On the phenomenological determination of the pion–nucleon σ-term

Martin Hoferichter

Institut für Kernphysik, Technische Universität Darmstadt

ExtreMe Matter Institute EMMI, GSI, Darmstadt

TECHNISCHE UNIVERSITÄT DARMSTADT

Bundesministerium für Bildung und Forschung

INT Workshop INT-14-57W

Nuclear Aspects of Dark Matter Searches

EMMI

Minerva Stiftung ARCHES Award for Research Cooperation and High Excellence in Science

Seattle, December 10, 2014

C. Ditsche, MH, B. Kubis, U.-G. Meißner, JHEP 1206 (2012) 043, JHEP 1206 (2012) 063

A. Crivellin, MH, M. Procura, PRD 89 (2014) 054021

MH, J. Ruiz de Elvira, B. Kubis, U.-G. Meißner, in preparation

Nuclear matrix elements: scalar couplings

Scalar couplings for *u*- and *d*-quark

$$f_u^N = \frac{\sigma_{\pi N}(1-\xi)}{2m_N} + \Delta f_u^N \qquad f_d^N = \frac{\sigma_{\pi N}(1+\xi)}{2m_N} + \Delta f_d^N$$

$$\Delta f_u^p = (1.0 \pm 0.2) \times 10^{-3} \qquad \Delta f_u^n = (-1.0 \pm 0.2) \times 10^{-3}$$

$$\Delta f_d^p = (-2.1 \pm 0.4) \times 10^{-3} \qquad \Delta f_d^n = (2.0 \pm 0.4) \times 10^{-3}$$

$$\sigma_{\pi N} = \frac{1}{2}\Big(\langle p|\hat{m}(\bar{u}u + \bar{d}d)|p\rangle + \langle n|\hat{m}(\bar{u}u + \bar{d}d)|n\rangle\Big) \qquad \hat{m} = \frac{m_d + m_u}{2} \qquad \xi = \frac{m_d - m_u}{m_d + m_u} = 0.36 \pm 0.04 \quad \text{(FLAG)}$$

- **Nuclear matrix elements**: $\langle N|m_q\bar{q}q|N\rangle = f_q^N m_N \qquad N \in \{p, n\}$
- For *u*- and *d*-quark: **$SU(2)$ ChPT + Feynman–Hellmann** Crivellin, MH, Procura 2014

  $\hookrightarrow$ free of $SU(3)$ assumptions Ellis et al. 2000, micrOMEGAs, Cheng 1989
- Couplings determined by **pion–nucleon σ-term** $\sigma_{\pi N}$, corrections by c_5 (strong proton–neutron mass difference)

Status of the phenomenological determination of $\sigma_{\pi N}$

- **Karlsruhe/Helsinki** partial-wave analysis KH80 Höhler et al. 1980s

  $\hookrightarrow$ comprehensive analyticity constraints, old data

- Formalism for the extraction of $\sigma_{\pi N}$ via the **Cheng–Dashen low-energy theorem**

  Gasser, Leutwyler, Locher, Sainio 1988, Gasser, Leutwyler, Sainio 1991

  $\hookrightarrow$ "canonical value" $\sigma_{\pi N} \sim 45\,\text{MeV}$, based on KH80 input

- **GWU/SAID** partial-wave analysis Pavan, Strakovsky, Workman, Arndt 2002

  $\hookrightarrow$ much larger value $\sigma_{\pi N} = (64 \pm 8)\,\text{MeV}$

- ChPT fits vary according to PWA input Fettes, Meißner 2000

  (same problem in different regularizations (w/ and w/o Δ) Alarcón et al. 2012)

Status of the phenomenological determination of $\sigma_{\pi N}$

- **Karlsruhe/Helsinki** partial-wave analysis KH80 Höhler et al. 1980s

  $\hookrightarrow$ comprehensive analyticity constraints, old data

- Formalism for the extraction of $\sigma_{\pi N}$ via the **Cheng–Dashen low-energy theorem**
  Gasser, Leutwyler, Locher, Sainio 1988, Gasser, Leutwyler, Sainio 1991

  $\hookrightarrow$ "canonical value" $\sigma_{\pi N} \sim 45\,\text{MeV}$, based on KH80 input

- **GWU/SAID** partial-wave analysis Pavan, Strakovsky, Workman, Arndt 2002

  $\hookrightarrow$ much larger value $\sigma_{\pi N} = (64 \pm 8)\,\text{MeV}$

- ChPT fits vary according to PWA input Fettes, Meißner 2000

  (same problem in different regularizations (w/ and w/o Δ) Alarcón et al. 2012)

- This talk: two new sources of information on low-energy πN scattering
  - Precision extraction of **πN scattering lengths** from **hadronic atoms**
  - **Roy-equation constraints**: analyticity, unitarity, crossing symmetry

Outline

1. Cheng–Dashen theorem, scalar form factor
2. Scattering lengths, isospin breaking, and πN coupling constant
3. Roy–Steiner equations for πN scattering
4. Conclusions

Extraction of $\sigma_{\pi N}$ from πN scattering

- **Scalar form factor** of the nucleon

$$\sigma(t) = \langle N(p')|\hat{m}(\bar{u}u + \bar{d}d)|N(p)\rangle \qquad t = (p' - p)^2 \qquad \sigma_{\pi N} = \sigma(0)$$

- **Low-energy theorem** Cheng, Dashen 1971

$$F_\pi^2 \bar{D}^+(\nu = 0, t = 2M_\pi^2) = \sigma(2M_\pi^2) + \Delta_R$$

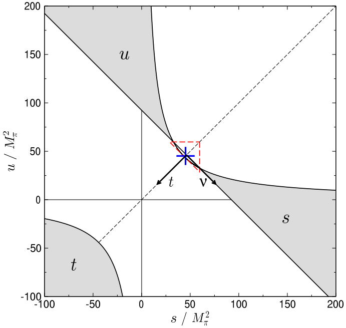
Extraction of $\sigma_{\pi N}$ from πN scattering

- **Scalar form factor** of the nucleon

$$\sigma(t) = \langle N(p')|\hat{m}(\bar{u}u + \bar{d}d)|N(p)\rangle \qquad t = (p'-p)^2 \qquad \sigma_{\pi N} = \sigma(0)$$

- **Low-energy theorem** Cheng, Dashen 1971

$$\underbrace{F_\pi^2 \bar{D}^+(\nu = 0, t = 2M_\pi^2)}_{F_\pi^2\left(d_{00}^+ + 2M_\pi^2 d_{01}^+\right) + \Delta_D} = \sigma(2M_\pi^2) + \Delta_R$$



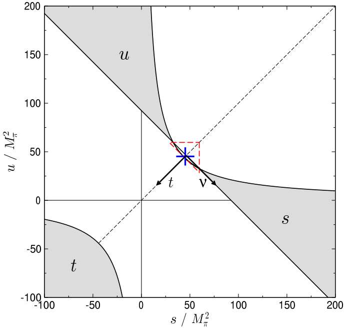
Extraction of $\sigma_{\pi N}$ from πN scattering

- **Scalar form factor** of the nucleon

$$\sigma(t) = \langle N(p')|\hat{m}(\bar{u}u + \bar{d}d)|N(p)\rangle \qquad t = (p'-p)^2 \qquad \sigma_{\pi N} = \sigma(0)$$

- **Low-energy theorem** Cheng, Dashen 1971

$$\underbrace{F_\pi^2 \bar{D}^+(\nu = 0, t = 2M_\pi^2)}_{F_\pi^2\left(d_{00}^+ + 2M_\pi^2 d_{01}^+\right) + \Delta_D} = \underbrace{\sigma(2M_\pi^2)}_{\sigma_{\pi N} + \Delta_\sigma} + \Delta_R$$



Extraction of $\sigma_{\pi N}$ from πN scattering

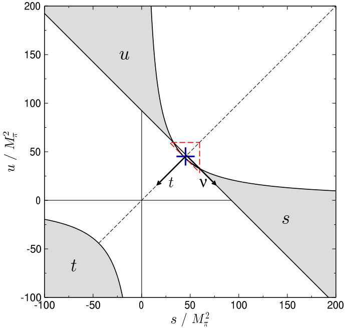
- **Scalar form factor** of the nucleon

$$\sigma(t) = \langle N(p')|\hat{m}(\bar{u}u + \bar{d}d)|N(p)\rangle \qquad t = (p'-p)^2 \qquad \sigma_{\pi N} = \sigma(0)$$

