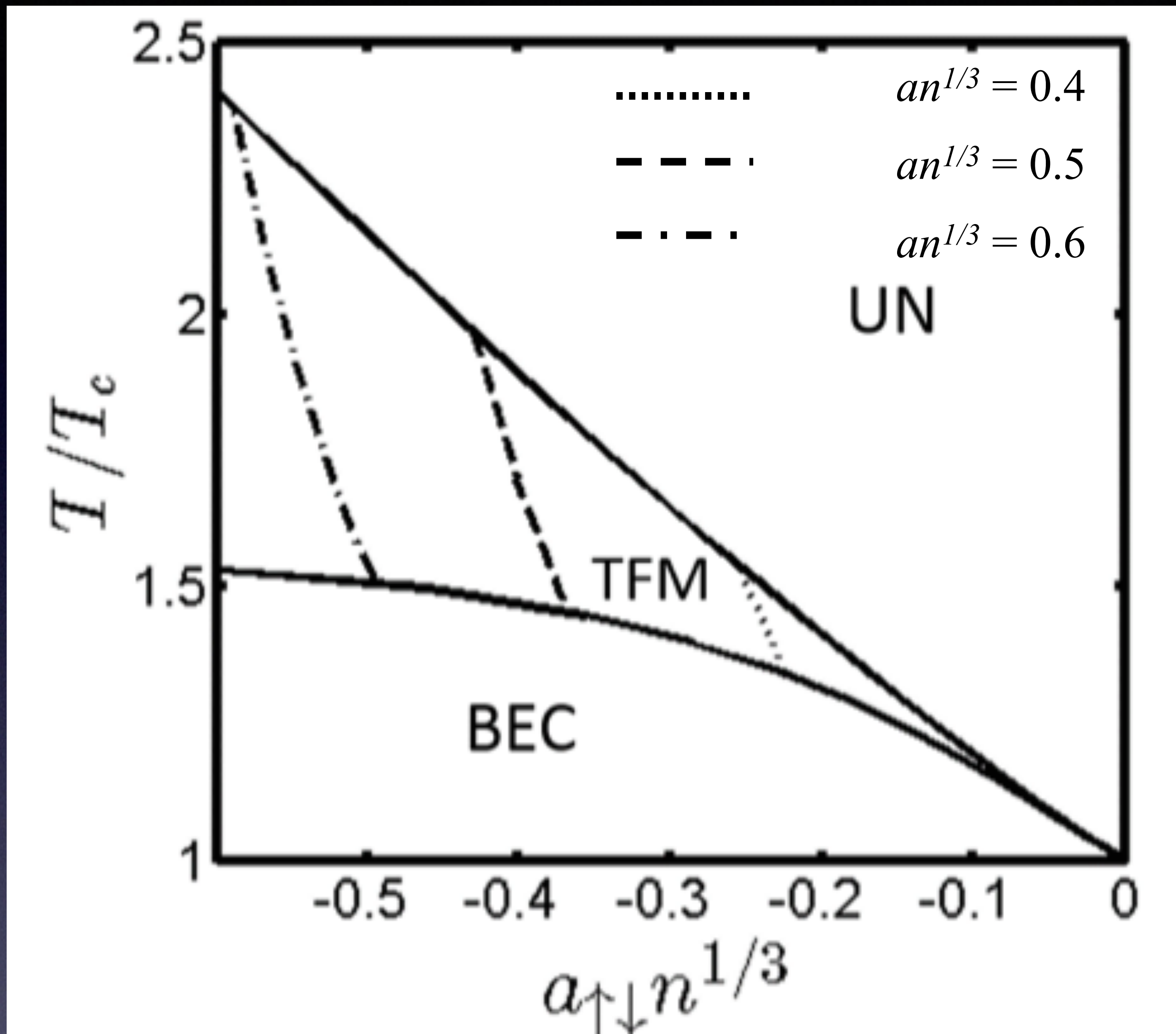
$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \epsilon_k + 2gn_\uparrow + g_{\uparrow\downarrow}n_\downarrow & g_{\uparrow\downarrow}n^*_{\uparrow\downarrow} \\ g_{\uparrow\downarrow}n_{\uparrow\downarrow} & \epsilon_k + 2gn_\downarrow + g_{\uparrow\downarrow}n_\uparrow \end{pmatrix}$$

$$\hat{H}_{\mathrm{HF}} = \sum_{\boldsymbol{k},j} E_j(\boldsymbol{k})\hat{b}^\dagger_j(\boldsymbol{k})\hat{b}_j(\boldsymbol{k}) - E_0$$

$$\langle \hat{b}^\dagger_j(k)\hat{b}_j(k)\rangle = \left[e^{\beta E_j(k)} - 1\right]^{-1}$$

Why is Ising FM different from XY ferromagnet?

Attractive Bosons



isothermal compressibility

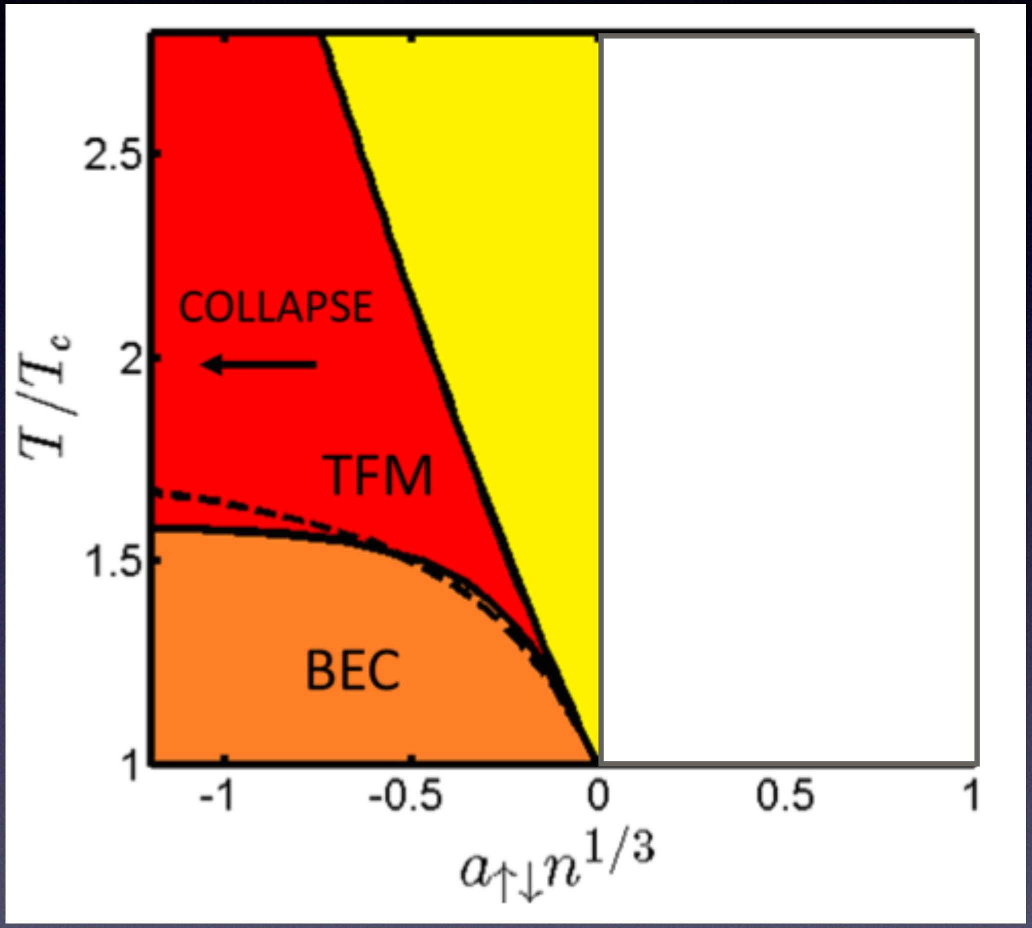
$$\beta_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

Intra-species density-density interactions stabilize the gas $g > 0$

New playground for exploring **attractive** interaction effects in bosonic gases!

PAIRING VS. MAGNETISM

3D phase diagram



XY magnetism: $n_{\sigma,\sigma'} = \frac{1}{V}\sum_k \langle \hat{a}^\dagger_{\boldsymbol{k},\sigma} \hat{a}_{\boldsymbol{k},\sigma'} \rangle$

]

BCS-pairing order: $\frac{1}{V}\sum_{\boldsymbol{k}} \langle \hat{a}_{-\boldsymbol{k}\uparrow} \hat{a}_{\boldsymbol{k}\downarrow} \rangle$

Gap equation for **bosons**

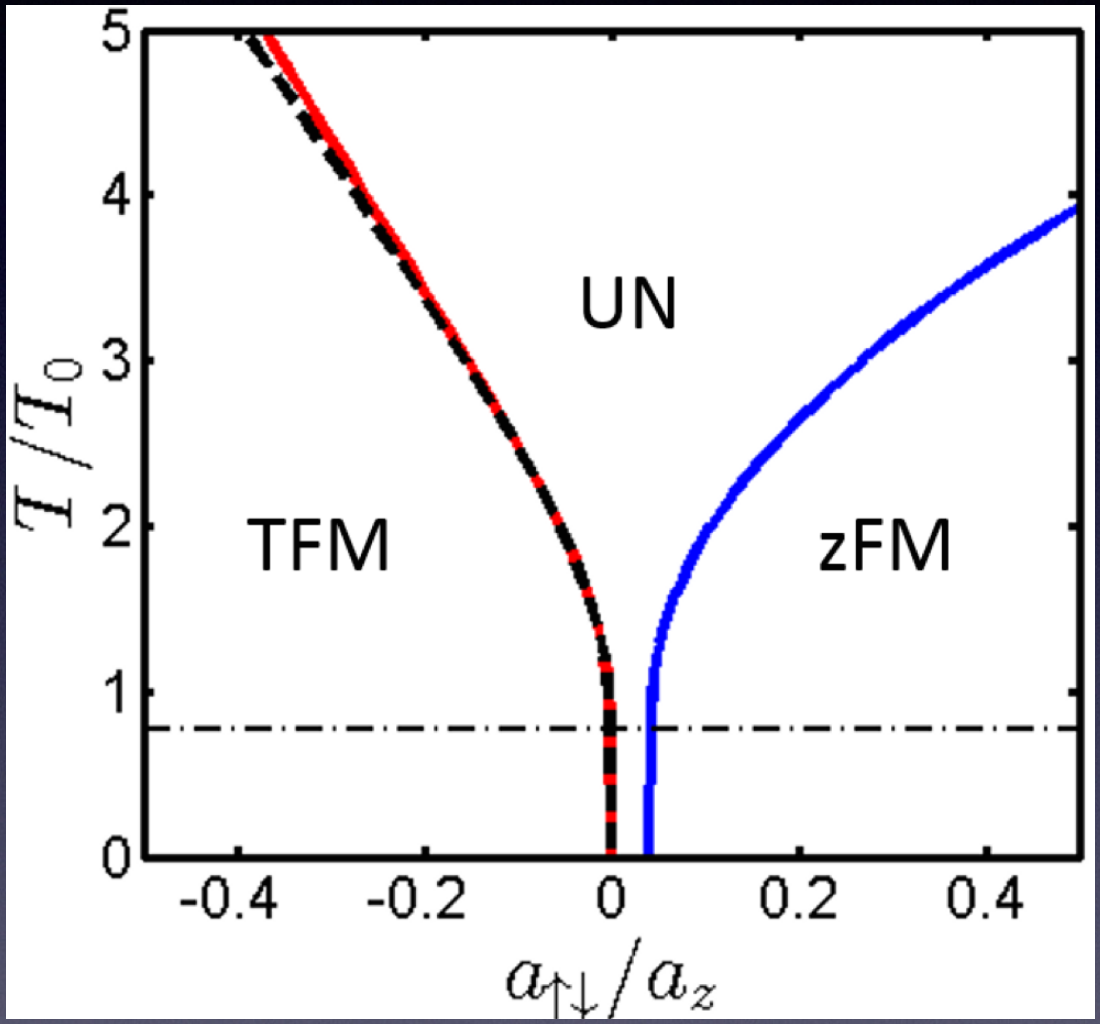
$$\frac{1}{g_{\uparrow\downarrow}} = -\frac{1}{V}\sum_{\boldsymbol{k}} \left[\frac{1}{E_k}\left(\frac{1}{e^{\beta E_k} - 1} + \frac{1}{2} \right) - \frac{1}{2\epsilon_k^0} \right]$$

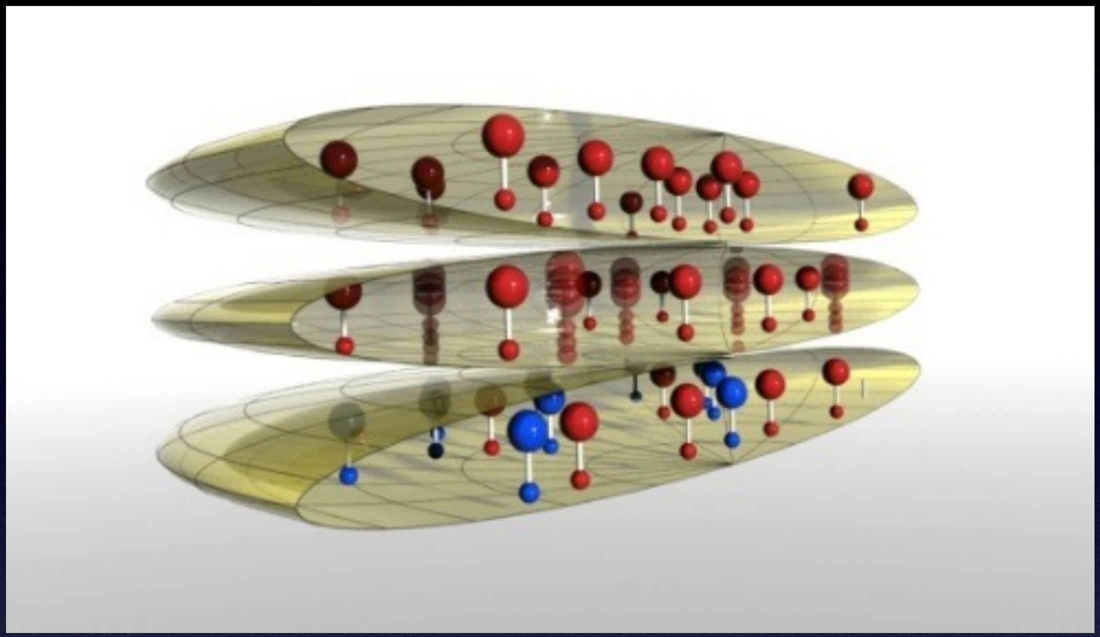
$$E_k = \sqrt{\epsilon_k^2 - g_{\uparrow\downarrow}^2 |\Pi_{\uparrow\downarrow}|^2}$$

In 3D Magnetism wins over pairing!

Pairing in 2D

Quasi-2D Finite Temperature Phase Diagram





3D gas strongly squeezed in the z-direction

$$g_{2D} = \frac{2\sqrt{2\pi}\hbar^2}{m} \frac{1}{a_z/a + (1/\sqrt{2\pi})\ln(B/\pi q^2 a_z^2)}$$

$$a_z = \sqrt{\frac{\hbar}{m\omega_z}}$$

Tendency towards pairing enhanced in lower dimensions: bound state

In 2D both orders have nearly the same Tc within Mean-field theory!
Beyond mean-field effects will determine which one will eventually win!

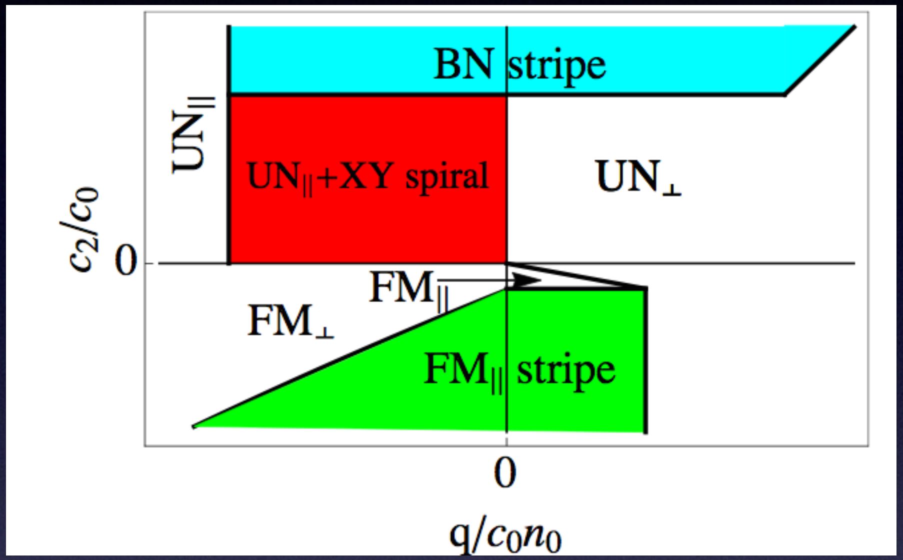
Open Questions

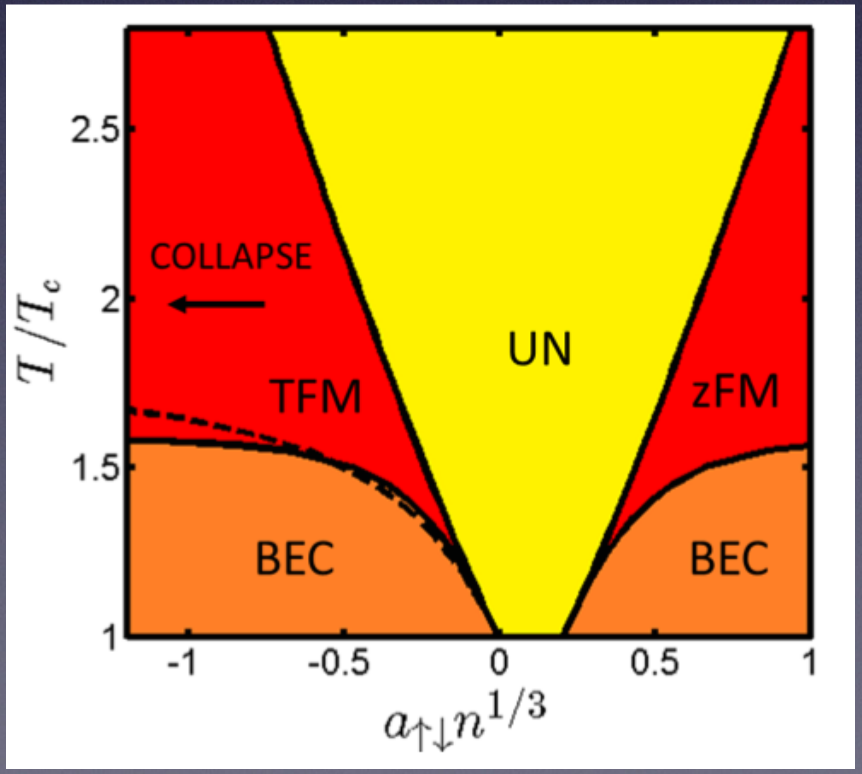
Excitation spectra in stripe phases?

How do these phases evolve in the Mott limit?

What happens at finite temperature?

Can SOC enhance pairing tendencies?



