Dark Matter without Astrophysics: Halo-independent methods at Direct Detection Experiments

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Based on work with A. Anderson, G. Jung, Y. Kahn, G. Kribs, J Liu, M. McCullough, T. Tait, P Sorensen and N. "break dancer" Weiner

Astrophysics independence

Comparing two positive results: Drees and Shan; 0803.4477, 0809.1295

vmin space: PJF, Liu, Weiner; 1011.1915

Applications to CDMS, CoGeNT, COUPP, CRESST, DAMA, LUX, XENON,light DM, etc; 1103.3481, 1106.0743,1106.6241, 1107.0715, 1107.0741,1110.5338,1304.6066, 1304.6183,....

Annual modulation: Herrero-Garcia et al.; 1112.1627, 1205.0134

Including detector effects: Gelmini and Gondolo; 1202.6359

Extension to iDM: Bozorgnia et al.; 1305.3575

Extension to general interactions and kinematics: Del Nobile et al.; 1306.7273

Unbinned analysis: PJF, Kahn, McCullough; 1403.6830

Wednesday, 10 December 14

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Astrophysics independence

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Direct Detection

with,

$$
\frac{d\sigma}{dE_R} = F_N^2(E_R) \frac{m_N}{\mu v^2} \bar{\sigma}(v, E_R)
$$

$$
\bar{\sigma}_i(v, E_R) = \begin{cases} \sigma_{i0} \\ \sigma_{i0} F_{\chi_i}^2(E_R) \\ \sigma_{i0}(v) F_{\chi_i}^2(E_R) \\ \sigma_{i0}(v, E_R) \end{cases}
$$

Usual case (SI elastic WIMP)

$$
\sigma_N = \frac{(Zf_p + (A-Z)f_n)^2}{f_p^2} \frac{\mu_{N\chi}^2}{\mu_{n\chi}^2} \sigma_p \qquad v_{min} = \sqrt{\frac{m_N E_R}{2\mu_{N\chi}^2}}
$$

methods are applied to the three anomalous events observed in the CDMS-Si detector and compared to the current constraints from XENON10 and LUX. Finally, in Sec. 4 conclusions from XENON 10 conclusions from XENON10 and LUX. Finally, in Sec. 4 conclusions from XENON 10 conclusions from XENON 10 conclusions fr **Theory**

$$
\frac{dR}{dE_R} = \frac{N_T \rho_\chi}{m_\chi} \int_{v_{\rm min}}^{v_{\rm max}} d^3 \vec{v} f(\vec{v}(t)) \frac{d\,\sigma|\vec{v}|}{dE_R}
$$

Comparisons between positive signals and null results are discussed in Sec. 2.3. In Sec. 2.4,

we describe how the halo-independent information for one specific DM mass may be specific DM mass may be simply

and unambiguously mapped to other DM masses, avoiding the proliferation of limit plots and

calculations. The reader only interested in a short explanation of how to apply the methods

can proceed directly to Sec. 2.5 where all necessary calculation steps for setting limits and

for interpreting signals are briefly set out. In Sec. 3 the new unbinned halo-independent

Expt.

$$
\frac{dR}{dE_R} = \frac{N_A \rho_X \sigma_n m_n}{2m_X \mu_{n\chi}^2} C_T^2(A, Z) \int dE_R' G(E_R, E_R') \epsilon(E_R') F^2(E_R') g(v_{min}(E_R'))
$$

where *m* is the DM mass, *mⁿ* the nucleon mass, *µn* the nucleon-DM reduced mass, *ⁿ* the

DM-nucleon scattering cross-section, ⇢ the local density, *N^A* is Avogadro's number, *F*(*ER*)

is the nuclear form factor which accounts for loss of coherence as the DM resolves sub-nuclear

The di↵erential event rate² at a direct detection experiment is

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$$
\n**Expt.**

\nresolution

\n
$$
\frac{dR}{dE_R} = \frac{N_A \rho_\chi \sigma_n m_n}{2m \mu^2} C_T^2(A, Z) \int dE'_R G(E_R, E'_R) \epsilon(E'_R) F^2(E'_R) g(v_{\text{min}}(E'_R))
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 dE_R

 $2m_\chi \mu^2_{n\chi}$

There are known knowns. These are things we know that we know. There are known calculations. That is to say, there are things and the methods of how to apply the methods of how to apply the methods. that we know we don't know. But there are also unknown unknowns. There are things we don't know we don't know. don't know we don't know.

Comparisons between positive signals and null results are discussed in Sec. 2.3. In Sec. 2.4,

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Expt.

