Dark Matter without Astrophysics:
Halo-independent methods at Direct Detection Experiments

Patrick Fox

Fermilab

Based on work with A. Anderson, G. Jung, Y. Kahn, G. Kribs, J Liu, M. McCullough, T. Tait, P Sorensen and N. "break dancer" Weiner

Astrophysics independence

Comparing two positive results: Drees and Shan; 0803.4477, 0809.1295

vmin space: PJF, Liu, Weiner; 1011.1915

Applications to CDMS, CoGeNT, COUPP, CRESST, DAMA, LUX, XENON, light DM, etc; 1103.3481, 1106.0743, 1106.6241, 1107.0715, 1107.0741, 1110.5338, 1304.6066, 1304.6183,....

Annual modulation: Herrero-Garcia et al.; 1112.1627, 1205.0134

Including detector effects: Gelmini and Gondolo; 1202.6359

Extension to iDM: Bozorgnia et al.; 1305.3575

Extension to general interactions and kinematics: Del Nobile et al.; 1306.7273

Unbinned analysis: PJF, Kahn, McCullough; 1403.6830

....

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Annual modulation: H... .0134

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A thriving industry

Direct Detection

$$\frac{dR}{dE_R} = \frac{N_T \rho_\chi}{m_\chi} \int_{v_{\min}}^{v_{\max}} d^3\vec{v}\, f(\vec{v}(t))\, \frac{d\,\sigma|\vec{v}|}{dE_R}$$

with,

$$\frac{d\sigma}{dE_R} = F_N^2(E_R)\frac{m_N}{\mu v^2}\bar{\sigma}(v, E_R)$$

$$\bar{\sigma}_i(v, E_R) = \begin{cases} \sigma_{i0} \\ \sigma_{i0} F_{\chi_i}^2(E_R) \\ \sigma_{i0}(v) F_{\chi_i}^2(E_R) \\ \sigma_{i0}(v, E_R) \end{cases}$$

Usual case (SI elastic WIMP)

$$\sigma_N = \frac{(Zf_p + (A-Z)f_n)^2}{f_p^2}\frac{\mu_{N\chi}^2}{\mu_{n\chi}^2}\sigma_p$$

$$v_{min} = \sqrt{\frac{m_N E_R}{2\mu_{N\chi}^2}}$$

Theory

$$\frac{dR}{dE_R} = \frac{N_T \rho_\chi}{m_\chi} \int_{v_{\rm min}}^{v_{\rm max}} d^3\vec{v}\, f(\vec{v}(t))\, \frac{d\,\sigma|\vec{v}|}{dE_R}$$

Expt.

$$\frac{dR}{dE_R} = \frac{N_A \rho_\chi \sigma_n m_n}{2 m_\chi \mu_{n\chi}^2} C_T^2(A,Z) \int dE'_R G(E_R, E'_R) \epsilon(E'_R) F^2(E'_R) g(v_{min}(E'_R))$$

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Expt.

resolution → efficiency

$$\frac{dR}{dE_R} = \frac{N_A \rho_\chi \sigma_n m_n}{2 m_\chi \mu_{n\chi}^2} C_T^2(A,Z) \int dE'_R G(E_R, E'_R) \epsilon(E'_R) F^2(E'_R) g(v_{min}(E'_R))$$

There are known knowns. These are things we know that we know. There are known unknowns. That is to say, there are things that we know we don't know. But there are also unknown unknowns. There are things we don't know we don't know.

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"In theory there is no difference between theory and practice. But in practice there is.--Yogi Berra"

The uncertain world of direct detection

- Nuclear form factors, SI/SD + >2 others [see talks from this workshop]
- Atomic physics, detector response.

e.g. Leff, keVee → keVnr

- Astrophysics

How is DM distributed in our neighbourhood?

So far we have only transited ~0.007pc, way below N-body resolution

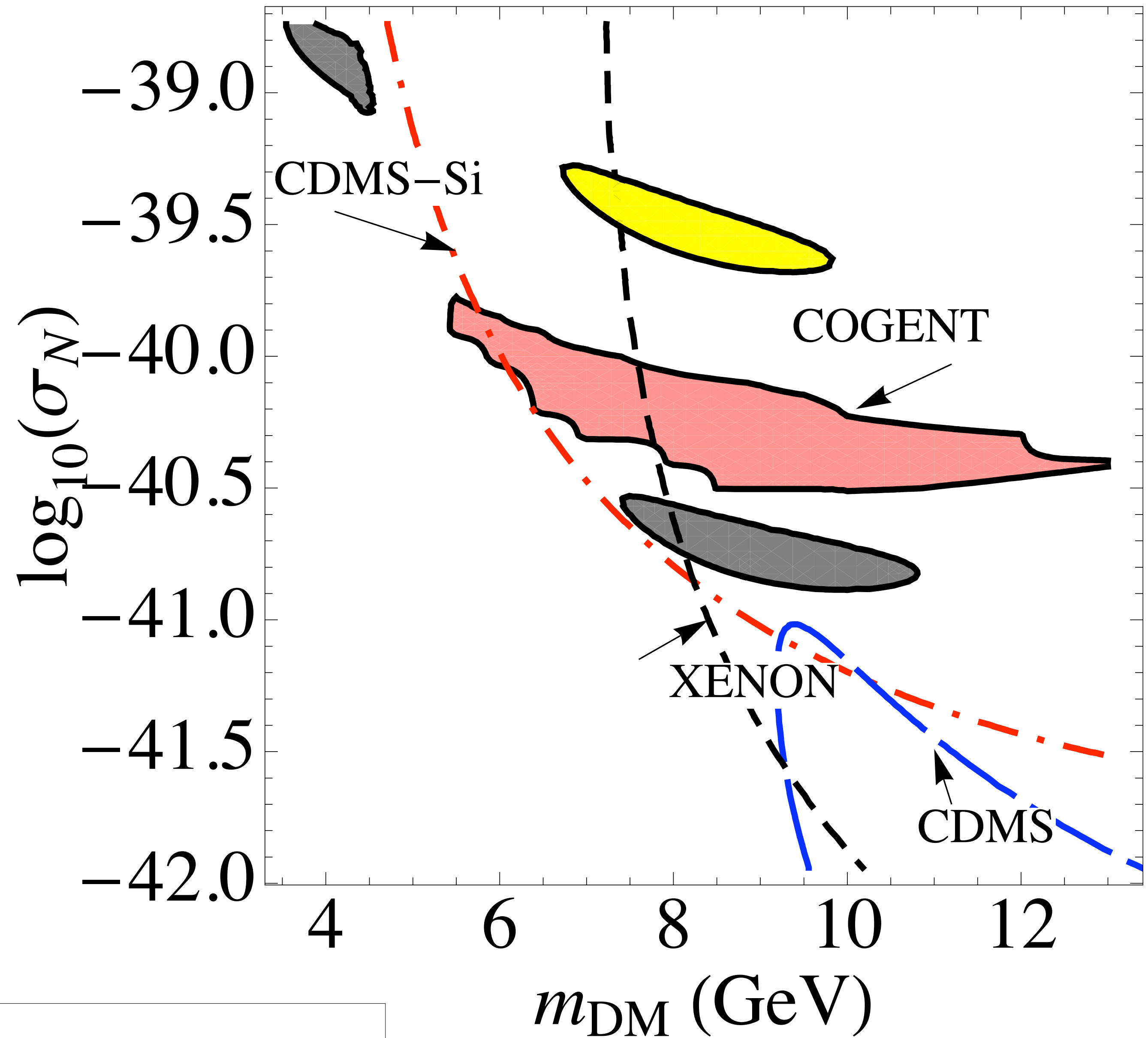
Baryons? SIDM?

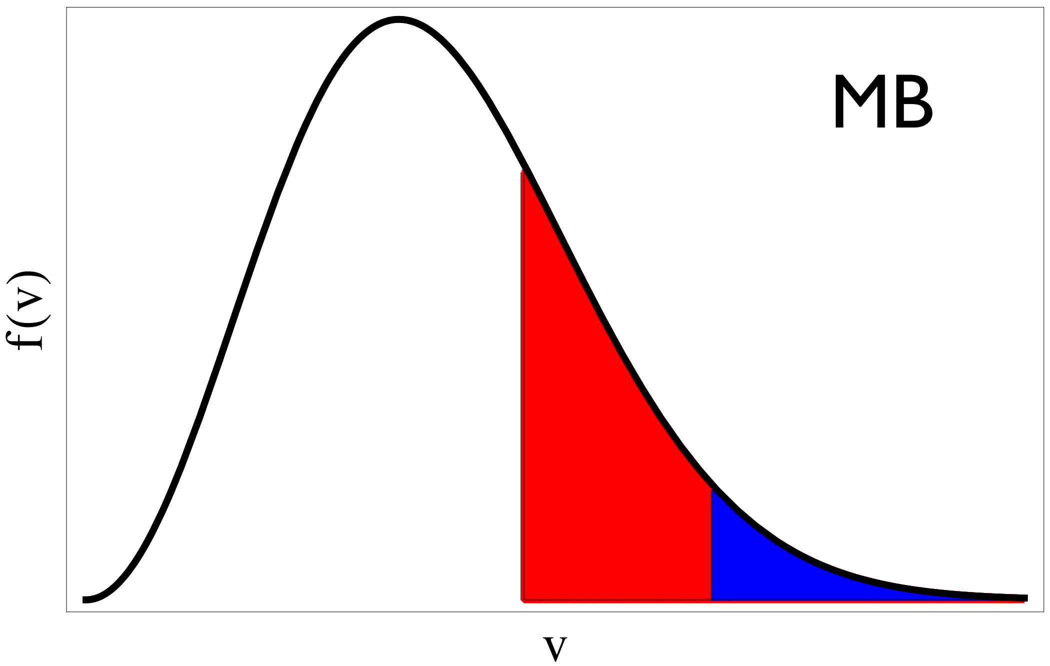
Dark disc? [Randall et al.]

Debris flow? [Lisanti et al.]

Gravitational focusing?

