Stability and Anomalous Compressibility of Resonant Bose Gases

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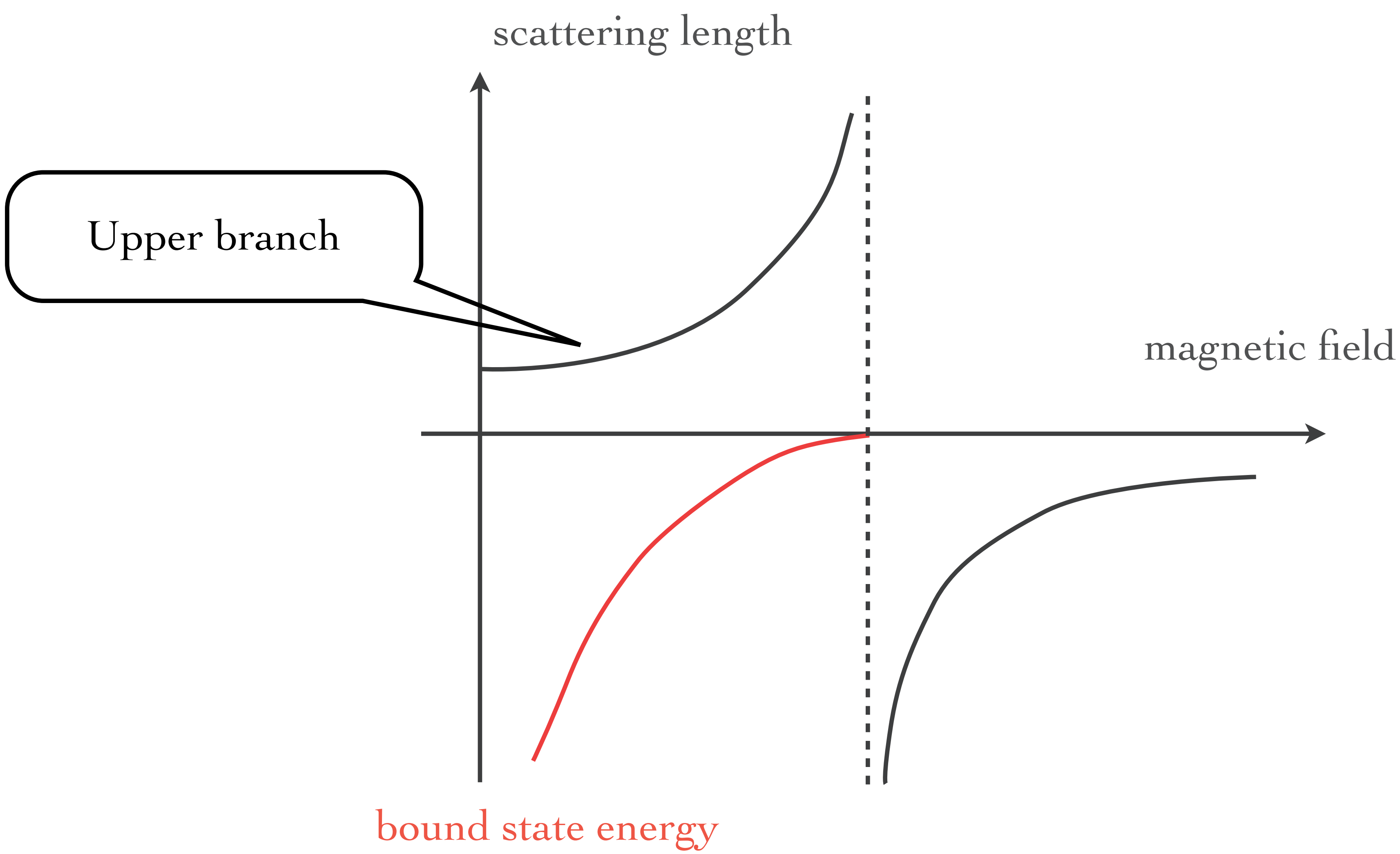
Outline

- Introduction
- Stability domain in “T-a” plane.
- Mechanism for stabilization
- Anomalous compressibility
- Density profile

