Heavy-Nucleus Structure for Dark Matter

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Nuclear Operators

EFT (Anand, Fitzpatrick, Haxton), to leading order in q:

$$C_{J=0M} \to \sum_{i=1}^{A} 1(i) \qquad L^5_{J=1M} \sim T^{\mathrm{el}\;5}_{J=1M} \to \sum_{i=1}^{A} \sigma_{1M}(i)$$

$$T^{\mathrm{mag}}_{J=1M} \to \frac{q}{m_N} \sum_{i=1}^{A} \ell_{1M}(i) \qquad L_{J=0M} \to \frac{q}{m_N} \sum_{i=1}^{A} \vec{\sigma}(i) \cdot \vec{\ell}(i)$$

$$L_{J=2M} \sim T^{\mathrm{el}}_{J=2M} \to \frac{q}{m_N} \sum_{i=1}^{A} \left[\vec{r}(i) \otimes \left(\vec{\sigma}(i) \times \frac{1}{i} \vec{\nabla} \right) \right]_{2M}$$

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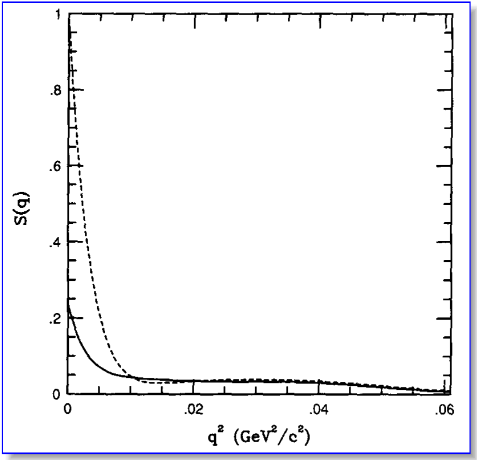
Chiral EFT, presumably, can give you handle on two-body corrections to these operators.

Heavy-Nucleus Proto-Calculation

^{131}Xe from 1991



"Spin-dependent" B-ino form factor:

- in a few oscillator shells,
- with spherical oscillator single-particle wave functions and unrelated interaction,
- in single-quasiparticle picture and with perturbative admixtures of three-quasiparticle states (to capture core polarization).

Much better version of this general framework is in common use now.

Current Paradigm for Heavy Nuclei: Nuclear DFT

Particularly Useful for One-Body Operators

$$
\begin{aligned}
E_{\text{Skyrme}}[\rho] = \sum_{t=0,1} \int d^3r \Big\{ & C_t^{\rho}[\rho_0]\,\rho_t^2(\vec{r}) + C_t^{\Delta\rho}\rho_t(\vec{r})\Delta\rho_t(\vec{r}) \\
& + C_t^{\tau}\left[\rho_t(\vec{r})\,\tau_t(\vec{r}) - \vec{j}_t^{\,2}(\vec{r})\right] + C_t^{s}[\rho_0]\,\vec{s}_t^{\,2}(\vec{r}) \\
& + C_t^{\Delta s}\vec{s}_t(\vec{r})\cdot\Delta\vec{s}_t(\vec{r}) + C_t^{T}\left[\vec{s}_0(\vec{r})\cdot\vec{T}_t(\vec{r}) - \overset{\leftrightarrow}{J}{}_t^{2}(\vec{r})\right] \\
& + C_t^{\nabla J}\left[\rho_t(\vec{r})\vec{\nabla}\cdot\vec{J}_t(\vec{r}) + \vec{s}_t(\vec{r})\cdot\vec{\nabla}\times\vec{j}_t(\vec{r})\right]
\end{aligned}
$$

with $\quad \rho_t(\vec{r}) = \rho_t(\vec{r},\vec{r}) \quad$ and

$$
\begin{aligned}
\vec{s}_t(\vec{r}) &= \vec{s}_t(\vec{r},\vec{r}), & \tau_t(\vec{r}) &= \vec{\nabla}\cdot\vec{\nabla}'\rho_t(\vec{r},\vec{r}\,')|_{\vec{r}=\vec{r}\,'} \\
\vec{T}_t(\vec{r}) &= \vec{\nabla}\cdot\vec{\nabla}'\vec{s}_t(\vec{r},\vec{r}\,')|_{\vec{r}=\vec{r}\,'} & \vec{j}_t(\vec{r}) &= -\tfrac{i}{2}(\vec{\nabla}-\vec{\nabla}')\,\rho_t(\vec{r},\vec{r}\,')|_{\vec{r}=\vec{r}\,'} \\
J_{t,ij}(\vec{r}) &= -\tfrac{i}{2}(\vec{\nabla}-\vec{\nabla}')_i\, s_{t,j}(\vec{r},\vec{r}\,')|_{\vec{r}=\vec{r}\,'} & \overset{\leftrightarrow}{J}{}_t^{2}(\vec{r}) &= \sum_{ij=xyz} J_{t,ij}^2 \\
\vec{J}_t(\vec{r}) &= -\tfrac{i}{2}(\vec{\nabla}-\vec{\nabla}')\times\vec{s}_t(\vec{r},\vec{r}\,')|_{\vec{r}=\vec{r}\,'}
\end{aligned}
$$