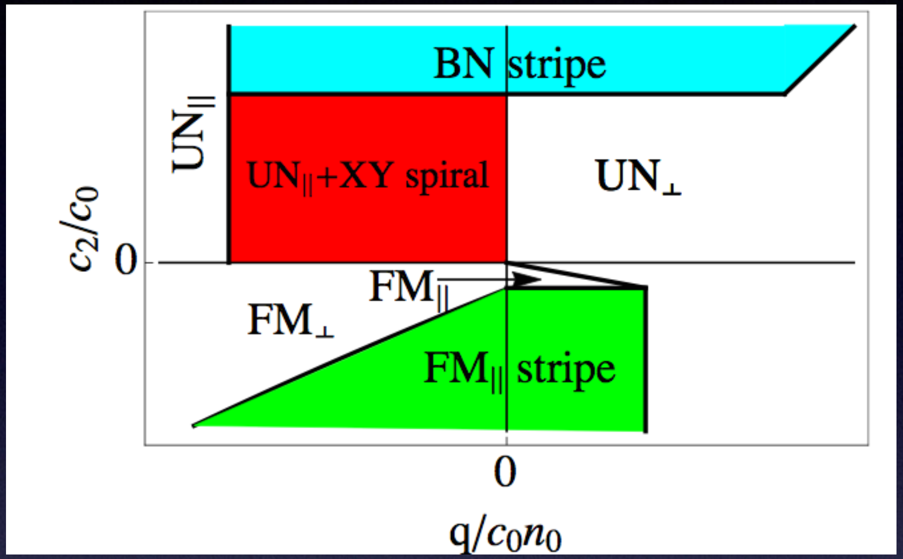


Conclusions

Phase Diagram of spin-1 SOC gas

Spin-dependent interactions stabilize stripes with tunable amplitude

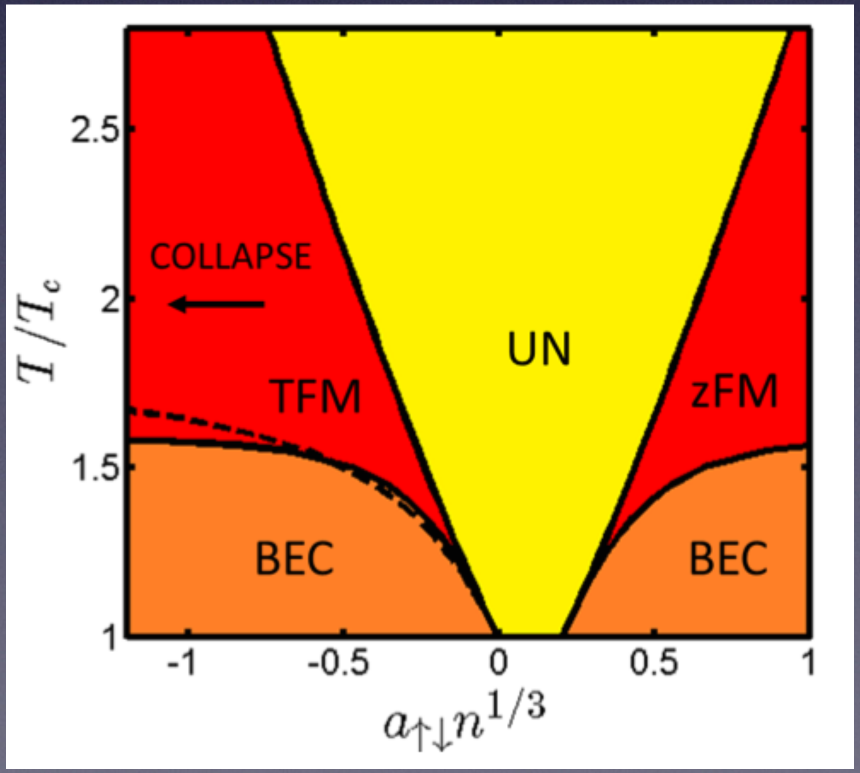
Spin-orbit coupling leads to coexistence of ferro and nematic order



Exciting physics at finite temperature

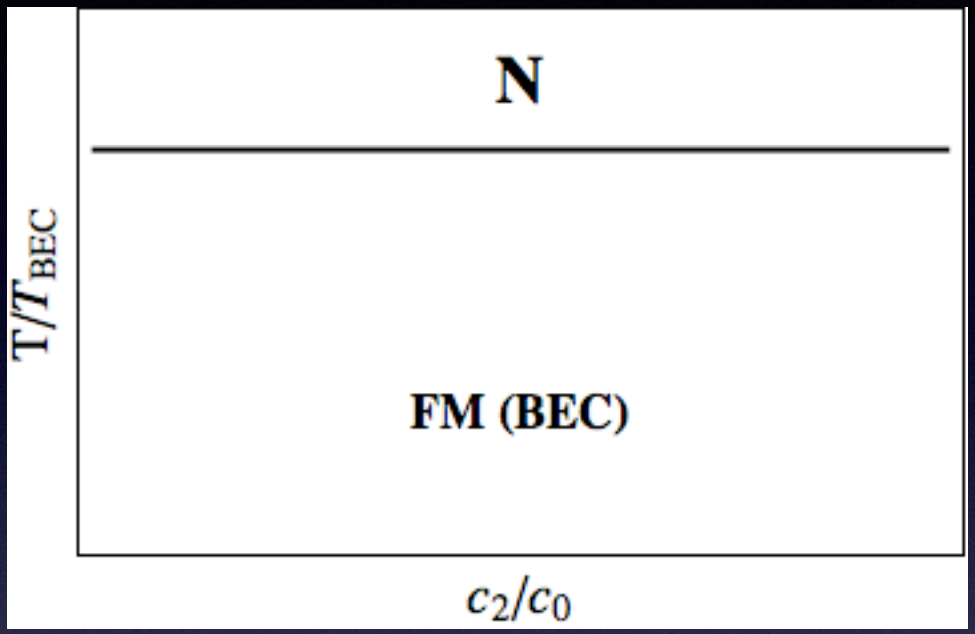
Bosonic analog of Stoner in spin-1/2 gas

Finite temperature pairing?

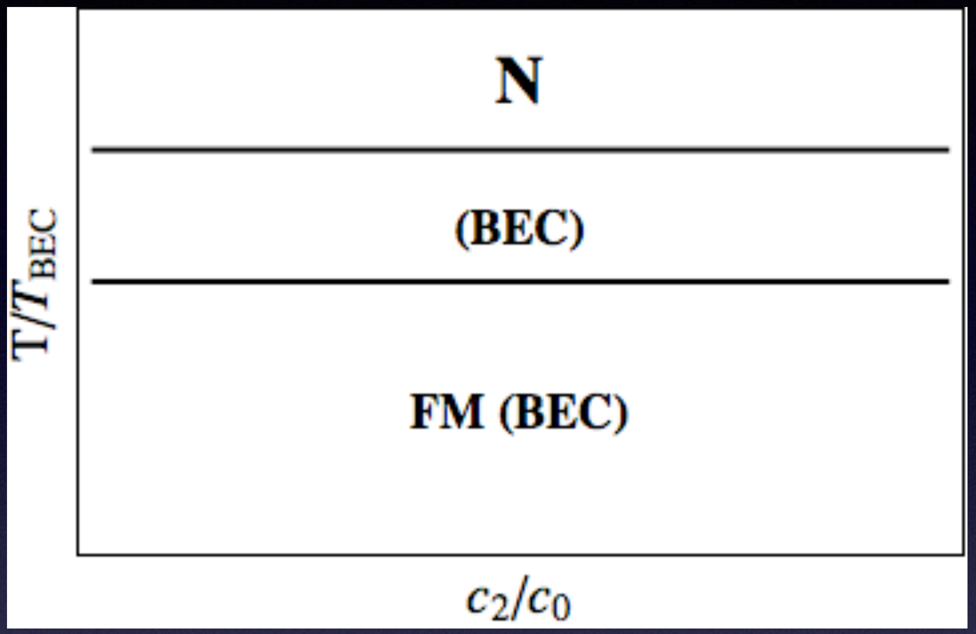


Scenarios

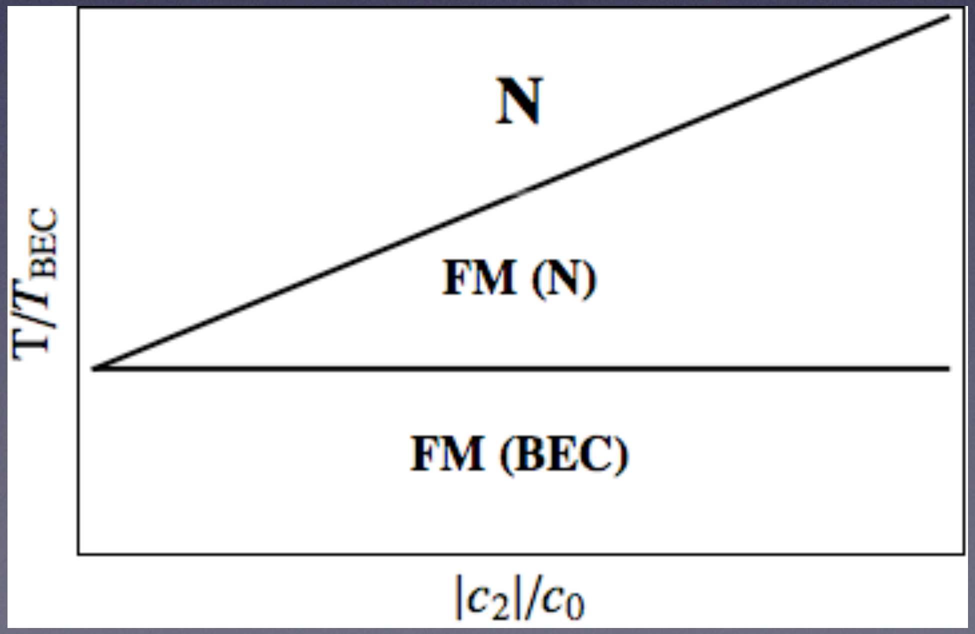
Magnetism/BEC have same T_C



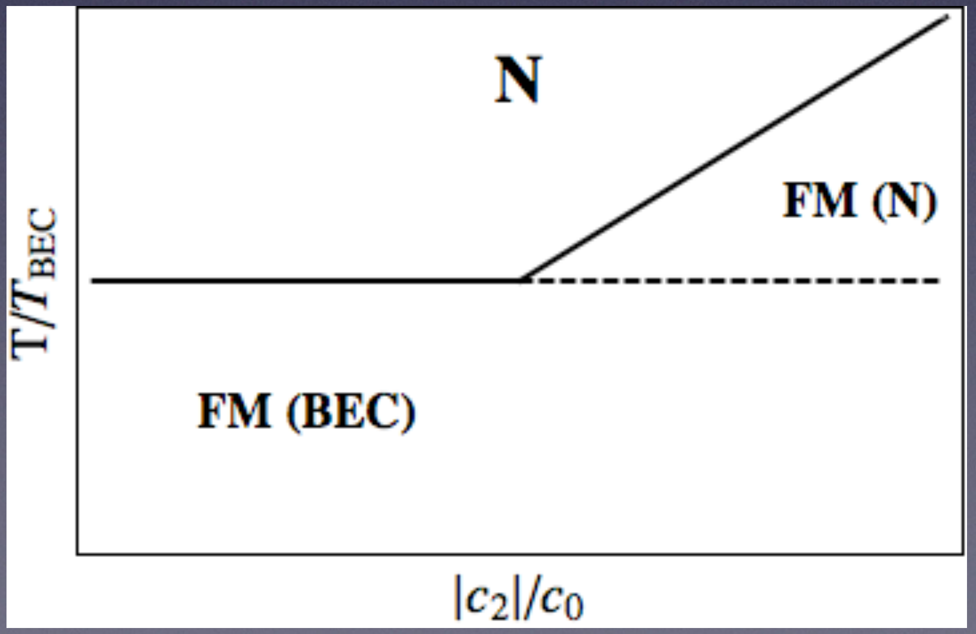
Magnetism disappears at lower T



Normal Ferromagnet occurs for arbitrarily weak interaction

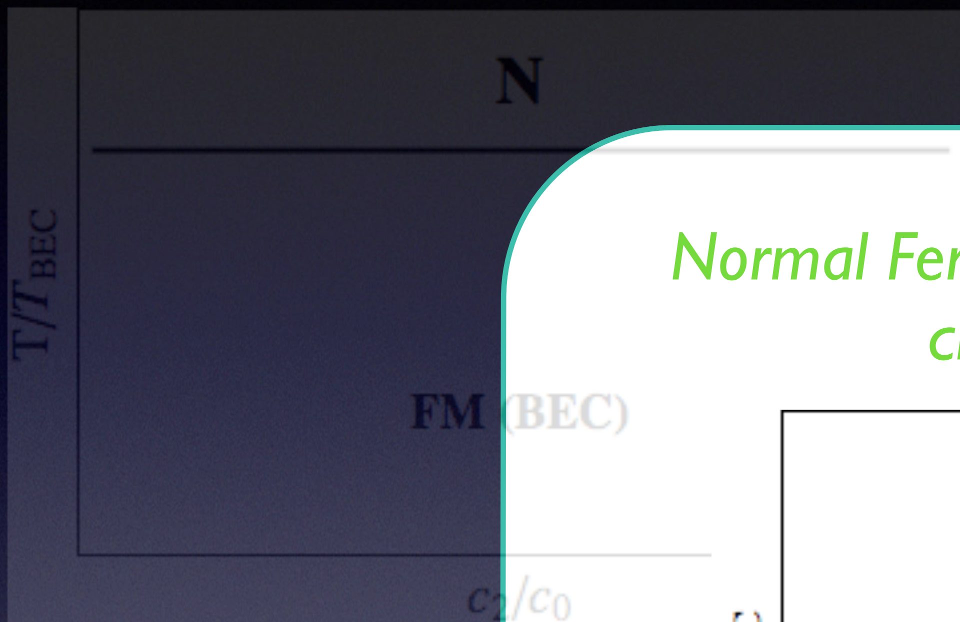


Normal Ferromagnet occurs for some critical interaction

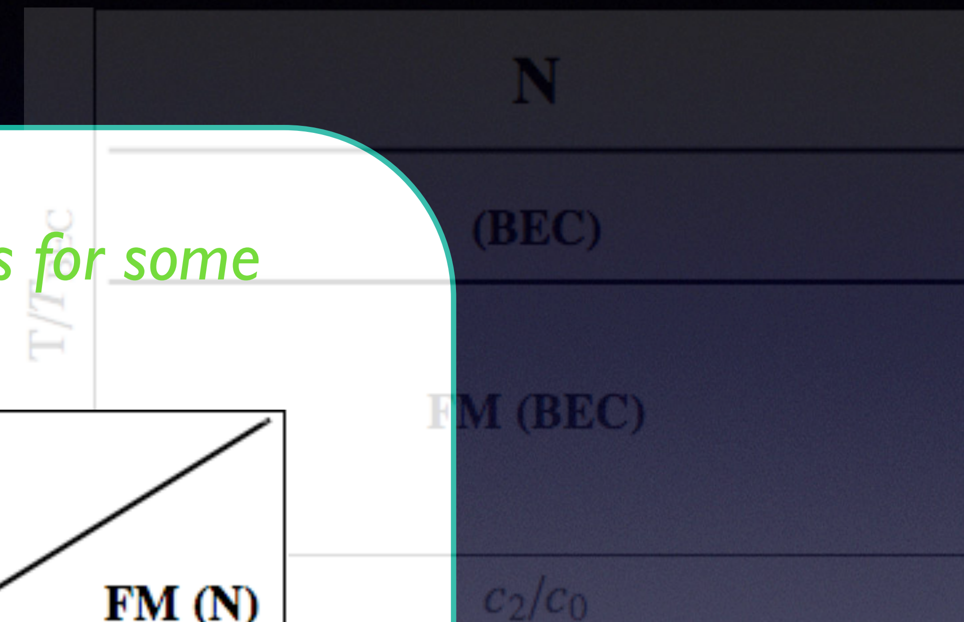


Winner!

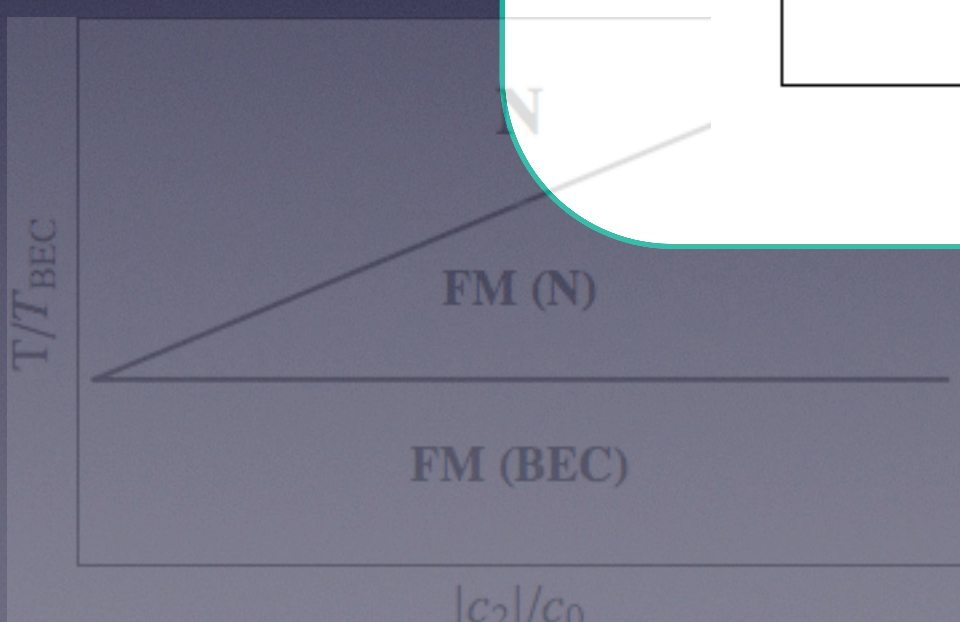
Magnetism/BEC have same T



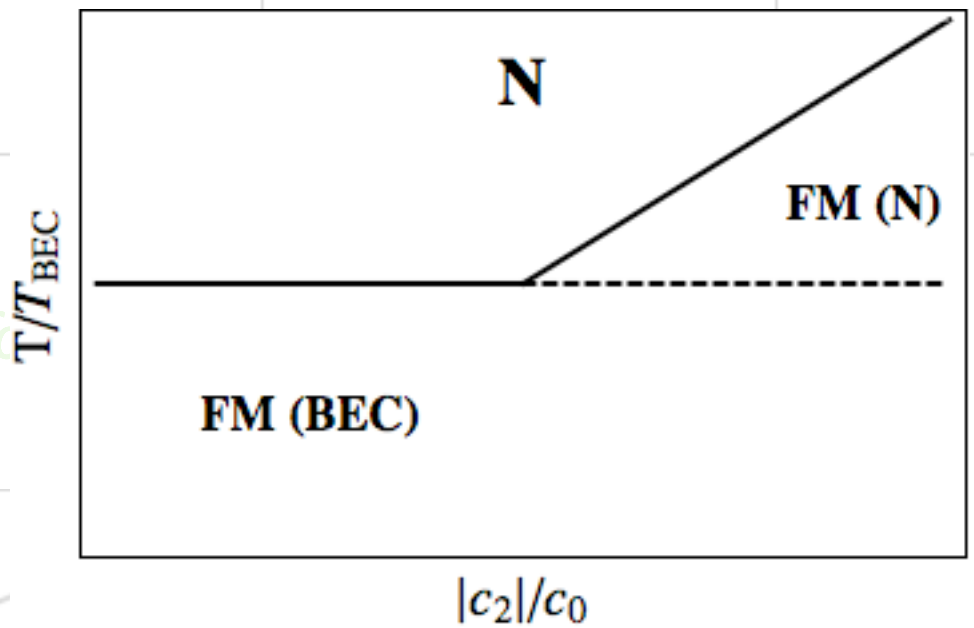
Magnetism disappears at lower T



Normal Ferromagnet occurs for weak interaction



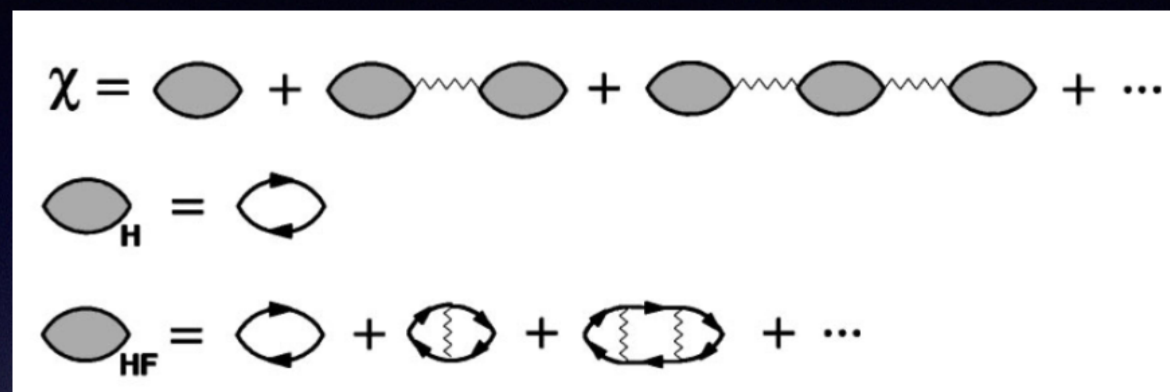
Normal Ferromagnet occurs for some critical interaction



Natu, Mueller PRA (2011)

Theory (RPA + Exchange)

Compute susceptibility of normal state to magnetism



Non-interacting susceptibility

$$\hat{\chi}^{RPA} = \left(\mathbb{1} - \hat{\chi}^0 \hat{V}\right)^{-1} \hat{\chi}^0$$

For short range interactions, exchange is a factor of 2!

