Bonus feature

TOPOLOGICAL STATE IN A ONE DIMENSIONAL FERMI GAS WITH ATTRACTIVE INTERACTIONS

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הקרן הלאומית למדע

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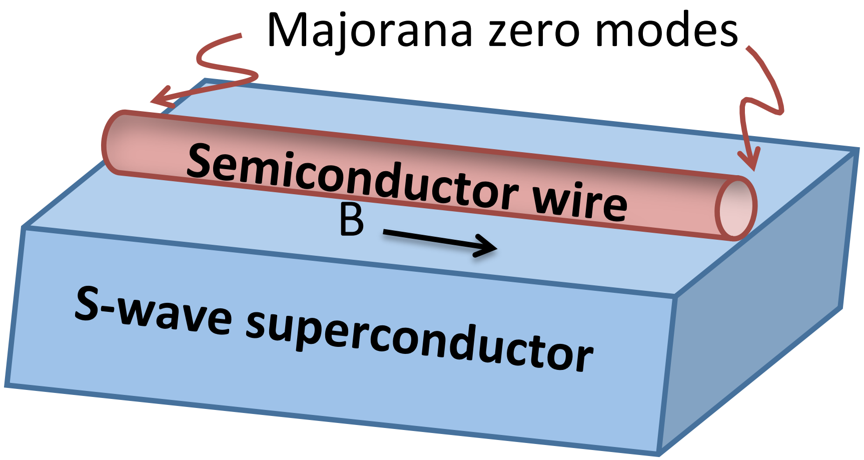
erc UQUAM

ultracold quantum matter

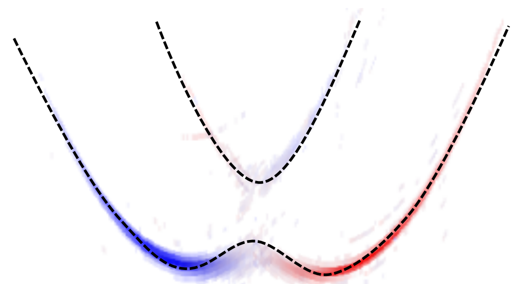
Semiconductor wire vs. SO coupled 1d Fermi gas

Lutchyn et al PRL 2010, Oreg et al PRL 2010
+ experiments at Delft, WIS, …

Wang, et. al. (Jing Zhang's) group PRL 2012
Cheuk et. al. (Zwierlein's group), PRL 2012



- Similar single particle dispersion



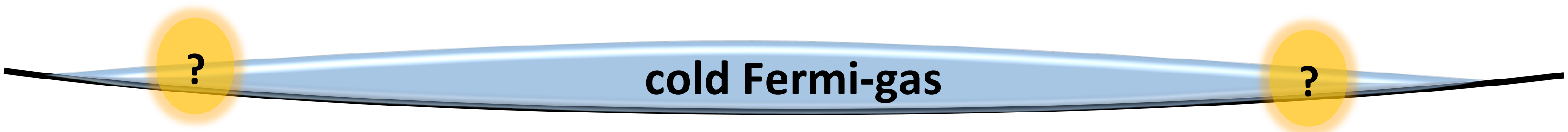
- Proximity coupling to SC
- Charge **not conserved**
- Fully **gapped**

- Intrinsic attractive interaction
- Charge **is conserved**
- **Gapless** phonon modes

Is there a topological state in the cold atomic system?
Protected zero modes? If yes, what is their experimental signature?

Essence of the problem: charge is a good quantum number

Fidkowski, Lutchyn, Nayak, Fisher PRB 2011; Sau, Halperin, Flensberg, Das Sarma PRB 2011



Majorana zero modes implies degeneracy of ground states with different particle number:

$$E(N+1) = E(N)$$

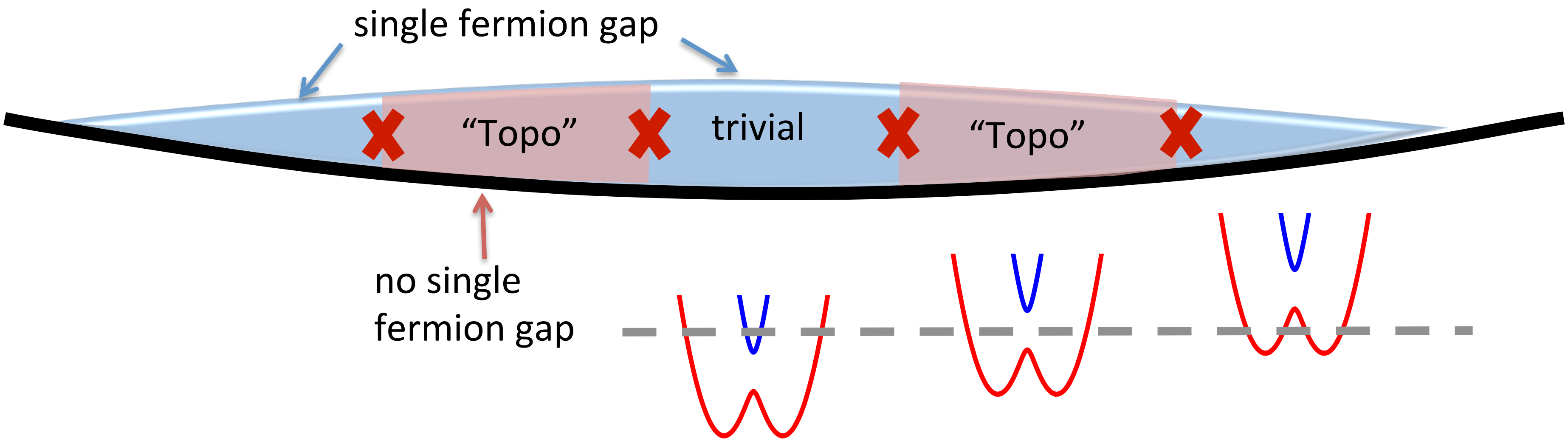
But in a system with compressibility κ:

$$E(N+1) - E(N) = \frac{1}{\kappa L}$$

Possible solution: 1d semiconductors coupled to 1d superconductor.
(Fidkowski et. al. 2011)

This talk:

$$\mathcal{H} = \int dx \Big[\psi^\dagger \Big(-\frac{\partial_x^2}{2m} + V(x) - \mu + \alpha\sigma^x i\partial_x - \delta_z \sigma^z \Big) \psi - U\, \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow \Big]$$



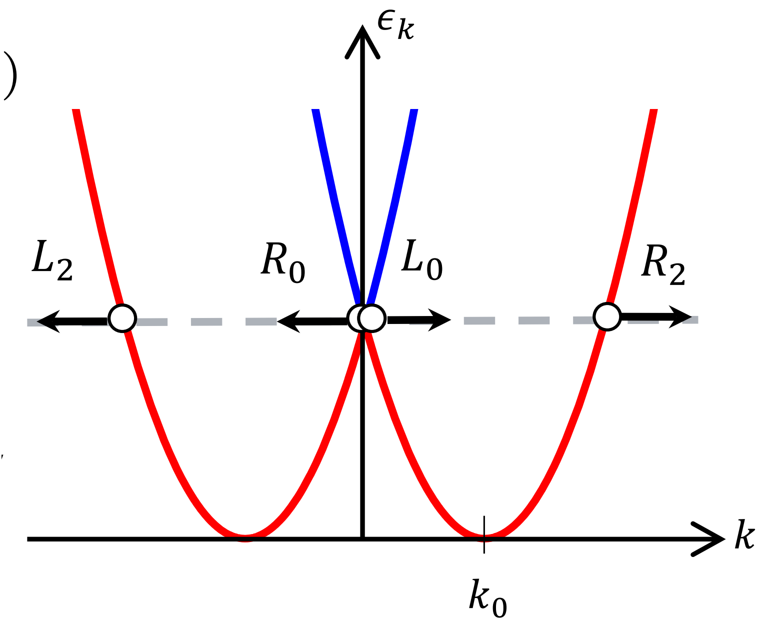
Z_2 subgroup of spin-symmetry remains: spin-parity = fermion parity.
Integerness of the spin (integer / half-integer) is a good quantum number

1. Topological (Majorana-like) ground state degeneracy associated with exchanging parity between "topo" domains

2. Observe through a novel topological pumping induced by slow sweep of the Zeeman field

Low energy description (Bosonization)

$$L_a \sim e^{i(\theta_a + \phi_a)} \qquad R_a \sim e^{i(\theta_a - \phi_a)}$$



Canonical transformation:

$$\phi_+ = \phi_0 + \phi_2 \qquad \theta_- = \theta_0 - \theta_2$$

$$\theta_+ = \theta_2 \qquad \phi_- = \phi_0$$

Charge density

$$\mathcal{H}_+ = \frac{u_+}{2\pi} \int dx \left[K_+ (\partial_x \theta_+)^2 + \frac{1}{K_+} (\partial_x \phi_+)^2 \right]$$

(gapless phonons)

$$\mathcal{H}_- = \frac{u_-}{2\pi} \int dx \left[K_- (\partial_x \theta_-)^2 + \frac{1}{K_-} (\partial_x \phi_-)^2 \right]$$

(spin-parity channel)

$$-g_i \int dx \cos(2\theta_-) - g_z \int dx \cos(2\phi_-)$$

Spin density

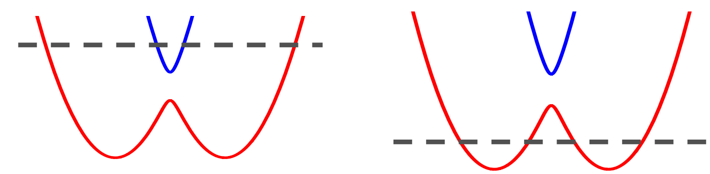
interaction

Zeeman field

Two phases of a spin-orbit coupled Fermi gas

$$\mathcal{H}_- = \frac{u_-}{2\pi}\int dx \left[K_-(\partial_x\theta_-)^2 + \frac{1}{K_-}(\partial_x\phi_-)^2\right] - g_i \int dx \cos(2\theta_-) - g_z \int dx \cos(2\phi_-)$$

1. Interaction dominated - paired

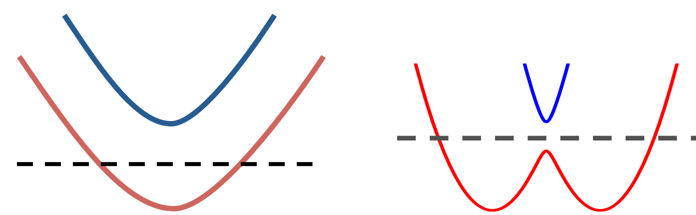


- The "spin- ½" field is pinned $\langle e^{i\theta_-}\rangle \approx \pm 1$

  (+1 and -1 are gauge equivalent, not distinct states)

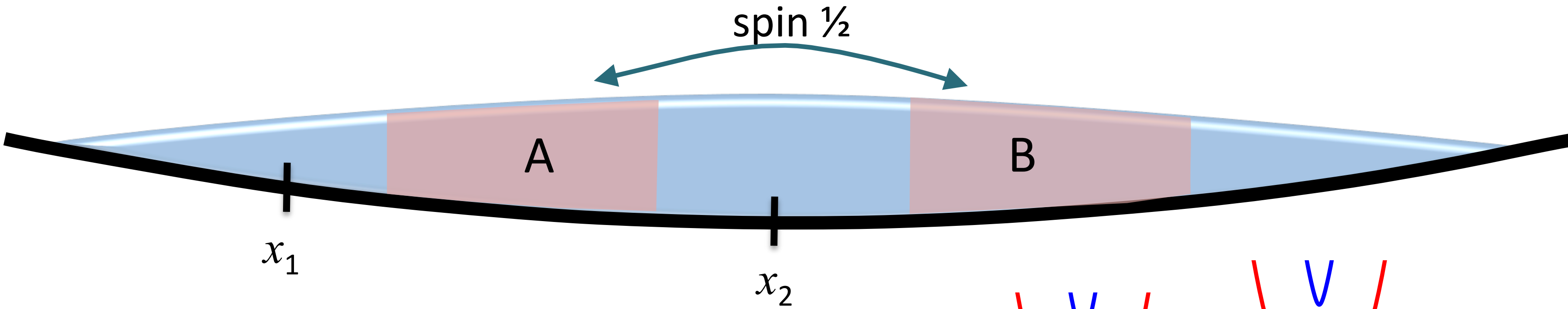
- Gap to single fermion excitations ("spin-gap")

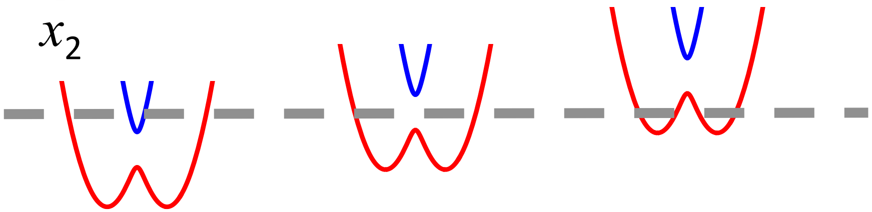
2. Zeeman dominated - unpaired



- $\langle e^{i\phi_-}\rangle \approx \pm 1$
- Gapless single fermion excitations

Degeneracy in a harmonic trap





Fermion parity of region A:

