Nucleon Matrix Elements From Lattice QCD

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Outline

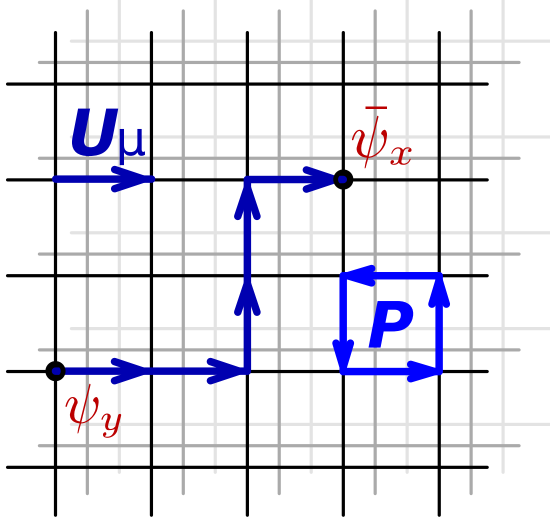
How well can we simulate nucleon structure on a lattice?

- Introduction
  - *lattice methodology and systematic errors*
- Nucleon Electromagnetic Form Factors
  - *Nucleon form factors, radii, magnetic moment*
- Nucleon Axial Form Factors
  - *Nucleon axial charge, axial radius, induced pseudoscalar form factor*
- Decomposition of the Nucleon Momentum and Spin
  - *Quark momentum fraction, spin and angular momentum*
- Summary

Nucleon Correlators and Matrix Elements

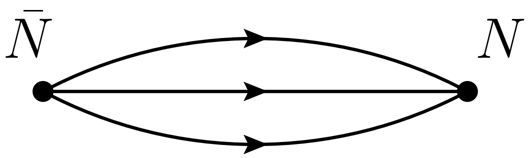
- Lattice QCD: numerical path integral on a 4D Euclidean grid

$$\langle\mathcal{O}\rangle = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar\psi\ \mathcal{O}[U,\psi,\bar\psi]\ e^{-S[U,\psi,\bar\psi]} \to \frac{1}{N}\sum_i^N \tilde{\mathcal{O}}[U_i]$$

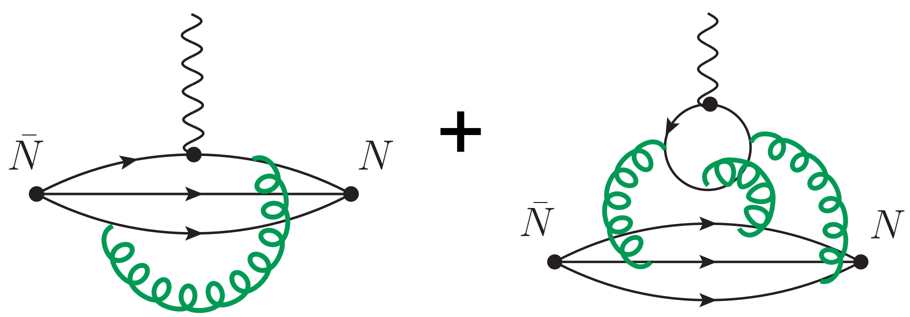


- Hadron correlators

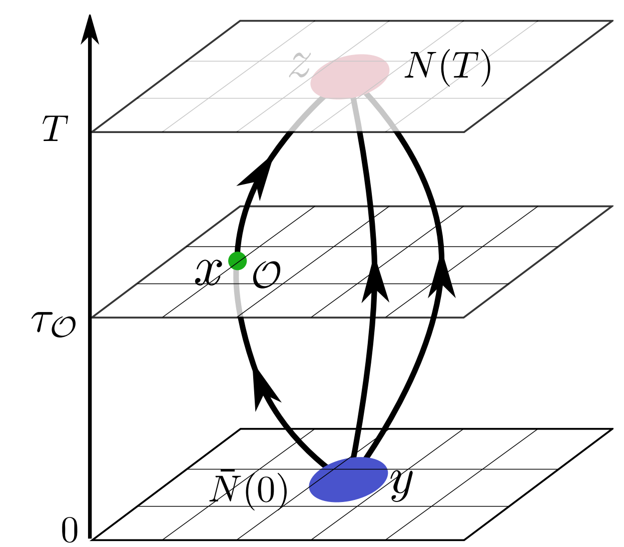
$$C_{2\mathrm{pt}}(T) = \langle N(T)\bar N(0)\rangle =$$



$$C^{\mathcal{O}}_{3\mathrm{pt}}(T) = \langle N(T)\mathcal{O}(\tau)\bar N(0)\rangle =$$



"connected" "disconnected"



- Matrix elements : C_{3pt}/C_{2pt} ratio or multi-exp. fits

$$R_{\mathcal{O}}(T,\tau;P,P') = \frac{\langle N(T)\mathcal{O}(\tau)\bar N(0)\rangle}{\langle N(T)\bar N(0)} \xrightarrow[T,\tau,(T-\tau)\to\infty]{} \langle P'|\mathcal{O}|P\rangle$$

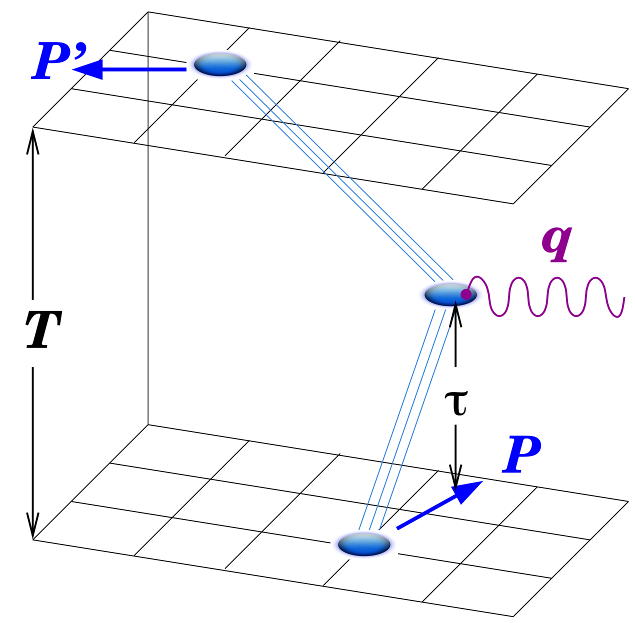
Systematic & Stochastic Error in M.E.

- Euclidean propagation "selects" the ground state:

$$\frac{1}{Z}\int \mathcal{D}U_\mu \,\mathcal{D}q\,\mathcal{D}\bar{q}\;[N(T)\mathcal{O}(\tau)\bar{N}(0)]$$

$$= \langle vac.|a_{N(\mathbf{P}')}\cdot \underbrace{e^{-a\mathcal{H}}\cdots e^{-a\mathcal{H}}}_{T-\tau}\cdot\mathcal{O}_{\mathbf{q}}\cdot\underbrace{e^{-a\mathcal{H}}\cdots e^{-a\mathcal{H}}}_{\tau}\cdot a^\dagger_{N(\mathbf{P})}|vac.\rangle$$

$$\xrightarrow[T\to\infty]{} Z_{00}e^{-M_N T}\Big[\langle P'|\mathcal{O}|P\rangle + \mathcal{O}\big(\underbrace{e^{-\Delta E_{10}T}, e^{-\Delta E_{10}\tau}, e^{-\Delta E_{10}(T-\tau)}}_{\text{excited states}}\big)\Big]$$



- Stochastic noise grows rapidly with *T*, especially with light pions [Lepage'89]:

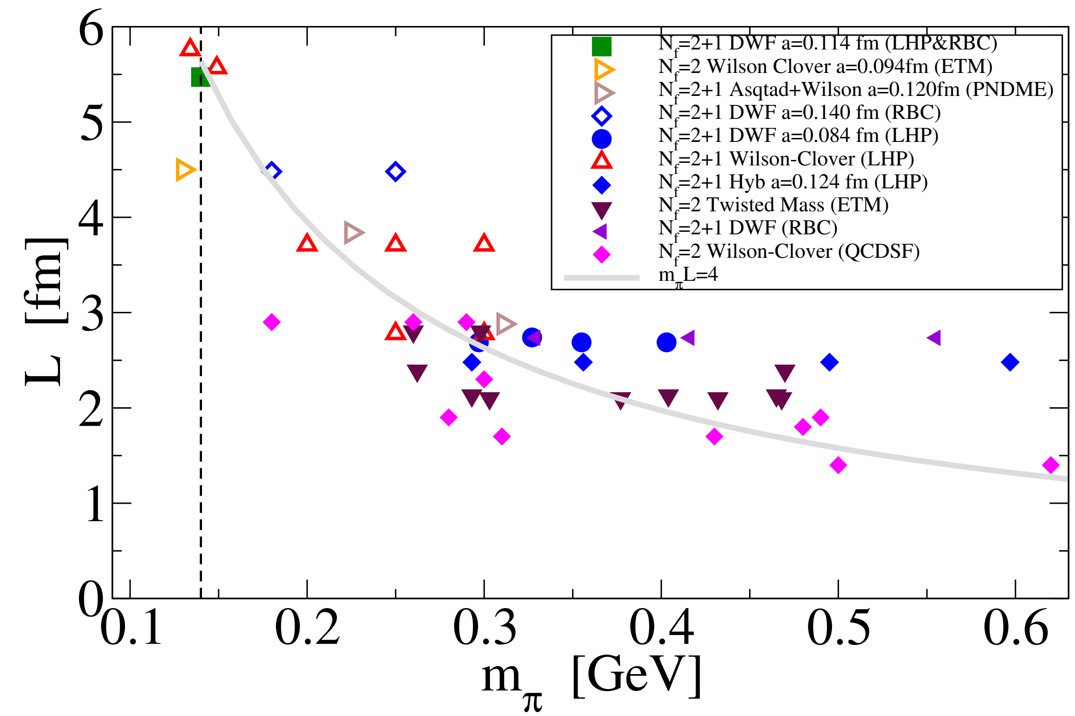
$$\text{Signal } \langle N(T)\bar{N}(0)\rangle \qquad \sim e^{-M_N T}$$

$$\text{Noise } \langle |N(T)\bar{N}(0)|^2\rangle - |\langle N(T)\bar{N}(0)\rangle|^2 \qquad \sim e^{-3m_\pi T}$$

$$\text{Signal/Noise} \qquad \sim e^{-(M_N-\frac{3}{2}m_\pi)T}$$

Other Sources of Systematic Errors

- unphysically heavy pion (quark) mass
- broken chiral symmetry of quarks
- finite volume
- discretization effects



Calculations currently in progress

- ✔ QCD at the physical point
- ✔ chiral-symmetric quarks (some groups)
- ✔ excited states are subtracted/removed
- ✘ isospin symmetry limit (N_{flav}=2+1)
- ✘ no electromagnetic interactions

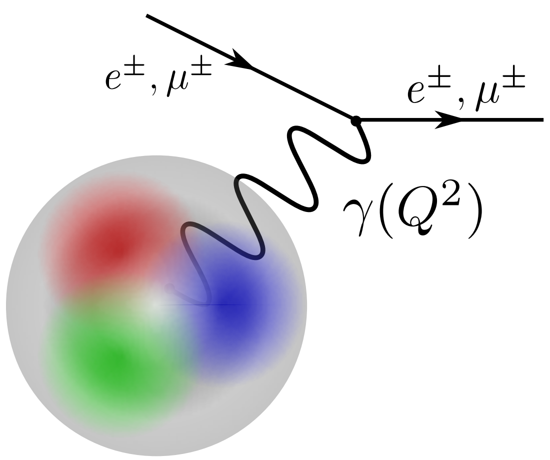
Physical chiral-symmetric quarks :

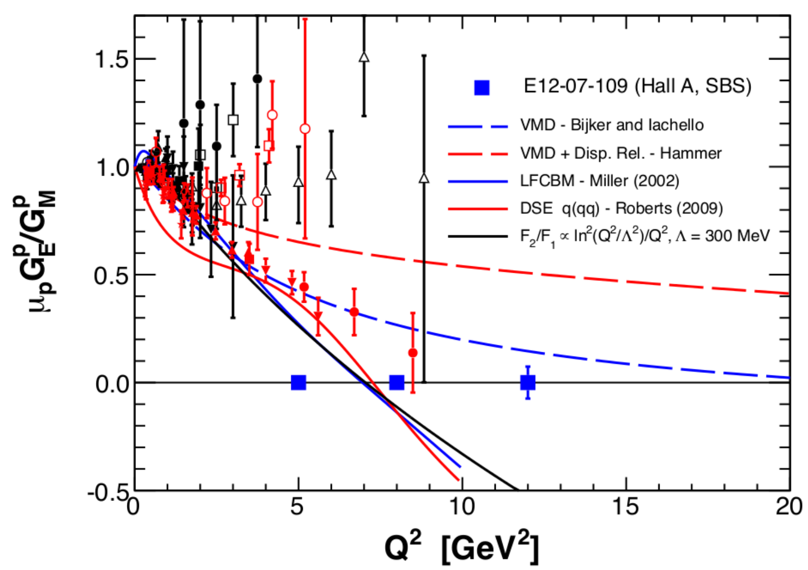
$$a \approx 0.113 \text{ fm} = (1.75 \text{ GeV})^{-1},$$
$$V_4 = 48^3 \times 96 = (5.4 \text{ fm})^3 \times 10.8 \text{ fm},$$
$$m_\pi L_x \approx 3.84$$

Nucleon Electromagnetic Form Factors

$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\Big[F_1(Q^2)\,\gamma^\mu + F_2\,(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$

- JLab@12GeV : explore form factors at Q^2>=10 GeV^2
  - (F_1 / F_2) scaling at Q^2 -> ∞
  - (G_E/G_M) dependence up to Q^2=18 GeV^2
  - *u*-, *d*-flavor dependence of form factors
- Proton radius puzzle: 7σ difference
  - JLab E12-11-106 (Hall B)
  - MUSE@PSI : e$^\pm$ / μ$^\pm$-scattering off the proton

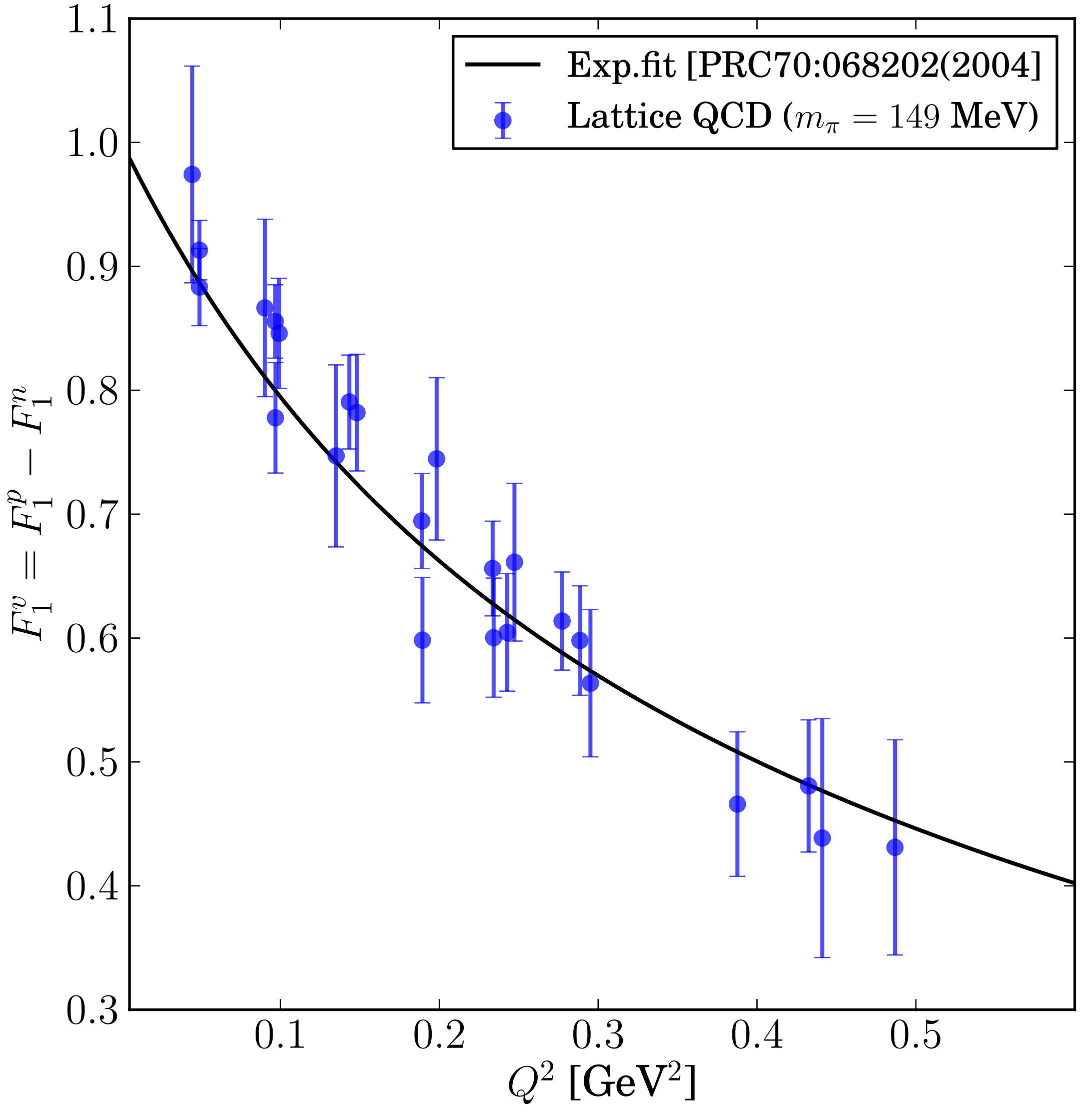


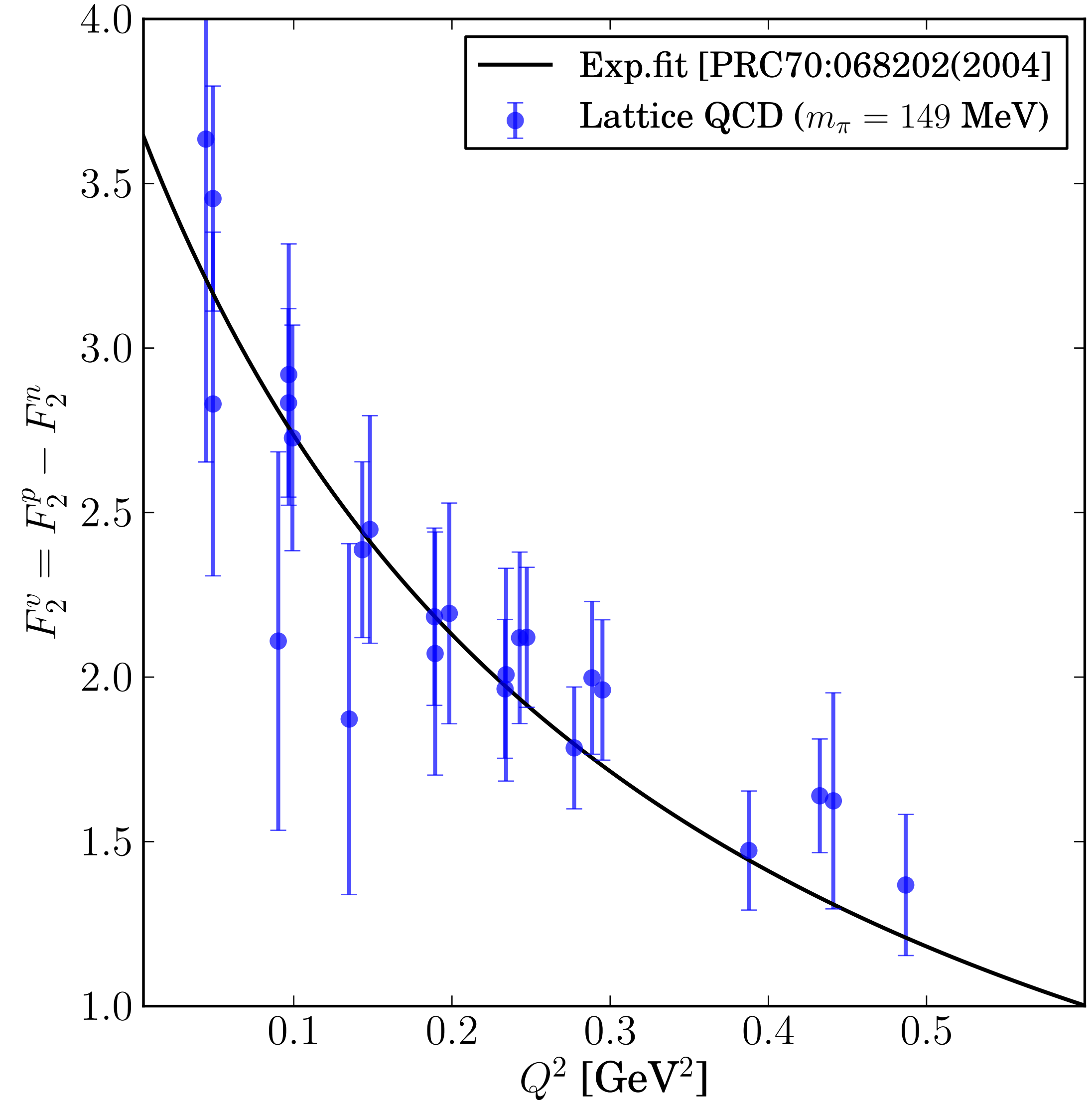


[Research Mgmt. Plan for SBS(JLab Hall A)]