Divergence in RPA at k = ω=0 indicates phase transition

Theory (RPA-X + Spin)

RPA EQUATION IS A MATRIX EQUATION

$$(\chi^{\mathrm{RPA}})^{\gamma\eta}_{\alpha\beta} = (\chi^0)^{\gamma\eta}_{\alpha\beta}\delta_{\alpha\eta}\delta_{\beta\gamma} + \sum(\chi^0)^{\gamma\eta}_{\eta\gamma}\mathbf{V}^{\gamma\eta}_{\mu\nu}(\chi^{\mathrm{RPA}})^{\nu\mu}_{\alpha\beta}$$

Direct + Exchange Matrix

$$V = \begin{pmatrix} 2(c_0+c_2) & 0 & 0 & 0 & c_0+c_2 & 0 & 0 & 0 & c_0-c_2 \\ 0 & 0 & 0 & c_0+c_2 & 0 & 0 & 0 & 2c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_0-c_2 & 0 & 0 \\ 0 & c_0+c_2 & 0 & 0 & 0 & 2c_2 & 0 & 0 & 0 \\ c_0+c_2 & 0 & 0 & 0 & 2c_0 & 0 & 0 & 0 & c_0+c_2 \\ 0 & 0 & 0 & 2c_2 & 0 & 0 & 0 & c_0+c_2 & 0 \\ 0 & 0 & c_0-c_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2c_2 & 0 & 0 & 0 & c_0+c_2 & 0 & 0 & 0 \\ c_0-c_2 & 0 & 0 & 0 & c_0+c_2 & 0 & 0 & 0 & 2(c_0+c_2) \end{pmatrix}$$

Non-interacting susceptibility MATRIX

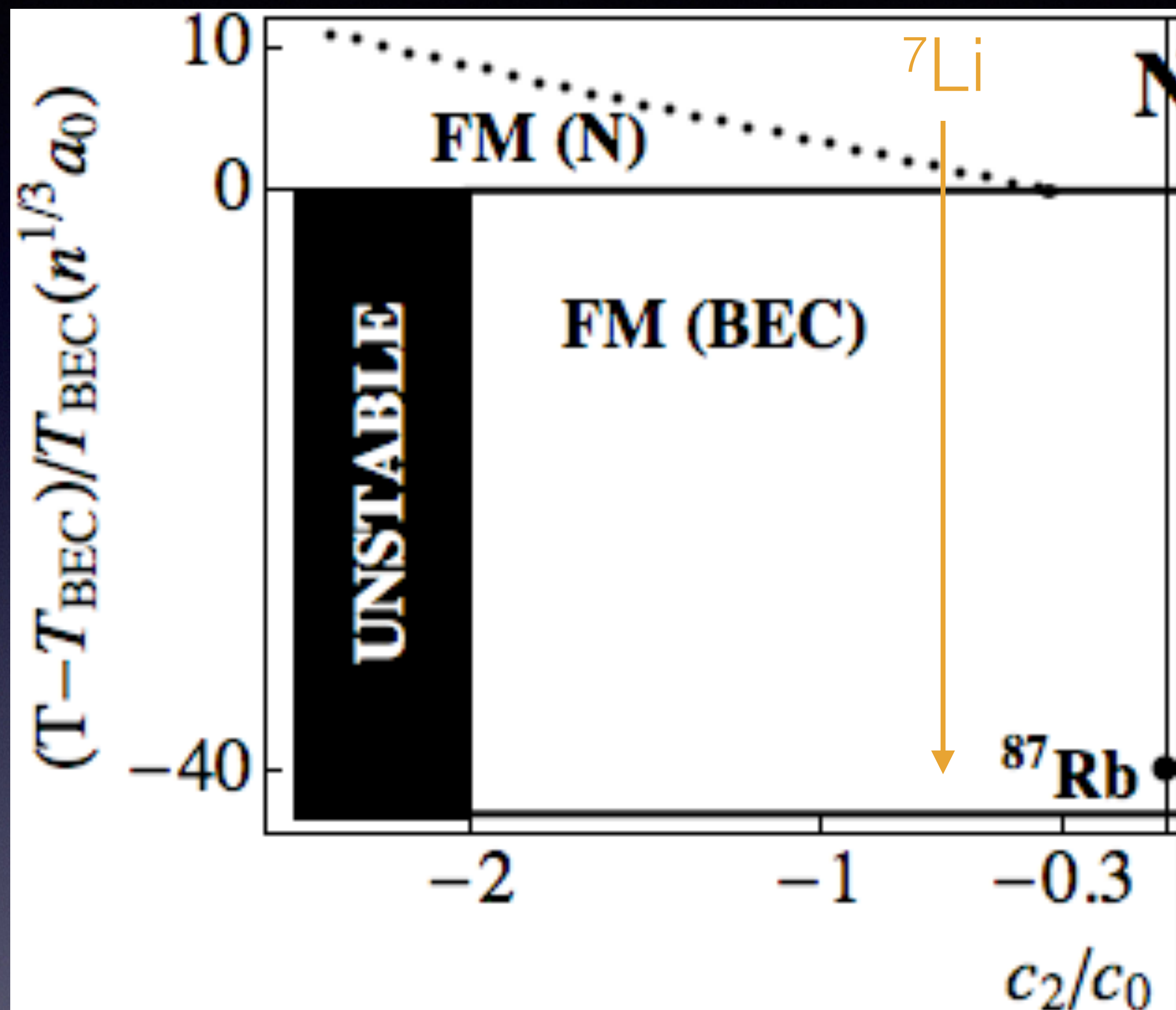
$$(\chi^0)^{\beta\alpha}_{\alpha\beta}(p,\omega) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{n(\epsilon_{k,\alpha}) - n(\epsilon_{k+p,\beta})}{\omega - (\epsilon_{k+p\beta} - \epsilon_{k\alpha})}$$

$$\chi_z^{\mathrm{RPA}}(\mathbf{k},0) = \frac{2(\chi^0)_{-1}^{1-1}{}_{1}(\mathbf{k},0)}{1-(c_0+3c_2)(\chi^0)_{-1}^{1-1}{}_{1}(\mathbf{k},0)}$$

Divergence in RPA at k = ω=0 indicates phase transition

Finite T Phase Diagram

Natu, Mueller PRA (2011)

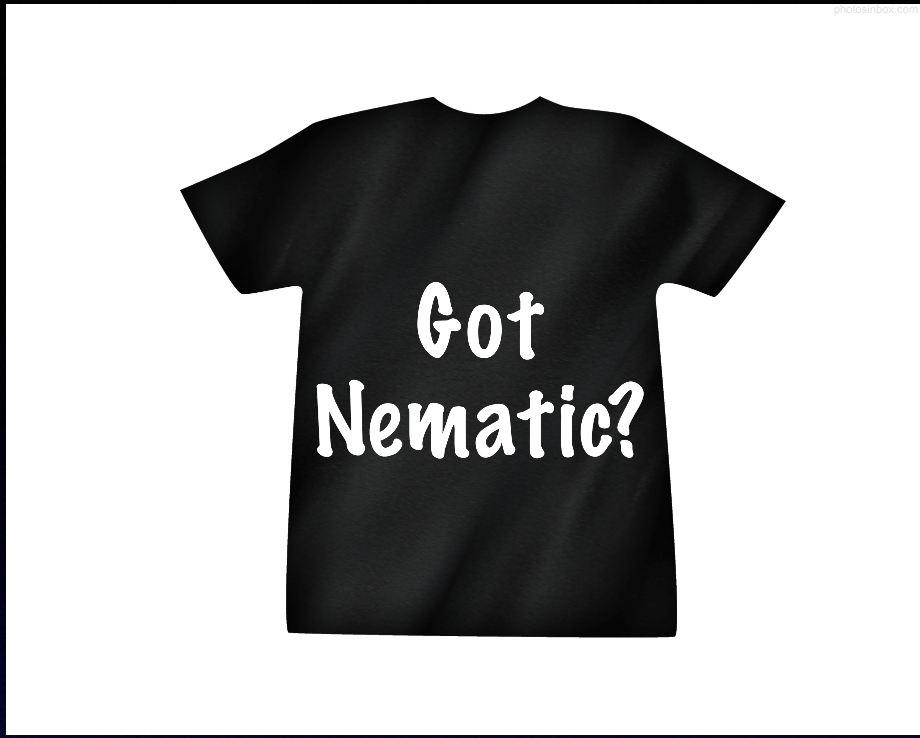


[redacted]

Critical temperature
Linearize RPA

$$t_{mag} = \frac{T_c^{mag} - T_{BEC}}{T_{BEC}} = 4.84\left(\frac{1}{3} - \frac{c_2}{c_0}\right) n^{1/3} a_0$$

Collapse: Isothermal compressibility $\partial n/\partial \mu < 0$



Is there a normal Nematic phase?

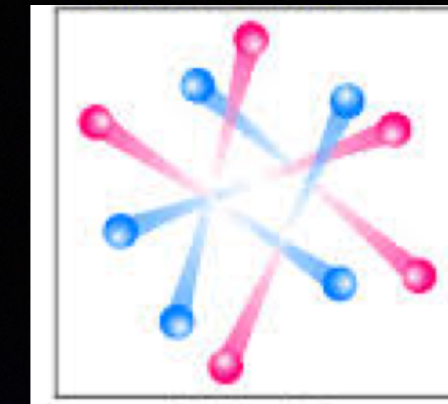
Not in spin-1 but in higher spin (spin-3 Cr for example)

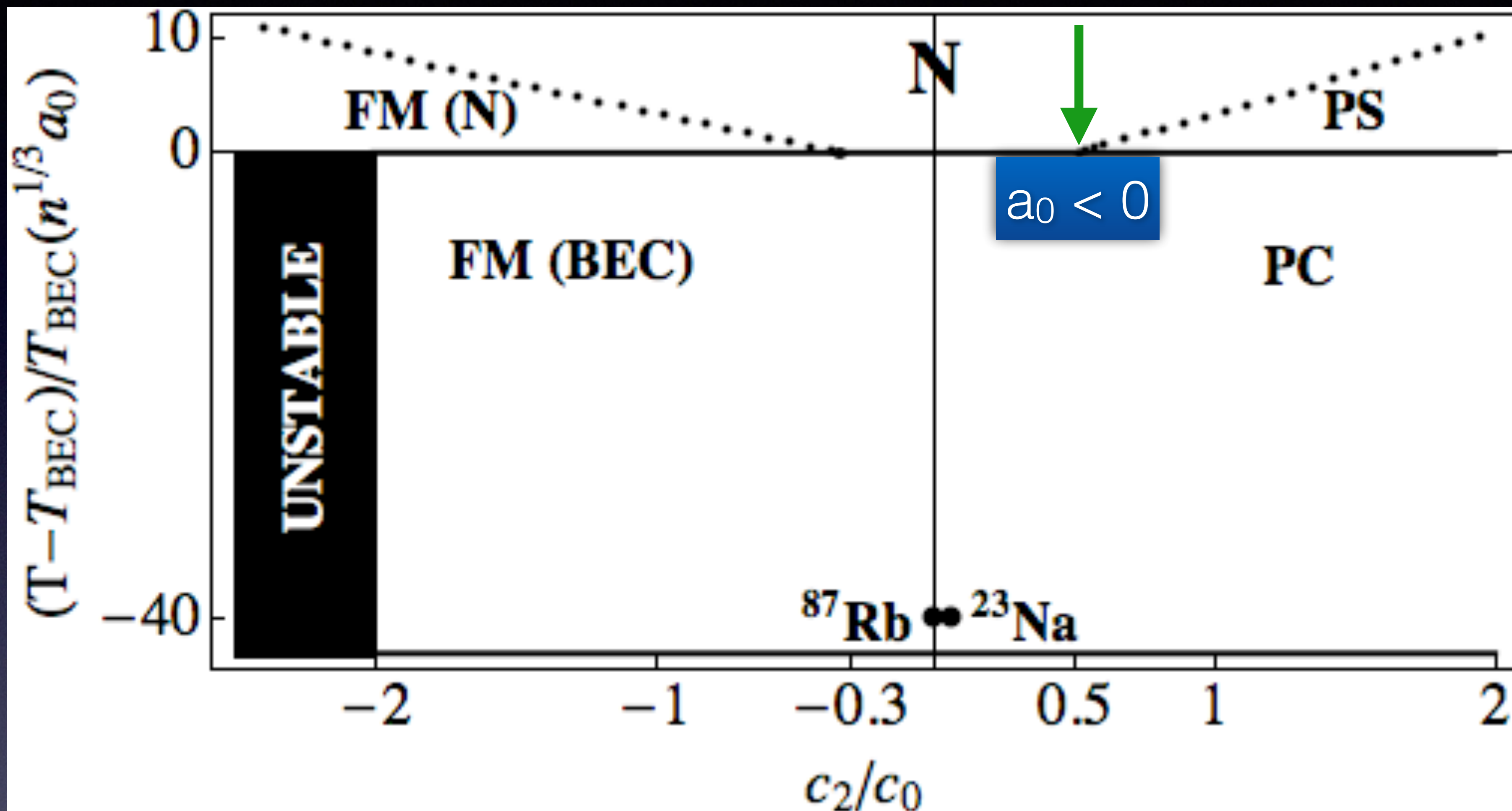
need terms of the form: $\langle \hat{S}_\mu \hat{S}_\nu \rangle^2$

$$\mathcal{H}_{\text{int}} = \frac{1}{2} \int d\mathbf{r} \psi_\alpha^\dagger \psi_\beta^\dagger \psi_\gamma \psi_\delta (c_0 \delta_{\alpha\delta} \delta_{\beta\gamma} + c_2 \mathbf{S}_{\alpha\delta} \cdot \mathbf{S}_{\beta\gamma})$$

Nematicity tied to single-particle order in Spin-1 continuum

But Pair Order!!





$(c_0 \propto a_0 + 2\, a_2)$

$(c_2 \propto a_2 - a_0)$

Stable paired state occurs when spin-0 scattering length attractive!

Nozieres, Saint James J. Phys (1982)

Critical temperature
Linearize RPA

$$t_{\rm pair} = \frac{T_c^{\rm pair} - T_{\rm BEC}}{T_{\rm BEC}} = 3.22\left(\frac{c_2}{c_0} - \frac{1}{2}\right) n^{1/3} a_0$$

PS: Boson pairing

$$\sum_k \langle a^\dagger_{\mu\mathbf{k}} a^\dagger_{\nu-\mathbf{k}}\rangle \neq 0$$

No single-particle order

Law Bigelow Pu Singlet

Neither state has single particle order

LBP Order parameter: $\langle a_{k=0\alpha} a_{k=0\beta} \rangle$

Law, Bigelow, Pu, PRL (1998)

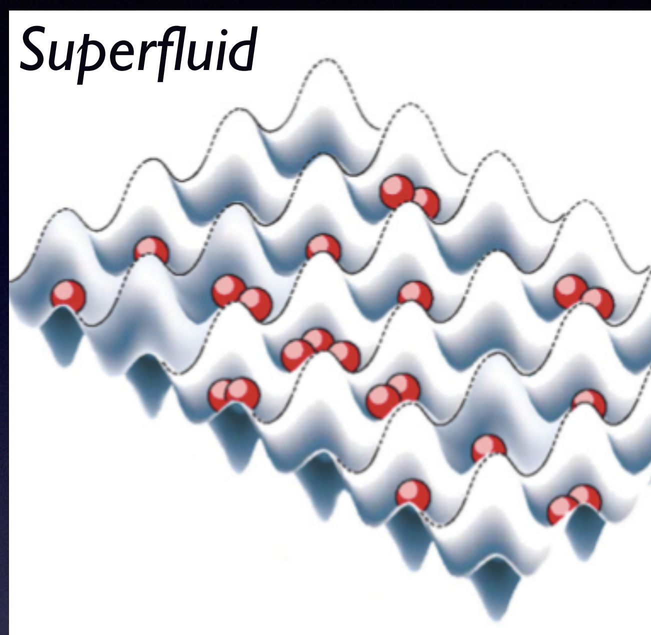
PS: Boson pairing $\sum_k \langle a^\dagger_{\mu\mathbf{k}} a^\dagger_{\nu-\mathbf{k}} \rangle \neq 0$

Nozieres, Saint James J. Phys (1982)

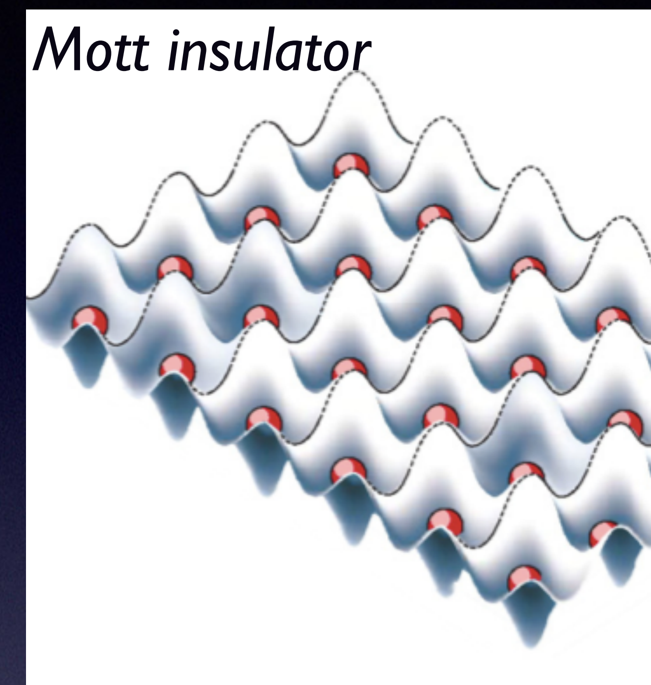
Experimentally LBP is destroyed over fragmented condensate

Boson pairing state still never observed!!

Large Spin stabilizes against collapse as there are more collision channels



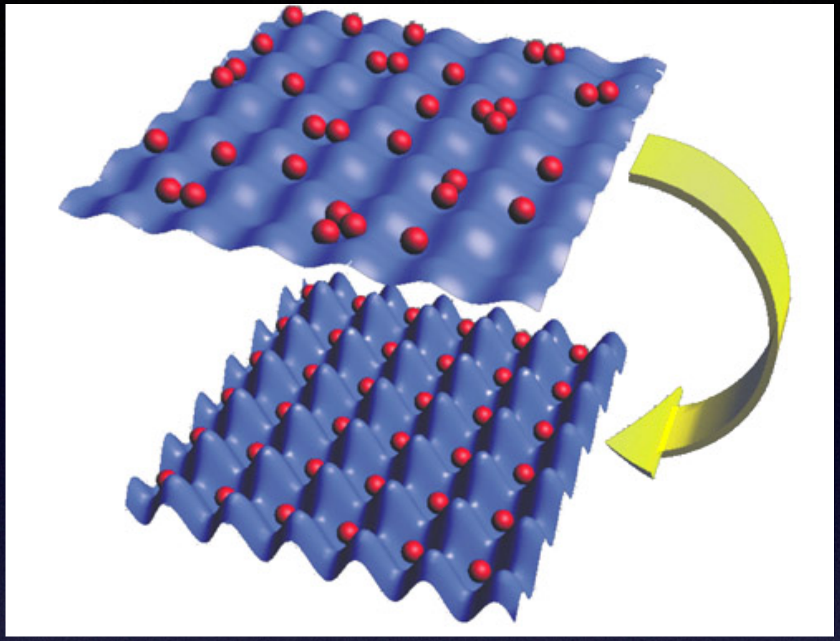
the lattice depth



Can strong interactions produce Mott phases with magnetic/ nematic order?

Spin-1 Optical Lattice

$$H - \mu\hat{N} = -t\sum_{\langle i,j\rangle,\alpha}\left(a^\dagger_{i\alpha}a_{j\alpha} + \mathrm{H.c}\right) + \frac{U_0}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) + \frac{U_2}{2}\sum_i\left(\mathbf{S}^2 - 2\hat{n}_i\right) - \mu\sum_i \hat{n}_i$$



Gutzwiller MFT

$$a^\dagger_{i\alpha}a_{j\alpha} \to \langle a^\dagger_{i\alpha}\rangle a_{j\alpha} + a^\dagger_{i\alpha}\langle a_{j\alpha}\rangle$$

$$|\Psi_{GS}\rangle = \otimes_{i=1}^{N_{\mathrm{site}}}|\phi_i\rangle$$

$$|\phi_i\rangle = \sum_{m_{-1},m_0,m_1} A_{m_{-1}m_0m_1}|m_{-1},m_0,m_1\rangle$$

Numerically minimize E

Natu, Pixley, Das Sarma arXiv:1502…

How does Order vanish?

Nature of transitions out of superfluid not obvious!!

