LEC 1: LATTICE GAUGE THEORIES ON QUANTUM EMULATORS AND QUANTUM COMPUTERS

Review: U.-J. Wiese, Ann. der Phy., 2013

1) INTERFACES BETWEEN AMO & HEP

a) WHY?

b) 3 TYPES → new algorithms → QUANTUM COMPUTING → QUANTUM EMULATORS

c) intermezzo: main MPS/TNs [SLIDES]

2) QUANTUM SIMULATORS / EMULATORS:

a) general idea, examples [SLIDES]

b) for gauge theories: QLMs

c) strategies for gauge invariant dynamics

→ I) energy

→ II) Zeno

→ III) encoding

3) QUANTUM SIMULATION OF A U(2) QLM (next lecture)

1) Motivation for AMO/HEP connection

I) quantum simulation/quantum technologies → finite density

→ real-time dynamics

II) novel scenarios for MANY-BODY physics:

→ new phases of matter

→ new (or old) theories

→ probes

2) Quantum simulators in a nutshell

$\dim \mathcal{H} \simeq e^{N} \Rightarrow$ intractable

we are interested in $\langle O(t) \rangle$

⇒ recreate $H$ of interest, & perform a measurement

→ similar to classical emulators [WINO ~~GALLERY~~ TUNNEL, HUMAN BRAIN PROJect]

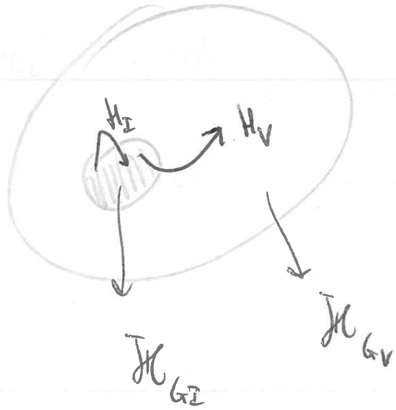
digital simulators [computers]

2) ENFORCING a gauge symmetry

$$[H, G_x] = 0$$

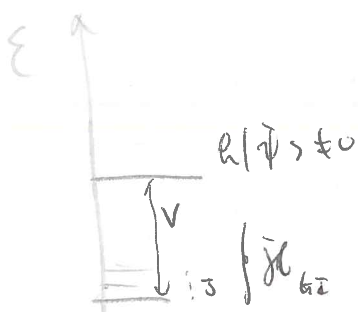
⇒ CONSTRAINT PROBLEM:

we want to constrain the dynamics within $\mathcal{H}_{GI}$




VARIOUS STRATEGIES

I) energy punishment (typical in cond. matt)




effective Hamiltonians in $J^2/V$

usually good for Abelian $(U(1), \mathbb{Z}_2)$ theories

II) DISSIPATIVE: use the quantum Zeno effect

$$H(t) = H_0 + H_1 + \underbrace{\sqrt{2k}\, \xi^{\varphi}_x\, G^{\varphi}_x}_{\text{CLASSICAL NOISE TERMS}} \Rightarrow \dot{\rho} = -i H_{eff}\, \rho + i \rho H_{eff}^{\dagger} + 2k \sum_i G_i^{a} \rho\, G_i^{a}$$

III) <u>exploit fundamental symmetries</u>

$\Rightarrow$ $U(N)$, $SU(N)$

IV) <u>analytical encoding</u> (e.g. Wilson's Schwinger model)

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# LEC 2: HOW TO BUILD A U(2)/SU(2) LATTICE GAUGE THEORY USING ULTRACOLD ATOMS

NON-ABELIAN IMPLEMENTATIONS

PRL 110, 125303 (2013)

Other NON-ABELIAN
PRL 112, 120406
PRL 110, 125304 (2013)

Nat Comm. 4, 2615 (2013)

1) DEFINITION OF The model

a) LATTICE, GAUSS LAW

b) HAMILTONIAN

c) Hilbert space

2) STRATEGY

a) GLOBAL -> LOCAL SYMMETRIES

b) ALKALINE-EARTH-LIKE ATOMS

c) perturbation Theory; determinant

3) DISCUSSION

-> No Time...

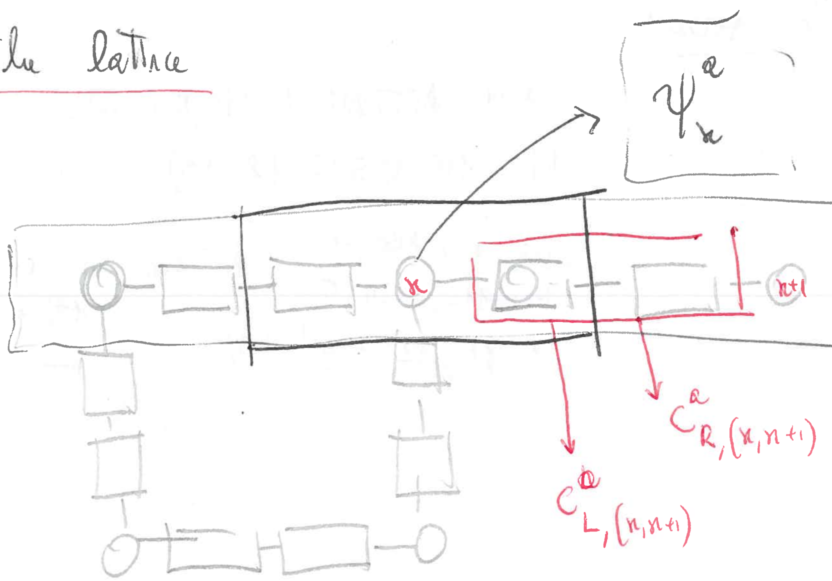
Talk is based on PRL 110, 125303 (2013); also, supplementary material!

