Topological bands with Chern number C=2 using dipolar exchange interaction

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Collaboration

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Sebastian Weber

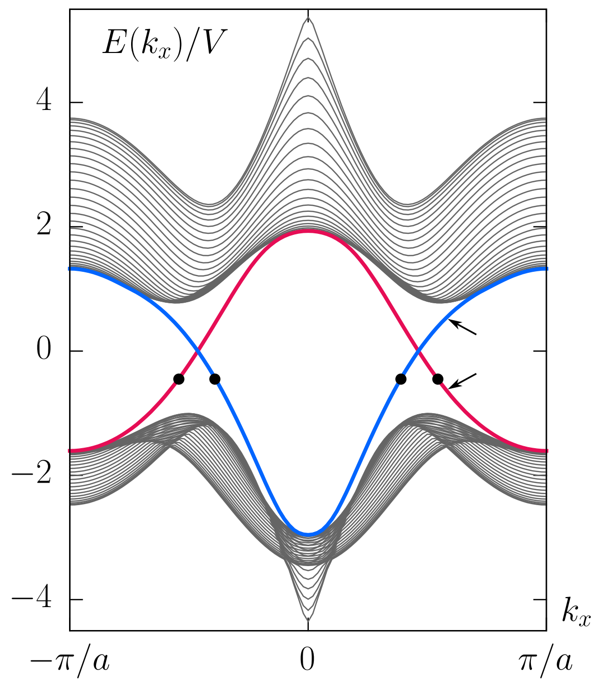
SFB TRR21:
Tailored quantum matter

Outline

Topological band structures with dipolar interactions

- realized with polar molecules and Rydberg atoms

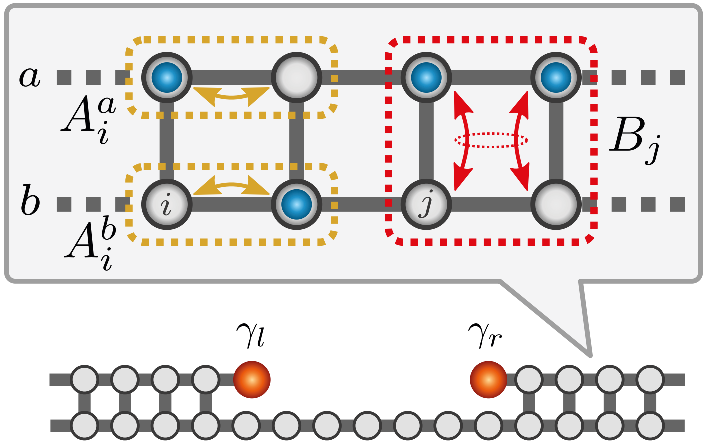
- Chern number C=2



Majorana Edge modes

- exact solvable system with fixed particle number

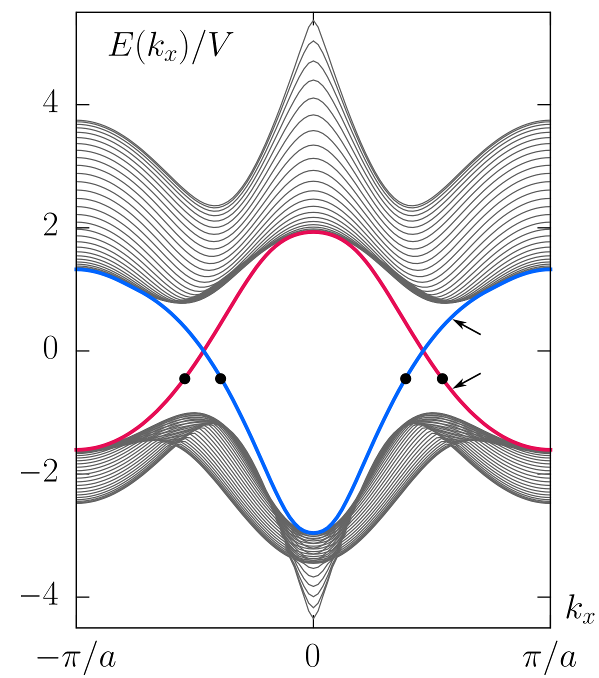
- double wire system



Topological band structure

singel particle band structure
independent on statistics of particles

- topological quantum numbers
- edge states



Requirements for topological states/phase

Fermions

- integer filling+ weak interactions → topological insulators
- flat bands + strong interactions → **fractional topological insulators**

Bosons

- flat bands + strong interactions → **bosonic fractional topological insulators**

Topological band structure

Homogeneous magnetic fields

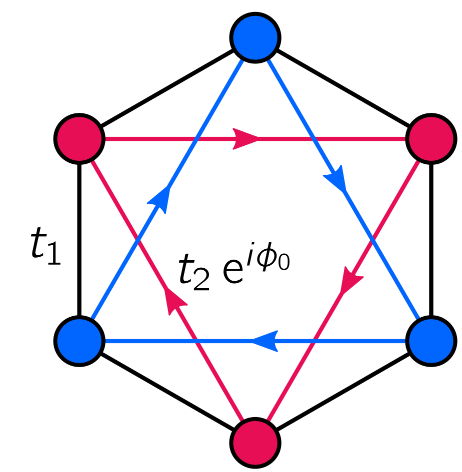
- integer quantum Hall effect
- Hofstadter butterfly

Time-reversal symmetry breaking

- Haldane model

- lattice shaking
- artificial gauge fields
- synthetic dimension

(Esslinger, Bloch, Spielman, Fallani,…



Here:

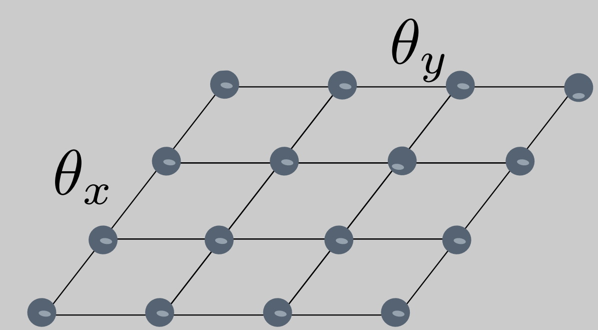
spin-orbit coupling + broken time reversal symmetry

characterized by Chern number

product state of lowest band

$$F(\theta_x, \theta_y) = \mathrm{Im}\left(\langle \partial_{\theta_y}\psi | \partial_{\theta_x}\psi\rangle - \langle \partial_{\theta_x}\psi | \partial_{\theta_y}\psi\rangle\right)$$

$$C = \frac{1}{2\pi}\int_0^{2\pi} d\theta_x \int_0^{2\pi} d\theta_y F(\theta_x, \theta_y)$$



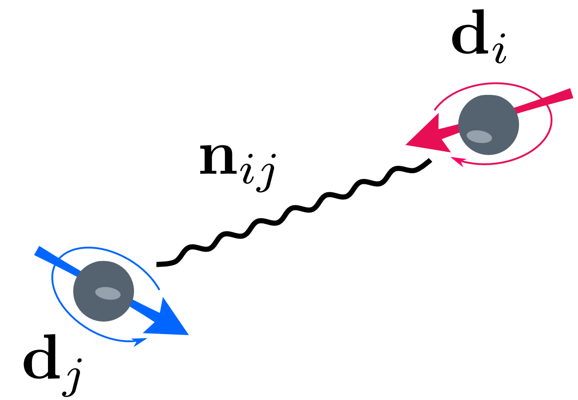
Dipolar interactions

Dipole-dipole interaction

- anisotropic interaction

dipole operator

$$V_{dd} = \frac{\mathbf{d}_i \cdot \mathbf{d}_j - 3\,(\mathbf{d}_i \cdot \mathbf{n}_{ij})\,(\mathbf{d}_i \cdot \mathbf{n}_{ij})}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$



- coupling between orbital degree of freedom
  and internal degree of freedom

"spin-orbit" coupling

Observation in cold gases

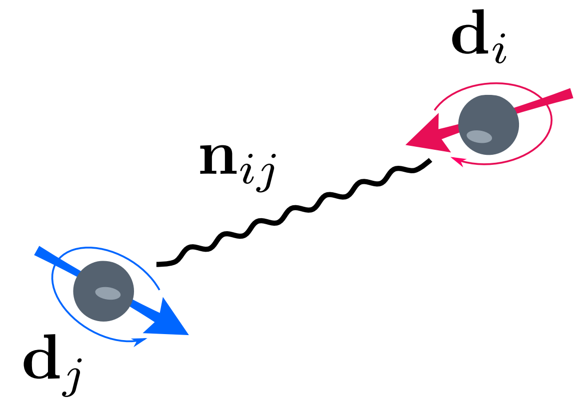
- Einstein de Haas effect
  (Y. Kawaguchi et al, PRL 2006)

- demagnetization cooling
  (M. Fattori et al, Nature Phys 2006)

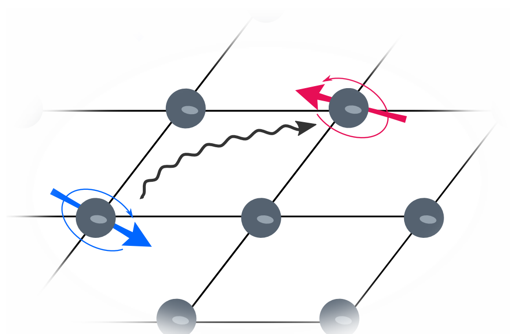
- pattern formation in spinor condensates
  (D. Stamper-Kurn, M. Ueda, RMP 2013).

Dipolar interactions

Exploit this spin orbit coupling
for the generation
of topological band structures



Requirements:



- particles in a 2D lattice
- internal “Spin” structure
- strong dipole-dipole interaction

Systems:

- **polar molecules**
- Rydberg atoms
- NV centers
- atoms with large magnetic dipole moments

Polar molecules in an optical lattice

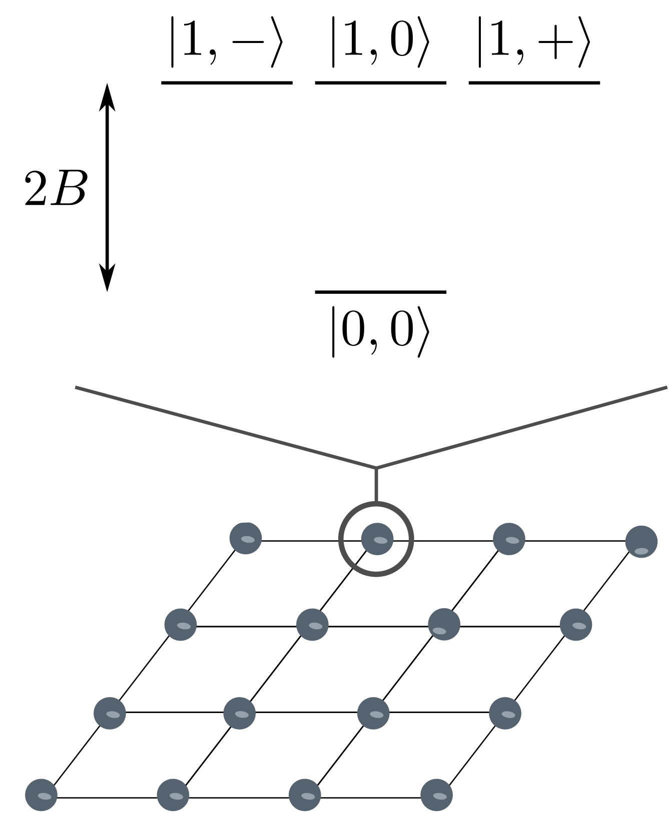
Internal Hamiltonian

- rigid rotor in an electric field

$$H_{\rm rot}^{(i)} = B\mathbf{N}_i^2 - \mathbf{d}_i\mathbf{E}(t)$$

$\mathbf{N}_i$: angular momentum

$\mathbf{d}_i$: dipole operator



External degree

- polar molecules in a 2D optical lattice → different geometries
- one polar molecule per lattice site → imperfections are allowed
- quenched dynamics

