Spin Textures of Dipolar Spinor Condensates from a Dirac String Perspective

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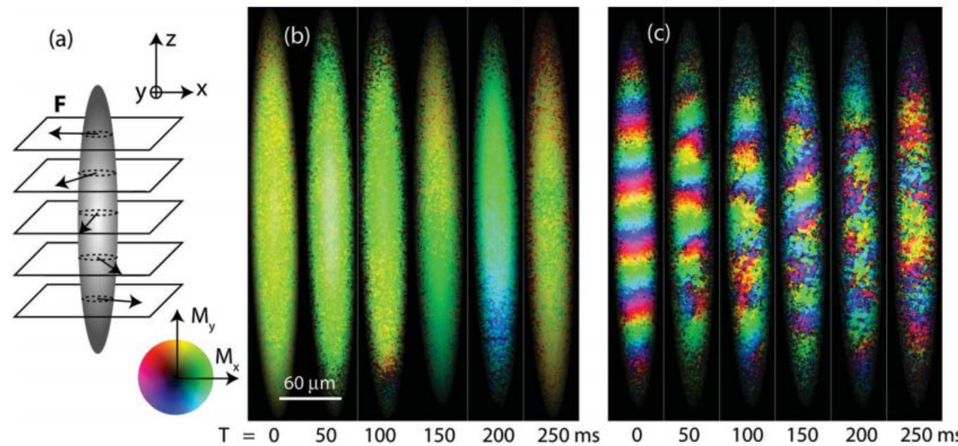
Spin textures in spinor condensates

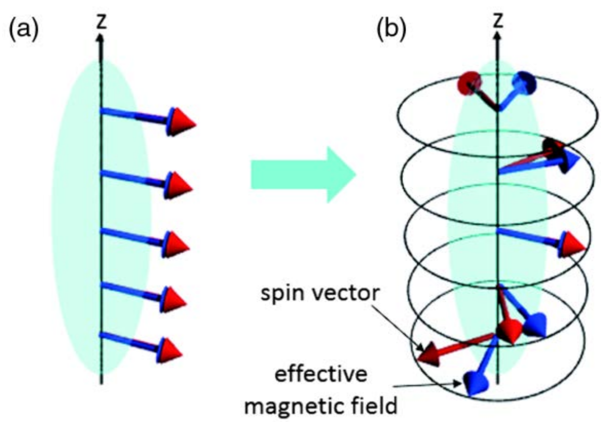
The dipole-dipole interaction (DDI) induces rich spin textures in spinor condensates (numerical & experimental):

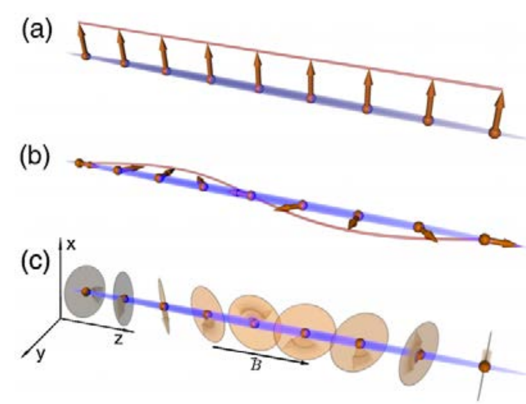
- Static (ground state)
- Dynamical



Vengalattore, et. al. PRL 100, 170403 (2008)

Eto, et. al. PRL 112, 185301 (2014)

