INT “Frontiers in Quantum Simulation with Cold Atoms” workshop
talk, UW Seattle, April 28, 2015

Chiral Bose and Fermi phases in optical lattices

W. Vincent Liu

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Acknowledgement

This talk is based on work:

- Nature Physics 7, 101 (2011) [Background and perspective (news & views), with M. Lewenstein]
- Nature Communications 5:3205 (2014a)
- Nature Communications 5:5064 (2014b)

People

Xiaopeng Li	(former Pitt student)
Bo Liu	(Pitt postdoc)

Pitt Student/Xiaopeng Li

- KITP Santa Barbara Grad Fellow (Spring 2013 term)
- 2013 APS March meeting, invited talk
- Moved in 10/2013, → U of Maryland, JQI Postdoc Fellowship

External collaborator

Arun Paramekanti	(Toronto)
Biao WU	(Peking U)
Exp: A. Hemmerich	(Hamburg)

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*DARPA-OLE-Rice/**Hulet** team (ended in 2014)*
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and
China NSF Overseas Scholar Collaborative Program
(2+4 years, through 12/**2018**; sponsor: **Peking Univ/Prof. Biao Wu**)

Outline

1. Highlights of recent research work – to stimulate discussion in the week

2. Some New Progress in Orbital Optical Lattices
   - ✧ Introduction
   - ✧ Boson: Chiral Bose liquid
   - ✧ Fermion: p-wave pair superfluidity without p-wave interaction

3. Conclusion

Selected recent results by our group

1. **Magnetic Skyrmions in electronic oxide STO/LAO interface:** Xiaopeng Li, WVL, Leon Balents, ***PRL*** (Feb 2014). Selected as Research Highlight by ***Nature Nanotechnology*** (April 2014)

2. **Prediction and Detection of p+ip chiral BEC**: Xiaopeng Li, A. Paramekanti, A. Hemmerich, WVL*, ***Nature Communications*** 5:3205 (Feb 2014) [*corresponding author] This talk!

3. **Chiral superfluidity with p-wave symmetry from an interacting s-wave atomic Fermi gas:** Bo LIU, X. Li, Biao WU, and WVL*, ***Nature Communications*** 5:5064 (Sep 2014) . [*corresponding author] This talk!

4. **Weyl superfluidity in a 3D dipolar Fermi gas.** Bo Liu, X. Li, L. Yin, and WVL, ***Phys. Rev. Lett.*** 114, 045302 (28 January 2015)

5. *Newly published:* **Spin-orbital exchange of interacting fermions on the p-band of optical lattice**: Z. Zhou, Erhai Zhao and WVL, ***Phys. Rev. Lett.*** 114, 100406 – Published 13 March 2015

6. *Newly published:* **Spontaneous quantum Hall effect in an atomic spinor Bose-Fermi mixture.** Zhi-Fang Xu, X. Li, P. Zoller, and WVL, ***Phys. Rev. Lett.*** 114, 125303 – Published 27 March 2015

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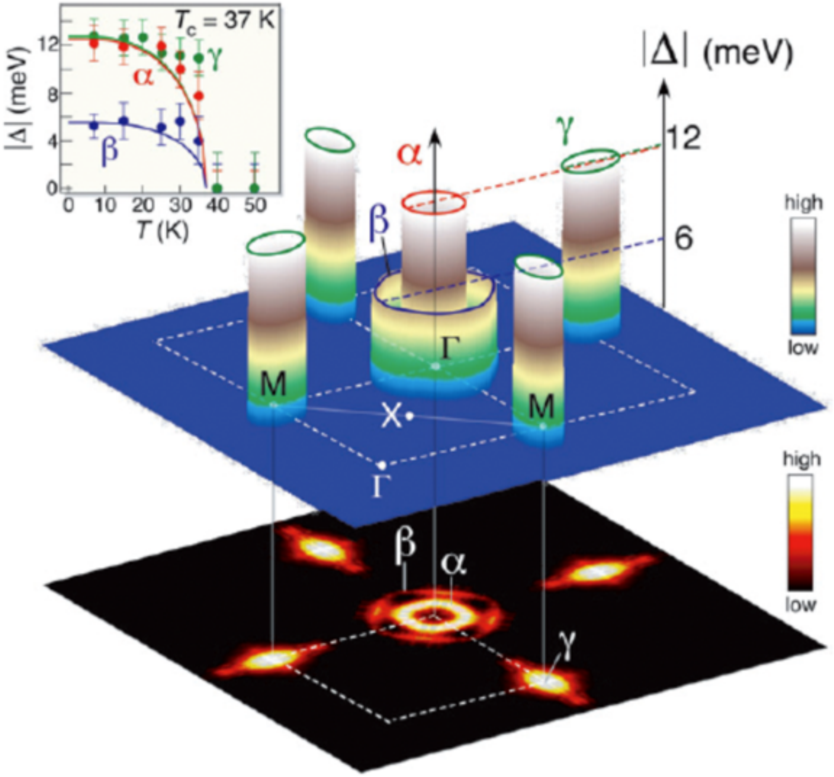
Orbital degrees of freedom in solids

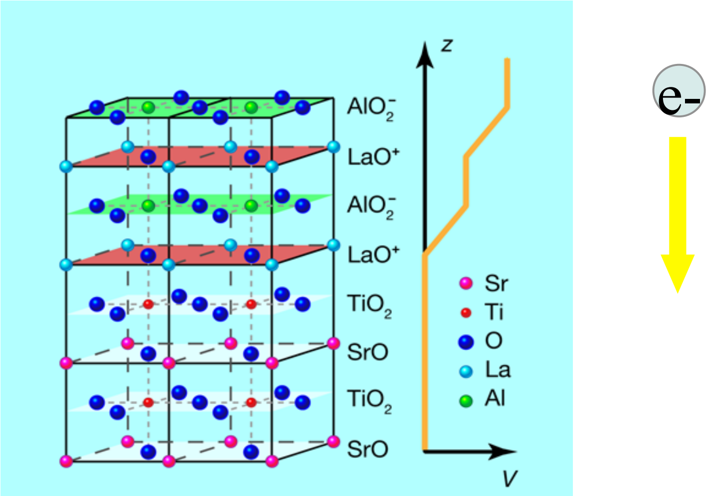
(skip all early studies of orbital physics, but focus on recent trends)

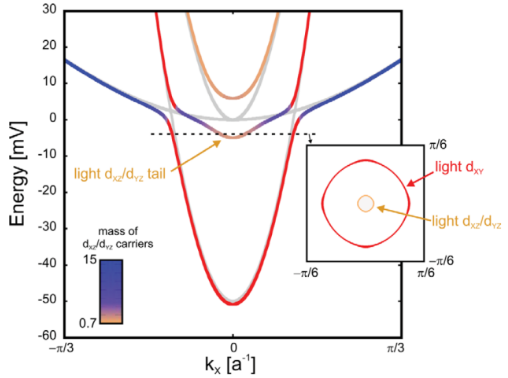
-iron-based superconductors



Michael R. Norman, Physics 1, 21 (2008)

-LAO-STO oxide heterostuctures

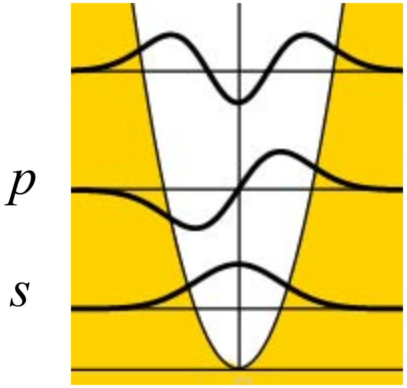


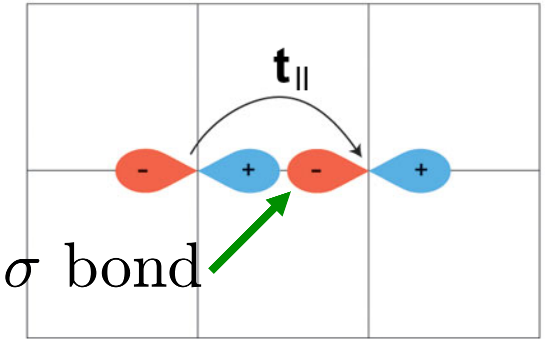


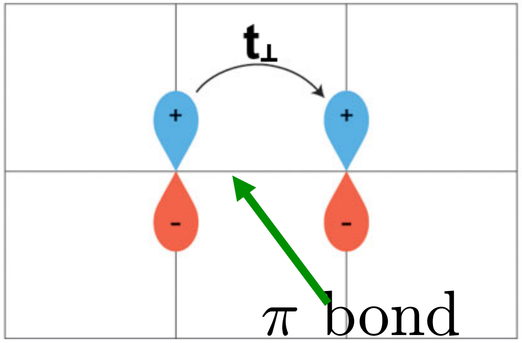
J. Kroha, PRL viewpoint, Physics (2011)

X. Li, WVL and Leon Balents, PRL (2014)

Orbital degrees of freedom in optical lattices



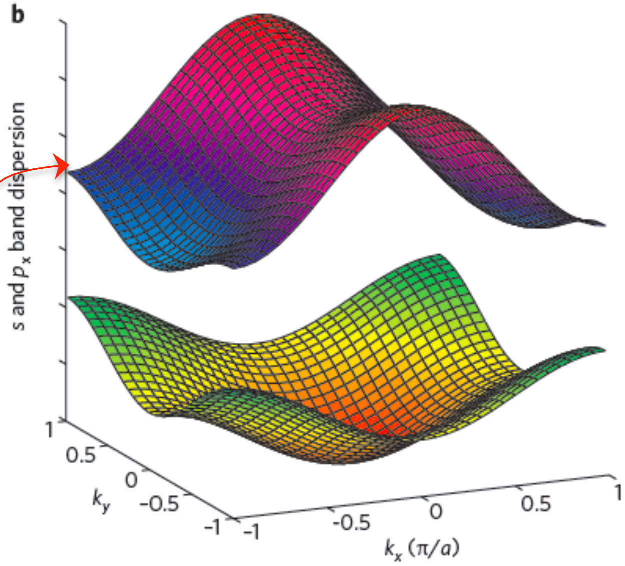




These orbitals "feel" crystal fields!