Insights from Strong Coupling

Perturbation theory in t/U_0

$$H_{JK} = \sum_{\langle i,j\rangle} \left(-J\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{2} J_2 (\mathbf{S}_i \cdot \mathbf{S}_j)^2 \right)$$

$$\begin{aligned}\frac{J_1}{t^2} &= \frac{2(15+20n+8n^2)}{15(U_0+U_2)} - \frac{16(5+2n)n}{75(U_0+4U_2)},\\ \frac{J_2}{t^2} &= \frac{2(15+20n+8n^2)}{45(U_0+U_2)} + \frac{4(1+n)(3+2n)}{9(U_0-2U_2)} + \frac{4n(5+2n)}{225(U_0+4U_2)}\end{aligned}$$

Bi-quadratic term arises purely from spin-1 algebra

Imambekov, Demler, Lukin PRA (2004)

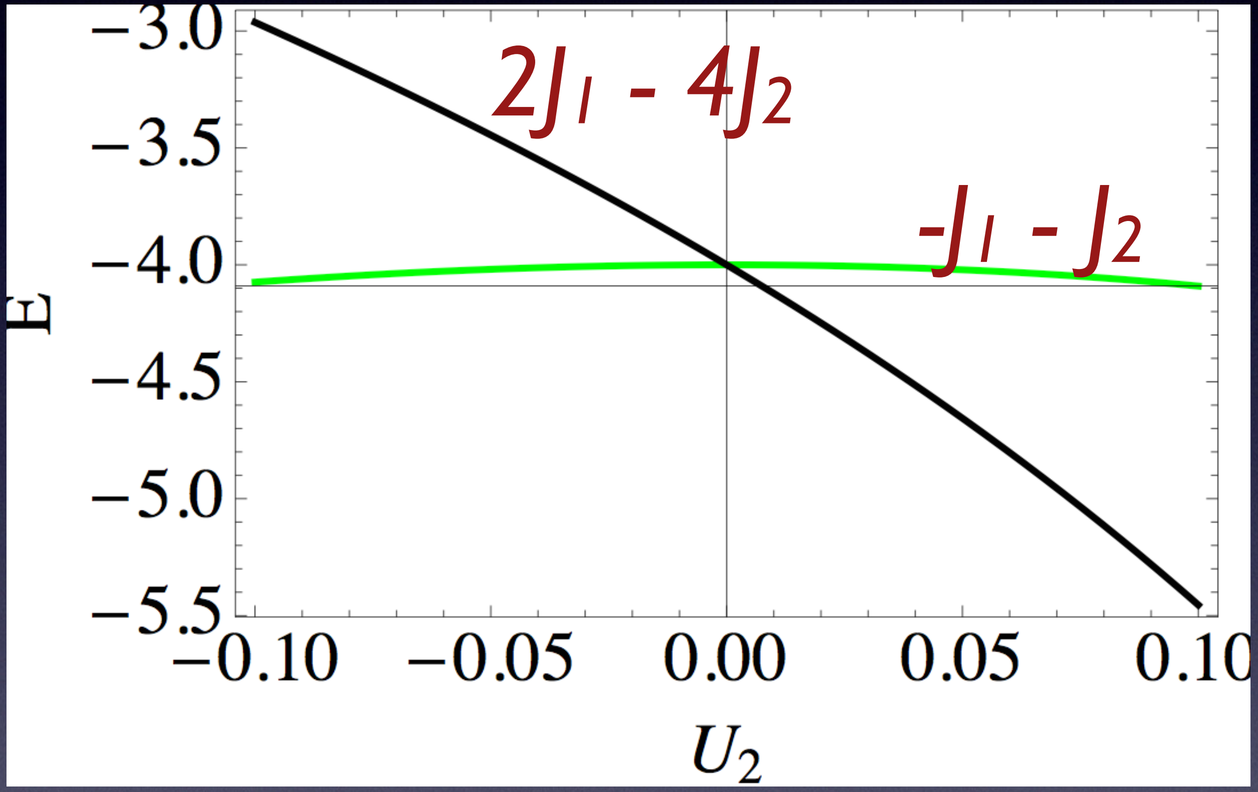
Insights from Strong Coupling

Perturbation theory in t/U_0

$$H_{JK} = \sum_{\langle i,j \rangle} \left(-J \mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{2} J_2 (\mathbf{S}_i \cdot \mathbf{S}_j)^2 \right)$$

Solve the 2-site problem exactly

S_{tot}	$\vec{S}_1 \vec{S}_2$	$(\vec{S}_1 \vec{S}_2)^2$	Energy
0	-2	4	$2J_1 - 4J_2$
1	-1	1	$J_1 - J_2$
2	1	1	$-J_1 - J_2$

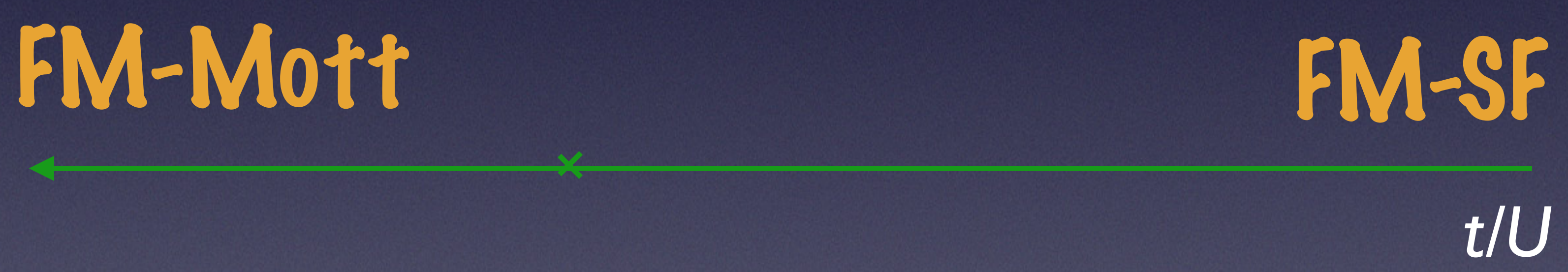


$U_2 < 0$: $J_1 > J_2$ FERROMAGNET

$U_2 > 0$: $J_1 < J_2$ SPIN -0 state

Imambekov, Demler, Lukin PRA (2004)

Favored Scenario



Ferromagnetic Interactions

Not featureless

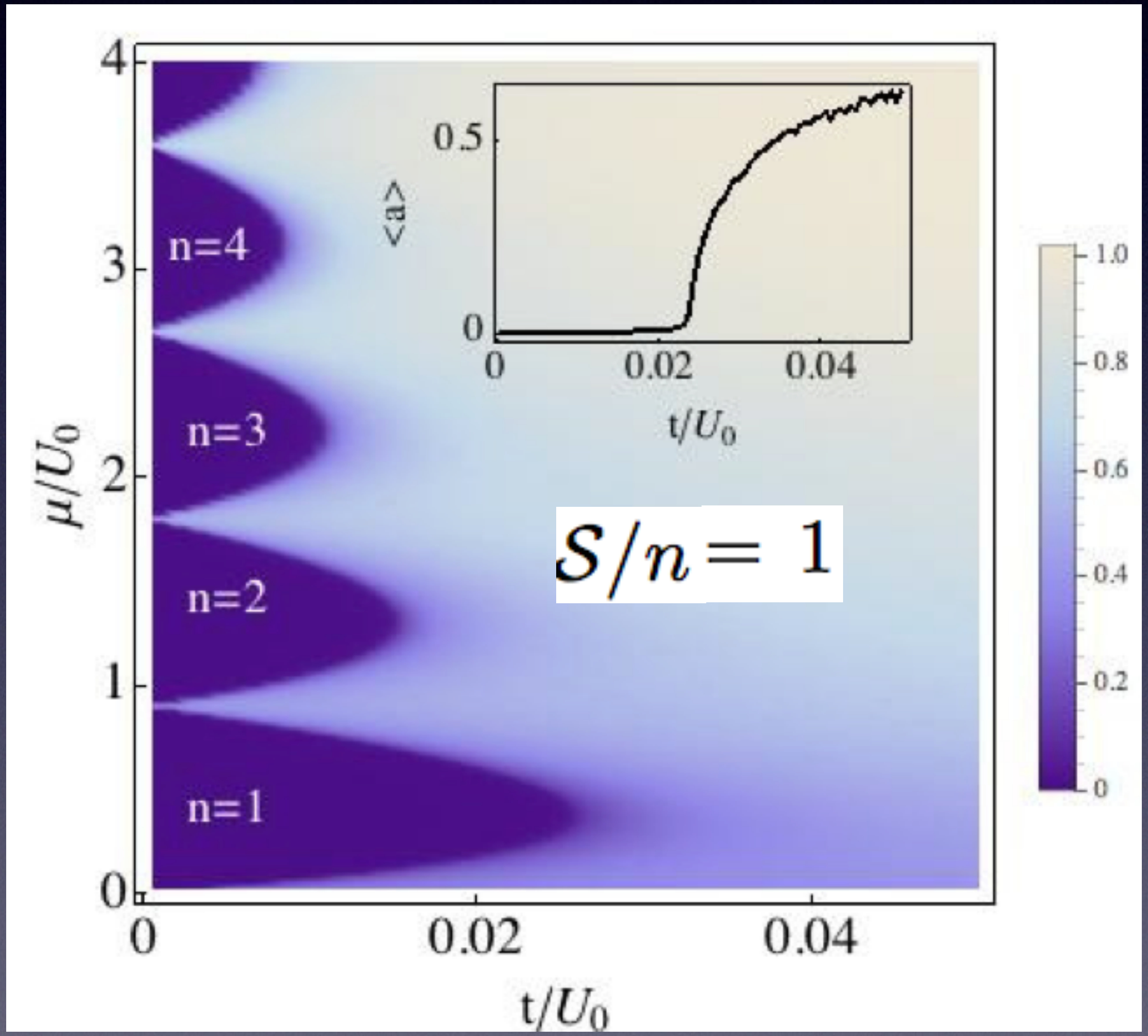
Long Range spin Order

$$H_{JK} = \sum_{\langle i,j \rangle} \left(-J\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{2} K (\mathbf{S}_i \cdot \mathbf{S}_j)^2 \right)$$

Quadratically dispersing spin modes

$$\begin{aligned} \omega_{\mathbf{k}} &= z(J+K) - (J+K)\gamma_{\mathbf{k}} \\ &\underset{\mathbf{k}\to 0}{\approx} (J+K)|\mathbf{k}|^2. \end{aligned}$$

Ferromagnetic Mott insulator

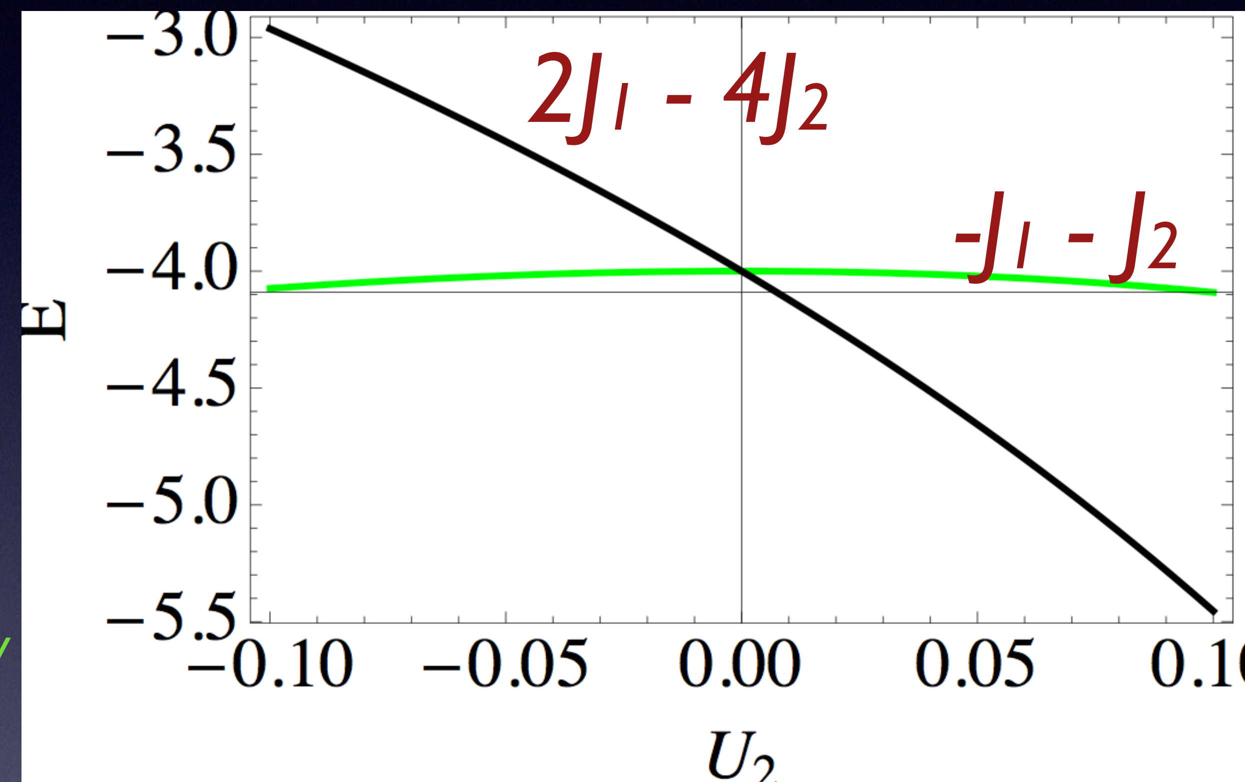


Natu, Pixley, Das Sarma arXiv:1502…

Anti-ferromagnetic Side

Perturbation theory in t/U_0

$$H_{JK} = \sum_{\langle i,j \rangle} \left(-J \mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{2} J_2 (\mathbf{S}_i \cdot \mathbf{S}_j)^2 \right)$$



Solve the 2-site problem exactly

S_{tot}	$\vec{S}_1\vec{S}_2$	$(\vec{S}_1\vec{S}_2)^2$	Energy
0	-2	4	$2J_1 - 4J_2$
1	-1	1	$J_1 - J_2$
2	1	1	$-J_1 - J_2$

U FERROMAGNET

$U_2 > 0$: $J_1 < J_2$ **SPIN -0 state**

Imambekov, Demler, Lukin PRA (2004)

Anti-Ferromagnetic Side

Local Nematic Order

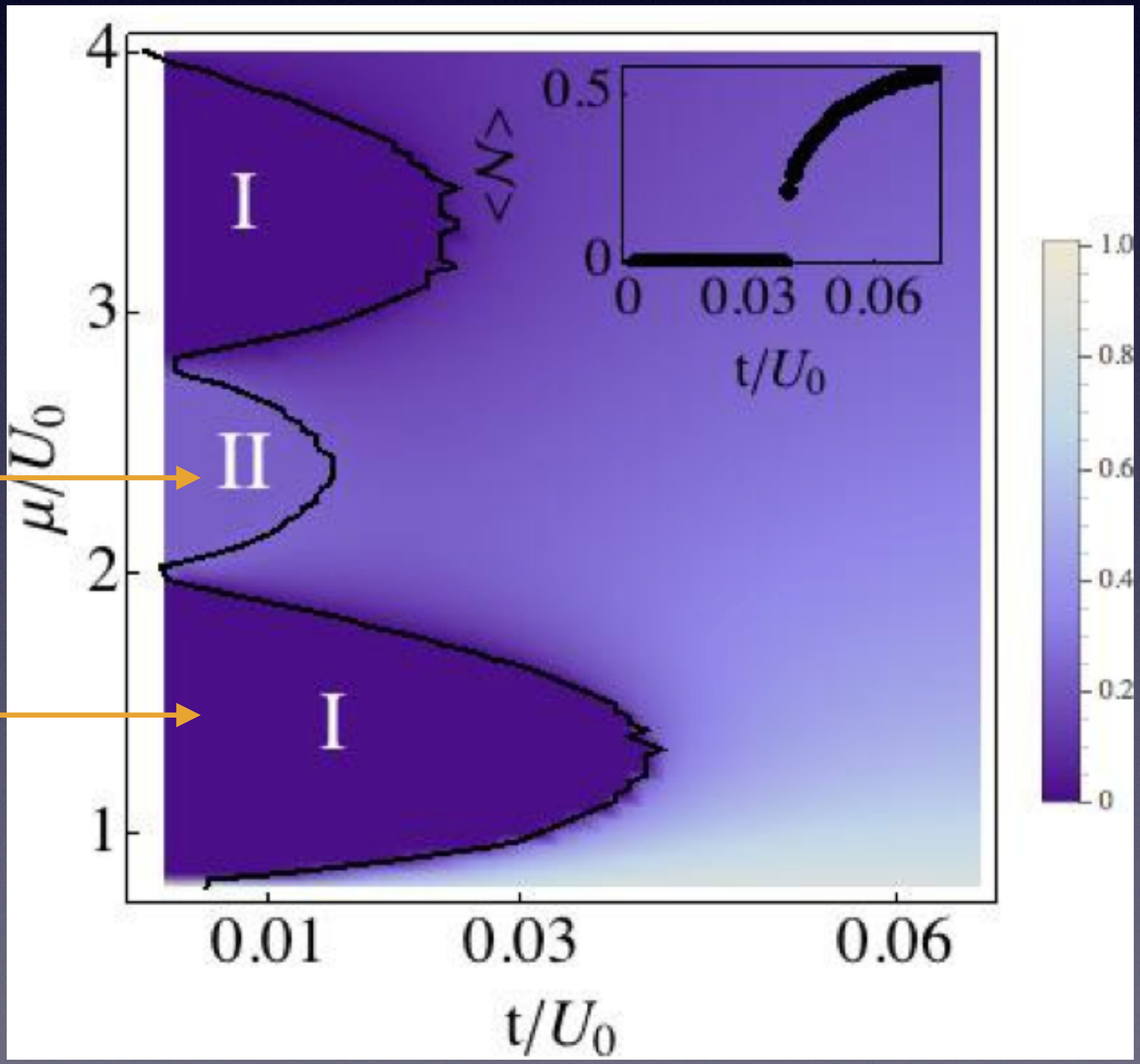
$$\mathcal{N}_{\alpha\beta} = \frac{1}{2}\langle S_\alpha S_\beta + S_\beta S_\alpha \rangle$$

First Order Mott-Superfluid transition

Interactions Stabilize Mott state with Residual nematic Order!

Odd Mott Lobes are NEMATIC!

Even Mott Lobes have NO nematicity



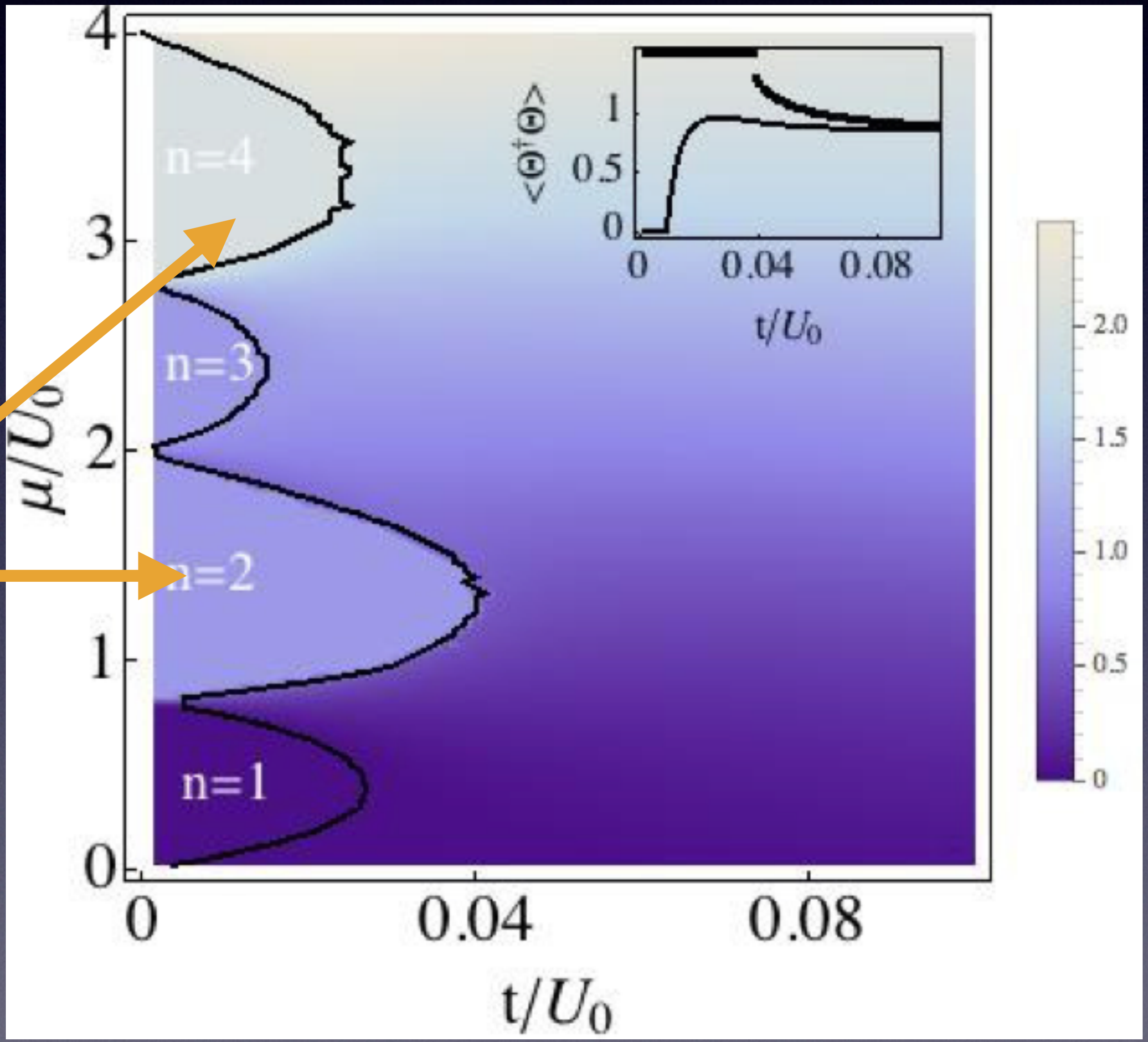
Anti-Ferromagnetic Side

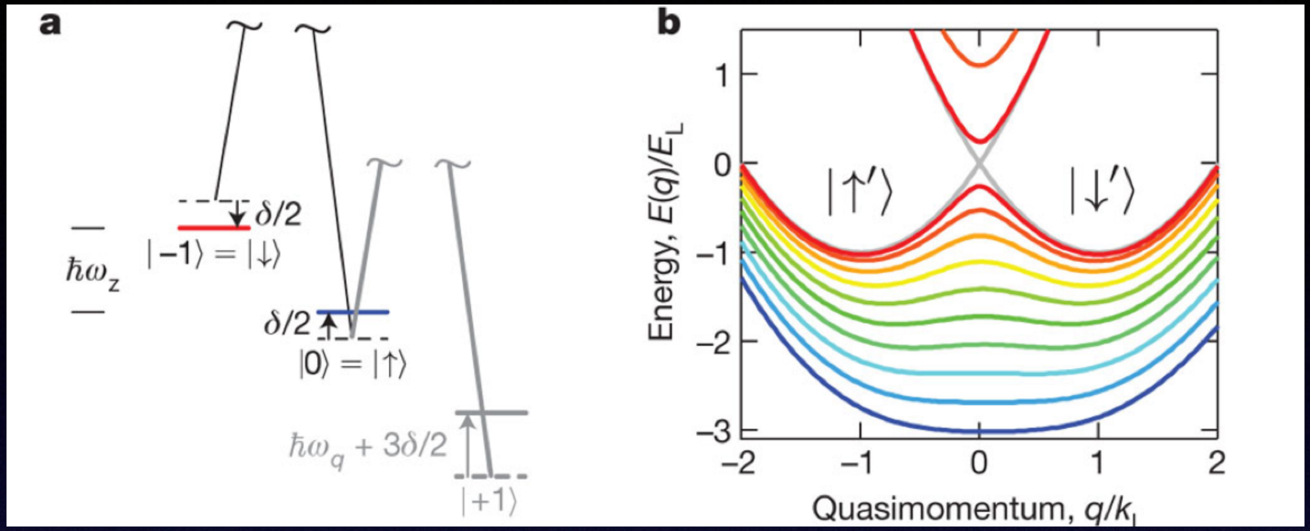
Local Singlet Order

$$\Theta_i^\dagger = 2a_{1i}^\dagger a_{-1i}^\dagger - a_{0i}^\dagger a_{0i}^\dagger$$

$$\langle \Theta_i^\dagger \Theta_i \rangle$$

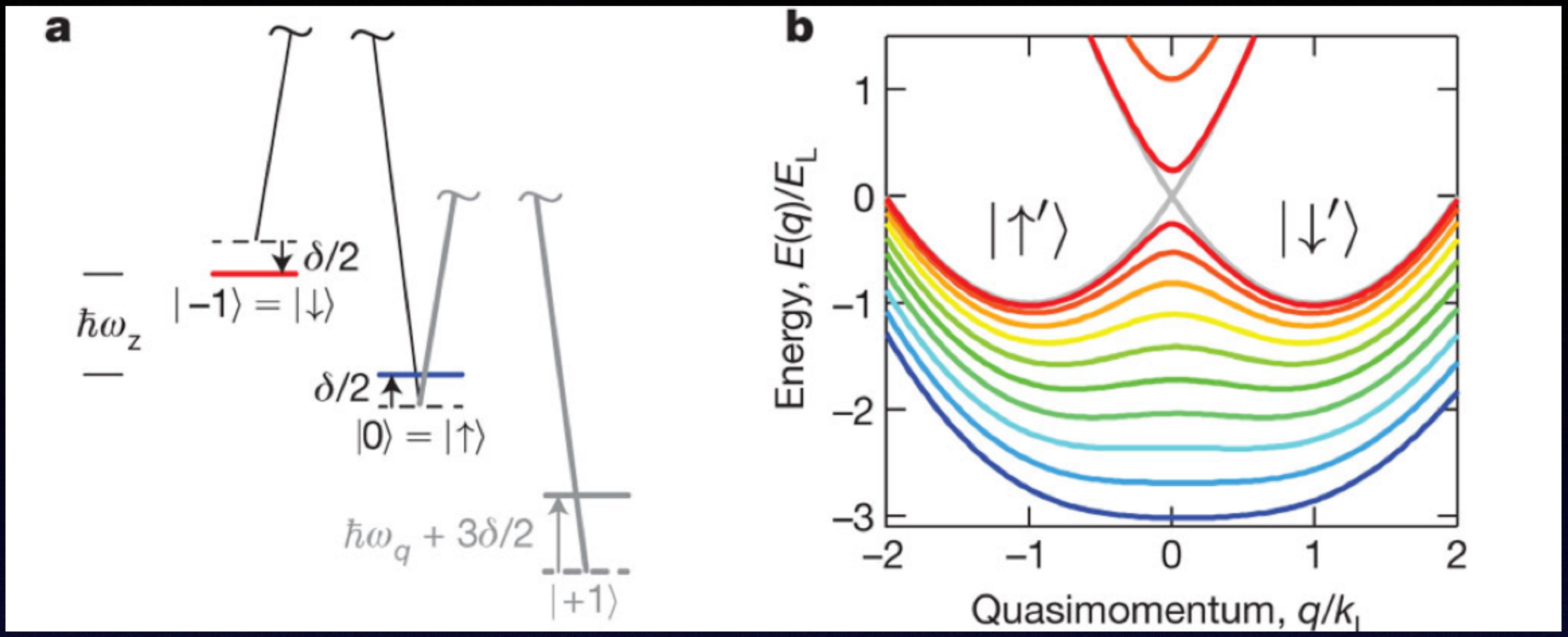
Even Mott Lobes have LOCAL Singlets!





