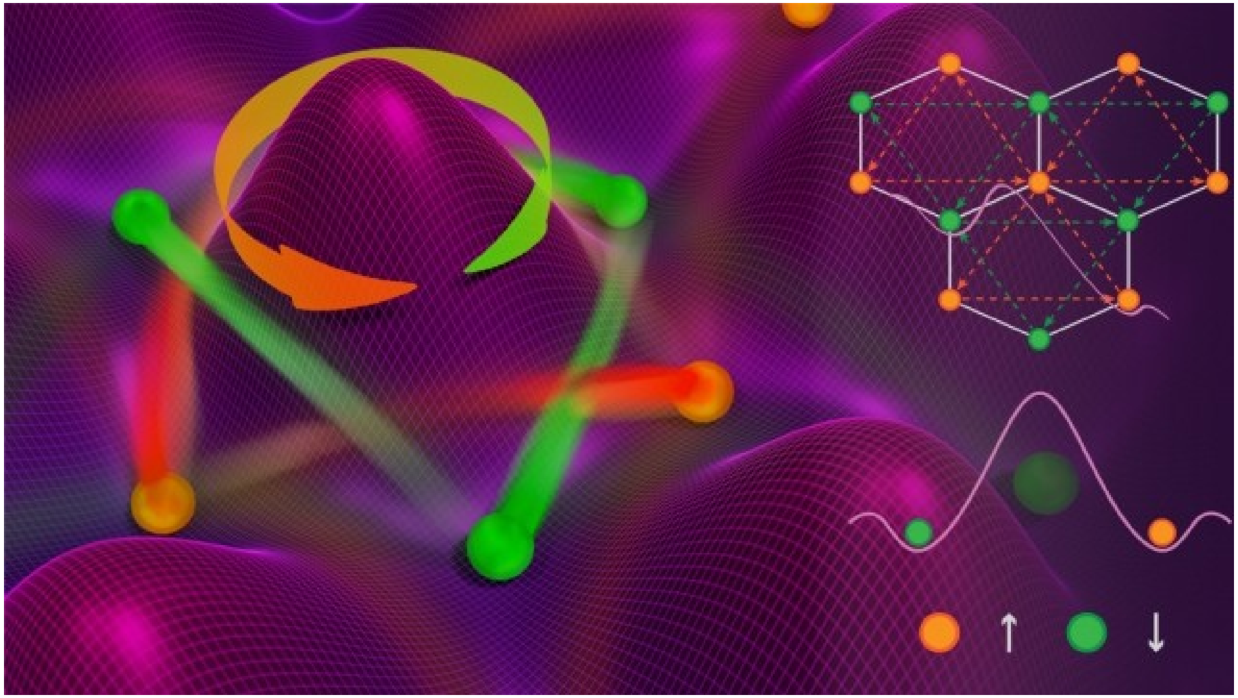
Frontiers in Quantum Simulations with Cold Atoms
April-09-2015, INT, Seattle

Loop currents and experimental signatures in optical lattices

Xiaopeng Li



[Figure credit: S. Kelley/JQI]

Acknowledgement

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Bo Liu (Pittsburgh)
Stefan Natu (Maryland)
W. Vincent Liu (Pittsburgh)
S. Das Sarma (Maryland)
Jed Pixley (Maryland)
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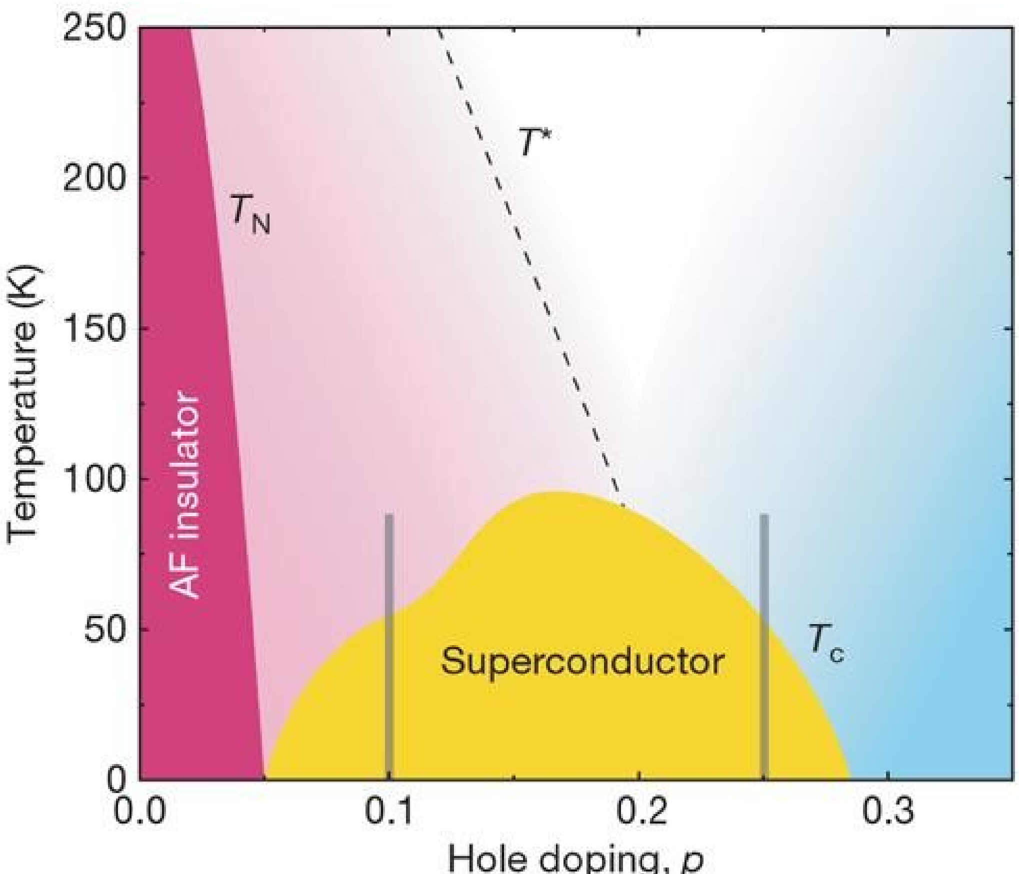
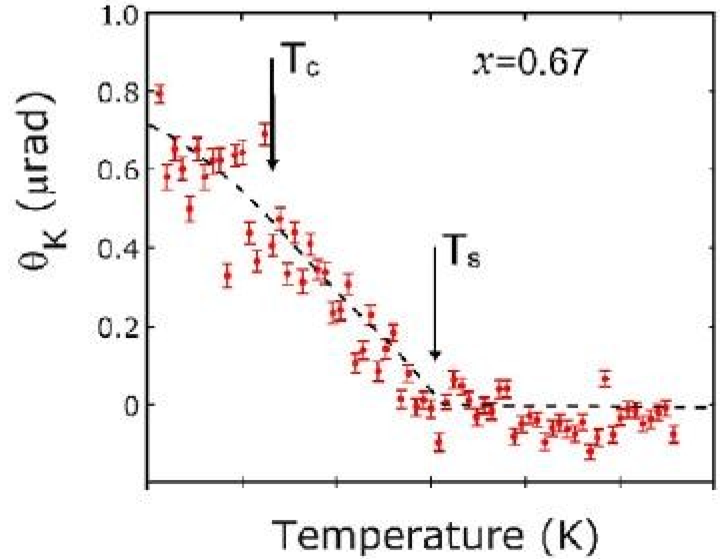
Outline

- Experimental observations of loop currents in optical lattices and cuprates
- Chiral spin condensate and spin loop currents in a hexagonal lattice
  [XL, S. Natu, A. Paramekanti, S. Das Sarma, Nature Communications (2014)]
- Spontaneous Quantum Hall effect with spinor Bose-Fermi mixture in a triangular lattice
  [Z.-F. Xu, XL, P. Zoller, W. Vincent Liu, PRL (2015)]
- Loop current order and spontaneous topological superfluids
  [Bo Liu, XL, Biao Wu, W. Vincent Liu, Nature Communications (2014)]
- Topological density waves with Rydberg dressed fermions
  [XL, S. Das Sarma, arXiv (2015), accepted to Nature Communications (2015)]

Loop currents in cuprates



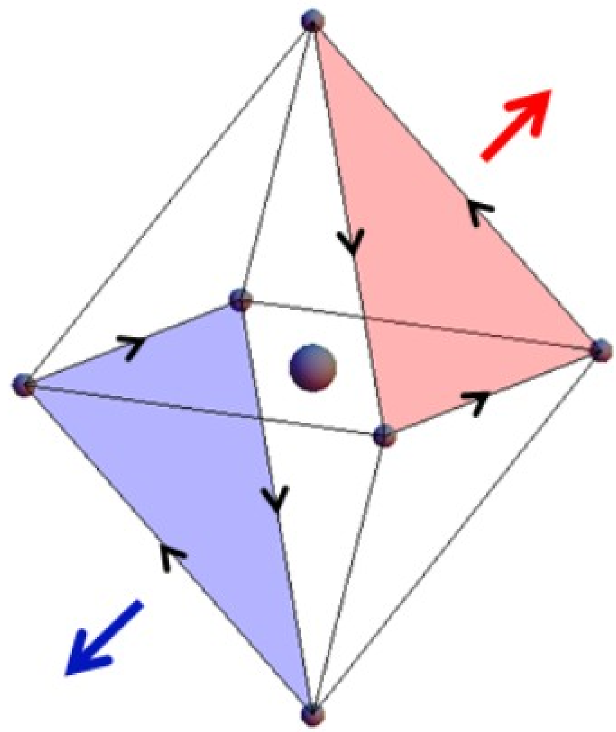
[Figure from Nature]

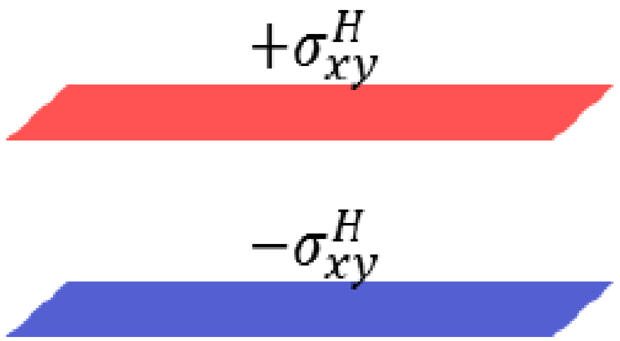


J. Xia et al., PRL (2008)
Z.-X. Shen et al., Science (2011)
H. Karapetyan et al., PRL (2012)
H. Karapetyan et al., PRL (2014)
….

Time-reversal symmetry is spontaneously broken. Electron loop current order is a promising candidate.

What is the loop current pattern?

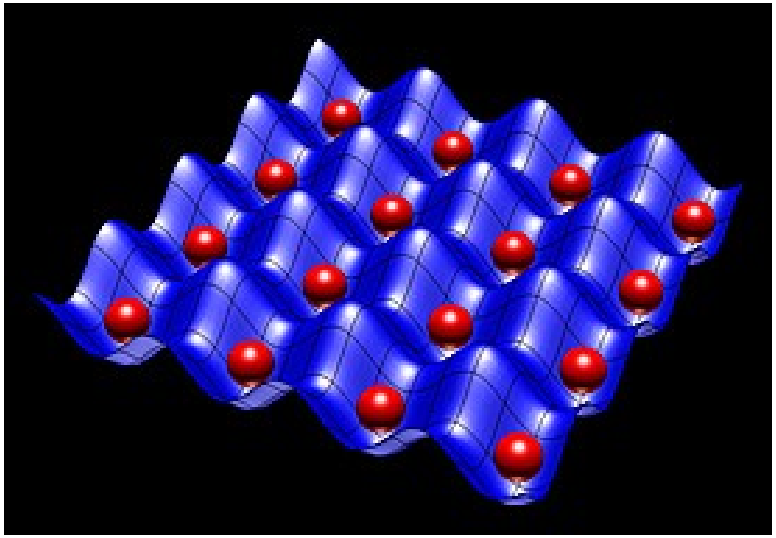




consistent with Kerr rotation

V. Yakovenko, arXiv (2014); Y. Li, PhD thesis (2010)

Loop currents in optical lattices?