Effective Interaction from Energy Functional

$$
\begin{aligned}
\bar{V}^{\text{eff}}_{\text{Skyrme}} = {} & (a_0 + b_0\,\vec{\sigma}\cdot\vec{\sigma}' + c_0\,\vec{\tau}\cdot\vec{\tau}' + d_0\,\vec{\sigma}\cdot\vec{\sigma}'\,\vec{\tau}\cdot\vec{\tau}')\,\delta(\vec{r}-\vec{r}') \\
& + (a_1 + b_1\,\vec{\sigma}\cdot\vec{\sigma}' + c_1\,\vec{\tau}\cdot\vec{\tau}' + d_1\,\vec{\sigma}\cdot\vec{\sigma}'\,\vec{\tau}\cdot\vec{\tau}') \\
& \qquad \times (\vec{k}^{\dagger 2}\,\delta(\vec{r}-\vec{r}') + \delta(\vec{r}-\vec{r}')\vec{k}^{2}) \\
& + (a_2 + b_2\,\vec{\sigma}\cdot\vec{\sigma}' + c_2\,\vec{\tau}\cdot\vec{\tau}' + d_2\,\vec{\sigma}\cdot\vec{\sigma}'\,\vec{\tau}\cdot\vec{\tau}')\,\vec{k}^{\dagger}\cdot\delta(\vec{r}-\vec{r}')\,\vec{k} \\
& + (a_3 + b_3\,\vec{\sigma}\cdot\vec{\sigma}' + c_3\,\vec{\tau}\cdot\vec{\tau}' + d_3\,\vec{\sigma}\cdot\vec{\sigma}'\,\vec{\tau}\cdot\vec{\tau}')\,\rho_{00}^{\alpha}(\vec{r})\,\delta(\vec{r}-\vec{r}') \\
& + \Big[e_3\,\rho_{10}(\vec{r})\,(\tau^{(0)} + \tau'^{(0)}) + g_3\,\vec{s}_{00}(\vec{r})\cdot(\vec{\sigma}+\vec{\sigma}') \\
& \qquad + m_3\,\vec{s}_{10}(\vec{r})\cdot(\vec{\sigma}\tau^{(0)} + \vec{\sigma}'\,\tau'^{(0)})\Big]\,\rho_{00}^{\alpha-1}(\vec{r})\,\delta(\vec{r}-\vec{r}') \\
& + \Big[f_3\,\rho_{10}^{2}(\vec{r}) + h_3\,\vec{s}_{00}^{\,2}(\vec{r}) + n_3\,\vec{s}_{10}^{\,2}(\vec{r})\Big]\,\rho_{00}^{\alpha-2}(\vec{r})\,\delta(\vec{r}-\vec{r}') \\
& + (a_4 + c_4\,\vec{\tau}\cdot\vec{\tau}')\,(\vec{\sigma}+\vec{\sigma}')\cdot\vec{k}^{\dagger}\times\delta(\vec{r}-\vec{r}')\,\vec{k}
\end{aligned}
$$

where

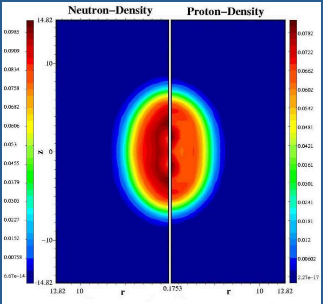
$$
\begin{aligned}
\vec{k} &= -\tfrac{i}{2}(\vec{\nabla} - \vec{\nabla}') \quad \text{acting to right,} \\
\vec{k}^{\dagger} &= \tfrac{i}{2}(\vec{\nabla} - \vec{\nabla}') \quad \text{acting to left.}
\end{aligned}
$$

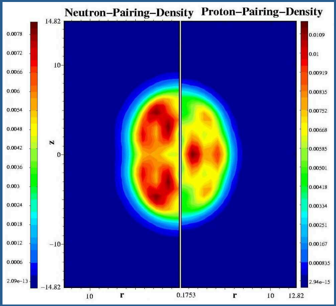
Basic Features of Framework

- All nucleons active, complete single-particle space used.
- Effective interaction (density dependent) and one-body mean field completely self consistent.
- Lots of work on determining good functionals, with improvements under active development.
- Phenomenology — masses, one-body observables such as density distributions — reproduced well.
- Some weaknesses that I won't mention.

