# **Bonus feature**

# **TOPOLOGICAL STATE IN A ONE DIMENSIONAL RMI GAS WITH ATTRACTIVE INTERACT**

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### Semiconductor wire vs. SO coupled 1d Fermi gas

Lutchyn et al PRL 2010, Oreg et al PRL 2010 + experiments at Delft, WIS, ...



- **Proximity coupling to SC**
- Charge **not conserved**
- **Fully gapped**

Wang, et. al. (Jing Zhang's) group PRL 2012 Cheuk et. al. (Zwierlein's group), PRL 2012



Similar single particle dispersion



- Intrinsic attractive interaction
- **Charge is conserved**
- **Gapless** phonon modes

Is there a topological state in the cold atomic system? Protected zero modes? If yes, what is their experimental signature?

### Essence of the problem: charge is a good quantum number

Fidkowski, Lutchyn, Nayak, Fisher PRB 2011; Sau, Halperin, Flensberg, Das Sarma PRB 2011



Possible solution: 1d semiconductors coupled to 1d superconductor. (Fidkowski et. al. 2011)



#### This talk: tractive interactions described by the following Hamilto- $\mathbf{t}$  interactions, naturally generated in atomic systems,  $\mathbf{t}$ with Feshbach resonances, would give rise to a Bardeen-

*H* = Z *dx †* ✓ @<sup>2</sup> *x* 2*m* <sup>+</sup> *<sup>V</sup>* (*x*) *<sup>µ</sup>* <sup>+</sup> ↵*<sup>x</sup>i*@*<sup>x</sup> z<sup>z</sup>* ◆ *U †* " *†* # # " *.* (1) Here annihilates a fermion with spin <sup>=</sup>"*,* #, *<sup>T</sup>* <sup>=</sup> ( "*,* #), *m* is the particle mass, ↵ the spin-orbit coupling strength, *µ* the chemical potential, *<sup>z</sup>* is an e↵ective Zeeman field, *V* (*x*) = *m*⌦<sup>2</sup>*x*<sup>2</sup>*/*2 is the parabolic trapping potential, and *U >* 0 is the interaction strength. Cooper-Schrie↵er (BCS) pairing gap equivalent to that induced by proximity to a bulk superconductor. But this is not the case in the one-dimensional system in question, where long range order superfluid order is impossible. Nevertheless, it was shown in Refs. [10, 11] that proximity coupling of two independent spin orbit coupled wires to a single one-dimensional superconducting wire with quasi-long range pairing correlations would retain a Majorana-like ground state degeneracy. The question remains if a single, isolated wire can sustain similar topological zero modes due to the intrinsic attractive in- *U †* " *†* # # " Here annihilates a fermion with spin <sup>=</sup>"*,* #, *<sup>T</sup>* <sup>=</sup> ( "*,* #), *m* is the particle mass, ↵ the spin-orbit coupling strength, *µ* the chemical potential, *<sup>z</sup>* is an e↵ective Zeeman field, *V* (*x*) = *m*⌦<sup>2</sup>*x*<sup>2</sup>*/*2 is the parabolic trapping potential, and *U >* 0 is the interaction strength. The parabolic trap potential can be thought of as a position dependent chemical potential. We consider filling the system to a point that the chemical potential in single fermion gap no single fermion gap "Topo" trivial "Topo" 

 $Z<sub>2</sub>$  subgroup of spin-symmetry remains: spin-parity = fermior Integerness of the spin (integer / half-integer) is a good quar Integentess of the spin (integent) han inte  $t$  matrix  $t$  the middle pairty. and continuously decreases to the flanks. In the flanks of  $\eta$  $Z_2$  subgroup of spin-symmetry remains: spin-parity = fermion parity. Integerness of the spin (integer / half-integer) is a good quantum number

