

The topological Haldane model

Tilman Esslinger ETH Zürich

- Haldane model
- Update on anti-ferromagnetic correlations
- Quantized conductance

Funding: ETH, EU (ERCadv SQMS, SIQS, TherMiQ), NCCR QSIT, SNF

www.quantumoptics.ethz.ch

Metals

Insulators

Superconductivity

Quantum Hall Effect

e^-

e^-

Quantum Magnetism

e^-

Topological Insulators

e^-

e^-

Quantum Phase Transitions

e^-

Dirac Fermions

e^-

e^-

Graphene

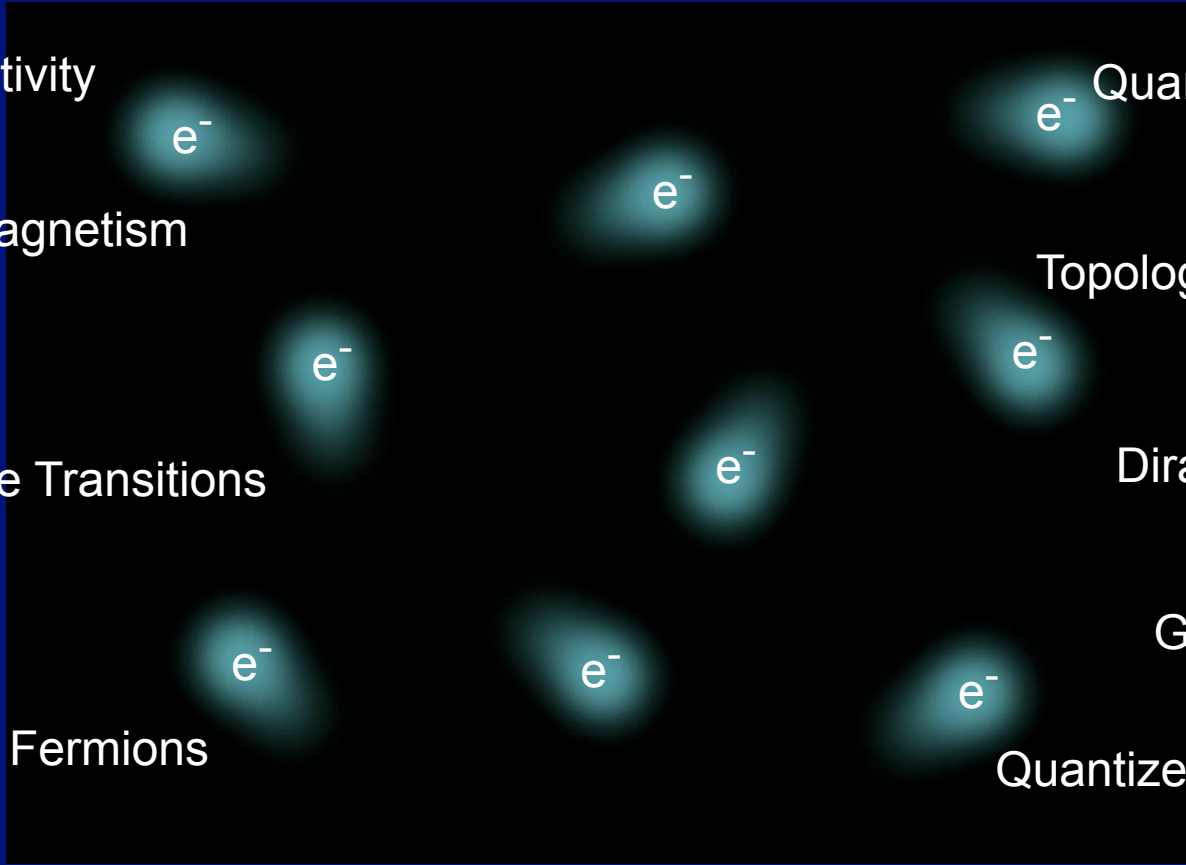
e^-

Majorana Fermions

Quantized Conductance

High T_c

Photovoltaic Quantum Hall effect



Why cold atoms?

- Different approach
- New regimes
- Surprises

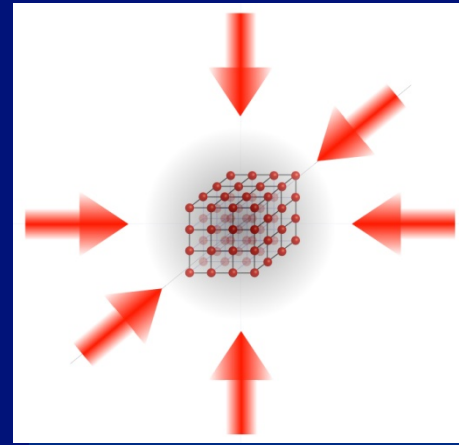
What can we do?

$$H = T + U + V_{\text{trap}}$$

Building the Hamiltonian



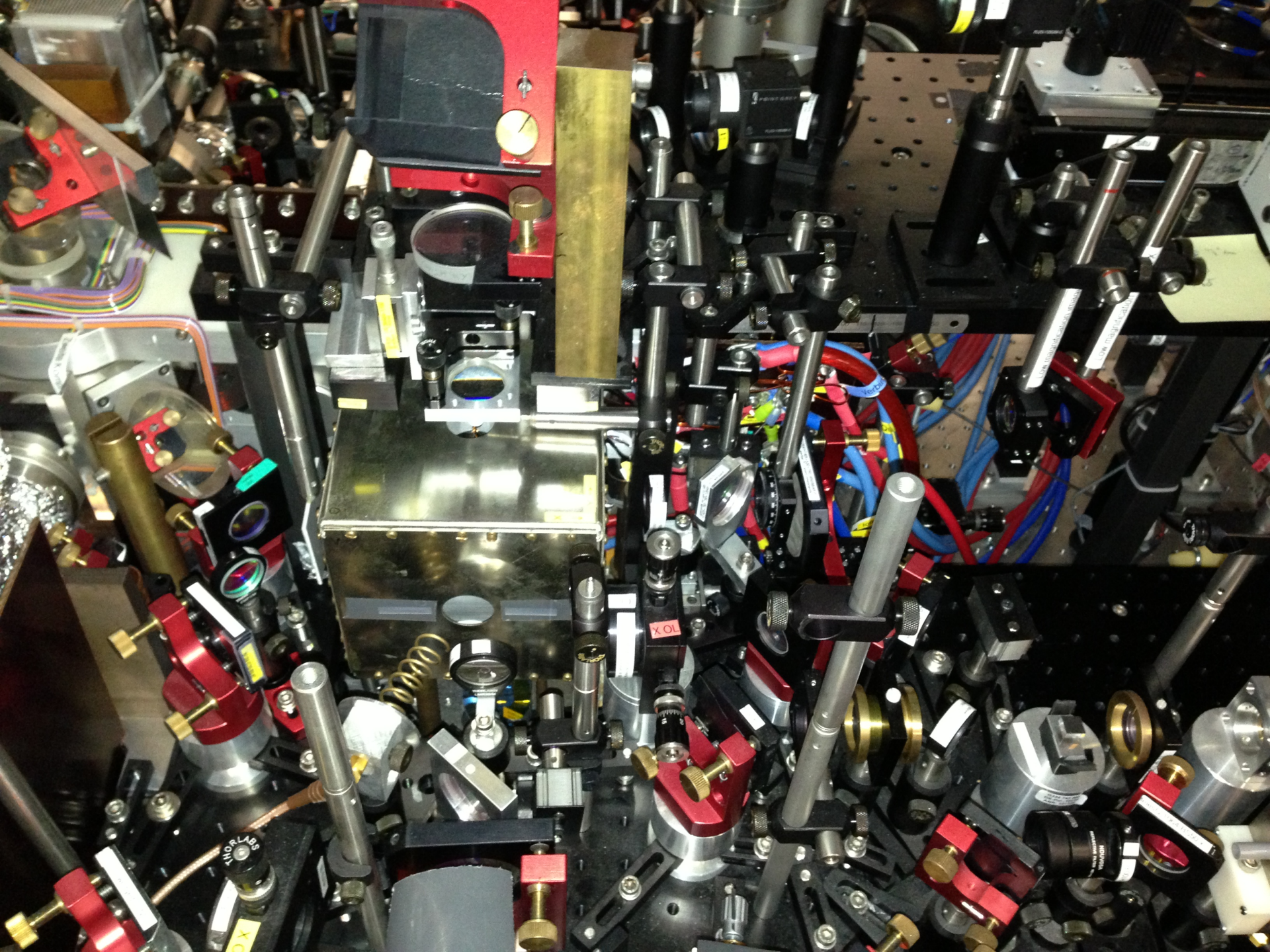
+



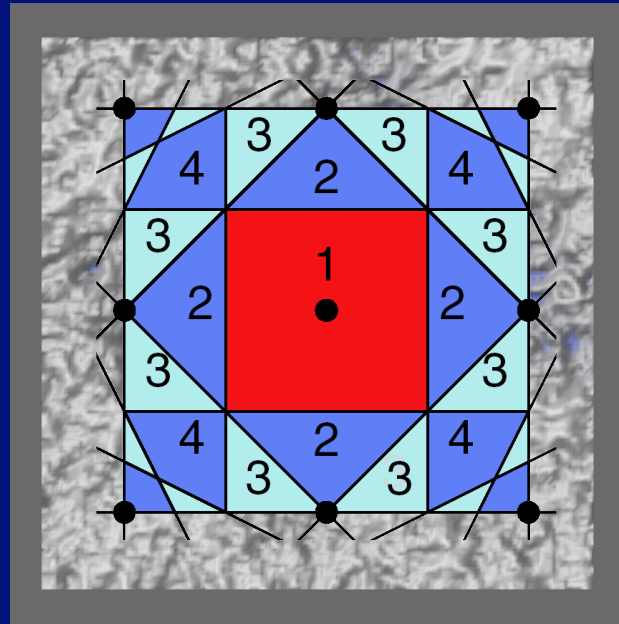
Quantum Gases (^4He)

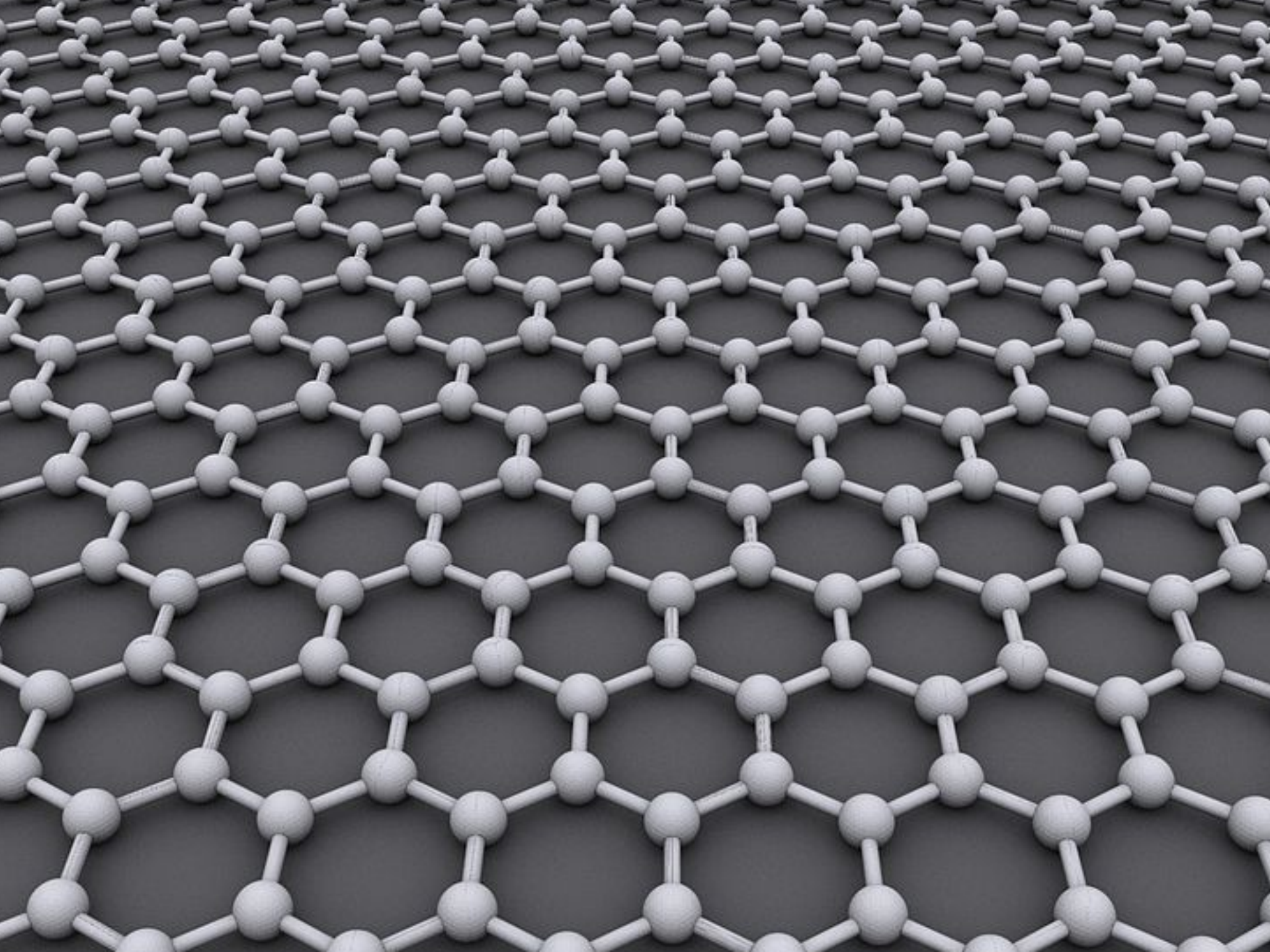
Optical Lattices

See also: Mainz/Munich, Hamburg, MIT, Illinois, Rice,...



Measuring

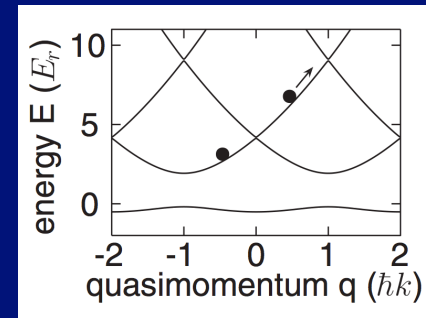




Bose gases in lattices with topological defects

BEC in Excited bands:

- 1D « Dirac point » (Weitz group, Bonn)
S. Kling *et al.*, Phys. Rev. Lett. 105, 215301 (2010)
T. Salger *et al.*, Phys. Rev. Lett. 107, 240401 (2011)
- Quadratic avoided band crossing (Hemmerich group, Hamburg)
M. Ölschläger *et al.*, Phys. Rev. Lett. 108, 075302 (2012)

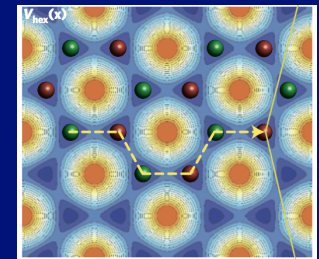


BEC in a Honeycomb lattice:

(Sengstock group, Hamburg)

P. Soltan-Panahi *et al.*, Nature Phys. 7, 434 (2011)

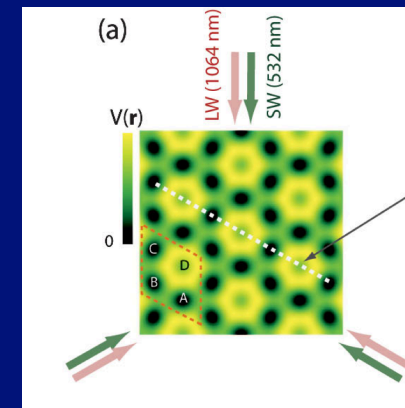
P. Soltan-Panahi *et al.*, Nature Phys. 8, 71 (2012)

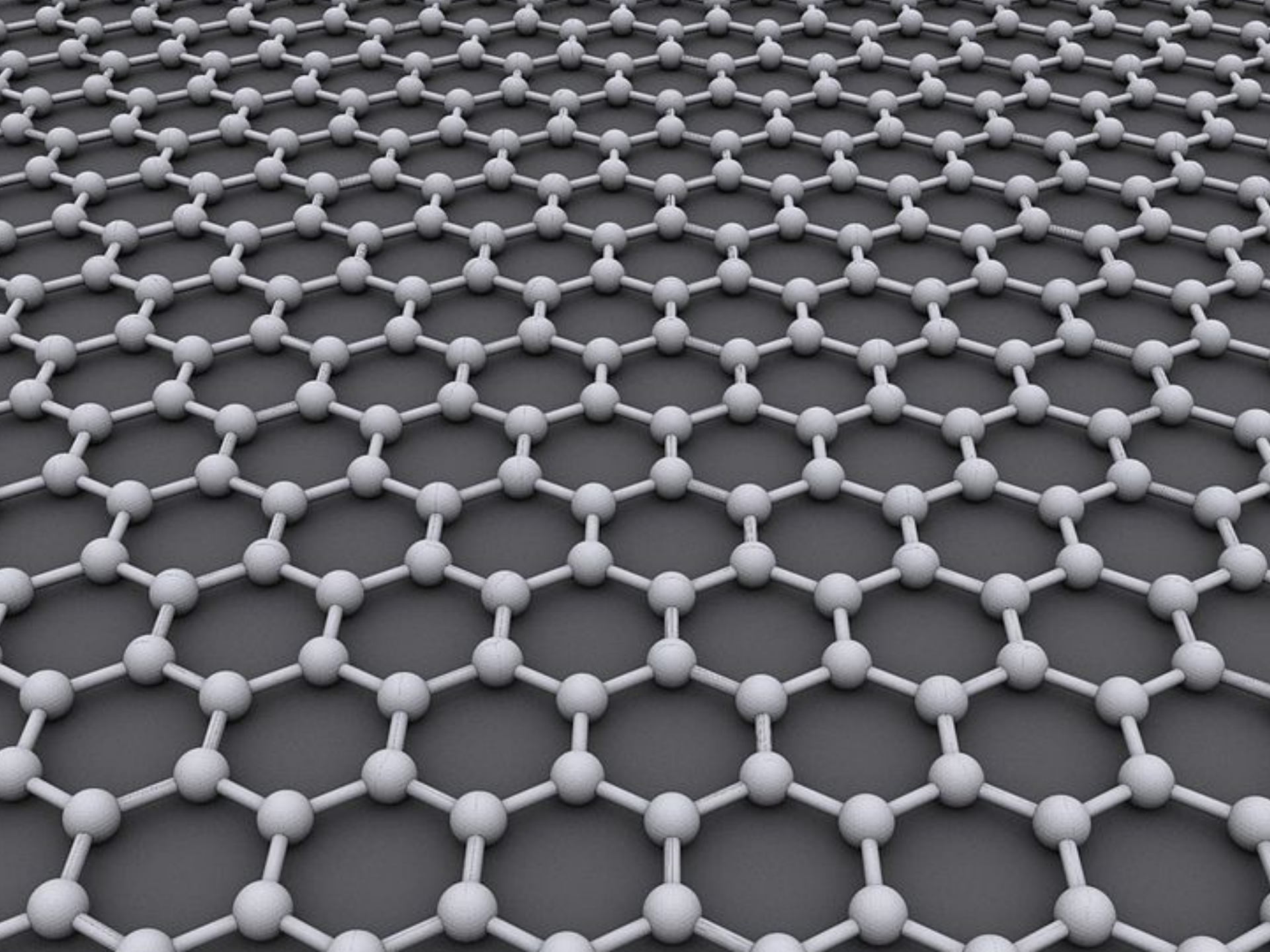


BEC in Kagome:

(Dan Stamper-Kurn, Berkeley)

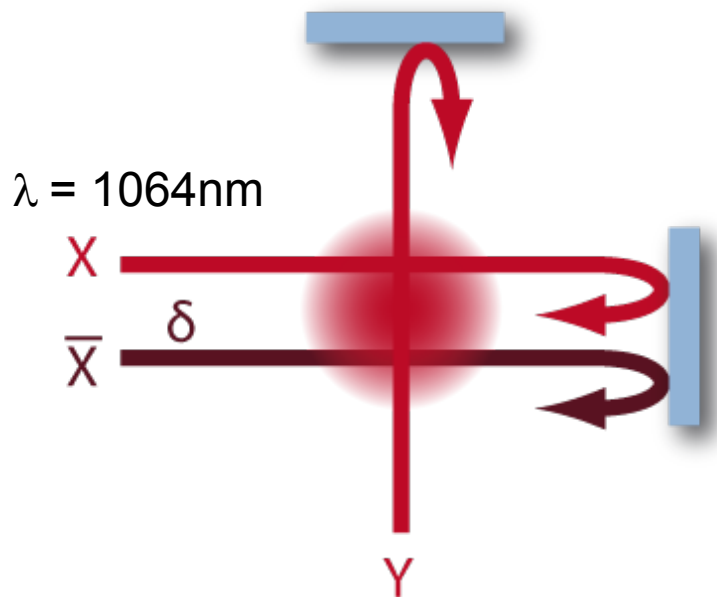
G.B. Jo *et al.* Phys. Rev. Lett. 108, 045305 (2012)



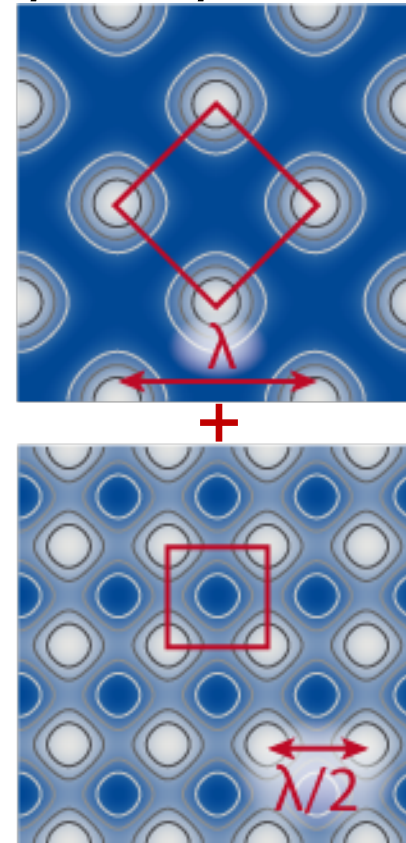


Tunable Geometry Optical Lattice

Setup



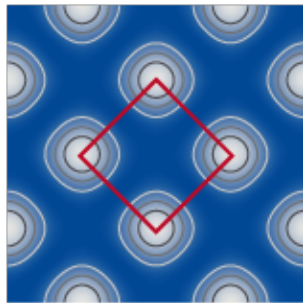
Optical potential



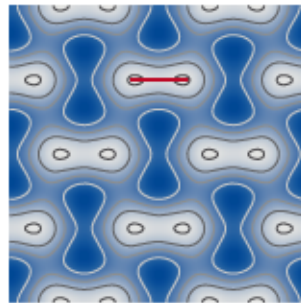
Other complex lattices: NIST, Munich, Hamburg, Berkeley

Tunable Geometry Optical Lattice

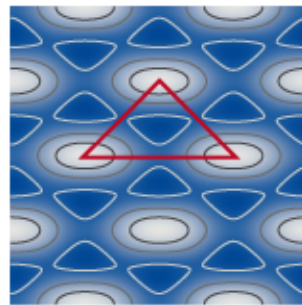
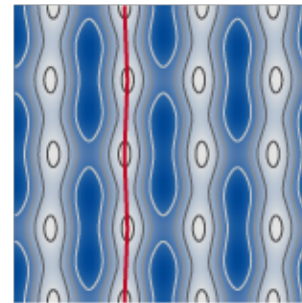
Chequerboard