Densities

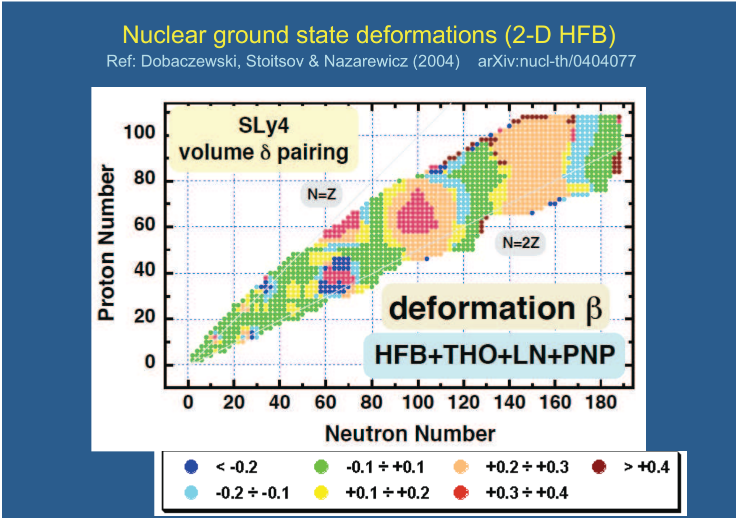
Zr-102: normal density and pairing density
HFB, 2-D lattice, SLy4 + volume pairing
Ref: Artur Blazkiewicz, Vanderbilt, Ph.D. thesis (2005)





HFB: $\beta_2^{(p)}$=0.43 exp: $\beta_2^{(p)}$=0.42(5) , J.K. Hwang et al., Phys. Rev. C (2006)

Applied Everywhere



Related Theories

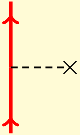
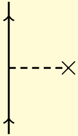
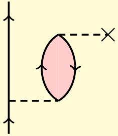
- Improved Skyrme DFT, derived in part from ab initio interactions
  - Density-matrix expansion leads to logarithmic dependence of energy on densities.
- Gogny DFT, based on finite-range density-dependent potential
- Density-independent effective interactions with three (and more) body terms (Bennaceur, Dobaczewski et al., Bender, Duguet et al.)

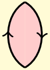
Can expect improvement in quality.

Odd Nuclei

Much of elastic response vanishes in even nuclei.

Core polarization in odd system:

 $\approx$  $+$ 

 $=$ RPA polarization propagator

Can do

- self-consistent odd-A calculation
- explicit RPA core polarization

Two are nearly equivalent and include most of the important corrections to single-particle picture.

Deformation and Angular-Momentum Restoration

If deformed state $|\Psi_K\rangle$ has good intr. $J_z = K$, angle average gives:

$$|J, M\rangle = \frac{2J+1}{8\pi^2} \int D^{J*}_{MK}(\Omega) R(\Omega)\, |\Psi_K\rangle\, d\Omega$$

Matrix elements:

$$\langle J, M|\, \hat{O}_m \,|J', M'\rangle \propto \int \int \sum_j d\Omega\, d\Omega' \times (\text{some D-functions})$$
$$\times \langle \Psi_K|\, R^{-1}(\Omega')\, \hat{O}_n\, R(\Omega)\, |\Psi_K\rangle$$

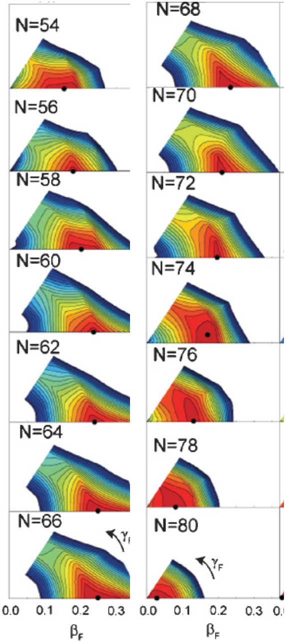
$$\xrightarrow[\Omega\approx\Omega']{\text{rigid defm.}} (\text{Geometric factor}) \times \underbrace{\langle \Psi_K|\hat{O}_0|\Psi_K\rangle}_{\langle\hat{O}\rangle_{\text{intr.}}}$$

For expectation value of, e.g., vector operator in $J = \frac{1}{2}$ state:

$$\langle \hat{O}\rangle = \langle \hat{O}_z\rangle_{J=\frac{1}{2}, M=\frac{1}{2}} \Longrightarrow \begin{cases} \langle\hat{O}\rangle_{\text{intr.}} & \text{spherical nucleus} \\ \frac{1}{3}\langle\hat{O}\rangle_{\text{intr.}} & \text{rigidly deformed nucleus} \end{cases}$$

Exact answer somewhere in between.

