Measuring local Green's functions: an STM for cold atoms

http://dqmp.unige.ch/giamarchi/

UNIVERSITÉ DE GENÈVE

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SCIENCE TO SHAPE THE FUTURE OF THE ARMY
ARMY RESEARCH OFFICE
1951

MaNEP
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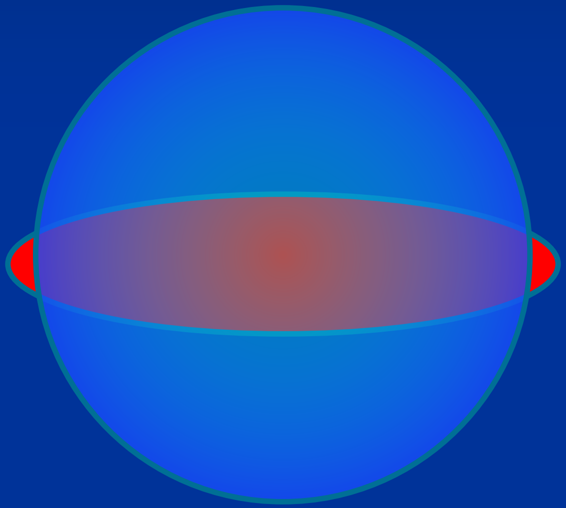
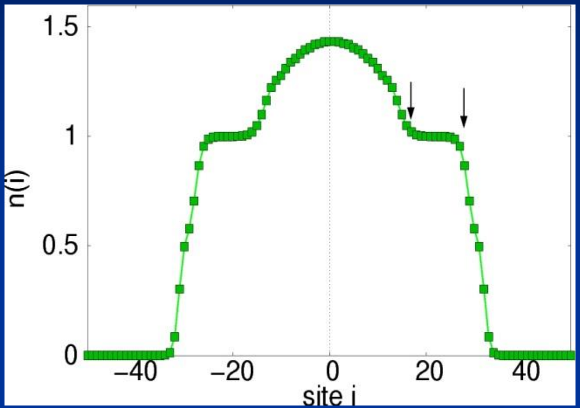
NETWORK

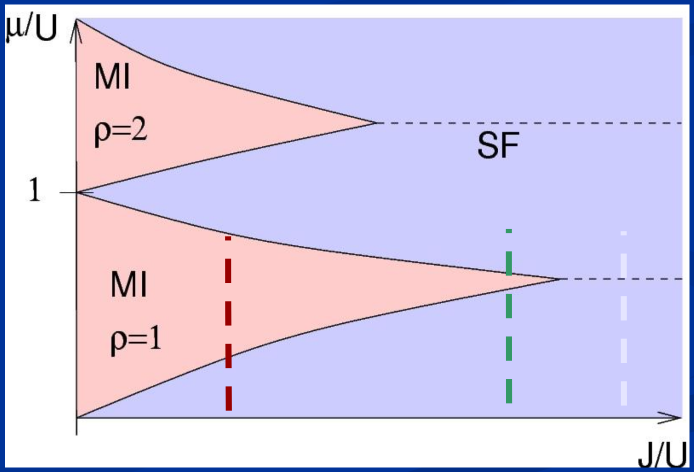
A. Kantian (Nordita)

U. Schollwoeck (Munich)

Cold atoms, Quantum simulators ?