[Figure from NIST]

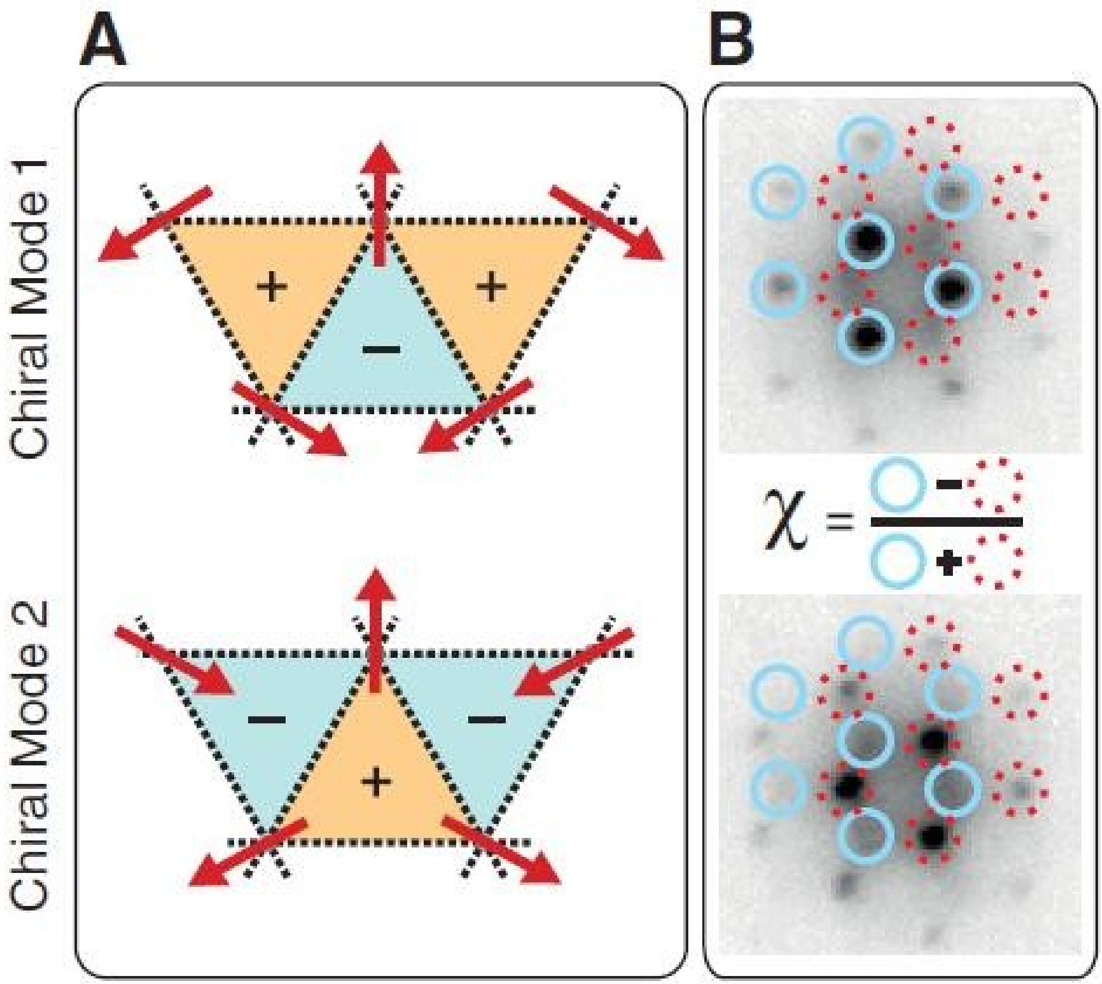
-current in a lattice model

$$J_{\mathbf{r}'\to\mathbf{r}} = -it_{\mathbf{r}\mathbf{r}'}\psi^\dagger_{\mathbf{r}}\psi_{\mathbf{r}'} + h.c.$$

For Bose-Einstein condensates, current means phase modulations in condensate wavefunctions.

*assumed that there is no pair hopping

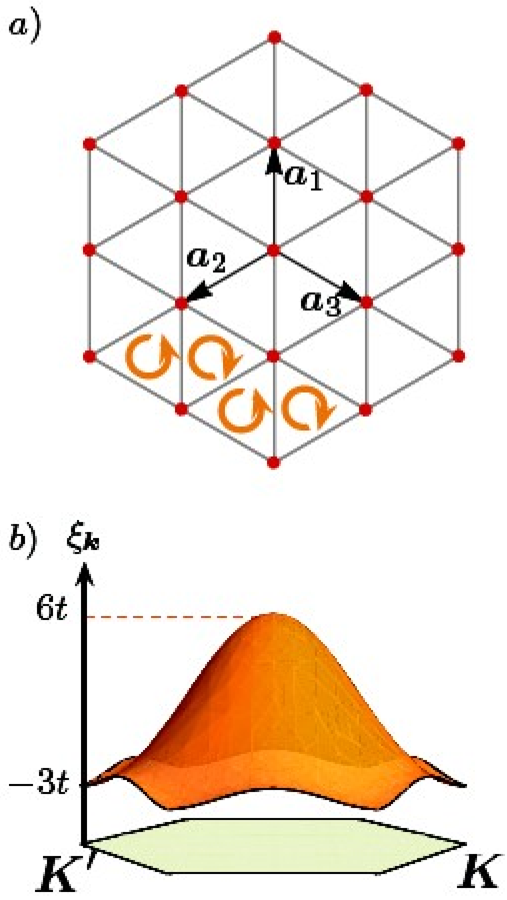
Pi-flux triangular lattice



J. Struck, K. Sengstock et al., Science (2010)

$$\langle \psi_{\mathbf{r}} \rangle = \sqrt{n_s} e^{i\mathbf{K}\cdot\mathbf{r}}$$

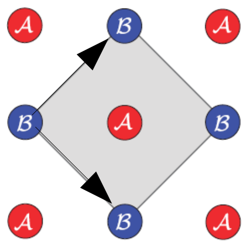
$$\mathbf{K} = (\pm \tfrac{2\pi}{3}, 0)$$

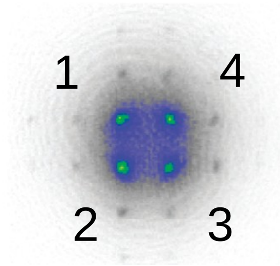


Measurement is simple when the band minima are not time-reversal invariant points.

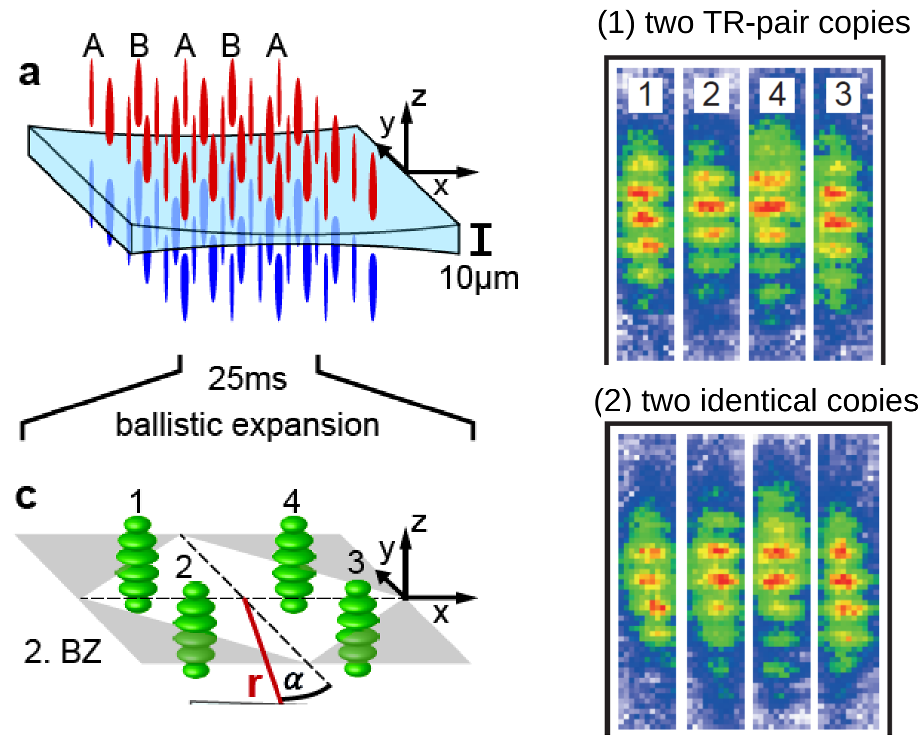
M. P. Zaletel, et al., PRB (2013)

2^{nd} band of Checkerboard lattice





G. Wirth, A. Hemmerich et al., Nat Phys (2011)



T. Kock, A. Hemmerich et al., PRL (2015)

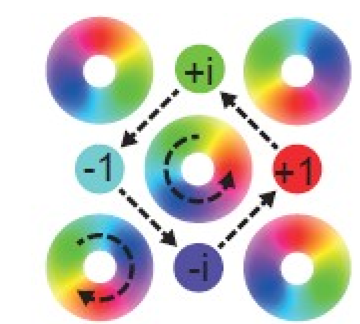
Band minima at time-reversal invariant points!

[Theory work:
A. Isacsson and S. Girvin, PRA (2005)
W. V. Liu, C. Wu, PRA (2006);
A. B. Kuklov, PRL (2006)
XL, Z.-X. Zhang, W.V. Liu, PRL (2012)
…]

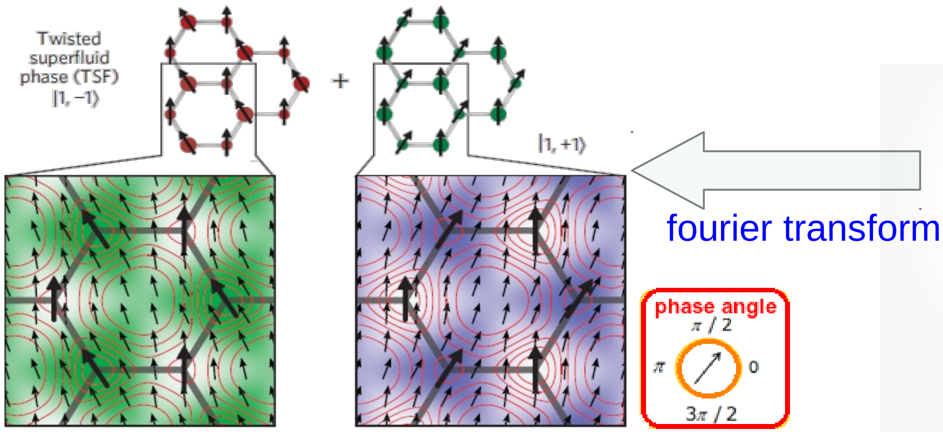
-Condensate wavefunction

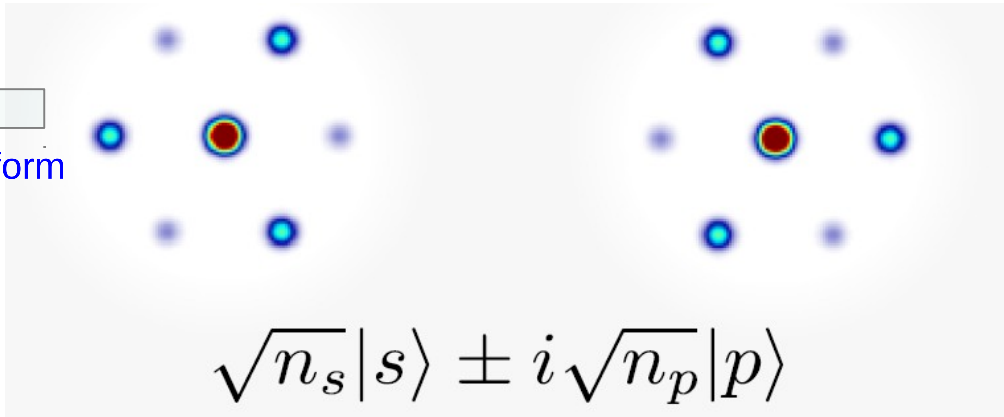
$$\langle \psi_{\mathbf{r}} \rangle = \sqrt{n_s}\left(e^{i\mathbf{K}_x\cdot\mathbf{r}} \pm ie^{i\mathbf{K}_y\cdot\mathbf{r}}\right)$$

$$\mathbf{K}_x = (\pi, 0) \quad \mathbf{K}_y = (0, \pi)$$



Spinor BEC in a Hexagonal lattice

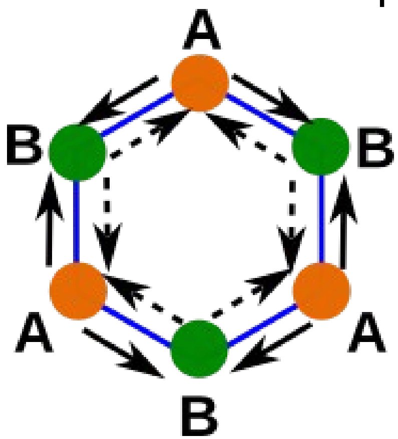




P. Soltan-Panahi K. Sengstock et al., NPHYS 8, 71-75 (2012)

Theory: S. Choudhury, E. Mueller, PRA (2013); L. Cao, K. Sengstock et al., arXiv (2014);
O. Jurgensen, K. Sengstock, D.S. Luhmann, arXiv (2015)