Nucleon Isovector (p-n) Form Factors

$$\langle P+q|\,\bar{q}\gamma^\mu q\,|P\rangle = \bar{U}_{P+q}\Big[F_1(Q^2)\,\gamma^\mu + F_2\,(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N}\Big]U_P$$





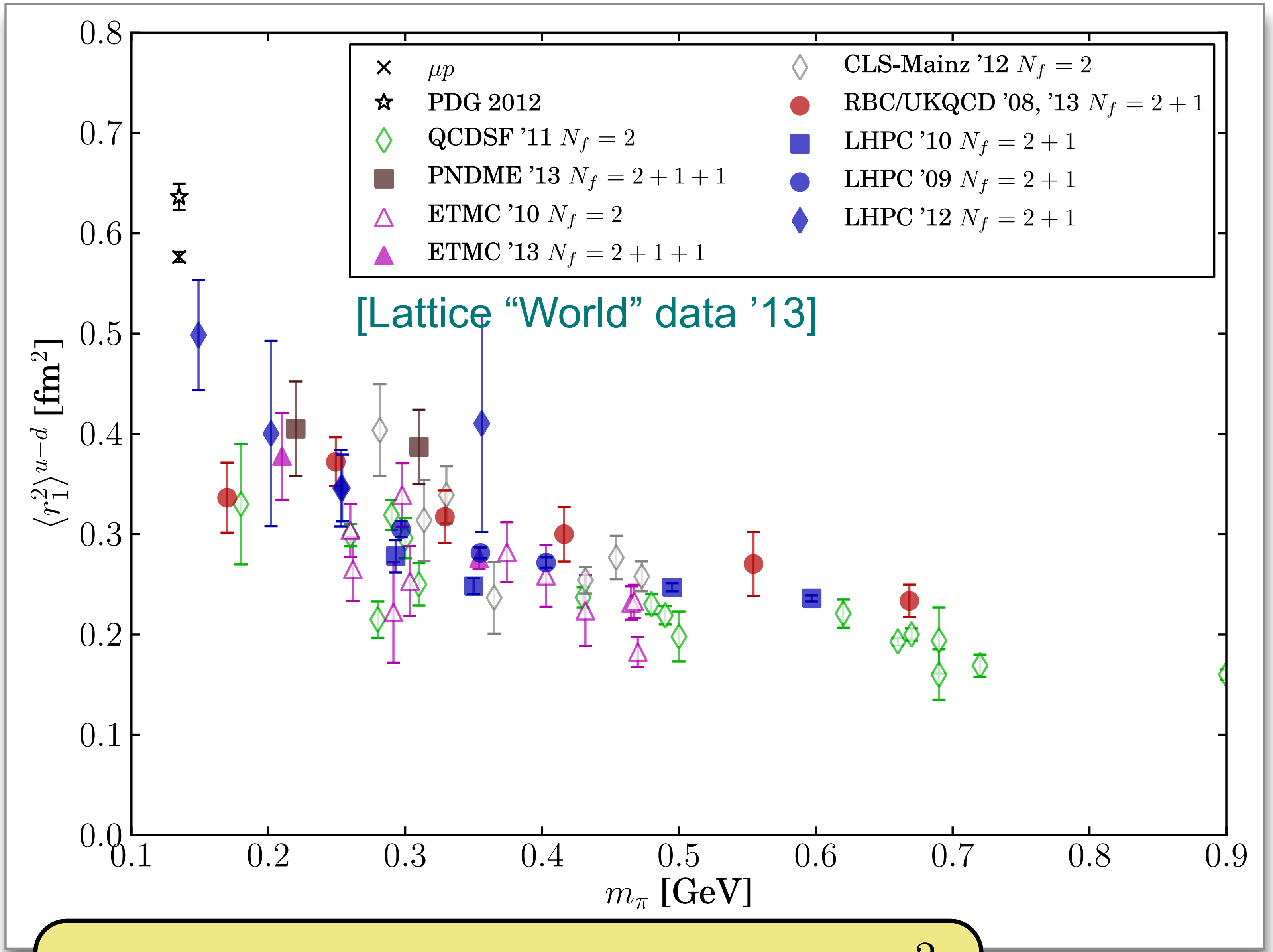
Nf=2+1 clover-imp.Wilson, m_π=149 MeV [J.R.Green, SNS et al (LHPC)]

Lattice Q^2 usually limited to $L^{-2} < Q^2 << a^{-2}$
(and high momenta are noisy)

Dirac Radius vs. m_π and Proton Size Puzzle

$$F_1^{u-d}(Q^2) \approx F(0)\left[1 - \frac{1}{6}Q^2 \langle r_1^2\rangle^{u-d} + \mathcal{O}(Q^4)\right]$$

(usually extracted from dipole fits in Q^2<0.5 GeV^2)



ChPT predicts divergence $\sim \log m_\pi^2$

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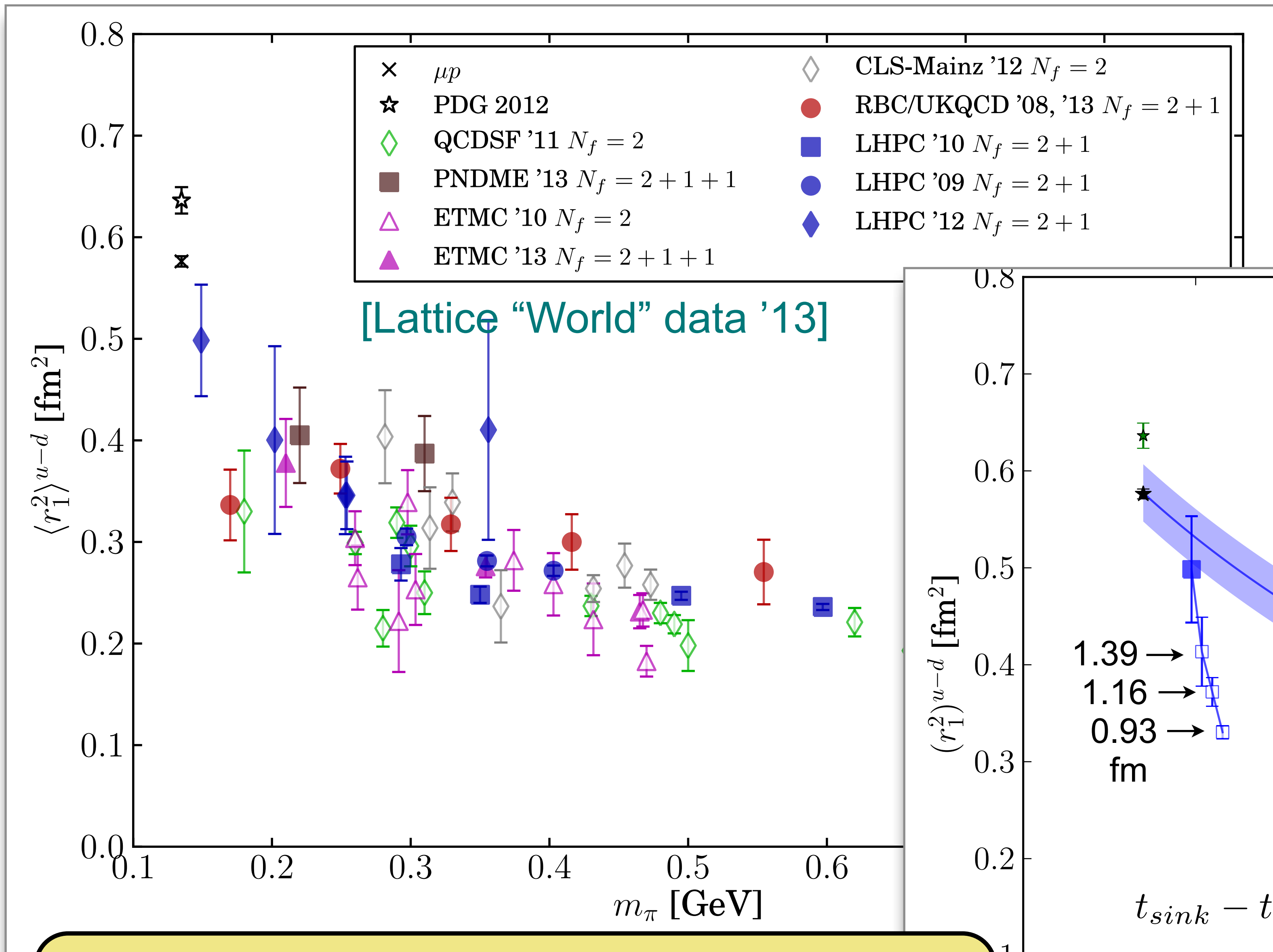
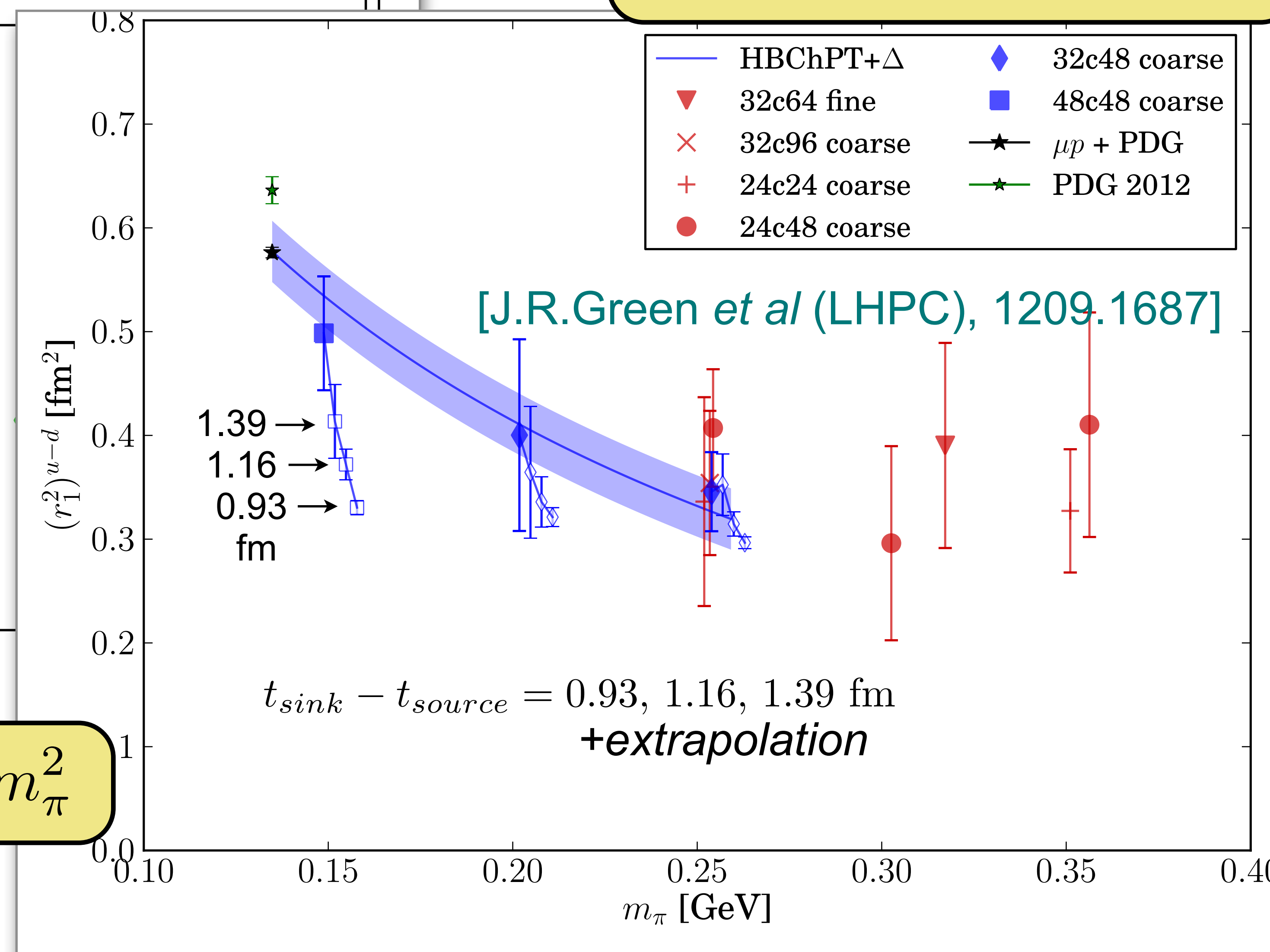


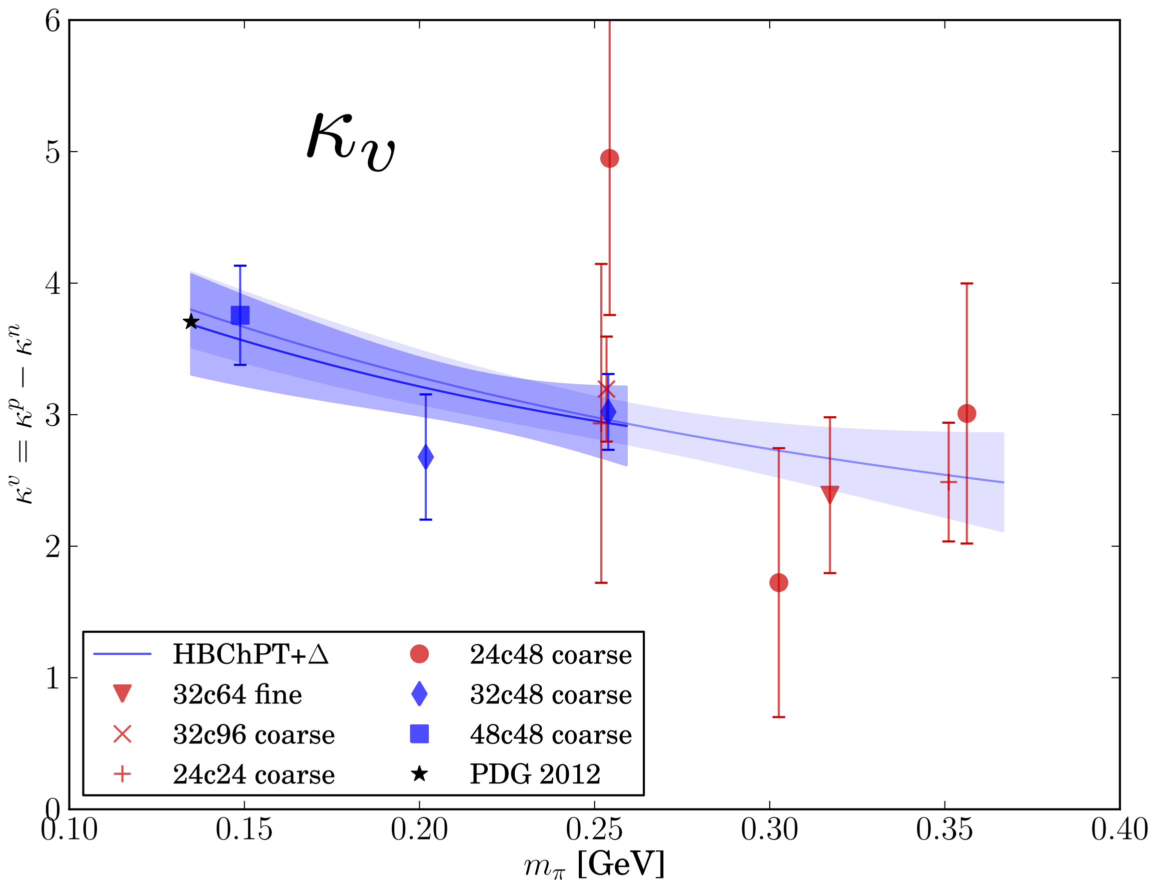
ChPT predicts divergence $\sim \log m_\pi^2$

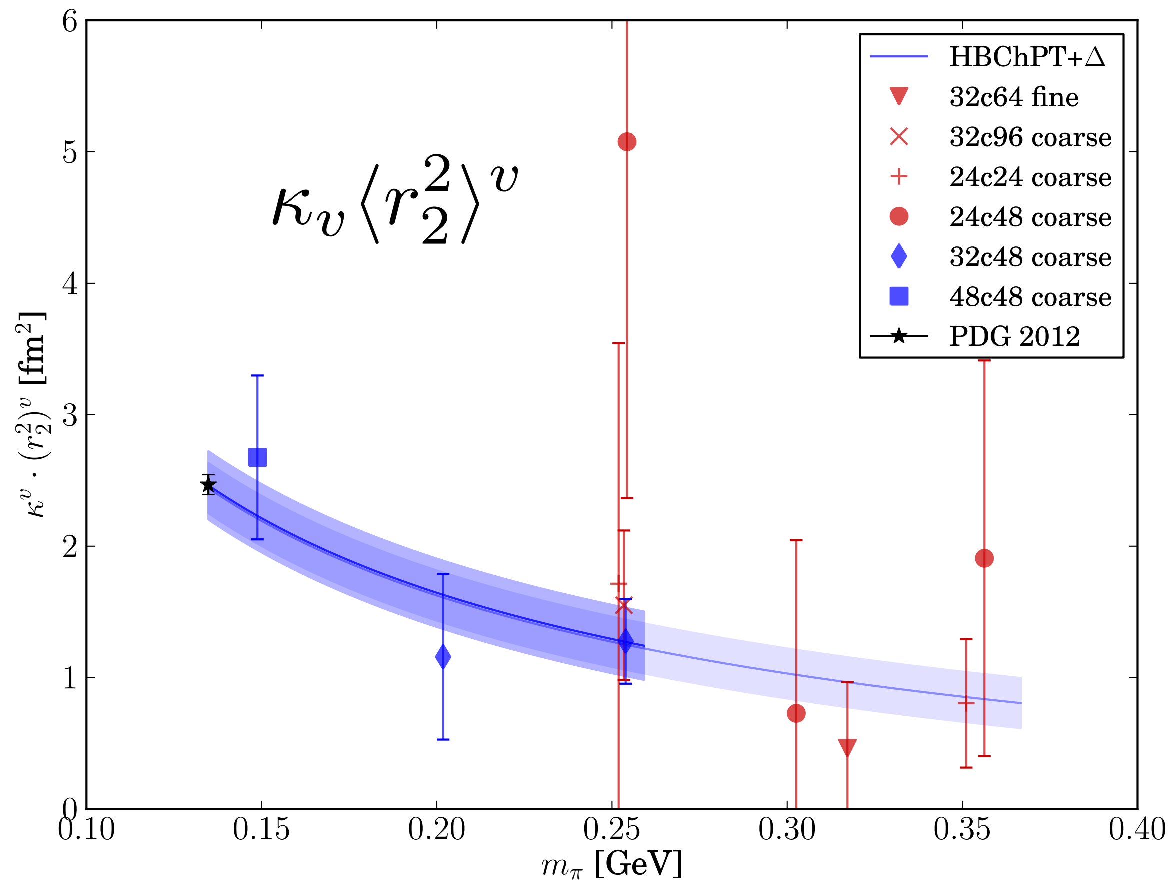
Excited states problem: Worse for m_π<200 MeV?