Dependence on details of velocity distribution





$$v_{min} = \sqrt{\frac{m_N E_R}{2\mu_{N\chi}^2}}$$

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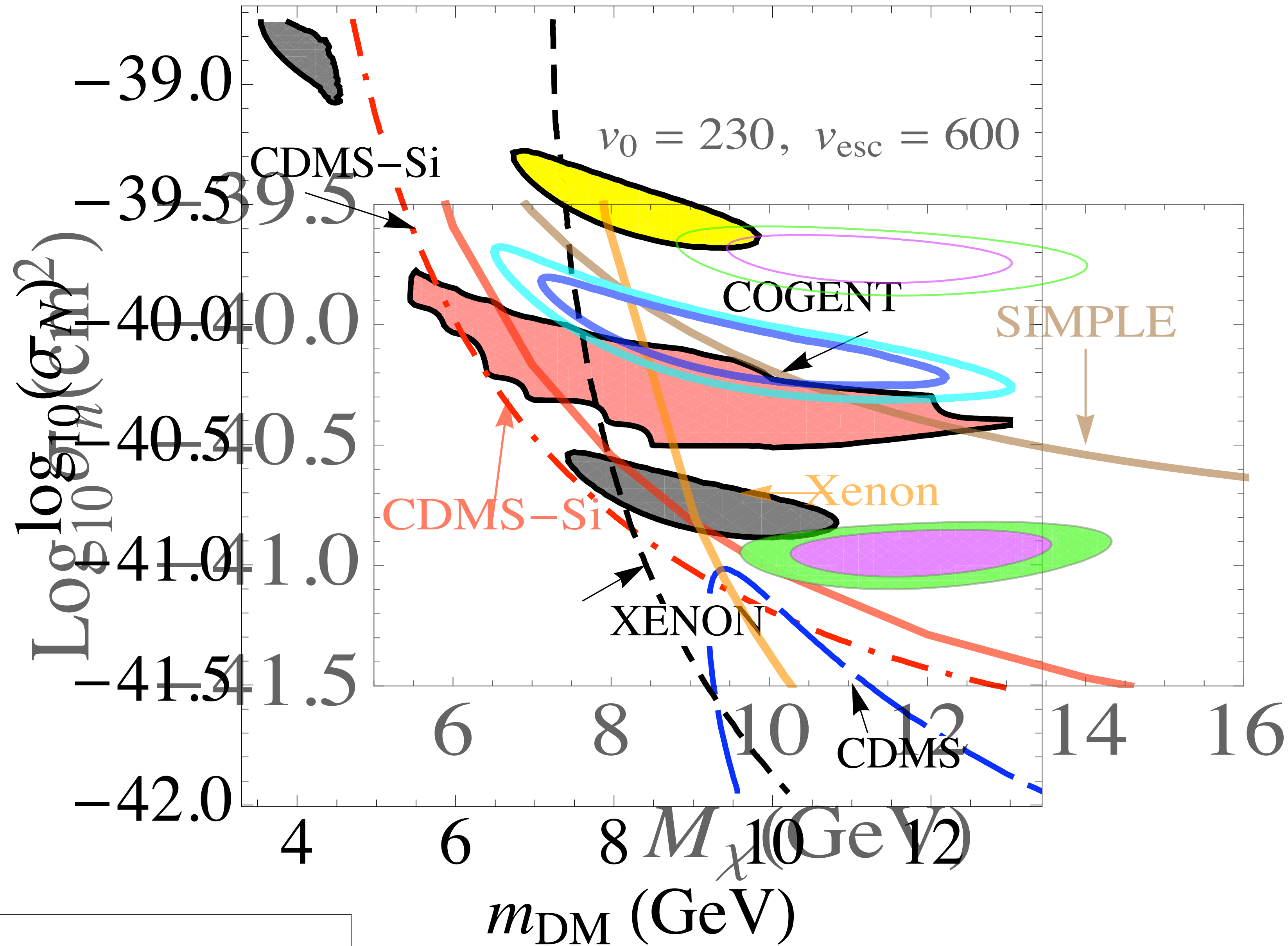

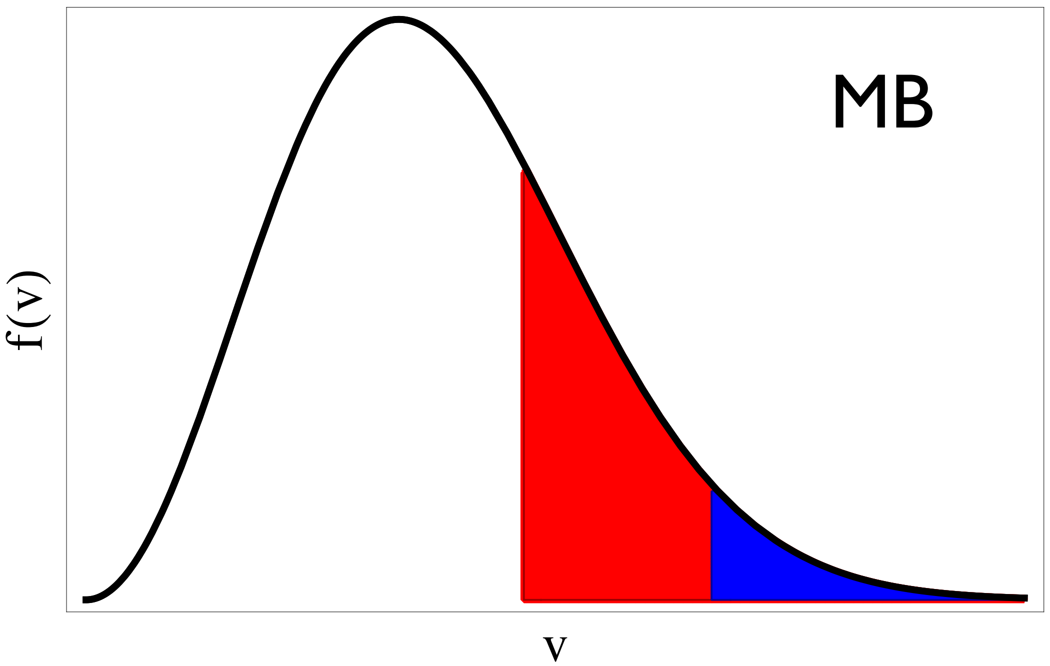

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Direct Detection

[PF, Kribs, Tait; see also, Drees and Shan, A. Peter, ...]

Is a signal a measurement of particle physics or astrophysics?

The only way we have of probing our **local** DM distribution

$$f_1(v_{\min}(E_R)) = -\frac{4\mu^2 E_R^2}{m_N^2 E_R^2 - \mu^2\delta^2}\frac{1}{\mathcal{N}\sigma_0(v_{\min}(E_R))F_\chi^2(E_R)}\left(\frac{d\mathcal{R}}{dE_R} - \mathcal{R}\frac{1}{F_\chi^2(E_R)}\frac{dF_\chi^2(E_R)}{dE_R}\right)$$

f-condition: $f(v) \geq 0$

(Deconvoluted) rate is a monotonically decreasing function, or there is non-standard particle physics e.g. inelastic or an increasing DM form factor

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"Deconvoluted" rate

$$\mathcal{R} \equiv \frac{1}{F_N^2(E_R)}\frac{dR}{dE_R}$$

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Neal is <u>not</u> a great break dancer...

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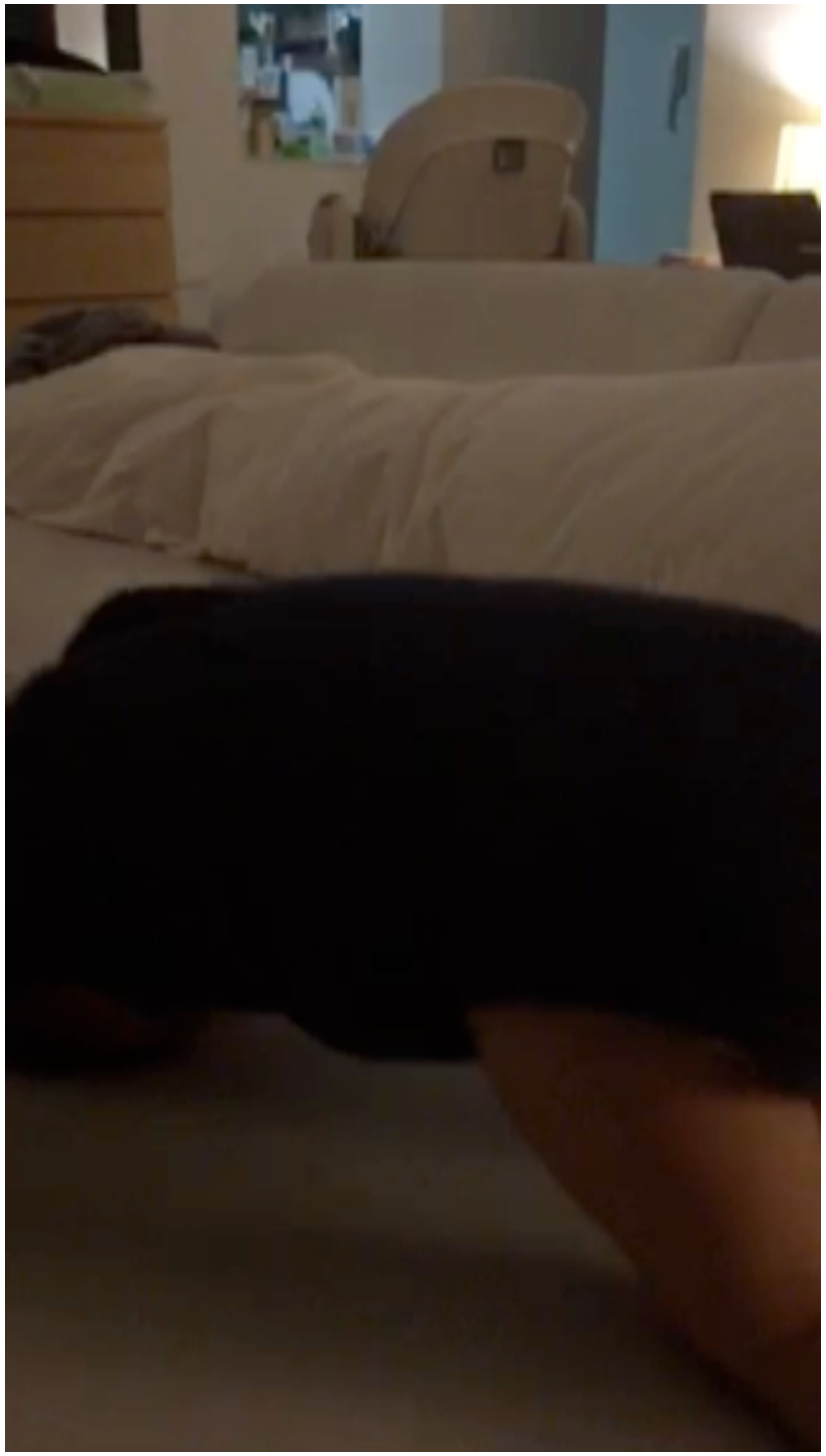

...but his son is!