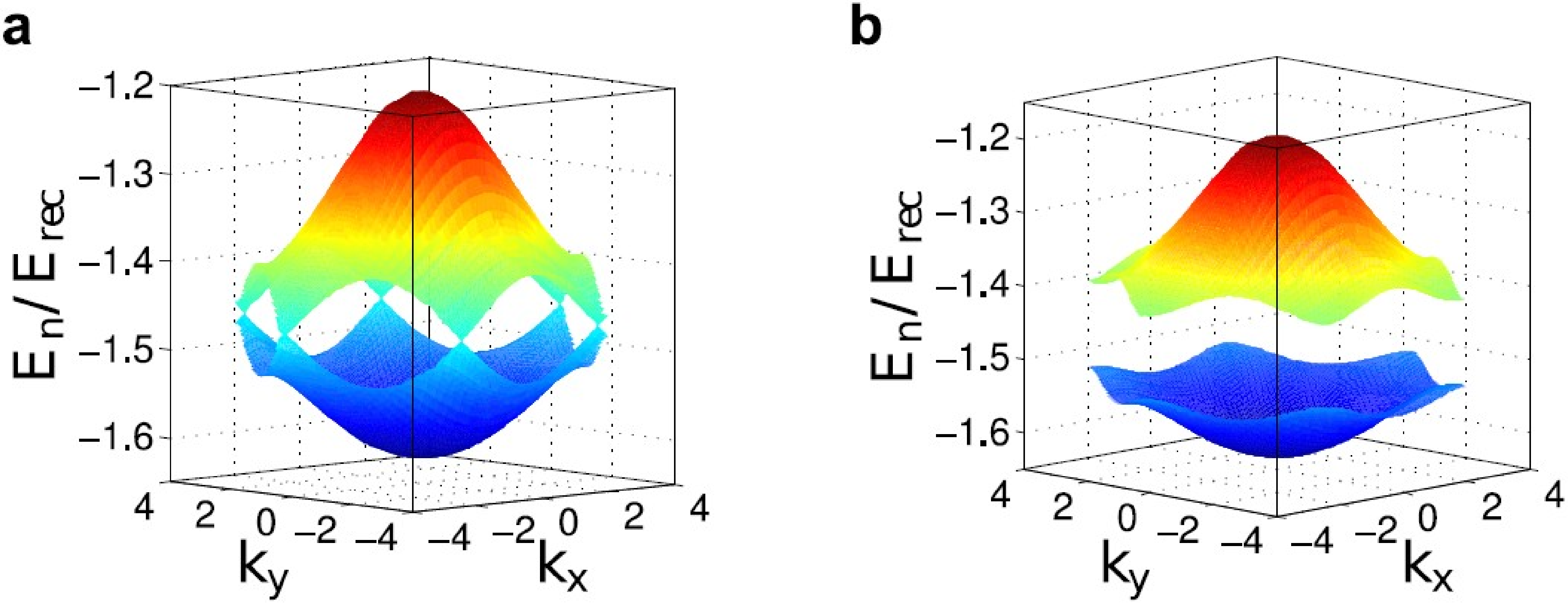
-current flow: not a loop current?



XL, 2013 March meeting presentation

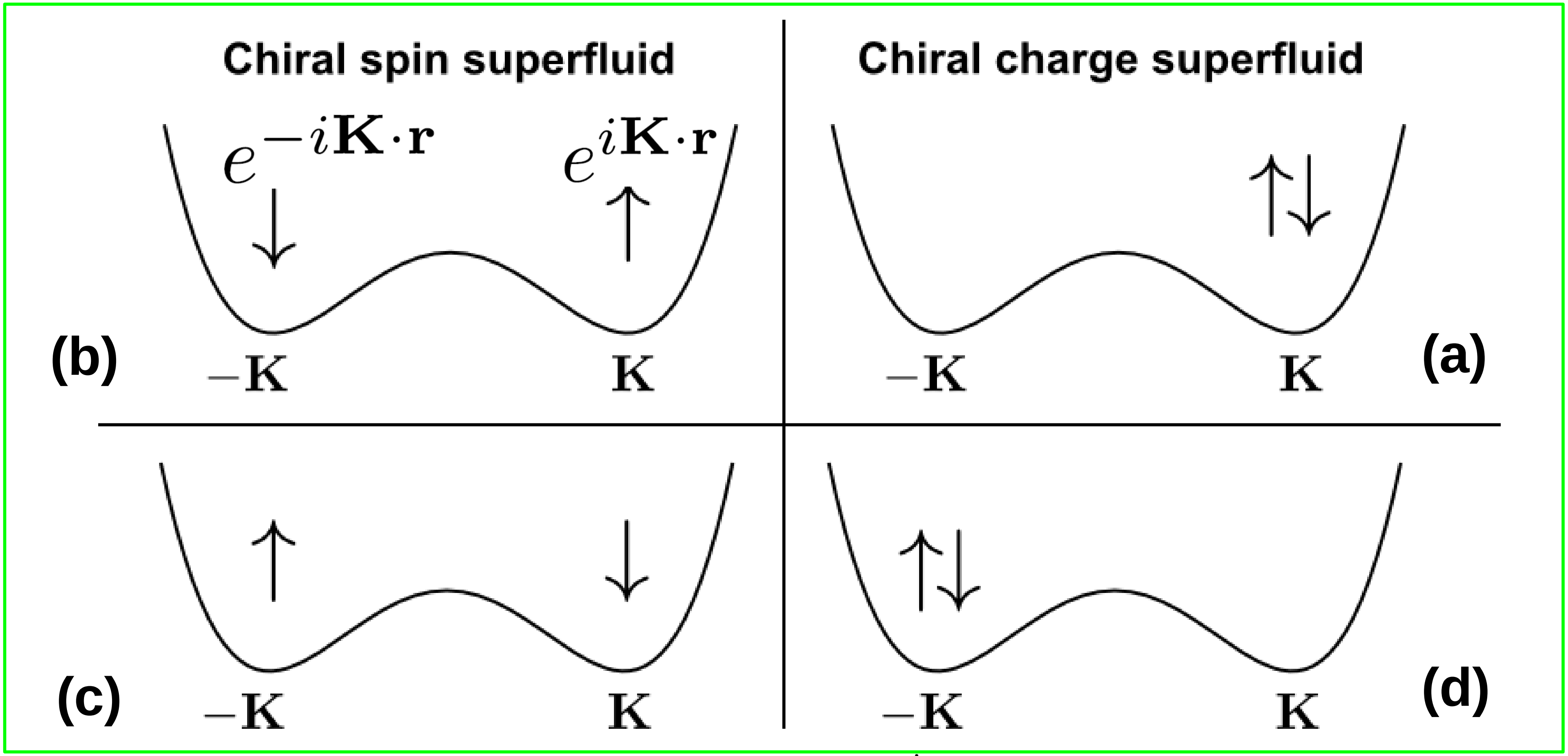
If we have spin Sz conservation, this current pattern cannot be dynamically stable. Its life time would be too short (1ms), in contrast to experimental observations (100ms).

Bandstructure of the hexagonal lattice



Question: What if some particles are left in the massive Dirac valleys of the 2[nd] band.

Spinor Bosons in a double-valley band

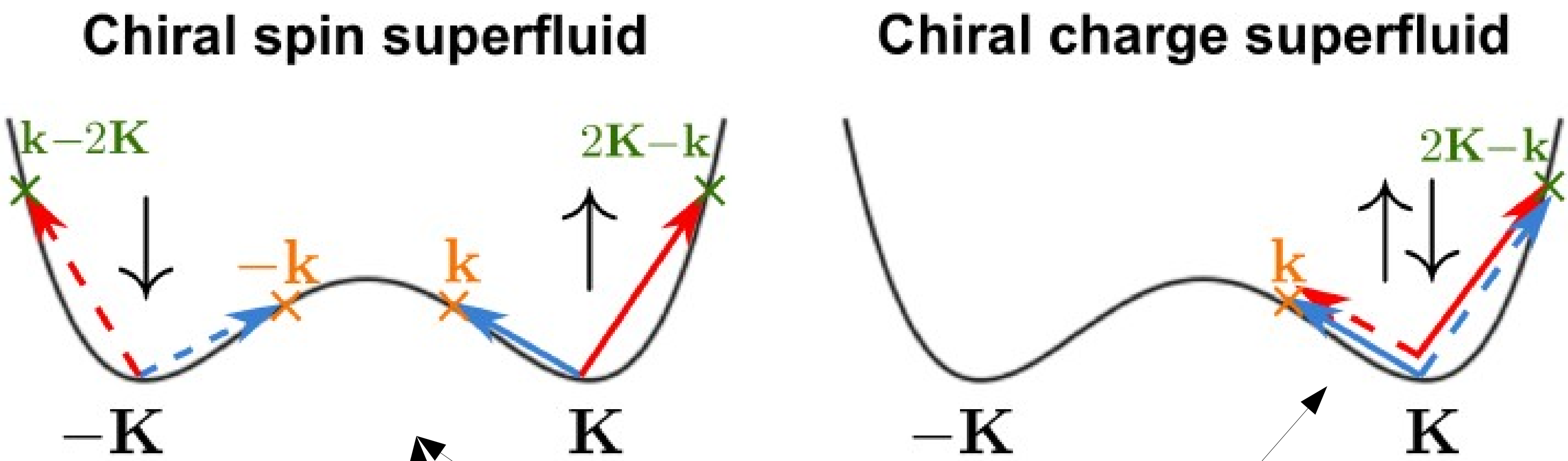


$$\varphi_{\downarrow\mathbf{r}} \rightarrow \varphi^*_{\downarrow\mathbf{r}}$$

$$E[\varphi_{\uparrow\mathbf{r}}, \varphi^*_{\downarrow\mathbf{r}}] = E[\varphi_{\uparrow\mathbf{r}}, \varphi_{\downarrow\mathbf{r}}]$$

XL, S. Natu, A. Paramekanti, S. Das Sarma, Nat Comms 5:5174 (2014)

Second order perturbation theory



$$\Delta E^{(2)}/N_s = -\int \frac{d^d\mathbf{k}}{(2\pi)^d}\rho_\uparrow\rho_\downarrow \left\{ \frac{|U_{\uparrow\downarrow}(\mathbf{k}-\mathbf{K})|^2}{\epsilon(\mathbf{k})+\epsilon(\mathbf{Q}-\mathbf{k})} - \frac{1}{2}\frac{|U_{\uparrow\downarrow}(\mathbf{k}-\mathbf{K})|^2}{\epsilon(\mathbf{k})+\epsilon(-\mathbf{k})} - \frac{1}{2}\frac{|U_{\uparrow\downarrow}(\mathbf{K}-\mathbf{k})|^2}{\epsilon(\mathbf{Q}-\mathbf{k})+\epsilon(\mathbf{k}-\mathbf{Q})} \right\}, \quad \mathbf{Q}=2\mathbf{K}$$

$$\Delta E^{(2)} = E^{(2)}_{\chi_c} - E^{(2)}_{\chi_s}$$

TRS: $T\phi_\sigma(\mathbf{k})T^{-1} = \phi_\sigma(-\mathbf{k})$

an anti-unitary transformation

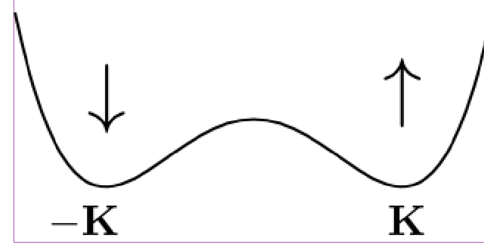
Universal quantum "order-by-disorder"

$$\Delta E^{(2)} = E^{(2)}_{\chi_c} - E^{(2)}_{\chi_s}$$

$$\Delta E^{(2)}/N_s = -\int \frac{d^d\mathbf{k}}{(2\pi)^d}\rho_\uparrow\rho_\downarrow \left\{ \frac{|U_{\uparrow\downarrow}(\mathbf{k}-\mathbf{K})|^2}{\epsilon(\mathbf{k})+\epsilon(\mathbf{Q}-\mathbf{k})} - \frac{1}{2}\frac{|U_{\uparrow\downarrow}(\mathbf{k}-\mathbf{K})|^2}{\epsilon(\mathbf{k})+\epsilon(-\mathbf{k})} - \frac{1}{2}\frac{|U_{\uparrow\downarrow}(\mathbf{K}-\mathbf{k})|^2}{\epsilon(\mathbf{Q}-\mathbf{k})+\epsilon(\mathbf{k}-\mathbf{Q})} \right\},$$

the universal winner!