Upper Branch Bose gases

- Upper branch of a Feshbach resonance — quantum gas of scattering atoms



Dilute gas theory

- Dilute limit $na^3 \ll 1$

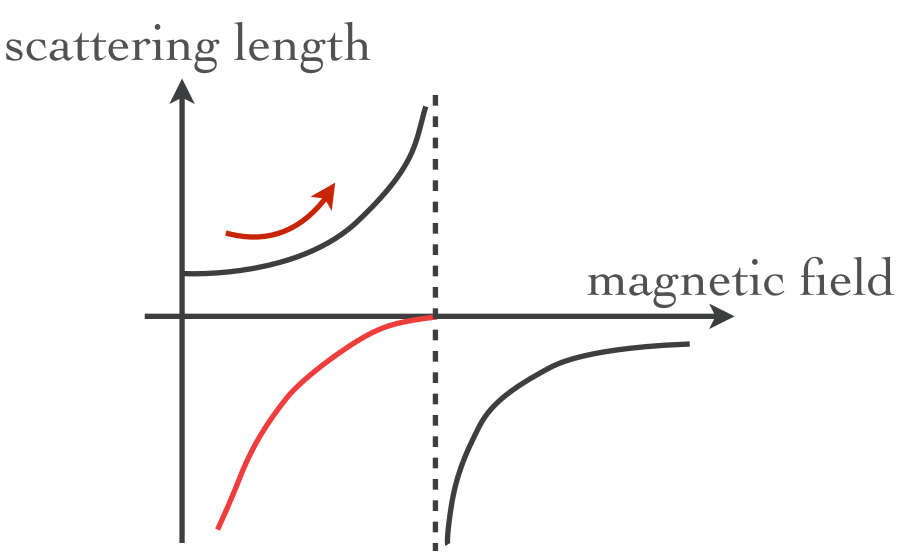
- Energy density

$$E = \frac{2\pi a}{m} n^2 \left[1 + \frac{128}{15\sqrt{\pi}} \sqrt{na^3} + C_1 na^3 \ln(na^3) + (C_2 + B) na^3 + \ldots \right]$$

HF LHY Brueckner, Sawada, Wu,... Braaten et al.

- Move towards resonance

$na^3 \sim 1$ or $\gg 1$



Go near resonance

- Experiments

  Papp et al. (2008), Pollack et al. (2009), Navon et al. (2011),

  Wild et al. (2012), Ha et al. (2013), Fletcher et al. (2013),

  Makotyn et al. (2014) …

- Theories

  Cowell et al. (2002), Song et al. (2009), Diederix et al. (2011),

  Borzov et al. (2012), Zhou et al. (2013), Pilati et al. (2005),

  Mashayekhi et al. (2013) …

Fermionization

$$\mu_{(na^3 \ll 1)} = 4\pi a n[1 + O(\sqrt{na^3})]$$
$$\mu_{(a \to \infty)} \sim n^{2/3}$$
$$\equiv \xi E_F$$

	Cowell et al. (2002)	Song et al. (2009)	Diederix et al. (2011)	Borzov et al. (2012)	Navon et al. (2011)
ξ	2.92	0.8	0.83	0.9	>0.44

Few-body loss

- Three-body loss rate, **Fedichev et al. (1996), Nielson et al. (1999), Esry et al. (1999), Bedaque et al. (2000)**

$$
\begin{aligned}
L_3 &\sim a^4, \text{ when } a \ll \lambda_T \\
L_3 &\sim T^{-2}, \text{ when } a \gg \lambda_T
\end{aligned}
$$

- Experiments **Rem et al. (2013), Fletcher et al. (2013)**

- Few-body loss is believed to be the reason that leads to short lifetime near unitarity, also the main difficulty of the adiabatic sweep.

Self-consistent approach

- Condensate with density *n_0* + non-condensed atoms with chemical potential *mu*.

$$\mu_c(n_0,\mu) = \frac{\partial E(n_0,\mu)}{\partial n_0}, \quad n = n_0 - \frac{\partial E(n_0,\mu)}{\partial \mu},$$

$$\mu = \mu_c(n_0,\mu),$$

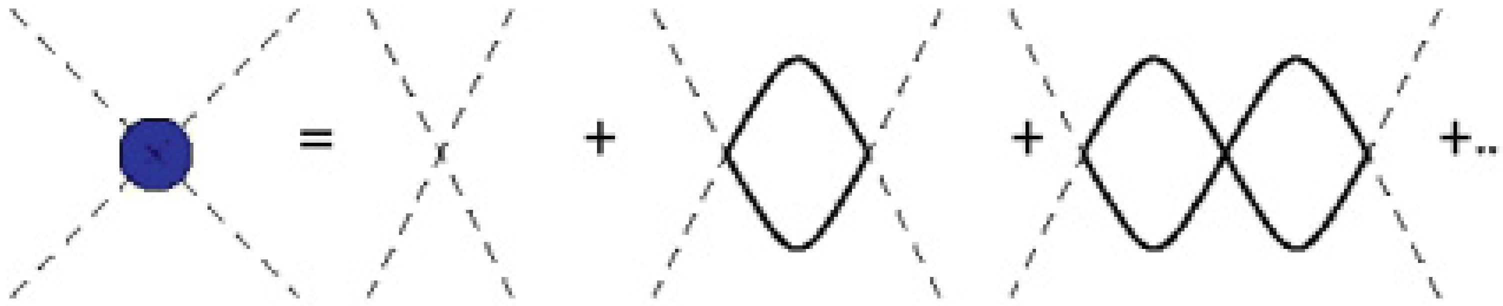
Hugenholtz et al.(1959)

