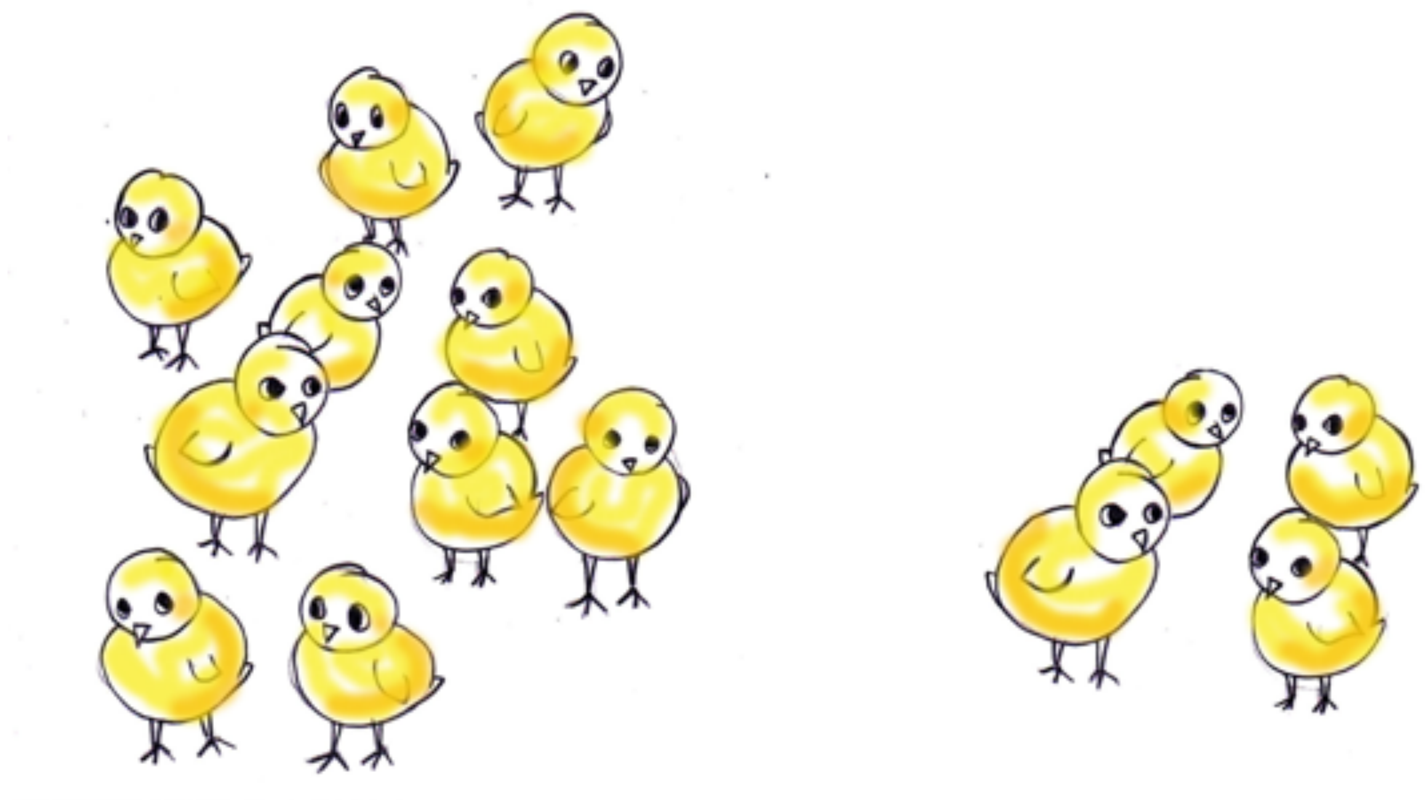
Few- and many-body physics of fermions in two dimensions

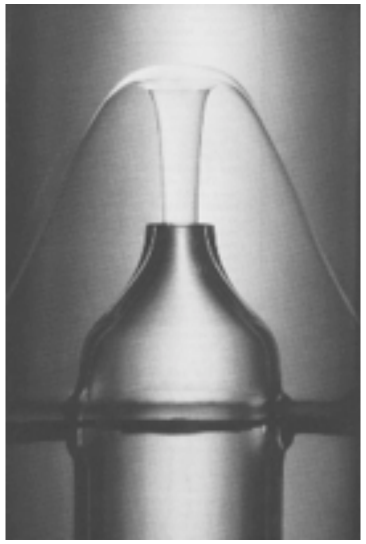


Sergej Moroz
CU Boulder

Chiral p+ip fermionic superfluids

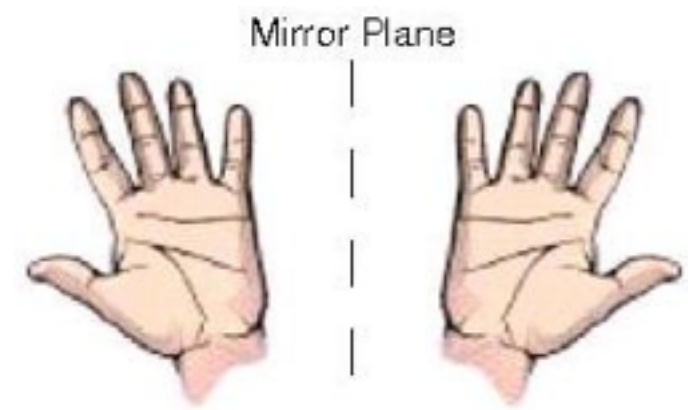
together with Carlos Hoyos and Dam Thanh Son

Superfluids



- T=0 state of a neutral many-body system
- No dissipation, quantum vortices, ...
- Old: ^{4}He and ^{3}He
- New: Bose and Fermi ultracold atoms

Chiral 2d superfluid



Spinless fermions in flatland:

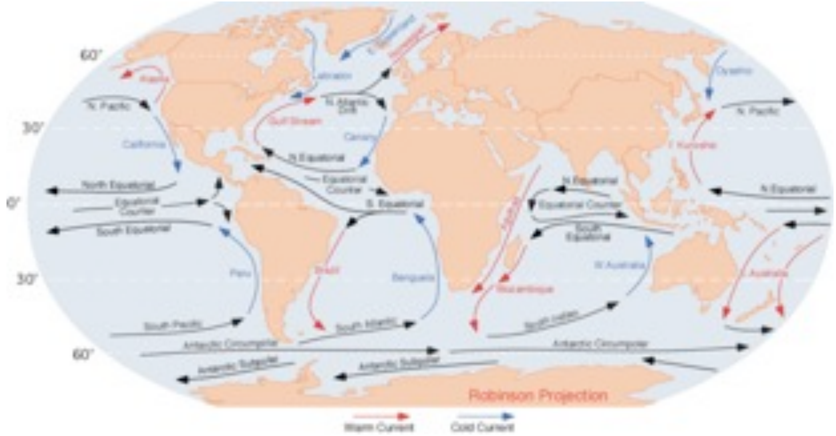
- Chiral condensate $\Delta_{\mathbf{p}} = (p_x \pm i p_y)\hat{\Delta}$ preferred
- Topological phase transition at $\mu = 0$
- Chiral Majorana mode on boundaries
- Toy model for a film of ^{3}He
- Moore-Read $\nu = 5/2$ QH state

Volovik, Read, Green,...

Symmetry breaking

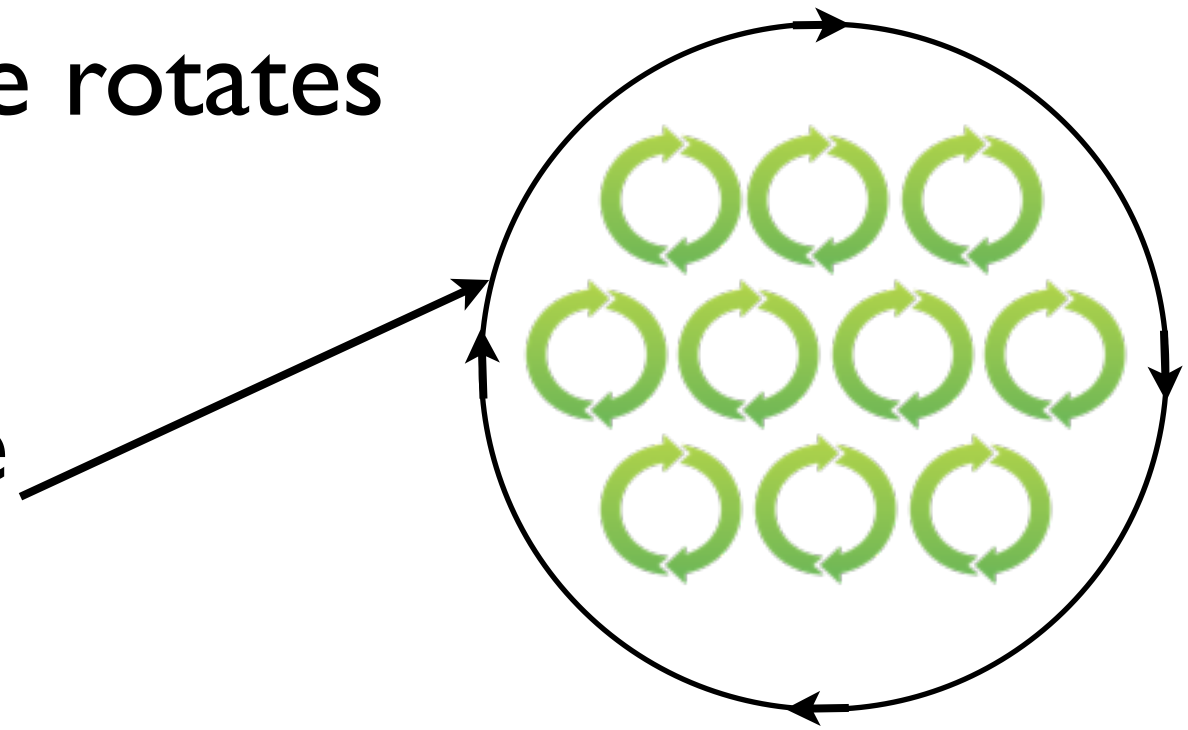
- Chiral condensate $\Delta_{\mathbf{p}} = (p_x \pm i p_y)\hat{\Delta}$
- SSB pattern: $U(1)_N \times SO(2)_L \to U(1)_V$
- Single gapless Goldstone mode
- Breaks parity and time reversal!

Edge current



- Chiral ground state rotates

edge particle
current



- Angular momentum of p+ip superfluid

$$L_{\text{GS}} = \int d^2x\,\epsilon_{kl}x^k J^l = 1/2 \underbrace{\int d^2x\,\rho}_{N}$$

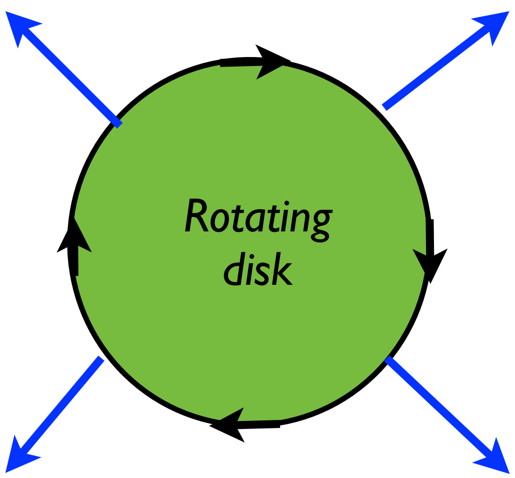
Hall viscosity

- Specific to 2d with broken P and T

Avron, Seiler, Zograf

- Non-dissipative effect

$$f^i_{\text{Hall}} = \eta_{\text{H}} \epsilon^{ij} \Delta v_j$$



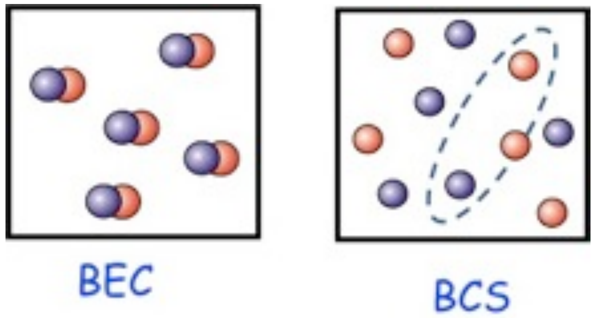
- Counts internal angular momentum density

$$\eta_{\text{H}} \sim \hbar / l^2$$

Read

Galilean-invariant examples:
IQHE, FQHE, p+ip SF

s-wave superfluid



Popov; Son, Wingate

$$S[\theta] = \int dt d\mathbf{x} \sqrt{g} P\left(X\right) \leftarrow \boxed{X = D_t\theta - \frac{g^{ij}}{2} D_i\theta D_j\theta}$$

pressure

$$D_\nu\theta \equiv \partial_\nu\theta - A_\nu$$

- Ideal superfluid hydrodynamics with

$$\rho \equiv dP/dX \qquad v_j \equiv -D_j\theta$$

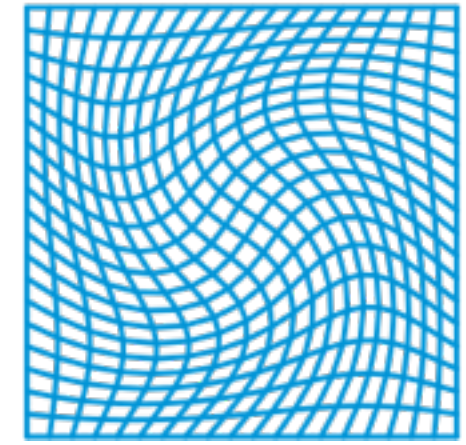
- Leading order in power-counting

$$\partial_\nu\theta \sim A_\nu \sim g_{ij} \sim O(1) \qquad [\partial_\nu O] = 1 + [O]$$