- 1. Topological (Majorana-like) ground state degeneract associated with exchanging parity between "topo" d  $\mathcal{L}$ usual case where is a small proximity induced s-case where is a small proximity induced s-case  $\mathcal{L}$ this question and characterize the emergent low energy and the second this system can experience of the spatial dependent construction of the spatial spatial dependent chem-<br>A consecutive dependent construction of the spatial dependent chem-alle dependent consecutive consecuti dssociated with exchanging parity between Topol domains  $u_{\rm max}$  and  $u_{\rm max}$  is a small proximity induced s-1. Topological (Majorana-like) ground state degeneracy associated with exchanging parity between "topo" domains
- 2. Observe through a novel topological pumping induced by slow sweep of the Zeeman field 2 Observe through a novel topologie  $\mathbf u$ midd $\mathbf v$ 2. Observe through a novel topological pumping

#### Low energy description (Bosonization)  $I_{OM}$  energy description (Boso LOW ENEISY UESCHIPUUN (DOSO the spin orientation of the helical modes. (c) Schematic departmentation of the energy spectrum showing the top Low energy description (DOSO



#### Two phases of a spin-orbit coupled Fermi gas ✓<sup>+</sup> = ✓<sup>2</sup> ✓ = ✓<sup>0</sup> ✓<sup>2</sup> I We pride to the temperature

$$
\mathcal{L}^{\mathcal{A}}(\mathcal{A})=\mathcal{L}^{\mathcal{A}}(\mathcal{A})
$$

2

(@*x*+)

$$
\mathcal{H}_{-} = \frac{u_{-}}{2\pi} \int dx \left[ K_{-} (\partial_{x} \theta_{-})^{2} + \frac{1}{K_{-}} (\partial_{x} \phi_{-})^{2} \right] - g_{i} \int dx \cos(2\theta_{-}) - g_{z} \int dx \cos(2\phi_{-})
$$

(2)

- and where *u<sup>±</sup>* and *K<sup>±</sup>* are the renormalized velocities and 1. Interaction dominated - paired
- **Cutting the "coin-1/4" field is ninned**  $\ell \cdot i\theta$  $\frac{1}{\sqrt{C}}$   $\sim \pm$ . (+1 and -1 are gauge equivalent, not distinct states) • The "spin- ½" field is pinned  $\qquad \langle e^{i\theta_-}\rangle \approx \pm 1$

<sup>2</sup> +

- Gap to single fermion excitations ("spin-gap") terms; it realizes one of two distinct phases separated by
- 2 Zeeman dominated unnaired depends on the two cosine terms is larger than  $\alpha$ 2. Zeeman dominated - unpaired
- $\cdot$   $\langle e^{i\phi_-} \rangle \approx \pm 1$
- The *'trivial'* phase is established when the interaction • Gapless single fermion excitations • Gapless single fermion excitations



### Degeneracy in a harmonic trap



$$
\theta_{-}(x_2) - \theta_{-}(x_1) = \pi \int_{x_1} dx \left[ n_{\uparrow}(x) - n_{\downarrow}(x) \right] = \pi \left( N_{A\uparrow} - N_{A\downarrow} \right)
$$

- **Degenerate ground states are related by transferring a** spin <sup>1</sup>/<sub>2</sub> (single fermion) between topological domains.
- Backscattering by local impurities splits the degeneracy Fidkowski et. al. PRB 2011; Sau et. al. PRB 2011  $\Delta E \sim V_1^*$  $\frac{1}{\sqrt{2}}$ ାର କରିଥିଲେ । ଏହା ବିଶ୍ୱ ମାଣ୍ଡି ମା  $_{1}^{*}V_{2}\frac{1}{|x_{1}-x_{2}|^{\alpha}}+c.c.$
- But with smooth potential only exponentially small splitting. **Vortex tunneling "measures" the fermion parity**

### Probing the topological state: quantized pumping in the trap

Slow sweep of the Zeeman field from high value to zero. Initial state with even particle number:



In the sweep we generate 0 or 2 gapped fermions!

Odd initial state: generate 1 fermion

Quantized excess energy per sweep for random even/odd initial state.