Isovector Magnetic Moment vs. m_π

$$F_2^{u-d}(Q^2) \approx \kappa_v \Big[1 - \frac{1}{6} Q^2 \langle r_2^2 \rangle^v + \mathcal{O}(Q^4)\Big]$$





m_π=149 MeV Nf=2+1 clover-imp.Wilson

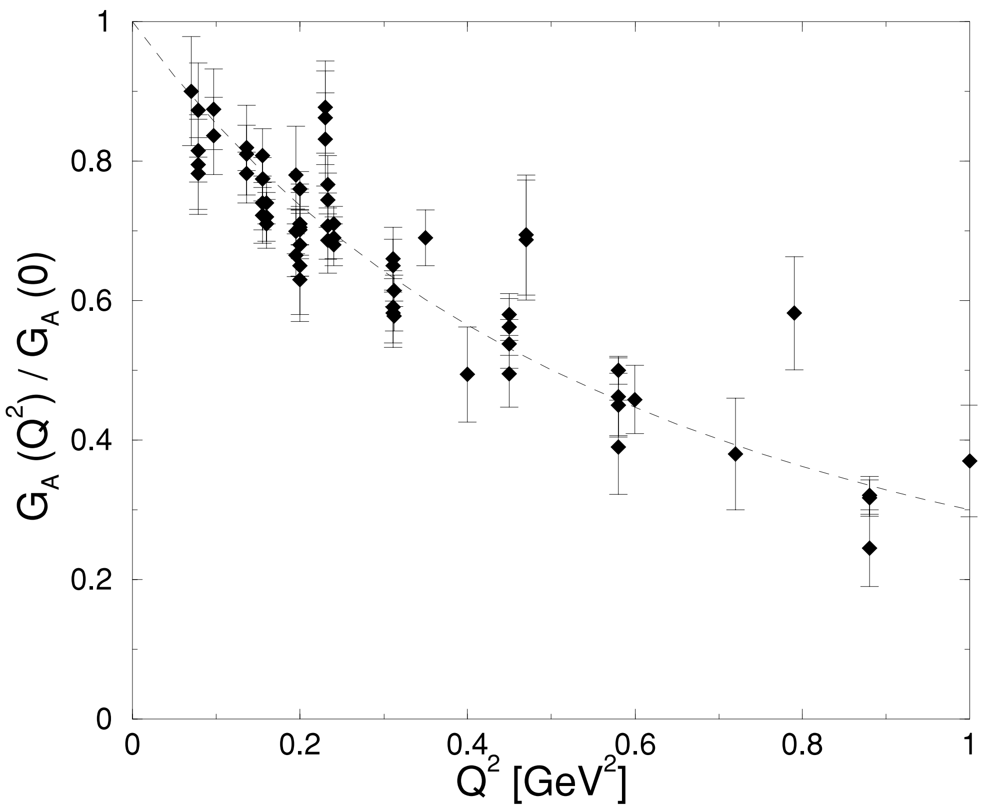
[J.R.Green, SNS et al (LHPC)]

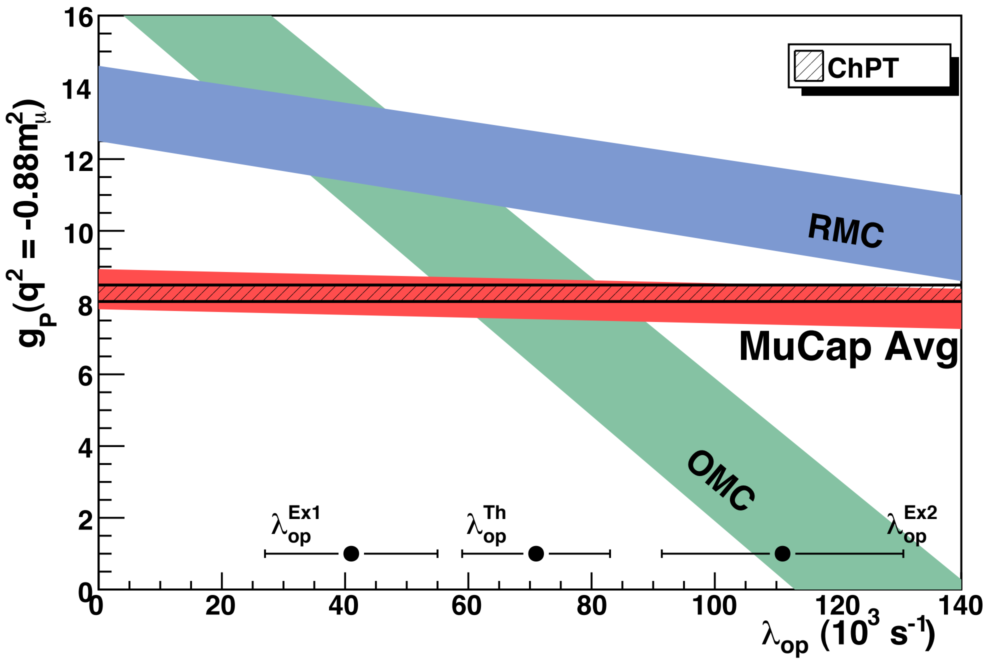
Larger L_s, smaller $Q^2_{\rm min}$ are desirable

Nucleon Axial Charge and Form Factors

$$\langle P+q|\,\bar{q}\gamma^\mu\gamma^5 q\,|P\rangle \;=\; \bar{U}_{P+q}\Big[G_A(Q^2)\,\gamma^\mu\gamma^5 + G_P(Q^2)\,\frac{\gamma^5 q^\mu}{2M_N}\Big]U_P$$

- Axial form factor $G_A(Q^2)$
  - Interaction with neutrinos: MiniBooNE
- Induced pseudoscalar form factor $G_P(Q^2)$
  - Charged pion electroproduction
  - Muon capture (MuCAP@UW): $g_P \sim G_P(Q^2 = 0.88\, m_\mu^2)$
- Strange axial form factor $G_A^s(Q^2)$: studied at MiniBooNE

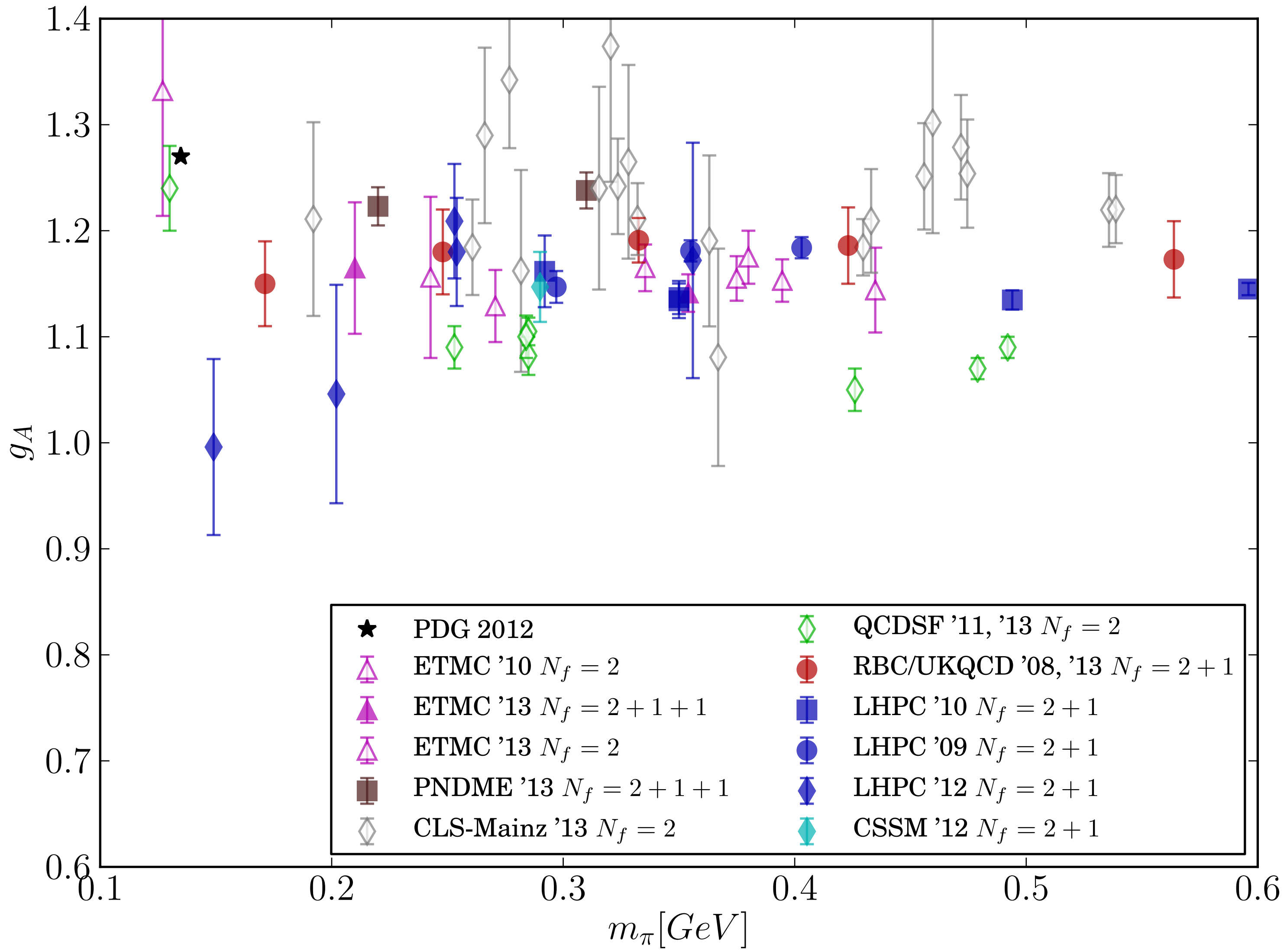




Nucleon Axial Charge

$$\langle N(p)|\bar{q}\gamma^\mu\gamma^5 q|N(p)\rangle = g_A\,\bar{u}_p\gamma^\mu\gamma^5 u_p\,,$$

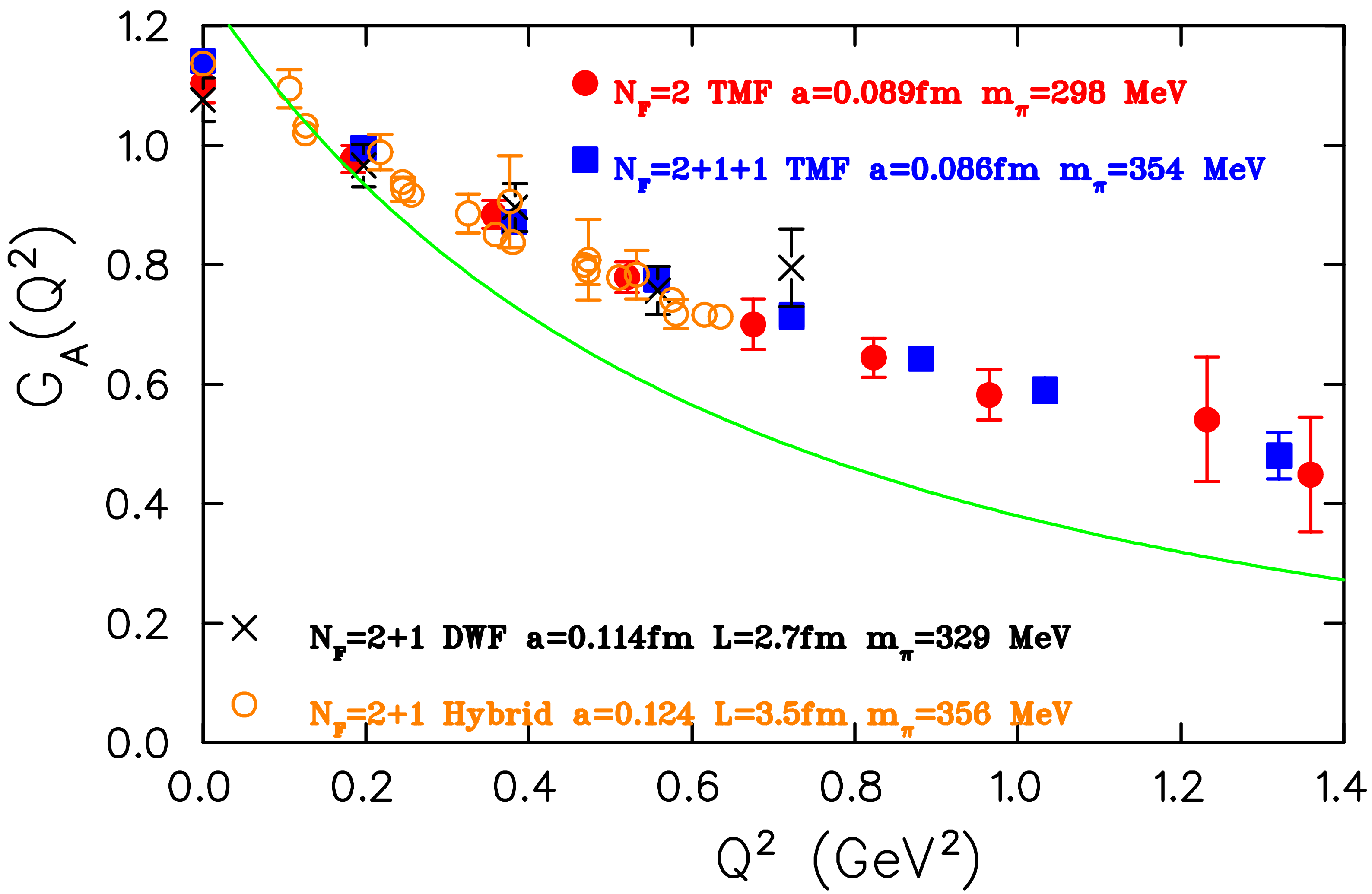
(beta-decay matrix element)



Lattice QCD underestimates g_A by 10-15%

Nucleon Axial Form Factor

$$\langle P+q|\,\bar{q}\gamma^\mu\gamma^5 q\,|P\rangle \;=\; \bar{U}_{P+q}\Big[G_A(Q^2)\,\gamma^\mu\gamma^5 + G_P(Q^2)\frac{\gamma^5 q^\mu}{2M_N}\Big]U_P$$



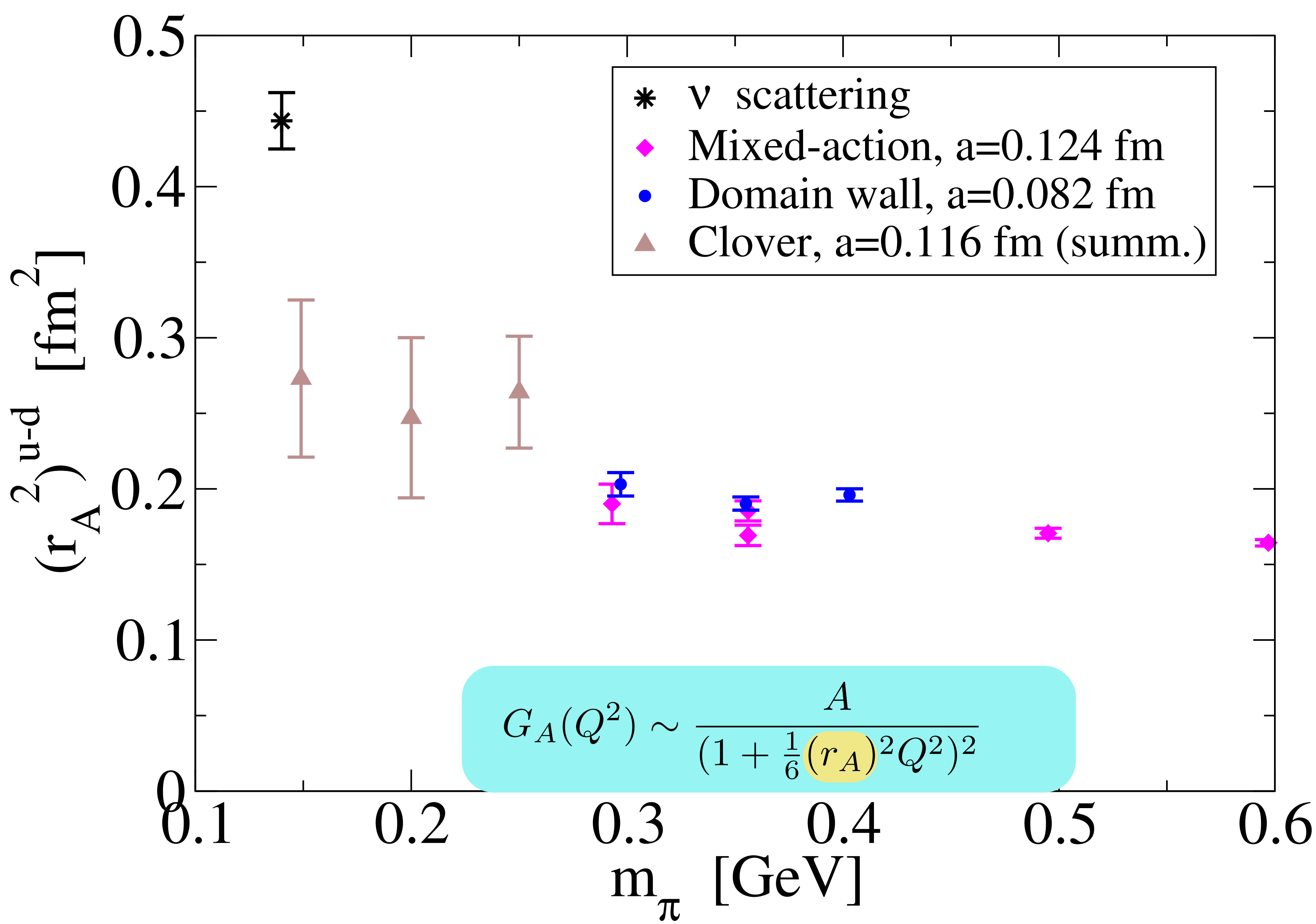
[C.Alexandrou (ETMC), 1303.5979]

Nucleon Axial Radius

- 5% discrepancy in exp. values of r_A (from $G_A(Q^2)$ dipole fits)

$$\sqrt{\langle r_A^2 \rangle_{\nu-\text{scatt.}}} = (0.666 \pm 0.014)\ \text{fm}$$

$$\sqrt{\langle r_A^2 \rangle_{el-prod}} = (0.639 \pm 0.010)\ \text{fm}$$

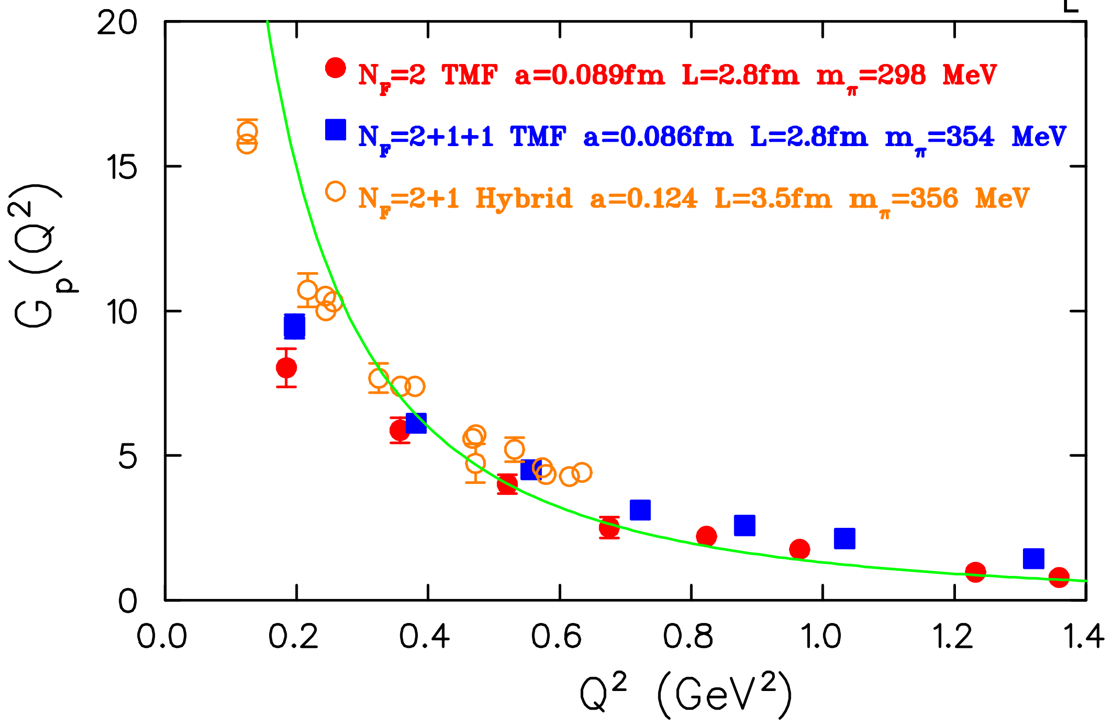


$$G_A(Q^2) \sim \frac{A}{(1 + \frac{1}{6}(r_A)^2 Q^2)^2}$$

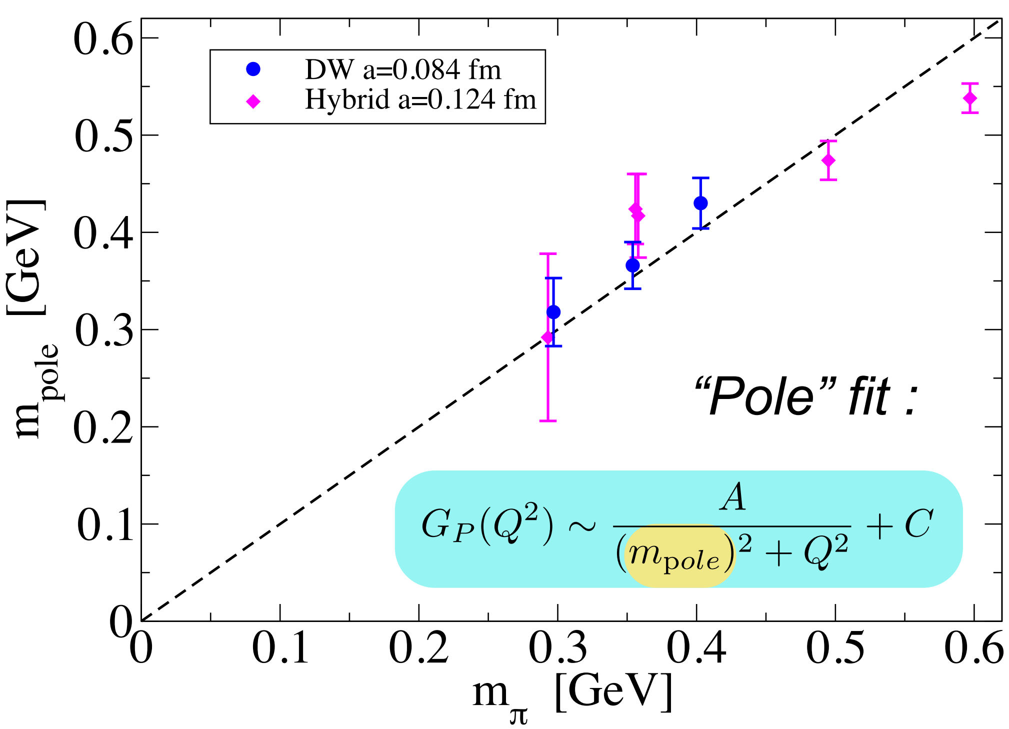
- Weak dependence on m_π and disagreement at m_π^{phys} : same problem as g_A ?
- Study required for volume dependence and exc.states.