- Cannot determine *anything* about DM from a single positive result

- Assuming an escape velocity can place a *lower* bound on DM mass, but no upper bound

- Luckily we have other complementary searches, and multiple direct detection expts.

Direct Detection without bias

[PF, Liu, Weiner]

$$\frac{dR}{dE_R} = \frac{N_T M_T \rho}{2 m_\chi \mu^2} \underbrace{\int_{v_{min}}^{v_{max}} d^3\vec{v} \frac{f(\vec{v}, \vec{v_E})}{v}}_{g(v)} \sigma(E_R)$$

$$\frac{dR}{dE_R} = \frac{N_T M_T F_N^2(E_r)}{2\mu^2} \frac{\rho\sigma}{m_\chi} g(v)$$

$$v_{min} = \sqrt{\frac{M_T E_R}{2\mu^2}}$$

Recoil energy uniquely determines **minimum** DM velocity

Direct Detection without bias

[PF, Liu, Weiner]

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$g(v)$

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Target specific

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Target specific — Target independent

$$v_{min} = \sqrt{\frac{M_T E_R}{2\mu^2}}$$

Recoil energy uniquely determines **minimum** DM velocity

Comparing experiments-vmin space

$$N_T = \kappa N_A m_p / M_T$$

Solve for g(v)

$$g(v_{min}) = \frac{2 m_\chi \mu^2}{N_A \kappa \, m_p \, \rho \, \sigma(E_R)} \frac{dR_1}{dE_1}$$

$$v_{min} = \sqrt{\frac{M_T E_R}{2\mu^2}}$$

$$\frac{dR_1}{dE_1} \Longleftrightarrow g(v_{min}) \Longleftrightarrow \frac{dR_2}{dE_2}$$

The master formula (SI):

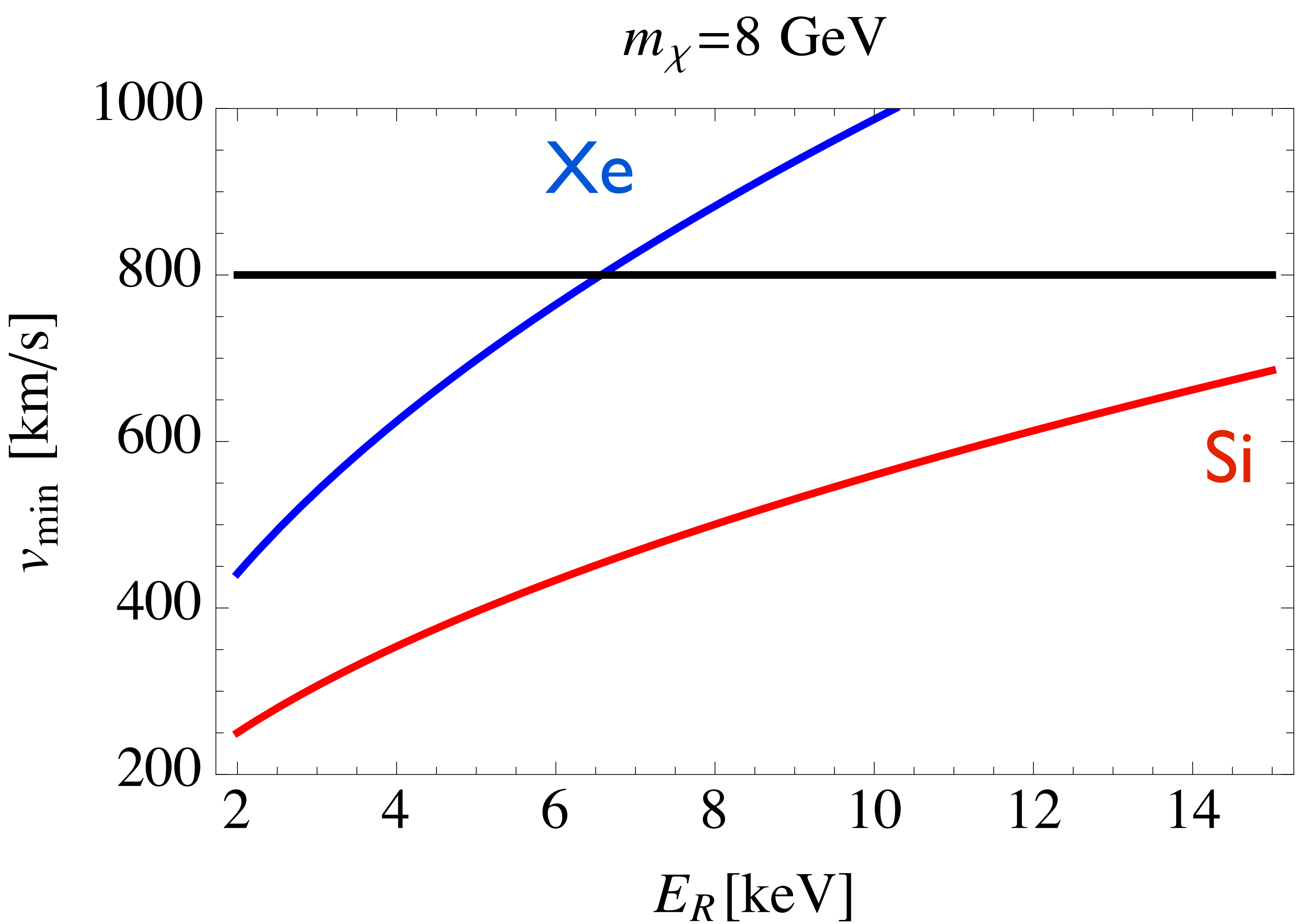
$$C_T^{(i)} = \kappa^{(i)} \left(f_p \, Z^{(i)} + f_n \left(A^{(i)} - Z^{(i)} \right) \right)^2$$

$$\frac{dR_2}{dE_R}(E_2) = \frac{C_T^{(2)}}{C_T^{(1)}} \frac{F_2^2(E_2)}{F_1^2\left(\frac{\mu_1^2 \, M_T^{(2)}}{\mu_2^2 M_T^{(1)}} E_2 \right)} \frac{dR_1}{dE_R} \left(\frac{\mu_1^2 \, M_T^{(2)}}{\mu_2^2 \, M_T^{(1)}} E_2 \right)$$

Using vmin space

Experiment 1 $\longleftrightarrow$ Experiment 2

$$[E^{(1)}_{low}, E^{(1)}_{low}] \Longleftrightarrow [v^{low}_{min}, v^{high}_{min}] \Longleftrightarrow [E^{(2)}_{low}, E^{(2)}_{high}]$$



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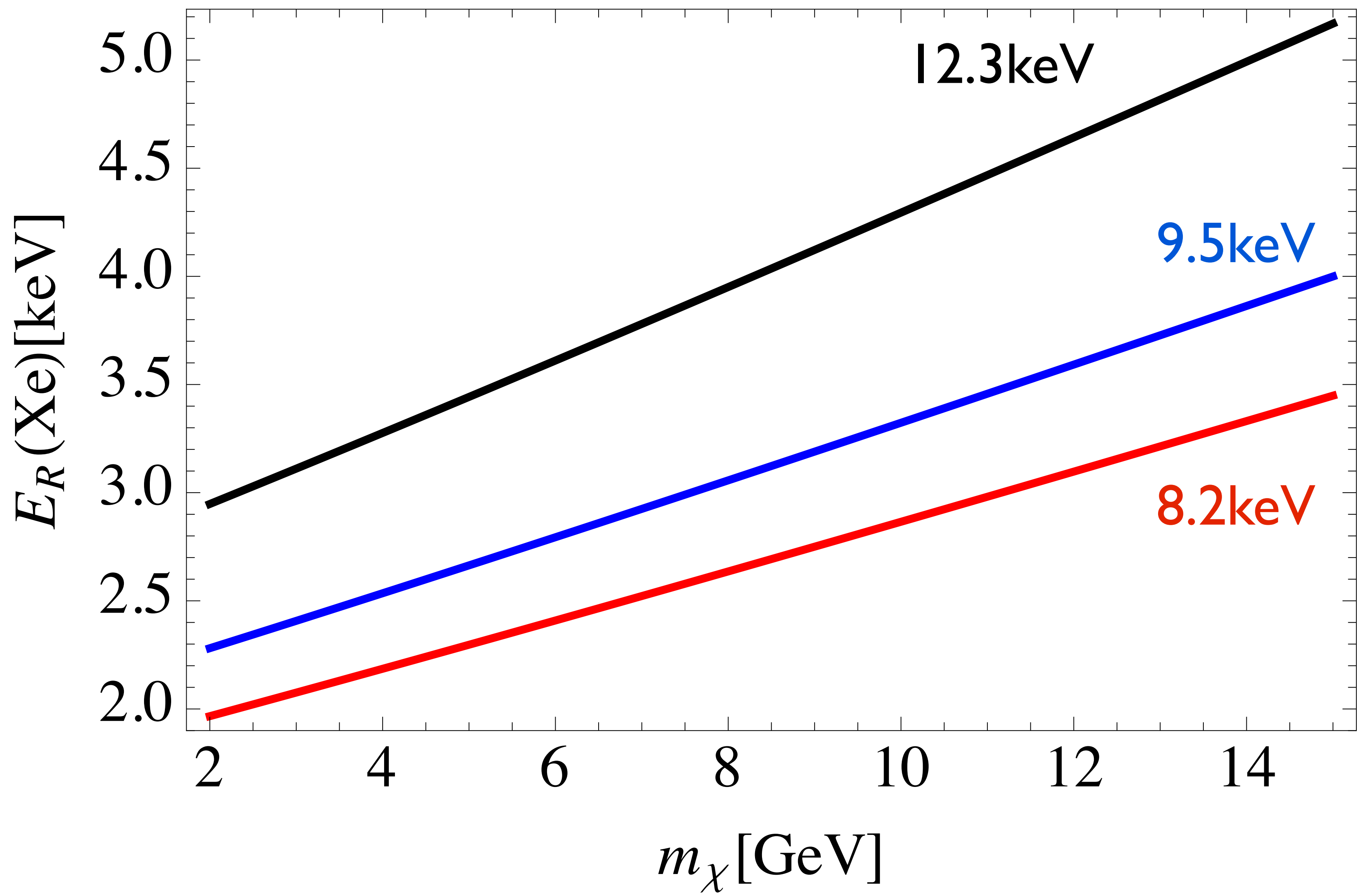