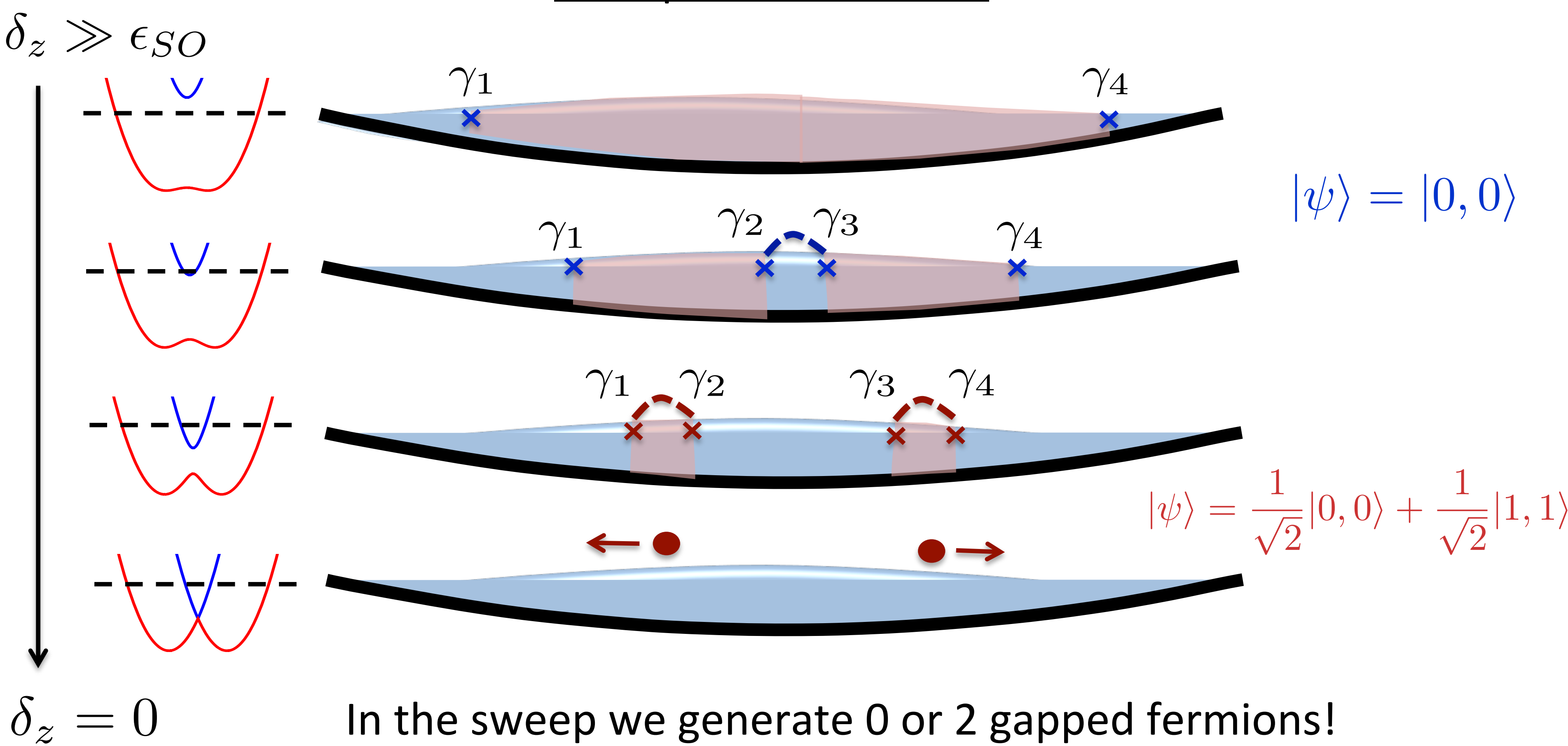
$$P_A = \langle \cos[\theta_-(x_2) - \theta_-(x_1)] \rangle = \pm 1$$

$$\theta_-(x_2) - \theta_-(x_1) = \pi \int_{x_1}^{x_2} dx \, [n_\uparrow(x) - n_\downarrow(x)] = \pi \, (N_{A\uparrow} - N_{A\downarrow})$$

- **Degenerate ground states are related by transferring a spin ½ (single fermion) between topological domains.**
- Backscattering by local impurities splits the degeneracy
  Fidkowski et. al. PRB 2011; Sau et. al. PRB 2011 $\Delta E \sim V_1^* V_2 \frac{1}{|x_1 - x_2|^\alpha} + c.c.$
- But with smooth potential only exponentially small splitting.

Probing the topological state: quantized pumping in the trap

Slow sweep of the Zeeman field from high value to zero.
Initial state with <u>even particle number:</u>

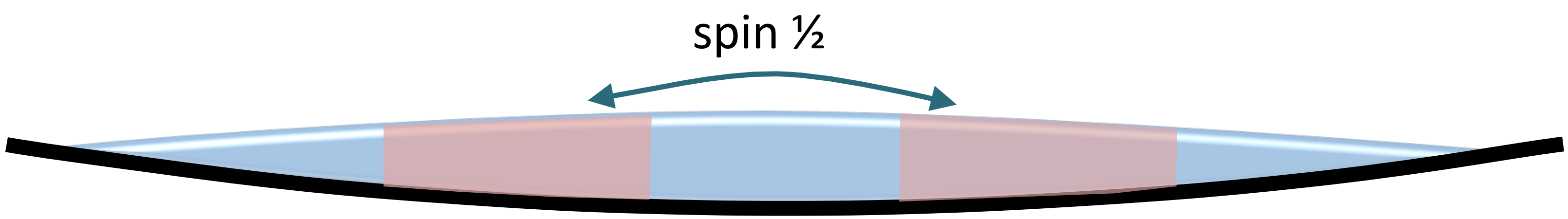


In the sweep we generate 0 or 2 gapped fermions!

<u>Odd initial state:</u> generate 1 fermion

Quantized excess energy per sweep for random even/odd initial state.

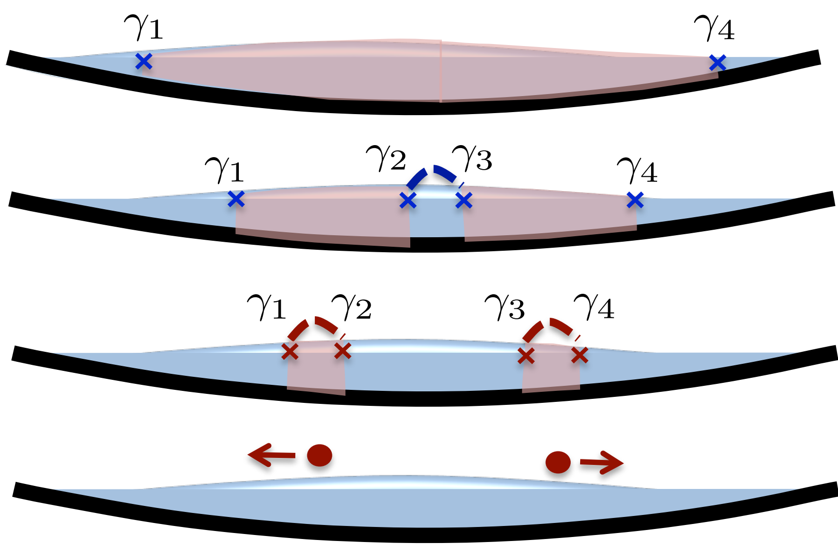
Summary of this part



- Zero modes naturally occur in a SO-coupled Fermi cold gas. But different from the Kitaev wire.

- Probed by a novel topological pump.

- Protected at low T by the spin gap in the non topological domains. (coupling to phonons is irrelevant at low energy)




FEATURE
PRESENTATION

Many-Body localization: new insights from theory and experiment

Ehud Altman, Weizmann Institute of Science

Collaborators: Ronen Vosk, Mark Fischer (WIS), David Huse (Princeton)

Experiment: Michael Schreiber, Sean Hodgman, Pranjal Bordia, Henrik Luschen, Ulrich Schneider, Immanuel Bloch (LMU)

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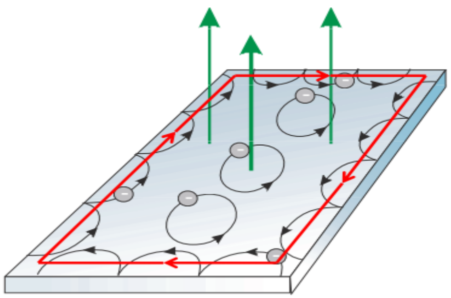
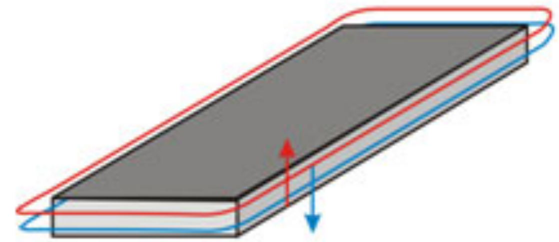
Conventional wisdom:
Quantum mechanics is manifest only close to the ground state

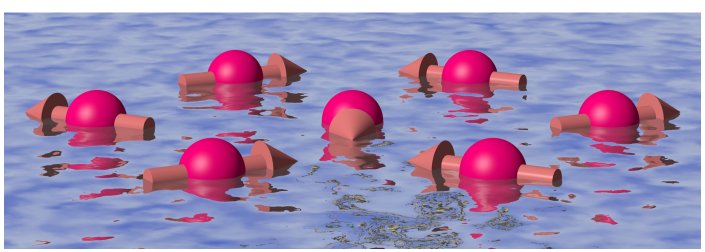
Quantum Hall effect:



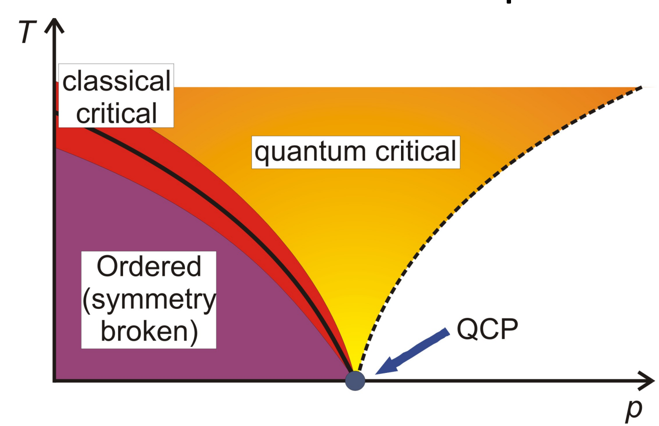
Topological insulators:



Fermi liquid:

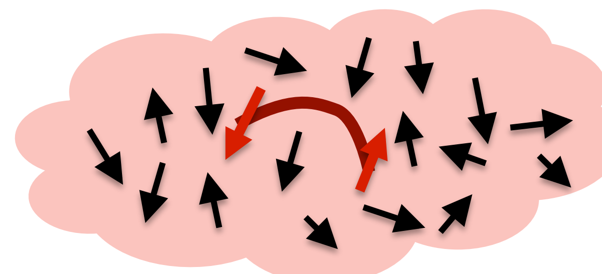


Quantum critical points



Ergodicity is the enemy of quantum mechanics

Many-body time evolution washes away quantum correlations.



Quantum information stored in local objects is rapidly lost as these get entangled with the rest of the system.

The only remaining structures of information are slow order parameter fields and conserved densities.

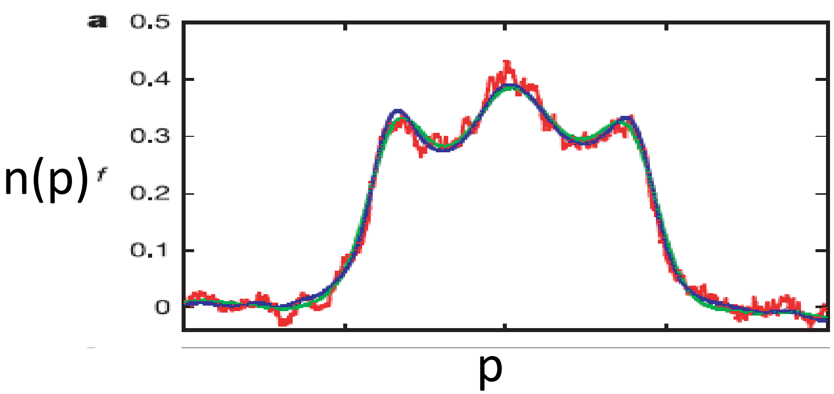
Classical hydrodynamic description (e.g. diffusion).

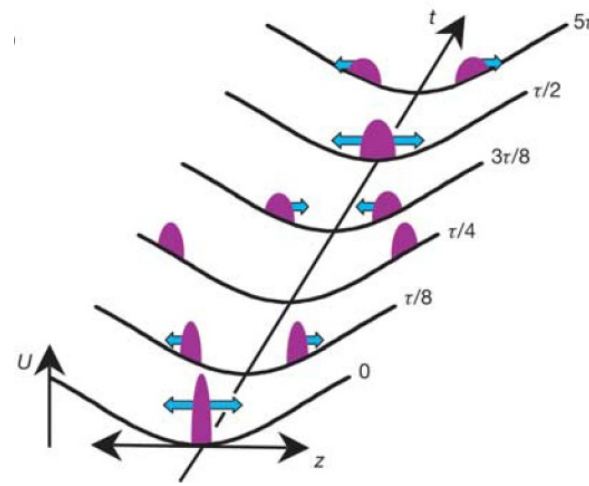
To see quantum phenomena at long times need breaking of ergodicity!

Well known example: integrability

Failure to thermalize due to constraints imposed by many conservation rules

Quantum newton's cradle experiment with cold atoms.
Weiss, Nature 2010

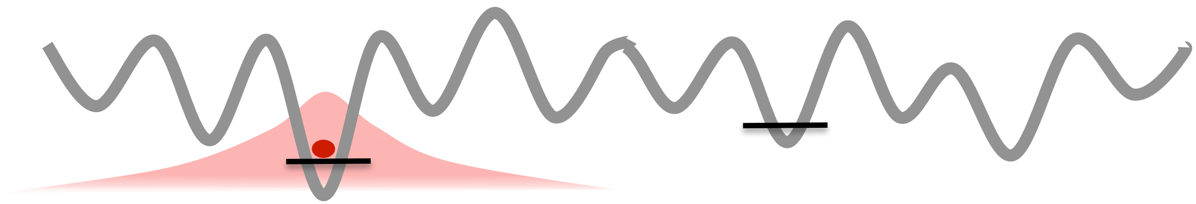


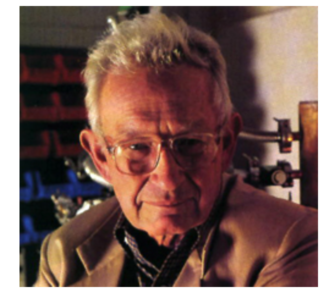


But integrability is fragile! Only special points in parameter space.
Are there more generic non-thermal states?

