Magnetic Crystals and Helical Liquids in Alkaline-Earth 1D Fermionic Gases

Leonardo Mazza

Scuola Normale Superiore, Pisa

Seattle – March 24, 2015

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  - Luca Taddia
  - Davide Rossini
  - Rosario Fazio

Italian Government – Regione Toscana – Scuola Normale Superiore, Pisa

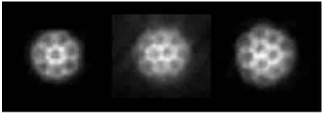
Outline

Outline

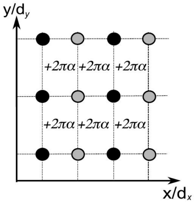
Synthetic Gauge Potentials with Cold Atoms

Coupling **neutral** atoms to a **magnetic** field

Rotating gases
Chevy, Dalibard, Ketterle, Foot, Cornell (2001-2003)



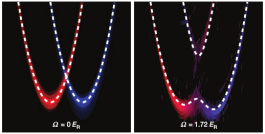
Optical lattices:
Light-induced potential & Shaking
Jaksch, Zoller, Gerbier, Dalibard, Juzeliunas, Öhberg, Lewenstein, Ruostekoski, Dunne, Javainen Bloch, Ketterle, Spielman



Artificial **spin-orbit** coupling for atomic gases

Rashba potential for effective spin-1/2 bosons and fermions

Spielman, Zwierlein, Zhang, Lewenstein, Zoller



Goals

1. quantum Hall effect
2. **novel strongly-correlated phases of matter**

Experimental and theoretical challenge:

to identify models and setups where the **interplay** of gauge potentials and interactions is crucial

Fractional Helical Liquids

PHYSICAL REVIEW B **89**, 115402 (2014)

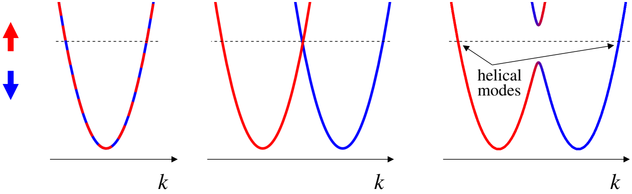
Fractional helical liquids in quantum wires

Yuval Oreg,[1] Eran Sela,[2] and Ady Stern[1]
[1]*Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot, 76100, Israel*
[2]*Raymond and Beverly Sackler School of Physics and Astronomy, Tel-Aviv University, Tel Aviv, 69978, Israel*
(Received 13 January 2014; published 4 March 2014)

Quantum wire:
Spin-$\frac{1}{2}$ 1D Fermi system (electrons)

Example: A non-interacting (integer) helical liquid

1. Free fermions
2. Free fermions + Rashba coupling ($\hat{z}$)
3. Free fermions + Rashba coupling ($\hat{z}$) + magnetic field ($\hat{x}$)



Result:
In presence of interactions the system develops **helical phases** at **fractional fillings**

$$\frac{k_F}{k_{SO}} = \frac{1}{2n+1} \quad \leftarrow$$ interplay of gauge potential and interactions

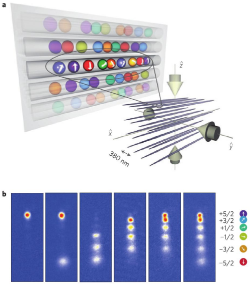
Earth-Alkaline-Like Gases

LETTERS
PUBLISHED ONLINE: 2 FEBRUARY 2014 | DOI: 10.1038/NPHYS2878

nature physics

A one-dimensional liquid of fermions with tunable spin

Guido Pagano[1,2], Marco Mancini[1,3], Giacomo Cappellini[1], Pietro Lombardi[1,3], Florian Schäfer[1], Hui Hu[4], Xia-Ji Liu[4], Jacopo Catani[1,5], Carlo Sias[1,5], Massimo Inguscio[1,3,5] and Leonardo Fallani[1,3,5]*



1D gas of Yb^{173}

Nuclear spin: $I = 5/2$

- $(2I + 1)$-component Fermi gas
- $SU(2I + 1)$-invariant contact interaction

Experimental probe of 1D SU(2) – SU(6) models

New physics beyond $SU(2)$ model of electron liquids accessible

Other works on SU(N) gases → Yb: Takahashi, Bloch — ^{87}Sr: J.Ye

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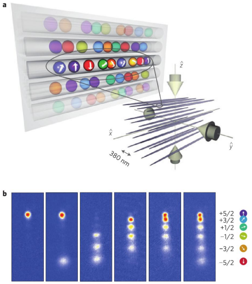

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This talk: $SU(2I + 1)$ Fermi gas

1. Rashba coupling
2. Orthogonal magnetic field
3. Interactions

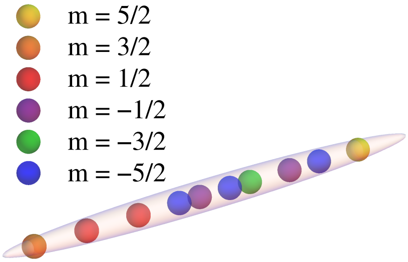
Helical phases? New exotic phases?

