Thermodynamic properties and transport coefficients of a Fermi gas around unitarity

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Superconductivity and superfluidity in Fermi systems

- ✓ Dilute atomic Fermi gases — $T_c \approx 10^{-9}$ eV
- ✓ Liquid ^{3}He — $T_c \approx 10^{-7}$ eV
- ✓ Metals, composite materials — $T_c \approx 10^{-3} – 10^{-2}$ eV
- ✓ Nuclei, neutron stars — $T_c \approx 10^{5} – 10^{6}$ eV
- QCD color superconductivity — $T_c \approx 10^{7} – 10^{8}$ eV

units (1 eV $\approx 10^4$ K)

Why would one want to study cold Fermi gases?

One reason:

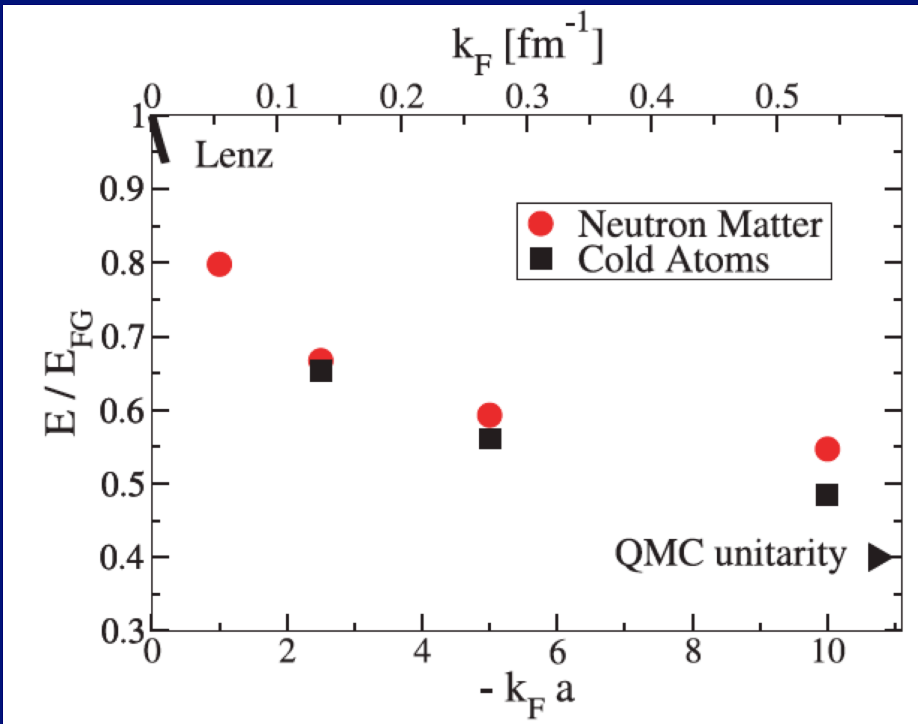
(for the nerds, I mean the hard-core theorists,
not for the phenomenologists)

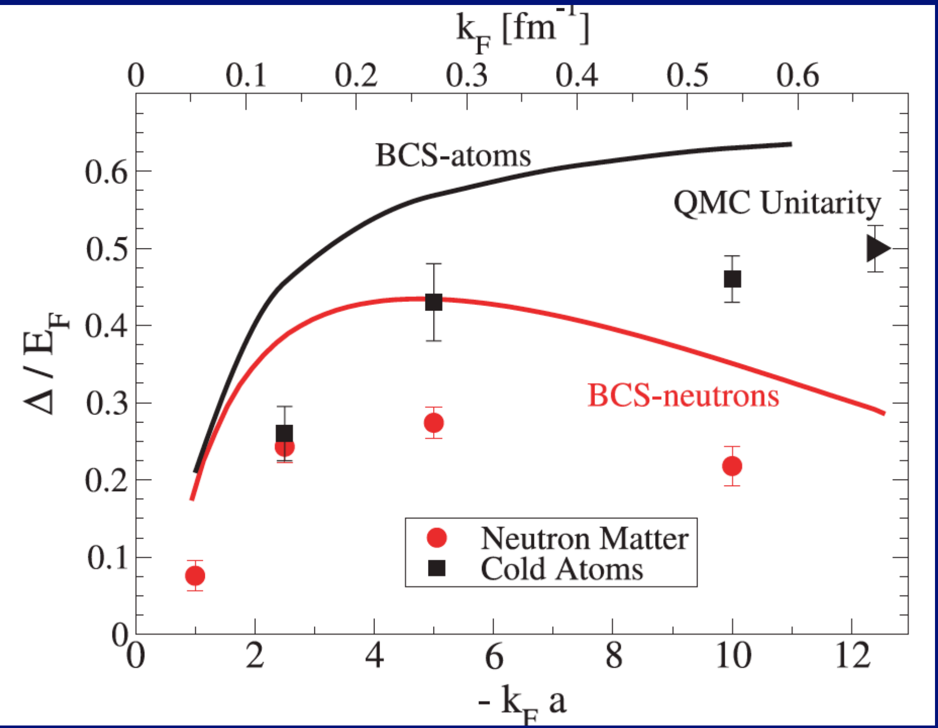
Bertsch's Many-Body X challenge, Seattle, 1999

What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering-length contact interaction?

Besides pure theoretical curiosity, this problem is relevant to neutron stars!

… and a few other systems!





Gezerlis and Carlson,
Phys. Rev. C 77, 032801(R) (2008)

What are the scattering length and the effective range?

$$k \text{ cotan } \delta_0 = -\frac{1}{a} + \frac{1}{2} r_{eff} k^2 + \cdots$$

$$\sigma = \frac{4\pi}{k^2} \sin^2 \delta_0 + \cdots = 4\pi \frac{a^2}{1 + k^2 a^2} + \cdots \underset{k \to 0}{=} 4\pi a^2 + \cdots$$

If the energy is small only the s-wave is relevant.

Let me consider at first as an instructive example the hydrogen atom.

The ground state energy could only be a function of:

- ✓ Electron charge
- ✓ Electron mass
- ✓ Planck's constant

and then trivial dimensional arguments lead to

$$E_{gs} = \frac{e^4 m}{\hbar^2} \times \frac{1}{2}$$

Only the factor ½ requires some hard work (Quantum Mechanics).

Let me now turn to dilute fermion matter

The ground state energy is given by such a function:

$$E_{gs} = f(N, V, \hbar, m, a, r_0)$$

Taking the scattering length to infinity and the range of the interaction to zero, we are left with:

$$E_{gs} = F(N, V, \hbar, m) = \frac{3}{5}\varepsilon_F N \times \xi$$

$$\frac{N}{V} = \frac{k_F^3}{3\pi^2}, \qquad \varepsilon_F = \frac{\hbar^2 k_F^2}{2m}$$

Pure number

What George Bertsch essentially asked in 1999 is:

What is the value of ξ ?! Is it positive?

But he wished to know the properties of the system as well:

The system turned out to be superfluid !

$$E_{gs} = \frac{3}{5}\varepsilon_F N \times \xi \qquad \Delta = \varepsilon_F \times \varsigma$$

$$\xi = 0.372(5), \qquad \zeta = 0.45(5)$$

Now these results are a bit unexpected.

✓ The energy looks almost like that of *a non-interacting system*!
(there are no other dimensional parameters in the problem)

✓ The system has *a huge pairing gap*!

✓ This system is a very strongly interacting one,
since the *elementary cross section is huge*!

The initial Bertsch's Many Body challenge has evolved over time and became the problem of *Fermions in the Unitary Regime.*
(this is part of the BCS-BEC crossover problem)

In cold old gases one can control the strength of the interaction!

The system is very dilute, but strongly interacting!

$$\boxed{n\, r_0^3 \ll 1} \qquad \boxed{n\, |a|^3 \gg 1}$$

n - number density

$$\boxed{r_0 \ll n^{-1/3} \approx \lambda_F/2 \ll |a|}$$

r_0 - range of interaction

a - scattering length

Finite Temperatures

Grand Canonical Path-Integral Monte Carlo

$$\hat{H} = \hat{T} + \hat{V} = \int d^3x \left[\psi_\uparrow^\dagger(\vec{x}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \psi_\uparrow(\vec{x}) + \psi_\downarrow^\dagger(\vec{x}) \left(-\frac{\hbar^2 \Delta}{2m} \right) \psi_\downarrow(\vec{x}) \right] - g \int d^3x \; \hat{n}_\uparrow(\vec{x}) \hat{n}_\downarrow(\vec{x})$$

$$\hat{N} = \int d^3x \; \left[\hat{n}_\uparrow(\vec{x}) + \hat{n}_\downarrow(\vec{x}) \right], \qquad \hat{n}_s(\vec{x}) = \psi_s^\dagger(\vec{x}) \psi_s(\vec{x}), \qquad s = \uparrow, \downarrow$$

Trotter expansion

$$Z(\beta) = \mathrm{Tr}\; \exp\left[-\beta \left(\hat{H} - \mu \hat{N} \right) \right] = \mathrm{Tr}\; \left\{ \exp\left[-\tau \left(\hat{H} - \mu \hat{N} \right) \right] \right\}^{N_\tau}, \qquad \beta = \frac{1}{T} = N_\tau \tau$$

$$E(T) = \frac{1}{Z(T)} \mathrm{Tr}\, \hat{H} \exp\left[-\beta \left(\hat{H} - \mu \hat{N} \right) \right]$$

$$N(T) = \frac{1}{Z(T)} \mathrm{Tr}\, \hat{N} \exp\left[-\beta \left(\hat{H} - \mu \hat{N} \right) \right]$$

No approximations so far, except for the fact that the interaction is not well defined!

Recast the propagator at each time slice and put the system on a 3D-spatial lattice, in a cubic box of side $L=N_s l$, with periodic boundary conditions

$$\exp\left[-\tau\left(\hat{H}-\mu\hat{N}\right)\right]\approx\exp\left[-\tau\left(\hat{T}-\mu\hat{N}\right)/2\right]\exp(-\tau\hat{V})\exp\left[-\tau\left(\hat{T}-\mu\hat{N}\right)/2\right]+O(\tau^3)$$

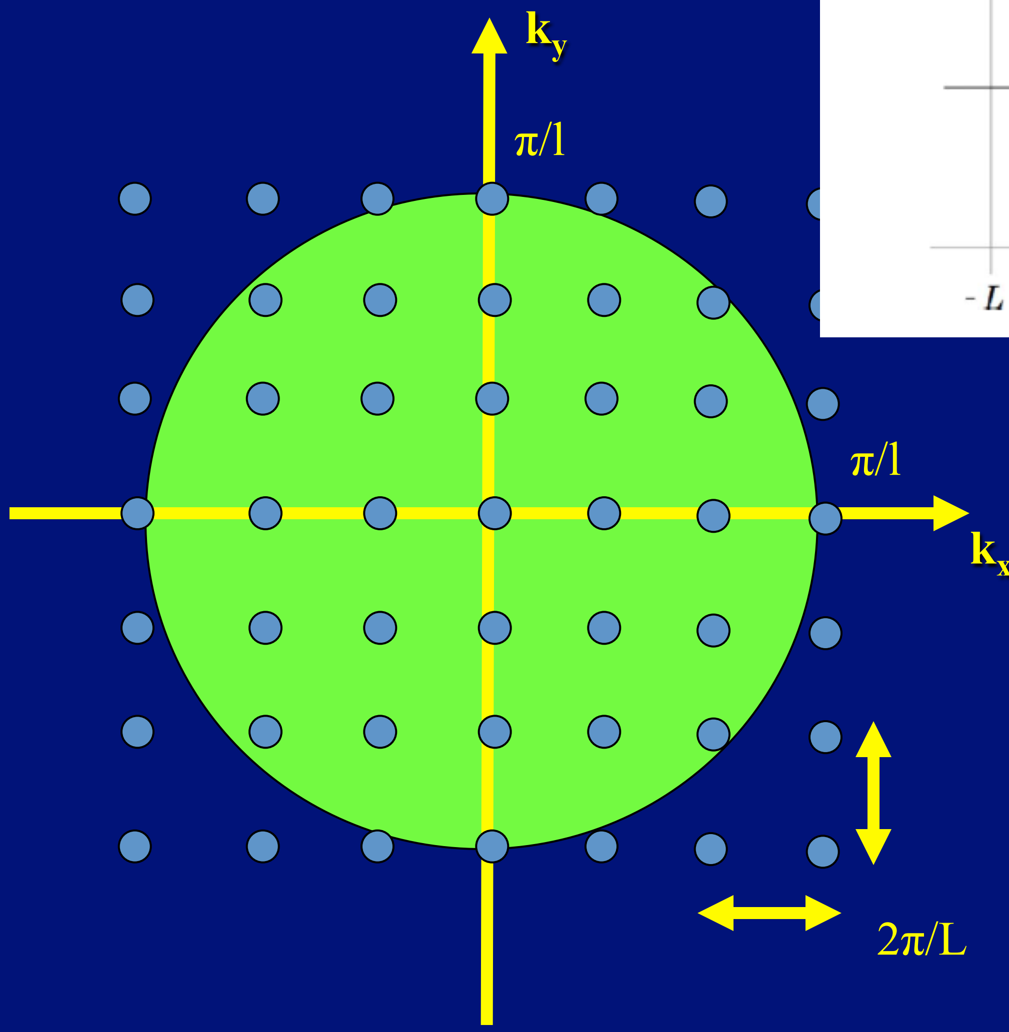
Discrete Hubbard-Stratonovich transformation

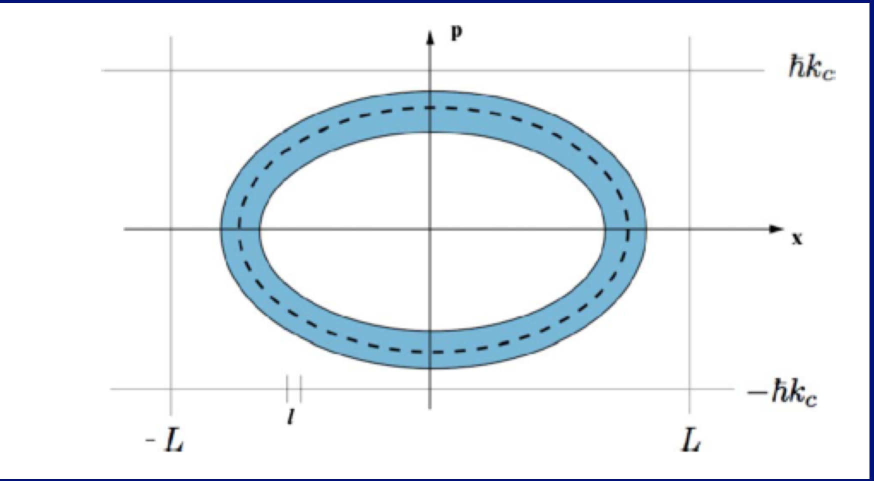
$$\exp(-\tau\hat{V})=\prod_{\vec{x}}\sum_{\sigma_{\pm}(\vec{x})=\pm 1}\frac{1}{2}\left[1+\sigma_{\pm}(\vec{x})A\hat{n}_{\uparrow}(\vec{x})\right]\left[1+\sigma_{\pm}(\vec{x})A\hat{n}_{\downarrow}(\vec{x})\right],\qquad A=\sqrt{\exp(\tau g)-1}$$

σ-fields fluctuate both in space and imaginary time

$$\frac{m}{4\pi\hbar^2 a}=-\frac{1}{g}+\frac{mk_c}{2\pi^2\hbar^2},\qquad k_c<\frac{\pi}{l}.\qquad \mathrm{r}_{eff}=\frac{4}{\pi k_c}$$

Running coupling constant g defined by lattice





$$\varepsilon_F,\ \Delta,\ T \ll \frac{\hbar^2\pi^2}{2ml^2}$$

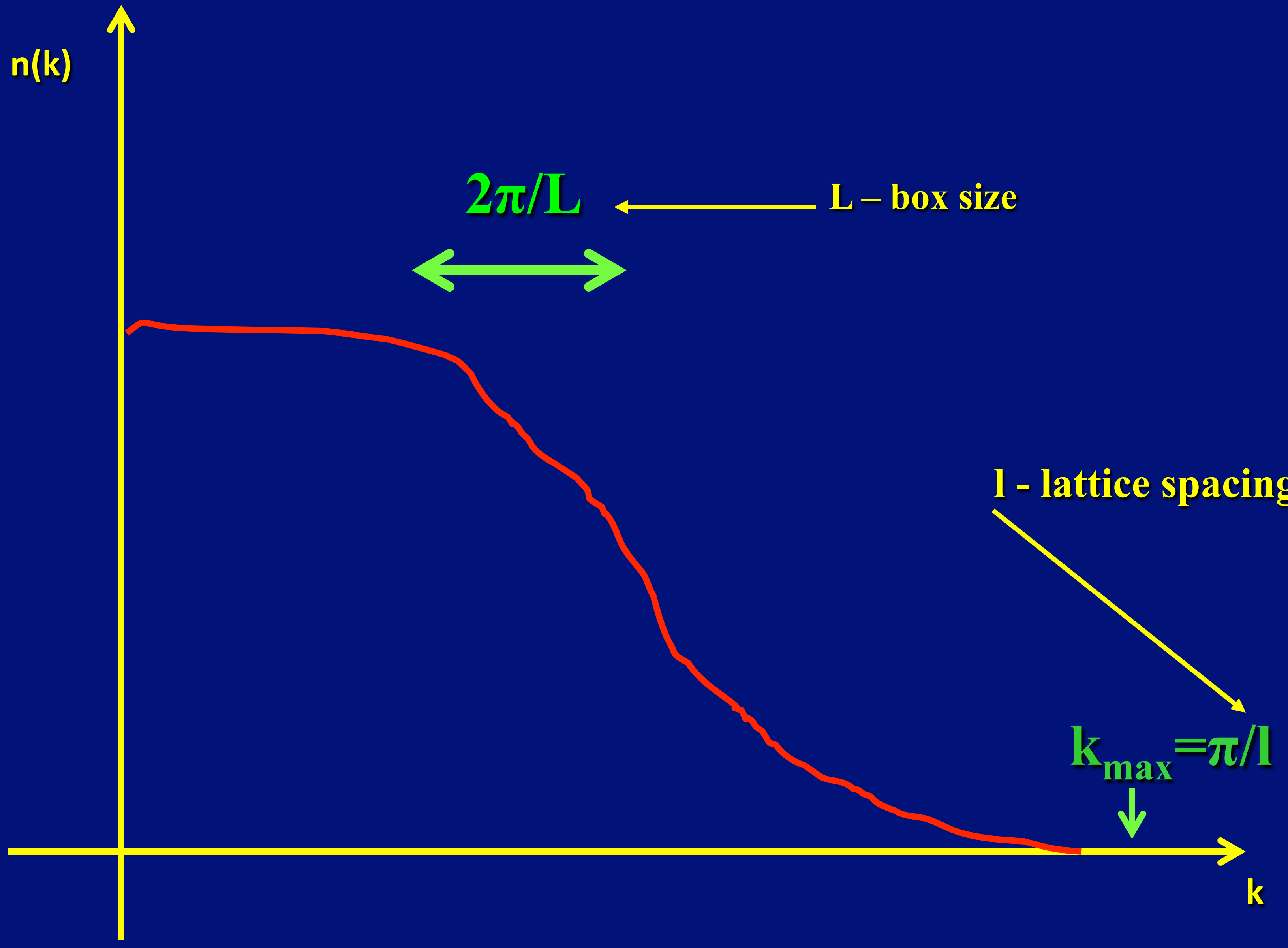
$$\delta\varepsilon > \frac{2\hbar^2\pi^2}{mL^2}$$

$$\varepsilon_F,\ \Delta \gg \frac{2\hbar^2\pi^2}{mL^2}$$

$$\xi_{coh} \ll L = N_s l$$

$$\delta p > \frac{2\pi\hbar}{L}$$

Momentum space



How to choose the lattice spacing and the box size?