Can Ferromagnetism/Nematicity coexist in these systems?

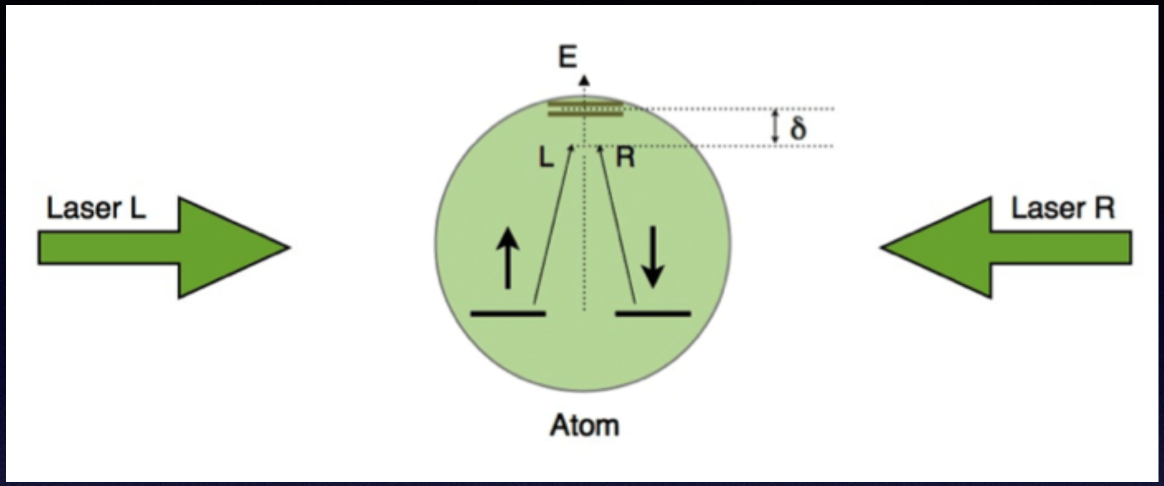
Other forms of Order?



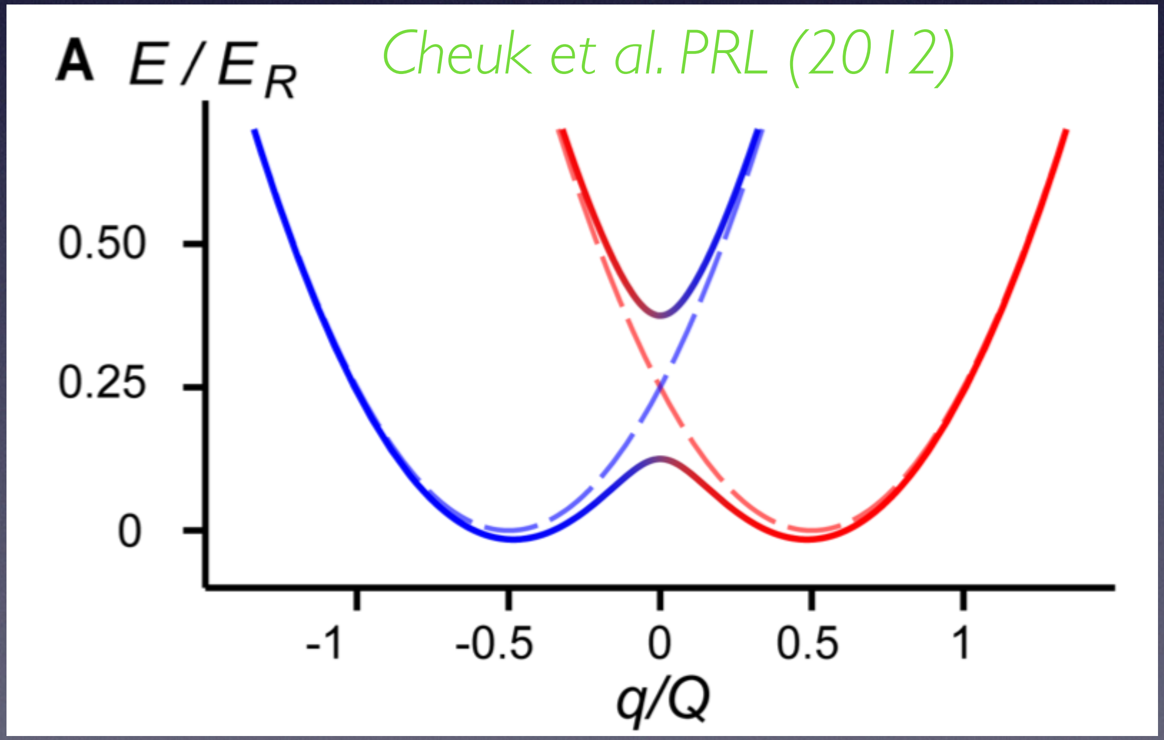
✓ *Can Ferromagnetism/Nematicity coexist in these systems?*

✓ *Other forms of Order?*

Spin-Orbit Coupling

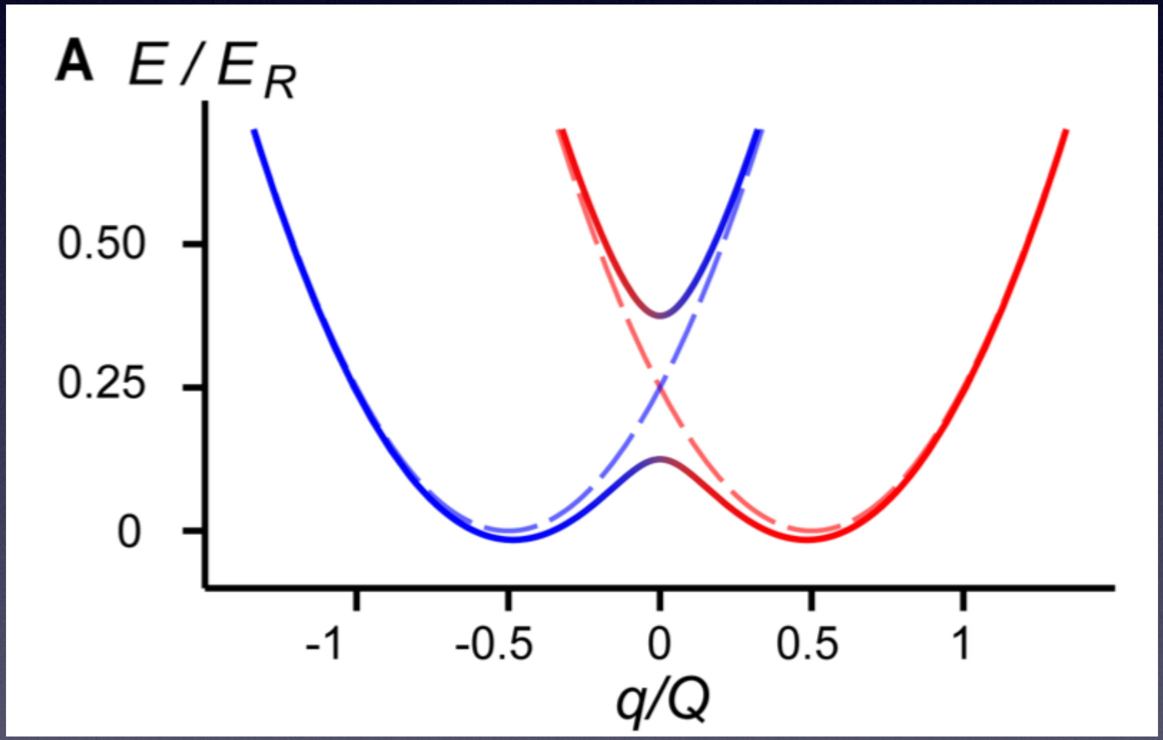


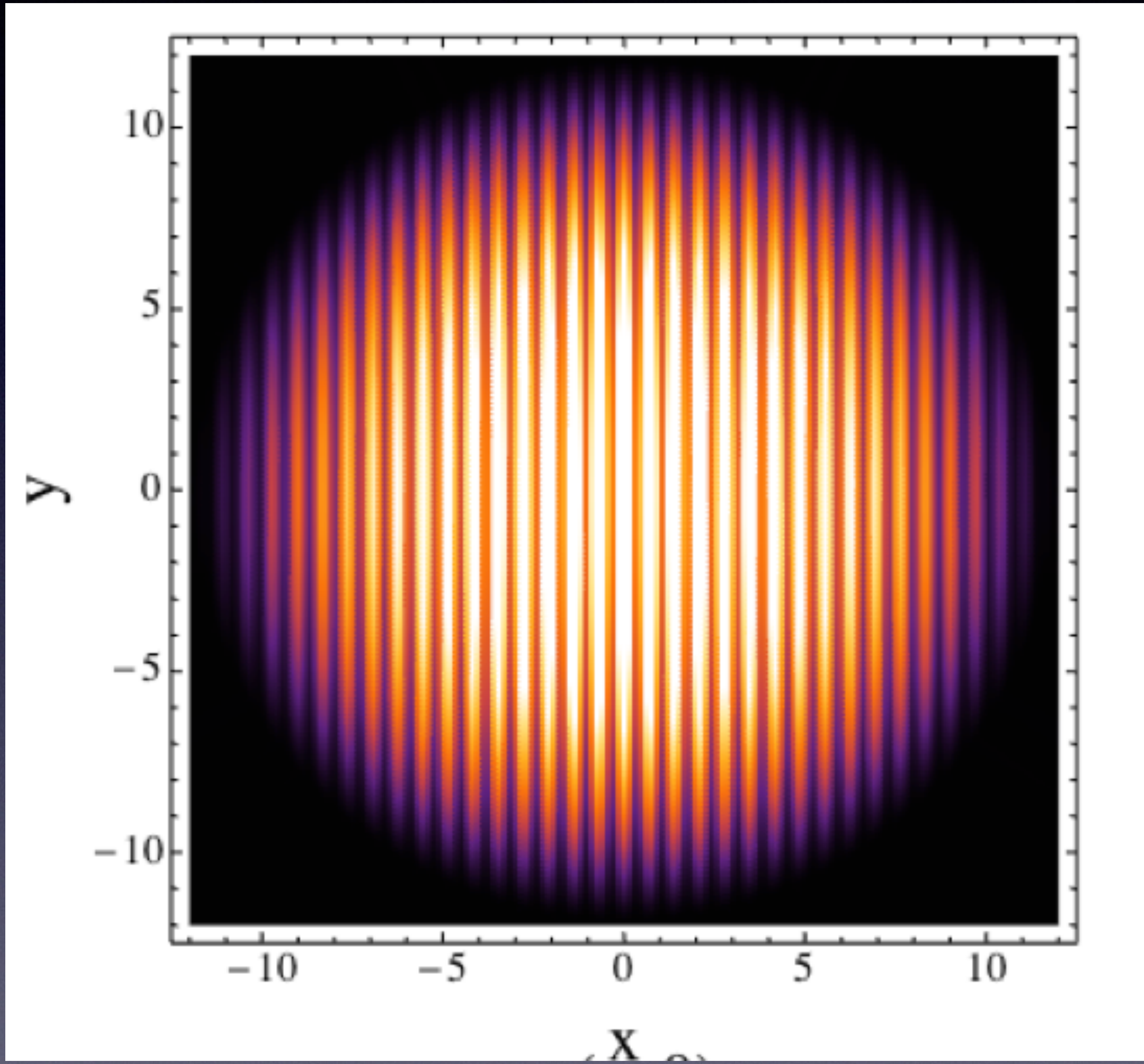
Mueller, Physics Viewpoint (2012)



SOC makes Stripes!

Single-particle term favors stripe formation!



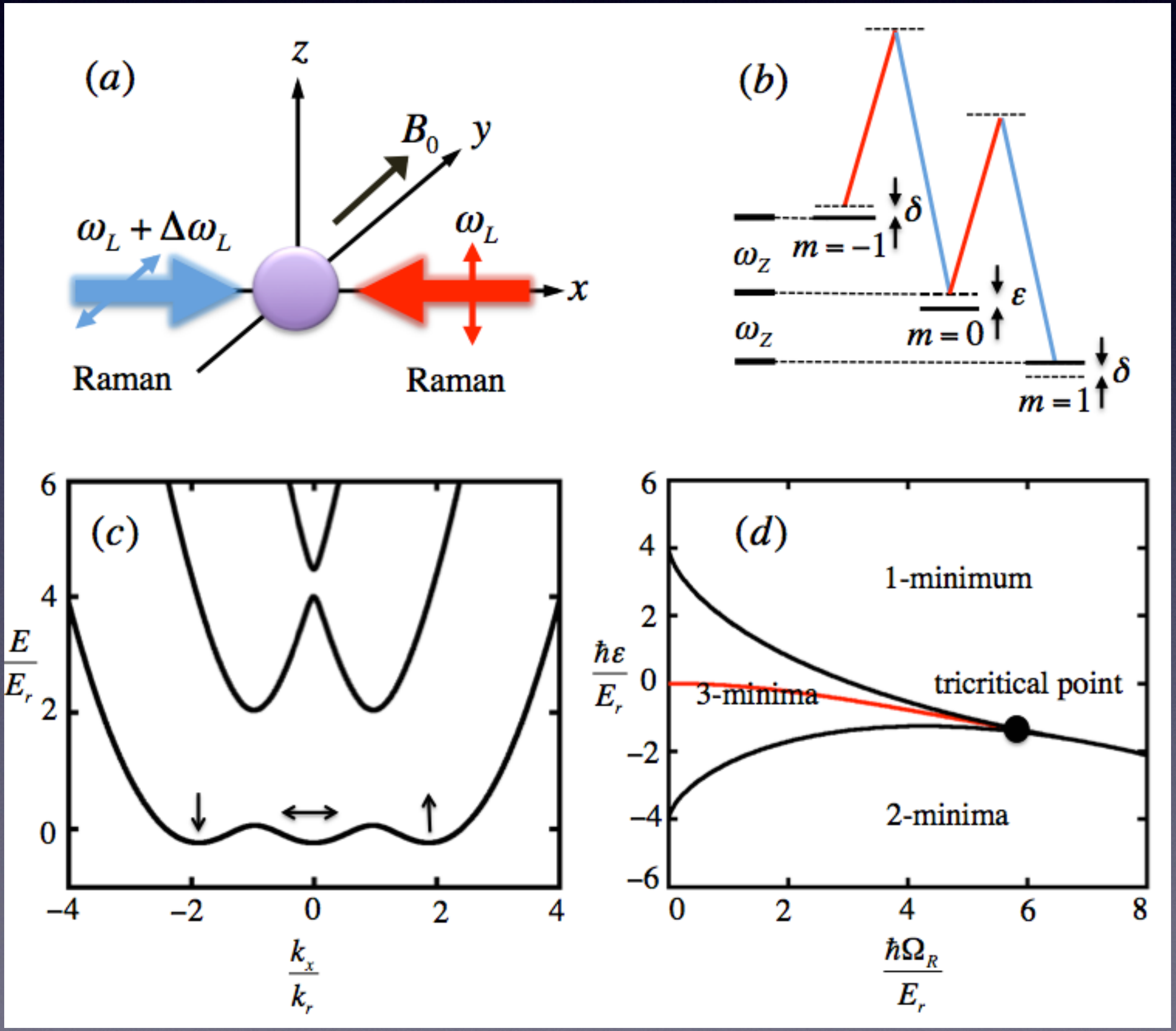


Ho and Zhang PRL (2011)

Single-particle physics

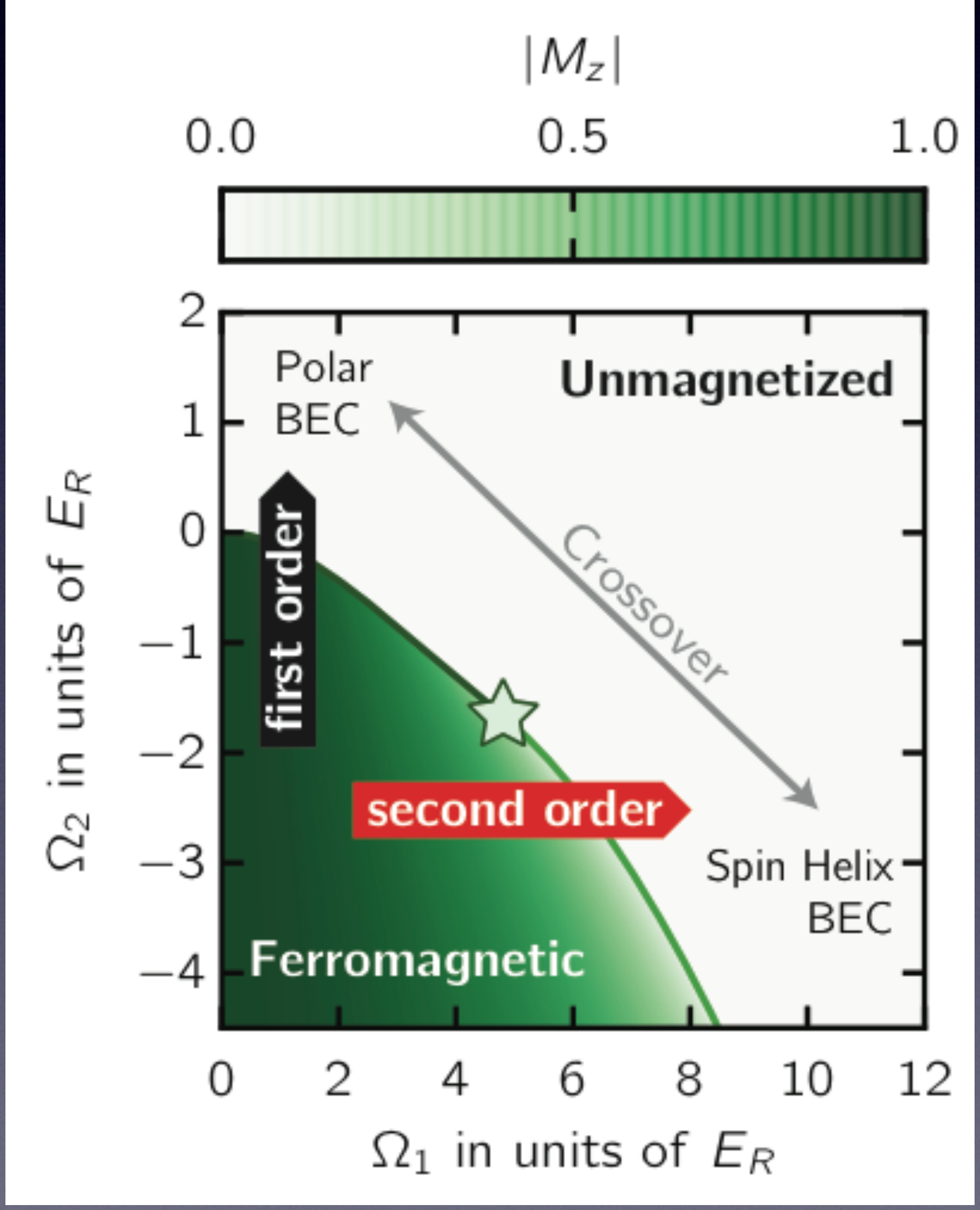
$$\mathcal{H}_{soc} = \frac{\hbar^2(k_x - k_0 S_z)^2}{2m} + \frac{\hbar^2 k_\perp^2}{2m} + \frac{\Omega}{2}S_x + \frac{\delta}{2}S_z + \frac{q}{2}S_z^2$$

