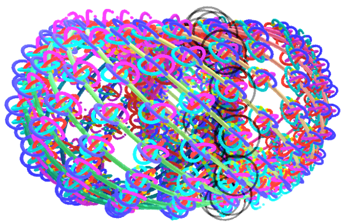
The Abdus Salam
International Centre
for Theoretical Physics

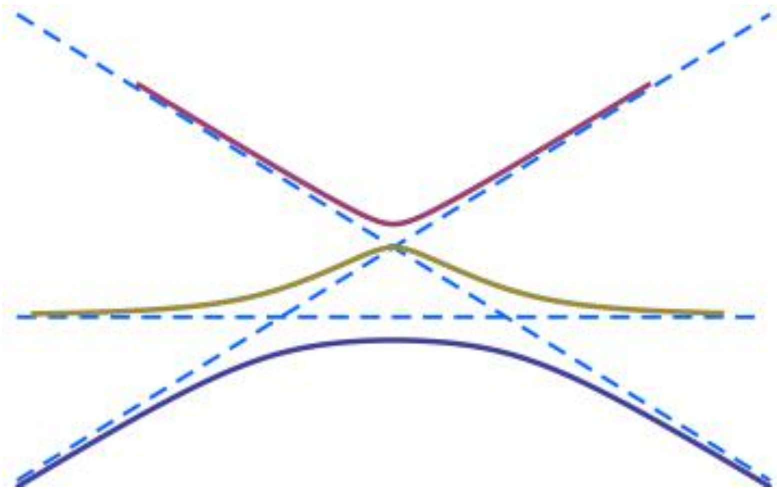
United Nations
Educational, Scientific and
Cultural Organization

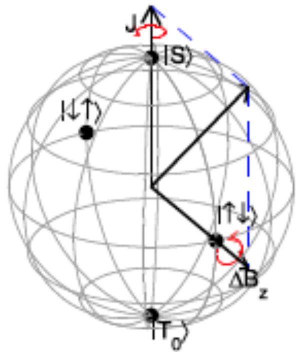
IAEA
International Atomic Energy Agency

M.N.Kiselev

Landau-Zener Interferometry in Multilevel Systems

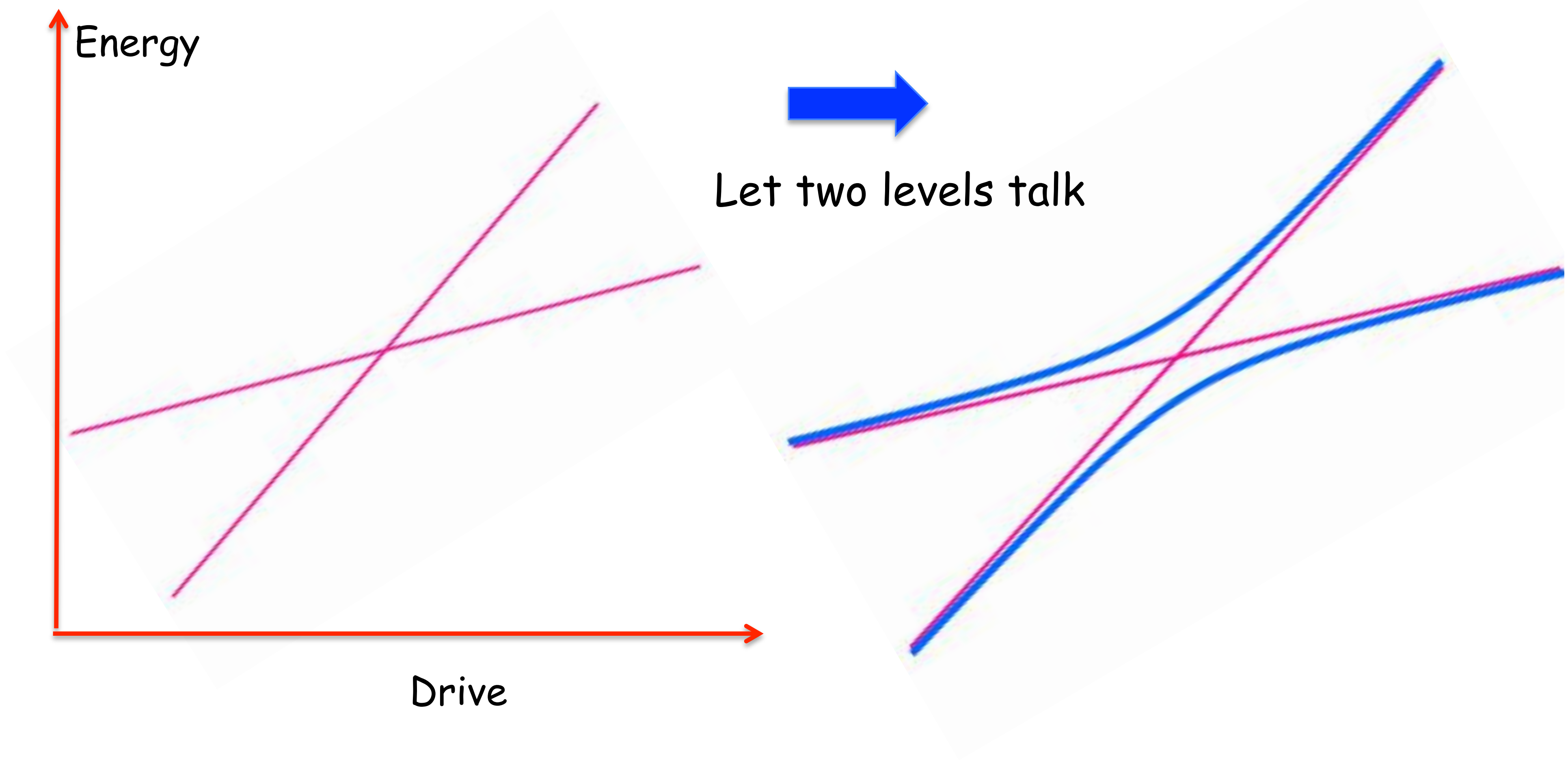






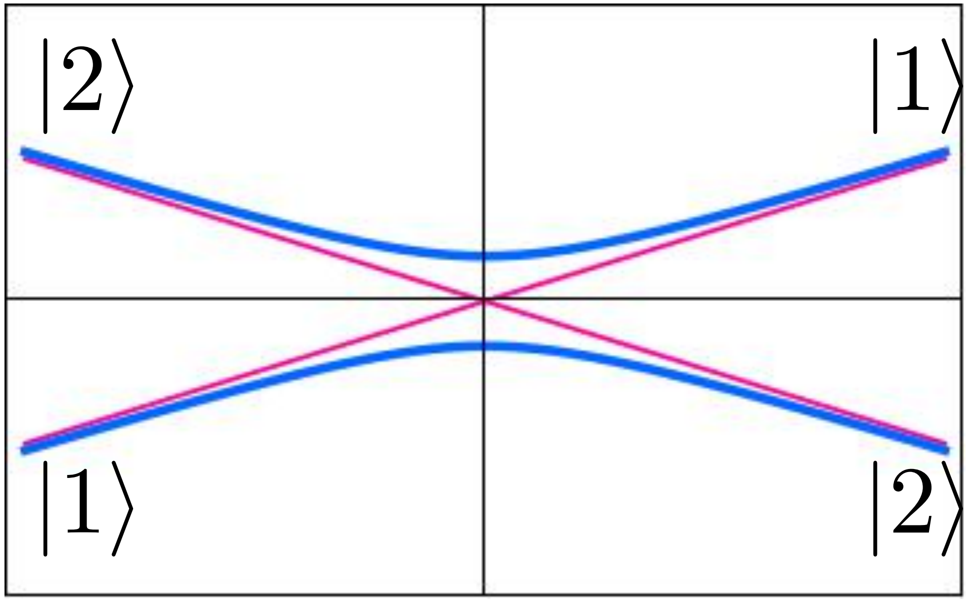
INT Seattle, April 23, 2015

Let us cross two levels once



“Drive” = energy, chemical potential, voltage, magnetic field etc

Two level crossing: the Hamiltonian



$$H = E(t)(|1\rangle\langle 1| - |2\rangle\langle 2|) + V_{12}(|1\rangle\langle 2| + |2\rangle\langle 1|)$$

$$H = vts^z + \Delta s^x$$

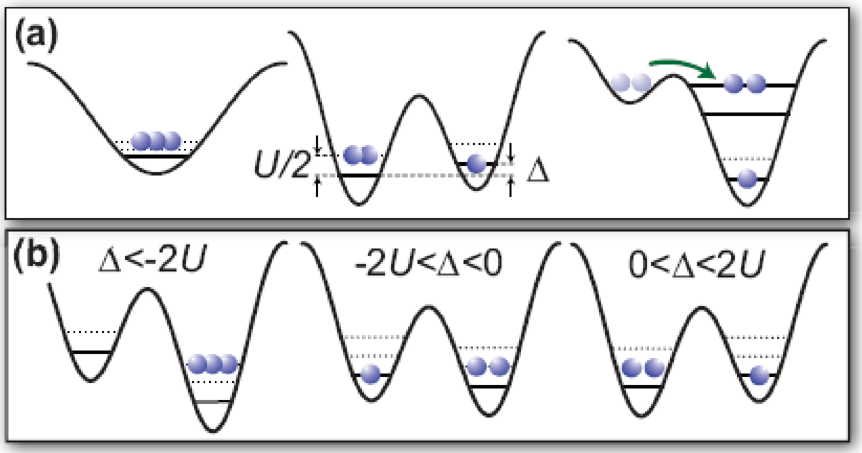
L.D. Landau, 1932
C. Zener, 1932,
E. Majorana, 1932
E.C.G. Stückelberg, 1932

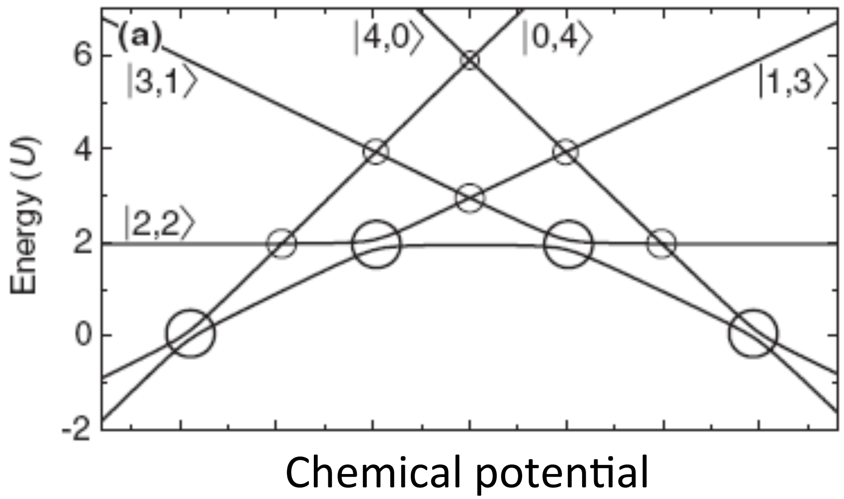
$$H = \vec{B}(t) \cdot \vec{s}$$

Level crossings in optical lattices

$$H = -J(\hat{a}_L^\dagger \hat{a}_R + a_R^\dagger a_L) - \frac{\Delta}{2}(\hat{n}_L - \hat{n}_R)$$

$$H_U = \frac{U}{2}[\hat{n}_L(\hat{n}_L - 1) + \hat{n}_R(\hat{n}_R - 1)]$$





$$H = vtS^z + \Delta S^x + D(S^z)^2$$

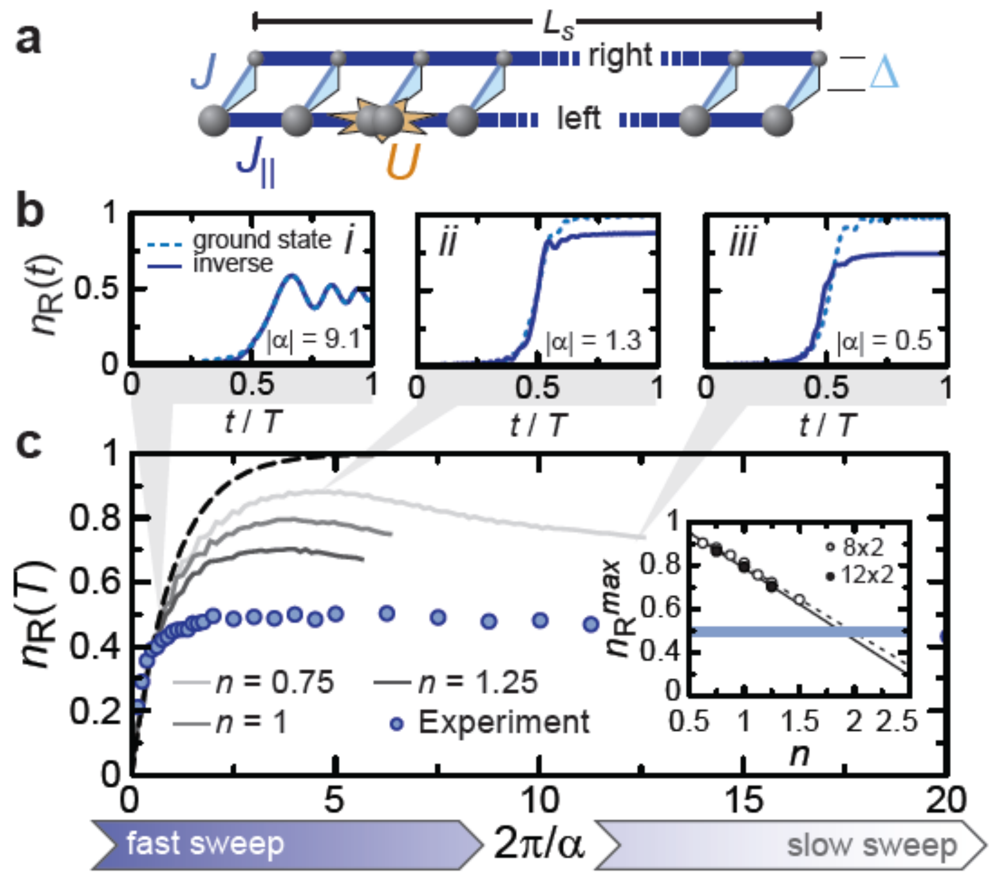
Spin S=2 model with quadrupole interaction

Munich group: I. Bloch et at, 2008

Landau-Zener sweeps and quenches in coupled chains

$$\hat{H}(t) = -\left(J \sum_i \hat{b}^\dagger_{i,\mathrm{L}} \hat{b}_{i,\mathrm{R}} + J_\| \sum_{i,\sigma} \hat{b}^\dagger_{i,\sigma} \hat{b}_{i+1,\sigma} \right) + h.c.$$

$$+ \frac{U}{2} \sum_{i,\sigma} \hat{n}_{i,\sigma}(\hat{n}_{i,\sigma} - 1) + \Delta(t) \sum_i \hat{n}_{i,\mathrm{R}},$$



linear sweep

$$\Delta(t > 0) = \begin{cases} \Delta_0 + \alpha\, t\,, \\ \Delta_f\,, \end{cases}$$

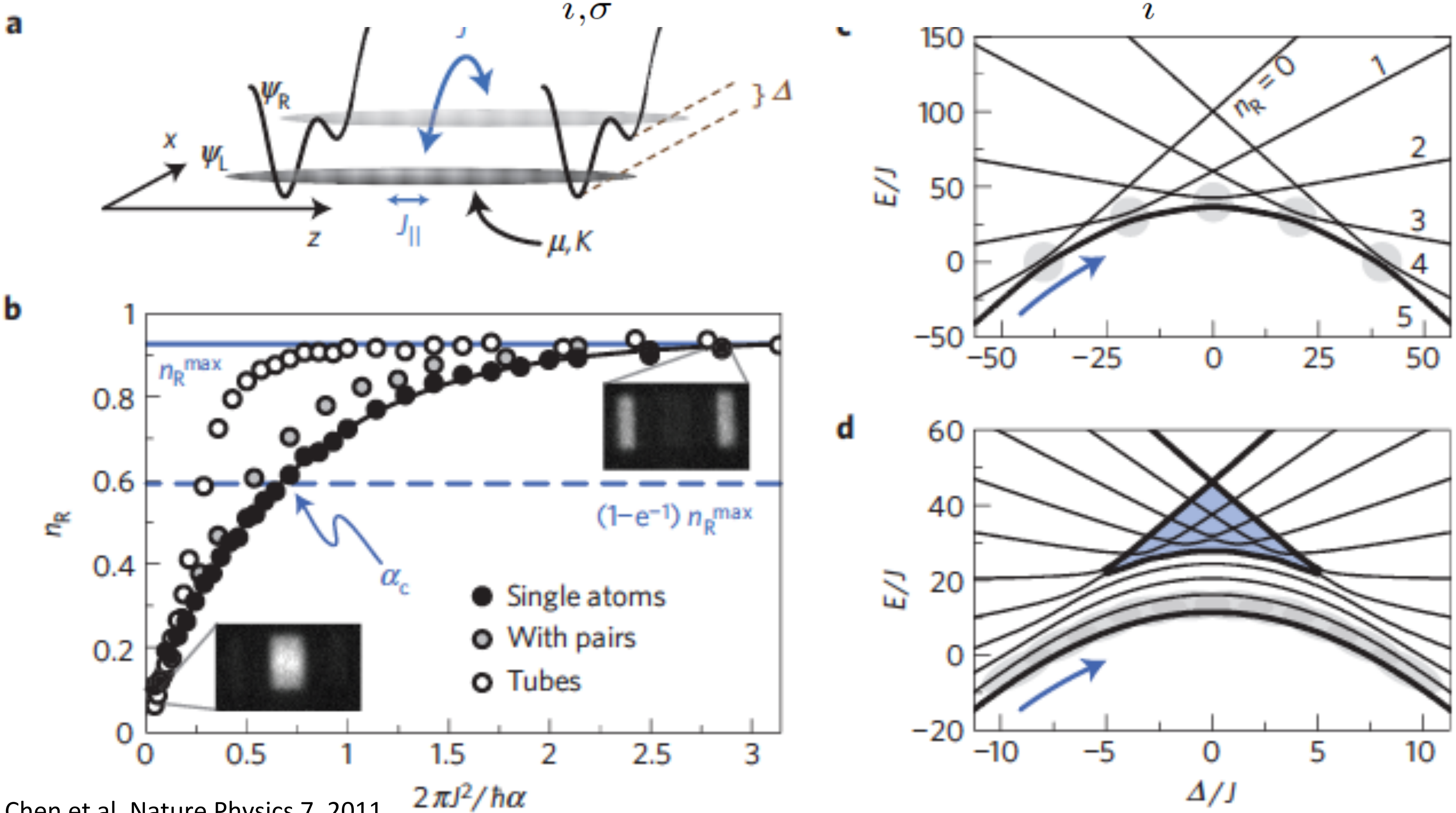
sudden quench

C. Kasztelan et al, Phys. Rev. Lett. 106, 155302 (2011)

Landau-Zener sweeps and quenches in coupled chains

$$\hat{H}(t) = -\left(J\sum_i \hat{b}^\dagger_{i,\mathrm{L}}\hat{b}_{i,\mathrm{R}} + J_\parallel \sum_{i,\sigma}\hat{b}^\dagger_{i,\sigma}\hat{b}_{i+1,\sigma}\right) + h.c.$$

$$+\frac{U}{2}\sum_{i,\sigma}\hat{n}_{i,\sigma}(\hat{n}_{i,\sigma}-1) + \Delta(t)\sum_i \hat{n}_{i,\mathrm{R}},$$