- **Low-energy theorem** Cheng, Dashen 1971

$$\underbrace{F_\pi^2 \bar{D}^+(\nu = 0, t = 2M_\pi^2)}_{F_\pi^2\left(d_{00}^+ + 2M_\pi^2 d_{01}^+\right) + \Delta_D} = \underbrace{\sigma(2M_\pi^2)}_{\sigma_{\pi N} + \Delta_\sigma} + \Delta_R$$

- Remainder $|\Delta_R| \lesssim 2\,\text{MeV}$ small Bernard, Kaiser, Meißner 1996



Extraction of $\sigma_{\pi N}$ from πN scattering

- **Scalar form factor** of the nucleon

$$\sigma(t) = \langle N(p')|\hat{m}(\bar{u}u + \bar{d}d)|N(p)\rangle \qquad t = (p'-p)^2 \qquad \sigma_{\pi N} = \sigma(0)$$

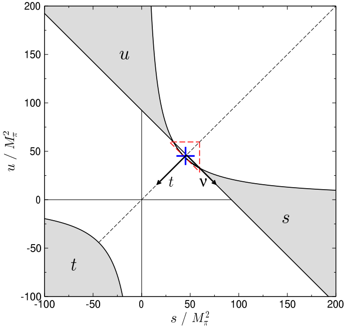
- **Low-energy theorem** Cheng, Dashen 1971

$$\underbrace{F_\pi^2 \bar{D}^+(\nu = 0, t = 2M_\pi^2)}_{F_\pi^2\left(d_{00}^+ + 2M_\pi^2 d_{01}^+\right) + \Delta_D} = \underbrace{\sigma(2M_\pi^2)}_{\sigma_{\pi N} + \Delta_\sigma} + \Delta_R$$

- Remainder $|\Delta_R| \lesssim 2\,\text{MeV}$ small Bernard, Kaiser, Meißner 1996
- **Dispersive approach** Gasser, Leutwyler, Sainio 1991

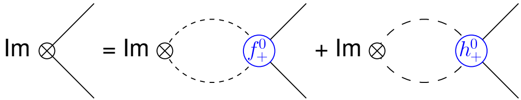
$$\Delta_D - \Delta_\sigma = (-3.3 \pm 0.2)\,\text{MeV}$$

but error only covers $\pi\pi$ phase shifts



Dispersion relation for the scalar form factor of the nucleon

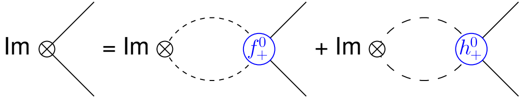
- Unitarity relation



$$\mathrm{Im}\,\sigma(t) = \frac{2}{4m^2-t}\left\{\frac{3}{4}\sigma_t^\pi\big(F_\pi^S(t)\big)^* f_+^0(t) + \sigma_t^K\big(F_K^S(t)\big)^* h_+^0(t)\right\}$$

Dispersion relation for the scalar form factor of the nucleon

- Unitarity relation



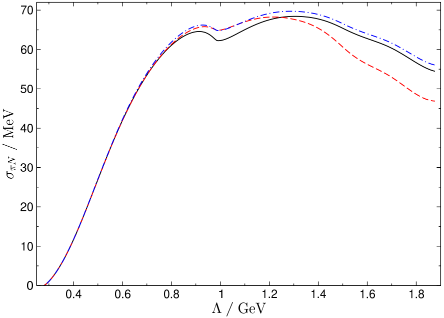
$$\mathrm{Im}\,\sigma(t) = \frac{2}{4m^2-t}\left\{\frac{3}{4}\sigma_t^\pi\left(F_\pi^S(t)\right)^* f_+^0(t) + \sigma_t^K\left(F_K^S(t)\right)^* h_+^0(t)\right\}$$

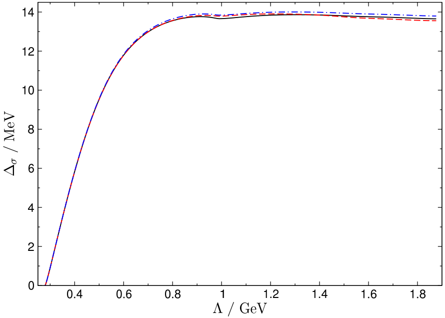
- Dispersion relation

$$\sigma(t) = \frac{1}{\pi}\int\limits_{4M_\pi^2}^{\infty} \mathrm{d}t'\,\frac{\mathrm{Im}\,\sigma(t')}{t'-t} = \sigma_{\pi N} + \frac{t}{\pi}\int\limits_{4M_\pi^2}^{\infty} \mathrm{d}t'\,\frac{\mathrm{Im}\,\sigma(t')}{t'(t'-t)}$$

  - **Unsubtracted**: $\sigma_{\pi N} = \sigma(0)$
  - **Once-subtracted**: $\Delta_\sigma = \sigma(2M_\pi^2) - \sigma_{\pi N}$

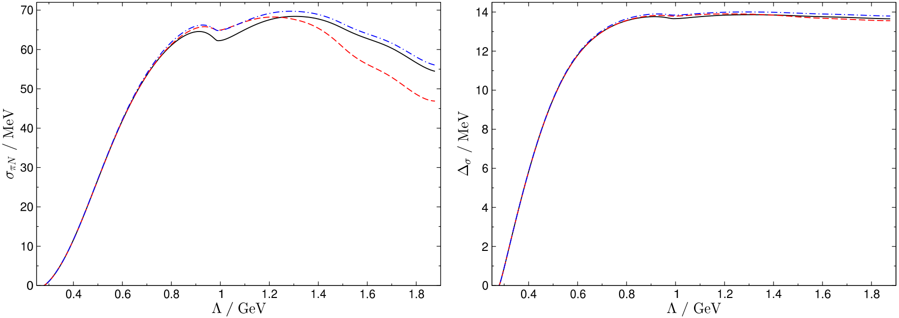
Convergence of the dispersive integral





- **Unsubtracted**: slow convergence
- **Once-subtracted**: stable result for $\Lambda \gtrsim 1$ GeV

Convergence of the dispersive integral



- **Unsubtracted**: slow convergence
- **Once-subtracted**: stable result for $\Lambda \gtrsim 1$ GeV
- Result for Δ_σ depends on pion–nucleon parameters (notation: $\bar{X}(\nu,t) = \sum_{n,m=0}^{\infty} x_{nm}\nu^{2n}t^m$)

$$\Delta_\sigma = (13.9 \pm 0.3)\,\text{MeV}$$

$$+ Z_1\left(\frac{g^2}{4\pi} - 14.28\right) + Z_2\left(d_{00}^+\,M_\pi + 1.46\right) + Z_3\left(d_{01}^+\,M_\pi^3 - 1.14\right) + Z_4\left(b_{00}^+\,M_\pi^3 + 3.54\right)$$

$$Z_1 = 0.36\,\text{MeV} \qquad Z_2 = 0.57\,\text{MeV} \qquad Z_3 = 12.0\,\text{MeV} \qquad Z_4 = -0.81\,\text{MeV}$$

Summary: σ-term corrections

- Scalar form factor

$$\Delta_\sigma = (13.9 \pm 0.3)\,\text{MeV}$$
$$+ Z_1\left(\frac{g^2}{4\pi} - 14.28\right) + Z_2\left(d_{00}^+\, M_\pi + 1.46\right) + Z_3\left(d_{01}^+\, M_\pi^3 - 1.14\right) + Z_4\left(b_{00}^+\, M_\pi^3 + 3.54\right)$$
$$Z_1 = 0.36\,\text{MeV} \qquad Z_2 = 0.57\,\text{MeV} \qquad Z_3 = 12.0\,\text{MeV} \qquad Z_4 = -0.81\,\text{MeV}$$

- πN amplitude

$$\Delta_D = (12.1 \pm 0.3)\,\text{MeV}$$
$$+ \tilde{Z}_1\left(\frac{g^2}{4\pi} - 14.28\right) + \tilde{Z}_2\left(d_{00}^+\, M_\pi + 1.46\right) + \tilde{Z}_3\left(d_{01}^+\, M_\pi^3 - 1.14\right) + \tilde{Z}_4\left(b_{00}^+\, M_\pi^3 + 3.54\right)$$
$$\tilde{Z}_1 = 0.42\,\text{MeV} \qquad \tilde{Z}_2 = 0.67\,\text{MeV} \qquad \tilde{Z}_3 = 12.0\,\text{MeV} \qquad \tilde{Z}_4 = -0.77\,\text{MeV}$$

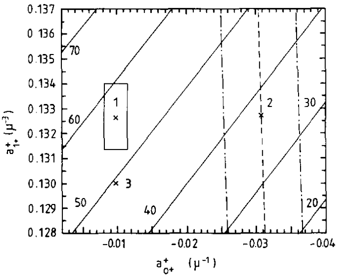
$\hookrightarrow$ most of the dependence on the πN parameters cancels in the difference!