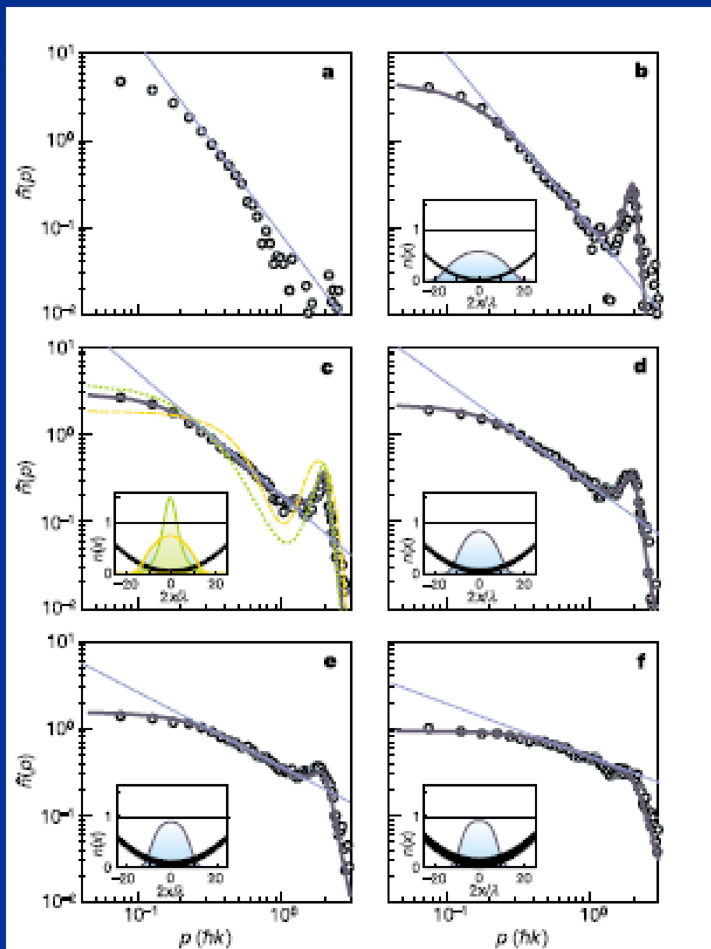


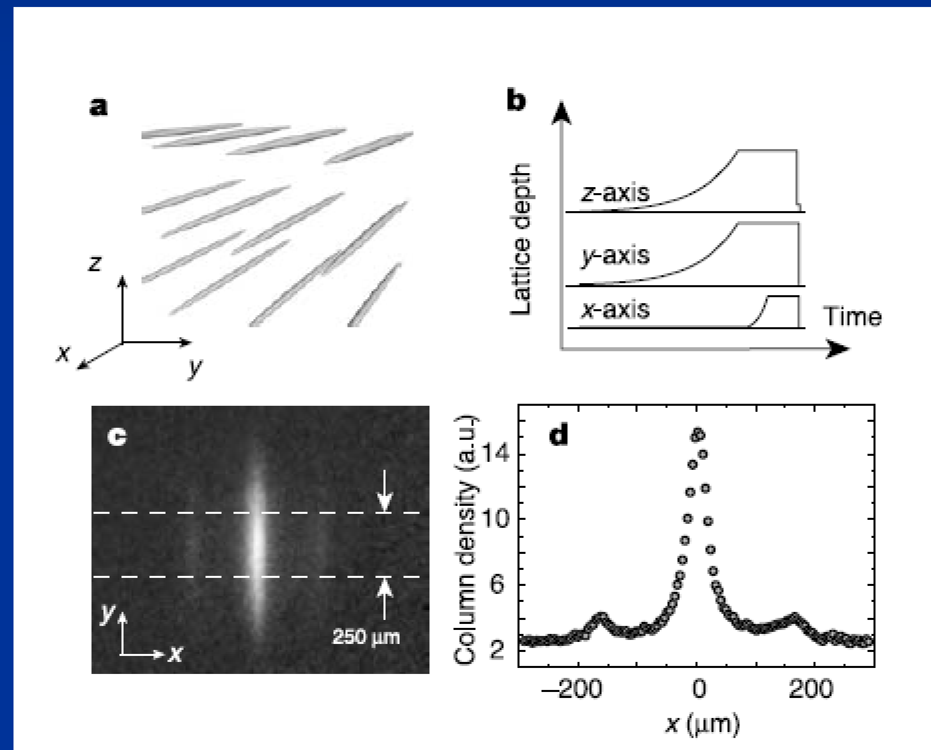
$$H = \int r^2 \rho(r)$$

- No homogeneous phase !

Optical lattices (dilute)

B. Paredes et al., Nature 429 277 (2004)





$$n(k) = \int dx e^{ikx} \langle \psi^\dagger(x)\psi(0) \rangle$$

Solutions ?