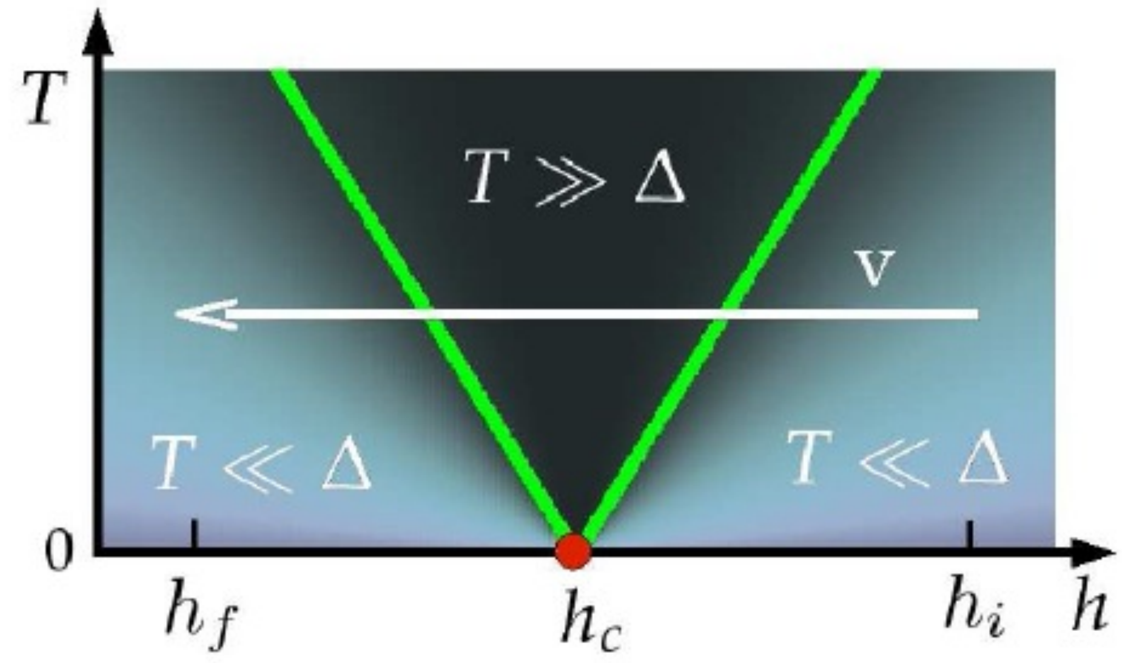


Y.-A. Chen et al, Nature Physics 7, 2011

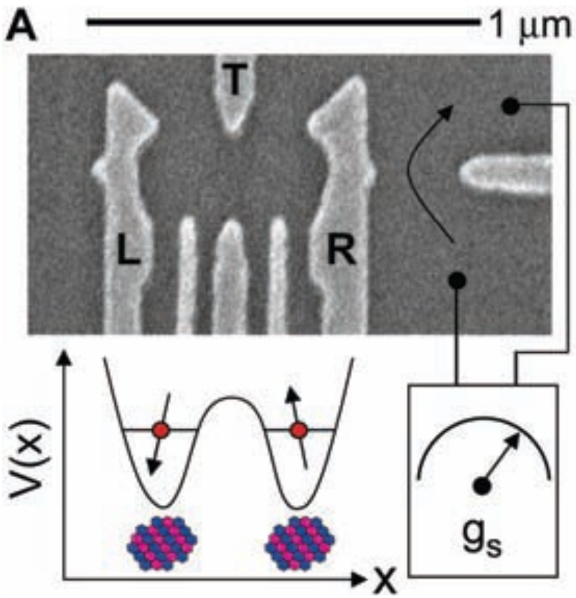
Quantum quenches

$$H = -h(t)\sum_i \sigma_i^x - \sum_i \sigma_i^z \sigma_{i+1}^z$$

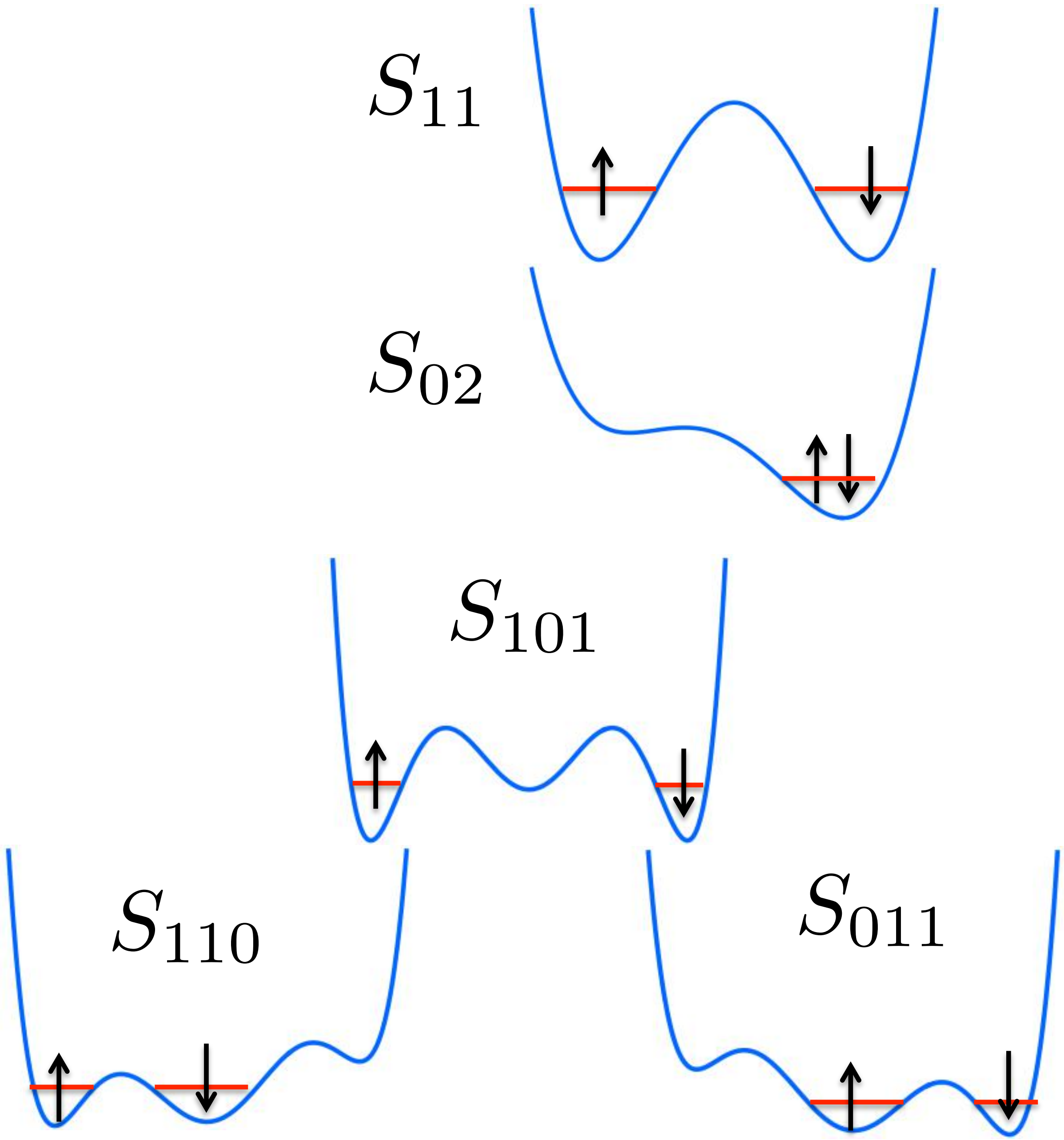
Quantum Ising model

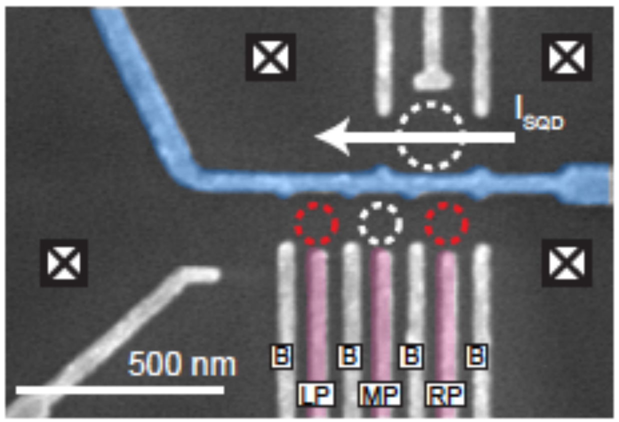


Level crossing in double and triple well potentials



Harvard group: J.Petta et al 2012

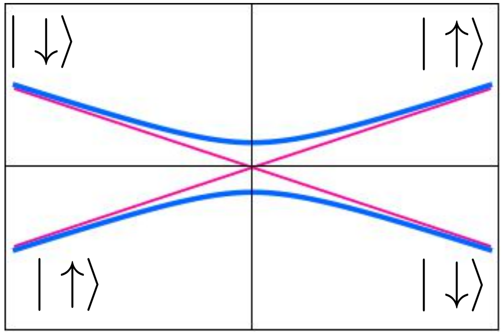




Delft group: L.Vandersypen et al 2012

Two level crossing: Zener times

Schrödinger dynamics



$$\delta = \frac{\Delta^2}{4v}$$

$$i\frac{d\Psi}{dt} = H\Psi$$

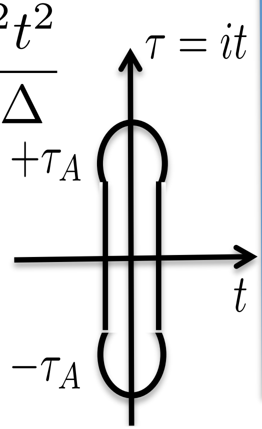
$$H = vts^z + \Delta s^x$$

Semi-classics

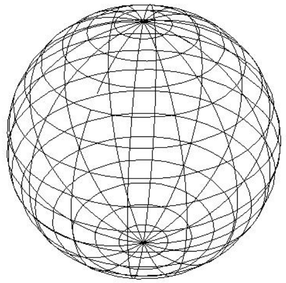
$$2\epsilon_\pm = \sqrt{\Delta^2 + v^2t^2} \approx \Delta + \frac{v^2t^2}{2\Delta}$$

$$\Delta\tau_A \gg 1 \qquad \tau_A = \frac{\Delta}{v}$$

$$P_{\uparrow\to\uparrow} = \exp(-2\pi\delta)$$



Bloch dynamics



$$i\frac{d\hat{\rho}}{dt} = [H\hat{\rho}]$$

$$H = \vec{B}(t)\cdot\vec{s}$$

$$\frac{d}{dt}\vec{n}(t) = -\vec{B}(t)\times\vec{n}(t)$$

$$\vec{n}(t) = \begin{pmatrix} 2\mathrm{Re}\rho_{12} \\ 2\mathrm{Im}\rho_{12} \\ \rho_{11} - \rho_{22} \end{pmatrix}$$

$$\mathrm{Tr}\hat{\rho}^2 = 1$$

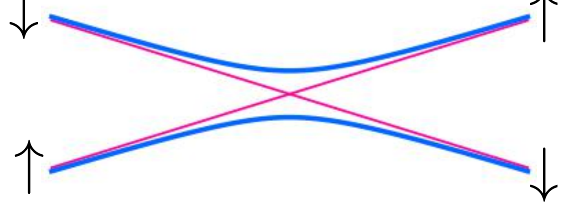
$$(\vec{n})^2 = 1$$

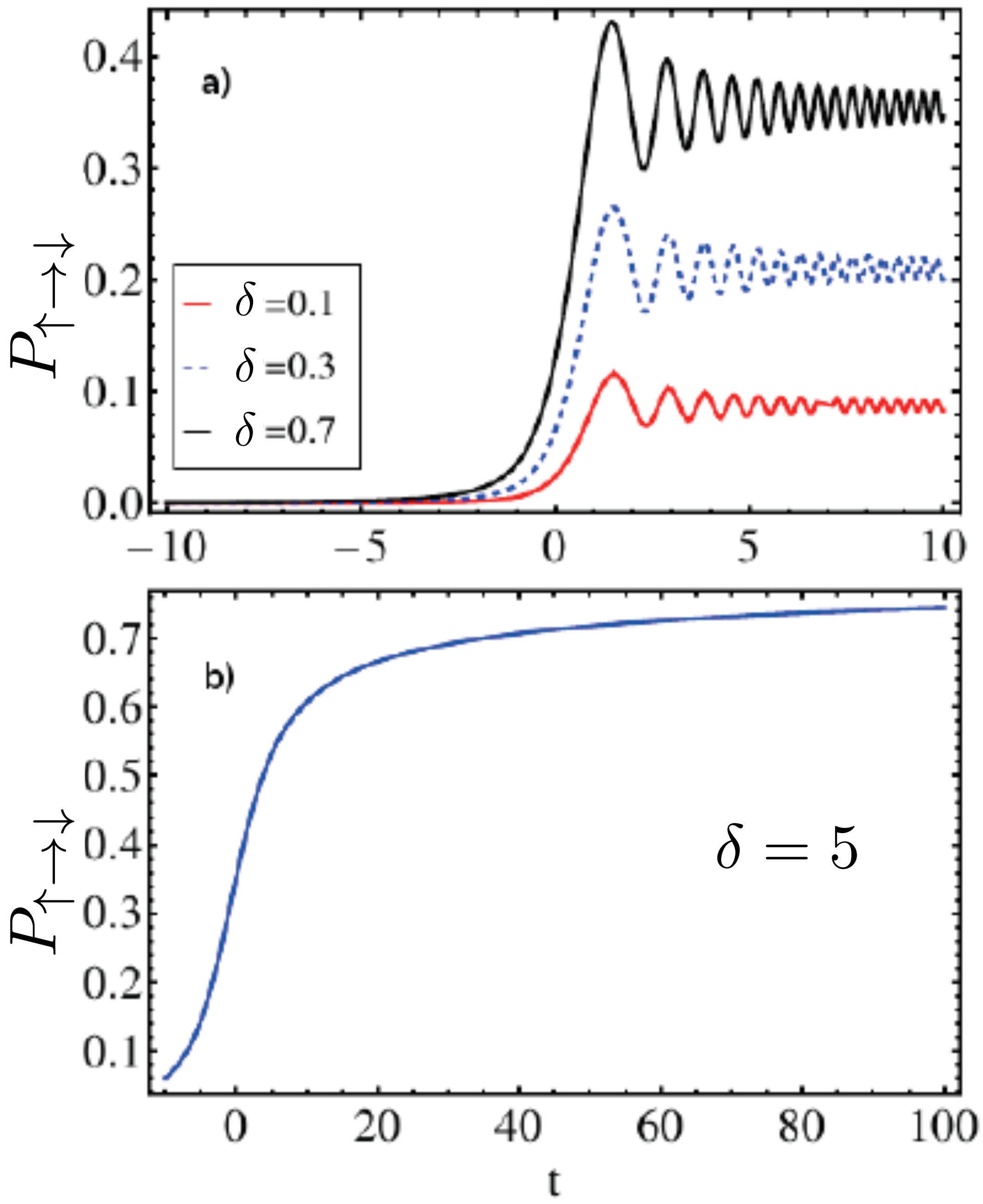
$$\frac{d}{dt}n^z(t) = -\Delta^2\int_{-\infty}^{t} dt_1 \cos\left[\frac{v}{2}(t^2 - t_1^2)\right] n^z(t_1)$$

$$\delta E \sim v\cdot\tau_{NA} \qquad \delta E\cdot\tau_{NA} \sim 1$$

$$\Delta\tau_{NA} \ll 1 \qquad \tau_{NA} = 1/\sqrt{v}$$

Zener times





$$\tau_c = \hbar/\Delta$$

$$\tau_{NA} = \sqrt{\hbar/v}$$ **Non-Adiabatic**

$$P_{\uparrow\to\downarrow} = 1 - \exp\left(-2\pi\delta\right)$$

$$\delta = \frac{\Delta^2}{4\hbar v}$$

$$\tau_A = \Delta/v$$ **Adiabatic**

$$\tau_Z = \max\left[\tau_A, \tau_{NA}\right]$$

Mullen, Ben-Jacob, Gefen, Schuss, 1989

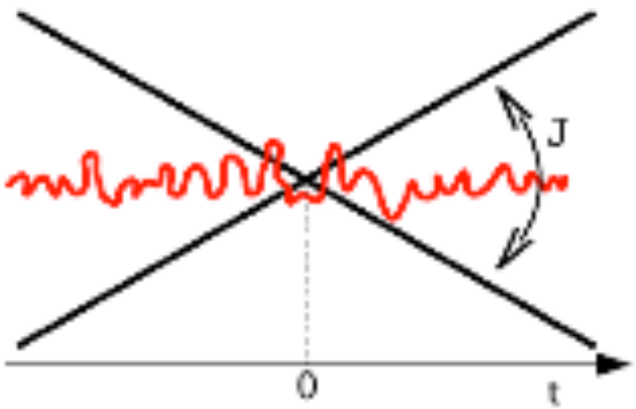
Fast noise in two-levels crossing

$$\frac{d}{dt}n_z(t) = -4\int_{-\infty}^{t} dt_1 \cos\left(\frac{v}{2}\left[t^2 - t_1^2\right]\right) f_+(t) f_-(t_1) n_z(t_1)$$

$$f_\pm(t) = f_x(t) \pm i f_y(t)$$

Initial condition $n_z(t \to -\infty) = 1$

$$F_\alpha(\tau) = \langle f_\alpha(t) f_\alpha(t+\tau)\rangle = J^2 \exp(-\gamma|\tau|)$$



