Status of scalar quark matrix elements from Lattice QCD

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Jefferson Lab

Outline

- *Nucleon matrix element calculations*
  - *Direct method - 3 point function*
  - *Indirect method - Feynman-Hellman Theorem*
- *Scalar Matrix Elements* $\langle N|m_q\bar{q}q|N\rangle$
  - $\bar{q}_l q_l$
  - $\bar{s}s$
  - $\bar{q}_h q_h$
- *Conclusions and questions*

Nucleon Matrix Elements

$$\langle N(\tau)|\mathcal{O}(t)|N(0)\rangle = \frac{1}{\mathcal{Z}} \int DU\, D\bar{\psi} D\psi e^{-S} N(\tau)\mathcal{O}(t)N^{\dagger}(0)$$

e.g. axial current in the nucleon $\quad \mathcal{O} = \frac{1}{2}\bar{q}\tau_a\gamma_\mu\gamma_5 q$

e.g. scalar matrix elements $\quad \mathcal{O} = m_q\bar{q}q$

NOTE: these involve 3-point correlation functions.
These are numerically difficult calculations.

Nucleon Matrix Elements

$$\langle N(\tau)|\mathcal{O}(t)|N(0)\rangle = \frac{1}{\mathcal{Z}}\int DU D\bar{\psi} D\psi e^{-S} N(\tau)\mathcal{O}(t)N^\dagger(0)$$

We also need to know the 2, 3, … body matrix elements

$$\langle NN(\tau)|\mathcal{O}(t)|NN(0)\rangle$$

$$\langle NNN(\tau)|\mathcal{O}(t)|NNN(0)\rangle$$

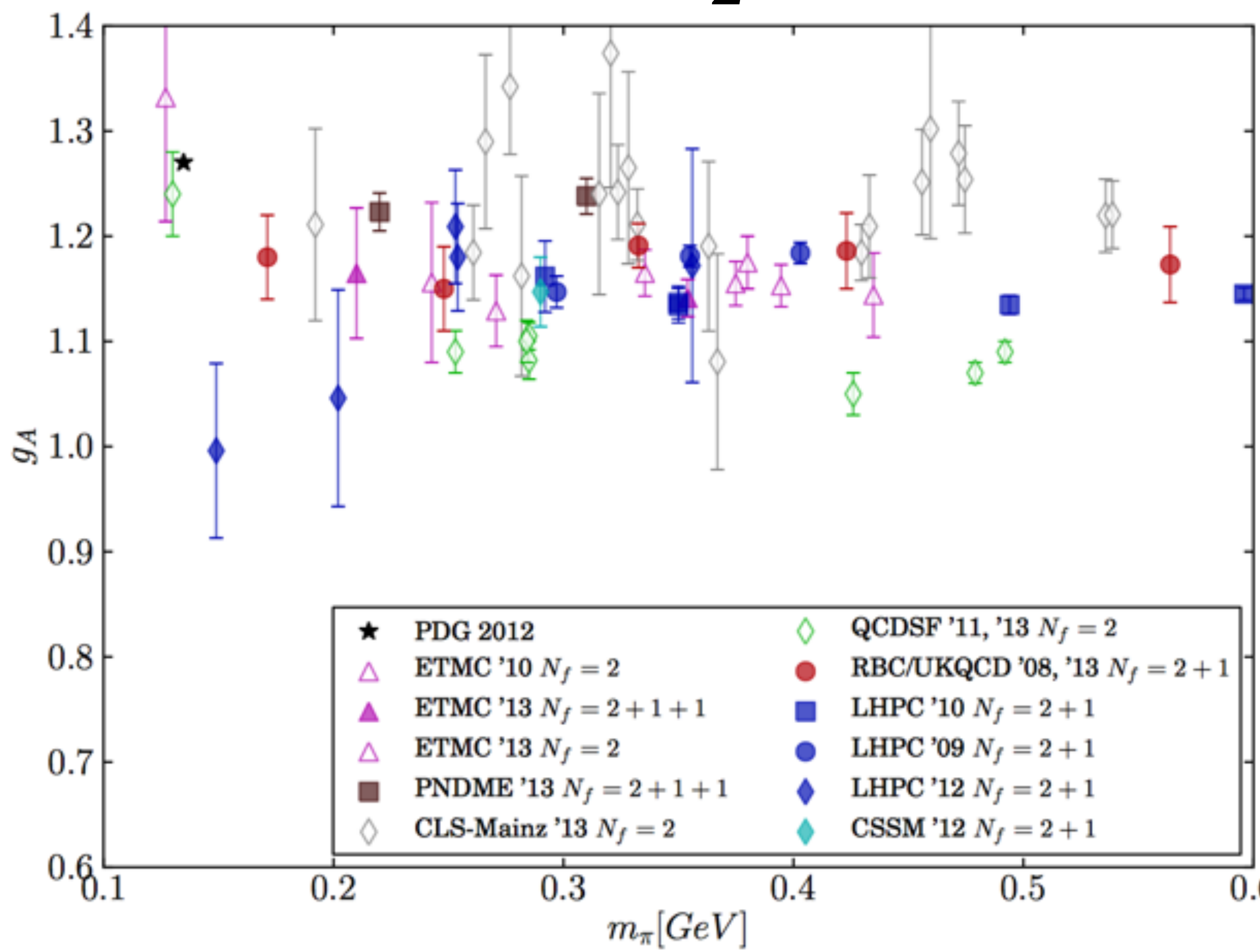
No reason (I know of) to expect these N-body matrix elements are significantly more important than in "standard" nuclear physics i.e. the precise nuclear matrix element will require knowledge of these but they are sub-corrections O(10%) see Will Detmold's talk

Nucleon Matrix Elements

$$\langle N(\tau)|\mathcal{O}(t)|N(0)\rangle = \frac{1}{\mathcal{Z}}\int DU\, D\bar{\psi} D\psi e^{-S} N(\tau)\mathcal{O}(t)N^{\dagger}(0)$$

e.g. axial current in the nucleon $\mathcal{O} = \frac{1}{2}\bar{q}\tau_a\gamma_\mu\gamma_5 q$

So far, we (LQCD) have failed to convincingly compute the most basic nucleon 3-point function - g_A



Nucleon Matrix Elements

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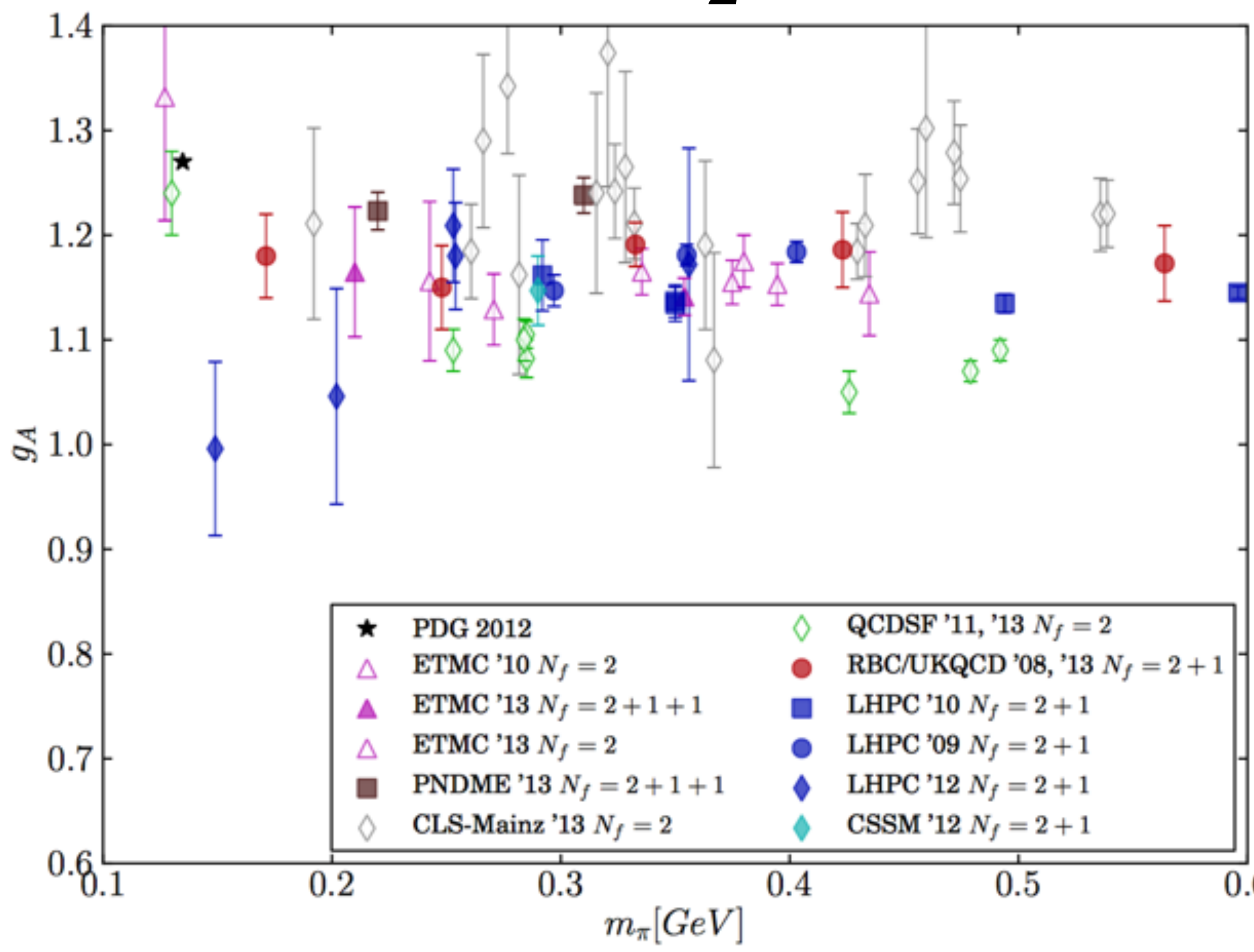
e.g. axial current in the nucleon $\mathcal{O} = \frac{1}{2}\bar{q}\tau_a\gamma_\mu\gamma_5 q$

We have good ideas about the systematics that need to be controlled, but as yet, there has been no convincing calculation

See Sergey Syritsyn's talk

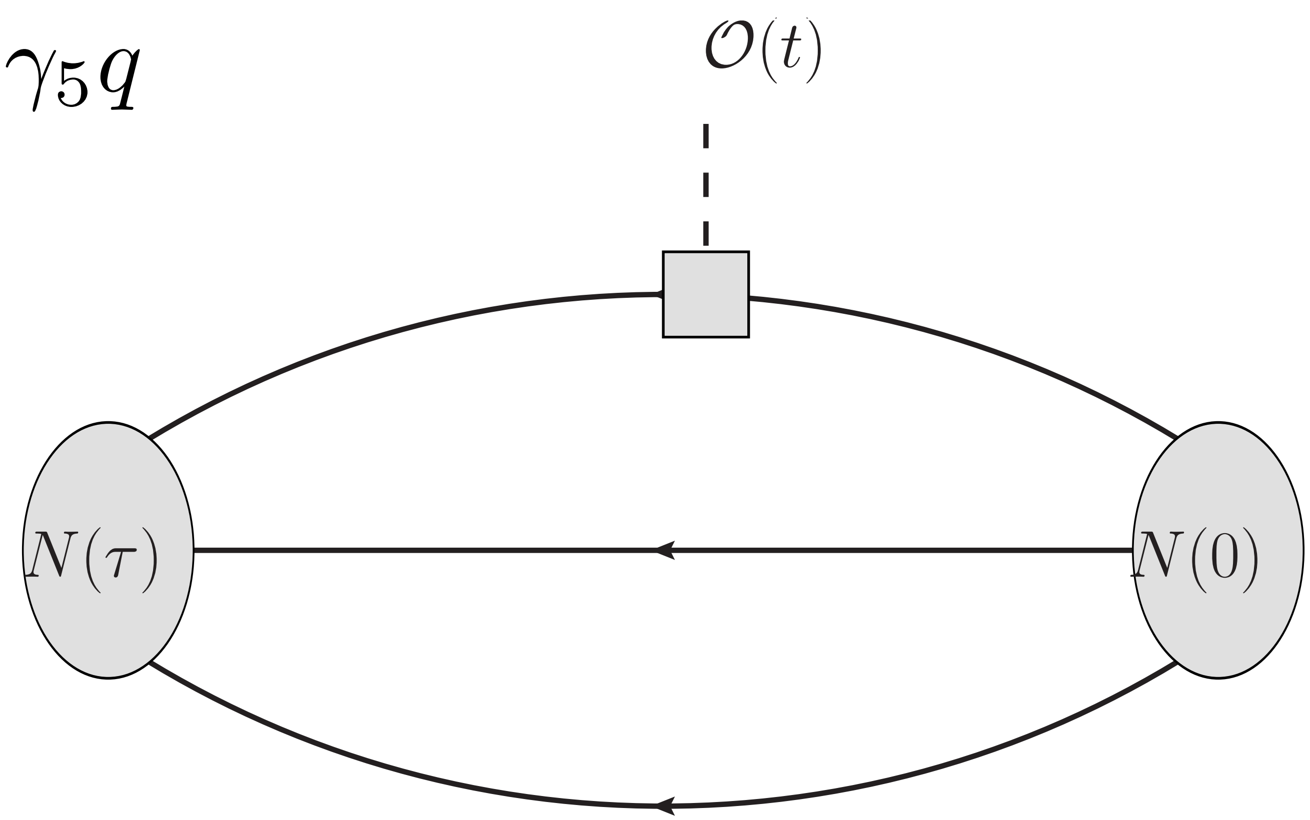


Nucleon Matrix Elements

$$\langle N(\tau)|\mathcal{O}(t)|N(0)\rangle = \frac{1}{\mathcal{Z}}\int DU\,D\bar{\psi}D\psi e^{-S}N(\tau)\mathcal{O}(t)N^\dagger(0)$$

nucleon axial current is numerically easier than scalar matrix elements - no disconnected diagrams