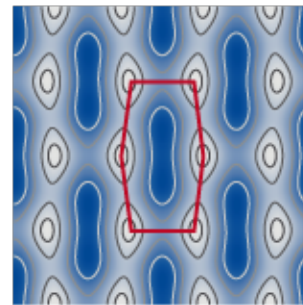
Dimer



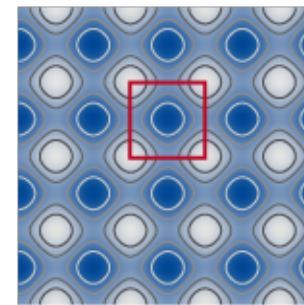
1D chains



Triangular

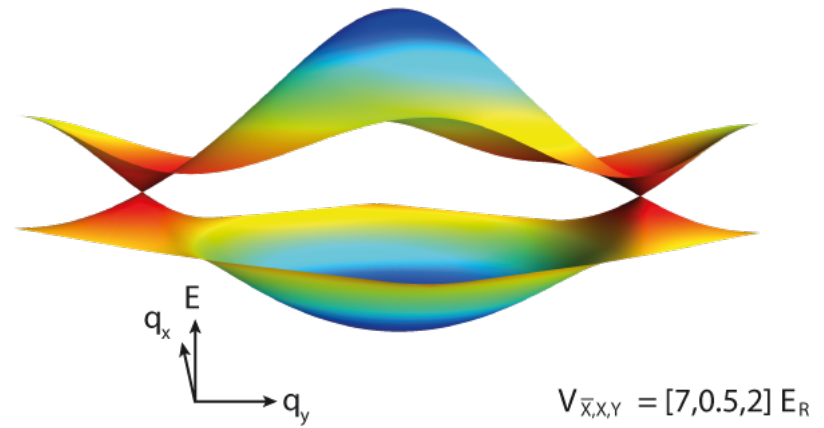
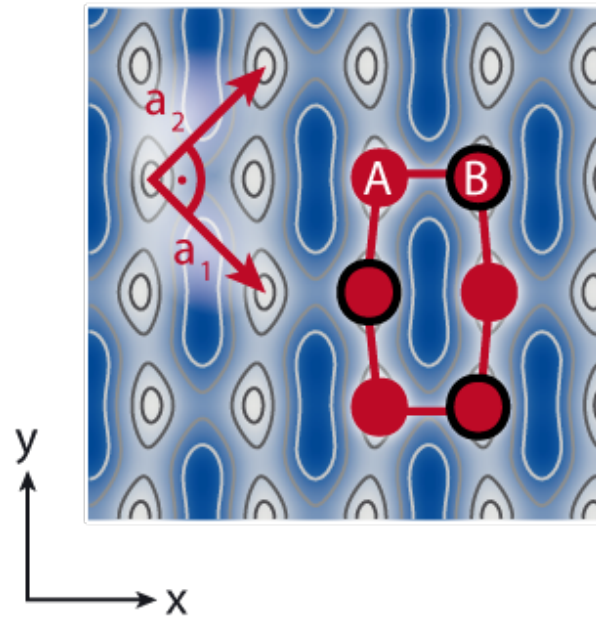


Honeycomb



Square

Honeycomb Lattice



Thanks to Dario Poletti

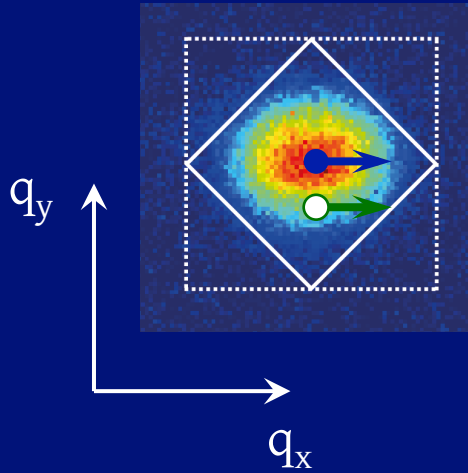
Probing the Dirac points

vanishing density of states

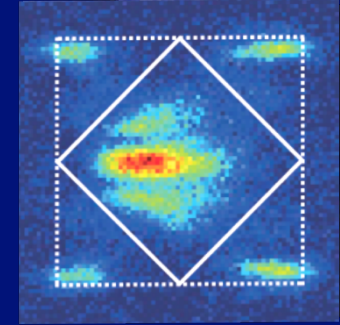
small energy scales

Bloch oscillation and interband transitions

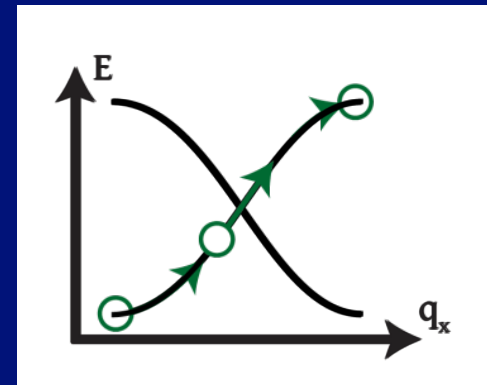
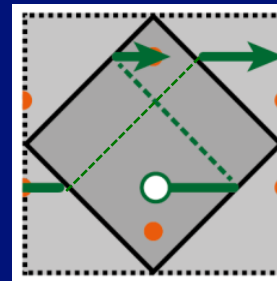
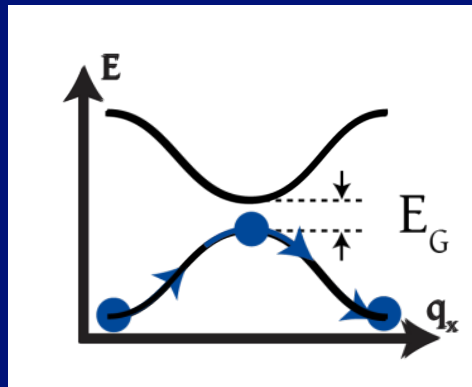
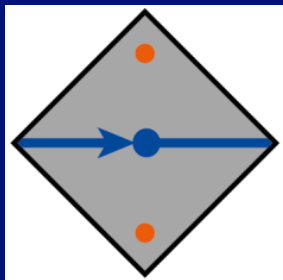
Starting point



+ magnetic gradient

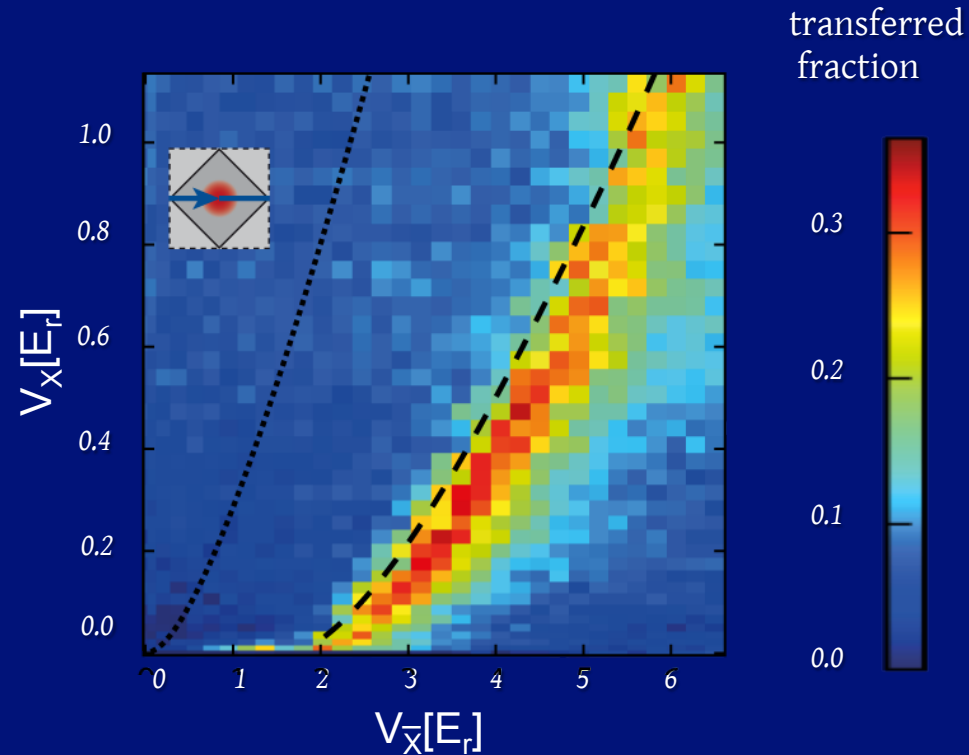
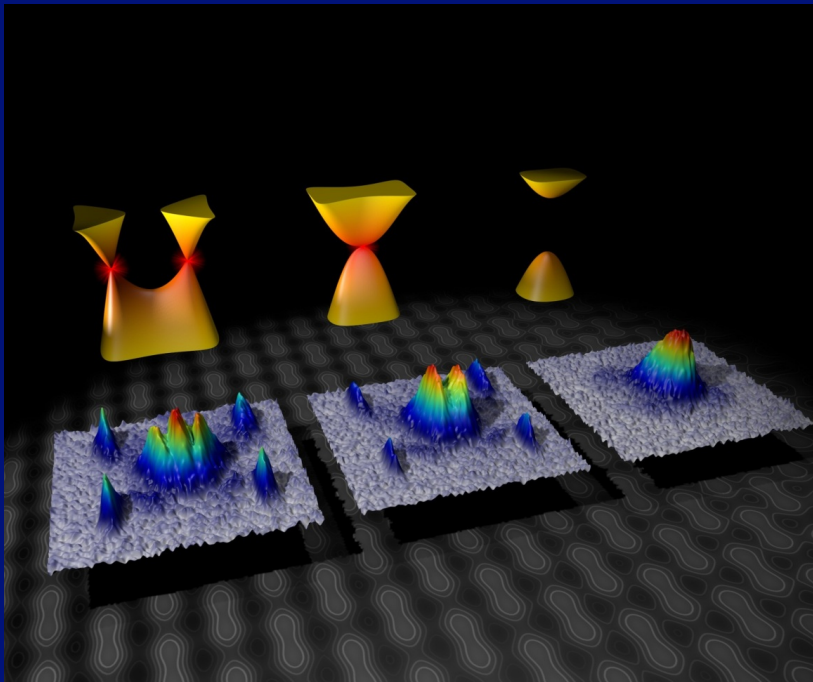


Transfer to 2nd band



Method in 1D: T. Salger et. al, Phys. Rev. Lett. 99, 190405 (2007)

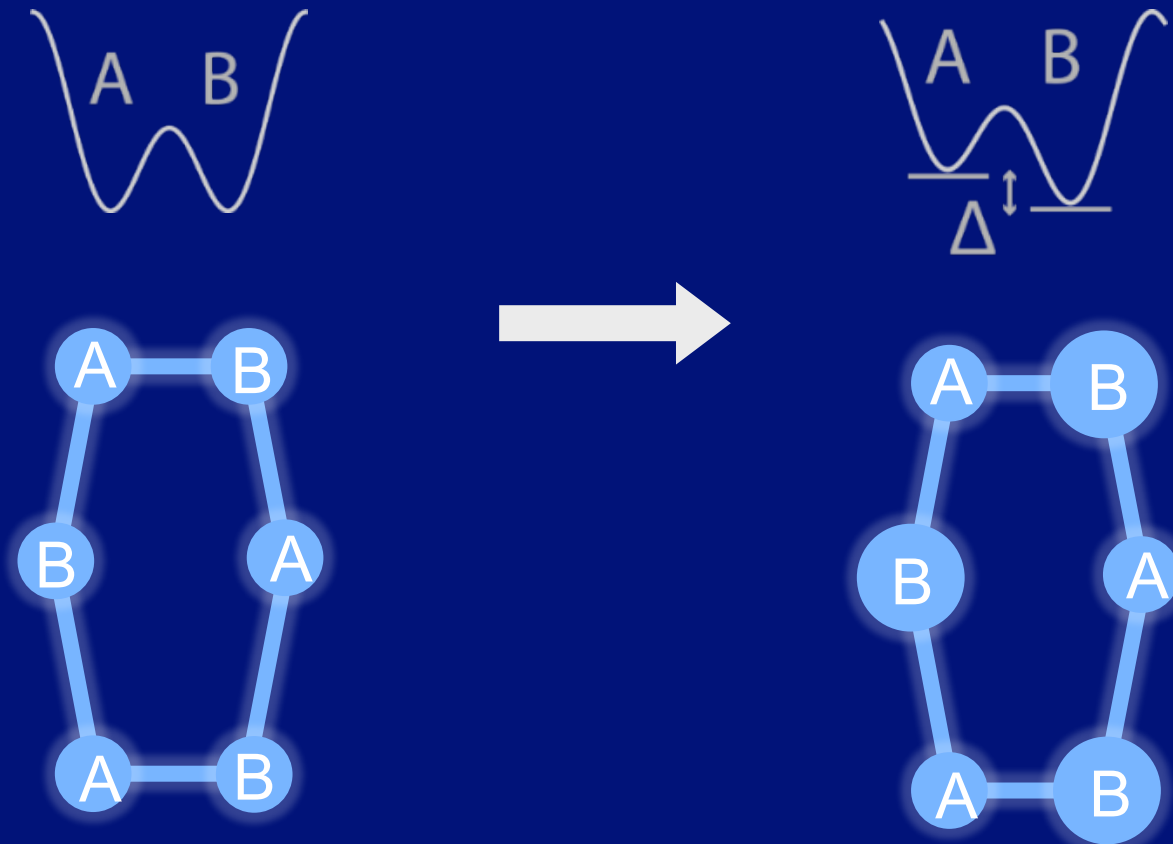
Merging and annihilating Dirac points



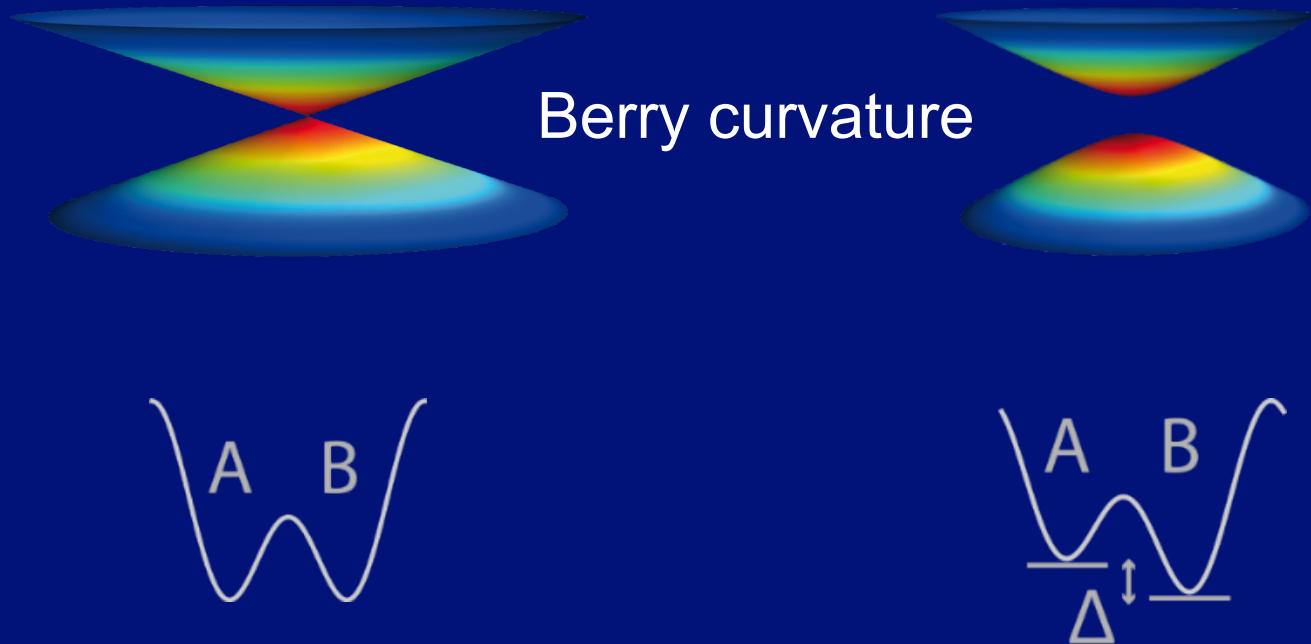
L. Tarruell, D. Greif, T. Uehlinger, G. Jotzu, and T. Esslinger, *Nature* 483, 302–305 (2012).
K. K. Gomes, W. Mar, W. Ko, F. Guinea & H. C. Manoharan, *Nature* 483, 306–310 (2012).
See also: L.-K. Lim, J.-N. Fuchs, G. Montambaux, *PRL* 108, 175303 (2012)



Breaking Inversion Symmetry

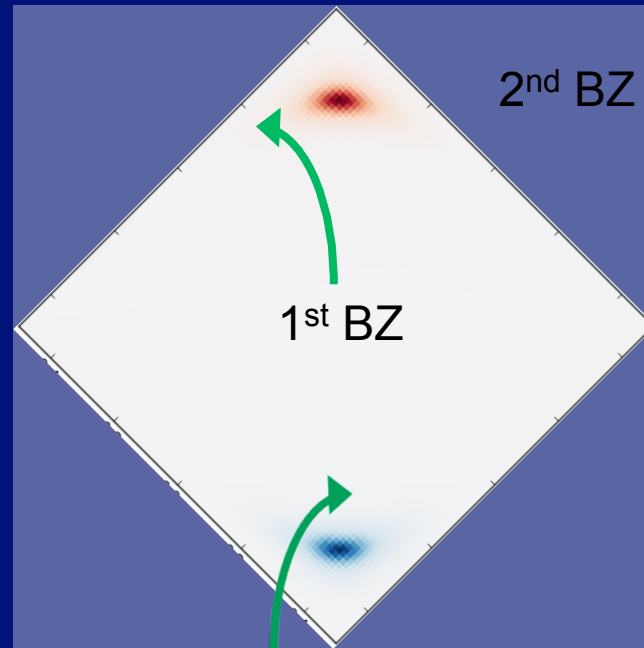


Breaking Inversion Symmetry



See also: L. Duca, Science 347, 288 (2015)

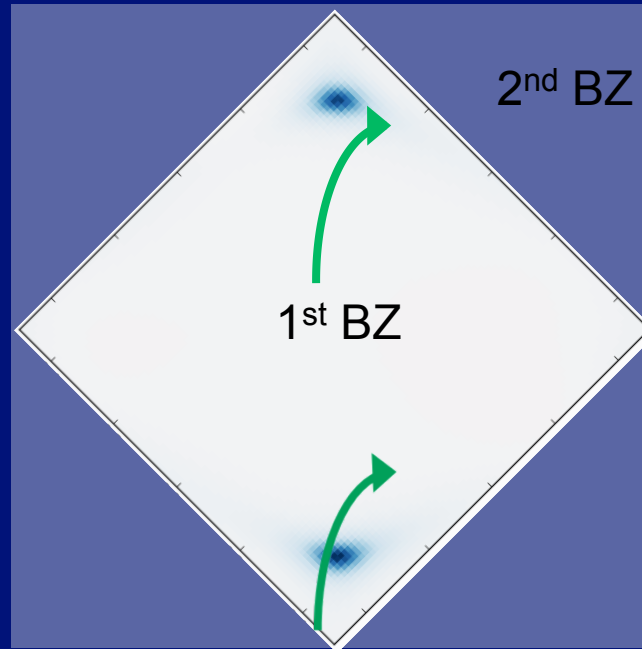
Berry Curvature and Transverse Drift



$$\dot{\mathbf{r}} = \frac{1}{\hbar} \partial_{\mathbf{k}} \epsilon(\mathbf{k}) - \dot{\mathbf{k}} \times \boldsymbol{\Omega}(\mathbf{k})$$
$$\hbar \dot{\mathbf{k}} = \mathbf{F}(\mathbf{r})$$

Chang and Niu, PRL 75, 1348 (1995)
Price and Cooper, PRA 85, 033620 (2012)

Berry Curvature and Transverse Drift



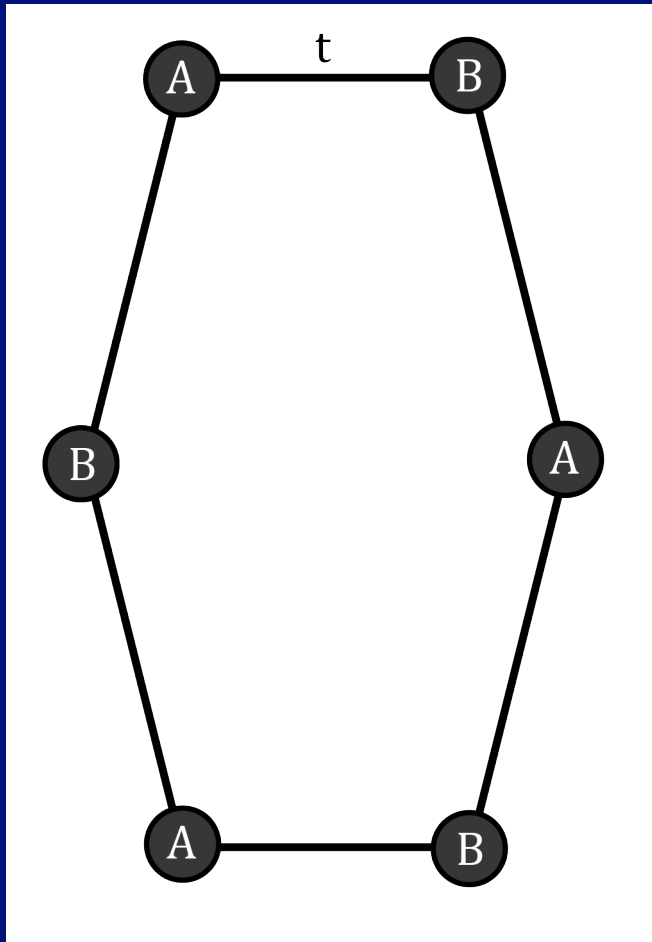
Like a Hall current

Topological Haldane model

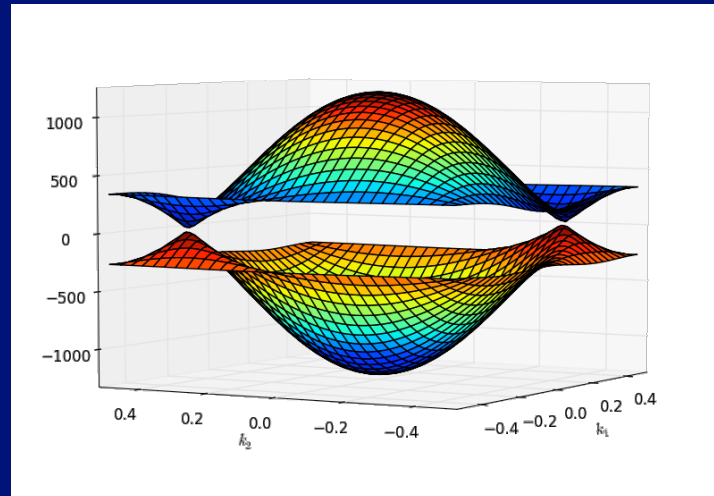
Proposal for Quantum Hall Effect *without* magnetic field!