Chiral spin superfluid



Chiral spin superfluid with the two spin components condensing at opposite valleys always has lower fluctuation energy. This universal quantum "order by disorder" selection rule only relies on the "Time-reversal" symmetry.

This momentum space antiferromagnetism can be generalized to multi-valley case with crystalline symmetries.

Logarithmic divergence and renormalized theory

In two dimensions, the bare perturbative result has a logarithmic divergence

$\int d^2\mathbf{k}\,\frac{1}{\mathbf{k}^2}$ → infrared log divergence

-renormalized theory

$\Delta E^{(2)}/N_s = -\frac{1}{2}\rho_\uparrow\rho_\downarrow \int_{\mathbf{k}} g^2(\mathbf{k})$ → effective scattering among quasi-particles

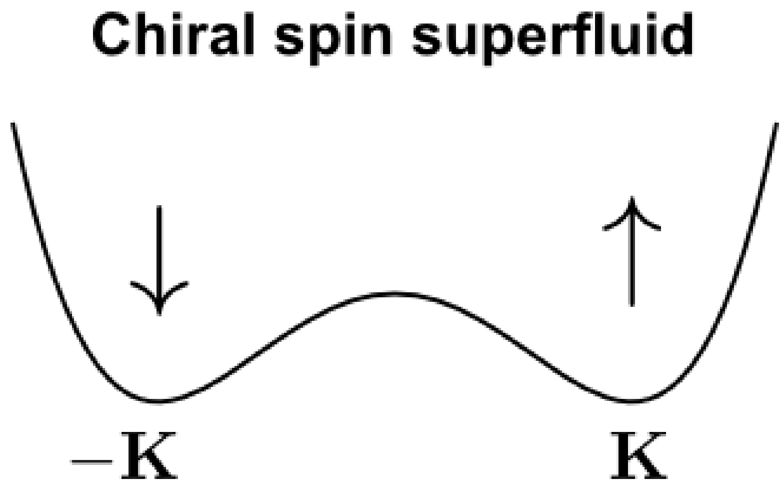
$$\times \left\{ \frac{2}{\varepsilon_\uparrow(\mathbf{k},\mathbf{Q}-\mathbf{k})+\varepsilon_\downarrow(\mathbf{k},\mathbf{Q}-\mathbf{k})} \right.$$

→ Bogoliubov spectra

$$- \frac{1}{\varepsilon_\uparrow(\mathbf{k},\mathbf{Q}-\mathbf{k})+\varepsilon_\downarrow(-\mathbf{Q}+\mathbf{k},-\mathbf{k})+\Delta\epsilon(\mathbf{k},\mathbf{Q}-\mathbf{k})-\Delta\epsilon(-\mathbf{Q}+\mathbf{k},-\mathbf{k})}$$

$$\left. - \frac{1}{\varepsilon_\downarrow(-\mathbf{Q}+\mathbf{k},-\mathbf{k})+\varepsilon_\uparrow(\mathbf{k},\mathbf{Q}-\mathbf{k})+\Delta\epsilon(-\mathbf{Q}+\mathbf{k},-\mathbf{k})-\Delta\epsilon(\mathbf{k},\mathbf{Q}-\mathbf{k})} \right\}$$

Universal Chiral spin superfluid



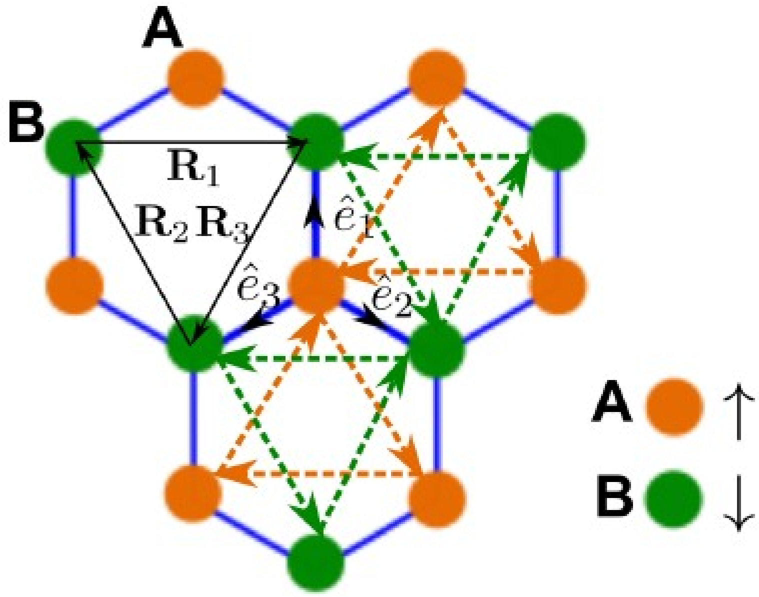
double-valley bandstructure

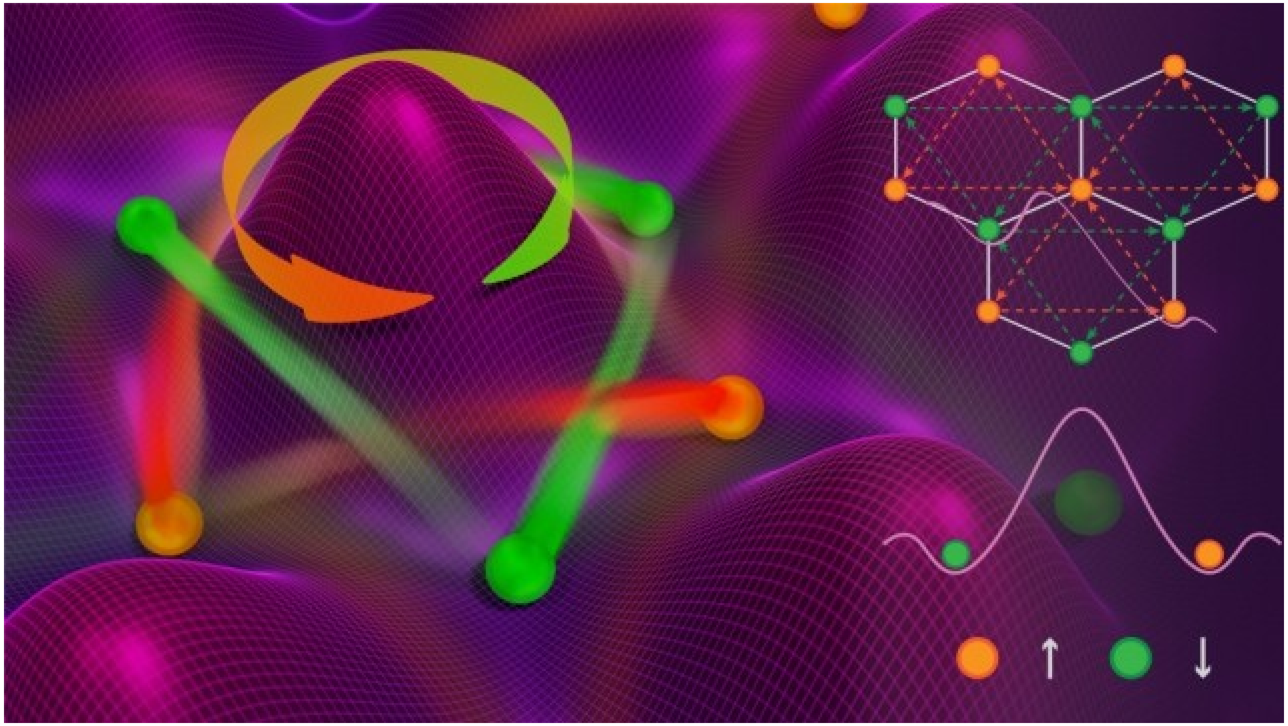
$$\int d^d\mathbf{k}\,\mathbf{k}\,[n_{\uparrow}(\mathbf{k}) - n_{\downarrow}(\mathbf{k})] \neq 0$$

With two component bosons loaded into a double-valley band, quantum fluctuations universally select the chiral spin superfluid through a quantum order by disorder mechanism.

XL, S. Natu, A. Paramekanti, S. Das Sarma, Nat Comms 5:5174 (2014)

Spin-loop current

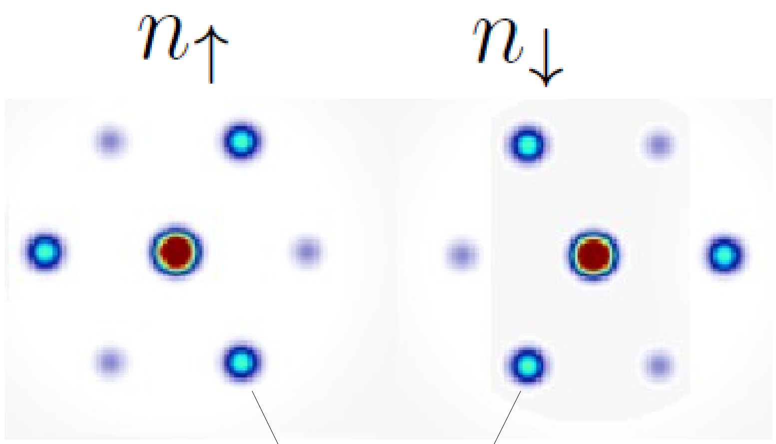




[Figure credit: S. Kelley/JQI]

Experimental signatures

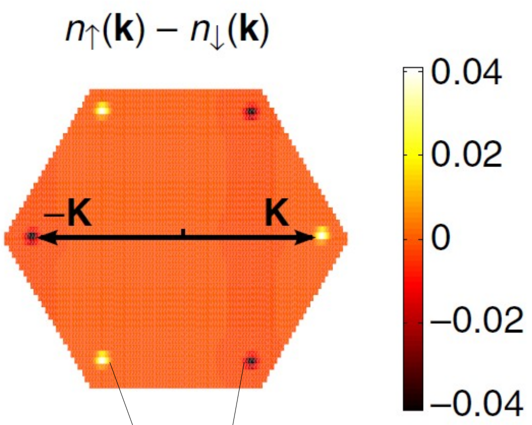
-experimental data



Lattice momentum = 0

P. Soltan-Panahi K. Sengstock et al., NPHYS 8, 71-75 (2012)

-our theory prediction

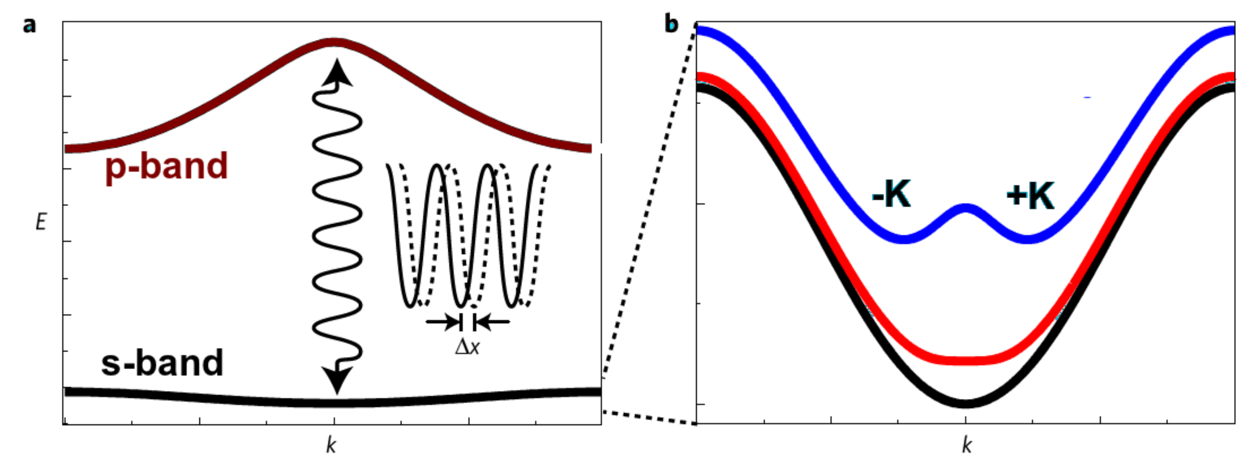


Dirac points

XL, S. Natu, A. Paramekanti, S. Das Sarma, Nat Comms (2014)