- Zero modes naturally occur in a SO-coupled Fermi cold gas. But different from the Kitaev wire.
- Probed by a novel topological pump.
- Protected at low  $T$  by the spin gap in the non topological domains. (coupling to phonons is irrelevant at low energy)



![](_page_9_Picture_0.jpeg)

# **Many-Body localization: new insights from theory and experiment**

Ehud Altman, Weizmann Institute of Science

MMMM

Collaborators: Ronen Vosk, Mark Fischer (WIS), David Huse (Princeton)

Experiment: Michael Schreiber, Sean Hodgman, Pranjal Bordia, Henrik Luschen, Ulrich Schneider, Immanuel Bloch (LMU)

Minerva foundation Minerva oundation

![](_page_10_Picture_5.jpeg)

![](_page_10_Picture_6.jpeg)

![](_page_10_Picture_7.jpeg)

### Conventional wisdom:

Quantum mechanics is manifest only close to the ground state

![](_page_11_Figure_3.jpeg)

![](_page_11_Picture_4.jpeg)

Quantum Hall effect: Topological insulators:

![](_page_11_Picture_6.jpeg)

#### Fermi liquid:

![](_page_11_Picture_8.jpeg)

### Quantum critical points

![](_page_11_Figure_10.jpeg)

### Ergodicity is the enemy of quantum mechanics

Many-body time evolution washes away quantum correlations.

Quantum information stored in local objects is rapidly lost as these get entangled with the rest of the system.

The only remaining structures of information are slow order parameter fields and conserved densities.

![](_page_12_Picture_4.jpeg)

![](_page_12_Picture_5.jpeg)

![](_page_12_Picture_6.jpeg)

Classical hydrodynamic description (e.g. diffusion).

To see quantum phenomena at long times need breaking of ergodicity!

## Well known example: integrability

Failure to thermalize due to constraints imposed by many conservation rules

Quantum newton's cradle experiment with cold atoms. Weiss, Nature 2010

![](_page_13_Figure_3.jpeg)

But integrability is fragile! Only special points in parameter space. Are there more generic non-thermal states?

![](_page_13_Picture_5.jpeg)

### **Anderson localization**

Single particle (Anderson 1958):

![](_page_14_Picture_2.jpeg)

![](_page_14_Picture_3.jpeg)

Vanishing probability of resonances.

Many particle states:  $U c^\dagger_\alpha c^\dagger_\beta c_\gamma c_\delta$ 

At high energies interaction connects between ~2<sup>L</sup> localized states! Can localization survive?

### Many-Body Localization

Basko, Aleiner, Altshuler (2005); Gornyi, Mirlin, Polyakov (2005): Insulating phase stable below a critical  $T$  or  $E$ ; metal above it.

System with bounded spectrum: Disorder tuned transition at  $T=\infty$ Oganesyan and Huse *(2007), Pal and Huse (2010)*

![](_page_15_Figure_3.jpeg)

Many questions:

Nature of the dynamics in the localized phase? At the critical point? Experiments?

# The eignestate perspective: Eigenstate thermalization hypothesis (ETH)

Deutsch 91, Srednicki 94

High energy eigenstate of an ergodic system appears thermal:

$$
\rho_A = \frac{1}{Z_A} e^{-\beta H_A}
$$

Von-Neuman (entanglement) entropy:

$$
S_A \equiv \text{tr}\left[\rho_A \ln \rho_A\right] \propto L^d
$$

Anderson localization is an example where ETH fails:

"Area law" entropy even in high  $S_A \propto L^{d-1}$ energy eigenstates

![](_page_16_Figure_8.jpeg)

![](_page_16_Picture_9.jpeg)

Many body localization  $=$  stability of such localized states to interactions

### A tale of two paradigms

#### Thermalization

![](_page_17_Picture_2.jpeg)

Quantum correlations in local d.o.f are rapidly lost as these get entangled with the rest of the system.

**Classical** hydro description of remaining slow modes (conserved quantities, and order parameters). ? 

#### Many-body localization

![](_page_17_Picture_6.jpeg)

Local quantum information persists indefinitely.

Need a fully **quantum** description of the long time dynamics!