Haldane, PRL **61**,2015-2018 (1988)

Topological Haldane model

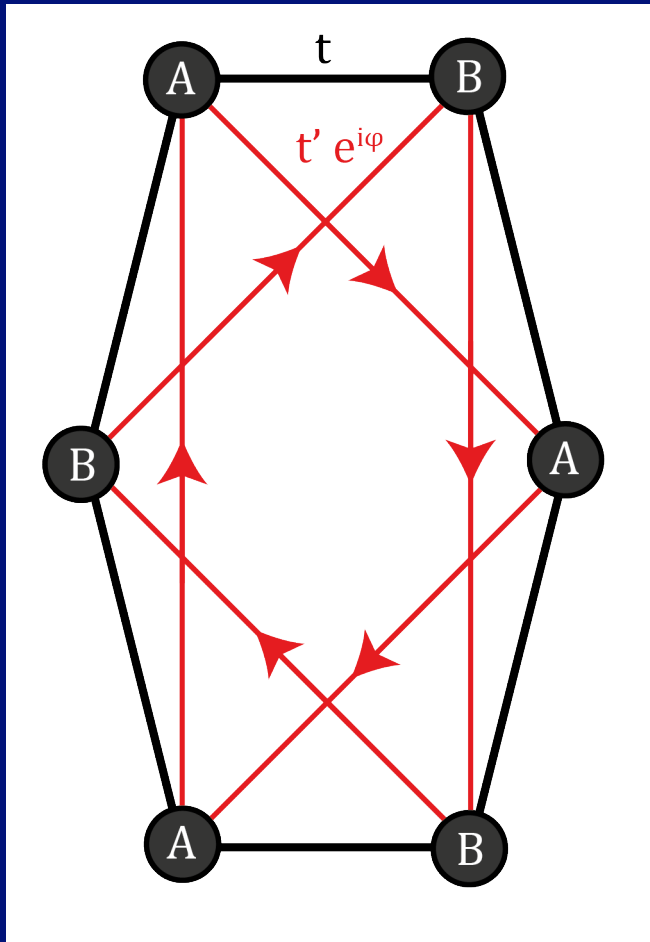


Start from a honeycomb lattice

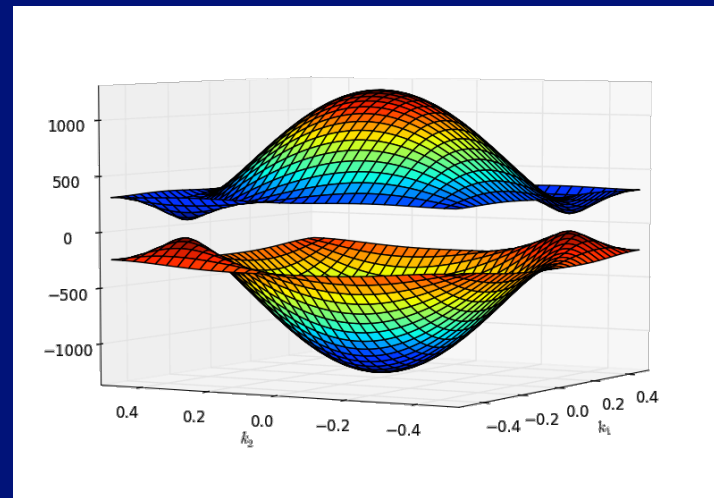


inversion and time-reversal symmetry

Topological Haldane model

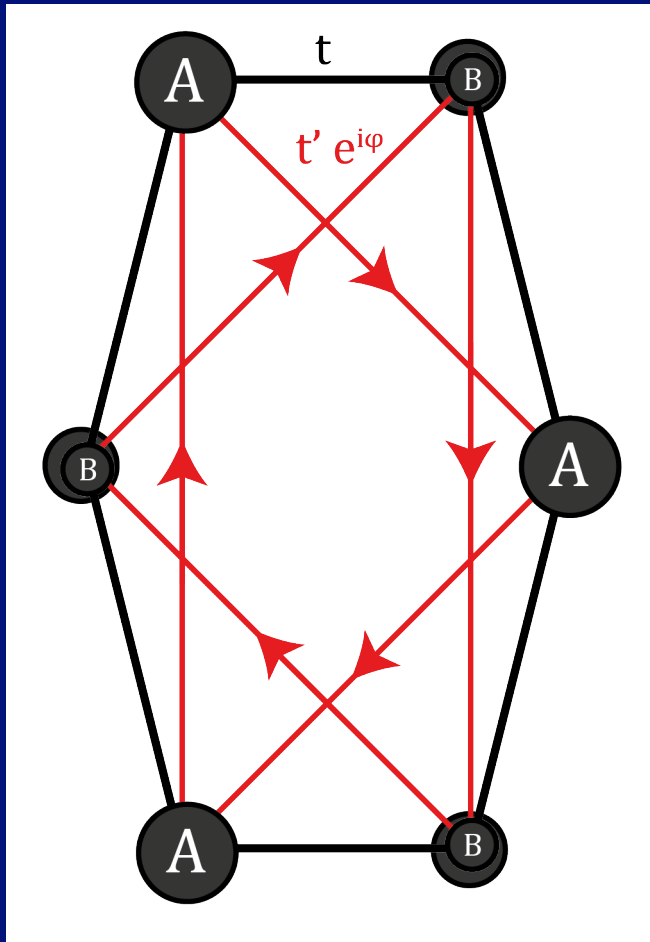


break time-reversal symmetry with complex next-nearest neighbour tunnellings

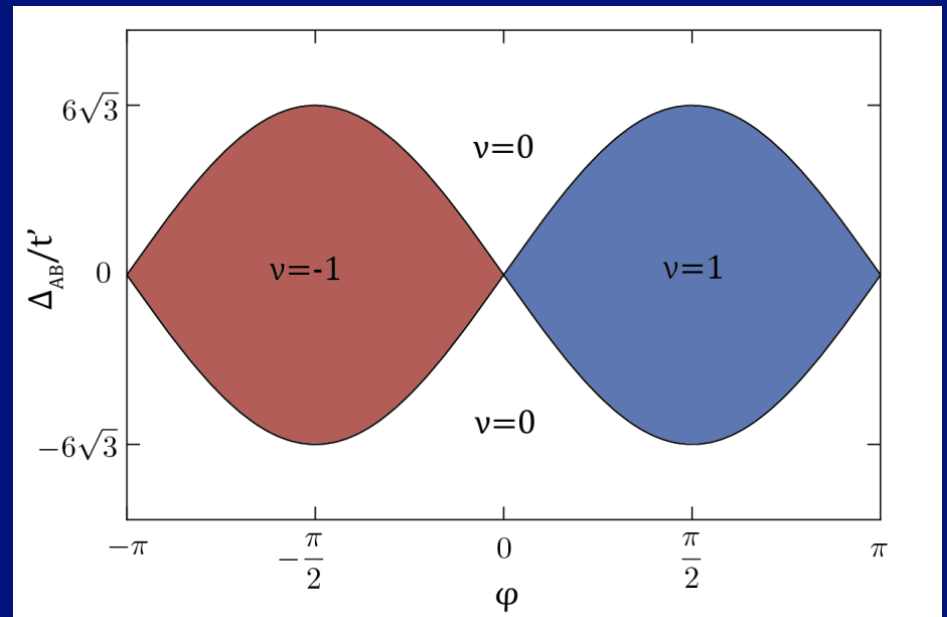


→ Topological Chern insulator, with non-zero Hall conductance

Topological Haldane model

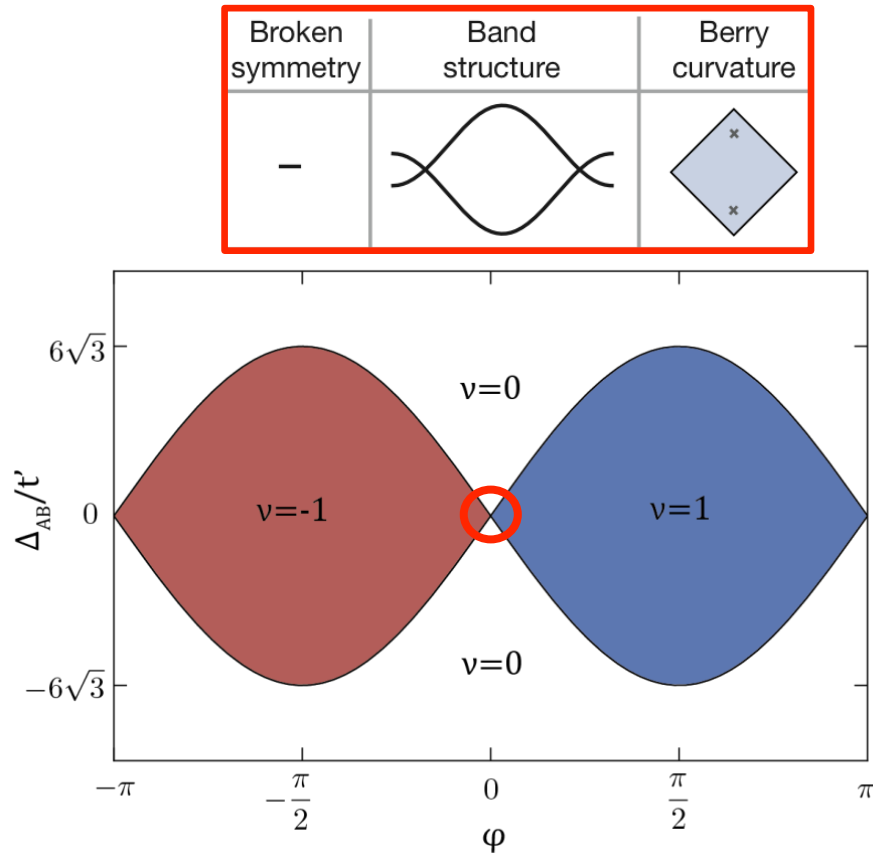


Additionally break time-reversal symmetry with energy offset



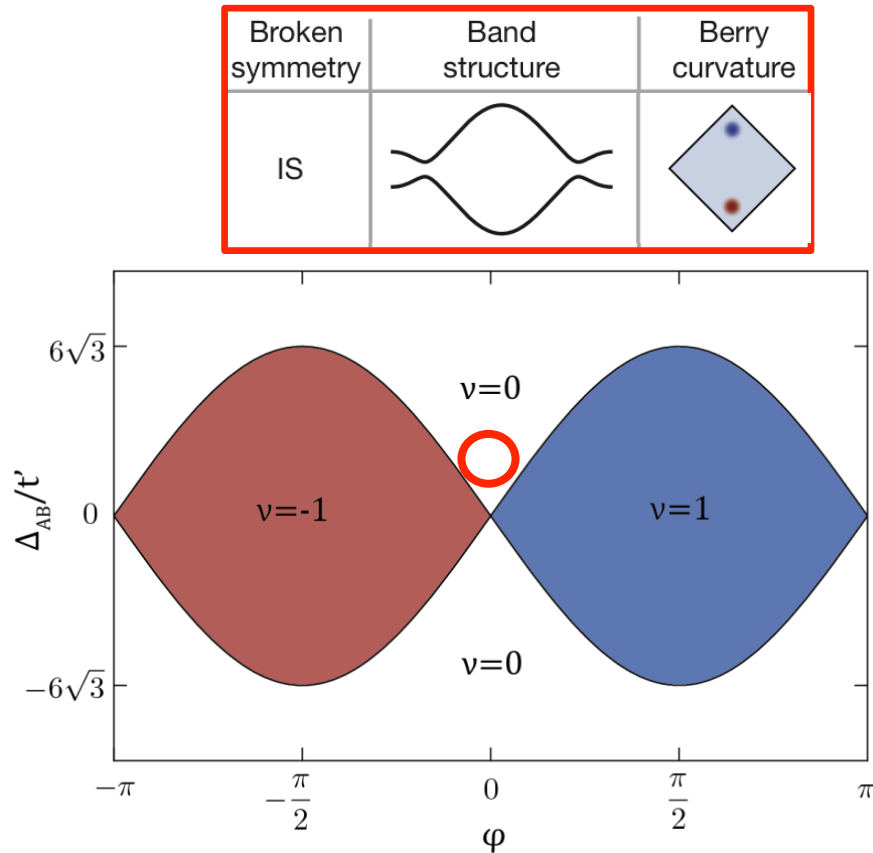
→ Distinct topological phases

Topological Haldane model



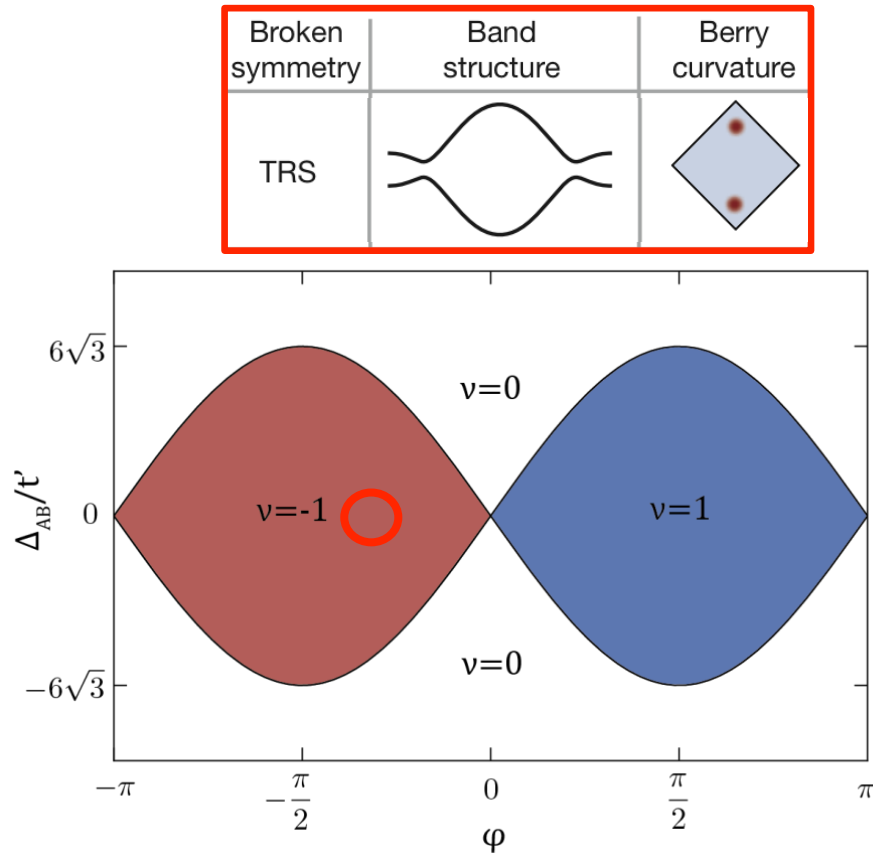
Haldane, PRL **61**,2015-2018 (1988)

Topological Haldane model



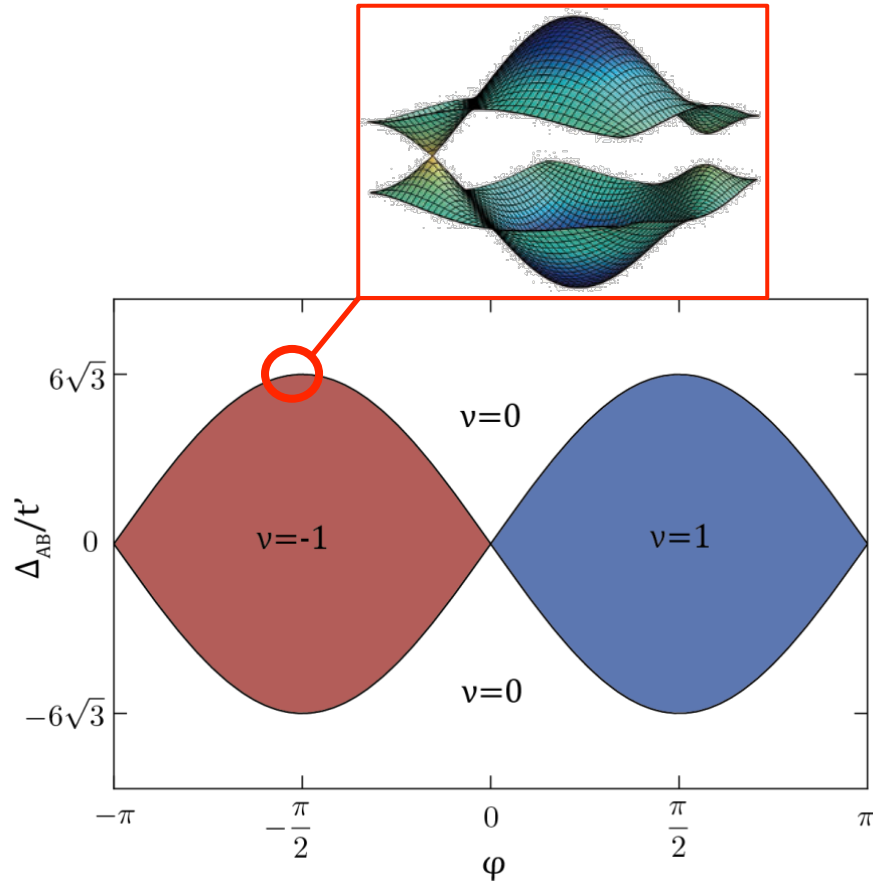
Haldane, PRL **61**,2015-2018 (1988)

Topological Haldane model



Haldane, PRL **61**,2015-2018 (1988)

Topological Haldane model



Haldane, PRL **61**,2015-2018 (1988)

How?

geometrical constant of order unity, and g is the Landé g factor for the electrons.

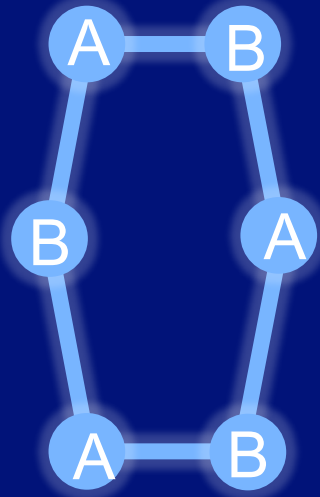
While the particular model presented here is unlikely to be directly physically realizable, it indicates that, at least in principle, the QHE can be placed in the wider context of phenomena associated with broken time-reversal invariance, and does not necessarily require external magnetic fields, but could occur as a consequence of magnetic ordering in a quasi-two-dimensional system.

Breaking time-reversal symmetry

Proposal for Photovoltaic Hall effect in graphene

T. Oka und H. Aoki, PRL **79**, 081406 (2009)

Breaking time-reversal symmetry



Other proposals to realize topological Hamiltonians:

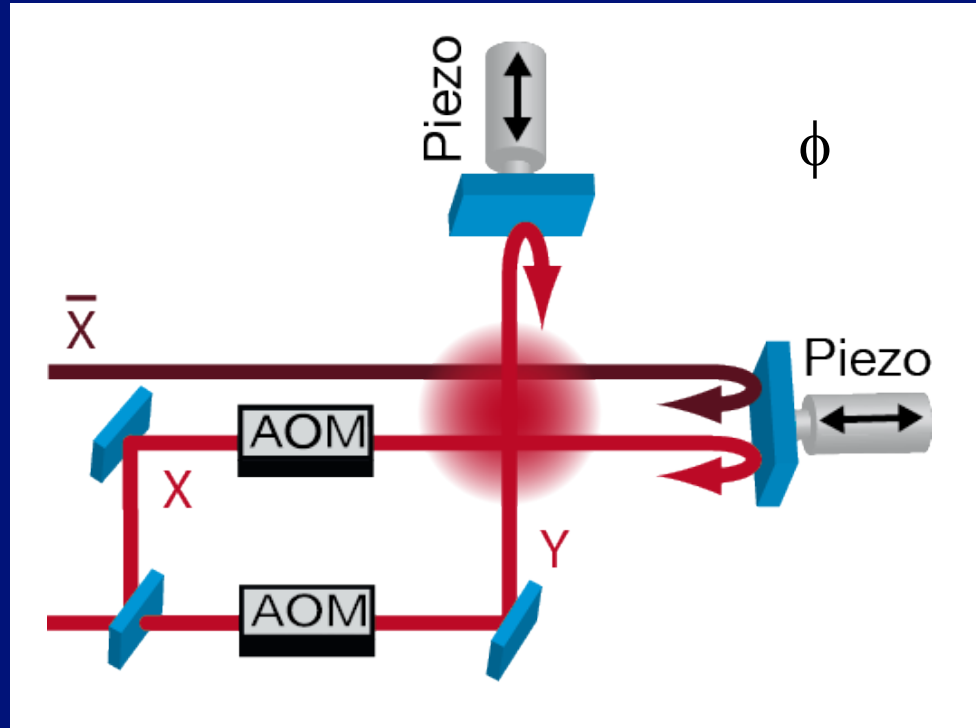
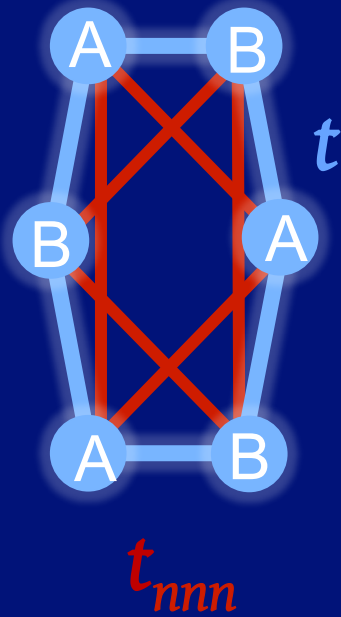
T. Kitagawa et al., Phys. Rev. B 82, 235114 (2010)

P. Hauke et al., Phys. Rev. Lett 109, 145301 (2012)

Realisation in photonic system: Rechtsman et. al Nature 496, 196–200 (2013)

Breaking time-reversal symmetry

Lattice Shaking



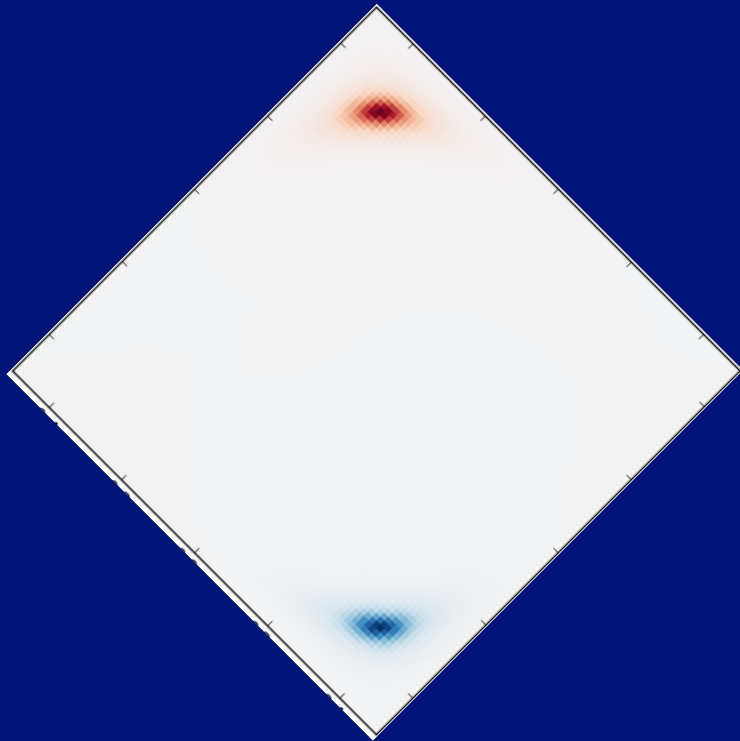
Lattice shaking: Pisa — Lignier, PRL **99**, 220403 (2007)

Hamburg/Barcelona — Struck, Science **333**, 996-9 (2011), PRL 108, 225304 (2012)

Chicago — Parker, Nat. Phys. **9**, 769-774 (2013)