Polar molecules in an optical lattice

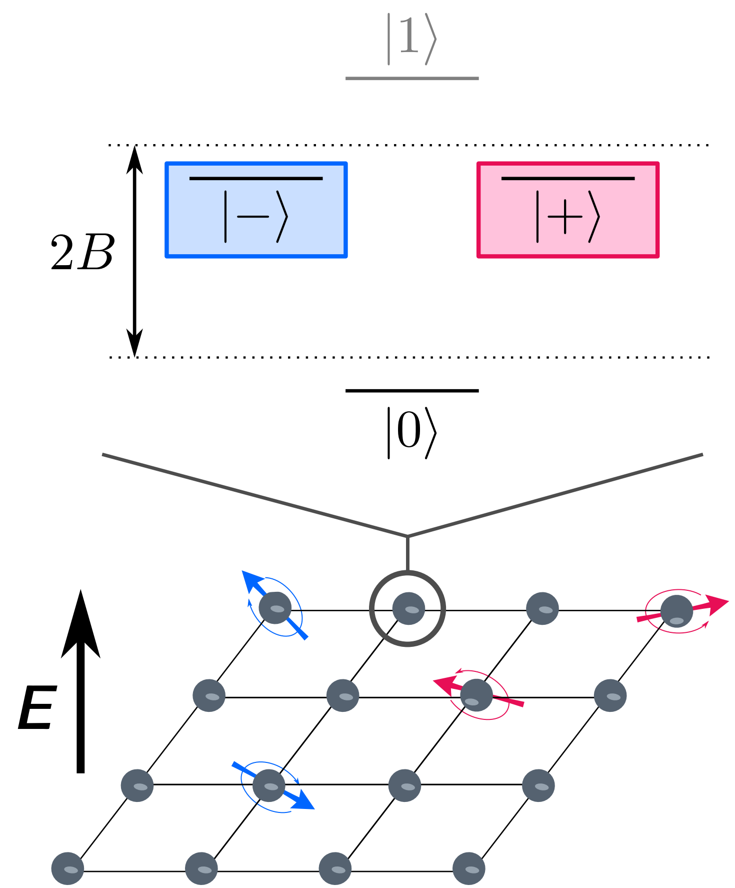
Internal Hamiltonian

- static external electric field

- select three internal states

$|0\rangle_i$: ground state

$|-\rangle_i \quad |+\rangle_i$: two excited states



Mapping onto two hard-core bosons:

- bosonic creation operators for excitations

$$|+\rangle_i = b_{i,+}^{\dagger}|0\rangle$$

$$|-\rangle_i = b_{i,-}^{\dagger}|0\rangle$$

Dipolar exchange interactions

Hamiltonian

- dipolar interaction restricted to the three internal levels

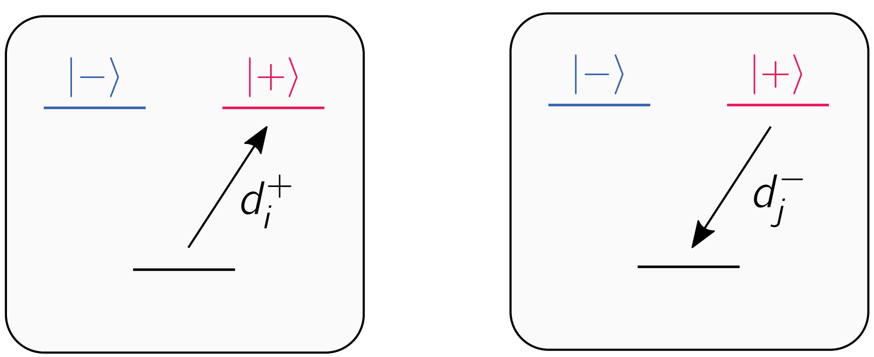
$$d^0 \equiv d^z \qquad d^+ \equiv d^x + i d^y \qquad d^- \equiv d^x - i d^y$$

$$H_{ij} = \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|^3}\left[d_i^0 d_j^0 + \frac{1}{2}\left(d_i^+ d_j^- + d_i^- d_j^+\right) - \frac{3}{2}\left(d_i^+ d_j^+ e^{-i2\phi_{ij}} + d_i^- d_j^- e^{i2\phi_{ij}}\right)\right]$$

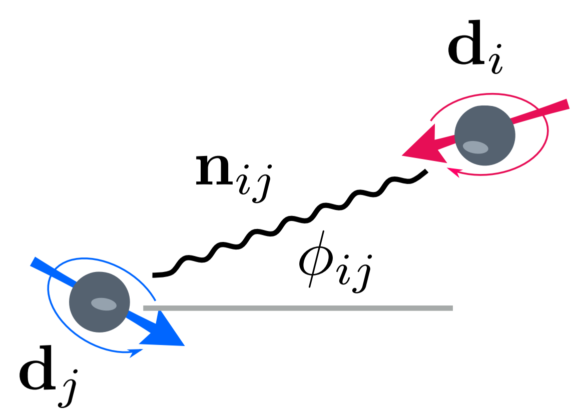
dipole interaction

exchange interaction

spin-orbit coupling



hopping of excitation



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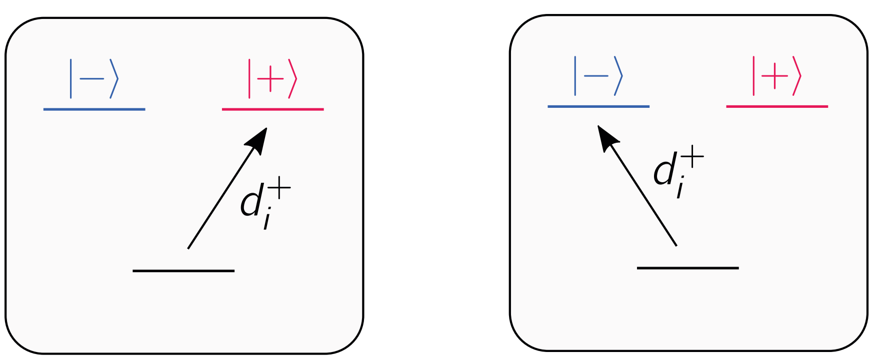
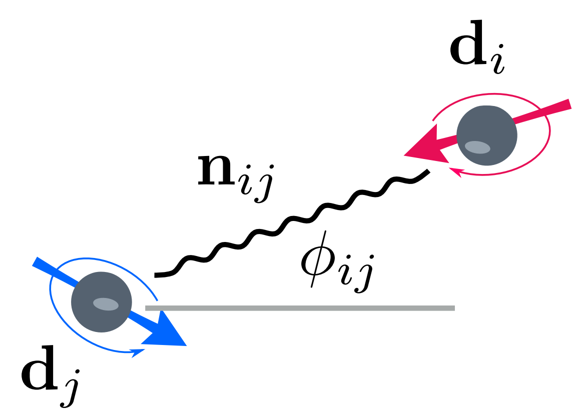
dipole interaction — exchange interaction — spin-orbit coupling



hopping with spin flip



Single excitation Hamiltonian

Hamiltonian

- single particle hoping with spin orbit interaction

$$\psi_i = \begin{pmatrix} b_{+,i} \\ b_{-,i} \end{pmatrix}$$

$$H = \sum_{i \neq j} \frac{a^3}{|\mathbf{r}_i - \mathbf{r}_j|^3} \psi_i^\dagger \begin{pmatrix} -t^+ & w e^{i2\phi_{ij}} \\ e^{-i2\phi_{ij}} & -t^- \end{pmatrix} \psi_i$$

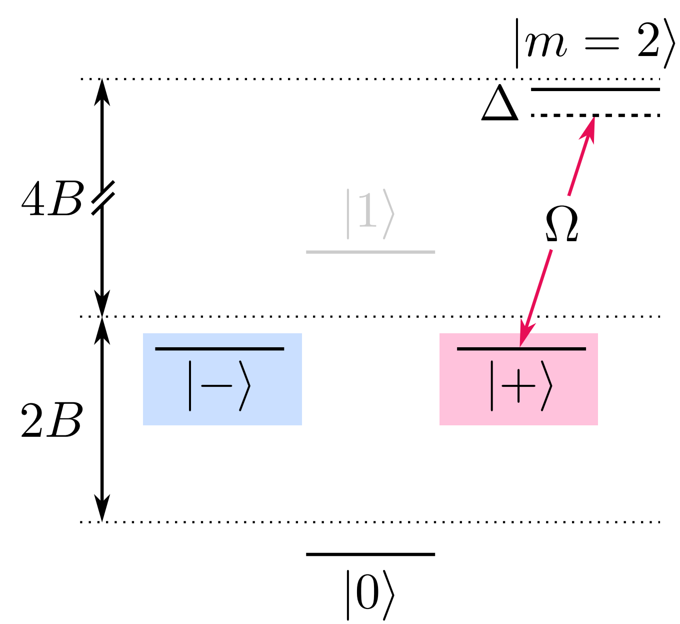
Time-reversal symmetry breaking

- different hopping for excitations

$$t^- \neq t^+$$

- ac stark shift on levels

$$\Delta H = \sum_i \psi_i^\dagger \begin{pmatrix} \mu & 0 \\ 0 & -\mu \end{pmatrix} \psi_i$$



Single excitation Hamiltonian

Hamiltonian

- single particle hoping with spin orbit interaction

$$\hat{\epsilon}_{\mathbf{k}} = \sum_{i \neq 0} \frac{1}{|\mathbf{r}_i|^3} e^{i\mathbf{k}\cdot\mathbf{r}_i + i2\phi_i}$$

$$H = \sum_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \begin{pmatrix} -t^{+}\epsilon_{\mathbf{k}} + \mu & w\hat{\epsilon}_{\mathbf{k}} \\ w\hat{\epsilon}^{*}_{\mathbf{k}} & -t^{-}\epsilon_{\mathbf{k}} + \mu \end{pmatrix} \psi_{\mathbf{k}}$$

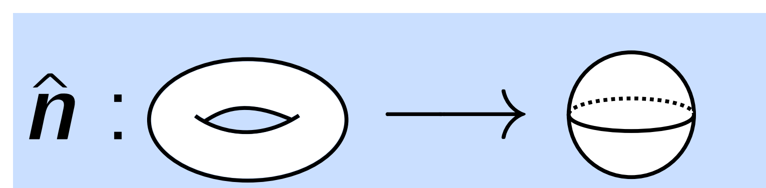
$$= \sum_{\mathbf{k}} \psi^{\dagger}_{\mathbf{k}} \left[n^{0}_{\mathbf{k}} + \mathbf{n}_{\mathbf{k}} \sigma \right] \psi_{\mathbf{k}}$$

$$\epsilon_{\mathbf{k}} = \sum_{i \neq 0} \frac{1}{|\mathbf{r}_i|^3} e^{i\mathbf{k}\cdot\mathbf{r}_i}$$