Anderson localization

Single particle (Anderson 1958):

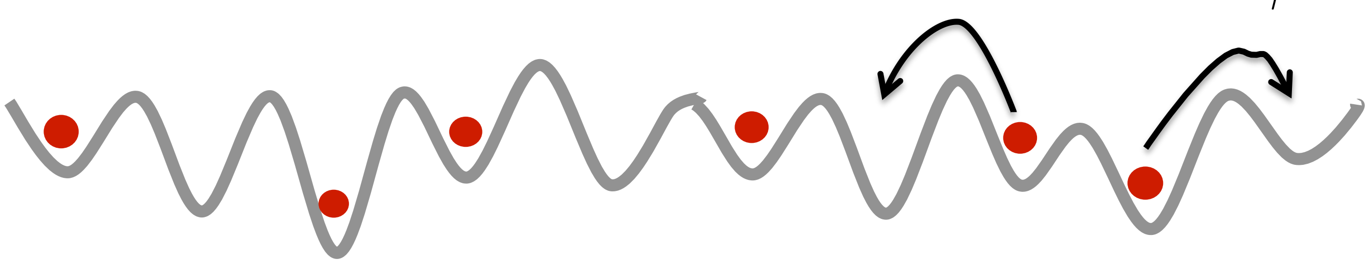




Vanishing probability of resonances.

Many particle states:

$$Uc^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta$$



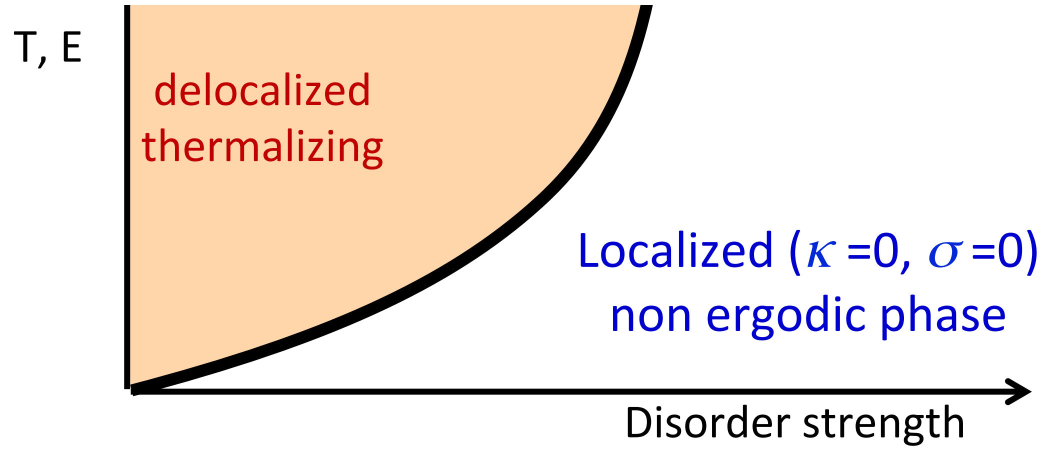
At high energies interaction connects between ~2^L localized states !

Can localization survive ?

Many-Body Localization

Basko, Aleiner, Altshuler (2005); Gornyi, Mirlin, Polyakov (2005):
Insulating phase stable below a critical T or E; metal above it.

System with bounded spectrum: Disorder tuned transition at $T=\infty$
Oganesyan and Huse *(2007), Pal and Huse (2010)*



Many questions:

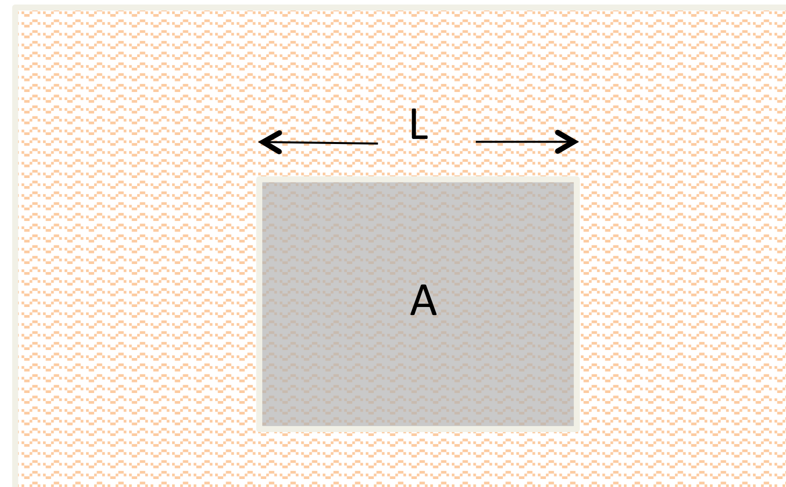
Nature of the dynamics in the localized phase?
At the critical point? Experiments?

The eignestate perspective:
Eigenstate thermalization hypothesis (ETH)

Deutsch 91, Srednicki 94

High energy eigenstate of an ergodic system appears thermal:

$$\rho_A = \frac{1}{Z_A} e^{-\beta H_A}$$

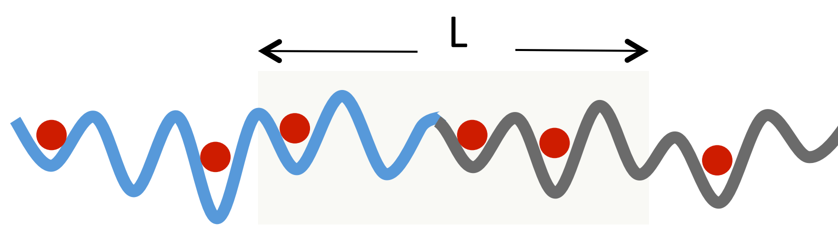


Von-Neuman (entanglement) entropy:

$$S_A \equiv \mathrm{tr}\left[\rho_A \ln \rho_A\right] \propto L^d$$

Anderson localization is an example where ETH fails:

"Area law" entropy even in high energy eigenstates



$$S_A \propto L^{d-1}$$

Many body localization = stability of such localized states to interactions

A tale of two paradigms

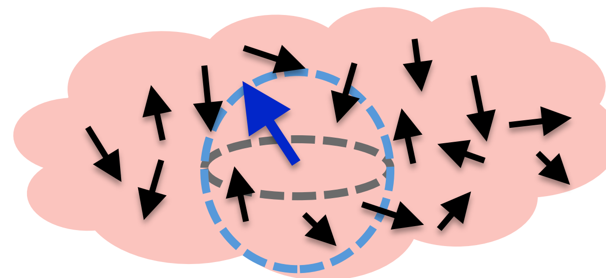
Thermalization

Quantum correlations in local d.o.f are rapidly lost as these get entangled with the rest of the system.

↓

Classical hydro description of remaining slow modes (conserved quantities, and order parameters).

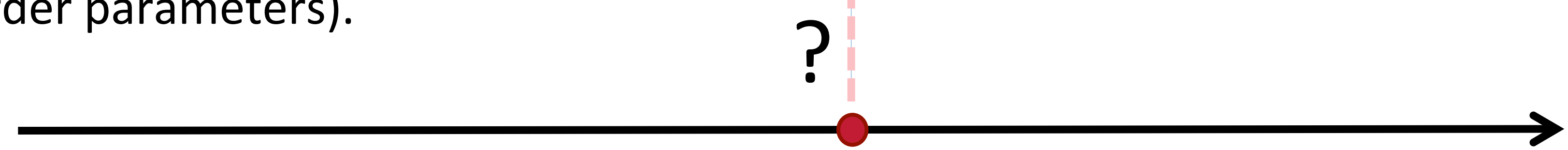
Many-body localization



Local quantum information persists indefinitely.

↓

Need a fully **quantum** description of the long time dynamics!

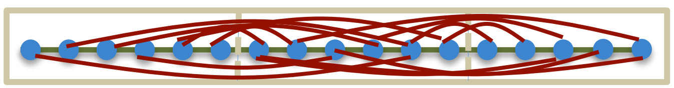


The many-body localization transition = elusive interface between quantum and classical worlds

The eigenstate perspective

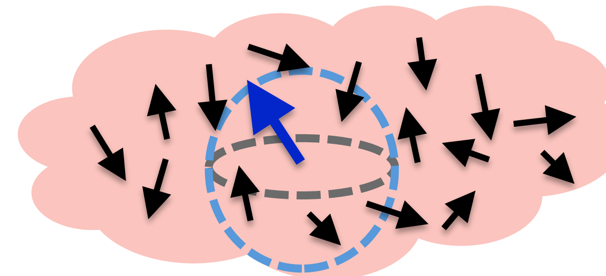
Thermalizing

Energy eigenstates are highly entangled:

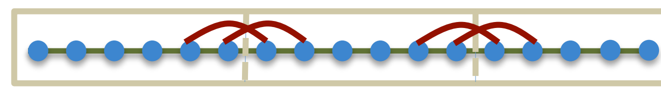


$S_A \sim L^d$ (volume law)

Many-body localized



Eigenstates have low entanglement



$S_A \sim L^{d-1}$ (area law)

?

Localization transition: fundamental change in entanglement pattern. More radical than in any other phase transition we know !

Outline

- **New insights from theory**
  - Dynamics in the many-body localized (MBL) phase
  - Phase transition from MBL to a thermal liquid

- **Confronting theory and experiment**

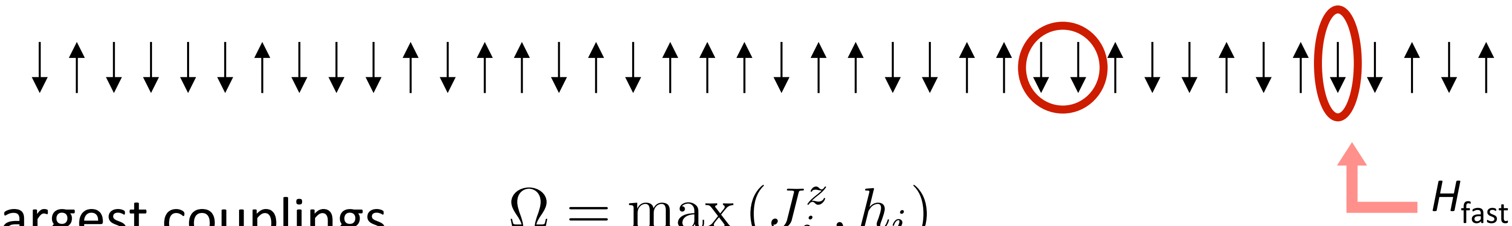
  Collaboration with I. Bloch's LMU group, U. Schneider and co.
  - MBL of fermions in quasi-periodic lattice
  - Outlook to future experiments

RG Solution of time evolution

R. Vosk and EA PRL (2013); PRL (2014)

$$H = \sum_i \left[J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots \right] \qquad e^{-iHt} \,|\, \Psi_0 \,\rangle$$

$|\Psi_0\rangle =$



Pick out largest couplings $\qquad \Omega = \max\left(J_i^z, h_i\right)$

Short times ($t \approx 1/\Omega$): System evolves according to H_{fast}
Other spins essentially frozen on this timescale.

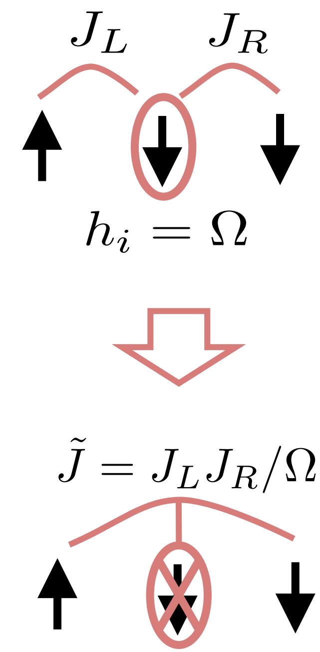
Longer times ($t >> 1/\Omega$): Eliminate fast modes (order Ω) perturbatively to obtain effective evolution for longer timescales.

Similar to standard strong disorder RG, Dasgupta-Ma (1980), D. Fisher (1992)
But here we target low frequency instead of low absolute energy

Outcome of RG: integrals of motion = (frozen spins)

R. Vosk and EA PRL (2013); PRL (2014); Pekker, Refael et. al. PRX (2014)

Example: strong transverse field



$$H = \sum_i \left[J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + V_i \sigma_i^x \sigma_{i+1}^x \right]$$

$$H_{\text{eff}} = e^{-iS} H e^{iS}$$

$$H_{\text{eff}} = \Omega \tilde{\sigma}_i^x + V_L \tilde{\sigma}_i^x \sigma_L^x + V_R \tilde{\sigma}_i^x \sigma_R^x + \frac{J_L J_R}{\Omega} \sigma_L^z \tilde{\sigma}_i^x \sigma_R^z$$

In this RG scheme degrees of freedom are not eliminated but rather frozen into *quasi-local integrals of motion*:

$$\tilde{\sigma}_i^x = Z \sigma_i^x + \text{exponential tail}$$

Effective Hamiltonian (fixed point theory)

Depends only on the quasi-local integrals of motion:

$$H_{FP} = \sum_i \tilde{h}_i \tilde{\sigma}_i^x + \sum_{ij} V_{ij} \tilde{\sigma}_i^x \tilde{\sigma}_j^x + \sum_{ijk} V_{ijk} \tilde{\sigma}_i^x \tilde{\sigma}_j^x \tilde{\sigma}_k^x + \ldots$$

$$V_{ij} \sim V e^{-|x_i - x_j|/\xi}$$

Independently postulated as a phenomenological description of the many-body localized phase.

Oganesyan & Huse (2013); Serbyn, Papic & Abanin (2013)

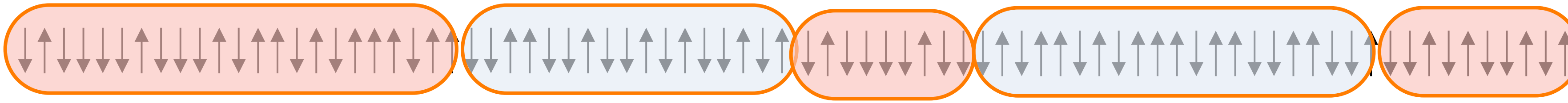
➡ Surprisingly rich dynamics in MBL phase:

- Slow log(t) growth of the entanglement entropy.
- Quantum coherence revealed by spin echoes
- Distinct localized phases (glassy, paramagnetic, topological …)

This scheme cannot access the localization phase transition!