Outline

The Model

- **Fermi gas with nuclear spin-I in 1D optical lattice**

$$\hat{\mathcal{H}}_0 = -t \sum_{i,m} \left[\hat{c}^{\dagger}_{j+1,m} \hat{c}_{j,m} + \text{H.c.} \right] + \hat{\mathcal{H}}_{\text{int}} \quad \longrightarrow \quad \text{SU}(2I+1) \text{ invariant model}$$



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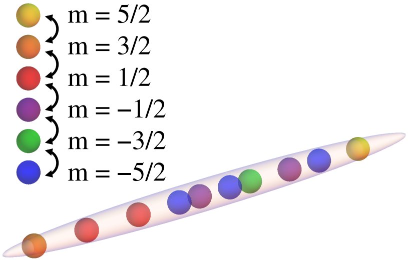

- **Raman coupling which locally flips the spin**

$$\hat{\mathcal{H}}_{\text{raman}} = \sum_j \sum_{m=-I}^{I-1} \left[\Omega_m e^{-i\gamma j} \hat{c}^\dagger_{j,m+1} \hat{c}_{j,m} + \text{H.c.} \right]$$

This term breaks SU$(2I+1)$ invariance

- Unitarily equivalent to a SU$(2I+1)$ Fermi gas
  with Rashba spin-orbit coupling $(\hat{z})$ and magnetic field $(\hat{x})$

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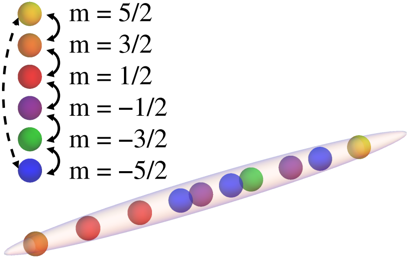

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$$+ \left[\Omega_{MP} e^{-i\gamma j} \hat{c}^\dagger_{j,-I} \hat{c}_{j,I} + \text{H.c.} \right]$$

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- Multiphoton couplings possible expecially for small I

See discussion on feasibility: Celi et al. PRL (2014)

Fully Gapped Phases

Fractional Insulators: $$\nu \equiv \frac{n}{\frac{\gamma}{2\pi}(2l+1)} = \frac{p}{q}$$

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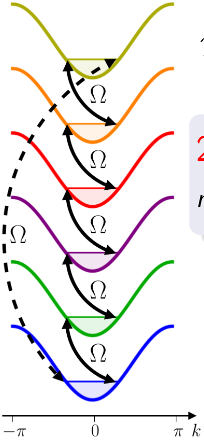

$$\hat{\mathcal{H}}_0 = -2t \sum_{k,m} \cos[k] \hat{c}^\dagger_{k,m} \hat{c}_{k,m}$$

$$\hat{\mathcal{H}}_{\text{raman}} = \Omega \sum_{k,m} \hat{c}^\dagger_{k,m} \hat{c}_{k+\gamma,m+1}$$

$$2k_F = \gamma$$

$$n = (2I+1)\frac{k_F}{\pi} \quad \Rightarrow \quad \nu = 1$$

- $q = 1$ can be observed without interactions

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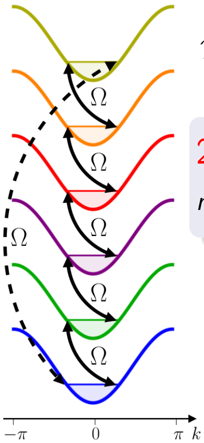

$$2k_F = \gamma$$

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- $q = 1$ can be observed without interactions
- $q > 1$ can be studied with:
  - bosonization (for q odd)
  - numerical methods: DMRG & MPS

Bosonization hint:

For high values of q longer-range interactions are necessary

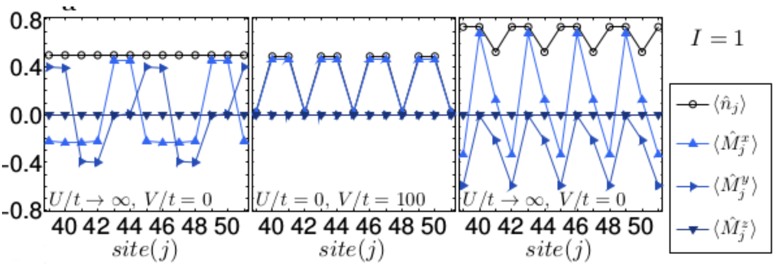
Methods borrowed from Kane, Teo, Lubensky, Mukhopadhyay, Stern, Oreg, Sela

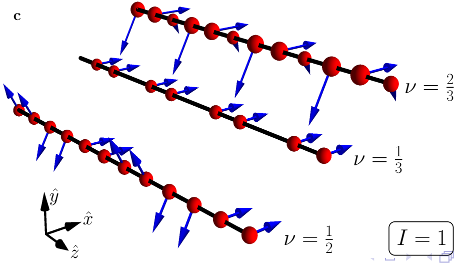
No numerical characterization so far (to the best of my knowledge!)

$$\hat{\mathcal{H}}_{\text{int}} = U \sum_j \sum_{m<m'} \hat{n}_{j,m} \hat{n}_{j,m'} + V \sum_j \hat{n}_j \hat{n}_{j+1}$$

DMRG simulations, 96 sites — (central region plot) — $\Omega_m = \Omega_{MP} = t$

$\nu = \frac{1}{2}$ $\nu = \frac{1}{3}$ $\nu = \frac{2}{3}$





Choosing the Proper Spin States

Trick: Let's work in the basis which diagonalizes $\hat{\mathcal{H}}_{\text{raman}}$

$$\hat{\mathcal{H}}_{\text{raman}} = \Omega \sum_j \sum_{m=-I}^{I-1} \left[e^{-i\gamma j} \hat{c}^\dagger_{j,m+1} \hat{c}_{j,m} + \text{H.c.} \right] + \left[e^{-i\gamma j} \hat{c}^\dagger_{j,-I} \hat{c}_{j,I} + \text{H.c.} \right]$$

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New Basis:

$$\hat{d}_{j,A} = \frac{1}{\sqrt{3}} \left(\hat{c}_{j,m=0} + \hat{c}_{j,m=1} + \hat{c}_{j,m=-1} \right)$$

$$\hat{d}_{j,B} = \frac{1}{\sqrt{3}} \left(\hat{c}_{j,0} + \omega \hat{c}_{j,1} + \omega^2 \hat{c}_{j,-1} \right)$$

$$\hat{d}_{j,C} = \frac{1}{\sqrt{3}} \left(\hat{c}_{j,0} + \omega^2 \hat{c}_{j,1} + \omega \hat{c}_{j,-1} \right)$$

$$\omega = e^{i2\pi/3}$$

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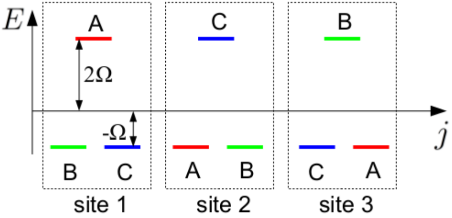
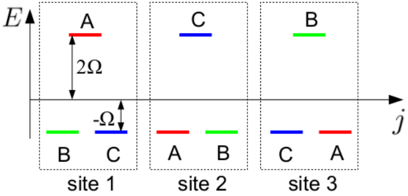
$\hat{\mathcal{H}}_{\text{raman}} =$



$\hat{\mathcal{H}}_0$ is SU(3) invariant

- hopping conserves the new species
- density-density interaction

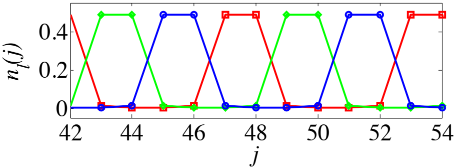


DMRG simulations, L=96, $\gamma = \frac{2\pi}{3}$

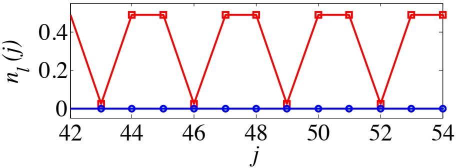
$$\nu = \frac{N/L}{\frac{\gamma}{2\pi}(2I+1)} = \frac{1}{2}, \quad \frac{1}{3}, \quad \frac{2}{3}$$

