Crystalline Confinement and fractional fluxes in Abelian Quantum Link and Dimer models

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JSTAT 2013(12), P12010 & Phys. Rev. B 90, 245143 (2014)

DESY, Zeuthen

INT Workshop on Frontiers in Quantum Simulation, Seattle

Introduction

- Lattice gauge theories $\rightarrow$ fundamental contribution towards understanding of strongly correlated systems.
- Most non-perturbative computations done in Euclidean space with Wilson formulation.
- Ultra-cold atoms toolbox $\rightarrow$ quantum dynamics of gauge theories.
- Questions of real-time evolution and finite baryon density.
- Alternate formulation of gauge theories (Horn,1981; Orland, Rohrlich, 1990; Chandrasekharan, Wiese, 1997) and QCD with domain wall fermions (Brower, Chandrasekharan, Wiese, 1999) are particularly relevant.
- These realize **continuous** gauge symmetries using **discrete** quantum link variables, having finite dimensional Hilbert space $\rightarrow$ extension of Wilson formulation of gauge theories.
- Excellent candidate models to be implemented in cold-atom systems.
- Allows construction of very efficient algorithms to study static properties.

Hamiltonian $U(1)$ LGT: Wilson formulation

- U(1) gauge invariant Hamiltonian:

$$H = \frac{g^2}{2}\sum_{x,i} e_{x,i}^2 - \frac{1}{2g^2}\sum_{\Box}(u_\Box + u_\Box^\dagger)$$

- $u = \exp(i\varphi); \;\; u^\dagger = \exp(-i\varphi); \;\; e = -i\partial_\varphi;$
  $\Rightarrow$ are operators in the Hamiltonian formulation, operating in an **infinite** dimensional Hilbert space on a single link
- U(1) gauge transformations generated by Gauss Law:

$$\begin{aligned} G_x &= \sum_i (e_{x,i} - e_{x-\hat{\imath},i}); \;\; [G_x, H] = 0 \\ V &= \prod_x \exp(i\alpha_x G_x); \;\; u'_{xy} = V u_{xy} V^\dagger = \exp(i\alpha_x) u_{xy} \exp(-i\alpha_y) \end{aligned}$$

- Commutation relations realizing gauge invariance:

$$[e, u] = u, \;\; [e, u^\dagger] = -u^\dagger$$

- $[u, u^\dagger] = 0$

Hamiltonian $U(1)$ LGT: Quantum Links

- U(1) gauge invariant Hamiltonian:

$$H = \frac{g^2}{2} \sum_{x,i} E_{x,i}^2 - \frac{1}{2g^2} \sum_{\Box} (U_{\Box} + U_{\Box}^{\dagger})$$

- $U = S^1 + iS^2 = S^+; \;\; U^{\dagger} = S^1 - iS^2 = S^-; E = S^3$
  $\Rightarrow$ are operators in the Hamiltonian formulation, operating in a **finite** dimensional Hilbert space on a single link
- U(1) gauge transformations generated by Gauss Law:

$$G_x = \sum_i (E_{x,i} - E_{x-\hat{i},i}); \;\; [G_x, H] = 0$$

$$V = \prod_x \exp(i\alpha_x G_x); \;\; U'_{xy} = V U_{xy} V^{\dagger} = \exp(i\alpha_x) U_{xy} \exp(-i\alpha_y)$$

- Commutation relations realizing gauge invariance:

$$[E, U] = U, \;\; [E, U^{\dagger}] = -U^{\dagger}$$

- $[U, U^{\dagger}] = 2E$

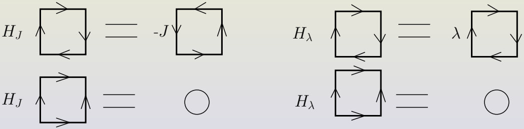
The (2+1)-d U(1) Quantum Link model

- Simplest Abelian pure gauge model: with spin $S = 1/2$
  $\rightarrow$ 2-dim Hilbert space per link

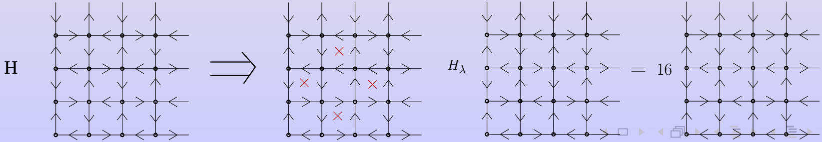
$$E|\uparrow\rangle = \frac{1}{2}|\uparrow\rangle; \quad E|\downarrow\rangle = -\frac{1}{2}|\downarrow\rangle; \quad U|\uparrow\rangle = 0; \quad U|\downarrow\rangle = |\uparrow\rangle; \quad U^\dagger|\uparrow\rangle = |\downarrow\rangle; \quad U^\dagger|\downarrow\rangle = 0$$