Theory of the many-body localization transition

Vosk, Huse and E.A. arXiv:1412.3117



Spin chain fragmented into puddles of different types: incipient insulators and incipient metals.
Modeled as coupled random matrices:

$$\Delta_i, \Gamma_i \quad \Rightarrow \quad g_i = \Gamma_i / \Delta_i$$

Δ_i Mean level spacing in the block

$\Gamma_i^{-1} = \tau_i$ Time for entanglement to spread across the block

$g_i \ll 1$ "insulating block"

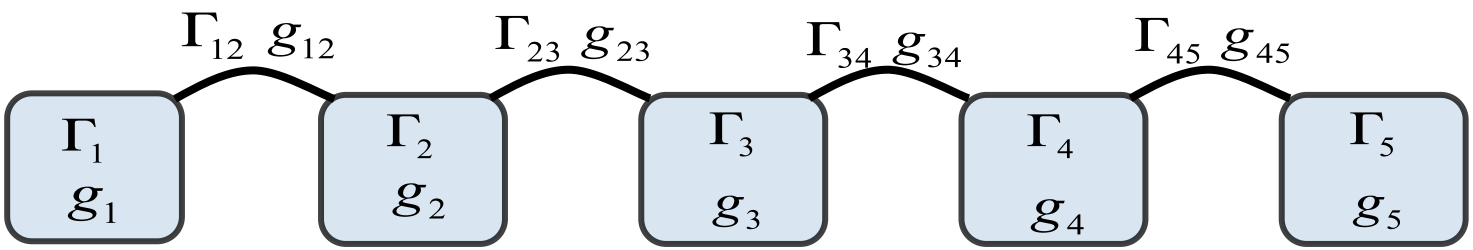
(Poisson level statistics)

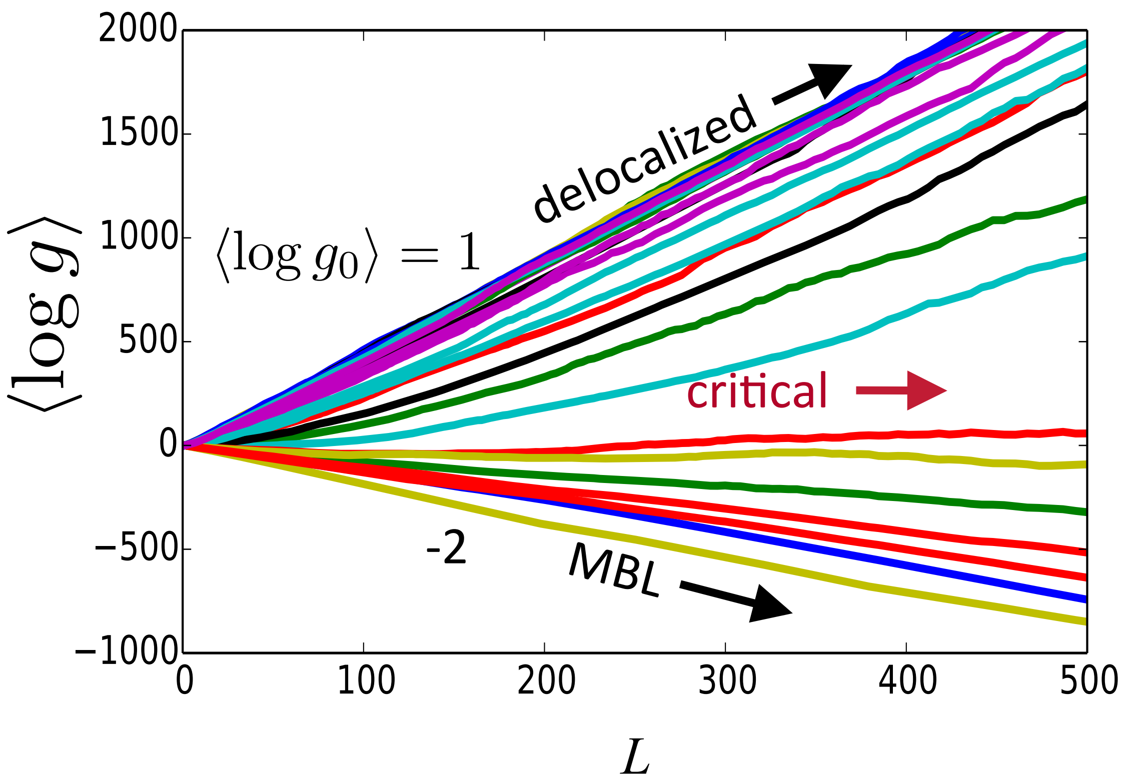
$g_i \gg 1$ "thermalizing block"

(Wigner-Dyson statistics)

RG flow: itteratively join matrices that entangle with each other at running cutoff scale. At the end of the flow we are left with one big block that is either insulating or thermalizing

Outcome of the RG flow



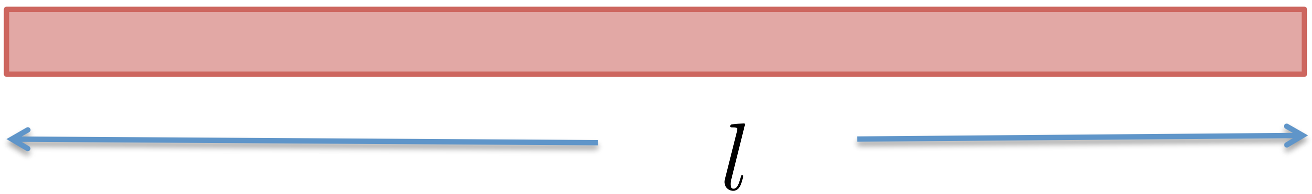


Next, characterize the transition:

(i) in terms of dynamics; (ii) in terms of eigenstate entanglement

RG results I – dynamical scaling for transport

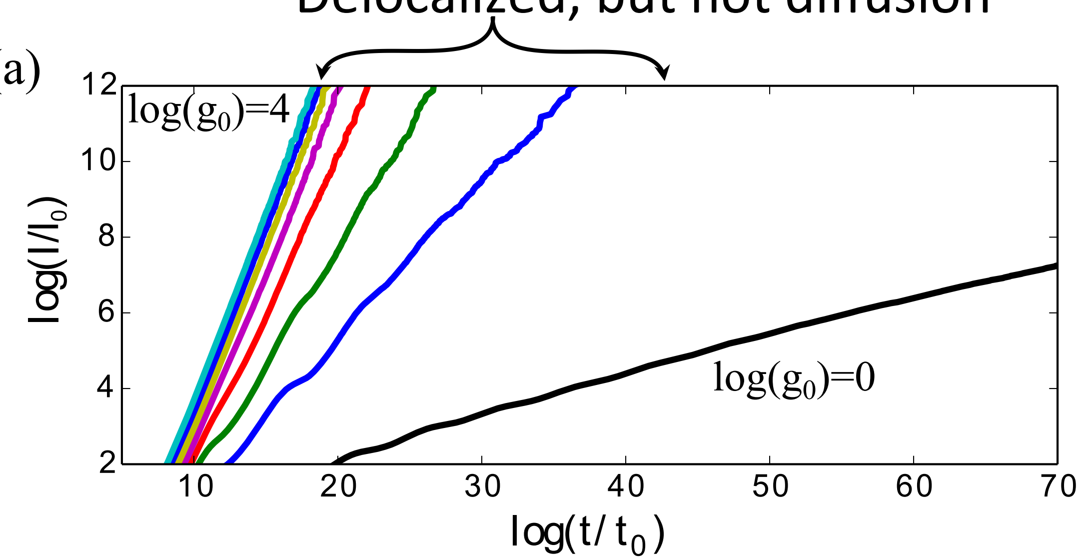
Relation between transport time τ_{tr} and length l of blocks:



Diffusion: $\tau_{tr} = l^2$ $l_{tr} = (D\tau)^{\cancel{1/2}}$ ← $\alpha < \frac{1}{2}$

Delocalized, but not diffusion

(a)

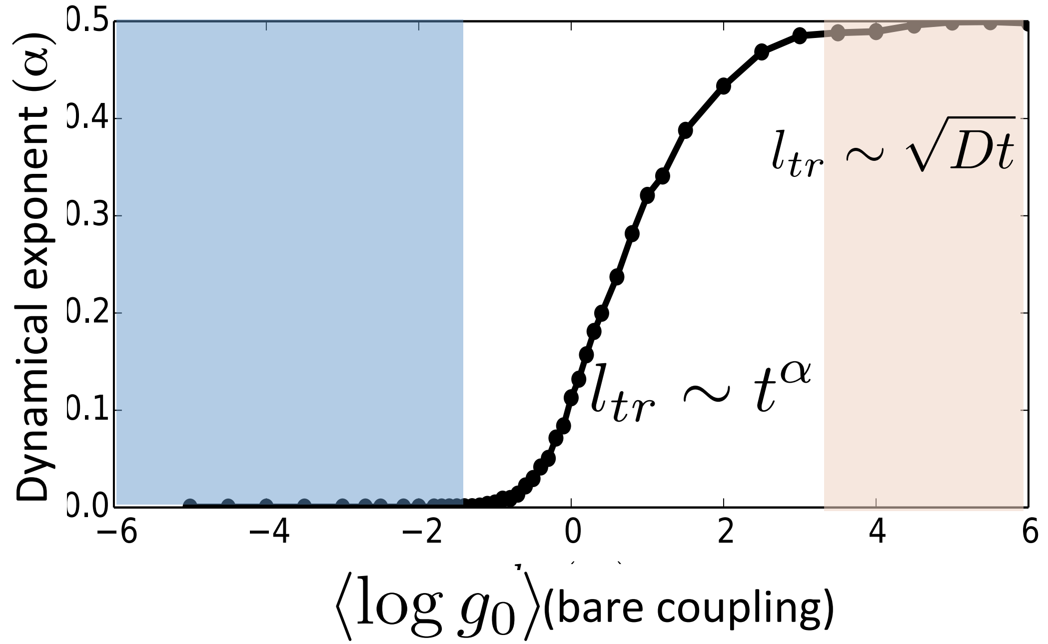


RG results I – dynamical scaling for transport

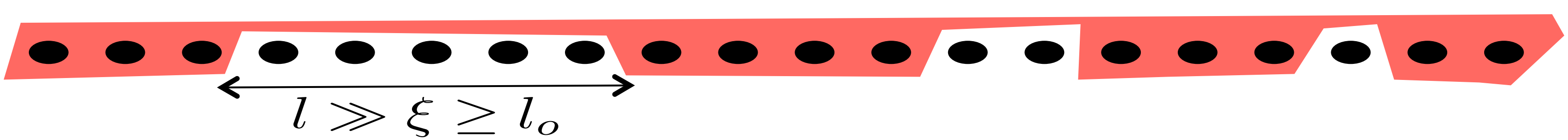
Relation between transport time τ_{tr} and length l of blocks:

Surprise! The transition is from localized to anomalous diffusion.

Seen also in recent ED studies:
Bar-Lev et al 2014; Agarwal et al 2014

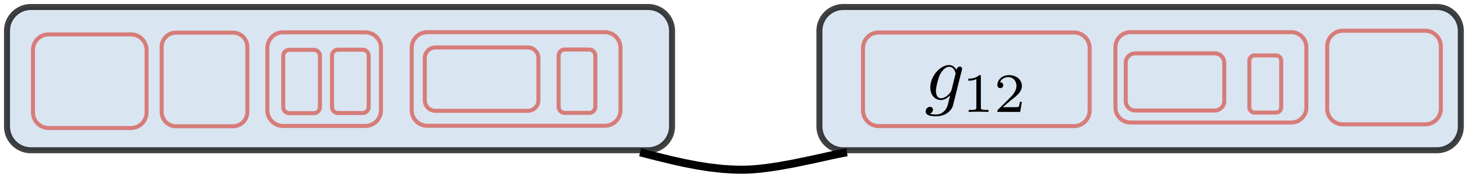


<u>Result of Griffiths effects</u>. long insulating inclusions inside the metal are exponentially rare but give exponentially large contribution to the transport time.

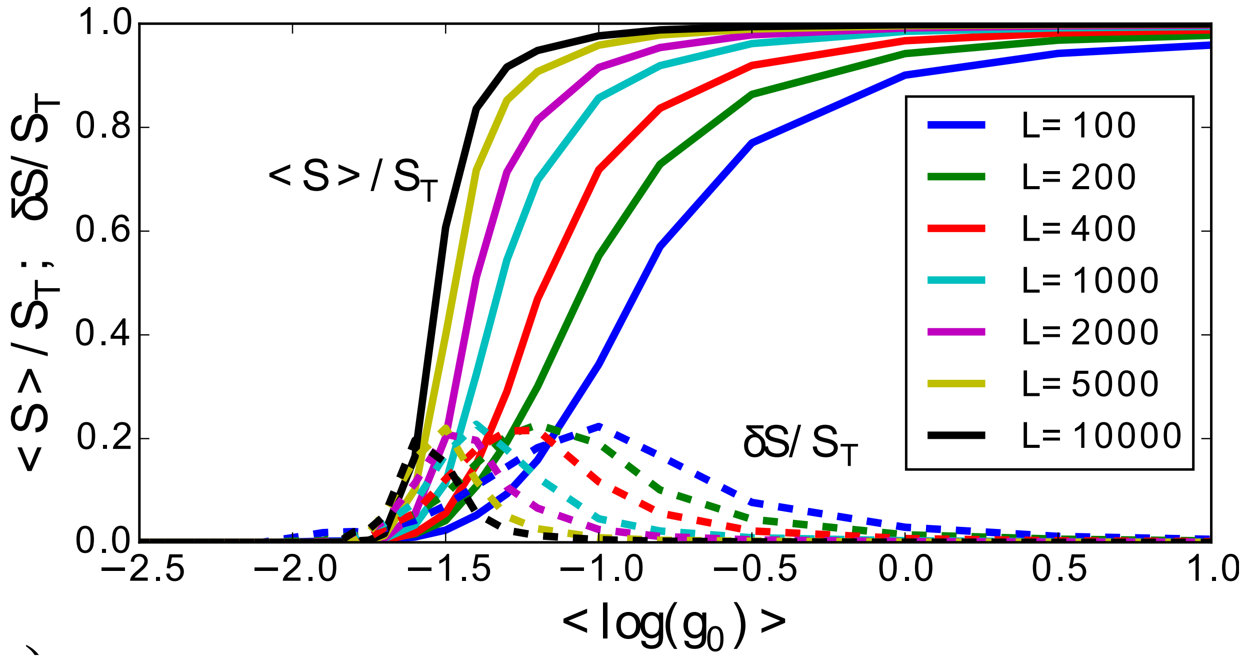


➡ Relaxation with slow power-law tails

Eigenstate entanglement turns out to be the natural scaling variable of this RG scheme !