Theory

resolution efficiency *dR* dE_R = $N_A\rho_\chi\sigma_n m_n$ $2m_\chi \mu^2_{n\chi}$ $C_T^2(A,Z)$ z
Z $dE_R' G(E_R, E_R') \epsilon(E_R') F^2(E_R') g(v_{min}(E_R'))$

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distance scales, 10 December 14 \overline{a} Wednesday, 10 December 14

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Z $dE_R' G(E_R, E_R') \epsilon(E_R') F^2(E_R') g(v_{min}(E_R'))$

"In theory there is no difference between theory and practice. DM-nucleon scattering cross-section, ⇢ the local density, *N^A* is Avogadro's number, *F*(*ER*) But in practice there is.--Yogi Berra"

The uncertain world of direct detection

•Nuclear form factors, SI/SD + >2 others •Atomic physics, detector response. e.g. Leff, keVee \rightarrow keVnr

[see talks from this workshop]

•Astrophysics How is DM distributed in our neighbourhood? So far we have only transited ~0.007pc, way below Nbody resolution Baryons? SIDM? Dark disc? Debris flow? Gravitational focusing? [Randall et al.] [Lisanti et al.]

Dependence on details of velocity distribution

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Direct Detection IV. DECONVOLUTED SCATTERING RATE AND LOCAL CONVOLUTED SCATTERING RATE AND LOCAL CONVOLUTED SCATTERING RATE AND *dR dE^R* rt Dete *F*² (*ER*) *dE^R dF*²

⇤

ⁱ ²

There is value is value information that can be extracted in $\mathcal{L}_\mathcal{F}$

from Eqs. (14) and (14). We know the velocity distribution \mathcal{L}_1

tion of dark matter must be positive for all *v*,

Is a signal a measurement of particle physics or t_{refr} $f(x)$ and $f(x)$ is out. This is one of the factor $f(x)$ ensemble of WIMPs with arbitrary cross sections. For a single WIMP with a factorizable cross section, with a factorizable cross section, \mathcal{L}_max astrophysics?

There is valuable information that can be extracted

from Eqs. (14) and (15). We know the velocity distribu-

The only way we have of probing our local DM distribution V. *f*-CONDITION

⌅

 $f_1(v_{\min}(E_R)) = -\frac{4\mu^2 E_R^2}{m_{\infty}^2 E_R^2}$ *R* $m_N^2 E_R^2 - \mu^2 \delta^2$ 1 $\mathcal{N}\sigma_0(v_\text{min}(E_R))F_\chi^2(E_R)$ $\sqrt{ }$ *dR* dE_R $R = \frac{1}{F^2(1)}$ $F_\chi^2(E_R)$ $dF_{\chi}^2(E_R)$ dE_R $\sum_{i=1}^{n}$

f -condition^{$f(y) > 0$} **f-condition:** $f(v) \ge 0$

tion of dark matter must be positive for all *v*,

This result allows us to gain information on the velocity

distribution of dark matter evaluated at the *minimum*

velocity to scatter for a given recoil energy *ER*. With

a range of *v*: *v*min(*E*min *^R*) *<v<v*min(*E*max *^R*). \vert (Deconvoluted) rate is a monotonically decreasing function, or \vert $f(x)$ for $f(x)$ results of $f(x)$ results of the written as $f(x)$ *i increasing DM form factor* physics e.g. inelastic or an there is non-standard particle physics e.g. inelastic or an
example of the position of the appearing in Eq. (15). The contract of the con
The contract of the contract o *there is non-standard particle physics e.g. inelastic or an increasing DM form factor*

Direct Detection IV. DECONVOLUTED SCATTERING RATE RATE CONVOLUTED SCATTERING CONVOLUTED SOMETIME CO ties de la reception de Sun, et al. (1971)
Earth *dR dE^R* rt Dete *F*² (*ER*) *dE^R dF*²

⇤

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$$
f_1(v_{\min}(E_R)) = -\frac{4\mu^2 E_R^2}{m_N^2 E_R^2 - \mu^2 \delta^2} \frac{1}{\mathcal{N}\sigma_0(v_{\min}(E_R)) F_\chi^2(E_R)} \left(\frac{d\mathcal{R}}{dE_R} - \mathcal{R} \frac{1}{F_\chi^2(E_R)} \frac{dF_\chi^2(E_R)}{dE_R} \right)
$$

$$
f_1(v) = \int d\Omega f(\vec{v}).
$$

f -condition^{$f(y) > 0$} $\mathbf{r} \cdot \mathbf{r} = \mathbf{r} \cdot \mathbf{r}$ *Idition: 1* r² *^N* (*ER*) **f-condition:** $f(v) \ge 0$

velocity to scatter for a given recoil energy *ER*. With

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⇤

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ⁱ ²

*dvⁱ vifi*1(*vi*)¯⇤*i*(*vi, ER*)*.* (11)