- E^2 contributes a constant for $S = 1/2$.

$$H = -J\sum_{\square}\left(U_\square + U_\square^\dagger\right) + \lambda\sum_{\square}\left(U_\square + U_\square^\dagger\right)^2$$



- Plaquettes are flipped only if they have flux in the right order; second term ($= H_\lambda$) counts the number of flippable plaquettes

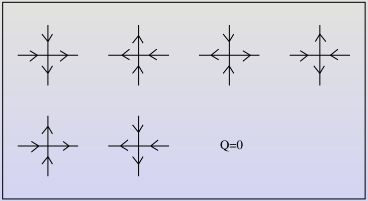


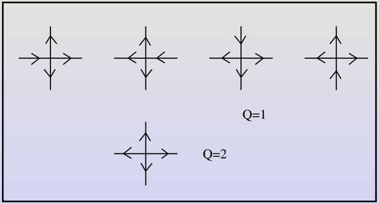
Gauss Law and Charge Sectors

To define the path integral $\mathcal{Z} = \text{Tr}\left(\exp(-\beta H)\mathcal{P}_{\mathcal{G}}\right)$,
the Gauss Law must be implemented :

$$\sum_i \left(E_{x,i} - E_{x-\hat{\imath},i}\right) = Q_x$$

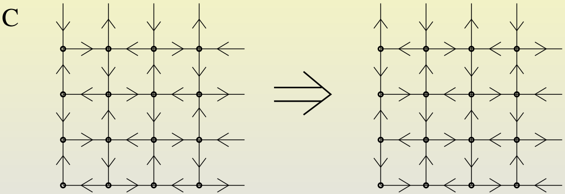
There is zero charge everywhere (charge-0 sector) unless external static charges are placed at vertices.



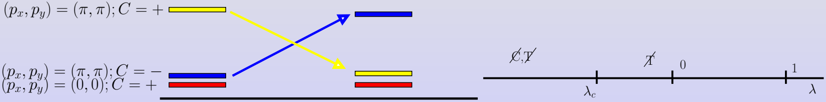


Symmetry breaking and phase transitions

- Discrete: Rotation by $\pi/2$, Reflection, **Charge Conjugation (C)**, **Translation($T = (T_x, T_y)$)**



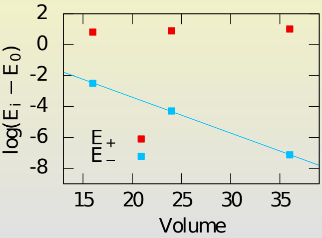
- Charge conjugation: ${}^{C}U = U^{\dagger}; \;\; {}^{C}E = -E$
- Symmetry breaking $\longrightarrow$ quantum phase transitions.

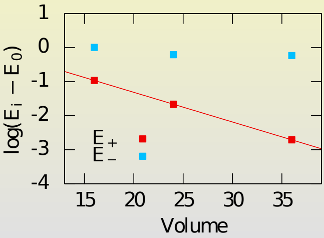


- Continuous: $U(1)$ center symmetries in x- and y-directions

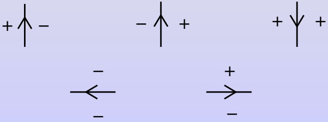
Diagnosis by Exact Diagonalization

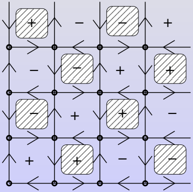
- ED on lattices of $4 \times 4, 4 \times 6, 6 \times 6, 6 \times 8, \ldots$ used to study the system. Quite large by ED standards: 6×6 has $\sim$ 16 million states.
- Volume scaling of the lowest energy states:





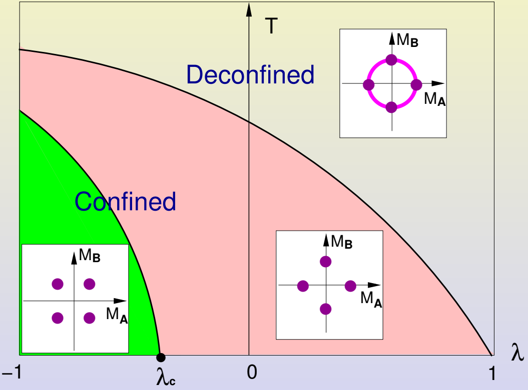
- 2-component order parameter (M_A, M_B) to analyze the symmetry breaking patterns

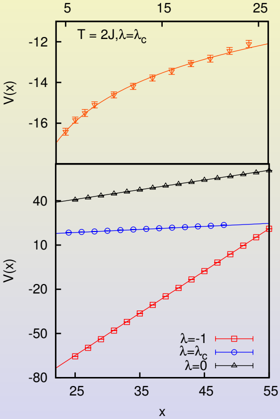




Phase diagram

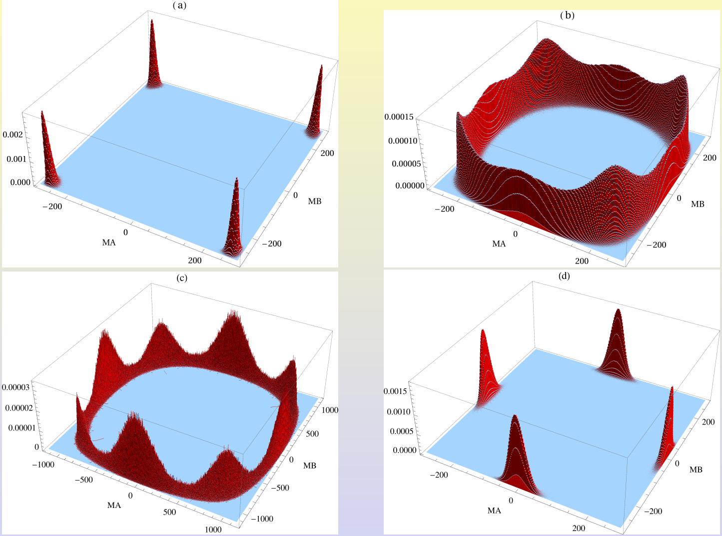
Explored with exact diagonalization and a newly developed cluster algorithm using dualization techniques.





An *approximate global SO(2) symmetry is emergent* at λ_c. A description in terms of a low-energy *effective theory suggests a weak 1st order transition*.

OP distributions from Monte-Carlo



(a) $L = 24a, \lambda = -1, T = 0$, (b) $L = 24a, \lambda \sim \lambda_c, T = 0$, (c) $L = 48a, \lambda \sim \lambda_c, T = 0$, (d) $L = 24a, \lambda = 0, T = 0$

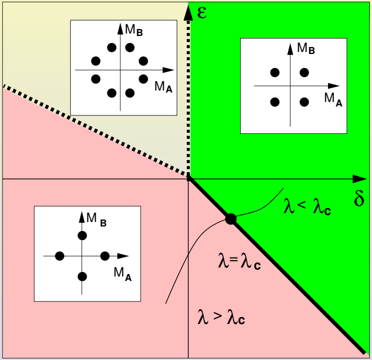
EFT description

- Near λ_c, ED shows (approximate) finite volume rotor spectra behavior: $E_m = \frac{m^2 c^2}{2\rho L_1 L_2}$, m even. Emergence of a $SO(2)$ symmetry which is spontaneously broken.
- EFT description around λ_c in terms of the unit vector field $\vec{e} = (\cos(\varphi), \sin(\varphi))$ representing the direction of (M_A, M_B).
- (M_A, M_B) indistinguishable from $(-M_A, -M_B) \Rightarrow \mathcal{R}P(1)$ model

$$S[\varphi] = \int d^3x \frac{1}{c} \left[\frac{\rho}{2} \partial_\mu \varphi \partial_\mu \varphi + \delta \cos^2(2\varphi) + \epsilon \cos^4(2\phi) \right]$$

♠ δ breaks the emergent SO(2) $\rightarrow Z(4)$,
♠ gives small Goldstone boson mass $Mc = 2\sqrt{2|\delta|/\rho}$
♠ higher order terms give finite string tension at λ_c