Full correction

$$\Delta_D - \Delta_\sigma = (-1.8 \pm 0.2)\,\text{MeV}$$

How to get d_{00}^{+} and d_{01}^{+}?

- Standard approach Gasser, Leutwyler, Locher, Sainio 1988: replace d_{00}^{+} and d_{01}^{+} in favor of **threshold parameters**: a_{0+}^{+} and a_{1+}^{+} (notation: $a_{\ell\pm 1/2}^{I=\pm}$)
  $\hookrightarrow$ corrections from PWA via DRs (D^{+} and E^{+})



- **Coupling constant**: $g^2/4\pi = 14.28/13.75$
- ***S*-wave**: $a_{0+}^{+} = -10/0 \times 10^{-3} M_\pi^{-1}$
- ***P*-wave**: a_{1+}^{+} needs to be known very precisely

	Born	a_{0+}^{+}	a_{1+}^{+}	D^{+}	E^{+}	Σ_d
KH80	-133	-7	$+352$	-91	-72	50
FA01	-127	0	$+351$	-88	-69	67
diff.	$+6$	$+7$	-1	$+3$	$+3$	17

$$\Sigma_d = F_\pi^2 (d_{00}^{+} + 2M_\pi^2 d_{01}^{+})$$

Hadronic atoms: constraints for πN

$$\tilde{a}^+ = a^+ + \frac{1}{4\pi(1+M_\pi/m_p)}\left\{\frac{4(M_\pi^2 - M_{\pi^0}^2)}{F_\pi^2}c_1 - 2e^2 f_1\right\}$$

- $\pi H/\pi D$: bound state of π^- and p/d, spectrum sensitive to threshold πN amplitude
- **Combined analysis** of πH and πD

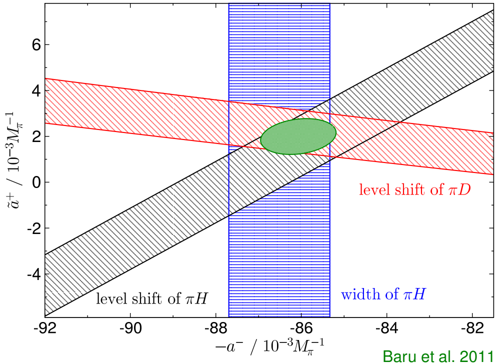
$$a^+ \equiv a^+_{0+} = (7.5 \pm 3.1) \times 10^{-3} M_\pi^{-1}$$
$$a^- \equiv a^-_{0+} = (86.0 \pm 0.9) \times 10^{-3} M_\pi^{-1}$$

- Large a^+ suggests even larger $\sigma_{\pi N}$, but

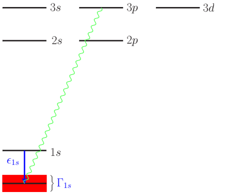
$$\frac{a_{\pi^- p} + a_{\pi^+ p}}{2} = (-1.1 \pm 0.9) \times 10^{-3} M_\pi^{-1}$$

↪ **Isospin breaking** in $\sigma_{\pi N}$ as large as 5 MeV?

↪ Revisit the **Cheng–Dashen** low-energy theorem



Baru et al. 2011



Cheng–Dashen theorem in the presence of isospin breaking

- Define "**isoscalar**" as

$$X^+ \to X^p \equiv \frac{1}{2}\big(X_{\pi^+p\to\pi^+p} + X_{\pi^-p\to\pi^-p}\big) \qquad X = D, d_{00}, d_{01}, a_{0+}, \ldots$$

  and "**isospin limit**" by proton and charged pion

- Assume virtual photons to be removed

  $\hookrightarrow$ scenario closest to actual πN PWAs

- Calculate **IV corrections** in $SU(2)$ ChPT, mainly due to $\Delta_\pi = M_\pi^2 - M_{\pi^0}^2$
  - For the σ-term, no difference at $\mathcal{O}(p^3)$

$$\sigma_{\pi N} = \sigma_p = \sigma_n = -4c_1 M_{\pi^0}^2 - \frac{3g_A^2 M_{\pi^0}^2}{64\pi F_\pi^2}\big(2M_\pi + M_{\pi^0}\big) + \mathcal{O}(M_\pi^4)$$

  - Slope of the scalar form factor

$$\Delta_\sigma^p = \sigma_p(2M_\pi^2) - \sigma_p = \frac{3g_A^2 M_\pi^3}{64\pi F_\pi^2} + \frac{g_A^2 M_\pi \Delta_\pi}{128\pi F_\pi^2}\Big(-7 + \sqrt{2}\log\big(3+2\sqrt{2}\big)\Big) + \mathcal{O}(M_\pi^4)$$

  and similarly for Δ_D^p

Cheng–Dashen theorem in the presence of isospin breaking

- Putting things together

$$\sigma_p = F_\pi^2\big(d_{00}^p + 2M_\pi^2 d_{01}^p\big) + \Delta_D - \Delta_\sigma + \big(\Delta_D^p - \Delta_D\big) - \big(\Delta_\sigma^p - \Delta_\sigma\big)$$
$$+ \sigma_p\big(2M_\pi^2\big) - F_\pi^2 \bar{D}_p\big(0, 2M_\pi^2\big)$$
$$= F_\pi^2\big(d_{00}^p + 2M_\pi^2 d_{01}^p\big) + \underbrace{\Delta_D - \Delta_\sigma}_{(-1.8\pm0.2)\,\text{MeV}} + \underbrace{\Delta_\text{R}}_{\lesssim 2\,\text{MeV}} + \underbrace{\frac{81 g_\text{A}^2 M_\pi \Delta_\pi}{256\pi F_\pi^2}}_{+3.4\,\text{MeV}} + \underbrace{\frac{e^2}{2}F_\pi^2\big(4f_1 + f_2\big)}_{(-0.4\pm2.2)\,\text{MeV}}$$

$\hookrightarrow$ indeed sizable correction from Δ_π, but “wrong” direction

- In the following, determine d_{00}^p and d_{01}^p by solving **Roy–Steiner equations**
- Constraints: scattering lengths from hadronic atoms (virtual-photon subtracted)

$$a_{0+}^{1/2} = (169.8 \pm 2.0) \times 10^{-3} M_\pi^{-1} \qquad a_{0+}^{3/2} = (-86.3 \pm 1.8) \times 10^{-3} M_\pi^{-1}$$

- First step: πN coupling constant

Goldberger–Miyazawa–Oehme sum rule

- Fixed-t dispersion relations at threshold $\Rightarrow$ **GMO sum rule**

$$\frac{g^2}{4\pi} = \left(\left(\frac{m_p + m_n}{M_\pi}\right)^2 - 1\right)\left\{\left(1 + \frac{M_\pi}{m_p}\right)\frac{M_\pi}{4}(a_{\pi^- p} - a_{\pi^+ p}) - \frac{M_\pi^2}{2} J^-\right\}$$

$$= 13.66 \pm 0.12 \pm 0.15$$

$$J^- = \frac{1}{4\pi^2}\int_0^\infty dk \frac{\sigma^{\text{tot}}_{\pi^- p}(k) - \sigma^{\text{tot}}_{\pi^+ p}(k)}{\sqrt{M_\pi^2 + k^2}}$$

- J^- known quite accurately Ericson et al. 2002, Abaev et al. 2007

- Other determinations:

	de Swart et al. 97	Arndt et al. 94	Ericson et al. 02	Bugg et al. 73	KH80
method	NN	πN	GMO	πN	πN
$g^2/4\pi$	13.54 ± 0.05	13.75 ± 0.15	14.11 ± 0.20	14.30 ± 0.18	14.28

- With KH80 scattering lengths $g^2/4\pi = 14.28$ is reproduced exactly!