Is a signal a measurement of particle physics or t_{refr} astrophysics? ensemble of WIMPs with arbitrary cross sections. For a single WIMP with a factorizable cross section, with a factorizable cross section, \mathcal{L}_max

The only way we have of probing our local DM distribution mately distribution V. *f*-CONDITION

⌅

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f_1(v_{\min}(E_R)) = -\frac{4\mu^2 E_R^2}{m_N^2 E_R^2 - \mu^2 \delta^2} \frac{1}{\mathcal{N}\sigma_0(v_{\min}(E_R))F_\chi^2(E_R)} \left(\frac{dR}{dE_R} - R \frac{1}{F_\chi^2(E_R)} \frac{dF_\chi^2(E_R)}{dE_R}\right)
$$
\n
$$
f_1(v) = \int d\Omega f(\vec{v}).
$$
\n
$$
\mathcal{R} \equiv \frac{1}{F_N^2(E_R)} \frac{dR}{dE_R}
$$
\n**f**-**conditional' rate**\n
$$
f(r) > 0
$$

f -condition^{$f(y) > 0$} $\mathbf{r} \cdot \mathbf{r} = \mathbf{r} \cdot \mathbf{r}$ *Idition: 1* r² *^N* (*ER*) **f-condition:** $f(v) \ge 0$ $\frac{1}{\sqrt{2}}$ increases the variable variable values of what has been considered values of what has been considered values of $\sqrt{2}$

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The contract of the contract o *i (Deconvoluted) rate is a monotonically decreasing nere is vi,*min *du v*_{*i*} *v*_{*i*} *v*_{*i*} *du ence <i>priyones e.g. menoscreasing DM form factor* $\overline{}$ there is non-standard barticle physics e σ inelastic or an $\overline{}$ torizable velocity and recoil energy dependence [c.f., the most general form written on the fourth line of \mathbf{I} *ⁱ mⁱ*). While there are imincreasing DM form factor *(Deconvoluted) rate is a monotonically decreasing function, or* there is non-standard particle physics e.g. inelastic ϵ *there is non-standard particle physics e.g. inelastic or an increasing DM form factor*

Neal is not a great break dancer...

Neal is not a great break dancer...

...but his son is!

- •Cannot determine *anything* about DM from a single positive result
- •Assuming an escape velocity can place a *lower* bound on DM mass, but no upper bound
- •Luckily we have other complementary searches, and multiple direct detection expts.

Direct Detection without bias

[PF, Liu, Weiner]

$$
\frac{dR}{dE_R} = \frac{N_T M_T \rho}{2m_{\chi} \mu^2} \int_{v_{min}}^{v_{max}} d^3 \vec{v} \frac{f(\vec{v}, \vec{v_E})}{v} \sigma(E_R)
$$

$$
\frac{dR}{dE_R} = \frac{N_T M_T F_N^2(E_r)}{2\mu^2} \frac{\rho \sigma}{m_\chi} g(v)
$$

$$
v_{min}=\sqrt{\frac{M_T E_R}{2\mu^2}}
$$

Recoil energy uniquely determines **minimum** DM velocity

Direct Detection without bias

[PF, Liu, Weiner]

$$
v_{min} = \sqrt{\frac{M_T E_R}{2\mu^2}}
$$

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$$
v_{min}=\sqrt{\frac{M_T E_R}{2\mu^2}}
$$

Recoil energy uniquely determines **minimum** DM velocity

Comparing experiments-vmin space *<i><u>nipa opéra vresir</u> dE^R* $\overline{}$ *NTM^T* ⇥ \bm{p} in space

high] = *^µ*²

^g(*vmin*) = ²*mµ*²

 $N_T = \kappa N_A m_p / M_T$ $N_T = \kappa N_A m_p / M_T$

scattering sites per kg with *N^A* Avogadro's number and the mass fraction of the detector

high]*.* (7)

 $N_T = \kappa N_A$
 $N_T = \kappa N_A$ Solve for g(v)

[*E*(2)

low, E(2)

$$
g(v_{min}) = \frac{2m_{\chi}\mu^2}{N_A\kappa m_p \rho \sigma(E_R)} \frac{dR_1}{dE_1}
$$

$$
\frac{dR_1}{dE_1} \iff g(v_{min}) \iff \frac{dR_2}{dE_2}
$$
 $v_{min} = \sqrt{\frac{M_T E_R}{2\mu^2}}$

[*E*(1)

low, E(1)

*dR*¹ ⇤2(*E*2) **The master formula (SI):** $w = w$ introduced a target specific coe $w = w$

(1)*µ*²

2

 $\overline{}$

*µ*2

¹ *^M*(2)