Lan, Ohberg PRA (2014)
Natu, Li, Cole PRA (to appear)



Experiments

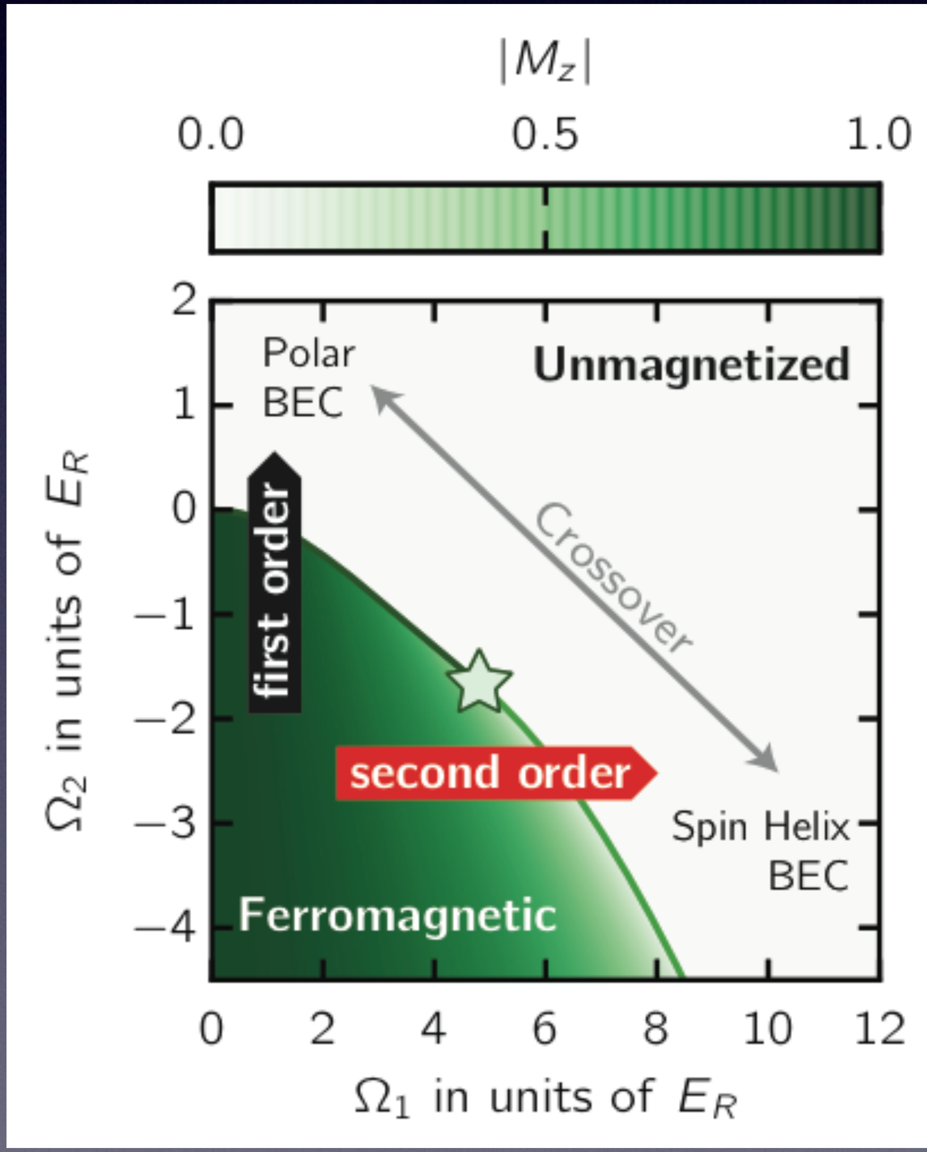
Model SINGLE-PARTICLE transitions from FM to spin Helix



Campbell et al. arXiv:1501.05984

Experiments

Model transition from FM to spin Helix



Campbell et al. arXiv:1501.05984

What are the interaction driven transitions??

Variational Ansatz

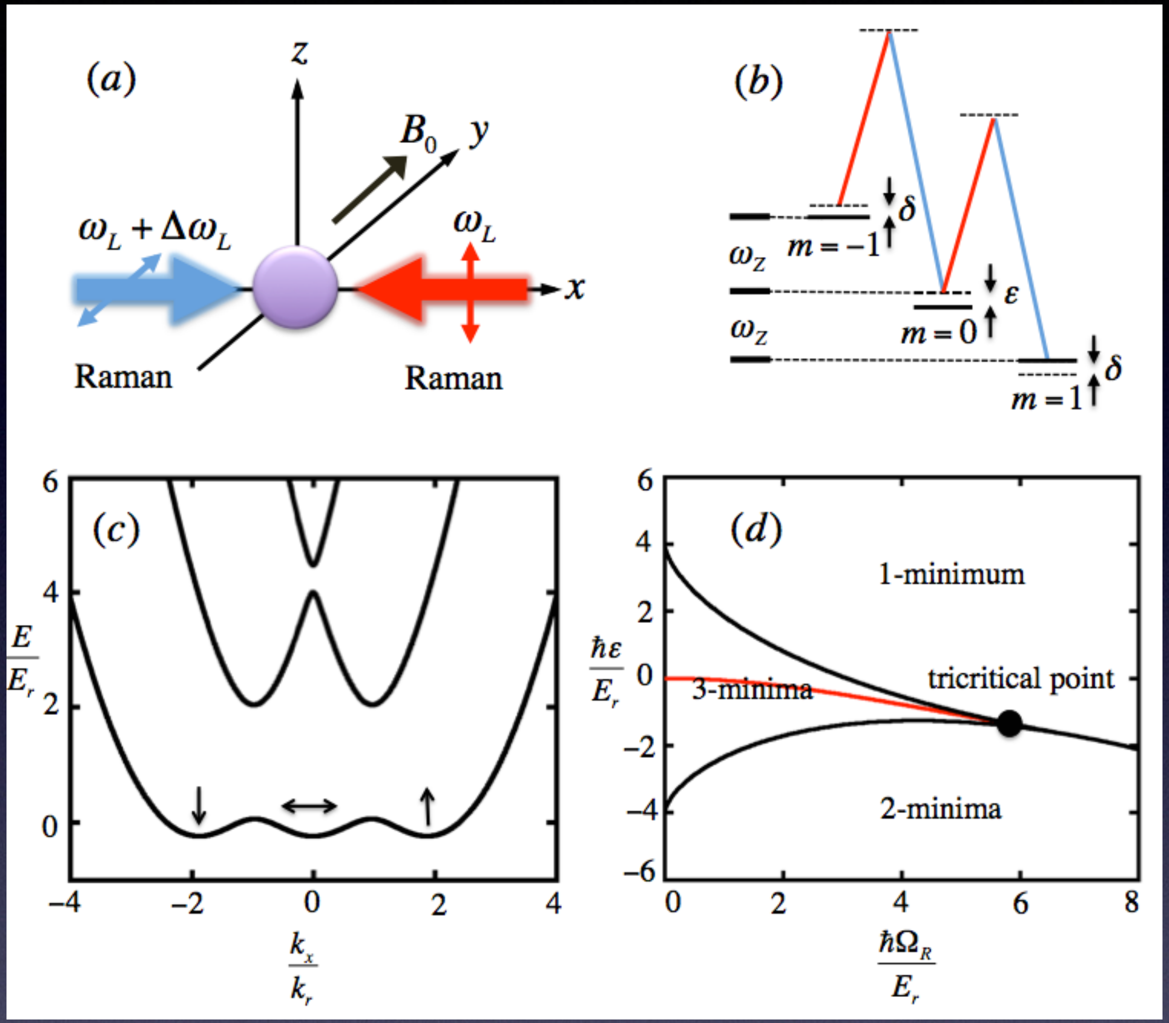
complex number

$$\psi = \sqrt{\frac{N}{V}}(\chi_+ e^{ik_1 x}\phi_+ + \chi_0\phi_0 + \chi_- e^{-ik_1 x}\phi_-)$$

3-component spinor

Minimize

$$\mathcal{H}_{int} = \frac{1}{2}\int d^3\mathbf{r}\ \psi^\dagger_\alpha \psi^\dagger_\beta \psi_\gamma \psi_\delta (c_0 \delta_{\alpha\delta}\delta_{\beta\gamma} + c_2 \mathbf{S}_{\alpha\delta}\cdot\mathbf{S}_{\beta\gamma})$$



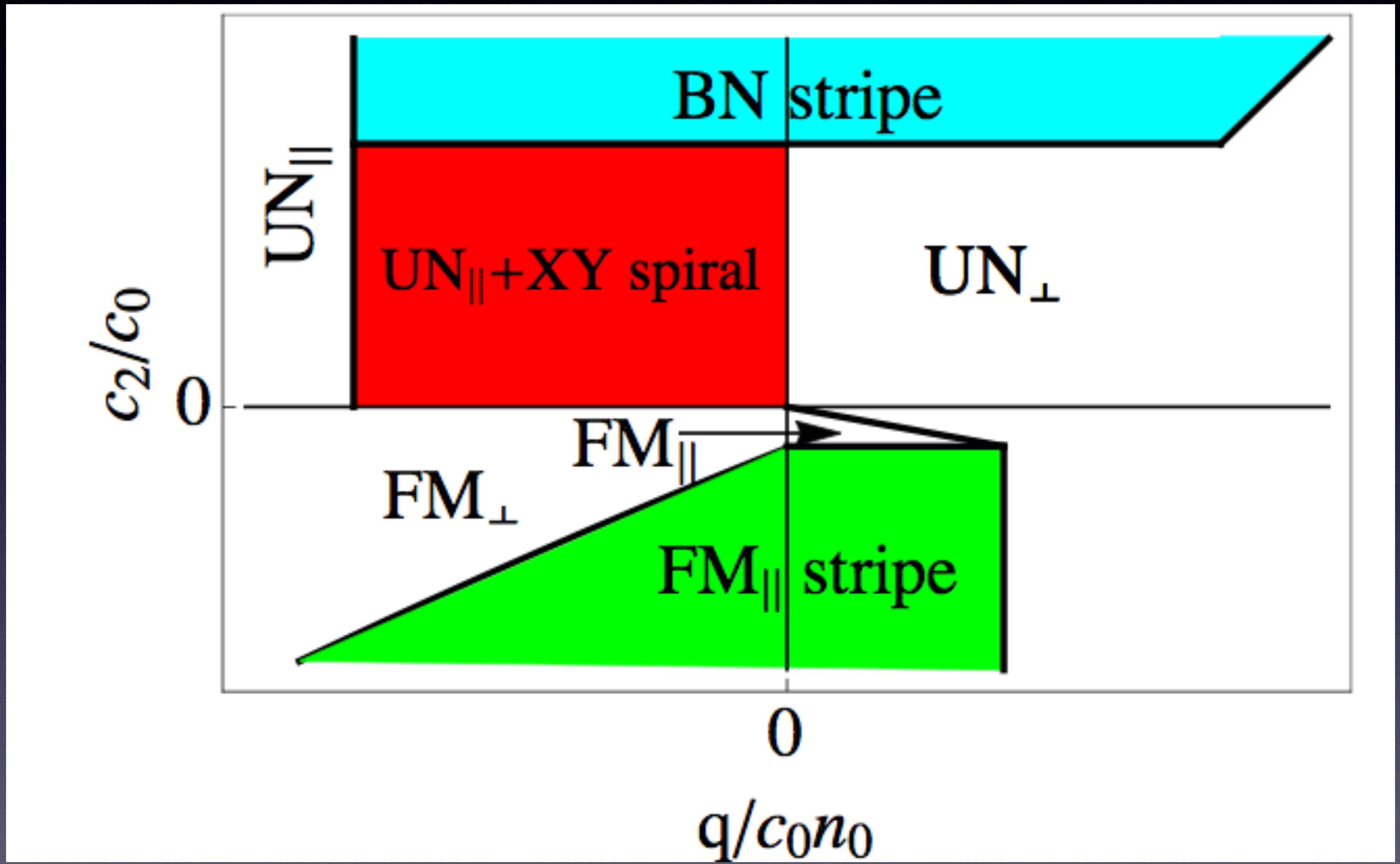
Li, Pitaevskii, Stringari PRL (2012)

Mo' Spin… Mo' Orders

TABLE I: Orders in spin-orbit coupled spin-1 gas.

Order	**Symbol**	**Order Parameter**
ferromagnetic	$FM_{\parallel/\perp}$	$\langle S^i \rangle \neq 0$
Uniaxial nematic	$UN_{\parallel/\perp}$	$\lambda_1 \neq 0, \lambda_2 = \lambda_3 = 0$
Biaxial nematic	BN	$\lambda_1 < \lambda_2 < \lambda_3$
Translation	stripe, XY spiral	$\langle S^i(\mathbf{r}) \rangle \sim \cos(k_1 r)$
		$n(r) \sim \cos(k_2 r)$

Phase Diagram

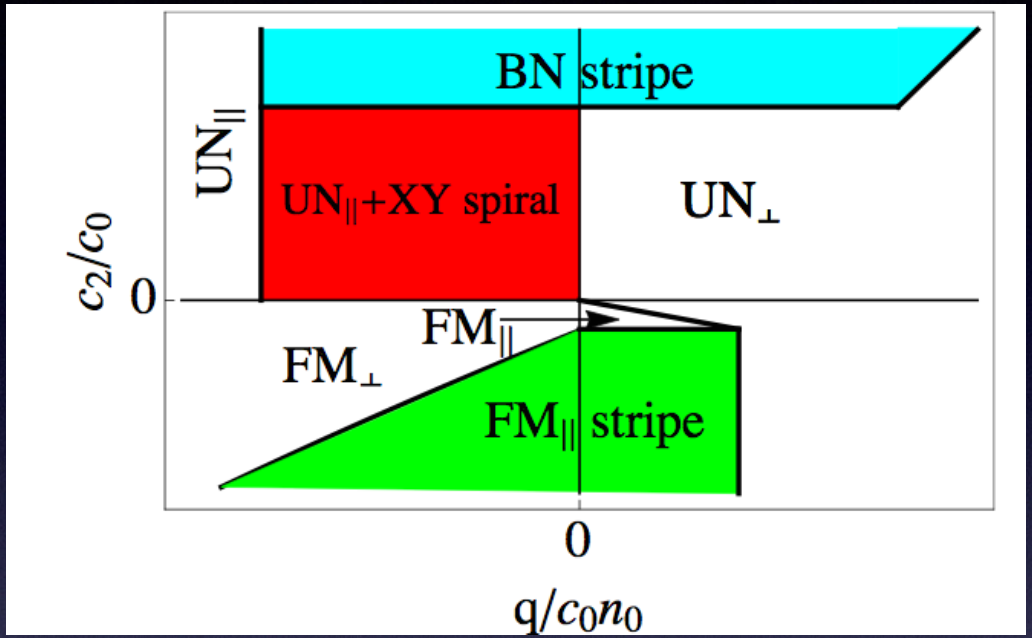


Stripes Phase competition between kinetic and interactions!

Natu, Li, Cole PRA (to appear)

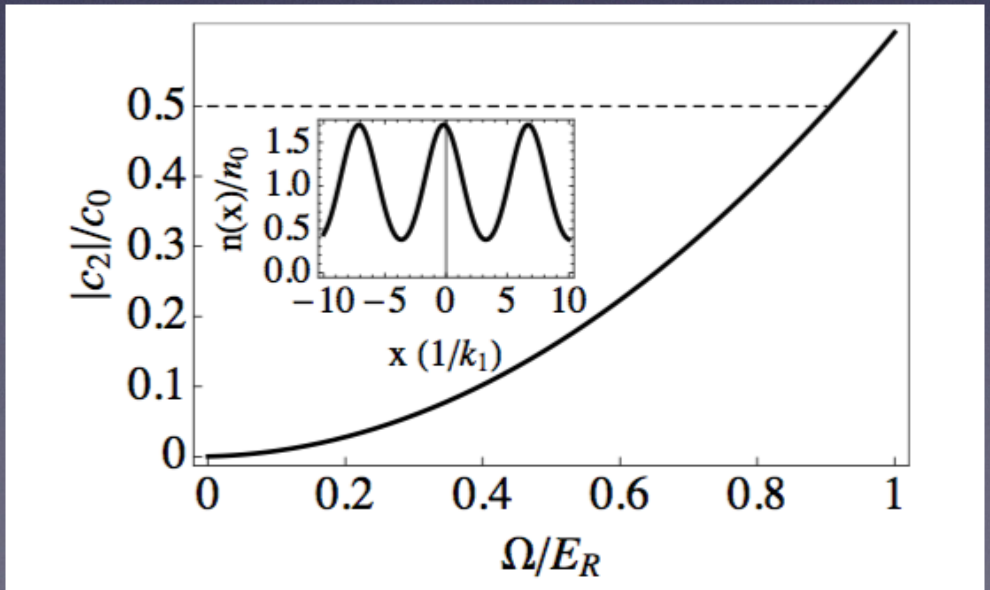
Stripe Phases

Raman coupling favors ferromagnet along x



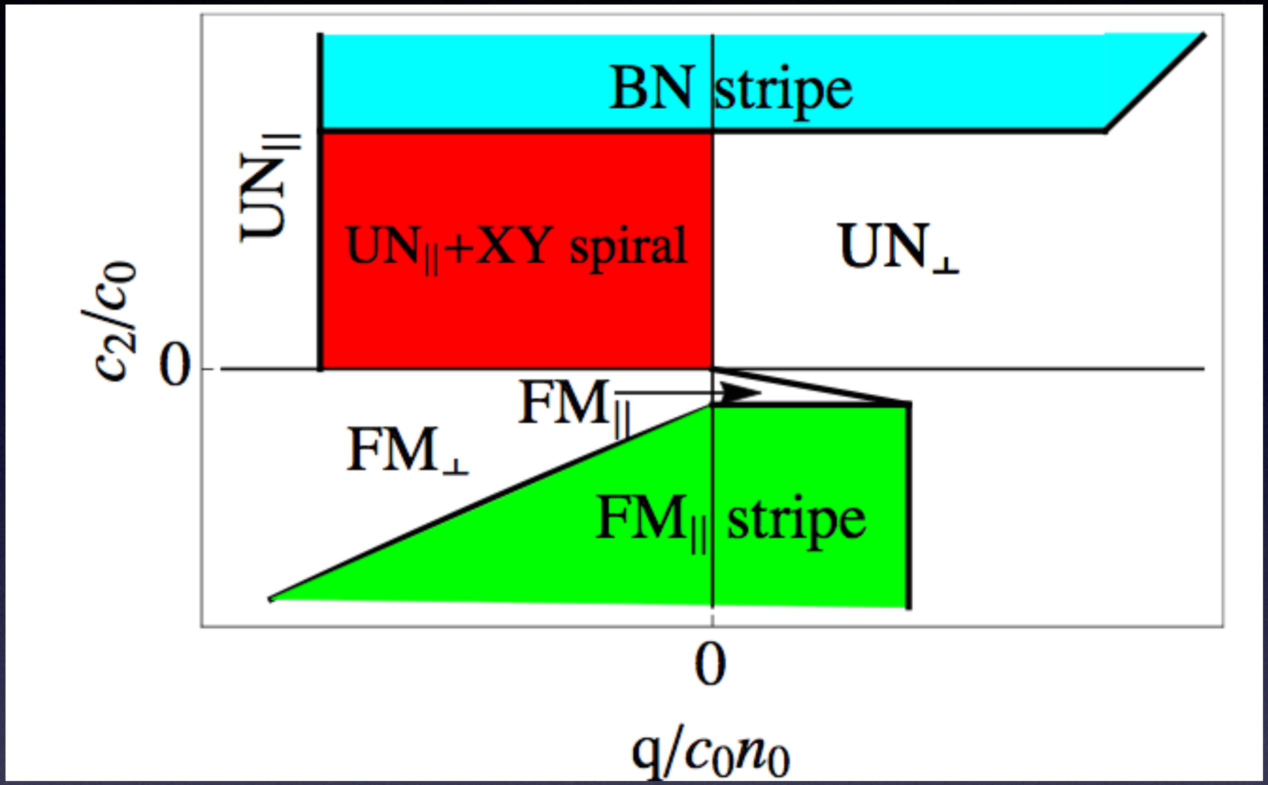
Ferromagnet along x costs density energy!

So Need large negative c2 to compensate!