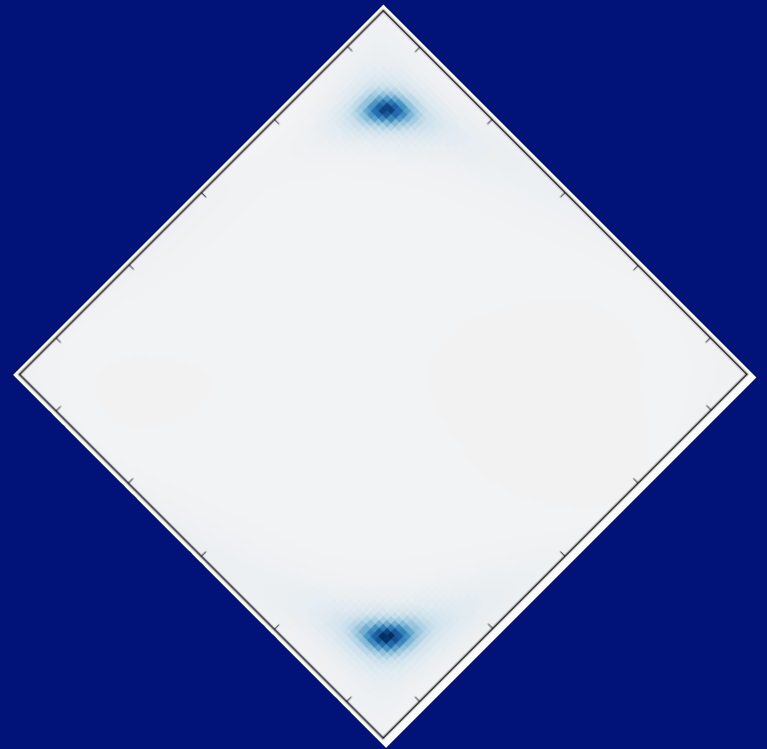
Berry Curvature

Trivial band insulator



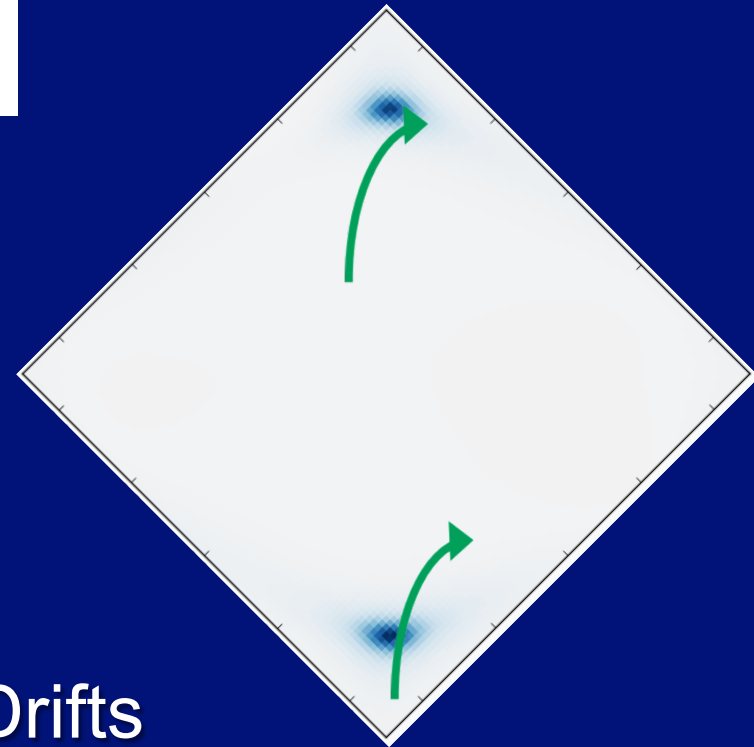
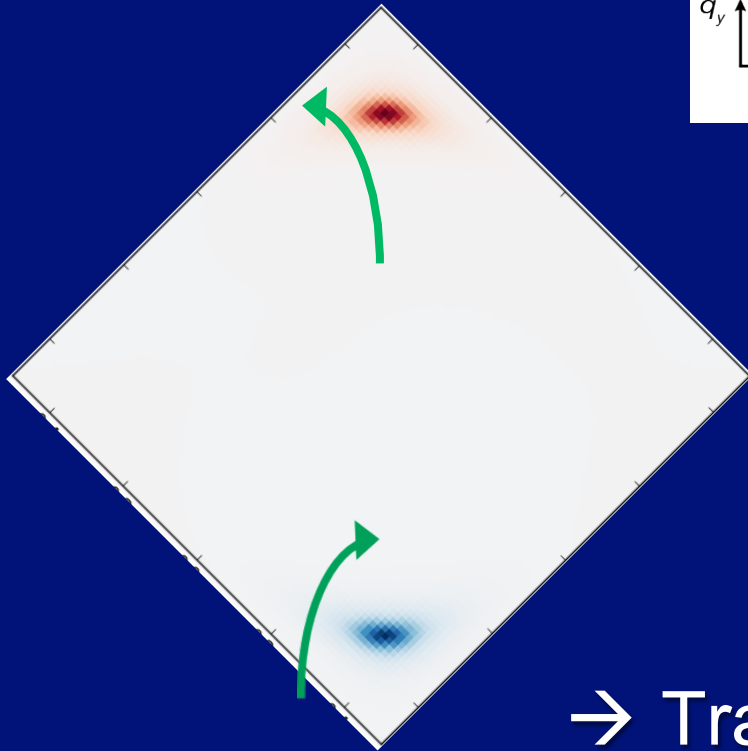
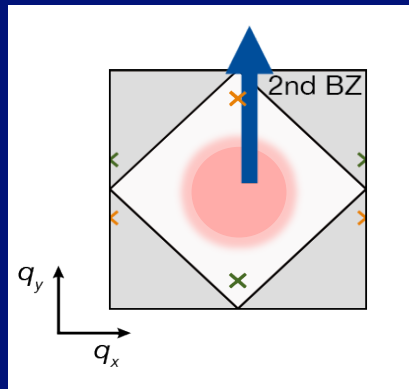
Chern number 0

Chern insulator



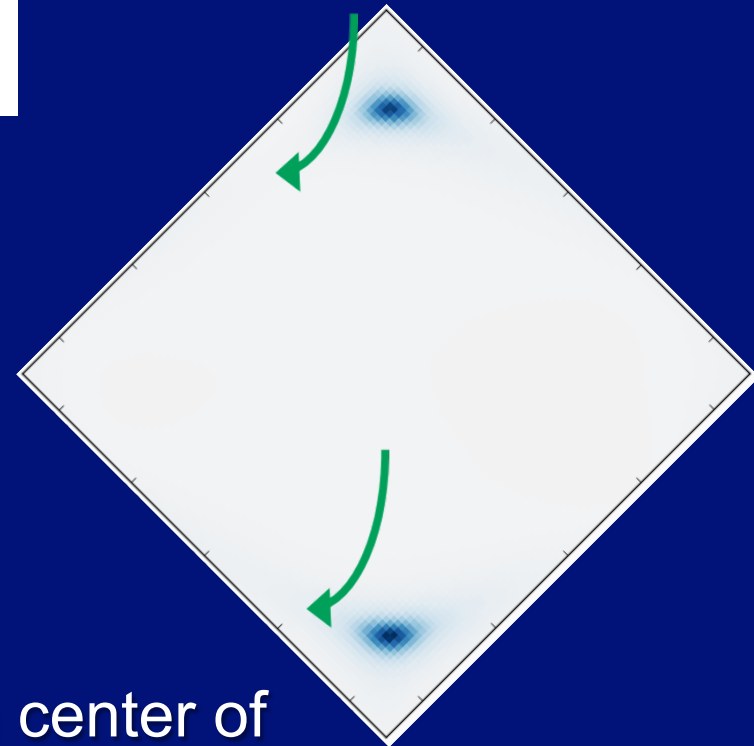
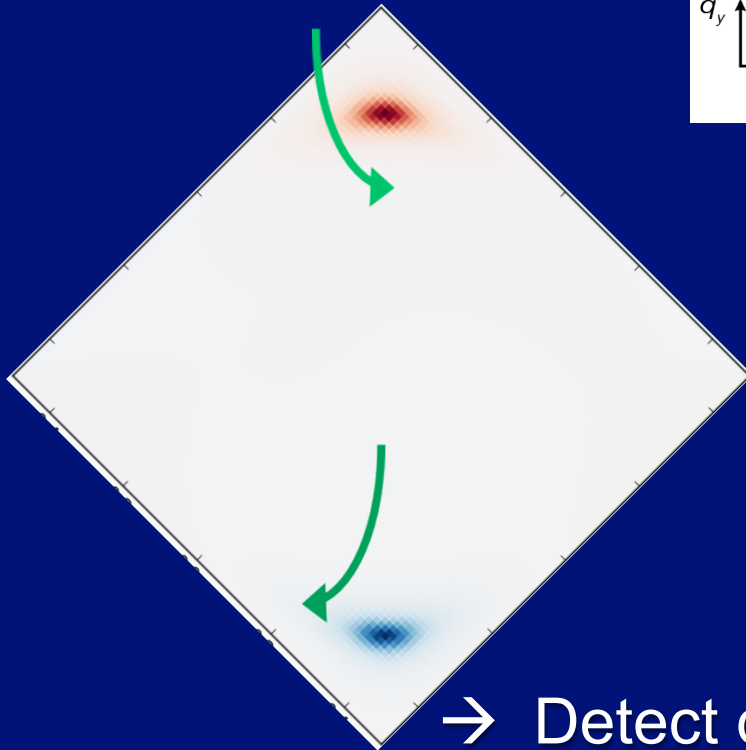
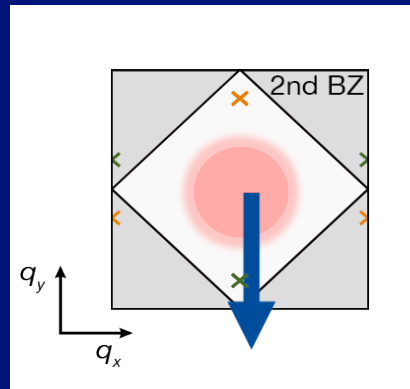
Chern number -1

Berry Curvature - Measurement



→ Transverse Drifts

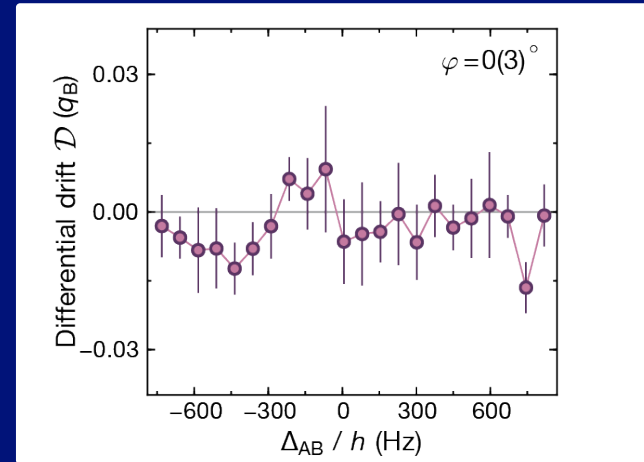
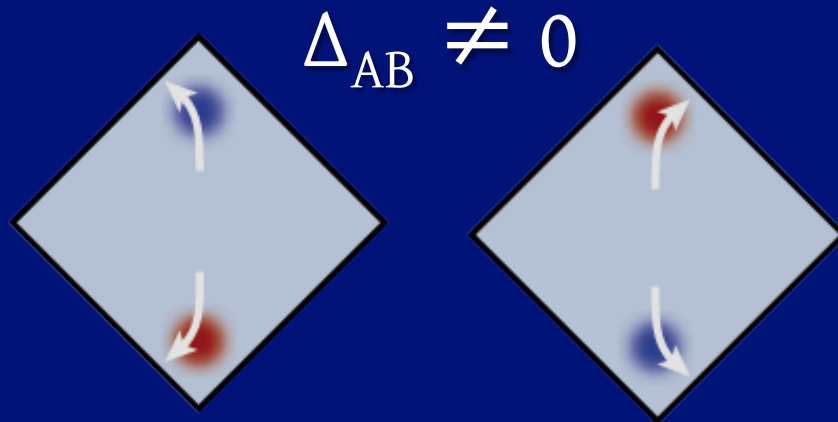
Berry Curvature - Measurement



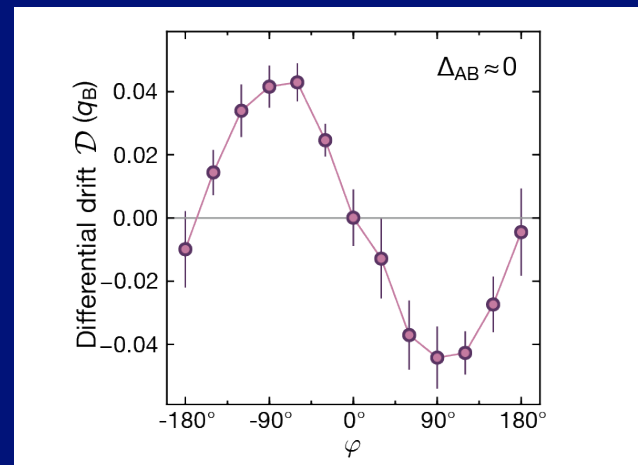
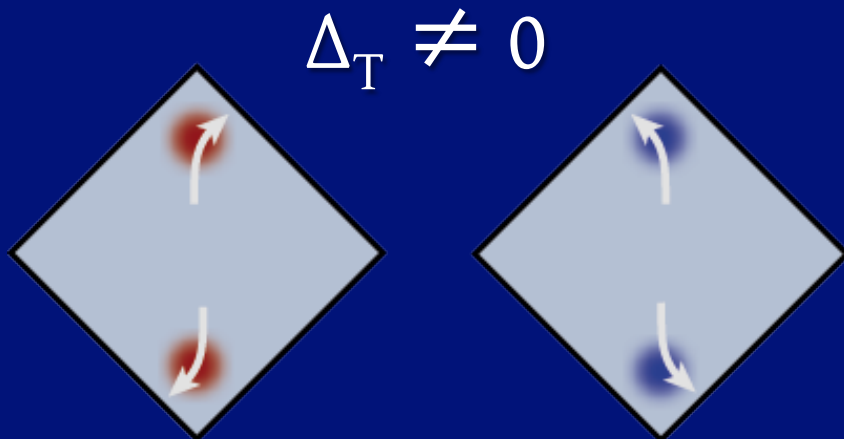
→ Detect difference in center of mass position after full Bloch cycle

Topological features of the system

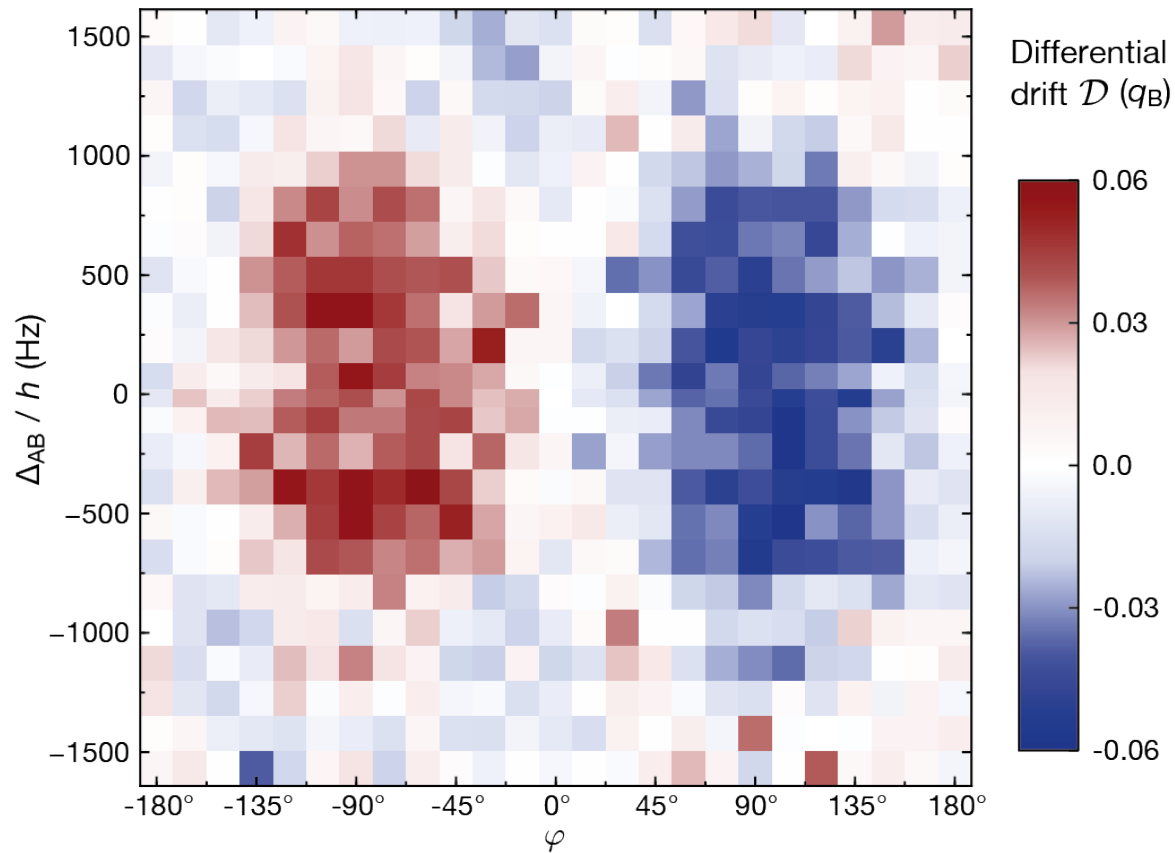
topologically trivial



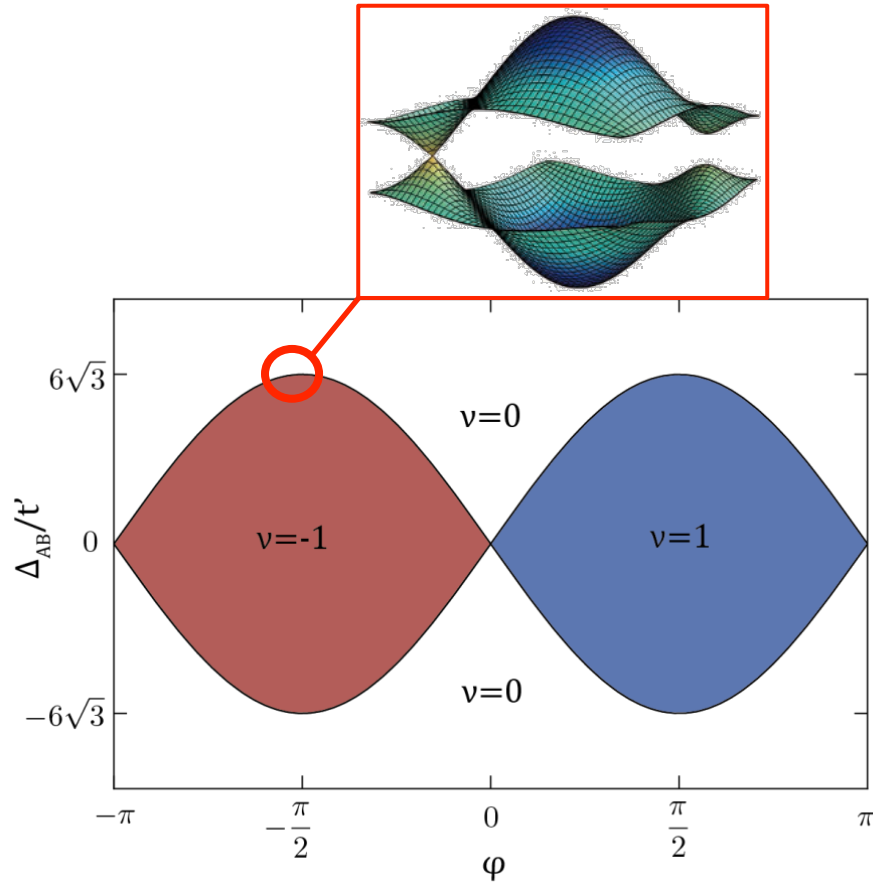
nonzero Chern number



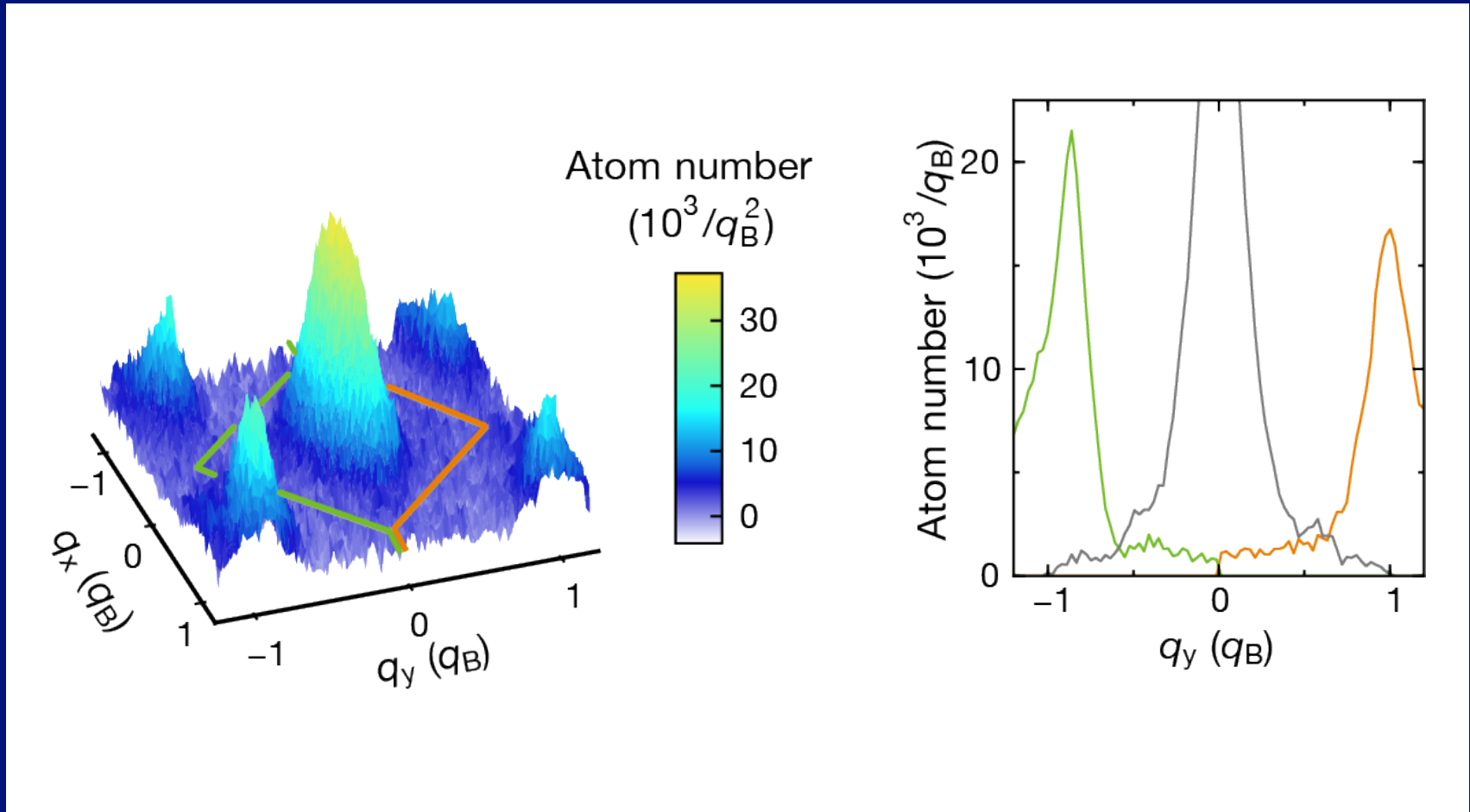
Observing Transverse Drifts



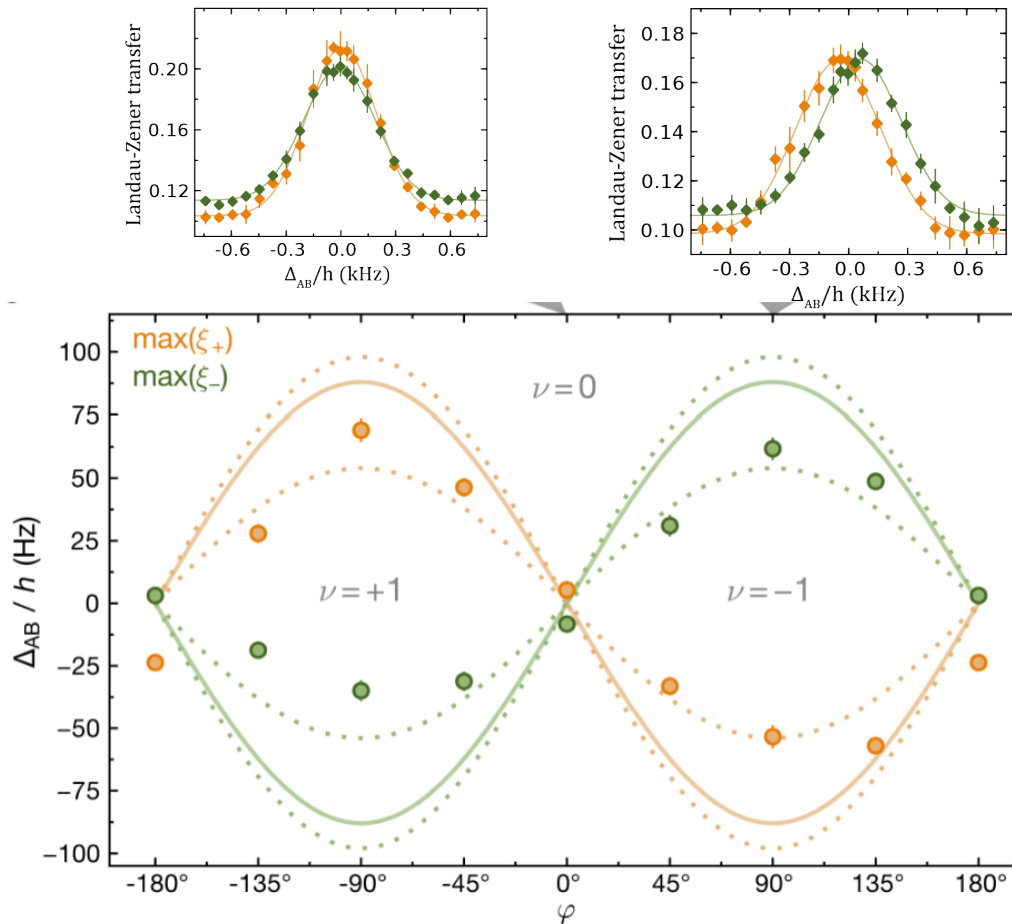
Mapping out the transition line



Mapping out the transition line



Mapping out the transition line



What about interactions?