Fast noise: $\tau_{LZ} \gg 1/\gamma$

Average the equation !

$$P_{\uparrow\to\downarrow} = \frac{1}{2}\left(1 - \exp\left(-\frac{4\pi\langle f_x(t) f_x(t)\rangle}{v}\right)\right)$$

Kayanuma, 1984
Pokrovsky et al 2003, 2004, 2010

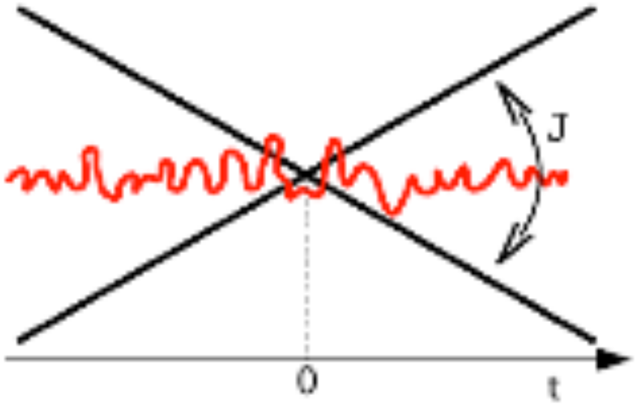
Slow noise in two-levels crossing

$$\frac{d}{dt}n_z(t) = -4\int_{-\infty}^{t} dt_1 \cos\left(\frac{v}{2}\left[t^2 - t_1^2\right]\right) f_+(t) f_-(t_1) n_z(t_1)$$

$$f_\pm(t) = f_x(t) \pm i f_y(t)$$

Initial condition $n_z(t \to -\infty) = 1$

$$F_\alpha(\tau) = \langle f_\alpha(t) f_\alpha(t+\tau)\rangle = J^2 \exp(-\gamma|\tau|)$$



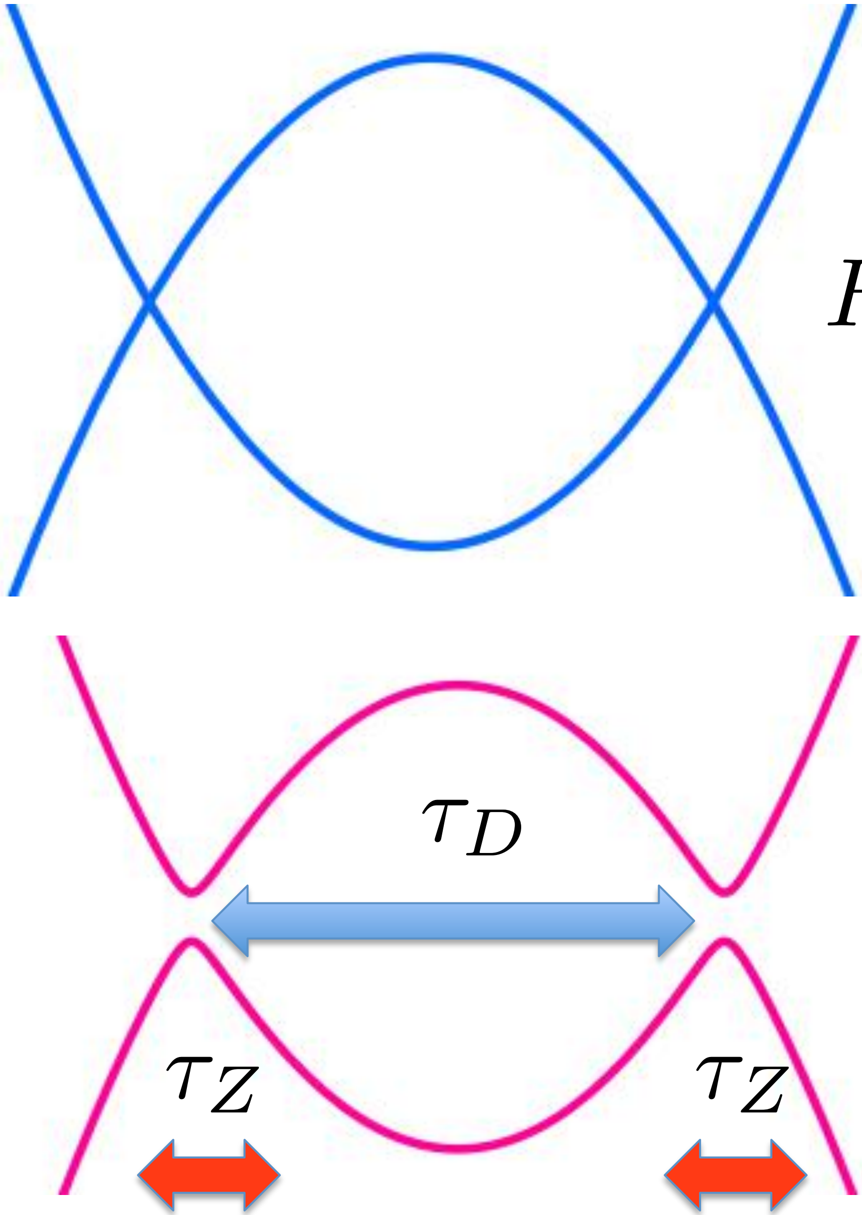
Kayanuma, 1984
MK et al 2010, 2013

Slow noise: $\tau_{LZ} \ll 1/\gamma$

Average the solution !

$$P_{\uparrow\to\downarrow} = 1 - \frac{1}{\sqrt{1+\frac{4\pi J^2}{v}}}$$

"Minimal" model for interferometer: crossing levels twice



$$\delta = \frac{\Delta^2}{4v} \quad v \sim \sqrt{\alpha\beta}$$

$$H = (\alpha t^2 - \beta)\sigma^z + \Delta\sigma^x$$

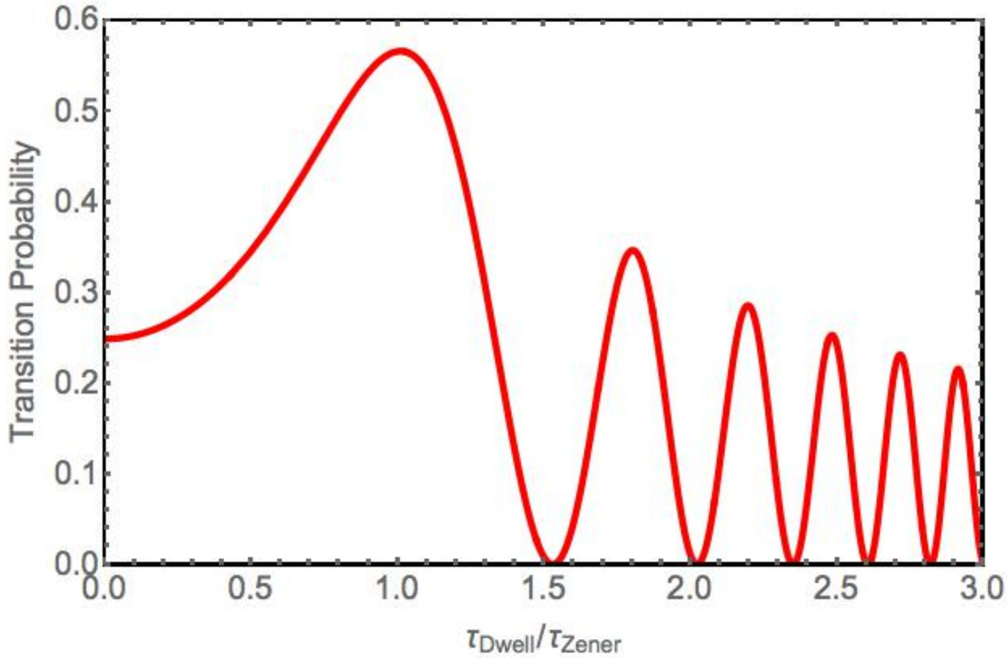
$$\tau_D \sim \sqrt{\frac{\beta}{\alpha}}$$

$$\tau_Z \sim \frac{1}{(\alpha\beta)^{1/4}}$$

Two LZ transitions are not separable if $\dfrac{\beta^3}{\alpha} < 1$

E. Shimshoni, Y. Gefen 1991

"Minimal" model for interferometer

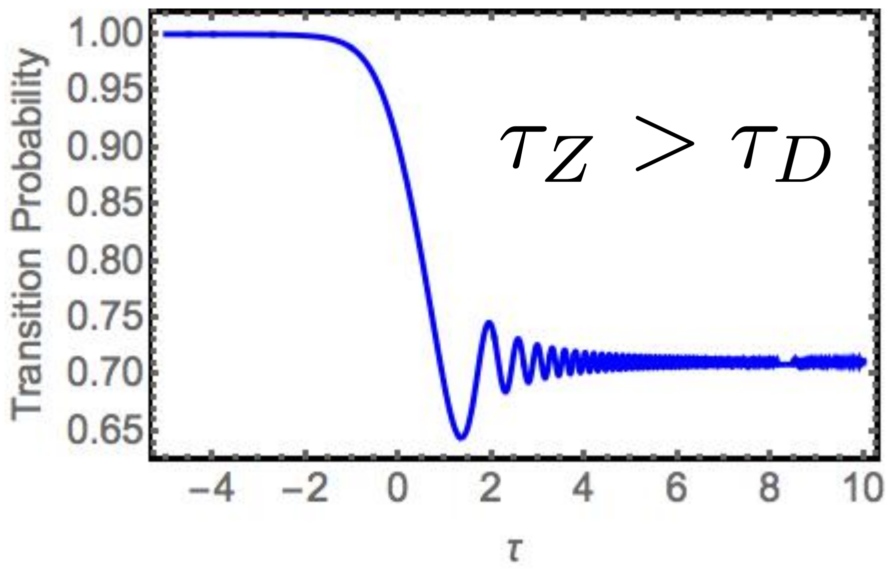


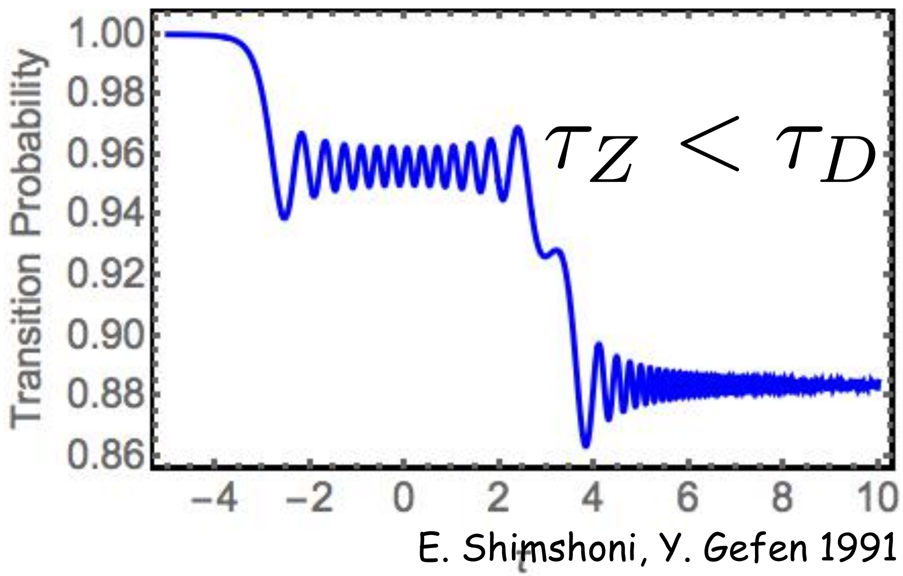
$$P_{LZ} = \exp(-2\pi\delta)$$

$$\bar{P}_{+} = 2P_{LZ}(1 - P_{LZ})$$

$$P_{+} = 4P_{LZ}(1 - P_{LZ}) \sin^2 \Phi_{St}$$

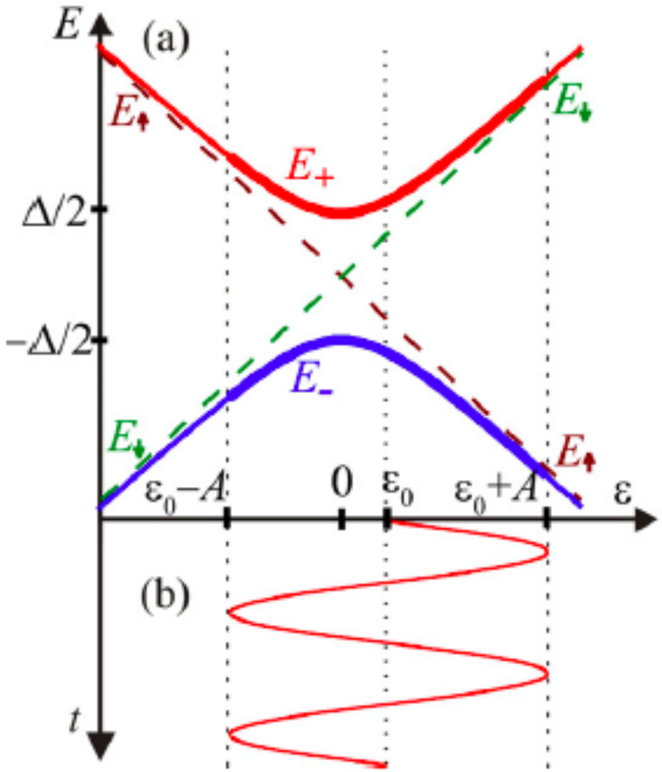
$$\Phi_{St} = \phi(\delta) + \xi(\delta, \tau_{LZ})$$