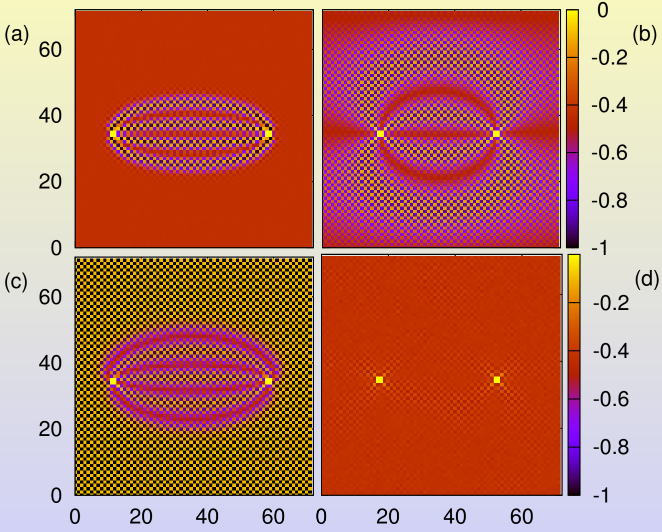
Mean Field Phase Diagram of the EFT



Solid line is 1st order, dotted lines are 2nd order.

Would need ”fine-tuning” to make the string tension vanish.

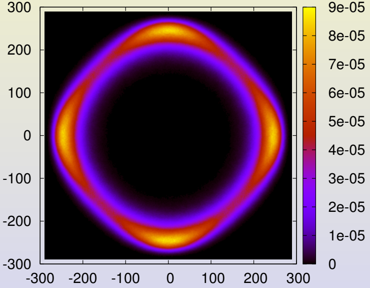
Crystalline confinement



Energy density $\langle H_J \rangle$ of two charges $Q = \pm 2$ placed along the axis on $L = 72a$ lattice

Deconfined Crystal?

Universality arguments predict the finite temperature transition to be of BKT type. Systematic investigation underway; hints of a high-temperature phase with broken T symmetry, which gets smoothly restored with increasing temperature.



Order parameter contour plots (M_A, M_B) for L=24a; $\lambda = 0$;

(left) βJ=1.4 and (right) βJ=0.8

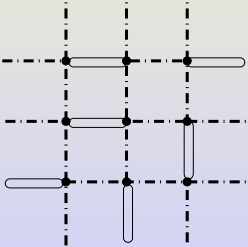
Selecting charge sectors: Quantum Dimer Models

Choose the sector of the Link model satisfying the (new) Gauss Law:

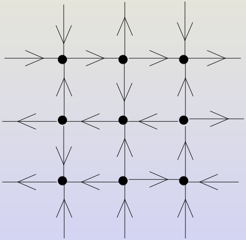
$$G_x|\Psi\rangle = (-1)^{x_1+x_2}|\Psi\rangle$$

Dimer number at a bond can be connected to the electric flux:

$$E_{xy} = (-1)^{x_1+x_2}(D_{xy} - \tfrac{1}{2})$$



Dimer Model config



U(1) QLM config

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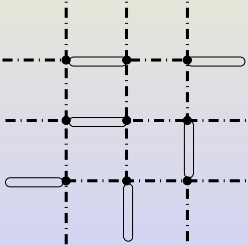
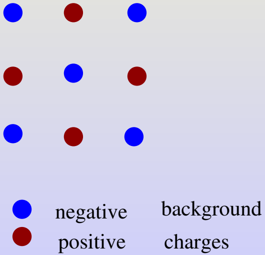
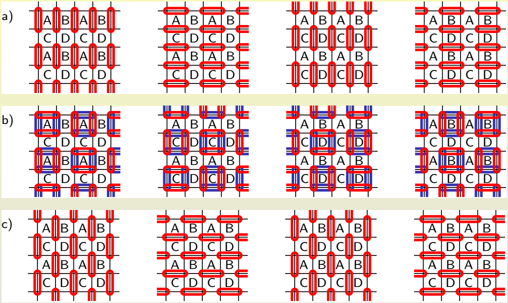


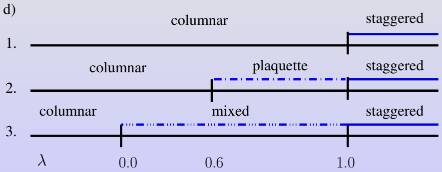
Dimer Model config



Candidate phases and the big question

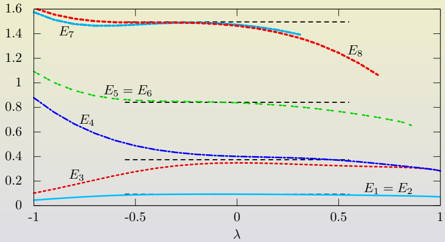


(a) Columnar phase (b) Plaquette phase (c) Staggered phase



Symmetries and results with ED

- Translations combined with charge conjugation: CT_x, CT_y
- $\pi/4$ rotation O around a lattice point
- rotation about a plaquette center combined with charge conj CO'
- $U(1)^2$ center symmetries