$$S_E(L/2) \sim \log_2 [g(L) + 1]$$



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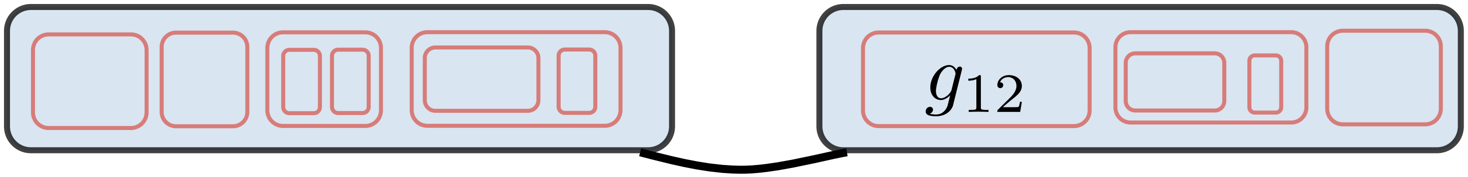
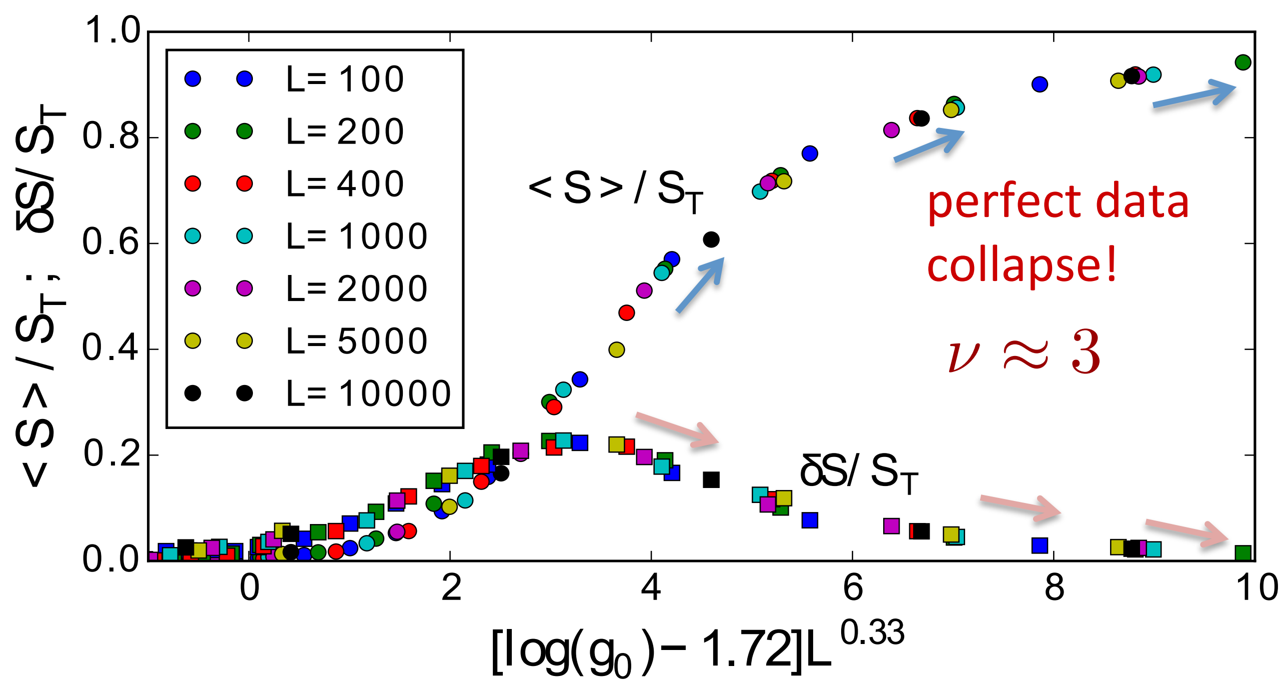
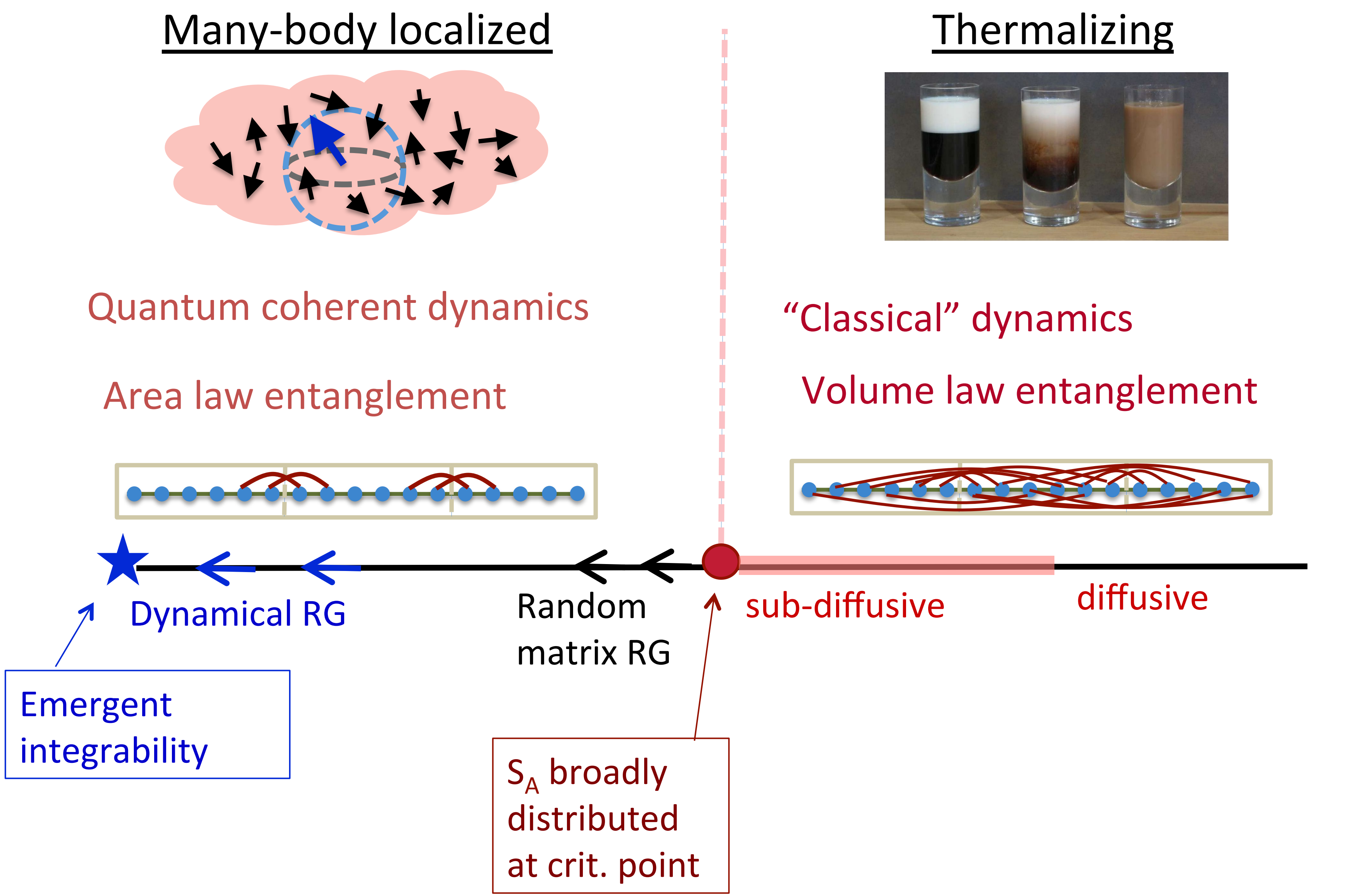


$$S_E(L/2) \sim \log_2\left[g(L)+1\right]$$

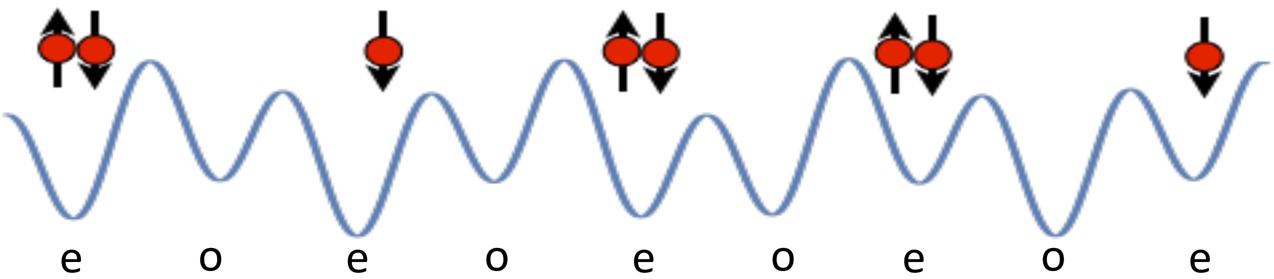


- Infinite randomness fixed point characterized by broad entanglement distribution
- Universal jump to full thermal entropy ⟶ the Griffiths phase is thermal


Many-body localized
Thermalizing
Quantum coherent dynamics
Area law entanglement
"Classical" dynamics
Volume law entanglement
Dynamical RG
Random matrix RG
sub-diffusive
diffusive
Emergent integrability
S_A broadly distributed at crit. point

Experimental study of MBL: fermions in a quasi-random optical lattice

arXiv:1501.05661



Collaboration with With:
Immanuel Bloch's group (Munich)
Mark Fischer and Ronen Vosk (Weizmann)



Michael Schreiber
Pranjal Bordia
Henrik Lüschen

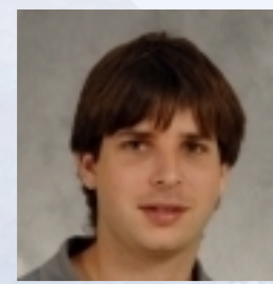
Sean Hodgman

Ulrich Schneider
Immanuel Bloch

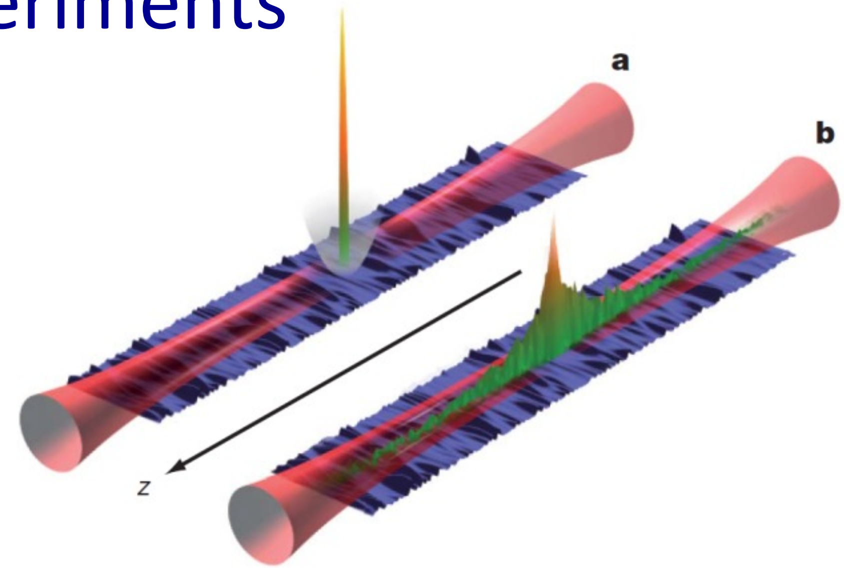
LMU & MPQ München

Mark Fischer
Ronen Vosk
EA

Weizmann



Previous experiments



Anderson localization:

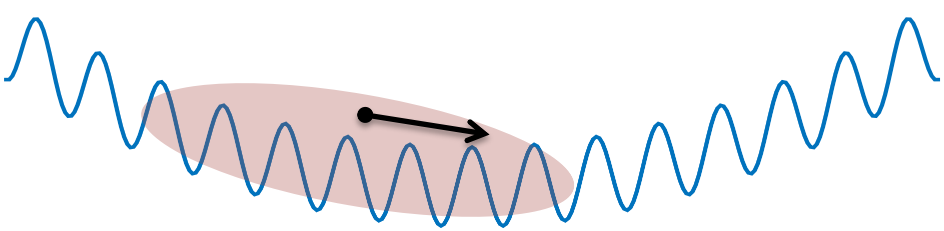
Expansion in disordered potential

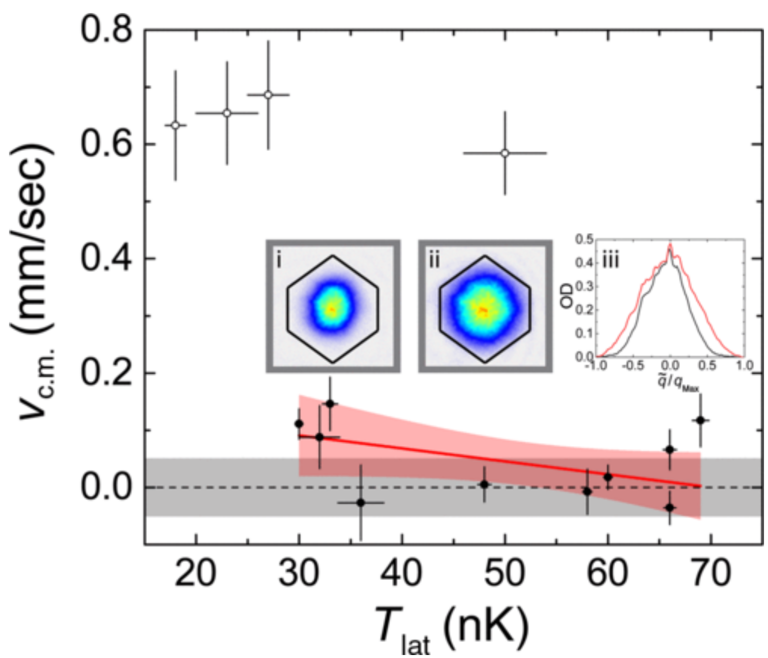
Billy et al. (Aspect) Nature (2008)
Roati (Inguscio) PRL (2008)

Many-body localization (?):

Transport in a trap (response to impulse)