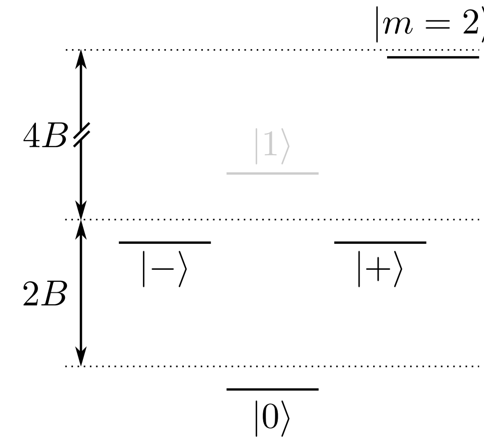
Topological character of band

- characterized by a Chern number

$$C = \int \frac{d\mathbf{k}}{v_0} \mathbf{n}_{\mathbf{k}} \cdot \left(\frac{\partial \mathbf{n}_{\mathbf{k}}}{\partial k_x} \wedge \frac{\partial \mathbf{n}_{\mathbf{k}}}{\partial k_y} \right) \in \mathcal{Z}$$



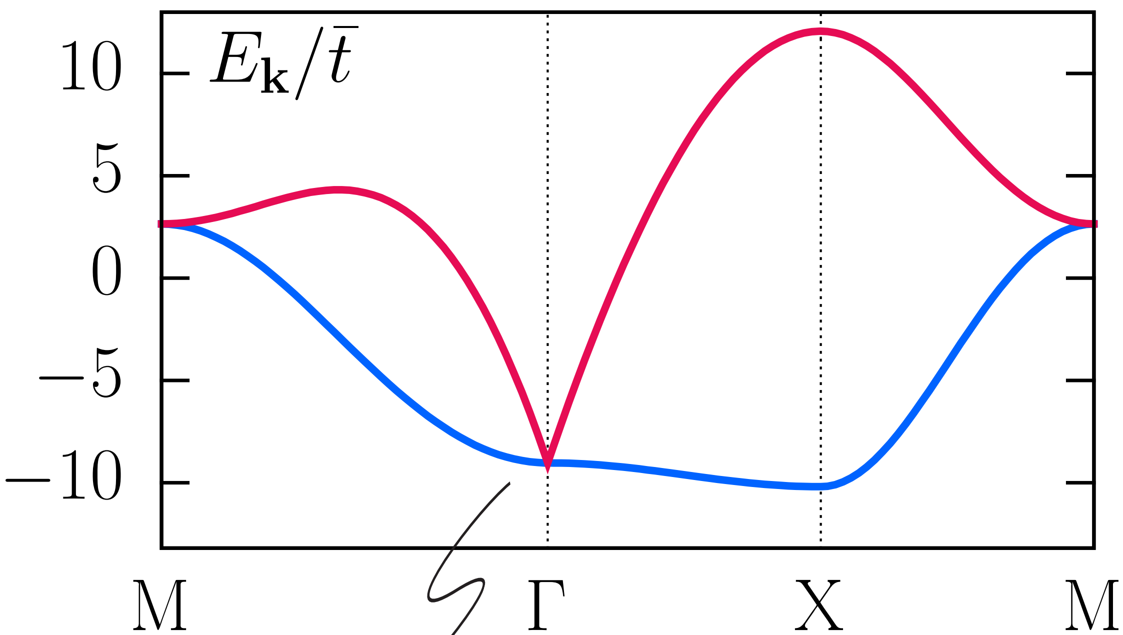
Topological band structure



With time reversal symmetry

- square lattice

- two bands with quadratic band touching points



Linear dispersion:

- consequence of slow dipolar decay of hopping

quadratic band touching point

Dirac points in rectangular lattice

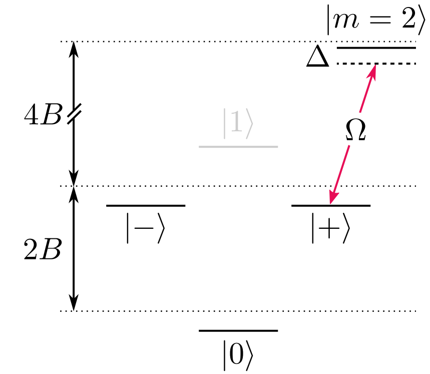
see also Syzranov *et al* Nat. Comm. 2014

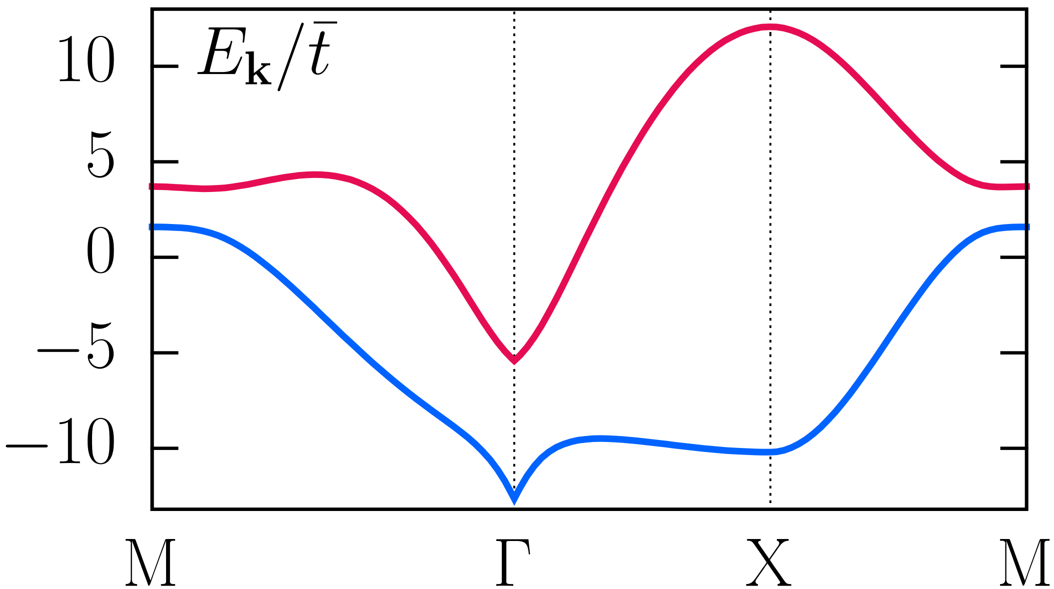
Topological band structure

Without time reversal symmetry

- square lattice

- gap openings

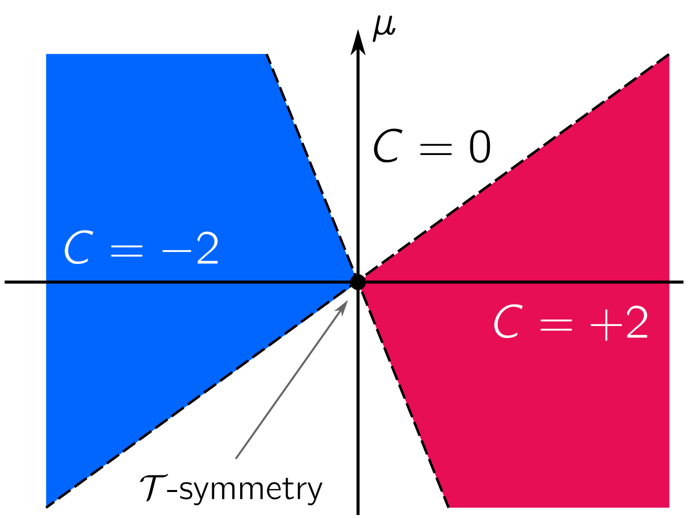




Topological bands

- Chern number

$$C = \int \frac{d\mathbf{k}}{v_0} \mathbf{n_k} \cdot \left(\frac{\partial \mathbf{n_k}}{\partial k_x} \wedge \frac{\partial \mathbf{n_k}}{\partial k_y} \right) \in \mathcal{Z}$$