 A_n and the energy mapping above, we have a rate mapping above, we have a rate mapping above, we have a rate mapping, μ

 $\bigcup_{T} - \kappa$ $\big(\int p Z \big|^{r} + \int n (A \big|^{r} - Z \big|^{r}) \big)$ d_a $\binom{i}{r} =$ $\kappa^{(i)}(f_p Z^{(i)} +$ $C_T^{(i)} = \kappa^{(i)} (f_p Z^{(i)} + f_n (A^{(i)} - Z^{(i)})$ *.* (10) $\big)^2$

In certain situations dierential rates may not be available and instead it is only possible

. (10)

, (11)

*E*²

$$
\frac{dR_2}{dE_R}(E_2) = \frac{C_T^{(2)}}{C_T^{(1)}} \frac{F_2^2(E_2)}{F_1^2 \left(\frac{\mu_1^2 M_T^{(2)}}{\mu_2^2 M_T^{(1)}} E_2\right)} \frac{dR_1}{dE_R} \left(\frac{\mu_1^2 M_T^{(2)}}{\mu_2^2 M_T^{(1)}} E_2\right)
$$

⇥

dE^R

*µ*2

² *M*(1)

(10) in a simple form

data for the dierential rate of DM scattering in their experiment, *dR*1*/dE^R* at energies *E*(1) this can be used to predict a rate at the used to predict a rate at energy $\mathcal{E}(\mathbf{z})$ **Using vmin space**

ⁱ at experiment 2, *dR*2*/dER*, or *vice versa* if

*min,*1*, vhigh*

*min,*2]

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 $Eynormont$ \perp $[E_{low}^{(1)}, E_{low}^{(1)}] \Longleftrightarrow [v_{min}^{low}, v_{min}^{high}] \Longleftrightarrow [E_{low}^{(2)}, E_{high}^{(2)}]$ Experiment 1 ↔ Experiment 2

ⁱ at experiment 2, *dR*2*/dER*, or *vice versa* if

*min,*1*, vhigh*

*min,*2]

data for the dierential rate of DM scattering in their experiment, *dR*1*/dE^R* at energies *E*(1) this can be used to predict a rate at the used to predict a rate at energy $\mathcal{E}(\mathbf{z})$ **Using vmin space** the detector is built for a single material, once limits and a best-fit velocity integrals have been calculated for a single DM mass *m*, it is simple to map

entire calculation must be repeated again, leading to a proliferation of plots when presenting

ⁱ at experiment 2, *dR*2*/dER*, or *vice versa* if

*min,*1*, vhigh*

*min,*2]

while preserv-

A more direct comparison of data than sigma-m plots Easy to derive from data

For a DM (and single target expts) pood only show for For eDM (and single target expts.) need only show for one mass

for particular parameters. The solid curves are for a threshold curve are for $3 k$ the red consistency of $g(v)$ determines allowed DM params. Ultimately allows for measurements of g(v) Consistency of g(v) determines allowed DM params.

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wi(*v, ER*)*fi*1(*v*) *,* (15)

form factors, the inversion result can be written as

\int Speed distribution is positive semidefinite $f(v) \geq 0$

where the velocity distributions of the WIMPs are Integral monotonically decreases $\frac{d}{dx}g(x)$

*f*1HvL

tion of dark matter must be positive for all *v*,

 $\begin{pmatrix} \text{function is a step} \\ \text{function } \Theta(y_1 - y_{\min}) \end{pmatrix}$ "Least" monotonic function is a step function $\Theta(v_1 - v_{\min})$

from Eqs. (14) and (14) and (15). We know the velocity distribution \mathcal{L}^{max}

Bounding $g(v)$

Using this, (1) becomes

the SI case, rather than simply ⇢*/m* as in the standard case where *g* is specified. For a

have on ⇢*/m*, or, for fixed ⇢ and *m*, precisely as before.

 $\frac{1}{2}$

$$
\frac{dR}{dE_R} = \frac{N_T M_T \rho}{2m_\chi \mu^2} \sigma(E_R) g_1 \Theta(v_1 - v_{min}(E_R))
$$

For a given WIMP mass *m*, the overall scaling is now proportional to e.g., ⇢*g*1*/m* in

| Use standard statistical techniques (max gap, Pmax, etc) to | α *pound puy₁* μ _{χ} $\bm{\mathsf{bound}}$ $\rho \sigma g_1/m_\chi$ $\overline{}$

> In short: to calculate the appropriate limits on *g*(*v*), one should use whatever technique one was intending to use for the standard analysis, but now replace the Maxwellian *g*(*v*) Most conservative possible limits

have on ⇢*/m*, or, for fixed ⇢ and *m*, precisely as before.