- Effective potential w.r.t. bosonic field. (Coleman & Weinberg)

- Diagrammatic summation $\rightarrow E(n_0,\mu)$

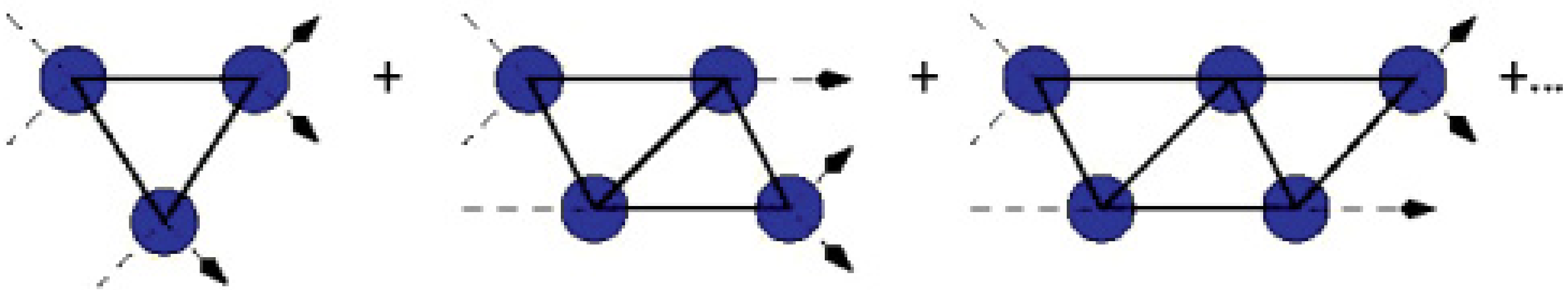
3D resonant Bose gases **Bozov et al. (2012)**

- irreducible 2-body diagram



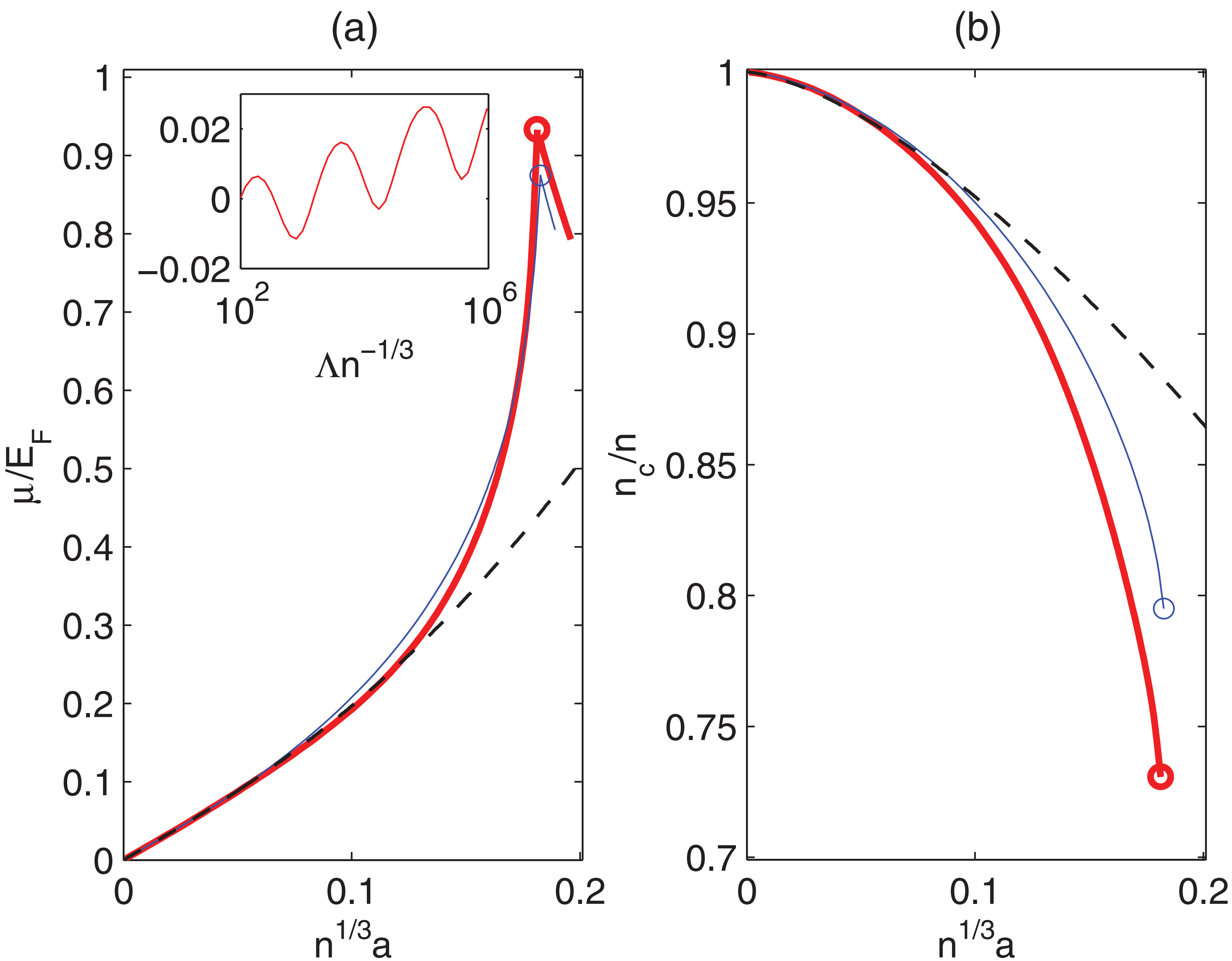
(a)