Little is known

What about interactions?

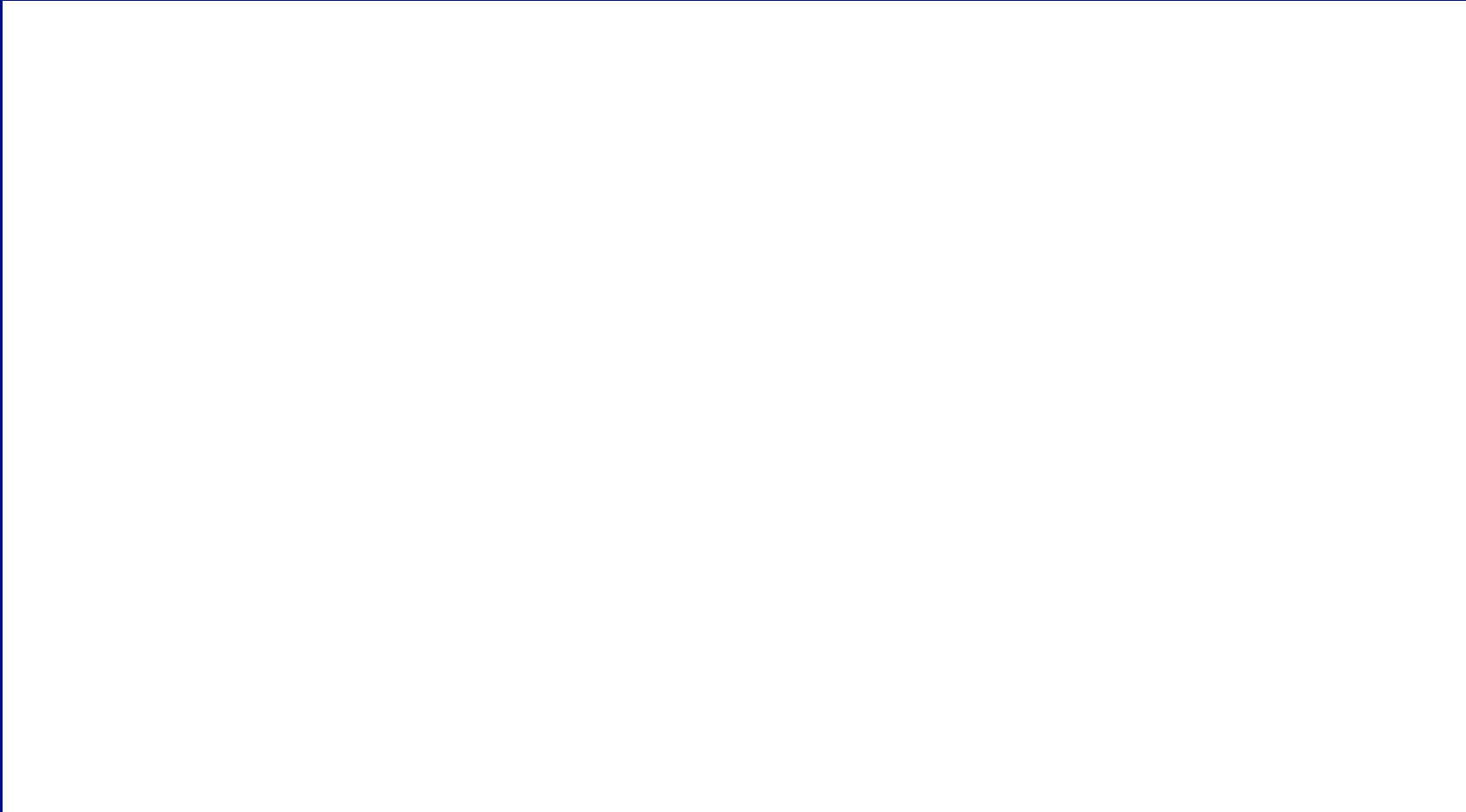
Loaded interacting gas into
coupled layers of Haldane models

→ Observed no significant heating

What about spin dependence?

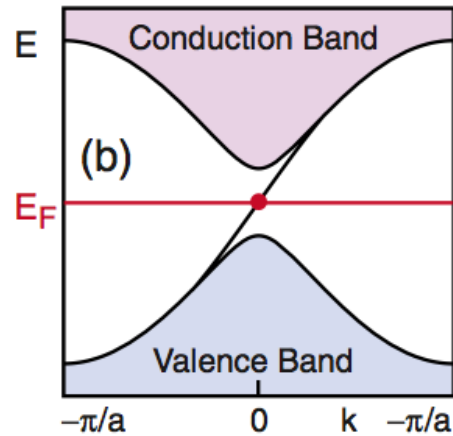


What about spin dependence?



Proposed in: G. Jotzu, M. Messer, R. Desbuquois, M. Lebrat,
T. Uehlinger, D. Greif, T. E., Nature 515, 237 (2014)

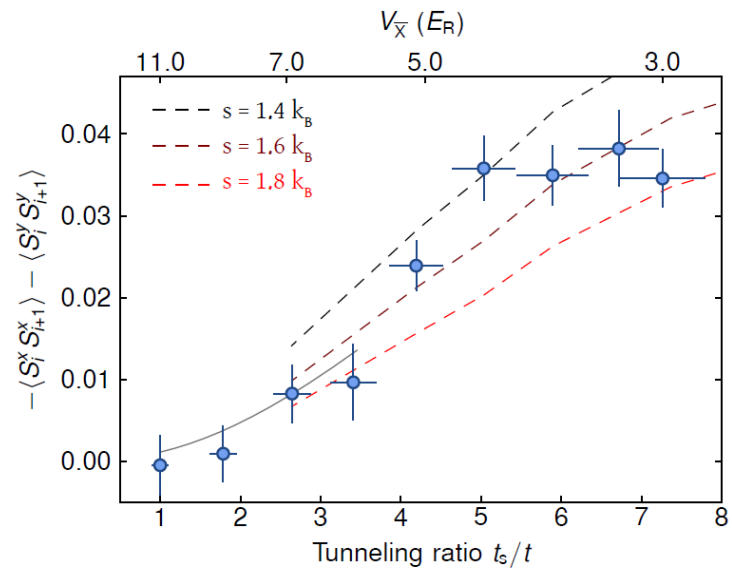
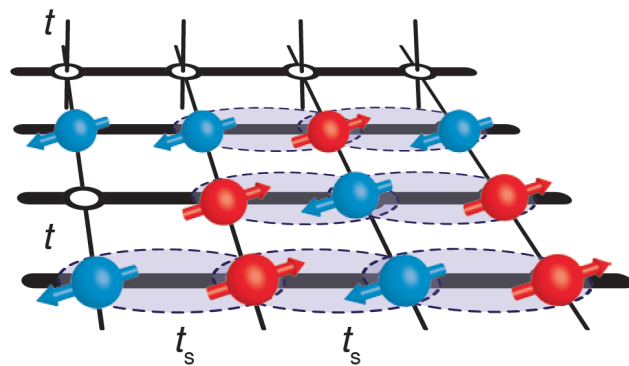
What about quantized edge currents?



Hasan, Kane RMP 82, 3045 (2010)

Update on anti-ferromagnetic correlations

Antiferromagnetic correlations in the anisotropic Hubbard model

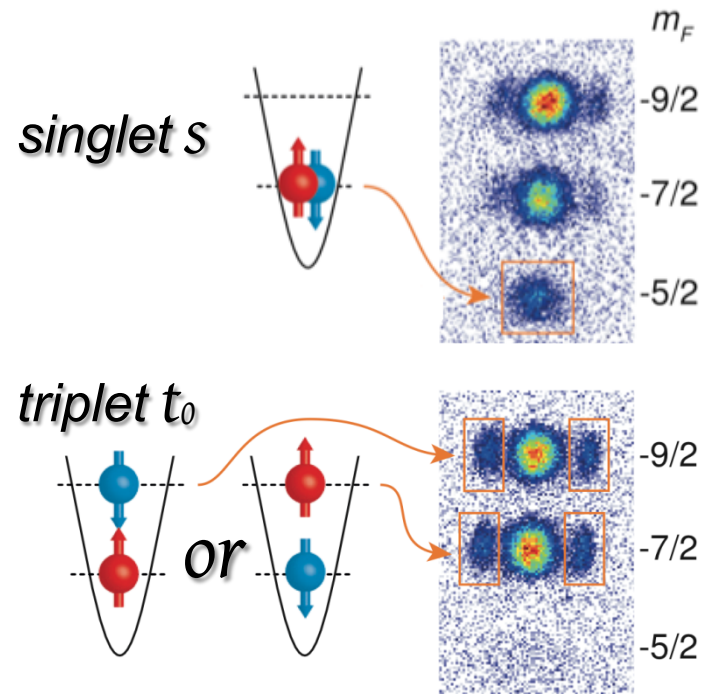
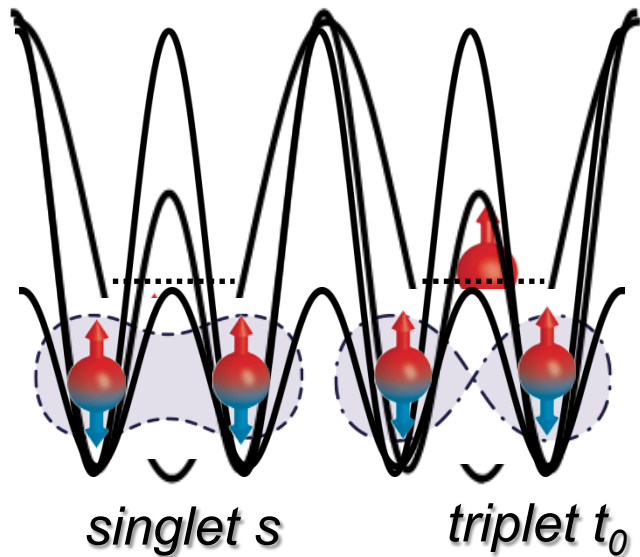


D. Greif, T. Uehlinger, G. Jotzu, L. Tarruell, T. Esslinger, Science 340, 6138 (2013)

J. Imriska et al., PRL 112, 115301 (2014), also: Hulet group, Nature (2015)

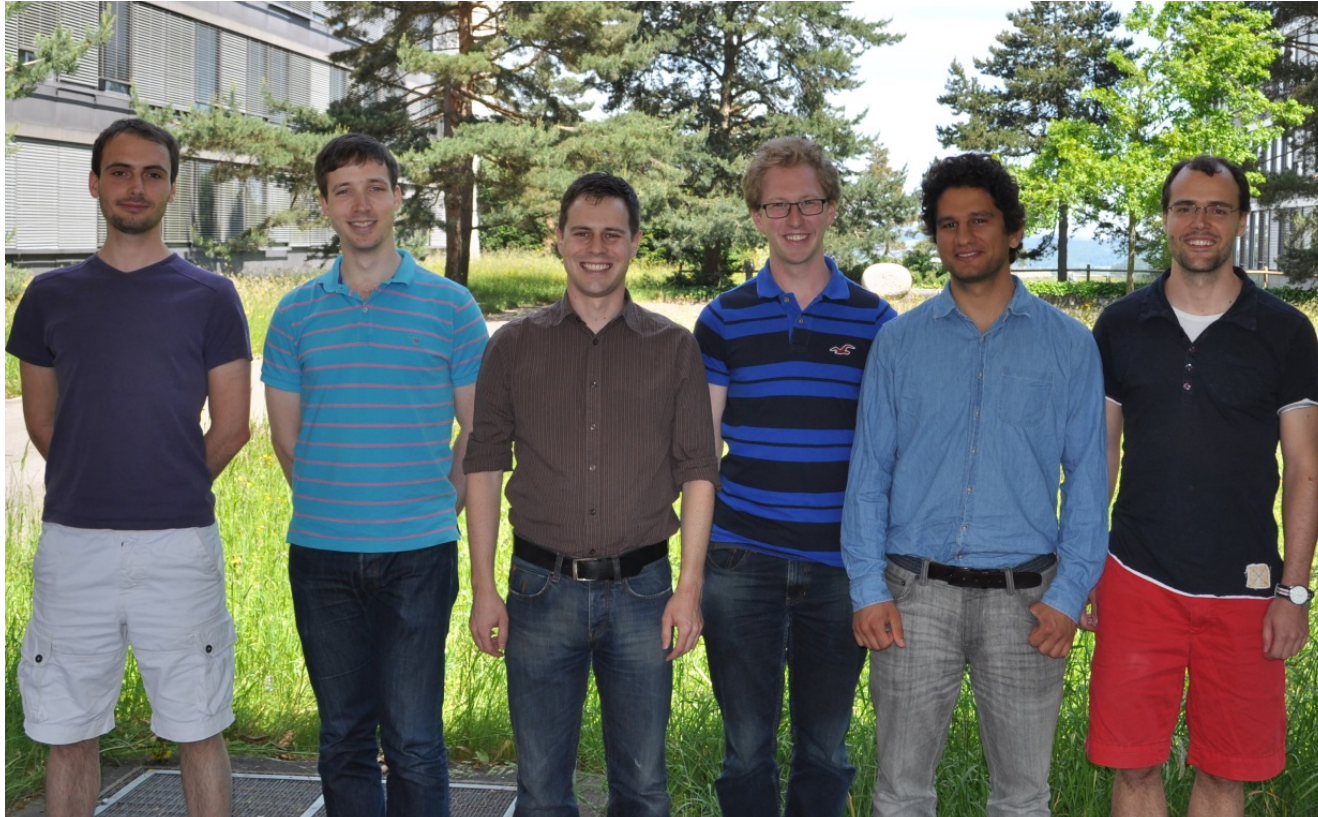
Detection protocol

$$-\langle S_i^x S_{i+1}^x \rangle - \langle S_i^y S_{i+1}^y \rangle = \frac{\rho_s - \rho_{t_0}}{2}$$



The Lattice team

+ Leticia Tarruell: now @ ICFO, Barcelona



Martin
Lebrat

Rémi
Desbuquouis

Thomas
Uehlinger

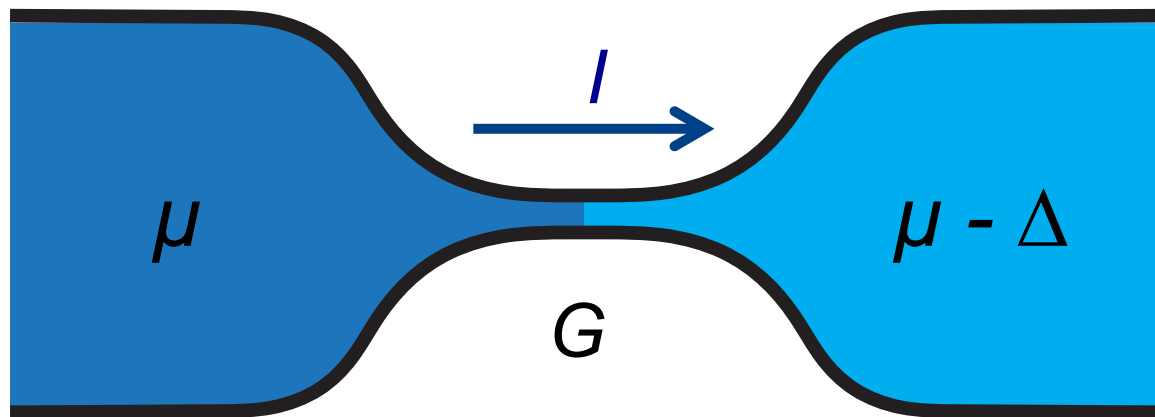
Michael
Messer

Gregor
Jotzu

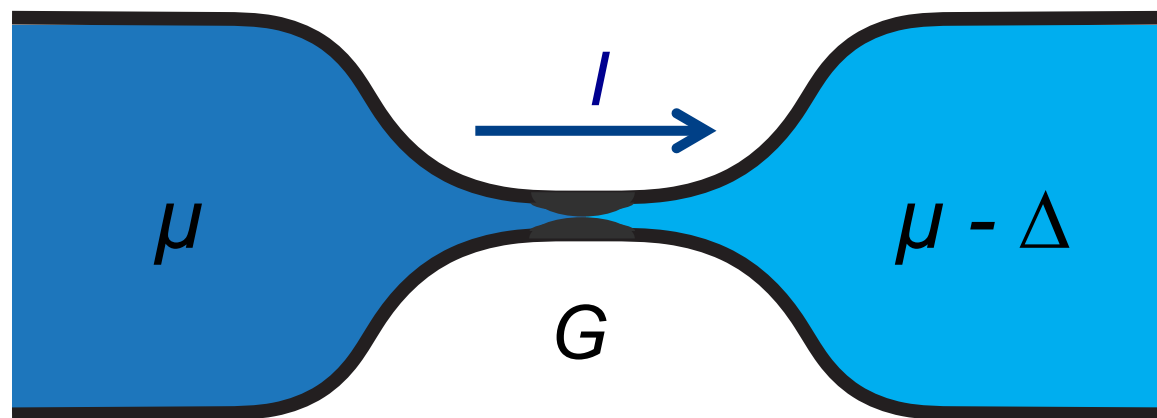
Daniel
Greif

Frederic Görg

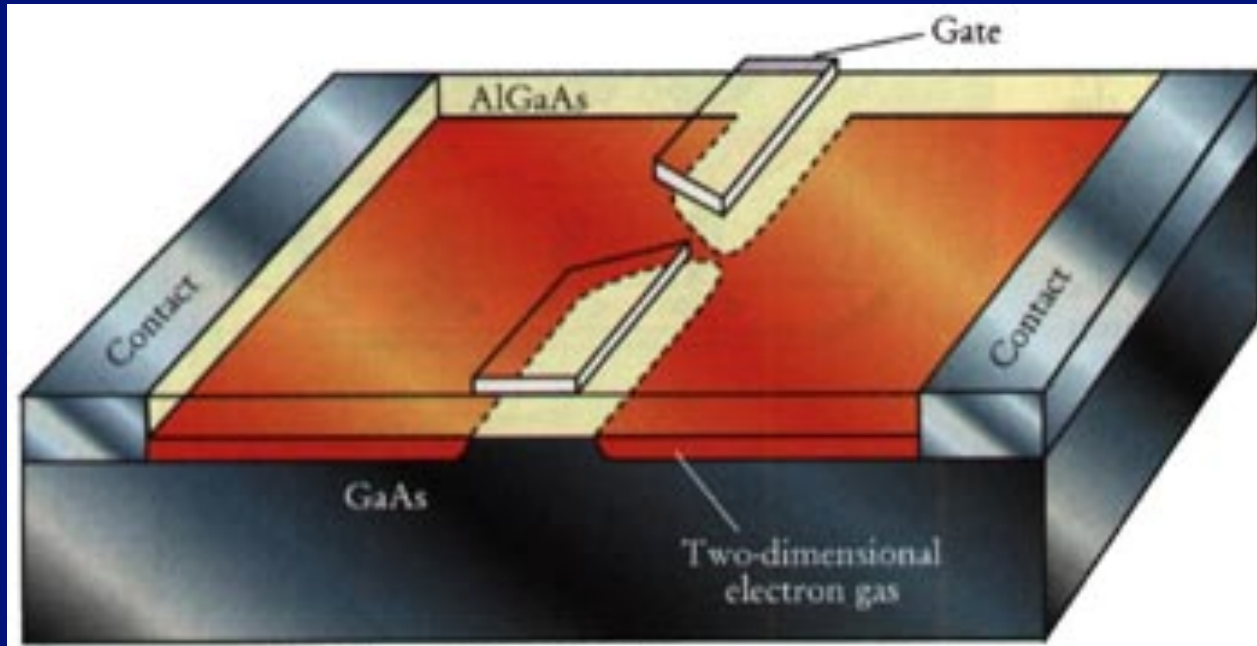
Transport between two Terminals



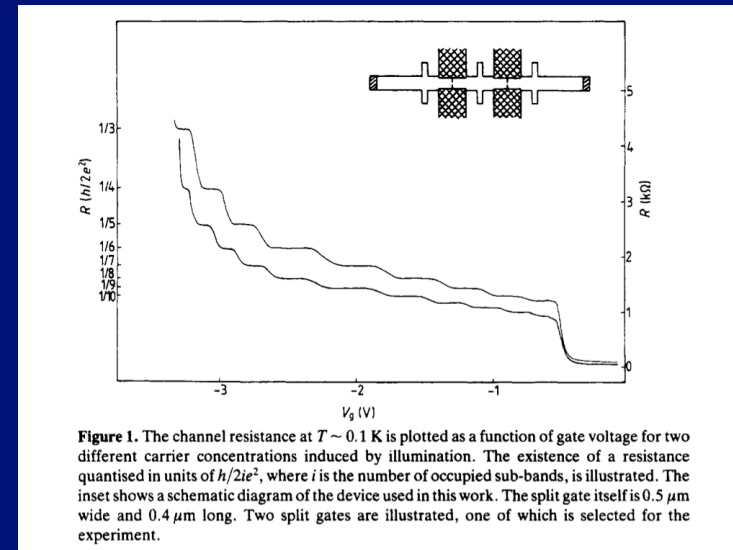
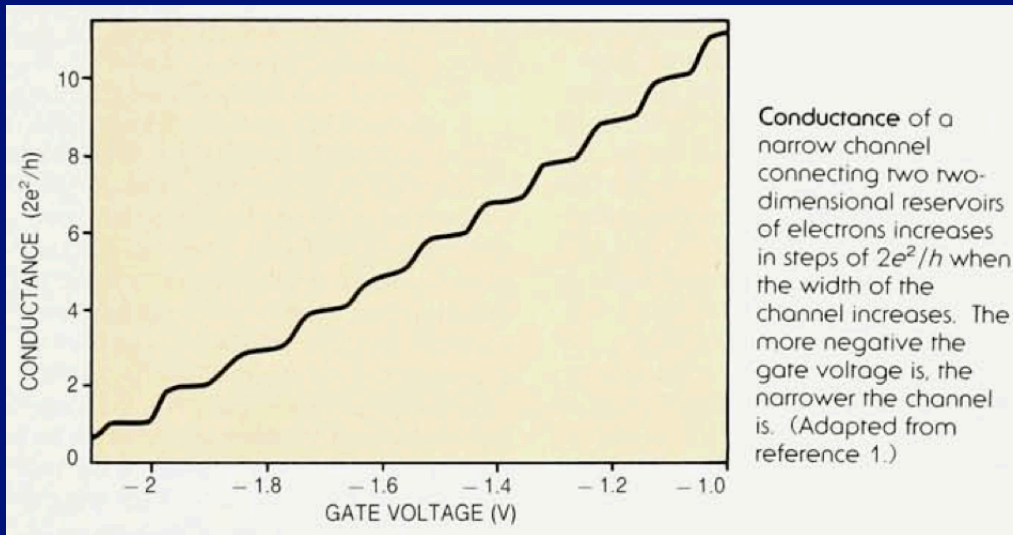
Transport between two Terminals