with the step function form. For any given *m*, one places a limit on ⇢*g*1*/m* as one would

Measuring g(v) We asuring $g(v)$ and $f(v)$ and $f(v)$ and $g(v)$

$$
g(v_{min}) = \frac{2m_{\chi}\mu^2}{N_A\kappa m_p \rho \sigma(E_R)} \frac{dR_1}{dE_1}
$$

*min,*1*, vhigh*

, (9)

*min,*2]

This then allows us to explicitly state the expected rate for experiment two, again ² *restricted the energy record events into measurements in vining i.e. E in the appropriate velocity range in the appropriate velocity* $\frac{1}{2}$ Analogous to the energy mapping above, we have a rate mapping, Convert binned recoil events into measurements in vmin space

> l pr Some potential problems:

*dE*¹ rticulariy a problem early on, with **O۱** *y* statis Particularly a problem early on, with low statistics *.* (10) Loss of information How to treat resolution, efficiencies [see eg Gelmini and Gondolo]

> 2*M*(1) *T*

*µ*2

<u>lobil</u> -1 6 / *Physics Procedia 00 (2014) 1–10* Del Nobile et al.

[PF, Kahn, McCullough] see also [Feldstein, Kahlhoefer]

dE^R , (2.7)

*d*3*v ,* (2.2)

. (2.8)

Address all these issues at once: *Extended (log) likelihood* $\begin{array}{ccc} \hline \multicolumn{3}{c} \multic$ at once:

dE^R

The di↵erential event rate² at a direct detection experiment is

L =

$$
\frac{dR}{dE_R} = \frac{N_A \rho_X \sigma_n m_n}{2m_X \mu_{n_X}^2} C_T^2(A, Z) \int dE'_R G(E_R, E'_R) \epsilon(E'_R) F^2(E'_R) g(v_{min}(E'_R))
$$

is the total number of events expected for a given set of parameters. We may compare di↵erent

compared to the current constraints from XENON10 and LUX. Finally, in Sec. 4 conclusions

Ξ

$$
L/2 = N_E - \sum_{i=1}^{N_O} \log \frac{dR_T}{dE_R}\bigg|_{E_R=E_i}
$$

where *f*(*v*) is the DM velocity distribution, and *v^E* is the Earth's velocity, both in the galactic

frame. We ignore the small time-dependence introduced by the Earth's motion around the

fitting procedure we have

Taking the method ut of the bin N_{\odot} and N_{\odot} in the idealized case of perfect energy resolution and perfect energy resolution and N_{\odot} *NO*! T_a ting the method is the normalization T_a is the DM Taking the method is a **let of the bin**
No and $\frac{1}{2}$, in the Early McCuloush, Taking the method Z *^Emax Emin dRTdE^R ,*

 $\textsf{Extended}$ $\begin{array}{ccc} & & & & & & & & \text{if} & \text{if$

 $\mathcal{L} =$

dE^R

 $\begin{bmatrix} \text{Extended} \\ L = -2\log(L) \end{bmatrix}$

Address $\mathcal{L} = \frac{c}{N} \prod \widehat{dE_R}\big|_{E_R = E_i}$:

The di↵erential event rate² at a direct detection experiment is

 $=1$

 dR_T

 $\overline{}$ $\overline{}$ $\overline{}$

 dE_R

L =

 \prod

i=1

N^O

extended and analysis methods of \mathbb{R}

NO!

 e^{-N_E}

dE^R

dE^R ER=*Eⁱ*

[PF, Kahn, McCullough] e^{-NE} \boldsymbol{I} $\boldsymbol{I$ N_{F} N_{Q} dR_{T} */defined* components and background compon (2.6) see also [Feldstein, Kahlhoefer]

dE^R , (2.7)

 $\frac{1}{2}$

*d*3*v ,* (2.2)

. (2.8)

(2.9)

(2.7)

Addres:
\n
$$
\mathcal{L} = \frac{e}{N_O!} \prod_{i=1}^{N} \frac{1}{dE_R} \bigg|_{E_R = E_i}.
$$
\n
$$
L = -2 \log(\mathcal{L})
$$
\n
$$
\frac{dR}{dE_R} = \frac{N_{\text{max}} \pi m n_0^2}{2m_{\chi} \mu_{n\chi}^2} C_T^2(A, Z) \int dE'_R G(E_R, E'_R) \epsilon(E'_R) F^2(E'_R) g(v_{min}(E'_R))
$$
\n
$$
N_O
$$

is the total number of events expected for a given set of parameters. We may compare di↵erent

compared to the current constraints from XENON10 and LUX. Finally, in Sec. 4 conclusions

dR^T

dE^R

Ξ

i=1

 $R \mid E_R = E$

 $E_R = E_i$

 \mathcal{L}

$$
L/2 = N_E - \sum_{i=1}^{N_O} \log \frac{dR_T}{dE_R}\bigg|_{E_R=E_i}
$$

where *f*(*v*) is the DM velocity distribution, and *v^E* is the Earth's velocity, both in the galactic

frame. We ignore the small time-dependence introduced by the Earth's motion around the

dR^T

fitting procedure we have

*L/*² ⁼ *^N^E* X

 $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

[PF, Kahn, McCullough] see also [Feldstein, Kahlhoefer]

dE^R , (2.7)