$\hookrightarrow$ discrepancy for $g^2/4\pi$ related to wrong scattering lengths

Roy equations $=$ Dispersion relations $+$ partial-wave expansion

$+$ crossing symmetry $+$ unitarity

- **Coupled system of integral equations** for partial waves $t^I_J(s)$ Roy 1971

$$t^I_J(s) = k^I_J(s) + \sum_{I'=0}^{2} \sum_{J'=0}^{\infty} \int_{4M_\pi^2}^{\infty} \mathrm{d}s' K^{II'}_{JJ'}(s,s') \,\mathrm{Im}\, t^{I'}_{J'}(s')$$

$\pi\pi$ Roy equations

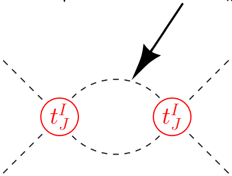
Roy equations $=$ **Dispersion relations** $+$ **partial-wave expansion**

$+$ **crossing symmetry** $+$ **unitarity**

- **Coupled system of integral equations** for partial waves $t^I_J(s)$ Roy 1971

$$\underbrace{t^I_J(s)}_{\frac{e^{2i\delta^I_J(s)}-1}{2i\sigma_\pi(s)}} = k^I_J(s) + \sum_{I'=0}^{2}\sum_{J'=0}^{\infty}\int_{4M_\pi^2}^{\infty} ds'\, K^{II'}_{JJ'}(s,s')\, \underbrace{\text{Im}\, t^{I'}_{J'}(s')}_{\frac{1}{\sigma_\pi(s)}\sin^2\delta^{I'}_{J'}(s')}$$

$$\sigma_\pi(s) = \sqrt{1-\frac{4M_\pi^2}{s}}$$



$\pi\pi$ Roy equations

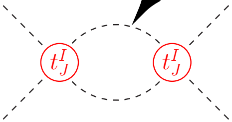
Roy equations $=$ **Dispersion relations** $+$ **partial-wave expansion**

$+$ **crossing symmetry** $+$ **unitarity**

- **Coupled system of integral equations** for partial waves $t_J^I(s)$ Roy 1971

$$\underbrace{t_J^I(s)}_{\frac{e^{2i\delta_J^I(s)}-1}{2i\sigma_\pi(s)}} = \underbrace{k_J^I(s)}_{\delta_0^I a_0^I+\cdots} + \sum_{I'=0}^{2}\sum_{J'=0}^{\infty}\int_{4M_\pi^2}^{\infty} \mathrm{d}s'\, K_{JJ'}^{II'}(s,s')\;\underbrace{\mathrm{Im}\; t_{J'}^{I'}(s')}_{\frac{1}{\sigma_\pi(s)}\sin^2\delta_{J'}^{I'}(s')}$$

$$\sigma_\pi(s) = \sqrt{1-\frac{4M_\pi^2}{s}}$$



free parameters a_0^0, a_0^2

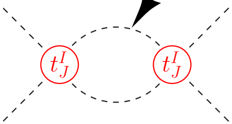
Roy equations $=$ **Dispersion relations** $+$ **partial-wave expansion**

$+$ **crossing symmetry** $+$ **unitarity**

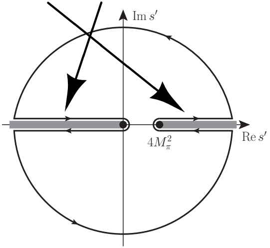
- **Coupled system of integral equations** for partial waves $t_J^I(s)$ Roy 1971

$$\underbrace{t_J^I(s)}_{} = \underbrace{k_J^I(s)}_{\delta_0^I a_0^I+\cdots} + \sum_{I'=0}^{2}\sum_{J'=0}^{\infty}\int_{4M_\pi^2}^{\infty} ds' \underbrace{K_{JJ'}^{II'}(s,s')}_{\frac{1}{\pi}\frac{\delta_{JJ'}\delta_{II'}}{s'-s-i\epsilon}+\bar{K}_{JJ'}^{II'}(s,s')} \underbrace{\operatorname{Im} t_{J'}^{I'}(s')}_{\frac{1}{\sigma_\pi(s)}\sin^2\delta_{J'}^{I'}(s')}$$

$$\sigma_\pi(s) = \sqrt{1-\frac{4M_\pi^2}{s}} \qquad \frac{e^{2i\delta_J^I(s)}-1}{2i\sigma_\pi(s)}$$



free parameters a_0^0, a_0^2



$\hookrightarrow$ **Self-consistency condition** for phase shifts

Flow of information and generalization to πN scattering

- Equations rigorously valid for a finite energy range

  $\hookrightarrow$ introduce **matching point s_m**

- Consider only partial waves for $J \leq J_{max}$
- **Input**
  - High-energy region: $\text{Im}\, t^I_J(s)$ for $s \geq s_m$ and all J
  - Higher partial waves: $\text{Im}\, t^I_J(s)$ for $J > J_{max}$ and all s
  - Inelasticities: $\eta^I_J(s)$ for $J \leq J_{max}$ and $4M_\pi^2 \leq s \leq s_m$
- **Output**
  - Self-consistent solution for phase shifts: $\delta^I_J(s)$ for $J \leq J_{max}$ and $4M_\pi^2 \leq s \leq s_m$
  - Constraints on subtraction constants

Flow of information and generalization to πN scattering

- Equations rigorously valid for a finite energy range

  $\hookrightarrow$ introduce **matching point s_m**

- Consider only partial waves for $J \leq J_\text{max}$
- **Input**
  - High-energy region: $\text{Im}\, t^I_J(s)$ for $s \geq s_\text{m}$ and all J
  - Higher partial waves: $\text{Im}\, t^I_J(s)$ for $J > J_\text{max}$ and all s
  - Inelasticities: $\eta^I_J(s)$ for $J \leq J_\text{max}$ and $4M_\pi^2 \leq s \leq s_\text{m}$
- **Output**
  - Self-consistent solution for phase shifts: $\delta^I_J(s)$ for $J \leq J_\text{max}$ and $4M_\pi^2 \leq s \leq s_\text{m}$
  - Constraints on subtraction constants
- Key challenges for πN:
  - **Crossing**: coupling between $\pi N \to \pi N$ (s-channel) and $\pi\pi \to \bar{N}N$ (t-channel)

    $\hookrightarrow$ hyperbolic dispersion relations Hite, Steiner 1973, Büttiker, Descotes-Genon, Moussallam 2004
  - Unitarity in the t-channel

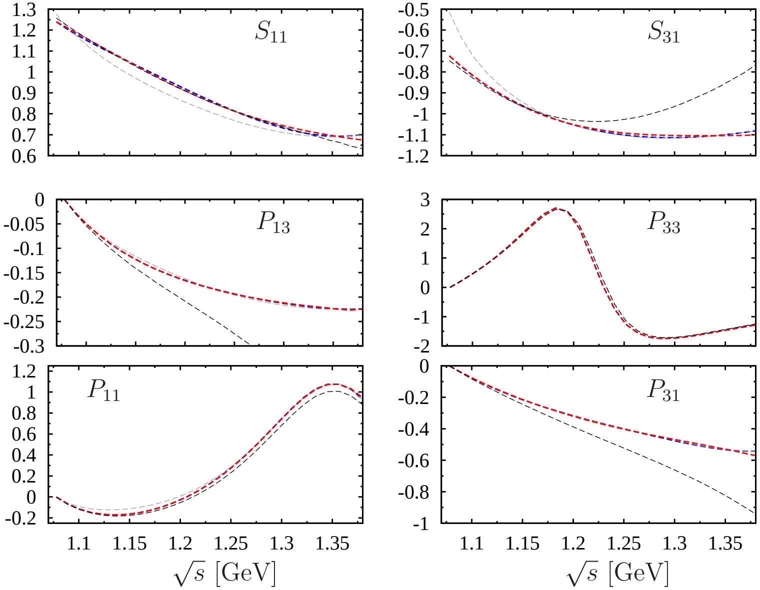
Roy–Steiner solution: strategy

- Introduce as many **subtractions** as necessary to match dof Gasser, Wanders 1999
- Minimize difference between LHS and RHS on a grid of points W_j

$$\chi^2_{\text{RS}} = \sum_{\ell, I_s, \pm} \sum_{j=1}^{N} \left(\frac{\text{Re}\, f^{I_s}_{\ell\pm}(W_j) - F\left[f^{I_s}_{\ell\pm}\right](W_j)}{\text{Re}\, f^{I_s}_{\ell\pm}(W_j)} \right)^2$$