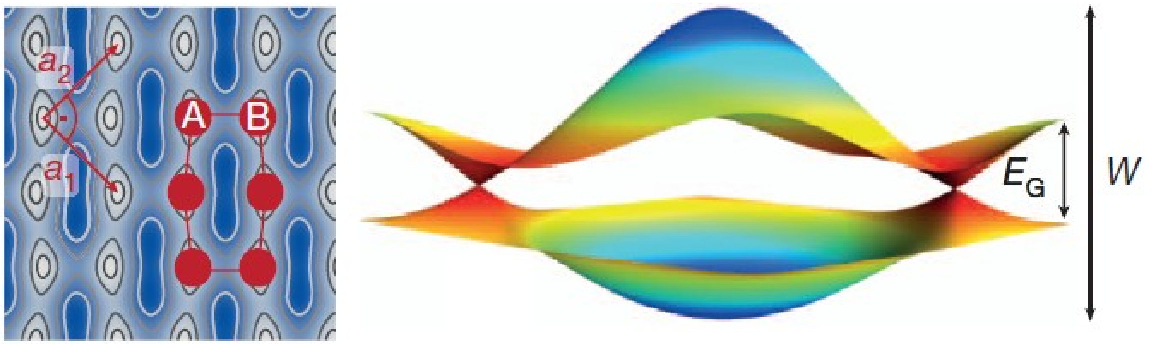
Relevance to other double-valley bands

C. Chin group (Chicago)
[C. Parker et al., Nat Phys (2013)]

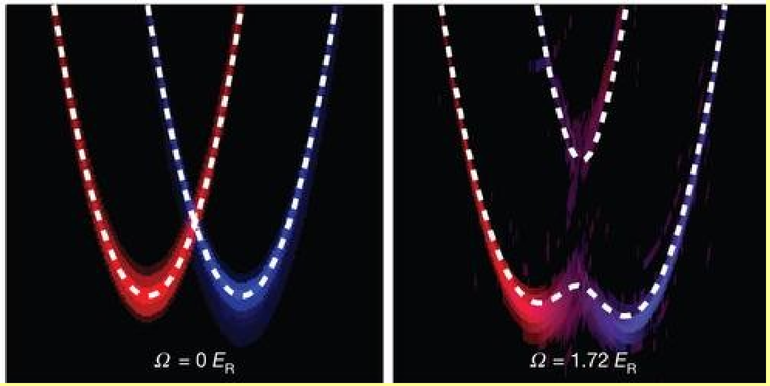


[Related theory work: XL, E. Zhao, W.Vincent Liu, Nat Comms (2013)]

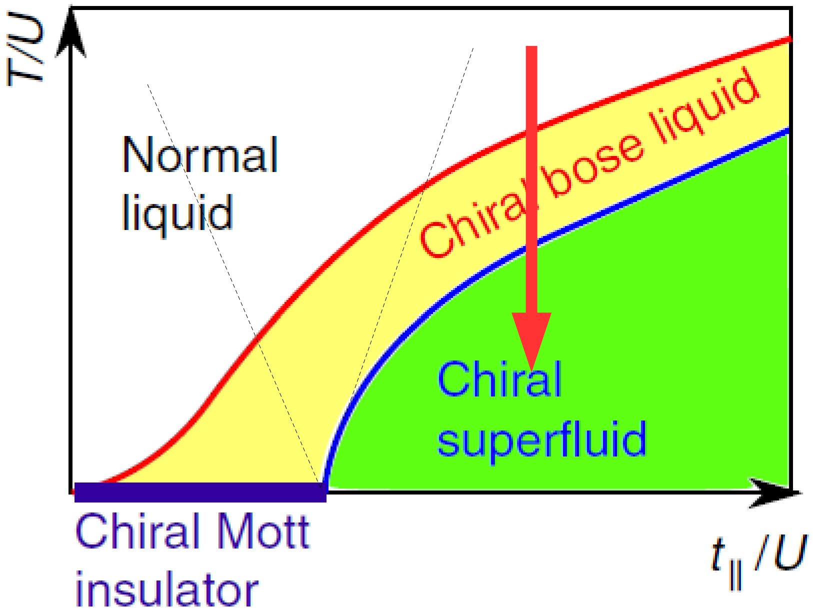
T. Esslinger group (ETH)
[L. Tarruell et al., Nature (2012)]

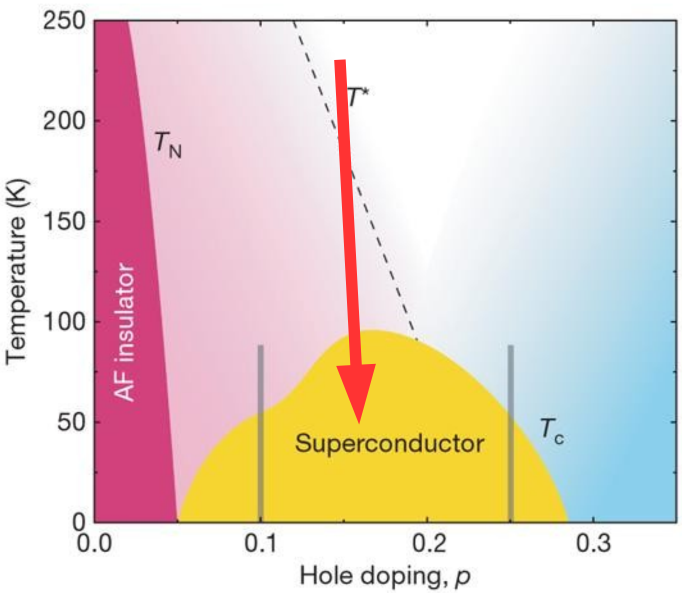


I. Spielman group (JQI/NIST)
[Y.J. Lin et al., Nature (2011)]



Finite temperature perspective





[Figure from Nature]

XL, A. Paramekanti, A. Hemmerich, W. V. Liu, Nat Comm 5:3205 (2014)

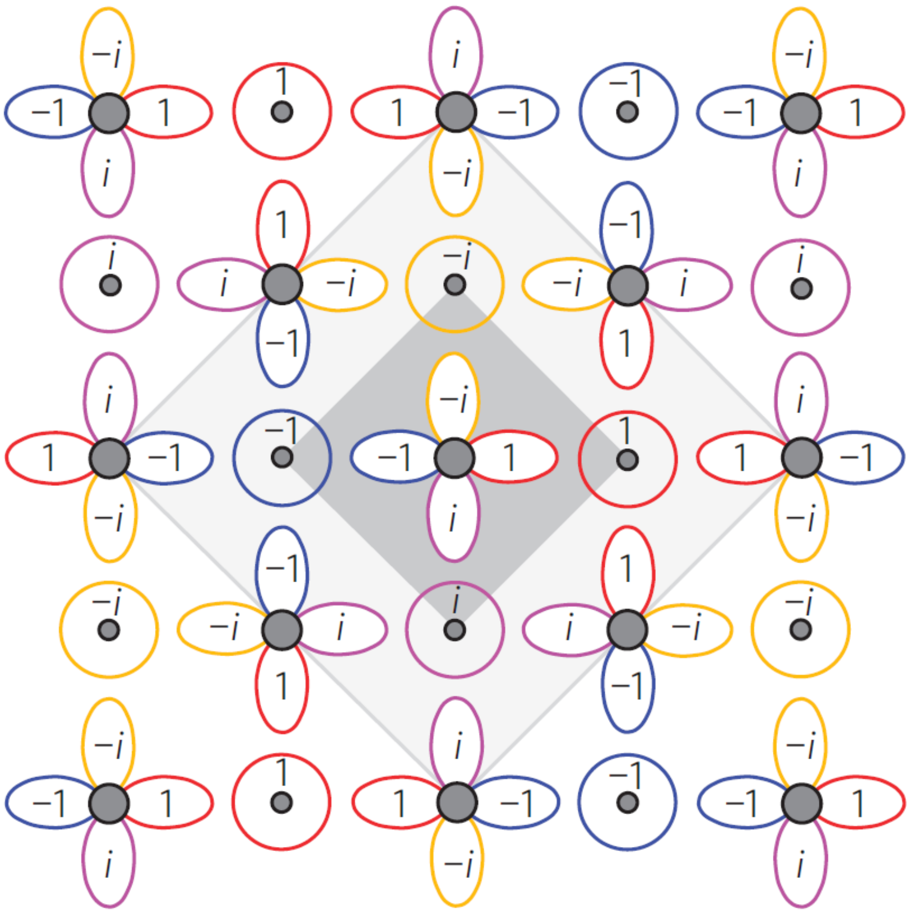
Zero temperature phase diagram by QMC: F. Hebert, Z. Cai, et al., PRB (2013)

Quantum simulations---Where are we with ultracold atoms?

BEC with spontaneous loop current order

- A generic state for bosons with valley degrees of freedom
- Time reversal symmetry is broken
- Easy to measure if the valley is not at time-reversal invariant point; harder otherwise, but has been measured
- No conclusive evidence for spin loop current yet
- Thermal phase transition, sharing features in cuprate phase diagram

Topological states with loop current order (fermions)



G. Wirth, A. Hemmerich et al., Nat Phys (2011)

If we have a Cooper pair condensatation of this pattern, the fermionic state then has all ingredients required by topological superfluids.

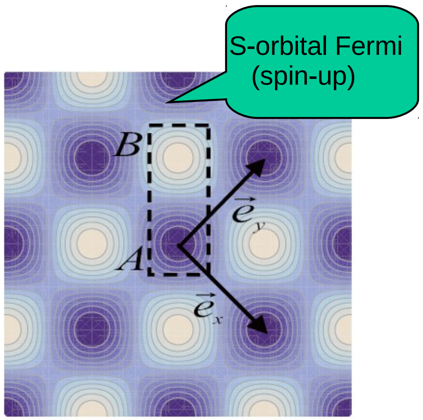
Cooper pairs with p-orbital symmetries

Lattice potential

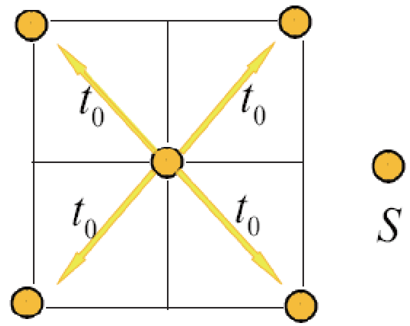
Tight binding model

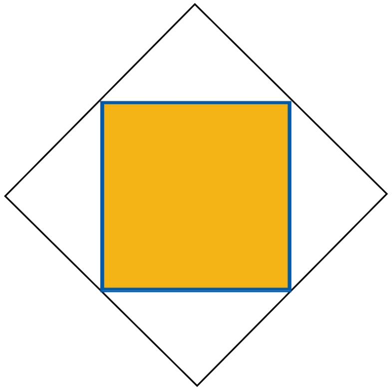
Fermi surface matched

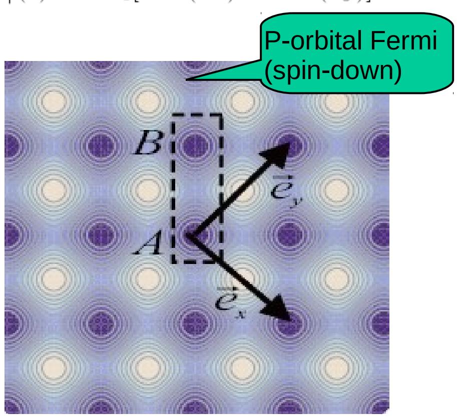
$(t_2/t_0 = 0, t_3/t_0 = 0)$



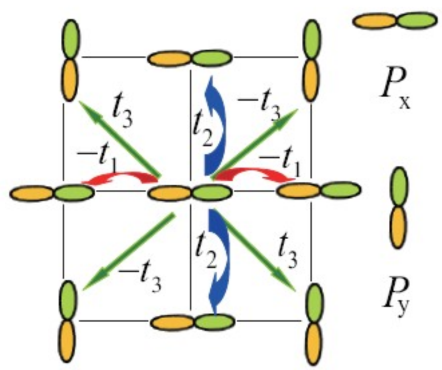
$V_\uparrow(\mathbf{r}) = -V_s[\cos^2(kx) + \cos^2(ky)]$