$$Z(T) = \int \prod_{\vec{x},\tau} D\sigma(\vec{x},\tau)\ \mathrm{Tr}\,\hat{U}(\{\sigma\})$$

$$\hat{U}(\{\sigma\}) = T_\tau \prod_{\tau} \exp\{-\tau[\hat{h}(\{\sigma\}) - \mu]\}$$

One-body evolution operator in imaginary time

$$E(T) = \int \frac{\prod_{\vec{x},\tau} D\sigma(\vec{x},\tau) \mathrm{Tr}\,\hat{U}(\{\sigma\})}{Z(T)} \frac{\mathrm{Tr}\left[\hat{H}\hat{U}(\{\sigma\})\right]}{\mathrm{Tr}\,\hat{U}(\{\sigma\})}$$

$$\mathrm{Tr}\,\hat{U}(\{\sigma\}) = \{\det[1+\hat{U}(\{\sigma\})]\}^2 = \exp[-S(\{\sigma\})] > 0$$

No sign problem!

$$n_\uparrow(\vec{x},\vec{y}) = n_\downarrow(\vec{x},\vec{y}) = \sum_{k,l<k_c} \varphi_{\vec{k}}(\vec{x}) \left[\frac{\hat{U}(\{\sigma\})}{1+\hat{U}(\{\sigma\})}\right]_{\vec{k}\vec{l}} \varphi^*_{\vec{l}}(\vec{y}), \qquad \varphi_{\vec{k}}(\vec{x}) = \frac{\exp(i\vec{k}\cdot\vec{x})}{\sqrt{V}}$$

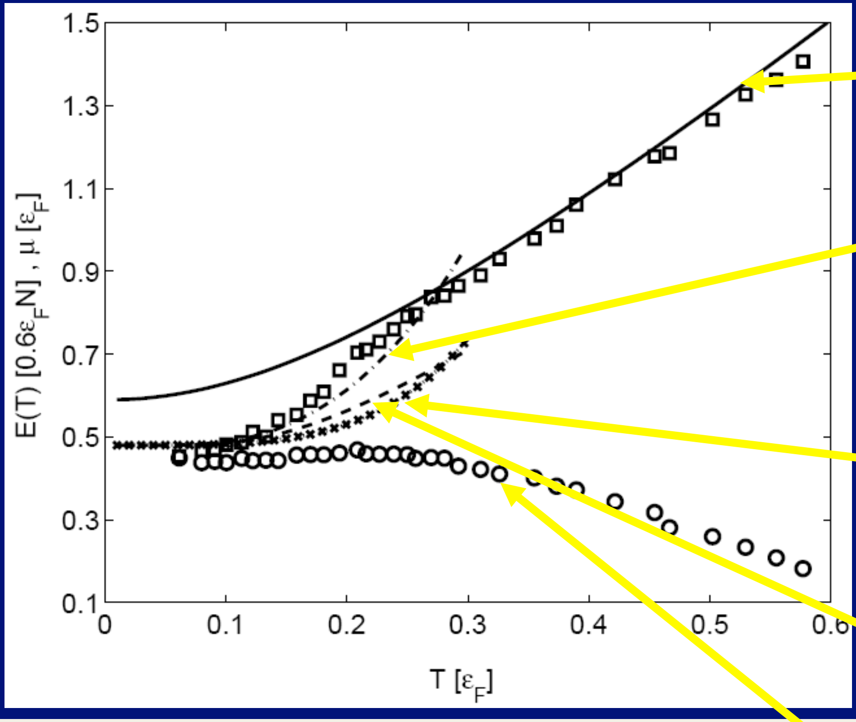
All traces can be expressed through these single-particle density matrices

One can thus determine as a function of T, V and chemical potential:

- ✓ **Total Energy**
- ✓ **Particle number**
- ✓ **Entropy of the system**
- ✓ **Pressure**
- ✓ **Spectrum of fermionic elementary excitations (pairing gap, pseudogap, effective mass, self-energy)**
- ✓ **Long range order, condensate fraction (onset of phase transition, critical temperature)**

$a = \pm\infty$

Bulgac, Drut, and Magierski
Phys. Rev. Lett. 96, 090404 (2006)



Normal Fermi Gas
(with vertical offset, solid line)

Bogoliubov-Anderson phonons and quasiparticle contribution (dot-dashed line)

Bogoliubov-Anderson phonons contribution only

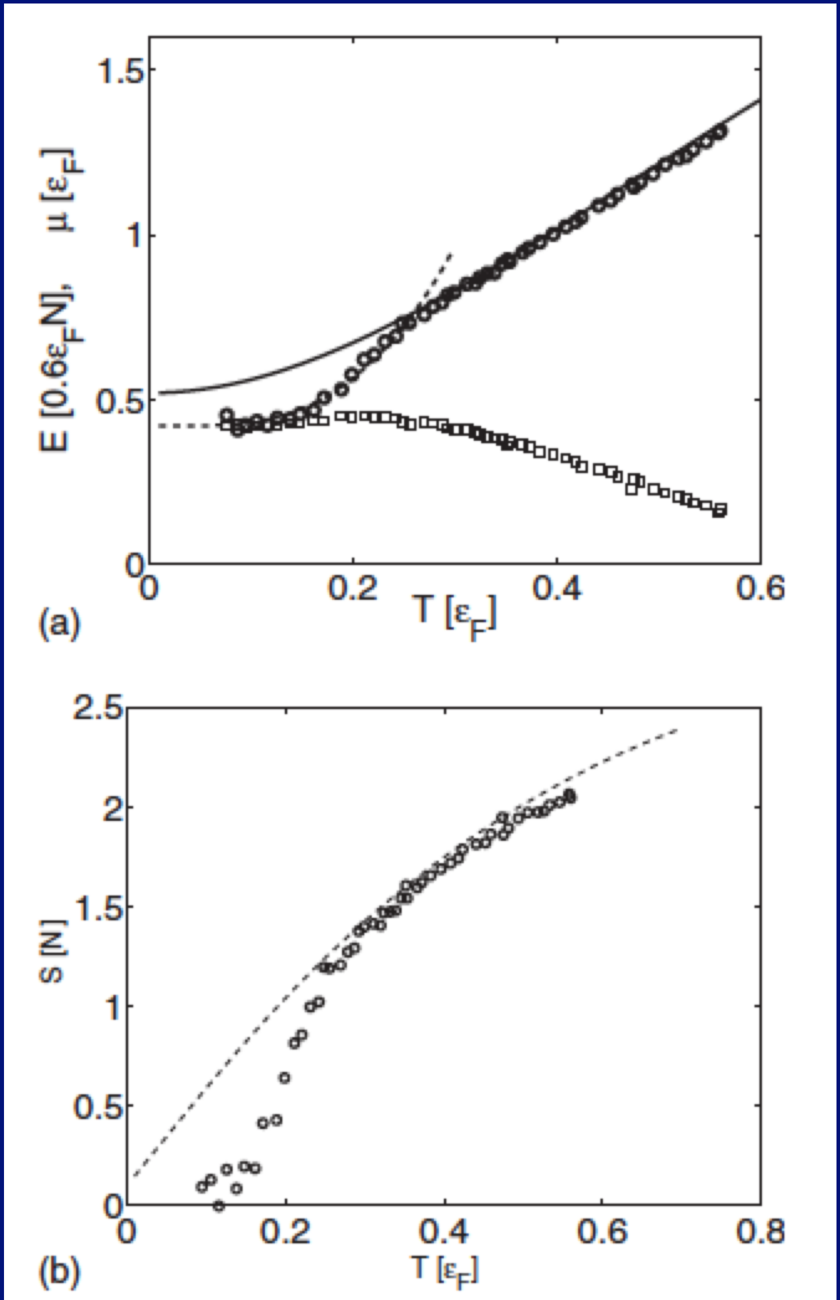
Quasi-particles contribution only (dashed line)

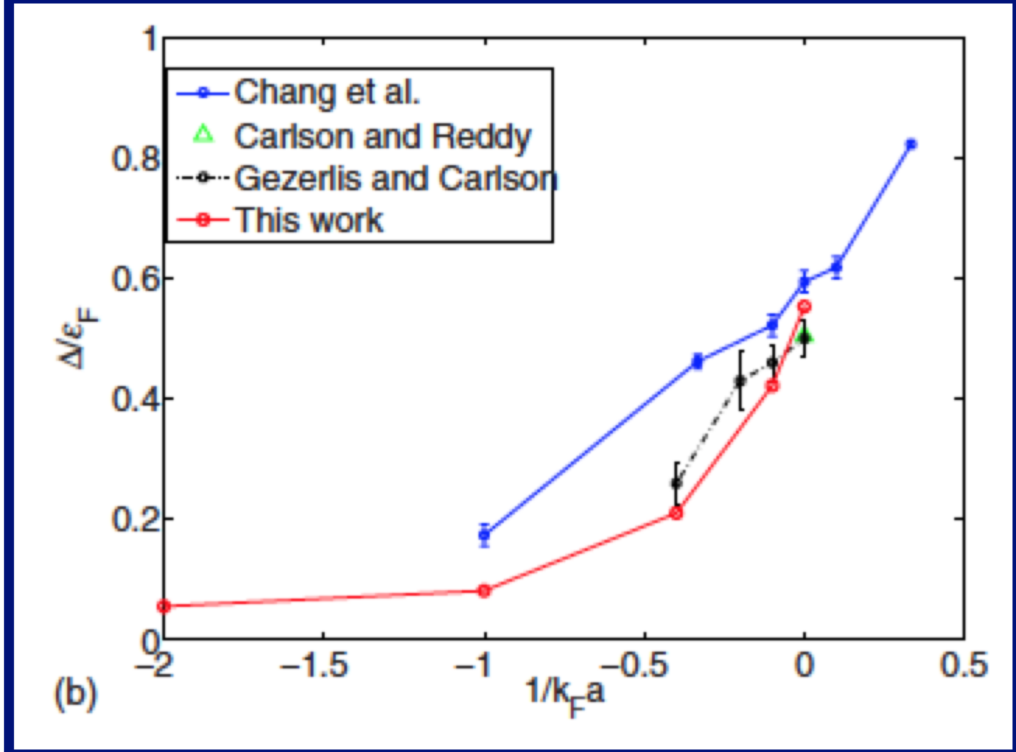
μ - chemical potential (circles)

$$E_{\text{phonons}}(T) = \frac{3}{5}\varepsilon_F N \; \frac{\sqrt{3}\pi^4}{16\xi_s^{3/2}}\left(\frac{T}{\varepsilon_F}\right)^4, \qquad \xi_s \approx 0.44$$

$$E_{\text{quasi-particles}}(T) = \frac{3}{5}\varepsilon_F N \; \frac{5}{2}\sqrt{\frac{2\pi\Delta^3 T}{\varepsilon_F^4}}\exp\left(-\frac{\Delta}{T}\right)$$

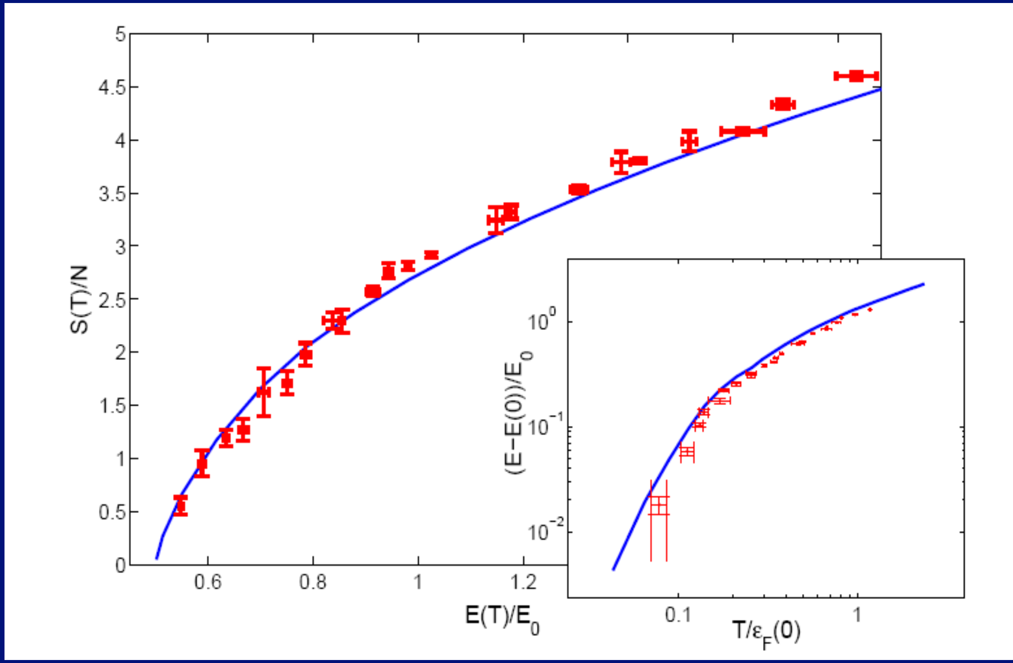
$$\Delta = \left(\frac{2}{e}\right)^{7/3}\varepsilon_F \exp\left(\frac{\pi}{2k_F a}\right)$$

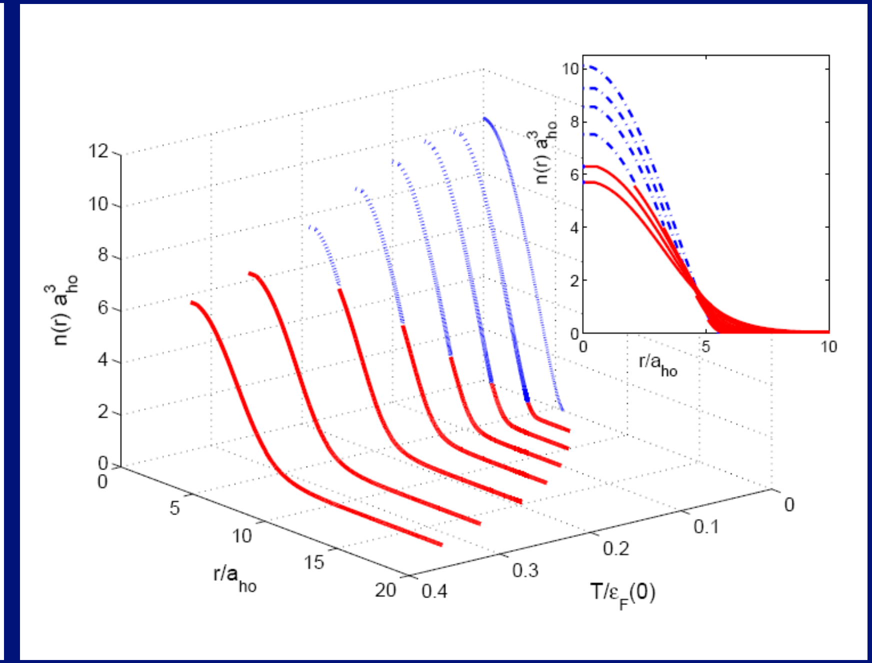

E [0.6εFN], μ [εF]
T [εF]
(a)
S [N]
T [εF]
(b)


Chang et al.
Carlson and Reddy
Gezerlis and Carlson
This work
Δ/εF
1/kFa
(b)

Experiment (about 100,000 atoms in a trap):

Measurement of the Entropy and Critical Temperature of a Strongly Interacting Fermi Gas,
Luo, Clancy, Joseph, Kinast, and Thomas, Phys. Rev. Lett. 98, 080402 (2007)





Full *ab initio* theory (no free parameters)

Bulgac, Drut, and Magierski, Phys. Rev. Lett. 99, 120401 (2007)

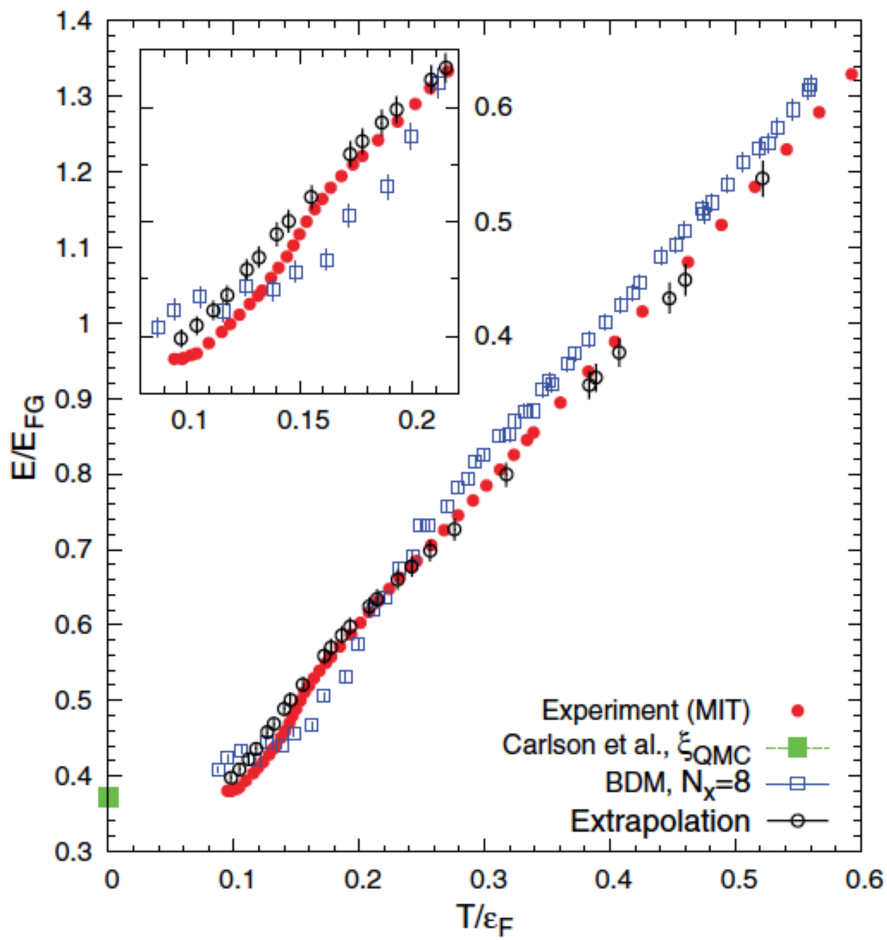


FIG. 2. (Color online) Energy E/E_{FG} (red dots), as obtained by Ku *et al.* [8]. Our AFQMC results extrapolated to infinite volume are shown by open black circles. The results for $N_x = 8$ (open blue squares) were obtained with the DMC algorithm in Ref. [9]. The green square shows the QMC result of Ref. [20] for ξ at $T = 0$. The inset shows the vicinity of the superfluid phase transition at $T_c/\epsilon_F \simeq 0.15$.