- Impose scattering lengths as constraints in the fit

Roy–Steiner solution: reproducing KH80



blue/red ⇔ LHS/RHS after fit

gray/black ⇔ LHS/RHS before fit

notation: $L_{2I_s 2J}$

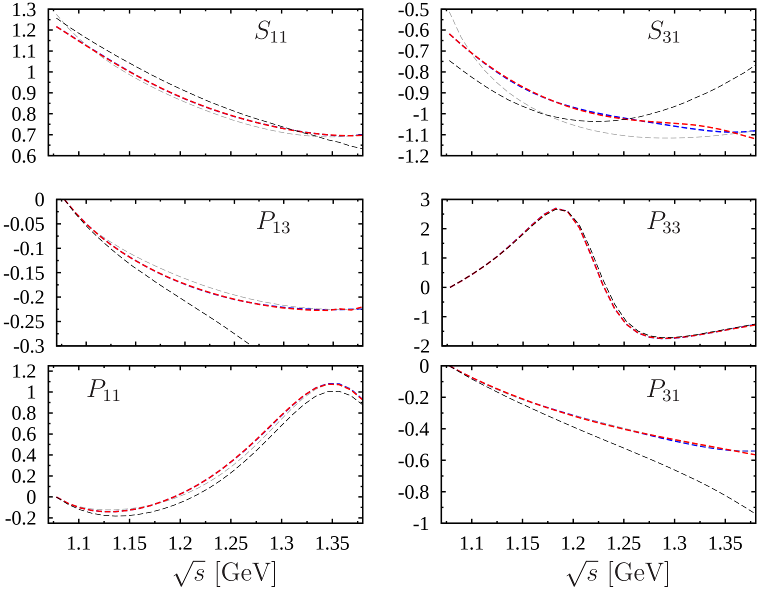
Roy–Steiner solution: reproducing KH80

- Resulting parameters with KH80 scattering lengths as constraint:

	d^+_{00} $[M_\pi^{-1}]$	d^+_{01} $[M_\pi^{-3}]$	$\Sigma_d = F_\pi^2 (d^+_{00} + 2M_\pi^2 d^+_{01})$ [MeV]
KH80	$-1.46(10)$	$1.14(2)$	$50(7)$
KH80 fit	-1.54	1.16	48

- KH80 **internally consistent**

Roy–Steiner solution: hadronic-atom fit



blue/red ⇔ LHS/RHS after fit

gray/black ⇔ LHS/RHS before fit

Roy–Steiner solution: lesson for the σ-term

- Resulting parameters:

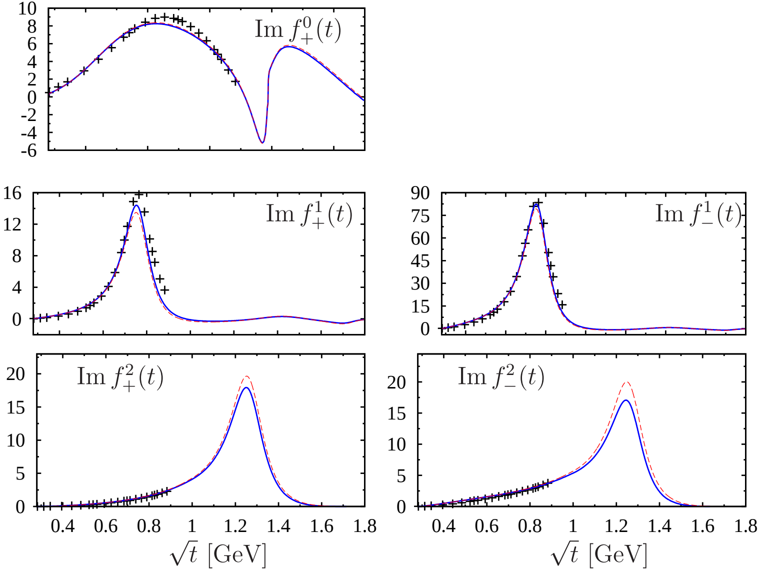
	d^+_{00} $[M_\pi^{-1}]$	d^+_{01} $[M_\pi^{-3}]$	$\Sigma_d = F_\pi^2(d^+_{00} + 2M_\pi^2 d^+_{01})$ [MeV]
KH80	$-1.46(10)$	$1.14(2)$	$50(7)$
KH80 fit	-1.54	1.163	48
hadronic-atom fit	-1.36	1.155	58

- **Modern input** for the scattering lengths does **increase** Σ_d, but by just about half the amount that Pavan, Strakovsky, Workman, Arndt 2002 found
- Compared to Gasser, Leutwyler, Locher, Sainio 1988: necessity for a^+_{1+} eliminated by Roy–Steiner **self-consistency** condition
- Resulting value for $\sigma_{\pi N} \sim 56/59$ MeV with/without IV correction
- Error estimate in progress:
  - Sensitivity to **input quantities**
  - Sensitivity to **truncations**

Conclusions

- Review of **standard procedure** to extract $\sigma_{\pi N}$ from πN scattering, and its **shortcomings**
- KH80 PWA self-consistent, but at odds with **hadronic-atom phenomenology**
- Roy–Steiner formalism **reproduces KH80** results with KH80 input
- With modern input for scattering lengths and coupling constant, $\sigma_{\pi N}$ **increases**
- **Precise definition** of $\sigma_{\pi N}$ non-trivial in presence of IV, **comparison to lattice**?
- Error propagation ongoing, more applications to come

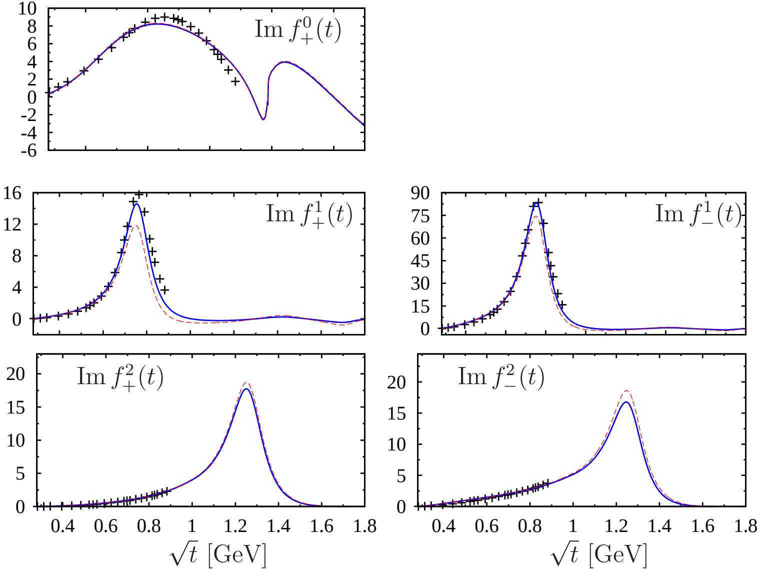
Roy–Steiner solution: t-channel for KH80 fit



black: KH80

blue/red
⇔
before/after fit

Roy–Steiner solution: t-channel for hadronic-atom fit



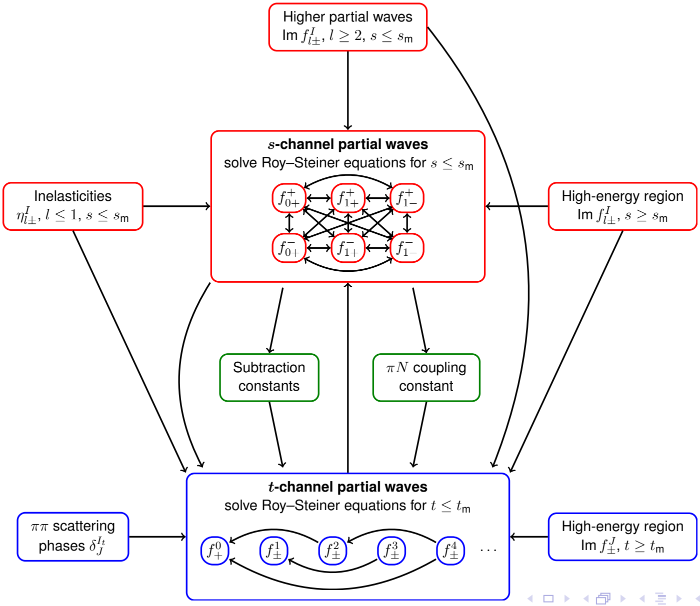
black: KH80

blue/red

⇔

before/after fit

Roy–Steiner equations for πN scattering: schematics



Scalar couplings for u- and d-quark

- Determination of **two-flavor couplings** Ellis et al. 2000, micrOMEGAs

$$y=\frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle}\qquad z=\frac{\langle N|\bar{u}u-\bar{s}s|N\rangle}{\langle N|\bar{d}d-\bar{s}s|N\rangle}\qquad \sigma_{\pi N}=\langle N|\hat{m}(\bar{u}u+\bar{d}d)|N\rangle$$

$$f_d^p=\frac{2\sigma_{\pi N}}{\big(1+\frac{m_u}{m_d}\big)m_p(1+\alpha)}\qquad \alpha=\frac{2z-(z-1)y}{2+(z-1)y}$$

$\hookrightarrow$ **Two-flavor** couplings from $\boldsymbol{SU(3)}$ quantities!