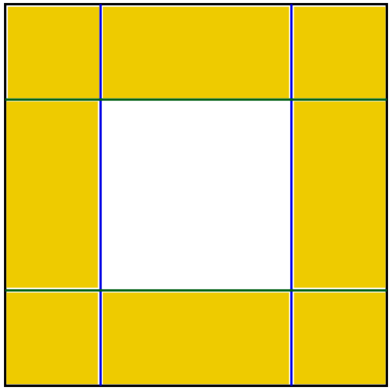






$V_\downarrow(\mathbf{r}) = -V_p[\cos^2(k(x+y)) + \cos^2(k(x-y))]$



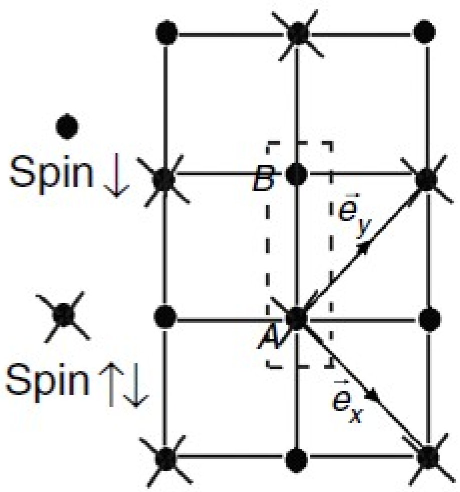


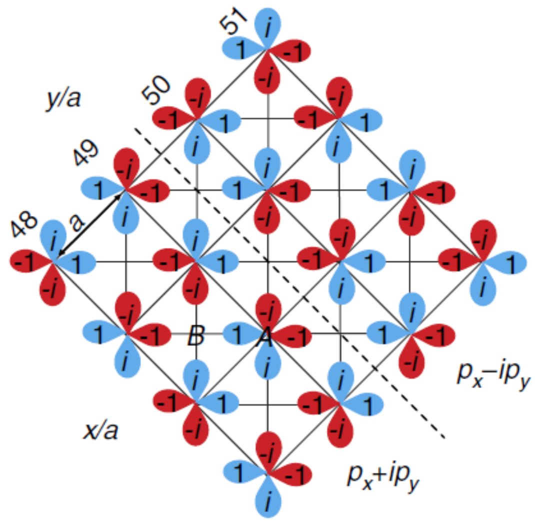
Bo Liu, XL, Biao Wu, W. Vincent Liu, Nat Comms (2014)

A cooper pair composed of s and p orbital fermions respects p-orbital symmetries.

Topological p+ip topological superfluids

-spin dependent checkerboard lattice





This lattice may have already been realized in JQI by Trey Porto's group.

Fermions in the current order backgroup form a topological superfluid, featuring protected chiral spin currents on the edges

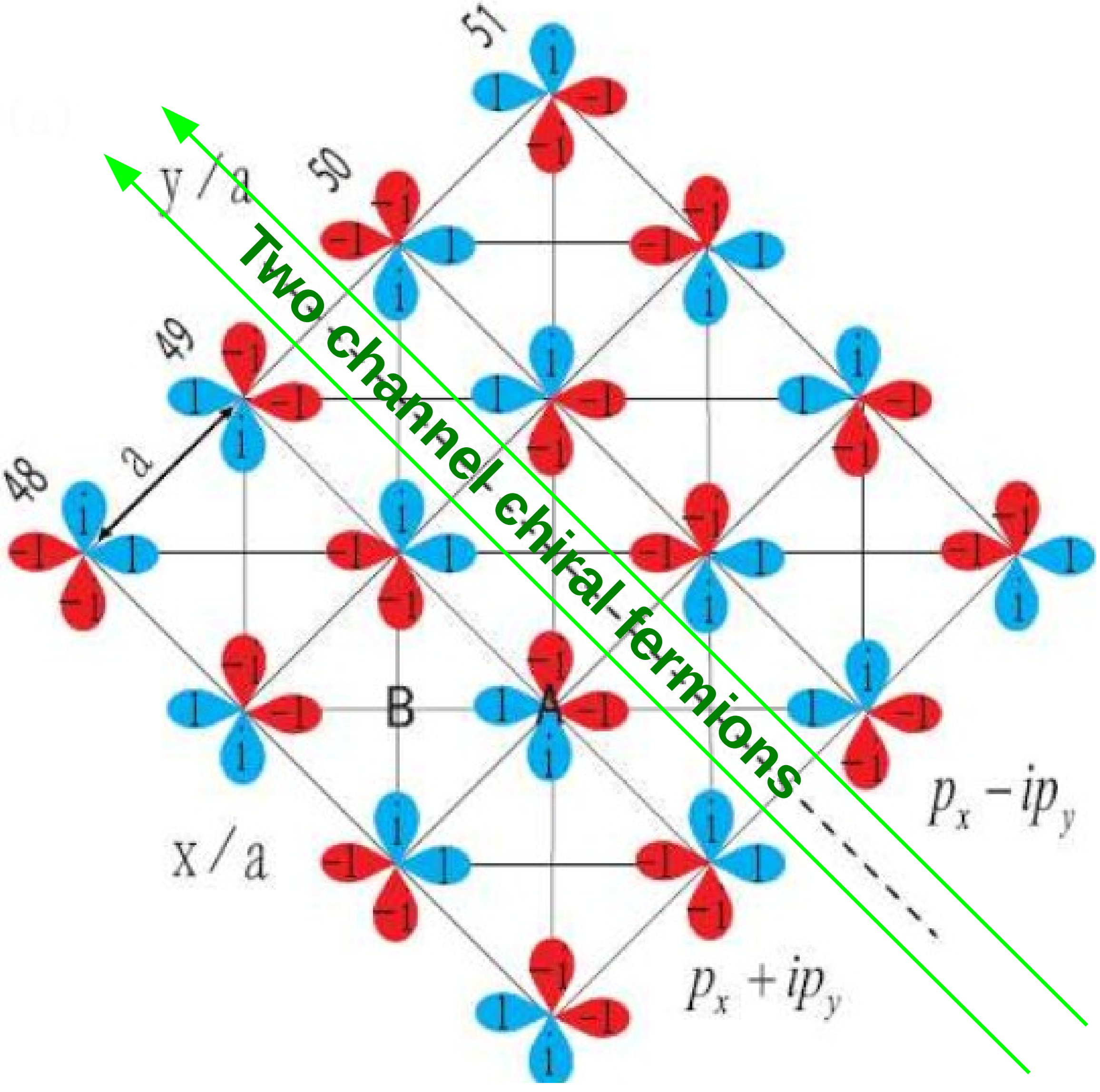
Cooper pairing fields:

$$\Delta_x(\mathbf{x}) \longrightarrow (-1)^{R_x+R_y} U \Psi^A_{P_x}(\mathbf{R}) \Psi^A_S(\mathbf{R})$$

$$\Delta_y(\mathbf{x}) \longrightarrow (-1)^{R_x+R_y} U \Psi^A_{P_y}(\mathbf{R}) \Psi^A_S(\mathbf{R})$$

Bo Liu, XL, Biao Wu, W.V. Liu, Nat Comms 5:5064(2014)

Chiral spin currents on the domain wall



Essential ingredients:

Fermi surface nesting->Spontaneous Loop currents->effective gauge fields-> Topological states

Atomic spinor Bose-Fermi mixture

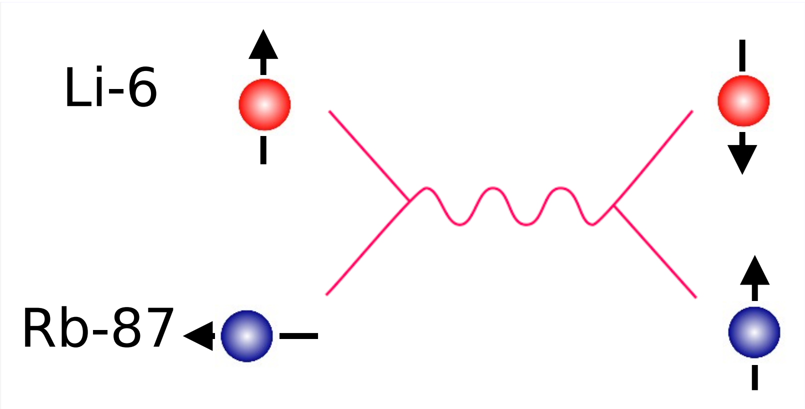
- **Local moments:**

  Spin-1 Rb-87 BEC: ferromagnetic interaction

- **Kondo coupling:**

  Spin-changing collision between spin-1 Rb-87 and spin-1/2 Li-6 atoms

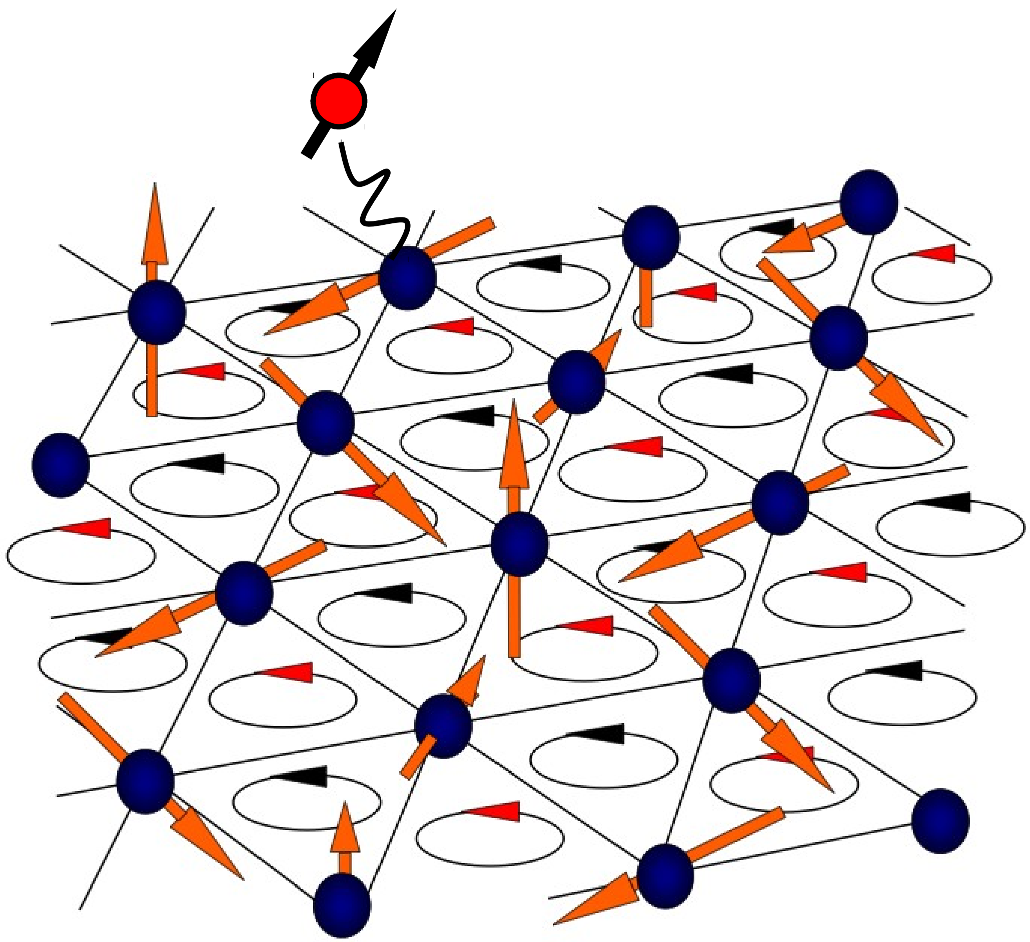
$$\hat{V}_{bf}(\mathbf{r}_1 - \mathbf{r}_2) = \left(g_{1/2}\mathrm{P}_{1/2} + g_{3/2}\mathrm{P}_{3/2} \right) \delta(\mathbf{r}_1 - \mathbf{r}_2)$$
$$= \left(g_d + g_s \mathbf{S} \times \mathbf{F} \right) \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

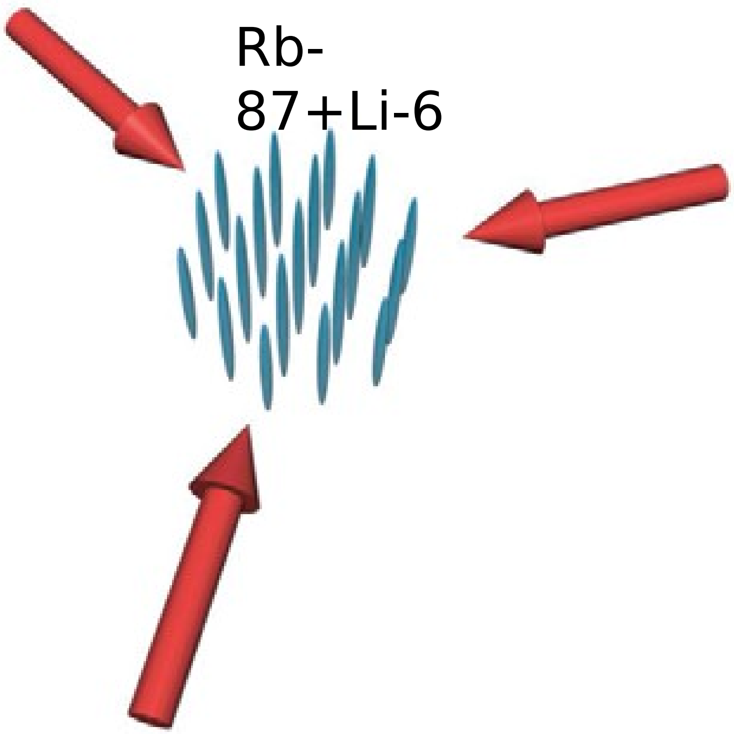


Spinor Bose-Fermi mixture is a natural platform to simulate Kondo lattice model (either quantum or classical).