Early theoretical work on p-band boson

- **A. Isacsson and S. M. Girvin**, PRA 72, 053604 (2005);
- **WVL and C. Wu**, Phys. Rev. A 74, 013607 (2006);
- **A.B. Kuklov**, PRL 97, 110405 (2006)



p-band

- **This talk**

Lewenstein & WVL, Nature Phys. (2011)

Chiral Bose p+ip phase driven by interaction

[WVL and C. Wu, PRA (2006)]

- Repulsive interaction favors spontaneous rotation order $H_{int} = \frac{1}{2}U\sum_{\mathbf{r}}[n_{\mathbf{r}}^2 - \frac{1}{3}\mathbf{L}_{\mathbf{r}}^2]$

Density field operator: $n_{\mathbf{r}} = \sum_{\mu} b^{\dagger}_{\mu\mathbf{r}} b_{\mu\mathbf{r}}$

Angular momentum operator: $L_{\mu\mathbf{r}} = -i\sum_{\nu\lambda} \epsilon_{\mu\nu\lambda} b^{\dagger}_{\nu\mathbf{r}} b_{\lambda\mathbf{r}}$

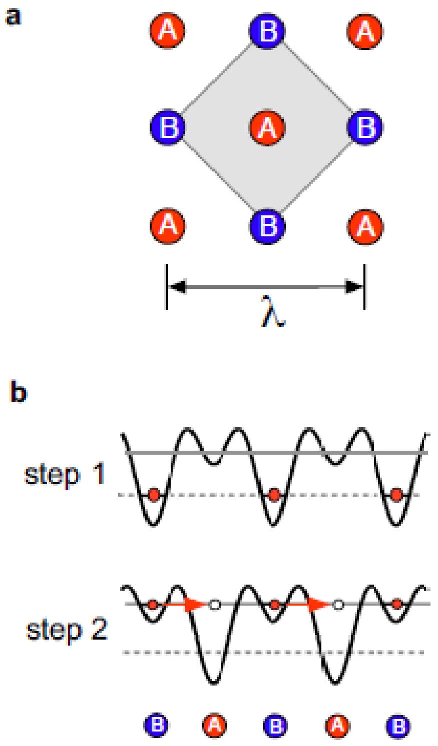
$\mu, \nu = x, y, z$ or $p_{\mathrm{x}}, p_{\mathrm{y}}, p_{\mathrm{z}}$

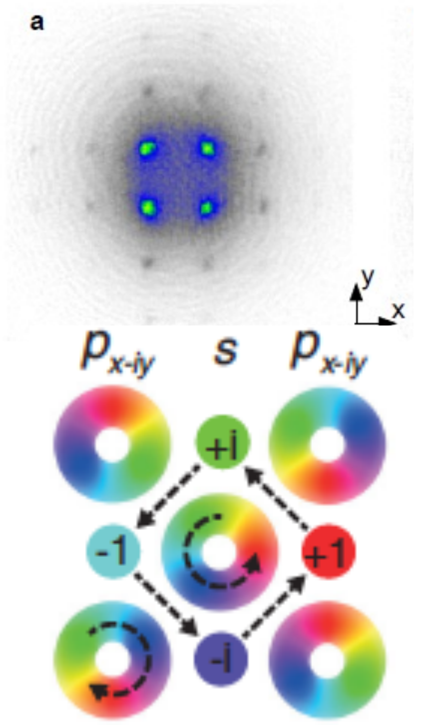
… ***leads to***

- $p_x + ip_y$ angular momentum ordered BEC (breaks T-symmetry)

Recall: Condense at Finite linear momentum

Experiment of p- and f-band bosons – double well lattices





complex $p_x \pm ip_y$

Hamburg/ A. Hemmerich group

First observation of p-band BEC with C4 symmetry and hence orbital degeneracy

- Early observation: finite momentum BEC, single p-band by **Bloch group** [T. Mueller, I. Bloch, et al, PRL, 2007]
- Even earlier p-band fermion observed in Feshbach crossing "accidentally" M. Köhl et al, PRL **94**, 080403 (2005)

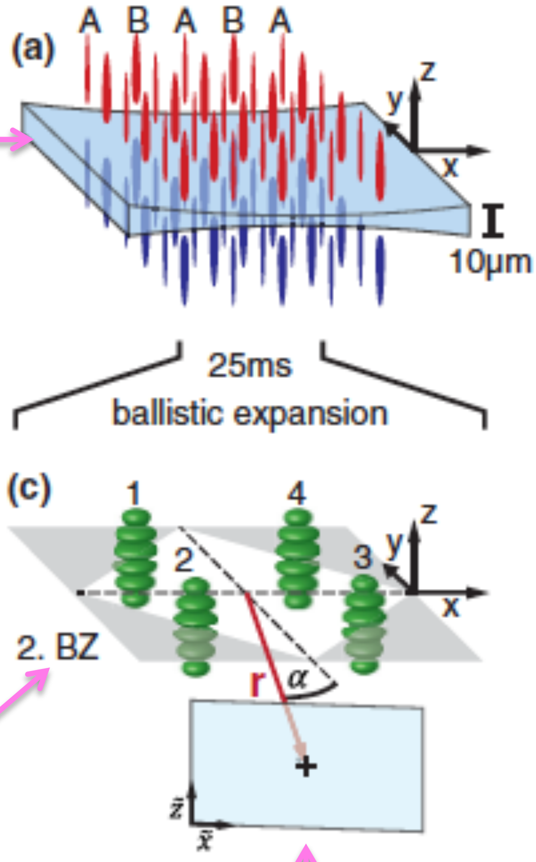
Data interpreted by Hamburg using theory by [WVL, C. Wu, PRA 2006]

- "P-band superfluidity+orbital order in chequerboard (double well) lattice", long life time [G. Wirth. M. Olschlager, A. Hemmerich, *Nature Physics* 2011]
- "F-band" [M. Olschlager, G. Wirth, A. Hemmerich, PRL 2011]
- Avoided band-crossing & Landau-Zener [Olschlager, Hemmerich, et al, PRL (2012)]
- Interacting chiral p+ip order [C. Morais Smith, A. Hemmerich, et al, New J. Physics (2013)]
- …
- "Observing Chiral Superfluid Order by Interference" [Kock, Mathey, Hemmerich et al, PRL, March 2015]

Hamburg interference experiment: Evidence of p+ip order firmed up

Kock, Mathey, Hemmerich et al,
PRL 114, 115301 (March 2015)

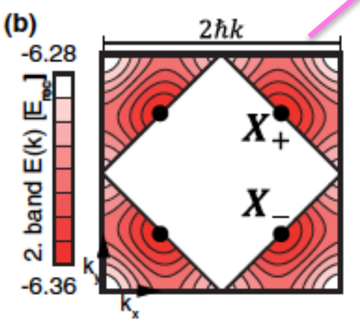
Optical barrier of $2E_R$ splits the system into two sub-gases → Young's double-slit



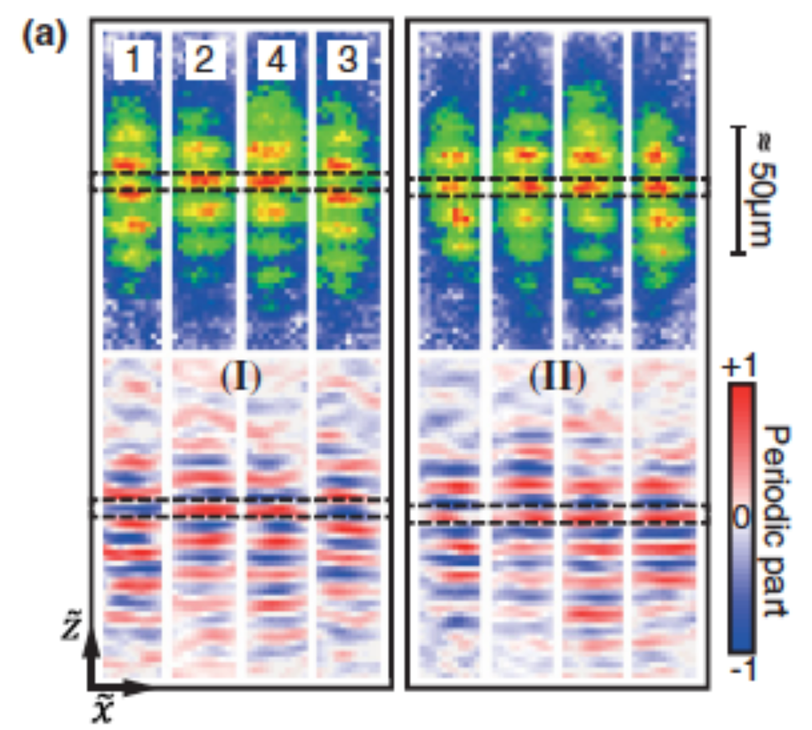
4 points in k-space:

$$1,3 = |p_x\rangle$$

$$2,4 = |p_y\rangle$$



Absorption imaging along Line of Sight



- Two classes of interference (I) vs (II)
- Evidence of ±π/2 phase difference between px and py components, i.e., px ±ipy