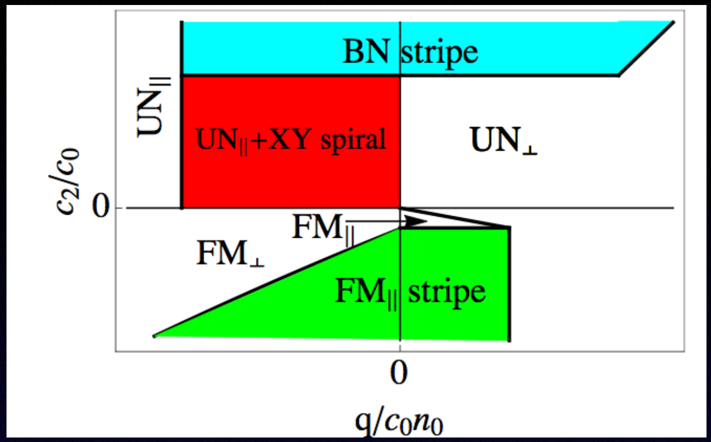
Coexistence of Ferromagnetism/Nematicity

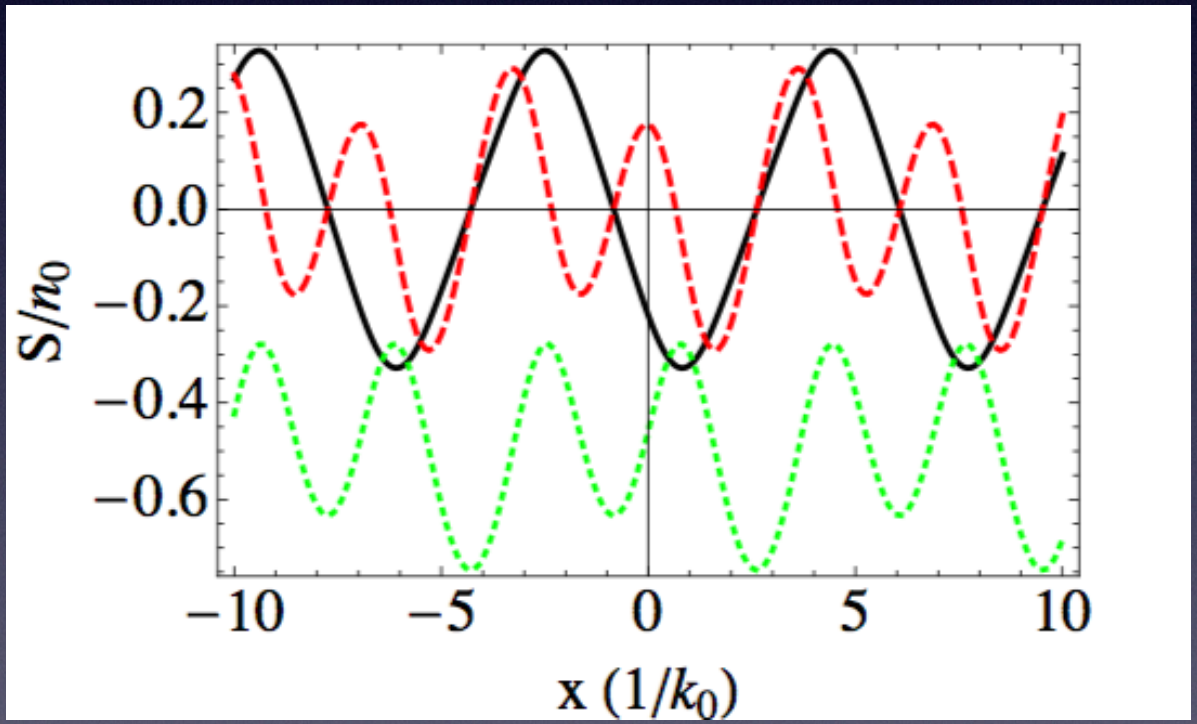
In presence of Raman field, Nematicity and Spiral spin order coexist with density wave order!

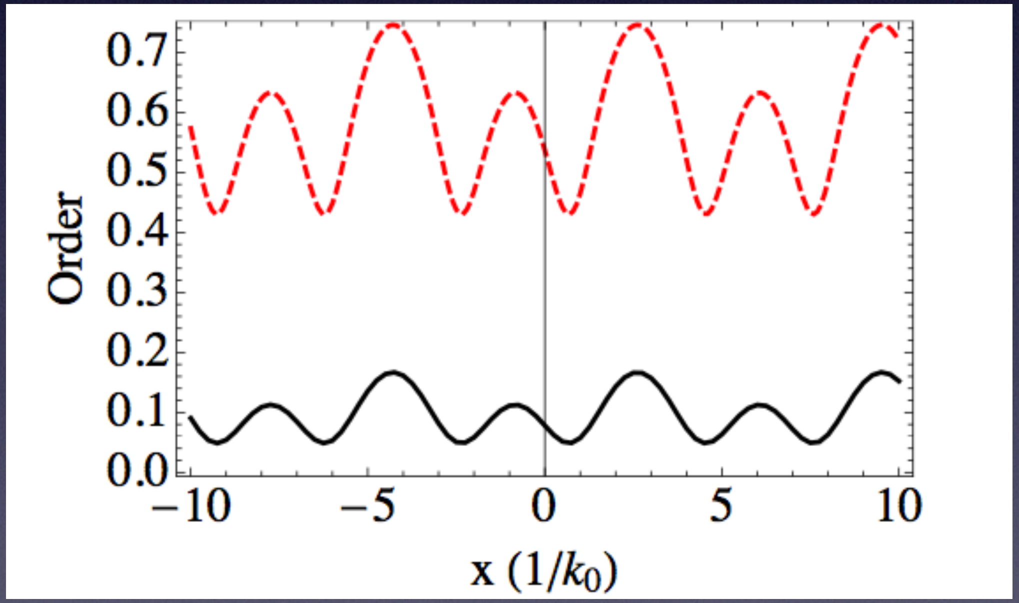


Ferronematics!

In presence of Raman field,
Nematicity and Spiral spin order coexist
with density wave order





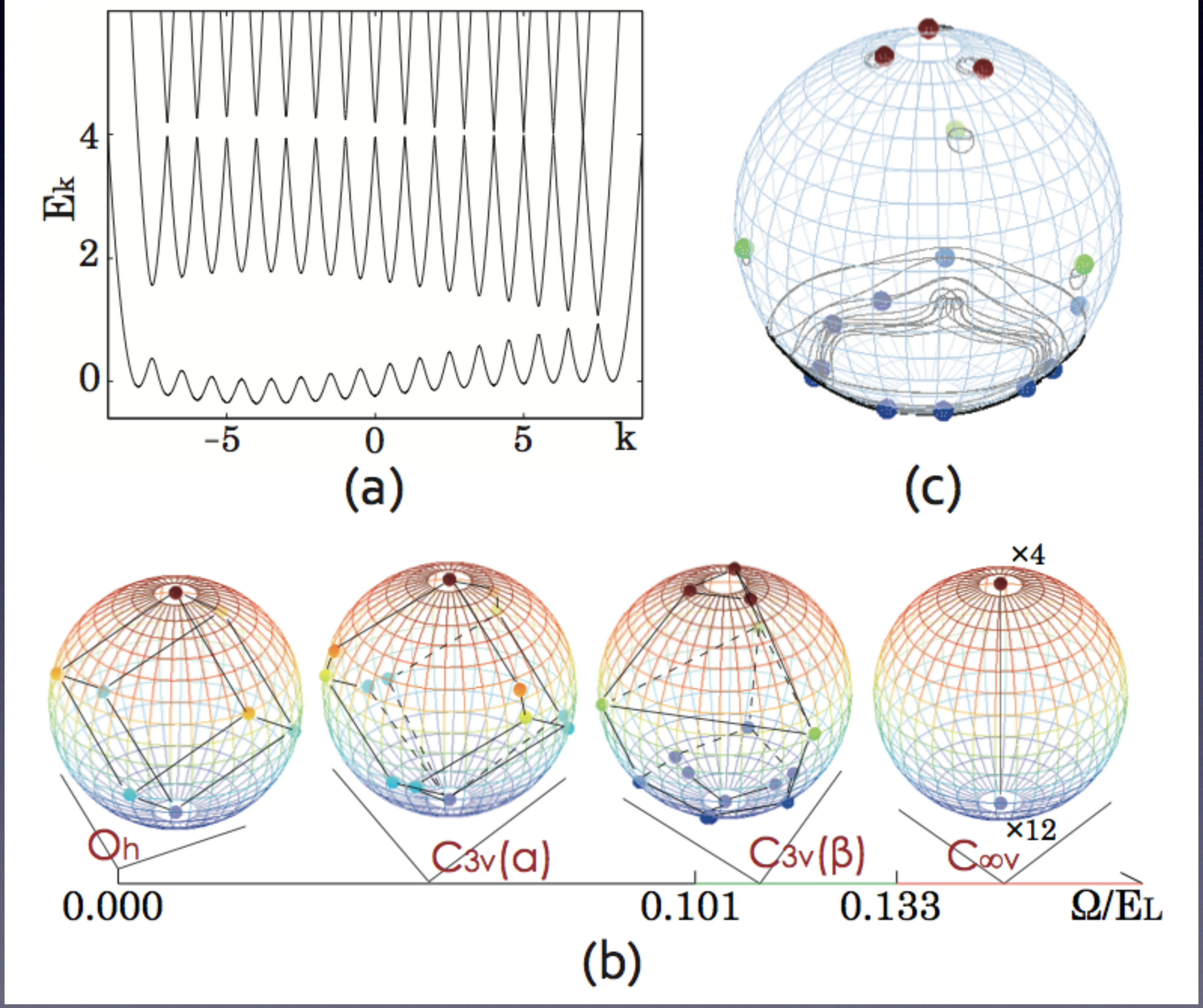


Spatially oscillating Biaxial nematic phase coexisting with FM
in presence of SOC!

Natu, Li, Cole PRA (to appear)

Mo' Spin... Mo' Orders

Spin-S atoms host generalizations of spin and nematic order!



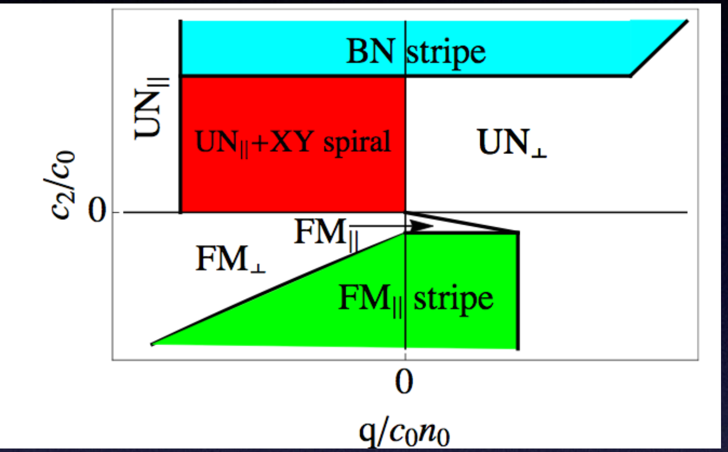
Spin-S viewed as 2S spin 1/2 on a Bloch sphere

Generalized platonic solids

Cui et al. PRA (R) (2013)

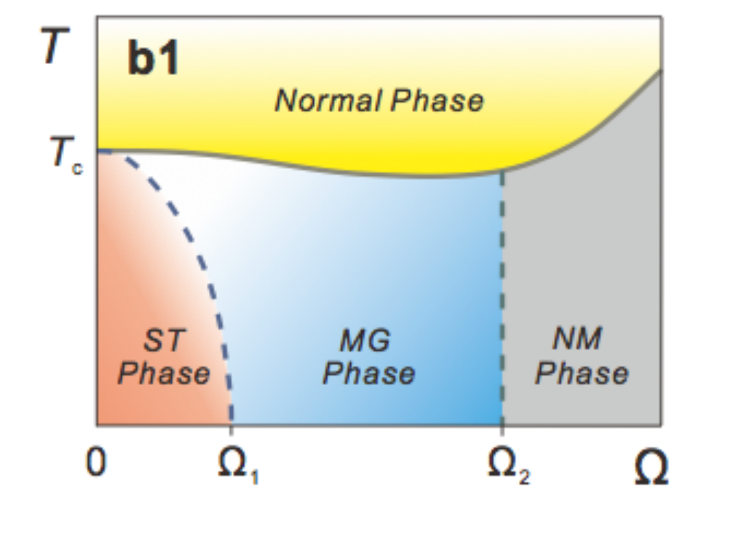
Ongoing Work/Open Questions

How do these phase evolve in the Mott limit?



Exotic Magnetic Hamiltonians at strong coupling?
High spin generalizations?

Hickey, Paramekanti, PRL (2014)

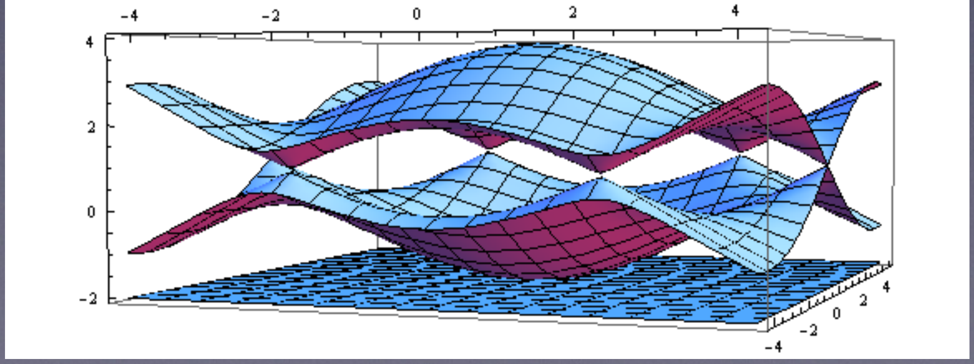


Magnetic/Nematic ordering at finite T?

Ji et al. Nat Phys (2014)

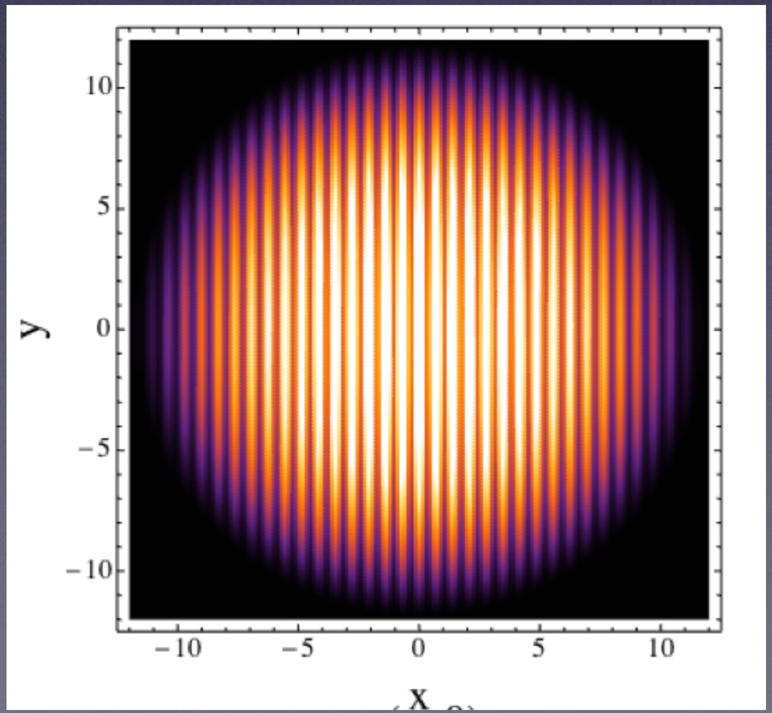
Bosons in entirely flat bands?

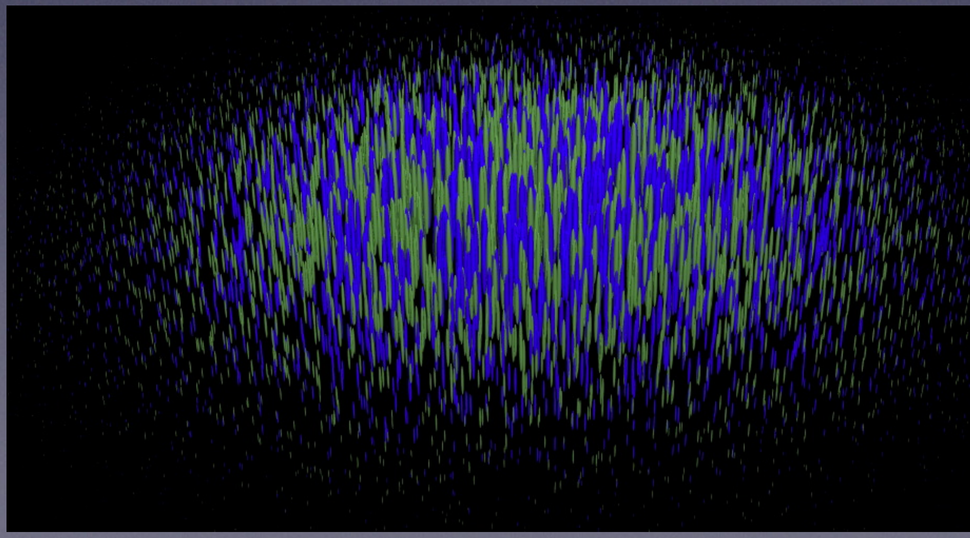
Sedrakyan et al. PRA (2012)



Conclusions

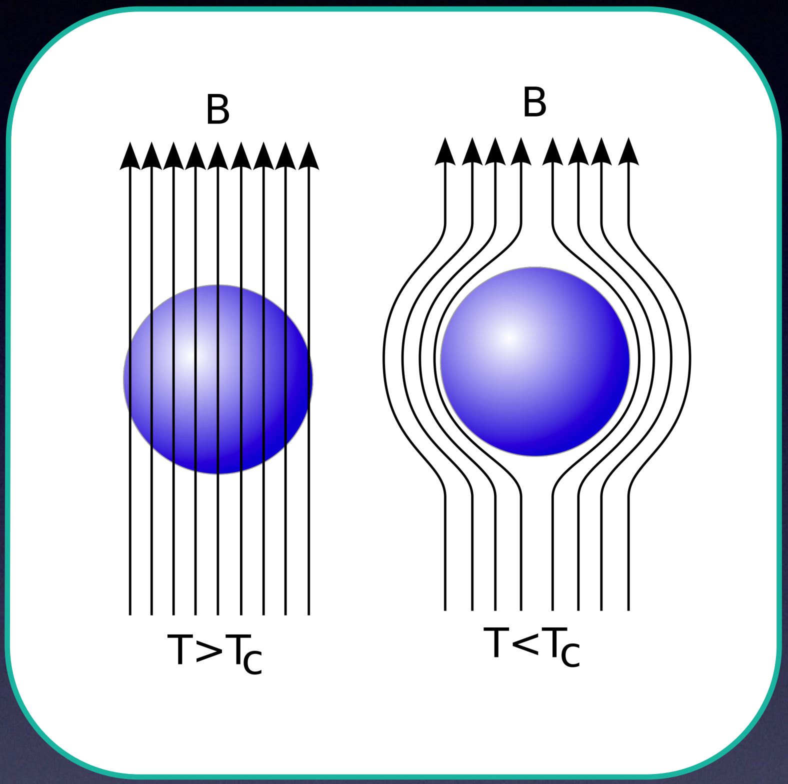
- *Bosons in single-particle degeneracies is a new frontier*
- *Interplay of several competing orders*
- *New playground for quantum magnetism/spin liquids?*