Z. F. Xu, XL, P. Zoller, and W. V. Liu, PRL (2015)

Triangular lattice



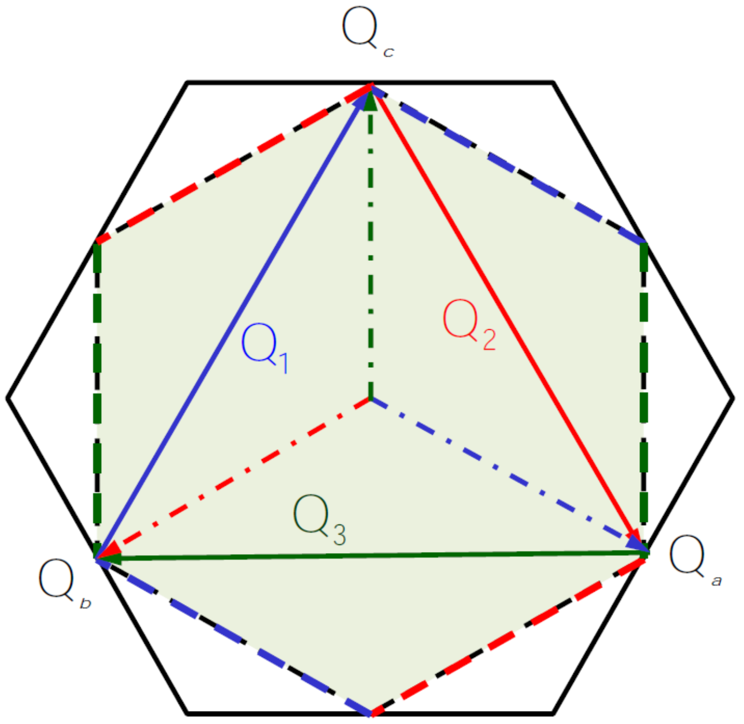


Lattice potential

Z. F. Xu, XL, P. Zoller, and W. V. Liu, PRL (2015)

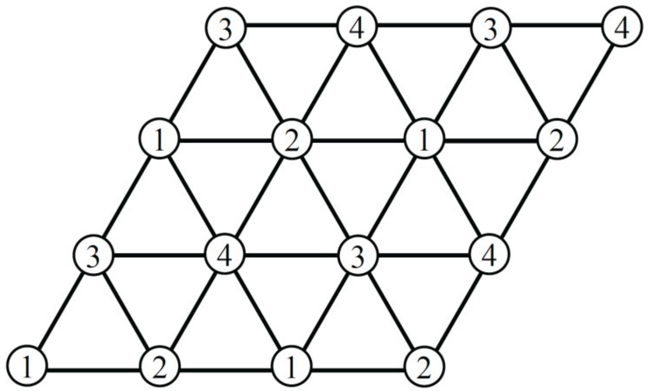
Fermi surface nesting and chiral magnetic ordering

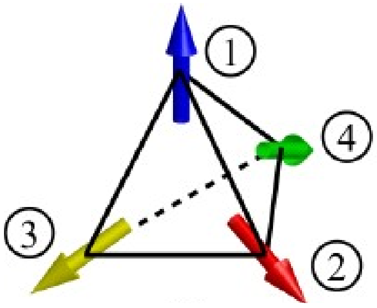
Fermi surface at ¾ filling



$$H = -t\sum_{<ij>} c^{\dagger}_{i\alpha} c_{j\alpha} - J \sum_i \vec{S}_i \cdot c^{\dagger}_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$$

$$\langle \vec{S}_i \rangle = \vec{S}_{\mathbf{Q}_1} e^{i\mathbf{Q}_1 \cdot \mathbf{r}_i} + \vec{S}_{\mathbf{Q}_2} e^{i\mathbf{Q}_2 \cdot \mathbf{r}_i} + \vec{S}_{\mathbf{Q}_3} e^{i\mathbf{Q}_3 \cdot \mathbf{r}_i}$$

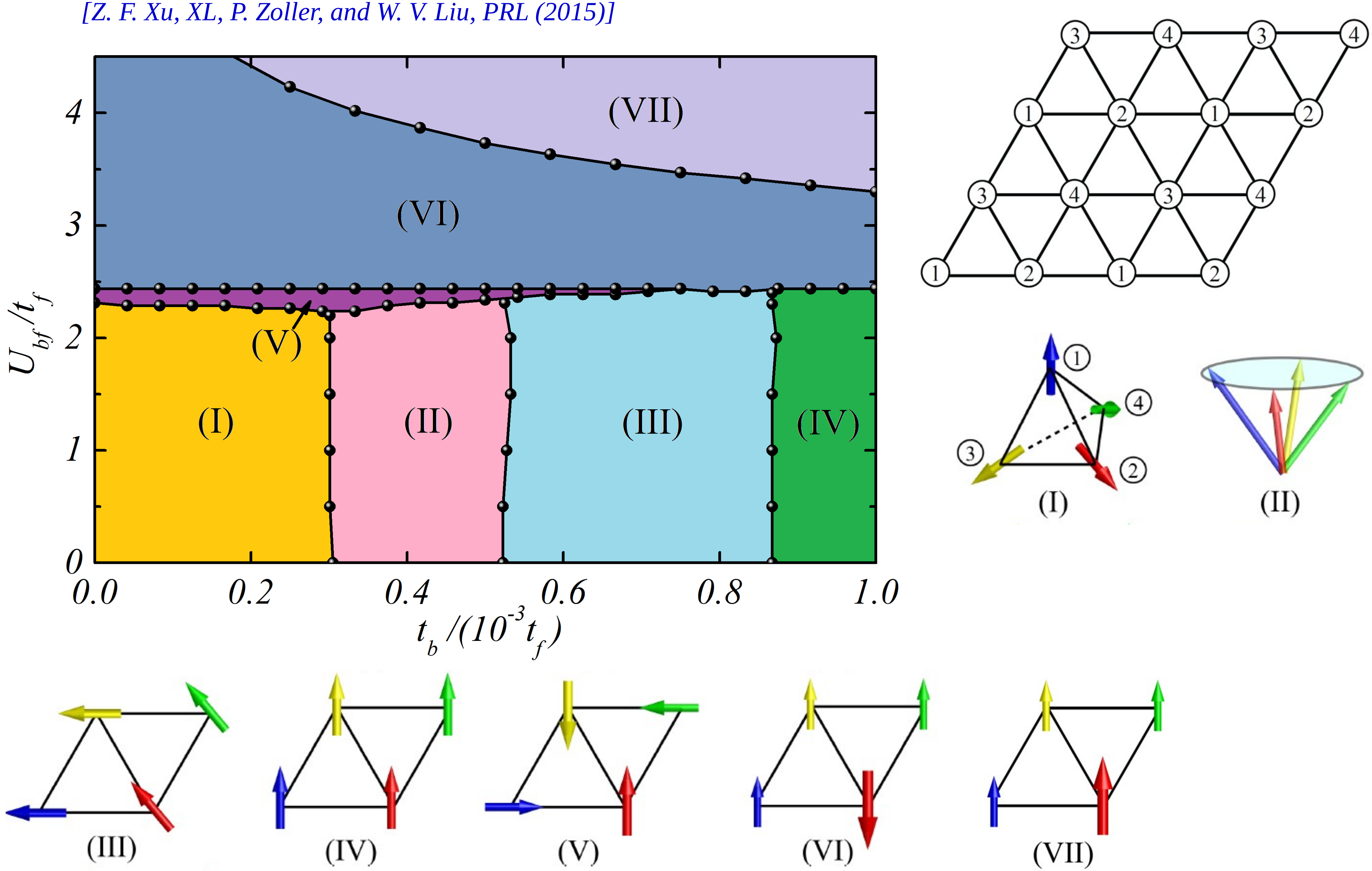




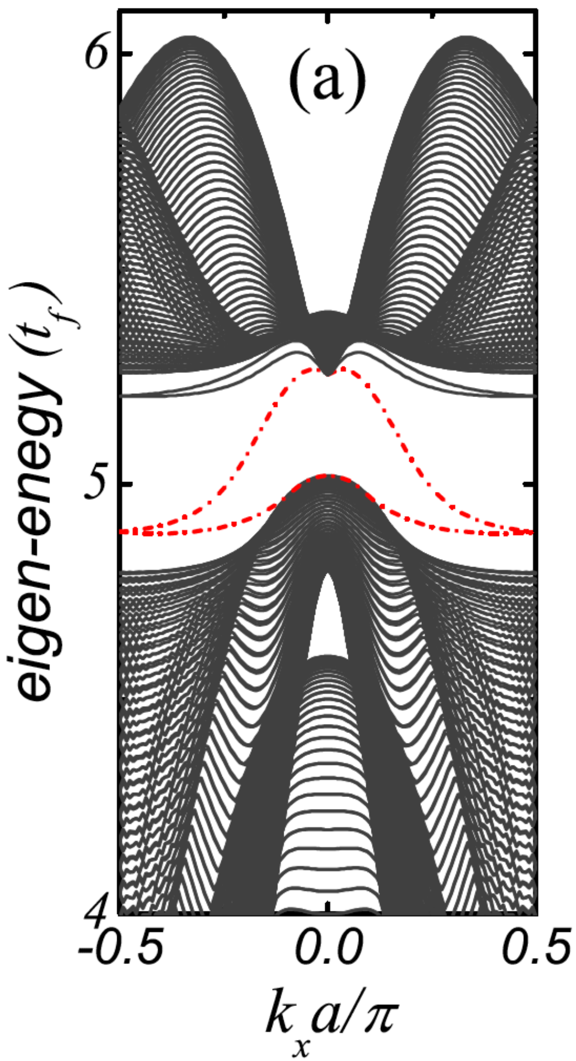
Ivar Martin and C. D. Batista, Phys. Rev. Lett. 101, 156402 (2008)

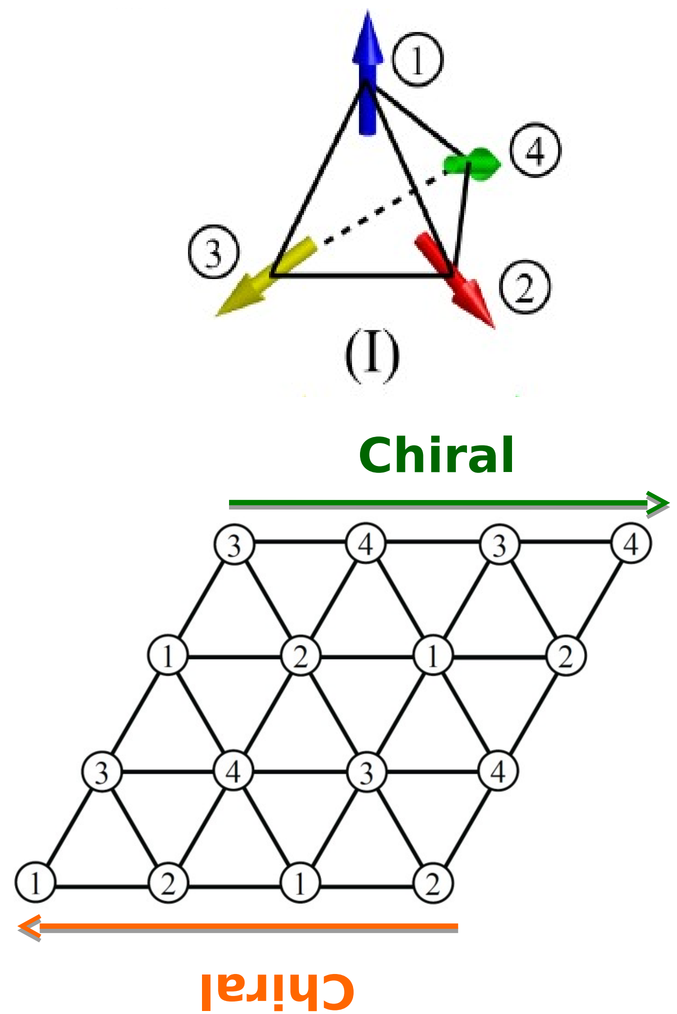
Rb-87: topological spin-textures in the ground state