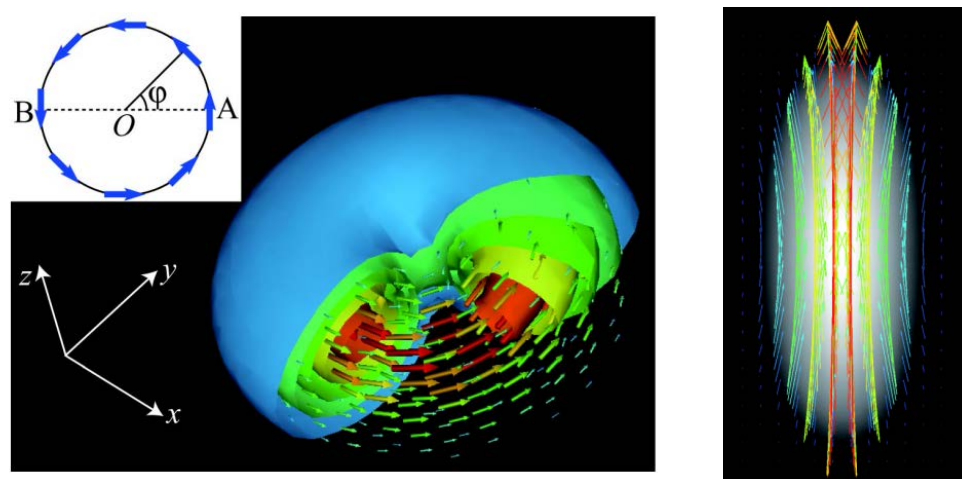




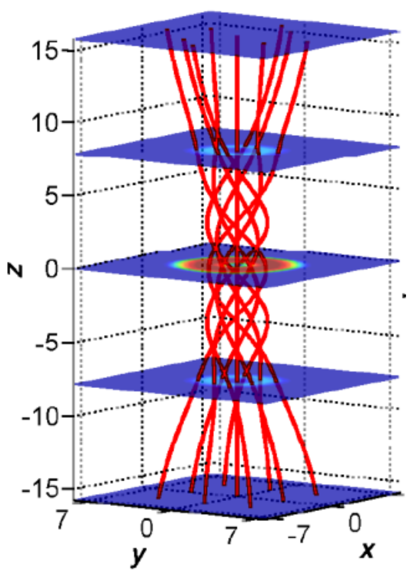
Kronjager et. al., PRL 105, 090402 (2010)

Spin textures in spinor condensates

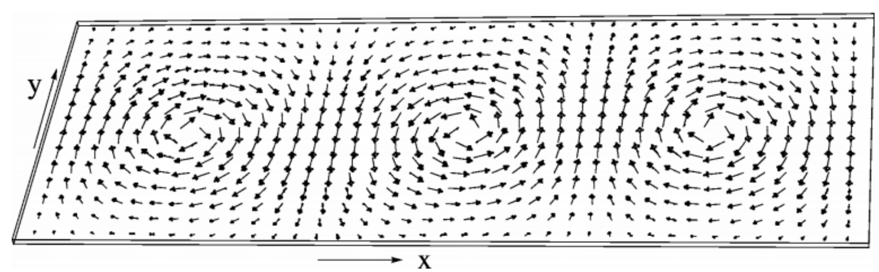
Numerical simulations are done for dipolar gases:



Takahashi, et. al. PRL 98, 260403 (2007)



Yi, Pu, PRL 97, 020401 (2006)



Zhang, Ho, J. Low Temp. Phys. 161, 325 (2010)

Mean field theory description of ferromagnetic condensates

The spinor condensate of spin F bosons :

$$\langle \Psi_m(\boldsymbol{r}) \rangle = \sqrt{n_0}\varphi_m(\boldsymbol{r}) \,,$$

- Local spin is defined as

$$\langle \mathbf{F}(\mathbf{r}) \rangle \;=\; \varphi^\dagger(\mathbf{r})\mathbf{F}\varphi(\mathbf{r})$$

The mean field energy is

Spin wave

DDI (long range)

$$H_0 = \frac{\alpha}{M}\int d^3\mathbf{r}(\nabla\boldsymbol{\mathcal{F}}(\mathbf{r}))^2 \;,$$

$$H_D = \frac{\lambda}{2}\int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \frac{\boldsymbol{\mathcal{F}}_1\cdot\boldsymbol{\mathcal{F}}_2 - 3(\boldsymbol{\mathcal{F}}_1\cdot\hat{\mathbf{r}}_{12})(\boldsymbol{\mathcal{F}}_2\cdot\hat{\mathbf{r}}_{12})}{4\pi r_{12}^3}$$