Part 2A:

Chiral Bose non-superfluid phase at finite temperature

Main finding: Chiral Bose liquid

Our collaborative Work:

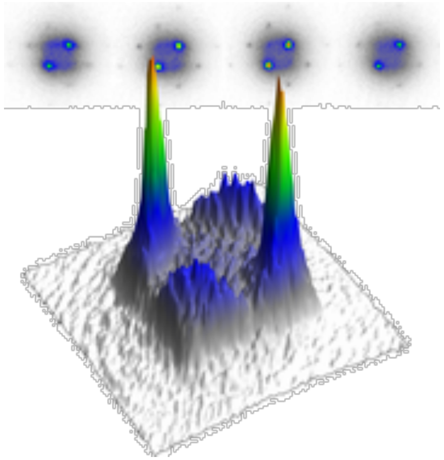
Xiaopeng Li	(Pitt student -> postdoc in JQI Maryland)
Arun Paramekanti	(U Toronto)
Andreas Hemmerich	(U Hamburg)
WVL	(U of Pitt)

Nature Communications 5:3205 (2014)

Experiments Revisited: finite temperature

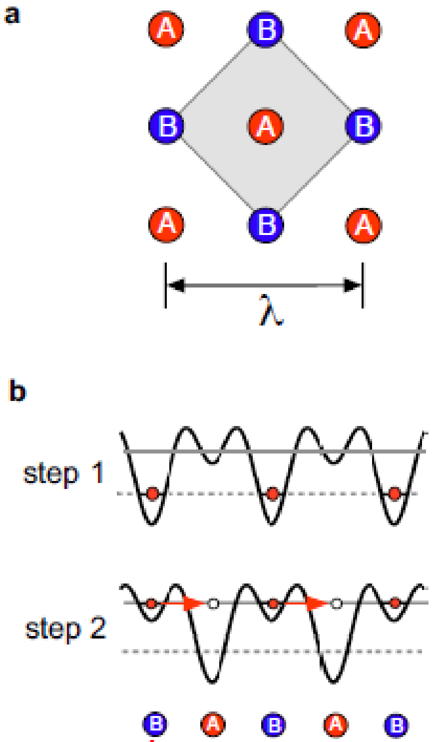
momentum distribution

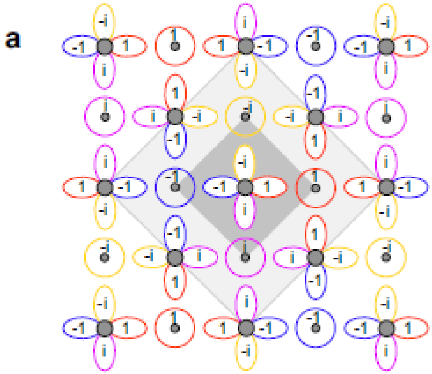
Note thermal background



A. Hemmerich et al., Nat. Phys (2011)

Experiments revisited.
Open questions at Finite temperature





A. Hemmerich et al., Nature Physiics (2011)

-Exotic features

- staggered px+ipy order: TRS breaking, condensed at finite momentum.
- superfluid: U(1) symmetry breaking

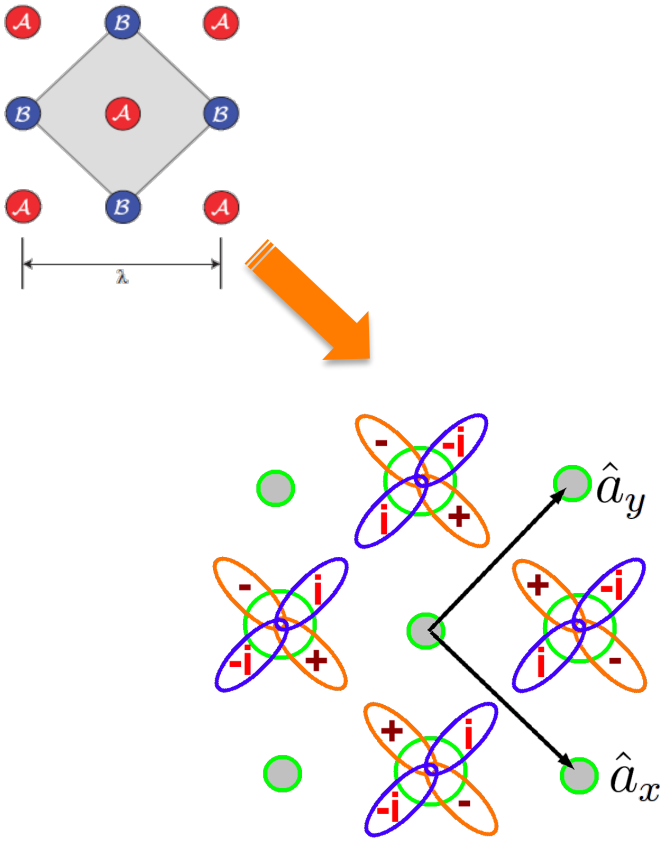
WVL, C. Wu, PRA (2006)
X. Li, E. Zhao, WVL, PRA (2011)
Z Cai, C. Wu, PRA (2011)

-Questions and Challenges

- How does this superfluid state melt under thermal fluctuations?
- Go beyond mean field (ground state): Orbital excitations …? Topological configurations (other than vortices)?

s+p-band model for Hamburg checkerboard lattice

-double wells, mixed
s and p orbitals



$$H = H_{\rm tun} + H_{\rm loc}$$

$$\begin{aligned} H_{\rm tun} &= -\frac{t}{\sqrt{2}} \sum_{\mathbf{r}} \left\{ \left[b_x^\dagger(\mathbf{r}) + b_y^\dagger(\mathbf{r}) \right] \left[b_s(\mathbf{r}_1) - b_s(\mathbf{r}_2) \right] \right. \\ & \quad \left. + \left[b_y^\dagger(\mathbf{r}) - b_x^\dagger(\mathbf{r}) \right] \left[b_s(\mathbf{r}_3) - b_s(\mathbf{r}_4) \right] + h.c. \right\} \quad (1) \\ H_{\rm loc} &= -\sum_{\mathbf{r}} \left[\mu_p n_p(\mathbf{r}) + \mu_s n_s(\mathbf{r}_1) \right] \\ & \quad + \sum_{\mathbf{r}} \frac{U_p}{2} \left\{ n_p(\mathbf{r}) \left[n_p(\mathbf{r}) - \frac{2}{3} \right] - \frac{1}{3} \mathcal{L}_z^2(\mathbf{r}) \right\} \\ & \quad + \sum_{\mathbf{r}} \frac{U_s}{2} n_s(\mathbf{r}_1) \left[n_s(\mathbf{r}_1) - 1 \right]. \quad (2) \end{aligned}$$

$$\mathbf{r}_\alpha = \mathbf{r} \pm \frac{\hat{a}_x \pm \hat{a}_y}{2}, \quad \alpha = 1, \ldots, 4$$

[Xiaopeng Li, Arun Paramekanti, Andreas Hemmerich & WVL, Nature Communications 2014]

Strong coupling & integer fillings: p_x+ip_y Mott insulator

(simple/easy/clean case in theory: s-band raised higher than p; filling n>=2)

Effective model reduced to 2D (classical) Ising:

$$\begin{aligned} H_{\rm loc} &= -\sum_{\mathbf{r}} \left[\mu_p n_p(\mathbf{r}) + \mu_s n_s(\mathbf{r}_1)\right] \\ &+ \sum_{\mathbf{r}} \frac{U_p}{2} \left\{ n_p(\mathbf{r}) \left[n_p(\mathbf{r}) - \frac{2}{3} \right] - \frac{1}{3} \mathcal{L}_z^2(\mathbf{r}) \right\} \\ &+ \sum_{\mathbf{r}} \frac{U_s}{2} n_s(\mathbf{r}_1) \left[n_s(\mathbf{r}_1) - 1 \right]. \end{aligned}$$

Hund's rule → two degenerate states of maximum angular momentum $|L_z|$

$$\mathcal{L}_z(\mathbf{r}) \equiv \sigma_z(\mathbf{r}) |\mathcal{L}_z(\mathbf{r})|$$

$$H^{\rm eff}_{\rm Ising} = \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \mathcal{J} \sigma_z(\mathbf{r}) \sigma_z(\mathbf{r}'), \qquad \mathcal{J} = \frac{3n^2(n+2)}{2(n+1)} \frac{t_\parallel t_\perp}{U} > 0.$$

$$\begin{pmatrix} |p_x + i p_y\rangle \\ |p_x - i p_y\rangle \end{pmatrix} \rightarrow \sigma_z = \begin{pmatrix} + \\ - \end{pmatrix}$$