$$\mathcal{O} = \frac{1}{2}\bar{q}\tau_a\gamma_\mu\gamma_5 q$$

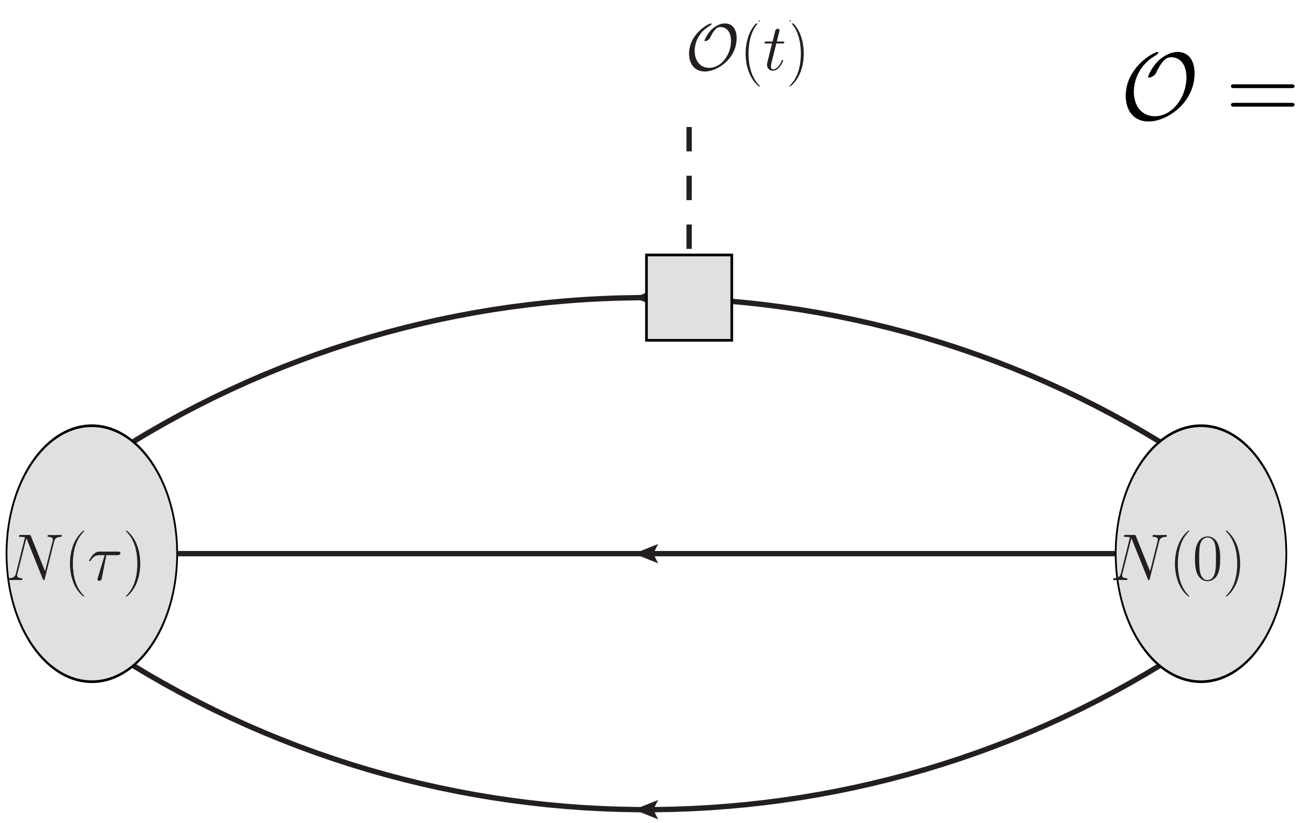


Nucleon Matrix Elements

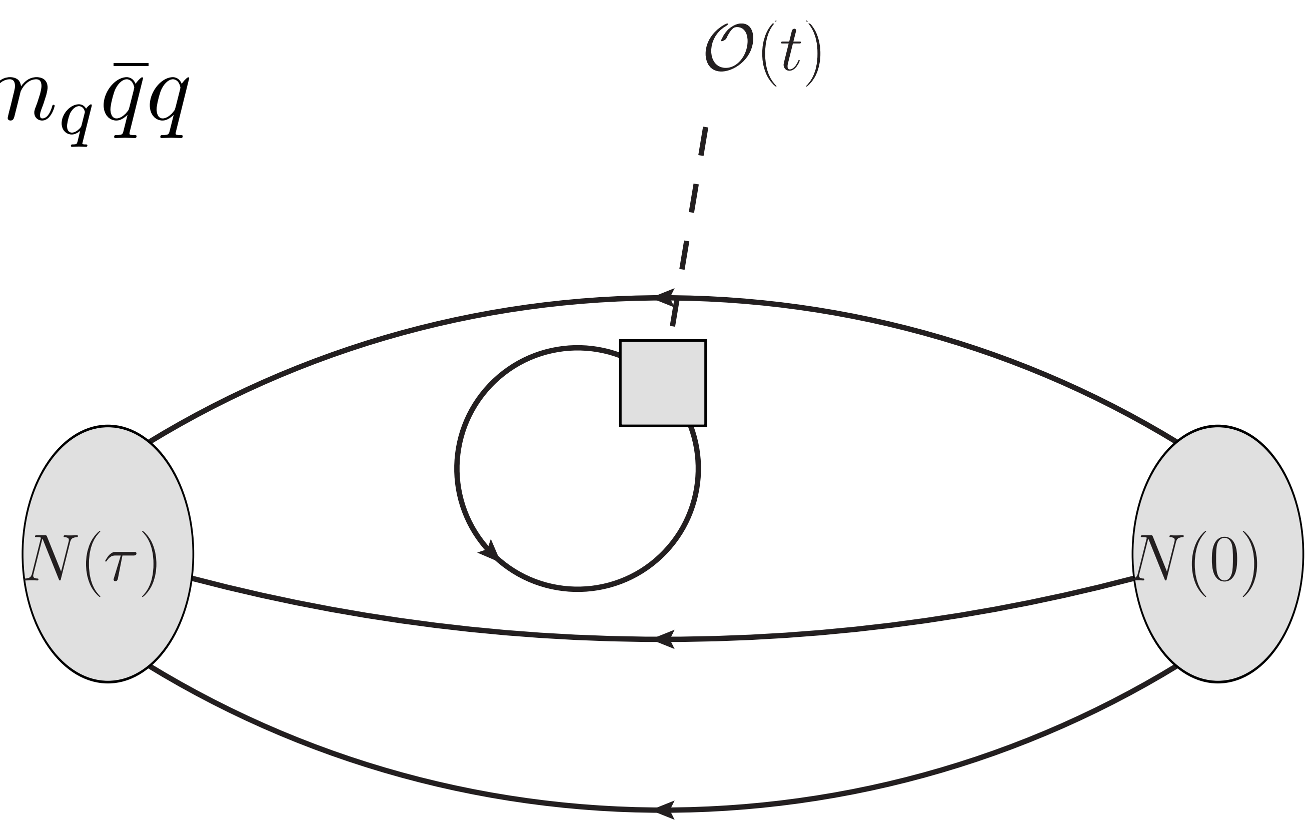
$$\langle N(\tau)|\mathcal{O}(t)|N(0)\rangle = \frac{1}{\mathcal{Z}}\int DU\,D\bar{\psi}D\psi e^{-S}N(\tau)\mathcal{O}(t)N^{\dagger}(0)$$

scalar matrix elements:

- $q = \{u,d\}$ *has connected and disconnected*
- $q = \{s,c\}$ *has only disconnected*



$$\mathcal{O} = m_q \bar{q} q$$



Nucleon Matrix Elements

$$\langle N(\tau)|\mathcal{O}(t)|N(0)\rangle = \frac{1}{\mathcal{Z}}\int DU\, D\bar{\psi} D\psi e^{-S} N(\tau)\mathcal{O}(t) N^\dagger(0)$$

scalar matrix elements: Fortunately, there is an alternate means to compute these matrix elements

Feynman-Hellman Theorem

Nucleon Matrix Elements

Feynman-Hellman Theorem

$$H = H_0 + \lambda H_1 \quad \rightarrow \quad \frac{\partial E_n}{\partial \lambda} = \langle n | \frac{\partial H}{\partial \lambda} | n \rangle$$

In our lattice QCD calculations, we can study the quark mass dependence of the nucleon and infer these matrix elements

$$m_q \frac{\partial m_N}{\partial m_q} = \langle N | m_q \bar{q} q | N \rangle$$

Nucleon Matrix Elements

Feynman-Hellman Theorem

$$m_q \frac{\partial m_N}{\partial m_q} = \langle N | m_q \bar{q} q | N \rangle$$

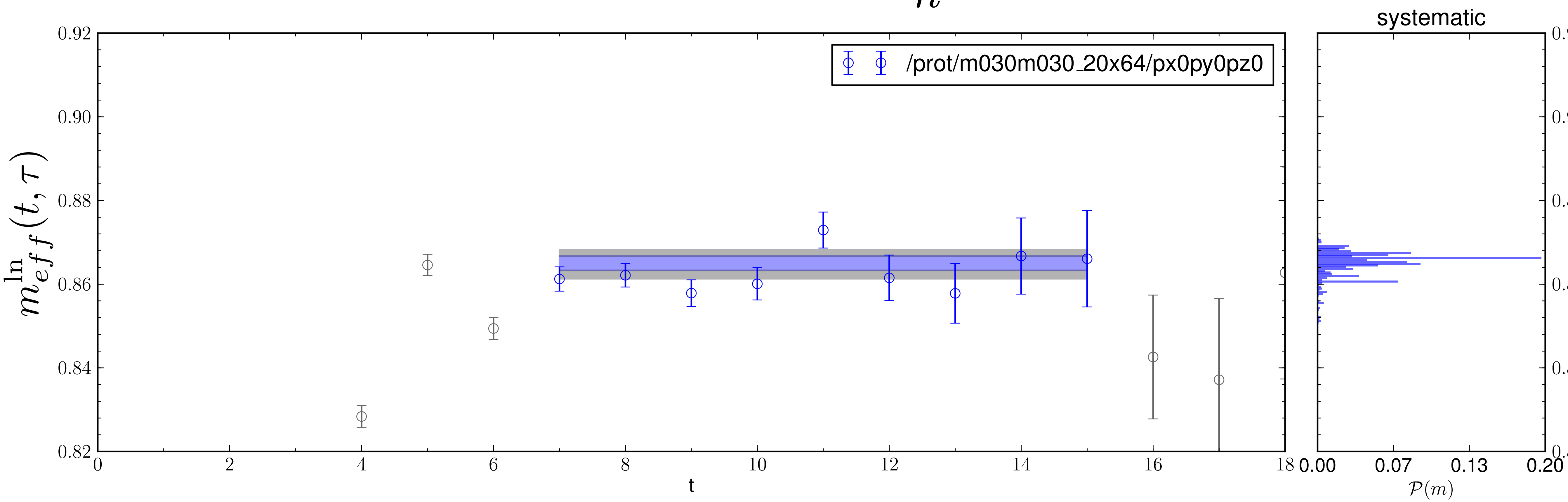
This is important: the thing we can do best with lattice QCD is spectroscopy.

$$\langle 0 | N(t) N^\dagger(0) | 0 \rangle = \sum_n Z_n e^{-E_n t}$$

Nucleon Matrix Elements

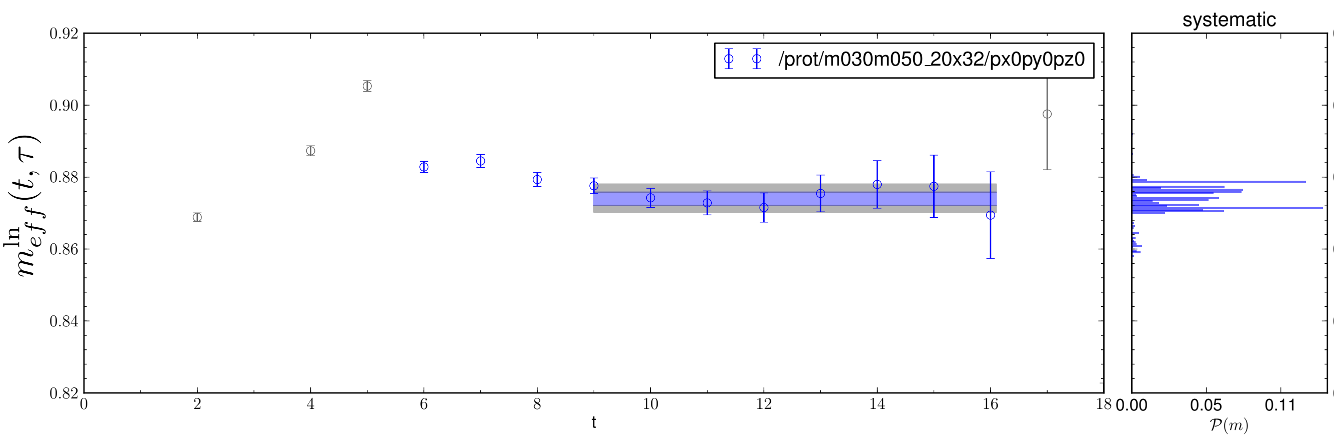
Feynman-Hellman Theorem

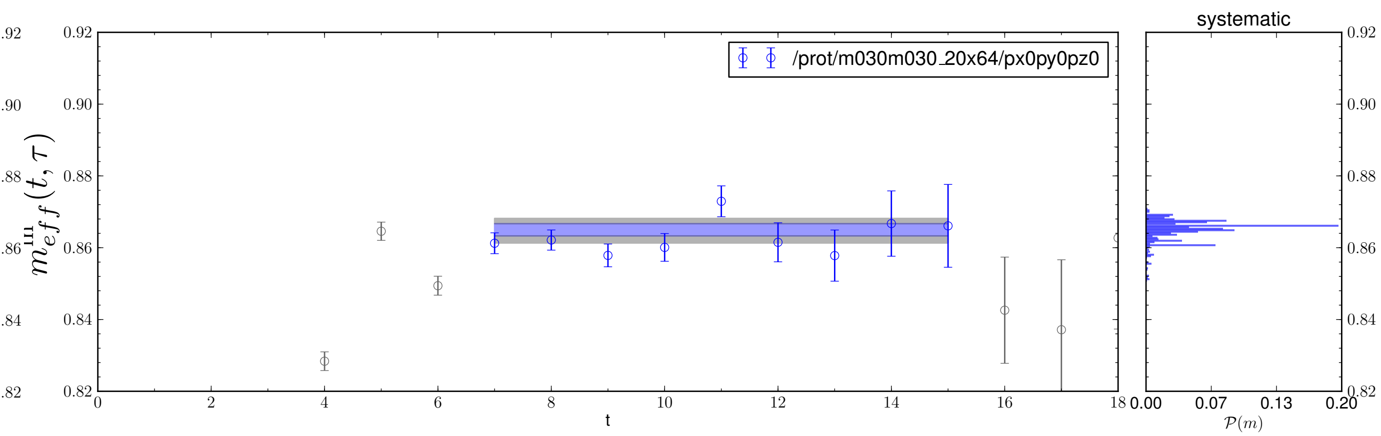
$$\langle 0|N(t)N^\dagger(0)|0\rangle = \sum_n Z_n e^{-E_n t}$$



Nucleon Matrix Elements

Feynman-Hellman Theorem



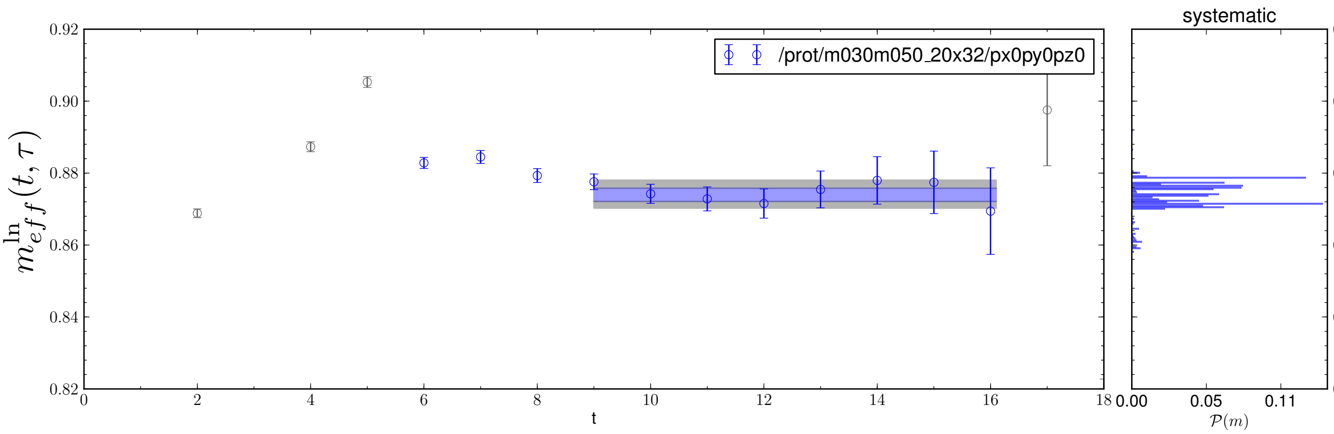


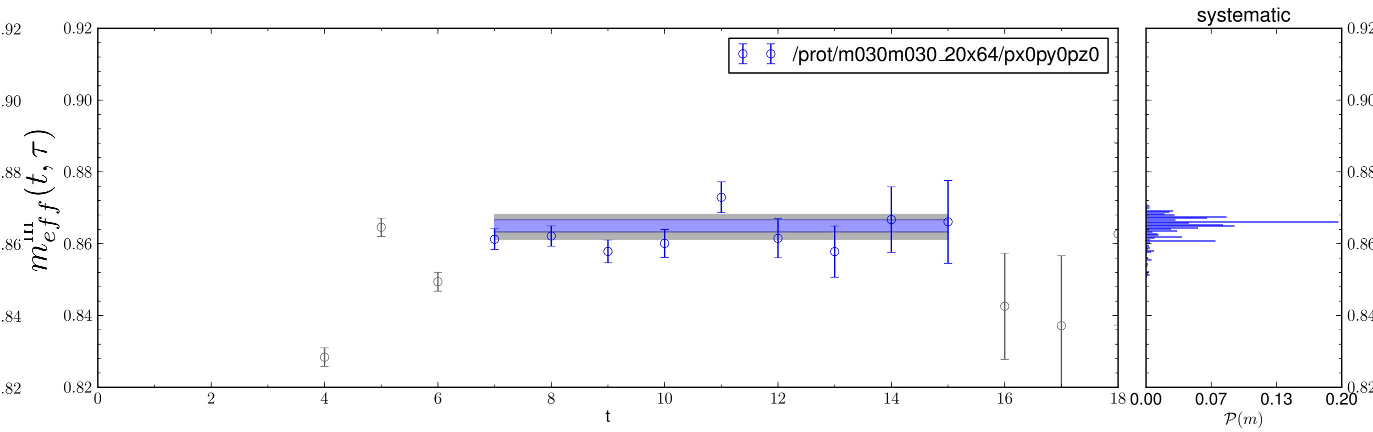
We can perform the lattice calculations with different values of the quark mass (with all other parameters held fixed) and approximate the derivative

