Stability and Anomalous Compressibility of Resonant Bose Gases

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Outline

- Introduction
- Stability domain in "T-a" plane.
- Mechanism for stabilization
- Anomalous compressibility
- Density profile

Upper Branch Bose gases

• Upper branch of a Feshbach resonance quantum gas of scattering atoms

Dilute gas theory

- Dilute limit $na^3 \ll 1$
- Energy density

$$
E = \frac{2\pi a}{m} n^2 \left[1 + \frac{128}{15\sqrt{\pi}} \sqrt{na^3} + C_1 na^3 \ln(na^3) + (C_2 + B)na^3 + \ldots \right]
$$

HF LHY Ehveckner, Sawada, Wu,... Braaten et al.

• Move towards resonance $na^3 \sim 1$ or $\gg 1$

Go near resonance

• Experiments

Papp et al. (2008), Pollack et al. (2009), Navon et al. (2011), Wild et al. (2012), Ha et al. (2013), Fletcher et al. (2013), Makotyn et al. (2014) …

• Theories

Cowell et al. (2002), Song et al. (2009), Diederix et al. (2011), Borzov et al. (2012), Zhou et al. (2013), Pilati et al. (2005), Mashayekhi et al. (2013) …

Fermionization

$$
\mu_{(na^3 \ll 1)} = 4\pi a n [1 + O(\sqrt{na^3})]
$$

\n
$$
\mu_{(a \to \infty)} \sim n^{2/3}
$$

\n
$$
\equiv \xi E_F
$$

Few-body loss

• Three-body loss rate, **Fedichev et al. (1996), Nielson et al. (1999), Esry et al. (1999), Bedaque et al. (2000)**

$$
L_3 \sim a^4
$$
, when $a \ll \lambda_T$
\n $L_3 \sim T^{-2}$, when $a \gg \lambda_T$

- Experiments **Rem et al. (2013), Fletcher et al. (2013)**
- Few-body loss is believed to be the reason that leads to short lifetime near unitarity, also the main difficulty of the adiabatic sweep.

Self-consistent approach Calf consistent annroach Jeli-Cursistent approach

• Condensate with density *n_0* + non-condensed atoms with chemical potential *mu*. • Condensate with density $n \Omega$ + non-condensed atoms with chemic

$$
\mu_c(n_0,\mu) = \frac{\partial E(n_0,\mu)}{\partial n_0}, \quad n = n_0 - \frac{\partial E(n_0,\mu)}{\partial \mu},
$$

$$
\mu = \mu_c(n_0,\mu),
$$
Hugenholtz et al.(1959)

- Effective potential w.r.t. bosonic field. (Coleman & Weinberg) \bullet Effective notential with hosonic field (Coleman 8. enceuve potential w.i.t. posonic neiu. (Oolenian d noncondensed atoms, as indicated in Eq. (3). Calculations of
- Diagrammatic summation $\sum_{n=0}^{\infty}$ • Diagrammatic summation $\rightarrow E(n_0,\mu)$

3D resonant Bose gases **Bozov et al. (2012)**

• irreducible 2-body diagram

• irreducible 3-body diagrams

Results in 3D

Running of coupling constant

Zhou et al. (2013)

• Running of 2-body coupling,

$$
g_2(\eta) = \frac{4\pi a}{m} \frac{1}{1 - \sqrt{2m\eta}a}
$$

• Green's function of non-condensed atoms,

$$
G(\varepsilon, \mathbf{p}) = \frac{1}{\varepsilon - p^2/(2m) - \Sigma + \mu + i0^+} \eta = \Sigma - \mu
$$

Few-body losses v.s. many-body instability

Bose gases at finite temperature

arXiv:1504.03434

- Questions & Motivations
	- Stability domain in "T-a" plane?
	- Mechanism(s) for stabilization?
	- Anomalous behavior near instability?
	- Experimental signature for instability?
- Finite temperature field theory

 $E(n_0, \mu) \rightarrow F(n_0, \mu)$

Phase diagram Dhoop diogram r ridde didgram

Stabilize Bose gases condensed atoms and *F*(*n*0*, µ*) is the free-energy dencondensed atoms and *F*(*n*0*, µ*) is the free-energy density [26, 29]. *F*(*n*0*, µ*) itself is calculated at a prefixed