*d*3*v ,* (2.2)

. (2.8)

Address all these issues at once: *Extended (log) likelihood* $\begin{array}{ccc} \hline \multicolumn{3}{c} \multic$ at once:

dE^R

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L =

$$
\frac{dR}{dE_R} = \frac{N_A \rho_X \sigma_n m_n}{2m_X \mu_{n_X}^2} C_T^2(A, Z) \int dE'_R G(E_R, E'_R) \epsilon(E'_R) F^2(E'_R) g(v_{min}(E'_R))
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Ξ

Taking the method out of the bin is the total number of events expected for a given set of parameters. We may compare di↵erent a good fit and grows with decreasing α fit. Discarding α fit. Discarding constants in α

Minimize L whilst preserving monotonicity $\overline{}$ Min

step functions

For perfect energy resolution **SLEP POSILIONS SIVEN DY** *R*)˜*g*(*vmin*(*E*⁰ step positions given by energies of events *vmin*(*Ei*)

Taking the method out of the bin 2 Halo-Independent Analysis Methods The dividendial event rate and discussed at a direct detection experiment of the state of th

$$
\frac{dR}{dE_R} = \frac{N_A \rho_X \sigma_n m_n}{2m_X \mu_{n\chi}^2} C_T^2(A, Z) \int dE_R' G(E_R, E_R') \epsilon(E_R') F^2(E_R') g(v_{min}(E_R'))
$$

where *m* is the DM mass, *mⁿ* the nucleon mass, *µn* the nucleon-DM reduced mass, *ⁿ* the

*d*3*v ,* (2.2)

, (2.3)

DM-nucleon scattering cross-section, ⇢ the local density, *N^A* is Avogadro's number, *F*(*ER*) In reality energy smearing means each step contributes at all distance scales, α and β and α *z* α *z* α *z* α β *z* α What is the form of g(v) that minimizes LL? *^R*) is the detector resolution function. The velocity integral is energies

 \mid Still series of N_O steps but now positions shift to the right frame. We ignore the small time-dependence introduced by the Earth's motion around the \mathcal{S} such the minimum DM the minimum DM velocity required to produce a nuclear \mathcal{S} Minimize over $2N_O$ parameters

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a proof to the contrary – for any physically reasonable resolution function, the *only* e↵ects **KKT - "Lagrange multipliers"** always assume some infinitesimal deviations which otherwise has no measurable has no measurable has no measura
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^R directly rather than *vmin*. Note however that since *vmin* is a monotonic

the functional *L*[˜*g*] with respect to the function ˜*g*(*E*0

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$e^{i\omega}$ which will be used in the arguments below, and states that if G is flat on some interval, it is flat on some interval, it is far on some function of *E*0 *R*, ˜*g*(*E*⁰ log-likelihood (2.8), written in the suggestive formula \mathcal{M}_1 in the suggestive formula \mathcal{M}_2

as a function of *E*0

$$
L[\tilde{g}] = \int dE'_R K(E'_R) \tilde{g}(E'_R) - \sum_{i=1}^{N_O} \log \left(\mu_i + \int dE'_R G(E_i, E'_R) K(E'_R) \tilde{g}(E'_R)\right)
$$

subject to inequality constraint: $d\tilde{g}/dE_R' \leq 0$ W_{max} is a view view view view view the log-likelihood minimizes \mathcal{S}/\mathcal{M} minimizes \mathcal{M} $R' \leq 0.$