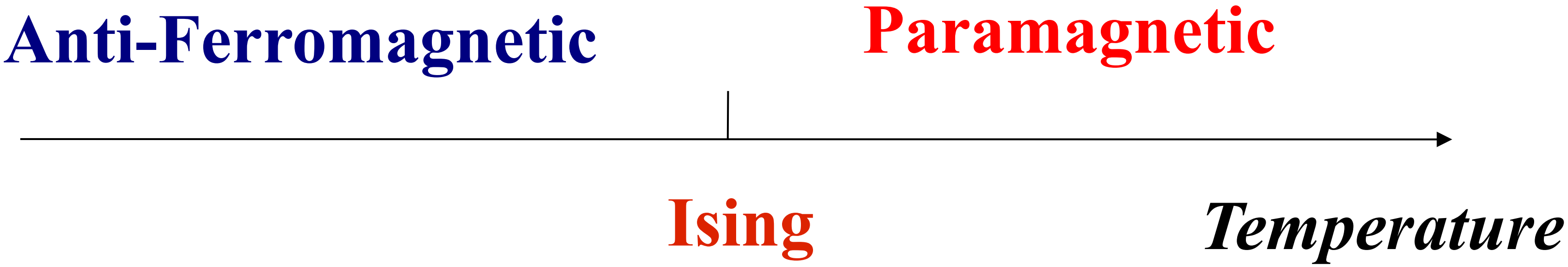
Results mapped from 2D Ising model:

- T=0 (ground state) and T< $T_{\rm Ising}$: long range order with staggered L_z order for integer filling $n \geq 2$
- T> T_I: Ising transition to a symmetry restored phase
- Critical T: $k_B T_I \approx 2.27 \mathcal{J}$ [L. Onsager, Phys. Rev. 65, 117 (1944)]

Effects of thermal fluctuations
---strong interaction regime

-super-exchange interaction in Mott states (filling>=2)

$$H_{\text{eff}} = \mathcal{J} \sum_{<\mathbf{r},\mathbf{r}'>} \sigma_y(\mathbf{r})\sigma_y(\mathbf{r}') \qquad \mathcal{J} > 0$$



*exact solution from Onsager

[Xiaopeng Li, Arun Paramekanti, Andreas Hemmerich & WVL, Nature Communications 2014]

Weak coupling, Finite temperature – three phases found: *Kosterlitz-Thouless superfluid, Chiral Bose liquid, and normal*

Theory: U(1)×U(1) Phase model with interaction:

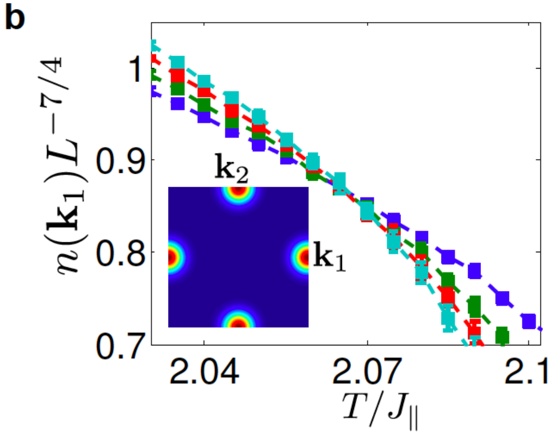
$$H^{\rm eff}_{\rm phase} = \sum_{\mathbf{r}} \left[\left\{ 2J_{||} \cos(\Delta_x \theta_x(\mathbf{r})) - 2J_{\perp} \cos(\Delta_y \theta_x(\mathbf{r})) \right\} + \ \{x \leftrightarrow y\} \right] - U \sum_{\mathbf{r}} \sin^2(\theta_x(\mathbf{r}) - \theta_y(\mathbf{r}))$$

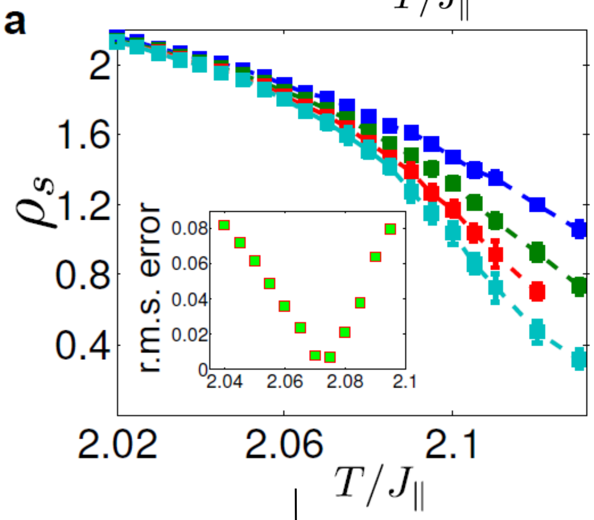
$b^\dagger_{x,y} \sim \sqrt{\rho/2} e^{i\theta_{x,y}}$ (p_x, p_y) components, coherent

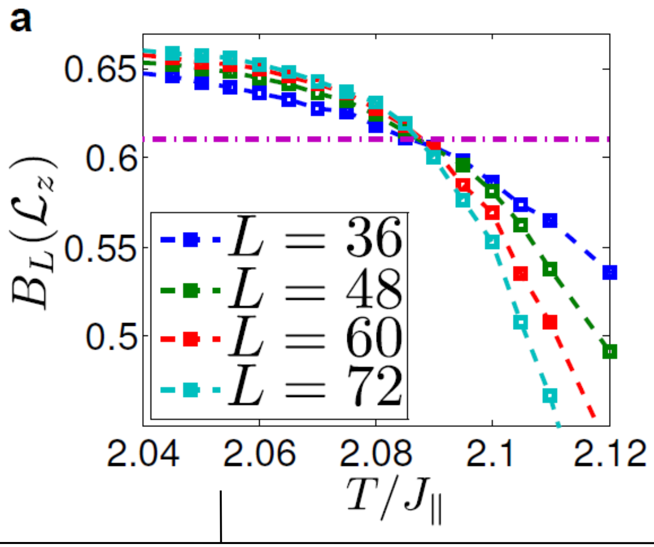
Solve by Monte Carlo simulations (Arun Paramekanti) … next slides

Weak interaction regime

Monte Carlo results: finite size scaling, two stage transition







KT **Ising** ***Temperature***

* $\rho_s(T_{\mathrm{BKT}}, L) = \rho_s(T_{\mathrm{BKT}}, \infty)(1 + \frac{1}{2\log L + c})$

$\mathcal{B}_L(\mathcal{L}_z) = 1 - \frac{\langle \mathcal{M}^4 \rangle}{3\langle \mathcal{M}^2 \rangle^2}$

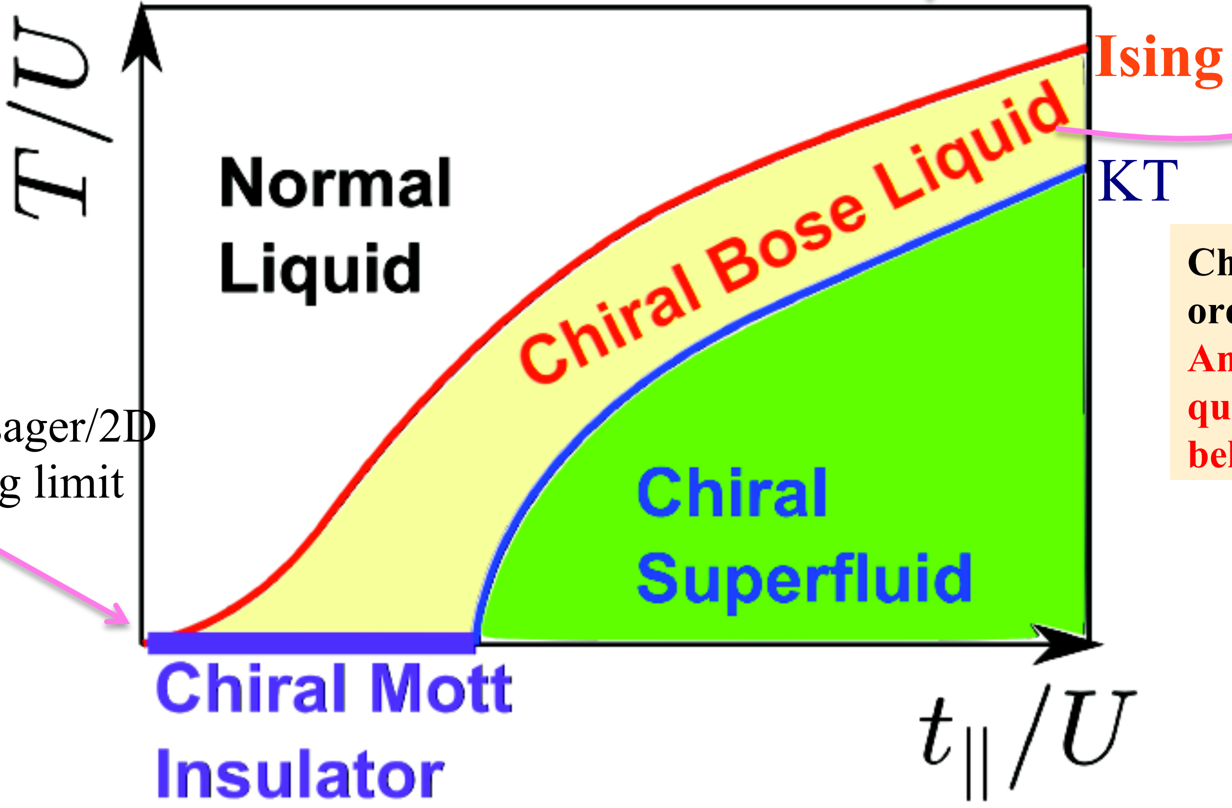
[Xiaopeng Li, Arun Paramekanti, Andreas Hemmerich & WVL, Nature Communications 2014]

Exotic “orbital” phases of bosons: Chiral “normal” liquid

prediction for Hamburg checkerboard lattice p-band experiments

Weak coupling/superfluid limit by Monte Carlo

Filling >=2

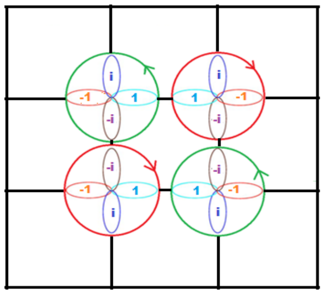


Onsager/2D Ising limit

Probable new state of matter at finite T?