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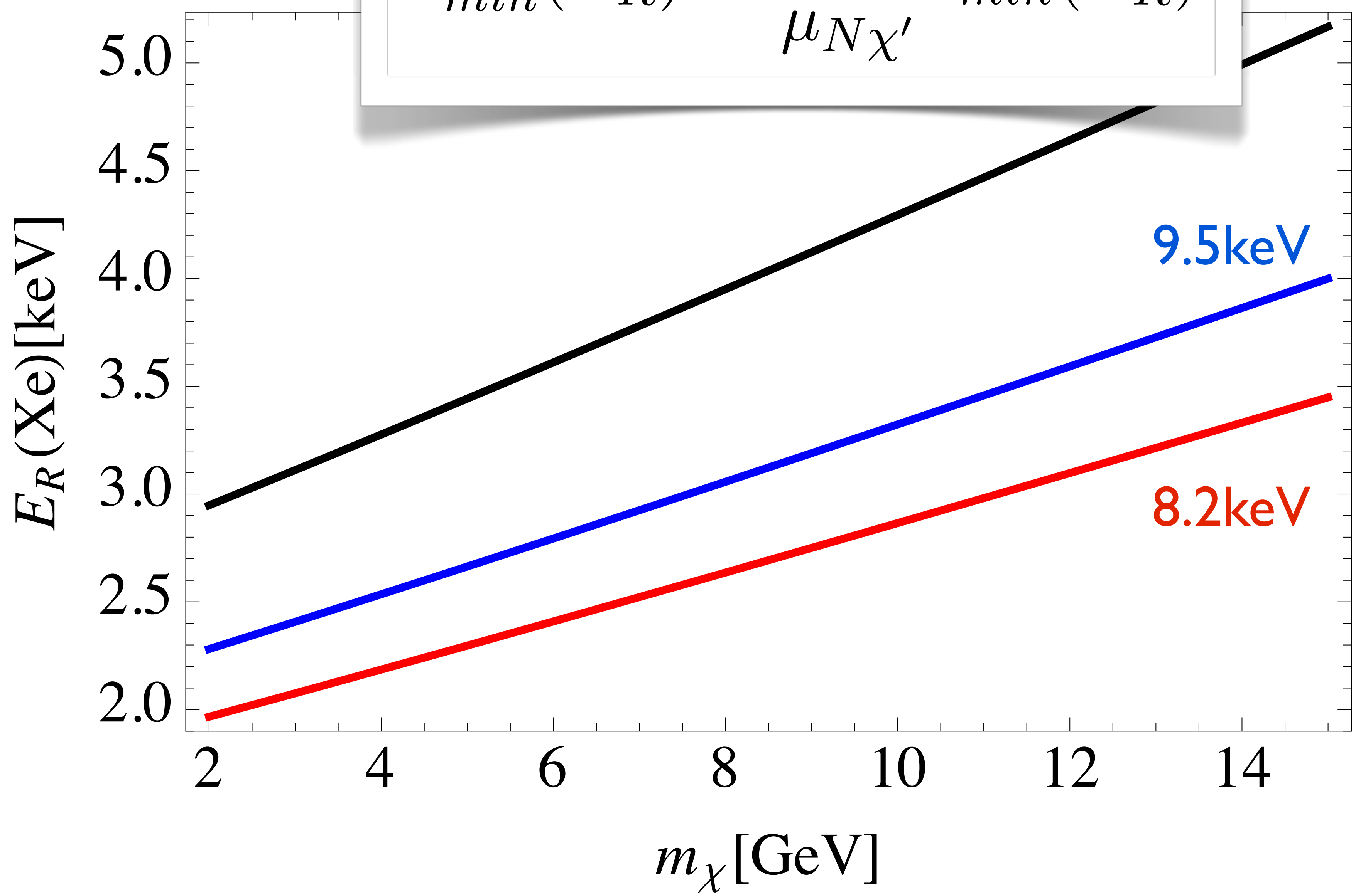
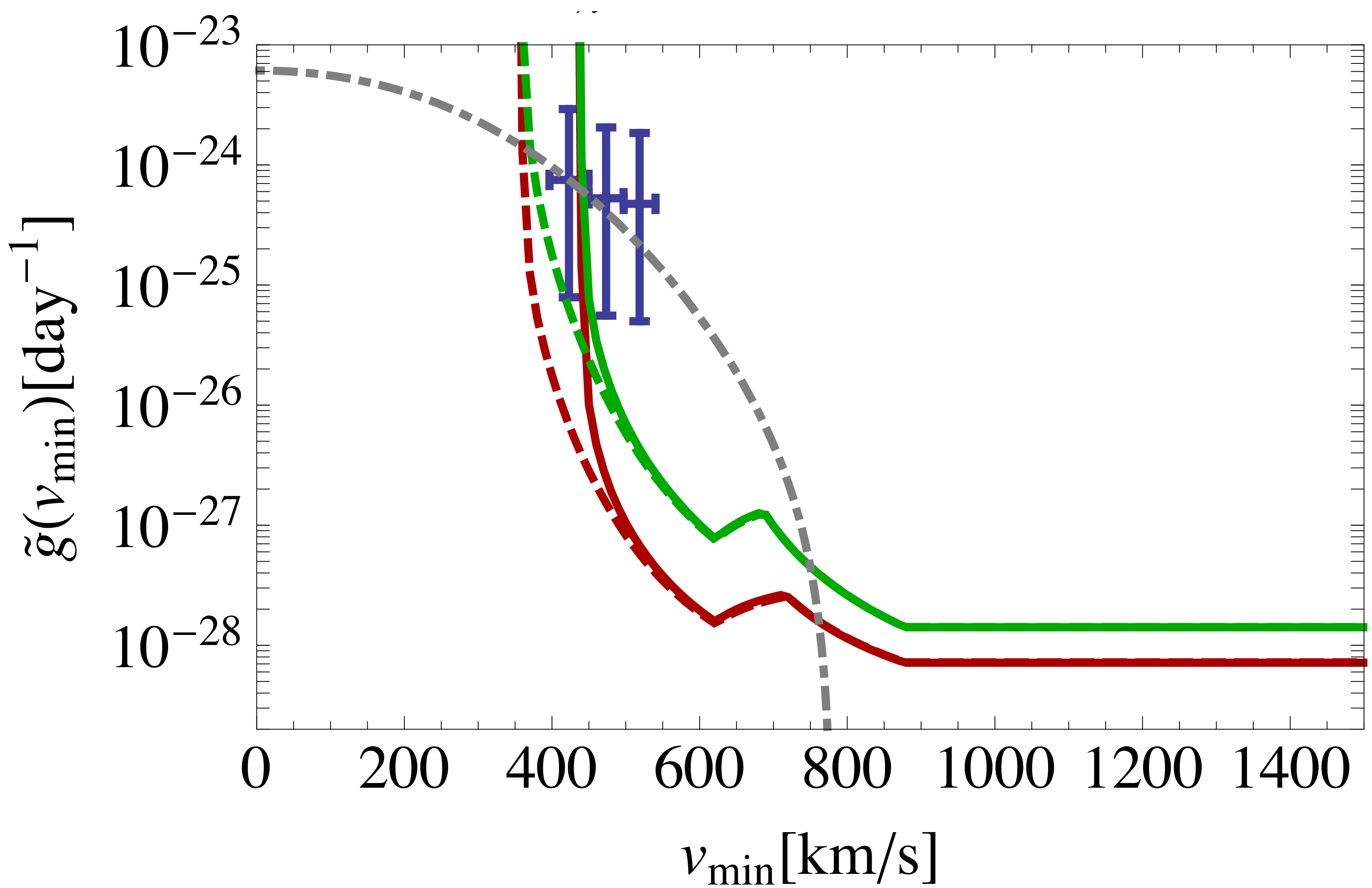
Experiment 1 $\longleftrightarrow$ Experiment 2

$$[E^{(1)}_{low}, E^{(1)}_{low}] \Longleftrightarrow [v^{low}_{min}, v^{high}_{min}] \Longleftrightarrow [E^{(2)}_{low}, E^{(2)}_{high}]$$

$$v'_{min}(E_R) = \frac{\mu_{N\chi}}{\mu_{N\chi'}} v_{min}(E_R)$$



Benefits of using vmin space



A more direct comparison of data than sigma-m plots

Easy to derive from data

For eDM (and single target expts.) need only show for one mass

Ultimately allows for measurements of g(v)

Consistency of g(v) determines allowed DM params.