- Nonlinear in Goldstones $[(\partial\theta)^n] = n[\partial\theta] = 0$

General coordinate invariance

Son, Wingate



- Put superfluid into curved space and turn on electromagnetic source
- Under diffeomorphism

Goldstone field
$$\theta = \mu t - \varphi$$

$$\delta\theta = -\xi^k \partial_k \theta$$

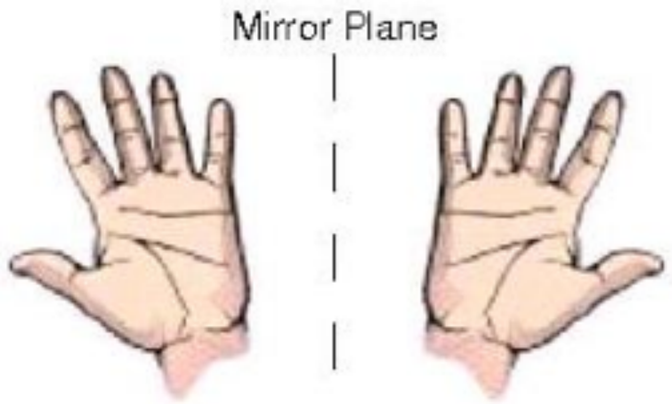
$$\delta A_t = -\xi^k \partial_k A_t - A_k \dot{\xi}^k$$

$$\delta A_i = -\xi^k \partial_k A_i - A_k \partial_i \xi^k + g_{ik} \dot{\xi}^k$$

$$\delta g_{ij} = -\xi^k \partial_k g_{ij} - g_{ik} \partial_j \xi^k - g_{kj} \partial_i \xi^k$$

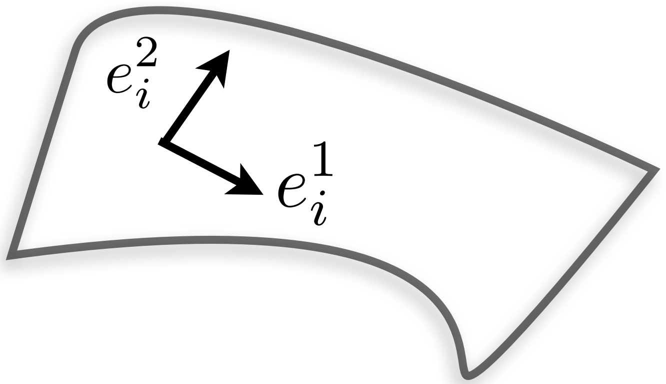
- Generalizes Galilean transformation

Chiral superfluid



- New gauge field needed $U(1)_N \times SO(2)_L \to U(1)_V$
- Orthogonal spatial vielbein:
- Spin connection:

$$\omega_t \equiv \frac{1}{2}\left(\epsilon^{ab} e^{aj} \partial_t e^b_j + B\right)$$
$$\omega_i \equiv \frac{1}{2}\epsilon^{ab} e^{aj} \nabla_i e^b_j = \frac{1}{2}\left(\epsilon^{ab} e^{aj} \partial_i e^b_j - \varepsilon^{jk} \partial_j g_{ik}\right)$$

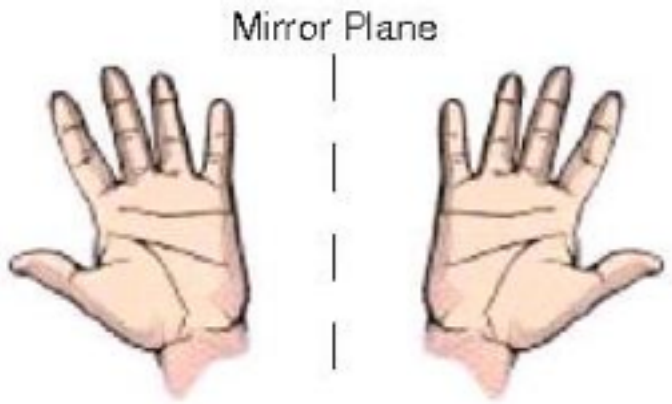


SO(2)$_L$: gauge field
diffeo: one-form

- Just introduce new covariant derivative

$$D_\nu \theta \equiv \partial_\nu \theta - A_\nu - s\omega_\nu$$

Chiral superfluid



- U(1) current:

$$J^i \equiv -\frac{1}{\sqrt{g}}\frac{\delta S}{\delta A_i} = \underbrace{\rho g^{ij} v_j}_{\text{convective}} + \underbrace{\frac{s}{2}\varepsilon^{ij}\partial_j \rho}_{\text{edge}}$$

- Stress tensor:

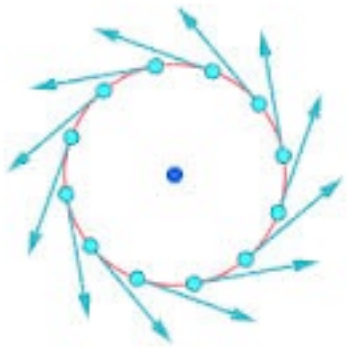
$$\Delta T^{ij}_{\text{ch}} \equiv \frac{2}{\sqrt{g}}\frac{\delta S_{\text{ch}}}{\delta g_{ij}}$$

$$\eta_H = -\frac{s}{2}\rho^{\text{GS}}$$

$$= (v^i J^j_{\text{edge}} + v^j J^i_{\text{edge}}) + T^{ij}_{\text{Hall}} - \frac{s^2}{4}\rho R g^{ij}$$

LO superfluid parity-violating hydrodynamics

Vorticity



$$\omega = \frac{1}{2}\epsilon^{ij}\partial_i v_j = \frac{\sqrt{g}}{2}\left(B + \frac{s}{2}R\right)$$

- Vorticity is sourced by magnetic field and curvature
- p-wave superfluid on a sphere without B

$$\int_{S^2} \omega = \pi$$

two quantum vortices

Linear response

- Electromagnetic $J^i = \sigma_H(\omega, \mathbf{p})\epsilon^{ij}E_j + \ldots$

$$\sigma_H(\omega, \mathbf{p}) = \frac{s\rho^{\text{GS}}}{2}\frac{-\mathbf{p}^2}{\omega^2 - c_s^2\mathbf{p}^2}$$

- Gravitational $\delta T^{xy} = -i\omega\frac{\eta_H(\omega)}{2}(h_{xx} - h_{yy}) + \ldots$

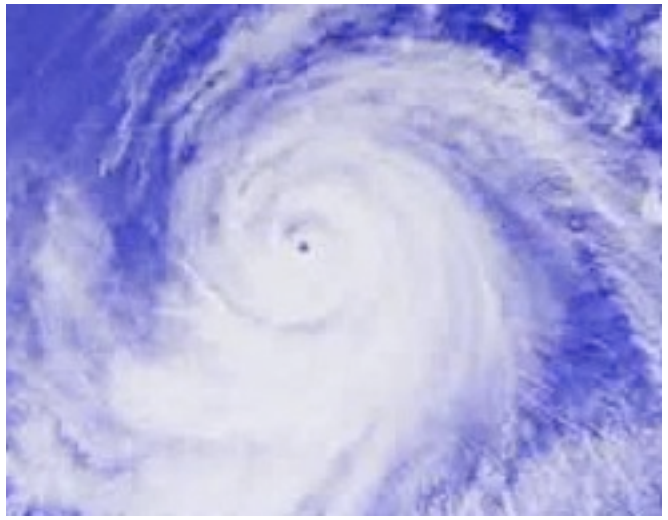
$$\eta_H(\omega) = -\frac{s}{2}\rho^{\text{GS}}$$

- Universal relation:

$$\eta_H(\omega) = \frac{\omega^2}{2}\frac{\partial^2}{\partial p_x^2}\sigma_H(\omega, \mathbf{p})\Big|_{\mathbf{p}=0}$$

Hoyos, Son; Bradlyn et al

Vortex solution



- Quantum vortex

$$v_r = 0 \qquad v_\phi = \frac{n}{2r} \qquad n \in \mathbb{Z}$$

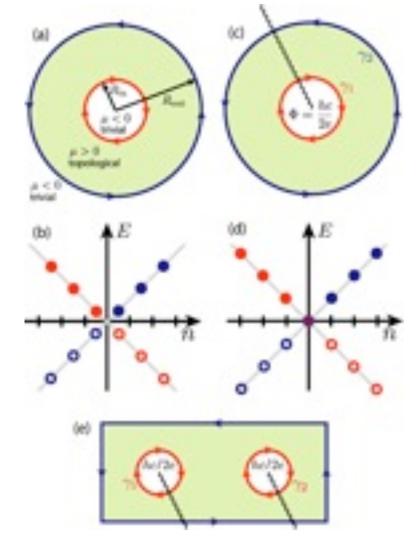
- Euler equation and its solution

$$\rho \frac{n^2}{4r^3} = \left[c_s^2 + \frac{sn}{2r^2} \right] \partial_r \rho \rightarrow \frac{\rho_\infty - \rho}{\rho_\infty} = \frac{n^2}{8 c_{s\infty}^2 r^2} + O(r^{-4})$$

- Vortex and anti-vortex are different

$$\frac{\Delta \rho}{\rho_\infty} = \frac{s}{16 c_{s\infty}^4 r^4} + O(r^{-6})$$

Gapless fermi modes



- Topological SF - gapless edge mode in BCS
- No explicit fermi modes in our EFT
- Non-analyticity of EoS at critical point
- This can appear only from integration of gapless modes

Edge modes are integrated out!

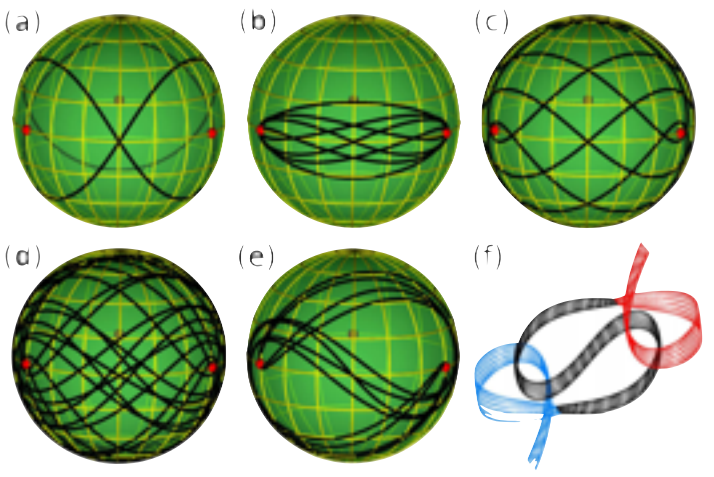
Conclusion

- Effective hydro theory for Galilean parity-violating superfluid
- Hall viscosity and edge current
- Extension to higher partial waves   Tada et al, Volovik 2014
- Better understanding of edge modes?