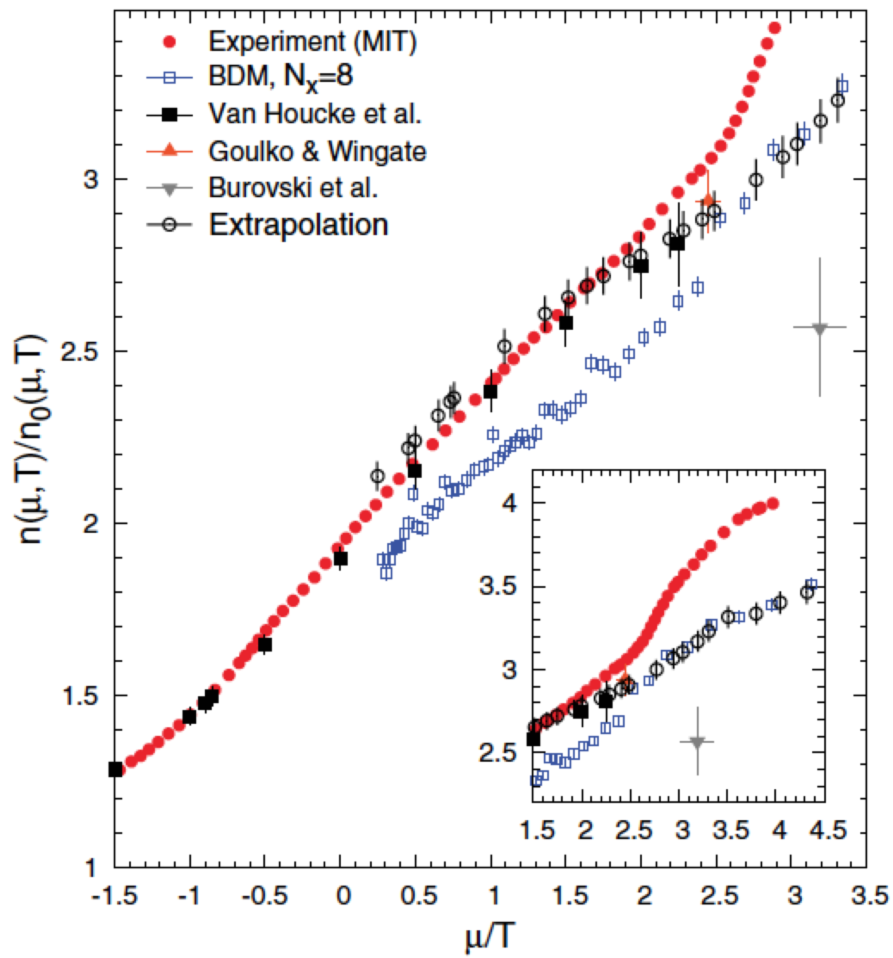
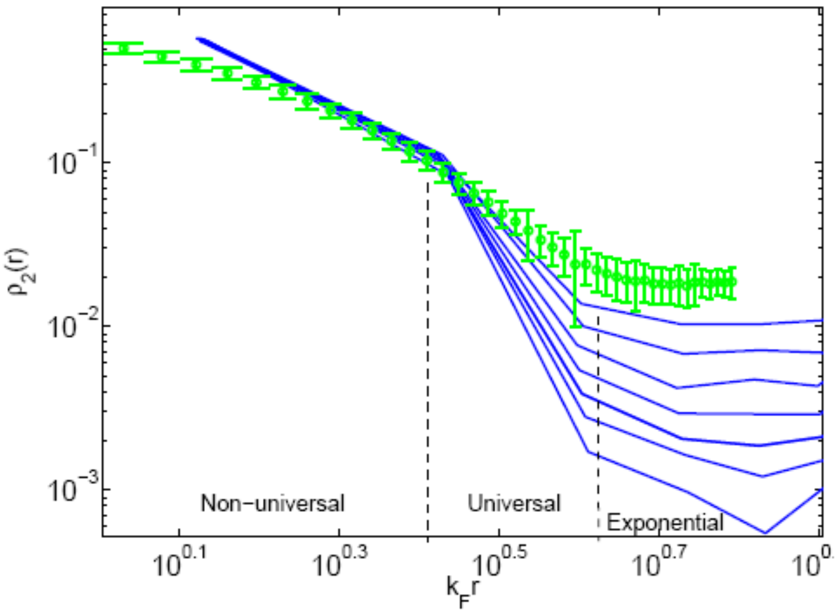
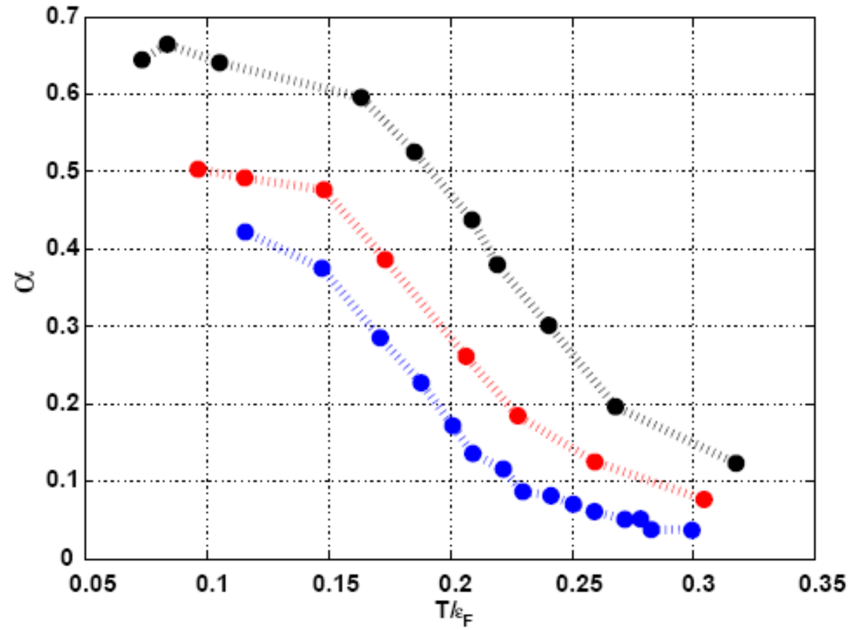


FIG. 4. (Color online) Density $n(\mu,T)$ of the UFG (red circles) as obtained by Ku *et al.* [8], normalized to the density $n_0(\mu,T)$ of a noninteracting Fermi gas. The notation for the AFQMC results is identical to Fig. 2. The diagrammatic MC results of Refs. [21,22] (solid up and down triangles) and the Bold Diagrammatic MC results of Ref. [23] are shown as well (solid squares). The inset shows the vicinity of the superfluid phase transition at $T_c/\epsilon_F \simeq 0.15$.

Long range order and condensate fraction



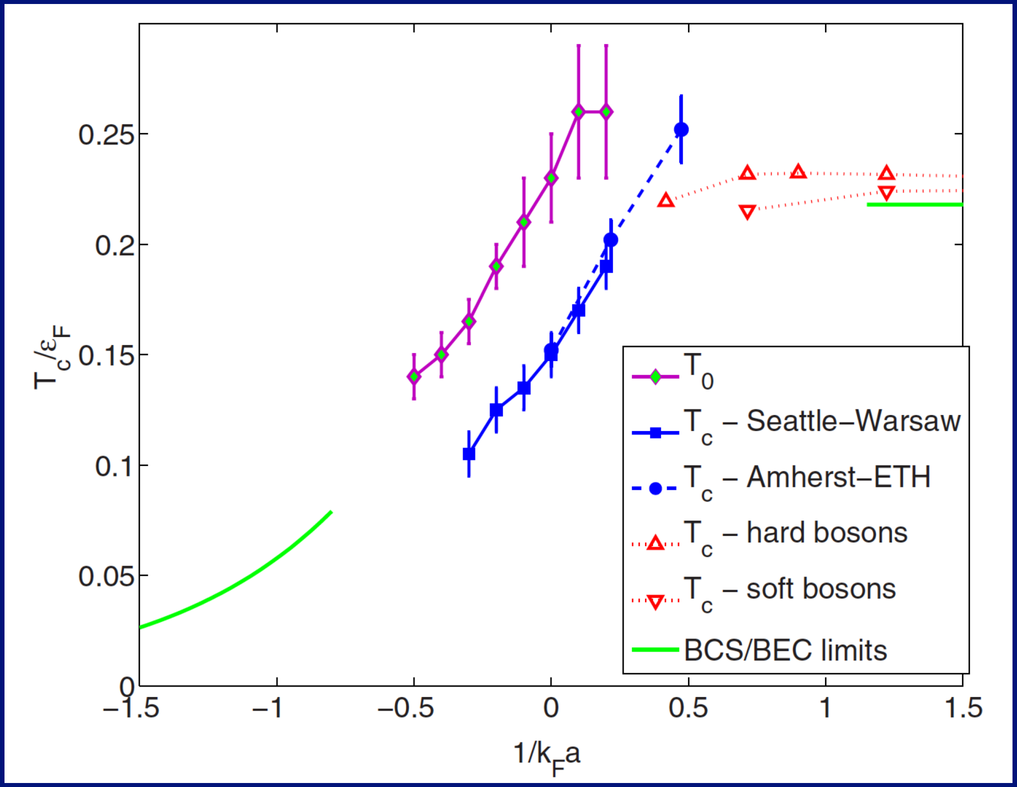


$$g_2(\vec{r}) = \left(\frac{2}{N}\right)^2 \int d^3\vec{r}_1 \int d^3\vec{r}_2 \left\langle \psi_\uparrow^\dagger(\vec{r}_1+\vec{r})\psi_\downarrow^\dagger(\vec{r}_2+\vec{r})\psi_\downarrow(\vec{r}_2)\psi_\uparrow(\vec{r}_2)\right\rangle$$

$$\alpha = \lim_{r\to\infty} \frac{N}{2} g_2(\vec{r}) - n(\vec{r})^2, \qquad n(\vec{r}) = \frac{2}{N}\int d^3\vec{r}_1 \left\langle \psi_\uparrow^\dagger(\vec{r}_1+\vec{r})\psi_\uparrow(\vec{r}_1)\right\rangle$$

Bulgac, Drut, and Magierski, Phys. Rev. A 78, 023625 (2008)

Critical temperature for superfluid to normal transition



Bulgac, Drut, and Magierski, Phys. Rev. A 78, 023625 (2008)

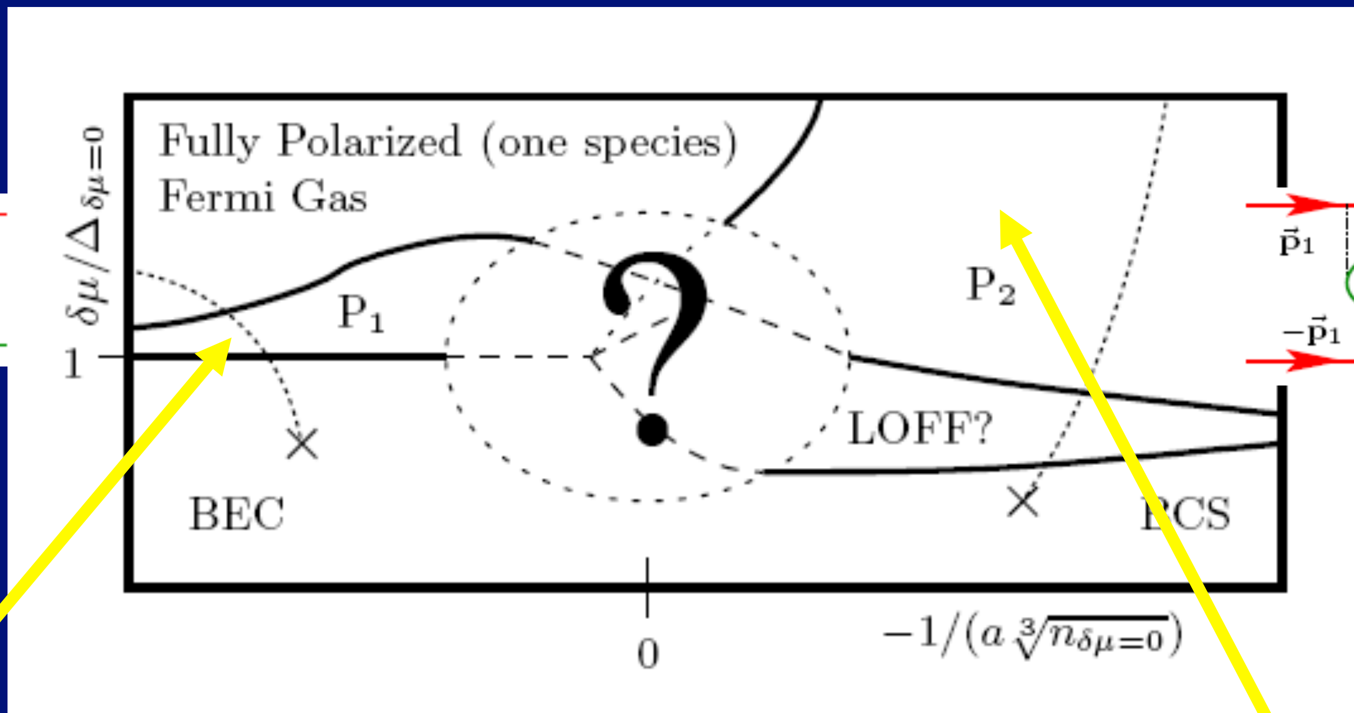
Amherst-ETH: Burovski et al. Phys. Rev. Lett. 101, 090402 (2008)

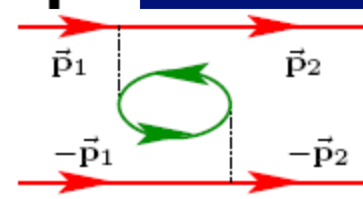
Hard and soft bosons: Pilati et al. PRL 100, 140405 (2008)

What is happening in spin imbalanced systems?

Induced P-wave superfluidity (*even though fermions interact in s-wave only)*

Two new superfluid phases where before they were not expected



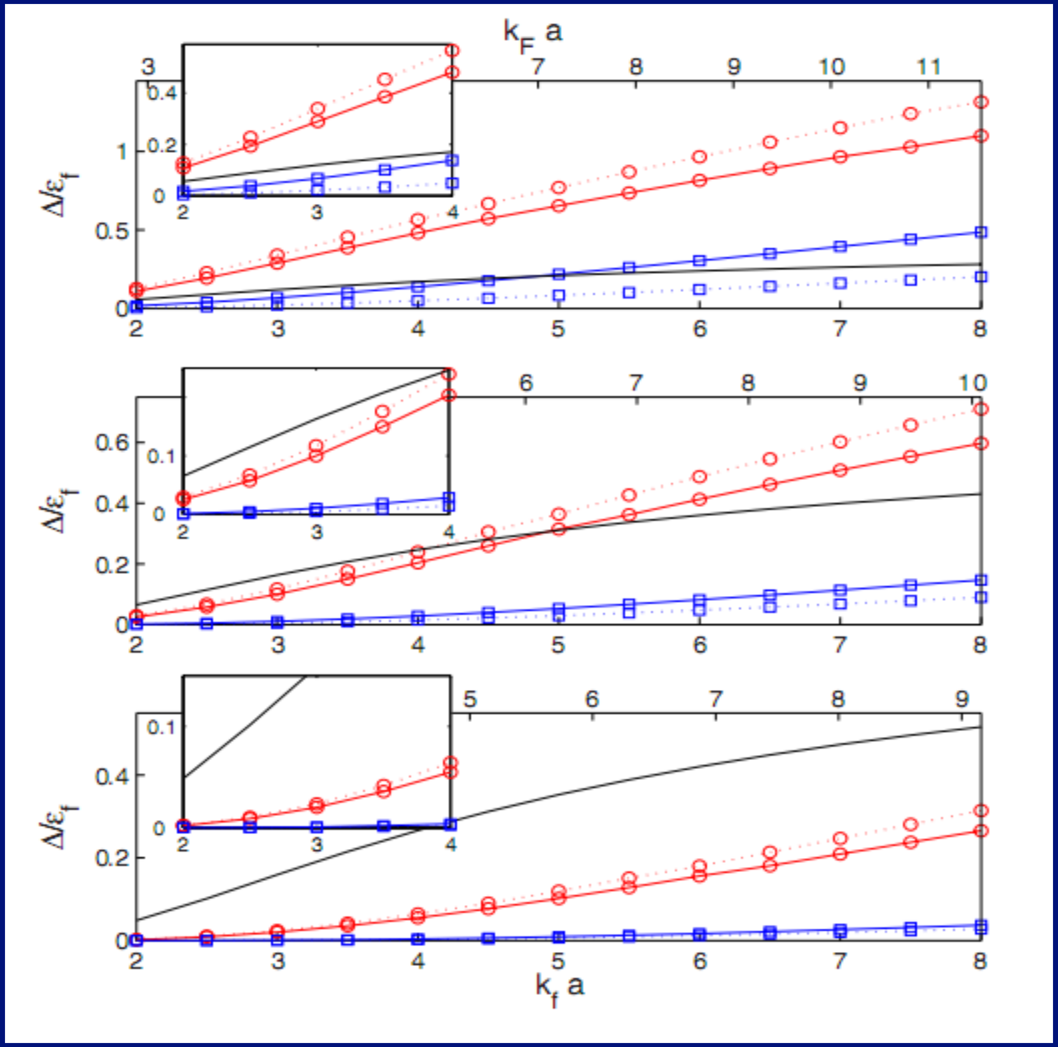


One Bose superfluid coexisting with one P-wave Fermi superfluid

Two coexisting P-wave Fermi superfluids

Bulgac, Forbes, and Schwenk, Phys. Rev. Lett. 97, 020402 (2006)

Going beyond the naïve BCS approximation



Eliashberg approx. (red)

BCS approx. (black)

Full momentum and frequency dependence of the self-consistent equations (red)

Bulgac and Yoon, Phys. Rev. A 79, 053625 (2009)

Dimensional arguments and Legendre transform for unitary Fermi gas

$$P(\mu_\uparrow,\mu_\downarrow)=\mu_\uparrow n_\uparrow+\mu_\downarrow n_\downarrow-\varepsilon\left(n_\uparrow,n_\downarrow\right)=\frac{2}{3}\varepsilon\left(n_\uparrow,n_\downarrow\right)$$

$$P(\mu_\uparrow,\mu_\downarrow)=\frac{2}{5}\beta\left[\mu_\uparrow h\left(\frac{\mu_\downarrow}{\mu_\uparrow}\right)\right]^{5/2},\qquad \beta=\frac{1}{6\pi^2}\left[\frac{2m}{\hbar^2}\right]^{3/2}$$

$$\varepsilon\left(n_\uparrow,n_\downarrow\right)=\frac{3}{5}\alpha\left[n_\uparrow g\left(\frac{n_\downarrow}{n_\uparrow}\right)\right]^{5/3},\qquad \alpha=\frac{\left(6\pi^2\right)^{2/3}\hbar^2}{2m}$$