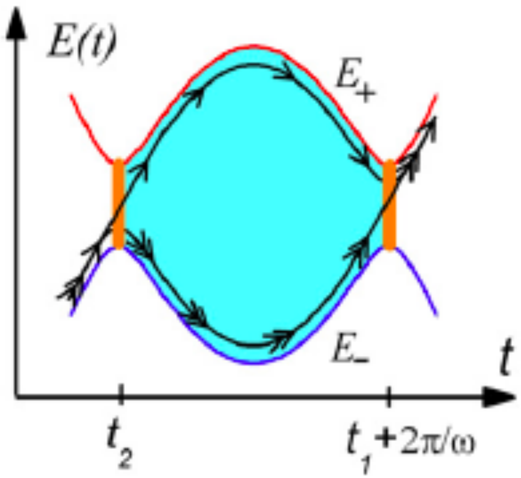


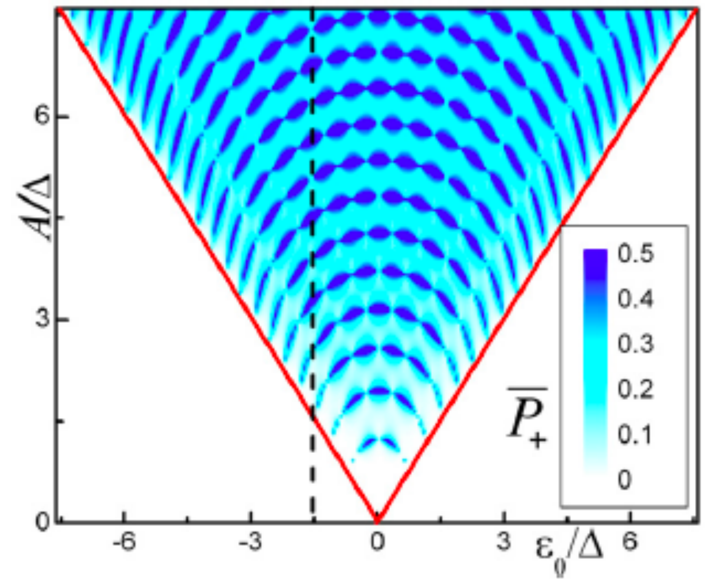


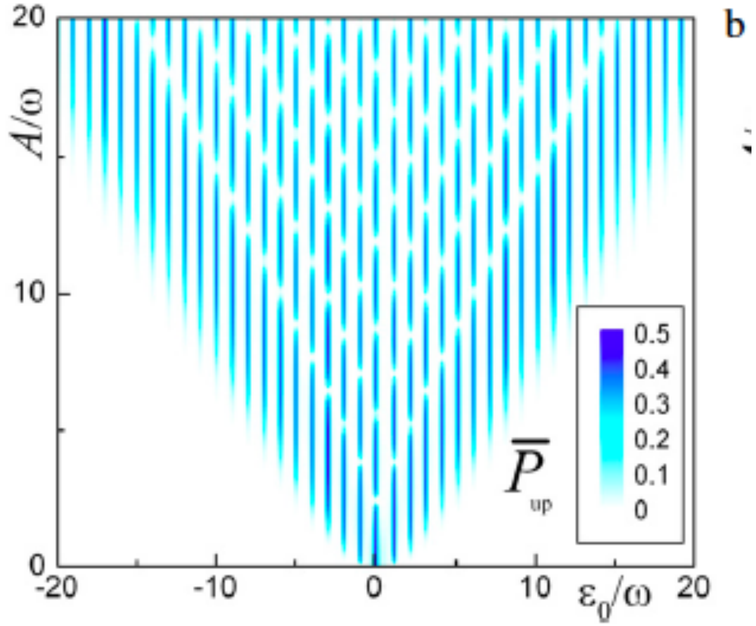
E. Shimshoni, Y. Gefen 1991

Stückelberg's oscillations



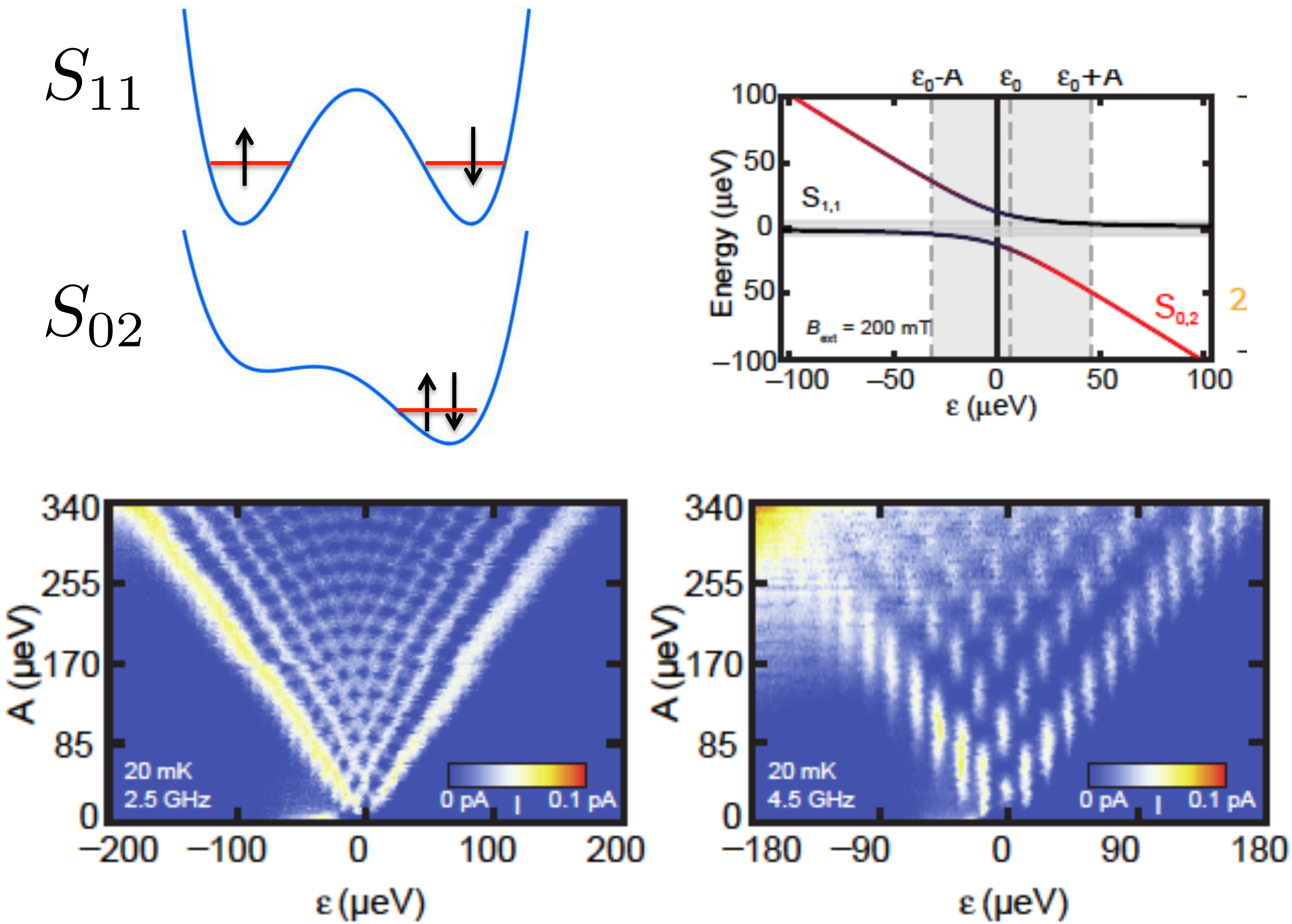






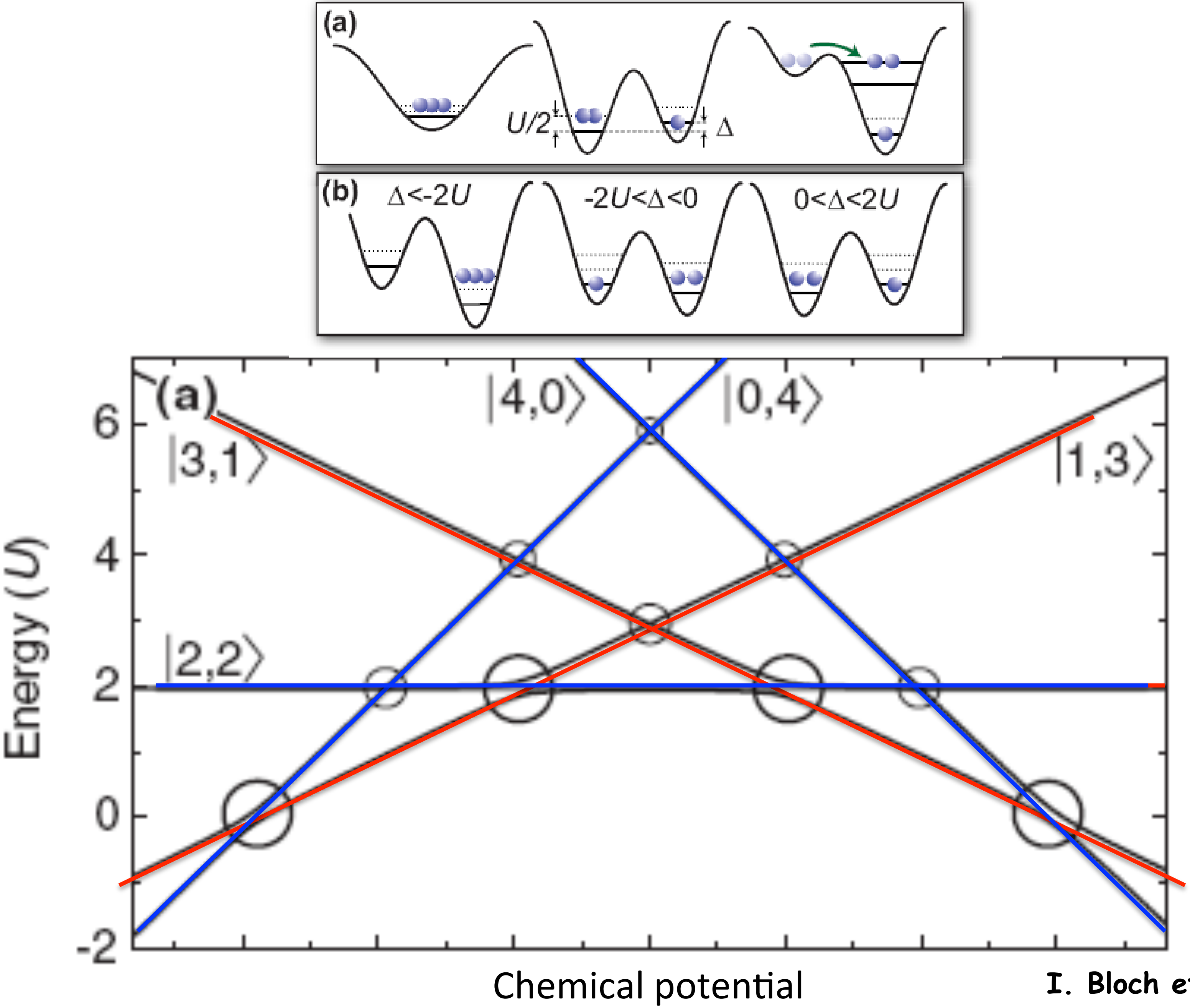
S. Shevchenko et al 2010

Stückelberg's oscillations in QD



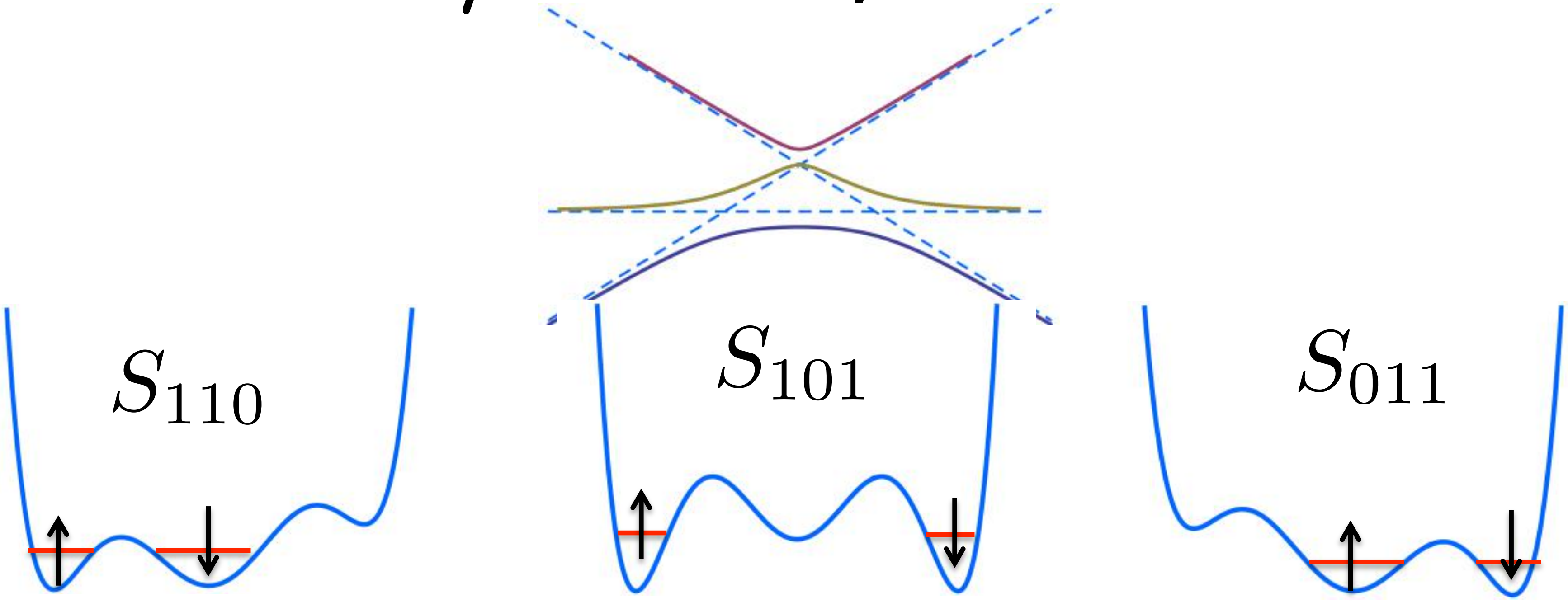
S. Ludwig et al 2012

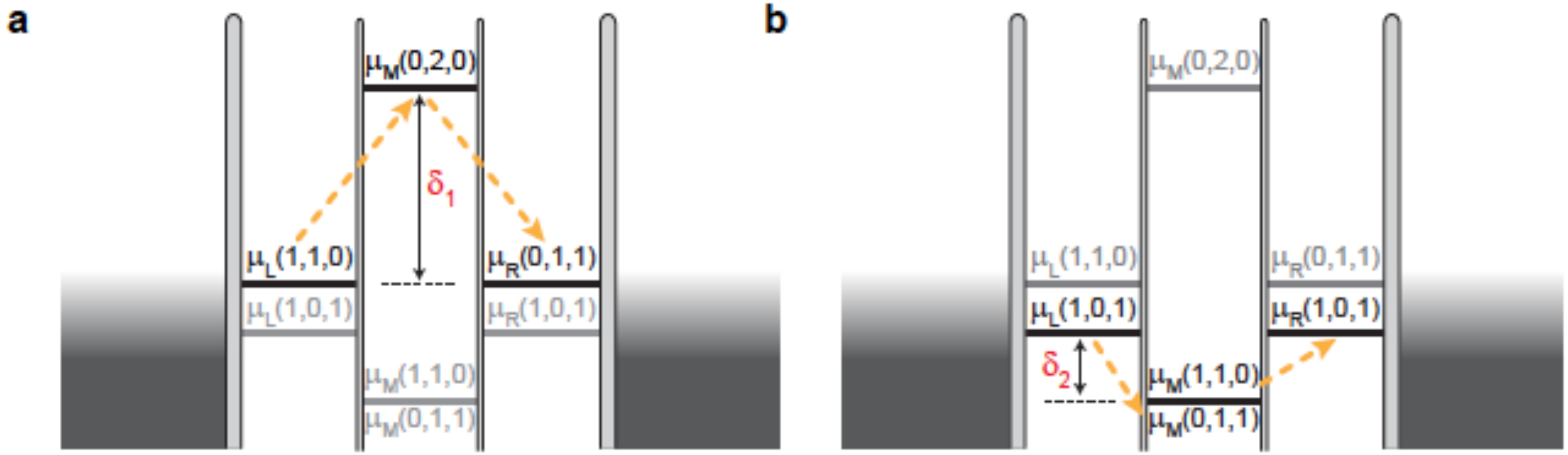
Do we always have only two levels to cross?



I. Bloch et at, 2008

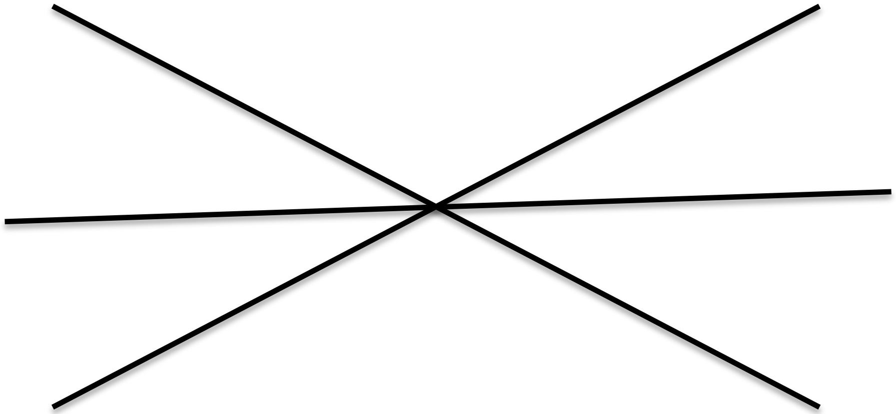
Do we always have only two levels to cross?