- Get rid of the trap !
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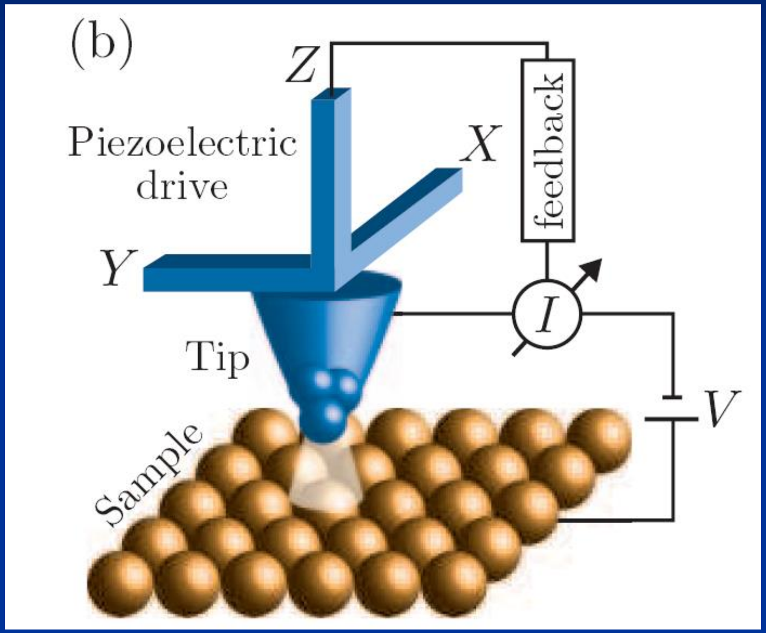
- Use local probes

Using trapped ions



STM

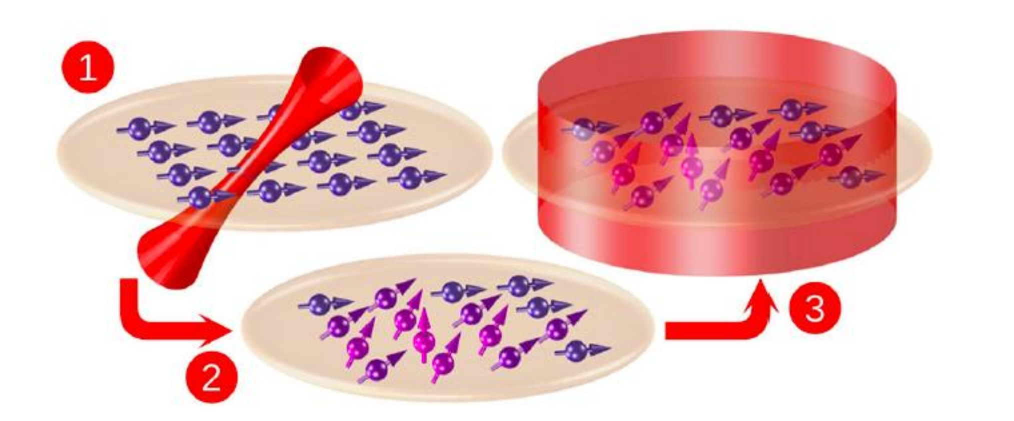
CAT

C. Kollath, M. Koehl, TG PRA 76 063602 (2007)
Physics Web
http://physicsweb.org/articles/news/11/4/13/1

Another route: local addressability

Many-Body Ramsey

M. Knapp, A. Kantian, TG, I. Bloch, M. Lukin, E. Demler PRL 111 147205 (2013)



$$\hat{H}_{\text{Heis}} = \sum_{i<j} J_{ij}^{\perp}(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) + J_{ij}^z \sigma_i^z \sigma_j^z.$$