- Charge ordering not given by the interaction only
- Magnetic ordering imprinted by $\hat{\mathcal{H}}_{\text{raman}}$
- Magnetic ordering typical of $\Omega/t \gg 1$ even at $\Omega/t = 1$

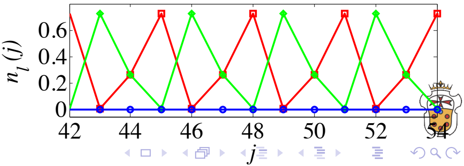
$\nu = 1/2$ $U/t = \infty,\ V/t = 0$



$\nu = 1/3$ $U/t = \infty,\ V/t = 100$

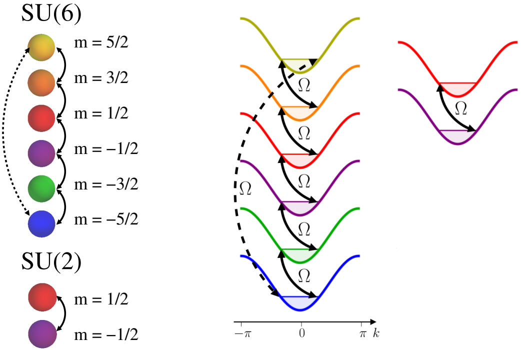


$\nu = 2/3$ $U/t = \infty,\ V/t = 0$



The case of SU(2)

Is it possible to have gapped phases for SU(2)?



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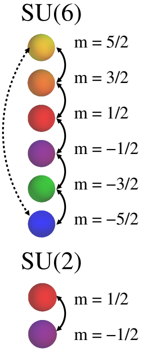
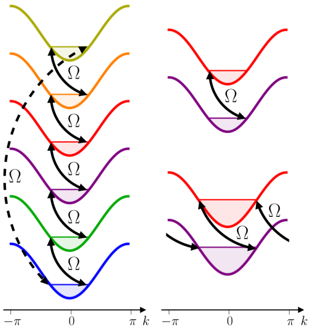


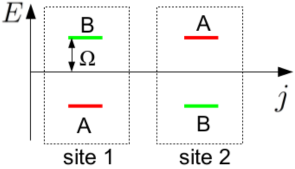


- No additional Raman coupling available
- Gapped phases possible via lattice effects

$$\gamma = 2k_F$$

$$k_F + \gamma = 2\pi - k_F$$

$$\boxed{\gamma = \pi \qquad k_F = \frac{\pi}{2}}$$



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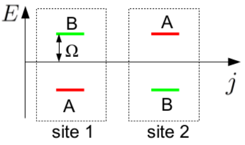
SU(2): Numerical Results for $\gamma = \pi$



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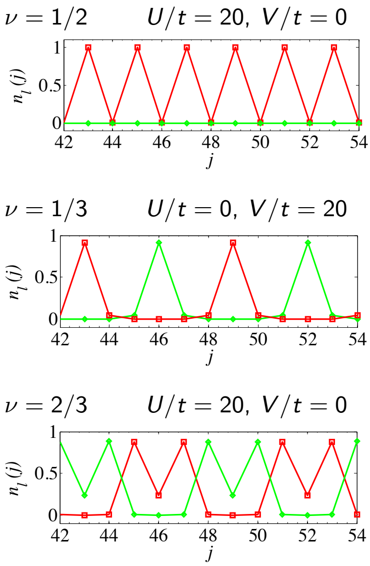

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Helical Liquids

Problem:
Multi-Raman couplings might be difficult to implement, especially for large I

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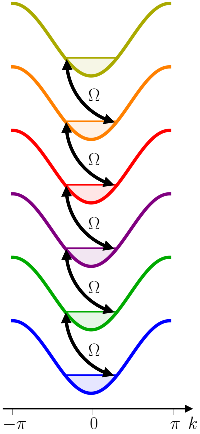

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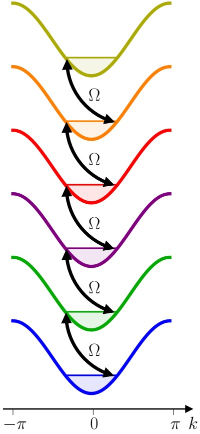

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- $q = 1$ can be understood without interactions
  - gapless modes with spin-momentum relation
- $q > 1$: interactions are needed, bosonization useful for q odd
- SU(2): $\gamma \neq \pi$ is necessary

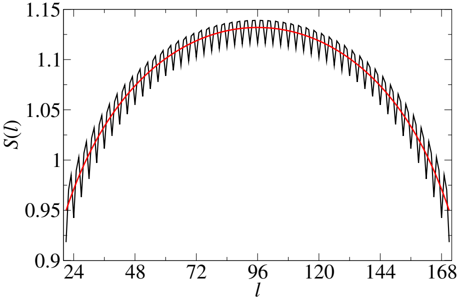
How to diagnose an helical liquid?