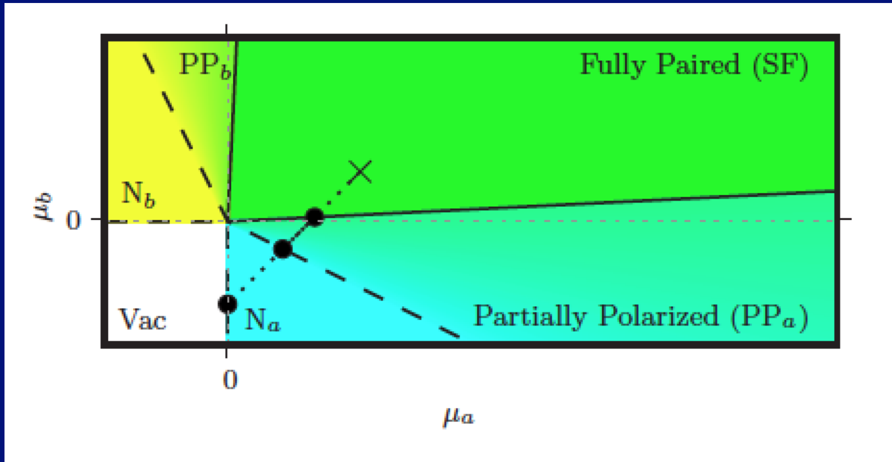
$$h(y)=\begin{cases}1, & y\le y_0<0\\ \dfrac{1+y}{(2\xi)^{3/5}}, & y_1\le y\le 1\end{cases}$$

$$h''(y)\ge 0$$

$$g(0)=1,\quad \mathrm{g}(1)=\left(2\xi\right)^{3/5}$$

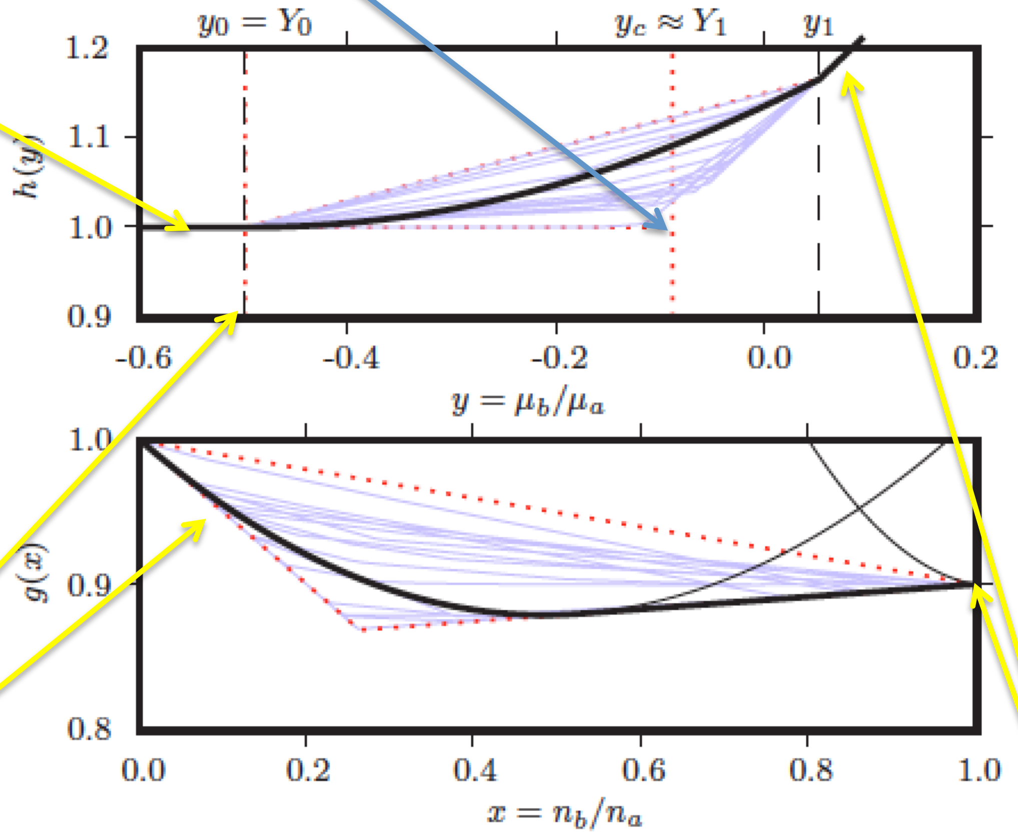
$$g''(x)\ge 0$$

$$g'(0)\le Y_0,\quad \mathrm{g}'(\mathrm{x})\in g(1)\left[\frac{Y_1}{1+Y_1},\frac{1}{2}\right]$$



Fully polarized

$$\frac{\mu_\uparrow - \mu_\downarrow}{2} = \Delta$$



From energy of one spin-down particle in a see of spin-ups

$$P_{SF} = \frac{4\beta}{5\xi^{3/2}}\left(\frac{\mu_\uparrow + \mu_\downarrow}{2}\right)^{5/2}$$

Unpolarized SF

FIG. 2. (Color online) Example of a function $h(y)$ and the corresponding function $g(x)$ shown as thick lines. Maxwell's construction for phase coexistence leads to a linear $g(x)$ for $x \in (0.5, 1.0)$, interpolating between the two pure phases shown with lighter lines. This corresponds to the kink and/or first-order phase transition at $y = y_1$ in $h(y)$. Various other sample functions are lightly sketched within the allowed (dotted) triangular region.

Asymmetric Superfluid Local Density Approximation

$$\Omega = -\int d^3r \left[\varepsilon(\vec{r}) - \mu_\uparrow n_\uparrow(\vec{r}) - \mu_\downarrow n_\downarrow(\vec{r}) - V_{ext}(\vec{r}) n_\uparrow(\vec{r}) - V_{ext}(\vec{r}) n_\downarrow(\vec{r}) \right]$$

$$\varepsilon(\vec{r}) = \frac{\hbar^2}{2m}\left[\alpha_\uparrow(\vec{r})\tau_\uparrow(\vec{r}) + \alpha_\downarrow(\vec{r})\tau_\downarrow(\vec{r})\right] + \frac{3\left(3\pi^2\right)^{2/3}\hbar^2}{10m}\left[n_\uparrow(\vec{r}) + n_\downarrow(\vec{r})\right]^{5/3}\beta(\vec{r}) + g_{eff}(\vec{r})\left|\upsilon(\vec{r})\right|^2$$

$$n_\uparrow(\vec{r}) = \sum_{E_n<0}\left|\mathrm{u}_n(\vec{r})\right|^2, \quad n_\downarrow(\vec{r}) = \sum_{E_n<0}\left|\mathrm{v}_n(\vec{r})\right|^2, \quad \upsilon(\vec{r}) = \frac{1}{2}\sum_{E_n}\mathrm{sgn}(E_n)\mathrm{u}_n(\vec{r})\mathrm{v}_n^*(\vec{r}),$$

$$\tau_\uparrow(\vec{r}) = \sum_{E_n<0}\left|\vec{\nabla}\mathrm{u}_n(\vec{r})\right|^2, \quad \tau_\downarrow(\vec{r}) = \sum_{E_n<0}\left|\vec{\nabla}\mathrm{v}_n(\vec{r})\right|^2,$$

$$\alpha_\uparrow(\vec{r}) = \alpha\left[\frac{n_\downarrow(\vec{r})}{n_\uparrow(\vec{r})}\right], \quad \alpha_\downarrow(\vec{r}) = \alpha\left[\frac{n_\uparrow(\vec{r})}{n_\downarrow(\vec{r})}\right], \quad \beta(\vec{r}) = \beta\left[\frac{n_\downarrow(\vec{r})}{n_\uparrow(\vec{r})}\right] = \beta\left[\frac{n_\uparrow(\vec{r})}{n_\downarrow(\vec{r})}\right],$$

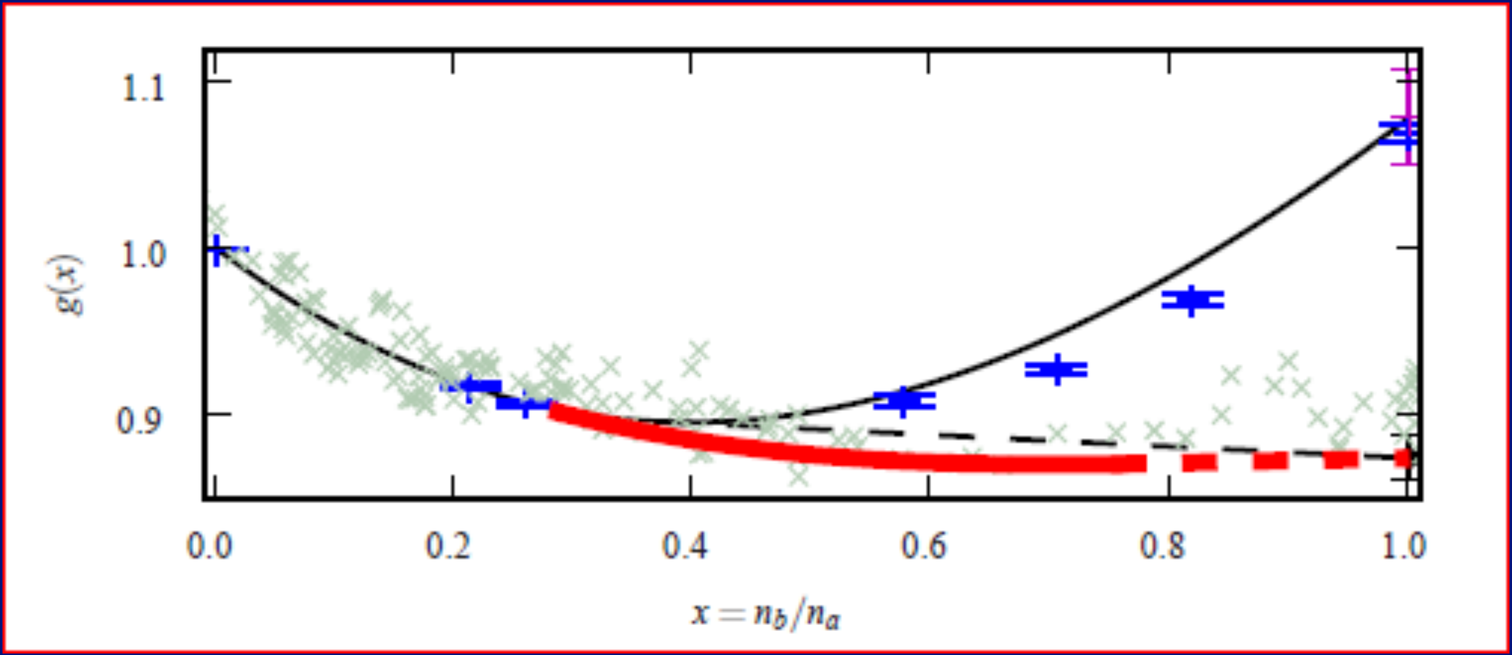
$$\begin{pmatrix} T_\uparrow(\vec{r}) + U_\uparrow(\vec{r}) - \mu_\uparrow & \Delta(\vec{r}) \\ \Delta^*(\vec{r}) & -T_\downarrow(\vec{r}) - U_\downarrow(\vec{r}) + \mu_\downarrow \end{pmatrix}\begin{pmatrix} \mathrm{u}_n(\vec{r}) \\ \mathrm{v}_n(\vec{r}) \end{pmatrix} = E_n\begin{pmatrix} \mathrm{u}_n(\vec{r}) \\ \mathrm{v}_n(\vec{r}) \end{pmatrix}$$

Normal State			
(N_a, N_b)	E_{FNDMC}	E_{ASLDA}	(error)
(3,1)	6.6 ± 0.01	6.687	1.3%
(4,1)	8.93 ± 0.01	8.962	0.36%
(5,1)	12.1 ± 0.1	12.22	0.97%
(5,2)	13.3 ± 0.1	13.54	1.8%
(6,1)	15.8 ± 0.1	15.65	0.93%
(7,2)	19.9 ± 0.1	20.11	1.1%
(7,3)	20.8 ± 0.1	21.23	2.1%
(7,4)	21.9 ± 0.1	22.42	2.4%
(8,1)	22.5 ± 0.1	22.53	0.14%
(9,1)	25.9 ± 0.1	25.97	0.27%
(9,2)	26.6 ± 0.1	26.73	0.5%
(9,3)	27.2 ± 0.1	27.55	1.3%
(9,5)	30 ± 0.1	30.77	2.6%
(10,1)	29.4 ± 0.1	29.41	0.034%
(10,2)	29.9 ± 0.1	30.05	0.52%
(10,6)	35 ± 0.1	35.93	2.7%
(20,1)	73.78 ± 0.01	73.83	0.061%
(20,4)	73.79 ± 0.01	74.01	0.3%
(20,10)	81.7 ± 0.1	82.57	1.1%
(20,20)	109.7 ± 0.1	113.8	3.7%
(35,4)	154 ± 0.1	154.1	0.078%
(35,10)	158.2 ± 0.1	158.6	0.27%
(35,20)	178.6 ± 0.1	180.4	1%

Superfluid State			
(N_a, N_b)	E_{FNDMC}	E_{ASLDA}	(error)
(1,1)	2.002 ± 0	2.302	15%
(2,2)	5.051 ± 0.009	5.405	7%
(3,3)	8.639 ± 0.03	8.939	3.5%
(4,4)	12.573 ± 0.03	12.63	0.48%
(5,5)	16.806 ± 0.04	16.19	3.7%
(6,6)	21.278 ± 0.05	21.13	0.69%
(7,7)	25.923 ± 0.05	25.31	2.4%
(8,8)	30.876 ± 0.06	30.49	1.2%
(9,9)	35.971 ± 0.07	34.87	3.1%
(10,10)	41.302 ± 0.08	40.54	1.8%
(11,11)	46.889 ± 0.09	45	4%
(12,12)	52.624 ± 0.2	51.23	2.7%
(13,13)	58.545 ± 0.18	56.25	3.9%
(14,14)	64.388 ± 0.31	62.52	2.9%
(15,15)	70.927 ± 0.3	68.72	3.1%
(1,0)	1.5 ± 0.0	1.5	0%
(2,1)	4.281 ± 0.004	4.417	3.2%
(3,2)	7.61 ± 0.01	7.602	0.1%
(4,3)	11.362 ± 0.02	11.31	0.49%
(7,6)	24.787 ± 0.09	24.04	3%
(11,10)	45.474 ± 0.15	43.98	3.3%
(15,14)	69.126 ± 0.31	62.55	9.5%

Bulgac, Forbes, and Magierski, Lecture Notes in Physics (2012)

EOS for spin polarized systems



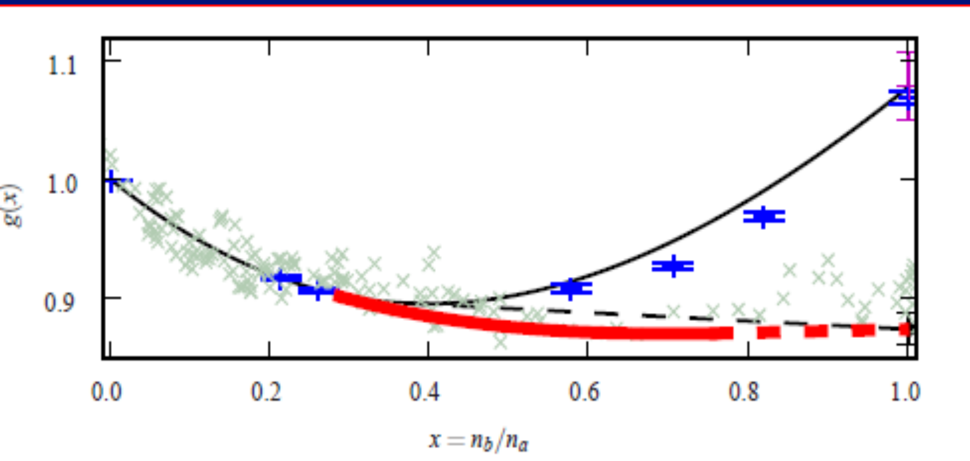
<u>**Red line: Larkin-Ovchinnikov phase (unitary Fermi supersolid)**</u>

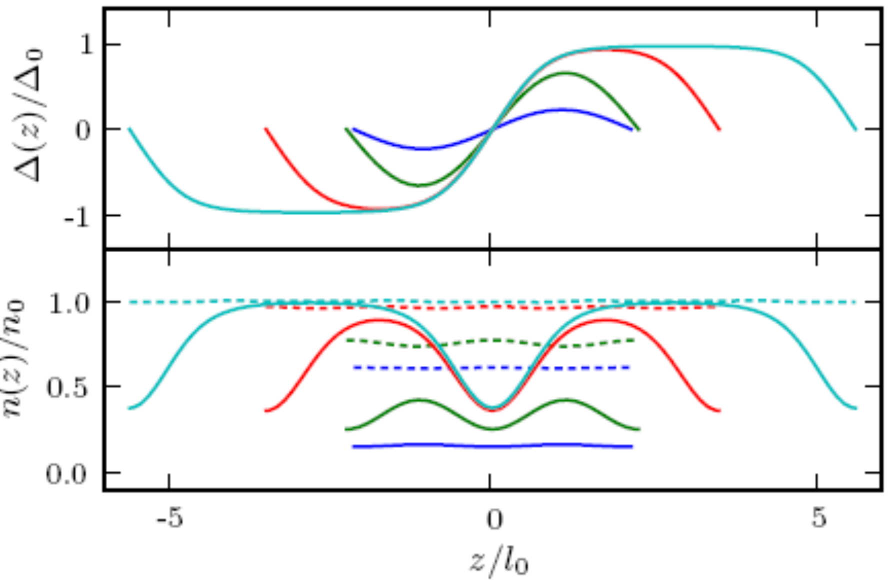
Black line: normal part of the energy density
Blue points: DMC calculations for normal state, Lobo et al, PRL <u>97</u>, 200403 (2006)
Gray crosses: experimental EOS due to Shin, Phys. Rev. A 77, 041603(R) (2008)

$$E(n_a,n_b) = \frac{3}{5}\frac{(6\pi^2)^{2/3}\hbar^2}{2m}\left[n_a g\left(\frac{n_b}{n_a}\right)\right]^{5/3}$$

Bulgac and Forbes,
Phys. Rev. Lett. <u>101</u>, 215301 (2008)

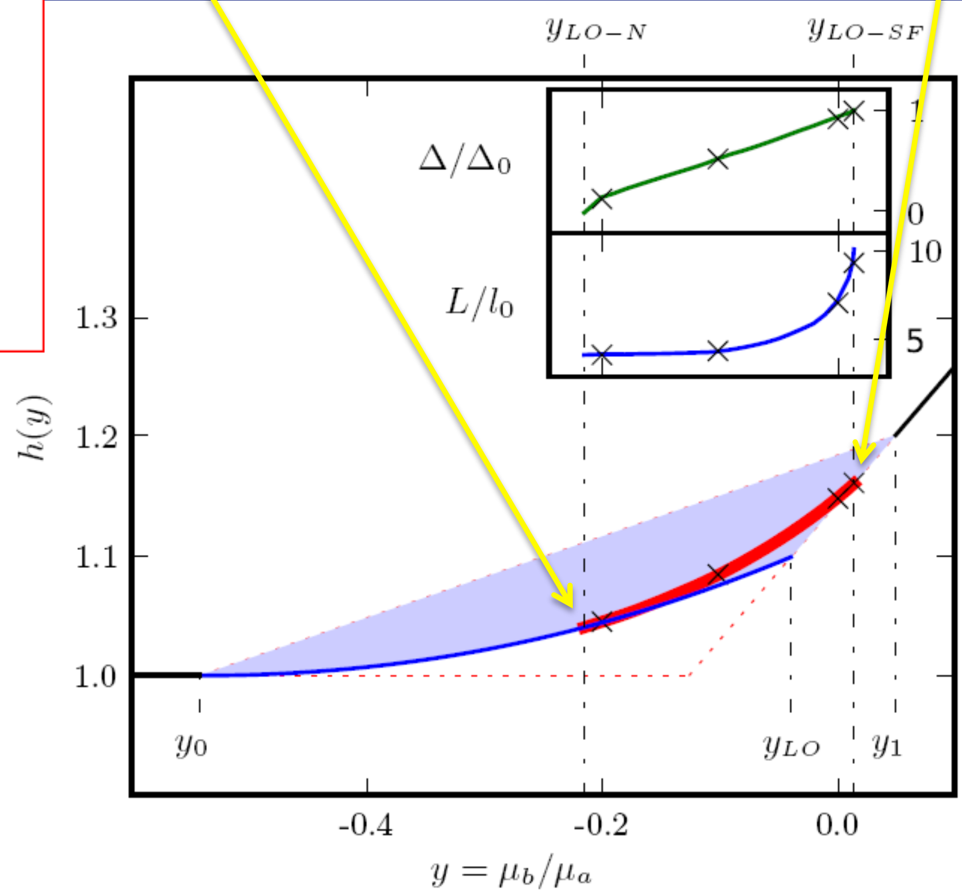
A Unitary Fermi Supersolid: the Larkin-Ovchinnikov phase