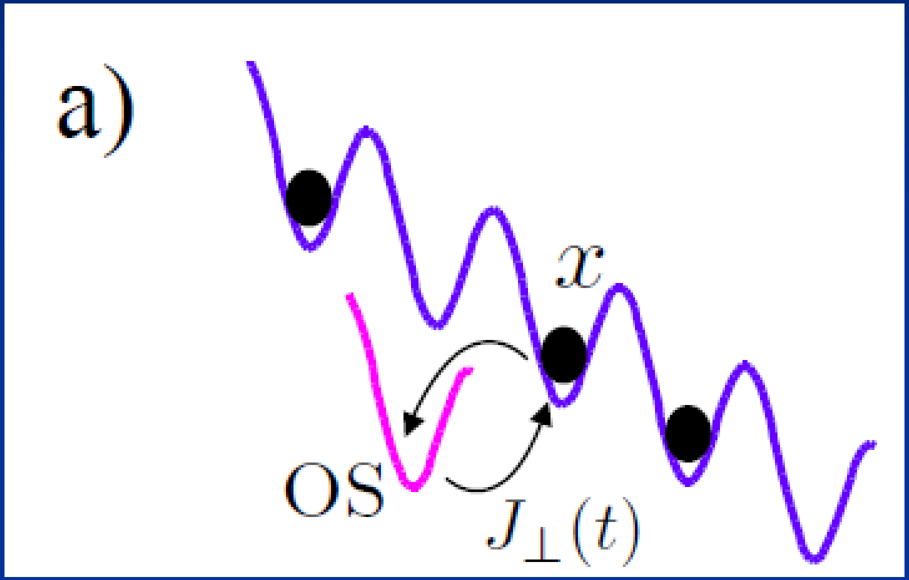
$$M_{ij}(\phi_1, \phi_2, t) = \sum_n \frac{e^{-\beta E_n}}{Z} \langle n | R_i^\dagger(\phi_1) e^{i\hat{H}t} R^\dagger(\phi_2) \sigma_j^z R(\phi_2) e^{-i\hat{H}t} R_i(\phi_1) | n \rangle.$$

$$M_{ij}(\phi_1, \phi_2, t) = -\frac{1}{4}\{\sin(\phi_1 - \phi_2)(G_{ij}^{xx} + G_{ij}^{yy}) - \cos(\phi_1 - \phi_2)(G_{ij}^{xy} - G_{ij}^{yx})\}.$$

Lattice-Assisted-Spectroscopy

A. Kantian, U. Schollwoeck, TG

Basic idea



$$J_\perp(t) = J_0 \sin(\omega t)$$

- Measure n(t)
- Start with either n=0 or n=1 on extra site

Linear response

$$\langle \hat{n}_{\rm OS}(t) \rangle_\omega = \langle \hat{n}_{\rm OS}(t) \rangle_{\cos\omega} + \langle \hat{n}_{\rm OS}(t) \rangle_{\sin\omega}$$

$$\langle \hat{n}_{\rm OS}(t) \rangle_\omega^{\rm init=0} = \frac{J_\perp^2}{\hbar^2} \int$$

$$\langle \hat{n}_{\rm OS}(t) \rangle_\omega^{\rm init=1} = 1 + \frac{J_\perp^2}{\hbar^2} \int d\omega' \tilde{\mathcal{K}}_t(\omega,\omega') \left[\, 2\mathcal{I}_x^{\rm em}(\omega' - \delta h(1,2)) - \mathcal{I}_x^{\rm ab}(\omega' - \delta h(1,0)) \,\right],$$

$$\tilde{\mathcal{K}}_t(\omega,\omega') = 2 \sum_{\sigma=\pm} \frac{(1 - \cos((\omega' + \sigma\omega)t))}{(\omega' + \sigma\omega)^2}$$

$$\mathcal{I}_x^{\rm em/ab}(\omega') = \sum_{n,m} |\langle n| \left[\hat{a}_x / \hat{a}_x^\dagger\right] |m\rangle|^2 \frac{e^{-\beta E_m}}{\mathcal{Z}} \delta(E_m - E_n - \omega').$$

Two regimes

- ``Short" times / ``Large" systems $1/t$ □ δE

  Linear growth in time

$$\langle \hat{n}_{\mathrm{OS}}(t) \rangle_\omega \to \frac{J_\perp^2}{\hbar^2} \left(\mathcal{I}_x(\omega) + \mathcal{I}_x(-\omega) \right) t$$