- $|\boldsymbol{F}(\boldsymbol{r})| = F_0 \,, \quad \boldsymbol{\mathcal{F}}(\boldsymbol{r}) = \boldsymbol{F}(\boldsymbol{r})/F_0 \quad .$
- $\alpha = n_0\hbar^2[F(F+1) - F_0^2]/4 \,, \qquad \lambda = \mu_0(g_F\mu_B)^2 n_0^2 F_0^2 \;.$

Spin chain as a Dirac String

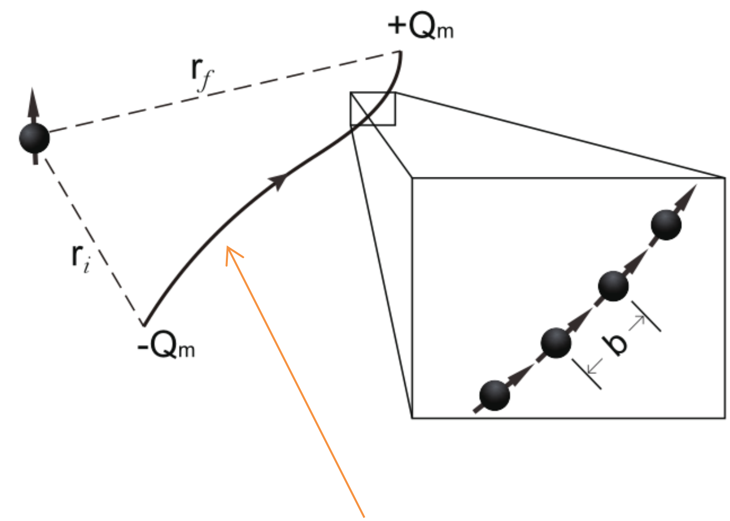
Claim : DDI is much simpler in the string picture.

- The DDI between a spin and a string of aligned spin is:

$$E_{\mathbf{m}_F} = \mu_0 Q_m \int dl \frac{r^2 \mathbf{m}_F \cdot \frac{d\mathbf{r}}{dl} - 3(\mathbf{m}_F \cdot \mathbf{r})(\mathbf{r} \cdot \frac{d\mathbf{r}}{dl})}{4\pi r^5}$$

$$= \mu_0 Q_m \left(\frac{\mathbf{r}_f}{4\pi r_f^3} - \frac{\mathbf{r}_i}{4\pi r_i^3} \right) \cdot \mathbf{m}_F = -\mathbf{m}_F \cdot \mathbf{B}_{str}$$

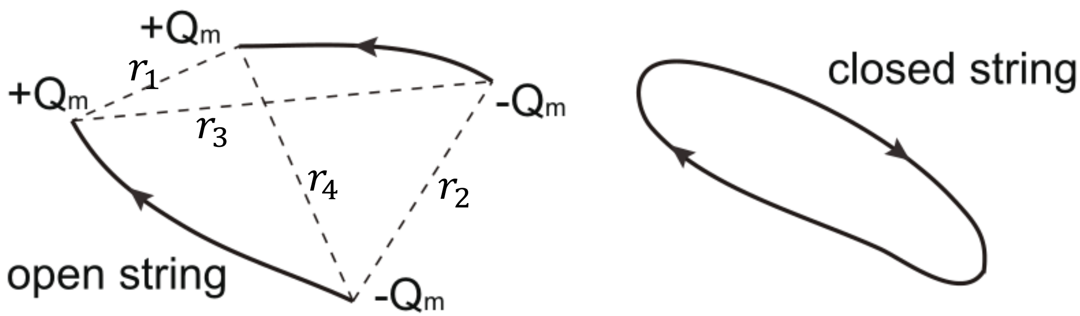
where $Q_m = g_F \mu_B F_0 / b$.



$$\boldsymbol{\mathcal{F}}(\boldsymbol{r}) = \frac{d\boldsymbol{r}}{dl}$$

The string is a **Dirac string** with monopole charges $\pm Q_m$ at the ends.