Second order phase transition

First order phase transition



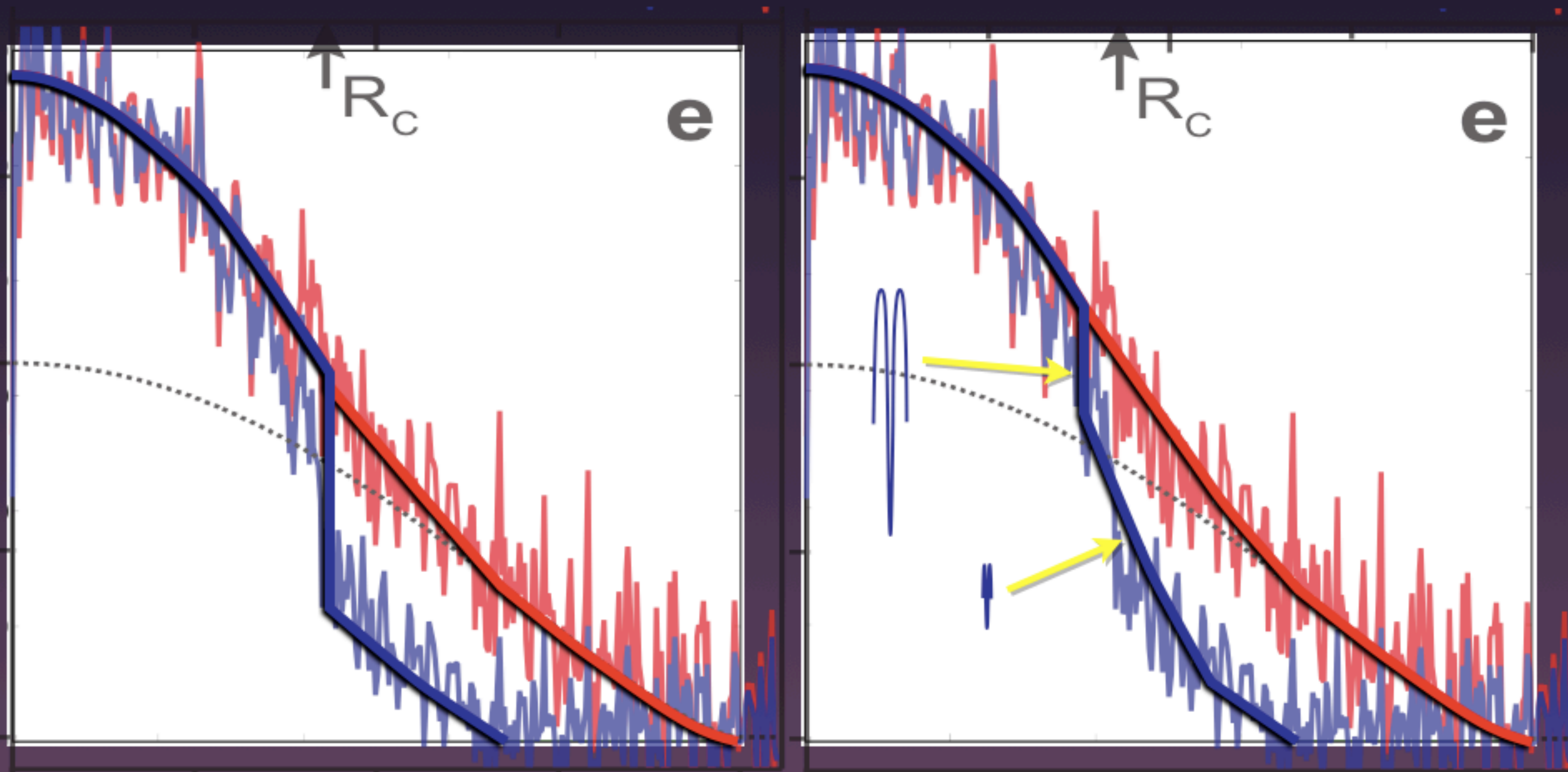
$$P[\mu_a,\mu_b] = \frac{2}{30\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$

Bulgac and Forbes
Phys. Rev. Lett. 101, 215301 (2008)

NB This is a gas system at the same time!

Observations: Inconclusive

- Need detailed structure or novel signature

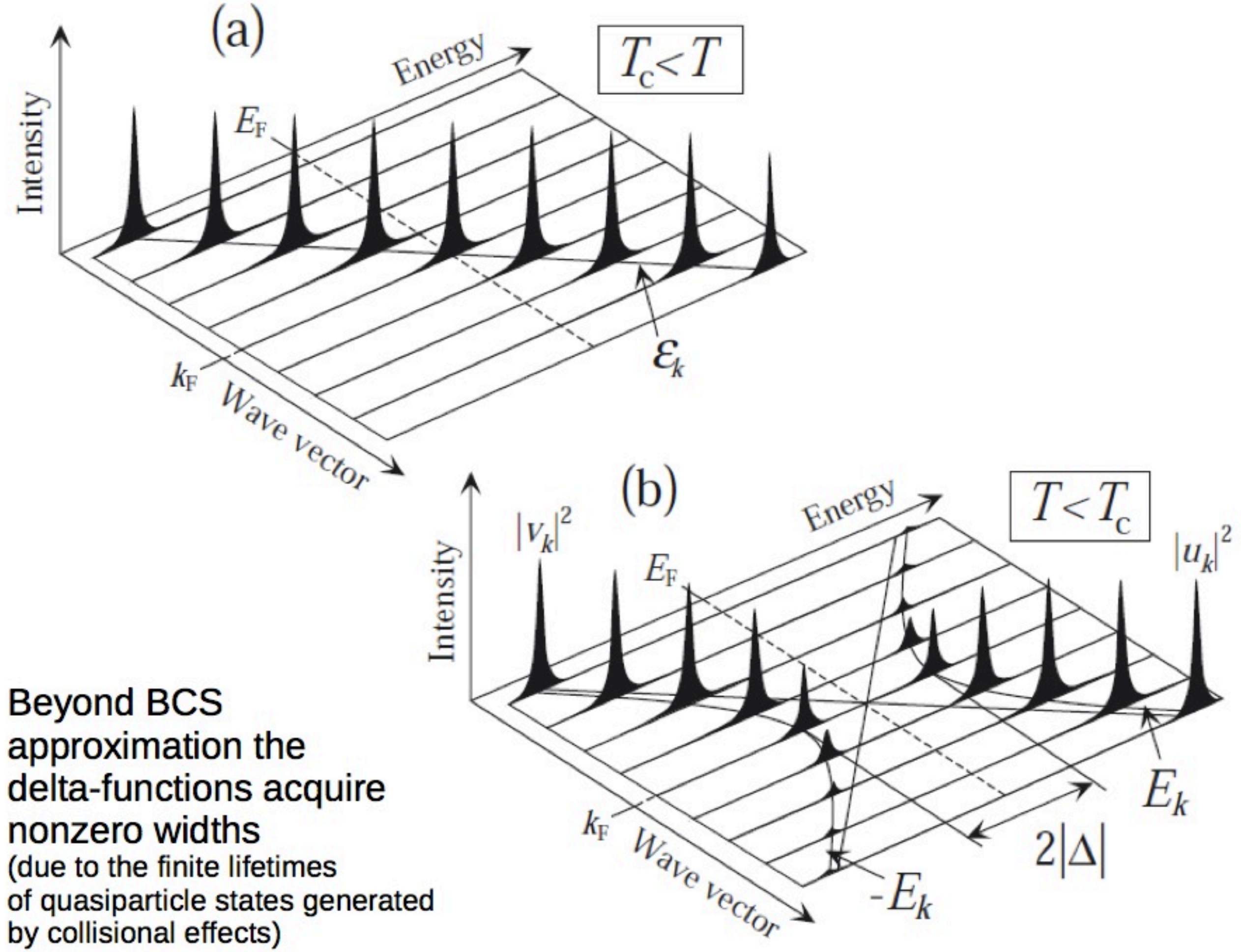


MIT Experimental data from Shin et. al (2008)

Courtesy of M.M. Forbes

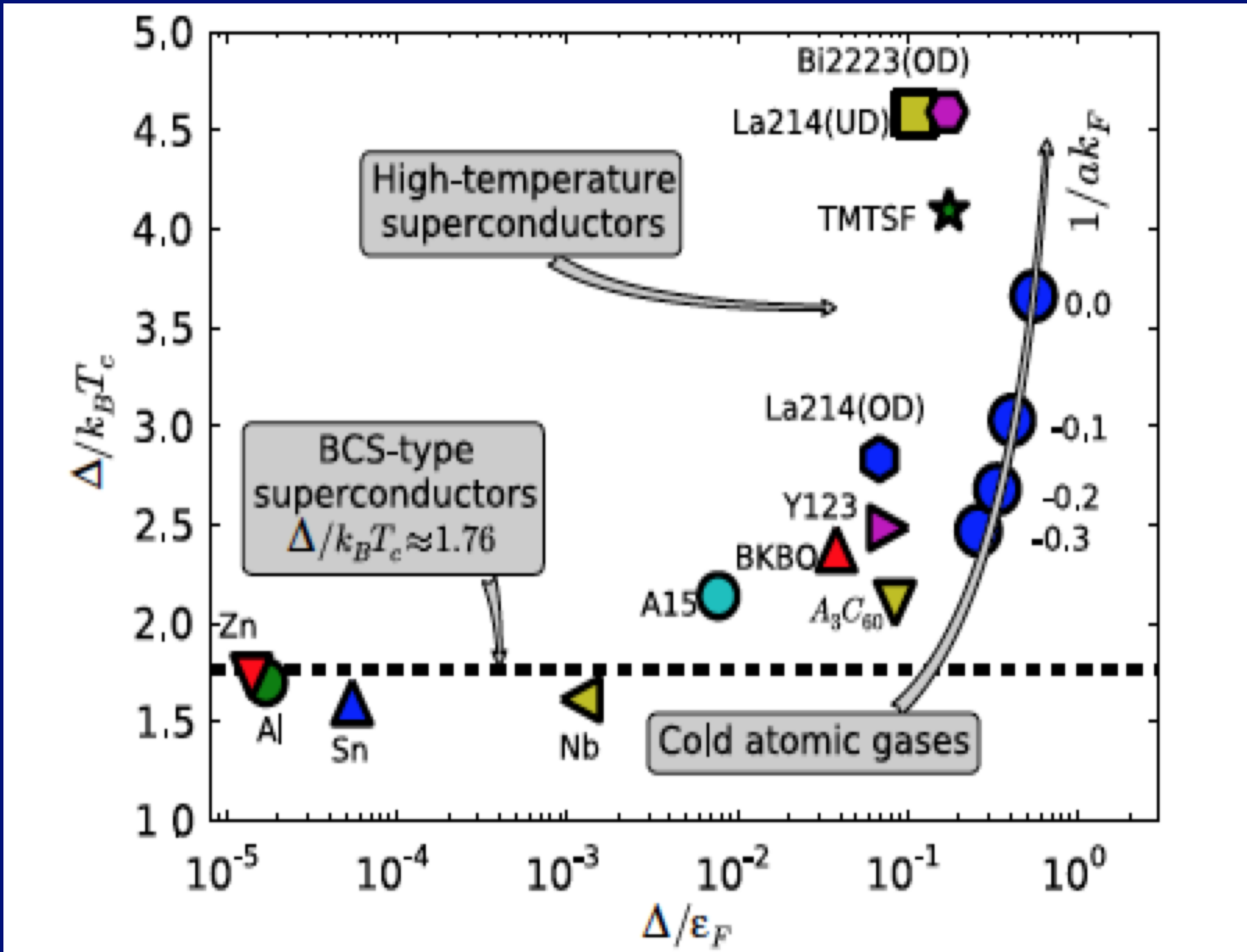
The temperature dependence of the spectral weight function

Spectral function – BCS case



Beyond BCS approximation the delta-functions acquire nonzero widths
(due to the finite lifetimes of quasiparticle states generated by collisional effects)

From a talk given by Wlazlowski at INT, Seattle, Spring 2011



High-temperature superconductors
BCS-type superconductors
$\Delta/k_BT_c \approx 1.76$
Cold atomic gases
Bi2223(OD)
La214(UD)
TMTSF
La214(OD)
Y123
BKBO
A15
A_3C_{60}
Zn
Al
Sn
Nb
$1/ak_F$
0.0
-0.1
-0.2
-0.3
Δ/k_BT_c
Δ/ε_F

Matsubara propagator, spectral function and linear response

$$G(\vec{p},\tau)=\frac{1}{Z}\mathrm{Tr}\left\{\exp\left[-\left(\beta-\tau\right)\left(H-\mu N\right)\right]\psi^{\dagger}(\vec{p})\exp\left[-\tau\left(H-\mu N\right)\right]\psi(\vec{p})\right\}$$

$$=-\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\,A(\omega,\vec{p})\frac{\exp(-\omega\tau)}{1+\exp(-\omega\beta)}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\,A(\omega,\vec{p})=1,$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega\,A(\omega,\vec{p})\frac{1}{1+\exp(\omega\beta)}=n(\vec{p}),$$

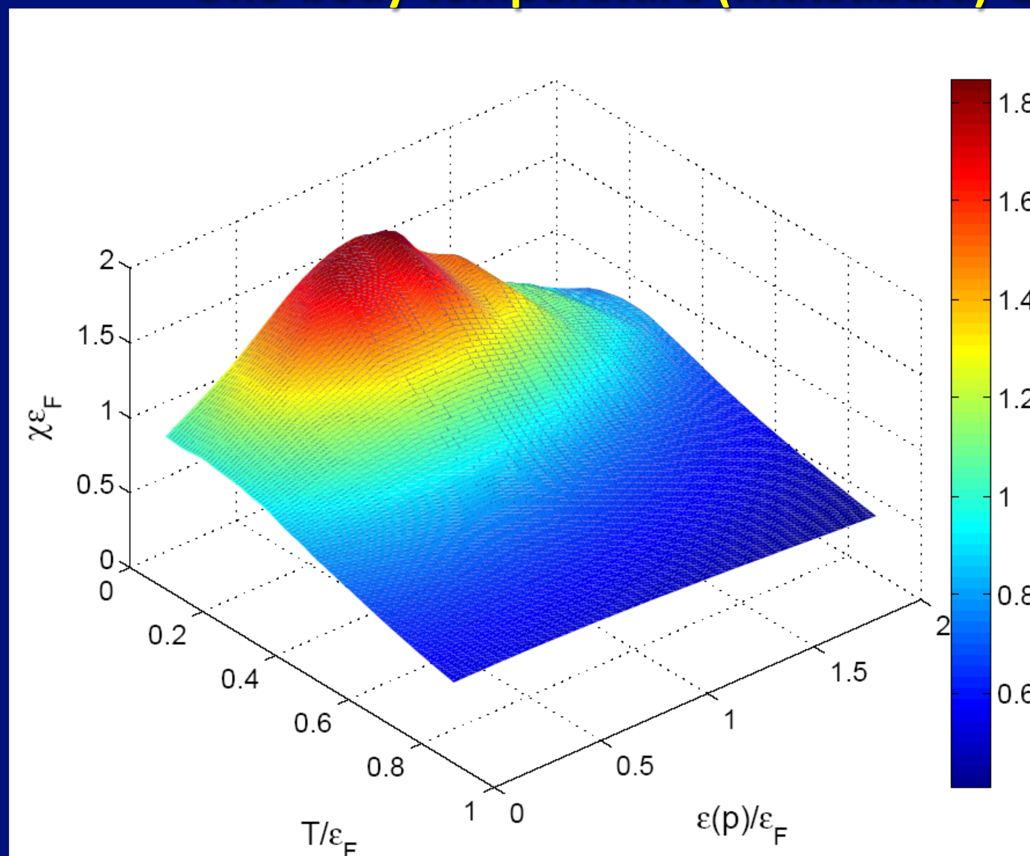
$$A(\omega,\vec{p})\geq 0$$

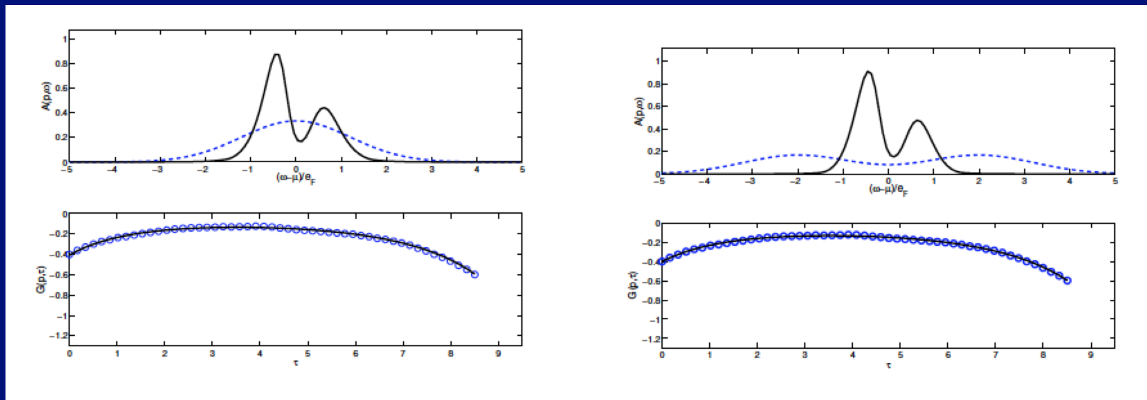
Response of the two-component Fermi gas in the unitary regime

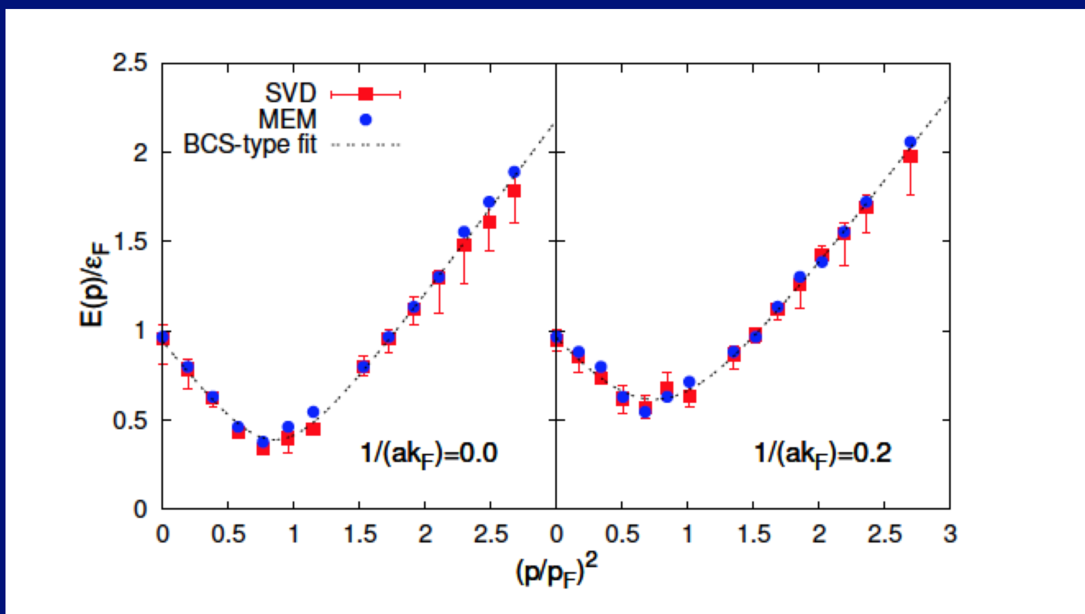
Bulgac, Drut, and Magierski, arXiv:0801:1504v3, PRL ,in press (2009)