1. Entanglement entropy: $c = 1$ (via Calabrese-Cardy formula)
2. Current pattern (experimentally observable!)

SU(3): Some Results

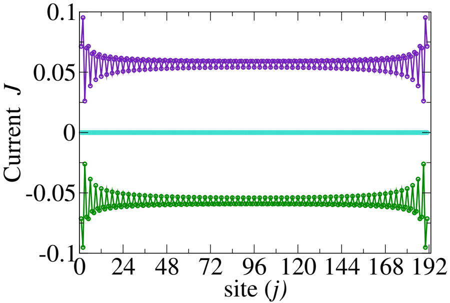
- L=192 and N=64
- $\gamma = \frac{2\pi}{3}$
- $\Omega/t = 1$ and $U/t \to \infty$

Entanglement Entropy



$S(\ell)$ fitted via $S(\ell) = A + \frac{c}{6} \ln \left[\frac{2L}{\pi} \sin \left(\frac{\pi \ell}{L} \right) \right]$
$A = 0.32...$ and $c = 1.0055...$

Currents Profile



- Clear helical-current patterns
- Bulk oscillations → zero for $L \to \infty$

Full phase diagram depending on interaction strength to be fully explored!

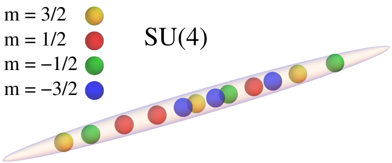
Outline

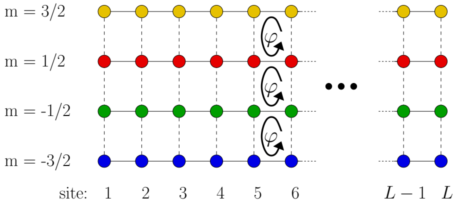
1. Introduction
2. Exotic Gapped Phases
3. Helical Liquids
4. Discussion and Conclusions

Relation to the Quantum Hall Effect

Spin-orbit coupled SU(N) 1D atomic gas is equivalent to
a spinless stripe model with magnetic field

Equivalence first pointed out by Celi et al. PRL (2014)





Multiphoton Raman coupling
is equivalent to the periodic boundary conditions for the stripe

Are the crystals and liquids we found related to the quantum Hall effect?

1. **Fillings**: $\nu = \dfrac{N}{\frac{\gamma}{2\pi}L(2I+1)} = \dfrac{N}{N_\Phi}$ $\rightarrow$ relation with Laughlin's states?

QHE in the Thin Torus Limit: $L_y \lesssim \ell_B$
Laughlin's states adiabatically connected to Charge Density Waves

Earth-alkaline(-like) gases accessing the Thin Torus Limit of the QHE

Relation to the Quantum Hall Effect

Are the crystals and liquids we found related to the quantum Hall effect?

1. **Helical states**: relation with the edge modes of the Laughlin's states?
   - What is the conductance associated to these states?
     First hints by Stern for SU(2) say that it is fractional
   - Is it possible to relate the bulk properties of the helical liquids with those of the crystals?

2. Is this **mathematically equivalent** to the "true" quantum Hall effect?
   - **NO! Extremely anisotropic interaction!**

     $$\hat{\mathcal{H}}_{\text{int}} = U \sum_j \sum_{m<m'} \hat{n}_{j,m} \hat{n}_{j,m'}$$

     - infinite-range in the synthetic direction
     - zero-range in the real direction

   No relation with the Coulomb interaction typical of quantum Hall

Conclusions

SU(N) Fermi gas in 1D

1. Rashba spin-orbit coupling
2. Perpendicular magnetic field
3. Interactions

- Insulating states
  - Crucial multi-photon coupling
  - Crystal phases
  - Simple understanding in terms of new spin basis
- Helical liquids
  - c=1 entanglement entropy
  - Current pattern
- Relations to the quantum Hall effect
  - Thin-torus limit of a non-standard quantum Hall effect

Barbarino, Taddia, Rossini, LM , Fazio, soon in the arXiv