Interactions between two strings



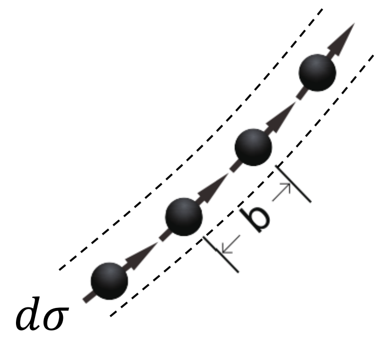
The DDI between two strings are just the monopole energies

$$E_Q = \frac{\mu_0 Q_m^2}{4\pi}\left(\frac{1}{r_1} + \frac{1}{r_2} - \frac{1}{r_3} - \frac{1}{r_4}\right).$$

- A closed string **does NOT interact** with any other strings.
- An open string lowers its monopole energy by connecting its ends (into a closed string), i.e., $E_{closed} < E_{open}$.

Monopole charges

In the continuum limit, a string has cross sectional area $d\sigma$.



The spacing between aligned spins is

$$b = (n_0 d\sigma)^{-1} \sim n_0^{-1/3} \ .$$

So the monopole charge on the string end is

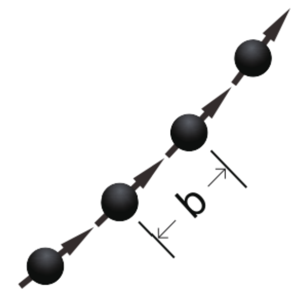
$$Q_m = \frac{g_F \mu_B F_0}{b} = g_F \mu_B n_0 F_0 d\sigma \ ,$$

proportional to $d\sigma$.

String self energy (string tension)

Though the bulk of string does not interact with other strings, it has a DDI self energy.

A spin is attracted by other spins in the same string:



$$E_{F_0} \approx -\mu_0 g_F^2 \mu_B^2 F_0^2 \sum_N^{\infty} 2 \left(\frac{1}{Nb}\right)^3 \sim -\frac{\lambda}{n_0} \ .$$

The total energy of the string is thus

$$E_{str}(L) = \frac{N E_{F_0}}{2} = \frac{L}{2d} E_{F_0} \sim -\lambda L d \sigma \ ,$$

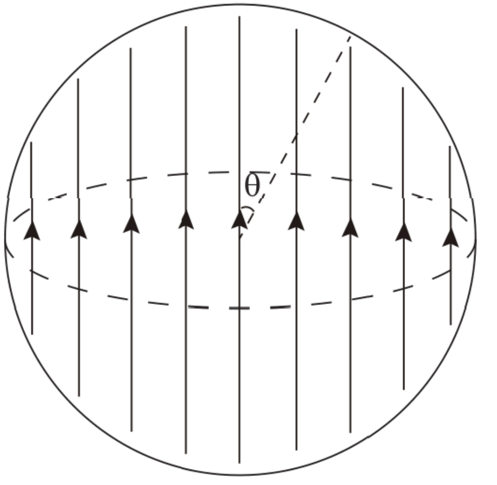
proportional to the volume of the string.

- The Dirac string has a **negative** string tension.

String self energy (string tension)

Determination of the exact value of the string self energy：

Calculate the DDI energy felt by the spin at the center.