DM-Detector Nuclei Often Have Complex Structure



$\Longleftarrow$ Potential-energy surface in β-γ plane

Nomura, Shimizu, Otsuka, PRC 81, 044307 (2010).

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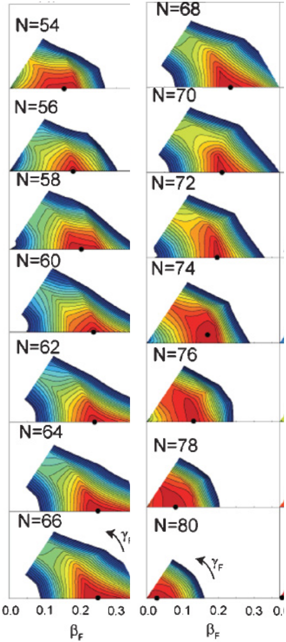

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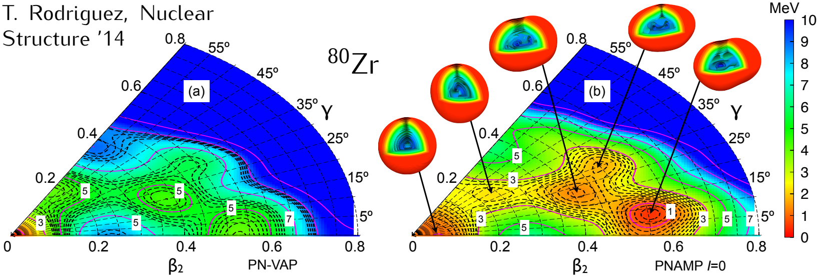
Similar softness, plus triaxial deformation and shape coexistence, in Ge isotopes.

DFT Extension: Generator Coordinate Method

T. Rodriguez, Nuclear Structure '14



Basic idea: Construct set of mean fields by constraining coordinate, e.g. $\langle Q_m \rangle \equiv \langle \sum_i r_i^2 Y_i^{2,m} \rangle$. Minimize

$$\langle H' \rangle = \langle H \rangle - \lambda_0 \langle Q_0 \rangle - \lambda_2 (\langle Q_2 \rangle)$$

for a whole range of $\langle Q_0 \rangle \propto \beta \cos \gamma$ and $\langle Q_2 \rangle \propto \beta \sin \gamma$. Then diagonalize H in space of A- and J-projected quasiparticle vacua with different β, γ.

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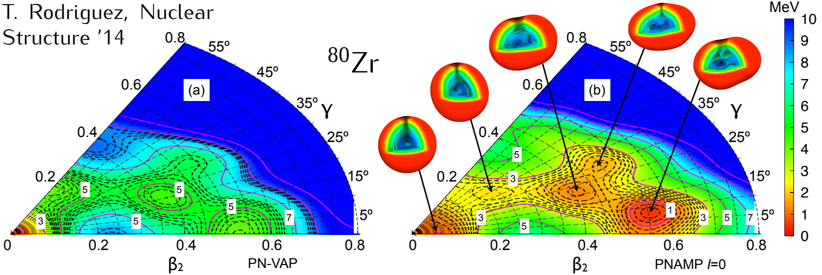

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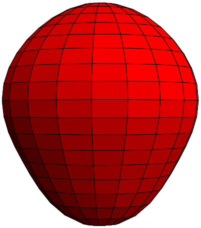
Density dependence spoils GCM in Skyrme DFT; need alternative effective interactions or regularization procedures.

Similar (but Harder): Schiff Moment of ^{225}Ra

No GCM, Just Single Odd-A Mean Field

$$S \propto \sum_i e_i r_i^2 z_i + \dots$$

V_{PT} = perturbative T-violating π-exchange potential



Calculated ^{225}Ra density

Ground state $|0\rangle$ has nearly-degenerate partner $|\bar{0}\rangle$ with opposite parity and same intrinsic structure, so:

$$\langle S \rangle \approx \frac{\langle 0 | \, S \, |\bar{0}\rangle \langle \bar{0} | \, V_{PT} \, |0\rangle}{E_0 - E_{\bar{0}}} + c.c. \quad \xrightarrow{\text{rigid limit}} \quad \frac{1}{3} \frac{\langle S \rangle_{\text{intr.}} \langle V_{PT} \rangle_{\text{intr.}}}{E_0 - E_{\bar{0}}}$$

Results vary with Skyrme functonal, but variation can be reduced by tying $\langle S \rangle_{\text{intr.}}$ to data.

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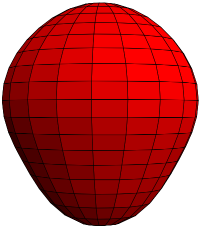

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Can tie dark-matter expectation values to other data, e.g. magnetic moments. spin-orbit splittings (**Q:** What else?)

Finally…

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The End

Thanks for your kind attention.