Challenge: How to measure p+ip order without superfluidity?
Answer: not this talk, but see quantum quench idea in paper below.

*** Chiral = p+ip**



X Li, A. Paramekanti, A. Hemmerich and WVL, ***Nature Communications*** (2014)
[Zero Temperature Phase diagram: F. Hebert, Z. Cai, et al., PRB (2013); Some early discussion on finite T: X. Li, E. Zhao, WVL, PRA, 2011]

Part 2B:

Chiral superfluidity with p-wave symmetry from an interacting s-wave atomic Fermi gas

Crucial difference: neither direct nor induced p-wave interaction needed

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Our collaborative Work:

Bo Liu	(Pitt Postdoc)
Xiaopeng Li	(Pitt student -> postdoc in JQI Maryland)
Biao WU	(Peking Univ, China)
WVL	(U of Pitt)

Nature Communications 5:5064 (Sep 2014)

Next …

Basic idea:

Concept of s+p cross-orbtial pairing ---

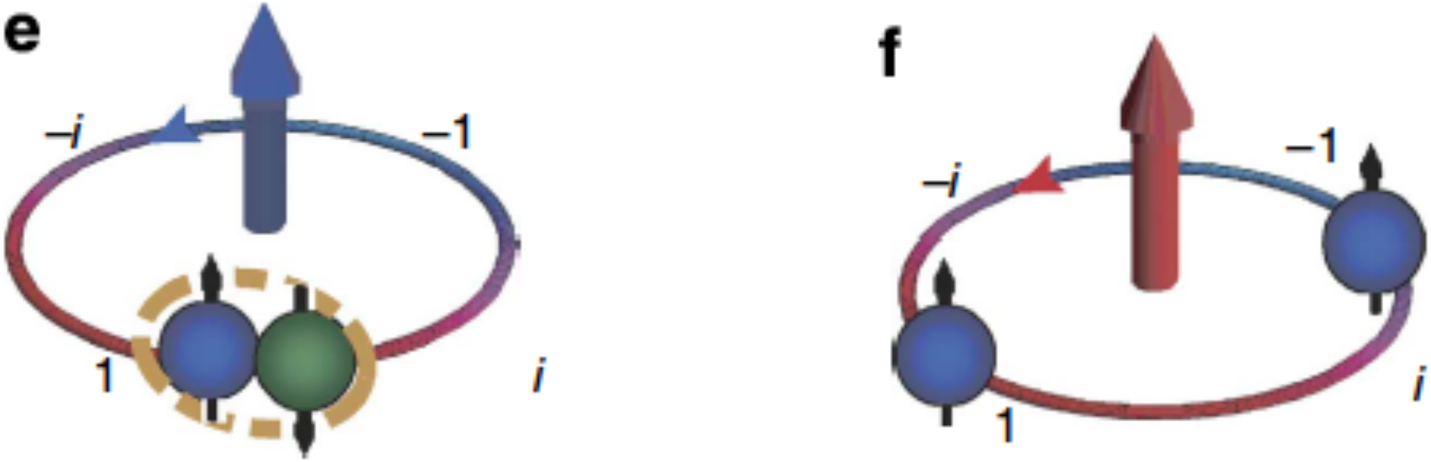
gives topological superconductivity;
does not require Spin-Orbital coupling, nor any form of induced p-wave interaction.

How?

summary

Chiral superfluidity with p-wave symmetry from an interacting s-wave atomic Fermi gas

[B. Liu, X. Li, B. Wu, & WVL, Nature Comm 5:5064 (30 Sep 2014)]



Center of mass *p+ip*

Conventional *p+ip*

Highlights

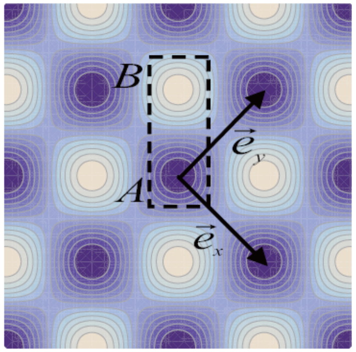
- **Center-of-mass p+ip topological superfluid phase discovered.**

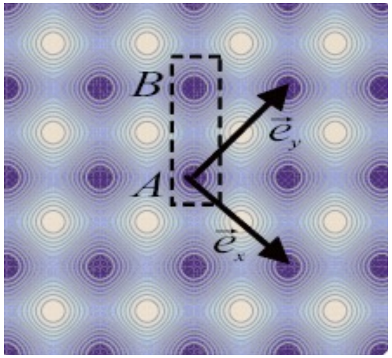
- **pure s-wave interaction**, no SOC (spin-orbital coupling).
- **High Tc for a p-wave**, potentially of order of E_F (fermi energy) as in s-wave Feshbach resonant atomic gases.