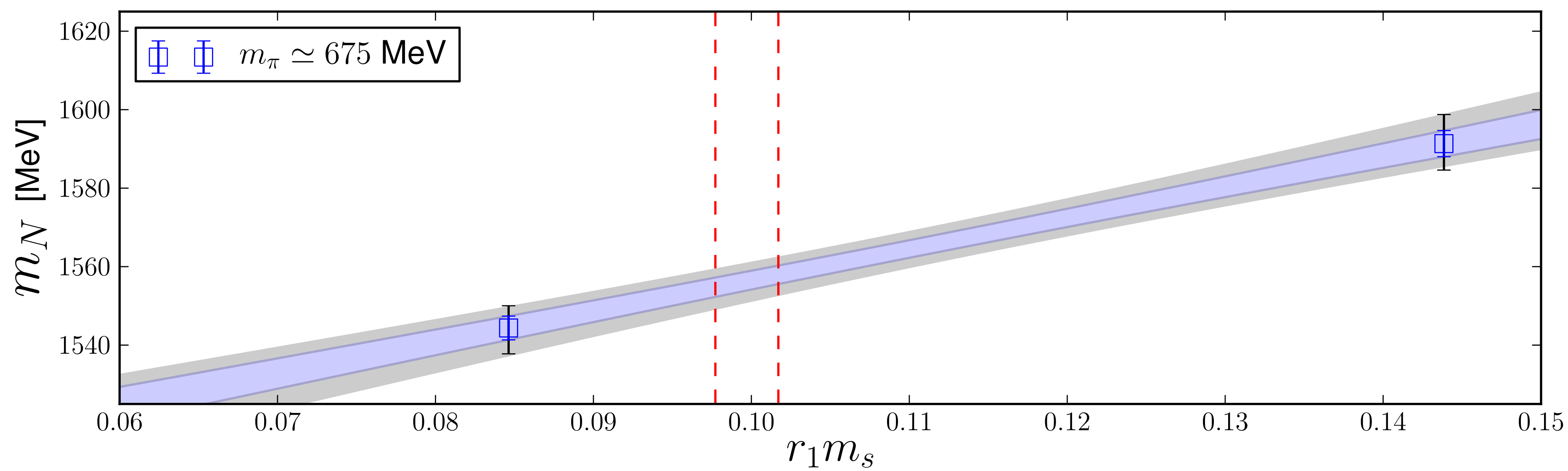
$$m_q\langle N|\bar{q}q|N\rangle[\mathrm{MeV}] = \frac{am_N^{(2)} - am_N^{(1)}}{am_q^{(2)} - am_q^{(1)}} \times am_q \times a^{-1}[\mathrm{MeV}]$$

Nucleon Matrix Elements

Feynman-Hellman Theorem



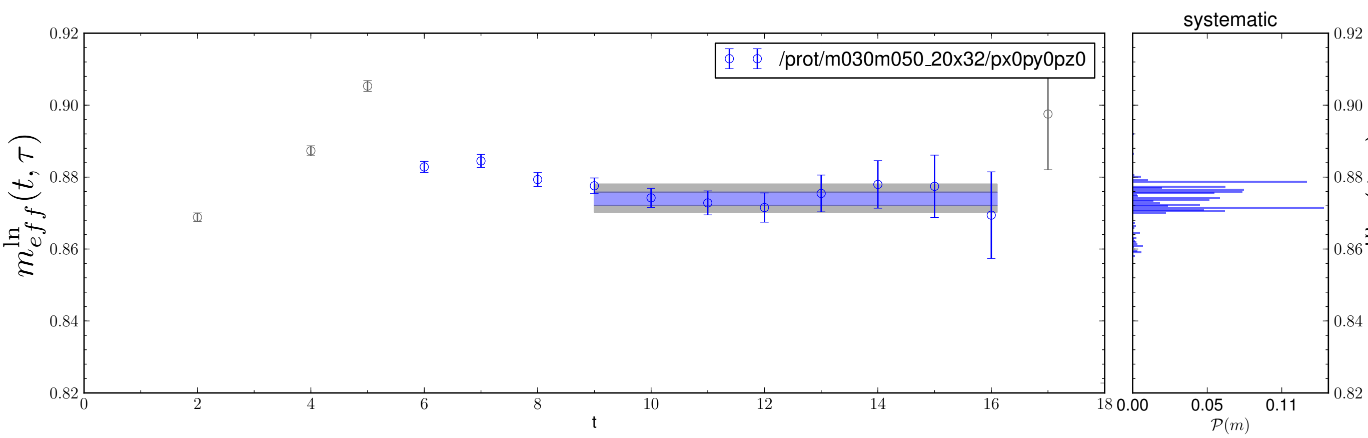


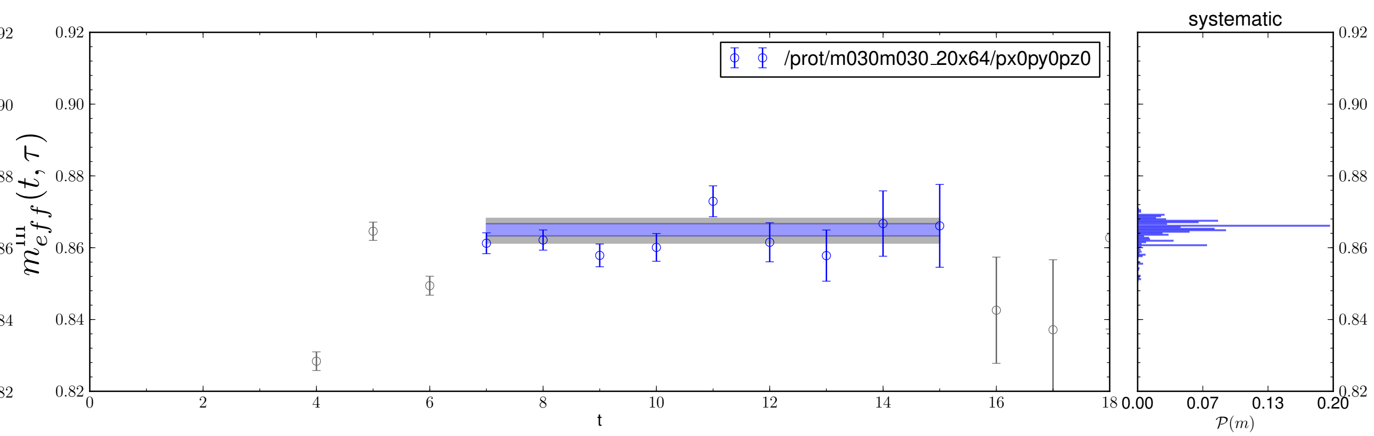


$$m_q \langle N|\bar{q}q|N\rangle[\text{MeV}] = \frac{am_N^{(2)} - am_N^{(1)}}{am_q^{(2)} - am_q^{(1)}} \times am_q \times a^{-1}[\text{MeV}]$$

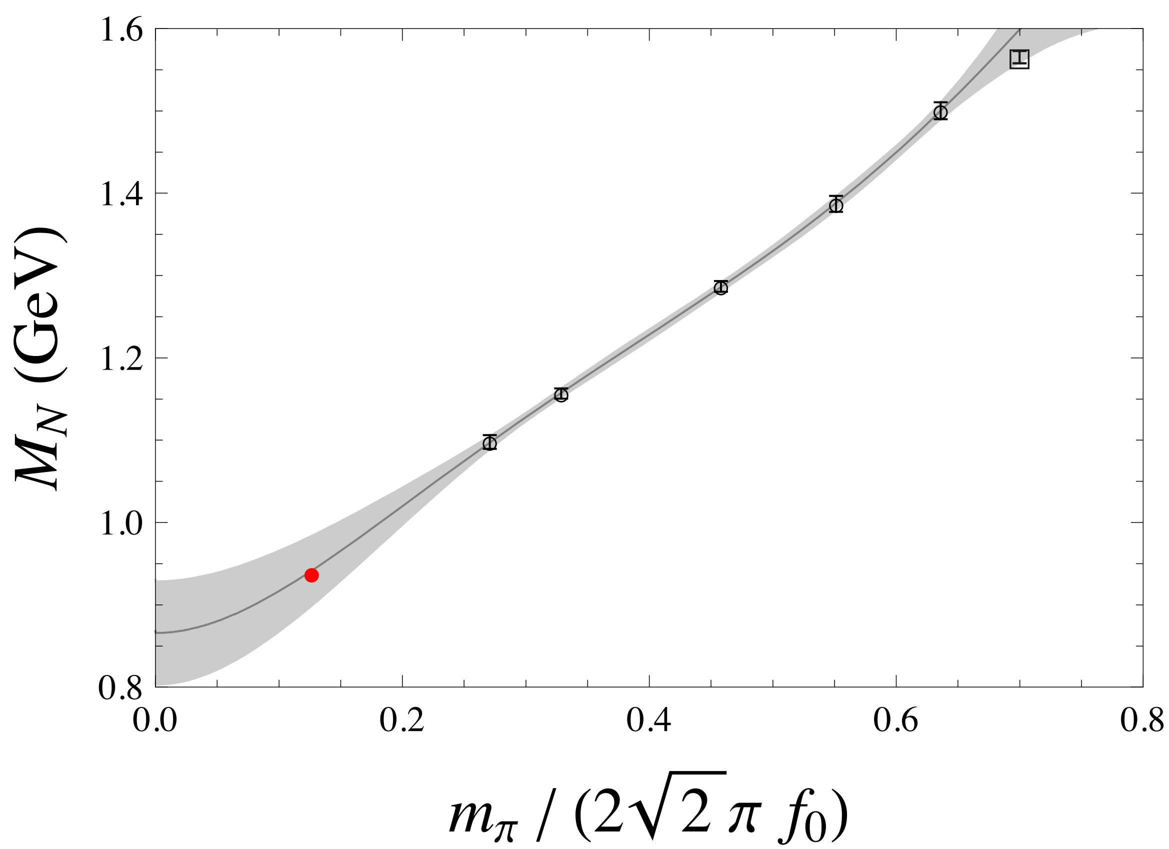
Nucleon Matrix Elements

Feynman-Hellman Theorem





NNLO – m_π^4, with g_A=1.2(1), $g_{\Delta N}$=1.5(3)



Or fit the light quark mass dependence and determine the derivative analytically

Light quark matrix element

$$\sigma_{\pi N} = \frac{m_u + m_d}{2}\langle N|\bar{u}u + \bar{d}d|N\rangle$$

$$= m_l \frac{\partial m_N}{\partial m_l} \qquad (2m_l = m_u + m_d)$$

$$\simeq \frac{m_\pi}{2}\frac{\partial m_N}{\partial m_\pi}$$

$$\sigma_{\pi N}[\mathrm{MeV}] = 45 \pm 5$$

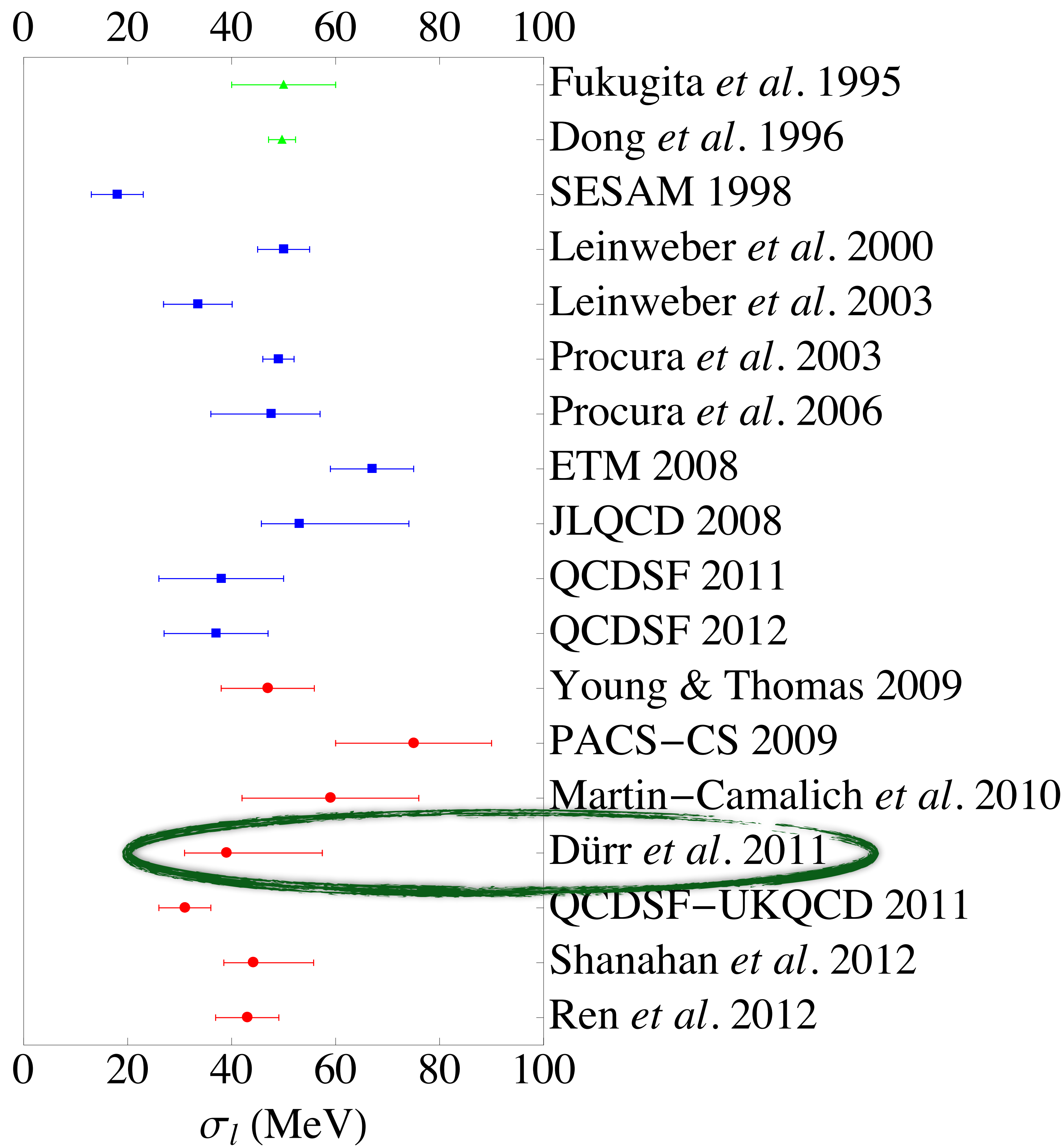
$$(30 - 80)$$

How believable is this value?