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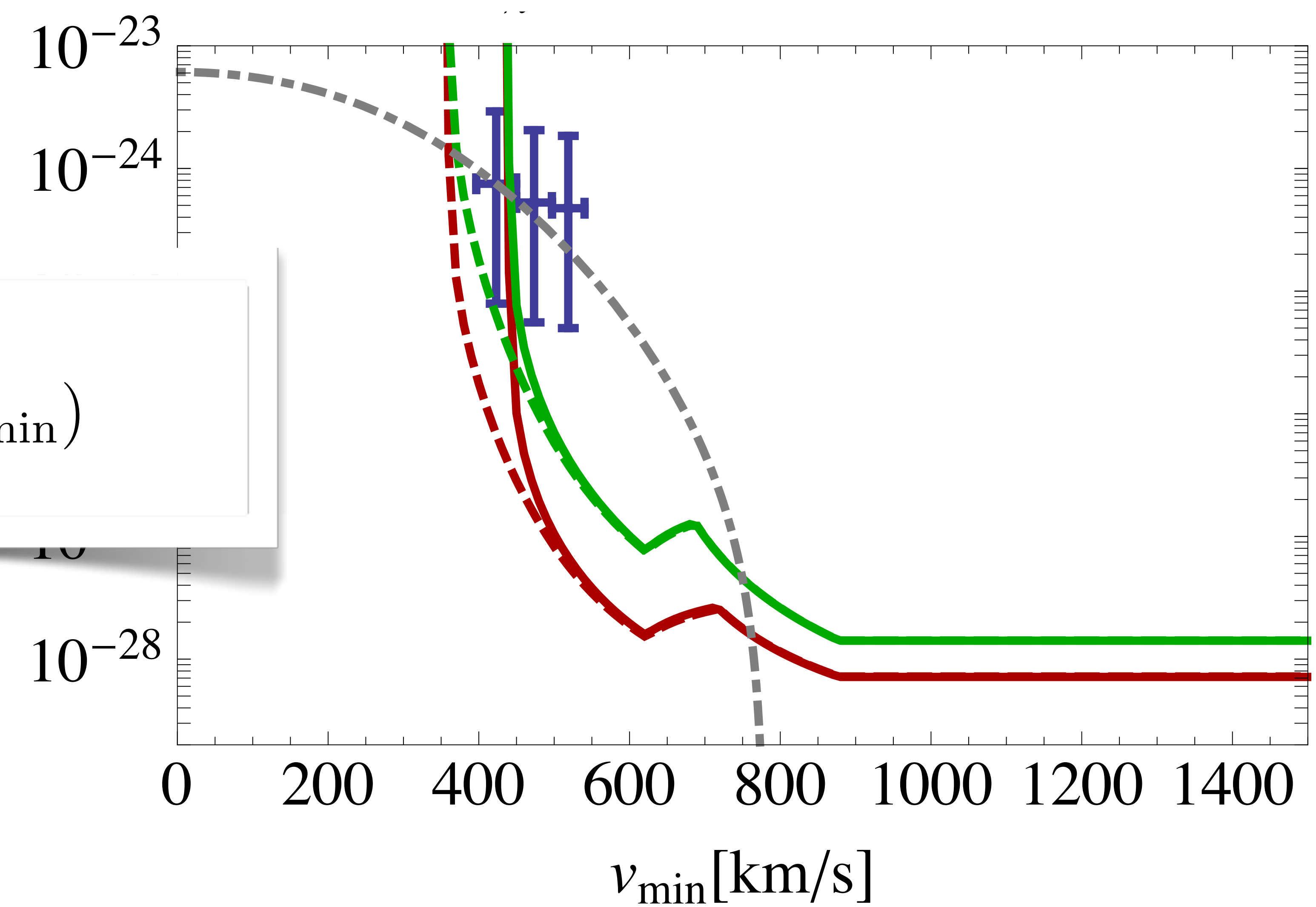

$$\tilde{g}(v_{\rm min}) = \frac{\rho \sigma_p}{m_\chi} g(v_{\rm min})$$

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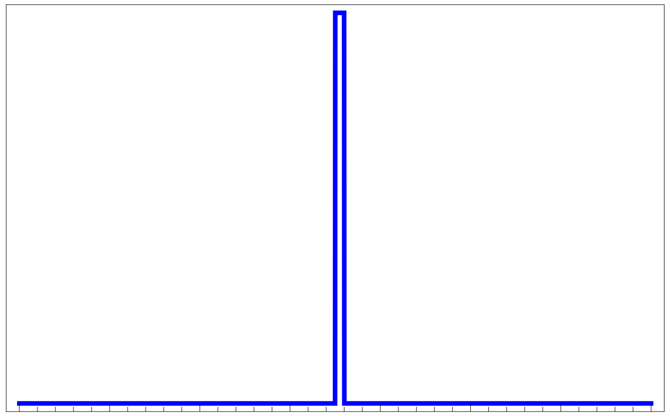
g(v)

Speed distribution is positive semidefinite $f(v) \geq 0$

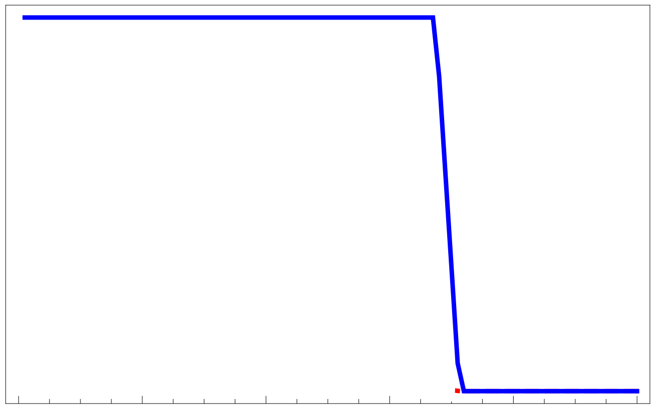
Integral monotonically decreases $\frac{d}{dv} g(v_{\rm min}) \leq 0$

"Least" monotonic function is a step function $\Theta(v_1 - v_{\rm min})$

$f(v)$



$g(v_{\rm min})$



Bounding g(v)

$$\frac{dR}{dE_R} = \frac{N_T M_T \rho}{2 m_\chi \mu^2} \sigma(E_R)\, g_1 \Theta(v_1 - v_{min}(E_R))$$

Use standard statistical techniques (max gap, Pmax, etc) to bound $\rho \sigma g_1 / m_\chi$

Most conservative possible limits

Measuring g(v)

$$g(v_{min}) = \frac{2 m_\chi \mu^2}{N_A \kappa \, m_p \, \rho \, \sigma(E_R)} \frac{dR_1}{dE_1}$$

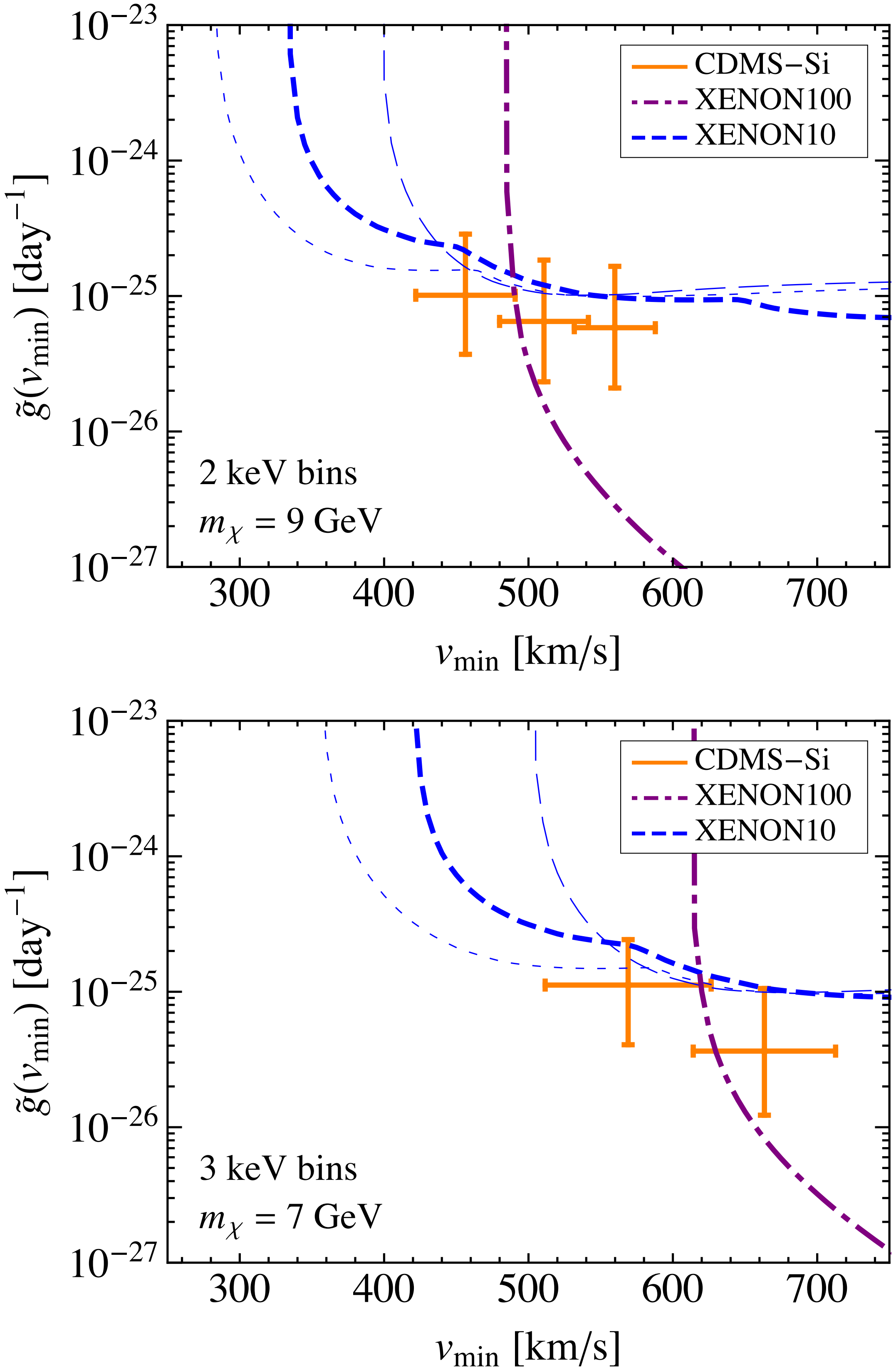
Convert binned recoil events into measurements in vmin space

Some potential problems:

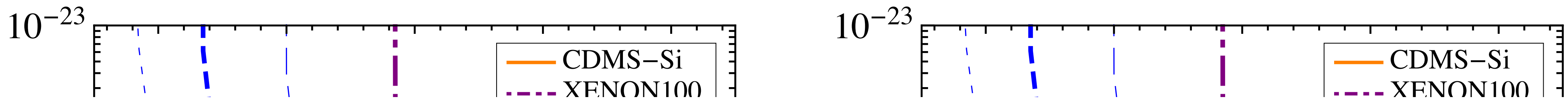
Loss of information

How to treat resolution, efficiencies [see eg Gelmini and Gondolo]

Particularly a problem early on, with low statistics



Frandsen et al.