The many-body  $localization$  transition  $\qquad \qquad \frac{1}{2}$ 

elusive interface between quantum and classical worlds 

### The eigenstate perspective

#### **Thermalizing**

![](_page_18_Picture_2.jpeg)

### Energy eigenstates are highly entangled:

 $S_A \sim L^d$ 

#### Many-body localized

![](_page_18_Picture_5.jpeg)

### Eigenstates have low entanglement

![](_page_18_Picture_7.jpeg)

(volume law)  $S_A \sim L^{d-1}$  (area law)

Localization transition: fundamental change in entanglement pattern. More radical than in any other phase transition we know !

<u>ገ</u>

# **Outline**

- New insights from theory
	- $-$  Dynamics in the many-body localized (MBL) phase
	- $-$  Phase transition from MBL to a thermal liquid

• Confronting theory and experiment

Collaboration with I. Bloch's LMU group, U. Schneider and co.

- $-$  MBL of fermions in quasi-periodic lattice
- Outlook to future experiments

### **RG Solution of time evolution**

 $\overline{D}$  in the non-equilibrium dynamics of  $\overline{D}$ chains with generic interactions with general states and several states with a structure interactions of the s<br>contractions with general states with a structure interactions with a structure interactions with a structure <br> R. Vosk and EA PRL (2013); PRL (2014)

*H* = X *i* ⇥ *Jz <sup>i</sup> <sup>z</sup> <sup>i</sup> <sup>z</sup> <sup>i</sup>*+1 + *hi<sup>x</sup> <sup>i</sup>* + *J<sup>x</sup> <sup>i</sup> <sup>x</sup> <sup>i</sup> <sup>x</sup> <sup>i</sup>*+1 <sup>+</sup> *...*⇤ (1) Here *J<sup>z</sup> <sup>i</sup>* , *h<sup>i</sup>* and *J<sup>x</sup> <sup>i</sup>* are uncorrelated random variables respect the *Z*<sup>2</sup> symmetry of the model. For simplicity dom systems[7]. The properties of the transition are elucidated by tracking the time evolution of two quantities: spin correlations and entanglement entropy. *<sup>C</sup>z*(*t*) = <sup>h</sup> *in <sup>|</sup> <sup>S</sup><sup>z</sup> |* <sup>0</sup>i = Pick out largest couplings ⌦ = max (*J<sup>z</sup> <sup>i</sup> , hi*) *H*fast

Short times ( $t \approx 1/\Omega$ ): System evolves according to  $H_{\text{fast}}$ Other spins essentially frozen on this timescale.

<u>Longer times</u> (*t* >>1/Ω): Eliminate fast modes (order Ω) perturbatively to obtain effective evolution for longer timescales.

Similar to standard strong disorder RG, Dasgupta-Ma (1980), D. Fisher (1992) But here we target low frequency instead of low absolute energy

### Outcome of  $RG:$  integrals of motion = (frozen spins)

 $h_i = \Omega$ 

 $\tilde{J} = J_L J_R / \Omega$ 

 $J_L$   $J_R$ 

R. Vosk and EA PRL (2013); PRL (2014); Pekker, Refael et. al. PRX (2014)

Example: strong transverse field

$$
H = \sum_{i} \left[ J_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + V_i \sigma_i^x \sigma_{i+1}^x \right]
$$

$$
H_{\text{eff}} = e^{-iS} H e^{iS}
$$

*i*

$$
H_{\text{eff}} = \Omega \tilde{\sigma}^x_i + V_L \, \tilde{\sigma}^x_i \sigma^x_L + V_R \, \tilde{\sigma}^x_i \sigma^x_R + \frac{J_L J_R}{\Omega} \sigma^z_L \tilde{\sigma}^x_i \sigma^z_R
$$

In this RG scheme degrees of freedom are not eliminated but rather frozen into *<u>quasi-local integrals of motion</u>:* 

$$
\tilde{\sigma}^x_i = Z \sigma^x_i + \text{exponential tail}
$$

### Effective Hamiltonian (fixed point theory)

Depends only on the quasi-local integrals of motion:

$$
H_{FP} = \sum_{i} \tilde{h}_i \tilde{\sigma}_i^x + \sum_{ij} V_{ij} \tilde{\sigma}_i^x \tilde{\sigma}_j^x + \sum_{ijk} V_{ijk} \tilde{\sigma}_i^x \tilde{\sigma}_j^x \tilde{\sigma}_k^x + \dots
$$
  

$$
V_{ij} \sim V e^{-|x_i - x_j|/\xi}
$$

Independently postulated as a phenomenological description of the many-body localized phase.