Super Efimov effect

together with Yusuke Nishida and Dam Thanh Son

Few-body problems



They are challenging but useful:

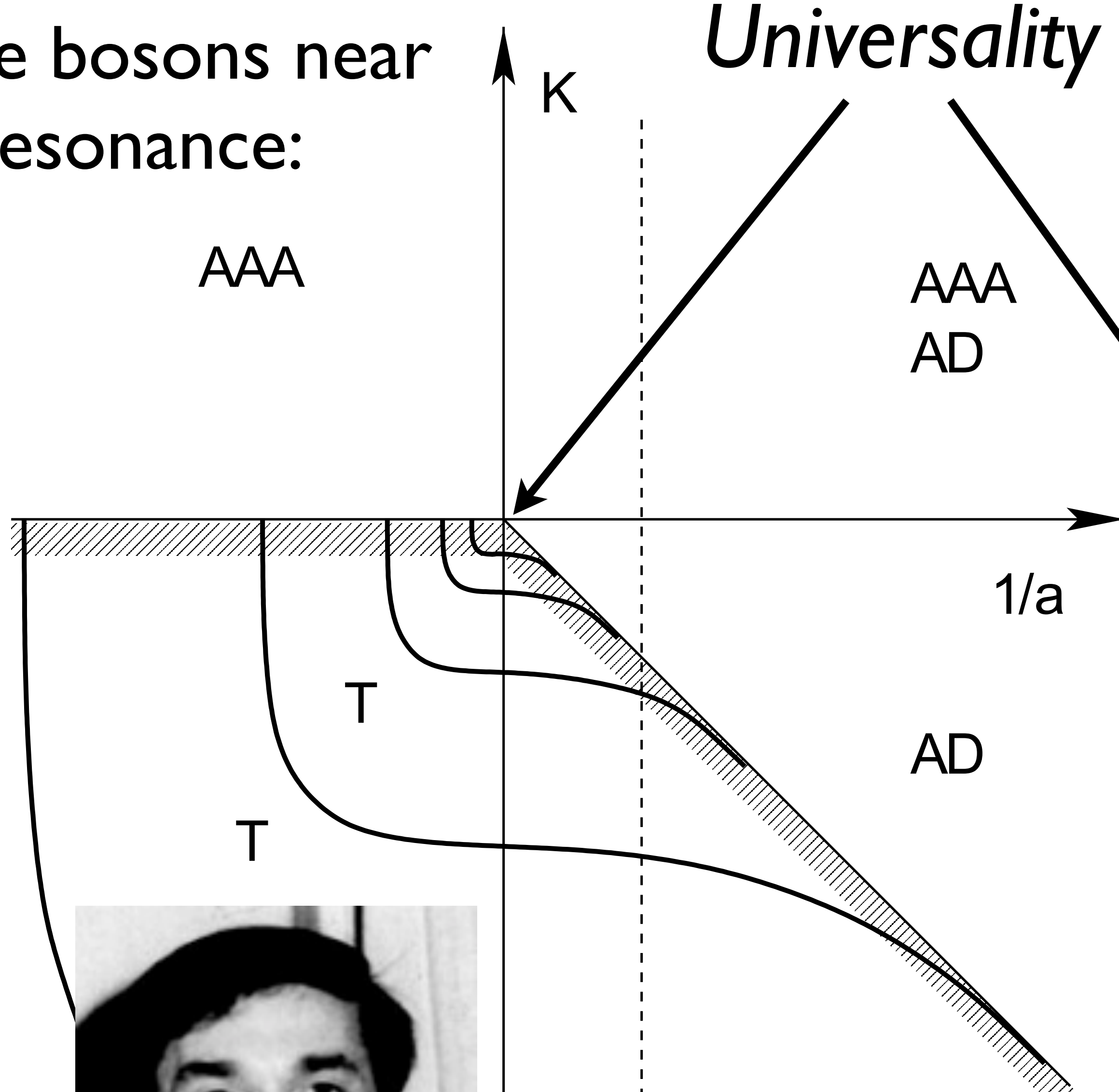
- Newton gravity $\longrightarrow$ perturbation theory, chaos
- Quantum atoms $\longrightarrow$ variational Hartree-Fock
- Quantum molecules $\longrightarrow$ Born-Oppenheimer

Efimov effect is "new" entry

Efimov problem

Three bosons near resonance:

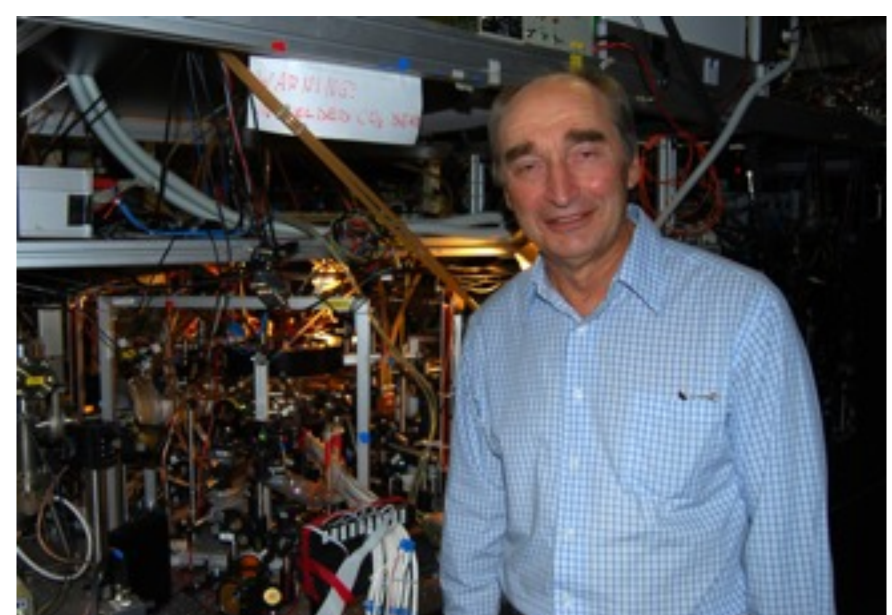
Universality



At resonance as $n \to \infty$

$$\frac{E_T^{(n+1)}}{E_T^{(n)}} \to e^{-2\pi/s_0}$$

40 years later



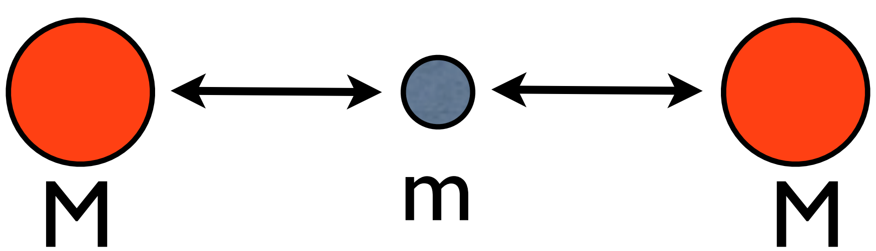
Basic intuition

- Efimov problem for heavy-heavy-light system

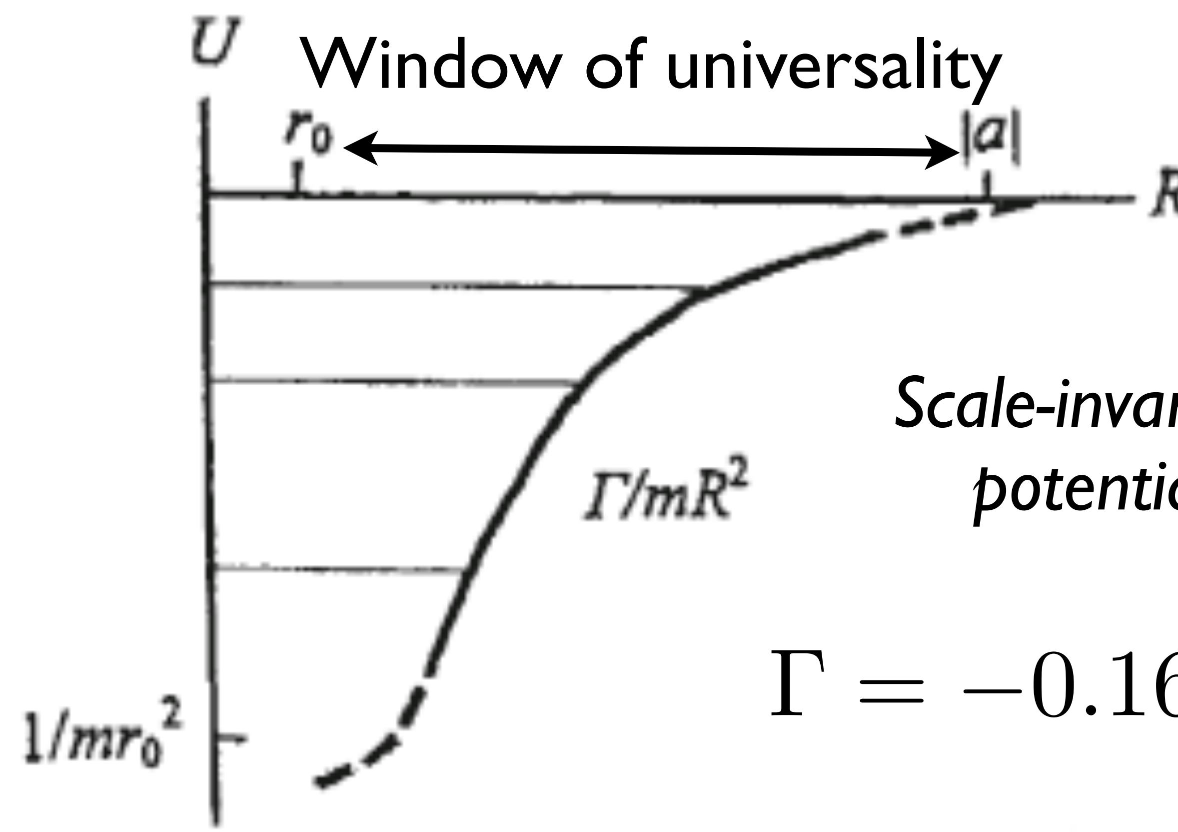
Efimov 1972
Amado&Noble 1972

- Born-Oppenheimer approximation: first freeze heavy particles

s-wave resonance
in 3d



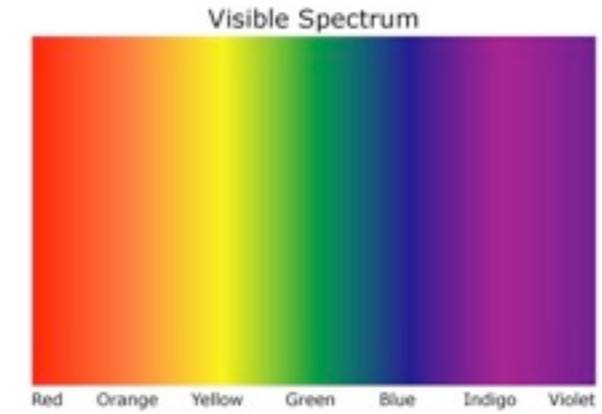
identical heavy
bosons



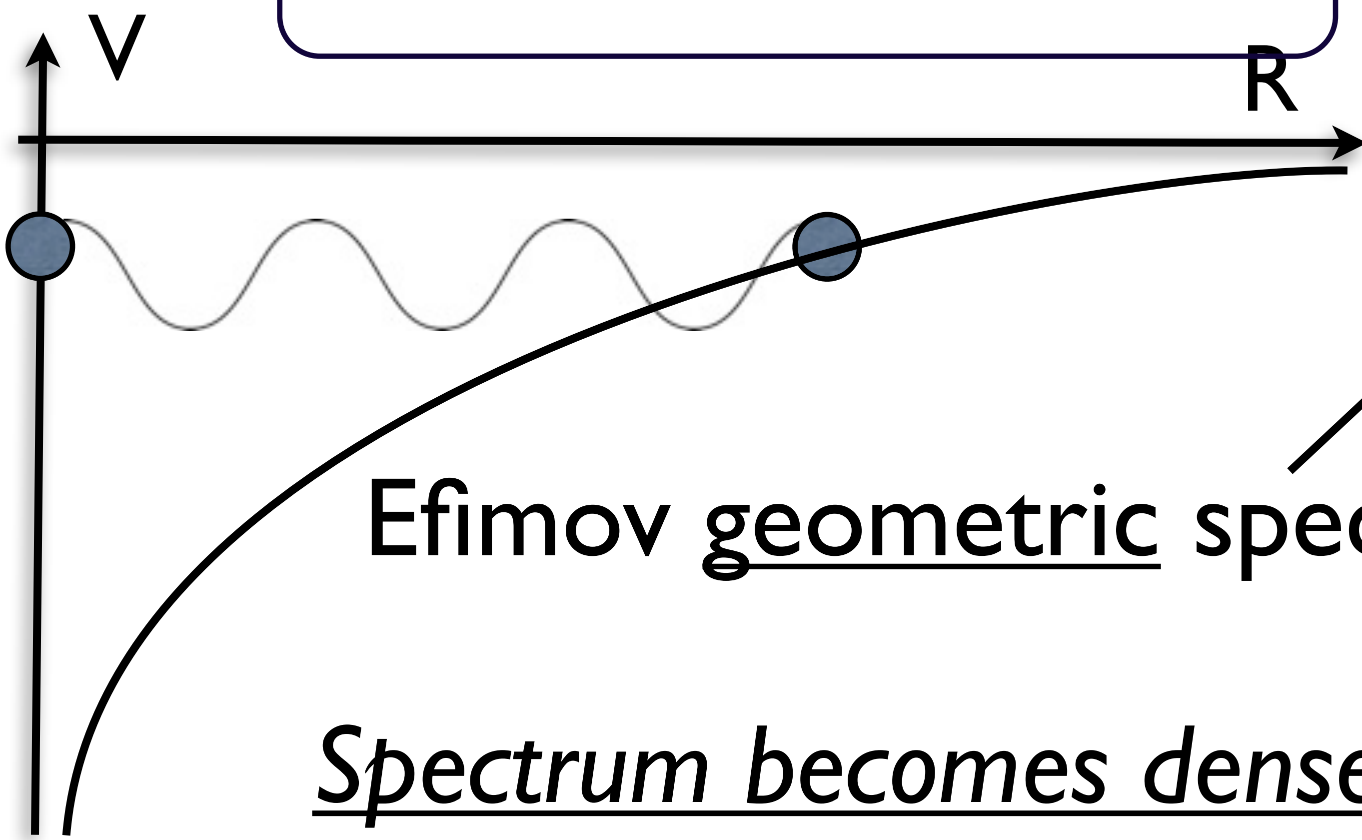
Scale-invariant potential

$$\Gamma = -0.16$$

Heavy-heavy spectrum



$$V(R) = \frac{\Gamma}{mR^2}$$



Landau&Lifshitz:
Fall to center for strong attraction

$$E_n \sim \exp\left(-\frac{2\pi n}{s} + \theta\right)$$

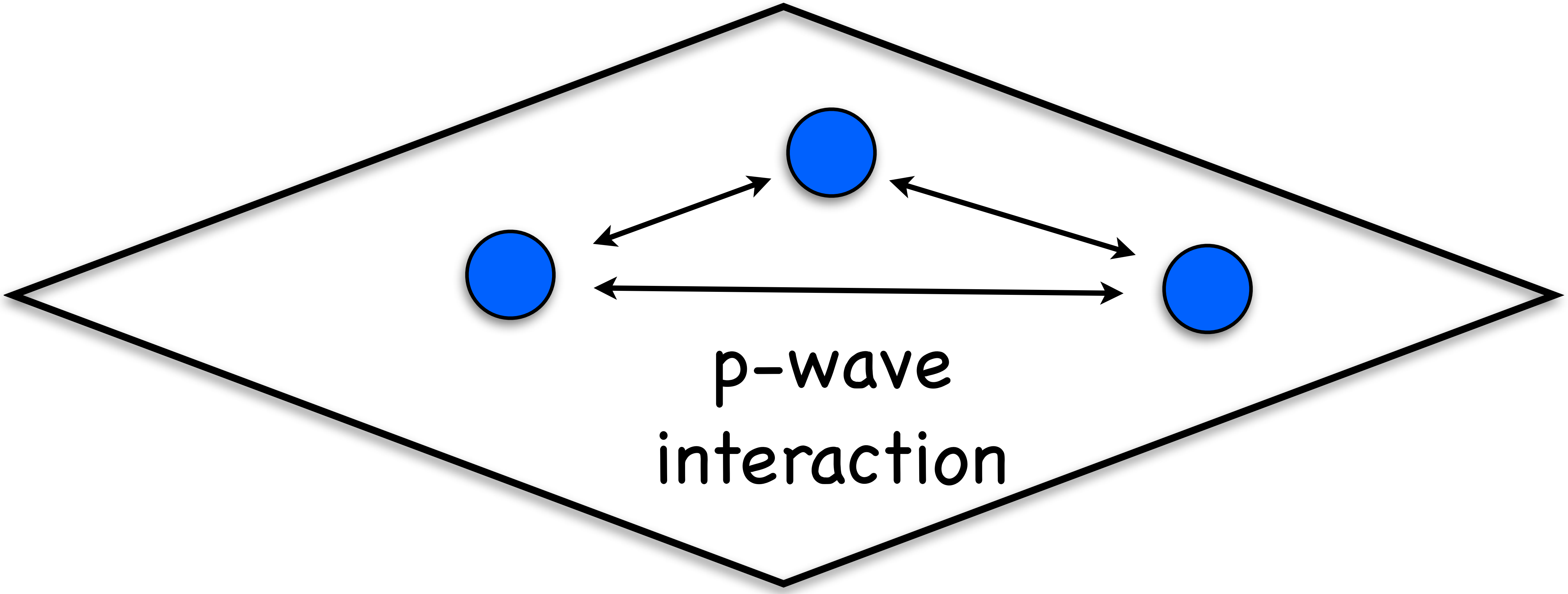
Efimov geometric spectrum $s = \sqrt{0.16M/m}$

Spectrum becomes denser as mass ratio grows!

Multiple Efimov states observed in Li-Cs mixture

Chicago and Heidelberg 2014

Super Efimov effect



Three-body quantum mechanics of <u>resonantly</u> interacting fermions in 2d

Super Efimov effect

At resonance near threshold:

Infinite tower of $l = \pm 1$ trimer bound states

$$E_3^{(n)} \propto \exp\left(-2e^{3\pi n/4+\theta}\right)$$

Super exponential scaling!

P-wave model in d=2

TAKE ACTION!

$$\mathcal{L} = \psi^\dagger\left(i\partial_t + \frac{\nabla^2}{2}\right)\psi + \phi_a^\dagger\left(i\partial_t + \frac{\nabla^2}{4} - \varepsilon_0\right)\phi_a$$

$$+g\,\phi_a^\dagger\psi(-i\nabla_a)\psi + g\,\psi^\dagger(-i\nabla_{-a})\psi^\dagger\phi_a$$

$$+v_3\,\psi^\dagger\phi_a^\dagger\phi_a\psi + v_4\,\phi_a^\dagger\phi_{-a}^\dagger\phi_{-a}\phi_a + v_4'\,\phi_a^\dagger\phi_a^\dagger\phi_a\phi_a$$

↑ spinless fermion

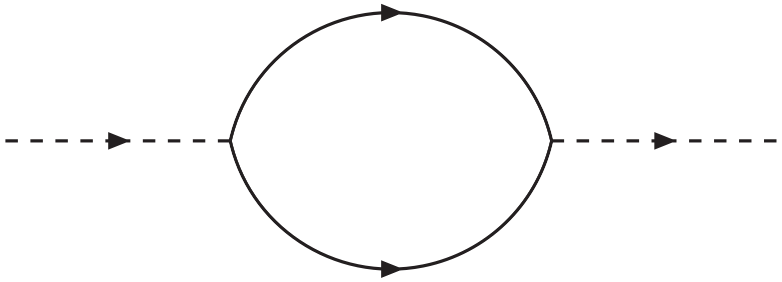
↑ composite $l = \pm 1$ boson

- P-wave resonance ⟷ zero energy bound state
- All dimensionless couplings are included

Super Efimov from RG

Two-body:

Perturbative counting is reliable!

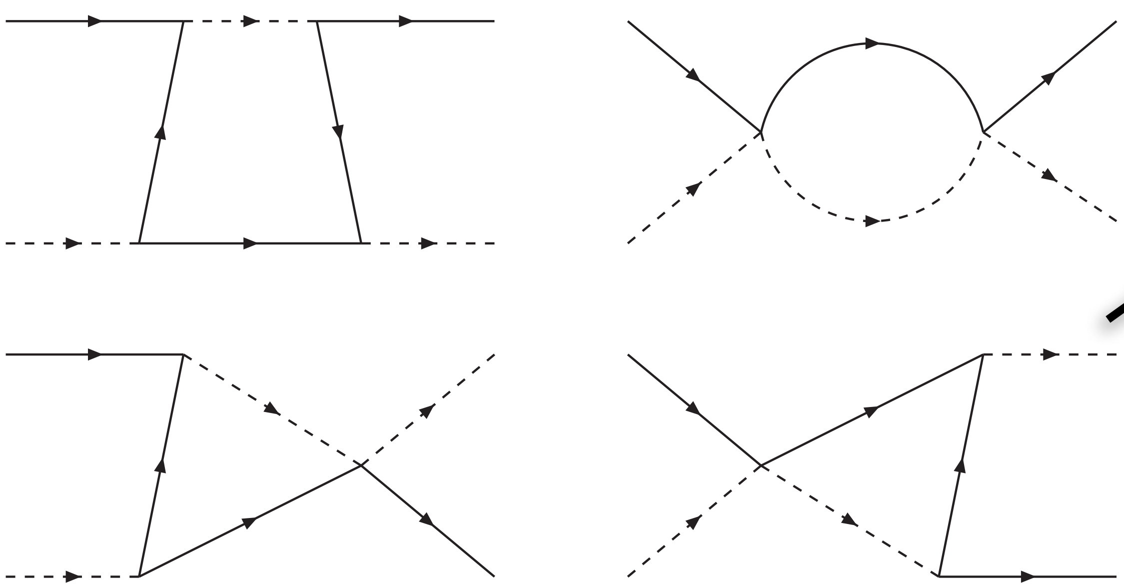


$s = \ln \Lambda / k$

irrelevant in IR

$$g^2(s) = \frac{1}{\frac{s}{\pi} + \frac{1}{g^2(0)}}$$

Three-body:



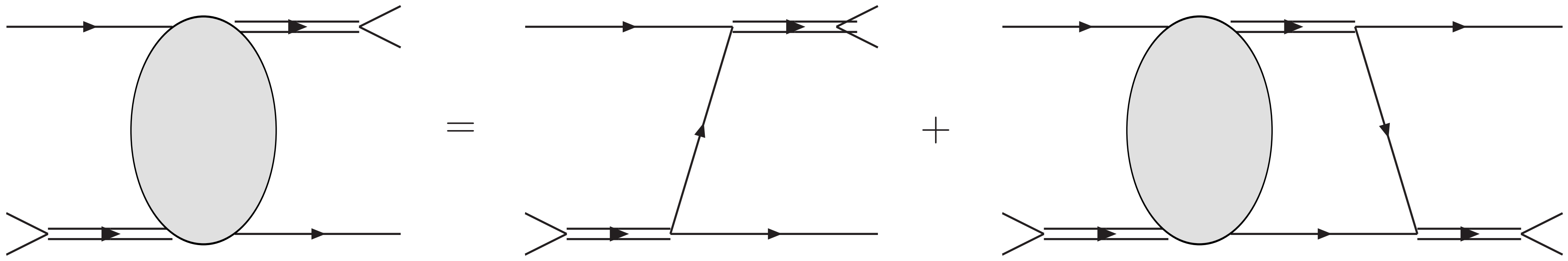
Double log periodic solution:

$$v_3(s) \to \frac{2\pi}{s}\left[1 - \cot\left(\frac{4}{3}(\ln s - \theta)\right)\right]$$

Divergences= trimer bound states

T-matrix solution

Near binding energy $T_{ab}(E;\vec{p},\vec{q}) \to Z_a(\vec{p})Z_b^*(\vec{q})/(E+\kappa^2)$



$\lambda_n \equiv \ln\ln\Lambda/\kappa_n$

<u>Analytic solution</u>

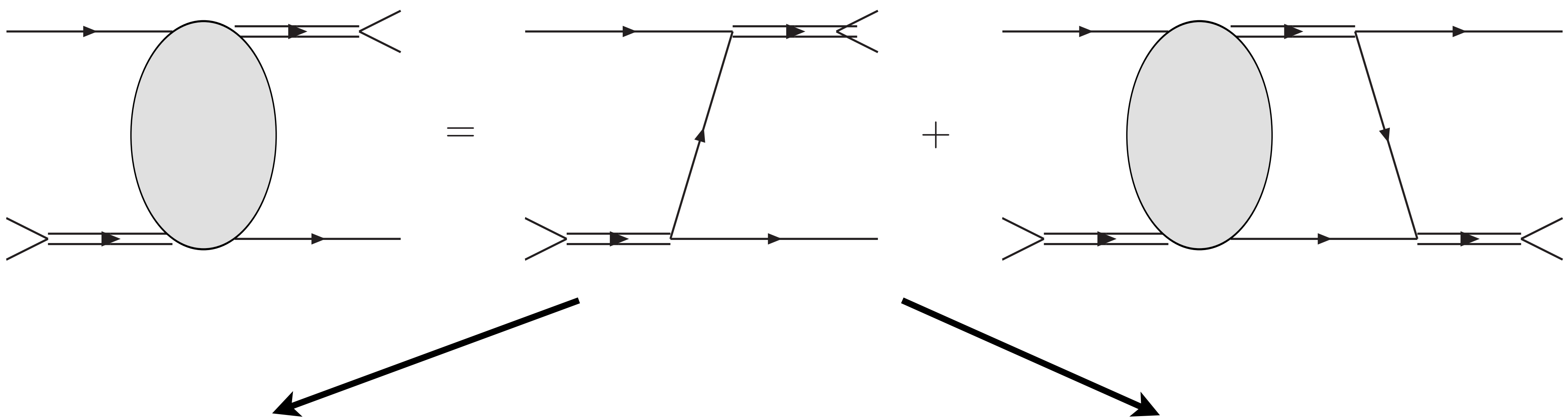
leading log approximation

<u>Numeric solution</u>

n	λ_n	$\lambda_n - \lambda_{n-1}$
0	0.5632	---
1	2.770	2.207
2	5.078	2.308
3	7.430	2.352
4	9.785	2.355
∞	---	2.35619