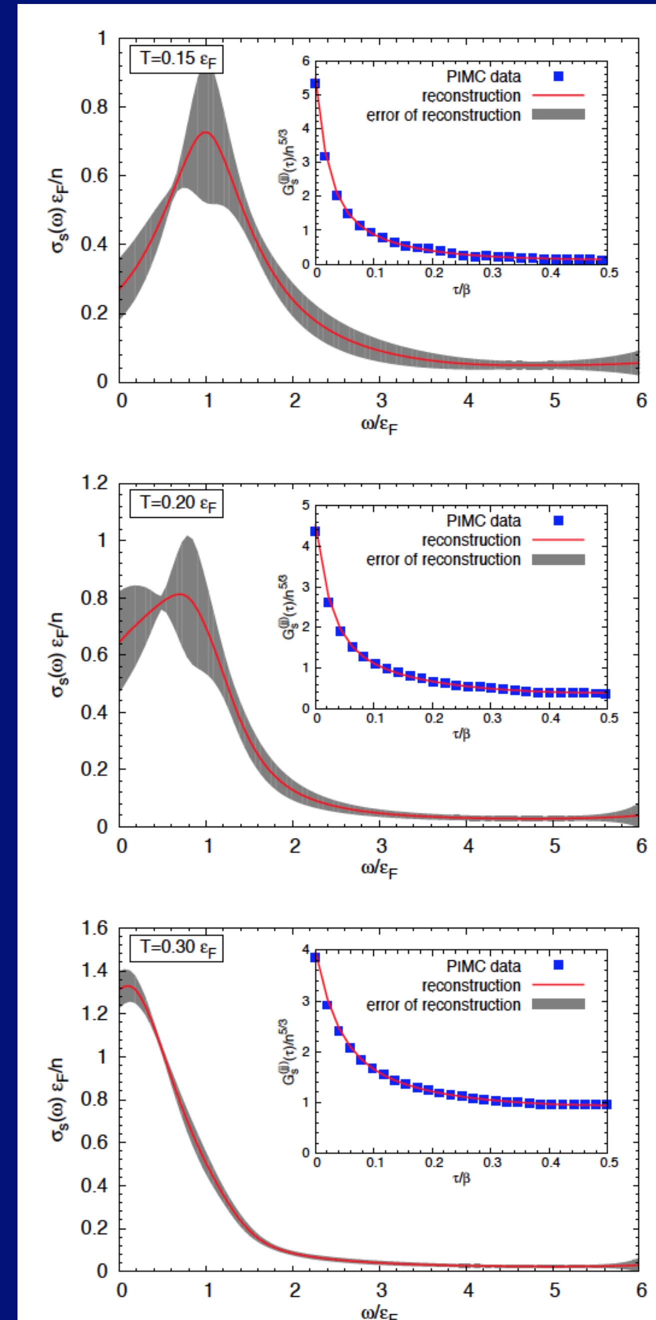
$$\chi(\vec{p}) = -T\frac{d}{dg}\frac{\mathrm{Tr}\{\exp[-\beta(\mathrm{H}-\mu\mathrm{N}+g\psi(\vec{\mathrm{p}}))]\psi^{\dagger}(\vec{\mathrm{p}})\}}{\mathrm{Tr}\{\exp[-\beta(\mathrm{H}-\mu\mathrm{N}+g\psi(\vec{\mathrm{p}}))]\}}\Bigg|_{g=0} = -\int_0^{\beta} d\tau\, \mathrm{G}(\vec{p},\tau)$$

One-body temperature (Matsubara) Green's function

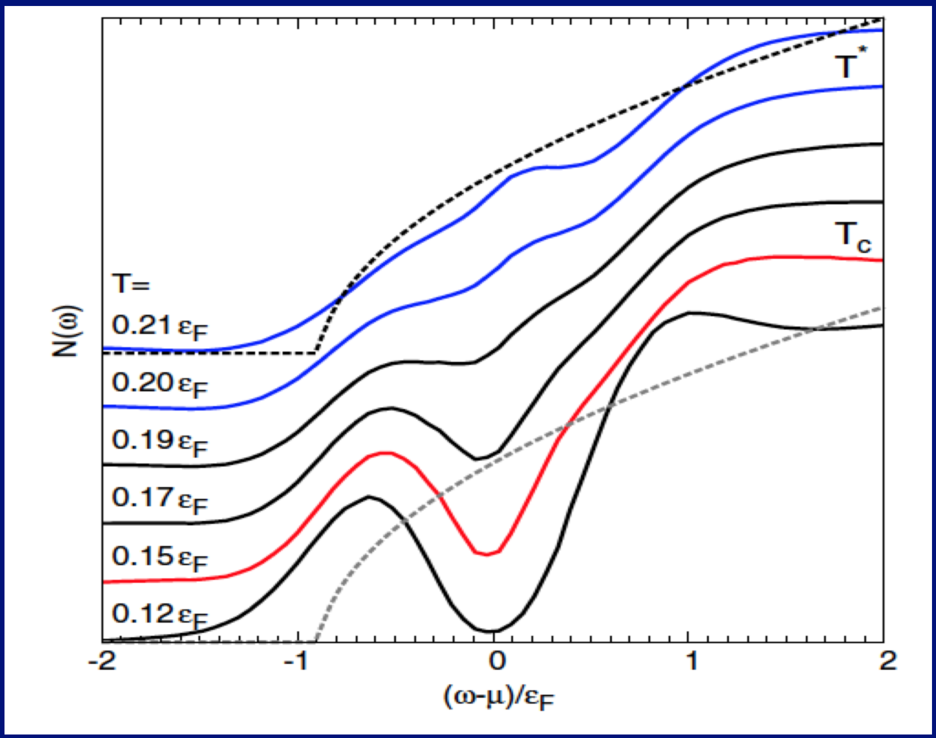


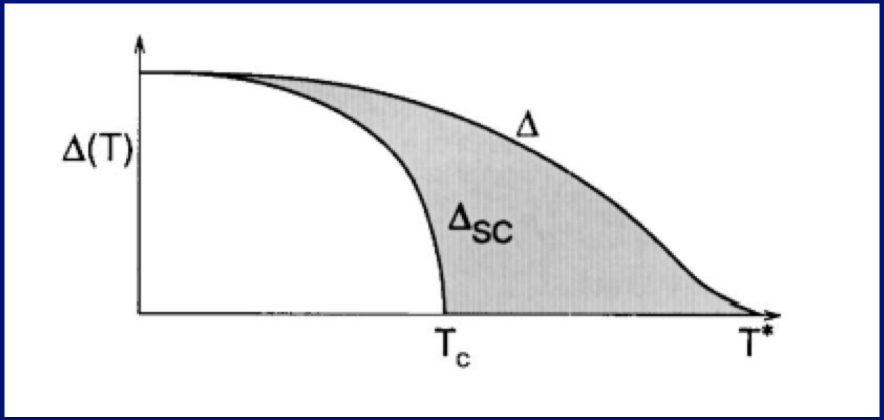


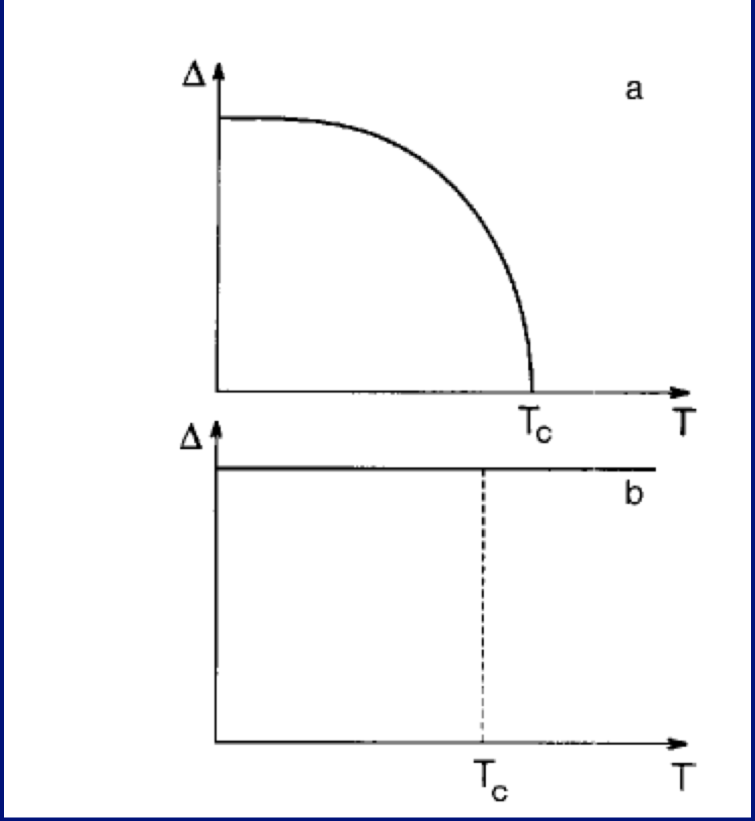




Singular value decomposition (SVD)
and maximum entropy method (MEM)
reconstruction of the spectral function

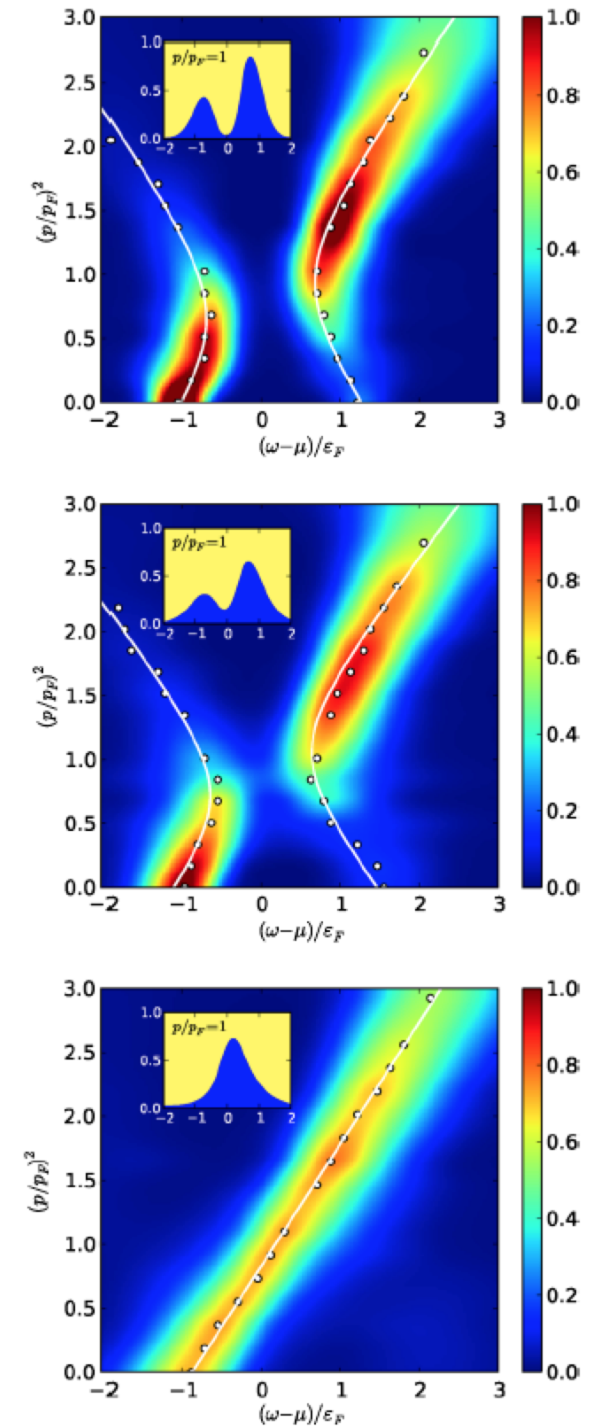






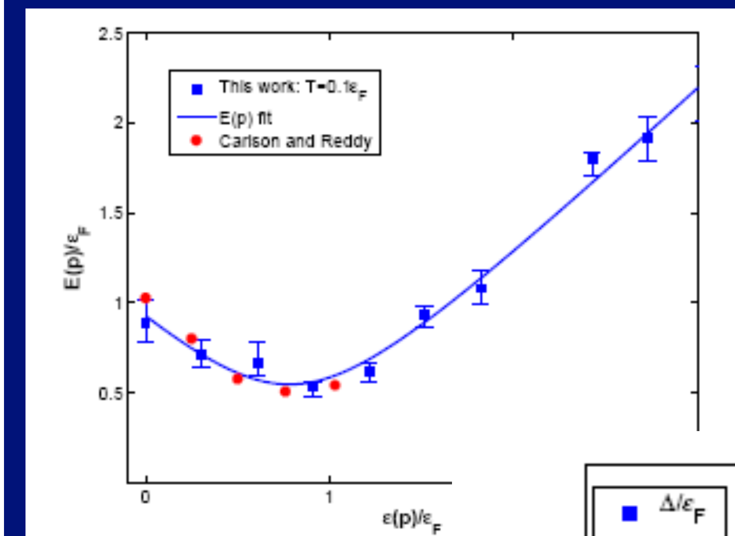
G. Wlazlowski, et al., Phys. Rev. Lett. 110, 090401 (2013)

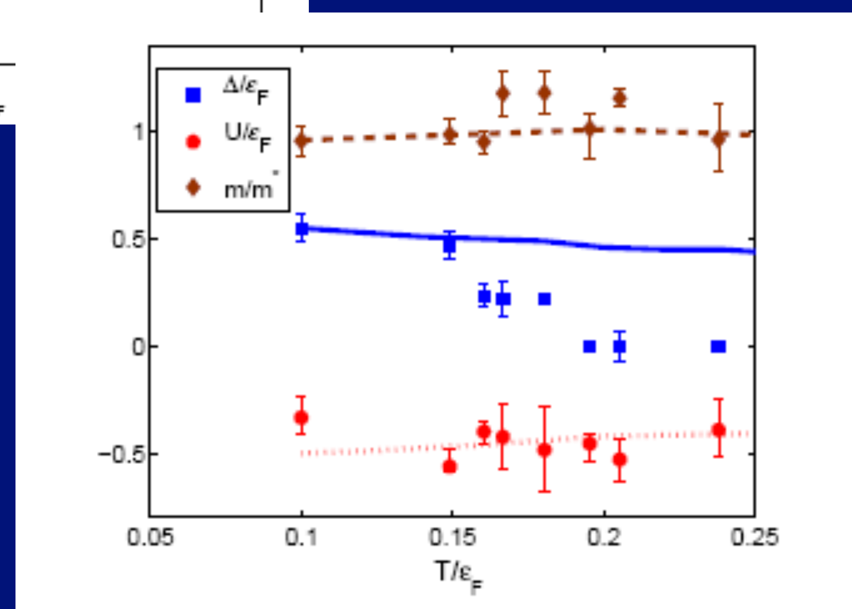
Chen et al, Low Temp. Phys. 32, 406 (2006)

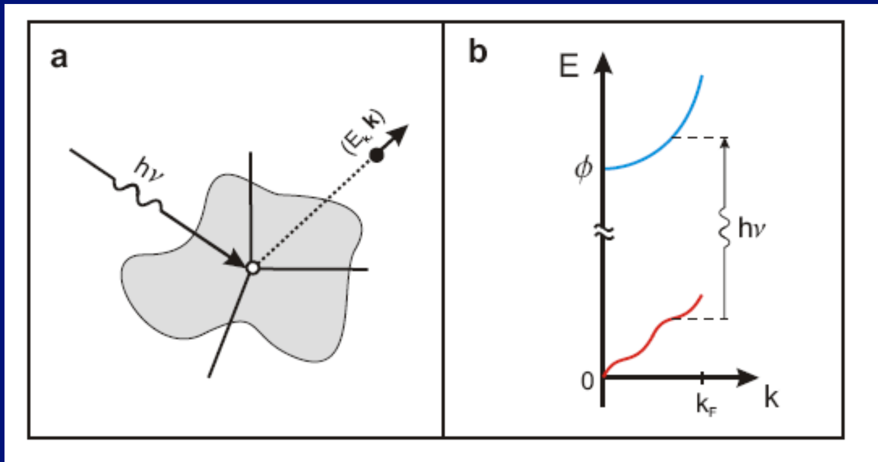


$$G(p,\tau) = \frac{1}{Z}\mathrm{Tr}\left\{\exp\left[-\left(\beta-\tau\right)\left(H-\mu N\right)\right]\psi^{\dagger}(p)\exp\left[-\tau\left(H-\mu N\right)\right]\psi(p)\right\}$$

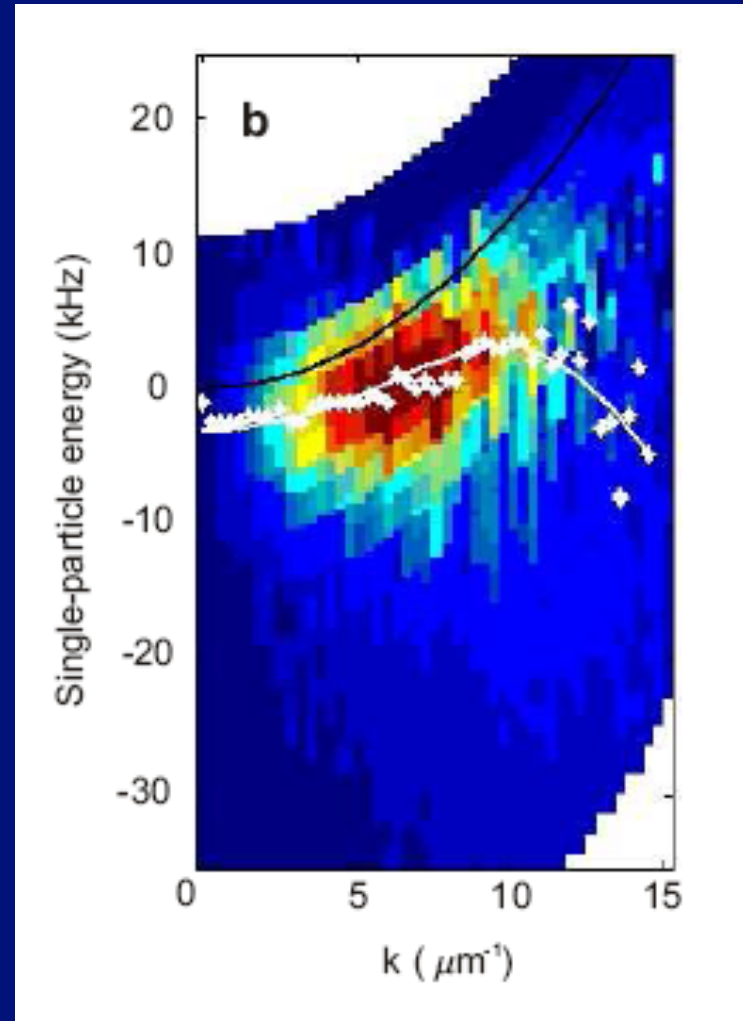
$$= -\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega A(p,\omega)\frac{\exp(-\omega\tau)}{1+\exp(-\omega\beta)}$$