The idea

---special geometry optical lattice

Spin dependent Lattice potential

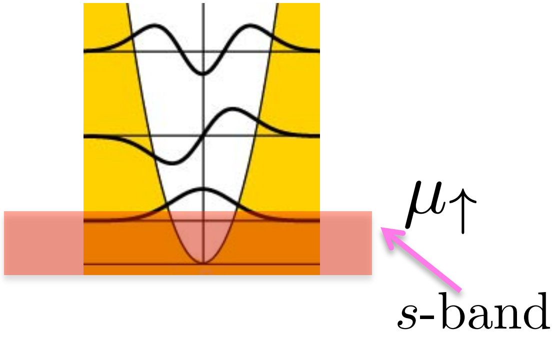


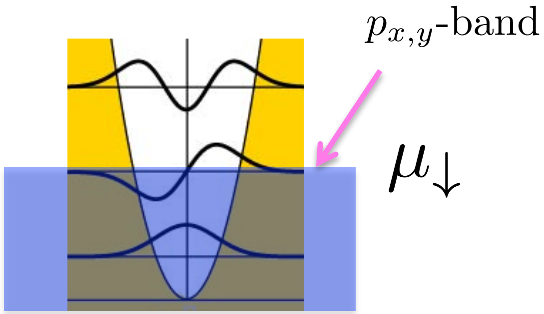


A sites: $\uparrow + \downarrow$

B sites: only $\downarrow$ fermions

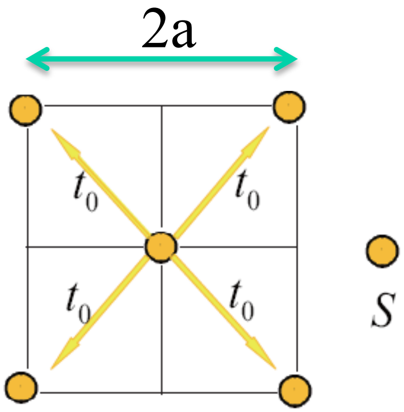
Spin imbalanced Chemical potentials

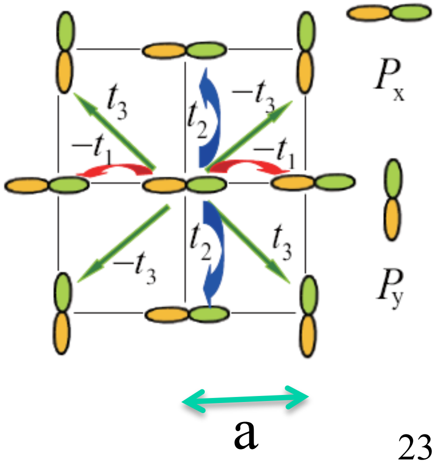




$$N_\downarrow > N_\uparrow$$

Tight binding model

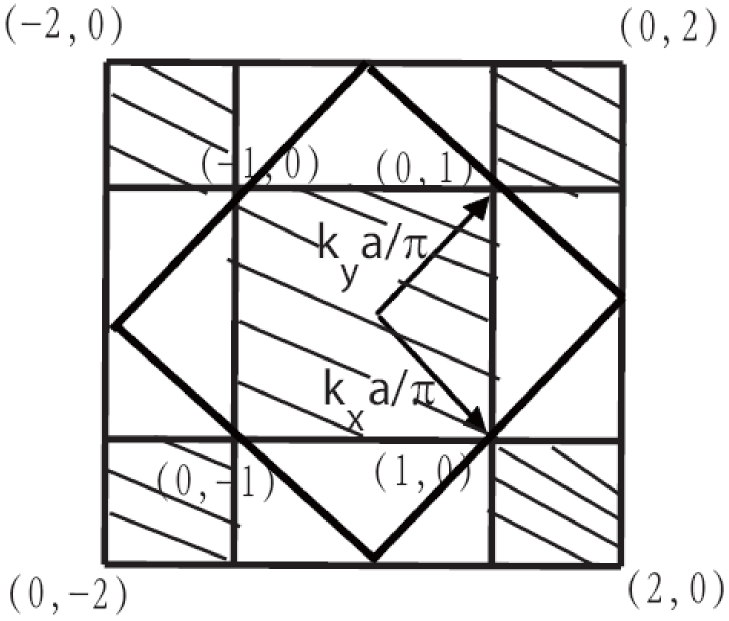




Fermi surface matched

$(t_2/t_0 = 0\,,\, t_3/t_0 = 0\,)$

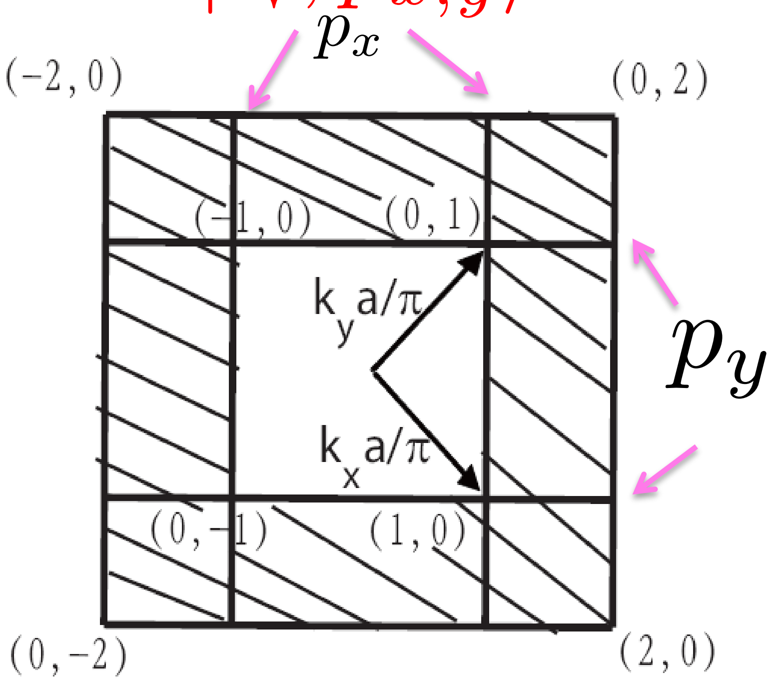
$$|\uparrow, s\rangle$$



Diamond = Brillouin zone for up

Momentum Units: $\dfrac{\pi}{2a}$

$$|\downarrow, p_{x,y}\rangle$$



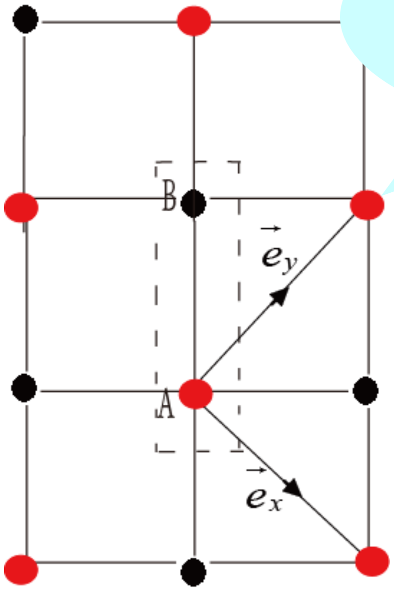
Whole square (biggest Square)
= Brillouin zone for down

- Two pairing order parameters → (px,py) Cooper pairs =p molecules (bosons)

$$\langle c_{\uparrow s} c_{\downarrow p_x} \rangle = \Delta_x \qquad \langle c_{\uparrow s} c_{\downarrow p_y} \rangle = \Delta_y$$

Model Hamiltonian

On-site interaction



Fermions interact only on red sites, by inter-spin s-wave.

Tuned by

Feshbach resonance

Our model

-No p-wave resonance;

-No long-range interaction;

-No spin-orbit coupling;

- but spin imbalance and possibly a spin-dependent optical lattice of novel geometry.

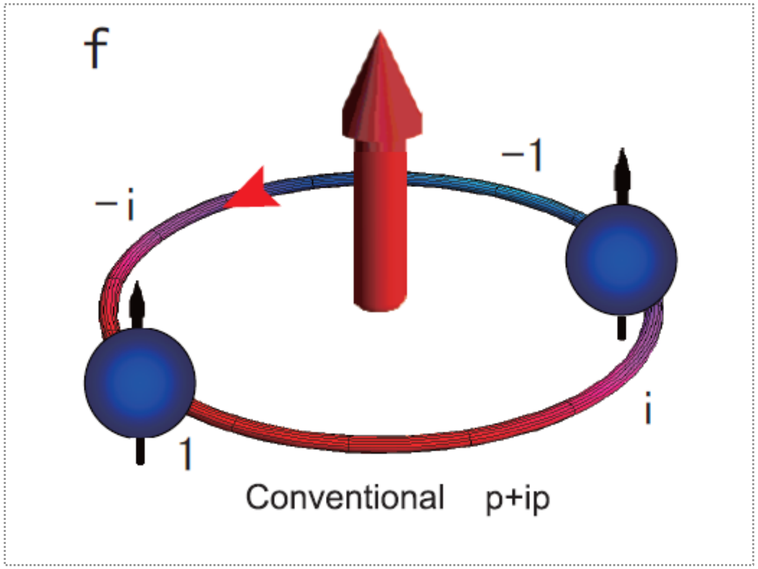
* The special lattice geometry is to make cooper paring favorable even with weak interactions. With strong interactions as in the resonance regime, such special lattice is expected to be unnecessary.

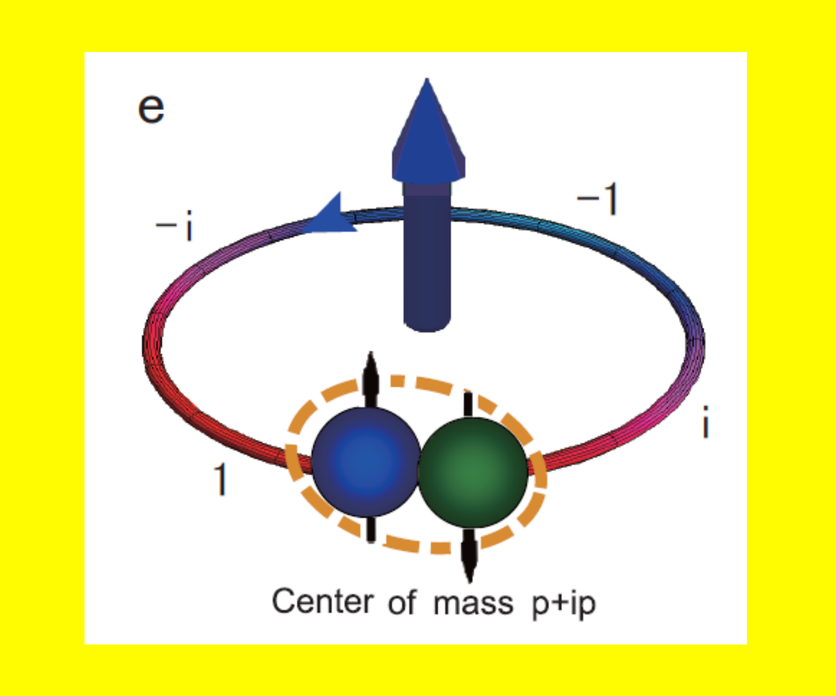
Chiral center-of-mass p-wave pairing of fermions