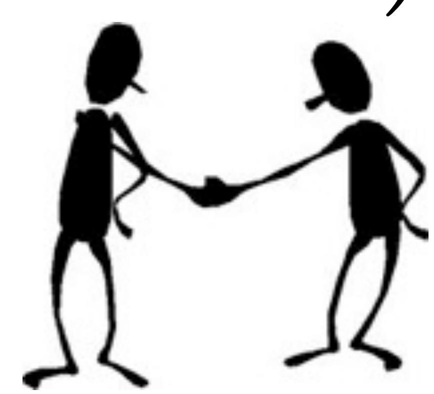
T-matrix solution

Near binding energy $T_{ab}(E;\vec{p},\vec{q}) \to Z_a(\vec{p})Z_b^*(\vec{q})/(E+\kappa^2)$

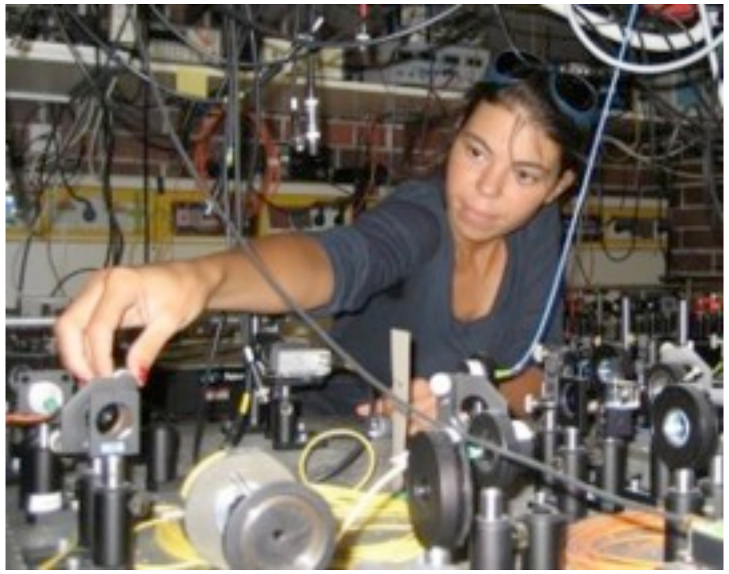


$$E_3^{(n)} \propto \exp\left(-2e^{3\pi n/4+\theta}\right)$$

Agreement with RG result



Experiments

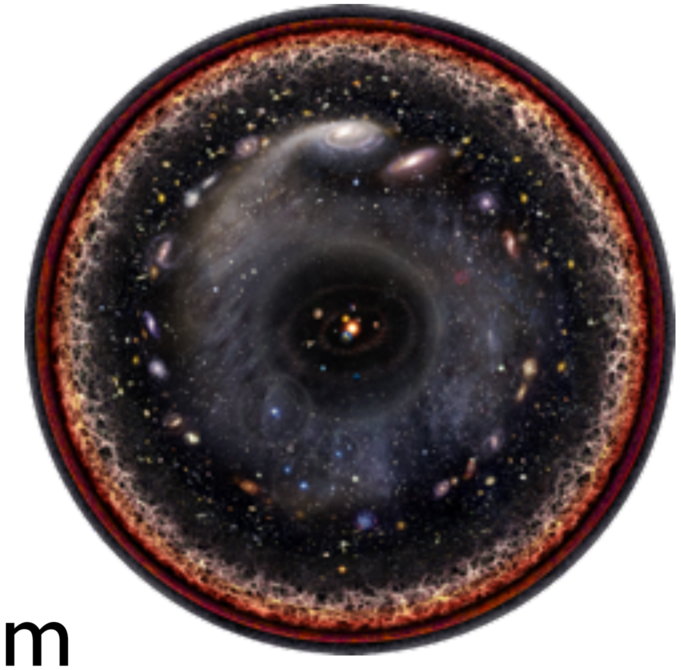


- Efimov physics in cold atom experiments *since 2006*
- Quasi 2d fermions near p-wave resonance *ETH 2005*
- Trimers sizes:

$$a_3^{(n)} \propto \exp\left(e^{3\pi n/4+\theta}\right)$$

$\theta = 0$

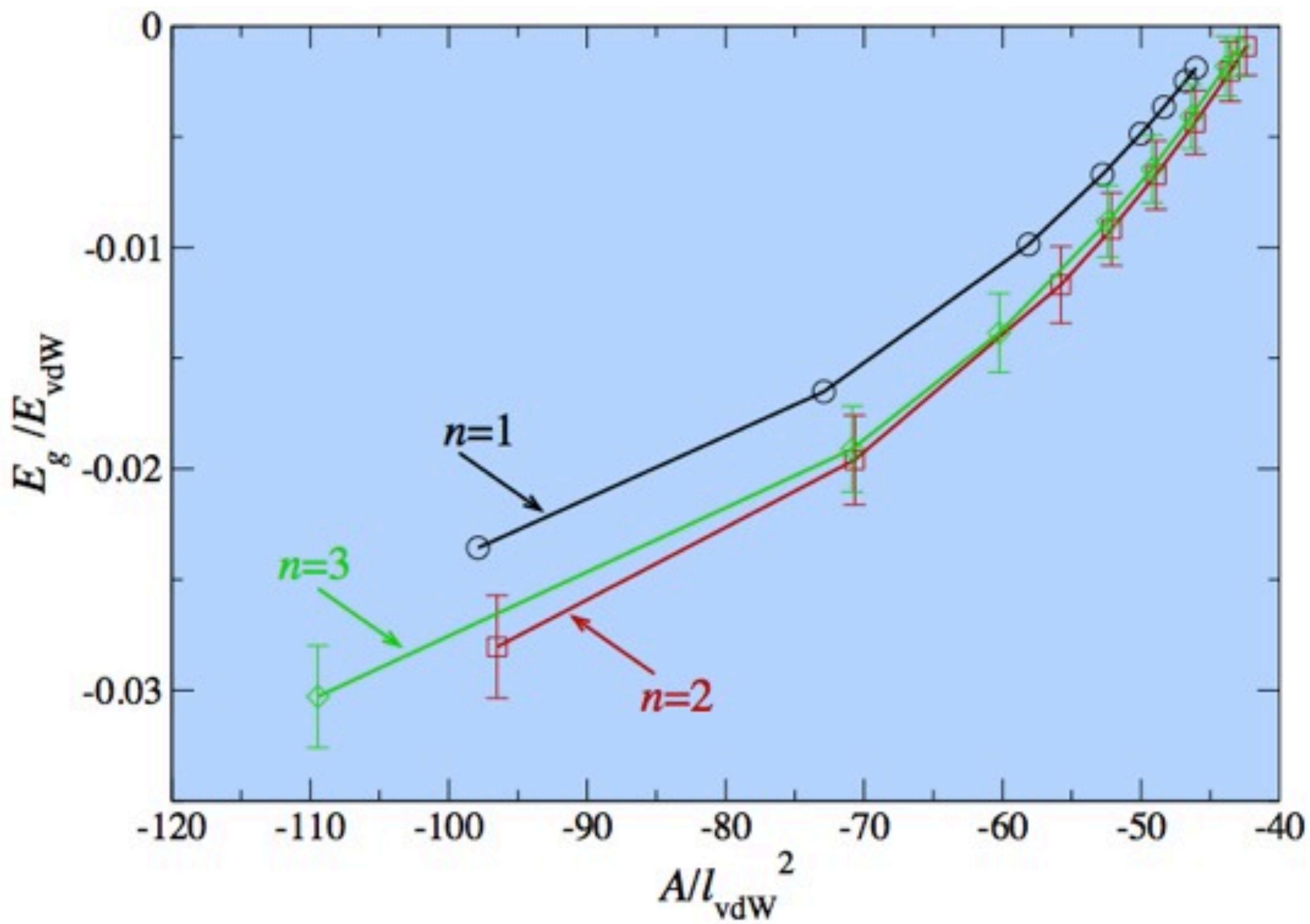
n	GS	1	2	3
size	Å	μ	10^{38}m	10^{499}m



Radius of observable Universe 10^{26}m

Super Efimov ground state $|0\rangle$

Ground state van der Waals universality

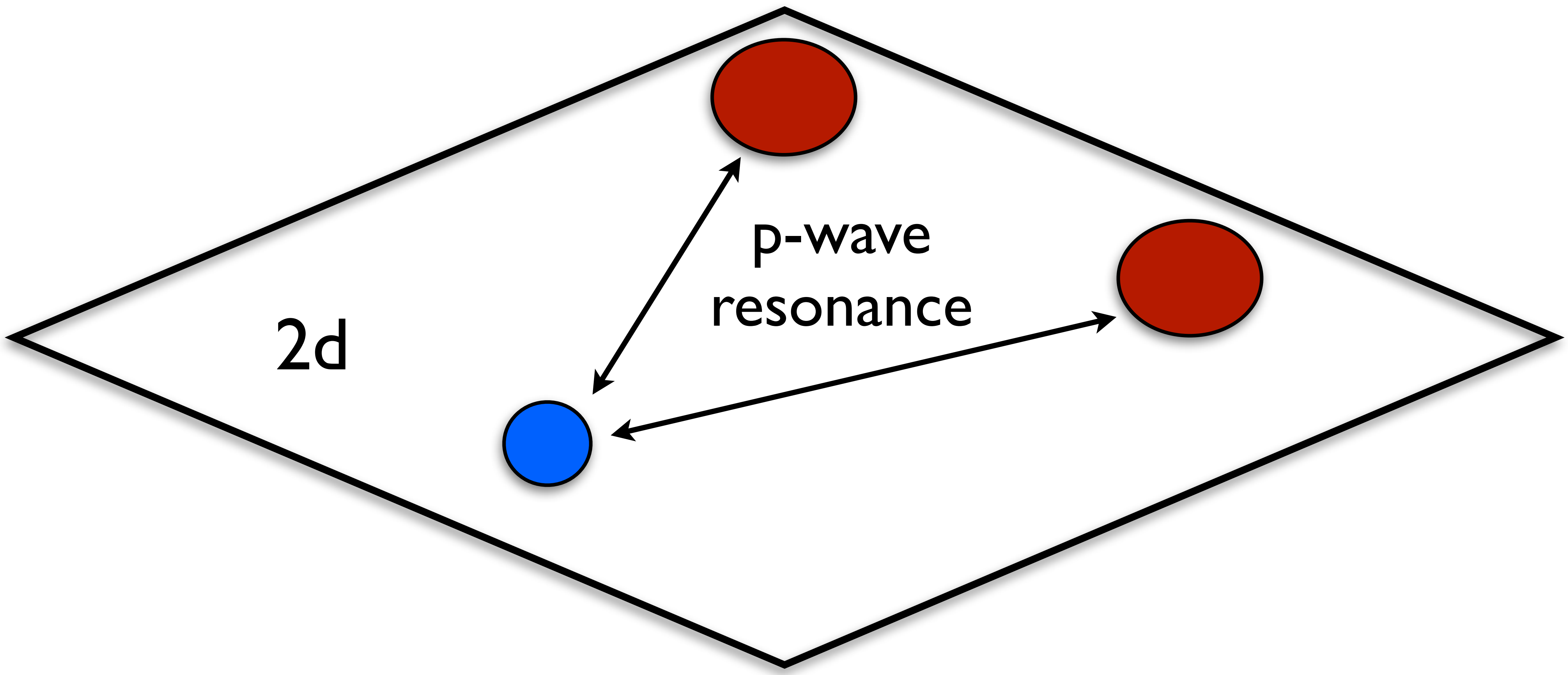


Gao, Wang, Yu 2014

Lenard-Jones tuned to p-wave resonance

Quasi two-dimensional confinement?