R. Young Lattice 2012
arXiv:1301.1765

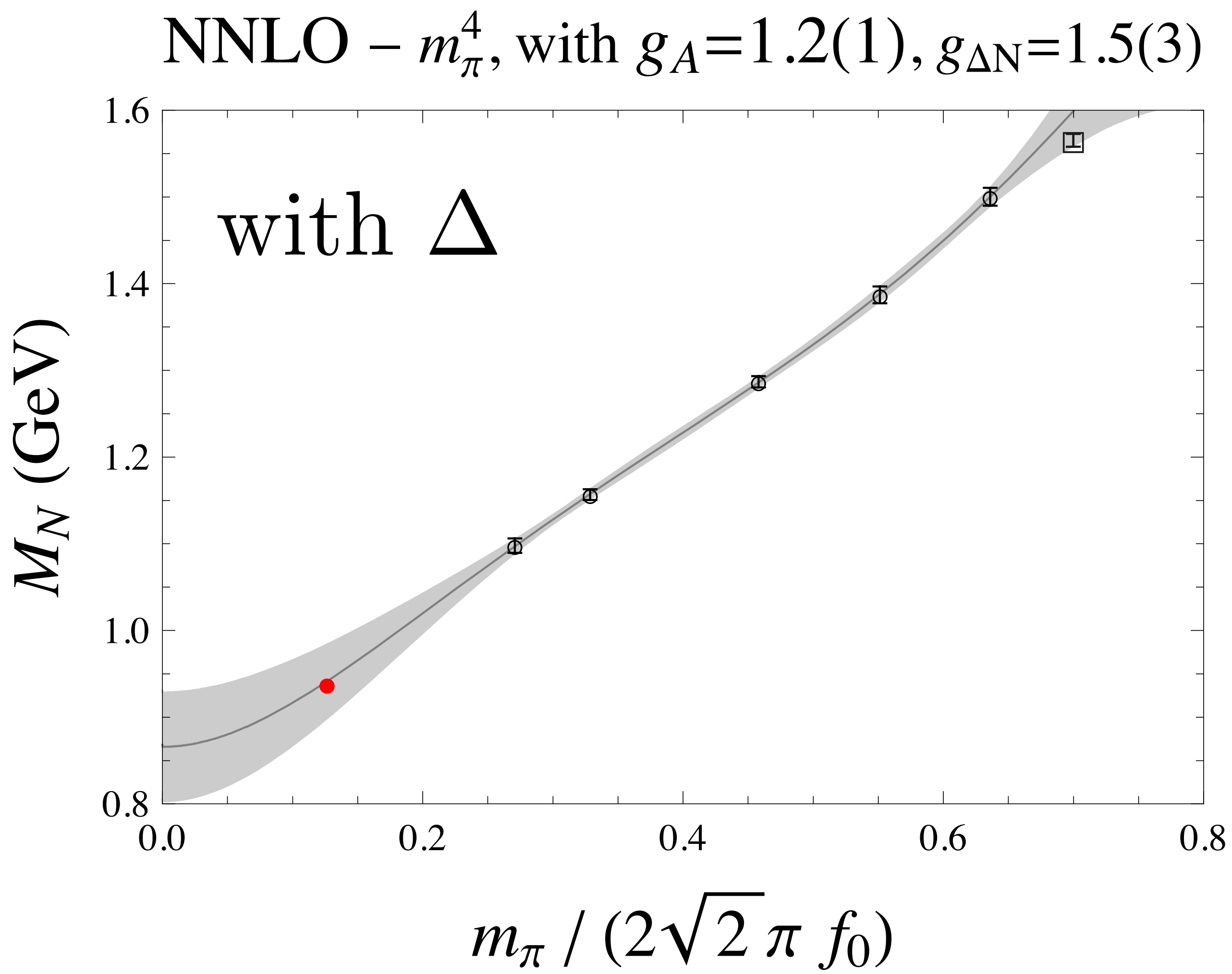
$$\sigma_{\pi N} = m_l \langle N | \bar{u}u + \bar{d}d | N \rangle$$

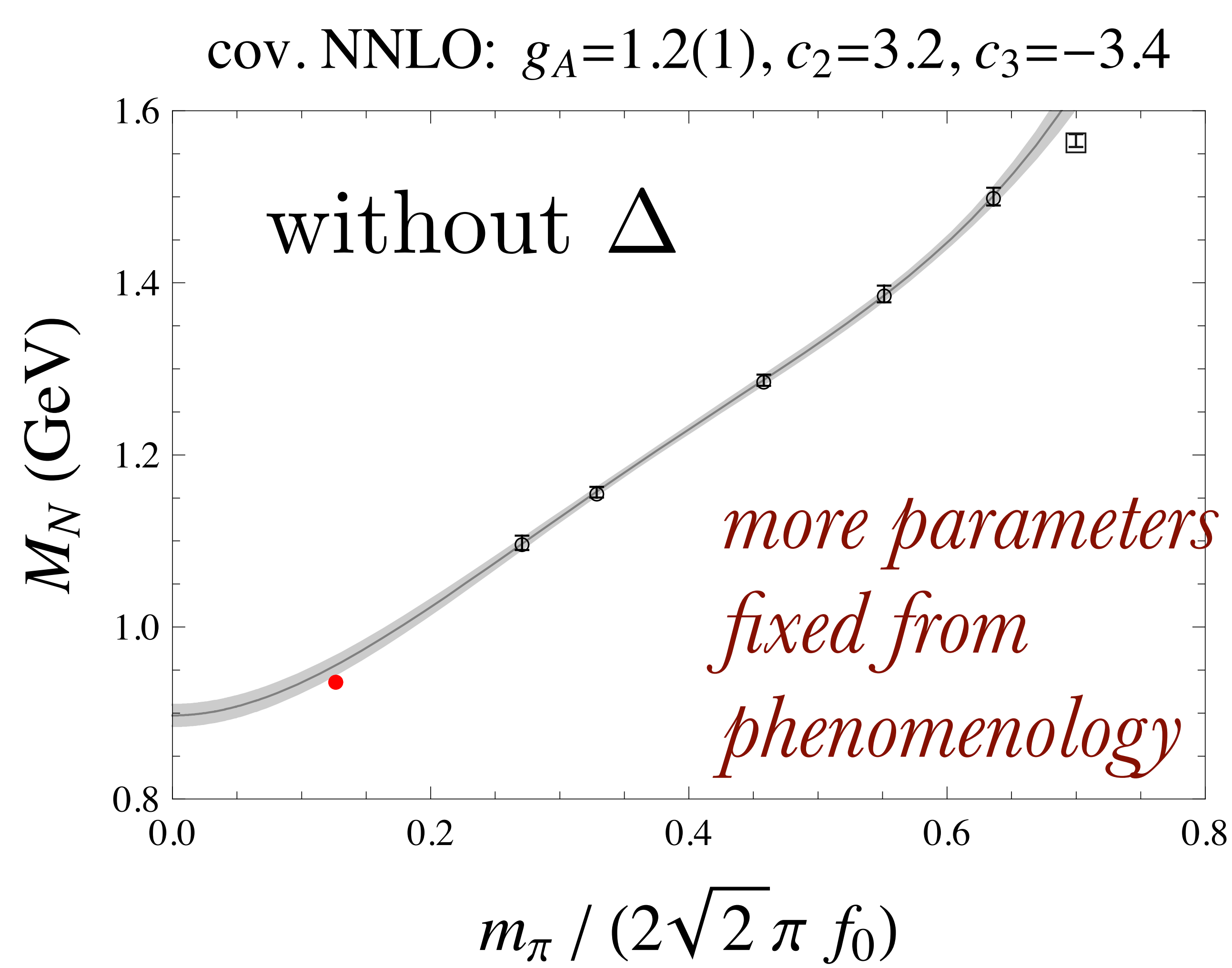
The only green value (from FLAG standards)



These colors have nothing to do with FLAG [taken from R Young review]

Light quark matrix element





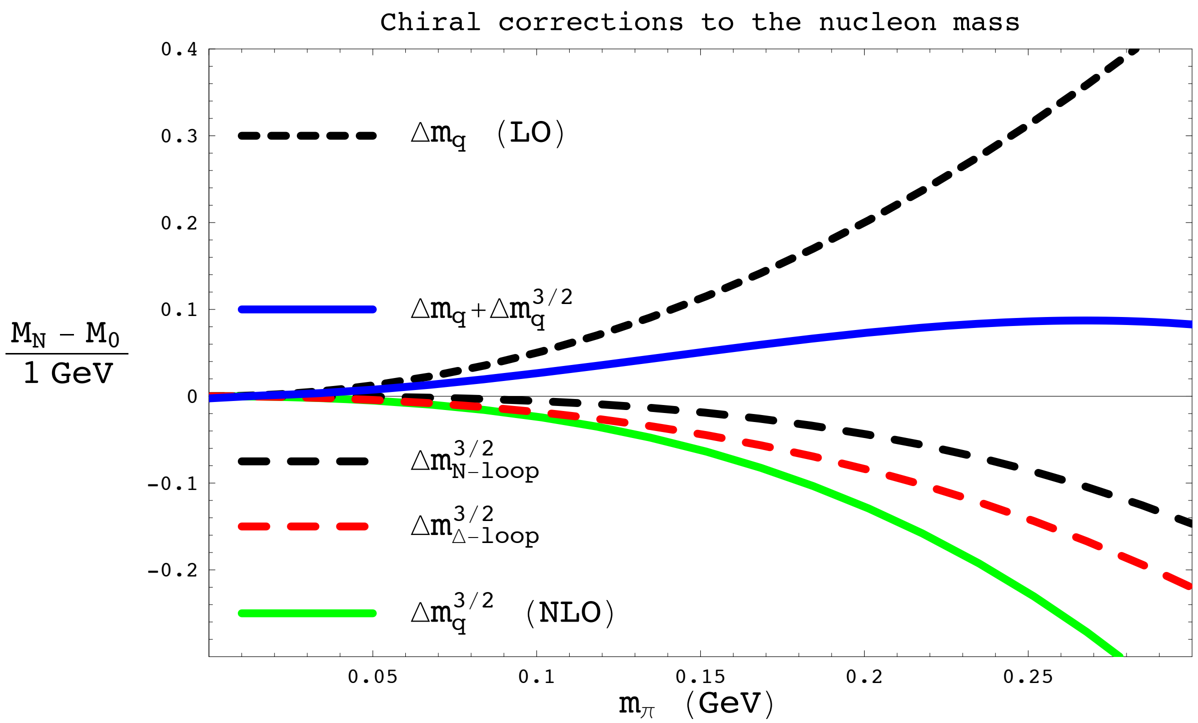
$$M_N = 941 \pm 42 \text{ MeV}$$
$$\sigma_{\pi N} = 84 \pm 17 \text{ MeV}$$

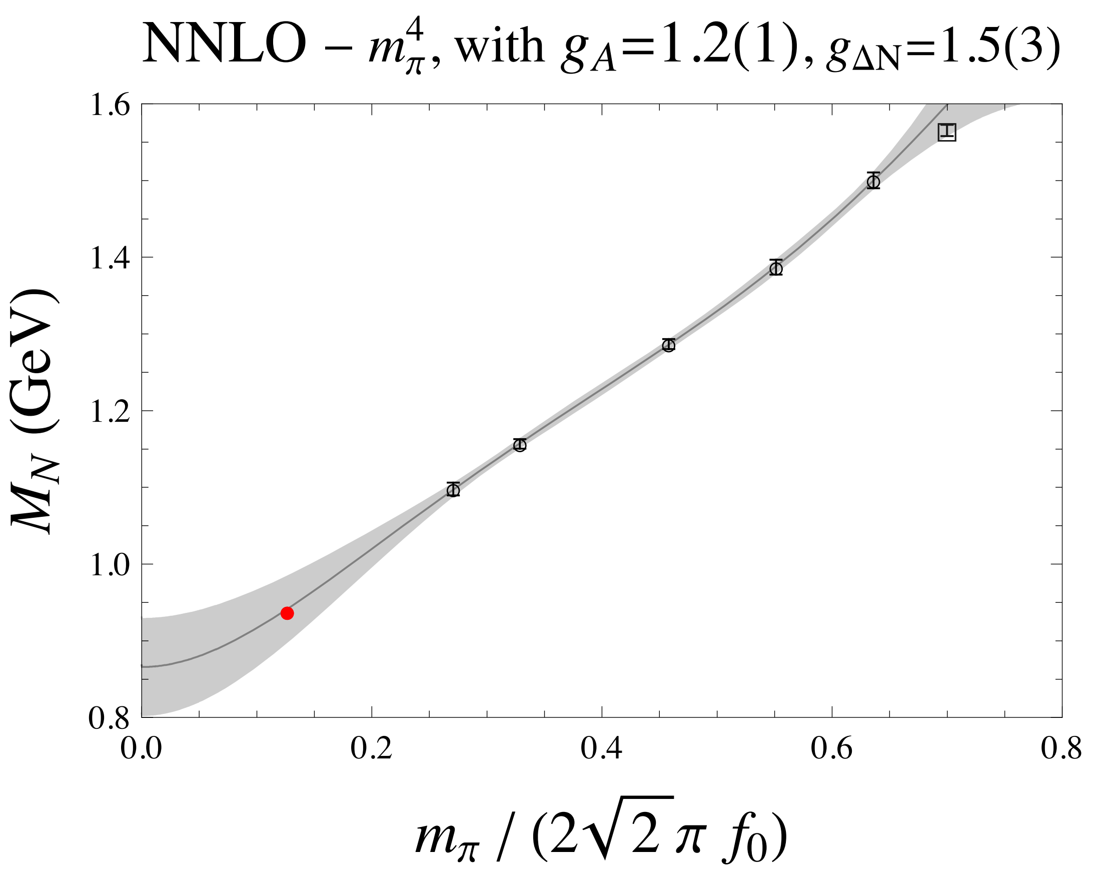
$$M_N = 956 \pm 12 \text{ MeV}$$
$$\sigma_{\pi N} = 42 \pm 14 \text{ MeV}$$

(statistical uncertainties only)

Light quark matrix element

$$M_N(m_\pi,\Delta) = M_0 + \sigma_N(\mu)\frac{m_\pi^2}{4\pi f_\pi} - 3\pi g_{\pi NN}^2\frac{m_\pi^3}{(4\pi f_\pi)^2} - \frac{8}{3}g_{\pi N\Delta}^2\frac{\mathcal{F}(m_\pi,\Delta,\mu)}{(4\pi f_\pi)^2} + \mathcal{O}\left(\frac{m_\pi^4}{(4\pi f_\pi)^3}\right)$$