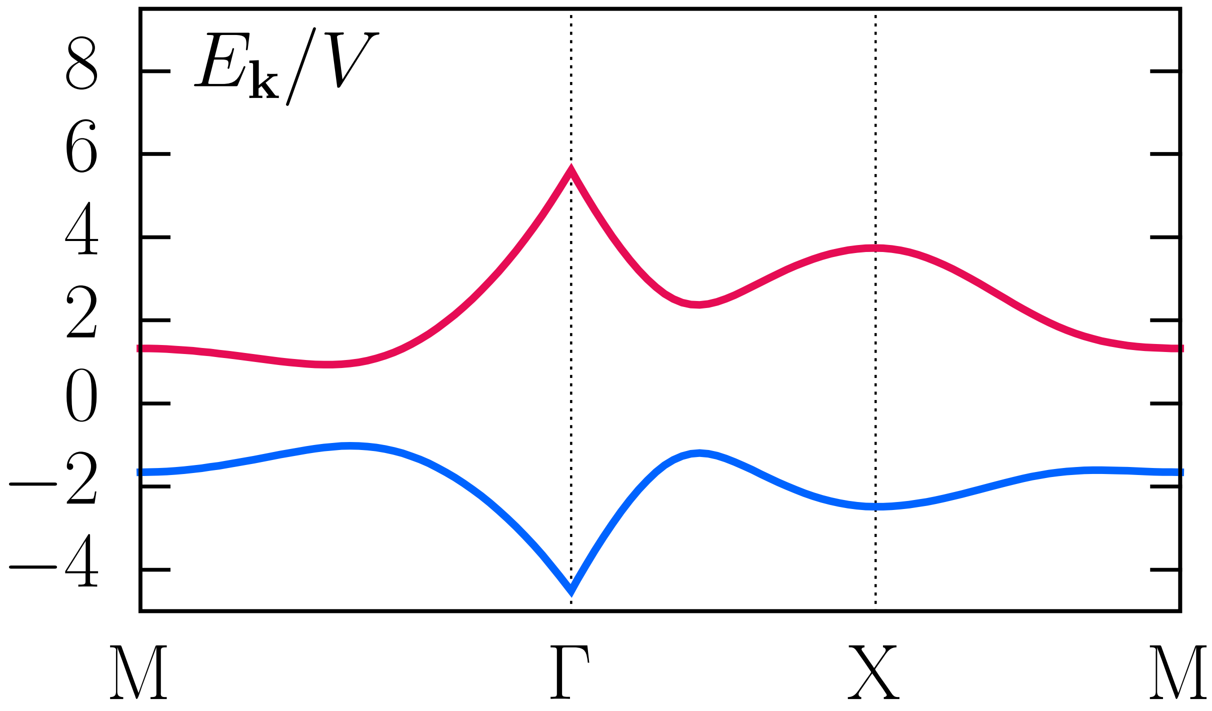


Topological band structure

Without time reversal symmetry

- optimal experimental parameters

- Chern number C=2

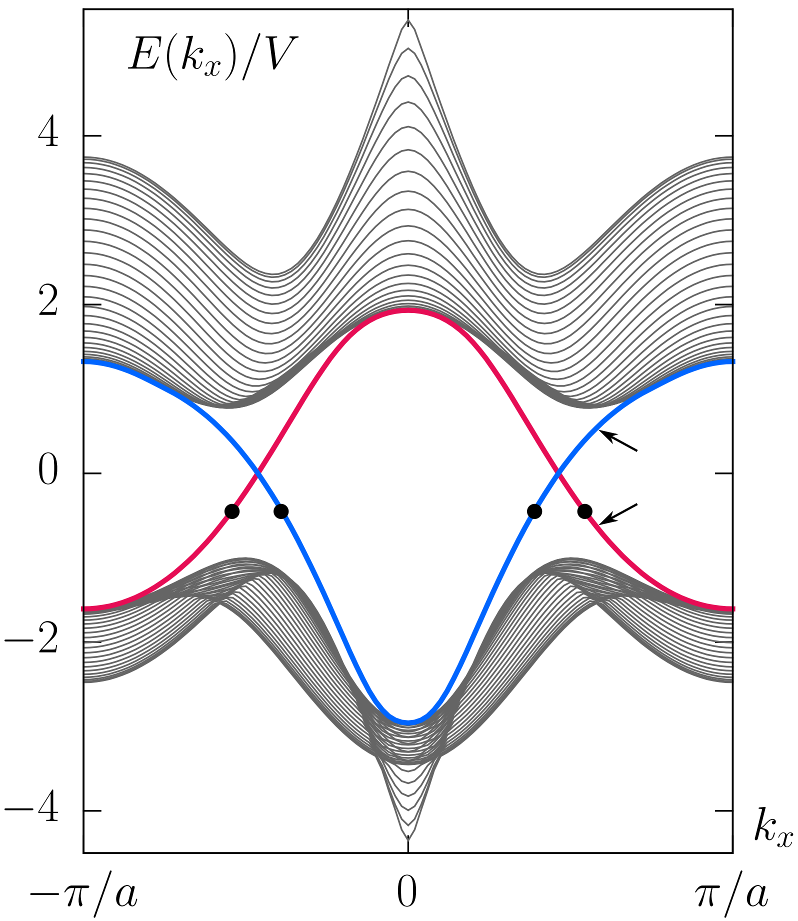


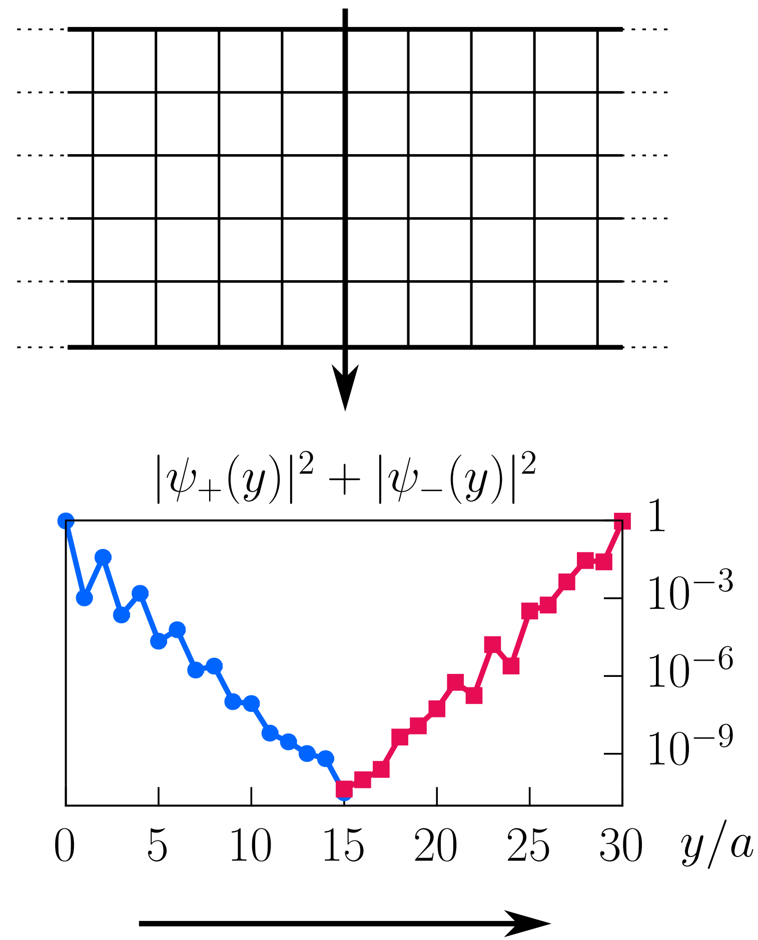
Edge states

Finite system in y-direction

- bulk edge correspondence (Hatsugai PRL 1993) → **C= 2 implies two edge states**

- exponential localization in presence of long-range hopping



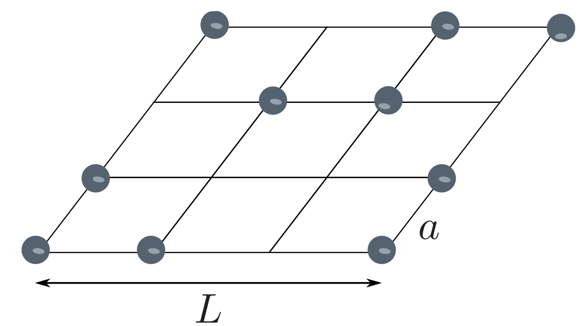


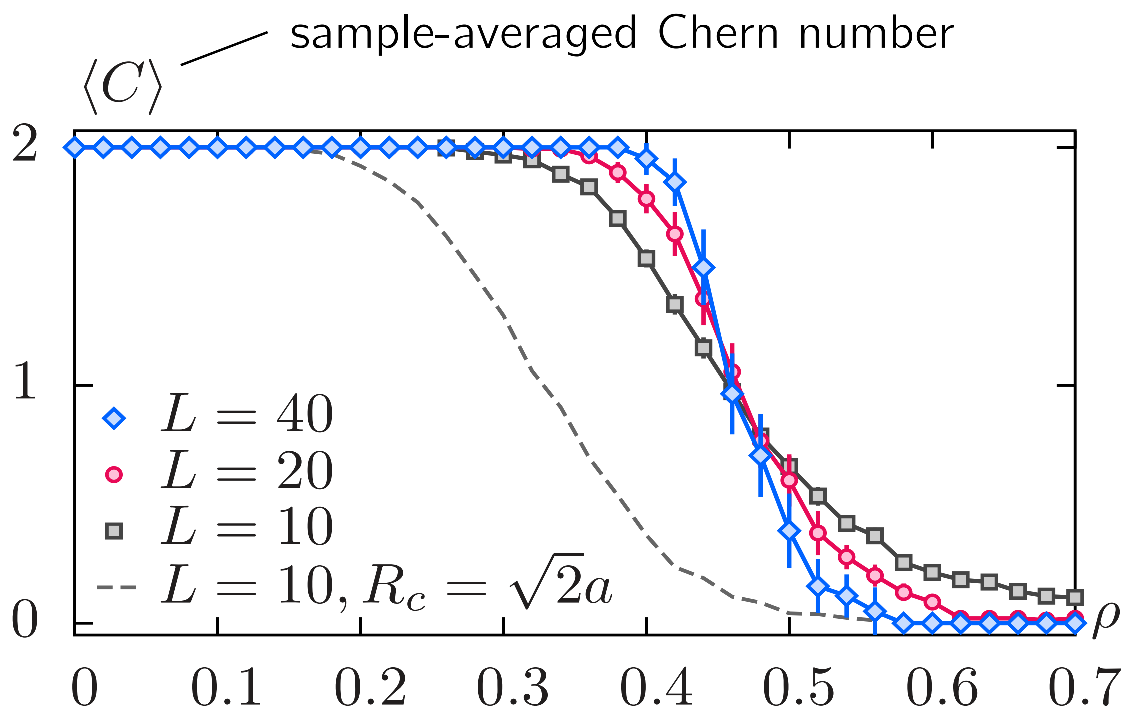
Stability under disorder

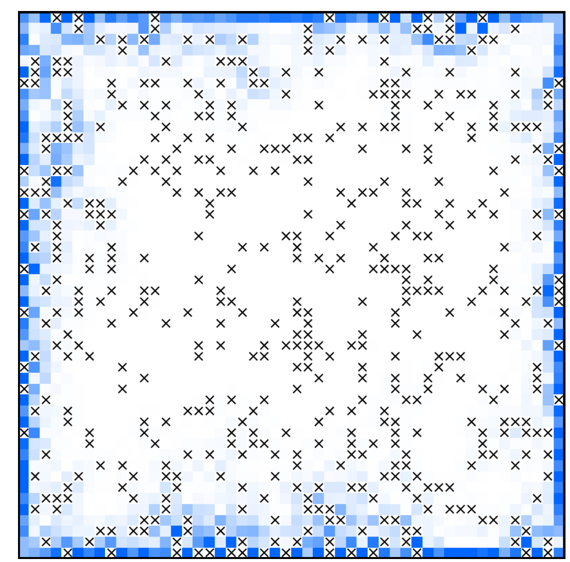
Disorder

- missing molecules in the lattice

- stabilized by long-range hopping







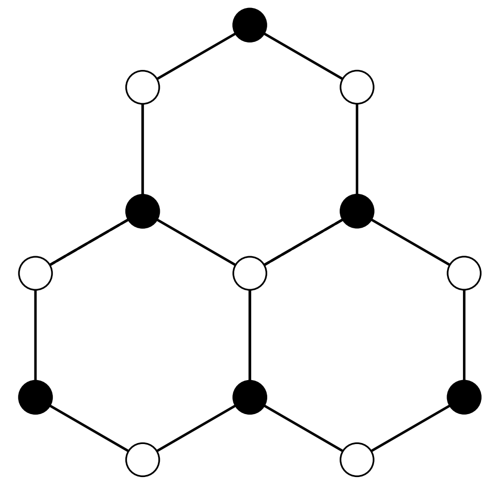
Flat topological bands

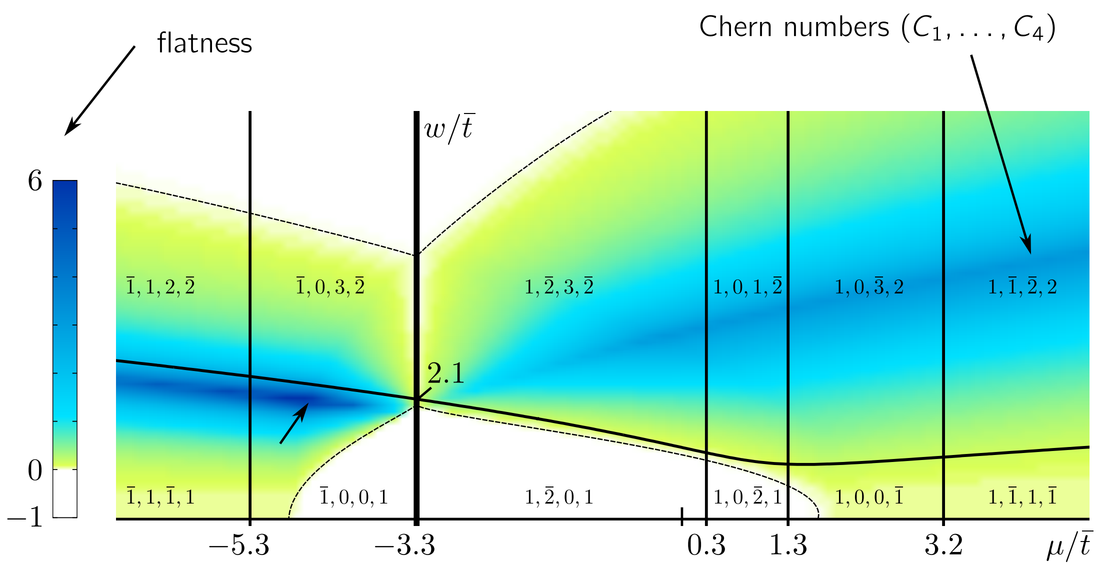
Honeycomb lattice

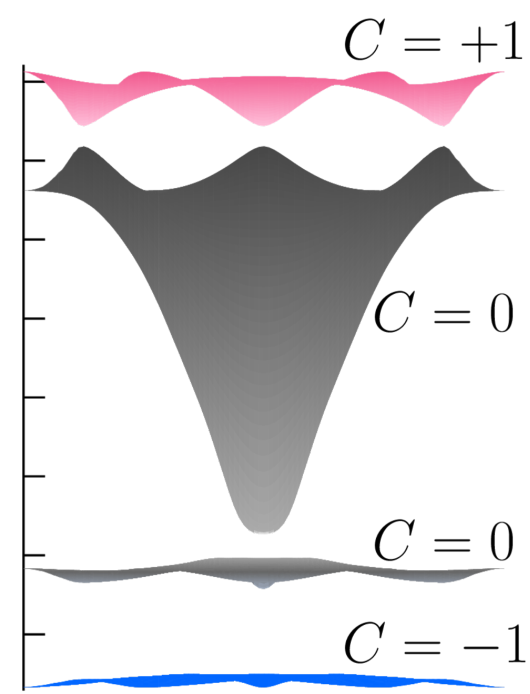
- much flatter bands accessible
- very rich topological structure