Mass imbalanced system



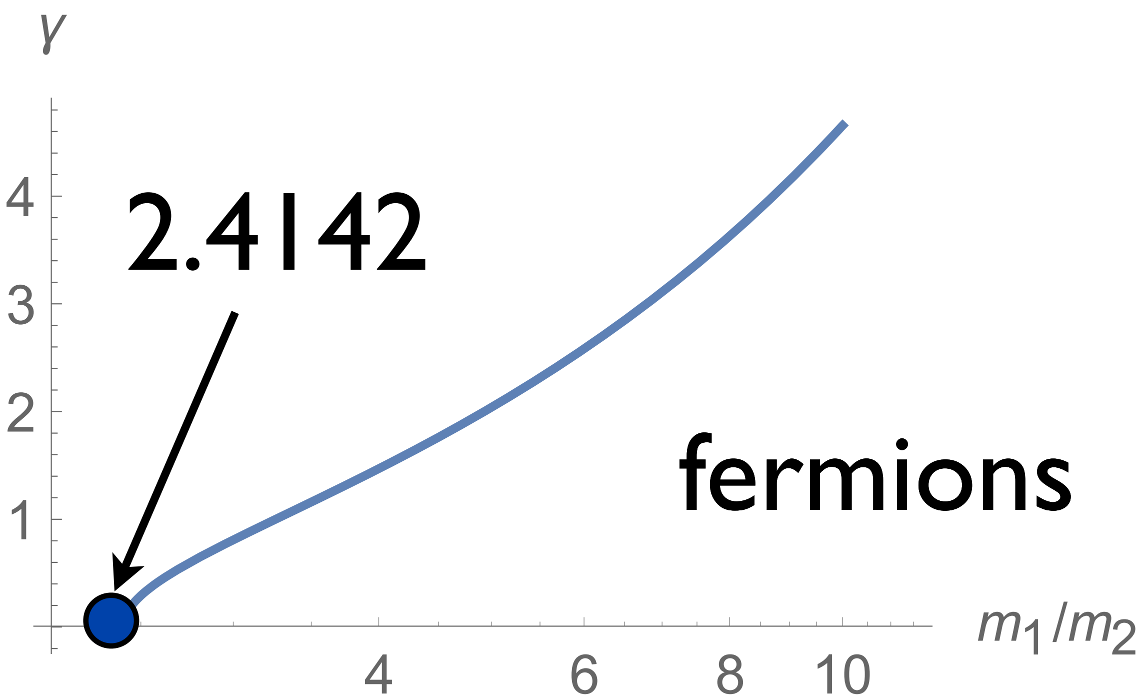
Identical particles are fermions or bosons

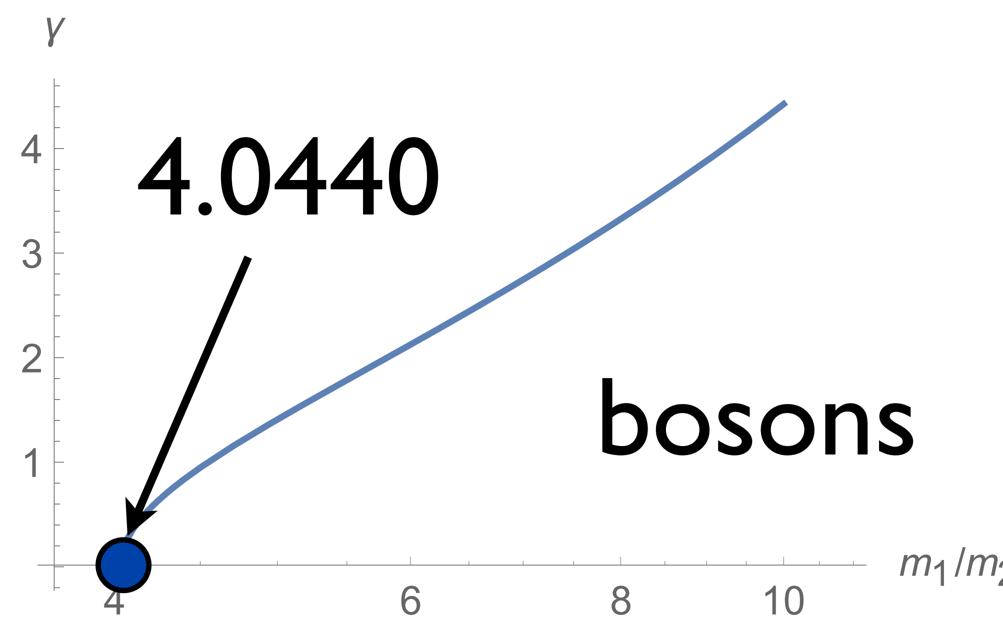
Non-resonant s-wave interactions allowed

Different masses

<u>Super Efimov spectrum</u>

$$E_n \propto \exp\left(-2e^{\pi n/\gamma+\theta}\right)$$





Much more interesting for experiments

Li-Cs mixture $\gamma \approx 10.7$

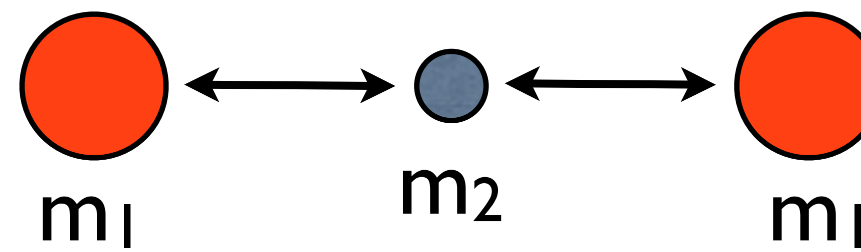
Born-Oppenheimer approximation

Induced heavy-heavy potential:

$$V(R) = -\frac{1}{m_2 R^2 \ln(R\Lambda)}$$

$$E_n^{(\mathrm{BO})} \propto \exp\left(-\frac{m_2 \pi^2}{2m_1} n^2\right)$$

p-wave resonance in 2d



identical heavy bosons/fermions

BO spectrum **differs** from super Efimov spectrum!

$$E_n \propto \exp\left(-2e^{2\frac{m_2}{m_1}\pi n + \theta}\right)$$

Failure of BO approximation

Petrov

Heavy particles time scale: $T_{\text{heavy}} \sim m_1 R^2$

Light particles time scale: $T_{\text{light}} \sim m_2 R^2 \ln(R\Lambda)$

Can not use adiabatic approximation if

$$T_{\text{light}} \gtrsim T_{\text{heavy}}$$

BO approximation breaks down for large distances

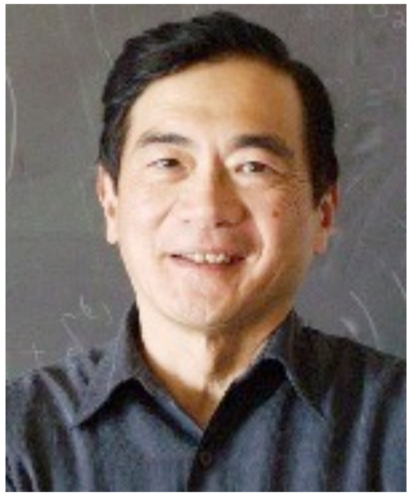
$$\boxed{R\Lambda \gtrsim e^{m_1/m_2}}$$

Conclusion

- Efimov physics is new few-body paradigm with experimental verification
- Super Efimov -- double exponential scaling
- Mixtures with high mass imbalance are favorable for experimental verification

Extra slides

Wen-Zee term



- Dual description of chiral superfluid

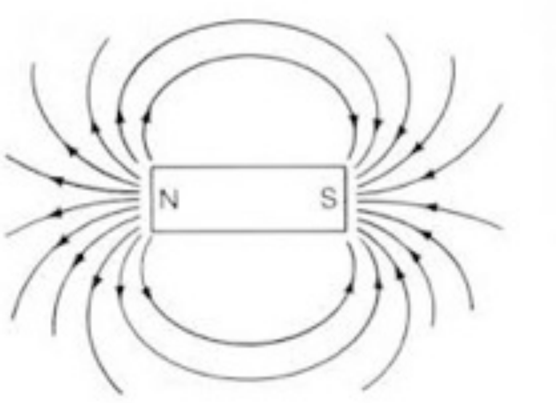
$$\mathcal{L}_{WZ} = -s\varepsilon^{\mu\nu\rho}\omega_\mu \partial_\nu a_\rho$$

- Encodes Hall viscosity and edge current
- Up to surface term: $\mathcal{L}_{WZ} = -s\left(a_t B_\omega - \varepsilon^{ij} a_i E_{\omega j}\right)$

$$B_\omega = \frac{1}{2}R, \quad E_{\omega i} = \frac{1}{2}\left[-\partial_t(\Gamma^k_{ij})\varepsilon^{jl} g_{kl} - \partial_i B\right]$$

Vielbein <u>eliminated</u> in dual electrodynamics

Gauge description



- Duality relation $J^\mu \equiv \varepsilon^{\mu\nu\rho}\partial_\nu a_\rho$
- Conservation of current: Bianchi identity
- Nonlinear dual electromagnetism in 2+1

$$\boxed{\mathcal{L}_{sf} = \frac{g^{ij}e_i e_j}{2b} - \epsilon(b) - \varepsilon^{\mu\nu\rho}A_\mu\partial_\nu a_\rho}$$

$$b \equiv \varepsilon^{ij}\partial_i a_j \qquad e_j \equiv \partial_t a_j - \partial_j a_t$$

Gauge description of s-wave superfluid

Do we need vielbeins?

Reminder: $D_\nu \theta \equiv \partial_\nu \theta - A_\nu - s\omega_\nu$

$$\omega_t \equiv \frac{1}{2}\Big(\epsilon^{ab} e^{aj} \partial_t e^b_j + B\Big)$$

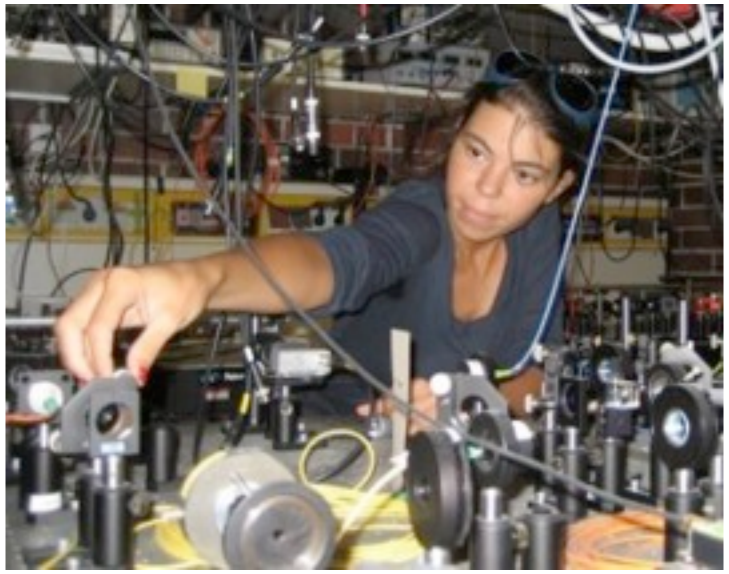
$$\omega_i \equiv \frac{1}{2}\epsilon^{ab} e^{aj} \nabla_i e^b_j = \frac{1}{2}\Big(\epsilon^{ab} e^{aj} \partial_i e^b_j - \varepsilon^{jk} \partial_j g_{ik}\Big)$$

We construct EFT of **bosons**, there are no fermions!

Can we write everything of terms of spatial metric?

$$g_{ij} = e^a_i e^a_j$$

Experiments



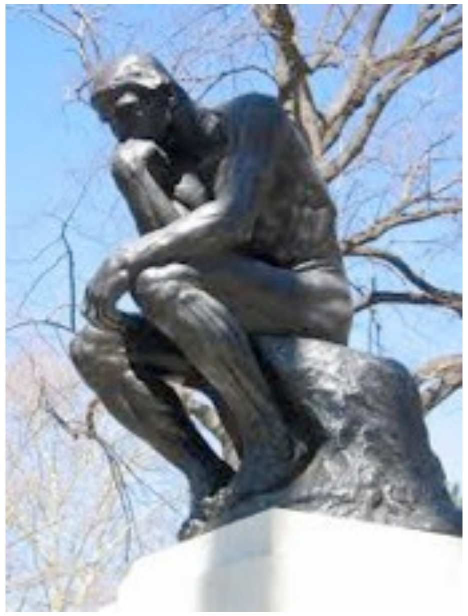
- Great success in three dimensions
- Quasi 2d fermions near resonance
- Trimers sizes:

n	GS	1	2	3
size	Å	μ	10^{38}m	10^{499}m

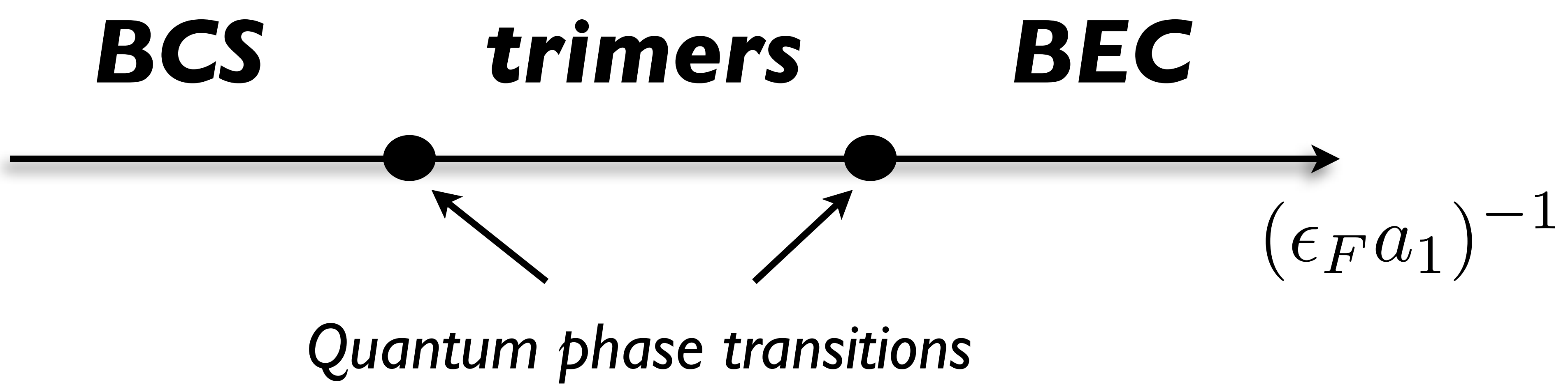
- No tuning possible in this theory!
- Problems with 3bd losses