- Quantum num. of ground state $(CT_x, CT_y) = (+,+)$
- $E_1 = E_2$ and have quantum numbers $(+,-),(-,+)$; E_3 has $(+,+)$
- For $\lambda \simeq -0.2$, energy gaps behave as $E_{1,2}, E_3 \sim \exp(-\alpha L_1 L_2)$
- For $-0.2 \leq \lambda \leq 0.8$, the state $(-,-)$ with energy $E_4 \approx E_3$ state almost degenerates with the $(+,+)$ state.
- For $\lambda \geq -0.2$, $E_{1,2} : E_{3,4} : E_{5,6} : E_{7,8} \approx 1:4:9:16$; approx rotor spectrum

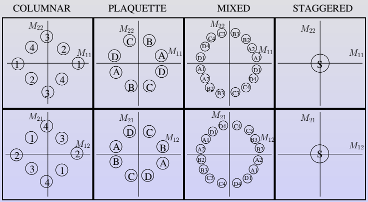
EFT considerations

Effective theory to describe the model around $-0.2 \leq \lambda \leq 1.0$

$$\mathcal{L} = \frac{\rho_t}{2}\partial_t\varphi\partial_t\varphi + \frac{\rho}{2}\partial_i\varphi\partial_i\varphi + \kappa(\partial_i\partial_i\varphi)^2 + \delta\cos^2(4\varphi)$$

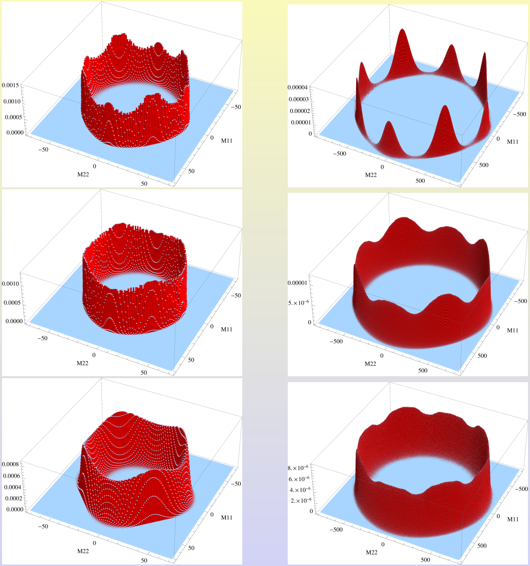
$$\begin{aligned}
M_{11} &= M_A - M_B - M_C + M_D = M_1\cos\varphi_1,\\
M_{22} &= M_A + M_B - M_C - M_D = M_1\sin\varphi_1,\\
M_{12} &= M_A - M_B - M_C - M_D = M_2\cos\varphi_2,\\
M_{21} &= -M_A + M_B - M_C - M_D = M_2\sin\varphi_2,
\end{aligned}$$

and $\varphi = \frac{1}{2}(\varphi_1 + \varphi_2 + \frac{\pi}{4})$, where the M_{ij} are the different order parameters to distinguish the different phases