- irreducible 3-body diagrams



(b)

Results in 3D



- Fermionization
- Many-body instability

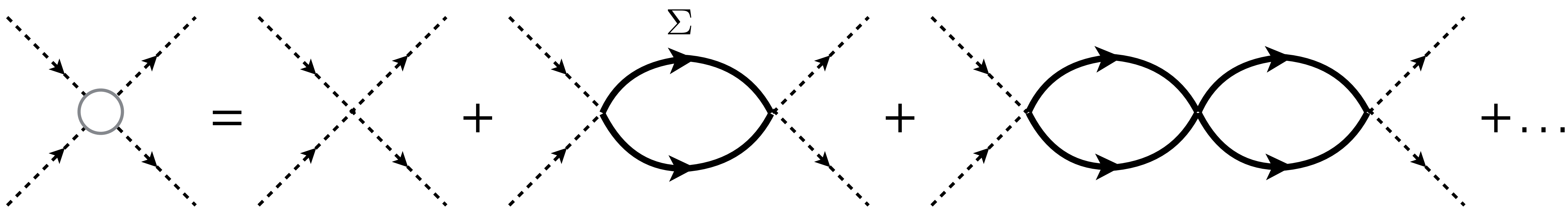
  $$n^{1/3}a \approx 0.18$$

- Efimov physics
- Smallness of 3-body effects

Running of coupling constant

Zhou et al. (2013)

- Running of 2-body coupling,



$$g_2(\eta) = \frac{4\pi a}{m} \frac{1}{1 - \sqrt{2m\eta}a}$$

- Green's function of non-condensed atoms,

$$G(\varepsilon, \boldsymbol{p}) = \frac{1}{\varepsilon - p^2/(2m) - \Sigma + \mu + \mathrm{i}0^+}$$

$$\eta = \Sigma - \mu$$

Few-body losses
v.s.
many-body instability

Few-body losses	Many-body instability
few-body recombination	running coupling constant
atom loss	thermodynamical instability
smooth	onset of instability