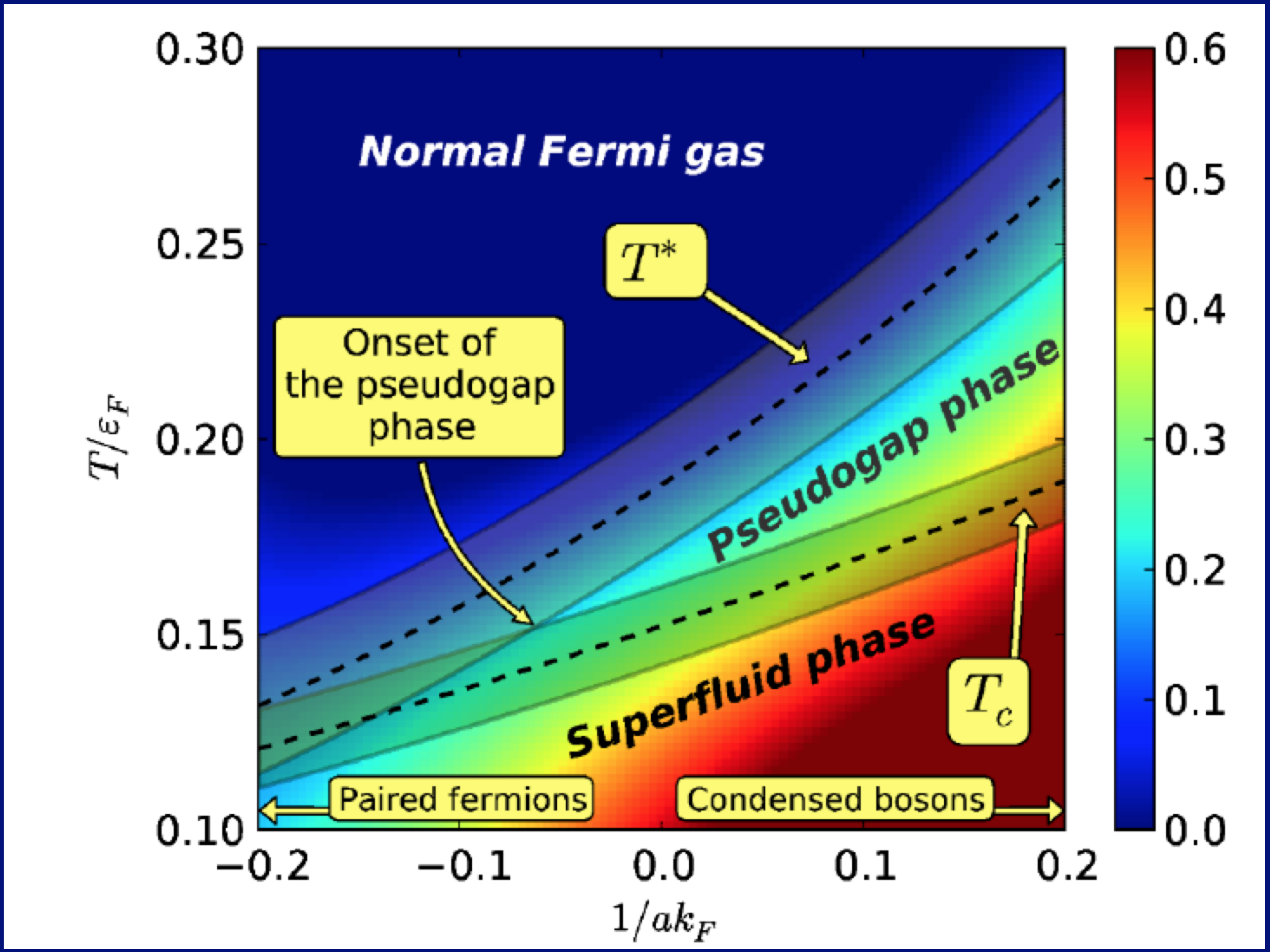




$$E(N) + h\nu = E(N-1) + E_k + \frac{\hbar^2 k^2}{2m} + \phi$$



Using photoemission spectroscopy to probe a strongly interacting Fermi gas
Stewart, Gaebler, and Jin, Nature, 454, 744 (2008)



Normal Fermi gas
T^*
Onset of the pseudogap phase
Pseudogap phase
Superfluid phase
T_c
Paired fermions
Condensed bosons
T/ε_F
$1/ak_F$
0.30
0.25
0.20
0.15
0.10
−0.2
−0.1
0.0
0.1
0.2
0.6
0.5
0.4
0.3
0.2
0.1
0.0

$$G_{\Pi}(\vec{q},\tau) = \frac{1}{V}\left\langle \Pi^{(xy)}_{\vec{q}}(\tau)\Pi^{(xy)}_{-\vec{q}}(0)\right\rangle$$

$$\Pi^{(xy)}_{0} = \sum_{\vec{p},\sigma} p_x p_y a^{\dagger}_{\sigma}(\vec{p}) a_{\sigma}(\vec{p})$$

$$\Pi^{(xy)}_{\vec{q}}(\tau) = e^{-\tau(H-\mu N)}\Pi^{(xy)}_{\vec{q}} e^{\tau(H-\mu N)}$$

$$G_{\Pi}(0,\tau) = \frac{1}{\pi}\int_0^{\infty} d\omega\ \eta(\omega)\omega \frac{\cosh\left[\omega\left(\tau-\beta/2\right)\right]}{\sinh\left(\omega\beta/2\right)}$$

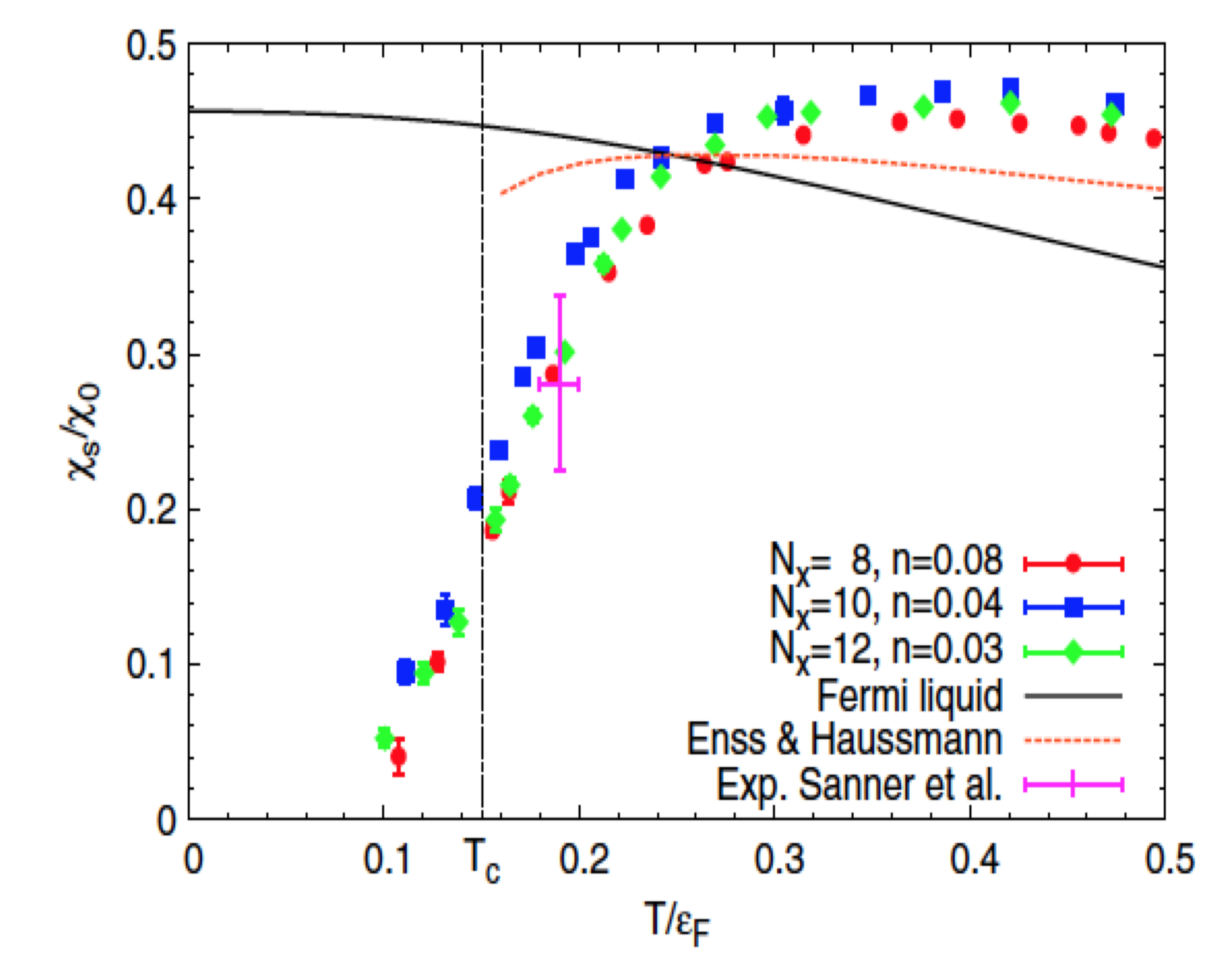
$$\int_0^{\infty} d\omega\left[\eta(\omega) - \frac{C}{15\pi\sqrt{\omega}}\right] = \frac{\varepsilon}{3} - \frac{C}{12\pi a},\quad \eta(\omega)\geq 0$$

$$n(p) \approx \frac{C}{p^4} \text{ when } p\to\infty$$

$$E = TS - pV + \mu N$$

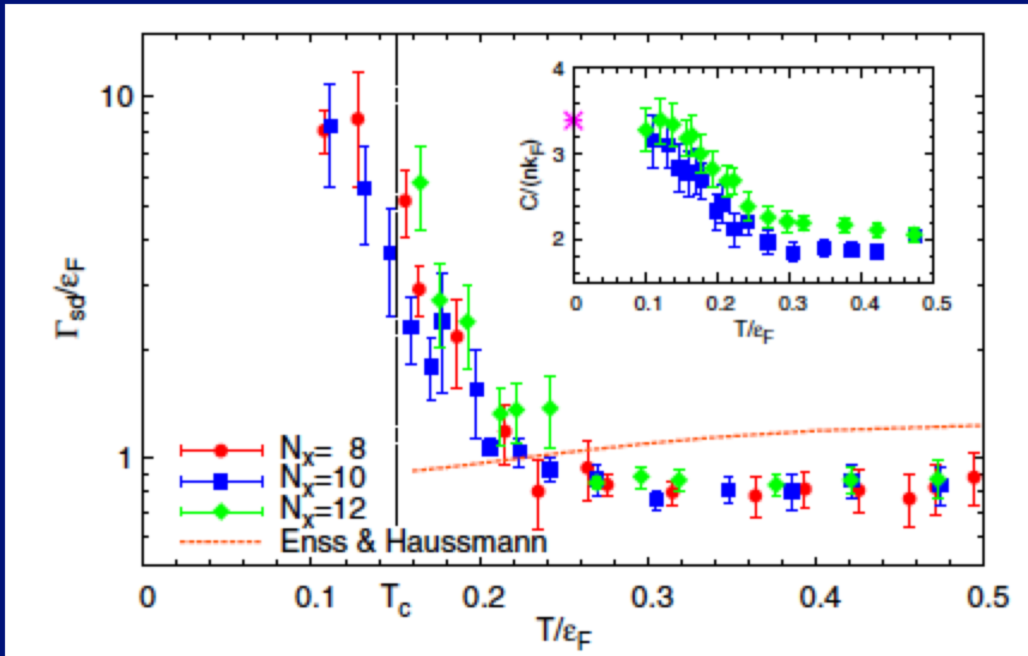
$$\frac{S(x,y)}{N} = \frac{\xi(x,y) - \varsigma(x,y) + \dfrac{C(x,y)y}{6\pi N k_F}}{\quad},$$

where $x = \dfrac{T}{\varepsilon_F}$, $y = \dfrac{1}{k_F a}$, $E = \dfrac{3}{5}\varepsilon_F\xi(x,y)$, $\mu = \varepsilon_F\varsigma(x,y)$



$$\chi_s = \lim_{p\to 0} \frac{1}{V}\int_0^\beta d\tau \left\langle s_z(\vec{p},\tau)s_z(-\vec{p},0)\right\rangle, \qquad s_z(\vec{p},\tau) = n_\uparrow(\vec{p},\tau) - n_\downarrow(\vec{p},\tau)$$

Spin susceptibility



$$\vec{j}_s = \vec{j}_\uparrow - \vec{j}_\downarrow = \sigma_s \vec{F}$$

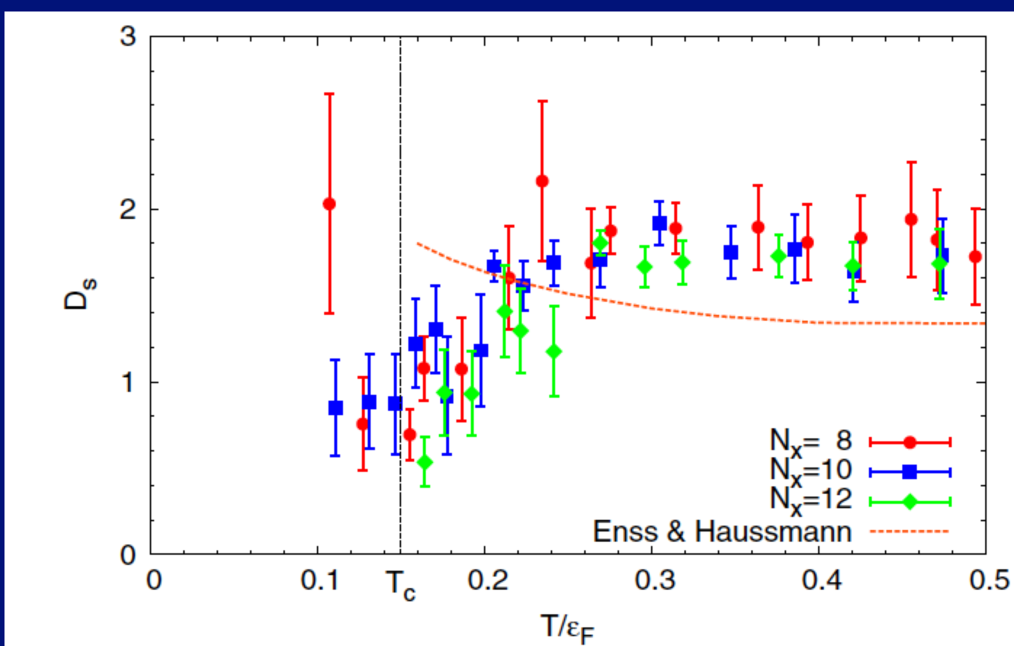
$$\Gamma_{sd} = \frac{n}{\sigma_s},\ \sigma_s \ge 0$$

$$G_s^{jj}(\vec{q},\tau) = \frac{1}{V}\left\langle \left[j^z_{q\uparrow}(\tau) - j^z_{q\downarrow}(\tau) \right]\left[j^z_{q\uparrow}(0) - j^z_{q\downarrow}(0) \right] \right\rangle$$

$$\vec{j}_s = -D_s \vec{\nabla}\left(n_\uparrow - n_\downarrow \right)$$

$$D_s = \frac{\sigma_s}{\chi_s} \quad \text{(Einstein relation)}$$

$$D_s \approx \mathrm{v}\lambda \sim 1 \quad \text{(kinetic theory)}$$



KSS conjecture

[Kovtun, Son, Starinets, PRL (2005)]

shear viscosity

entropy density

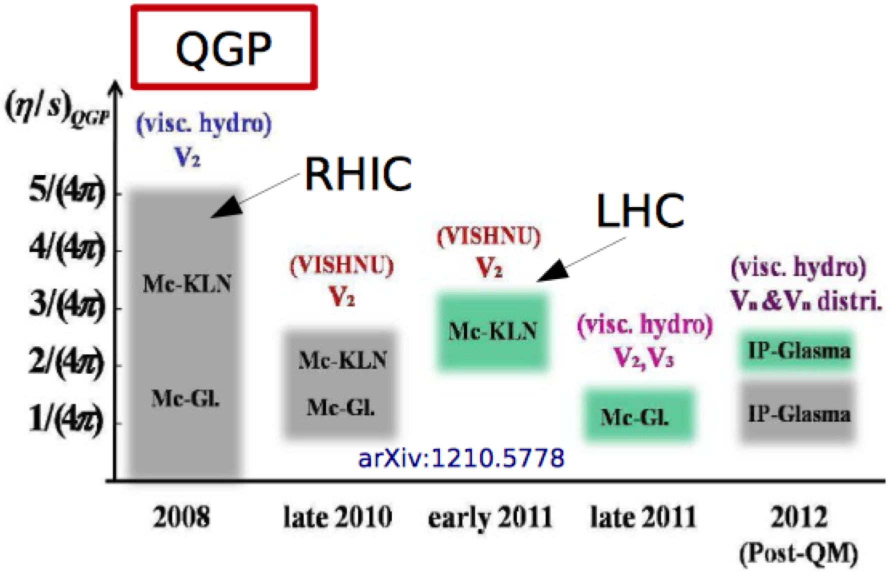
$$\frac{\eta}{s} \geq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

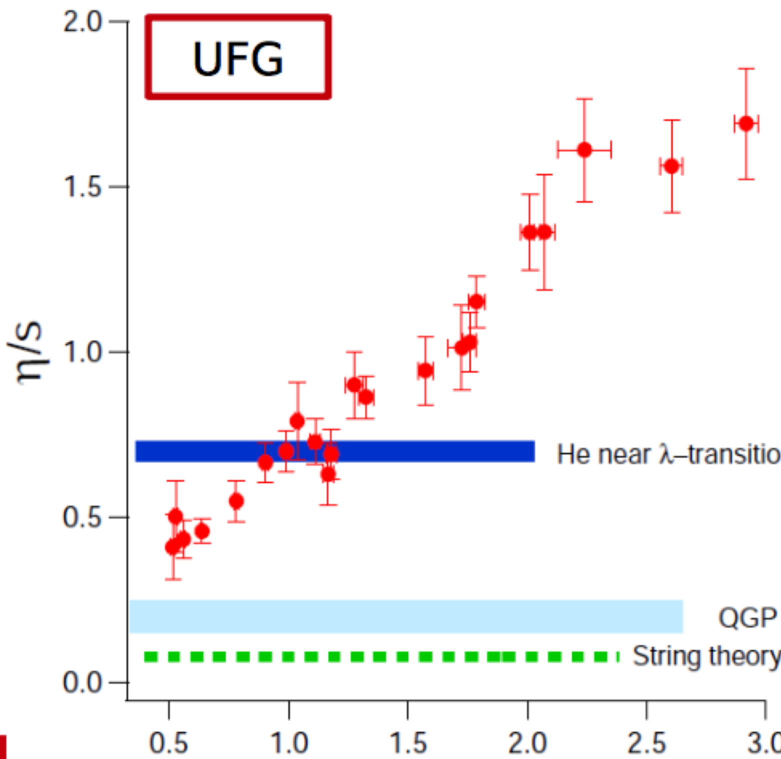
Minimum defines a “perfect” fluid

Bound has been proposed on the basis of string theory.

Valid for large class of (string) theories.

Saturated for the case of strongly coupled theory.





J. Low Temp. Phys. 150, 567 (2008)

Slide from a talk given by G. Wlazlowski

Shear viscosity of a unitary Fermi gas
(the only complete ab initio calculation in a Fermi system)

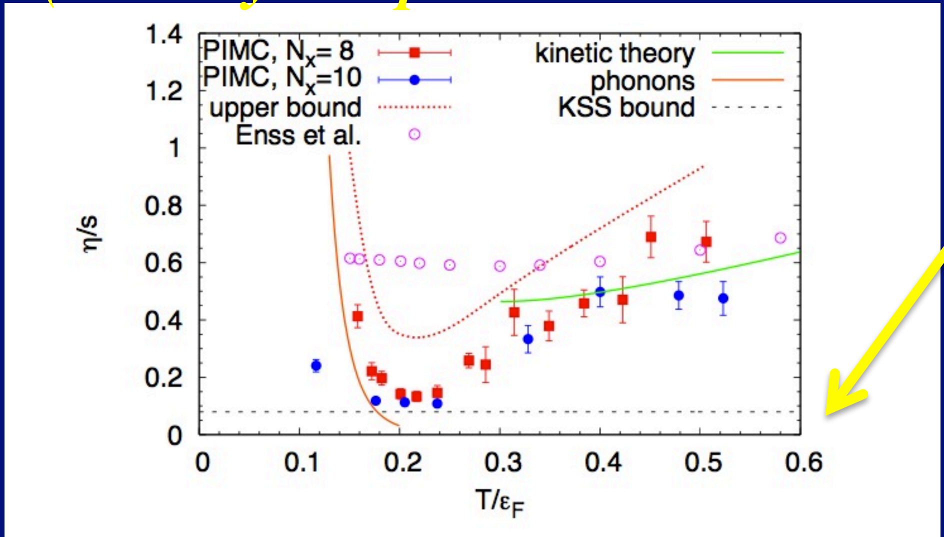


FIG. 3: (Color online) The ratio of the shear viscosity to the entropy density η/s as a function of dimensionless temperature for 8^3-lattice (red) squares and 10^3-lattice (blue) circles. The error bars only presents the stability of the combined (SVD and MEM) analytic continuation procedure with respect to the change of algorithm parameters, and do not include systematic errors of the entropy determination. By (red) dotted line conservative estimation for the upper bound is depicted. Result of the T-matrix theory are plotted by open (purple) circles [15]. In the high and low temperatures regime known asymptotics are depicted: for $T > 0.3\varepsilon_F$ by (green) line prediction of the kinetic theory and for $T < 0.2\varepsilon_F$ by (brown) line contribution from phonon excitations [13]. By dashed (black) line the KSS bound is plotted.