$$C \in \{0, \pm 1, \pm 2, \pm 3, \pm 4\}$$

- even richer for Kagame lattice



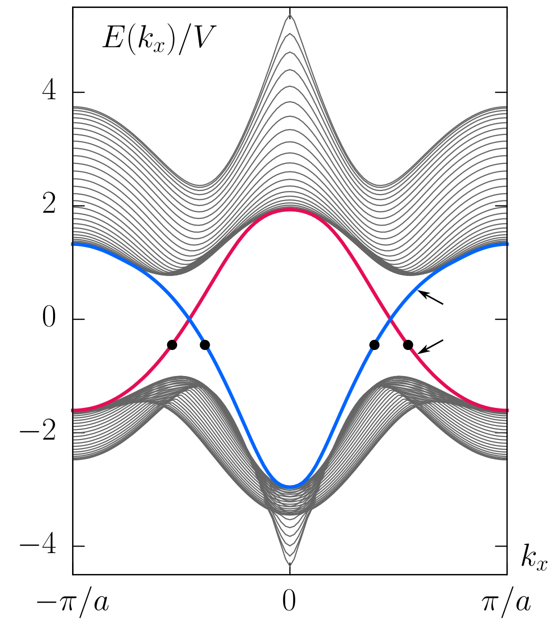




Outlook on topological bands

Dipolar interaction provides natural spin orbit coupling

- topological band structures
  with Chern number C=2



Robust to disorder

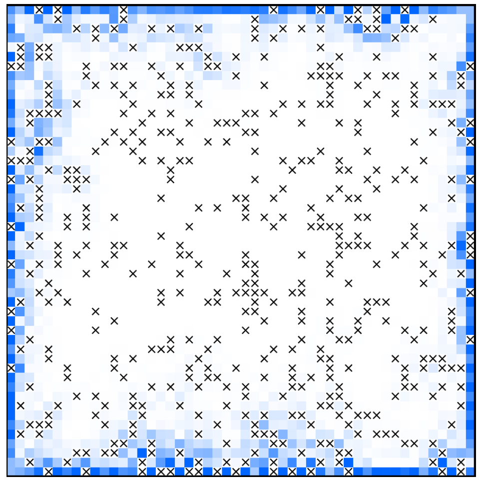
- topological nature is very robust
  to missing particles in the lattice

Towards bosonic fractional Chern insulators

- strongly interacting system

- is flatness high enough for
  topological phases

- candidate expected at 2/3 filling

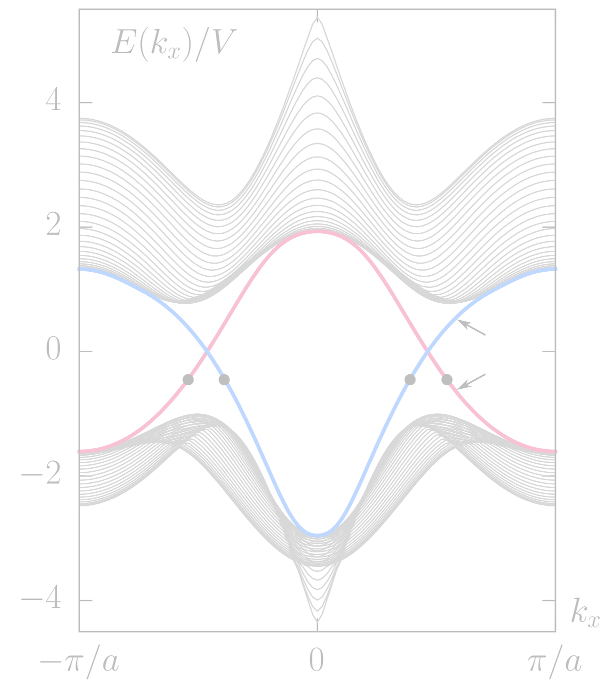


Outline

Topological band structures with dipolar interactions

- realized with polar molecules and Rydberg atoms

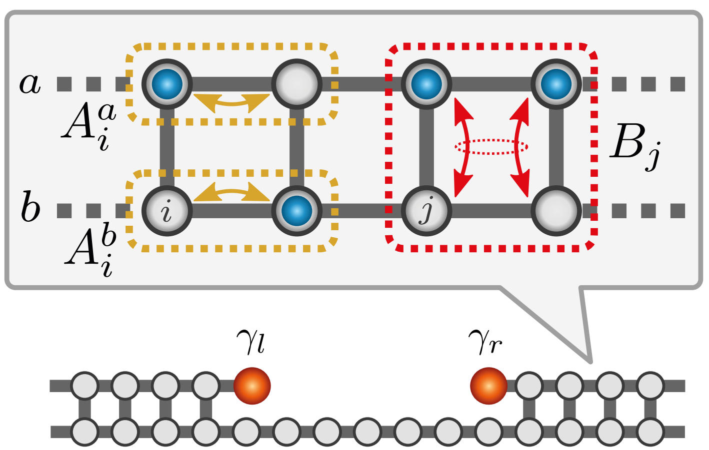
- Chern number C=2



Majorana Edge modes

- exact solvable system with fixed particle number

- double wire system



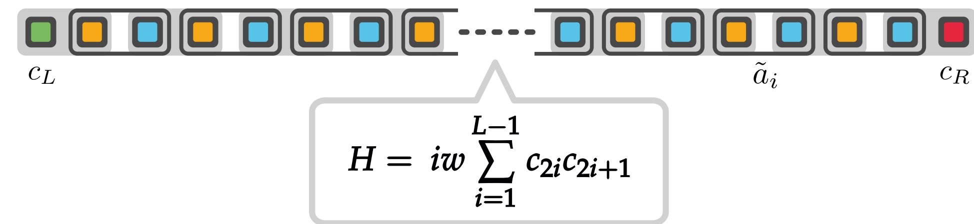
Kitaev's Majorana chain

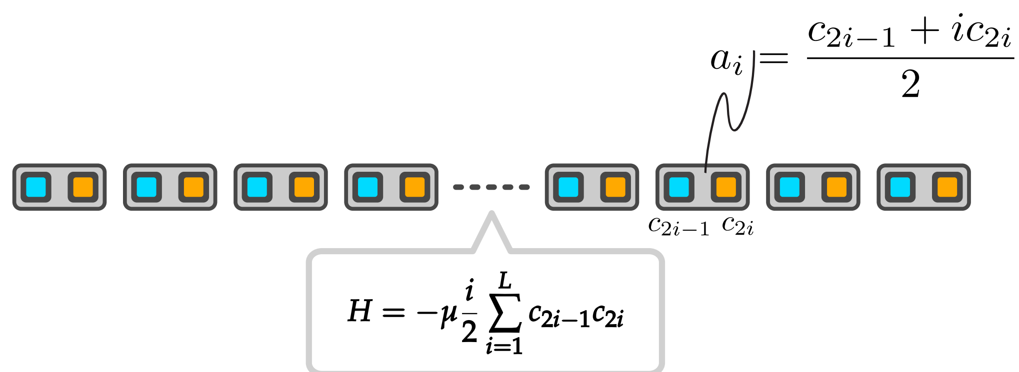
Kitaev's Majorana chain

- fermions on a 1D lattice
  with supefluid pairing term

$$H = -\sum_{i=1}^{L-1}\left[w a_i^\dagger a_{i+1} - \Delta a_i a_{i+1} + \text{h.c.}\right]$$

$$-\mu \sum_{i=1}^{L} a_i^\dagger a_i$$



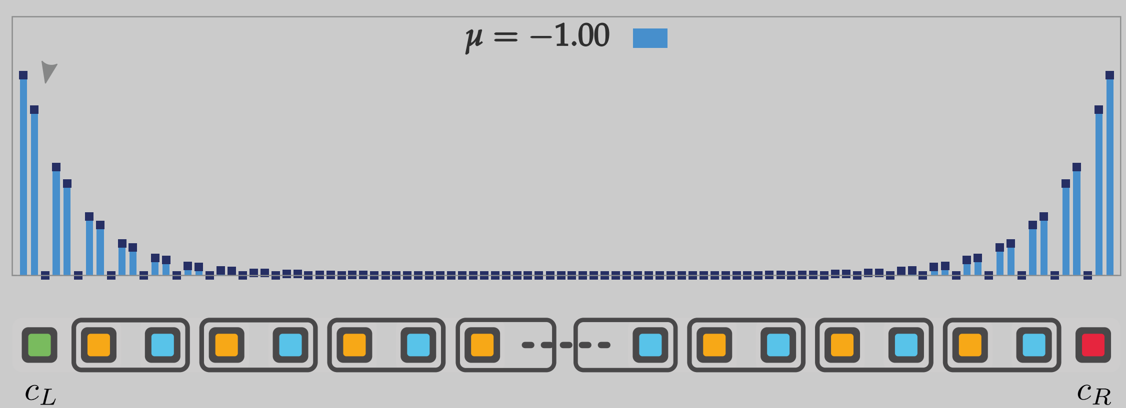


- exact solution by introducing
  Majorana fermions

$$a_i = \frac{c_{2i-1} + ic_{2i}}{2}$$

Topological state

- robust ground state
  degeneracy
- non-local order
  parameter
- localized edge states



Why should we care

Topological invariant edge states

- ground state degeneracy is robust
  to local perturbations

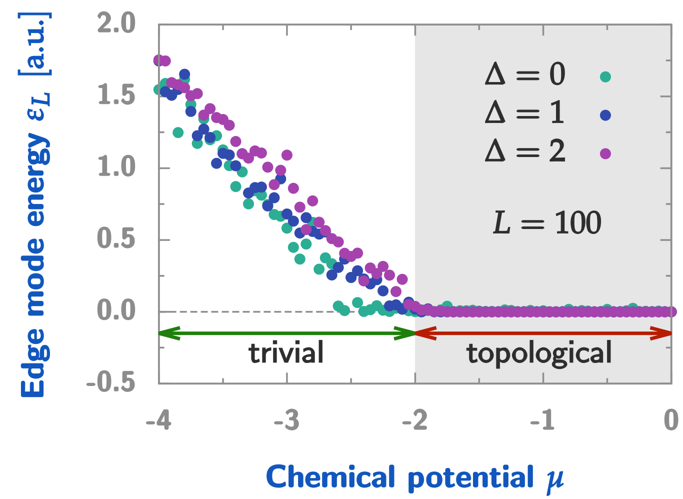
→ robust quantum memory?