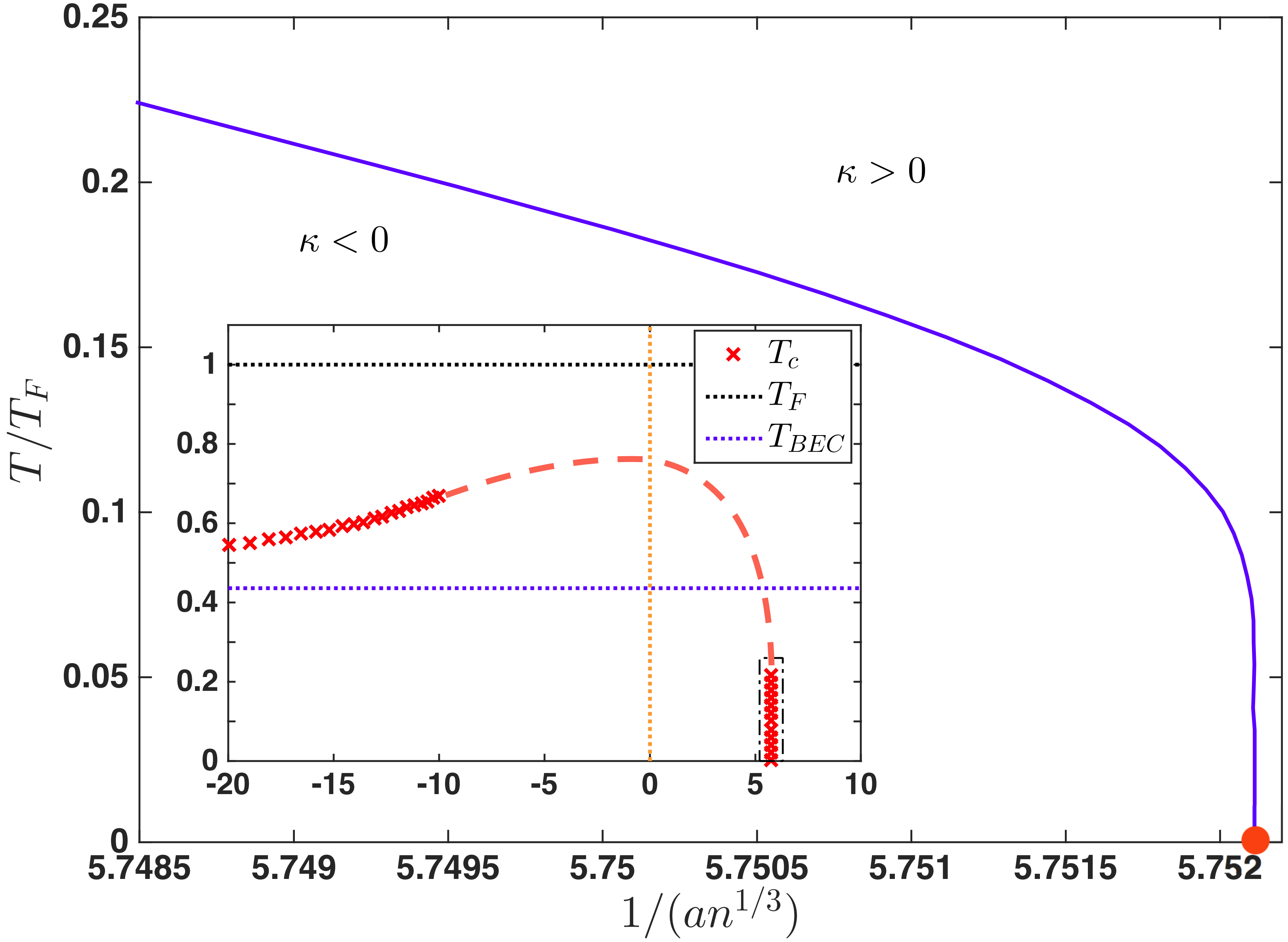
Bose gases at finite temperature

arXiv:1504.03434

- Questions & Motivations
  - Stability domain in “T-a” plane?
  - Mechanism(s) for stabilization?
  - Anomalous behavior near instability?
  - Experimental signature for instability?
- Finite temperature field theory