- Even more breathtaking
  - z from **LO fits** to baryon masses Cheng 1989
  - **Isospin violation** within this framework

$\hookrightarrow$ do the calculation based on $\boldsymbol{SU(2)}$ **ChPT** Crivellin, MH, Procura 2014

Scalar couplings for *u*- and *d*-quark

- Expansion of the **nucleon mass** including **isospin violation** Meißner, Steininger 1998

$$m_{p/n} = m_0 - 4c_1 M_{\pi^0}^2 \pm 2Bc_5(m_d - m_u) - \frac{e^2 F_\pi^2}{2}(f_1 \pm f_2 + f_3) - \frac{g_A^2(2M_{\pi^\pm}^3 + M_{\pi^0}^3)}{32\pi F_\pi^2} + \mathcal{O}(M_\pi^4)$$

- Feynman–Hellmann + Gell-Mann–Oakes–Renner

$$f_u^N = \frac{m_u}{m_N}\frac{\partial m_N}{\partial m_u} = B\frac{m_u}{m_N}\frac{\partial m_N}{\partial M_{\pi^0}^2} = -\frac{2B}{m_N}m_u\left[2c_1 \pm c_5 + \frac{3g_A^2(2M_{\pi^\pm} + M_{\pi^0})}{128\pi F_\pi^2}\right]$$

$$f_d^N = \frac{m_d}{m_N}\frac{\partial m_N}{\partial m_d} = B\frac{m_d}{m_N}\frac{\partial m_N}{\partial M_{\pi^0}^2} = -\frac{2B}{m_N}m_d\left[2c_1 \mp c_5 + \frac{3g_A^2(2M_{\pi^\pm} + M_{\pi^0})}{128\pi F_\pi^2}\right]$$

- Expressed in terms of $\sigma_{\pi N}$

$$m_N f_u^N = \frac{\sigma_{\pi N}}{2}(1-\xi) \pm Bc_5(m_d - m_u)\left(1 - \frac{1}{\xi}\right)$$

$$m_N f_d^N = \frac{\sigma_{\pi N}}{2}(1+\xi) \pm Bc_5(m_d - m_u)\left(1 + \frac{1}{\xi}\right)$$

$$\sigma_{\pi N} = \frac{1}{2}\Big(\langle p|\hat{m}(\bar{u}u + \bar{d}d)|p\rangle + \langle n|\hat{m}(\bar{u}u + \bar{d}d)|n\rangle\Big) \qquad \xi = \frac{m_d - m_u}{m_d + m_u} = 0.36 \pm 0.04 \quad \text{(FLAG)}$$

Scalar couplings for *u*- and *d*-quark

- Numbers

$$f_u^N = \frac{\sigma_{\pi N}(1-\xi)}{2m_N} + \Delta f_u^N \qquad f_d^N = \frac{\sigma_{\pi N}(1+\xi)}{2m_N} + \Delta f_d^N$$

$$\Delta f_u^p = (1.0 \pm 0.2) \times 10^{-3} \qquad \Delta f_u^n = (-1.0 \pm 0.2) \times 10^{-3}$$

$$\Delta f_d^p = (-2.1 \pm 0.4) \times 10^{-3} \qquad \Delta f_d^n = (2.0 \pm 0.4) \times 10^{-3}$$

- Isospin violation

$$f_{u,d}^p - f_{u,d}^n = 2Bc_5(m_\text{d} - m_\text{u})\left(1 \mp \frac{1}{\xi}\right) \qquad (m_\text{p} - m_\text{n})^\text{str} = 4Bc_5(m_\text{d} - m_\text{u})$$

- $m_p - m_n$ involves $\mathcal{O}(p^4)$ chiral log with large coefficient $(6g_A^2 + 1)/2 \sim 5$

  $\hookrightarrow$ within uncertainties when included consistently

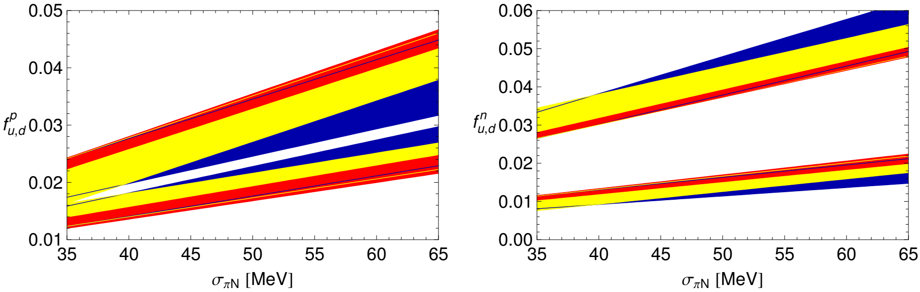
- Compare with $SU(3)$ approach micrOMEGAs 2013

$$f_u^p - f_u^n = (1.9 \pm 0.4) \times 10^{-3} \qquad f_u^p - f_u^n\Big|_{SU(3)} = 4.3 \times 10^{-3}$$

$$f_d^p - f_d^n = (-4.1 \pm 0.7) \times 10^{-3} \qquad f_d^p - f_d^n\Big|_{SU(3)} = -8.2 \times 10^{-3}$$

$\hookrightarrow$ **Isospin violation** overestimated by a **factor 2**

Scalar couplings for *u*- and *d*-quark



- Upper/lower ⇔ down-/up-coupling
- Color coding
  1. Red: $SU(2)$ approach
  2. Yellow: $SU(3)$ approach, y from $\sigma_{\pi N}$
  3. Blue: $SU(3)$ approach, y from lattice

Strangeness coupling

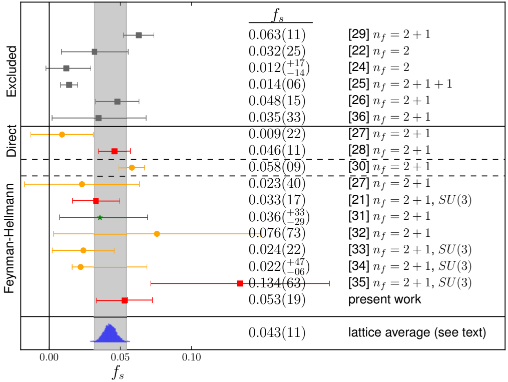
- f_s^N from $\sigma_{\pi N}$ via $SU(3)$ ChPT, but **large uncertainties**

$$m_N f_s^N = \frac{m_s}{2\hat{m}}\left(\sigma_{\pi N} - \sigma_0\right)$$

- **Lattice** average

$$f_s^N = 0.043 \pm 0.011$$

$\hookrightarrow$ large strangeness content seems unlikely



Junnarkar, Walker-Loud 2013

Heavy quarks

- **Trace anomaly** of the energy-momentum tensor

$$m_N = \langle N|\theta^\mu_\mu|N\rangle = \Big\langle N\Big|\sum_{q\leq n_f} m_q\bar{q}q + \frac{\beta^{n_f}_{\text{QCD}}}{2g_s}G^a_{\mu\nu}G^{\mu\nu}_a\Big|N\Big\rangle \qquad \beta^{n_f}_{\text{QCD}} = -\Big(11-\frac{2n_f}{3}\Big)g_s\frac{\alpha_s}{4\pi}$$