Oganesyan & Huse (2013); Serbyn, Papic & Abanin (2013)

![](_page_22_Picture_5.jpeg)

Surprisingly rich dynamics in MBL phase:

- Slow log(t) growth of the entanglement entropy.
- Quantum coherence revealed by spin echoes
- Distinct localized phases (glassy, paramagnetic, topological ...)

#### This scheme cannot access the localization phase transition!

## Theory of the many-body localization transition

Vosk, Huse and E.A. arXiv:1412.3117

![](_page_23_Figure_2.jpeg)

Spin chain fragmented into puddles of different types: incipient insulators and incipient metals. Modeled as coupled random matrices:

$$
\boxed{\Delta_i,\Gamma_i\;\;\Rightarrow\;g_i=\Gamma_i/\Delta_i}
$$

$$
\Delta_i
$$
 **Mean level spacing in the block**

 $\Gamma_i^{-1} = \tau_i \hspace{0.5cm}$  Time for entanglement to spread across the block

 $g_i \ll 1$  "insulating block"  $g_i \gg 1$  "thermalizing block"

(Poisson level statistics) (Wigner-Dyson statistics)

RG flow: itteratively join matrices that entangle with each other at running cutoff scale. At the end of the flow we are left with one big block that is either insulating or thermalizing

### Outcome of the RG flow

![](_page_24_Figure_1.jpeg)

Next, characterize the transition:

(i) in terms of dynamics; (ii) in terms of eigenstate entanglement

### RG results  $I -$  dynamical scaling for transport

Relation between transport time  $\tau_{tr}$  and length *l* of blocks:

![](_page_25_Figure_2.jpeg)

### RG results  $I -$  dynamical scaling for transport

Relation between transport time  $\tau_{tr}$  and length *l* of blocks:

Surprise! The transition is from localized to anomalous diffusion.

Seen also in recent ED studies: Bar-Lev et al 2014; Agarwal et al 2014

![](_page_26_Figure_4.jpeg)

Result of Griffiths effects. long insulating inclusions inside the metal are exponentially rare but give exponentially large contribution to the transport time.

![](_page_26_Picture_6.jpeg)

![](_page_26_Picture_7.jpeg)

### Eigenstate entanglement turns out to be the natural scaling variable of this RG scheme !

![](_page_27_Figure_1.jpeg)

#### Eigenstate entanglement turns out to be the natural scaling variable of this RG scheme !  $\overline{\mathbf{v}}$ er  $\sim$   $\sim$  $\overline{v}$ in a shekara

![](_page_28_Figure_1.jpeg)

- Infinite randomness fixed point characterized by broad entanglement distribution
- Universal jump to full thermal entropy  $\rightarrow$  the Griffiths phase is thermal

![](_page_29_Figure_0.jpeg)

## Experimental study of MBL: fermions in a quasi-random optical lattice

arXiv:1501.05661

![](_page_30_Figure_2.jpeg)

Collaboration with With: Immanuel Bloch's group (Munich) Mark Fischer and Ronen Vosk (Weizmann)

![](_page_31_Picture_0.jpeg)

![](_page_31_Picture_1.jpeg)

![](_page_31_Picture_2.jpeg)

Michael Schreiber Pranjal Bordia Henrik Lüschen

Sean Hodgman Ulrich Schneider Immanuel Bloch 

**LMU & MPQ München** 

![](_page_31_Picture_6.jpeg)

![](_page_31_Picture_7.jpeg)

Mark Fischer Ronen Vosk EA **Weizmann** 

![](_page_31_Picture_9.jpeg)

![](_page_31_Picture_10.jpeg)

![](_page_31_Picture_11.jpeg)

### Previous experiments

### Anderson localization:

Expansion in disordered potential

Billy et al. (Aspect) Nature (2008) Roati (Inguscio) PRL (2008)