- Direct calculation:

$$E_d = \frac{\lambda}{n_0}\int_0^R 2\pi r^2 dr \int_0^\pi d\theta \sin\theta \frac{1-3\cos^2\theta}{4\pi r^3} = 0$$

- Calculation using Dirac string picture:

$$E_d = \underbrace{E_{F_0}}_{\text{self energy}} + \underbrace{\frac{2\lambda}{n_0}\int_0^R 2\pi\rho d\rho \frac{\sqrt{R^2-\rho^2}}{4\pi R^3}}_{\text{from monopoles}} = E_{F_0} + \frac{\lambda}{3n_0}\ .$$

⇨

$$\boxed{E_{str}(L) = \frac{L}{2d}E_{F_0} = -\frac{1}{6}\lambda L d\sigma\ .}$$

The kinetic energy

Now consider the kinetic energy

$$H_0 = \frac{\alpha}{M} \int d^3\mathbf{r} (\nabla \boldsymbol{\mathcal{F}}(\mathbf{r}))^2 \ .$$

Dividing ∇ into $\nabla_\perp + \nabla_\parallel$:

The $\nabla_\parallel$ part modifies the self energy to

$$E_{str}(L) = d\sigma \int_0^L dl \left[-\frac{1}{6}\lambda + \frac{\alpha}{M} \left(\frac{d^2\mathbf{r}}{dl^2} \right)^2 \right] \ .$$

curvature

The $\nabla_\perp$ part is a local interaction between strings:

$$E_I(L_1, L_2) = d\sigma_1 d\sigma_2 \frac{3\alpha}{4M} \int_0^{L_1} dl_1 \int_0^{L_2} dl_2 \delta^3(\mathbf{r}_{12}) \left(\frac{\boldsymbol{\mathcal{F}}_1 - \boldsymbol{\mathcal{F}}_2}{r_{12}} \right)^2 \left[3 - 5 \frac{(\boldsymbol{\mathcal{F}}_1 \cdot \mathbf{r}_{12})(\boldsymbol{\mathcal{F}}_2 \cdot \mathbf{r}_{12})}{r_{12}^2} \right]$$

two nearby strings tends to be parallel

Duality

The spin picture is dual to the Dirac string picture:

spin picture: kinetic energy H_0 + long range interaction H_D

Dirac string picture: $E_I(L_1, L_2)$ + $E_{str}(L)$ + E_Q

local interaction, free string energy, monopole energy

A classical negative tension string theory with U(1) gauge group

- Strong dipolar condensates ↔ 'weakly' interacting string gas
- Quantum version ↔ a quantum string theory?

Ground state principles

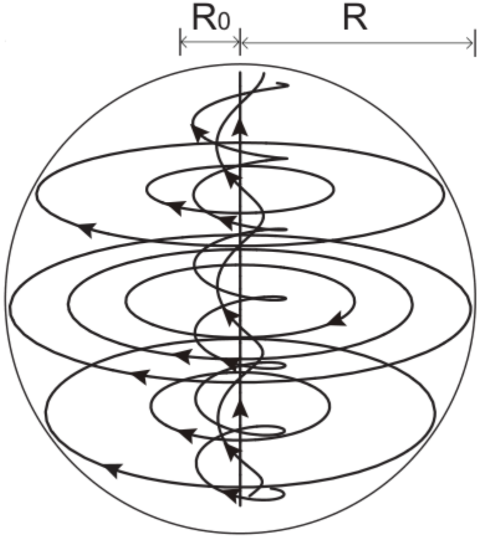
Principles for the ground state:

1) As many closed strings with as small curvatures as possible.
2) Nearby strings as parallel as possible.

The ground state texture depends strongly on the trap geometry.

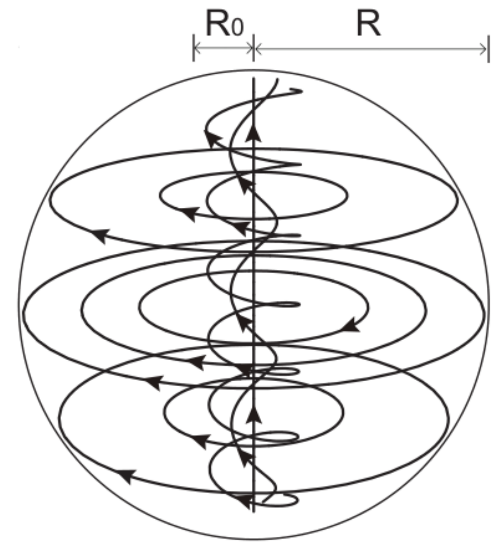
a) 3D spherical traps.