Quantum Point Contacts



Quantized Conductance



B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, C. T. Foxon, Phys. Rev. Lett. 60, 848 (1988);

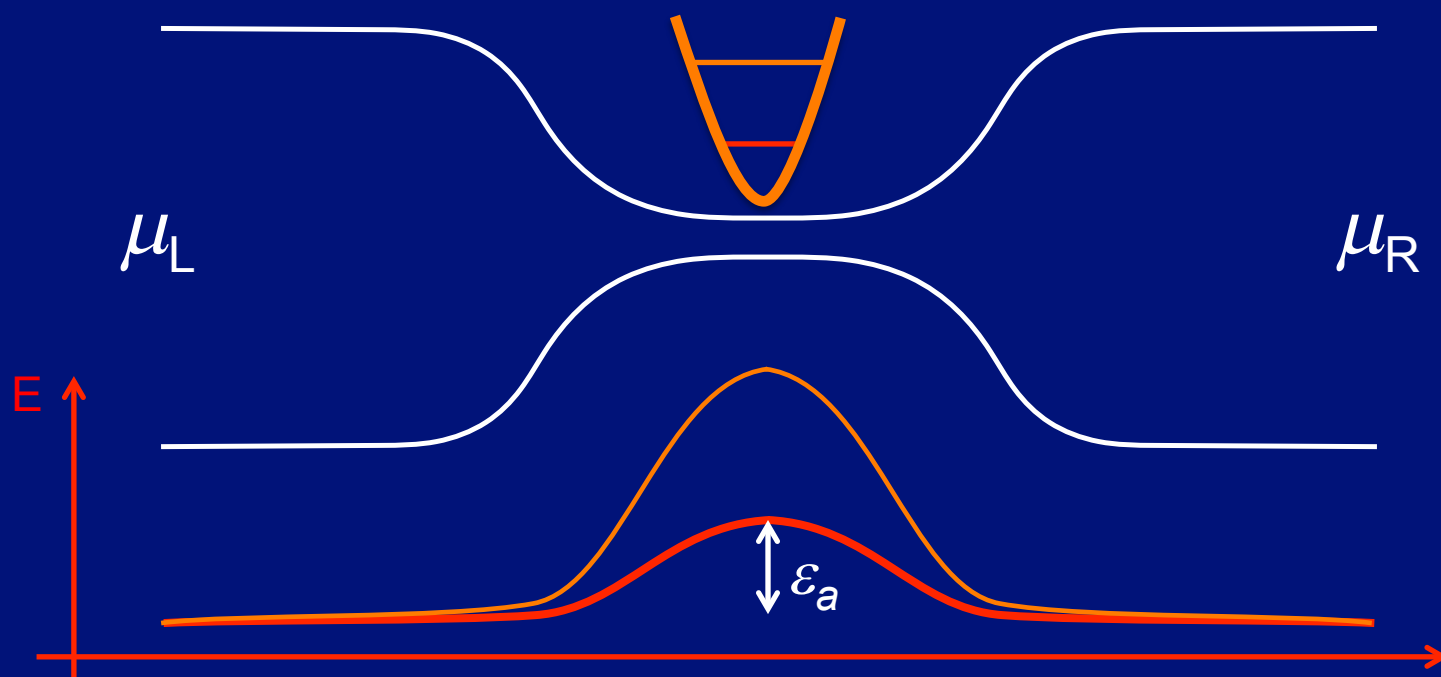
D A Wharam, T J Thornton, R Newbury, M Pepper, H Ahmed, J E F Frost, D G Hasko, D C Peacock, D A Ritchie and G A C Jones, J. Phys. C: Solid State Phys. 21 L209

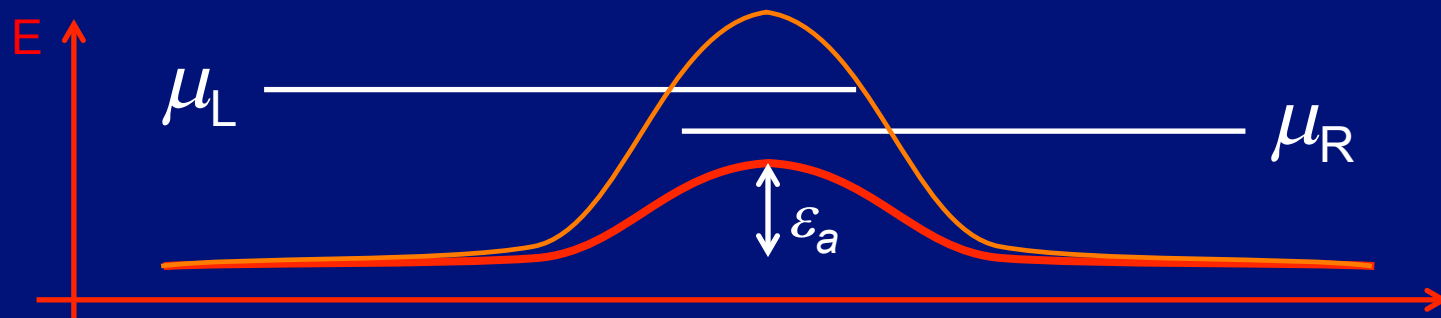
Why?

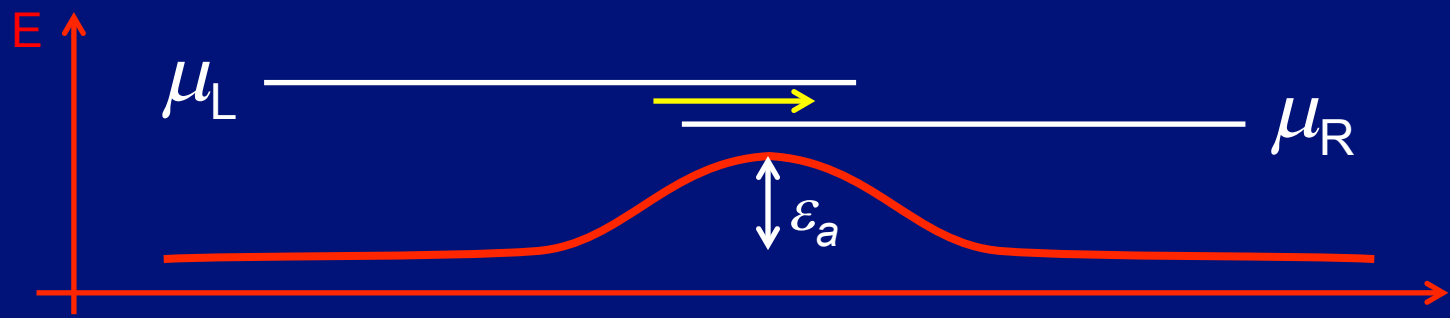
Left Reservoir

Constriction

Right Reservoir

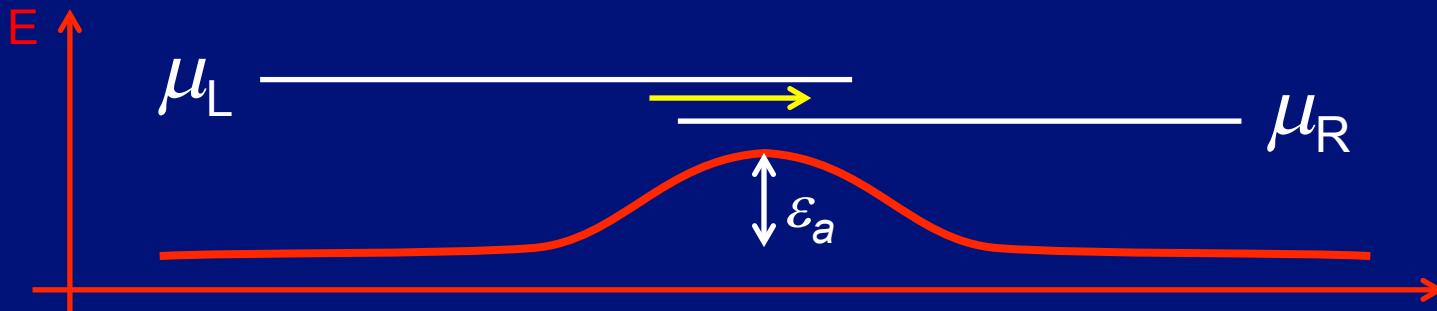






Current for $T=0$, $T_a=1$:

$$I_a = \int_{\varepsilon_F}^{\varepsilon_F + \Delta} d\varepsilon g_a(\varepsilon) v_a(\varepsilon) T_a(\varepsilon)$$



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Current for $T=0$, $T_a=1$:

$$I_a = \int_{\varepsilon_F}^{\varepsilon_F + \Delta} d\varepsilon g_a(\varepsilon) v_a(\varepsilon) T_a(\varepsilon) = \frac{\Delta}{h}$$

velocity: $v_a(\varepsilon) = \frac{\hbar k_a}{m} = \sqrt{2(\varepsilon - \varepsilon_a) / m}$

density of states:
(right movers) $g_a(\varepsilon) = \frac{1}{2\pi} \frac{dk_a}{d\varepsilon} = \frac{1}{2\pi\hbar v_a(\varepsilon)}$

Current for $T=0$, $T_a=1$:

$$I_a = \int_{\varepsilon_F}^{\varepsilon_F + \Delta} d\varepsilon g_a(\varepsilon) v_a(\varepsilon) T_a(\varepsilon) = \frac{\Delta}{h}$$

Conductance $G = \frac{1}{h}$

Multimode Conductance

$$G = \frac{1}{h} \sum_n \frac{1}{\Delta\mu} \int_{E_n}^{\infty} dE [f_L(E) - f_R(E)]$$

Landauer, Büttiker, Imry

Quantized Conductance in Neutral Matter?

Cold atoms proposal: Thywissen, J. H., Westervelt, R. M. & Prentiss, M. Quantum point contacts for neutral atoms. Phys. Rev. Lett. 83, 3762–3765 (1999).

Quantized Conductance in Neutral Matter?

On the Feasibility of Detecting Quantized Conductance in Neutral Matter

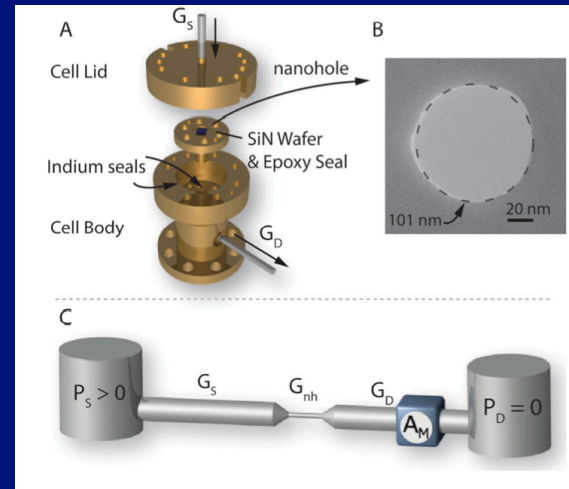
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(Received May 18, 2005; revised July 18, 2005)

When an electrochemical potential difference (i.e., a voltage) is applied across a metal wire whose transverse dimensions are on the order of the electron's Fermi wavelength, the conductance $G \equiv I/\Delta V$ becomes quantized in units of $2e^2/h$. We present calculations that show that when a chemical potential difference $\Delta\mu_3$ is applied across an array of small apertures whose sizes are comparable to the Fermi wavelength of ^3He in a ^3He - ^4He mixture, the mass conductance $G \equiv \left(\frac{I_3}{\Delta\mu_3/m_3^*}\right)$ will be quantized in units of $2m_3^*/h$ where m_3^* is the ^3He effective mass. We show that the mass conductance will be quantized for a 0.1% mixture passing through 10 nm diameter pores at temperatures below 25 mK. The phenomenon should be observable in a filter material made by nuclear track etching.

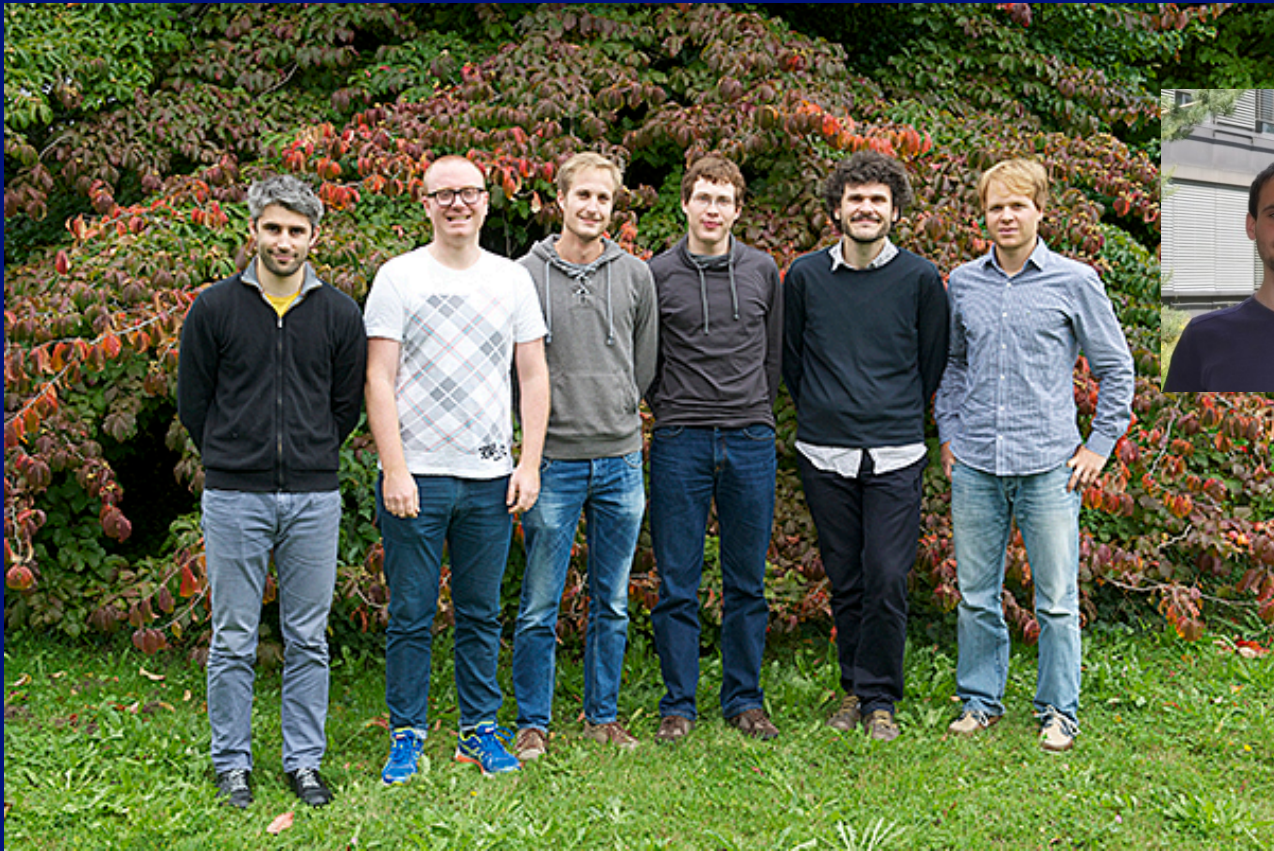


M. Savard, C. Tremblay-Darveau, and G. Gervais, PRL 103, 104502 (2009)

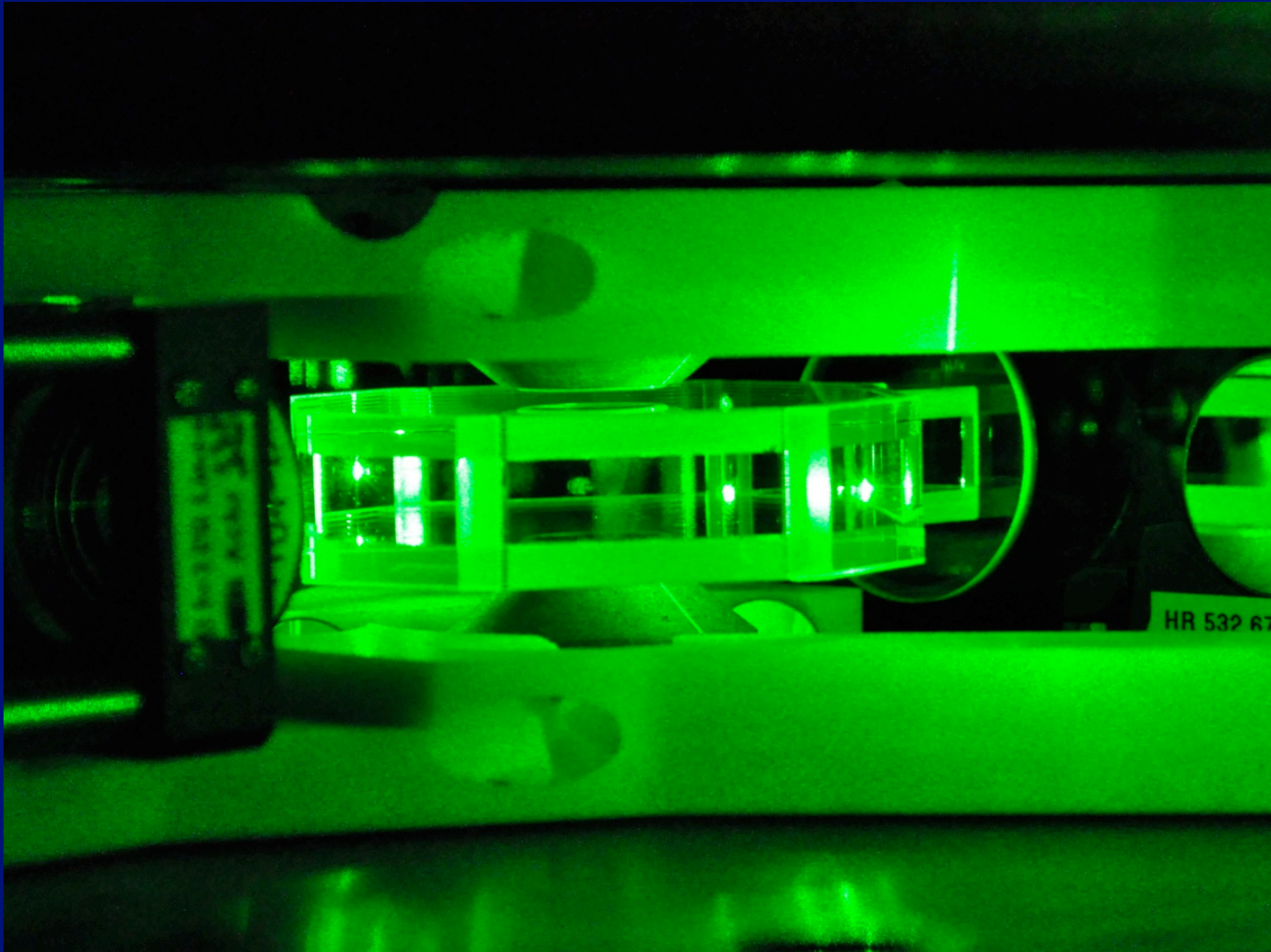
See also: G. Lambert, G. Gervais, and W.J. Mullin, Low Temp. Phys. 34, 249 (2008).

Quantized Conductance in Neutral Matter?