# 1) U(N) / SU(N) QLMs

## a) The lattice

MATTER FIELDS

$$\{\psi^a_x, \psi^{b\dagger}_y\} = \delta_{ab}\delta_{xy}$$




RISHONS - "GAUGE FIELDS"

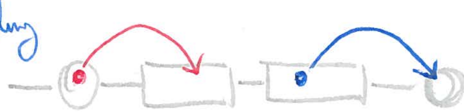
## b) The Hamiltonian (minimal version)

$$H^{SU(N)} = -t\sum_{x=1}^{L-1}\Bigg[\overbrace{\Bigg(\underbrace{\sum_{a=1}^{N}\psi^{a\dagger}_x\, c^a_{L,(x,x+1)}}_{\sum_a Q^a_{x,L}}\Bigg)\Bigg(\sum_{b=1}^{N}\underbrace{c^{b\dagger}_{R,(x,x+1)}\,\psi^b_{x+1}}_{\sum_b Q^b_{x+1,R}}\Bigg)}^{\psi^{a\dagger}_x U^{ab}_{xy}\psi^b_y} + h.c.\Bigg]$$

COSTITUENT QUARK OPERATORS

$$+\underbrace{m\sum_{x=1}^{L}(-1)^x\Bigg(\sum_{a=1}^{N}\psi^{a\dagger}_x\,\psi^a_x\Bigg)}_{\text{MASS TERM}} + \underbrace{\varepsilon\Bigg(\sum_{x=1}^{L-1}\Bigg[\Bigg(\prod_{a=1}^{N}c^{a\dagger}_{L,(x,x+1)}\Bigg)\Bigg(\prod_{b=1}^{N}c^{b}_{R,(x,x+1)}\Bigg)\Bigg] + h.c.}_{\propto\ \det U_{xy}}$$

1 correlated tunneling



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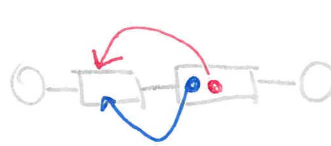


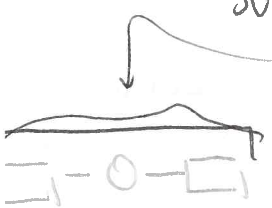
## c) Hilbert space & GAUGE INVARIANCE

$$|\Psi\rangle \;\Big|\; G^a_x |\Psi\rangle = 0$$

$\rightarrow$ SU(N) Gell-Mann matrices

where:

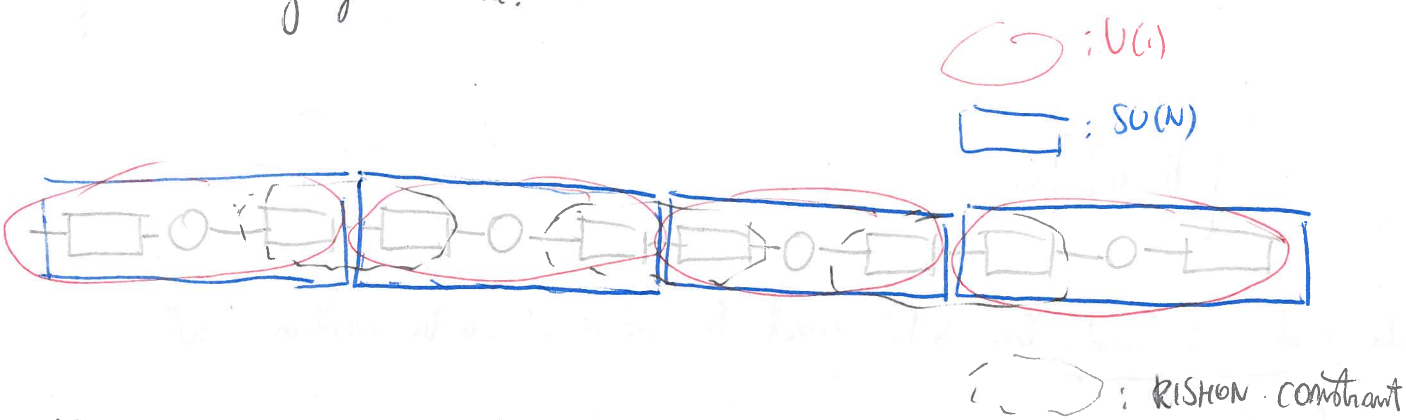
$$SU(N):\quad G^a_x = \psi^{i\dagger}_x \lambda^a_{ij} \psi^j_x + c^{i\dagger}_{L,(x,x+1)} \lambda^a_{ij} c^j_{L,(x,x+1)} + c^{i\dagger}_{R,(x-1,x)} \lambda^a_{ij} c^j_{R,(x-1,x)}$$