- Circular strings optimizes the curvature.
- Circles are parallel to each other.
- Within a certain region R_0, circular strings have too large curvatures, and give way to spiral open strings (like a meron).



Ground state principles

The total energy consists mainly of：



- String self energy (tension + curvature)
- Monopole energy. Monopoles are distributed within R_0 with a density

$$n_A \approx \pm Q_m/d\sigma = \pm g_F \mu_B n_0 F_0.$$

$$E_{3D} \approx -\frac{\lambda}{6}V + \int_{\rho \geq R_0}^{R} d^3\mathbf{r}\frac{\alpha}{M}\frac{1}{\rho^2} + \frac{2\mu_0(\pi R_0^2 n_A)^2}{4\pi R_0}$$
$$\approx -\frac{\lambda}{6}V + \frac{4\pi\alpha}{M}R\log\frac{R}{R_0} + \frac{\pi}{2}\lambda R_0^3 \ ,$$

Minimizing E_{3D} yields

$$\boxed{R_0 = (8\alpha/3\lambda M)^{1/3}R^{1/3}.}$$

3D traps

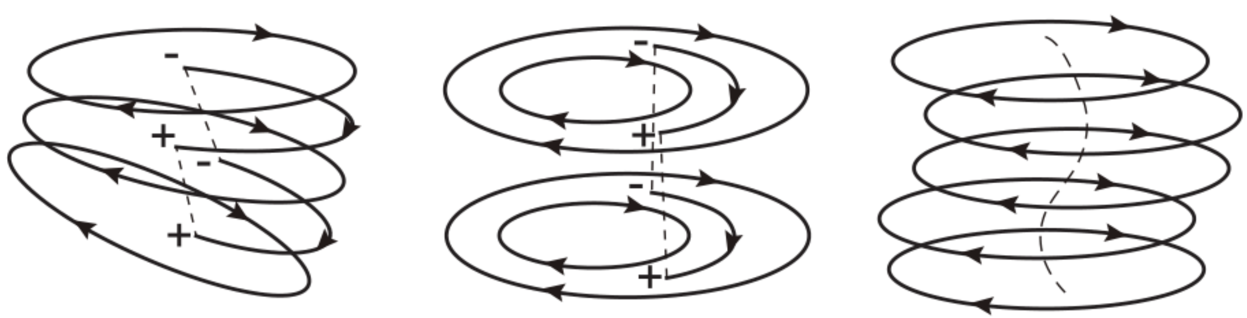
$$R_0 = (8\alpha/3\lambda M)^{1/3} R^{1/3}.$$

So the 3D spin texture arises roughly when $R > R_0$, i.e.,

$$R > (8\alpha/3\lambda M)^{1/2}.$$

The texture is stable in energy.

Possible perturbations (dynamics to be studied):



Quasi-1D traps

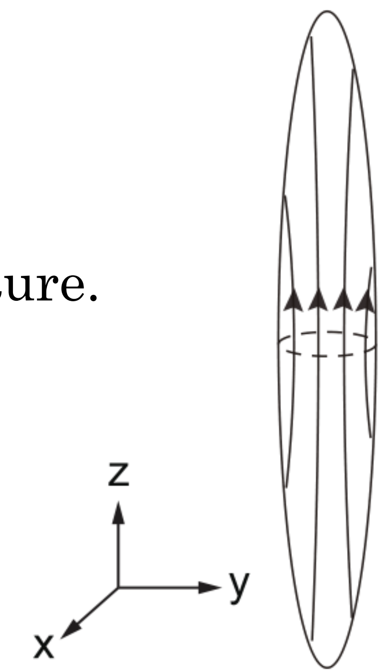
Consider traps bounded by

$$x^2 + y^2 + (z/A)^2 = R^2.$$

b) Quasi-1D cigar traps ($A \gg 1$) .