Levinsen, Cooper, Gurarie

Many-body thoughts



- Broad res with $r_0 \to 0$: ideal mixture
- At least two parameters needed at T=0
- Trimer phase near resonance



Linear response

- Electromagnetic $J^i = \sigma_H(\omega, \mathbf{p})\epsilon^{ij}E_j$

$$\sigma_H(\omega, \mathbf{p}) = \frac{s\rho^{\mathsf{GS}}}{2}\frac{-\mathbf{p}^2}{\omega^2 - c_s^2\mathbf{p}^2}$$

- Gravitational $\delta T^{xy} = -i\omega\frac{\eta_H(\omega)}{2}(h_{xx} - h_{yy})$

$$\eta_H(\omega) = -\frac{s}{2}\rho^{\mathsf{GS}}$$

- Universal relation: $\boxed{\eta_H(\omega) = \frac{\omega^2}{2}\frac{\partial^2}{\partial p_x^2}\sigma_H(\omega, \mathbf{p})\Big|_{\mathbf{p}=0}}$

Hoyos, Son; Bradlyn et al

Few-body universality

- Low energies, short-range interactions

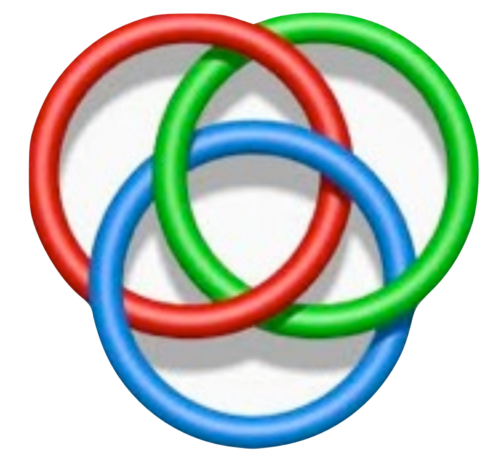
$$\downarrow$$

in 3d: scattering length a, ...

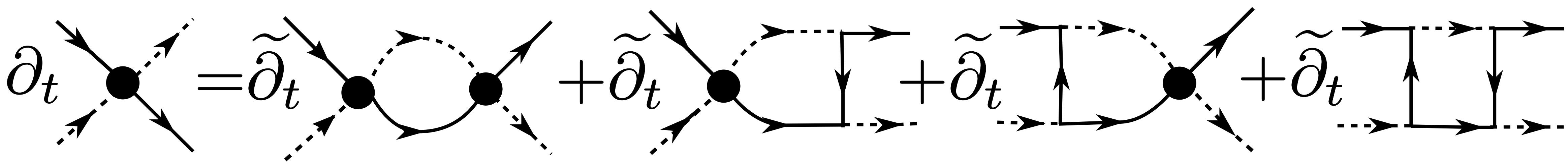
- <u>Universal regime</u>: a $\gg$ other length scales
- Two-body bound state near resonance

$$E_{\mathsf{D}} = \frac{\hbar^2}{ma^2} \quad \text{for} \quad a > 0$$

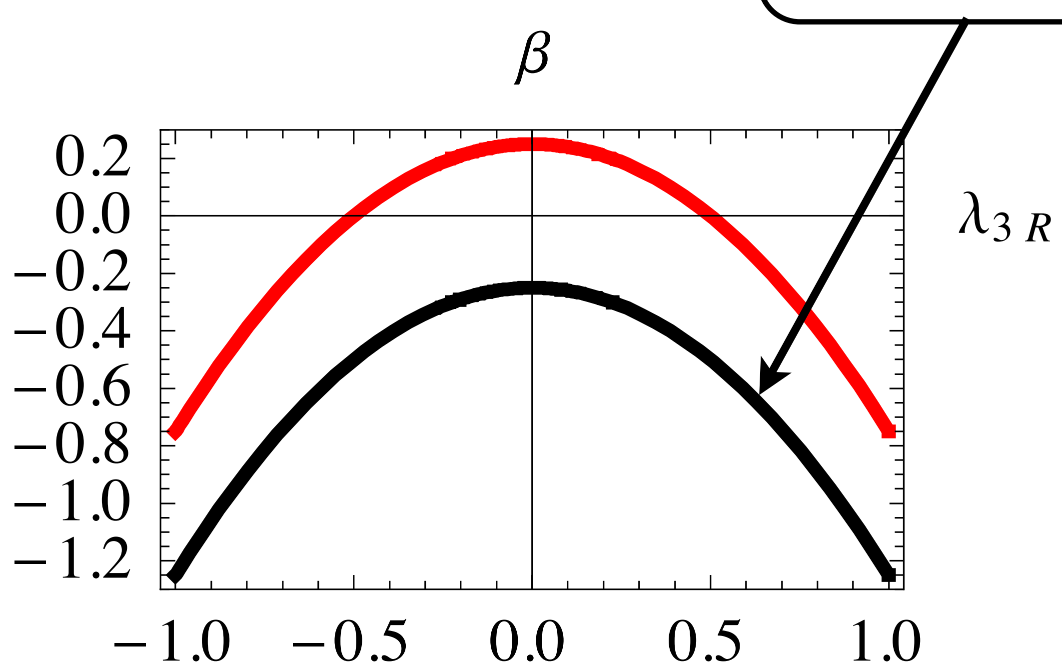
Efimov effect from RG

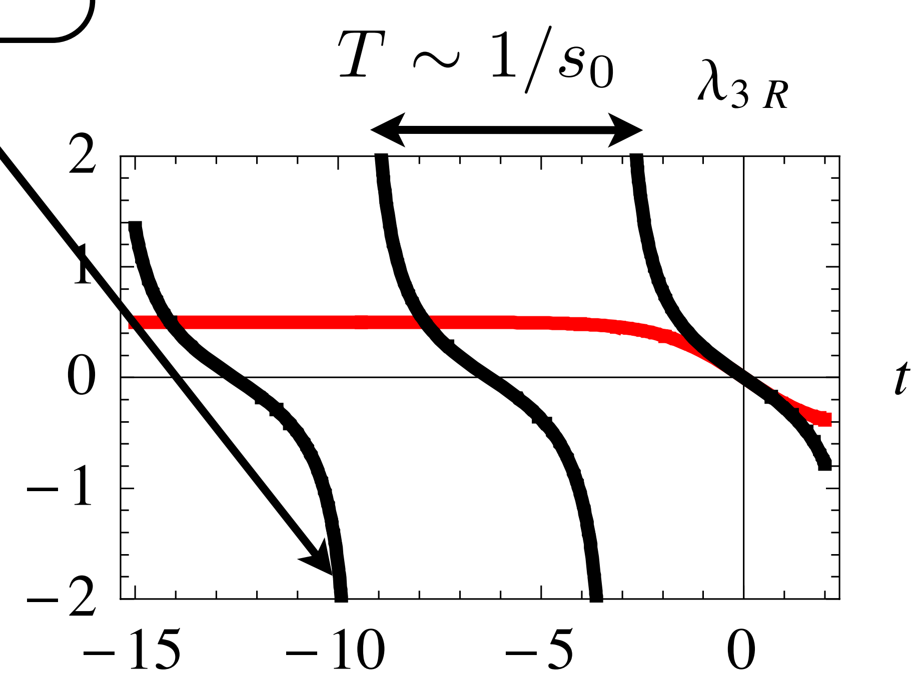


Flow of atom-dimer vertex: RG=one-loop diagrams



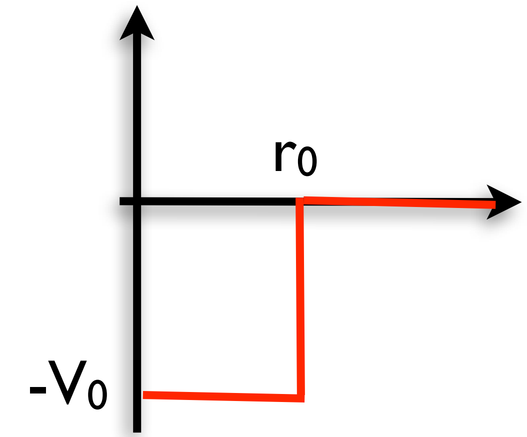
bosons in 3d





limit cycle

Two-body solution



- Spherical well: $$\frac{dJ_l(kr)/dr}{J_l(kr)} = \frac{dK_l(\kappa r)/dr}{K_l(\kappa r)}$$

$$l = 0$$

No threshold

$$\kappa \sim e^{-\frac{2}{V_0 r_0^2}}$$

Cooper-like form

$$l = 1$$

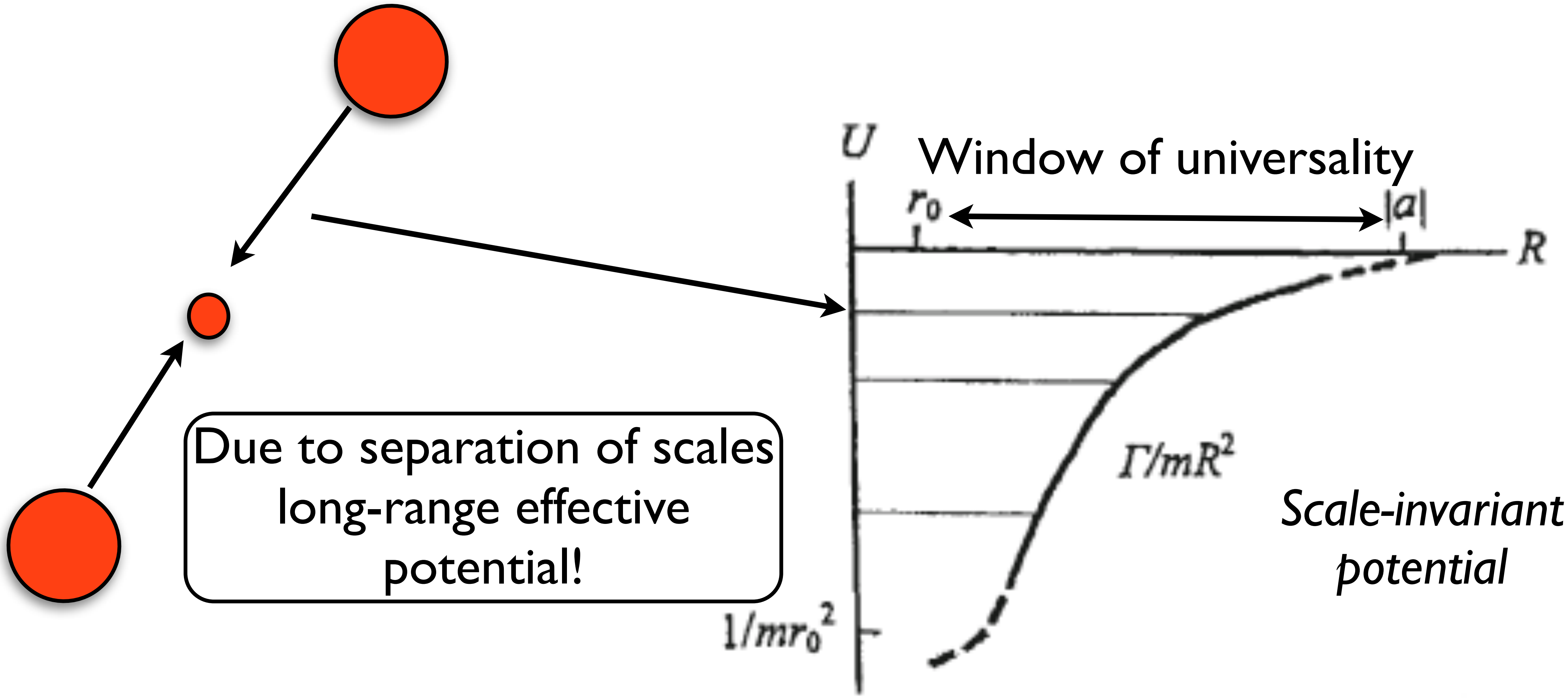
Finite threshold!