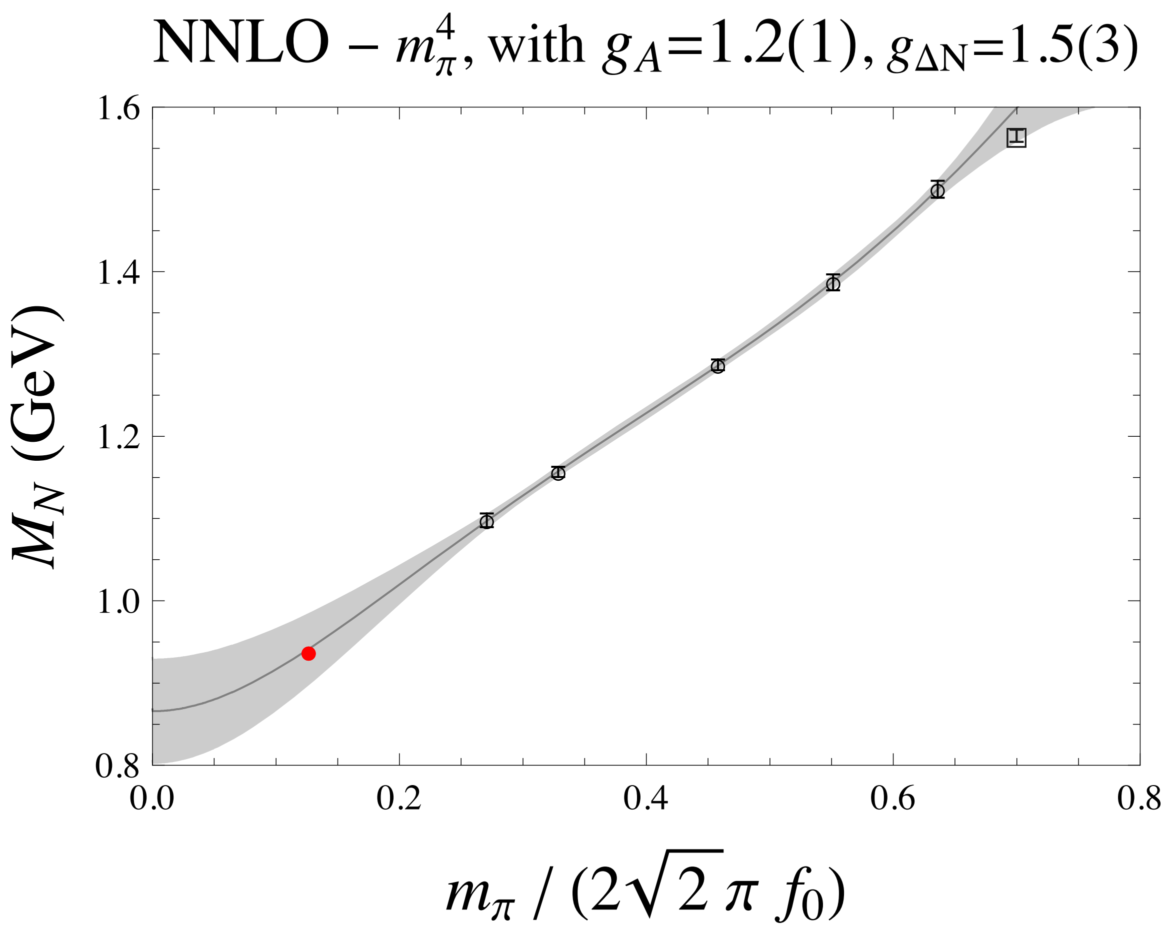


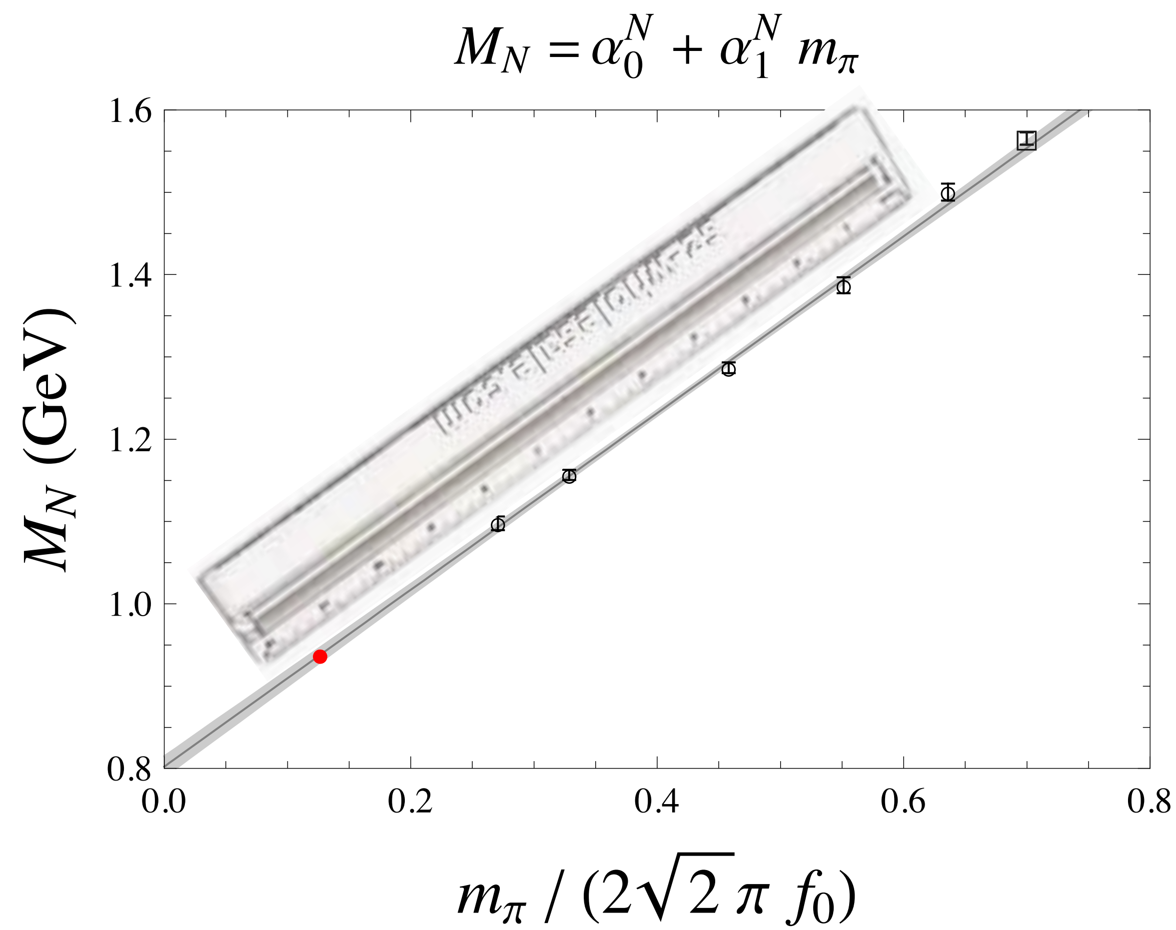
for the nucleon mass, the delta loop a little more than doubles the nucleon loop.

the nucleon masses rises uniformly as a function of the pion mass

including explicit delta dof forces the leading term to be larger to compensate the fit

Light quark mass dependence of M_N





NNLO Heavy Baryon Fit

$$M_N = 941 \pm 42 \pm 17 \text{ MeV}$$

Ruler Approximation

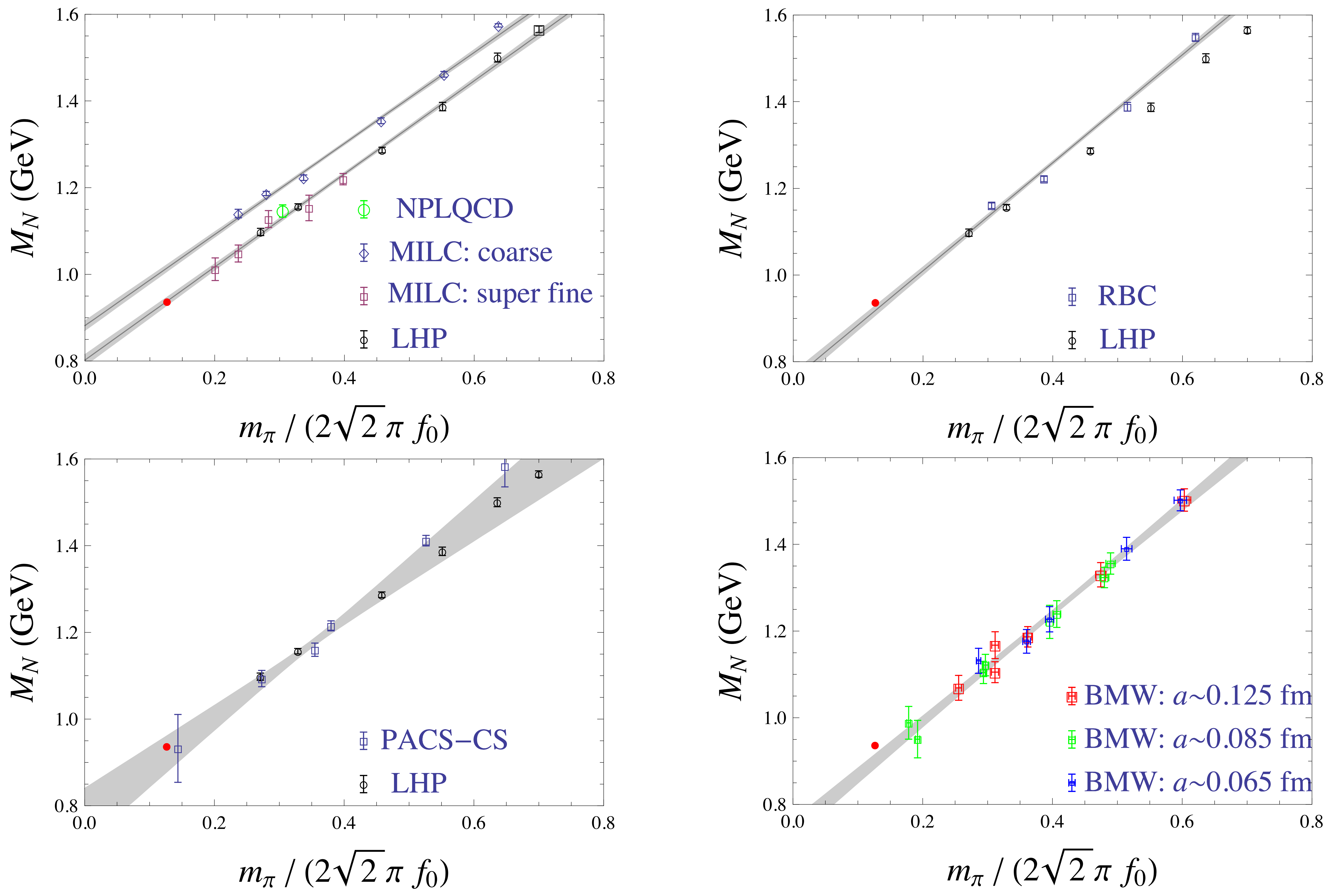
$$M_N = \alpha_0^N + \alpha_1^N m_\pi$$
$$= 938 \pm 9 \text{ MeV}$$

I am not advocating this as a good model for QCD!

LHP Collaboration arXiv:0806.4549

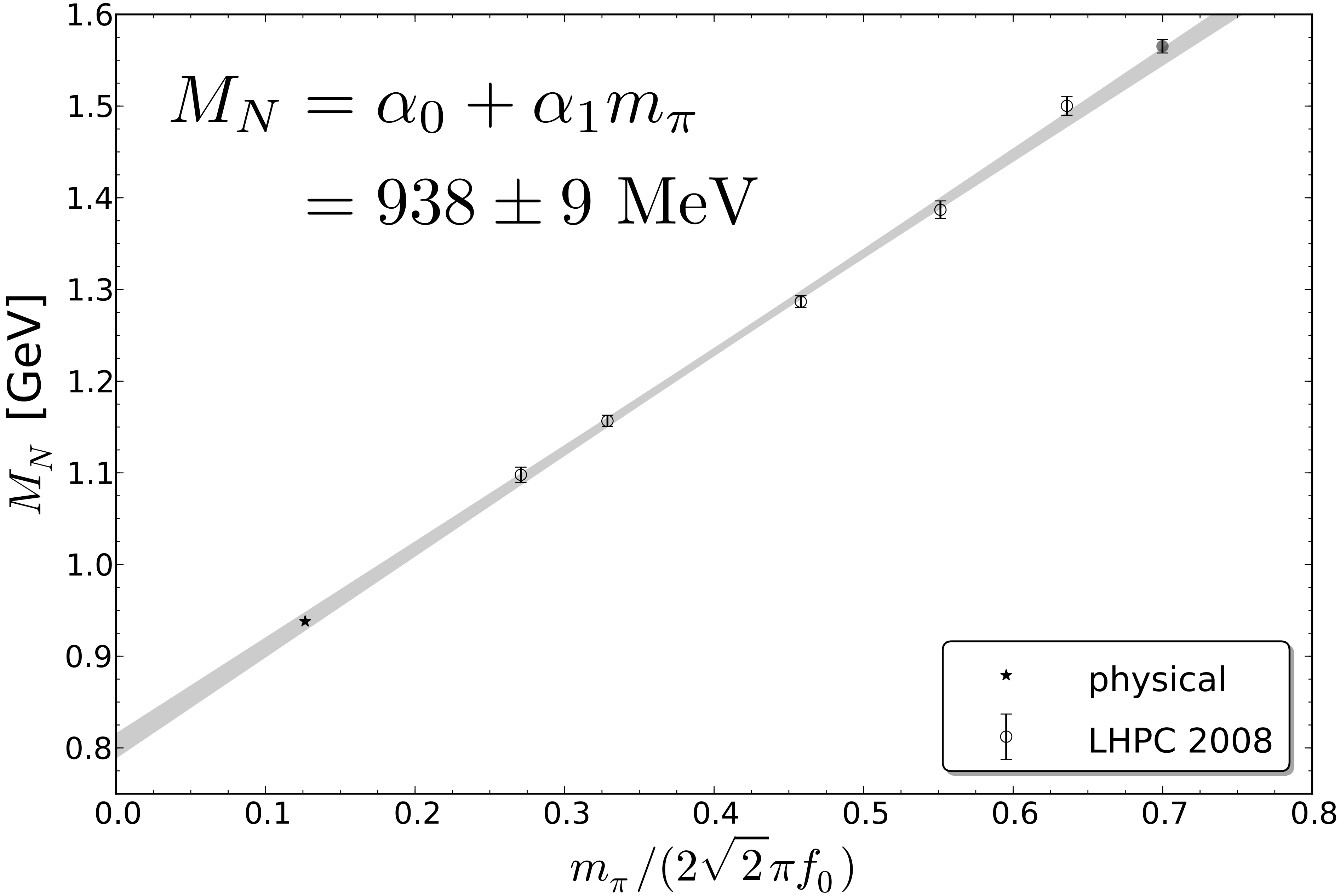
Baryons in lattice QCD — Light quark mass dependence of M_N

MILC: $M_N = \alpha_0^N + \alpha_1^N m_\pi$ — Latt 2008, arXiv:0810.0663 — $M_N = \alpha_0^N + \alpha_1^N m_\pi$



Chiral Dynamics
2012 arXiv:1304.6341

What is the status now (2012)?

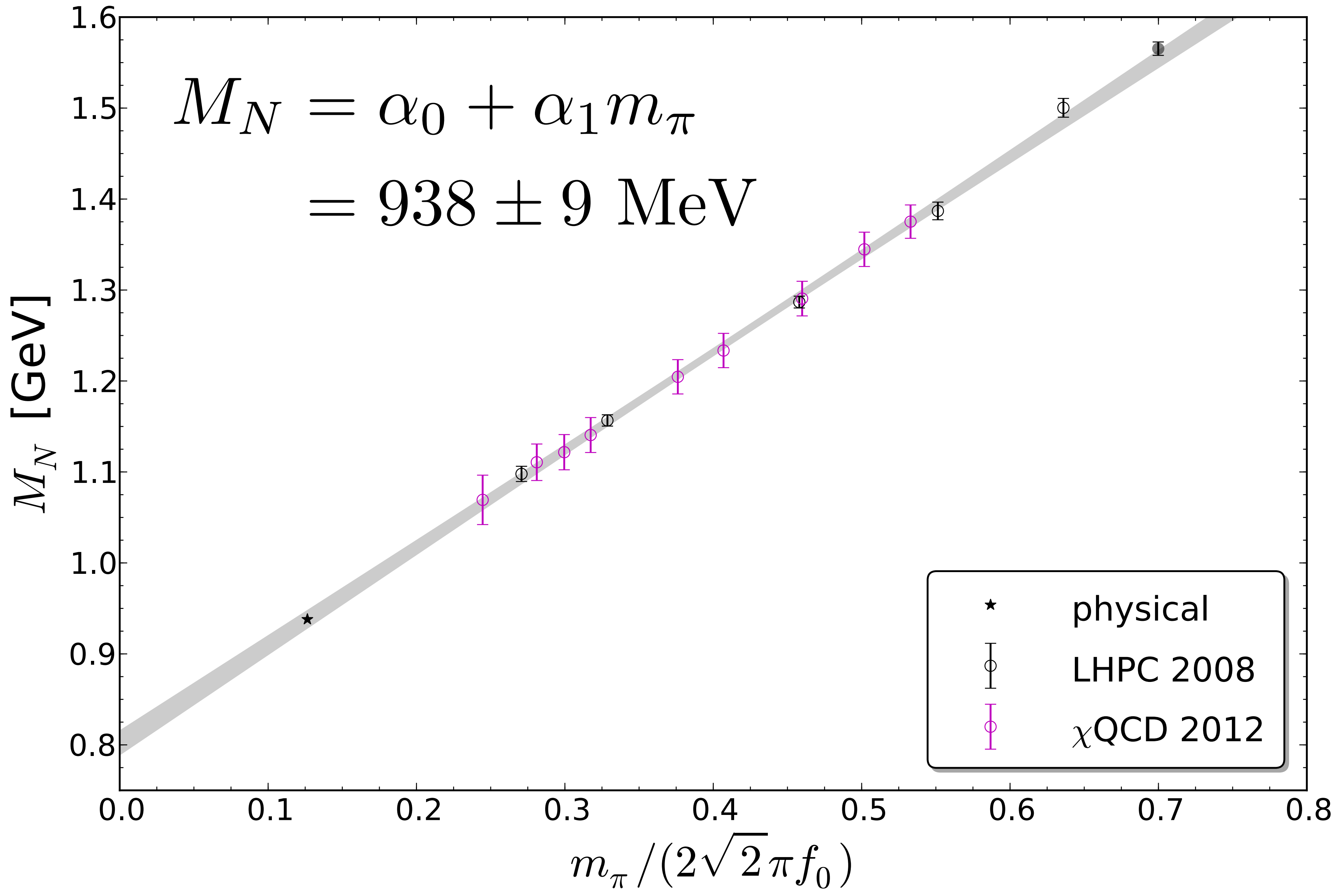


Physical point NOT included in fit

Baryons in lattice QCD — Light quark mass dependence of M_N

Chiral Dynamics 2012 arXiv:1304.6341

What is the status now (2012)?