Nucleon Pseudoscalar Form Factor $G_P(Q^2)$

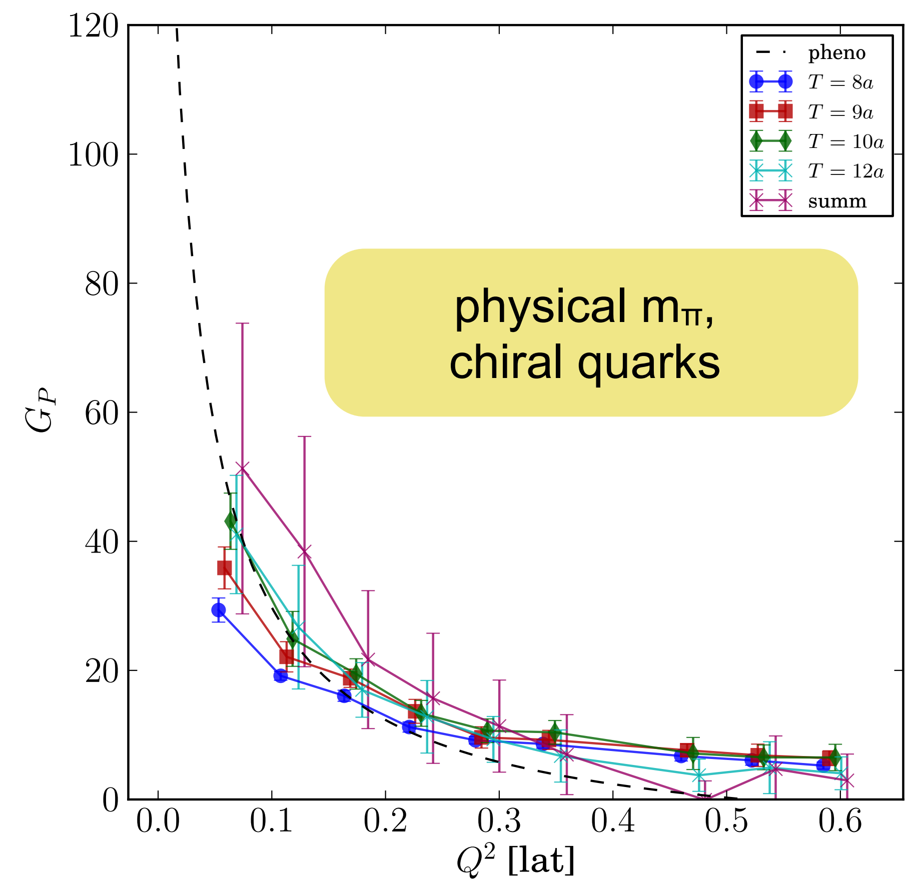
$$\langle P+q|\,\bar{q}\gamma^\mu\gamma^5 q\,|P\rangle = \bar{U}_{P+q}\Big[G_A(Q^2)\,\gamma^\mu\gamma^5 + G_P(Q^2)\frac{\gamma^5 q^\mu}{2M_N}\Big]U_P$$



- Is G_P dominated by the pion pole ?



- G_P at the physical point : large excited states contrib.



Quark Momentum, Angular Momentum and Spin

Proton spin puzzle:

1989 EMC experiment found $$\Delta\Sigma = \sum_q (\Delta q + \Delta\bar{q}) = 0.2\ldots0.3$$

Spin sum rule:

$$J_{\text{glue}} + \sum_q J_q = \frac{1}{2}\,,$$

$$J_q = \frac{1}{2}\Delta\Sigma_q + L_q$$

Quark Spin:

$$\langle N(p)|\bar{q}\gamma^\mu\gamma^5 q|N(p)\rangle = (\Delta\Sigma_q)\left[\bar{u}_p\gamma^\mu\gamma^5 u_p\right]$$

Quark Momentum fraction (<*x*>$_q$) and Angular momentum (J_q) [X.Ji'96]:

$$\langle x\rangle_q = A_{20}^q(0) \qquad J_{q,glue} = \frac{1}{2}\left[A_{20}^{q,glue}(0) + B_{20}^{q,glue}(0)\right]$$

where A_{20} , B_{20} are E.-M. tensor form factors:

$$\langle N(p+q)|\, T_{\mu\nu}^{q,glue}\,|N(p)\rangle \to \left\{A_{20},\, B_{20},\, C_{20}\right\}(Q^2)$$

$$T_{\mu\nu}^q = \bar{q}\,\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}\,q$$

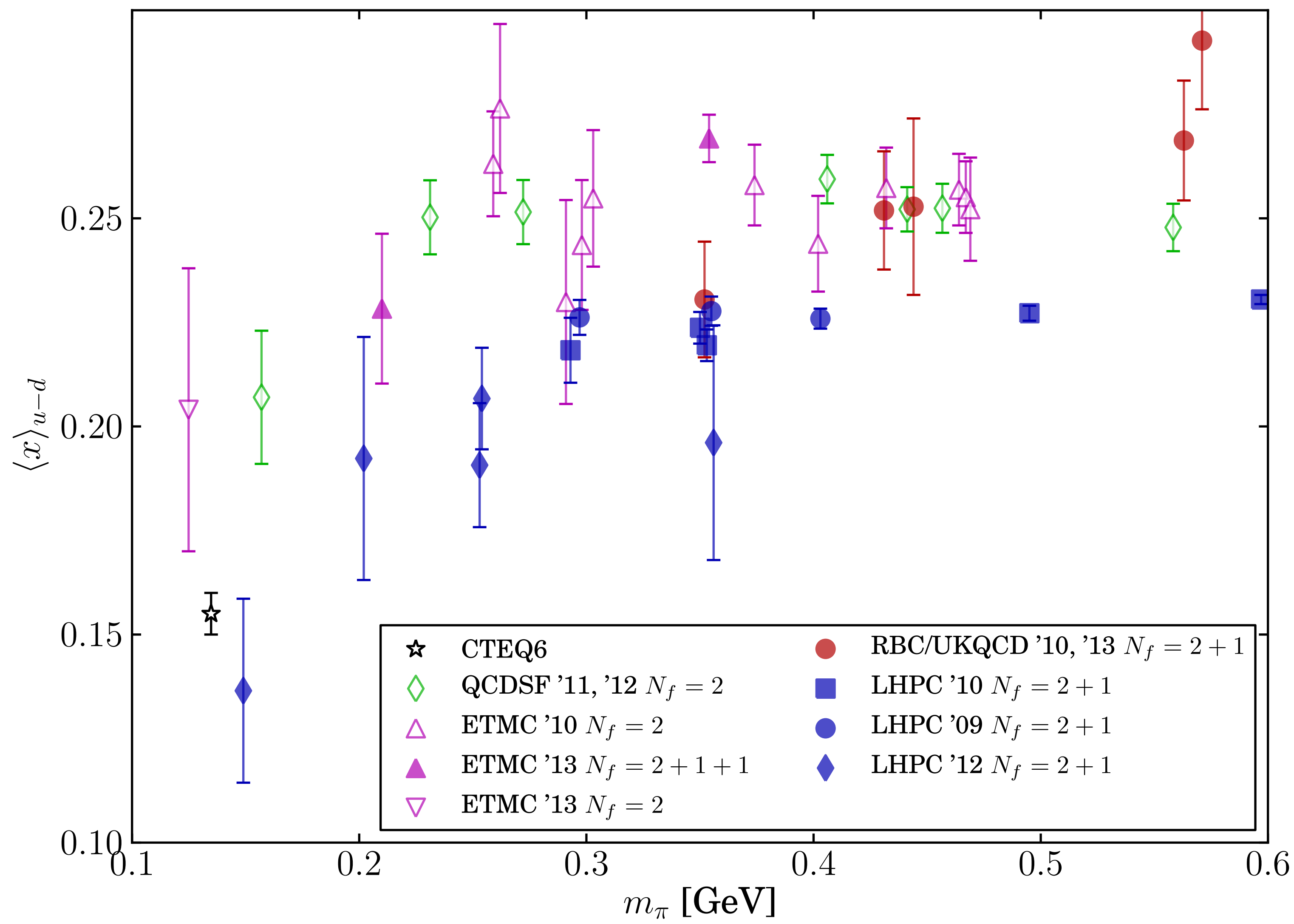
$$T_{\mu\nu}^{\text{glue}} = G_{\mu\lambda}^a G_{\nu\lambda}^a - \frac{1}{4}\delta_{\mu\nu}(G_{\mu\nu})^2$$

Quark Momentum Fraction (Isovector)

$$\langle x \rangle_{u-d} = \int dx\, x\, \big(u(x) + \bar{u}(x) - d(x) - \bar{d}(x)\big)$$

Phenomenology: $\langle x \rangle_{u-d}^{\overline{MS}(2\text{ GeV})} = 0.155(5)$

$$\langle N(p)| \bar{q}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}} q |p\rangle = \langle x \rangle_q\, \bar{u}_p \gamma_{\{\mu} p_{\nu\}} u_p$$

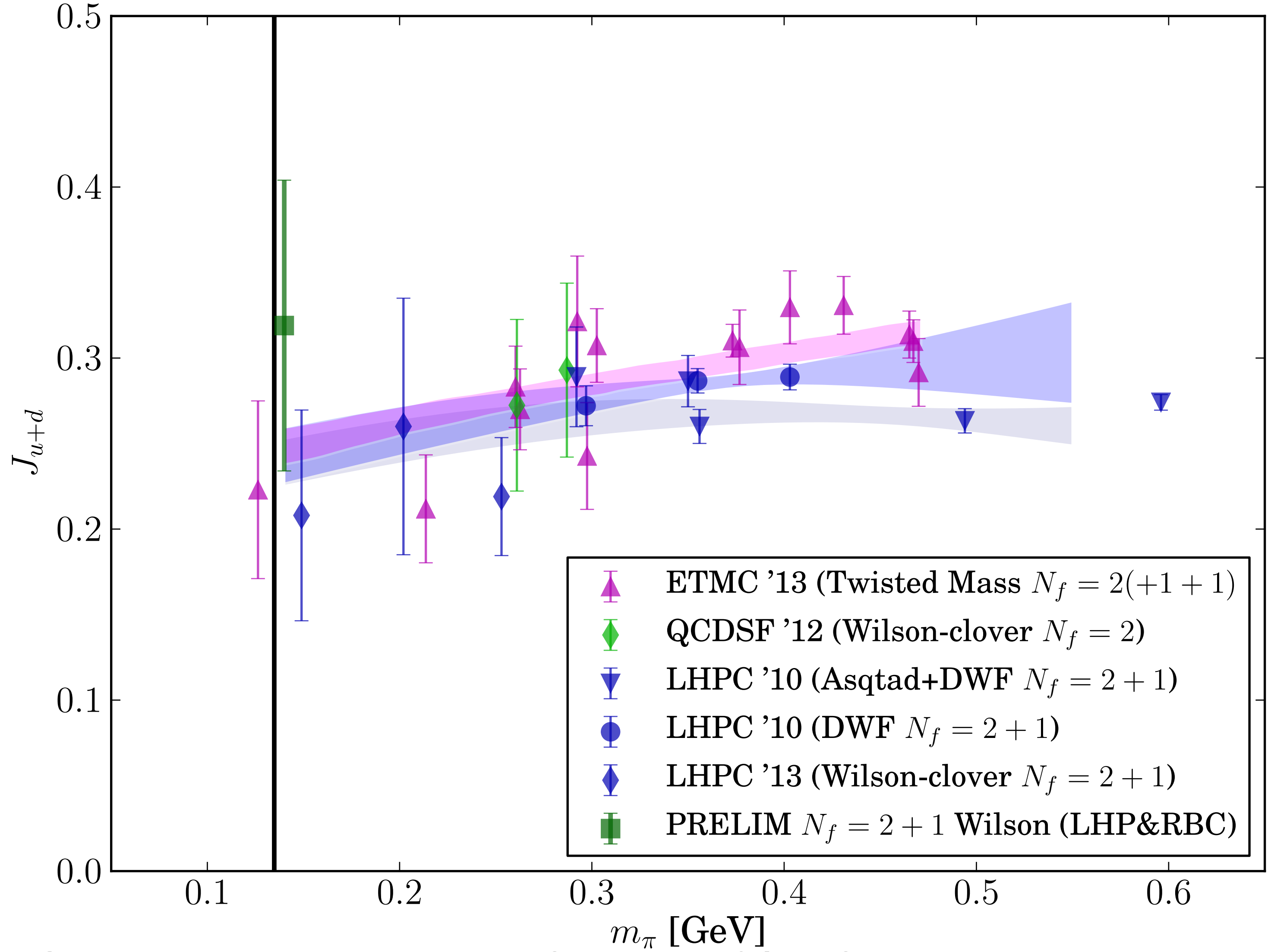


- disconnected diagrams **cancel**
- Lattice QCD “benchmark” quantity
- Significant excited states contributions

Light Quark Angular Momentum (u+d)

[X. Ji PRL'96] $J_q^3 = \langle N | \int d^3x \, M^{012} | N \rangle \qquad M_q^{\alpha\mu\nu} = x^\mu T_q^{\alpha\nu} - x^\nu T_q^{\alpha\mu}$

$$J_q = \frac{1}{2}\left[A_{20q}(0) + B_{20q}(0)\right]$$

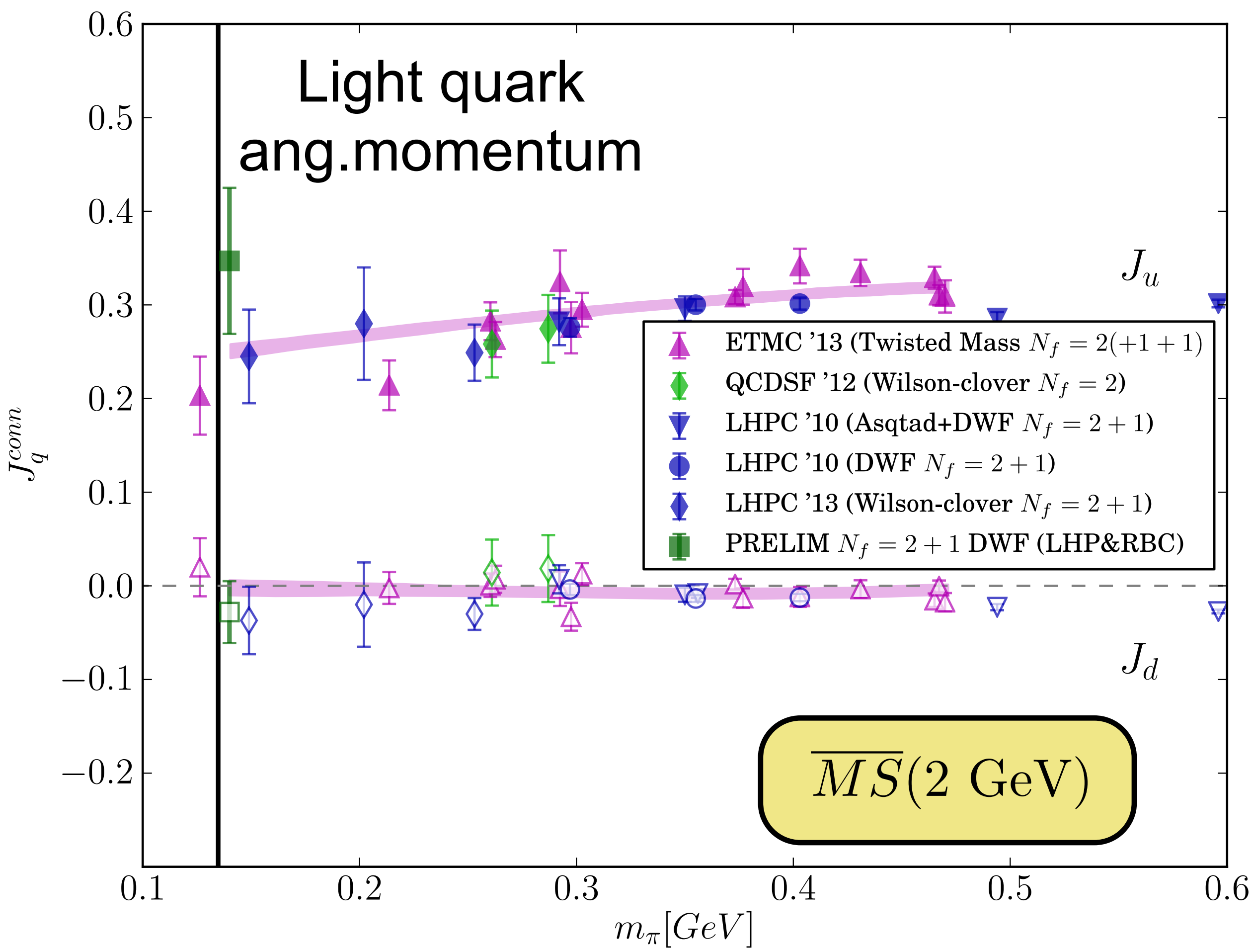


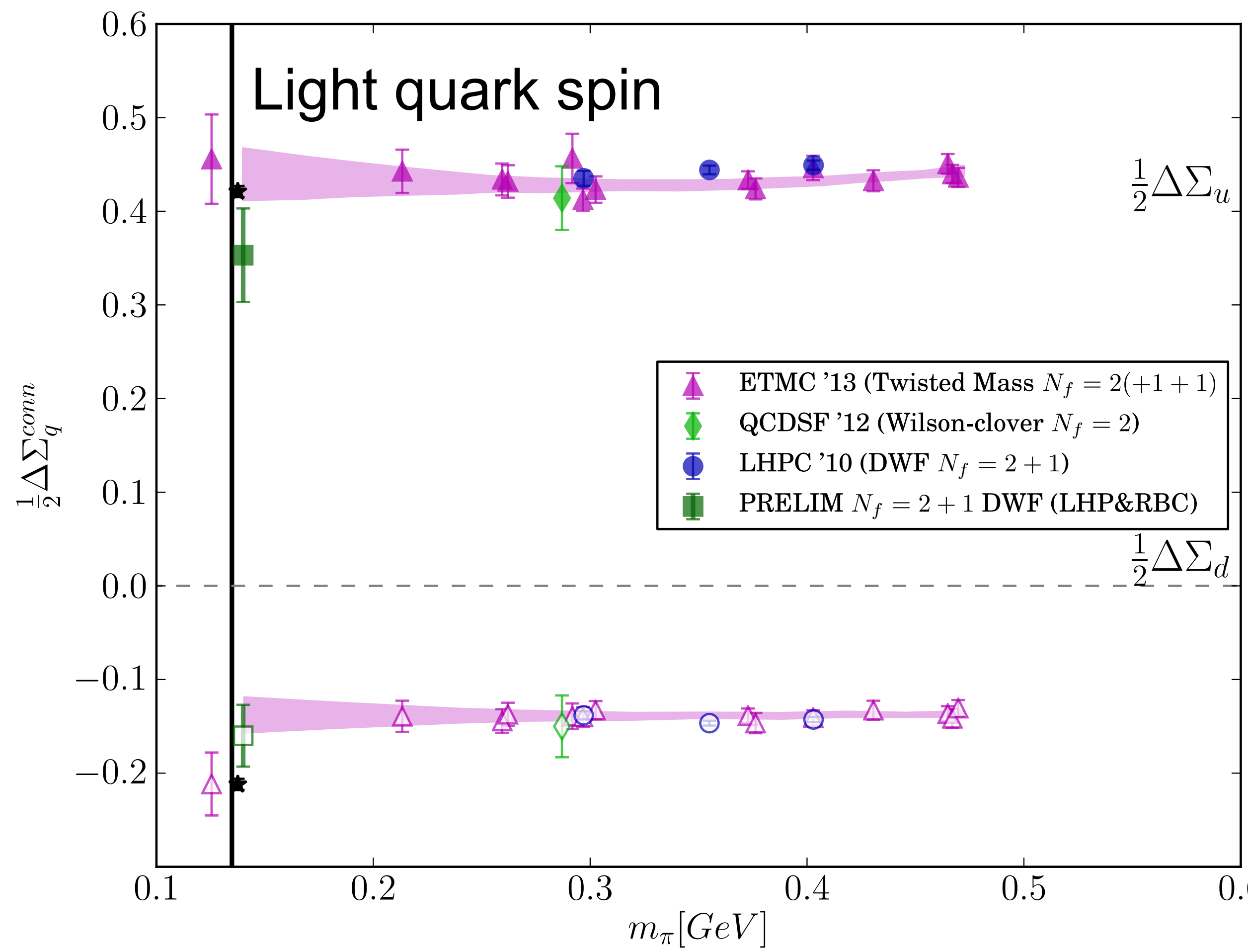
Glue must account for ~50% of the nucleon spin