Lower limit for "perfect liquid"

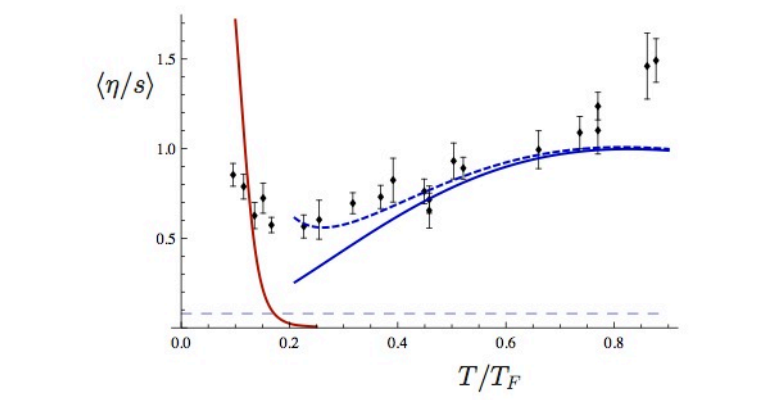
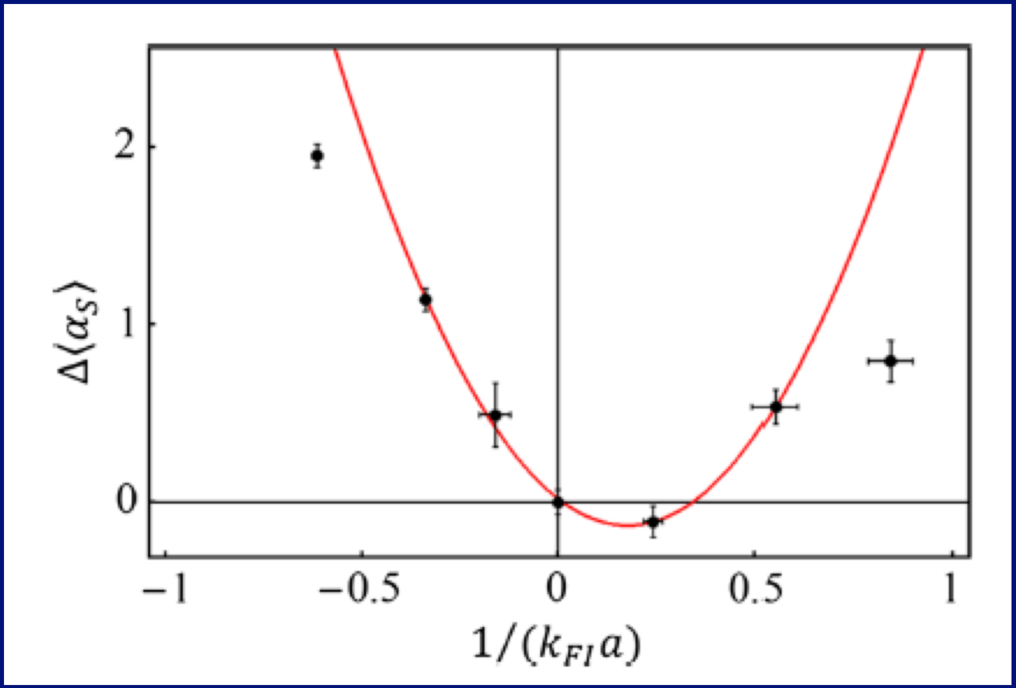


Fig. 11 Trap average $\langle\alpha_s\rangle = \langle\eta/s\rangle$ extracted from the damping of the radial breathing mode. The data points were obtained using equ. (91) to analyze the data published by Kinast et al. [9]. The thermodynamic quantities (S/N) and E_0/E_F were taken from [22]. The solid red and blue lines show the expected low and high temperature limits. Both theory curves include relaxation time effects. The blue dashed curve is a phenomenological two-component model explained in the text.

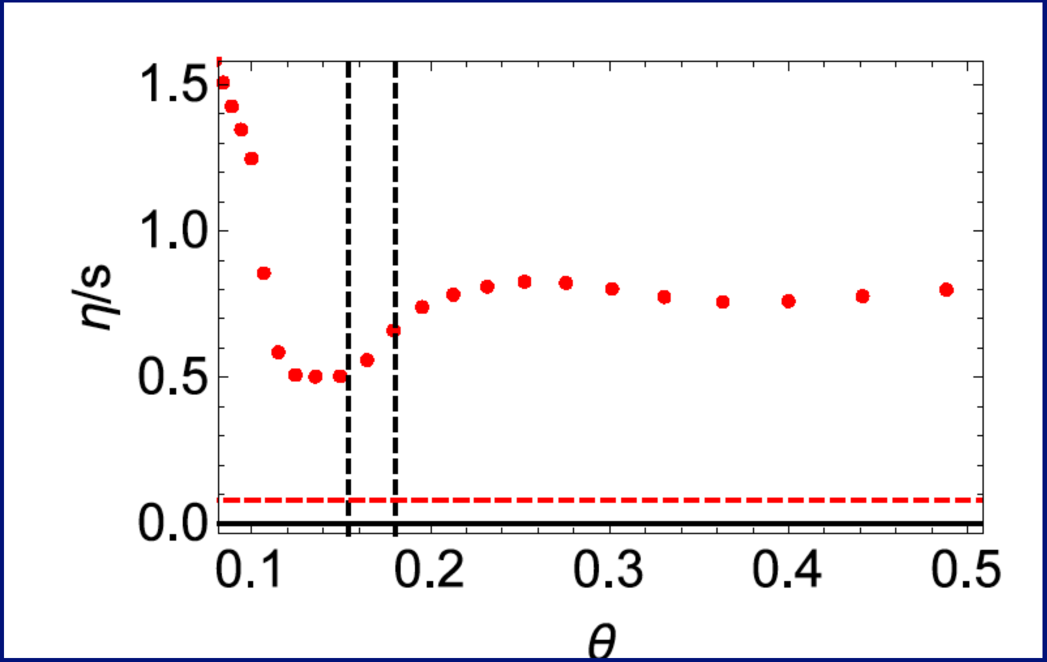
Schaefer and Chafin, chapter in *BCS-BEC crossover and Unitary Fermi Gas,* Lect. Notes in Phys. ed. Zwerger, Springer (2012)

Wlazlowski, Magierski, and Drut, Phys. Rev. Lett. 109, 020406 (2012)
Enss, Haussmann, and Zwerger, Ann. Phys. 326, 770 (2011)
Kovtun, Son, and Starinets (KSS), Phys. Rev. Lett. 94, 111601 (2005)

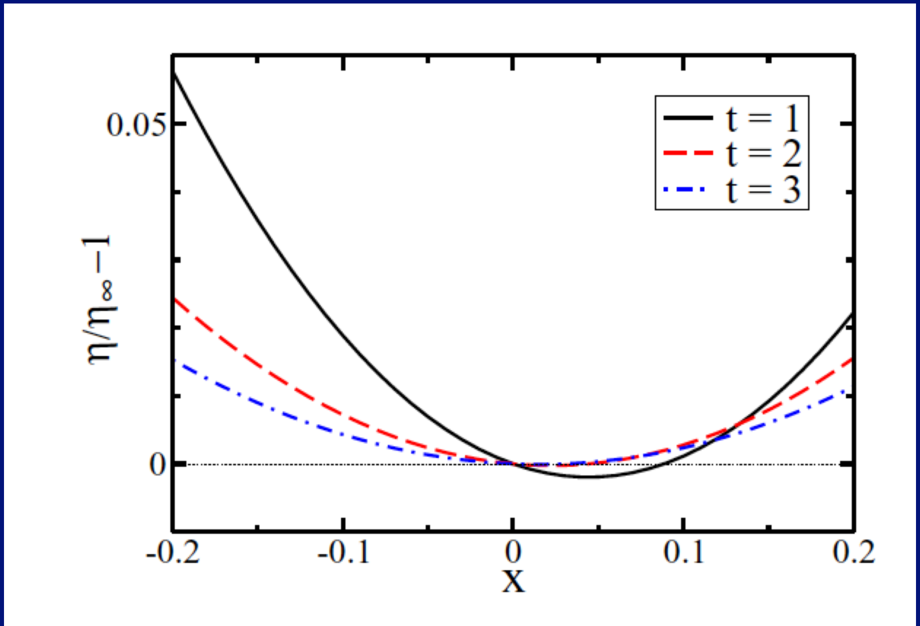


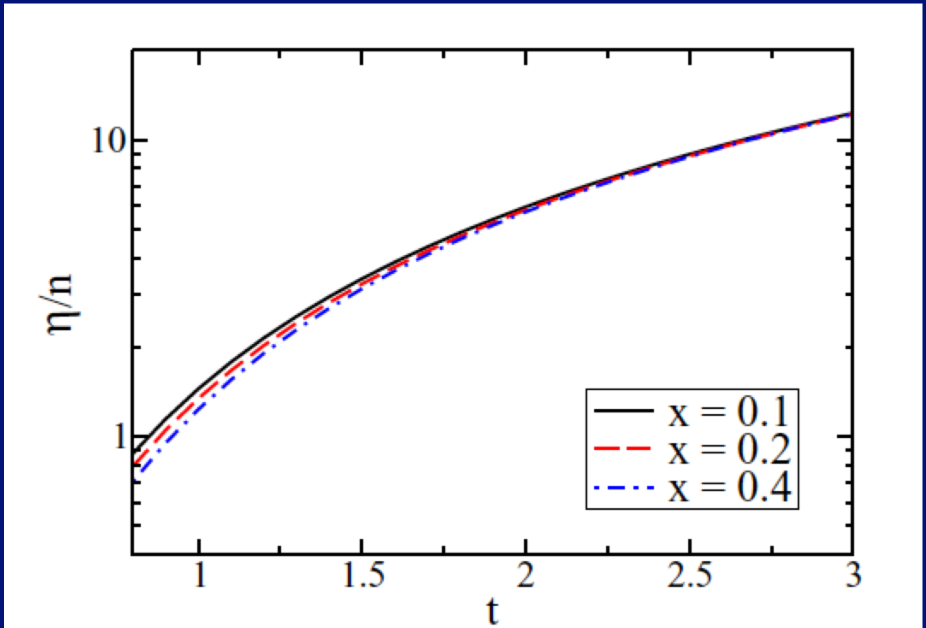
Elliott, Joseph, and Thomas
Phys. Rev. Lett. 113, 020406 (2014)

$$\eta = \alpha\hbar n$$



Joseph, Elliott, and Thomas,
arXix:1410.4835





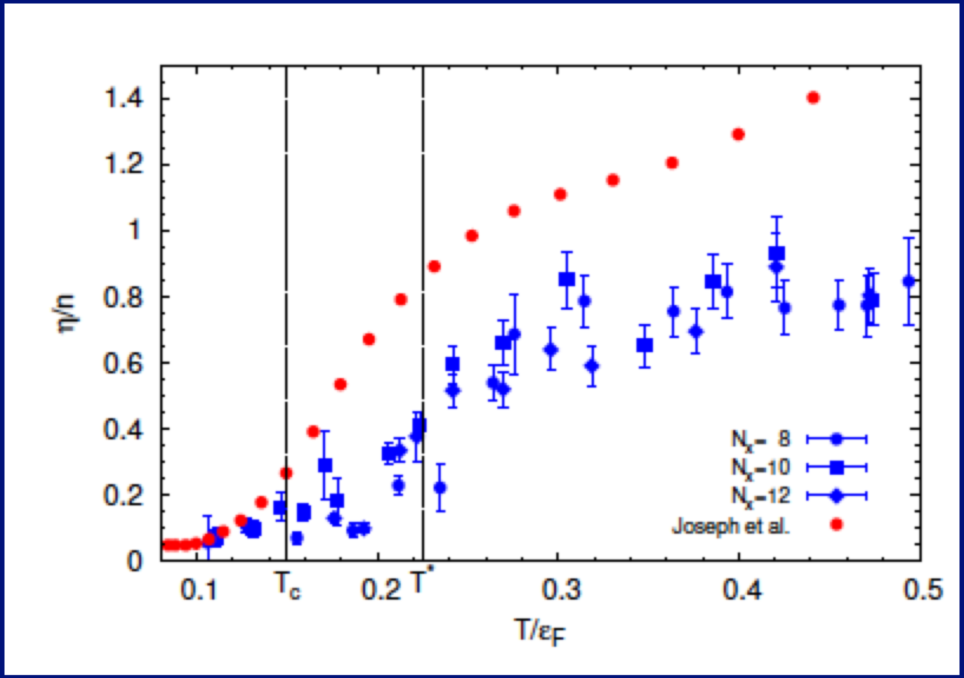
$$t = \frac{T}{\varepsilon_F},\ x = \frac{1}{k_F a}$$

$$z = \frac{n\lambda^3}{2}\ \text{(fugacity)}$$

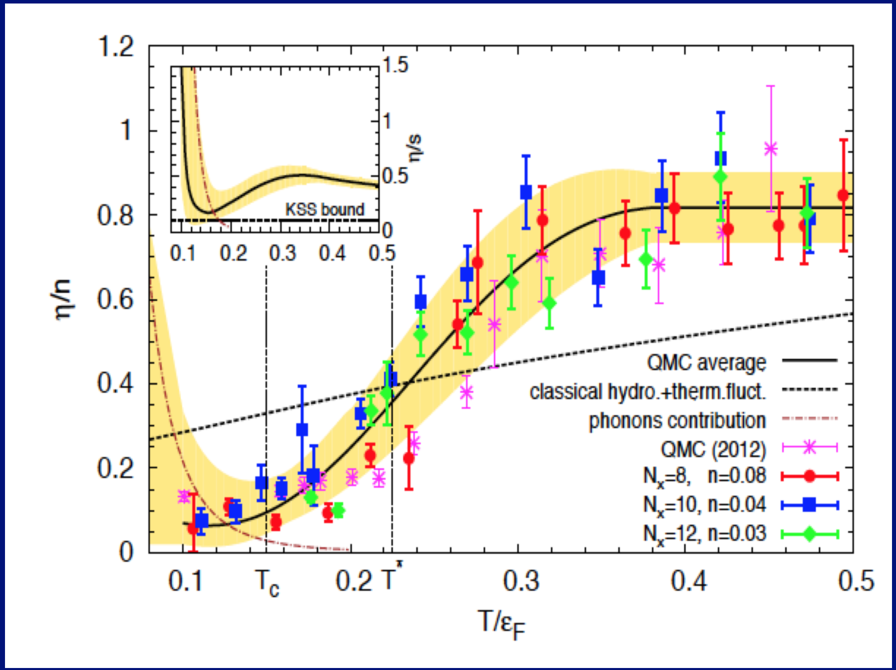
Expansion to order $O\left(\left(\frac{\lambda}{a}\right)^2\right)$ and $O\left(z\frac{\lambda}{a}\right)$

High temperature kinetic theory

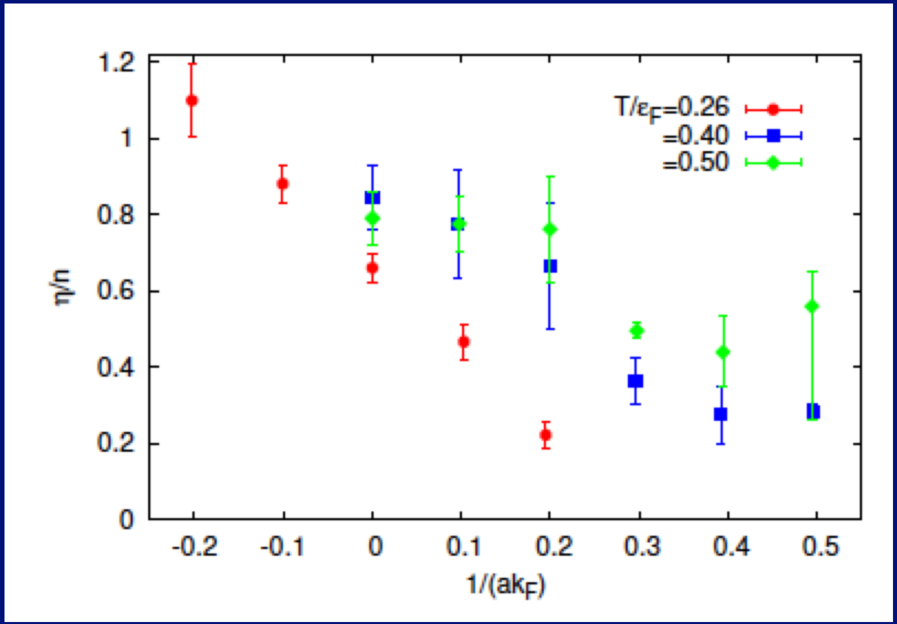
Bluhm and Schäfer, Phys. Rev. A 90, 063615 (2014)



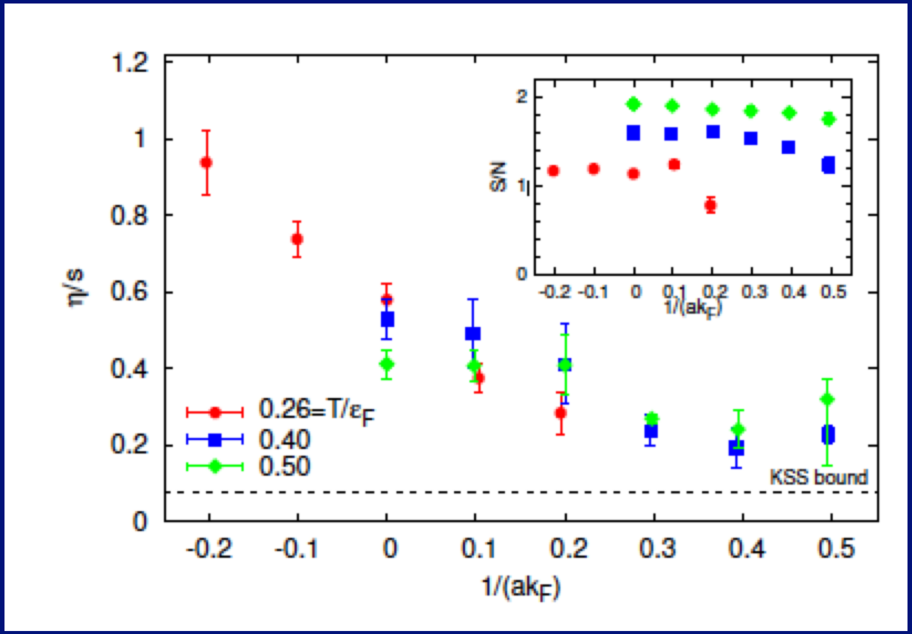
η/n
T/εF
Tc
T*
Nx= 8
Nx=10
Nx=12
Joseph et al.



η/n
T/εF
Tc
T*
η/s
KSS bound
QMC average
classical hydro.+therm.fluct.
phonons contribution
QMC (2012)
Nx=8, n=0.08
Nx=10, n=0.04
Nx=12, n=0.03



η/n
1/(akF)
T/εF=0.26
=0.40
=0.50



η/s
1/(akF)
0.26=T/εF
0.40
0.50
KSS bound
S/N

References (QMC only):

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G. Wlazlowski, P. Magierski, and J.E. Drut, Phys. Rev. Lett. 109, 020406 (2012)

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