s+p_x and s+p_y Cooper pairs condense! ⟶ **“p+ip” superconductor.**

B. Liu, X. Li, B. Wu, WVL, Nature Comm (2014)

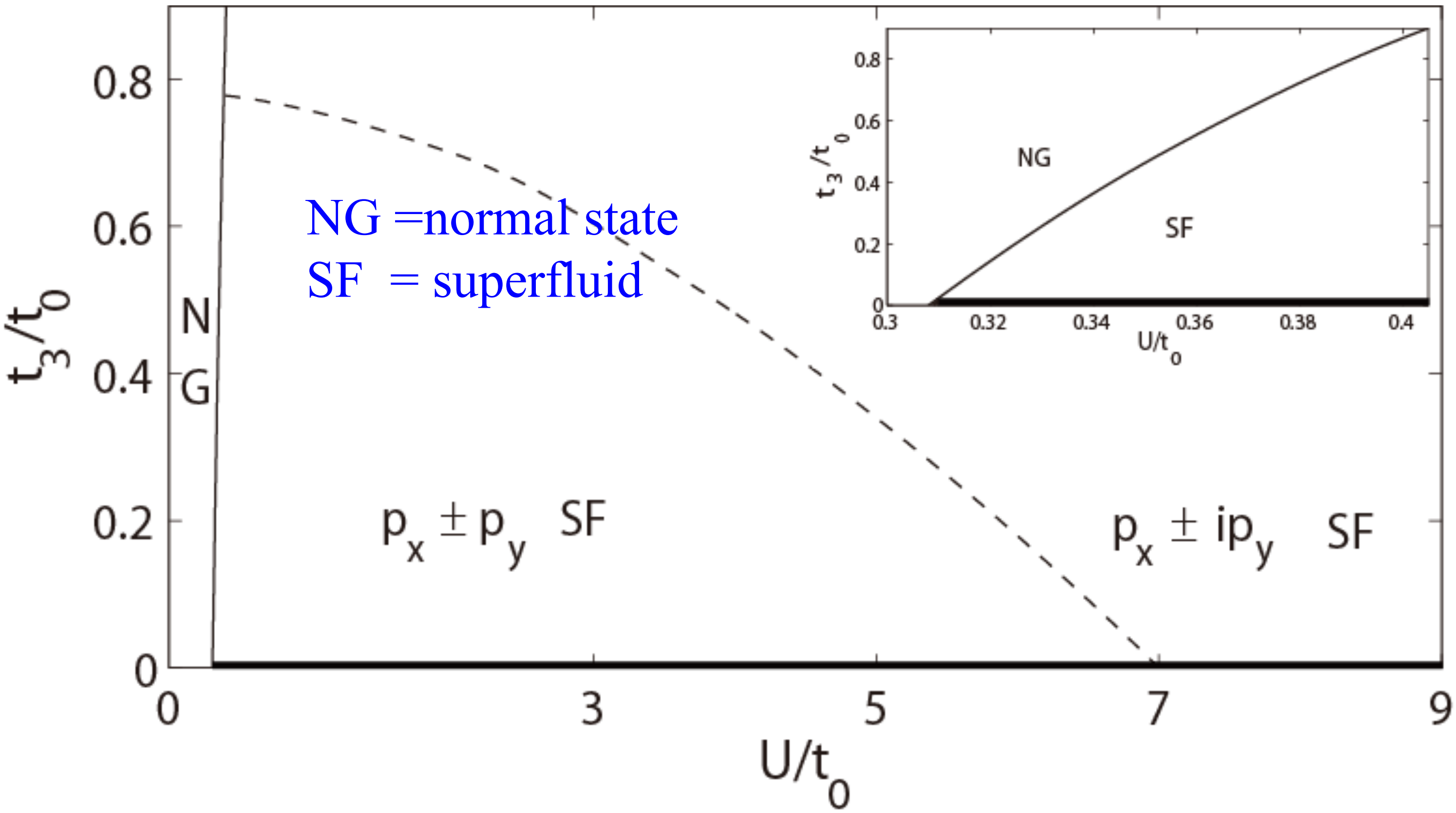
-our center of mass topological p+ip





Zero-temperature phase diagram

Obtained by Feyman diagram expansion + symmetry based Ginzburg-Landau theory



NG =normal state
SF = superfluid

There is a wide parameter region supporting $p_x \pm ip_y$ SF.

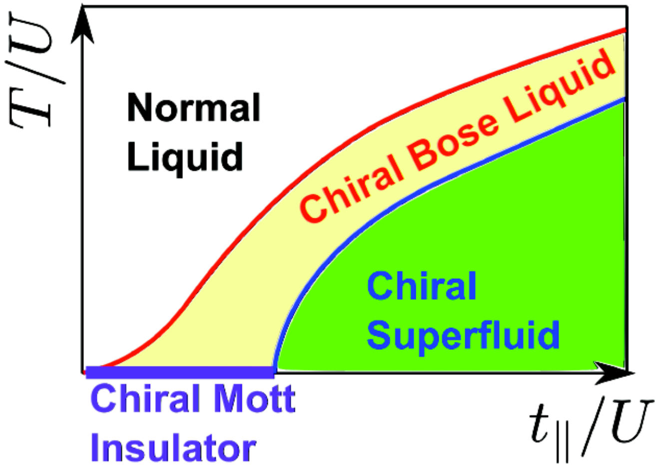
What is next? --- Work in progress with R. Hulet group

- Rice/R. Hulet group experiment achieved (to the best estimate)
  - Spin polarized Fermi gas (Li-6 atoms), in 3D optical lattices
  - Already very cool, down to Antiferromagnetic transition T, t^2/U; we only need low T down to a fraction of tunneling energy scale enough
  - s-wave Feshbach resonance

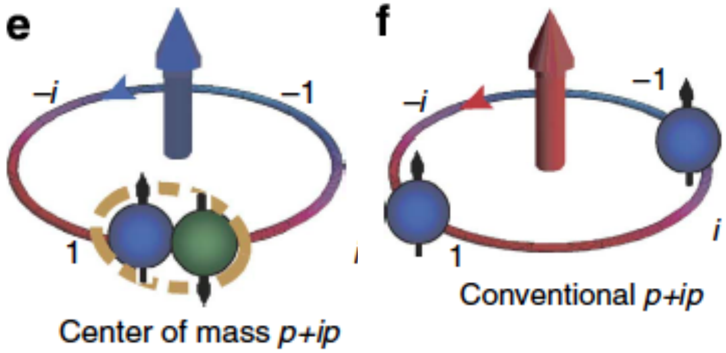
- Theoretical proposal:
  - Tune lattice depths to make quasi-1D [so to have just one p-band, e.g., keep $p_{y,z,}$ bands significantly higher than p_x band]. (Rice/Hulet group has quasi-1D tubes of atoms already – 1D Fermi gases – see Xiwen GUAN talk)
  - Further tune spin polarization, so to make the two spins in s and p bands, respectively

Conclusion---Orbital Optical Lattice Physics

Chiral Bose liquid at finite T



Center-of-mass topological p-wave superconductivity



next two weeks at INT --- Discussion about Other recent work

Magnetic Skyrmions in electronic oxides: *Phys. Rev. Lett* (2014), with X Li and L Balents

Weyl superfluidity: *Phys. Rev. Lett.* 114, 045302 (28 January 2015)

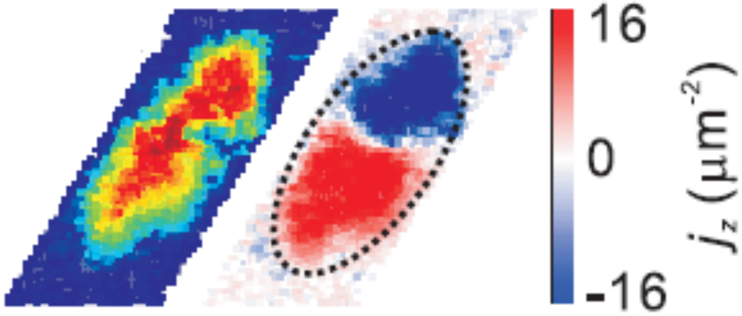
Spin-orbital exchange of interacting p-band fermions: *Phys. Rev. Lett.* (2015), with Erhai Zhao et al

Spontaneous QHE in spinor Bose-Fermi mixtures. *Phys. Rev. Lett.* (2015), with Xu, Li and P. Zoller

A Domain Wall connecting two TRS pairs

Ferromagnetic domain formation is common in Ferro-Magnetic (FM) solid state materials [many references …]

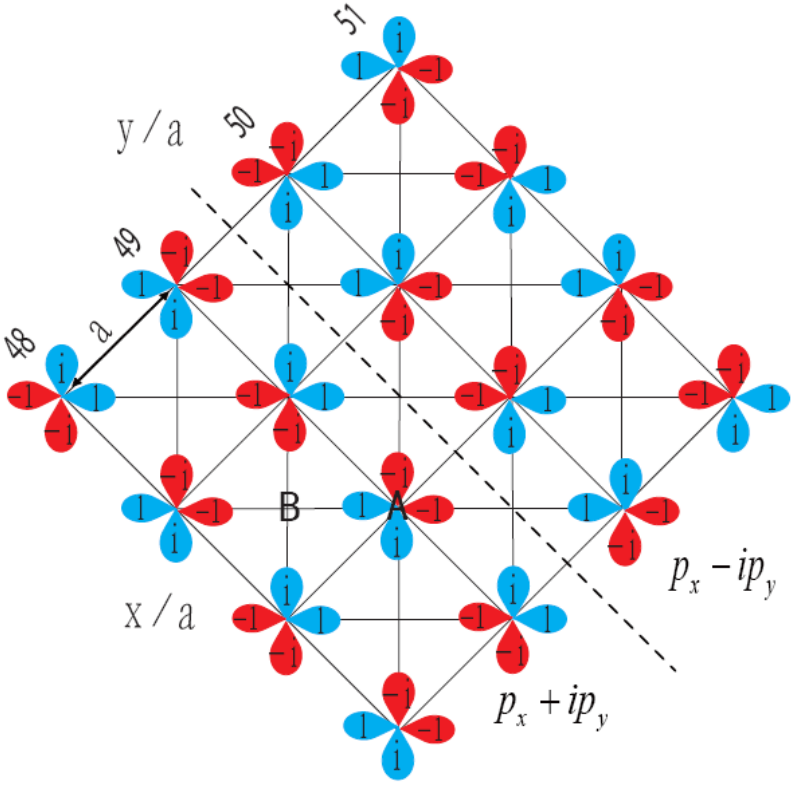
Example: Ferromagnetic phase domains in momentum space seen in atomic gases



Dynamic Ferromagnetic transition
Shown in k-space. About 25,000 Cs atoms. TOF observes condensate on one of the minima of the double wells.