- Closed strings are always having too large curvature.
- Open strings along z direction yields minimal monopole charges.

⇨ z direction polarized (flare) texture



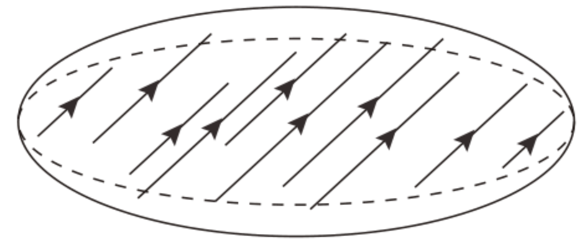
3D – quasi 1D crossover at $A \sim (\lambda M/\alpha)^{1/3} L_c^{2/3}$

Quasi-2D traps

c) Quasi-2D pancake traps ($A \ll 1$).

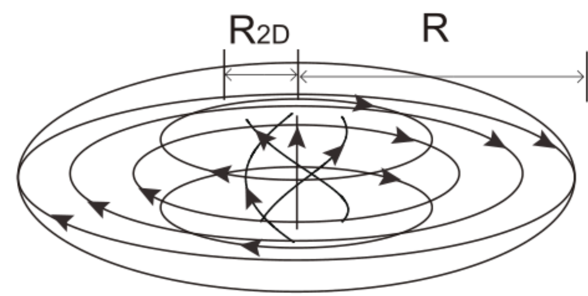
There are two competing phases:

(I) in-plane polarized phase (no curvature):



$$E_{2D}^{(I)} \approx -\frac{\lambda}{6}V + \underbrace{w\lambda R^3 A^2 \log\frac{1}{A}}_{\text{monopole energy}}$$

(II) meron texture (minimal monopoles)