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$e^{i\omega}$ which will be used in the arguments below, and states that if G is that if G is flat on some interval, it is found in the interval, *must vall prefactors (form factor, eff. etc)* satisfies a delta function $\frac{N_O}{N}$ of $\frac{C}{N}$ $\mathcal{L} = \int dL_R \mathbf{R} \left(E_R \right) g(E_R) = \sum \log \left(\mu_i + \int dE_R \mathbf{G}(E_i, E_R) \mathbf{R} \left(E_R \right) g(E_R) \right)$ $\sum_{i=1}^{n}$ /*J R*, ˜*g*(*E*⁰ *^R*. Consider now the expression for the log-likelihood (2.8), written in the suggestive form in the suggestive form in the suggestive form \mathcal{L} $L[\tilde{g}] = \int dE_R' \, K(E_R') \tilde{g}(E_R') - \sum_{n=1}^{N_O}$ *N^O i*=1 $\log\left(\mu_i + \right)$ Z $dE'_R G(E_i, E'_R)K(E'_R) \tilde{g}(E'_R)$ ◆ Minimize, all prefactors (form factor, eff. etc) *i*=1 log ✓ *µⁱ* + $P\subset Y$ [log $\left(\mu_i + \int dE'_R G(E_i, E'_R) K(E'_R) \tilde{g}(E'_R)\right)$] We can now view the log-likelihood minimization as a variational problem: minimize

 σ entiact to inequality constraint: $d\tilde{a}/dE' < 0$ W_{max} is a view view view view view the log-likelihood minimizes \mathcal{S}/\mathcal{M} minimizes \mathcal{M} subject to inequality constraint: $d\tilde{g}/dE^{\prime}_B$ $R' \leq 0.$

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^R) is also monotonic function of *E*⁰

Property (i) simply states that the resolution function is normalized and doesn't change the total number of events. Property (ii) states that *G* has a single peak where the detected $\frac{\partial E_R}{\partial \theta}$ is small. *R* anywhere, unless 0

^R)*,* (A.1)

. (A.2)

^R. Consider now the expression for the

R), subject to the monotonicity con-

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Minimize,
\n
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In each of the physicists, but is well-known in each of the physicists, and related fields; and related fields the solution is given by the solution of the Kuhn-Tucker conditions $\mathbf{S} = \mathbf{S} \mathbf{S}$ the solution is given by the Karush-Kuhn-Tucker conditions \mathbb{Z} and \mathbb{Z} and \mathbb{Z} the Karush-Kuhn-Tucker conditions \mathbb{Z} concept of Lagrange multipliers. In a similar fashion to import the similar fashion to include the constraint o

 Γ conforce monotonicity with a Lagrange multiplier, we can impose the inequality constraint by introducing and ℓ To enforce monotonicity with a Lagrange multiplier, we can impose the inequality constraint by introducing and α **To enforce monotonicity** \overline{C}

$$
L[\tilde{g}] \rightarrow L[\tilde{g}] + \int dE_R' \frac{d\tilde{g}}{dE_R'} q(E_R')
$$

$$
\frac{\delta L}{\delta \tilde{g}} - \frac{dq}{dE_R'} = 0 ,
$$

$$
\frac{d\tilde{g}}{dE_R'} \leq 0 ,
$$

$$
q(E_R') \geq 0 ,
$$

$$
\int dE_R' \frac{d\tilde{g}}{dE_R'} q(E_R') = 0 .
$$

 $E_{\rm eff}$ is the familiar equation resulting from varying from varying the modified functional with respect to \sim

to ˜*g*, and Eq. (A.4) is the desired monotonicity constraint. Eq. (A.6) is a complementarity

condition which ensures that the shift in *L* vanishes on the solution, just as the extra Lagrange

multiplier term vanishes on the solution in the case of equality constraints. When combined

^R) = 0.

with Eqs. (A.4) and (A.5), Eq. (A.6) enforces that at every point *E*0

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*dE*0 **Fither saturate constraint (usual Lagrange multiplier) or q=0** multiplier term vanishes on the solution in the solution in the solution in the case of equality constraints. W
When combined in the solution in the solution in the case of equality combined in the case of equality constra

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*dE*0

Assume $g(E)$ is <u>not flat</u> over some range. Then,

Thus, Eq. (A.3) becomes *L/g*˜ = 0, or taking the functional derivative explicitly,⁷

$$
\sum_{i=1}^{N_O} \frac{G(E_i, E'_R)}{\gamma_i} = 1,
$$

satisfy the control of the

 $\frac{\delta L}{\delta \tilde{g}}-\frac{dq}{dE_{R}^{\prime}}$

z

interval (*a, b*). Then by Eq. (A.6), *q* = 0 on (*a, b*), and hence *dq/dE*0

 dE_R'

 $d\tilde{g}$

 dE_R'

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 $d\tilde{g}$

 dE_R'

 $= 0 \; , \; \; \vert$

 ≤ 0 , \vert

 $q(E_R') \ge 0$,

 $q(E_R')=0$.