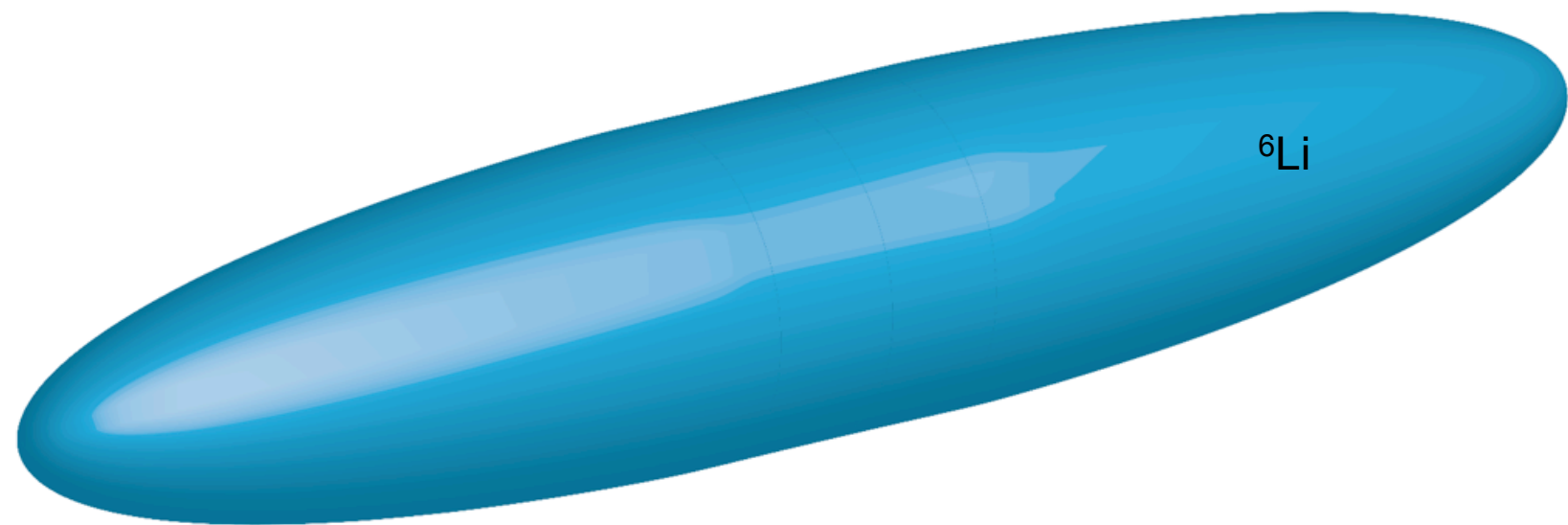
- Method to measure conductance
- Ballistic channel
- Quantum degenerate Fermi gas
- Resolve individual conduction channels
- Adiabatic regime
- Applicability of Landauer theory
(mean free path $>$ trap)

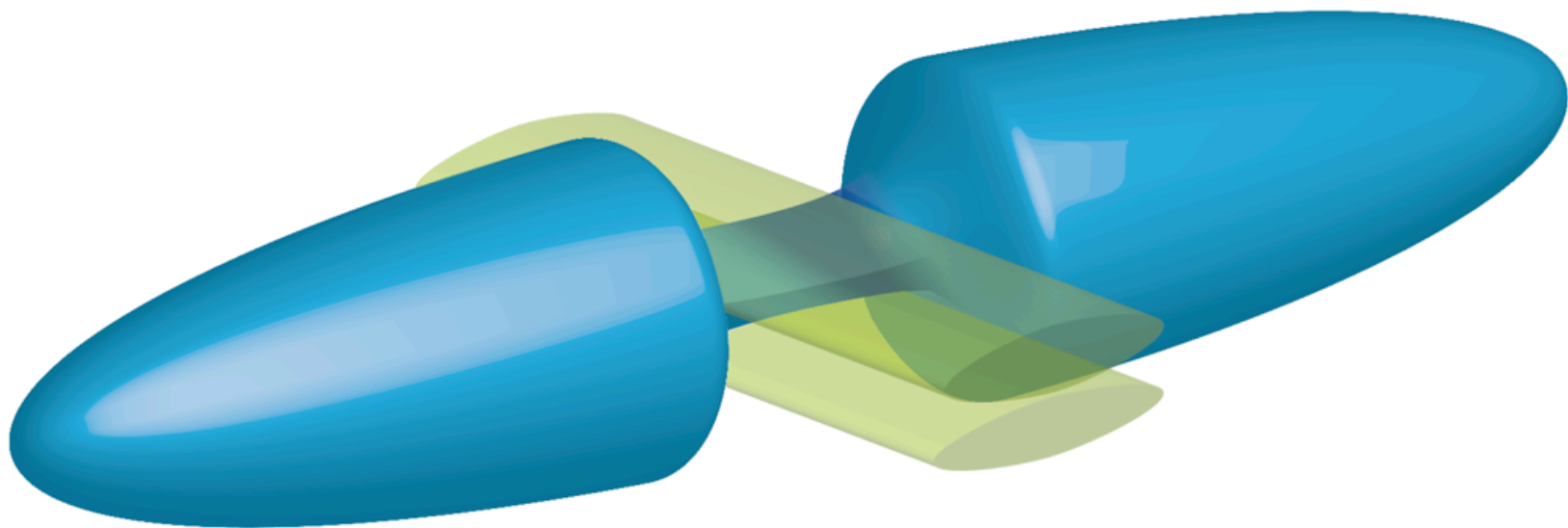


**Jean-Philippe Brantut, David Stadler, Dominik Husmann,
Samuel Häusler, Charles Grenier, Sebastian Krinner
Martin Lebrat**

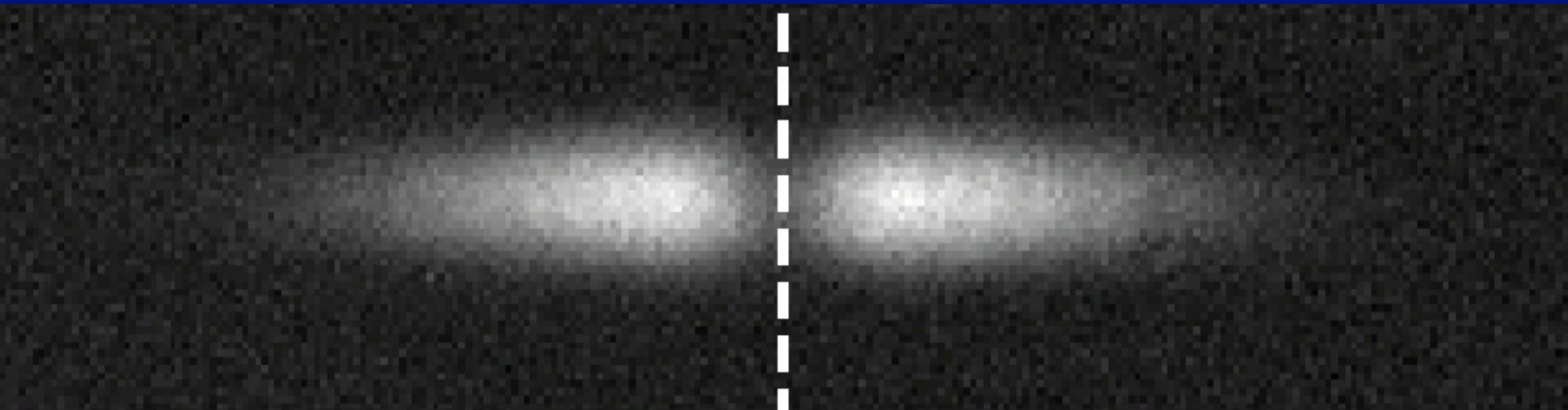


Thanks to Henning Moritz now @ Hamburg





Fermi battery



reservoir

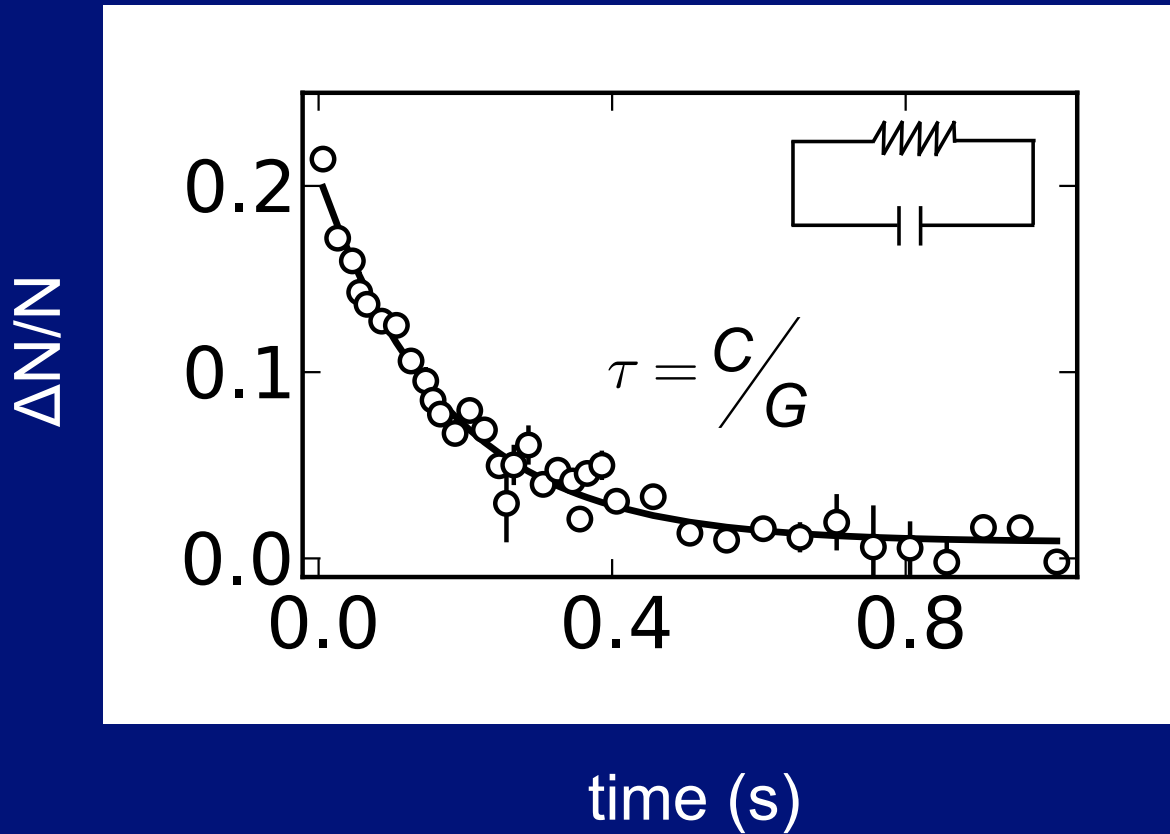
channel

reservoir

N_L

N_R

Battery discharge

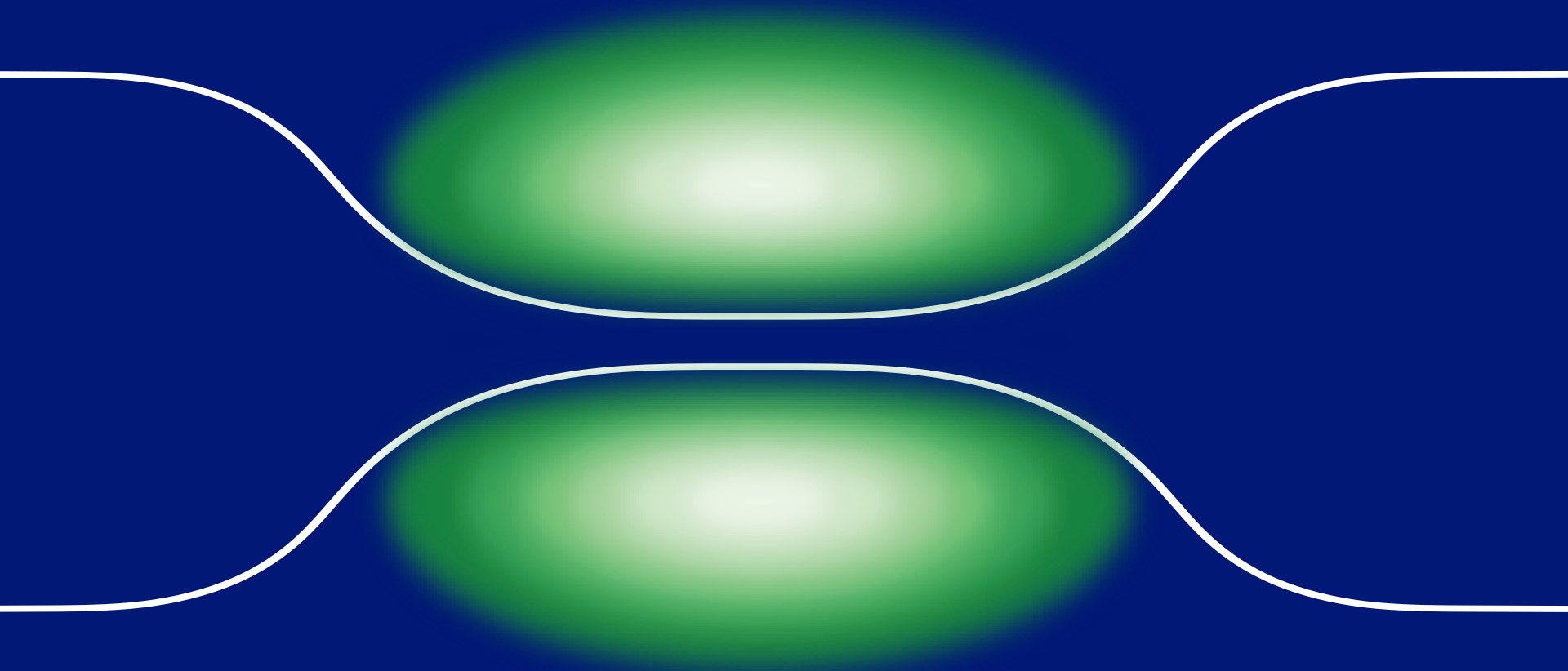


$$\frac{d}{dt}\Delta N = -\frac{G}{C}\Delta N$$

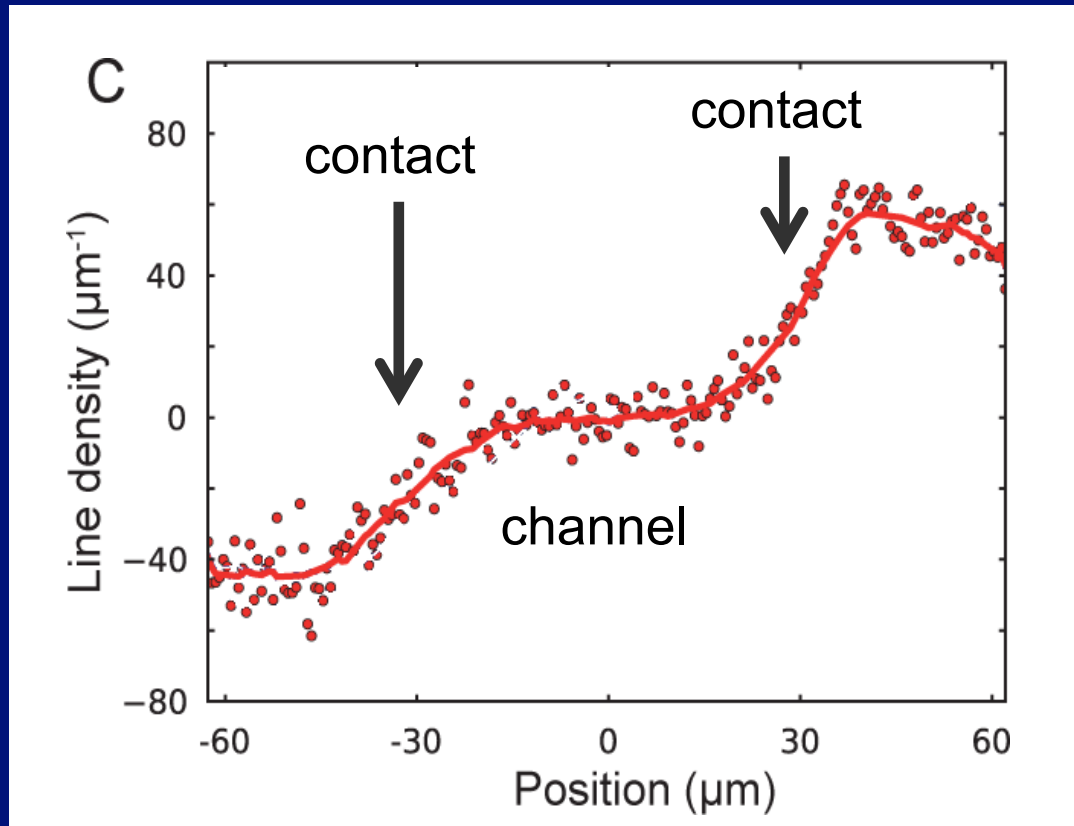
G : conductance

C : compressibility $\partial N/\partial \mu$

Resistance?

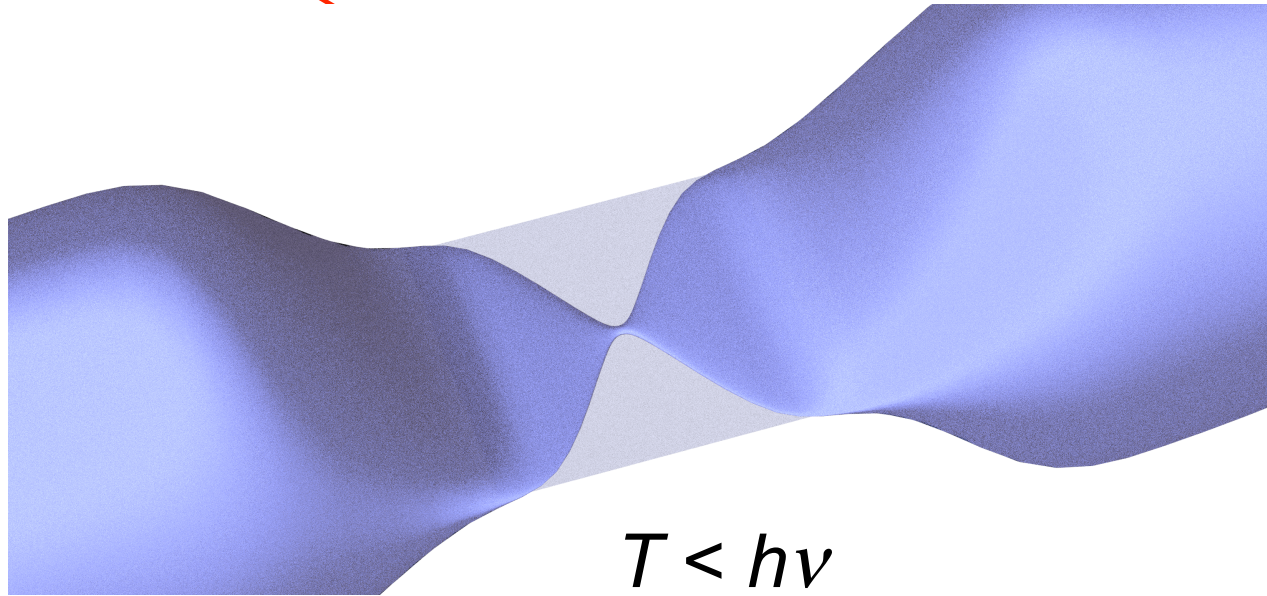


Ballistic Multimode Channel



J.-P. Brantut, J. Meineke, D. Stadler, S. Krinner, T. Esslinger, *Science* 337, 1069 (2012)

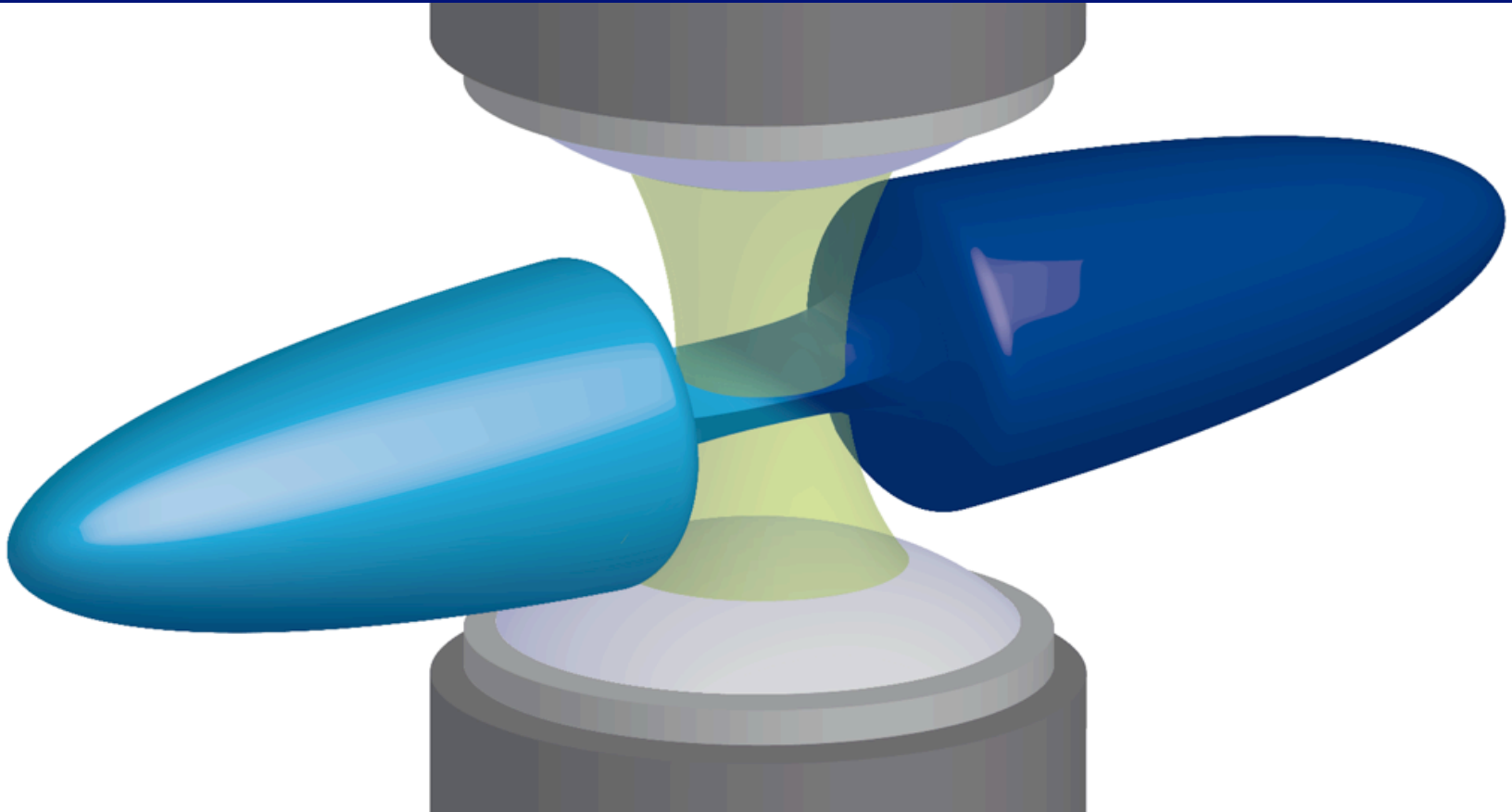
Dreams and worries

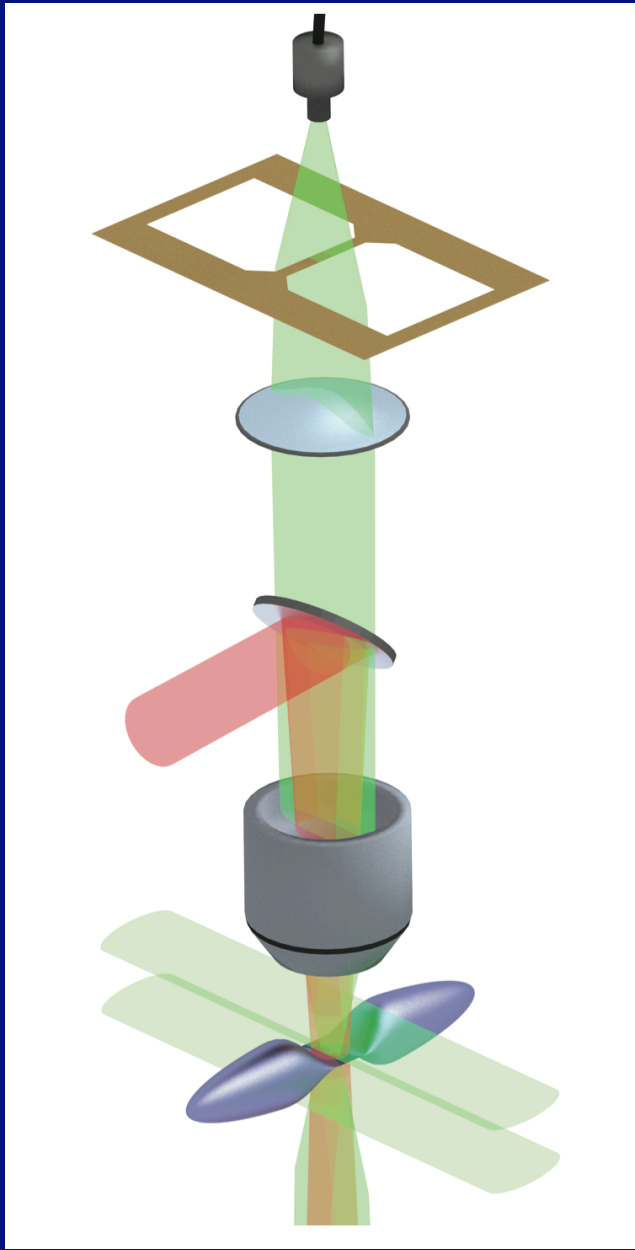


$$T < h\nu$$

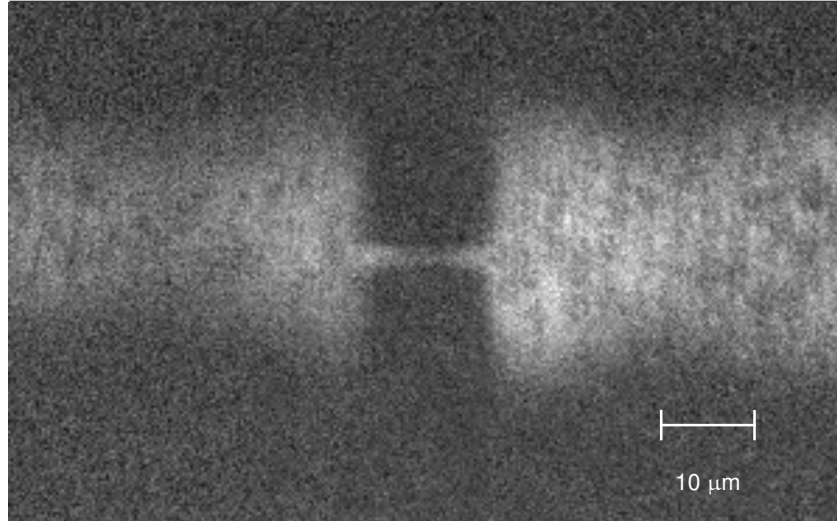
10 atoms in channel

Single Mode Channel?