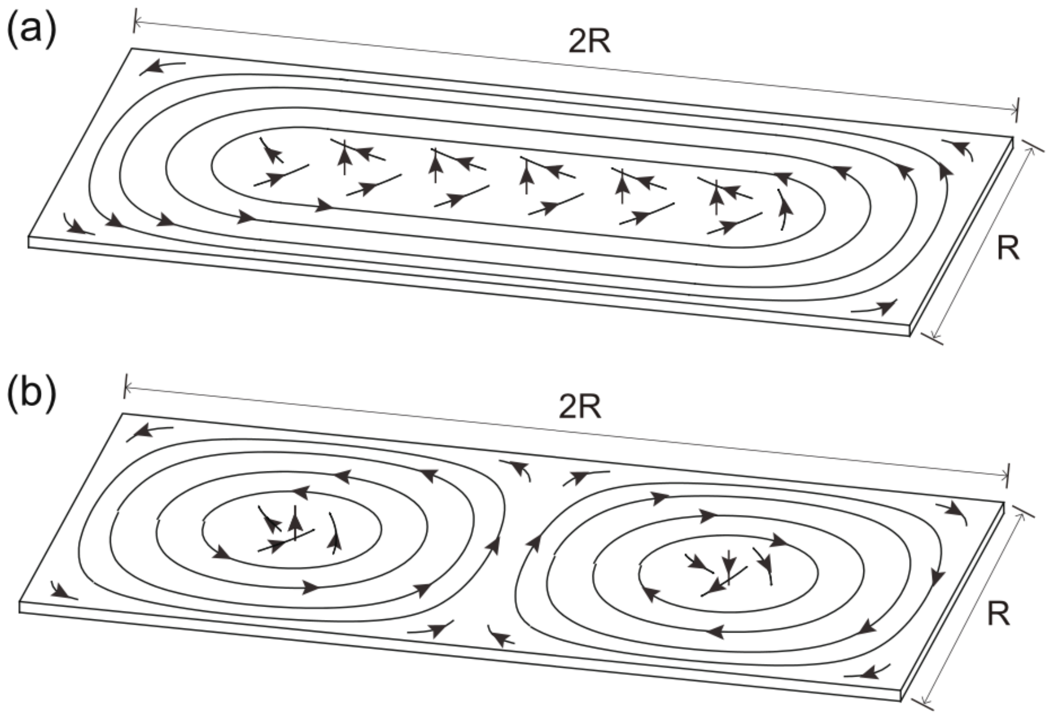


$$E_{2D}^{(II)} \approx -\frac{\lambda}{6}V + \frac{4\pi\alpha}{M}z_h \log\frac{R}{R_{2D}} + \frac{\pi\lambda}{2}z_h R_{2D}^2 \;.\xrightarrow{\text{minimize}}\; R_{2D} = (4\alpha/\lambda M)^{1/2}$$

Phase (I) occurs for $R > R_c$, Phase (II) occurs for $R < R_c$.

Quasi-2D traps with different aspect ratios

Multiple merons will arise when the aspect ratio ≠ 1 .



kinetic energy $\propto z_h R/R_{2D}$

kinetic energy $\propto z_h \log(R/R_{2D})$

Numerical simulation： Zhang, Ho, J. Low Temp. Phys. 161, 325 (2010)

Summary & future work

- In the Dirac string picture, strong dipolar condensates are weakly, locally interacting strings.
- The ground state favors closed, small curvature, parallel strings.
- Ground state energy is easy to estimate.

Future questions:

- Dynamics & response to external fields of Dirac strings
- Quantum version $\leftrightarrow$ quantum string theory?

  Making $\boldsymbol{S} = d\boldsymbol{r}/dl$ an operator, etc. …

- Antiferromagnetic / other spinor condensates

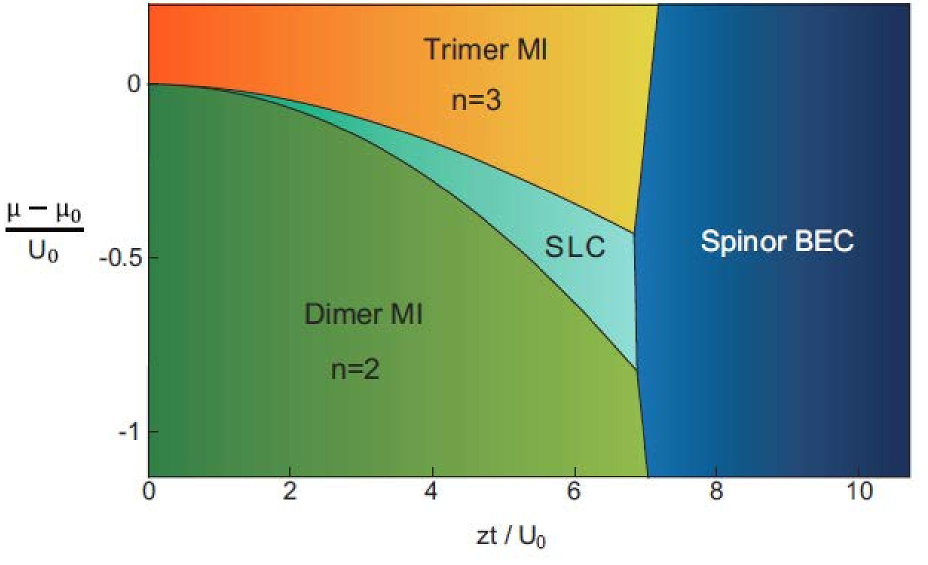
Advertisement

- Spin liquid condensate of spinful bosons

B. Lian, S. C. Zhang, Phys. Rev. Lett. **113**, 080402

Definition：

A superfluid of spinful bosons at zero temperature that breaks U(1) symmetry, but preserves SU(2) symmetry and has a non-zero spin gap.



(Will be presented in the poster section of the international conference.)