- ``long" times / ``small" systems $1/t$ □ δE

  quadratic growth in time/Rabi oscillations

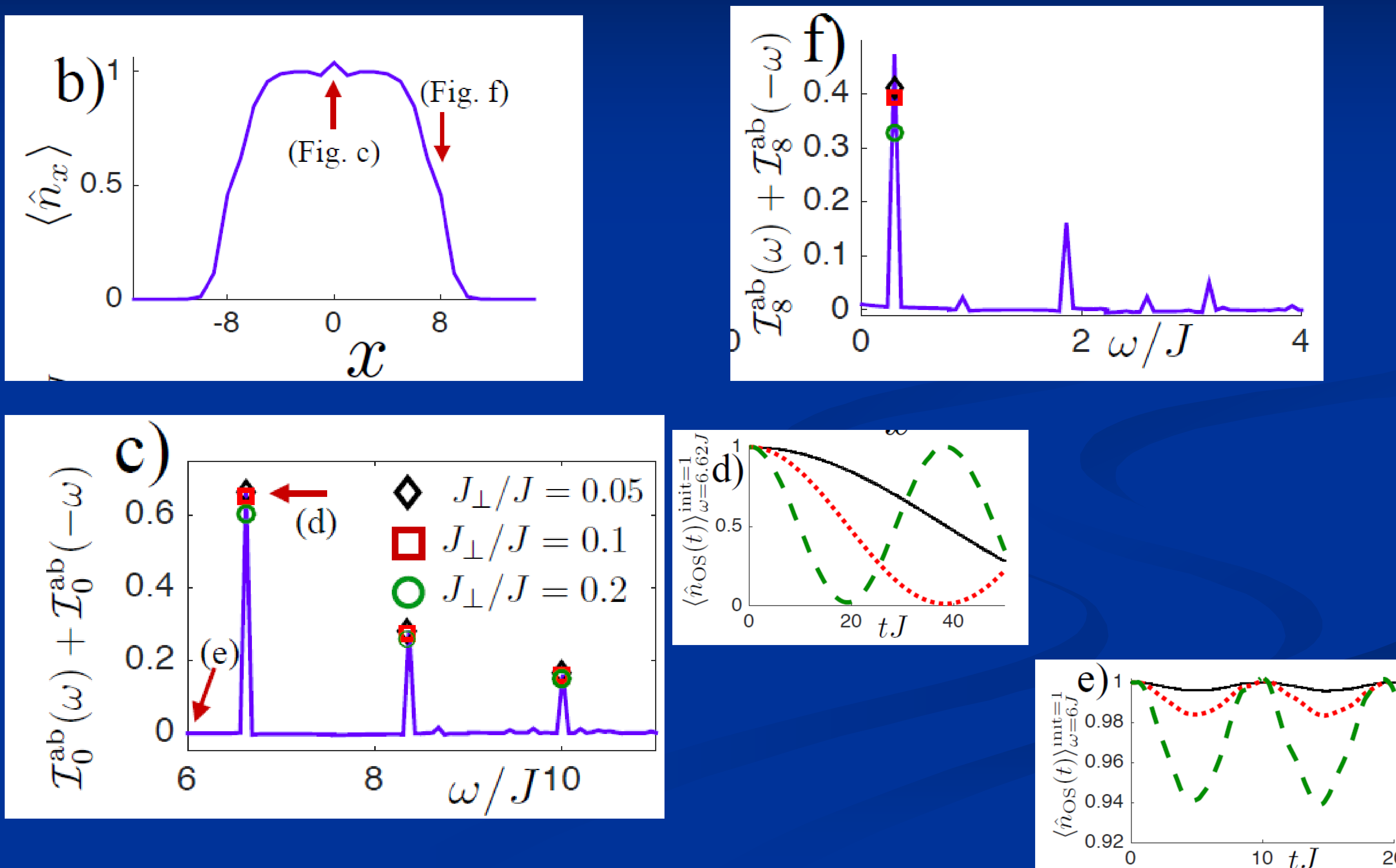
$$\frac{J_\perp^2}{\hbar^2} \left(\sum_{n,m:E_m - E_n = \pm\omega} |\langle n|\hat{O}|m\rangle|^2 \frac{e^{-\beta E_m}}{\mathcal{Z}} \right) t^2$$

Good in practice ?