Kondov et al. (DeMarco) preprint 2013

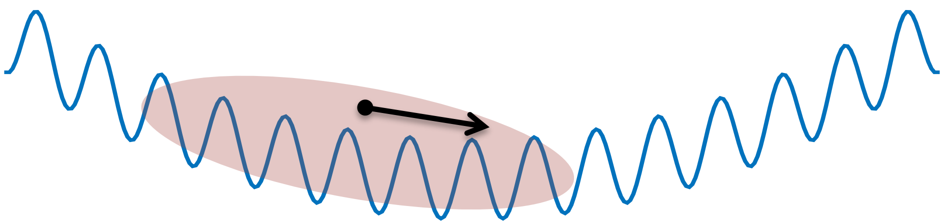


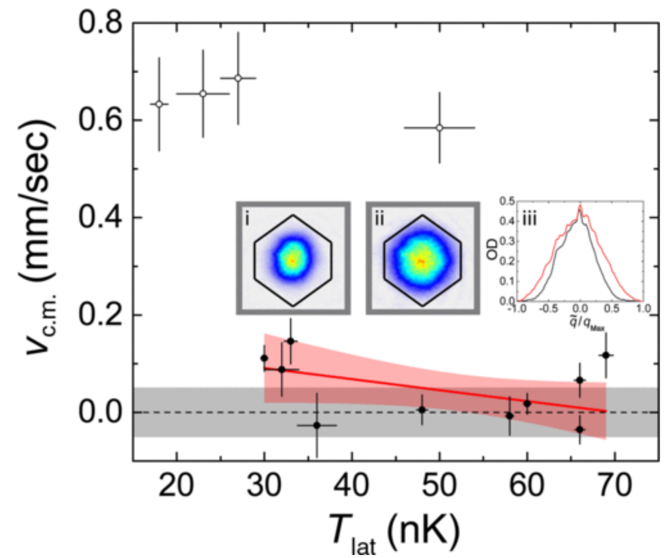


Previous experiments

Many body localization (?) :

In-trap transport (following impulse)





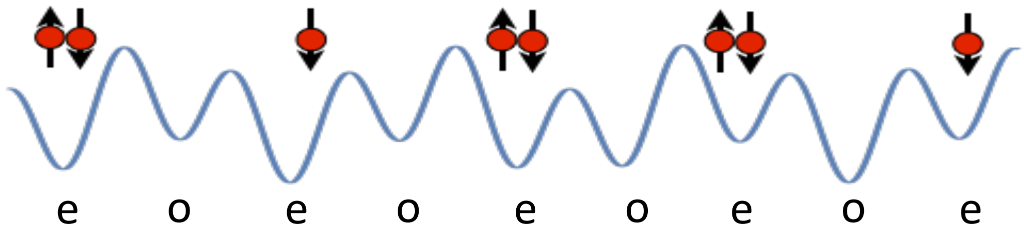
The Problems with such a global probe:

- Very slow probe (finite size time scale by definition)
- Sensitive to inhomogeneity.
  e.g. *Mott shells can block transport*

Our solution: Use a fast local observable

Quantum quench protocol

1. Fermions in optical lattice prepared in period-2 CDW

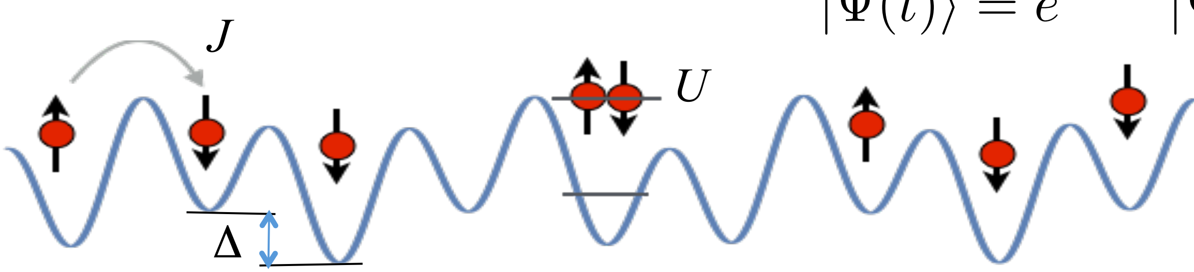


2. Evolve the state with the 1d lattice Hamiltonian:

$$\hat{H} = -J\sum_{i,\sigma}\left(\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i+1,\sigma} + h.c.\right) + \Delta\sum_{i,\sigma}\cos(2\pi\beta i + \phi)\hat{c}^{\dagger}_{i,\sigma}\hat{c}_{i,\sigma} + U\sum_{i}\hat{n}_{i,\uparrow}\hat{n}_{i,\downarrow}.$$

Incommensurate potential

$$|\Psi(t)\rangle = e^{-i\hat{H}t}|\Psi(0)\rangle$$



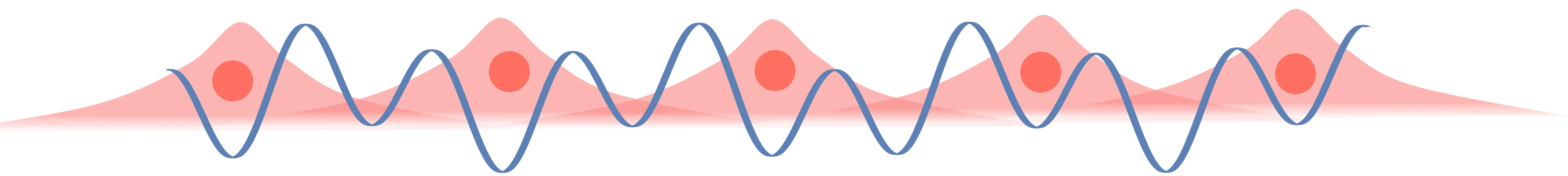
Numerics suggest that this model shows generic MBL (Iyer et. al. PRB 2013)

What to measure?

Relaxation of the CDW with time:

$$\mathcal{I} = \frac{1}{N}\sum_{j=1}^{L}(-1)^j \langle n_j \rangle = \frac{\langle N_e - N_o \rangle}{N_e + N_o}$$

time: $0 \longrightarrow t$



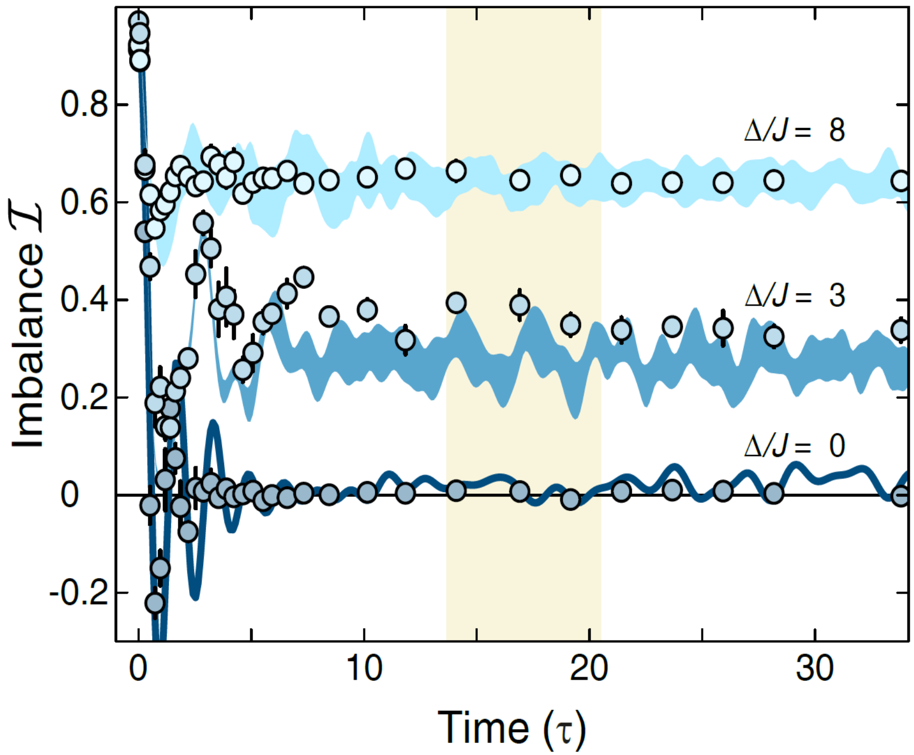
If the system is localized, the CDW operator has finite overlap with an integral of motion.

➡ $\mathcal{I}(t)$ will relax to a non-vanishing value

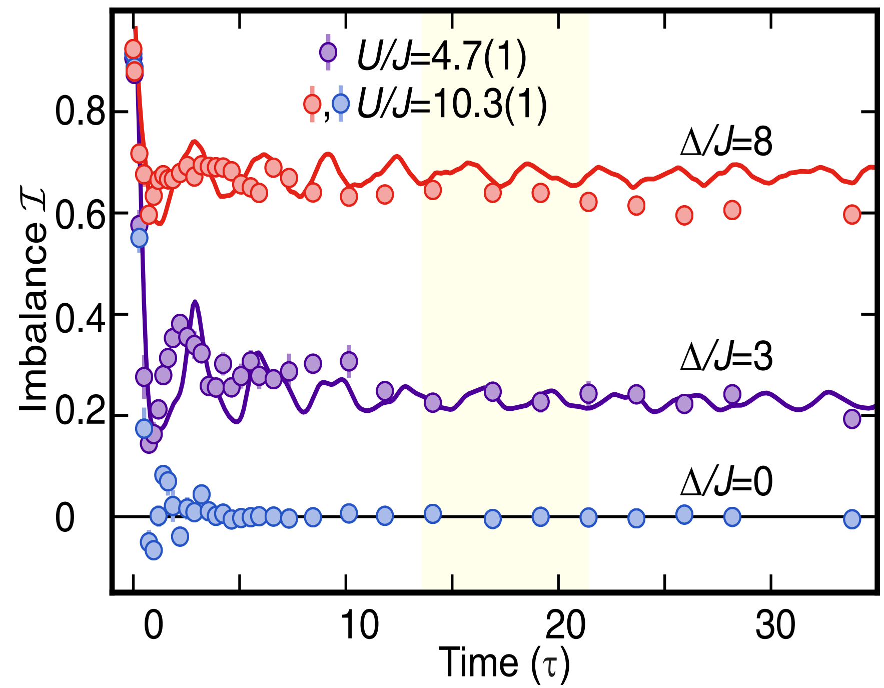
Macroscopic order parameter of the MBL phase

Experimental results

Non interacting Aubry-André:



With interactions:

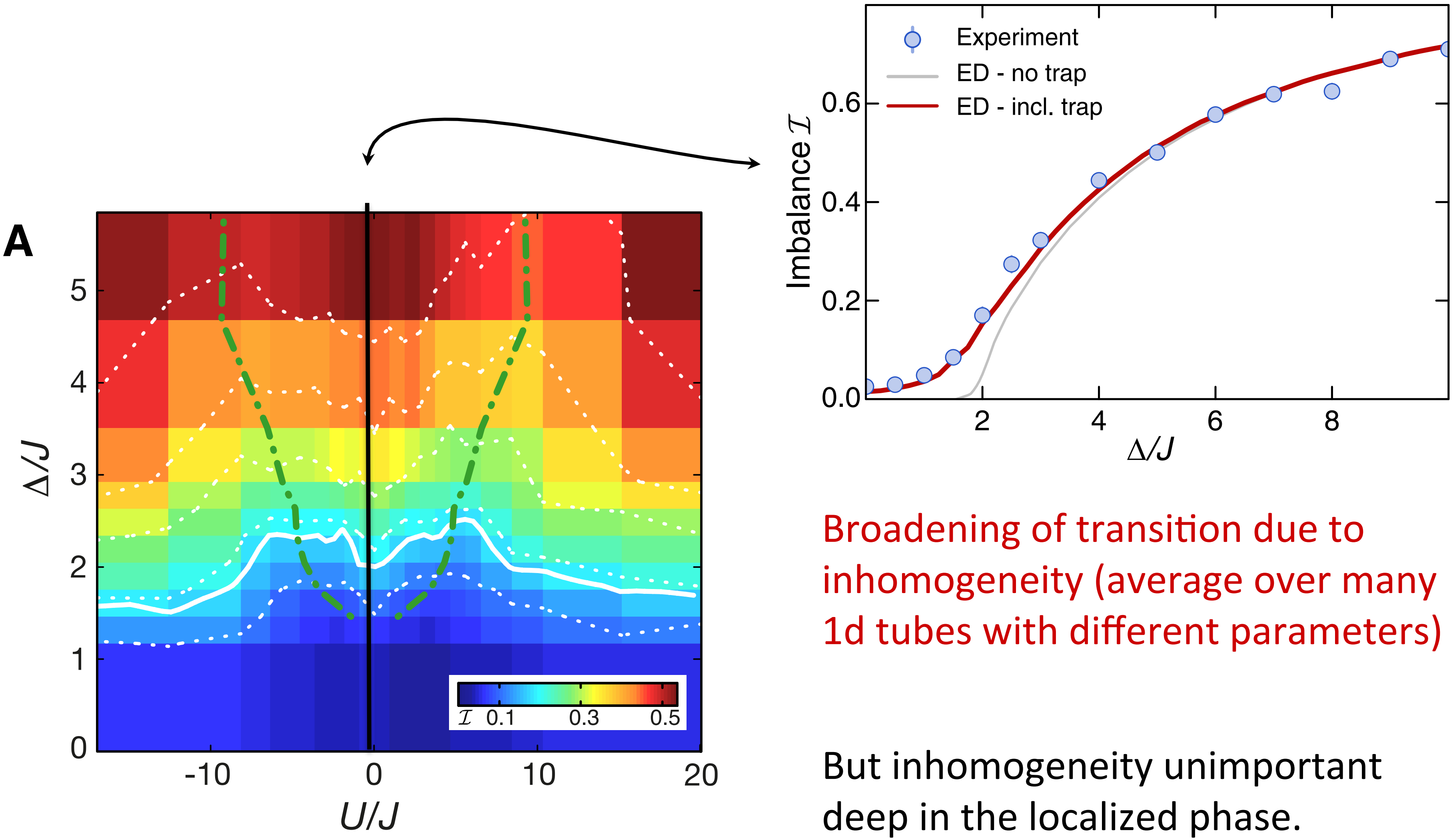


Ergodicity is broken, as expected, for Δ/J>2

Ergodicity is broken even with interactions!

Direct signature of MBL!