χQCD Collaboration uses Overlap Valence fermions on Domain-Wall (RBC-UKQCD) sea fermions

Chiral Dynamics 2012 arXiv:1304.6341

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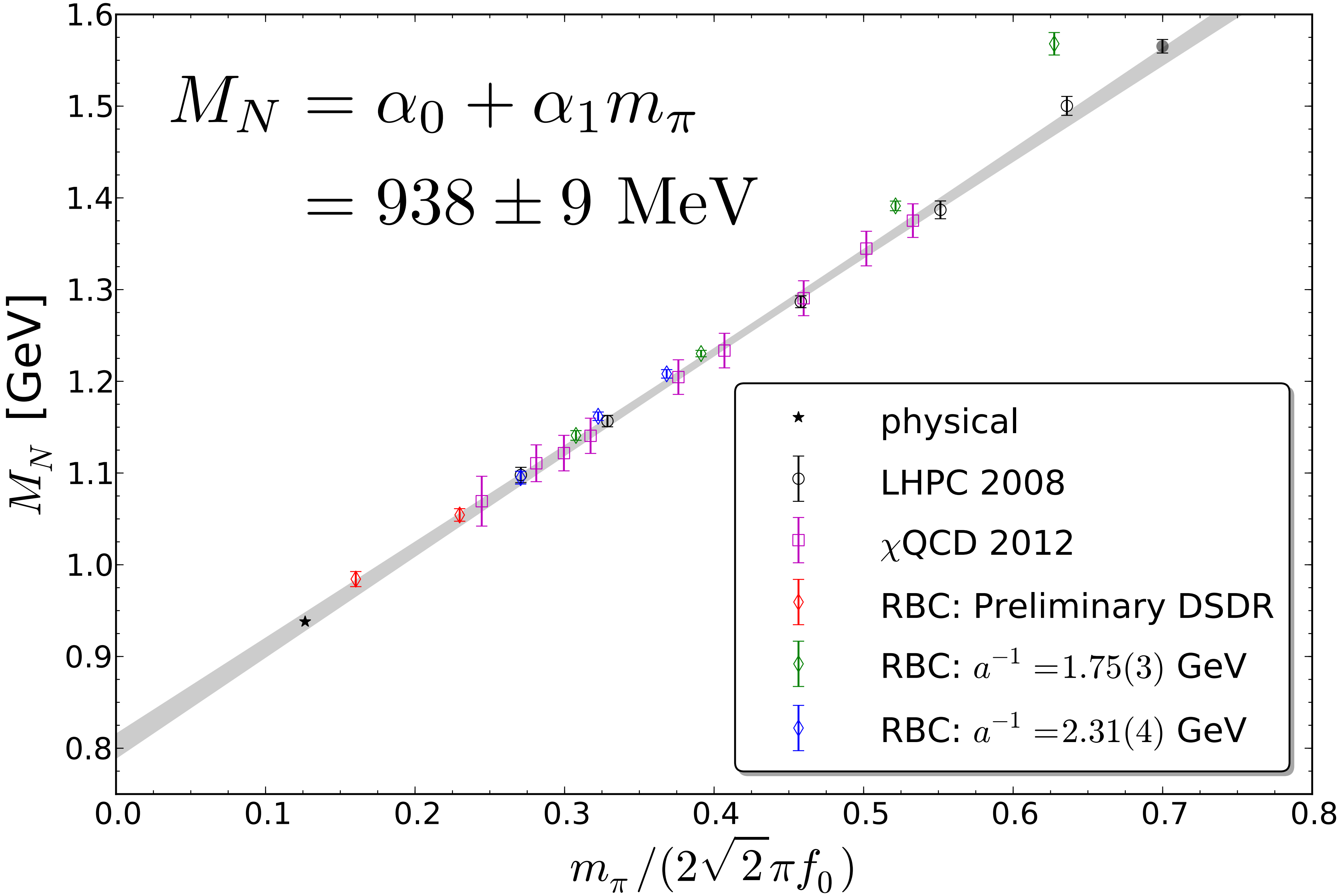
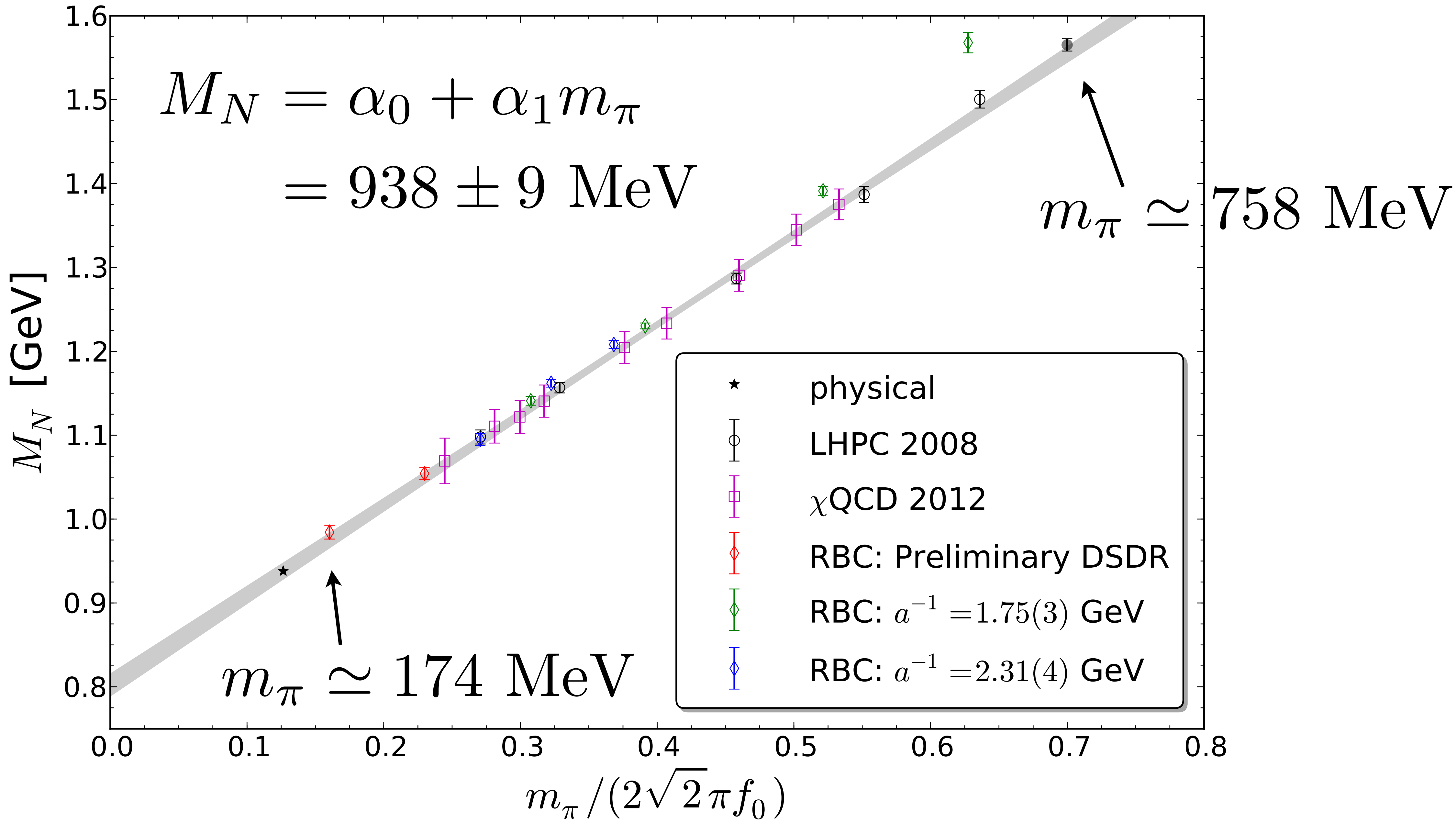


RBC-UKQCD Collaboration uses Domain-Wall valence and sea fermions

Light quark mass dependence of M_N

Chiral Dynamics
2012 arXiv:1304.6341

What is the status now (2012)?



Taking this seriously yields

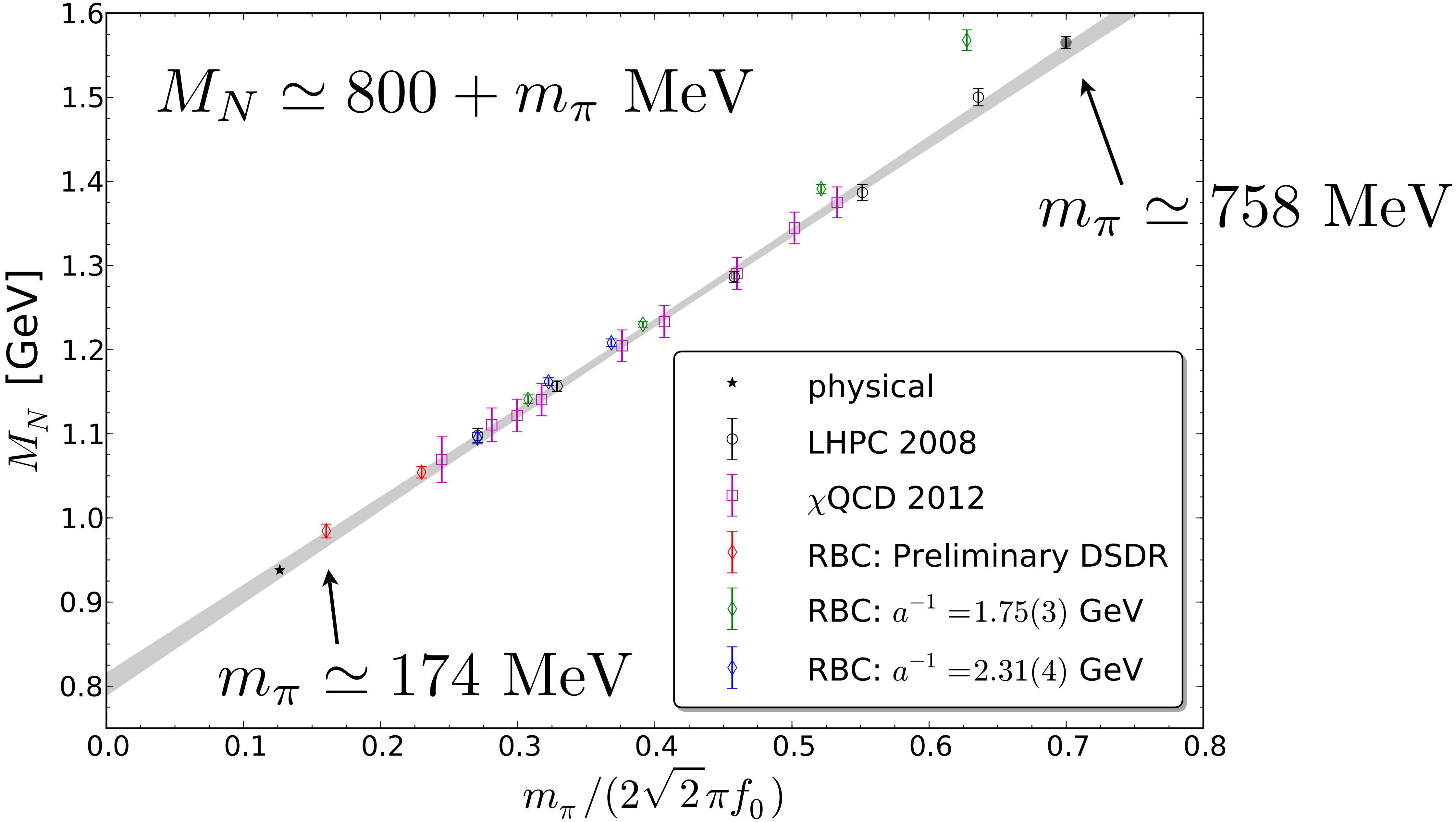
$$\sigma_{\pi N} = 67 \pm 4 \text{ MeV}$$

I am not advocating this as a good model for QCD!

Light quark mass dependence of M_N

Chiral Dynamics 2012 arXiv:1304.6341

What is the status now (2012)?



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Light quark matrix element

There is a larger uncertainty in the light-quark scalar matrix element in the nucleon than is often appreciated - at least the determination from lattice QCD

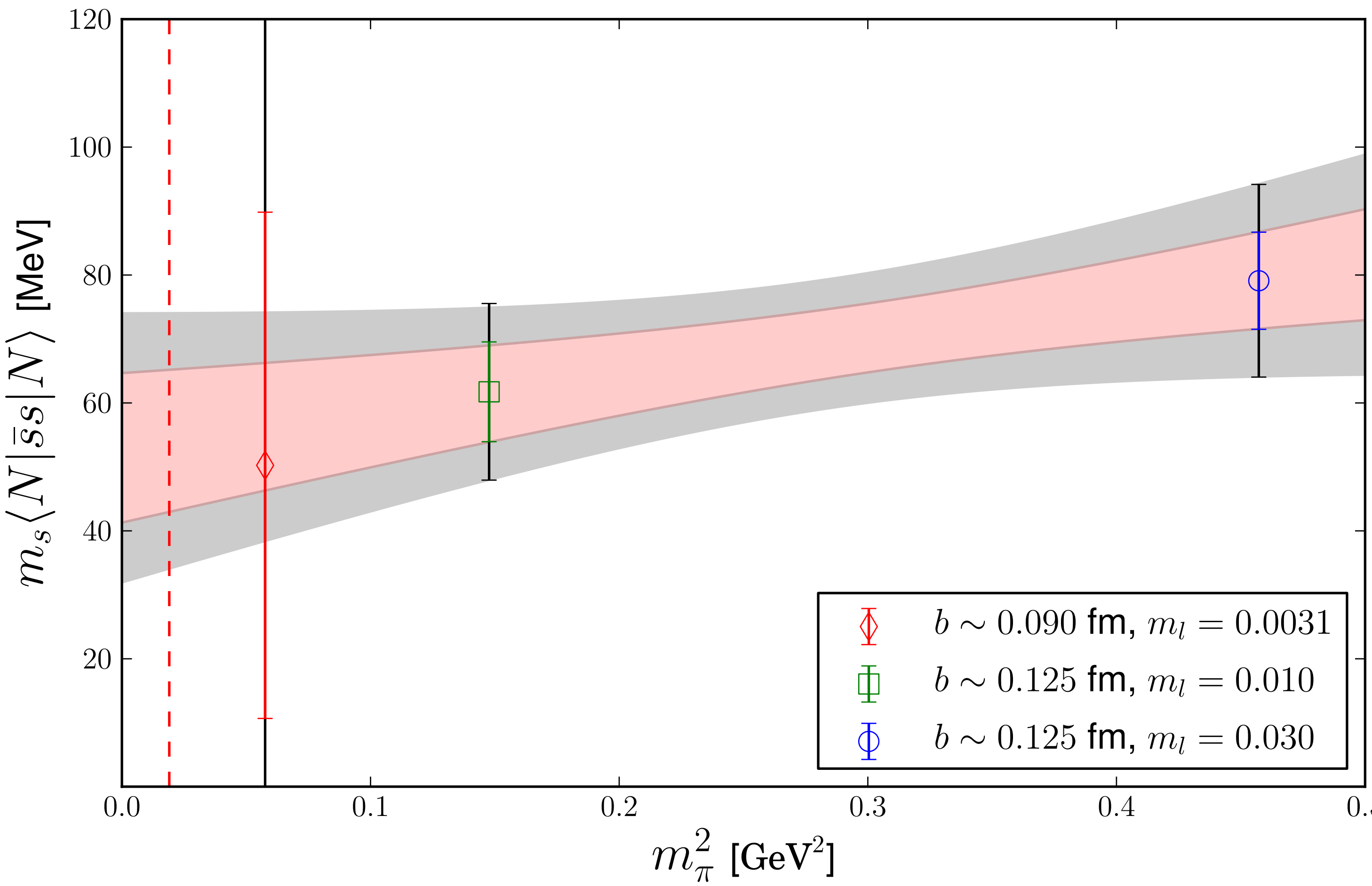
$$30 \lesssim \sigma_{\pi N} \lesssim 70 \text{ MeV}$$