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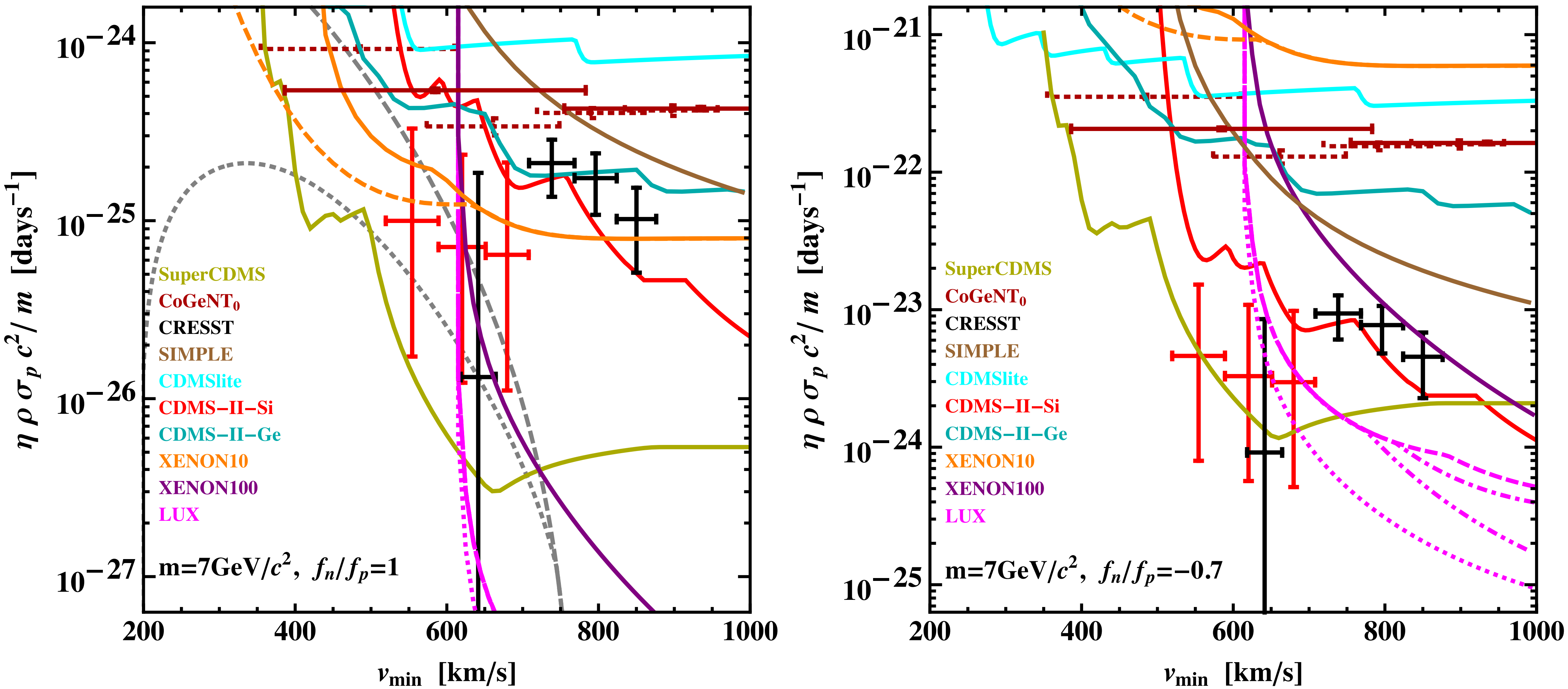

Frandsen et al.

Taking the method out of the bin

[PF, Kahn, McCullough]
see also [Feldstein, Kahlhoefer]

Address all these issues at once:
Extended (log) likelihood

$$\frac{dR}{dE_R} = \frac{N_A \rho_\chi \sigma_n m_n}{2 m_\chi \mu_{n\chi}^2} C_T^2(A,Z) \int dE'_R G(E_R, E'_R) \epsilon(E'_R) F^2(E'_R) g(v_{min}(E'_R))$$

$$L/2 = N_E - \sum_{i=1}^{N_O} \log \left. \frac{dR_T}{dE_R} \right|_{E_R = E_i}$$

Taking the meth... ut of the bin

[PF, Kahn, McCullough]
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Address... :

Extende...

$$\mathcal{L} = \frac{e^{-N_E}}{N_O!} \prod_{i=1}^{N_O} \frac{dR_T}{dE_R}\bigg|_{E_R=E_i}$$

$$L = -2\log(\mathcal{L})$$

$$\frac{dR}{dE_R} = \frac{N ...}{2m_\chi \mu_{n\chi}^2} C_T^2(A,Z) \int dE'_R G(E_R, E'_R)\epsilon(E'_R)F^2(E'_R)g(v_{min}(E'_R))$$

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$$L/2 = N_E - \sum_{i=1}^{N_O} \log \left. \frac{dR_T}{dE_R} \right|_{E_R = E_i}$$

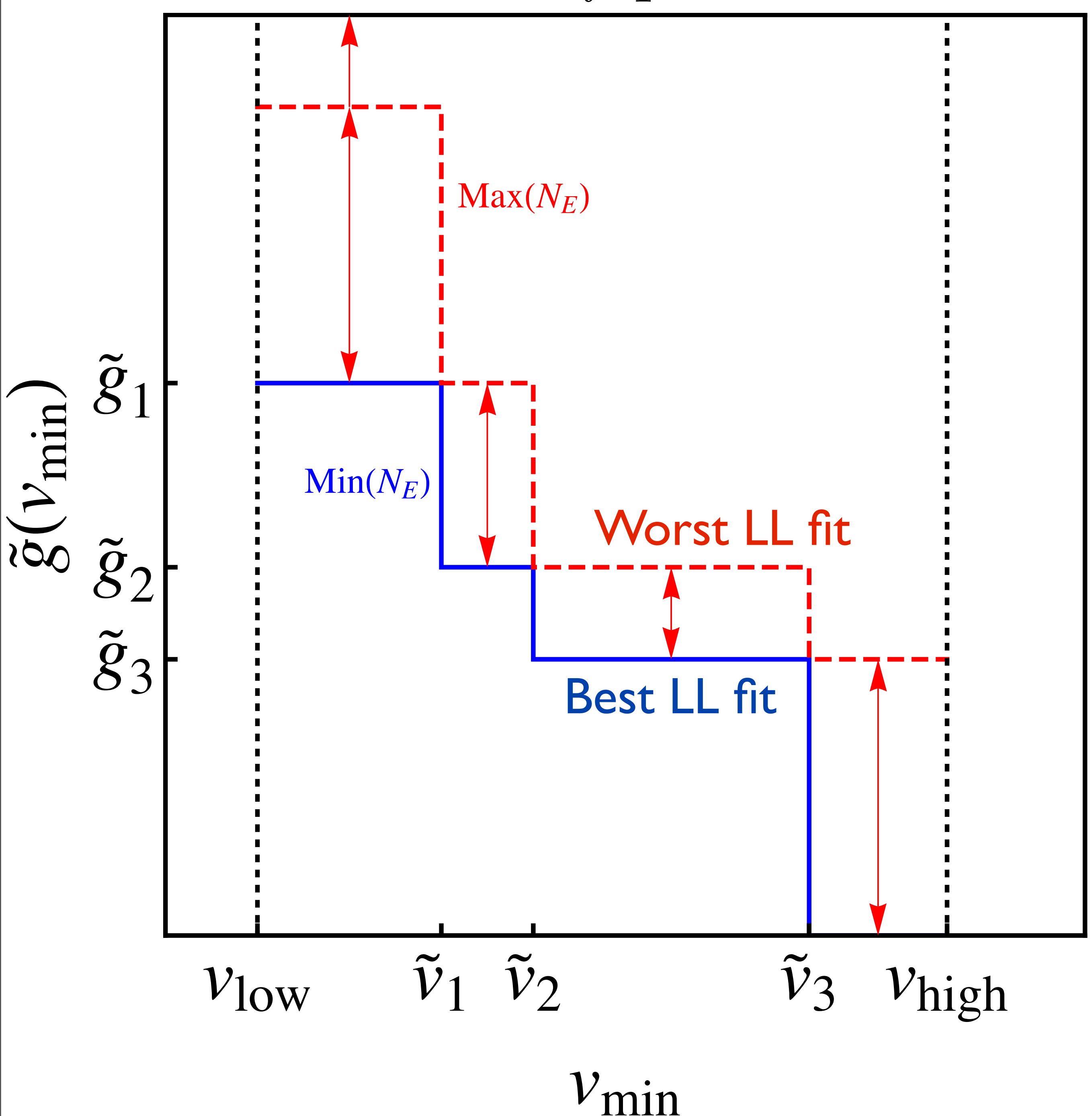
total number of events

$$N_E = \int_{E_{min}}^{E_{max}} \frac{dR_T}{dE_R} dE_R$$

Taking the method out of the bin

Minimize L whilst preserving monotonicity

$$L/2 = N_E - \sum_{i=1}^{N_O} \log \frac{dR_T}{dE_R}\bigg|_{E_R=E_i}$$



Best fit is given by a series of N_O step functions

For perfect energy resolution step positions given by energies of events $v_{min}(E_i)$

Minimize L over step heights

Confidence region for parameters

Taking the method out of the bin

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In reality energy smearing means each step contributes at all energies

What is the form of g(v) that minimizes LL?

Still series of N_O steps but now positions shift to the right

Minimize over $2N_O$ parameters

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KKT - "Lagrange multipliers"

General resolution function (think Gaussian)

$G(E_R, E'_R)$ to have the following properties:

(i) $\int G(E_R, E'_R) dE'_R = 1$ for any E_R.

(ii) As a function of E'_R for fixed E_R, $G(E_R, E'_R)$ has a single local maximum at $E'_R = E_R$ and no other local extrema.

(iii) For $E_R \neq E'_R$, either $G(E_R, E'_R) = 0$ or $\partial G(E_R, E'_R)/\partial E'_R \neq 0$.

Minimize,

$$L[\tilde{g}] = \int dE'_R\, K(E'_R)\tilde{g}(E'_R) - \sum_{i=1}^{N_O} \log\left(\mu_i + \int dE'_R\, G(E_i, E'_R)K(E'_R)\tilde{g}(E'_R)\right)$$

subject to inequality constraint: $d\tilde{g}/dE'_R \leq 0$

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(i) $\int G(E_R, E'_R) dE'_R = 1$ for any E_R.

(ii) As a function of E'_R for fixed E_R, $G(E_R, E'_R)$ has a single local maximum at $E'_R = E_R$ and no other local extrema.

(iii) For $E_R \neq E'_R$, either $G(E_R, E'_R) = 0$ or $\partial G(E_R, E'_R)/\partial E'_R \neq 0$.