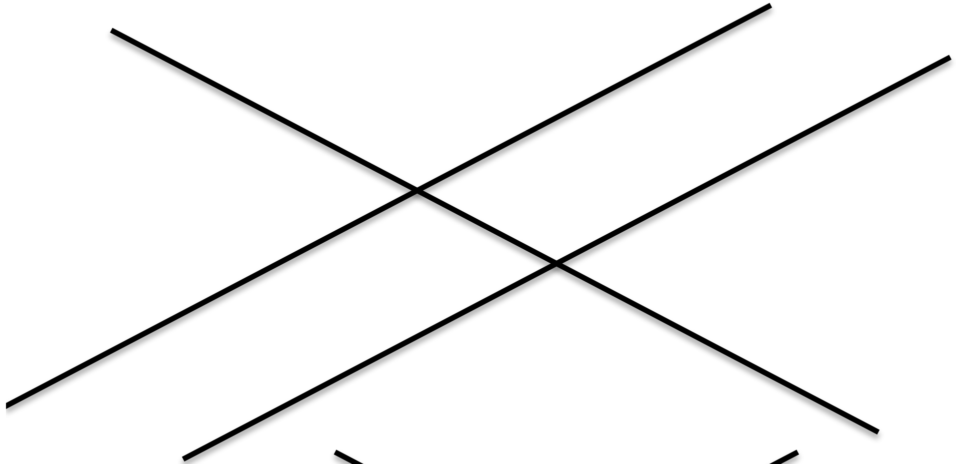


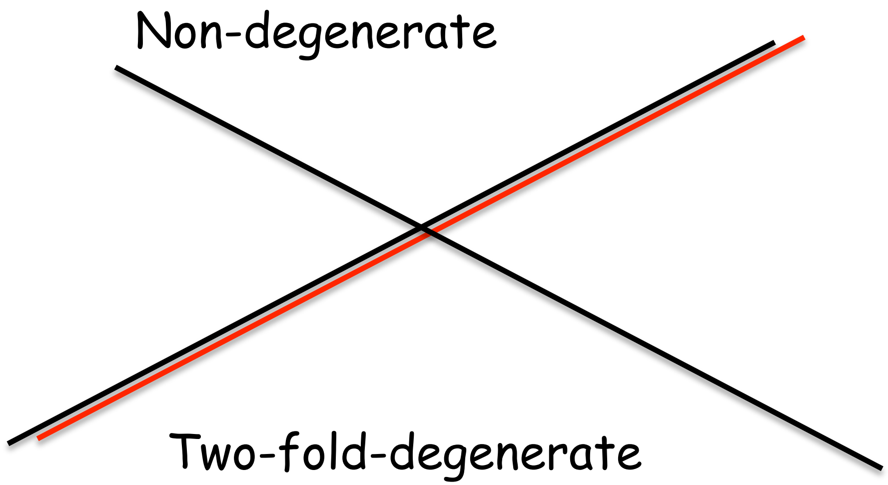
L.Vandersypen et al 2012

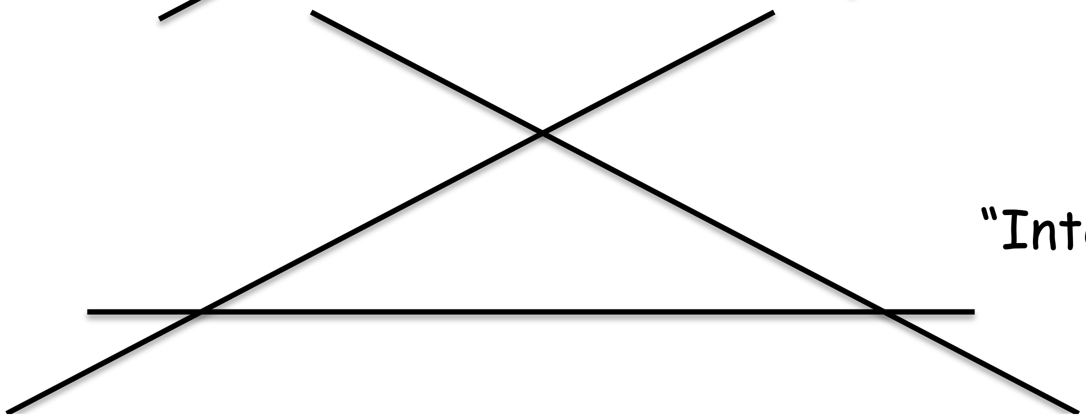
Let us cross three levels



S=1 Landau-Zener transition
if and extra particle-hole symmetry
is assumed

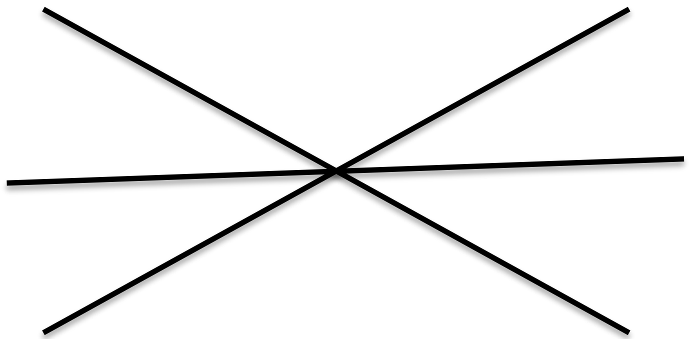






"Interacting" Landau - Zener

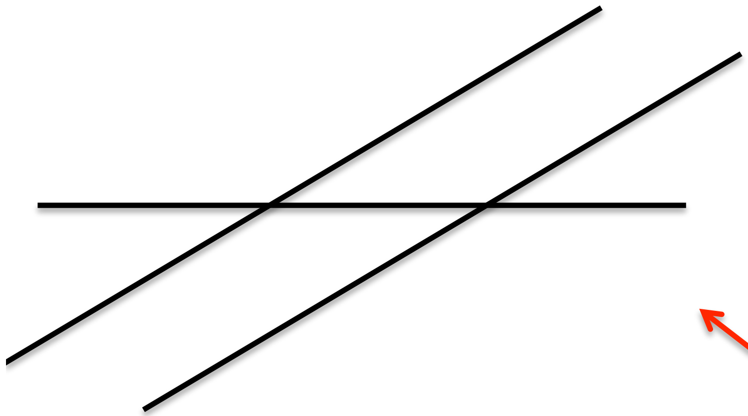
Three level crossings: the Hamiltonian



3 x 3 matrices

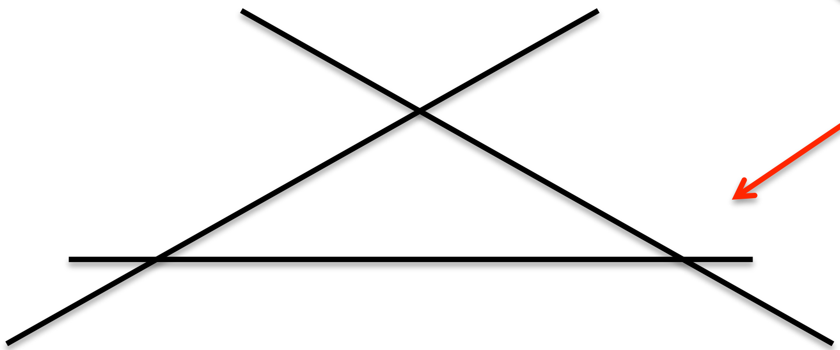
$$H = vtS^z + \Delta S^x$$

SU(2) S=1 Landau-Zener transition



$$H = vt(S^z)^2 + \Delta S^x + hS^z$$

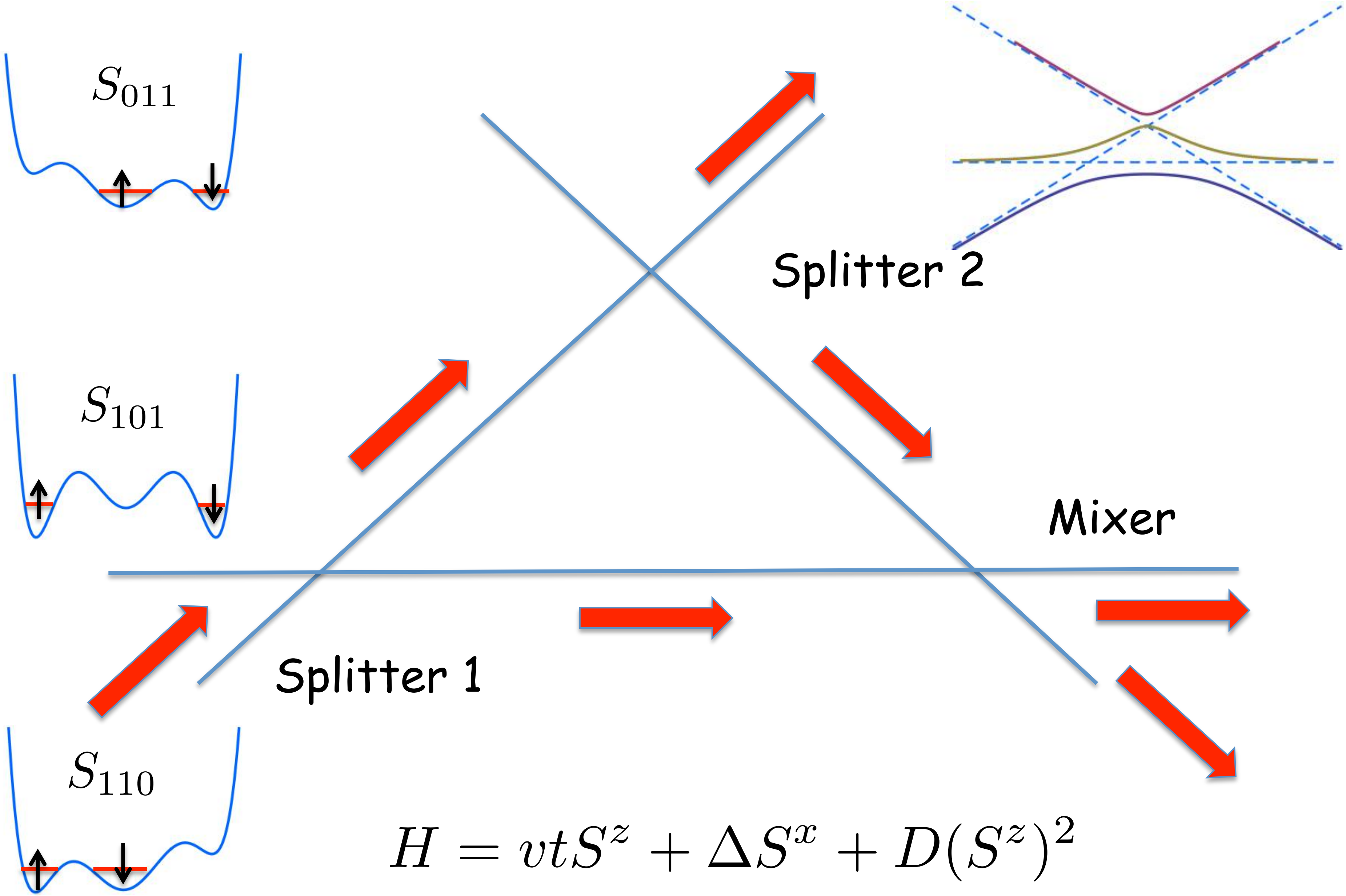
If $h \neq 0$, the 2-fold level degeneracy is lifted out



SU(2) LZ with quadrupole interaction
= linear SU(3) LZ transitions

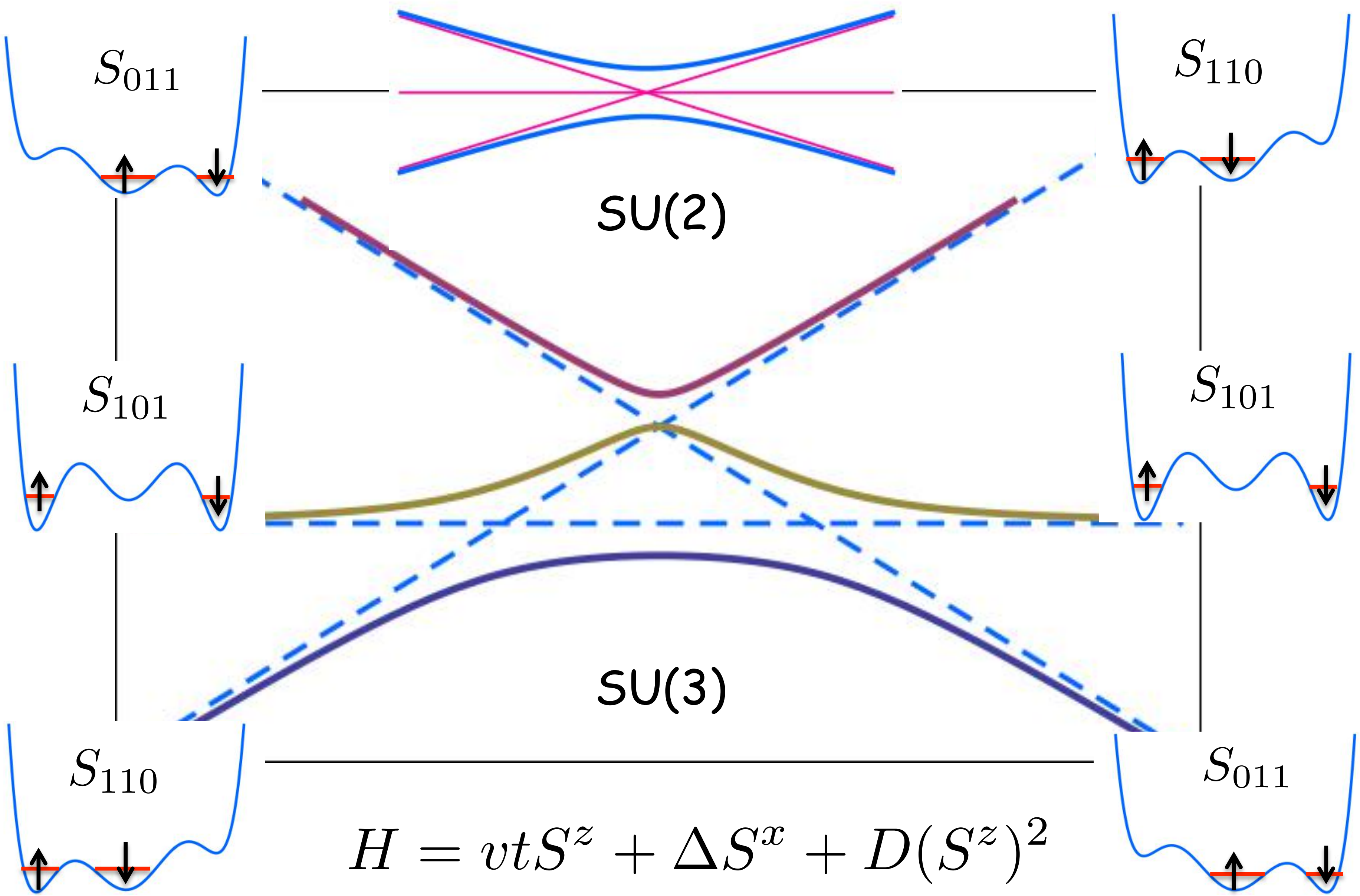
$$H = vtS^z + \Delta S^x + D(S^z)^2$$

"Minimal" model of LZ interferometer

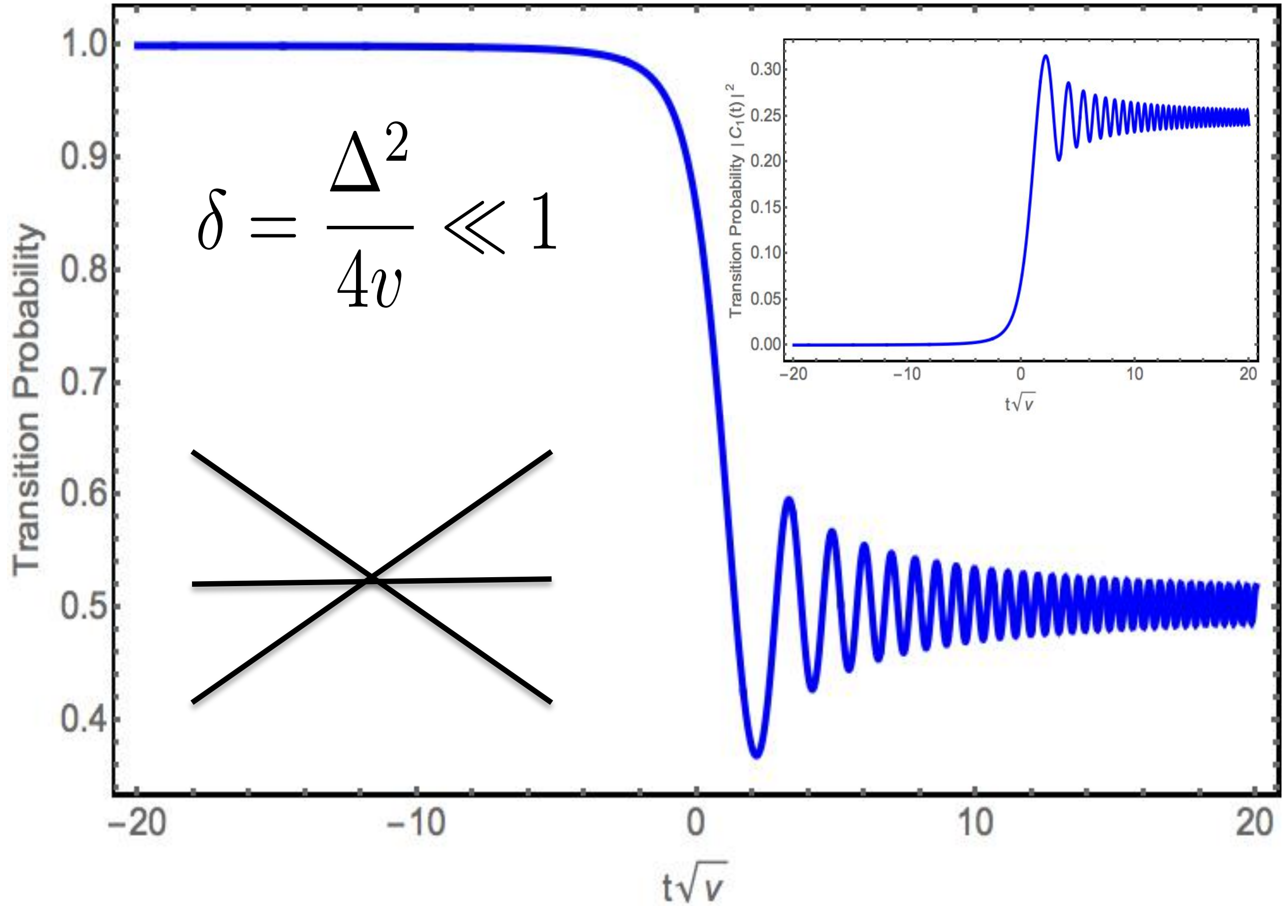


$$H = vtS^z + \Delta S^x + D(S^z)^2$$

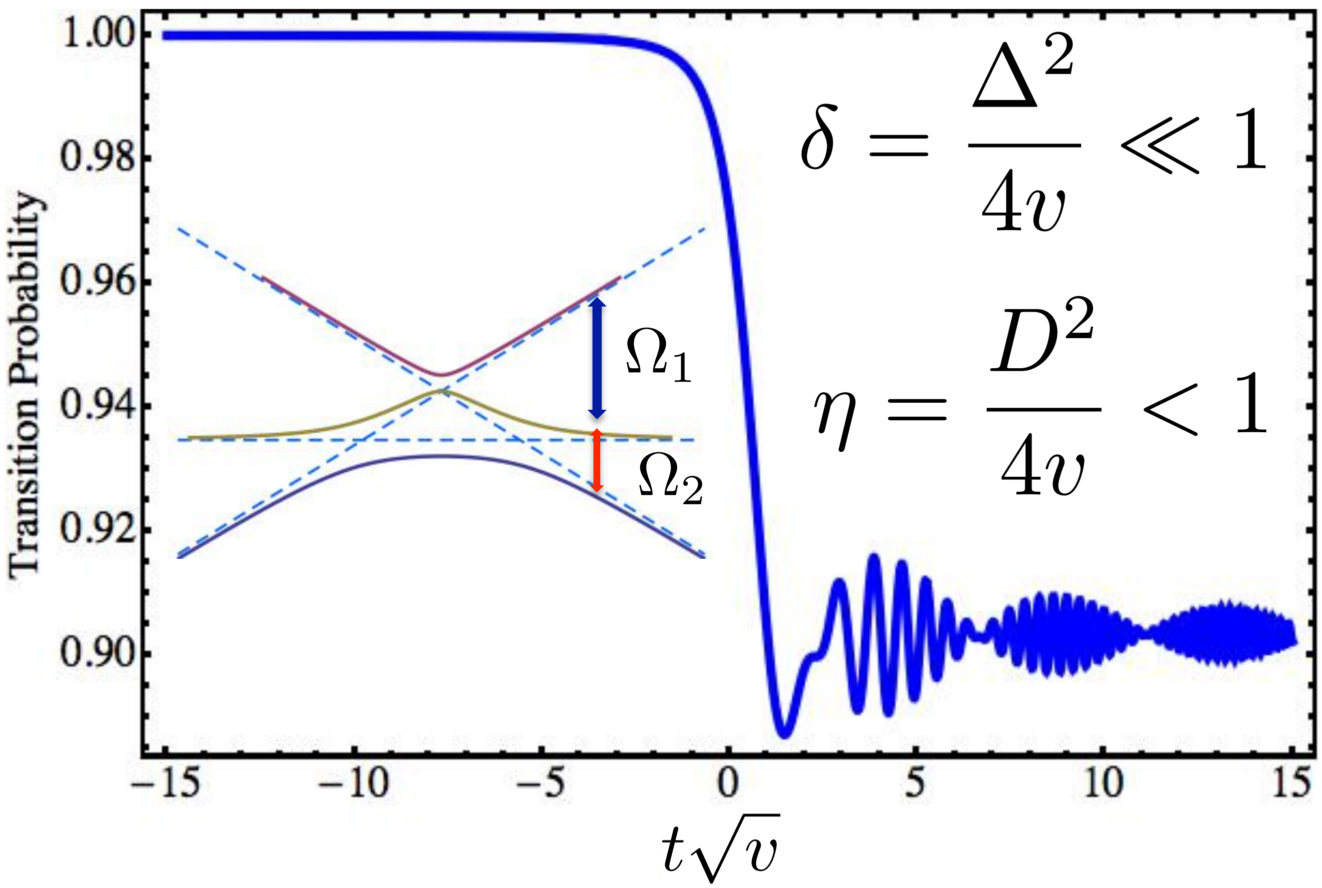
(A)Diabatic states for 3-levels crossing