[C. Parker, Cheng Chin,, et al., Nature Physics (2013)]

A domain wall decorated superconducting background

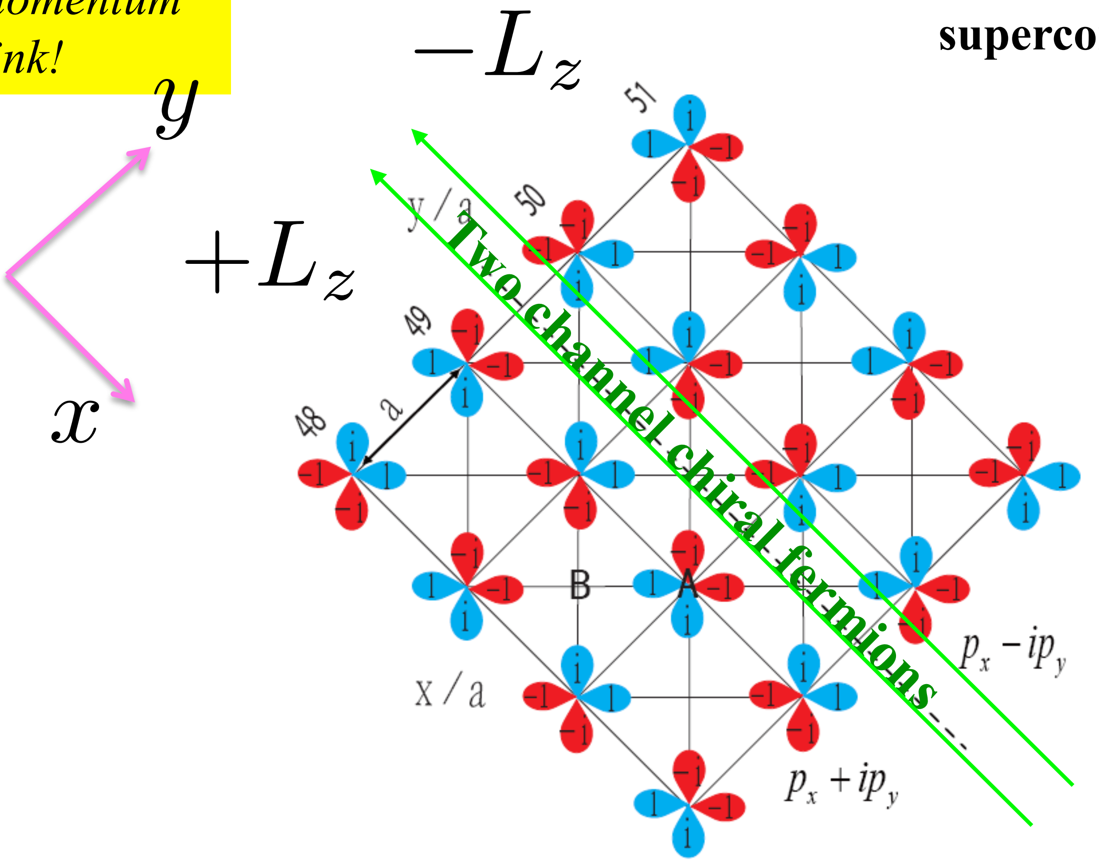


A Domain Wall connecting two TRS pairs

Ferromagnetic domain formation is common in Ferro-Magnetic (FM) solid state materials [many references …]

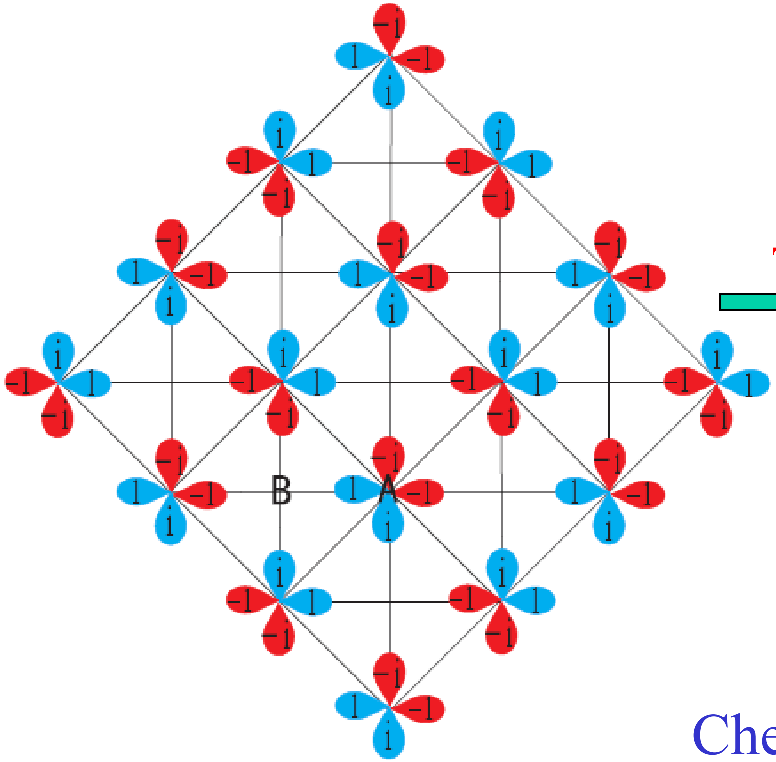
… when Orbital Angular momentum shows a kink!

A domain wall decorated superconducting background



Time-Reversal-Symmetry breaking, Ferro-orbital order with superfluidity → Chiral topological SF

Ferro-orbital order
coexists with superfluid



TRS-breaking →

Nonzero Chern number

$$p_x + ip_y \rightarrow C_+ = 1$$

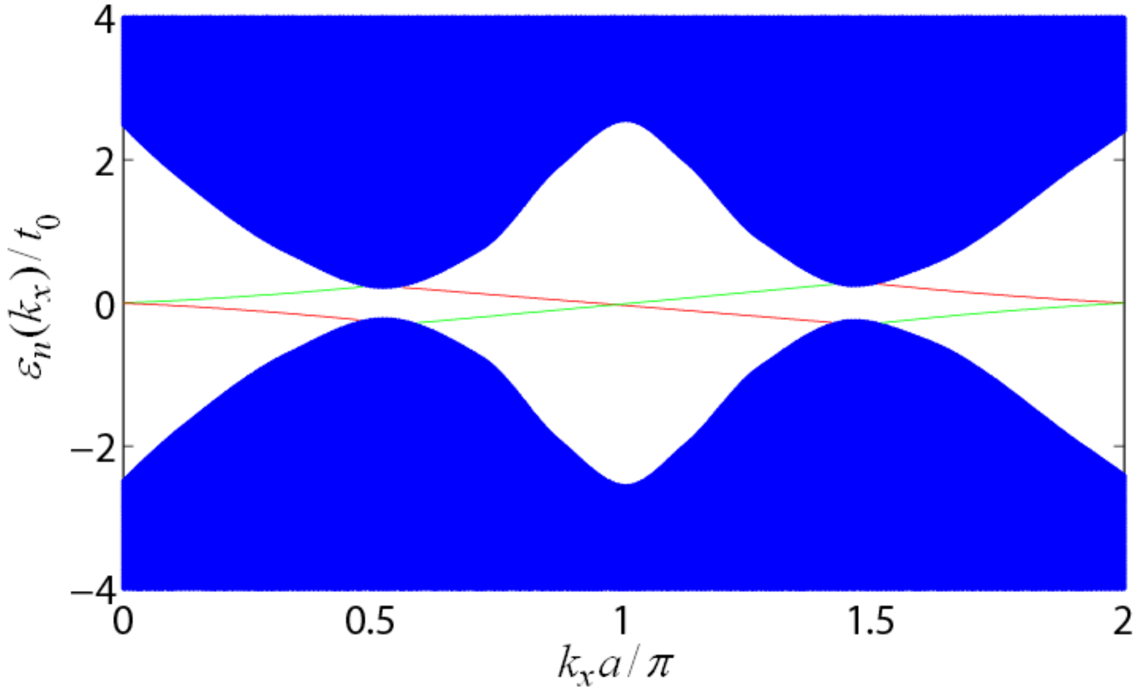
$$p_x - ip_y \rightarrow C_- = -1$$

Bogoliubov-de Gennes
Bloch bands

Chern number

$$C = \sum_{n,\text{occupied}} \int_{BZ} \frac{d^2\vec{k}}{\pi} \, \mathrm{Im}\langle \partial_{k_x}\phi_n(\vec{k}) | \partial_{k_y}\phi_n(\vec{k}) \rangle$$

Gapless chiral fermions



The in-gap states can be probed by radio-frequency spectroscopy.

Green = domain wall fermions
Red= edge states (boundary of the system)

Topological nature:

- Two chiral modes on the domain wall
- *At* $k_x a = 0$ or π, zero energy Majorana fermions

Number of right- (left-) moving edge-modes = |C1-C2|

In our case: $|\Delta C| = 2$

Chern number difference

F. D. M. Haldane, et al., PRL (2008).

Key features of the s+p band pairing

- Center-of-mass p-wave
- Topological and chiral
- High critical Tc: as high as *s-wave Feshbach resonant* BCS-BEC crossover superfluid
- Alternative to the mechanisms of p-wave Feshbach resonance, or induced one (e.g., by spin-orbital coupling SOC)
- Predicted for spin polarized Fermi gas on lattice:

$$N_\downarrow < N_\uparrow$$