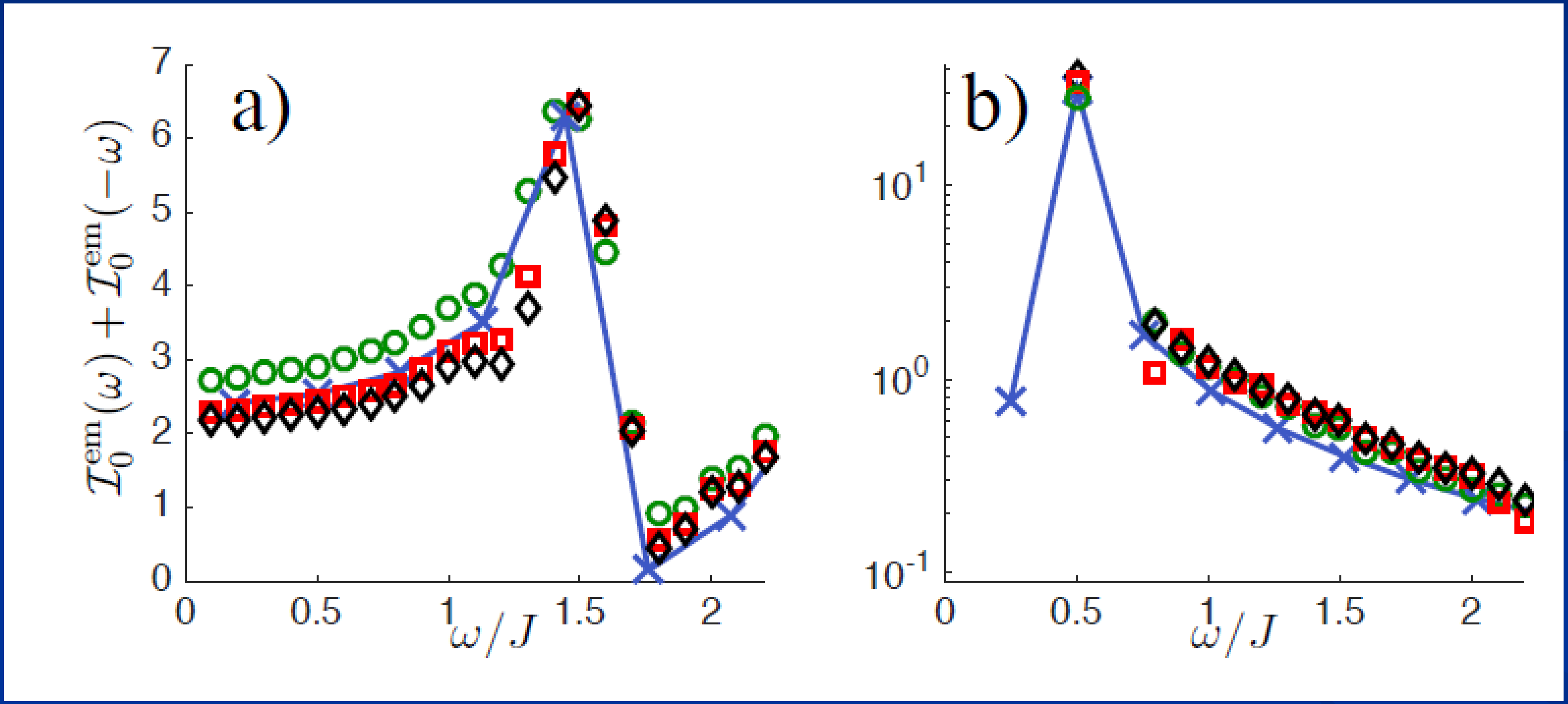
- Test with a full t-DMRG solution

$$\begin{aligned}\hat{H} &= -\sum_{x=1}^{L}(J+(-1)^x\delta J)(\hat{a}_x^\dagger \hat{a}_{x+1} + \text{h.c.}) \\ &\quad +\frac{U}{2}\sum_x \hat{n}_x(\hat{n}_x - 1) + V_\text{p} a^2 \sum_x (x-(L+1)/2)^2 \hat{n}_x \qquad (5)\end{aligned}$$