Three level crossing: the splitter

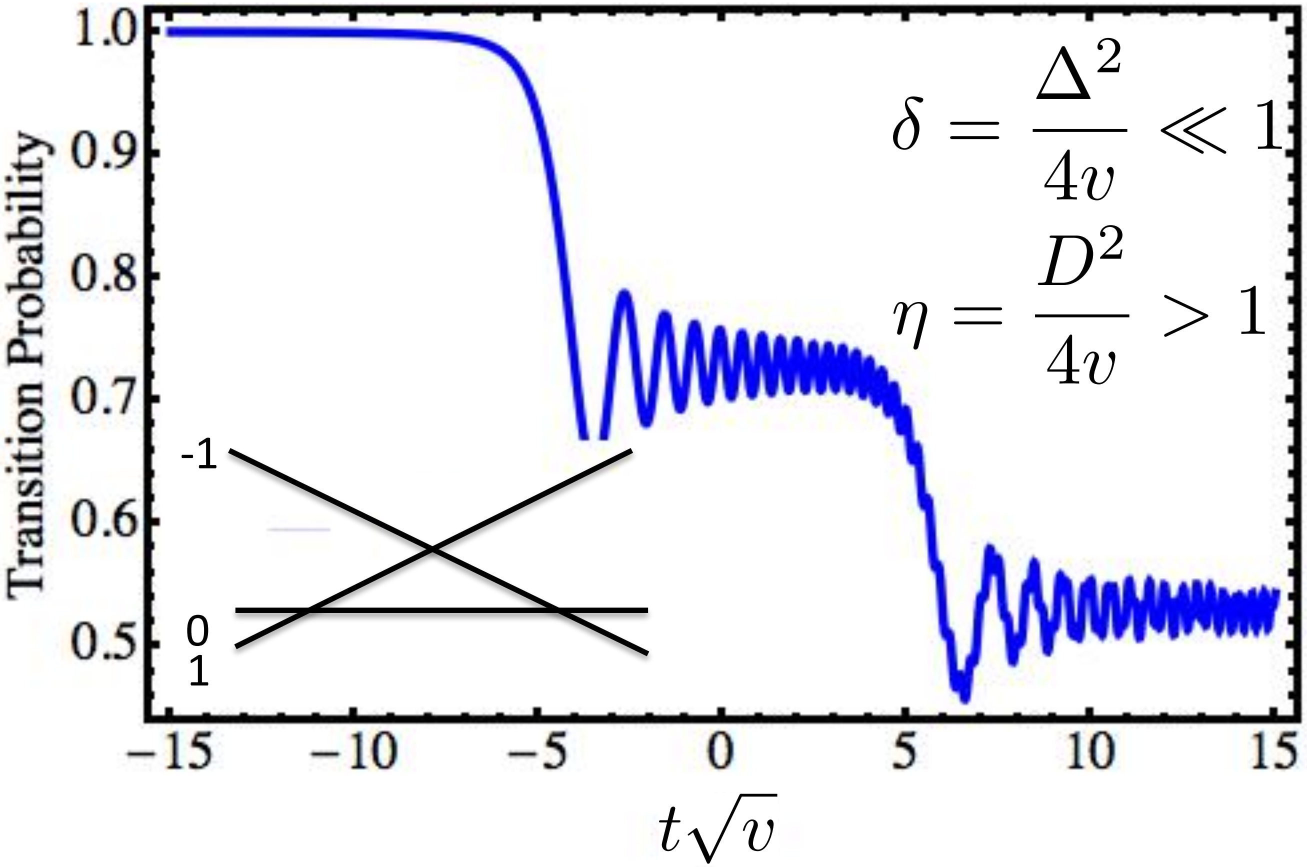


Triangular LZ interferometer : the beats

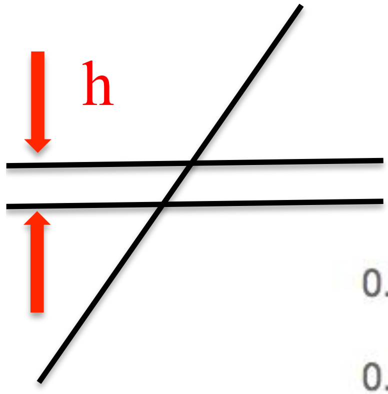


What is the period of the beats ?

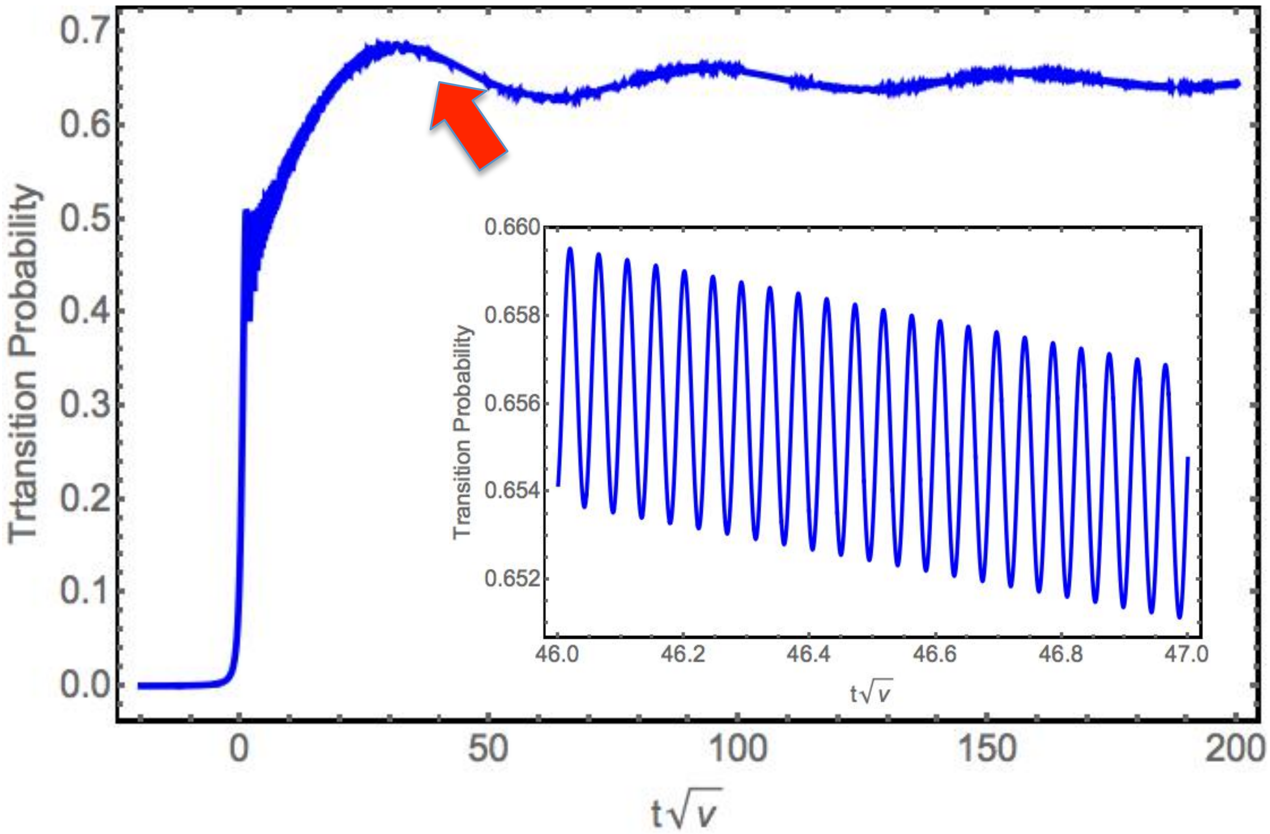
Triangular LZ interferometer : the steps



What is the time scale for the steps ?

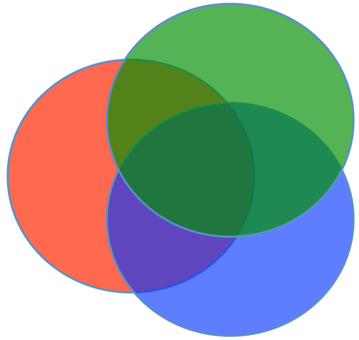


LZ interferometer : slow oscillations



How does the long period scale with splitting ?

Basis of SU(3): Gell-Mann matrices



$$3 \times SU(2) \begin{cases} \vec{s}_1 = \frac{1}{2}(\lambda_1 \lambda_2 \lambda_3) \\ \vec{s}_2 = \frac{1}{2}(\lambda_4 \lambda_5 \lambda_+) \\ \vec{s}_3 = \frac{1}{2}(\lambda_6 \lambda_7 \lambda_-) \end{cases}$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_+ = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_- = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\lambda_\pm = (\sqrt{3}\lambda_8 \pm \lambda_3)/2$$

Correspondence between SU(2) and SU(3)

SU(2)		SU(3)
Pauli Matrices $\sigma^\alpha, \quad \alpha = 1-3$		Gell-Mann Matrices $\lambda^\alpha, \quad \alpha = 1-8$
$n^\alpha = tr(\rho \cdot \sigma^\alpha)$	Bloch vector	$n^\alpha = tr(\rho \cdot \lambda^\alpha)$
$(\vec{n})^2 = 1$	Surface	$\begin{cases} (\vec{n})^2 = 1 \\ \vec{n} \cdot \vec{n} * \vec{n} = 1 \end{cases}$

Equation of Motion for the Density Matrix = Bloch equation

$$i\frac{d}{dt}n^\alpha = tr([H, \rho] \cdot \sigma^\alpha) \qquad\qquad i\frac{d}{dt}n^\alpha = tr([H, \rho] \cdot \lambda^\alpha)$$

$$H = \vec{B}(t) \cdot \vec{s} \qquad \boxed{\frac{d}{dt}\vec{n} = -\vec{B} \wedge \vec{n}} \qquad H = \vec{B}(t) \cdot \vec{\lambda}$$

$$(\vec{B} \wedge \vec{n})^\alpha = \epsilon^{\alpha\beta\gamma} B^\beta n^\gamma \qquad\qquad (\vec{B} \wedge \vec{n})^\alpha = f^{\alpha\beta\gamma} B^\beta n^\gamma$$

$$\epsilon^{\alpha\beta\gamma} = \frac{1}{4i} tr([\sigma^\alpha, \sigma^\beta] \cdot \sigma^\gamma) \qquad\qquad f^{\alpha\beta\gamma} = \frac{1}{4i} tr([\lambda^\alpha, \lambda^\beta] \cdot \lambda^\gamma)$$

Welcome to the 8-dimensional world !

Three level crossing: the equations

$$\frac{d}{dt}\vec{n} = -\vec{B}\wedge\vec{n}$$

$$\left\{\begin{aligned} \frac{dQ(t)}{dt} &= -\Delta^2\int_{-\infty}^{t} dt_1\left(Kr^-(t,t_1)S(t_1)+Kr^+(t,t_1)Q(t_1)\right)+2\Delta\Phi_-(t)\\ \frac{dS(t)}{dt} &= -3\Delta^2\int_{-\infty}^{t} dt_1\left(Kr^+(t,t_1)S(t_1)+Kr^-(t,t_1)Q(t_1)\right)+6\Delta\Phi_+(t)\\ W(t) &= \Delta\int_{-\infty}^{t} dt_1\left(Ki^+(t,t_1)S(t_1)+Ki^-(t,t_1)Q(t_1)\right)+\Phi_0(t)\end{aligned}\right.$$

$$\Phi_\pm(t) = -\frac{\Delta}{3}\int_{-\infty}^{t}dt_1\int_{-\infty}^{t_1}dt_2\left[Kr^{2\Omega^0}(t_1,t_2)Kr^\pm(t,t_1)-Ki^{2\Omega^0}(t_1,t_2)Ki^\pm(t,t_1)\right]\frac{d}{dt_2}S(t_2)$$
$$-4\Delta^2\int_{-\infty}^{t}dt_1\int_{-\infty}^{t_1}dt_2\left[Kr^{2\Omega^0}(t_1,t_2)Ki^\pm(t,t_1)+Ki^{2\Omega^0}(t_1,t_2)Kr^\pm(t,t_1)\right]W(t_2)$$

$$\Phi_0(t) = \frac{\Delta}{3}\int_{-\infty}^{t}dt_1\int_{-\infty}^{t_1}dt_2\left[Kr^{2\Omega^0}(t_1,t_2)Ki^+(t,t_1)+Ki^{2\Omega^0}(t_1,t_2)Kr^+(t,t_1)\right]\frac{d}{dt_2}S(t_2)$$
$$-4\Delta^2\int_{-\infty}^{t}dt_1\int_{-\infty}^{t_1}dt_2\left[Kr^{2\Omega^0}(t_1,t_2)Kr^+(t,t_1)-Ki^{2\Omega^0}(t_1,t_2)Ki^+(t,t_1)\right]W(t_2)$$

$$Kr^\xi(t,t_1) = \mathrm{Re}\left[\exp\left(i(\xi(t)-\xi(t_1)\right)\right] \qquad Ki^\xi(t,t_1) = \mathrm{Im}\left[\exp\left(i(\xi(t)-\xi(t_1)\right)\right]$$

$$Kr^\pm(t,t_1) = Kr^{\Omega^+}(t,t_1)\pm Kr^{\Omega^-}(t,t_1)$$
$$Ki^\pm(t,t_1) = Ki^{\Omega^+}(t,t_1)\pm Ki^{\Omega^-}(t,t_1)$$

$$\Omega^0(t) = vt^2$$

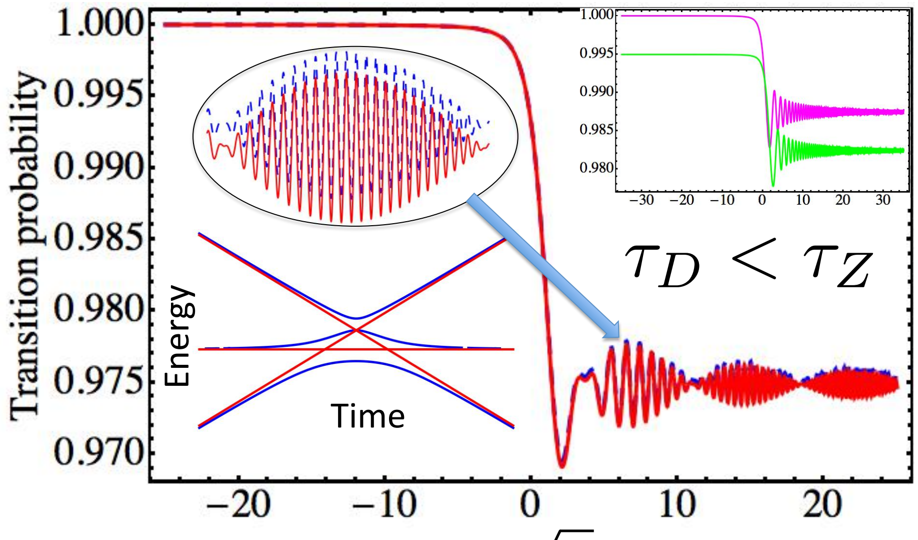
IC: $S(-\infty) = Q(-\infty) = 1,\quad W(-\infty) = 0$

$$\Omega^\pm(t) = v\left(t\pm\frac{D}{v}\right)^2$$

Diabatic Probabilities

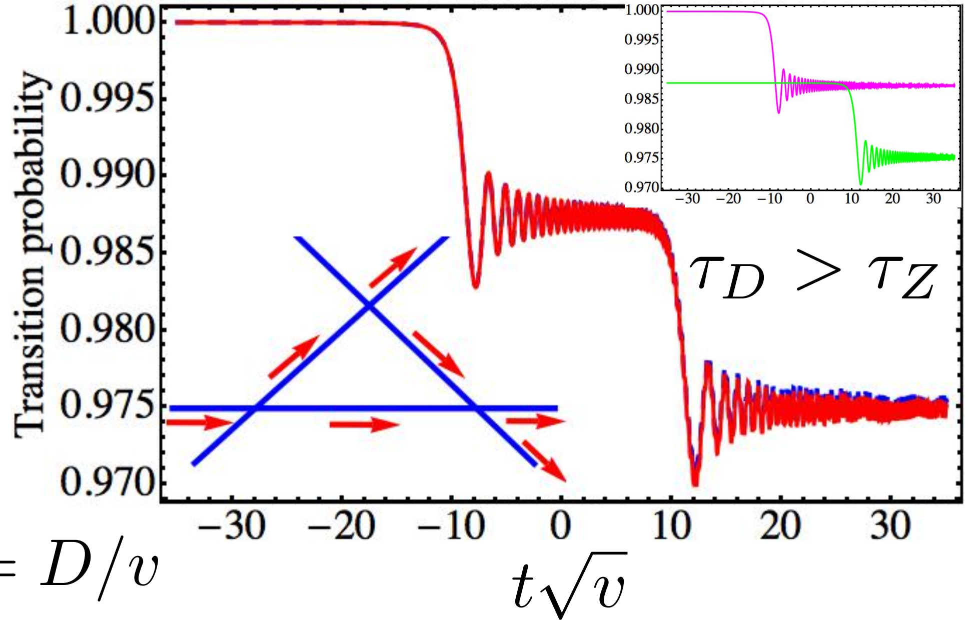
$$\rho_{11} = \frac{1}{3}\left(1+\frac{S}{2}+\frac{3Q}{2}\right)$$
$$\rho_{22} = \frac{1}{3}(1-S)$$
$$\rho_{33} = \frac{1}{3}\left(1+\frac{S}{2}-\frac{3Q}{2}\right)$$