changing the sign of delta $\Rightarrow$ columnar to plaquette phase

Results from QMC

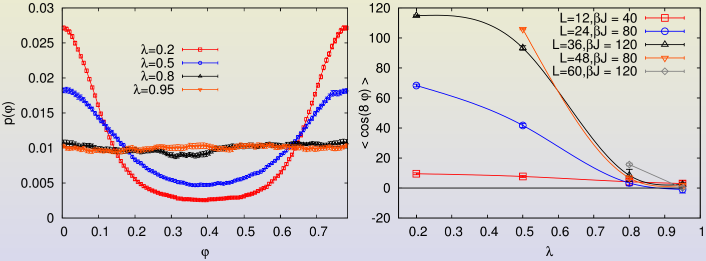


left: L=12a, right: L=48a; top to bottom: $\lambda = 0.5, 0.8, 0.9$

Evidence for columnar phase

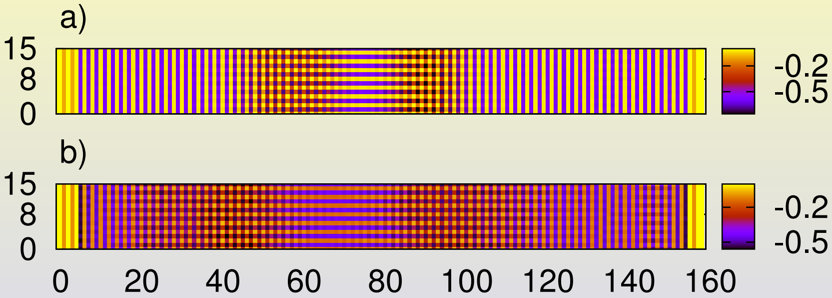
Study the angular histogram of the probability density:

$$\langle\cos(8\varphi)\rangle = \int_{-\pi}^{\pi} d\varphi\, p(\varphi)\cos(8\varphi)$$



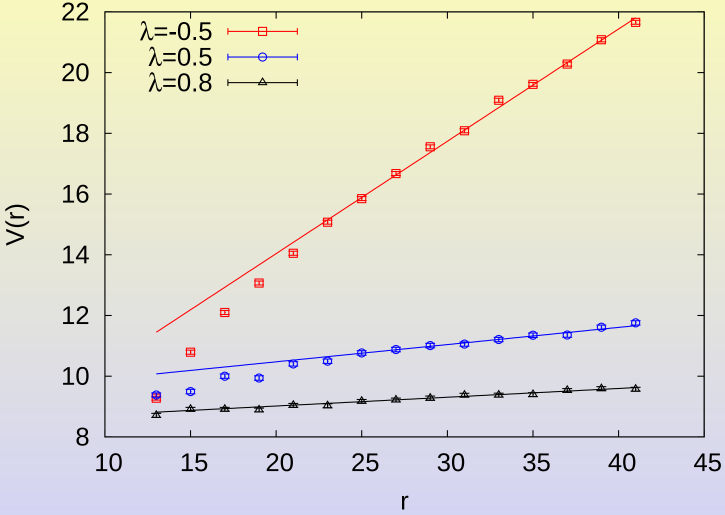
left: Lattice size $L = 24a$, $\beta J = 80$

Interface dynamics



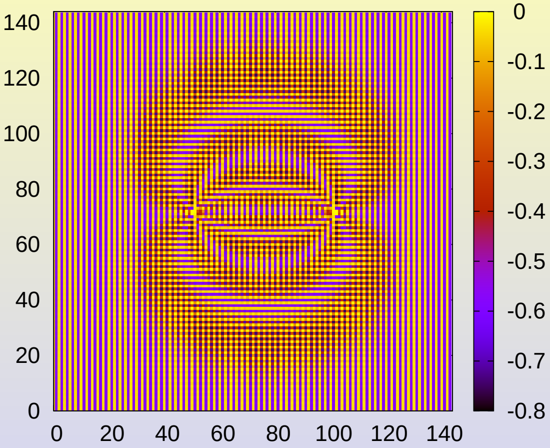
the Plaquette phase exists as an interface separating the two columnar phases. Lattice 16×160, $\beta J = 100, \lambda = -0.5(a)$ and $\beta J = 500, \lambda = 0.7(b)$

Static Potential



Potential between two static charges $Q = \pm 2$ separated by distance r along the lattice axis for $\beta J = 100$ and $L = 100a$.

Fractional Fluxes



Energy density $-J\langle U_\square + U_\square^\dagger \rangle$ in the presence of two charges ± 2 (separated by $r = 49a$ for $\lambda = -0.2$ and $\beta J = 72$ on $L = 144a$

Conclusions

- Although quantum simulating QCD is still far away, many of the simpler models have similar physical phenomena. Very useful for insight into the physics of QCD.
- Proposal for the construction of quantum simulators for the quantum link and quantum dimer models have been presented by colleagues from Innsbruck [see arXiv: 1404.5326 (Quantum Spin Ice) and Annals of Physics 351, 634 (2014) (for QLM)]
- Exciting time when theory and experiment meet together for the lattice gauge theories!
- As pointed out earlier, non-Abelian extensions (all the way upto QCD!) exist, which makes the development of this area exciting.
- Interesting challenges coming up next: demonstrate dimensional reduction in gauge theories (connection with spin-liquids).

Thank You for your attention!