### Many-body localization (?):

Transport in a trap (response to impulse) Kondov et al. (DeMarco) preprint 2013

![](_page_32_Picture_7.jpeg)

![](_page_32_Figure_8.jpeg)

### Previous experiments

### Many body localization (?) :

In-trap transport (following impulse)

### **The Problems with such a global probe:**

- Very slow probe (finite size time scale by definition)
- Sensitive to inhomogeneity. e.g. Mott shells can block transport

### **Our solution: Use a fast local observable**

![](_page_33_Figure_8.jpeg)

#### Quantum quench protocol pared to the lattice constant, then the CDW vanishes as *<sup>I</sup>* / <sup>1</sup>*/*⇠<sup>2</sup> [43]. In the contrast to previous experiments, which studies are effect of  $\sim$  $\mu$ geneity of the trapped system.

1. Fermions in optical lattice prepared in period-2 CDW parameter acts as a purely local probe, directly captures the ergodicity captures the ergodicity captures the e 1. Fermions in optical lattice prepared in period-2 CDW Our system can be described by the one-dimensional fermionic

![](_page_34_Figure_2.jpeg)

2. Evolve the state with the 1d lattice Hamiltonian: *i,*

![](_page_34_Figure_4.jpeg)

Numerics suggest that this model shows generic MBL (Iver et. al. PRB 2013) Numerics suggest that this model shows generic MBL ( Iyer et. al. PRB 2013)

### What to measure?

Relaxation of the CDW with time:

$$
\mathcal{I} = \frac{1}{N} \sum_{j=1}^{L} (-1)^j \langle n_j \rangle = \frac{\langle N_e - N_o \rangle}{N_e + N_o}
$$

time: 0 *t* 

![](_page_35_Figure_4.jpeg)

If the system is localized, the CDW operator has finite overlap with an integral of motion.

![](_page_35_Figure_6.jpeg)

Macroscopic order parameter of the MBL phase

### Experimental results

![](_page_36_Figure_1.jpeg)

as expected, for  $\Delta/J>2$ 

Ergodicity is broken even with interactions!

**Direct signature of MBL!** 

#### Experimental phase diagram *L* APLITIILITIC and the error bars show the standard deviation of the mean. The shaded region indicates the time window used to characterise the stationary in the station  $\mathcal{L}$ rest of the analysis.

![](_page_37_Figure_1.jpeg)

![](_page_37_Figure_2.jpeg)

Figure 3: Stationary values of the imbalance *I* as a function of disorder Broadening of transition due to show the experimental data, along with Exact Diagonalization (ED) calculations inhomogeneity (average over many 1d tubes with different parameters)

But inhomogeneity unimportant deep in the localized phase.

### Dependence on initial state

![](_page_38_Figure_1.jpeg)

Small U: no dependence on doublon fraction. Large U: isolated doublons localize easily because  $J_D \sim J^2/U$ 

M. Schreiber *et al.* arXiv:1501.05661

### Phase diagram

![](_page_39_Figure_1.jpeg)

#### Measurement times are sufficiently long to have the logarithmic growth of entanglement  $\mathbf{A}$  , see  $\mathbf{a}$ , we fig. 3 for  $\mathbf{b}$ , plotted in Fig. 3  $\mathbf{a}$ , plotted in Fig. interaction and in Fig. 2 in F states with the finite for  $\alpha$  . Above the critical contract of  $\alpha$ point of the homogeneous Aubry-Andre model at ´ */J* = 2 [37], howionthy *long* to hove the iciently long to have the is finite, such that we observe a weaker effect. We find the localization entangiement iff<br>|
|
| V  $\overline{\phantom{a}}$  $\sim$ d $\sim$

can be seen in the contour lines ( $\frac{1}{2}$   $\frac{1}{4}$   $\frac{1}{4$ 

 $DIMDC$  coloulation. DMRG calculation: Va

![](_page_40_Figure_2.jpeg)