- Integrating out the heavy quarks: $n_f = 3$

$$m_N = \Big\langle N\Big|\sum_{q=u,d,s} m_q\bar{q}q - \frac{9}{8\pi}\alpha_s G^a_{\mu\nu}G^{\mu\nu}_a\Big|N\Big\rangle$$

- **Heavy-quark** contribution Shifman, Vainshtein, Zakharov 1978

$$f^N_Q = \frac{1}{m_N}\langle N|m_Q\bar{Q}Q|N\rangle = -\frac{1}{m_N}\frac{2}{3}g_s\frac{\alpha_s}{4\pi}\frac{1}{2g_s}\langle N|G^a_{\mu\nu}G^{\mu\nu}_a|N\rangle = -\frac{\alpha_s}{12\pi m_N}\langle N|G^a_{\mu\nu}G^{\mu\nu}_a|N\rangle$$

$$f^N_Q = \frac{2}{27}\Big(1-\sum_{q=u,d,s}f^N_q\Big)$$

$\hookrightarrow$ fixed in terms of **light flavors**

Second correction: Δ_D

t-channel expansion

$$\bar{D}^+(\nu=0,t) = 4\pi\bigg\{-\frac{1}{p_t^2}\bar{f}_+^0(t) + \frac{5}{2}q_t^2\bar{f}_+^2(t) - \frac{27}{8}p_t^2 q_t^4\bar{f}_+^4(t) + \frac{65}{16}p_t^4 q_t^6\bar{f}_+^6(t) + \cdots\bigg\}$$

- Insert t-channel RS equations for Born-term-subtracted amplitudes $\bar{f}_+^J(t)$

$$\bar{D}^+(\nu=0,t) = d_{00}^+ + d_{01}^+ t - 16t^2\int\limits_{4M_\pi^2}^{\infty} \mathrm{d}t'\,\frac{\operatorname{Im} f_+^0(t')}{t'^2(t'-4m^2)(t'-t)} + \{J\geq 2\} + \{s\text{-channel integrals}\}$$

- $\Delta_D = F_\pi^2\Big(\bar{D}^+(\nu=0,t=2M_\pi^2) - d_{00}^+ - 2M_\pi^2 d_{01}^+\Big)$ from evaluation at $t=2M_\pi^2$

$$\begin{aligned}\Delta_D &= (12.1\pm 0.3)\,\text{MeV}\\ &\quad+\tilde{Z}_1\bigg(\frac{g^2}{4\pi}-14.28\bigg)+\tilde{Z}_2\Big(d_{00}^+\,M_\pi+1.46\Big)+\tilde{Z}_3\Big(d_{01}^+\,M_\pi^3-1.14\Big)+\tilde{Z}_4\Big(b_{00}^+\,M_\pi^3+3.54\Big)\\ \tilde{Z}_1 &= 0.42\,\text{MeV}\qquad \tilde{Z}_2 = 0.67\,\text{MeV}\qquad \tilde{Z}_3 = 12.0\,\text{MeV}\qquad \tilde{Z}_4 = -0.77\,\text{MeV}\end{aligned}$$

Origin of the cancellation

- Dominant contribution from dispersive integral over $f_+^0(t)$

$$\Delta_\sigma = \frac{3M_\pi^2}{\pi} \int\limits_{4M_\pi^2}^{\infty} dt' \frac{\sigma_{t'}^\pi \big(F_\pi^S(t')\big)^* f_+^0(t')}{t'(t'-2M_\pi^2)(4m^2-t')} + \cdots$$

$$\Delta_D = 64F_\pi^2 M_\pi^4 \int\limits_{4M_\pi^2}^{\infty} dt' \frac{\operatorname{Im} f_+^0(t')}{t'^2(t'-2M_\pi^2)(4m^2-t')} + \cdots = 64F_\pi^2 M_\pi^4 \int\limits_{4M_\pi^2}^{\infty} dt' \frac{\sigma_{t'}^\pi \big(t_0^0(t')\big)^* f_+^0(t')}{t'^2(t'-2M_\pi^2)(4m^2-t')} + \cdots$$

- Largest contribution around $t' = 4M_\pi^2$

$$\frac{\Delta_\sigma}{\Delta_D} \to \frac{3M_\pi^2}{\pi} \frac{\big(F_\pi^S(t')\big)^* t'}{64F_\pi^2 M_\pi^4 \big(t_0^0(t')\big)^*} \to \frac{3M_\pi^4}{\pi} \frac{32\pi F_\pi^2 t'}{64F_\pi^2 M_\pi^4(2t'-M_\pi^2)} \to \frac{6}{7} = \frac{18}{21} \qquad \text{ChPT: } \frac{\Delta_\sigma}{\Delta_D} = \frac{18}{23} + \mathcal{O}(M_\pi)$$

- This explains
  - Δ_σ and Δ_D of similar size
  - Strong curvature generated by $\pi\pi$ rescattering

    $\hookrightarrow$ sensitivity to $\pi\pi$ phase shift reduced in the difference Gasser, Leutwyler, Sainio 1991
  - Spectral functions depend similarly on $f_+^0(t)$ $\hookrightarrow$ sensitivity to πN parameters reduced

Spin-independent WIMP–nucleon scattering

- Effective Lagrangian

$$\mathcal{L}_{\text{eff}} = C_{qq}^{SS} \frac{m_q}{\Lambda^3} \bar{\chi}\chi\, \bar{q}q + C_{qq}^{VV} \frac{1}{\Lambda^2} \bar{\chi}\gamma^\mu\chi\, \bar{q}\gamma_\mu q + C_{gg}^{S} \frac{\alpha_s}{\Lambda^3} \bar{\chi}\chi\, G_{\mu\nu}G^{\mu\nu}$$

- WIMP χ **Dirac fermion** and **SM singlet**
- Integrate out heavy quarks

$$C_{gg}^{L} \to C_{gg}^{L} - \frac{1}{12\pi} \sum_{Q=c,b,t} C_{QQ}^{SL}$$

- **Spin-independent cross section** at vanishing momentum transfer

$$\sigma_N^{\text{SI}} = \frac{\mu_\chi^2}{\pi\Lambda^4} \left| \frac{m_N}{\Lambda} \left(\sum_{q=u,d,s} C_{qq}^{SS} f_q^N - 12\pi\, C_{gg}^{S} f_Q^N \right) + \sum_{q=u,d} C_{qq}^{VV} f_{V_q}^N \right|^2$$

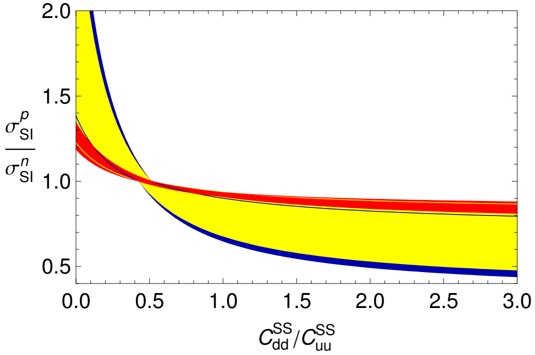
$$\mu_\chi = \frac{m_\chi m_N}{m_\chi + m_N} \qquad f_{V_u}^p = f_{V_d}^n = 2f_{V_d}^p = 2f_{V_u}^n = 2$$

Spin-independent WIMP–nucleon scattering

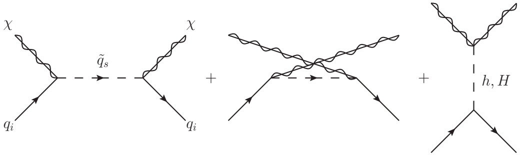
$$\sigma_N^{\text{SI}} = \frac{\mu_\chi^2}{\pi\Lambda^4}\left|\frac{m_N}{\Lambda}\left(\sum_{q=u,d,s} C_{qq}^{SS} f_q^N - 12\pi\, C_{gg}^{S} f_Q^N\right) + \sum_{q=u,d} C_{qq}^{VV} f_{V_q}^N\right|^2$$

- Maximal amount of **isospin violation** induced by scalar operators

$\hookrightarrow$ $C_{ss}^{SS} = C_{gg}^{S} = C_{qq}^{VV} = 0$, take $\sigma_{\pi N} = 50\,\text{MeV}$