SU(3) beats and steps: non-adiabatic passage



$t\sqrt{v}$

$\tau_D = D/v$



Blue – numerical solution of SE. Red – perturbative analytic solution of BE.

$$P_{2\to 2}(t) \approx 1 - \frac{\pi\Delta^2}{4v}\left[F(t - D/v) + F(t + D/v)\right]$$

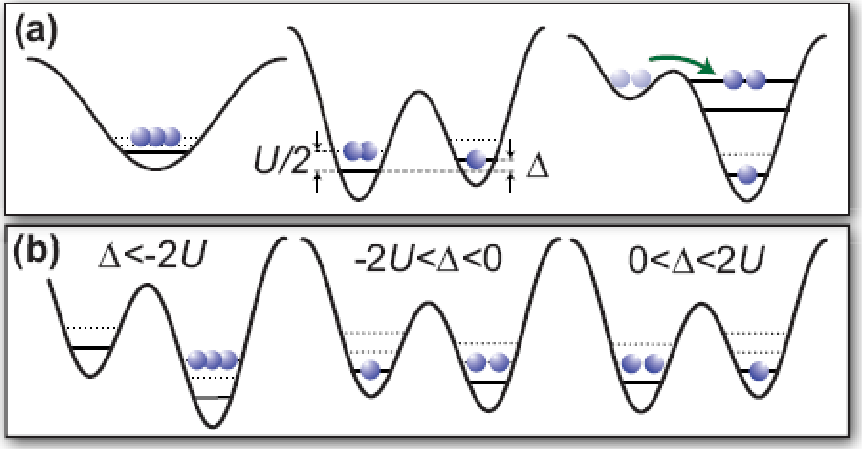
$$\sim \sin\left(\frac{\pi}{2}Dt\right)$$

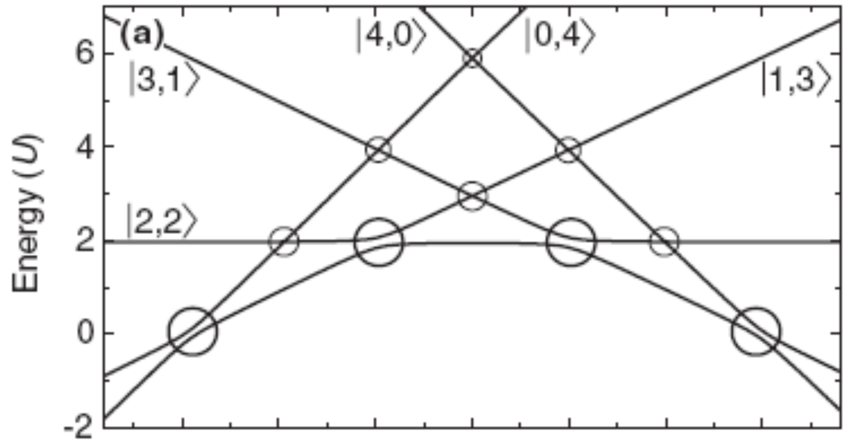
Period $\quad T \sim \frac{1}{D} \gg \frac{1}{\sqrt{v}}$

Fresnel Integrals

$$F(t) = \frac{1}{2}\left[\left(\frac{1}{2} + C\left(\sqrt{\frac{v}{\pi}}t\right)\right)^2 + \left(\frac{1}{2} + S\left(\sqrt{\frac{v}{\pi}}t\right)\right)^2\right]$$

Bloch's experiment on interacting blockade and Landau-Zener (PRL 2008)





$$SU(3) \rightarrow SU(5)$$

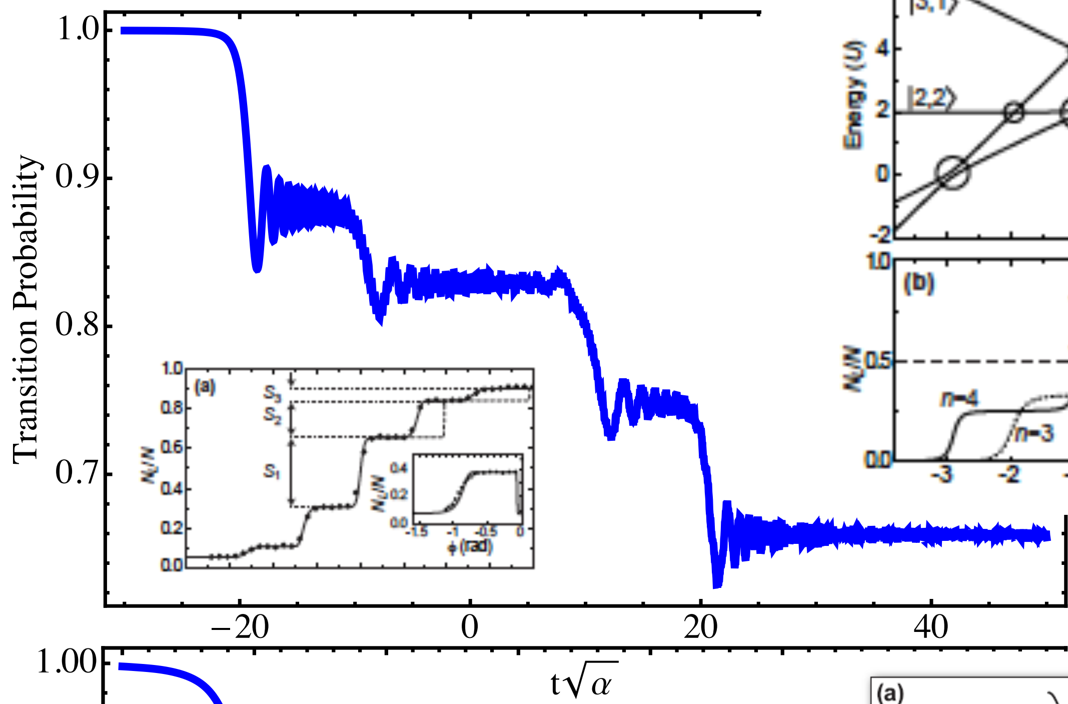
$$H = vtS^z + \Delta S^x + D(S^z)^2$$

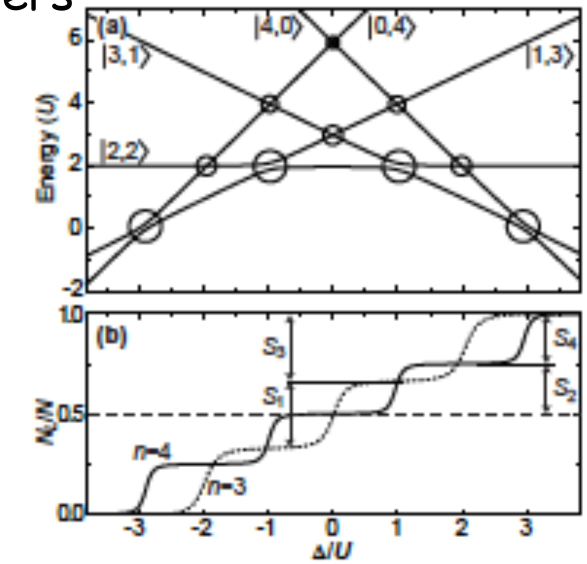
$$H(t) = \begin{pmatrix} 2vt+4D & \Delta & 0 & 0 & 0 \\ \Delta & vt+D & \Delta\frac{\sqrt{6}}{2} & 0 & 0 \\ 0 & \frac{\sqrt{6}}{2}\Delta & 0 & \Delta\frac{\sqrt{6}}{2} & 0 \\ 0 & 0 & \frac{\sqrt{6}}{2}\Delta & -vt+D & \Delta \\ 0 & 0 & 0 & \Delta & -2vt+4D \end{pmatrix}$$

Spin S=2 model with quadrupole interaction

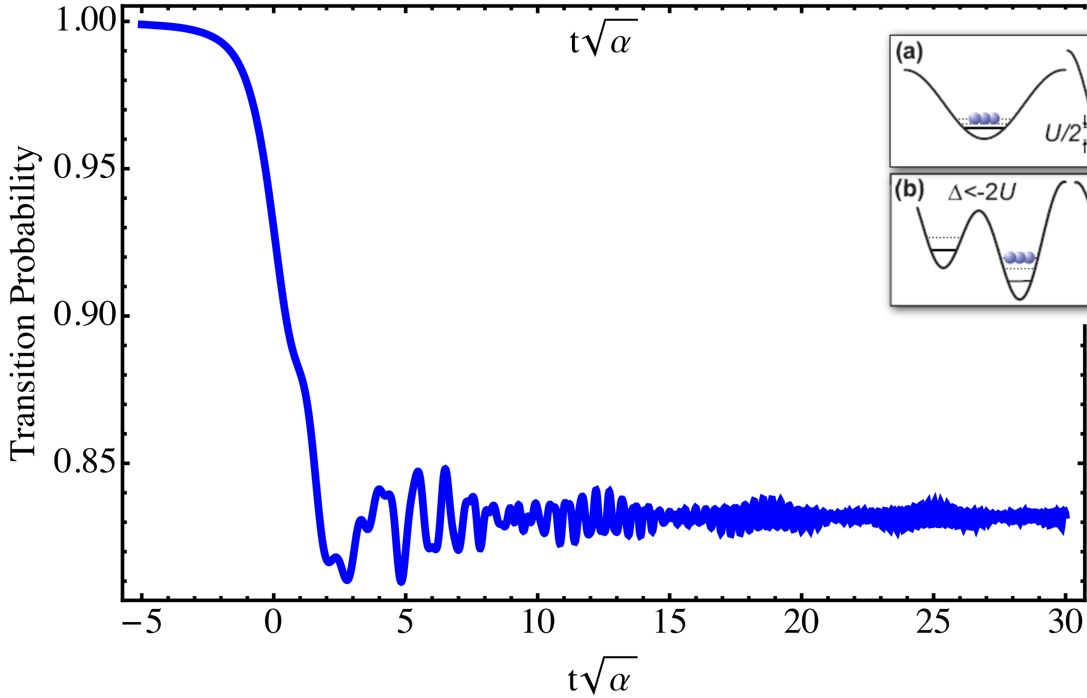
System of coupled interferometers

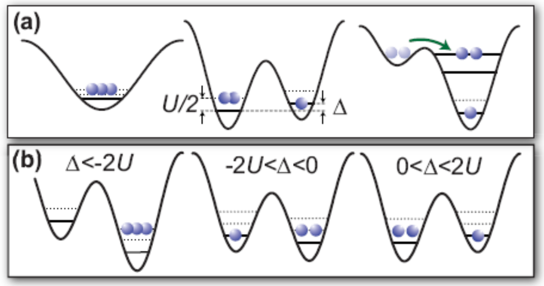
$$\eta = \frac{D^2}{4v} > 1$$



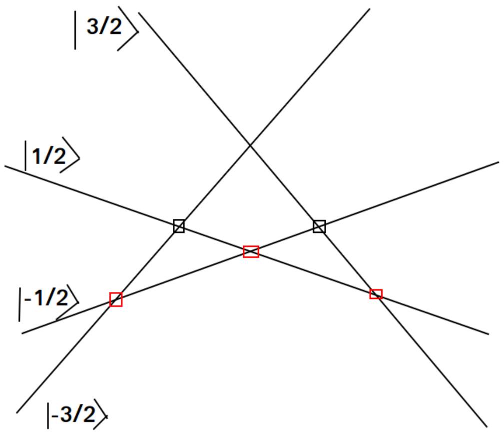


$$\eta = \frac{D^2}{4v} < 1$$

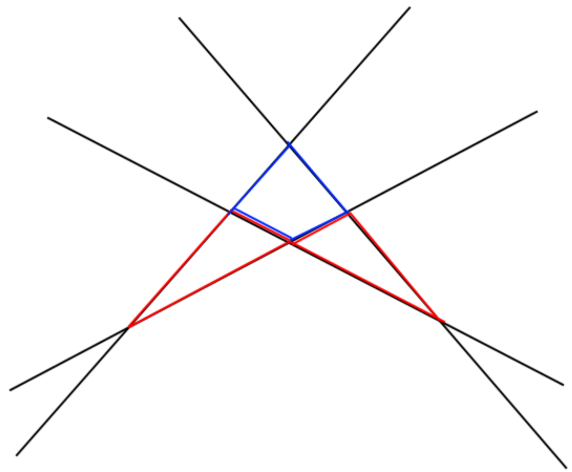




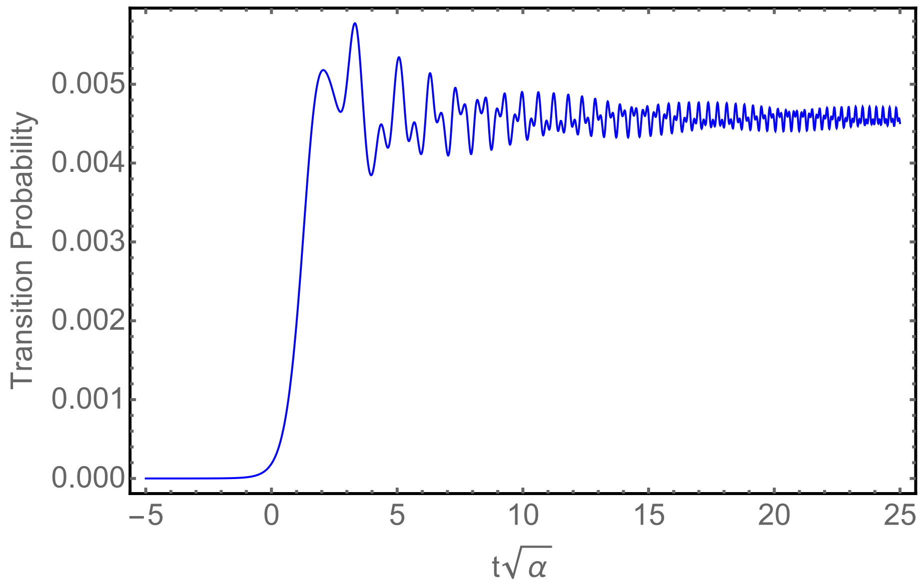
"Minimal" model for coupled interferometers



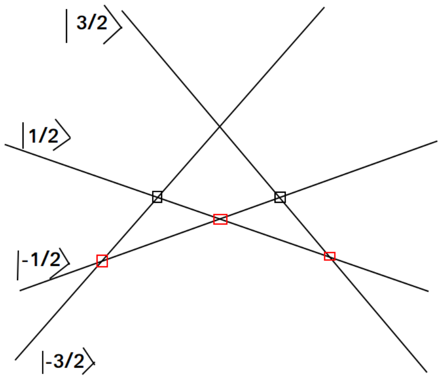
SU(4)

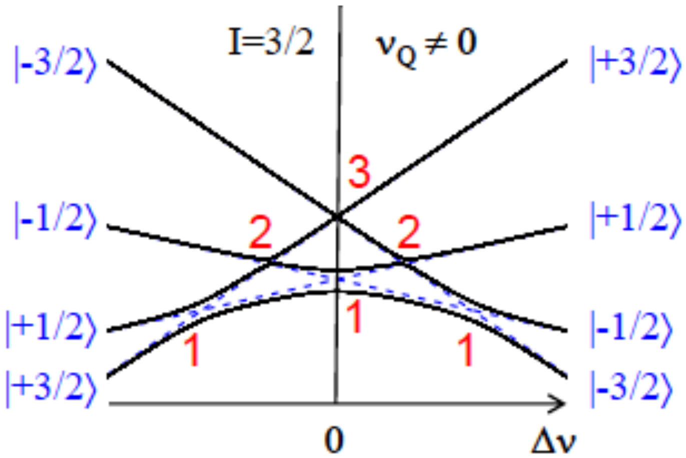


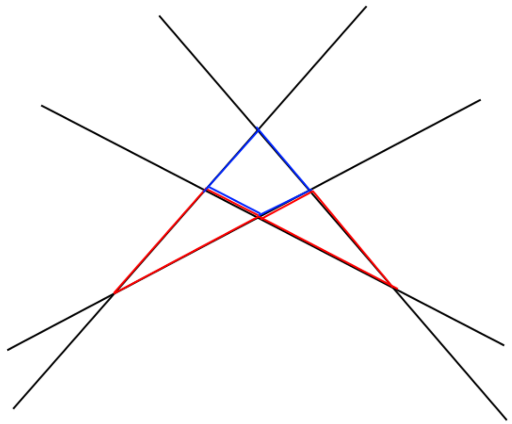
$$H = vtS^z + \Delta S^x + D(S^z)^2 + \gamma\left[(S^x)^2 - (S^y)^2\right]$$



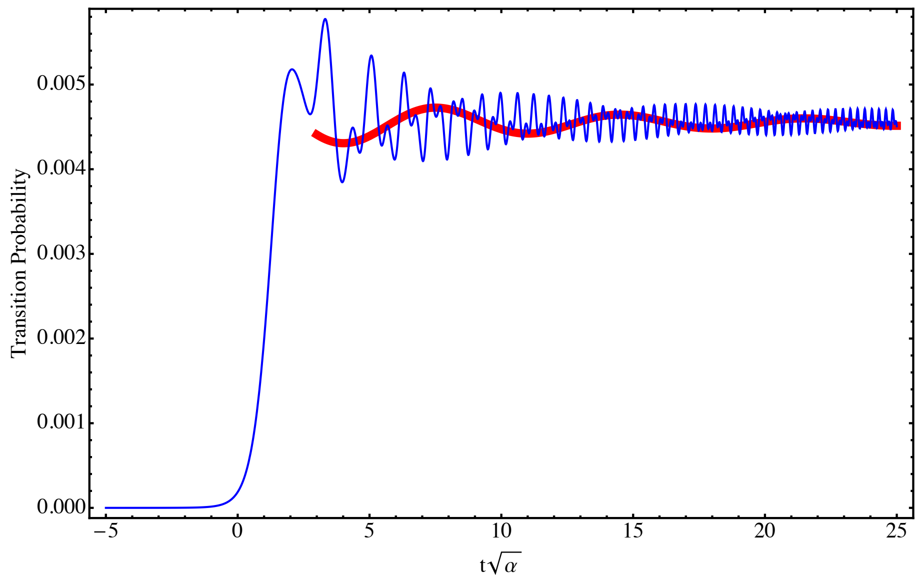
Inverse engineering of dynamical Hamiltonians