Experimental phase diagram

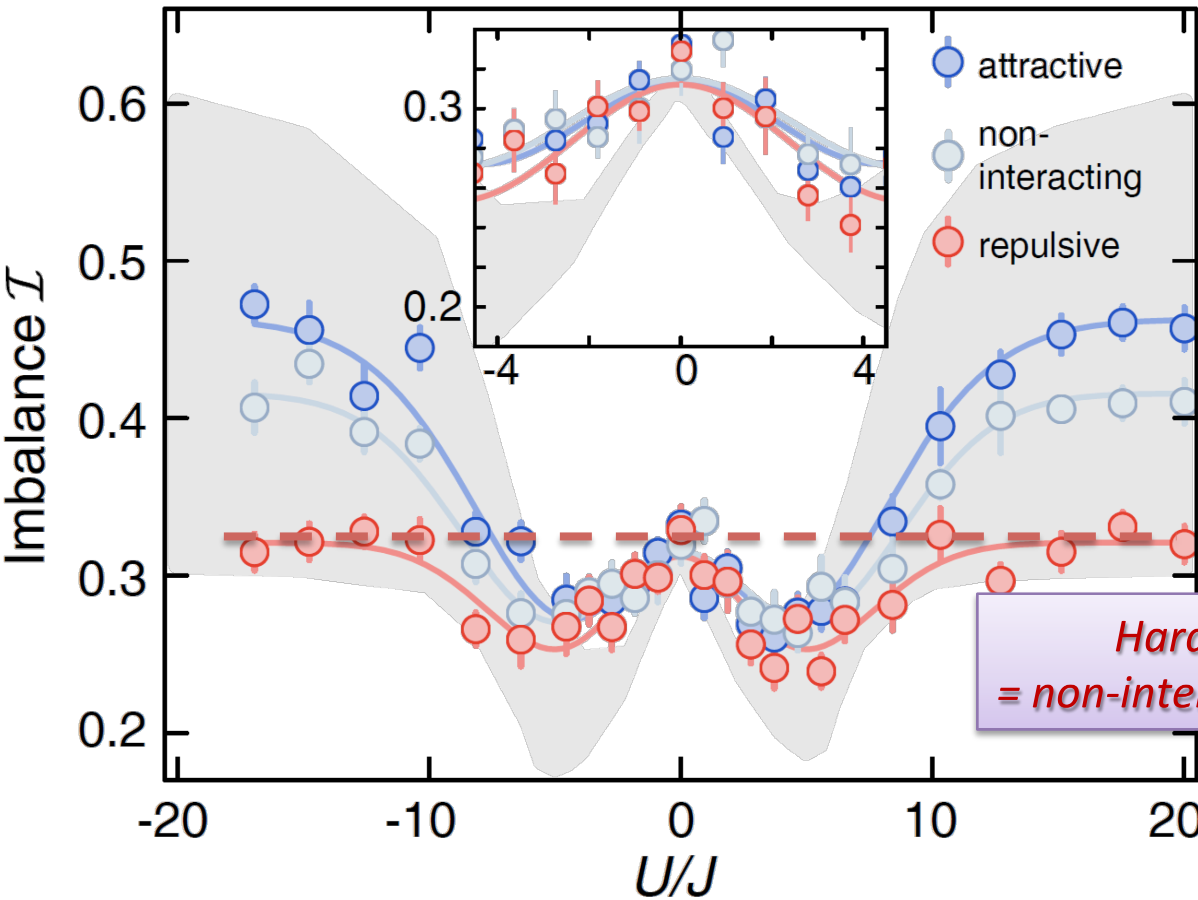


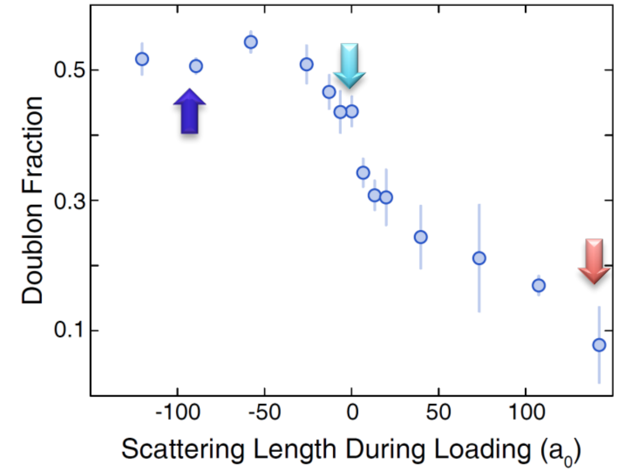
Broadening of transition due to inhomogeneity (average over many 1d tubes with different parameters)

But inhomogeneity unimportant deep in the localized phase.

Dependence on initial state

- Δ/J=5 : Localized phase





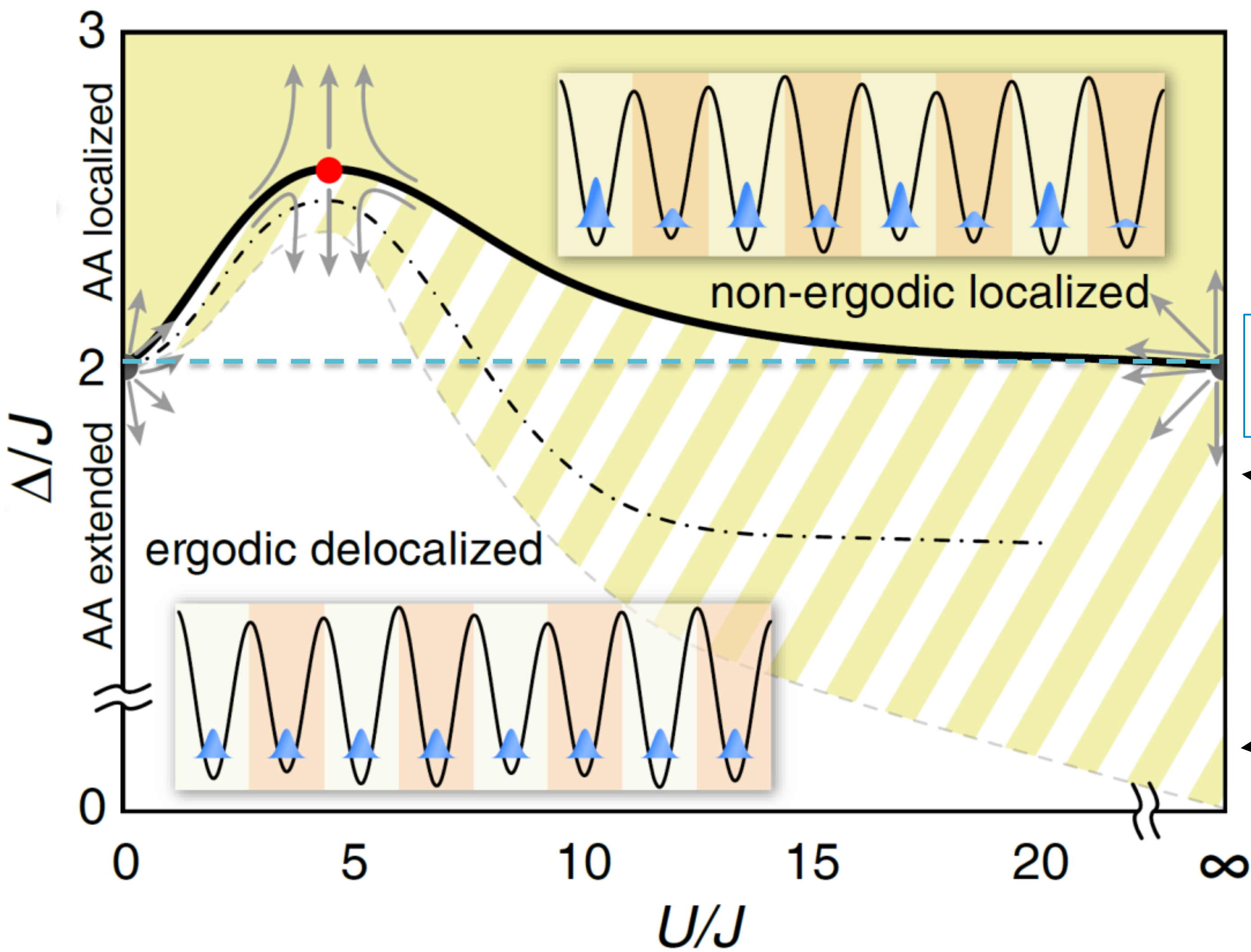
Hard-core limit
= non-interacting Fermions

Small U: no dependence on doublon fraction.

Large U: isolated doublons localize easily because $J_D \sim J^2/U$

M. Schreiber *et al.* arXiv:1501.05661

Phase diagram



No doublons
Hard-core limit

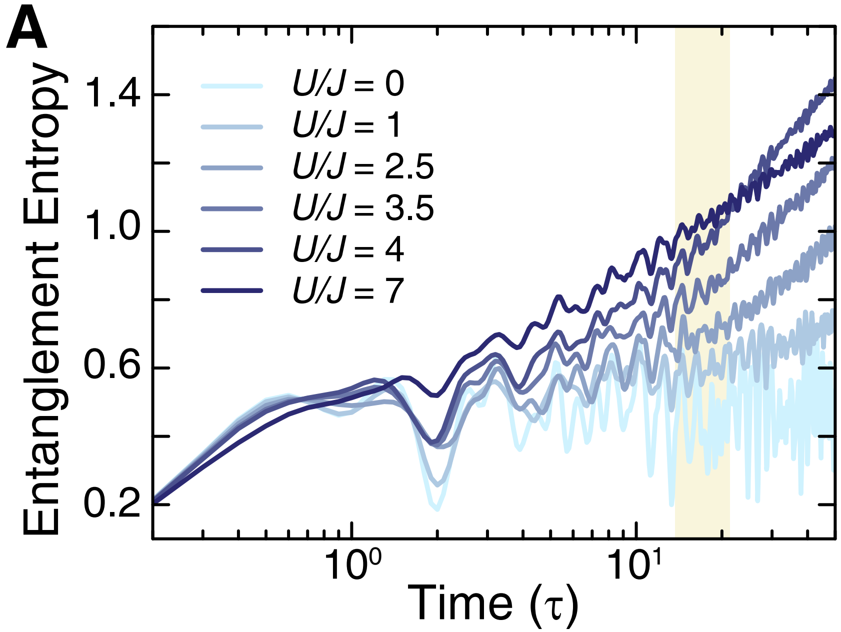
Regime with
mobility edge

All doublons

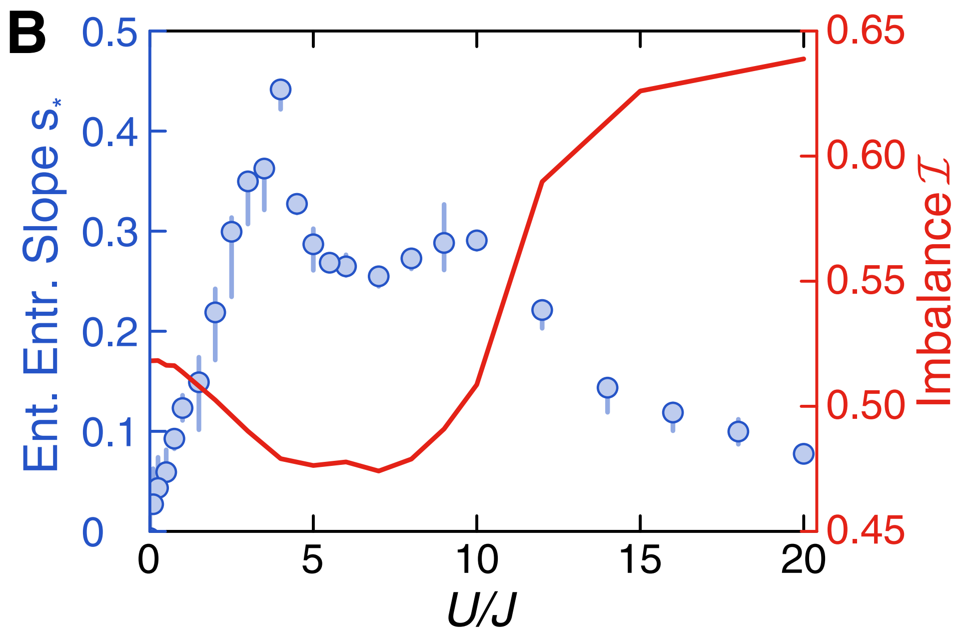
Measurement times are sufficiently long to have the logarithmic growth of entanglement

DMRG calculation:

$$S(t) = S_0 + s_* \log(t)$$



Variation of the slope with interaction U correlated with variation of the imbalance



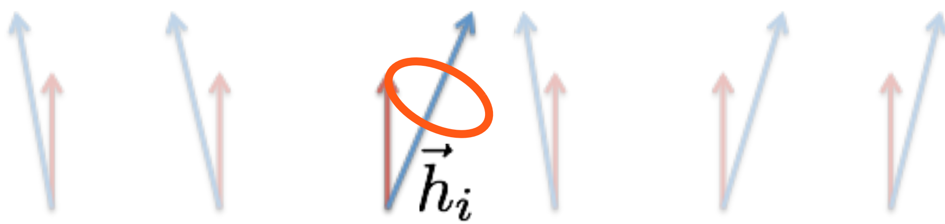
Is there an observable with a direct relation to the entanglement entropy?

fluctuations: effective model

Backup slide

- effective model: $\sigma_i^z = (n_{2i} - n_{2i+1}) \Rightarrow \mathcal{I} = \sum_i \langle \sigma_i^z \rangle$

$$\mathcal{H} = \sum_i \vec{h}_i \cdot \vec{\sigma}_i$$



$$\langle \sigma_i^z(t) \rangle = \cos^2\theta_i + \sin^2\theta_i \cos(\omega_i t)$$

$$\Rightarrow \langle \delta\mathcal{I}(t)^2 \rangle_T = \left\langle \frac{1}{L^2} \sum_{i,j} \sin^2\theta_i \sin^2\theta_j \cos(\omega_i t)\cos(\omega_j t) \right\rangle_T$$

T ← average over $T \gg 1/\omega_i$

$$\Rightarrow \delta\mathcal{I}_{\text{rms}} \equiv \sqrt{\langle \delta\mathcal{I}(t)^2 \rangle_T} \sim 1/\sqrt{L}$$