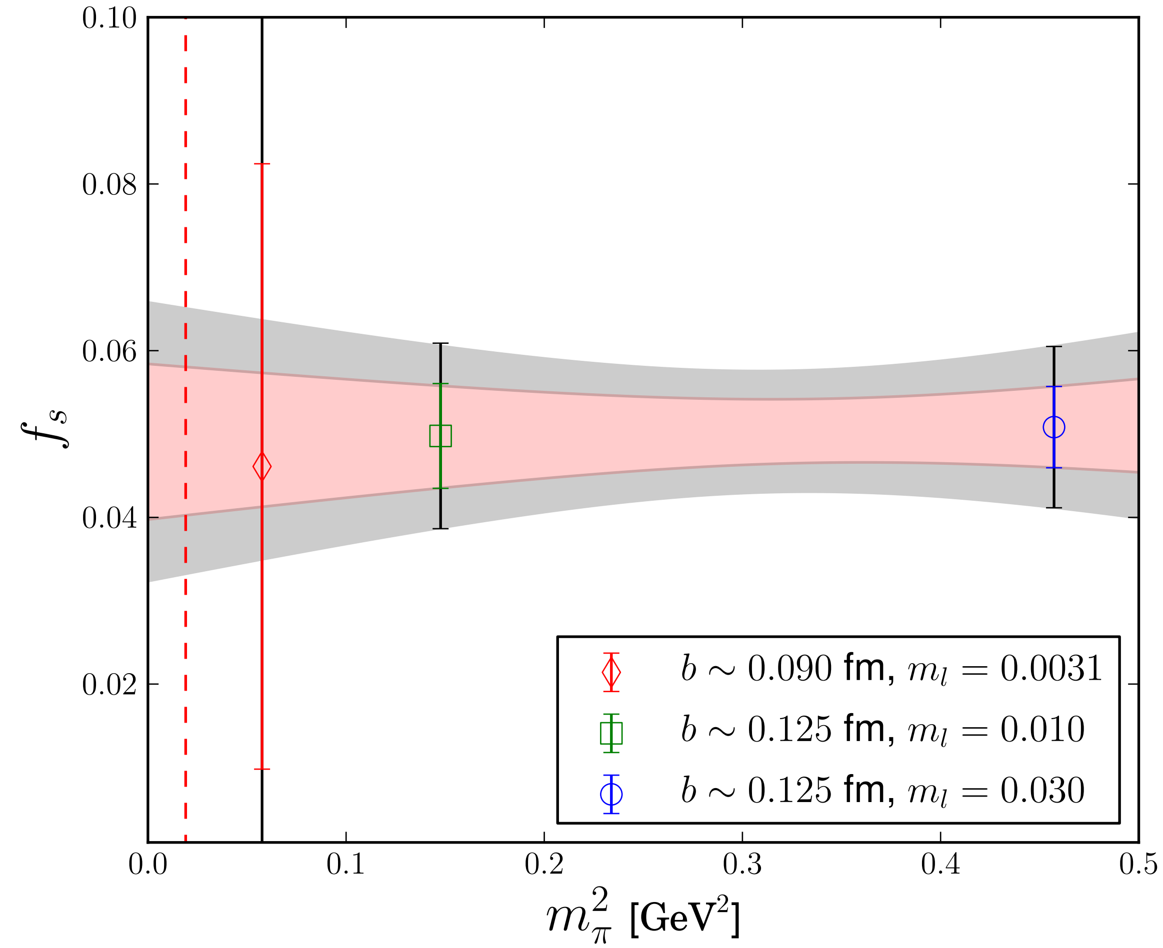
(conservatively)

See talk by Martin Hoferichter for phenomenological determination

strange content of the nucleon

P. Junnarkar and AWL
PRD 87 (2013)

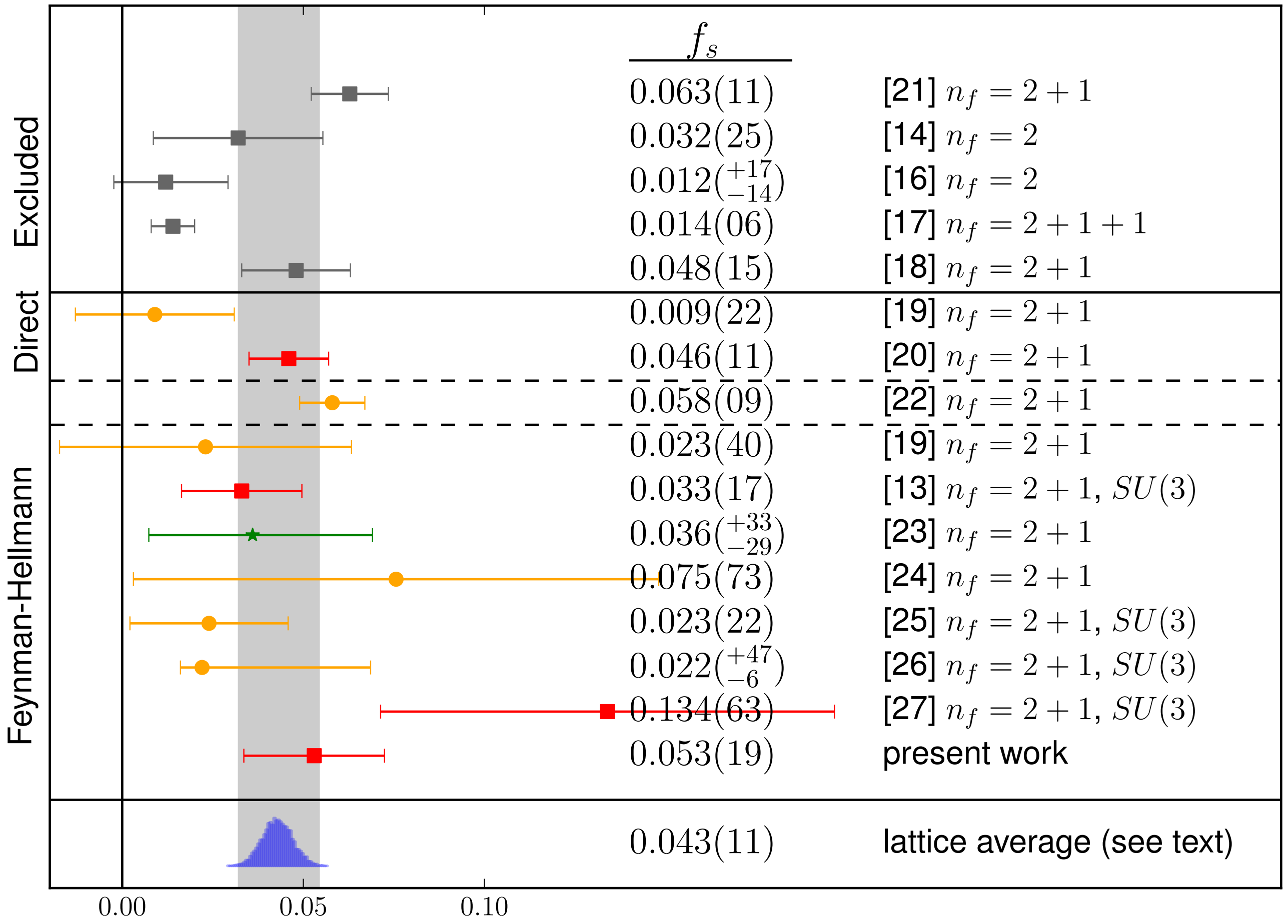




Nucleon strange content seems to have very mild light-quark mass dependence

strange content of the nucleon

P. Junnarkar and AWL
PRD 87 (2013)



$$f_s = m_s \langle N|\bar{s}s|N\rangle / m_N$$

colors inspired by FLAG

good ok bad

absence of many good results leads one to consider all results

but we don't want to treat all results as equal...

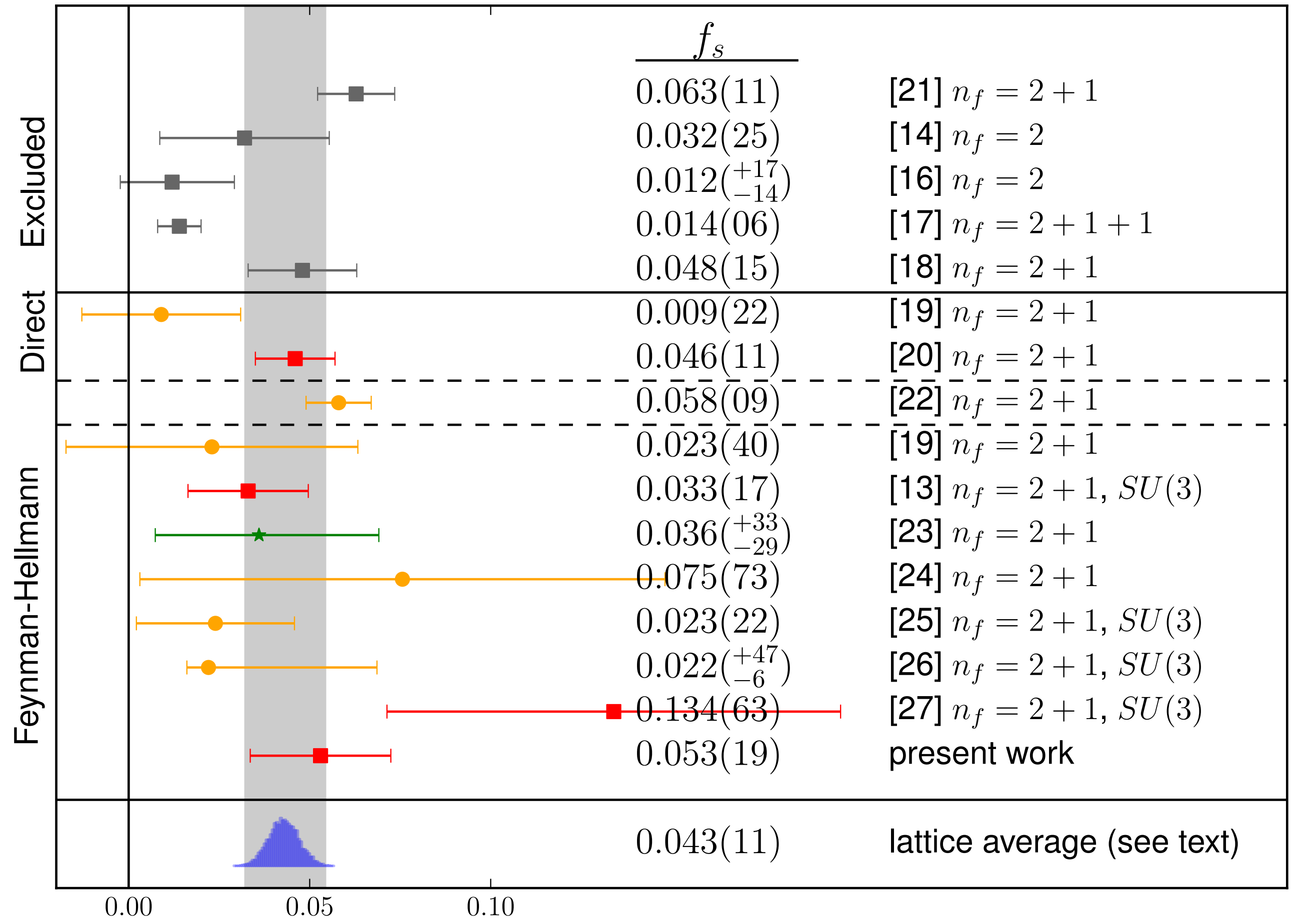
in fact, there is a second layer of badness, reliance upon SU(3) baryon Chiral Perturbation Theory

SU(3) baryon Chiral Perturbation Theory is NOT quantitatively reliable

AWL with LHPC PRD79 (2009)
PACS-CS PRD80 (2009)
AWL with NPLQCD PRD81 (2010)

strange content of the nucleon

P. Junnarkar and AWL
PRD 87 (2013)



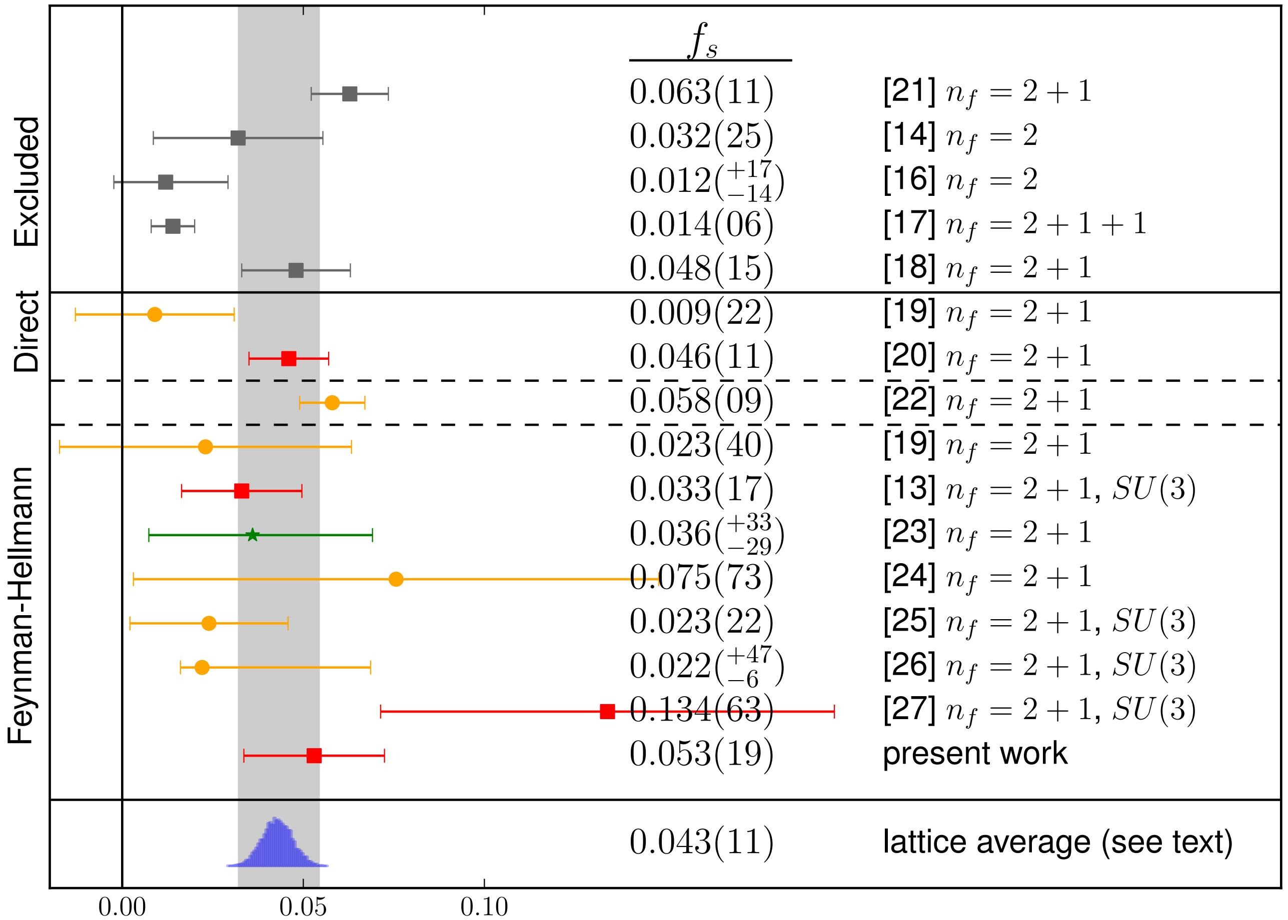
$$f_s = m_s\langle N|\bar{s}s|N\rangle/m_N$$

all numbers included have been extrapolated to the physical pion mass

excluded numbers were:
- improved and included
- not extrapolated to physical pion mass
- only u,d dynamical quarks

strange content of the nucleon

P. Junnarkar and AWL
PRD 87 (2013)



$$f_s = m_s \langle N|\bar{s}s|N\rangle / m_N$$

simple weighted average produces unbelievably small uncertainty

also - these results tend to be dominated by systematic uncertainties, which are not Gaussian

i) For each lattice result, generate random, uniform distribution

$$\text{uniform}(f_i - \sigma_i^-, f_i + \sigma_i^+, N_{dist} = 10^4)$$

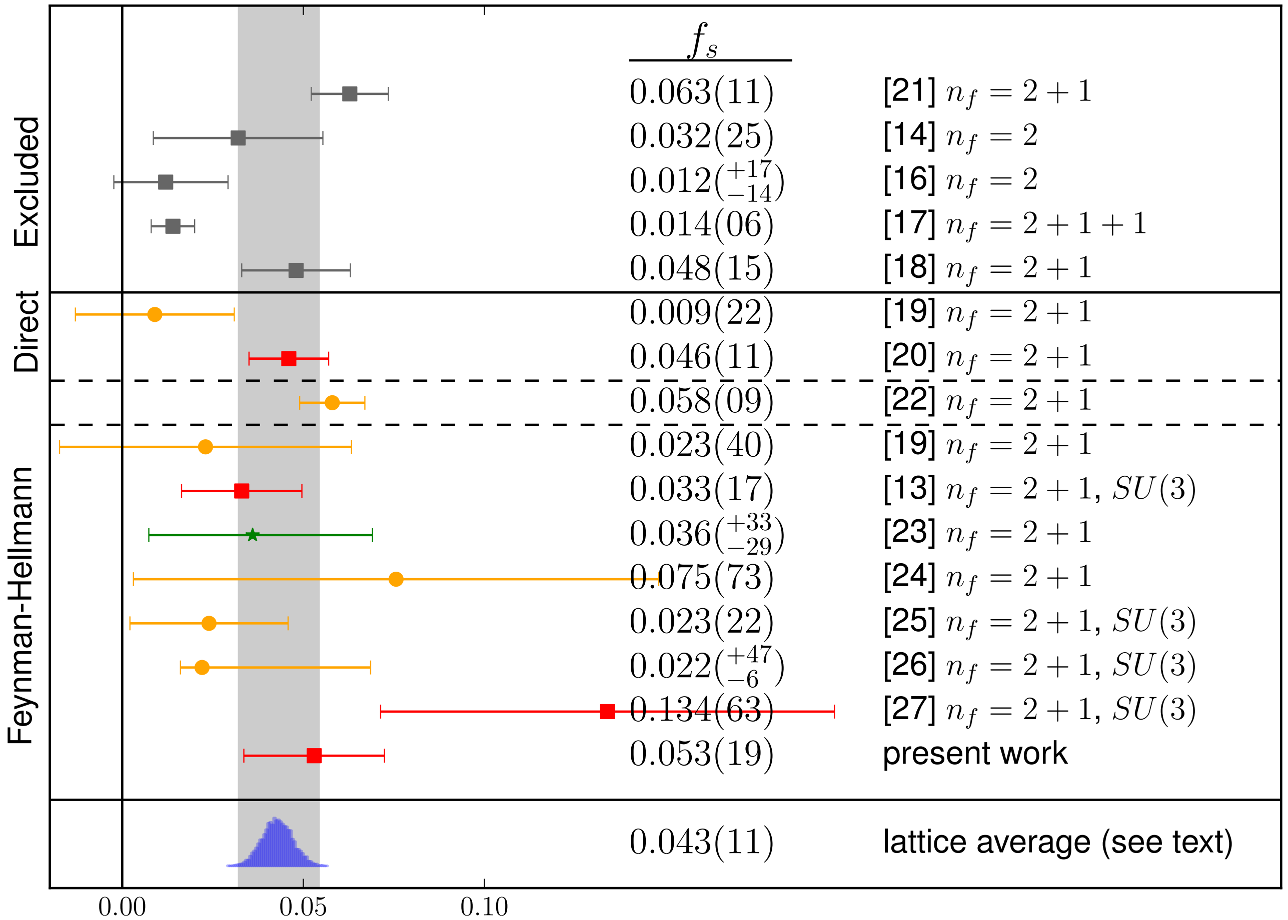
ii) For each random sample, perform a weighted sum over the lattice results with weight $w_i = y_i/\sigma_i$ $\quad y_i = \{3, 2, 1\} \times \{1, .5\}$

reliance upon SU(3) (generous)