Is there an observable with a direct relation to the entanglement entropy? n a direct relation to have been always a substitution of the entropy of the entropy of the entropy of the entr  $\mathcal{F}_{\mathcal{A}}$  figure 5: Growth of entanglement entropy and corresponding slope. A:  $\mathcal{A}$ strengths and */J* = 5. For long times, logarithmic growth characteristic of interacting MBL states is visible. The experimentally used evolution times individually used evolution times in

#### fluctuations: effective model **Backup** slide

$$
\sigma_i^z = (n_{2i} - n_{2i+1}) \longrightarrow \mathcal{I} = \sum_i \langle \sigma_i^z \rangle
$$
  
\n
$$
\mathcal{H} = \sum_i \vec{h}_i \cdot \vec{\sigma}_i
$$
  
\n
$$
\langle \sigma_i^z(t) \rangle = \cos^2 \theta_i + \sin^2 \theta_i \cos(\omega_i t)
$$
  
\n
$$
\langle \delta \mathcal{I}(t)^2 \rangle_T = \left\langle \frac{1}{L^2} \sum_{i,j} \sin^2 \theta_i \sin^2 \theta_j \cos(\omega_i t) \cos(\omega_j t) \right\rangle_T
$$
  
\n
$$
\delta \mathcal{I}_{\text{rms}} \equiv \sqrt{\langle \delta \mathcal{I}(t)^2 \rangle_T} \sim 1/\sqrt{L}
$$
  
\n
$$
\sum_{\text{over } T \gg 1/\omega_i} \langle \delta \mathcal{I}(T) \rangle_T
$$

#### fluctuations: effective model **Backup** slide

e effective model:  
\n
$$
\sigma_i^z = (n_{2i} - n_{2i+1}) \longrightarrow \mathcal{I} = \sum_i \langle \sigma_i^z \rangle
$$
\n
$$
\mathcal{H} = \sum_i \vec{h}_i \cdot \vec{\sigma}_i + \sum_{i,j} V_{ij} \tilde{\sigma}_i^z \tilde{\sigma}_j^z + \dots
$$
\nnumber of  
\nfrequencies  
\n
$$
N_f \sim e^{S(t)} \underbrace{\left(\prod_{i} \sum_{j} \sigma_i \right)}_{l(t)} \sim S(t) \sim s_* \log(t) \longrightarrow
$$
\n
$$
\langle \sigma_i^z(t) \rangle = \cos^2 \theta_i + \sin^2 \theta_i \frac{1}{N_f} \sum_{n=1}^{N_f} \cos \omega_n t
$$
\n
$$
\delta \mathcal{I}_{\text{rms}}(t) \sim \frac{1}{\sqrt{L}} \left(\frac{1}{Ut}\right)^{\frac{1}{2}s_*}
$$

### . Fluctuations of the imbalance:

![](_page_43_Figure_1.jpeg)

### Temporal fluctuations of the imbalance

Expect them to decay as:  $1/(Ut)$ 

 $1/(Ut)^{\xi_0}$ 

![](_page_44_Figure_2.jpeg)

*z**n d e e t e t e t e e towth yet are measurable!<br><i>I I* require experiment with single site<br>*Properiment with single site*<br>*Properiment with single site* But will require experiment with s Temporal fluctuations carry similar information as the entanglement growth yet are measurable! But will require experiment with single site resolution remporal nucluations carry similar the international growth you are meat

Vasseur, Parameswaran and Moore 2014, Serbyn, Panic and Abanin 2014 bard model on the quasi-periodic lattice using time dependent DMRG. Vasseur, Parameswaran and Moore 2014, Serbyn, Papic and Abanin 2014

## Very long time behavior

- At very long times, both atom number and Imbalance decay to zero.
	- Photon scattering / Light induced collisions
	- Coupling between tubes / influence of higher bands
	- Other sources?

Slow decay of the imbalance at long times:

![](_page_45_Figure_6.jpeg)

# **Outlook**

Much more to be done!

- Control coupling to environment
- Address critical point: finite time scaling
- Measure local observables: fluctuaitons
- Two and three dimensions
- True disorder
- Measure dynamic response
- Topological-localized states (?)

![](_page_46_Figure_9.jpeg)

![](_page_46_Figure_10.jpeg)