Spin-independent neutralino–nucleon scattering



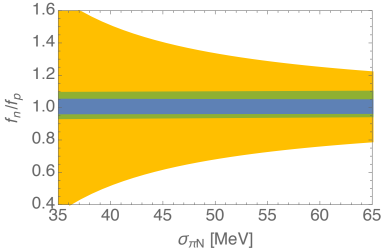
- Consider the MSSM, all particles but χ and heavy (*CP*-even) Higgs *H* decoupled
- Write cross section as

$$\sigma_N^{SI} = \frac{4\mu_\chi^2}{\pi} f_N^2 \qquad f_N = \frac{m_N}{\Lambda^2}\bigg(\sum_{q=u,d,s} C_{qq}^{SS} f_q^N - 12\pi\, C_{gg}^S f_Q^N\bigg)$$

- Just SM Higgs *h*

  ↪ amount of isospin violation $\lesssim$ 5%

  ↪ less than in $SU(3)$ approach Ellis et al. 2008



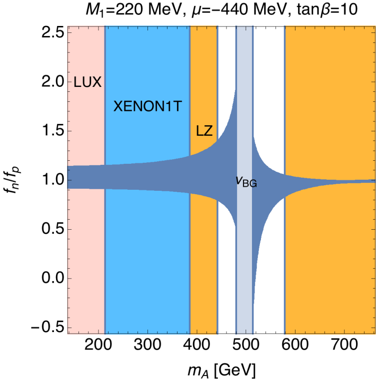
Crivellin, MH, Procura, Tunstall, in preparation

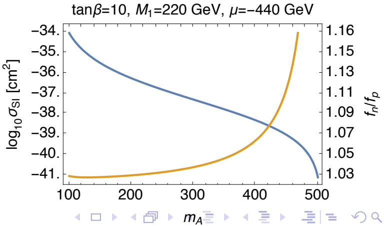
Spin-independent neutralino–nucleon scattering with heavy Higgs

- Include the *CP*-even **heavy Higgs *H***

  ↪ cancellations between h and H

  ↪ **blind spots**
- In vicinity of blind spots, **isospin violation** is **enhanced**
- Direct-detection limits drive parameter space towards blind spots
- MSSM conventions:
  - M_1: soft SUSY-breaking mass of the bino $\tilde{B}$
  - μ: Higgsino mass parameter
  - $\tan\beta$: ratio of Higgs vevs
  - m_A: mass of the heavy Higgs





Removing Coulomb effects: the *pp* scattering length

- Consider first a more familiar example: ***pp* scattering**
  1. Split total phase shift into **pure Coulomb** σ^C + **remainder** δ_{pp}^C
  2. δ_{pp}^C related to strong amplitude $T_{pp}(k)$ by

$$k\big(\cot\delta_{pp}^C - i\big) = -\frac{4\pi}{m}\frac{e^{2i\sigma^C}}{T_{pp}(k)} \qquad\qquad k = |\mathbf{k}|$$

  3. **Modified effective range expansion** Bethe 1949

$$k\Big[C_\eta^2\big(\cot\delta_{pp}^C - i\big) + 2\eta H(\eta)\Big] = -\frac{1}{a_{pp}^C} + \frac{1}{2}r_0 k^2 + \cdots$$

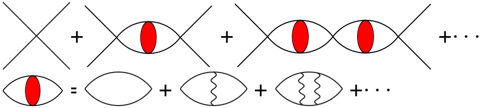
$$C_\eta^2 = \frac{2\pi\eta}{e^{2\pi\eta}-1} \qquad \eta = \frac{\alpha m}{2k} \qquad H(\eta) = \psi(i\eta) + \frac{1}{2i\eta} - \log(i\eta) \qquad \psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

  4. Removal of **residual Coulomb** effects **scale-dependent**

$$\frac{1}{a_{pp}} = \frac{1}{a_{pp}^C} + \alpha m\bigg[\log\frac{1}{\alpha M r_0} - 0.33\bigg] \qquad \text{Jackson, Blatt 1950}$$

$$\frac{1}{a_{pp}} = \frac{1}{a_{pp}^C} + \alpha m\bigg[\log\frac{\mu\sqrt{\pi}}{\alpha M} + 1 - \frac{3}{2}\gamma_E\bigg] \qquad \text{Kong, Ravndal 1999}$$

Removing Coulomb effects: the *pp* scattering length



- Difference due to **Coulomb-dressed bubble sum**

$$\frac{1}{a_{pp}} = \frac{1}{a^C_{pp}} + \alpha m\left[\log\frac{1}{\alpha M r_0} - 0.33\right]$$ Jackson, Blatt 1950

$$\frac{1}{a_{pp}} = \frac{1}{a^C_{pp}} + \alpha m\left[\log\frac{\mu\sqrt{\pi}}{\alpha M} + 1 - \frac{3}{2}\gamma_E\right]$$ Kong, Ravndal 1999

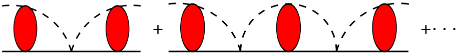
- a_{pp} supposed to correspond to strong part of the potential, but **Coulomb-nuclear interference** depends on short-distance part of the nuclear force
- Numbers for **singlet channel**
  - $a^C_{pp} = (-7.8063 \pm 0.0026)\,\text{fm}$ Bergervoet et al. 1988
  - $a_{np} = (-23.749 \pm 0.008)\,\text{fm}$ Koester, Nistler 1975
  - $a_{nn} = (-18.8 \pm 0.5)\,\text{fm}$ González et al. 2006
  - $a_{pp} = (-17.3 \pm 0.4)\,\text{fm}$ Miller et al. 1990

$\hookrightarrow$ $a^C_{pp} - a_{pp}$ **huge effect**!

Back to πN scattering

- **Deser formula**: shift and $a_{\pi^- p}$ in NREFT

$$\epsilon_{1s} = -2\alpha^3\mu_H^2 a_{\pi^- p}\big(1 + 2\alpha(1-\log\alpha)\mu_H a_{\pi^- p} + \cdots\big)$$



- **ChPT convention** for scattering length Lyubovitskij, Rusetsky 2000

$$e^{-2i\sigma^C} T_{\pi^- p} = \frac{\pi\alpha\mu_H a_{\pi^- p}}{k} - 2\alpha\mu_H\big(a_{\pi^- p}\big)^2 \log\frac{k}{\mu_H} + a_{\pi^- p} + \mathcal{O}(k,\alpha^2)$$

- **Compare with mERE**: expand first in α, then in k

$$e^{-2i\sigma^C} T_{\pi^- p} = \frac{\pi\alpha\mu_H a^C_{\pi^- p}}{k} - 2\alpha\mu_H\big(a^C_{\pi^- p}\big)^2 \left(\gamma_E + \log\frac{k}{\alpha\mu_H}\right) + a^C_{\pi^- p} + \mathcal{O}(k,\alpha^2)$$

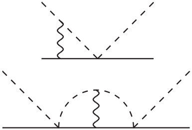
$\hookrightarrow$ same $\log\alpha$ as in Deser formula!

ChPT vs. mERE scattering length

$$\underbrace{a_{\pi^- p}}_{86.1\pm1.8} = a^C_{\pi^- p} + \underbrace{2\alpha\mu_H\big(a^C_{\pi^- p}\big)^2(\log\alpha - \gamma_E)}_{-0.5} + \mathcal{O}(\alpha^2)$$

Subtraction of virtual-photon effects

- Application in **dispersion relations** $\hookrightarrow$ analytic properties
- Effects calculable in ChPT, e.g.
  - Coulomb pole $\sim 1/k$ at NLO, $\mathcal{O}(p^3)$
  - $\log k$ first at two loops, $\mathcal{O}(p^5)$



- Subtract virtual-photon contributions
  - **Finite terms** $\Rightarrow$ fine
  - **UV divergent photon loops** $\Rightarrow$ need to separate mass-difference and virtual-photon contributions to LECs $\Rightarrow$ scale dependence
- How large are these effects?
  - Full: $a_{\pi^- p} - a_{\pi^+ p} = (172.8 \pm 1.6) \times 10^{-3} M_\pi^{-1}$
  - Virtual photons: $a^{\gamma}_{\pi^- p} - a^{\gamma}_{\pi^+ p} = (2.1 \pm 1.8) \times 10^{-3} M_\pi^{-1}$
  - Virtual-photon subtracted: $a^{\not\gamma}_{\pi^- p} - a^{\not\gamma}_{\pi^+ p} = (170.7 \pm 2.4) \times 10^{-3} M_\pi^{-1}$

$\hookrightarrow$ much smaller than in a_{pp}