[Z. F. Xu, XL, P. Zoller, and W. V. Liu, PRL (2015)]



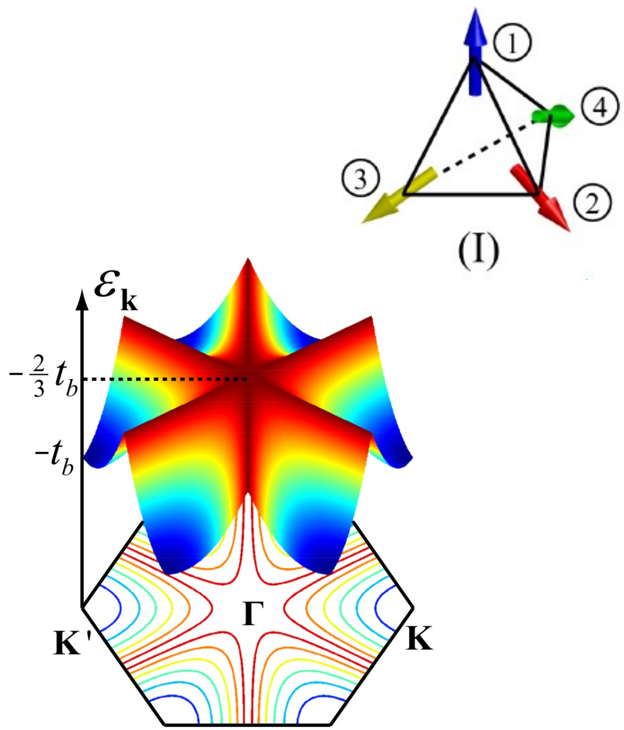
Li-6: Quantum Hall state



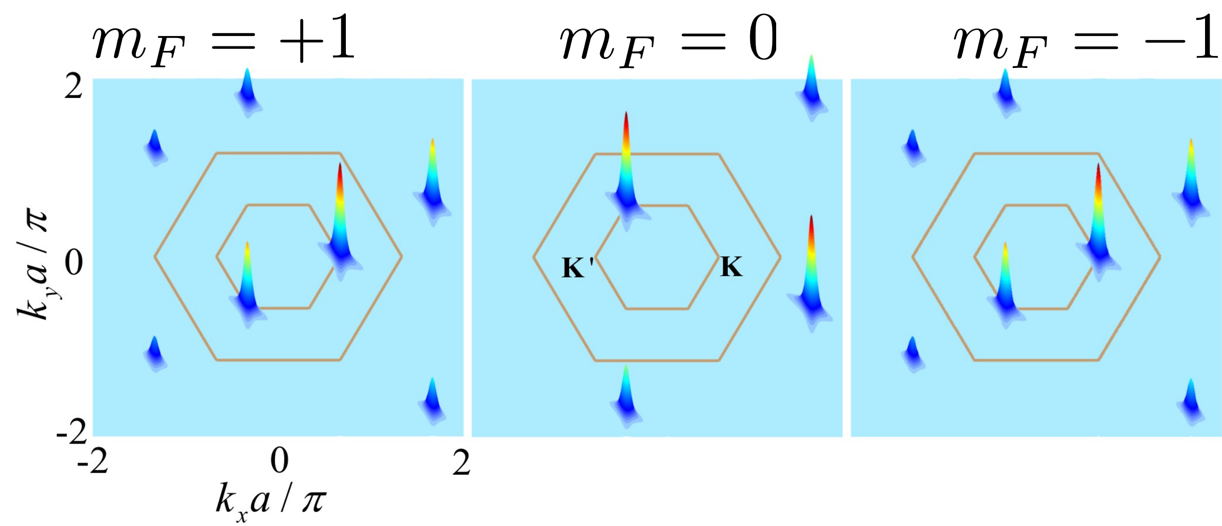


Z. F. Xu, XL, P. Zoller, and W. V. Liu, PRL (2015)

Rb-87: Chiral superfluid



As a consequence of emergent gauge fields, Bosons condense at finite momenta



Expected time-of-flight picture

Z. F. Xu, XL, P. Zoller, and W. V. Liu, PRL (2015)

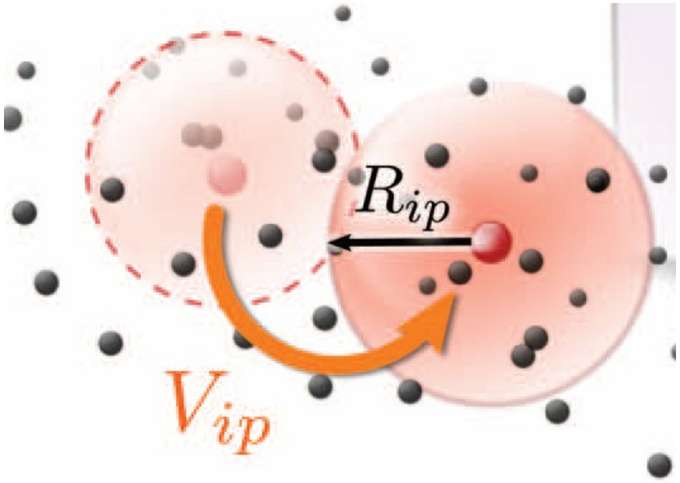
Systems to look for topological loop currents

- Multi-band Fermion systems (parity mixing)
- Bose-Fermi mixture
- Fermions with non-local interactions, say by Rydberg dressing

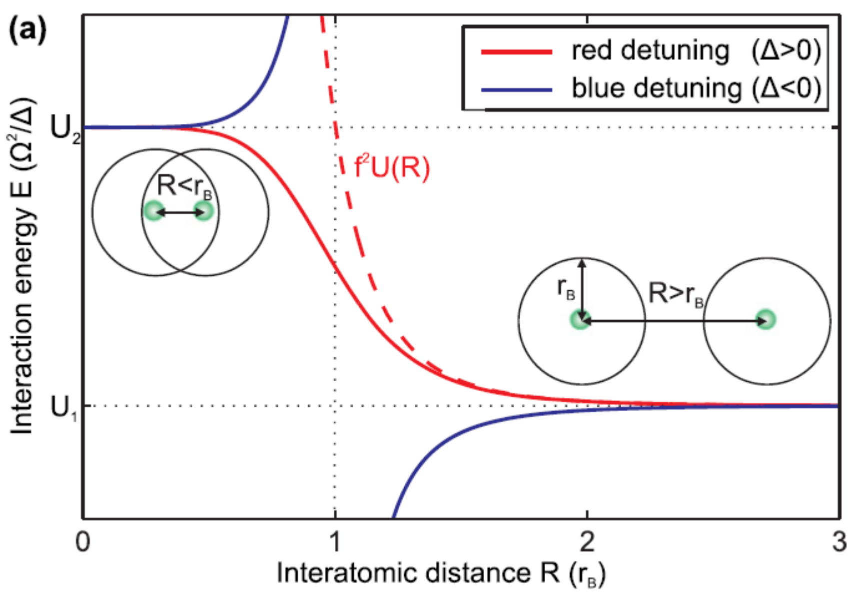
Common feature:

Fermi surface nesting->Spontaneous Loop currents->effective gauge fields-> Topological states

Rydberg dressing and non-local interactions



[Figure from G. Gunter et al., Science (2013)]



J. B. Balewski et al., NJP (2014)
N. Henkel R. Nath, T. Pohl, PRL (2010)
G. Pupillo, P. Zoller et al., (2010)

$$\begin{array}{cccc} |rr\rangle & |rg\rangle & |gr\rangle & |gg\rangle \end{array}$$

$$\begin{bmatrix} -2\Delta + V(\mathbf{x}) & \Omega & \Omega & 0 \\ \Omega & -\Delta & 0 & \Omega \\ \Omega & 0 & -\Delta & \Omega \\ 0 & \Omega & \Omega & 0 \end{bmatrix}$$

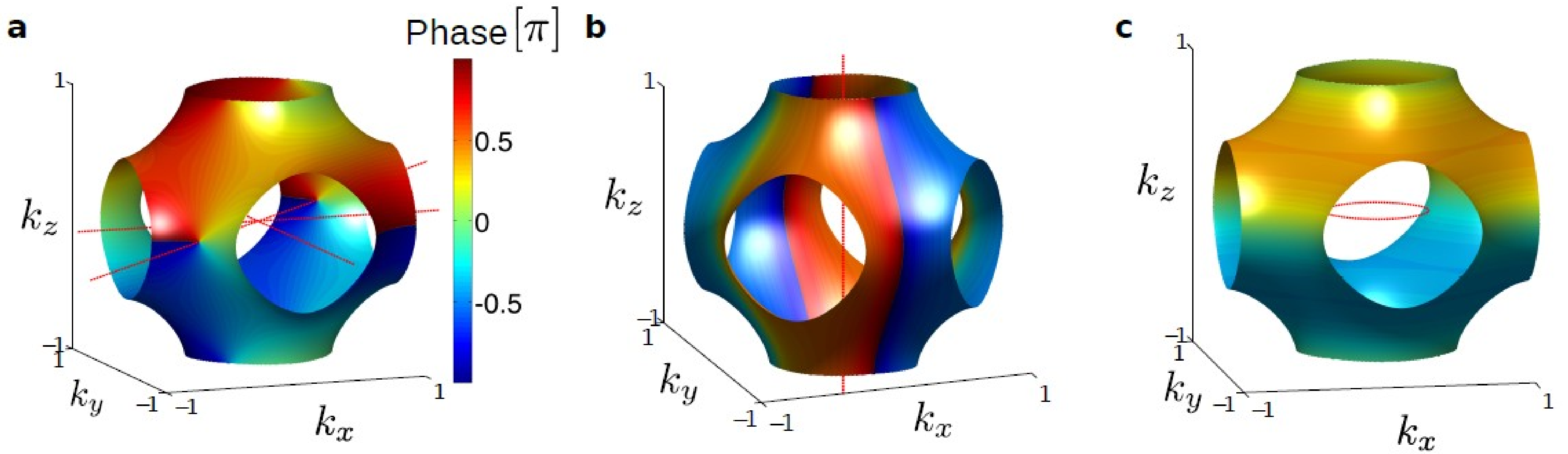
$$V_{\text{eff}}(\mathbf{r}) = \frac{V_6}{1+(|\mathbf{r}|/r_c)^6}$$

Topological density waves

$$H_{\mathrm{BdG}}(\mathbf{k}) = \begin{bmatrix} \epsilon_{\mathbf{k}} & \Delta_{\mathbf{k}} \\ \Delta_{\mathbf{k}}^* & -\epsilon_{\mathbf{k}} \end{bmatrix}$$

$$\rho_{\mathbf{k}} = \langle \psi^\dagger(\mathbf{k}+\mathbf{Q})\psi(\mathbf{k})\rangle$$

$$\Delta_{\mathbf{k}} = \int \frac{d^3\mathbf{q}}{(2\pi)^3}\left[\tilde{V}(\mathbf{Q}) - \tilde{V}(\mathbf{k}-\mathbf{q})\right]\rho_{\mathbf{q}}$$



XL, S. Das Sarma, arXiv (2015), accepted to Nature Communications

Summary

- Experimental observations of loop currents in optical lattices and cuprates
- Chiral spin condensate and spin loop currents in a hexagonal lattice
  [XL, S. Natu, A. Paramekanti, S. Das Sarma, Nature Communications (2014)]
- Spontaneous Quantum Hall effect with spinor Bose-Fermi mixture in a triangular lattice
  [Z.-F. Xu, XL, P. Zoller, W. Vincent Liu, PRL (2015)]
- Loop current order and spontaneous topological superfluids
  [Bo Liu, XL, Biao Wu, W. Vincent Liu, Nature Communications (2014)]
- Topological density waves with Rydberg dressed fermions
  [XL, S. Das Sarma, arXiv (2015), accepted to Nature Communications]