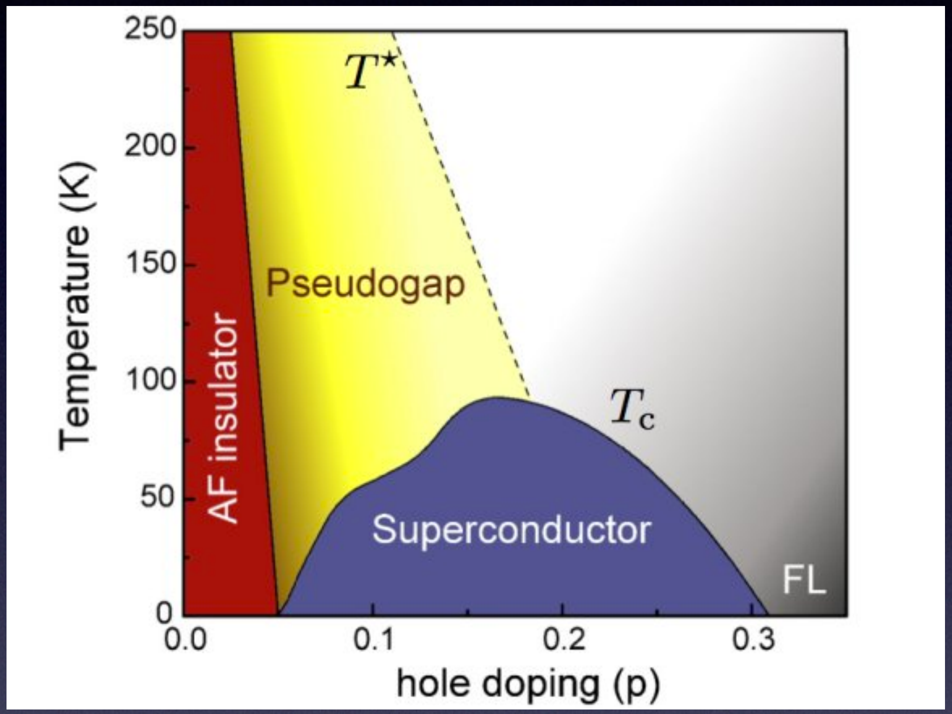


Competing/Coexisting Orders

weak coupling



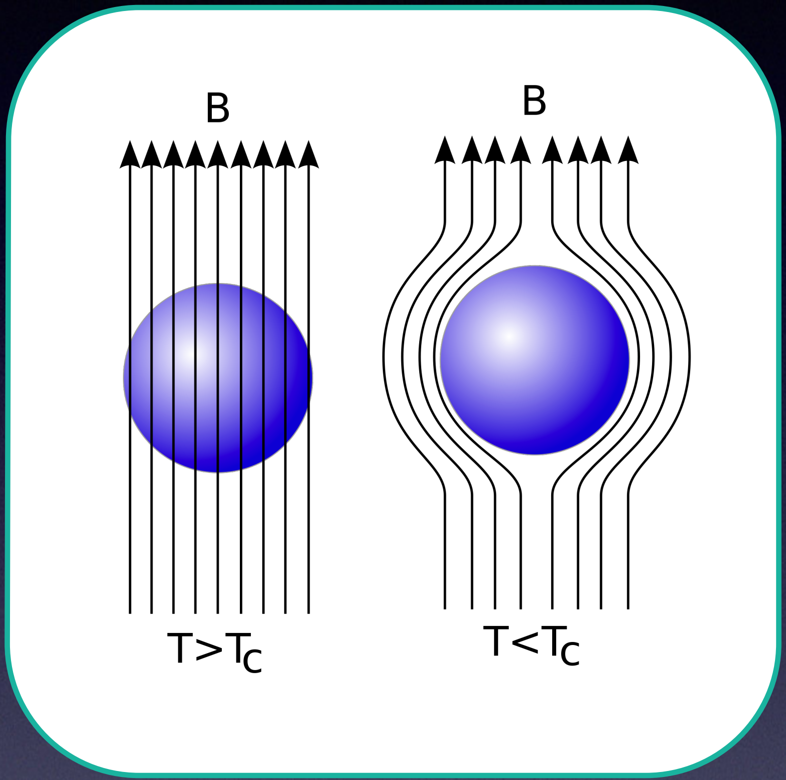
strong coupling



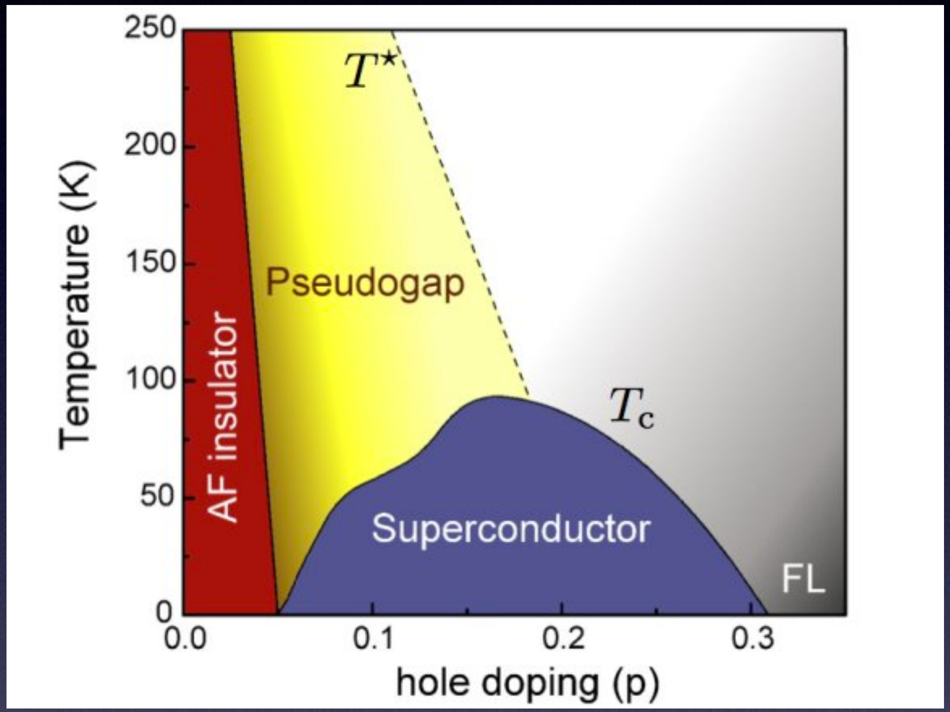
Interplay between magnetism and superconductivity?

Competing/Coexisting Orders

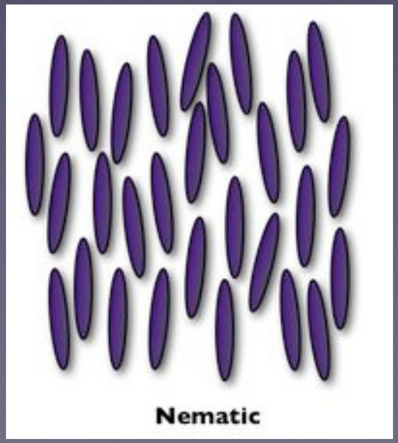
weak coupling

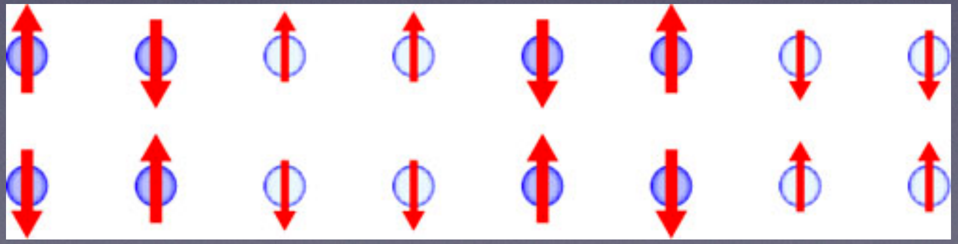


strong coupling

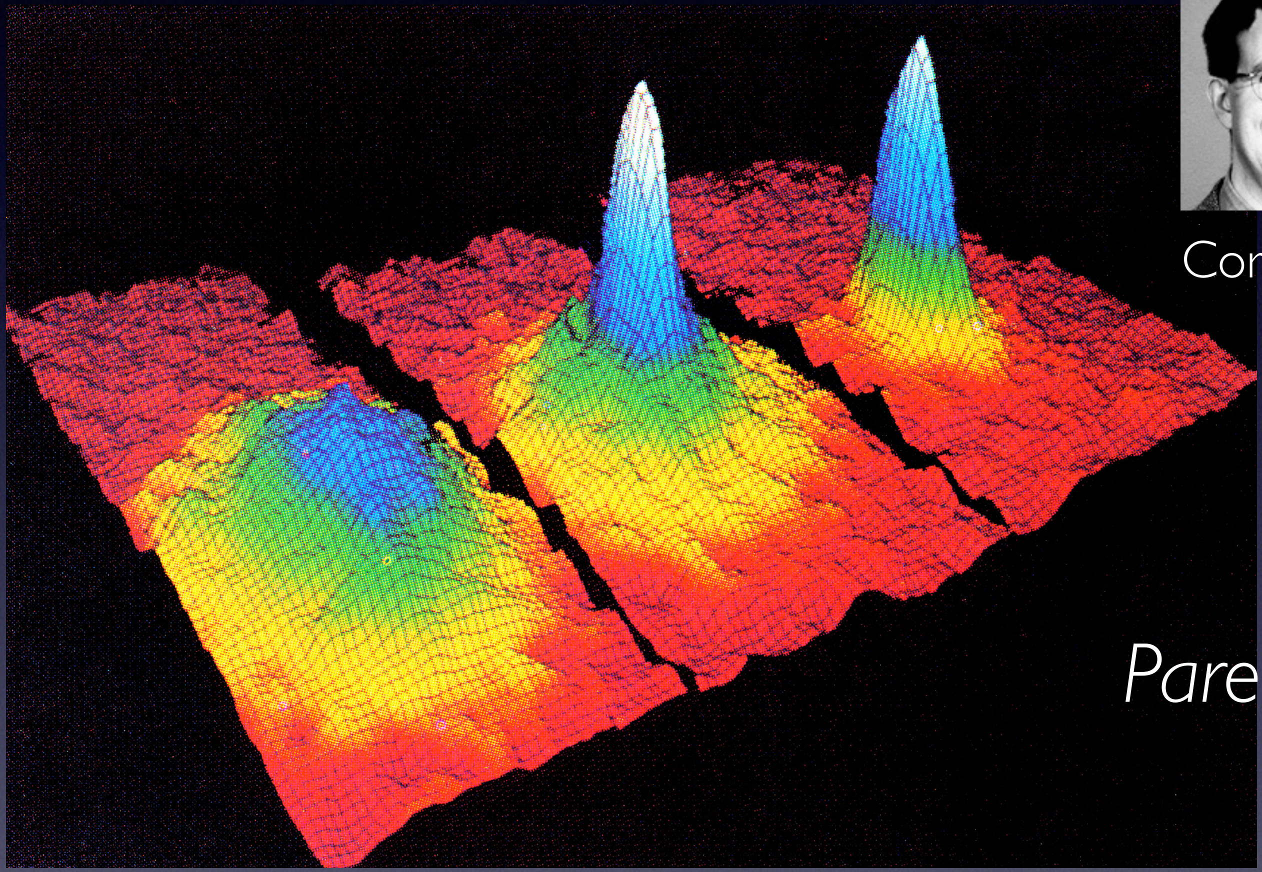


Other orders: liquid crystallinity, density-wave order….





What about Bosons?



2001

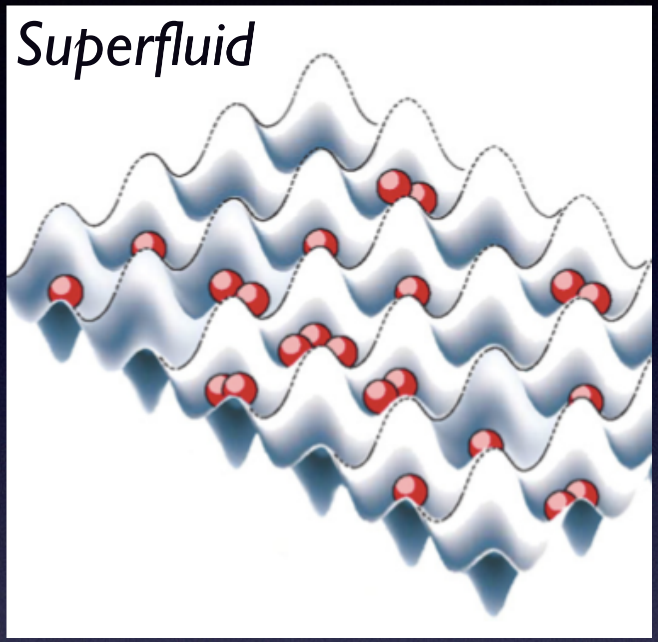
Cornell Wieman Ketterle

Parent T =0 phase remarkably ROBUST!

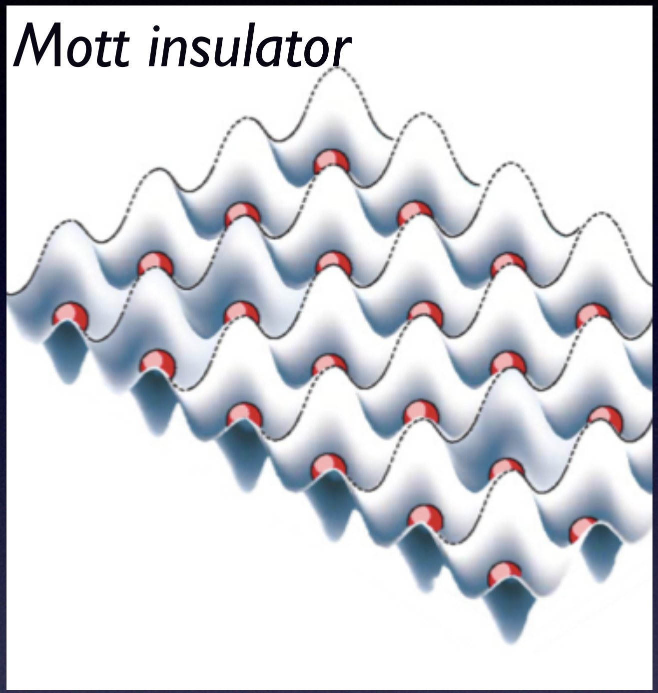
Can magnetism/nematicity/density wave order etc. compete with Bose condensation?

Destroying BEC

Strong interactions



the lattice depth



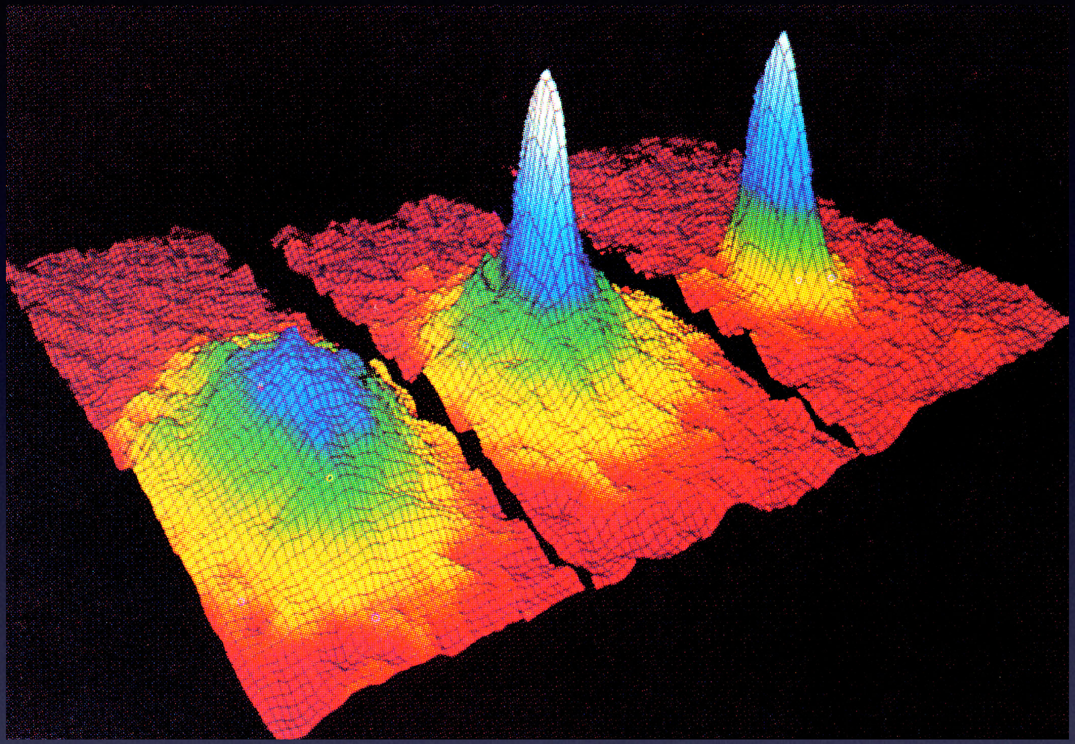
Fisher et al. PRB (1989)
Greiner et al. (2002)

Can strong interactions produce something other than a featureless Mott insulator?

Destroying BEC

Finite Temperature

Superfluid

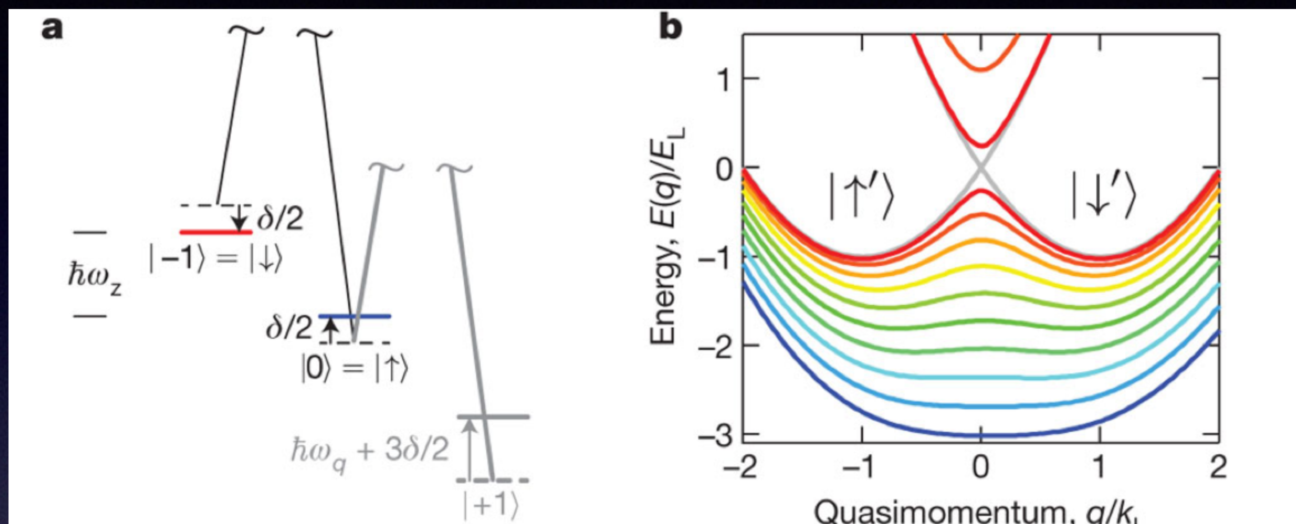


Can finite temperature produce something other than a boring normal phase?

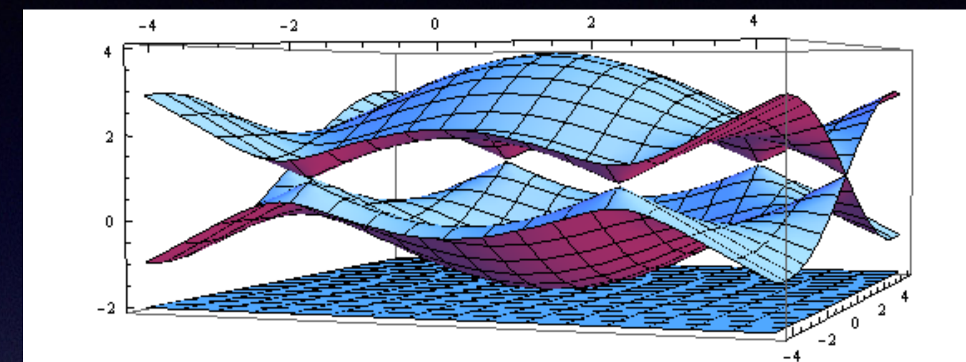
A New Hope*

Single Particle Degeneracies

Raman coupling

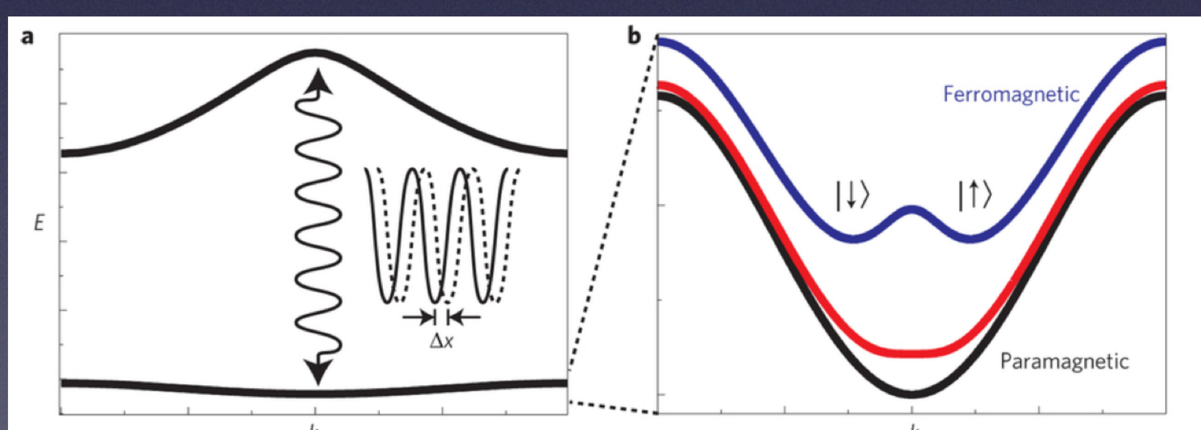


Flat Band lattices



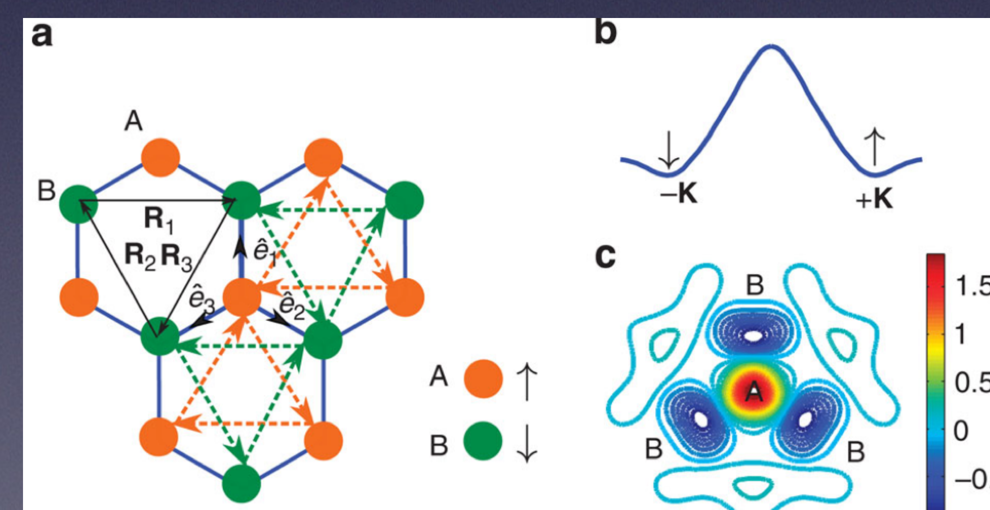
Kagome/Lieb lattices
Experiments ongoing!

Lattice Shaking



Parker et al. Nature Physics (2013)
Also Sengstock, Stamper-Kurn groups

Spin-dependent lattices/Artificial Gauge fields



Fate of interacting bosons in the presence of single-particle degeneracies?

* apologies to George Lucas