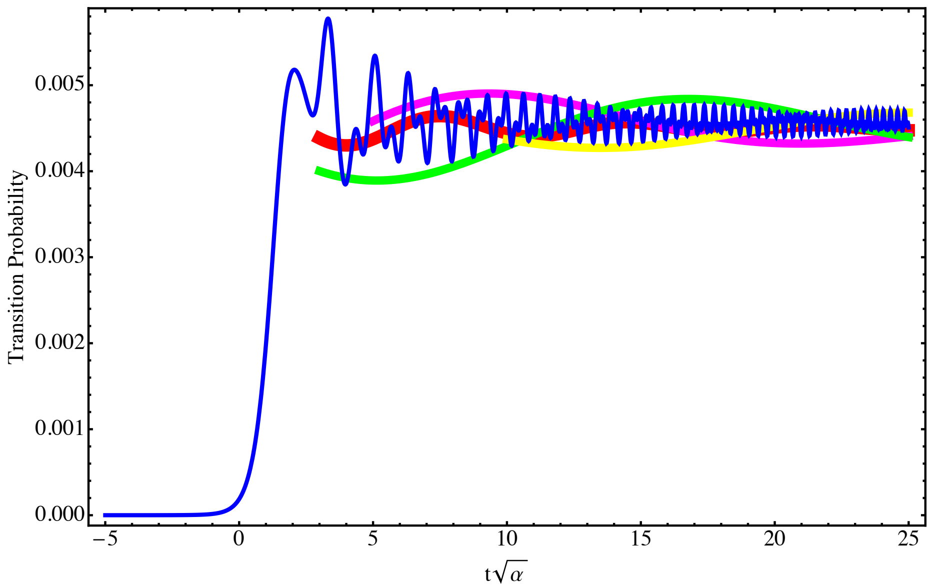






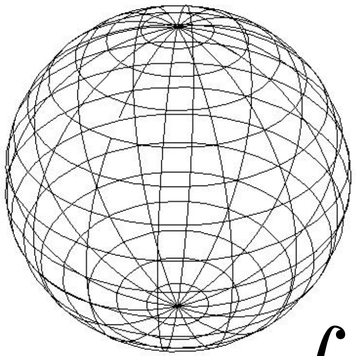
$$H = vtS^z + \Delta S^x + D(S^z)^2 + \gamma\left[(S^x)^2 - (S^y)^2\right]$$





Multi level crossing and Berry phase

$SU(2)$



S_2

$$S = S_B + S_{WZ}$$

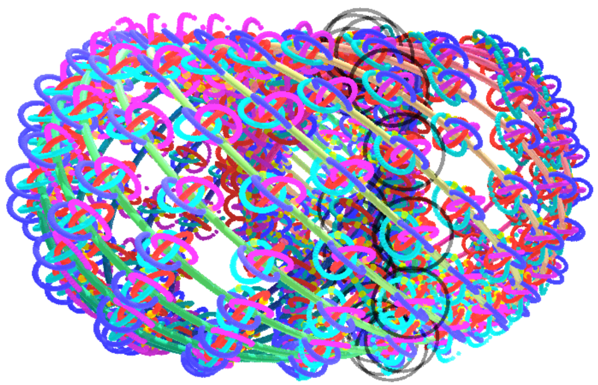
$$S_B = -\oint_C \vec{B}(t)\cdot\vec{n}$$

$$S_{WZ} = 2\pi i \oint_C dt \int_0^1 d\xi \frac{1}{8\pi}\epsilon^{\alpha\beta\gamma}\epsilon_{\mu\nu} n^\alpha \partial_\mu n^\beta \partial_\nu n^\gamma$$

$$\vec{n}(t,\xi=0) = (0,0,1) \qquad \vec{n}(t,\xi=1) = \vec{n}(t)$$

$$\frac{d}{dt}\vec{n} = -\vec{B}\wedge\vec{n}$$

$SU(3)$

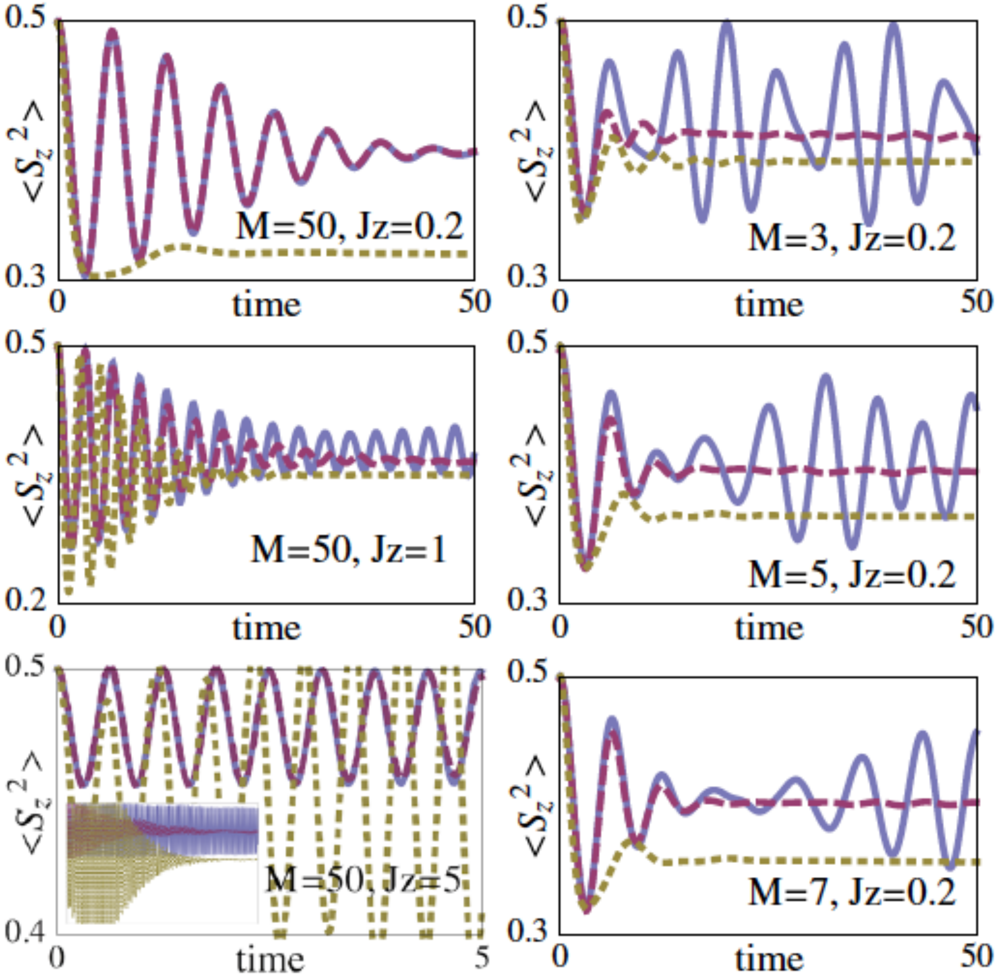


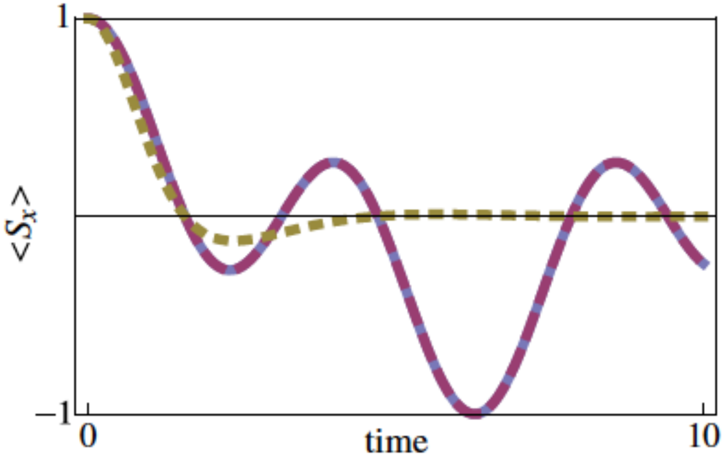
CP^2

$$(\vec{B}\wedge\vec{n})^\alpha = \epsilon^{\alpha\beta\gamma}B^\beta n^\gamma$$

$$(\vec{B}\wedge\vec{n})^\alpha = f^{\alpha\beta\gamma}B^\beta n^\gamma$$

SU(N) quantum spin dynamics





$$\hat{H} = \sum_n \hat{H}_I^{(n)} + \hat{H}_C,$$

$$\hat{H}_I = -\vec{B}\,\hat{S} + (U/2)\hat{S}_z^2.$$

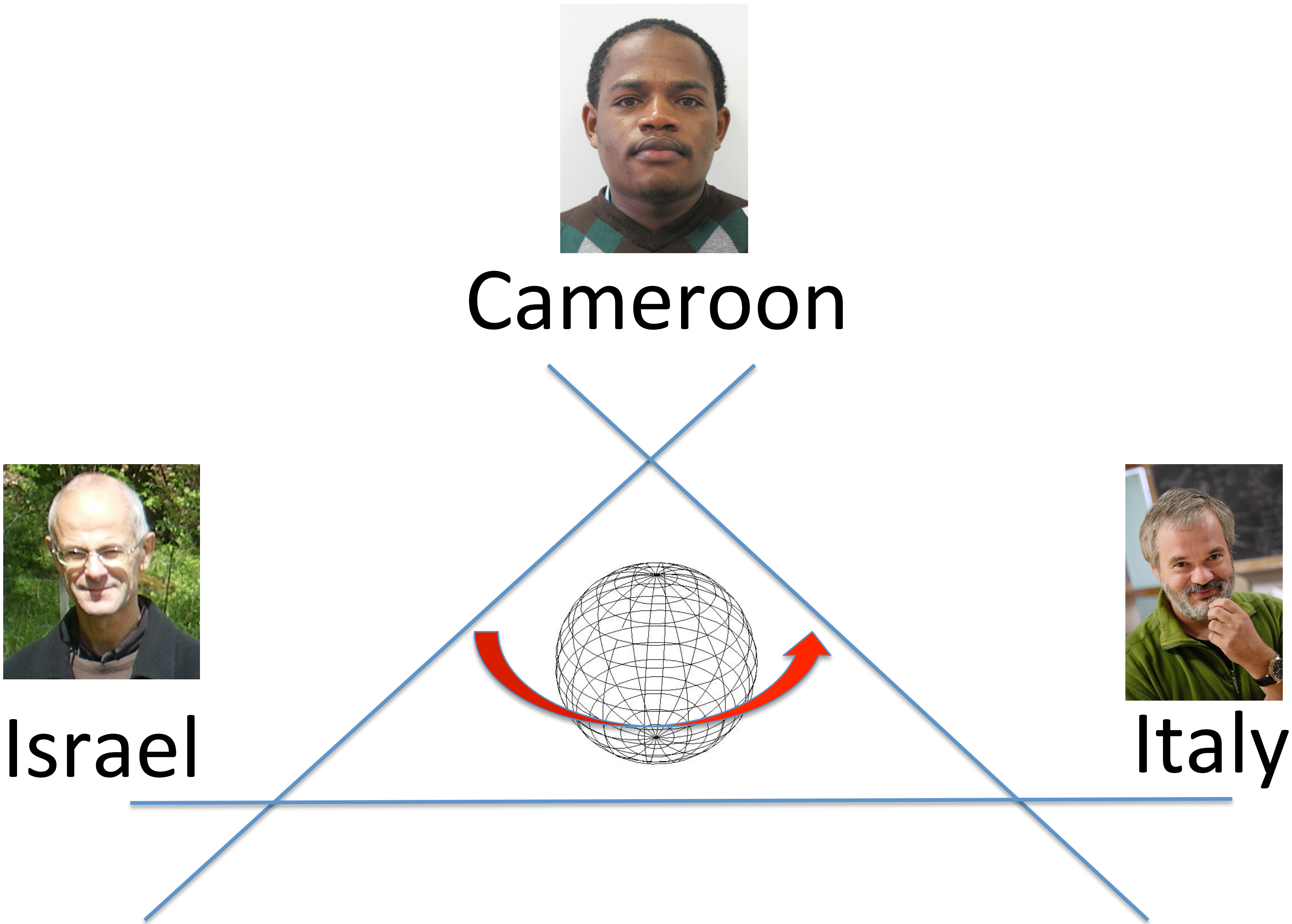
$$\hat{H}_C = -J\sum_{n\neq m}(\hat{S}_x^n\hat{S}_x^m + \hat{S}_y^n\hat{S}_y^m).$$

Controllable way for 1/N expansion?

S. Davidson and A.Polkovnikov PRL 2015

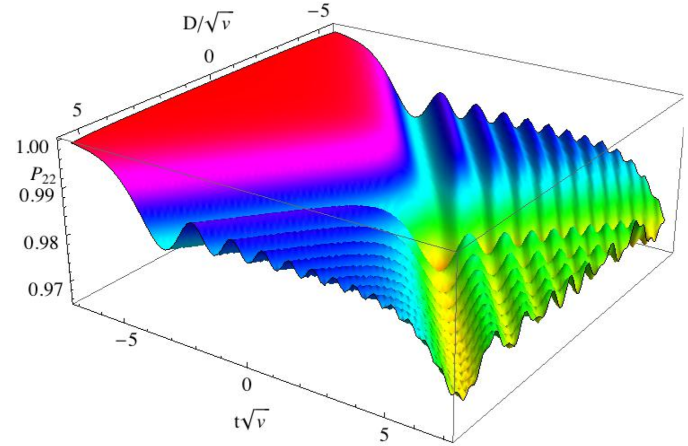
Perspectives (to do list)

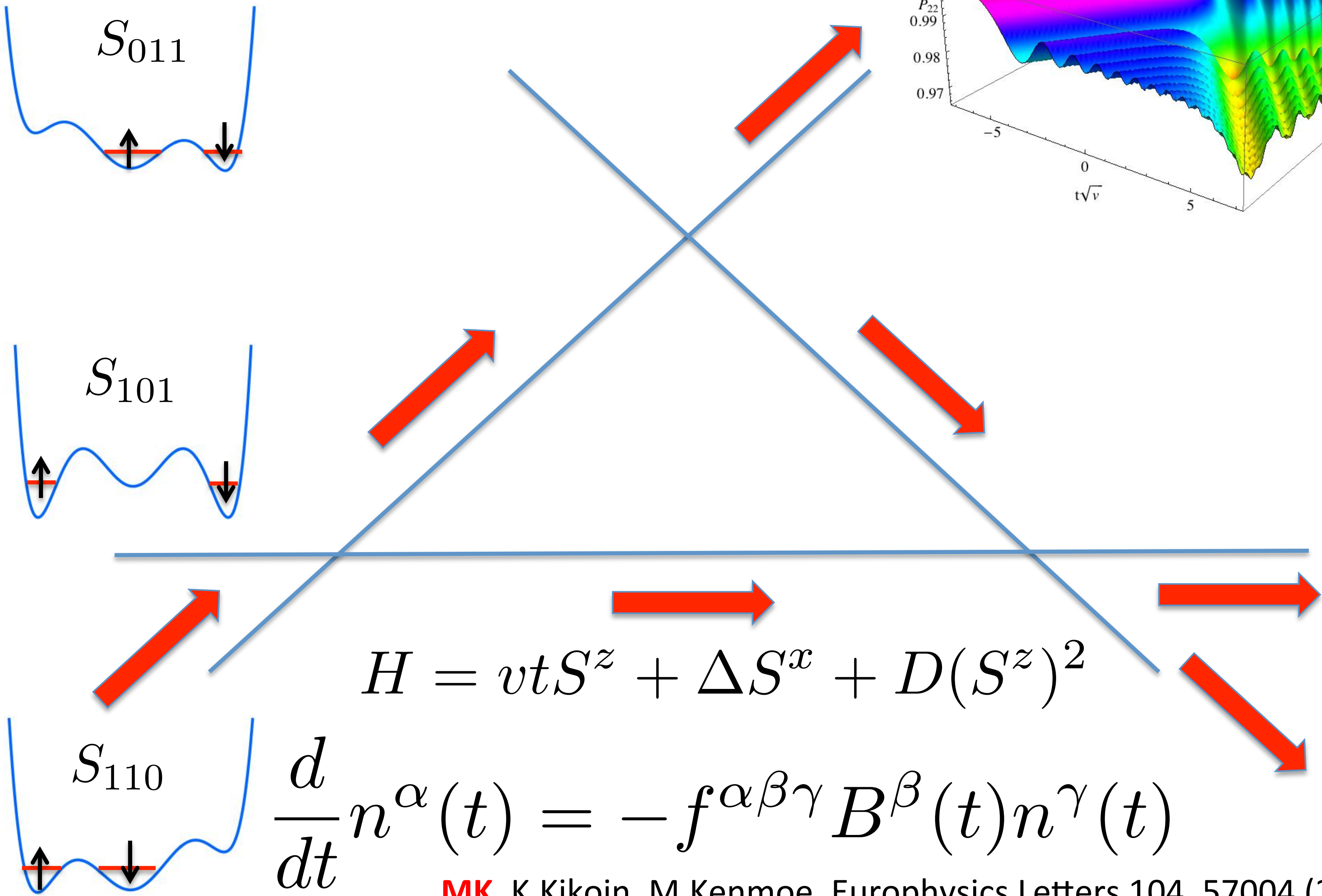
- Dissipative two-level crossing : SU(4) LZ model
- “Longitudinal” and “transverse” relaxations in BE
- Loschmidt echo and statistics of works for LZ transition
- Quantum quenches to- and from- degenerate GS
- Fast and slow noise in SU(3) LZ: a random magnetic field
- “Parabolic” SU(3) LZ interferometry: superconducting qubits
- Periodically driven SU(3) systems
- Singlet/Triplet transitions in DQD and TQD: SO(4) LZ model
- Berry phases in SU(3)/U(2): an exotic AB effect?



SU(3) constructive interference

Conclusions





$$H = vtS^z + \Delta S^x + D(S^z)^2$$

$$\frac{d}{dt} n^\alpha(t) = -f^{\alpha\beta\gamma} B^\beta(t) n^\gamma(t)$$

MK, K.Kikoin, M.Kenmoe, Europhysics Letters 104, 57004 (2013)

A. Adhikari, MK, K.Kikoin (2015)