Minimize,

all prefactors (form factor, eff. etc)

$$L[\tilde{g}] = \int dE'_R\, K(E'_R)\tilde{g}(E'_R) - \sum_{i=1}^{N_O} \log\left(\mu_i + \int dE'_R\, G(E_i, E'_R) K(E'_R)\tilde{g}(E'_R)\right)$$

subject to inequality constraint: $d\tilde{g}/dE'_R \leq 0$

KKT - "Lagrange multipliers"

General resolution function (think Gaussian)

$G(E_R, E'_R)$ to have the following properties:

(i) $\int G(E_R, E'_R) dE'_R = 1$ for any E_R. **normalised**

(ii) As a function of E'_R for fixed E_R, $G(E_R, E'_R)$ has a single local maximum at $E'_R = E_R$ and no other local extrema. **single peaked**

(iii) For $E_R \neq E'_R$, either $G(E_R, E'_R) = 0$ or $\partial G(E_R, E'_R)/\partial E'_R \neq 0$.

not flat anywhere, unless 0

Minimize,

all prefactors (form factor, eff. etc)

$$L[\tilde{g}] = \int dE'_R\, K(E'_R)\tilde{g}(E'_R) - \sum_{i=1}^{N_O} \log\left(\mu_i + \int dE'_R\, G(E_i, E'_R) K(E'_R)\tilde{g}(E'_R)\right)$$

subject to inequality constraint: $d\tilde{g}/dE'_R \leq 0$

Karush-Kuhn-Tucker

To enforce monotonicity

$$L[\tilde{g}] \to L[\tilde{g}] + \int dE'_R \, \frac{d\tilde{g}}{dE'_R} q(E'_R)$$

$$\begin{aligned} \frac{\delta L}{\delta \tilde{g}} - \frac{dq}{dE'_R} &= 0 \; , \\ \frac{d\tilde{g}}{dE'_R} &\leq 0 \; , \\ q(E'_R) &\geq 0 \; , \\ \int dE'_R \, \frac{d\tilde{g}}{dE'_R} q(E'_R) &= 0 \; . \end{aligned}$$

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↓

Either saturate constraint (usual Lagrange multiplier) or q=0

KKT

$$\frac{\delta L}{\delta \tilde{g}} - \frac{dq}{dE'_R} = 0 \,,$$
$$\frac{d\tilde{g}}{dE'_R} \leq 0 \,,$$
$$q(E'_R) \geq 0 \,,$$
$$\int dE'_R \, \frac{d\tilde{g}}{dE'_R} q(E'_R) = 0 \,.$$

Assume g(E) is not flat over some range. Then,

$$\sum_{i=1}^{N_O} \frac{G(E_i, E'_R)}{\gamma_i} = 1,$$

with

$$\gamma_i = \mu_i + \int dE''_R \, G(E_i, E''_R) K(E''_R) \tilde{g}(E''_R)$$

g(v) must be flat except at individual points i.e. steps

KKT

$$\frac{\delta L}{\delta \tilde{g}} - \frac{dq}{dE'_R} = 0 \ ,$$
$$\frac{d\tilde{g}}{dE'_R} \leq 0 \ ,$$
$$q(E'_R) \geq 0 \ ,$$
$$\int dE'_R \, \frac{d\tilde{g}}{dE'_R} q(E'_R) = 0 \ .$$

Assume g(E) is <u>not flat</u> over some range. Then,

depends on E'

$$\sum_{i=1}^{N_O} \frac{G(E_i, E'_R)}{\gamma_i} = 1,$$

const.

with

$$\gamma_i = \mu_i + \int dE''_R \, G(E_i, E''_R) K(E''_R) \tilde{g}(E''_R)$$

g(v) must be <u>flat</u> except at individual points i.e. steps

KKT

$$\frac{\delta L}{\delta \tilde{g}} - \frac{dq}{dE'_R} = 0 \; ,$$
$$\frac{d\tilde{g}}{dE'_R} \leq 0 \; ,$$
$$q(E'_R) \geq 0 \; ,$$
$$\int dE'_R \, \frac{d\tilde{g}}{dE'_R} q(E'_R) = 0 \; .$$

Assume g(E) is <u>not flat</u> over some range. Then,

depends on E' → $\sum^{N_O}$ …

Reducio ad absurdum

… ← const.

with

$$\gamma_i = \mu_i + \int dE''_R \, G(E_i, E''_R) K(E''_R) \tilde{g}(E''_R)$$

g(v) must be <u>flat</u> except at individual points i.e. steps

KKT

$$\frac{\delta L}{\delta \tilde{g}} - \frac{dq}{dE'_R} = 0 \,, \quad \frac{d\tilde{g}}{dE'_R} \leq 0 \,, \quad q(E'_R) \geq 0 \,, \quad \int dE'_R \, \frac{d\tilde{g}}{dE'_R} q(E'_R) = 0 \,.$$

Can determine the positions of the steps

$$\frac{dq}{dE'_R} = K(E'_R)\left(1 - \sum_{i=1}^{N_O} \frac{G(E_i, E'_R)}{\gamma_i}\right)$$

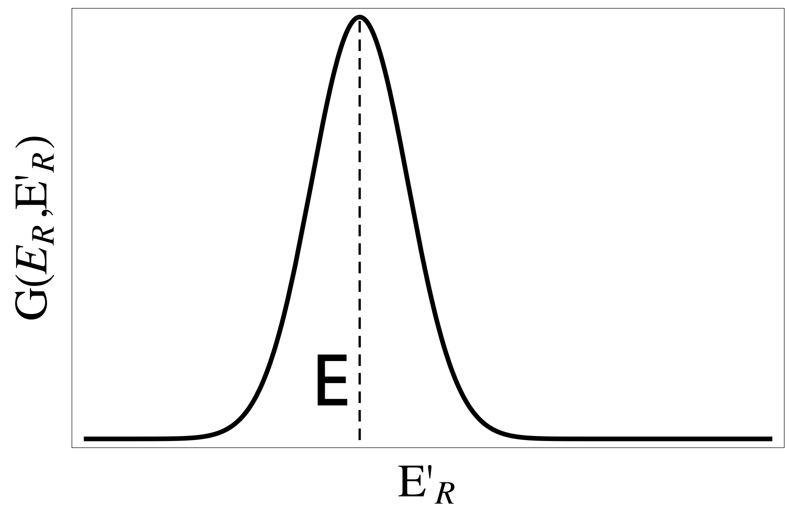
At step energies $(\tilde{E}_j)$ positivity of q requires

$$\left.\frac{dq}{dE'_R}\right|_{\tilde{E}_j} = 0, \quad \left.\frac{d^2 q}{dE'^2_R}\right|_{\tilde{E}_j} \geq 0$$

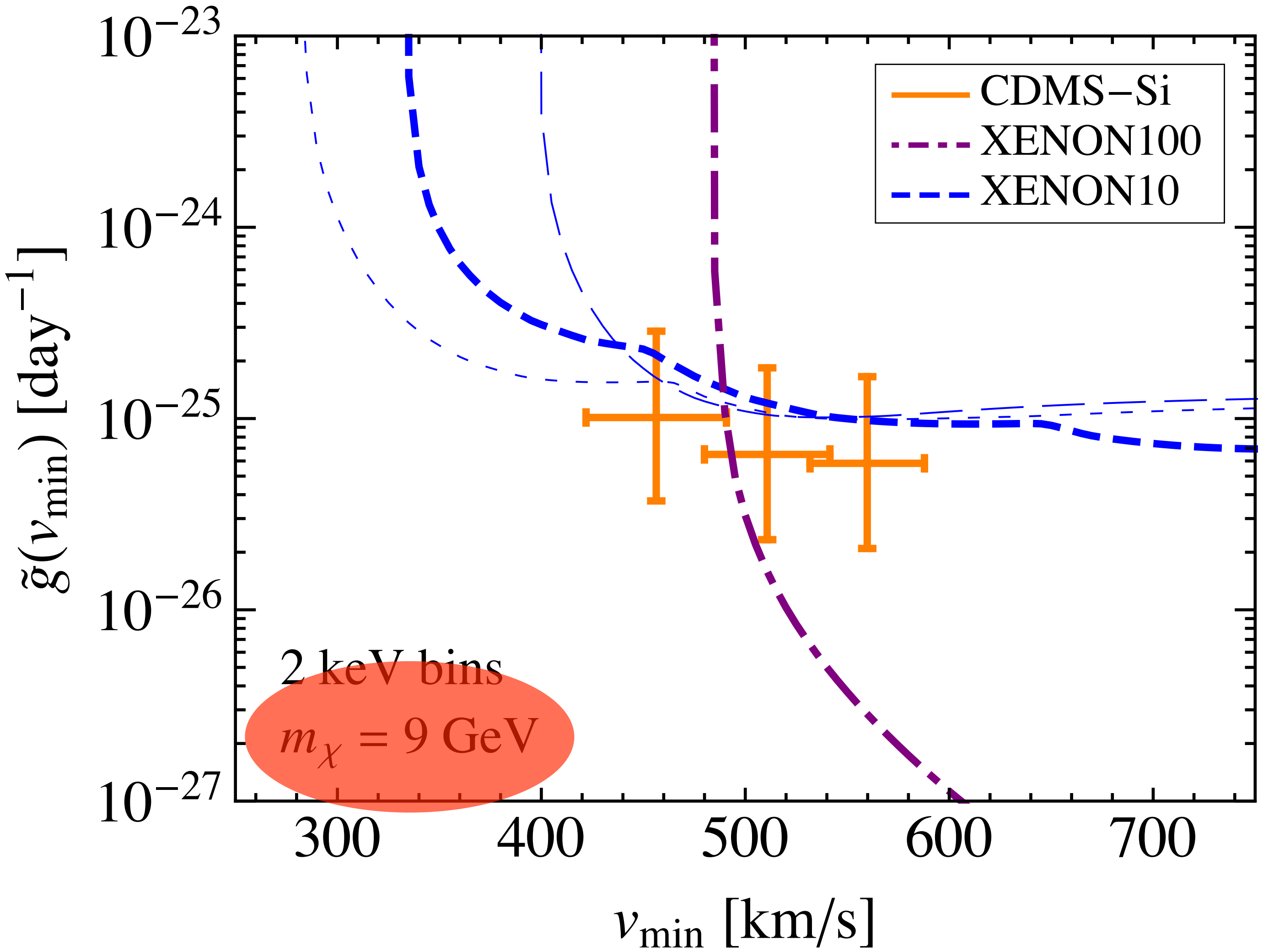
Thus,

$$-\sum_{i=1}^{N_O} \frac{1}{\gamma_i} \left.\frac{\partial G(E_i, E'_R)}{\partial E'_R}\right|_{E'_R = \tilde{E}_j} \gtrsim 0$$