$$U(1) \Rightarrow G^U_x = \psi^{a\dagger}_x \psi^a_x + c^{a\dagger}_{L,(x,x+1)} c^a_{L,(x,x+1)} + c^{a\dagger}_{R,(x-1,x)} c^a_{R,(x-1,x)}$$

↳ simple interpretation: $\boxed{\text{\# of particles per block is conserved}}$

pictorial view on gauge invariance:




In addition, there is a "RISHON" CONSTRAINT:

$$\tilde{N}_{x,x+1} = c^{a\dagger}_{L,(x,x+1)} c^a_{L,(x,x+1)} + c^{a\dagger}_{R,(x,x+1)} c^a_{R,(x,x+1)}$$

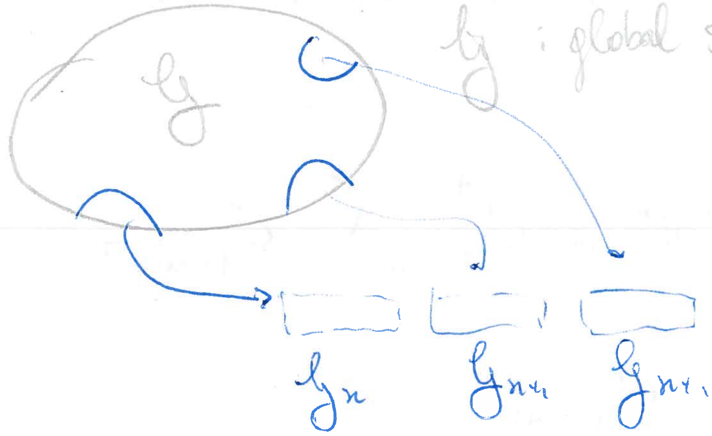
which is imposed

## 2) IMPLEMENTATION STRATEGY: "DIVIDE et IMPERA"

filippo II di Macedonia

### a) GAUGE SYMMETRY

$\mathcal{G}$ : global symmetry of some sort




The symmetry is now LOCAL

each block has a set of operators $K_x$ which satisfy

$$[K_x, G^a_x] = 0$$

so are gauge invariant [we assume they are fermion bilinears]

Now, whatsoever

$$H = H\{K_x\}$$

satisfies

$$[H, G^a_x] = 0$$

N.B.: the vishon symmetry has to be additionally introduced with another mechanism, discussed below.

c) full model & RISHON CONSTRAINT

$$H_0 = U\left(n_{lin} - \bar{m}\right)^2 \qquad (1)$$

$\bar{m} = 1, \dots 2N-1$ for $U(N)$ all TABLEAUS

$n_{link} = n^e_{L,(x,x+1)} + n^e_{R,(x,x+1)}$

$\bar{m} = N$ for $SU(N)$

$$H_1 = -J \sum_x \left\{ \psi^{\dagger a}_x \left( c^a_{L,(x,x+1)} + c^a_{R,(x+1,x+2)} \right) + h.c. \right\} \qquad (2)$$

Comments; a) in (1), we need

$U^{ab}_{LR,\dots} = U_{LR,\dots} \Rightarrow SU(N)$ scattering symmetry

notice that, if $U_{LL} \neq U_{LR}$, this automatically generates an additional ABELIAN electric field, an also a NON-ABELIAN ONE

b) in addition to $H_{U(N)}$, we get also local terms (some of them exactly cancel)

c) up to now, only $U(N)$ is realized; we need to introduce by hand the $\det U_{xy}$ using an assisted pair tunneling.

schematics

b) We need a SU(N) global symmetry

$\Rightarrow$ Alkaline-earth-like fermions

$^{171,173}Yb$ , $^{87}Sr$

$\Rightarrow$ SU(N) ($N \leq 10$) scattering symmetry

There are

$$m_F = \{-I, \dots I\} = 2I+1 \quad \text{Zeeman states}$$

only differ for the nuclear SPIN $\Rightarrow$ NO ELECTRONIC SPIN $(J=0)$


I) $$\frac{a_s[m_F, m_F'] - \langle a_s \rangle}{\langle a_s \rangle} \leq 10^{-9}$$ quite good

II) NO SPIN-CHANGING COLLISIONS

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FINAL PROOF: ~~som~~ perturbation theory in $U/J \gg 1$ gives the

U(N)/SU(N) QLM (+ few other terms which can also be compensated for)