[Balitsky, Ji PRL'97]: $2J_g^{\text{MSbar(1 GeV)}} = 0.5 - 0.7$

(*) disconnected contributions are not included

Light Quark Angular Momentum: Decomposition

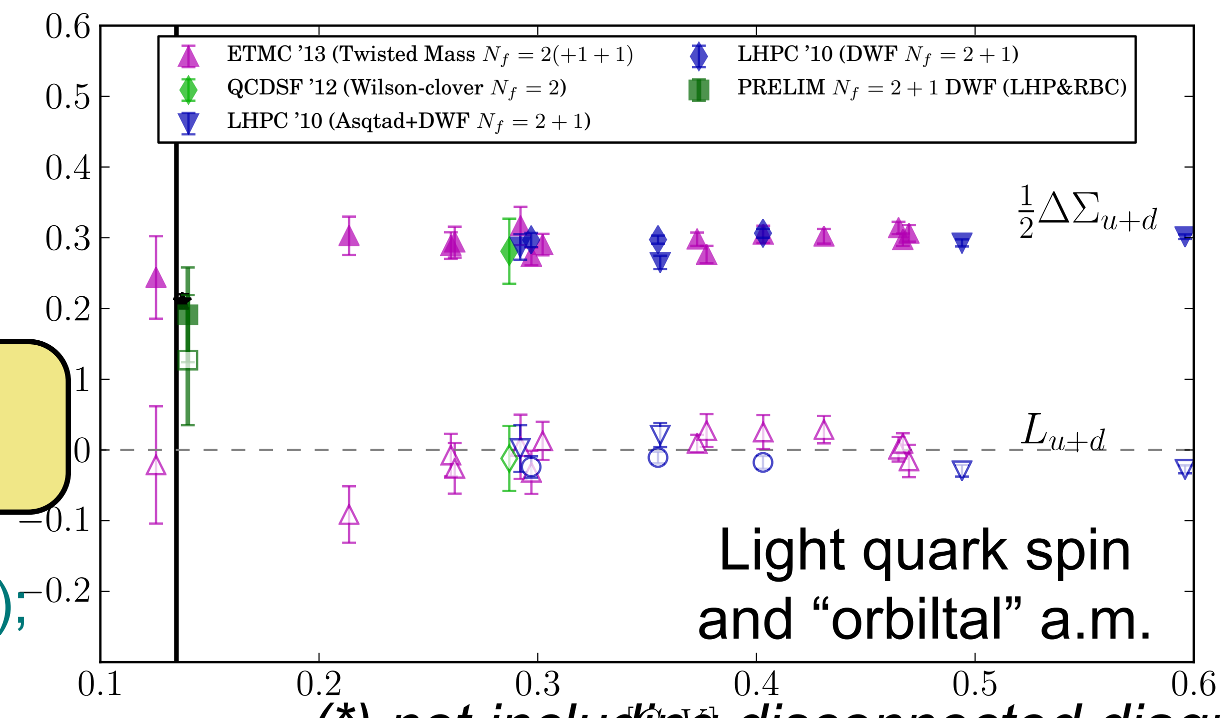




$$J_u \approx 40 - 50\%$$

$$|J_d| \lesssim 10\%$$

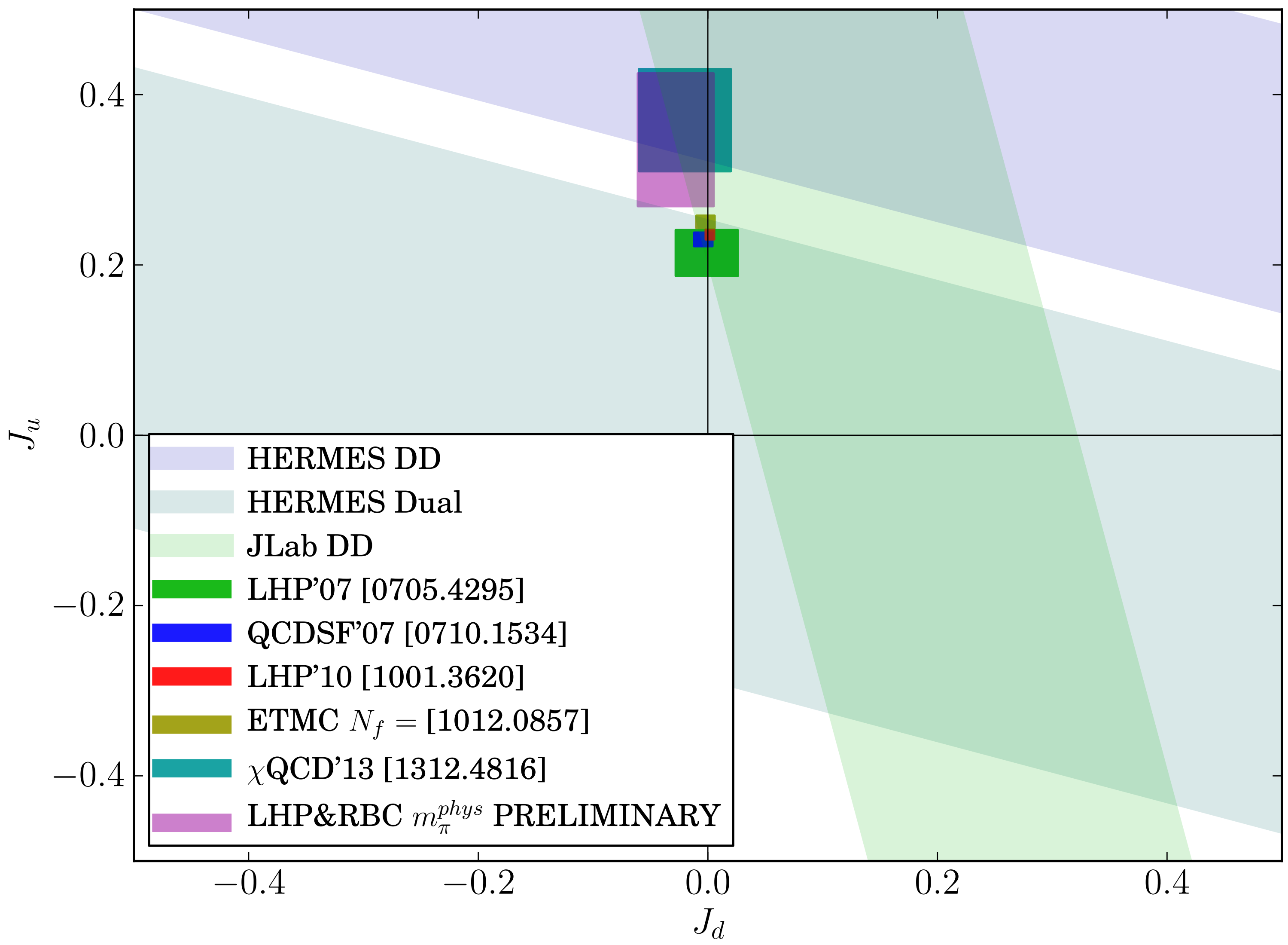
$$|L_{u+d}| \ll \frac{1}{2}\Delta\Sigma_{u+d}$$



First in [Ph.Hagler et al, PRD77:094502 '08 (LHPC); D.Brommel et at, arXiv:0710.1534(QCDSF)]

(*) not including disconnected diagrams!

Quark Angular Momentum



Phenomenological bands from HERMES & JLab
[Airapetian et al, JHEP 06, 006 (2008)]

The most precise LQCD values : from ChPT extrapolations of $m_\pi \gtrsim 300$ MeV data

(*) disconnected contributions are not included

Summary

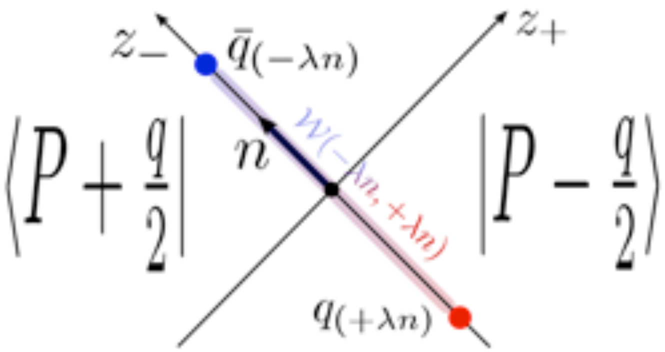
- Realistic calculations of nucleon structure on a lattice
  multiple lattice groups pursue calculations with physical light quarks
- Nucleon electromagnetic form factors agree with experiment
  lattice QCD results may be important for the "proton radius puzzle"
- Nucleon axial charge and radius : persistent disagreement
  axial charge : 10-15% ; axial radius : x(1/2)
- Lattice QCD predicts peculiar structure of light quark angular momentum
  full angular momentum $|J_u| \gg |J_d|$, *total orbital angular momentum* $|L_{u+d}| \ll |L_{u,d}|$

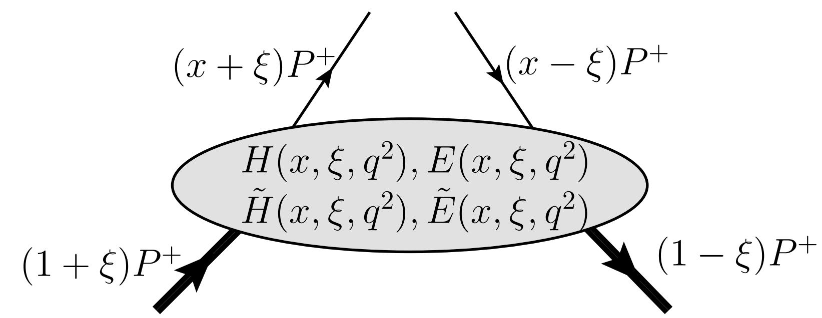
BACKUP

Quark GPDs

Generalized Parton Distributions probe quarks with

$$\mathcal{O}^{[\gamma^5]}(x) = \int \frac{d\lambda}{2\pi}\, e^{ix(2\lambda n\cdot P)}\, \bar{q}_{(-\lambda n)} \Big[\not{n}\, [\gamma^5]\, \mathcal{W}(-\lambda n, \lambda n) \Big] q_{(\lambda n)}$$





Helicity-independent and dependent operator matrix elements -> GPDs :

$$\langle P+q/2|\mathcal{O}(x)|P-q/2\rangle = \bar{u}_{P+q/2}\big[\mathcal{H}(x,\xi,q^2)\,\not{n} + \mathcal{E}(x,\xi,q^2)\,\frac{i\sigma^{\mu\nu}n_\mu q_\nu}{2m}\big]u_{P-q/2}$$

$$\langle P+q/2|\mathcal{O}^{\gamma_5}(x)|P-q/2\rangle = \bar{u}_{P+q/2}\big[\tilde{\mathcal{H}}(x,\xi,q^2)\,\not{n}\gamma_5 + \tilde{\mathcal{E}}(x,\xi,q^2)\,\frac{(n\cdot q)\gamma_5}{2m}\big]u_{P-q/2}$$

- forward case (*ξ=0, q=0*) : regular PDFs
- no gauge link (λ=0) : vector & axial-vector current

General non-forward kinematics (*q≠0*) :
Distribution of partons in the transverse plane ($\mathbf{b}_\perp$)

Reviewed in detail [Diehl, Phys.Rept.388:41 ’03]

Generalized Form Factors (quarks)

Mellin moments of the LC operator produce local operators :

$$\mathcal{O}_n = \int x^{n-1}\, dx\, \mathcal{O}(x) \longrightarrow \bar{q}\big[\not{n}(i\overleftrightarrow{D}\cdot n)^n\big]q = \mathcal{O}_{\{\mu_1\cdots\mu_n\}}\, n_{\mu_1}\cdots n_{\mu_n}$$

and may be computed on a lattice

$$\mathcal{O}_{\{\mu_1\cdots\mu_n\}} = \bar{q}\Big[\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}} - \langle traces\rangle\Big]q$$

GPDs $\mathcal{H}(x,\xi,q^2)$, $\mathcal{E}(x,\xi,q^2)$ are reduced to Generalized Form Factors

$$\int x^{n-1}\, dx\, \mathcal{H}(x,\xi,q^2) \longrightarrow \sum_{i=0}^{[n/2]}(2\xi)^{2i} A_{n,2i}(q^2) \quad \big[+(2\xi)^n C_n(q^2)\,,\ \text{even } n\big]\,,$$

$$\int x^{n-1}\, dx\, \mathcal{E}(x,\xi,q^2) \longrightarrow \sum_{i=0}^{[n/2]}(2\xi)^{2i} B_{n,2i}(q^2) \quad \big[-(2\xi)^n C_n(q^2)\,,\ \text{even } n\big]\,,$$

$$\int x^{n-1}\, dx\, \tilde{\mathcal{H}}(x,\xi,q^2) \longrightarrow \sum_{i=0}^{[n/2]}(2\xi)^{2i} \tilde{A}_{n,2i}(q^2)\,,$$

$$\int x^{n-1}\, dx\, \tilde{\mathcal{E}}(x,\xi,q^2) \longrightarrow \sum_{i=0}^{[n/2]}(2\xi)^{2i} \tilde{B}_{n,2i}(q^2)\,,$$

n=1: vector & axial form factors
n=2: A_{20}, B_{20}, C_{20} : energy-mom.
(quark momentum and J_q)

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$$\mathcal{O}_{\{\mu_1\cdots\mu_n\}} = \bar{q}\Big[\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}} - \langle traces\rangle\Big] q$$

GPDs $\mathcal{H}(x,\xi,q^2)$, $\mathcal{E}(x,\xi,q^2)$ are reduced to Generalized Form Factors

$$\int x^{n-1}\, dx\, \mathcal{H}(x,\xi,q^2) \longrightarrow \sum_{i=0}^{[n/2]} (2\xi)^{2i} A_{n,2i}(q^2) \quad \big[+ (2\xi)^n C_n(q^2)\,,\ \text{even } n\big]\,,$$

$$\int x^{n-1}\, dx\, \mathcal{E}(x,\xi,q^2) \longrightarrow \sum_{i=0}^{[n/2]} (2\xi)^{2i} B_{n,2i}(q^2) \quad \big[- (2\xi)^n C_n(q^2)\,,\ \text{even } n\big]\,,$$

$$\int x^{n-1}\, dx\, \tilde{\mathcal{H}}(x,\xi,q^2) \longrightarrow \sum_{i=0}^{[n/2]} (2\xi)^{2i} \tilde{A}_{n,2i}(q^2)\,,$$

$$\int x^{n-1}\, dx\, \tilde{\mathcal{E}}(x,\xi,q^2) \longrightarrow \sum_{i=0}^{[n/2]} (2\xi)^{2}$$

n=1: vector & axial form factors
n=2: A_{20}, B_{20}, C_{20} : energy-mom.
(quark momentum and J_q)

Experiments do not have direct access to n>1 GFFs:
- not full region of *x* is measured
- DVCS access GPDs only at $x = \xi$

Twist-2 Operators on a Hypercubic Lattice

Mellin moments of GPDs : symmetric, trace=0 quark-bilinear operators :

- In continuum: Lorentz symmetry preserves ops. from mixing
- On a lattice: Hypercubic group has only 20 irreps

$$\begin{aligned}
n&=1 & \bar q\gamma_\mu q &\to \mathbf{4}_1^- \\
n&=2 & \bar q\big[\gamma_{\{\mu} i\overleftrightarrow{D}_{\nu\}} - \langle\mathrm{Tr}\rangle\big]q &\to \mathbf{3}_1^+ \oplus \mathbf{6}_3^+ \\
n&=3 & \bar q\big[\gamma_{\{\mu} i\overleftrightarrow{D}_{\nu} i\overleftrightarrow{D}_{\rho\}} - \langle\mathrm{Tr}\rangle\big]q &\to \mathbf{8}_1^- \oplus \mathbf{4}_1^- \oplus \mathbf{4}_2^- \\
n&=4 & \bar q\big[\gamma_{\{\mu} i\overleftrightarrow{D}_{\nu} i\overleftrightarrow{D}_{\rho} i\overleftrightarrow{D}_{\sigma\}} - \langle\mathrm{Tr}\rangle\big]q &\to \mathbf{1}_1^+ \oplus \mathbf{3}_1^+ \oplus \mathbf{6}_3^+ \oplus \mathbf{2}_1^+ \oplus \mathbf{1}_2^+ \oplus \mathbf{6}_1^+ \oplus \mathbf{6}_2^+ \\
&\dots
\end{aligned}$$

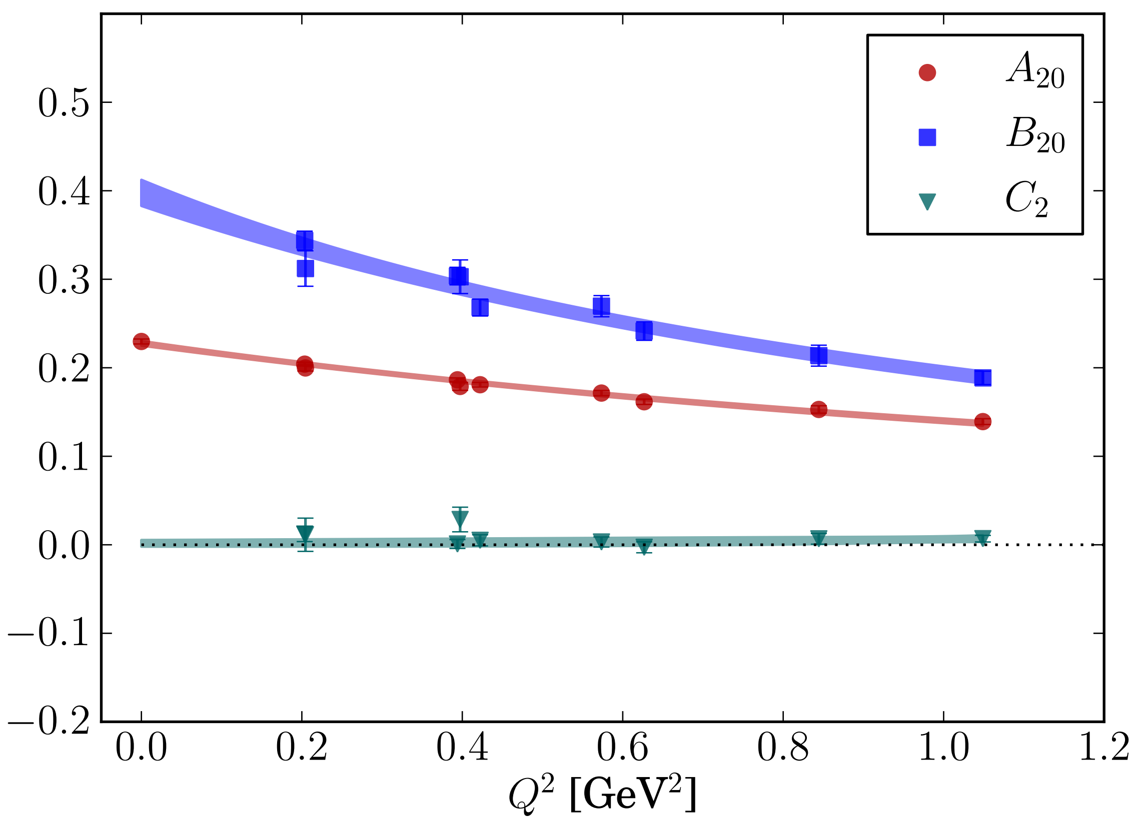
Mixing coefficients $\sim \Lambda_{\mathrm{UV}}^{d_1-d_2} = \left(\frac{1}{a}\right)^{d_1-d_2}$