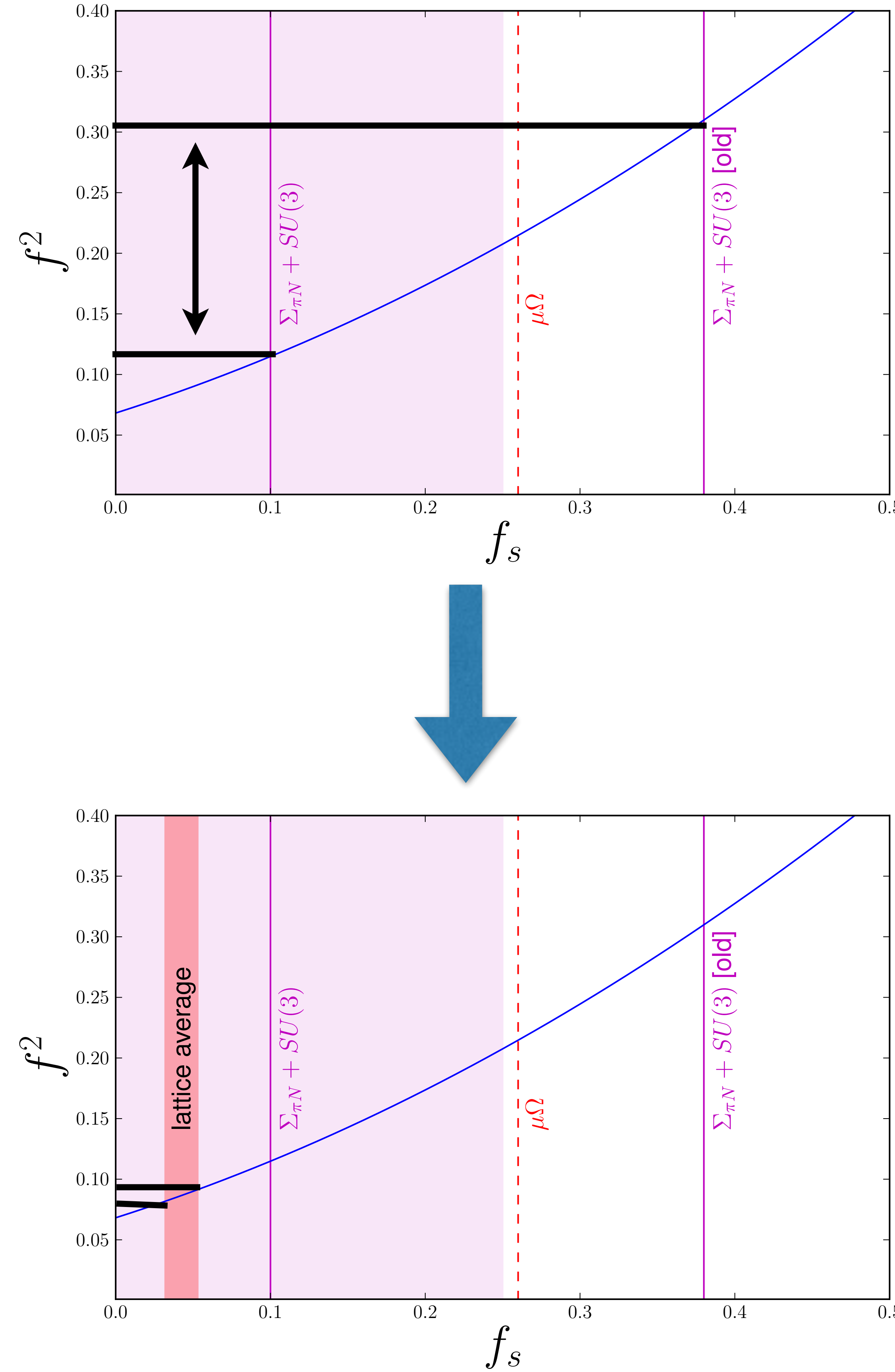
iii) quote 99% confidence interval of resulting distribution as uncertainty (easy way to chop off potential outliers in "uniform" distribution)

strange content of the nucleon

P. Junnarkar and AWL
PRD 87 (2013)



$$f_s = m_s \langle N | \bar{s}s | N \rangle / m_N$$



$$\sigma \propto |f|^2$$

$$f = \frac{2}{9} + \frac{7}{9} \sum_{q=u,d,s} f_q$$

Heavy quark matrix elements

P Junarkar & AWL arXiv:1301.1114

$$\langle N|\theta^\mu_\mu|N\rangle = m_N \bar{\psi}_N \psi_N$$

$$= \langle N| \sum_{q=u,d,s,\ldots} (1-\gamma_m) m_q \bar{q} q + \frac{\beta(n_f)}{2} G^2 |N\rangle$$

$$f_c = 0.08896(1 - x_{uds})$$
$$f_b = 0.08578(1 - x_{uds})$$
$$f_t = 0.08964(1 - x_{uds})$$

$$f_q = \frac{\langle N|m_q \bar{q} q|N\rangle}{m_N}$$

$$x_{uds} = f_u + f_d + f_s$$

Shifman, Vainshtein and Zakharov PLB78 1978
A Kryjevski PRD70 2004 [hep-ph/0312196]
Solon and Hill, arXiv:1409.8290

$$m_c \langle N|\bar{c}c|N\rangle = 76(^{+11}_{-19}) \text{ MeV}$$

Heavy quark matrix elements

P Junarkar & AWL arXiv:1301.1114

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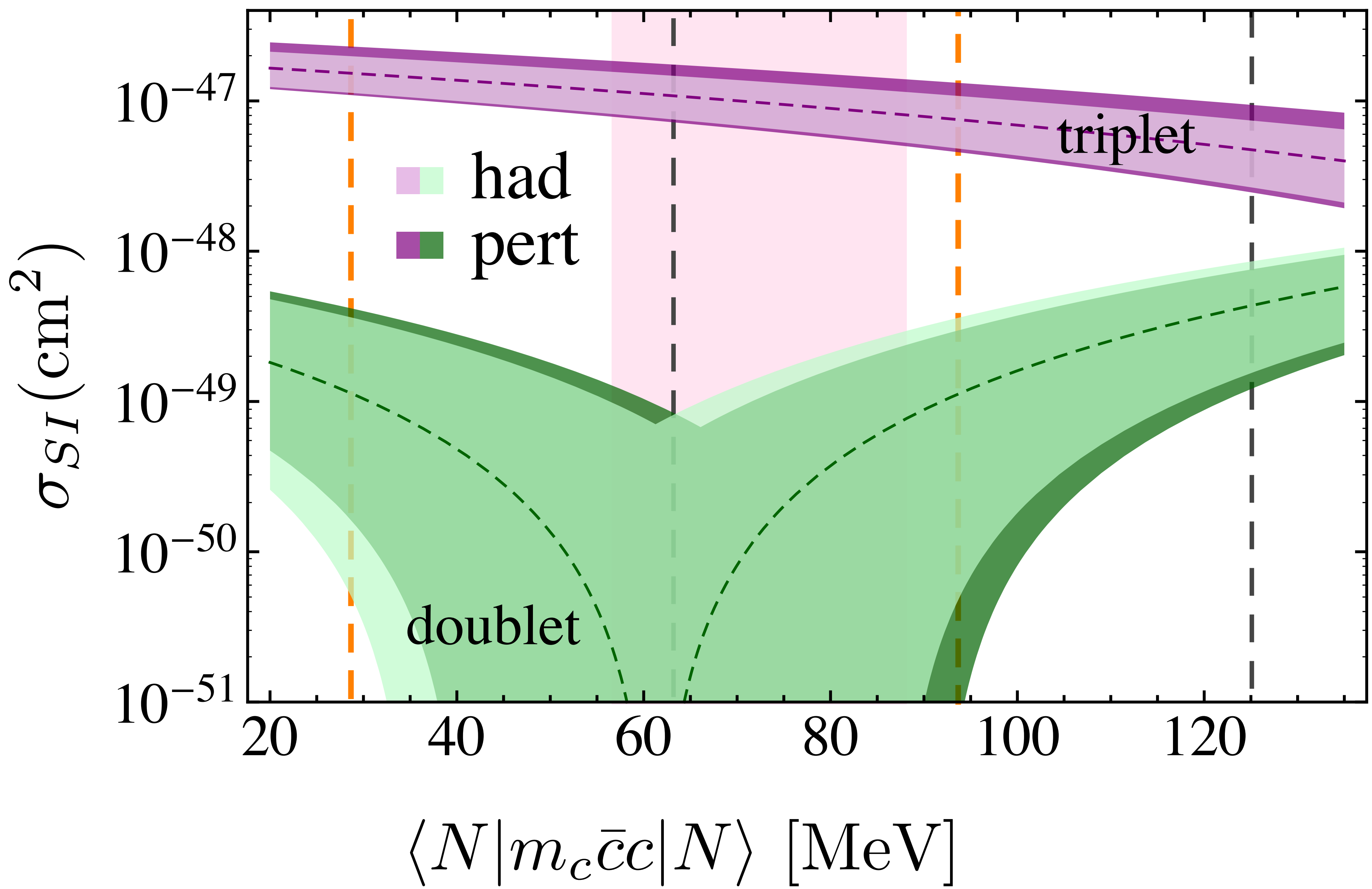
Shifman, Vainshtein and Zakharov PLB78 1978
A Kryjevski PRD70 2004 [hep-ph/0312196]
Solon and Hill, arXiv:1409.8290

$$m_c \langle N|\bar{c}c|N\rangle = 76(^{+1}_{-2})\ \mathrm{MeV}$$

(thanks Mikhail Solon for finding this mistake)
I do not believe the updated precision - I am not sure what the pQCD uncertainty on this is

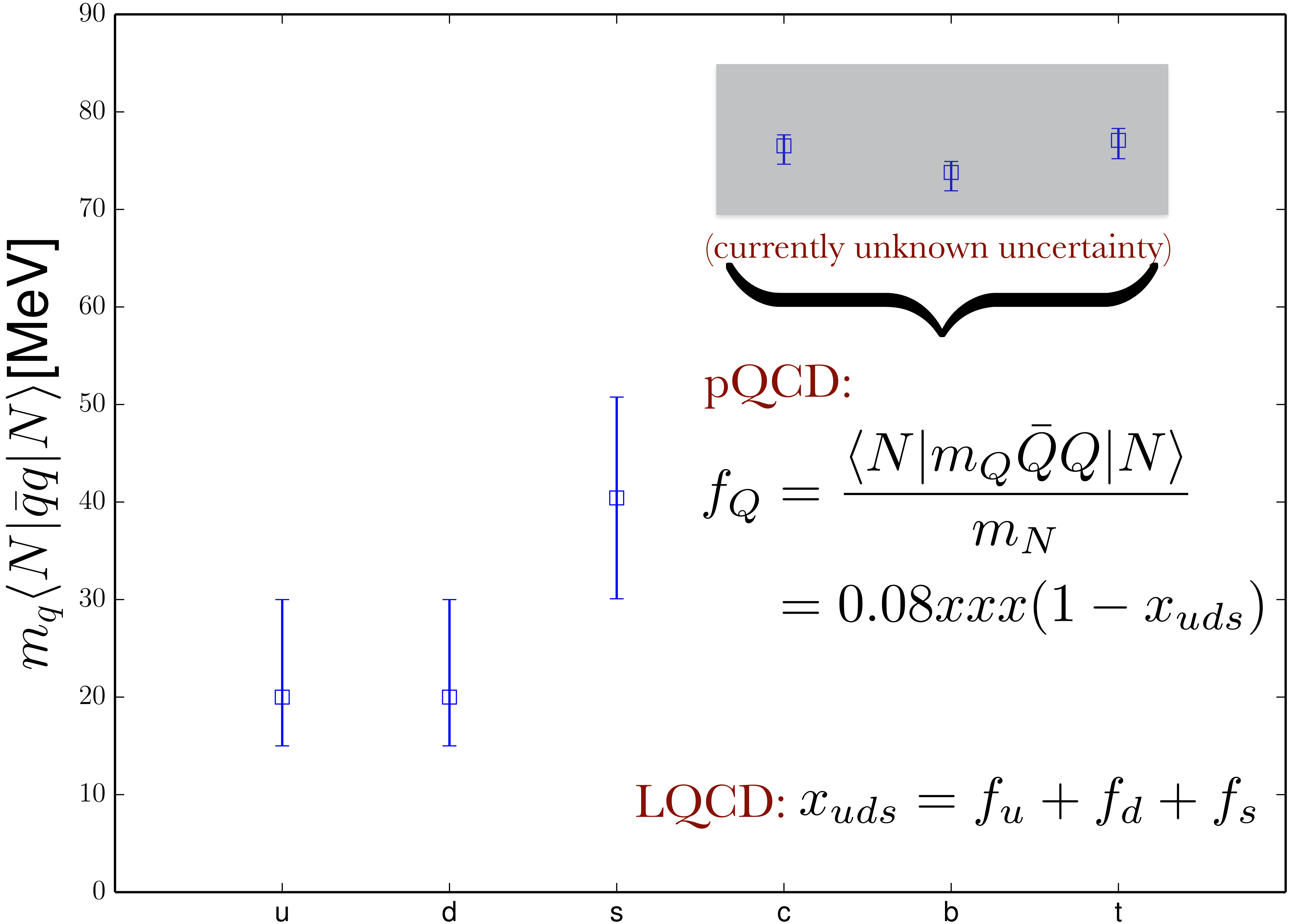
Heavy quark matrix elements



Hill & Solon
PRL 112 (2014)
arXiv:1309.4092

Factors of 2 normally do not seem to matter much in this business, but in some cases - there are large cancelations which are sensitive to the precise values

How much does each quark contribute to the mass of the nucleon?



Conclusions

- Lattice QCD calculations of scalar matrix elements are nearing "averageability", but still a bit premature
- SU(3) baryon Chiral Perturbation Theory should NOT be used for quantitative studies
- Other nucleon matrix element calculations lag significantly compared to scalar quark content
- lattice QCD calculations of scalar charm content beginning, but in preliminary stage: charm particularly interesting to compare LQCD to pQCD values
- We can also begin looking at $m_q \langle N(p+q)|\bar{q}q|N(p)\rangle$ but more challenging

Thank You