$$\psi(r) = \frac{\kappa}{\sqrt{2\pi}} \frac{K_1(\kappa r)}{\sqrt{\ln(\kappa r_0)}}$$

local dimers for $r_0 \rightarrow 0$

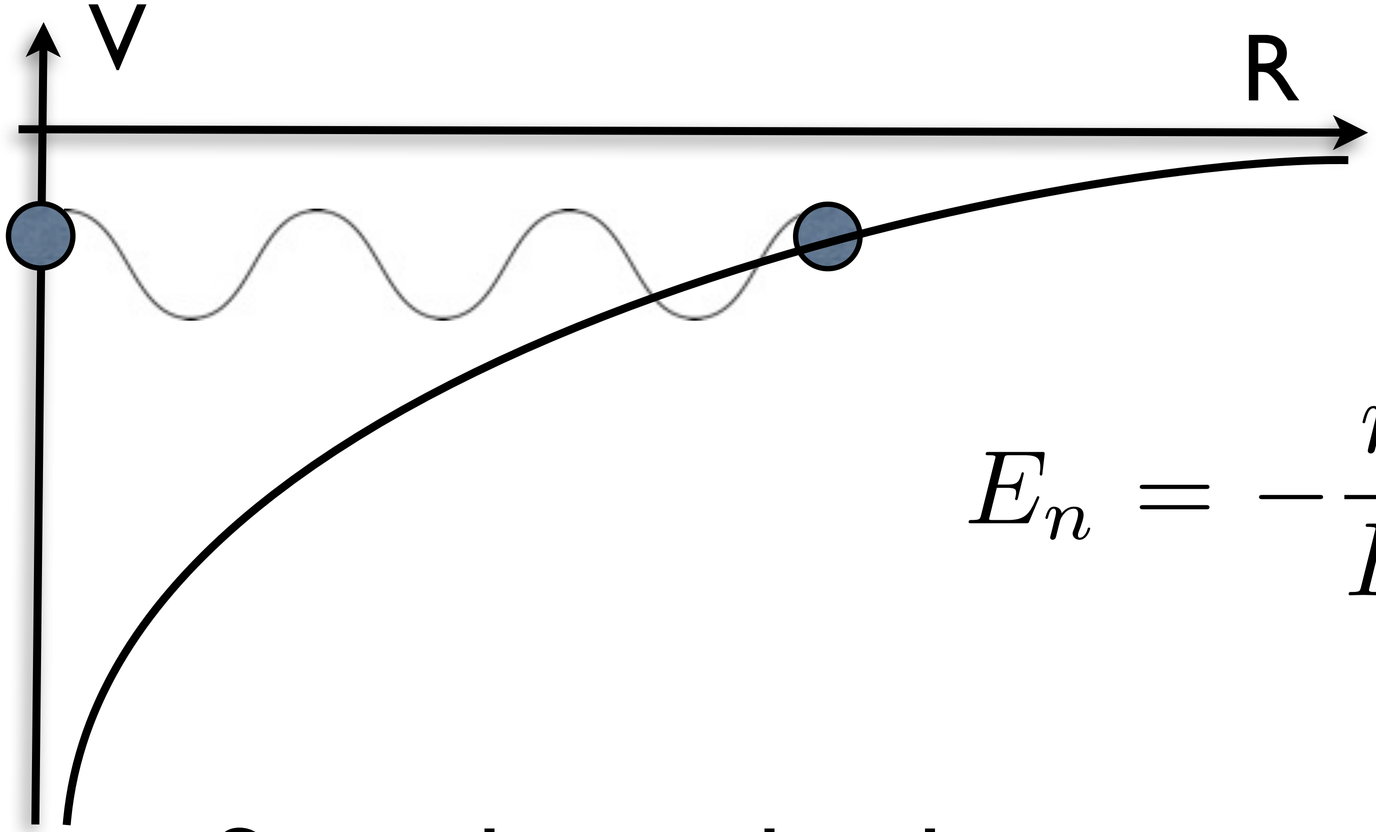
Basics intuition

- How can short-range forces create infinite number of bounds states?
- Born-Oppenheimer approximation:



Effective potential

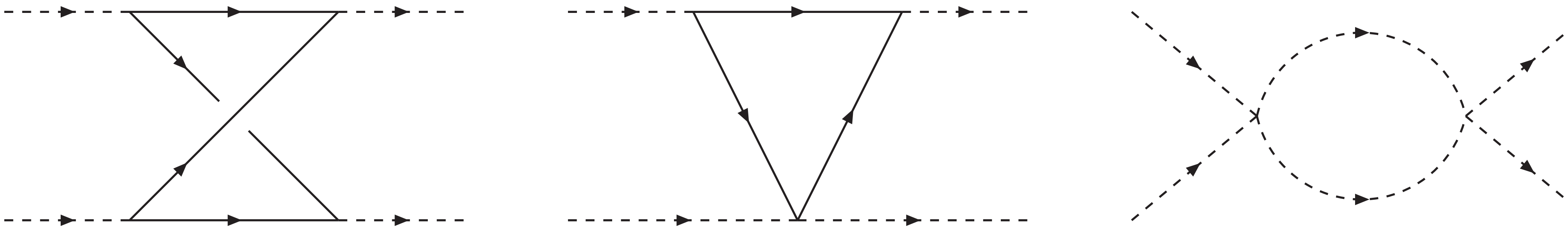
$$V(R) = -\frac{1}{4R^2} - \frac{1/4 + r^2}{(R \ln \frac{R}{R0})^2}$$



$$E_n = -\frac{r^2}{R_0^2} \frac{\exp\left(- 2e^{\frac{\pi}{r} n + \theta} \right)}{e^{2\left(\frac{\pi}{r} n + \theta\right)}}$$

Semiclassical solution

Tetramer states



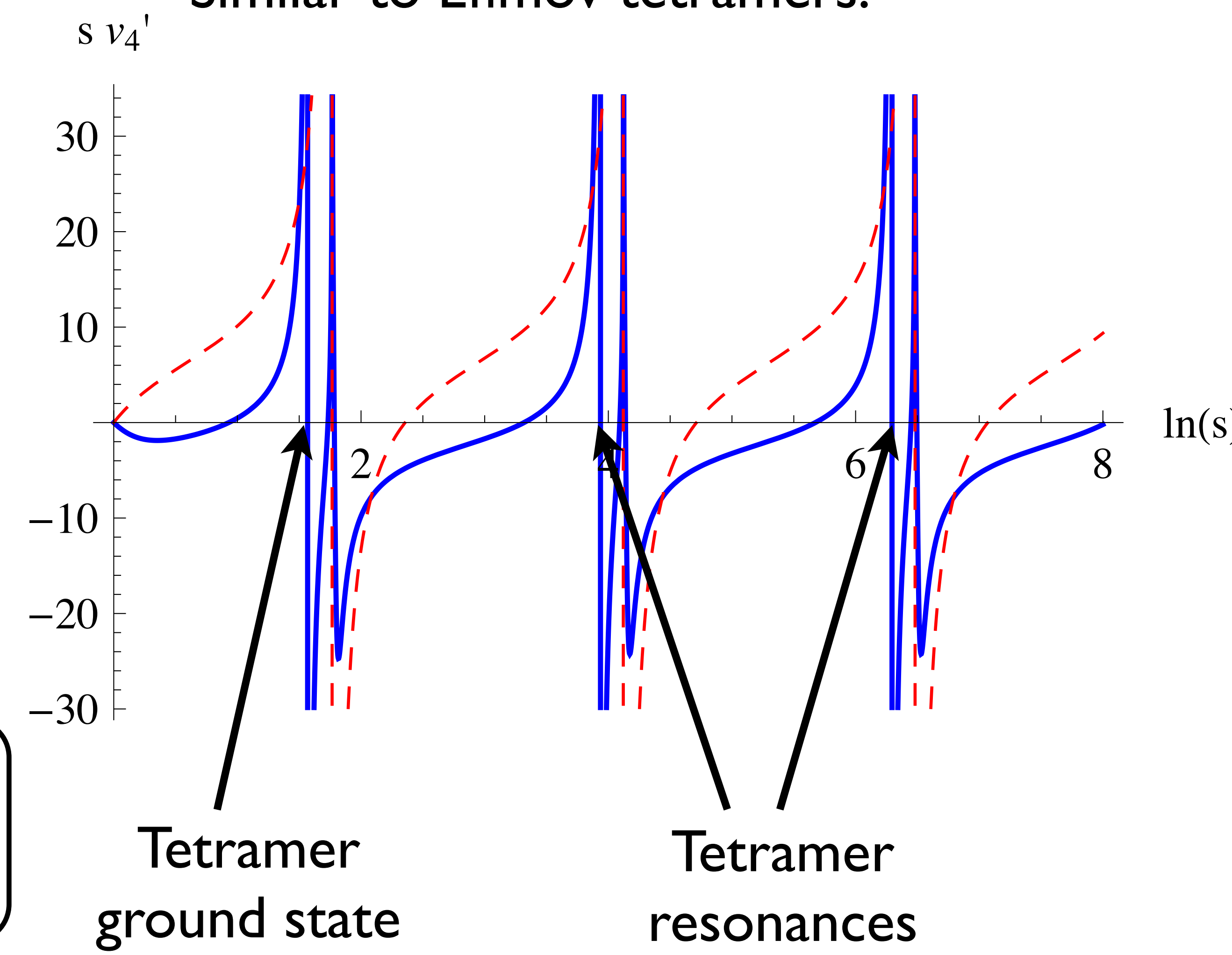
I=2 sector:

$$\frac{dv_4'}{ds} = -\frac{4g^4}{\pi} + \frac{2g^2 v_3}{\pi} - \frac{2g^2 v_4'}{\pi} + \frac{2v_4'^2}{\pi}$$

- Numerical solution necessary
- Singularities understood analytically

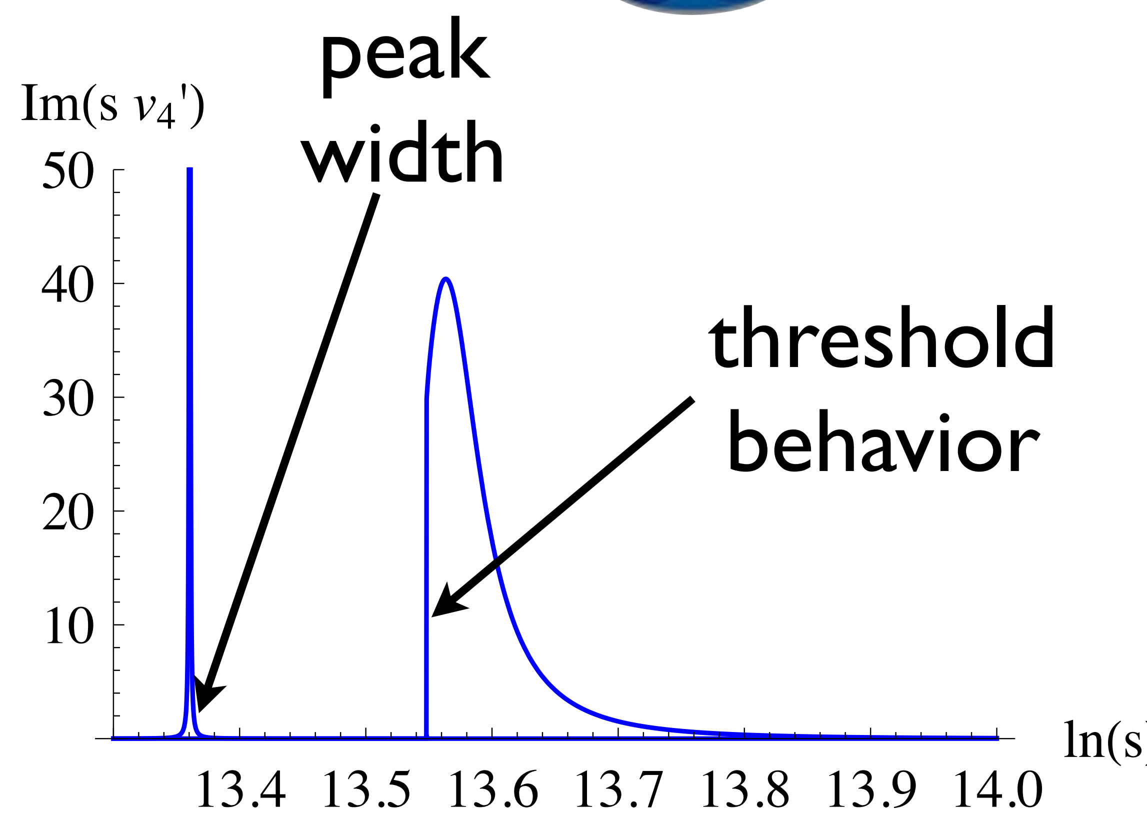
$$E_4^{(n)} \propto \exp\left(-2e^{3\pi n/4+\theta-0.188}\right)$$

Similar to Efimov tetramers:



Open questions

- Decay of tetramers



- Higher-body clusters: RG $\longrightarrow$ there are no
- Beyond mean-field phase diagram