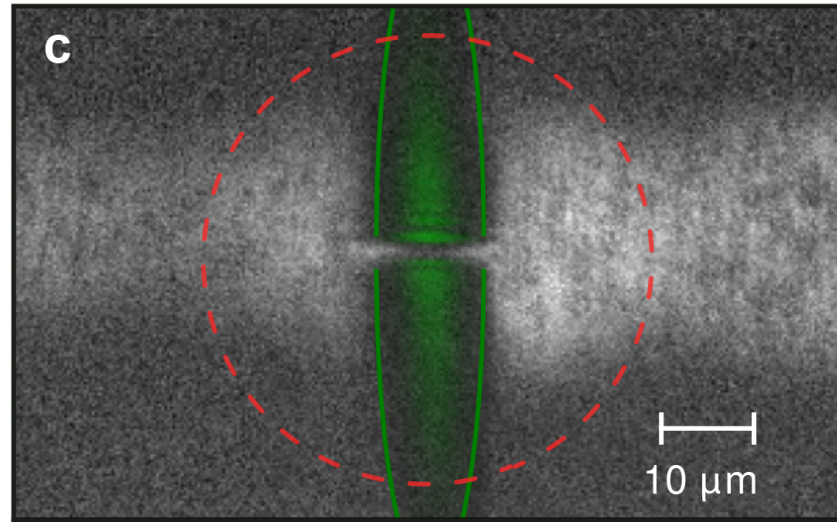


Quantum Point Contact

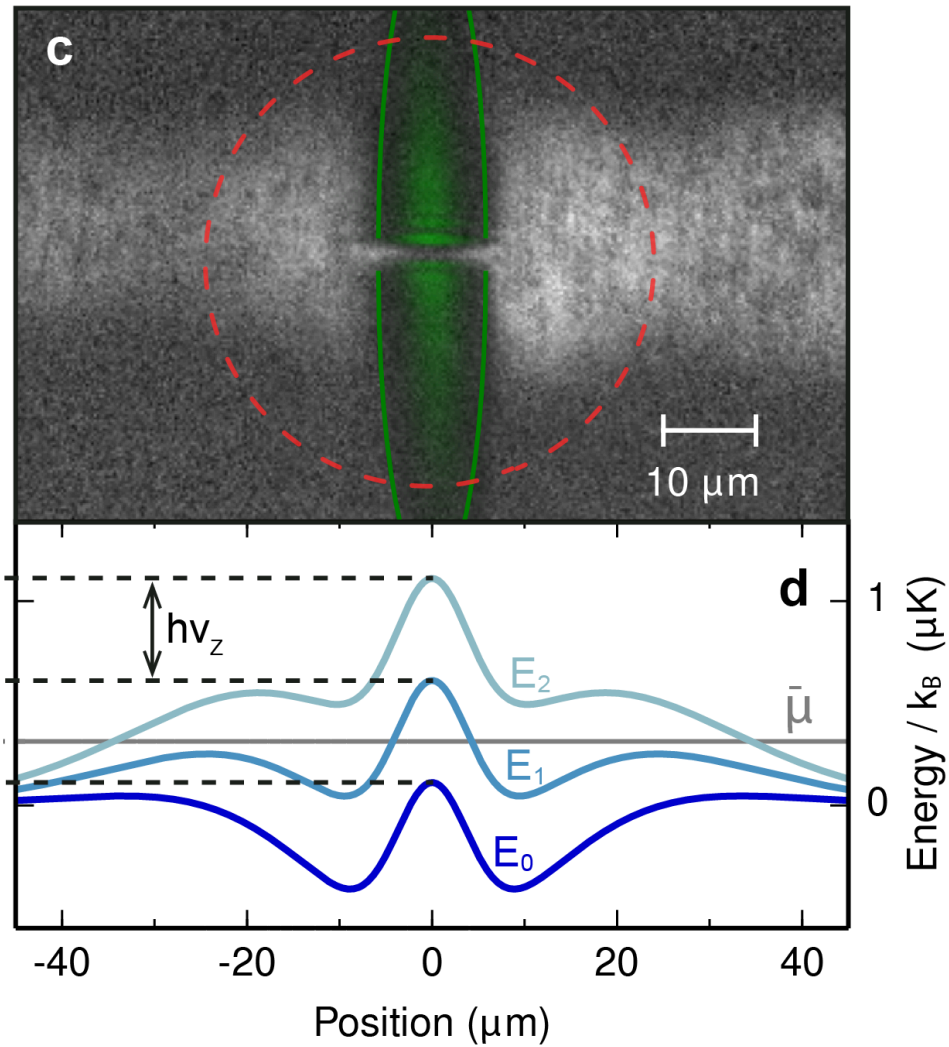


Fermi wavelength of $2.2 \mu\text{m}$

Quantum Point Contact

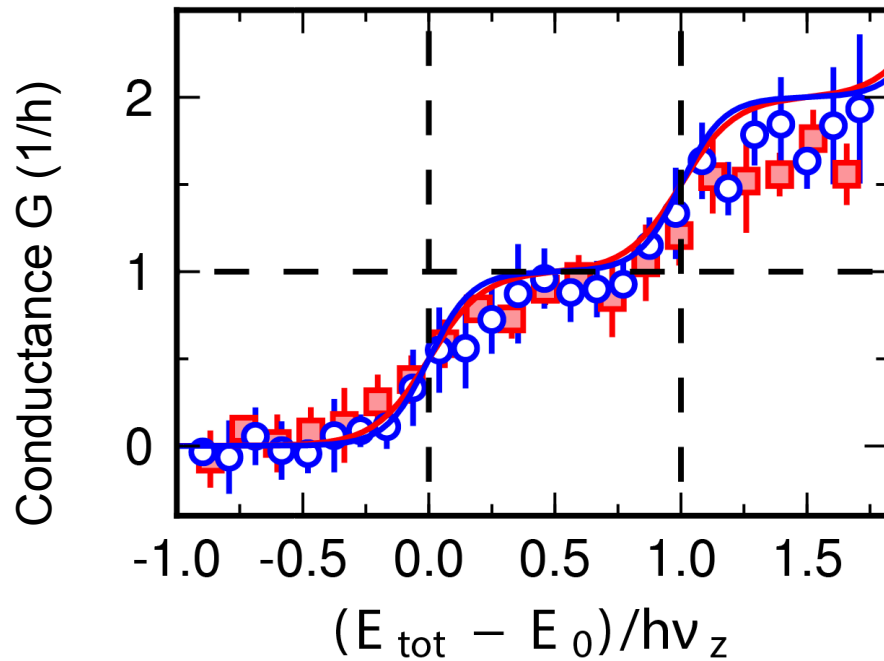


Quantum Point Contact



Quantum Point Contact

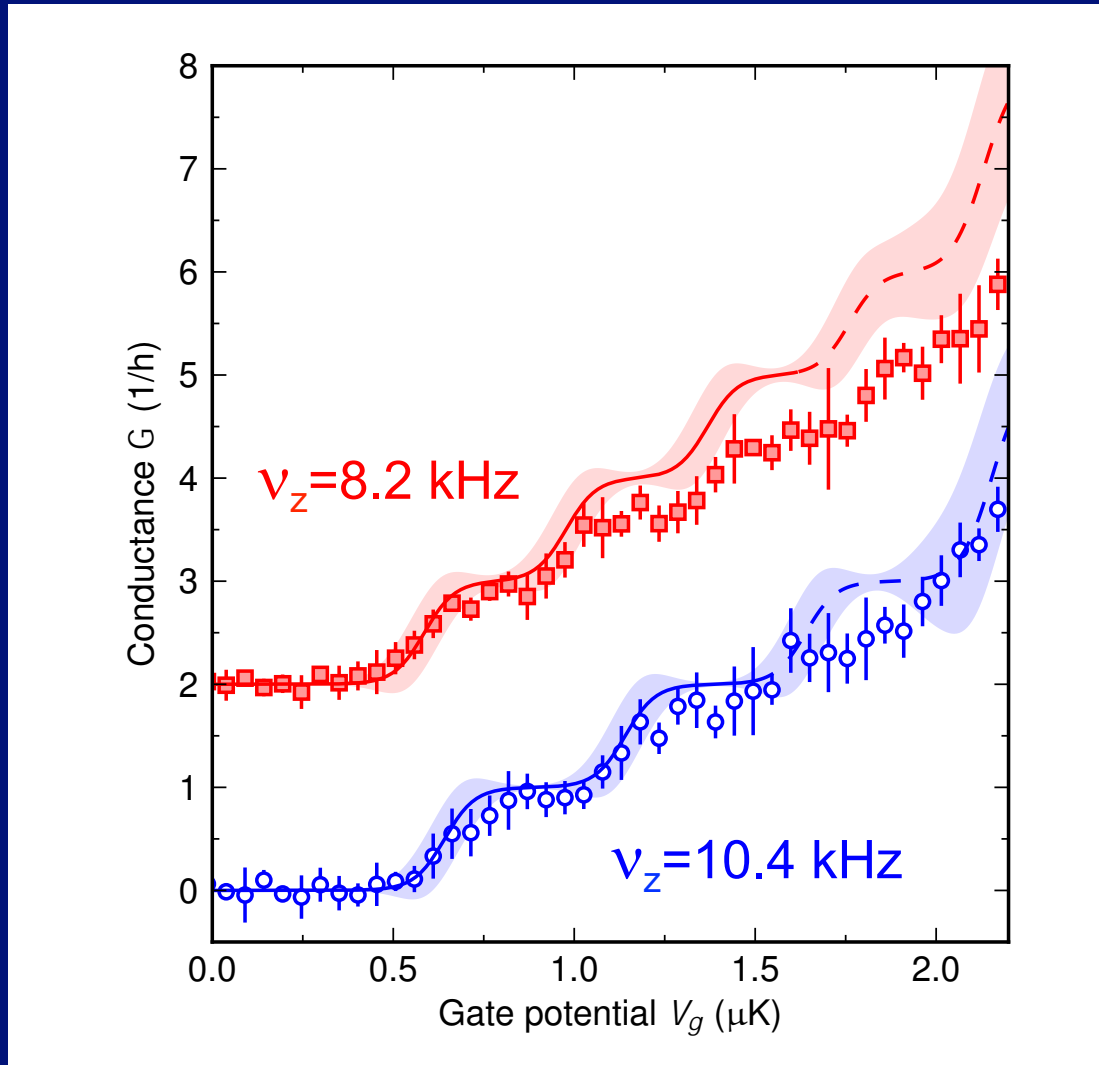
Measure number imbalance after 1.5 s



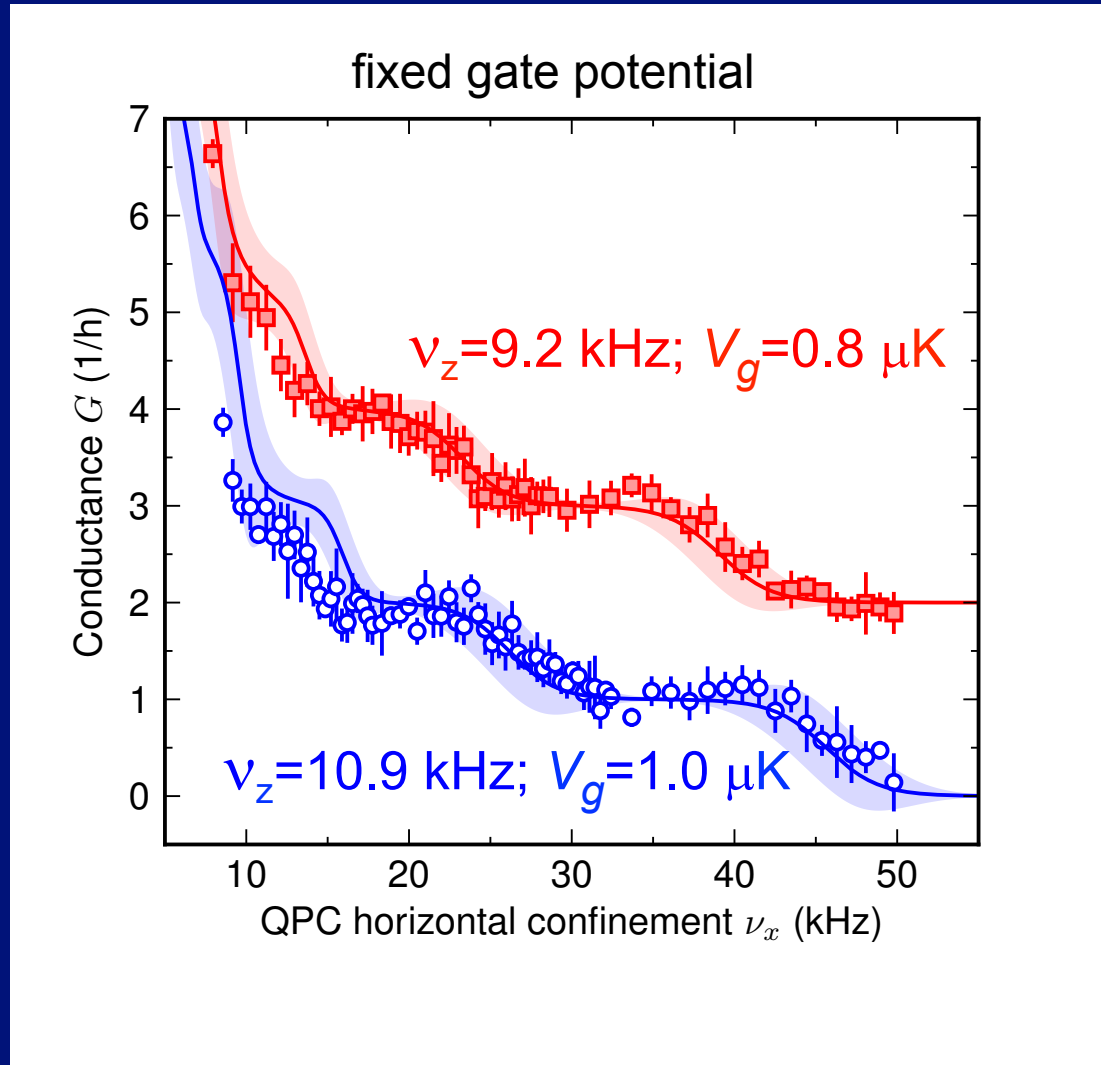
$\nu_x = 31.8$ kHz
 $\nu_z = 10.4$ kHz
 $\nu_z = 8.2$ kHz

$T = 35$ nK ($= 0.1 T_F$) $< h\nu_z \approx 500$ nK

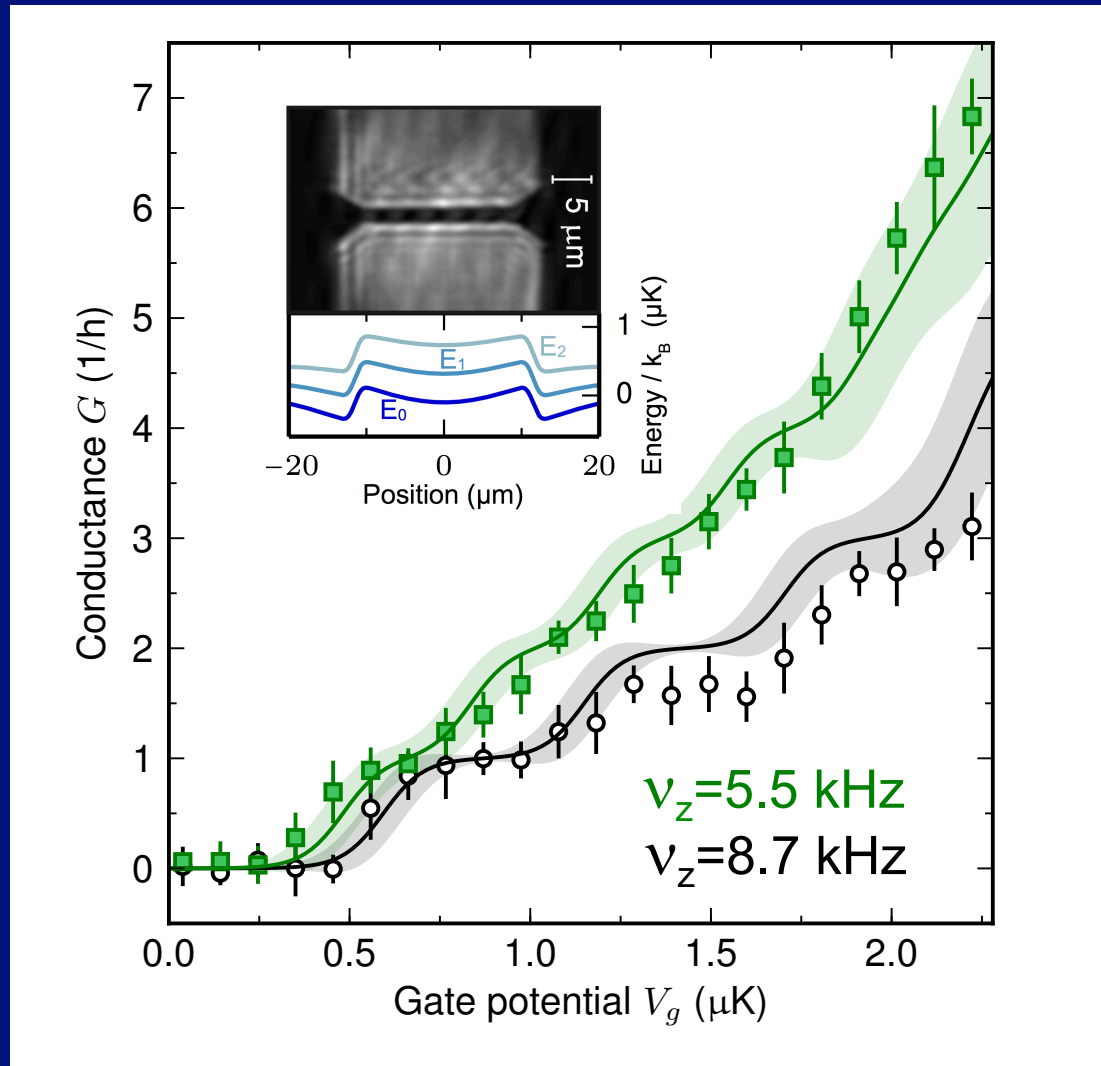
Quantum Point Contact



Quantum Point Contact



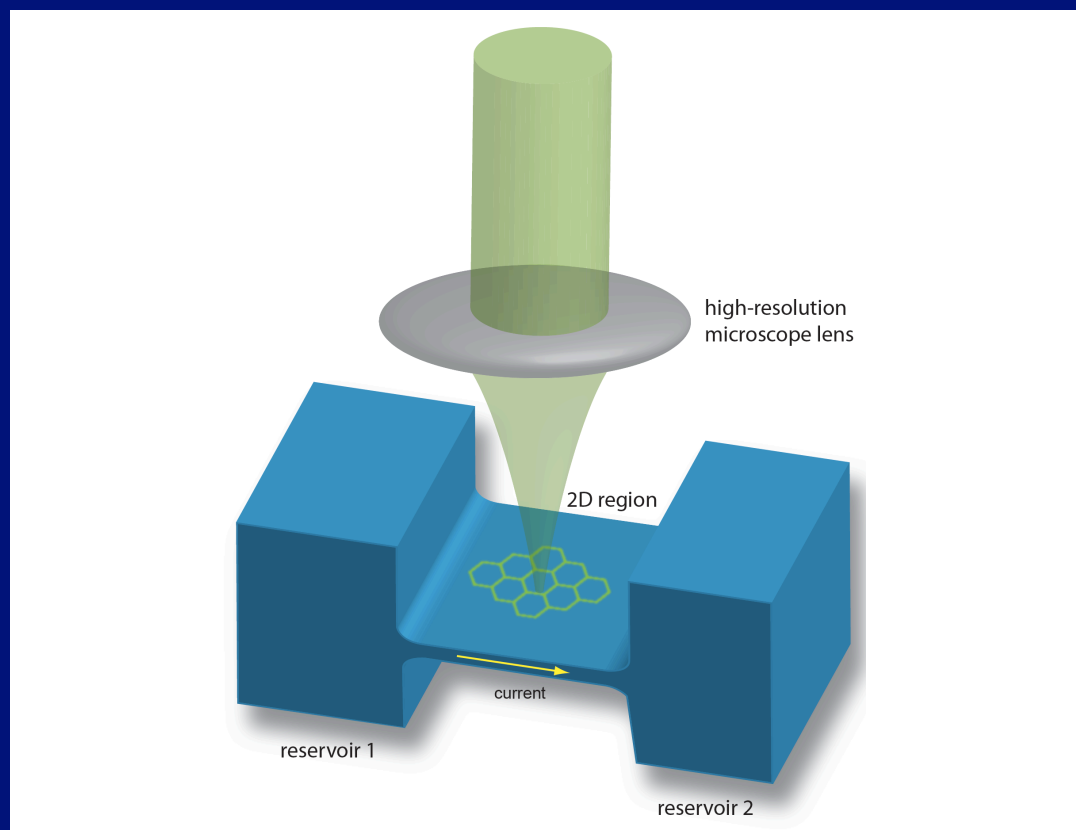
Quantum Wire



Conclusions

- Ballistic and adiabatic channels can be produced
- Isolated system
- Ultraballistic regime, mean free path \gg system
- Tunable Interactions
- Spin transport
- Precision measurement of h ???

Outlook: Quantum Simulation of Devices



Thanks !

Funding: ETH, SNF, NCCR QSIT, EU SIQS, TherMiQ, ERCadv SQMS

Quantum Gases in Optical Lattices

Daniel Greif
Thomas Uehlinger
Gregor Jotzu
Michael Messer
Rémi Desbuquois
Frederic Görg
(Leticia Tarruell)



Lithium Microscope

Jean-Philippe Brantut
Sebastian Krinner
Dominik Husmann
Martin Lebrat
Samuel Häusler
Shuta Nakajima
(David Stadler)

BEC and Cavity

Ferdinand Brennecke
(Rafael Mottl)
Tobias Donner
Renate Landig
Lorenz Hruby
Andrea Morales
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Impact experiment

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Theory discussions: Ehud Altmann, Gianni Blatter, Georg Bruun, Nigel Cooper, Eugene Demler, Antoine Georges, Thierry Giamarchi, Gian Michele Graf, Sebastian Huber, Corinna Kollath, Dario Poletti, Christian Rüegg, Manfred Sigrist, Wilhelm Zwerger, ...