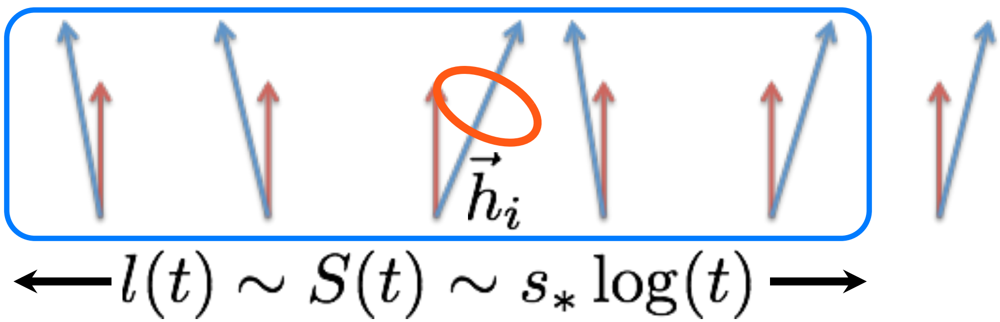
fluctuations: effective model

Backup slide

- effective model: $\sigma_i^z = (n_{2i} - n_{2i+1}) \Rightarrow \mathcal{I} = \sum_i \langle \sigma_i^z \rangle$

$$\mathcal{H} = \sum_i \vec{h}_i \cdot \vec{\sigma}_i + \sum_{i,j} V_{ij} \tilde{\sigma}_i^z \tilde{\sigma}_j^z + \ldots$$

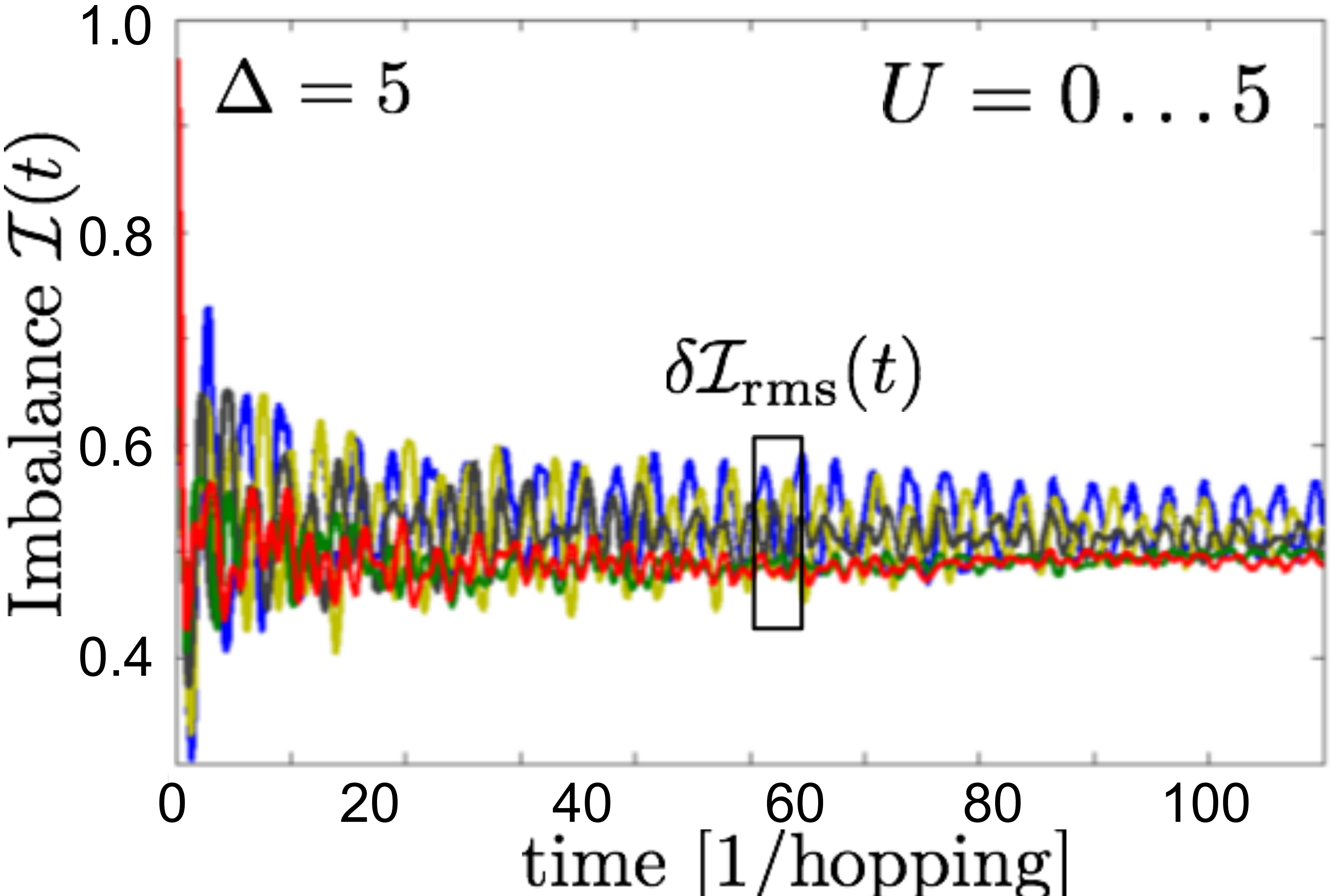
number of frequencies $N_f \sim e^{S(t)}$



$$\Rightarrow \langle \sigma_i^z(t) \rangle = \cos^2\theta_i + \sin^2\theta_i \frac{1}{N_f} \sum_{n=1}^{N_f} \cos \omega_n t$$

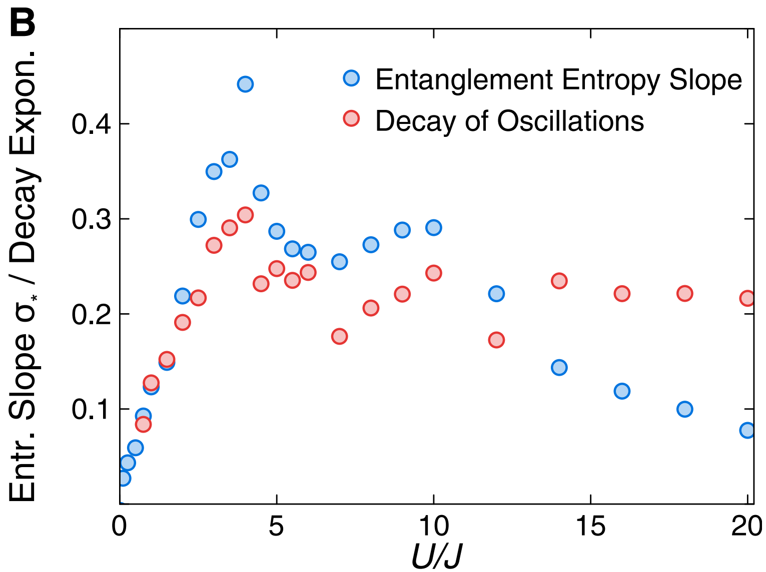
$$\Rightarrow \delta\mathcal{I}_{\rm rms}(t) \sim \frac{1}{\sqrt{L}} \left(\frac{1}{Ut}\right)^{\frac{1}{2} s_*}$$

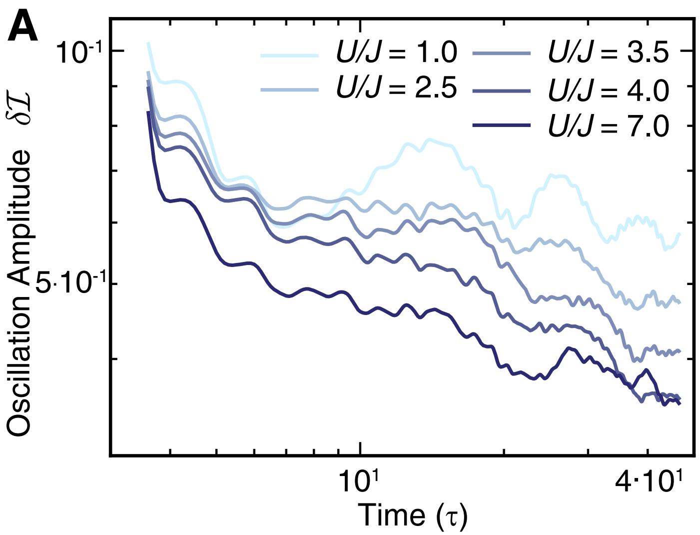
- Fluctuations of the imbalance:



Temporal fluctuations of the imbalance

Expect them to decay as: $1/(Ut)^{\xi_0}$





Temporal fluctuations carry similar information as the entanglement growth yet are measurable!

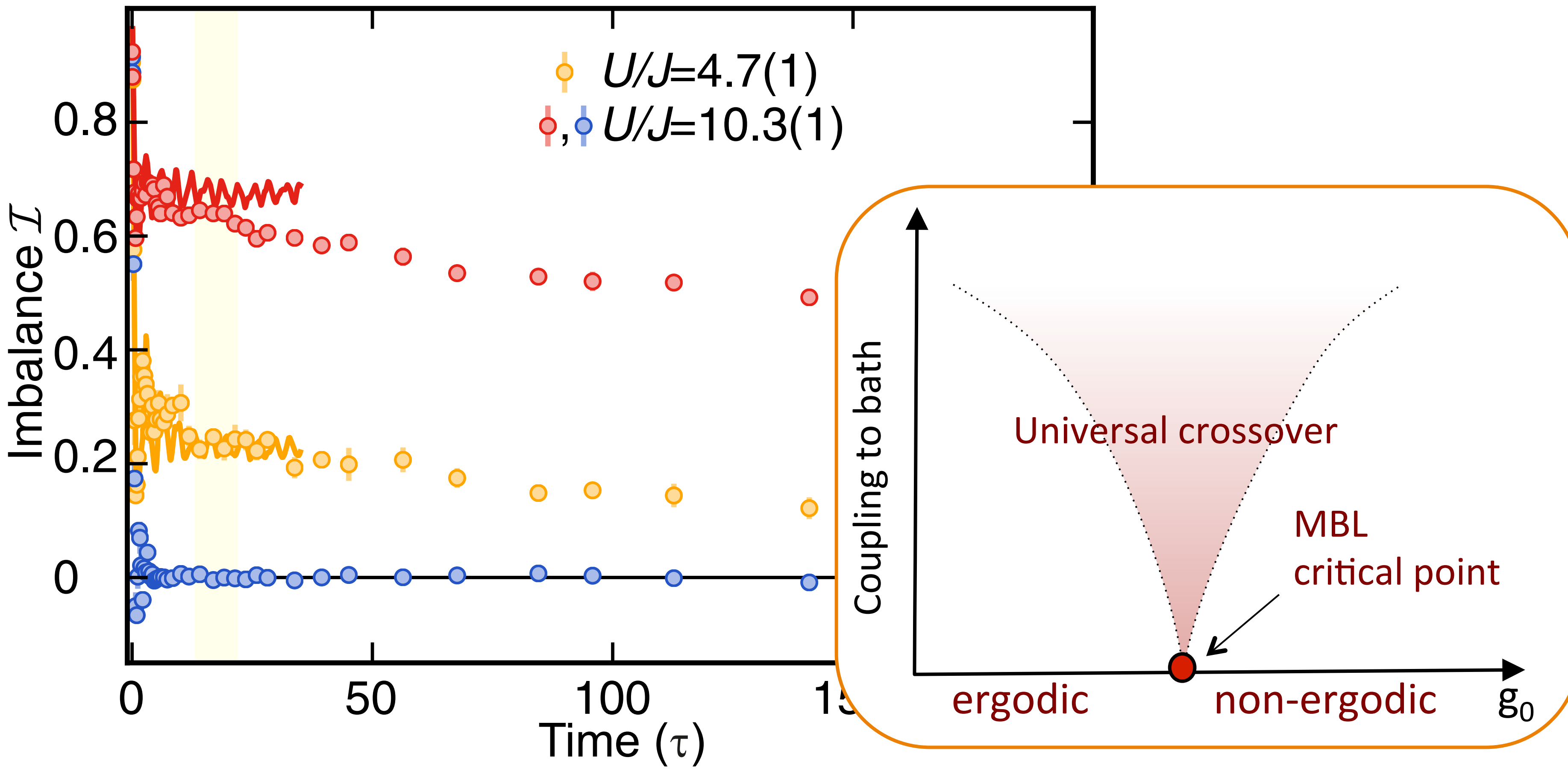
But will require experiment with single site resolution

Vasseur, Parameswaran and Moore 2014, Serbyn, Papic and Abanin 2014

Very long time behavior

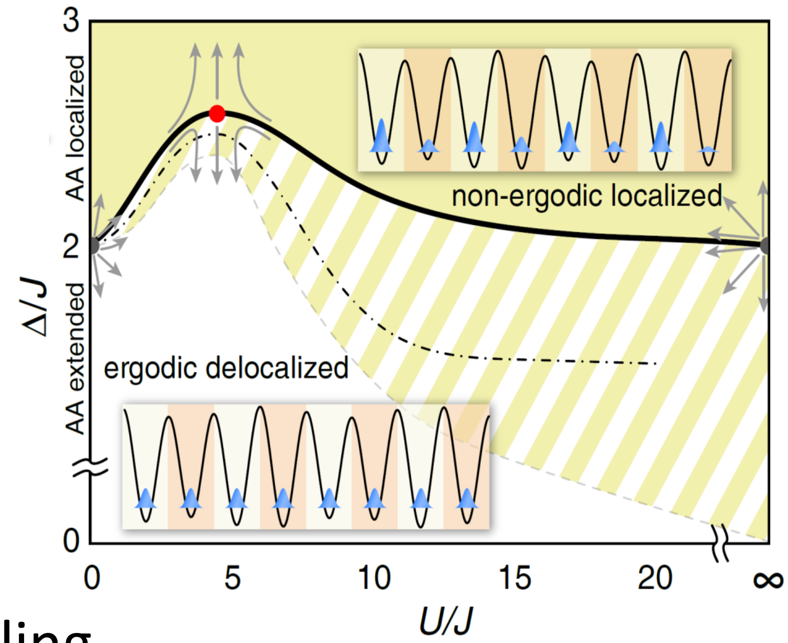
- At *very long times*, both atom number and Imbalance decay to zero.
    - Photon scattering / Light induced collisions
    - Coupling between tubes / influence of higher bands
    - Other sources?

Slow decay of the imbalance at long times:



Outlook

Much more to be done!



- Control coupling to environment
- Address critical point: finite time scaling
- Measure local observables: fluctuaitons
- Two and three dimensions
- True disorder
- Measure dynamic response
- Topological-localized states (?)