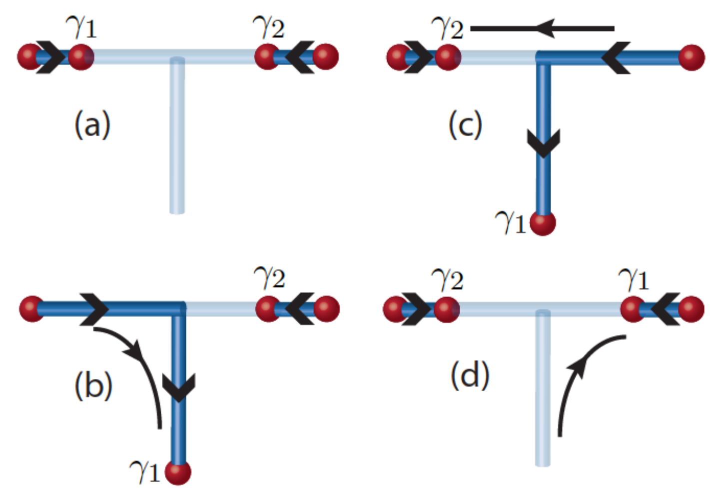
Non-abelian anyons

- localized edge modes obey
  non-abelian braiding statistics

→ topological quantum computation

- Novel state of matter
- Toy model of a topological phase





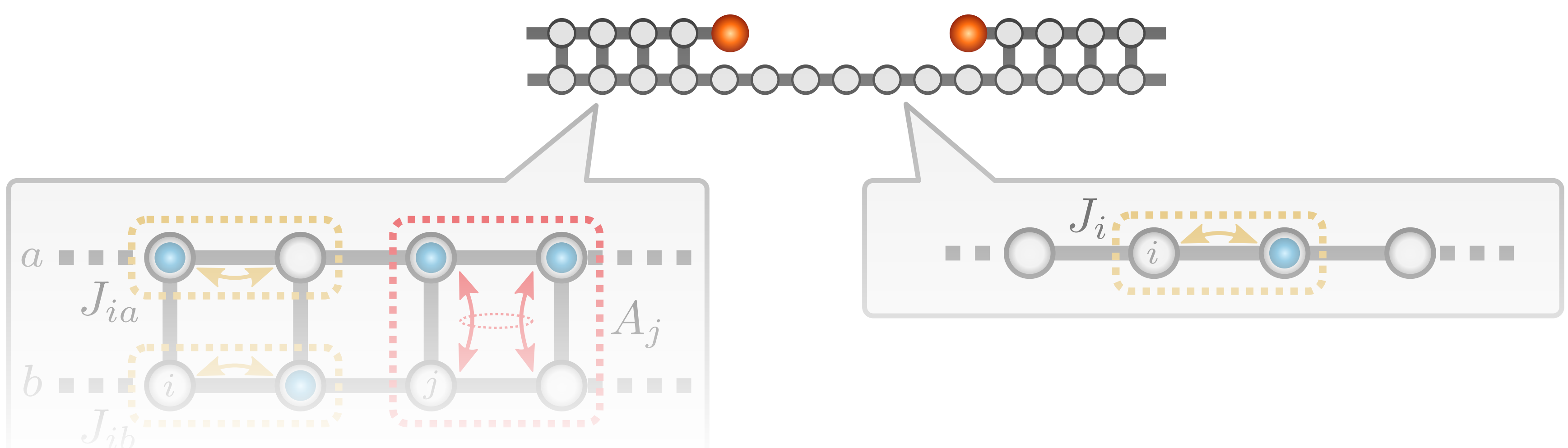
(Alicea et al Nat. Phys. 2011)

Beyond mean-field

$$H = -\sum_{i=1}^{L-1}\left[w a_i^\dagger a_{i+1} - \Delta a_i a_{i+1} + \mathrm{h.c.}\right]$$

- mean-field coupling
- violates particle conservation

exist a particle conserving theory
with Majorana modes in one-dimension?



Beyond mean-field

→ M. Cheng and H.-H. Tu (2011). Physical Review B, 84(9), 094503.
Majorana edge states in interacting two-chain ladders of fermions.

→ J. D. Sau et al. (2011). Physical Review B, 84(14), 144509.
Number conserving theory for topologically protected degeneracy in one-dimensional fermions.

→ L. Fidkowski et al. (2011). Physical Review B, 84(19), 195436.
Majorana zero modes in one-dimensional quantum wires without long-ranged superconducting order.

→ J. Ruhman et al. (2014). arXiv:1412.3444
Topological States in a One-Dimensional Fermi Gas with Attractive Interactions.

Bosonization

→ C. V. Kraus et al. (2013). Physical Review Letters, 111(17), 173004.
Majorana Edge States in Atomic Wires Coupled by Pair Hopping.

Numerical

→ G. Ortiz et al. (2014). arXiv:1407.3793
Many-body characterization of topological superconductivity: The Richardson-Gaudin-Kitaev chain.

Long-range

Here: Sort-range interacting theory
exact ground state
Majorana edge modes

Microscopic model

Hamiltonian

- double wire system

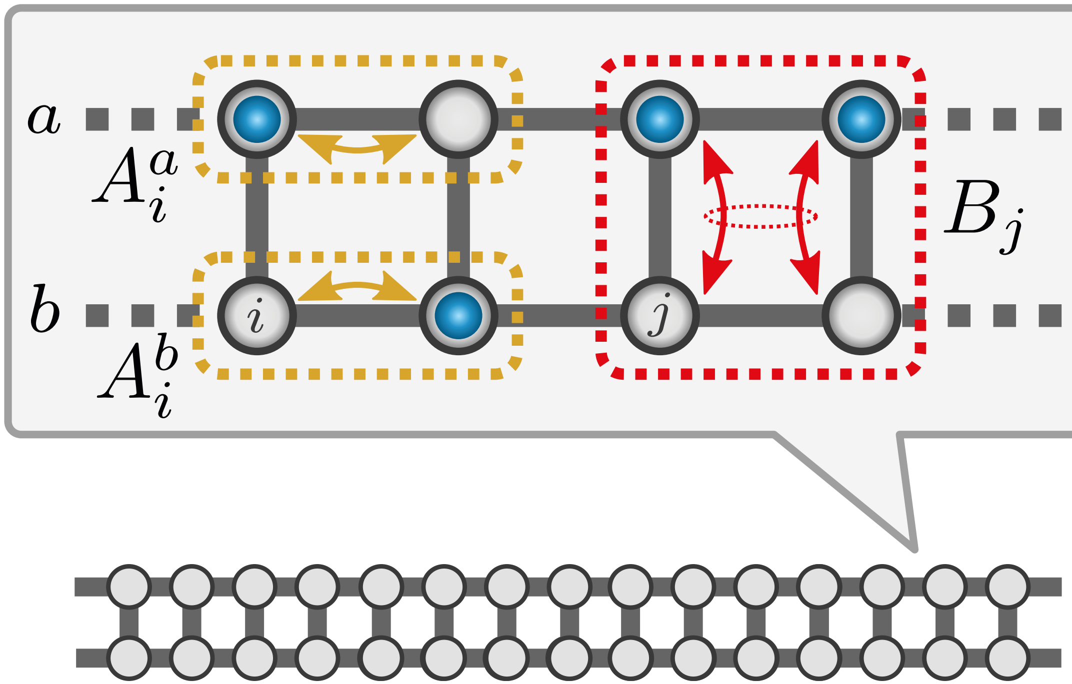
$$H = H_a + H_b + H_{ab}$$

- intra-chain contribution

$$H_a = \sum_i A_i^a \left(1 + A_i^a\right)$$

- inter-chain contribution

$$H_{ab} = \sum_i B_i \left(1 + B_i\right)$$



Symmetries

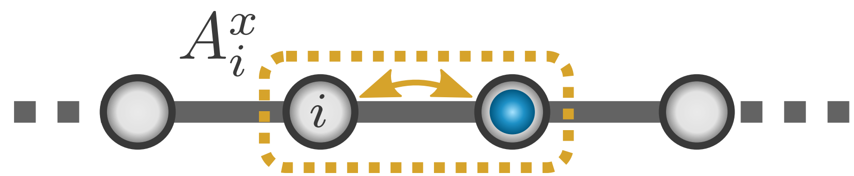
- total number of particles N
- time reversal symmetry T
- sub-chain parity P

Microscopic model

Inter-chain Hamiltonian

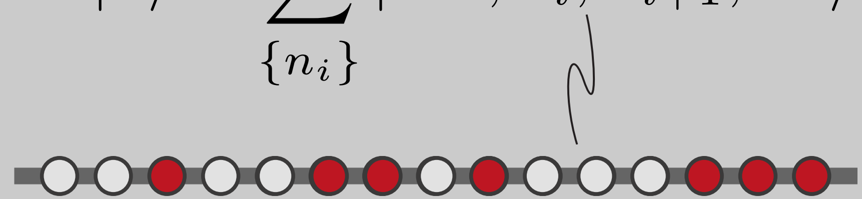
$$H_a = \sum_i A_i^a \left(1 + A_i^a\right)$$

$$A_i^a = a_i^\dagger a_{i+1} + a_{i+1}^\dagger a_i$$



- positive Hamiltonian
- zero-energy state is ground state

$$|n\rangle = \sum_{\{n_i\}} |\ldots, n_i, n_{i+1}, \ldots\rangle$$



equal weight superposition of all possible distribution of n fermions

Inter-chain Hamiltonian (expanded)

$$H_i^a = a_i a_{i+1}^\dagger + a_{i+1} a_i^\dagger + n_i^a \left(1 - n_{i+1}^a\right) + n_{i+1}^a \left(1 - n_i^a\right)$$

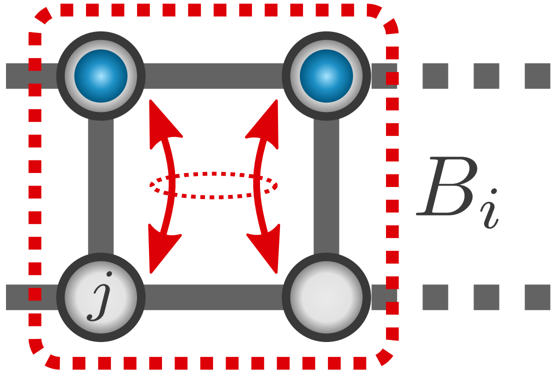
Microscopic model

Intra-chain Hamiltonian

$$H_{ab} = \sum_i B_i \left(1 + B_i\right)$$

$$B_i = a_i^\dagger a_{i+1}^\dagger b_i b_{i+1} + b_i^\dagger b_{i+1}^\dagger a_i a_{i+1}$$

pair-hopping between chains



- positive Hamiltonian
- zero-energy state is ground state
- fixed total number of particles

$$|\psi\rangle = \sum_n |n\rangle |N-n\rangle$$

equal weight superposition of all possible distribution of N fermions between the two wires

Intra-chain Hamiltonian (expanded)

$$H_{ab}^i = a_i^\dagger a_{i+1}^\dagger b_i b_{i+1} + b_i^\dagger b_{i+1}^\dagger a_i a_{i+1} + n_i^a n_{i+1}^a \left(1-n_i^b\right)\left(1-n_{i+1}^b\right) + n_i^b n_{i+1}^b \left(1-n_i^a\right)\left(1-n_{i+1}^a\right)$$