Steps shift to the right



Varying the DM mass



$$v'_{min}(E_R) = \frac{\mu_{N\chi}}{\mu_{N\chi'}} v_{min}(E_R)$$

$$\tilde{g}' = \frac{\mu^2_{n\chi'}}{\mu^2_{n\chi}} \tilde{g}$$

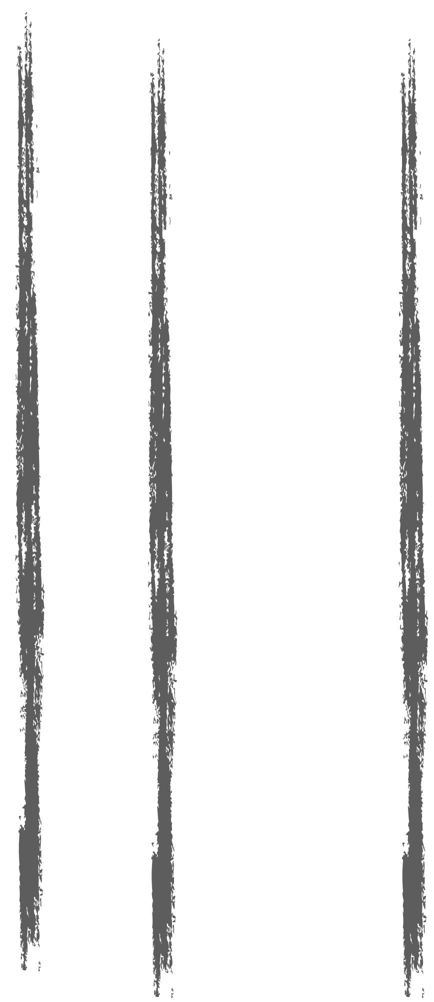
<u>One plot</u> is sufficient to determine results for all masses

Unbinned application

[PF, Kahn, McCullough]

Unbinned application

[PF, Kahn, McCullough]

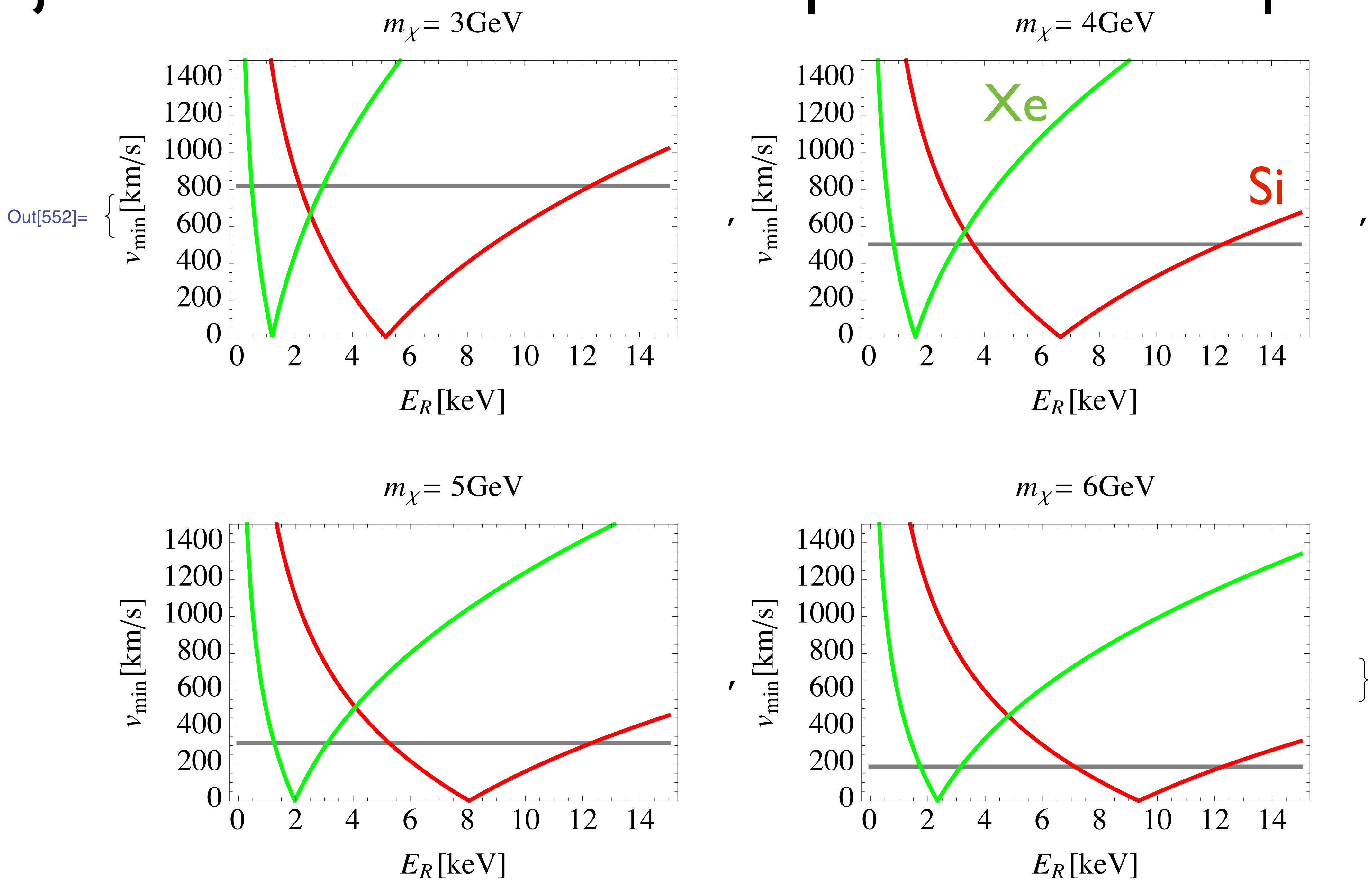


Extension to inelastic DM

[see also Bozorgnia et al; del Nobile et al]

$$v_{\min} = \left| \delta + \frac{m_{\rm N}\, E_{\rm R}}{\mu} \right| \frac{1}{\sqrt{2\, E_{\rm R}\, m_{\rm N}}}$$

Projection to and from vmin space more complicated



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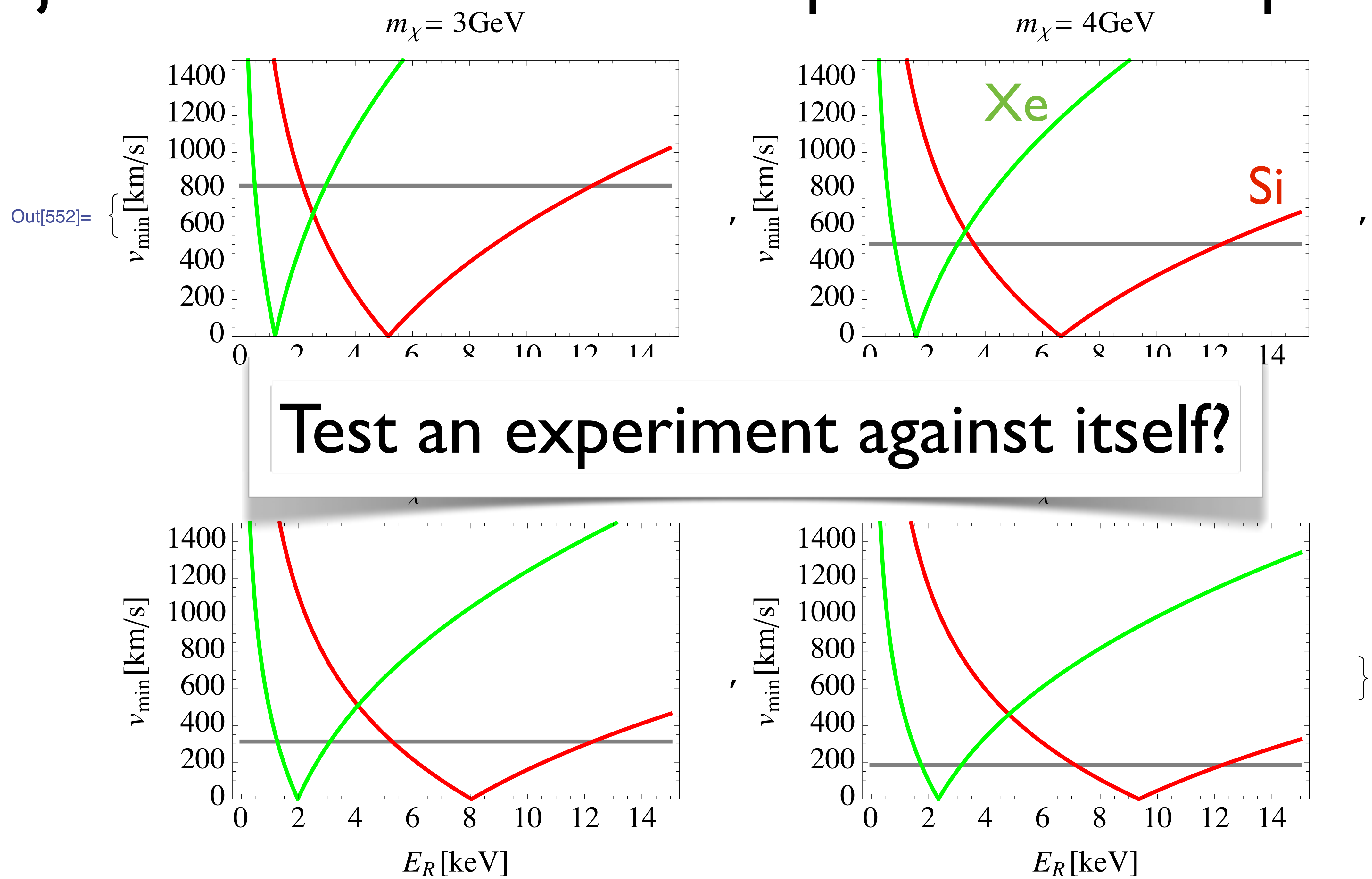

Test an experiment against itself?

Conclusions

- Should analyse data independent of astro uncertainties
- With multiple experiments should compare g(v), tests consistency
- Presenting experimental results in g(v) *very* useful
  - One plot contains all information, for all masses
- Find region of consistent parameter space
- 

Unbinned approach using likelihood techniques
  - Maximal use of information
  - Independent of astrophysics, expts. agree/disagree?
- Application: CDMS-Si is at odds with LUX
- Ultimately may be able to extract f(v) by differentiating deconvoluted rate