For *n*=2 : $\mathcal{O}^{\mathrm{lat}} = \mathcal{O}^{\mathrm{phys}} + O(a^2)$

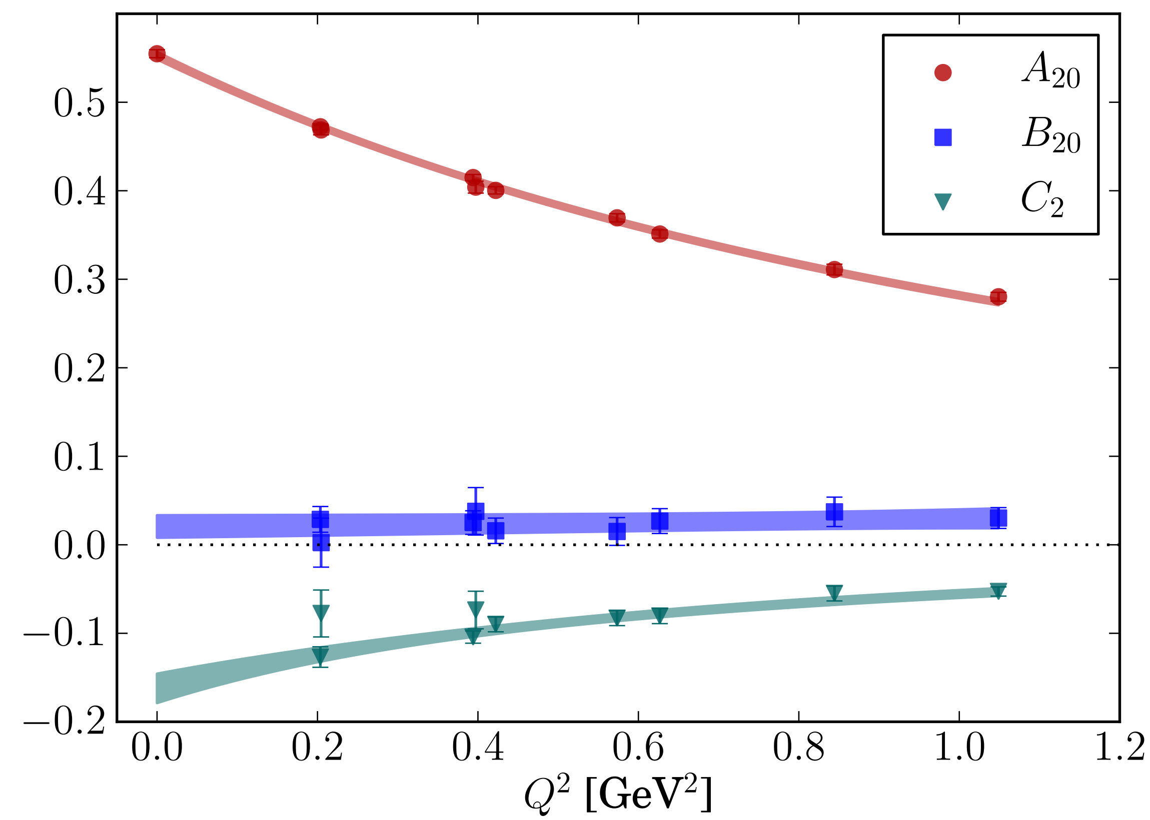
For higher n>4, need subtraction with non-perturbative mixing coefficients

Unpolarized n = 2 GFFs

isovector (*u-d*)

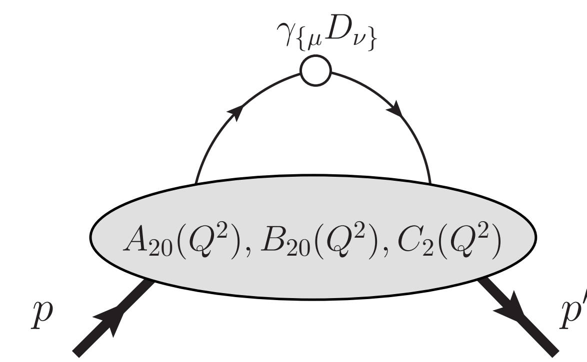


isoscalar (*u+d*)



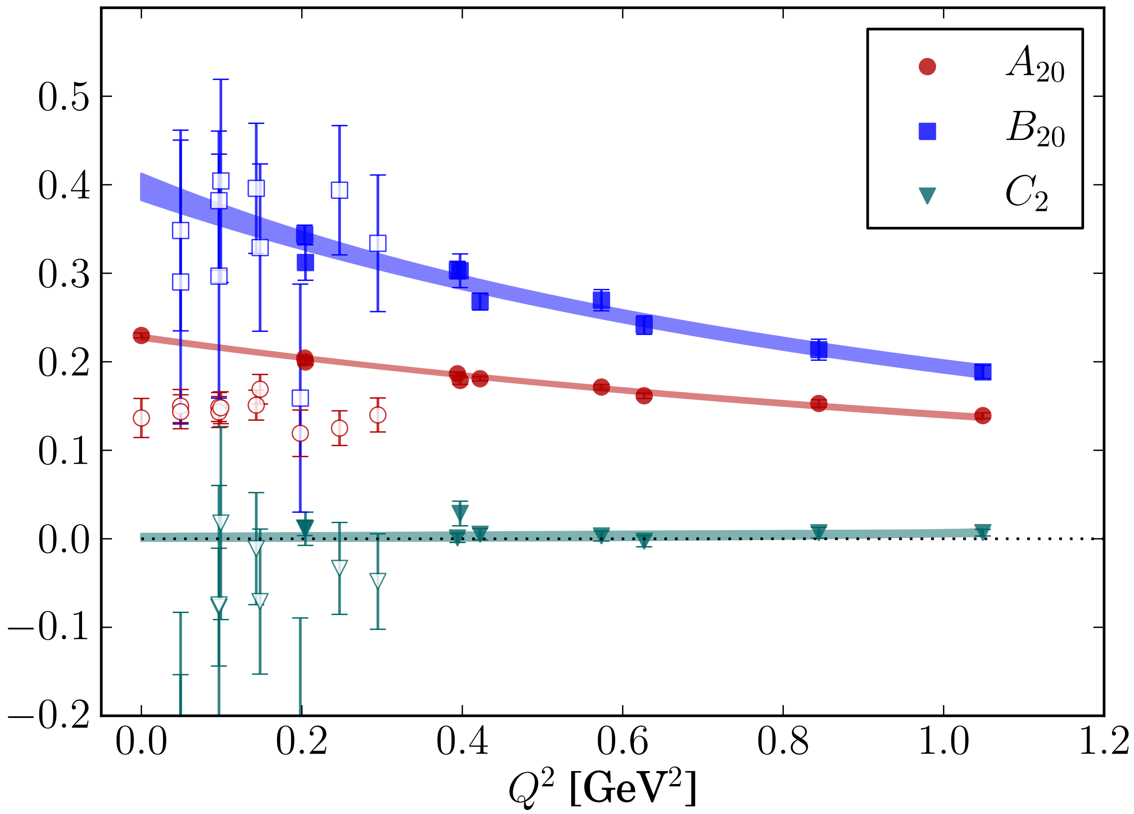
- m_π=350 MeV ;
- $|C_2^{u-d}|$≈0 : little ξ-dependence in the isovector-channel
- large-N_c counting hierarchy:

$$|A_{20}^{u+d}| \gg |A_{20}^{u-d}| \qquad (\sim N_c^2,\, N_c),$$
$$|B_{20}^{u-d}| \gg |B_{20}^{u+d}| \qquad (\sim N_c^3,\, N_c^2),$$
$$|C_2^{u+d}| \gg |C_2^{u-d}| \qquad (\sim N_c^2,\, N_c)$$

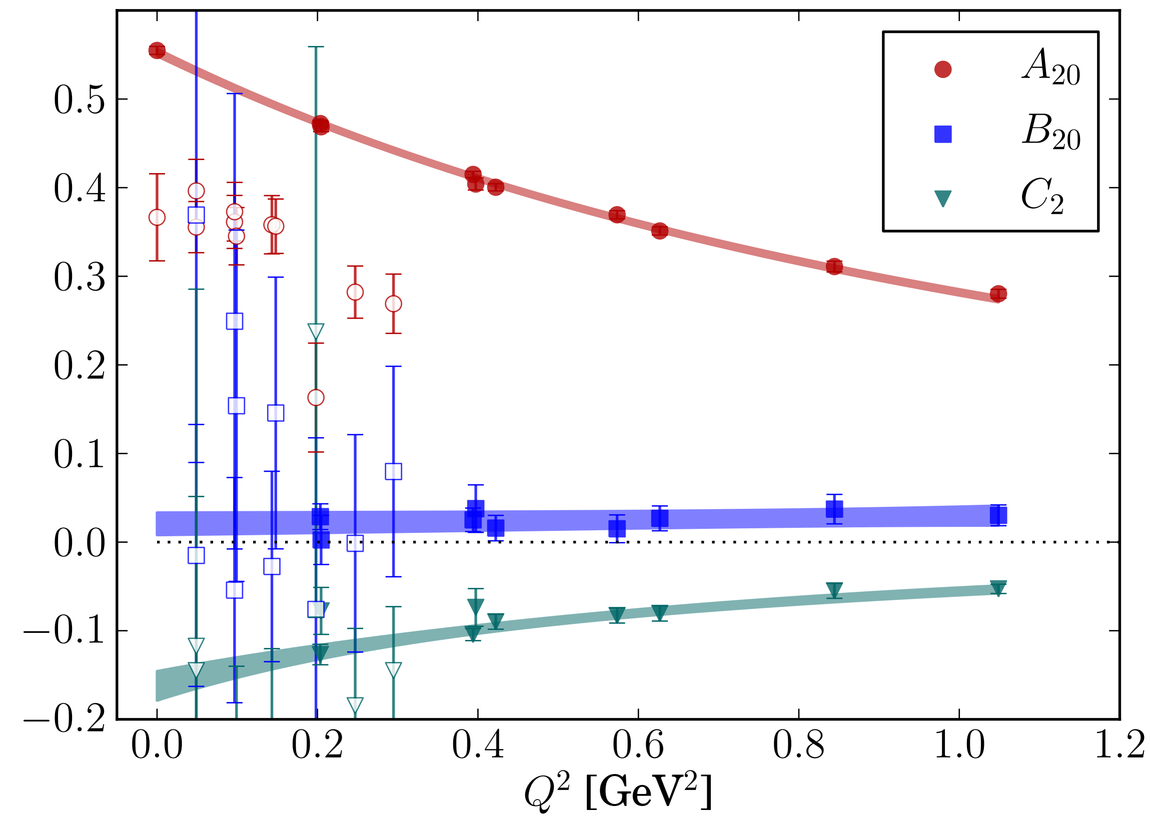


Unpolarized n = 2 GFFs

isovector (*u*-*d*)



isoscalar (*u*+*d*)

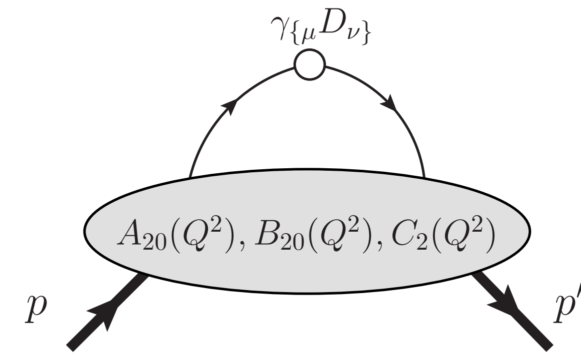


- m_π=350 MeV ; open symbols : m_π=149 MeV
- $|C_2^{u-d}|\approx 0$: little ξ-dependence in the isovector-channel
- large-N_c counting hierarchy:

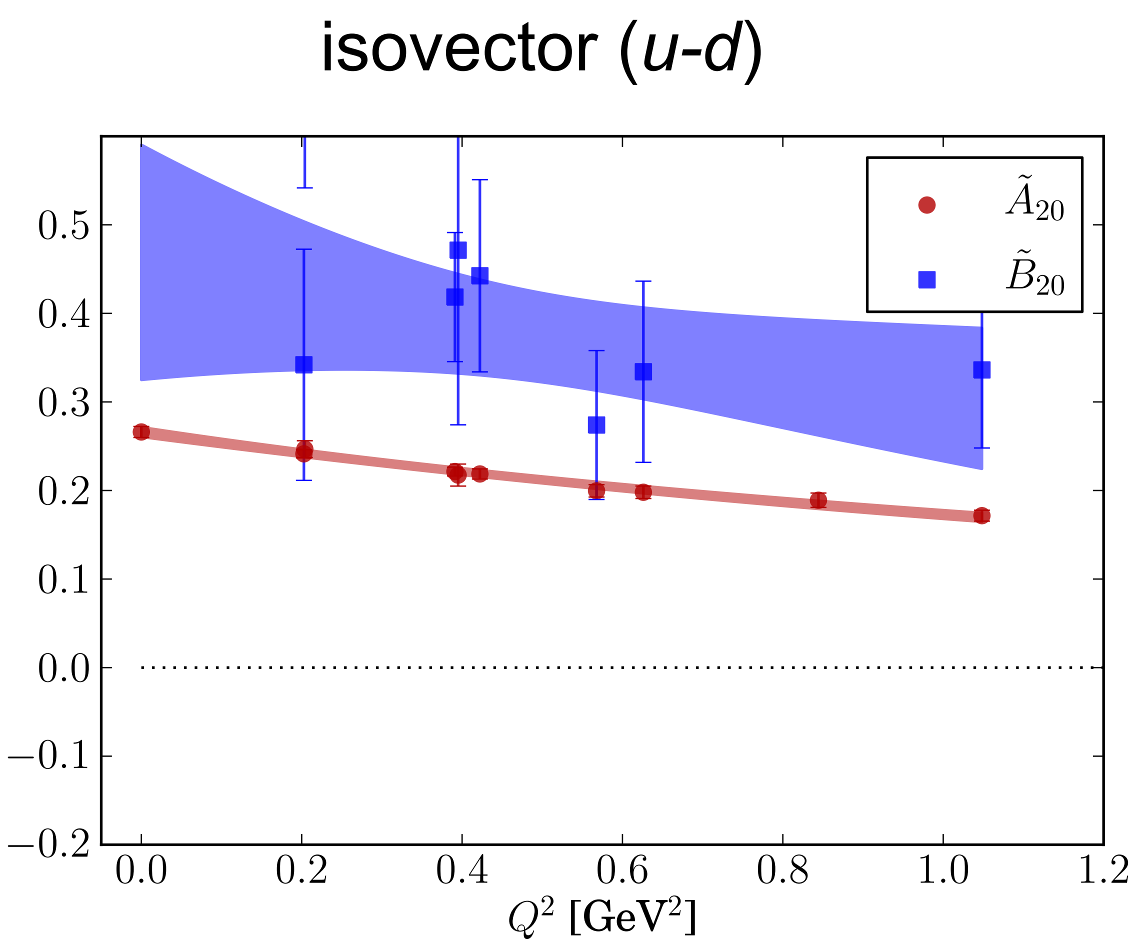
$$|A_{20}^{u+d}| \gg |A_{20}^{u-d}| \qquad (\sim N_c^2,\ N_c),$$

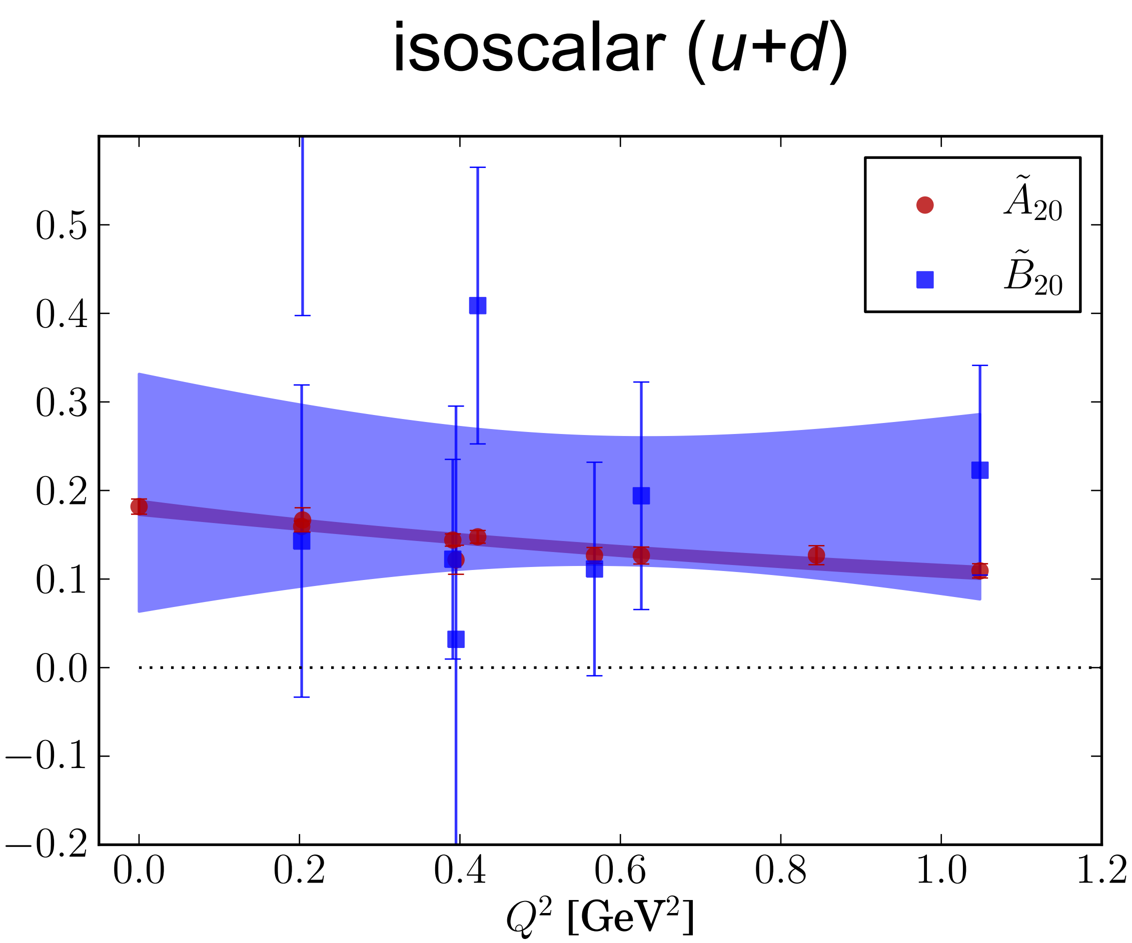
$$|B_{20}^{u-d}| \gg |B_{20}^{u+d}| \qquad (\sim N_c^3,\ N_c^2),$$