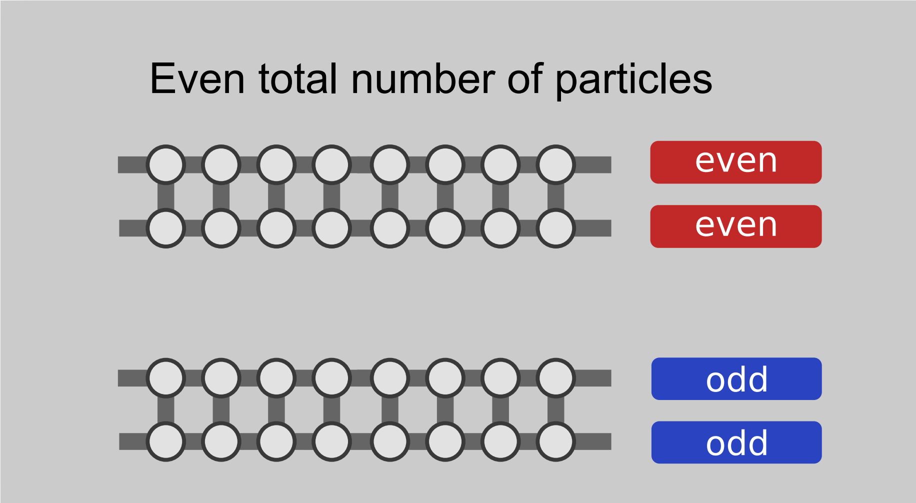
Ground state degeneracy

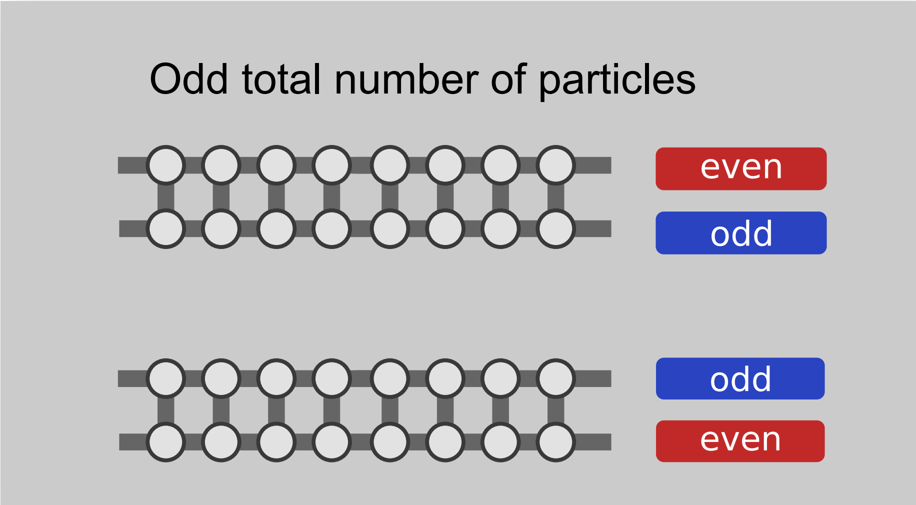
Two-open chains

- two-fold ground state
  degeneracy

$$|\psi_{\text{even}}\rangle = \sum_{n \in \text{even}} |n\rangle |N-n\rangle$$

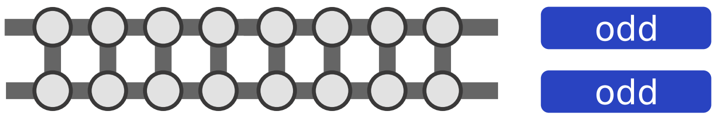
$$|\psi_{\text{odd}}\rangle = \sum_{n \in \text{odd}} |n\rangle |N-n\rangle$$





Two- closed chains

- only one zero energy state for
  total even number of particles



Wire networks

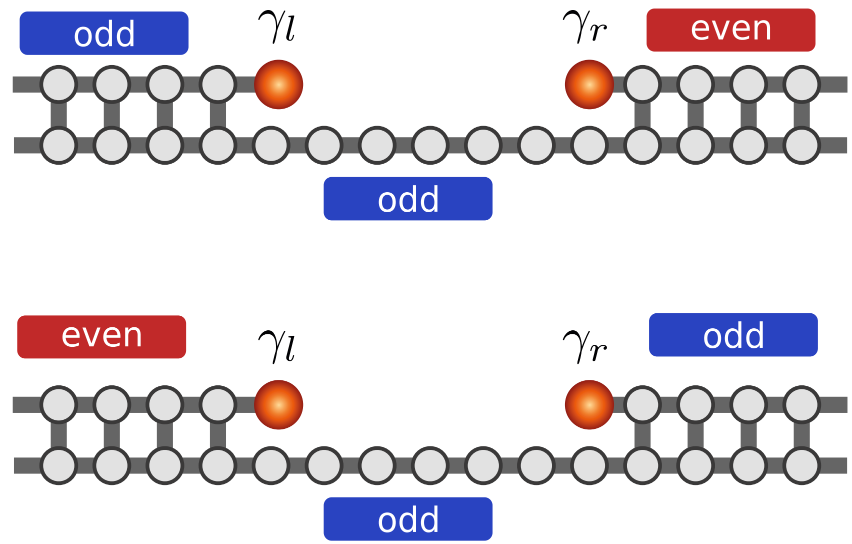
Networks of wires

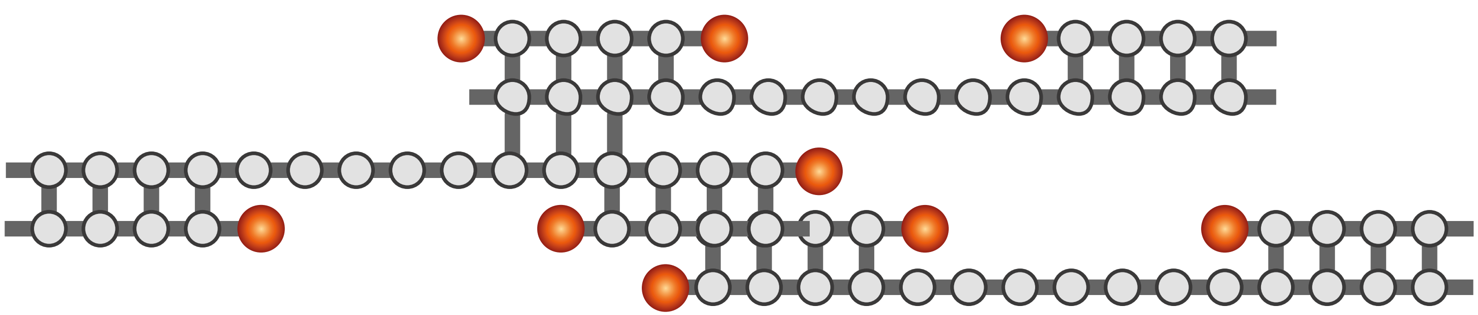
- exact ground states for arbitrary networks

- degeneracy consistent with majorana modes at edges

$$2^{E/2-1}$$

number of edges



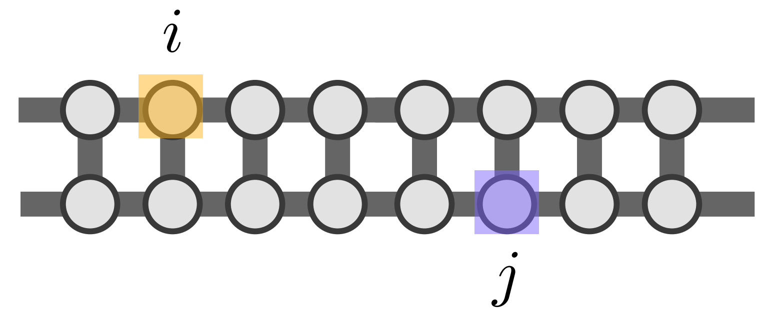


Ground state properties

Density-density correlations

- independent on ground state

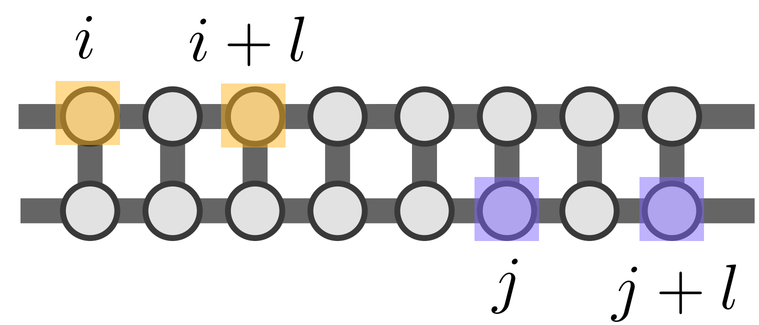
$$\langle n_i^{\sigma} n_j^{\sigma'} \rangle = \rho^2 \qquad i \neq j$$



Superfluid correlations

$$\langle a_i^\dagger a_{i+l}^\dagger a_j a_{j+l} \rangle = \rho(1-\rho)$$

- long-range order with infinity correlation length

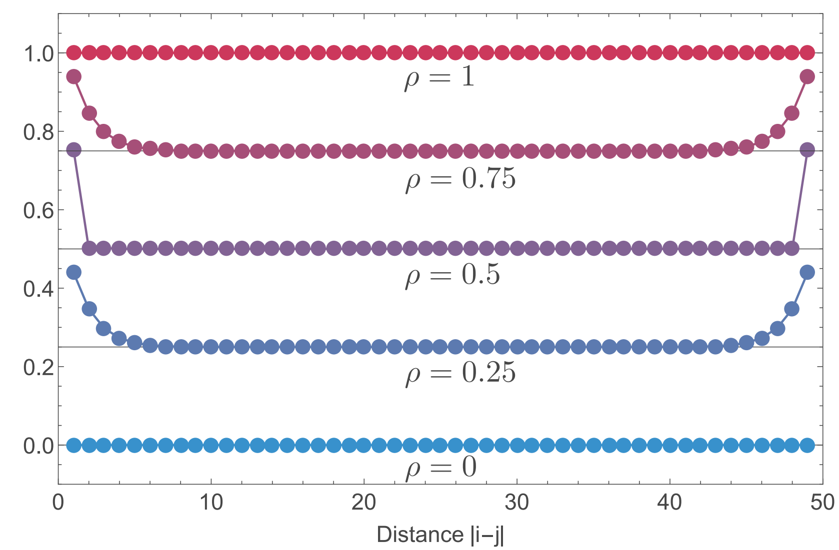


Green's function

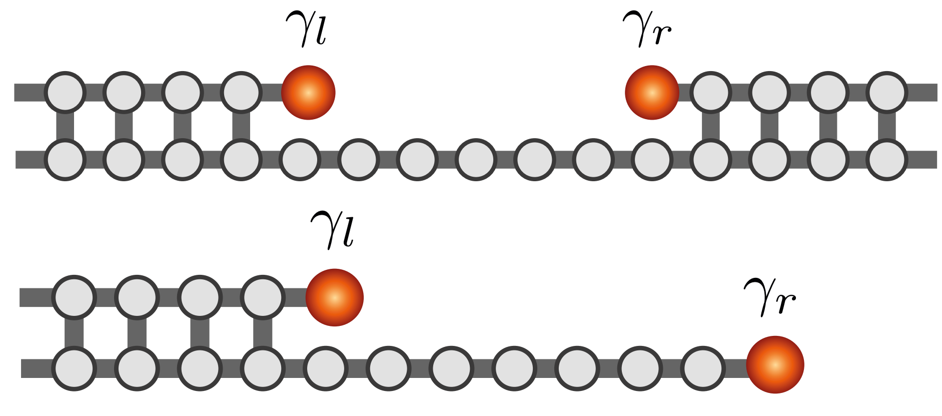
$$\langle a_i^\dagger a_j \rangle$$

- exponential decay
- revival at the edge

→ existence of edge modes



Ground state properties



Stability of ground state degeneracy of edge states

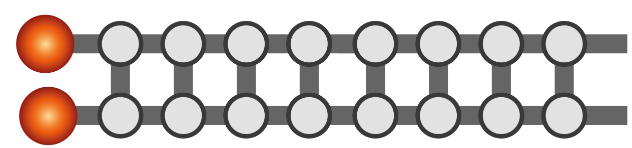
- stable under all local perturbations
- splitting decays exponentially