^R) had a nonzero derivative everywhere on some open

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where

through *G*(*Ei, E*⁰

 $\overline{}$

This proves that ˜*g*(*E*0

$$
\gamma_i = \mu_i + \int dE_R'' G(E_i, E_R'') K(E_R'') \tilde{g}(E_R'')
$$

is the total di↵erential event rate at *Ei*. But the left-hand side of Eq. (A.7) depends on *E*⁰

R)

*dE*0

$\sigma(\nu)$ must *R*) while the right-hand side is constant; by property (iii), this is impossible.⁸ *g*(v) must be <u>flat</u> except at individual points i.e. steps g(v) must be flat except at individual points i.e. steps

¹ ^X

i

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 $\overline{}$

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of step functions.

function ˜*g*(*E*0

be necessary.

function ˜*g*(*E*0

be necessary.

be necessary.

Can determine the positions of the steps $\sqrt{E_R^2 - \frac{E_R^2}{E_R^2}}$ di↵erential equation for *q*: *dq dE*0 *SITIONS* f the s fe_Ds

dq

 dE_R'

tivity condition (A.5), we must also have

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^R) had a nonzero derivative everywhere on some open

R ⇡

The solution to this equation depends on the *i*, which in turn depend on the full solution α $\bm{\kappa}$ are called the $\bm{\kappa}$ $\bm{\kappa}$ and $\bm{\kappa}$ is the sequation of the sequation of the sequation of the sequation of $\bm{\kappa}$ condition which ensures that the shift in \mathcal{L} vanishes on the solution, \mathcal{L} in \mathcal{L} as the extra Lagrange extra Lagran \mathbf{m} or \mathbf{q} is \mathbf{q} is equality combined combined combined combined combined combined constraints. When combined combined combined constraints. When combined combined combined combined combined combined combi At step energies (\tilde{E}_j) positivity of q requires At step energies (E_j) positivity of q requires pos *d*2*q*

^R) will be peaked at *E*⁰

 $= K(E_R)$

 $\sqrt{ }$

 $1 - \sum$

N^O

 $G(E_i,E_R')$

!

 γ_i

i=1

$$
\left.\frac{dq}{dE'_R}\right|_{\widetilde{E}_j}=0,\quad \left.\frac{d^2q}{dE''_R}\right|_{\widetilde{E}_j}\geq 0
$$

at the roots *E*e*^j* of *q*. Taking the derivative of Eq. (A.9), and using the assumption *dK/dE*⁰ 0, the condition on the second derivative becomes the second derivative becomes the second derivative becomes Thus,

Thus,

$$
-\sum_{i=1}^{N_O} \frac{1}{\gamma_i} \left. \frac{\partial G(E_i, E'_R)}{\partial E'_R} \right|_{E'_R = \widetilde{E}_j} \gtrsim 0
$$

Ste *i* @*G*(*Ei, E*⁰ *R*) @*E*0 *R* = *R* $\frac{1}{2}$ $\$

^R = *Ei*, so for *E*⁰

By property (ii), *G*(*Ei, E*⁰

By property (ii), *G*(*Ei, E*⁰

best-fit velocity integrals have been calculated for a single DM mass *m*, it is simple to map the DM mass *mass* and mass *mass* a direction of a direc Let us first consider the energy of a scattering event. The minimum DM velocity Warwing the DM mass integral between any neighboring events, the total number of events predicted between any **Varying the DM mass**

$$
v'_{min}(E_R) = \frac{\mu_{N\chi}}{\mu_{N\chi'}} v_{min}(E_R)
$$

min(*ER*) for a DM mass *m*⁰

min axes for *m*⁰

XENON10

XENON100

³⁰⁰ ⁴⁰⁰ ⁵⁰⁰ ⁶⁰⁰ ⁷⁰⁰ ¹⁰-²⁷

–8–

for *µ*2 *n*0 *µ*2 It to determine results for all masses One plot is sufficient to determine results for all masses

10-²⁶

Unbinned application

[PF, Kahn, McCullough]

Unbinned application

[PF, Kahn, McCullough]

 $\left| \Gamma \right|$

 $\frac{1}{0}$

states of slightly dividend for a recoil of energy \mathbf{F} then: **Extension to inelastic DM**

[see also Bozorgnia et al; del Nobile et al]

$$
v_{\min} = \left| \delta + \frac{m_{\rm N} E_{\rm R}}{\mu} \right| \frac{1}{\sqrt{2 E_{\rm R} m_{\rm N}}}
$$

, (4.1)

Wednesday, 10 December 14

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, (4.1)

Conclusions

- •Should analyse data independent of astro uncertainties •With multiple experiments should compare g(v), tests consistency
- •Presenting experimental results in g(v) *very* useful
	- •One plot contains all information, for all masses
- •Find region of consistent parameter space
- SNew Z Unbinned approach using likelihood techniques •Maximal use of information
	- •Independent of astrophysics, expts. agree/disagree?
- •Application: CDMS-Si is at odds with LUX
- •Ultimately may be able to extract f(v) by differentiating deconvoluted rate