Bosonic atoms in a parabolic trap



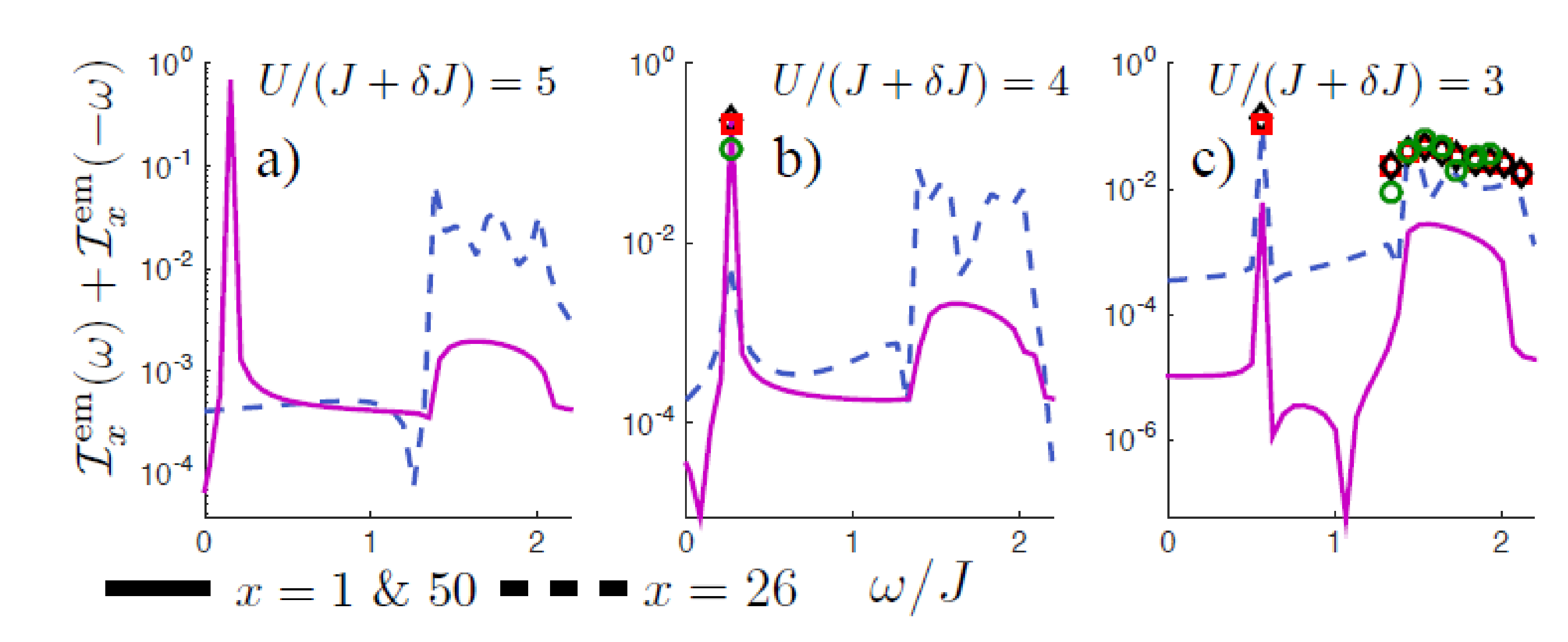
Large systems



Jperp/J = 0.01 (black), 0.1 (red), 0.2 (red)

U=10J (a) U=2J (b)

Edge states in SSH model



Conclusions

- Possibility to measure local Green's function using local density measurements
- Excellent possibilities with realistic sizes and times
  (comparison with t-DMRG calculations)
- Various possible extensions (non-local correlations, etc.)
- Other methods and correlations