Stability of ground state degeneracy for open wires

- Protected by either time-reversal symmetry or subchain parity

$a_i^\dagger b_i + b_i^\dagger a_i$: stable under time reversal hopping

$i a_i^\dagger b_i - i b_i^\dagger a_i$: finite overlap between two ground states



Excitation spectrum

Low-energy excitations

- Goldstone mode due to broken U(1) symmetry

- exact wave function for single phase kink excitation

$$|k, \psi\rangle = \sum_j e^{ikj} \left[(-1)^{n_j^a} + (-1)^{n_j^b} \right] |\psi\rangle$$

- quadratic excitation spectrum

$$\epsilon_k = 4 \sin^2 k/2 \sim k^2$$

System is in a critical state

- vanishing compressibility

- Goldstone mode with quadratic dispersion

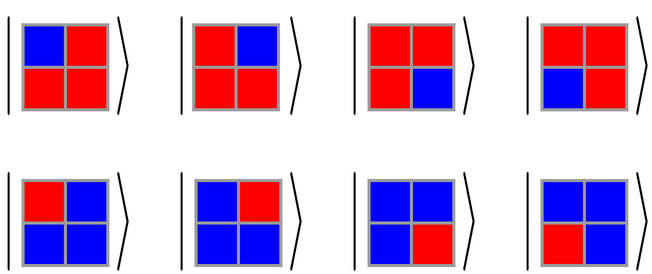
Non-abelian Braiding statistics

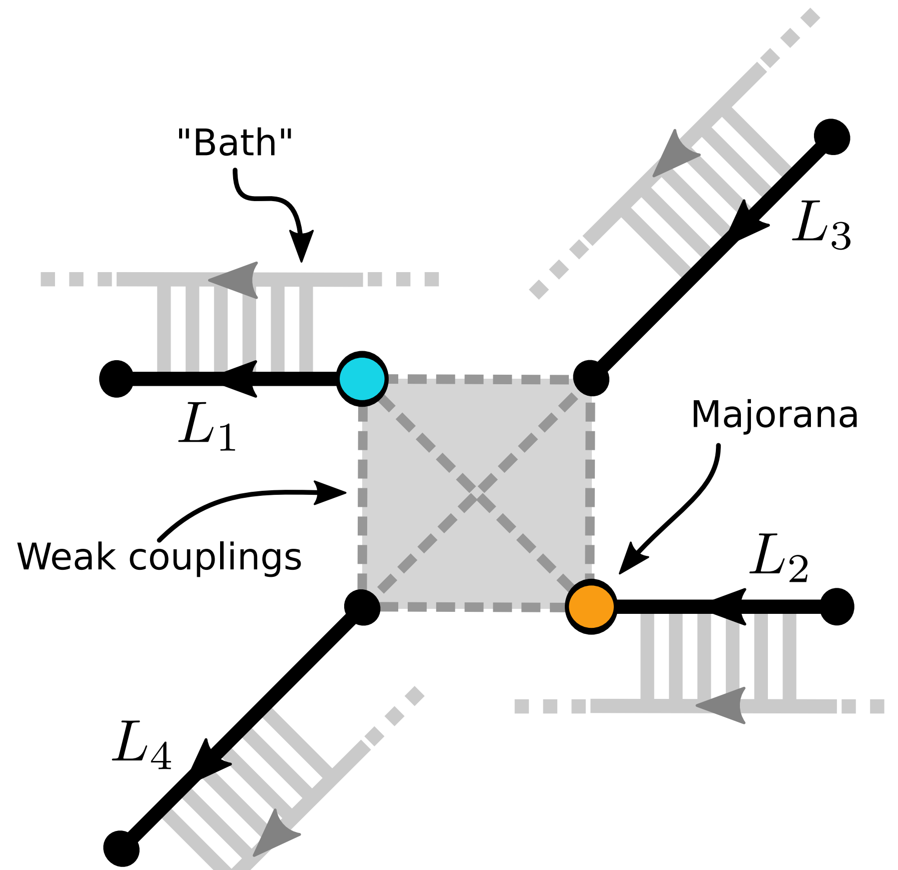
Setup for braiding of
two edge states

- wire network with two edges
- restriction to the low energy sector
- very weak coupling terms: adiabatic switching between them

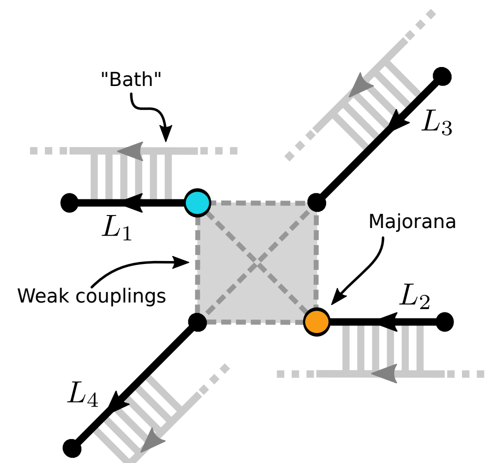
8 relevant states

- characterized by subchain parity



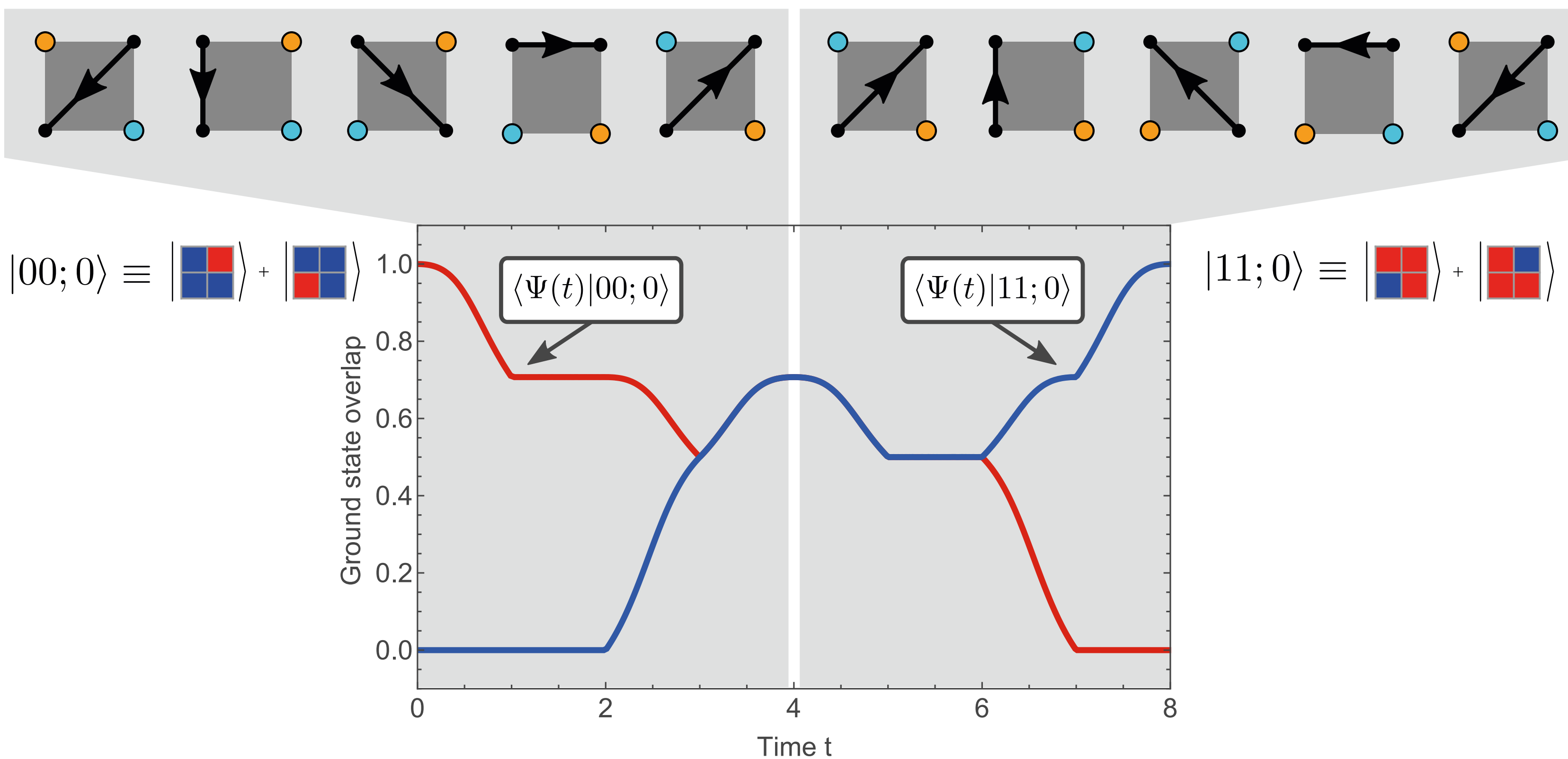


Non-abelian Braiding statistics

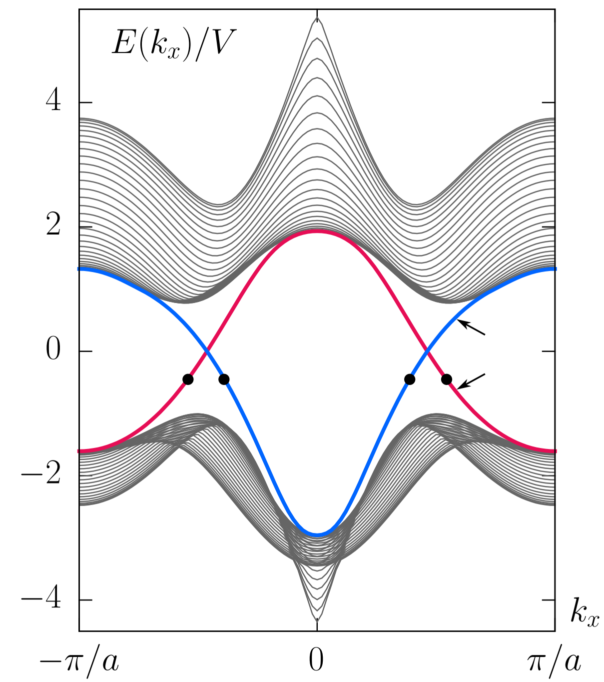


Adiabatic switching of coupling

- transformation of the ground state according to the non-abelian statistic of Majorana modes



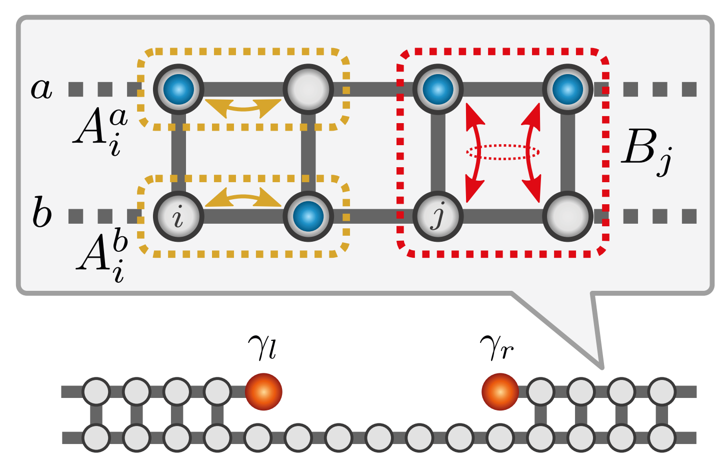
Conclusion



Topological band structure with dipolar exchange interactions

- spin-orbit coupling natural present in dipolar system

- existence of topological band structures



Majorana Edge modes

- exact solvable system with fixed particle number

- analytical demonstration of Majorana edge modes