$$E(n_0, \mu) \to F(n_0, \mu)$$

Phase diagram

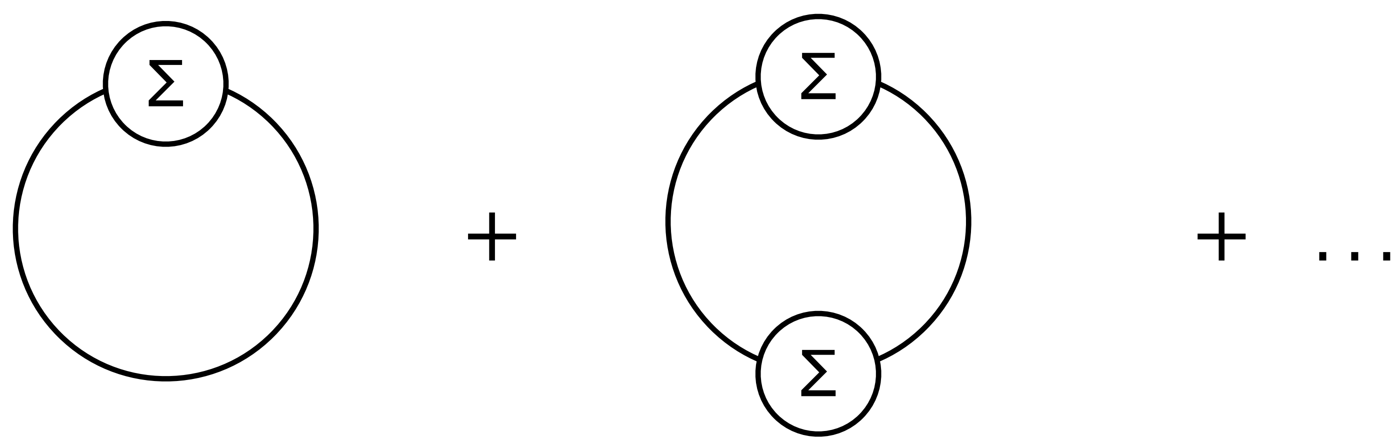


- Compressibility

$$\frac{1}{\kappa} = \frac{\partial \mu}{\partial n}$$

Stabilize Bose gases

- Thermal pressure



$$F_1 = T \int \frac{d^3q}{(2\pi)^3} \ln(1 - e^{-\beta(\epsilon_k + \eta)})$$

$$\eta = \Sigma - \mu$$

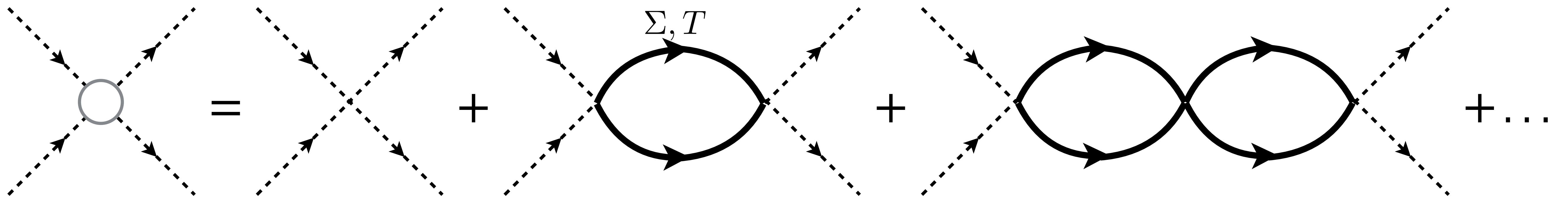
Stabilize Bose gases

- Finite temperature running of coupling constant

$$F_2 = \frac{1}{2} g_2(\eta) n_0^2$$

$$g_2^{-1}(\eta) = \frac{1}{4\pi a} - \frac{\sqrt{2\eta}}{4\pi} + \int \frac{d^3q}{(2\pi)^3} \frac{2n_B(q^2/2+\eta)}{q^2+2\eta}$$

bosonic enhancement



Three-body loss, lifetime

- Estimation of 3-body loss rate near instability

$$L_3 \approx 203.7a^4 f(a\Lambda_*)$$ Bedaque et al. (2000)