$$|C_{2}^{u+d}| \gg |C_{2}^{u-d}| \qquad (\sim N_c^2,\ N_c)$$

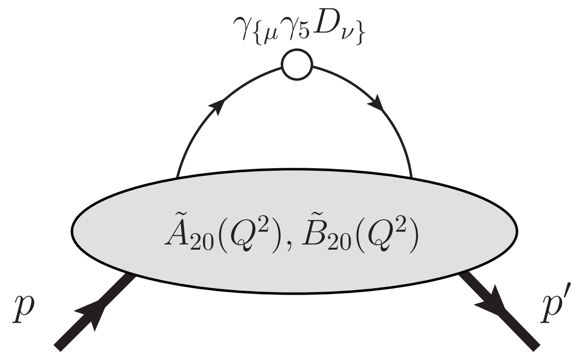


Polarized n = 2 GFFs



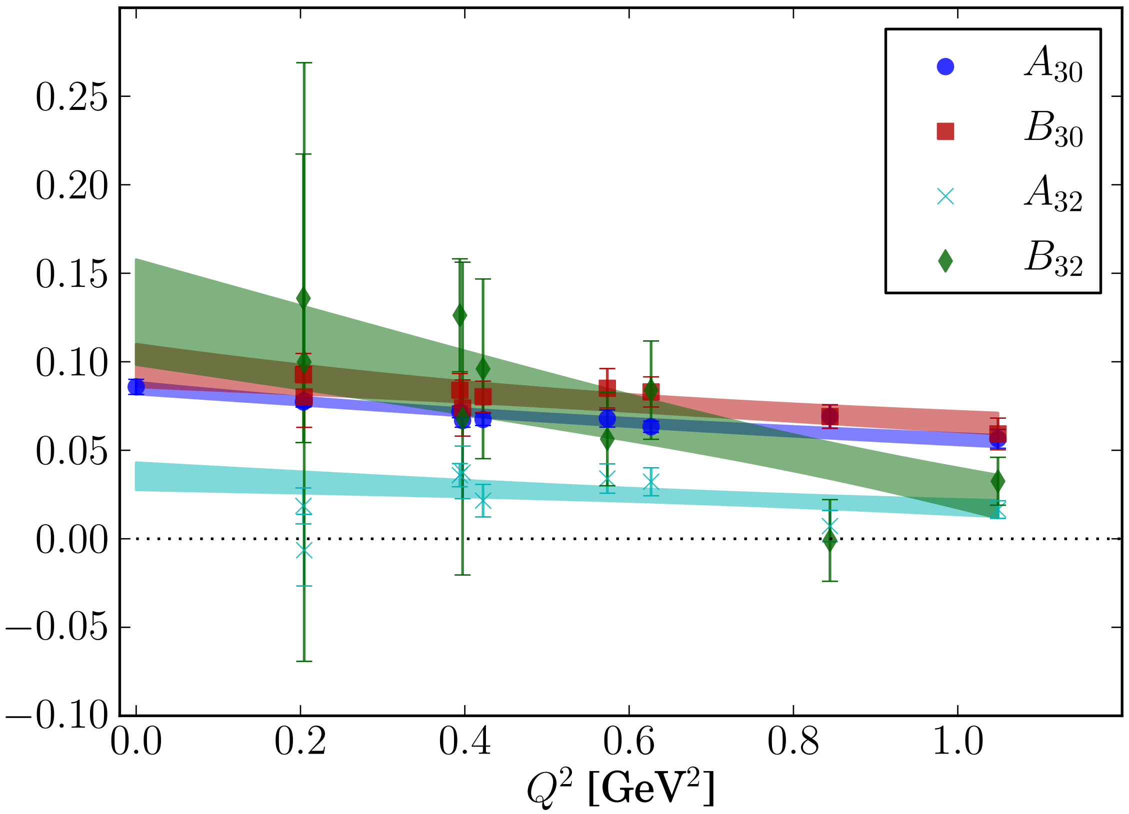


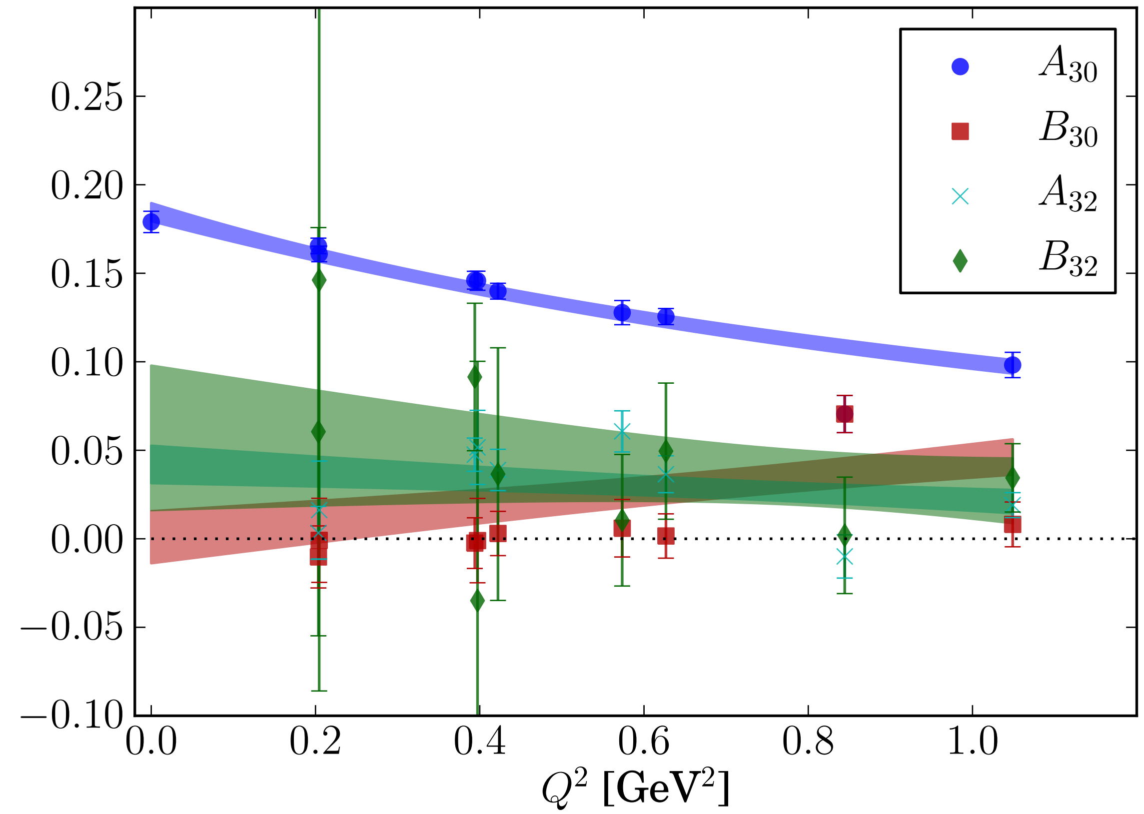
- m_π=350 MeV [LHP collaboration]
- noisy signal for $\tilde{B}_{20}(Q^2)$



Unpolarized n = 3 GFFs

isovector (*u-d*) isoscalar (*u+d*)



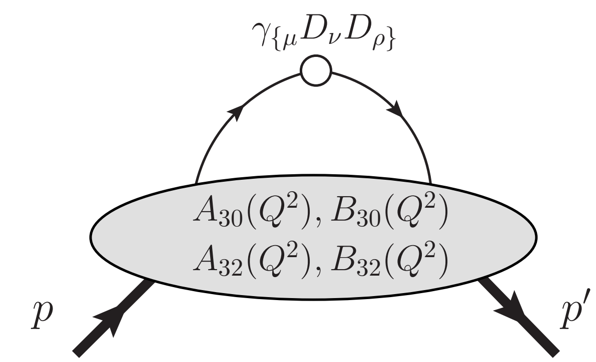


- m_π=350 MeV [LHP collaboration]

- Observable ξ-dependence ~*O(1)* in *n*=3,

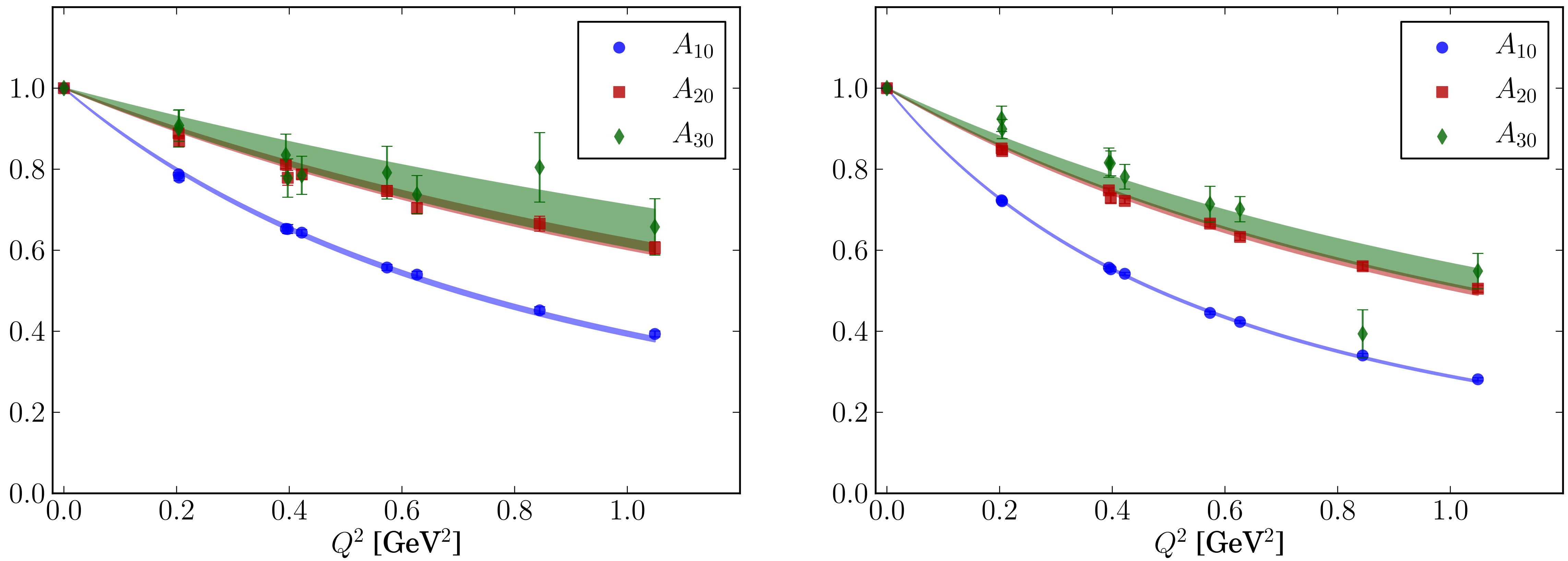
$$\int x^2\,dx\,\mathcal{H}(x,\xi,Q^2) = A_{30}(Q^2) + (2\xi)^2 A_{32}(Q^2),$$

$$\int x^2\,dx\,\mathcal{E}(x,\xi,Q^2) = B_{30}(Q^2) + (2\xi)^2 B_{32}(Q^2)$$



Comparison of unpolarized n = 1, 2, 3 GFFs

isovector (*u*-*d*) isoscalar (*u*+*d*)



- m_π=350 MeV [LHP collaboration]

- Slope of $A_{n0}(Q^2)$ form factors gives transverse radii (in the $\mathbf{b}_\perp$-plane):

$$A_{n0}(Q^2) = A_{n0}(0)\left[1 - \frac{1}{4}\langle r^2_{\perp,n}\rangle Q^2 + \mathcal{O}(Q^4)\right]$$

$$\langle r^2_{\perp,n}\rangle = \frac{\int d^2\vec{b}_\perp (\vec{b}_\perp)^2 \int dx\, x^{n-1}\mathcal{H}(x,\vec{b}_\perp)}{\int d^2\vec{b}_\perp \int dx\, x^{n-1}\mathcal{H}(x,\vec{b}_\perp)}$$

$$\langle r^2_{\perp,1}\rangle > \langle r^2_{\perp,2}\rangle \gtrsim \langle r^2_{\perp,3}\rangle$$

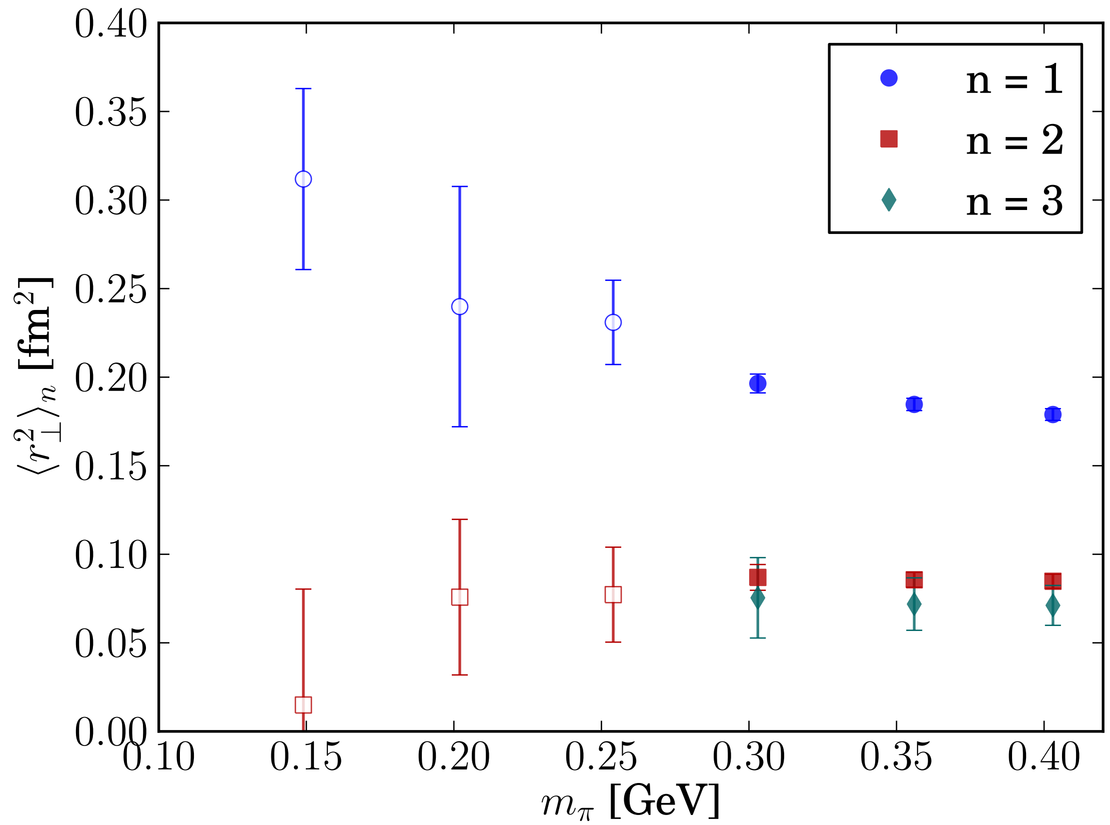
$$\mathcal{H}(x,\vec{b}^2_\perp) \xrightarrow{x\to1} \delta(\vec{b}^2_\perp)$$

x^n-moments of GPDs shrink with *n*

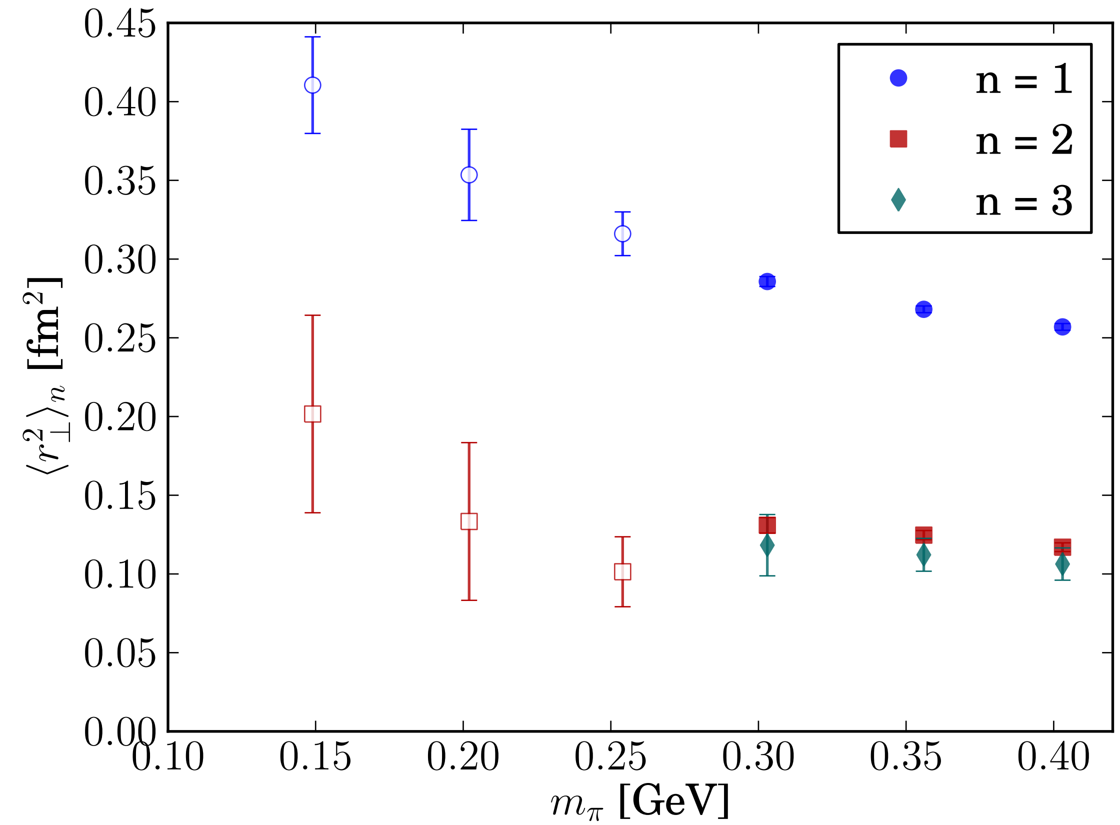
First in [Ph.Hagler *et al*, PRL 93:112001 '04]

Comparison of unpolarized n = 1, 2, 3 GFFs

isovector (*u-d*)



isoscalar (*u+d*)



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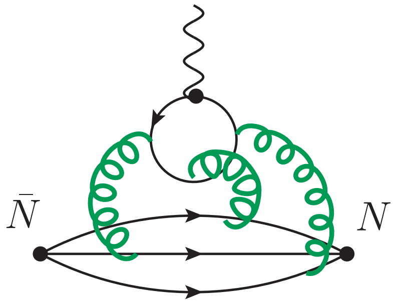
$$\langle r^2_{\perp,1}\rangle > \langle r^2_{\perp,2}\rangle \gtrsim \langle r^2_{\perp,3}\rangle$$

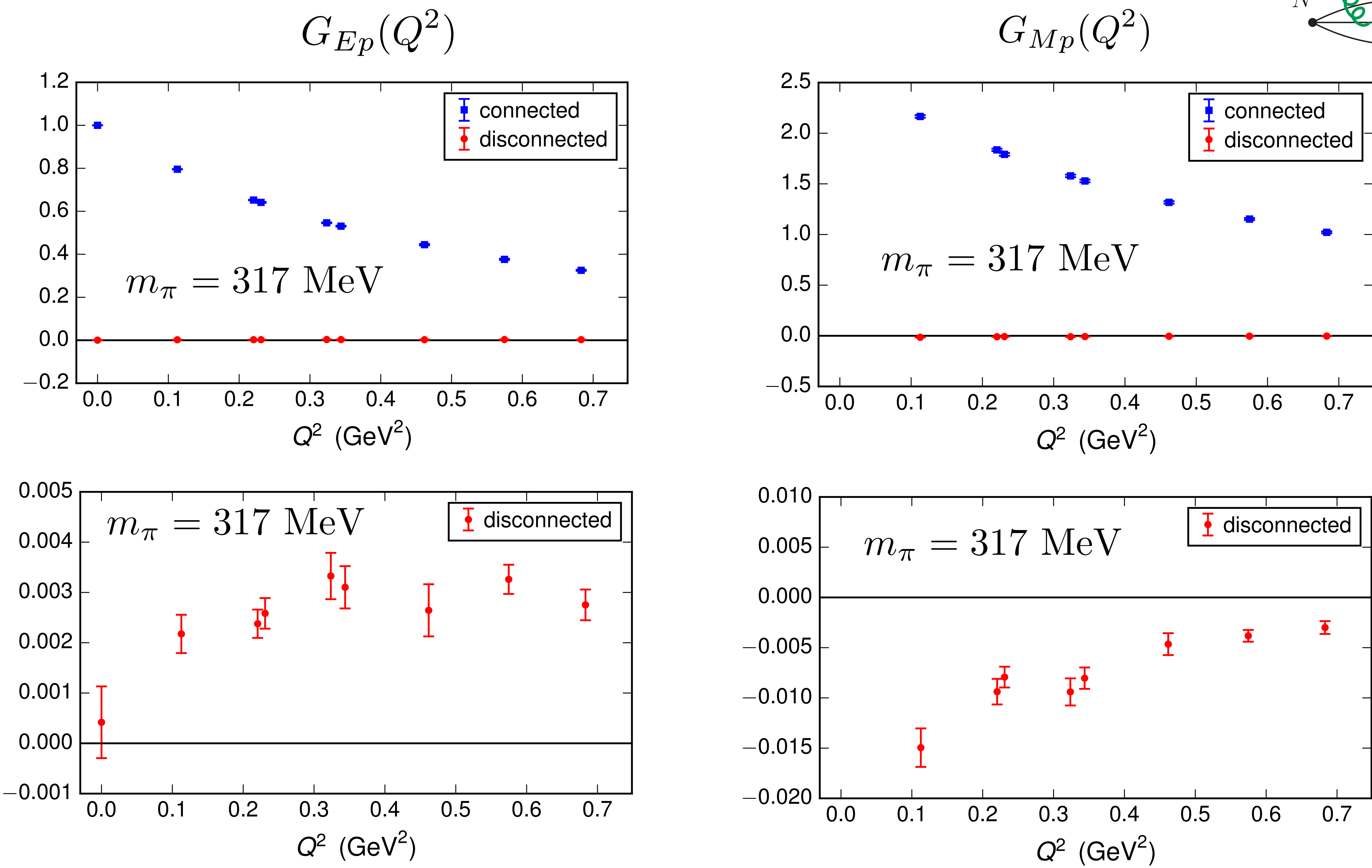
First in [Ph.Hagler *et al*, PRL 93:112001 '04]

“Disconnected” EM Form Factors

Disconnected contributions with hierarchical probing ~ 0.5%

[S.Meinel, Lattice 2014]