• Thermal pressure *intermal pressure*

$$
F_1 = T \int \frac{d^3q}{(2\pi)^3} \ln(1 - e^{-\beta(\epsilon_k + \eta)})
$$

$$
\eta = \Sigma - \mu
$$

Stabilize Bose gases mal pressure provided by *F*¹ enhances the ther- α stabilize bose gases $\overline{}$ *g*2(⌘)*n*² ⁰*,* (3)

 $\mathbb{F}_{\mathbb{F}_{q}}$ = temperature running of coupling constant energy of the condensed atoms and can be written atoms and can be written atoms and can be written atoms and c
The condensed atoms and can be written atoms and can be written at the condensed atoms and can be written at t temperature running of coupling constant

$$
F_2=\frac{1}{2}g_2(\eta)n_0^2
$$

5.7485 5.749 5.7495 5.75 5.7505 5.751 5.7515 5.752

$$
g_2^{-1}(\eta) = \frac{1}{4\pi a} - \frac{\sqrt{2\eta}}{4\pi} \left[+ \int \frac{d^3q}{(2\pi)^3} \frac{2n_B(q^2/2 + \eta)}{q^2 + 2\eta} \right]
$$

bosonic enhancement

Three-body loss, lifetime smooth variation of *L*³ with *a* is fundamentally di↵erent from the sharp sign can be sharp that the many sign conset of the many sign conset of the many-sign conset of the many-Three-body loss, lifetime pare the few- and many-body e↵ects, we carry out an body long lifatima smooth variation of *L*³ with *a* is fundamentally di↵erent

• Estimation of 3-body loss rate near instability parameter and many-pare the fewestimation of *L*³ near the quantum critical point. In the estimation of *L*³ near the quantum critical point. In the • Lstimation of 3-body loss rate near instability body loss rate near instability

 $L_3 \approx 203.7 a^4 f(a \Lambda_*)$ **Bedaque et al. (2000)** $L_3 \approx 2.903.7a^4 f(a)$ $\mathcal{L}_{\mathcal{Q}}$, and $\mathcal{L}_{\mathcal{Q}}$ (and \mathcal{Q}) and \mathcal{Q} ا
ا\0ا ام ام د \mathcal{L} periodic function of \mathcal{L} and \mathcal{L} and \mathcal{L} and \mathcal{L} ν (ν ever, the traditional perturbation expansion based on $L_3 \approx 203.(a^{\text{-}}f(a\Lambda_*)$ Bedaque et al. bare interactions was demonstrated to converge quite $L_3 \approx 203.7a^4 f(a\Lambda_\ast)$ **Bedaque at al. (2000)** do the shown temperature is to be 2007.

• Near instability where Moor inotability the short-distance details of the interaction of the interaction of the interaction of the interaction of the i **•** Near instability

 $an^{1/3} \approx 0.174$ ergyscale set by the three-body loss rate can be estimated by the three-body loss rate can be estimated by \sim $\mu \approx 0.1331$ F $L_3 n^2 \approx 0.0246 T_F$ $\alpha m^{1/3} \sim 0.174$ $\frac{u}{\sqrt{2}}$ \sim $\frac{0.11 \pm}{0.11 \pm}$ $\mu \approx 0.735 T_F$, σ many σ many σ in before, in before, in before the three-body in before the three-body in before the three-body in σ $L_3 n^2 \approx 0.0246 T_F$ $\frac{1}{\sqrt{2}}$ $an^{1/\sigma} \approx 0.174$ $\mu\approx0.735 T_F$ ergy set by the three-body loss rate can be estimated by the three-body loss rate can be estimated by $\mathcal{L}_{\mathcal{A}}$

Anomalous compressibility ³

Density profile

• Flat top in density
Profile induced by profile induced by the rapid drop of compressibility as a precursor of instability. stability is shown in Fig. 2 (a) for *T* = 0*.*2*T^F* and prome induced by can see a drop of compressibility of compressibility of contract of changes in the changes of changes in the cha compressionity as a provardor or
inctability m becaming the e m

Conclusions

- Phase diagram in T-a plane
- Finite temperatures stabilize Bose gases via two mechanisms
- Many-body instability sets in before losses become significant.
- Anomalous compressibility along critical line
- Flat top in density profile as a precursor of the onset of many-body instability

Thank you!