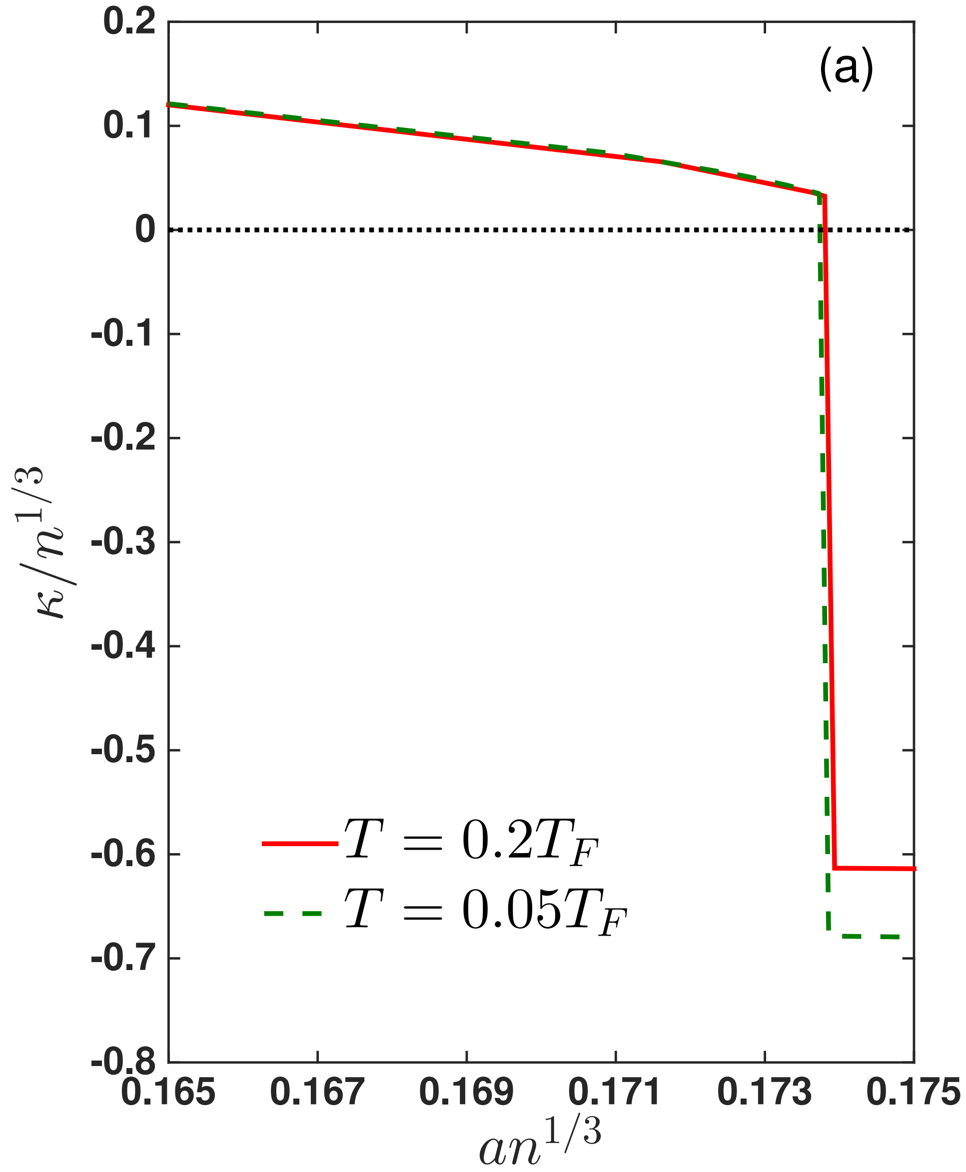
- Near instability

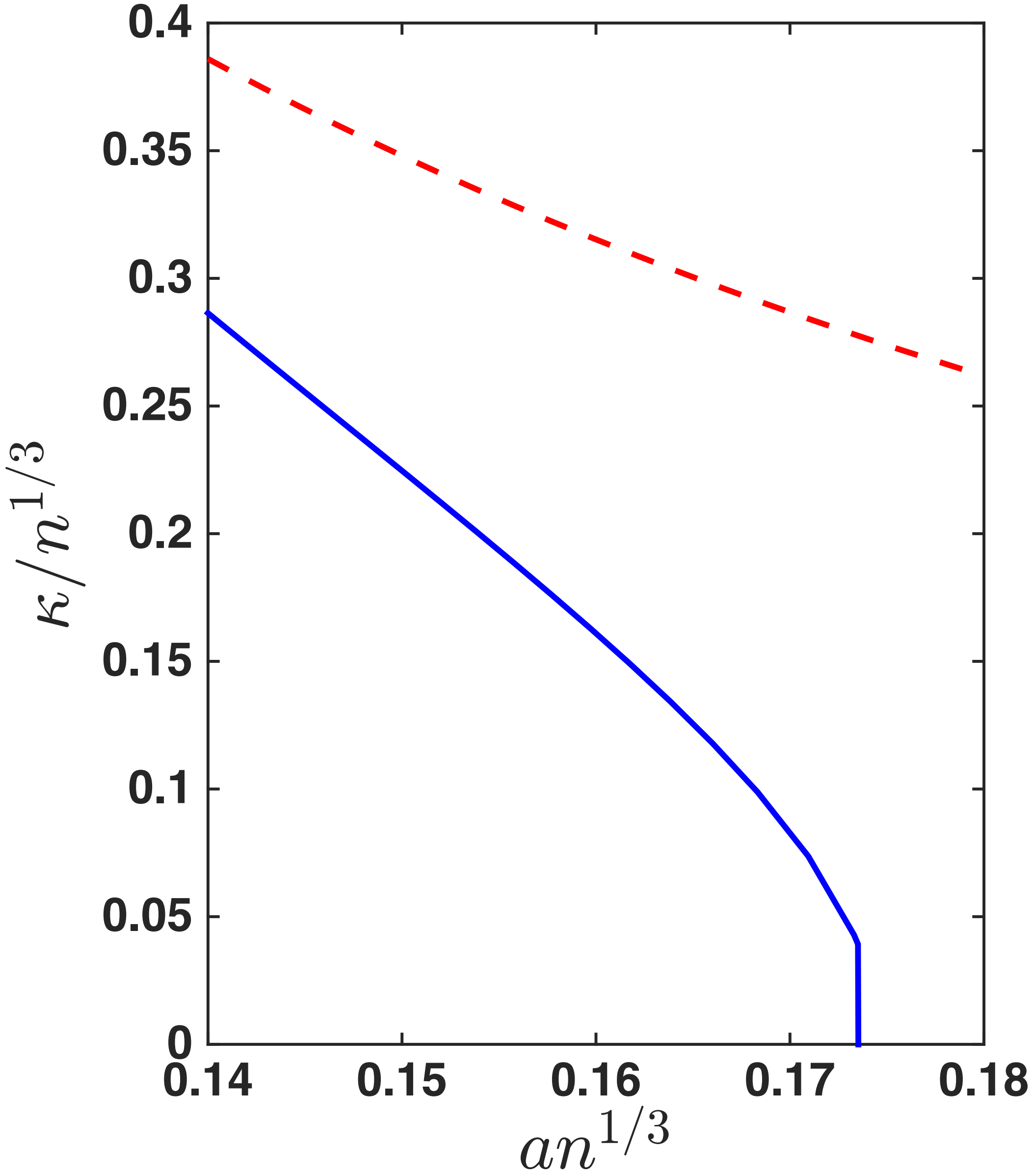
$$an^{1/3} \approx 0.174$$

$$\mu \approx 0.735T_F$$

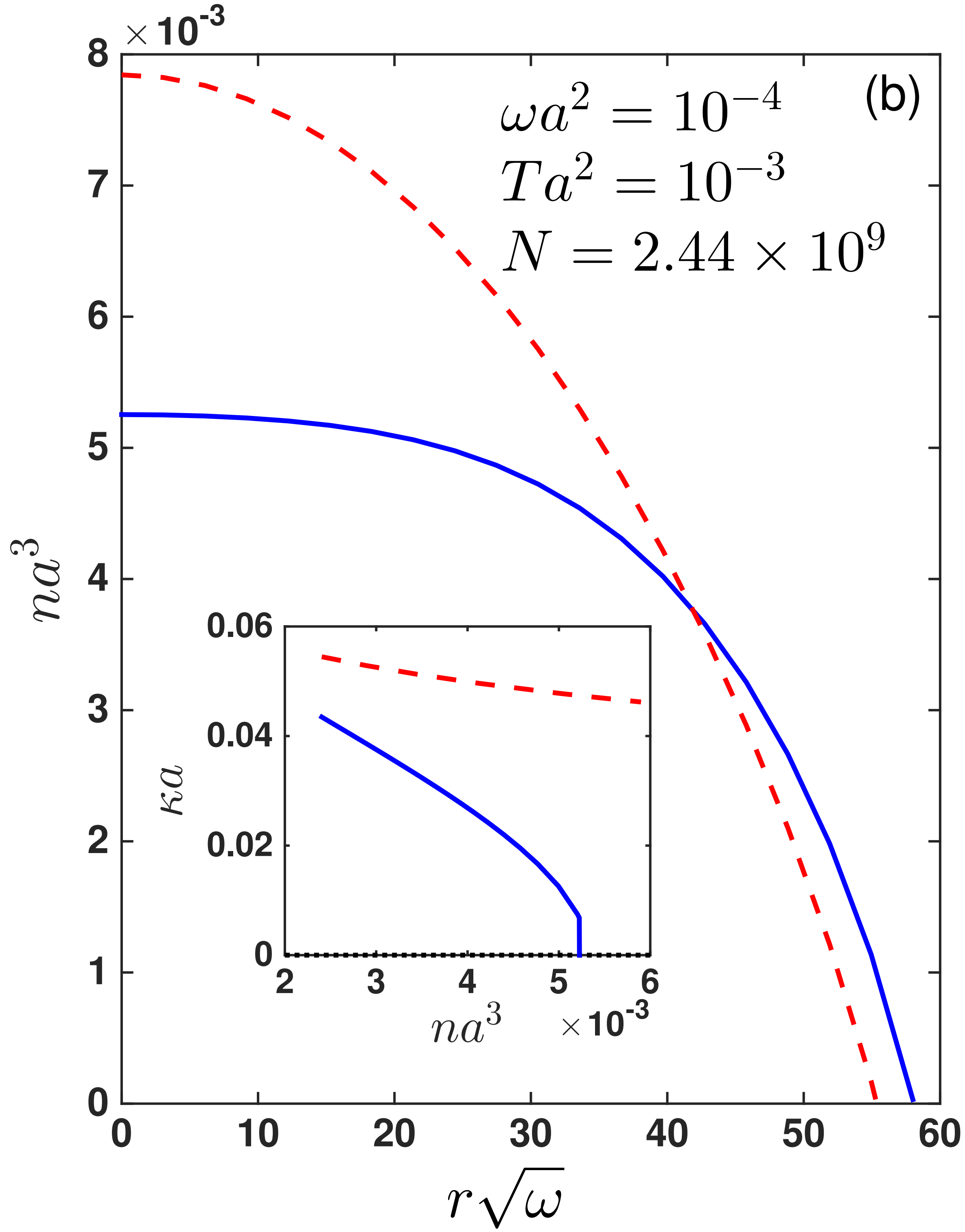
$$L_3 n^2 \approx 0.0246T_F$$

Anomalous compressibility





Density profile



- Flat top in density profile induced by the rapid drop of compressibility as a precursor of instability.

Conclusions

- Phase diagram in T-a plane
- Finite temperatures stabilize Bose gases via two mechanisms
- Many-body instability sets in before losses become significant.
- Anomalous compressibility along critical line
- Flat top in density profile as a precursor of the onset of many-body instability

Thank you!