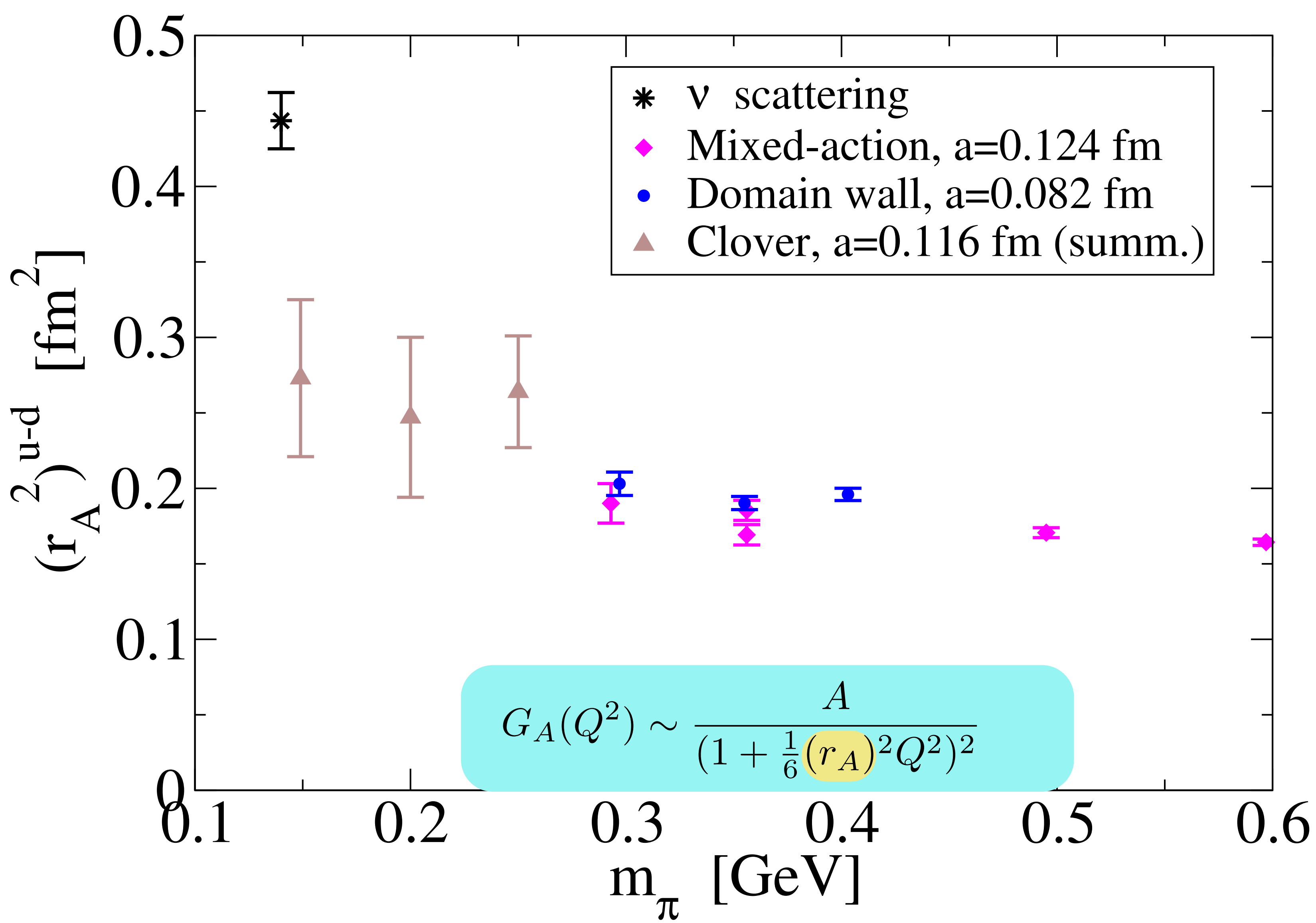




Nucleon Axial Radius

- 5% discrepancy in exp. values of r_A (from $G_A(Q^2)$ dipole fits)

$$\sqrt{\langle r_A^2\rangle_{\nu-\mathrm{scatt.}}} = (0.666 \pm 0.014)\ \mathrm{fm}$$
$$\sqrt{\langle r_A^2\rangle_{el-prod}} = (0.639 \pm 0.010)\ \mathrm{fm}$$

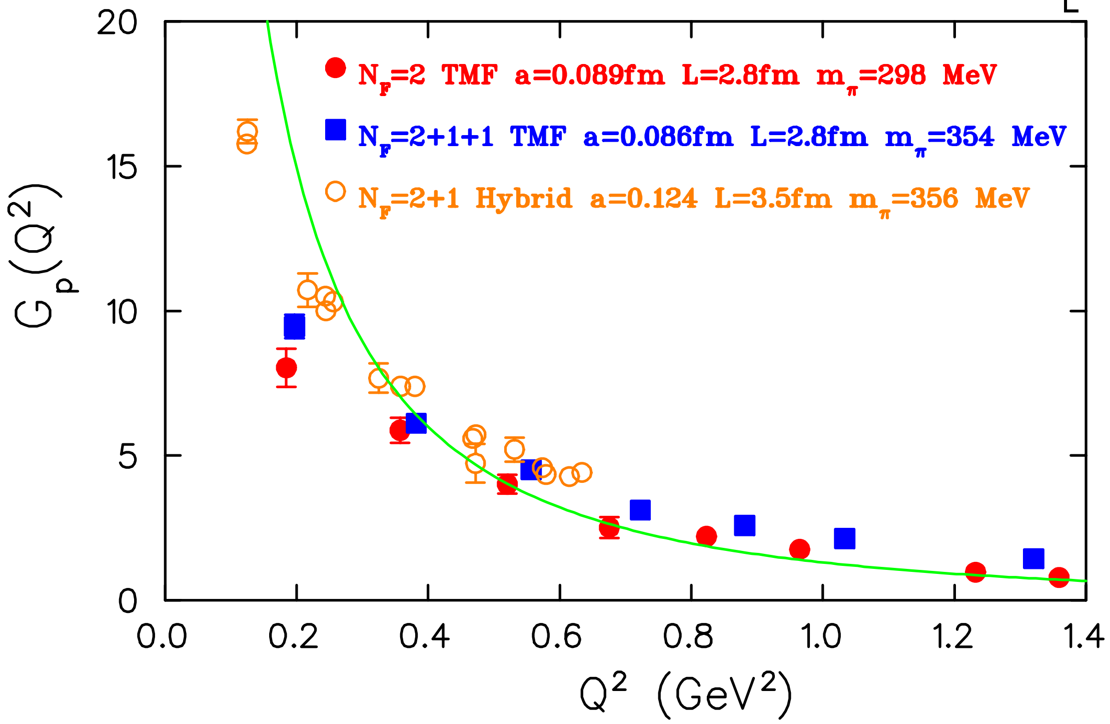


$$G_A(Q^2) \sim \frac{A}{(1+\frac{1}{6}(r_A)^2 Q^2)^2}$$

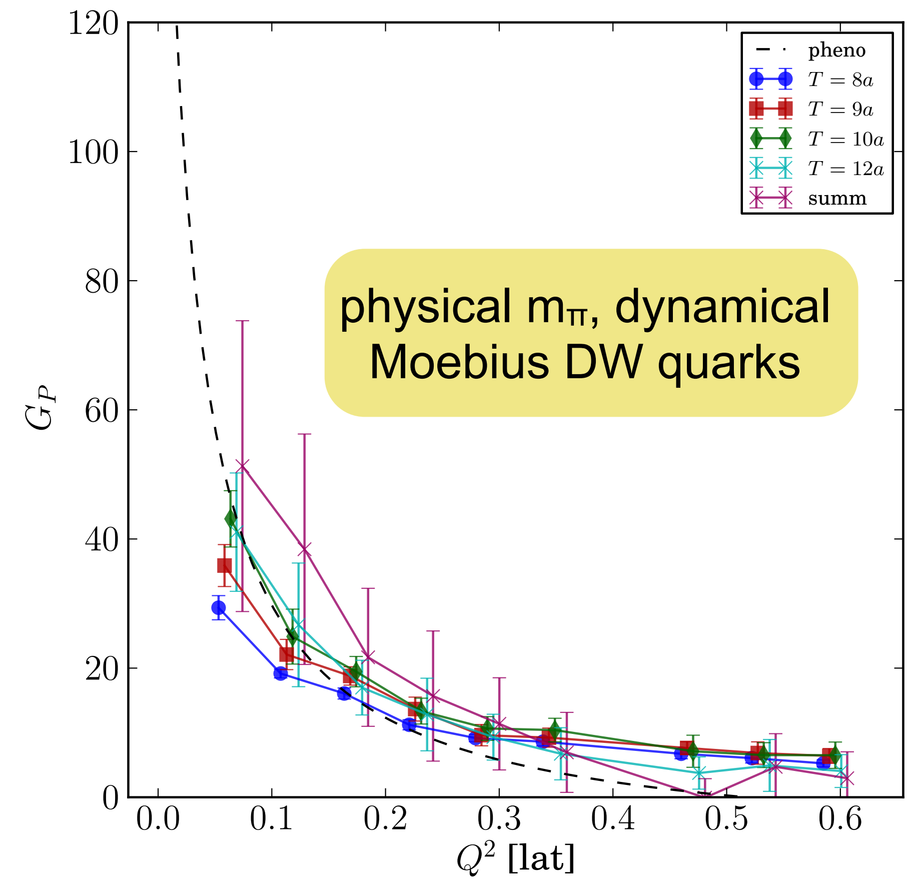
- Weak dependence on m_π and disagreement at m_π^{phys} : same problem as g_A ?
- Study required for volume dependence and exc.states.

Nucleon Pseudoscalar Form Factor $G_P(Q^2)$

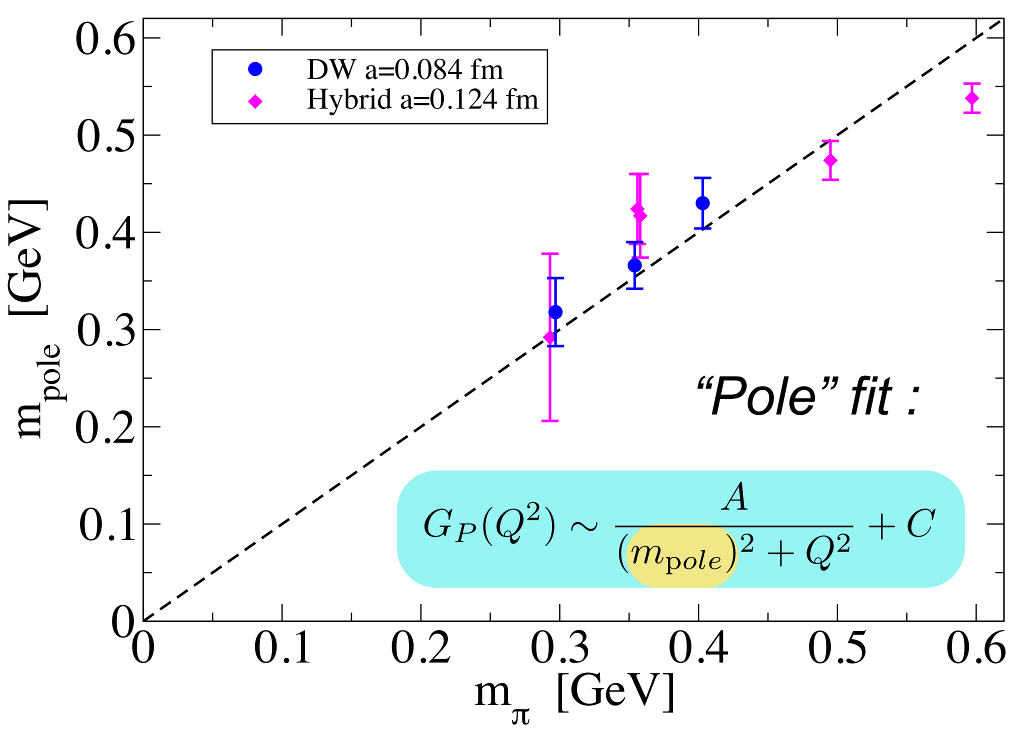
$$\langle P+q|\,\bar{q}\gamma^\mu\gamma^5 q\,|P\rangle \;=\; \bar{U}_{P+q}\Big[G_A(Q^2)\,\gamma^\mu\gamma^5 + G_P(Q^2)\frac{\gamma^5 q^\mu}{2M_N}\Big]U_P$$



- G_P at the physical point : substantial excited states



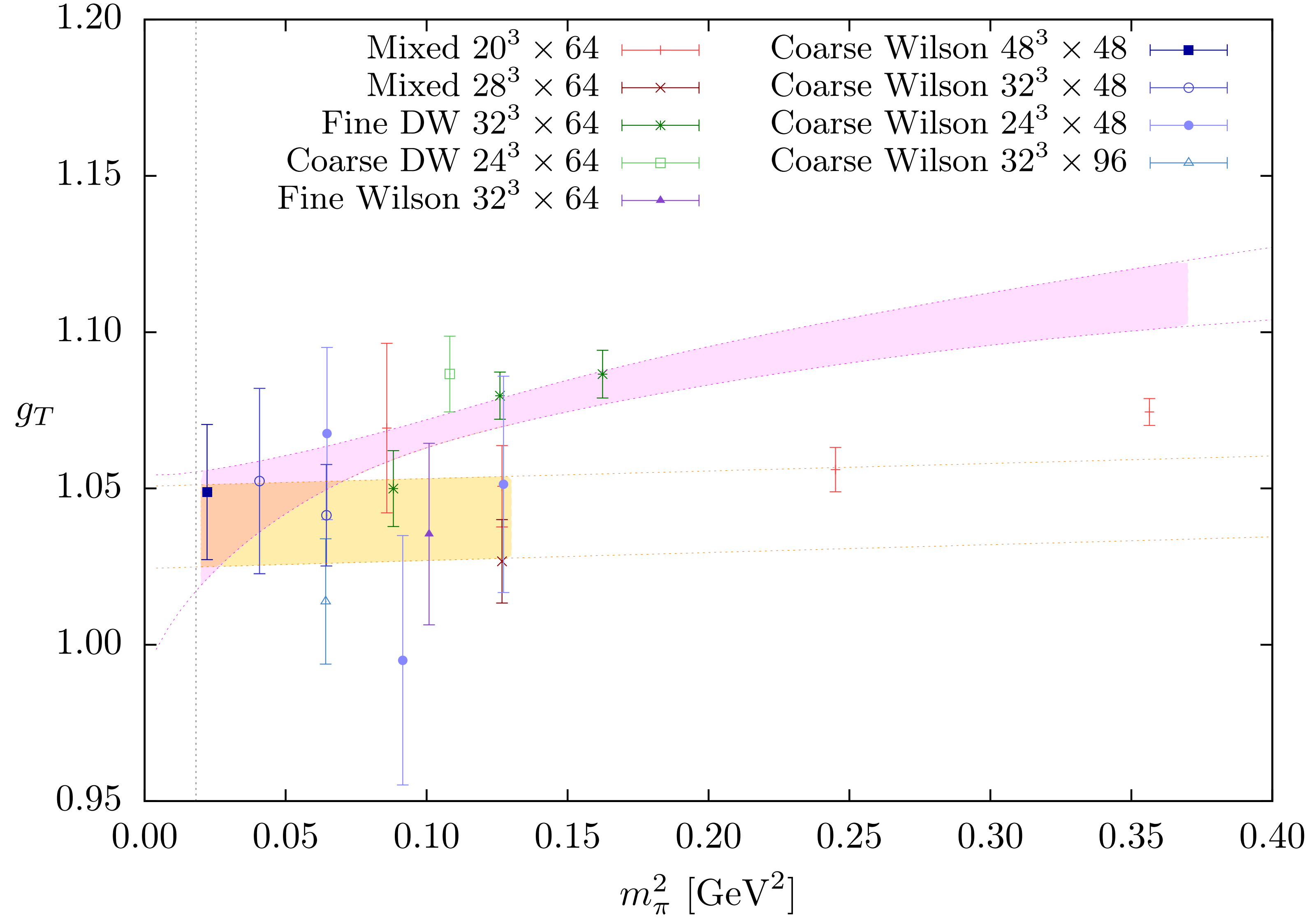
- Is G_P dominated by the pion pole ?



Nucleon Tensor Charge (u-d)

$$\langle N(P)|\bar{u}\sigma_{\mu\nu}u - \bar{d}\sigma_{\mu\nu}d|N(P)\rangle = g_T \bar{u}_P \sigma_{\mu\nu} u_P$$

sensitivity of BSM searches in ultracold neutron decays



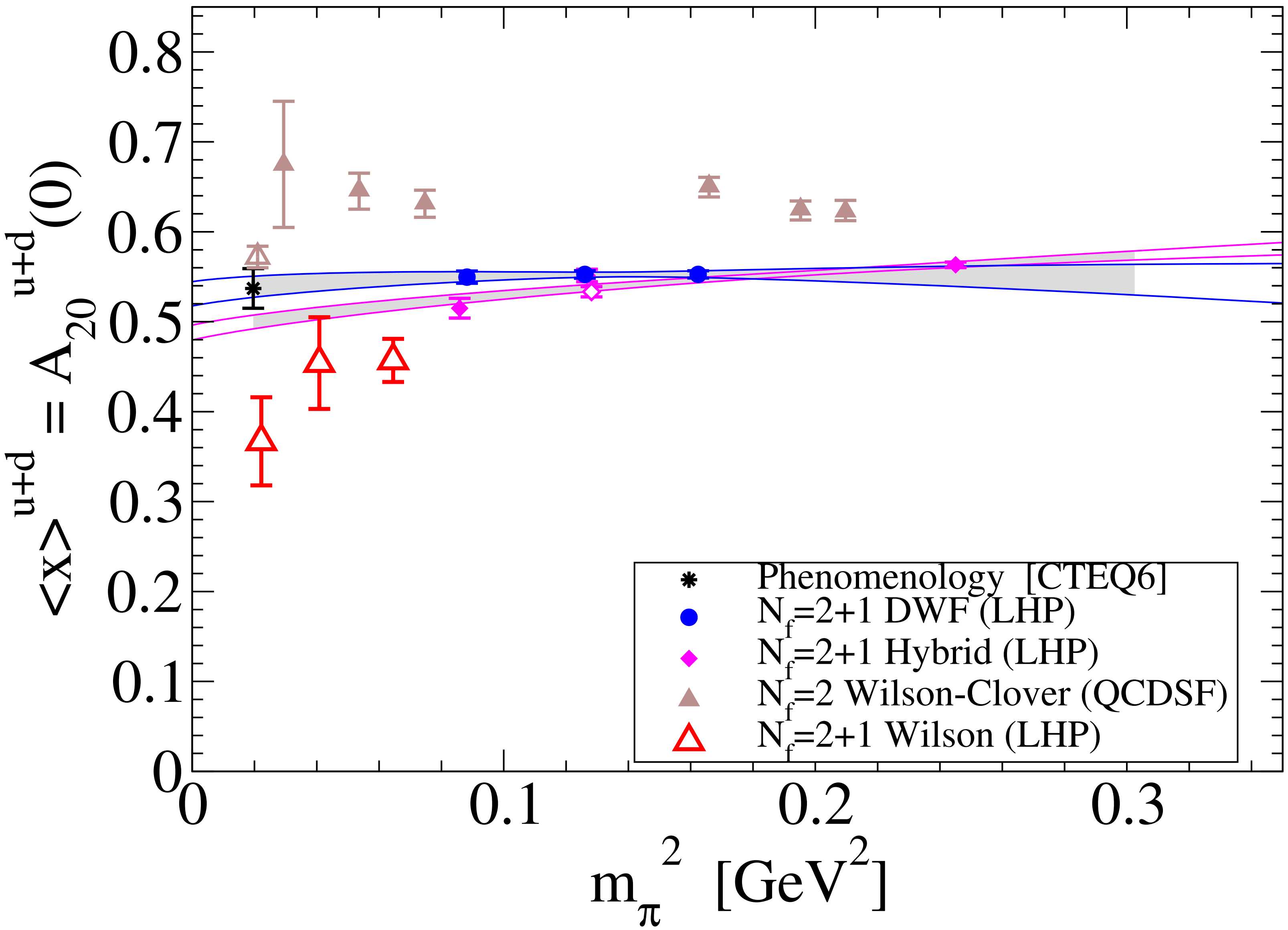
$$g_T = 1.049(23)$$

[LHP collab., PRD86:114509 '12]

Light Quark Momentum Fraction (u+d)

$$\langle x \rangle_{u+d} = \int dx \left(u(x) + \bar{u}(x) + d(x) + \bar{d}(x) \right)$$

$$\langle N(p) | \bar{q} \gamma_{\{\mu} \overleftrightarrow{D}_{\nu\}} q | p \rangle = \langle x \rangle_q \; \bar{u}_p \gamma_{\{\mu} p_{\nu\}} u_p$$



(*) disconnected contributions are not included