

Dark nuclei

William Detmold
MIT

Dark nuclei

- Dark nuclear physics of QCD like theory with $N_c=2$ and $N_f=2$
- Based on two recent papers in collaboration with [Matthew McCullough](#) & [Andrew Pochinsky](#)
 - *Dark Nuclei I: Cosmology and Indirect Detection* —1406.2276
 - *Dark Nuclei II: Nuclear Spectroscopy in Two-Colour QCD* —1406.4116
- Motivation
 - Understand what is “nuclear physics” in a general context
What are generic features, what is special?
 - Dark matter model building: interesting new phenomenology

- Two-colour QCD with two flavours of fundamental fermions
 - Numerically feasible (simpler than QCD)
 - Emergent complexity: novel phenomenological aspects
- Single hadron aspects already considered in DM context [Lewis et al., Neil & Buckley, Hietanen et al.]
- Also lattice investigations of quenched $N_c=4$ QCD and other theories in this context [LSD collaboration]
 - Sigma terms, polarisabilities,...

Symmetries of two-colour QCD

- Global flavour symmetry $SU(2)_L \times SU(2)_R$ enlarges to $SU(4)$
- Pseudo-reality of $SU(2)$ - left and right handed quarks can be combined into multiplets

$$\Psi = \begin{pmatrix} u_L \\ d_L \\ -i\sigma_2 C \bar{u}_R^\top \\ -i\sigma_2 C \bar{d}_R^\top \end{pmatrix} \quad \Psi \xrightarrow{SU(4)} \exp \left(i \sum_{j=1}^{15} \theta_j T_j \right) \Psi$$

- Strong interactions result in condensate that spontaneously breaks the global symmetry: $SU(4) \rightarrow Sp(4) \sim SO(5)$ [Peskin 1980]
- Numerical calculations have significant explicit symmetry breaking: $m_u = m_d \sim \Lambda_{\text{QC2D}}$

Spectrum

- Simplest colour singlets

- “Pions”: $\pi^- \sim \bar{u}\gamma_5 d$, $\pi^0 \sim \bar{u}\gamma_5 u + \bar{d}\gamma_5 d$, $\pi^+ \sim \bar{d}\gamma_5 u$ $J^P=0^-$
- (anti-)“Nucleons”: $u d$, $\bar{u}\bar{d}$ $J^P=0^+$

} Degenerate
SO(5) multiplet

- “Rhos”: $\rho^- \sim \bar{u}\gamma_\mu d$, $\rho^0 \sim \bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d$, $\rho^+ \sim \bar{d}\gamma_\mu u$ $J^P=1^-$
- (anti-)“Deltas”: $u\gamma_\mu\gamma_5 d$, $\bar{u}\gamma_\mu\gamma_5\bar{d}$ $J^P=1^+$

} Degenerate
SO(5) multiplet

- Axial vector, scalar, tensor mesons + associated baryons

- Single hadron spectrum studied by [Hietanen et al. 1404.2794]

- Pion multiplet are pseudoGoldstone bosons of χ SB: $SU(4) \rightarrow Sp(4)$

- Rho stable for masses considered

- Colour singlets can combine
 - Two-, three-, ... particle scattering states
 - “Nuclei” for sufficiently attractive interactions—not *a priori* obvious
- Two “pions” combine to give 25 states: $\mathbf{5} \otimes \mathbf{5} = \mathbf{1} \oplus \mathbf{10} \oplus \mathbf{14}$
 - $J=0$ systems, contains $B=2, 1, 0, -1, -2$ states
- “pion”+ “rho”: $J=1$ systems with same flavour breakdown

$$\mathbf{D}^\mu = \begin{pmatrix} S_+^\mu & D_{2,0}^\mu & D_{1,0}^\mu & D_{1,-1}^\mu & D_{1,1}^\mu \\ \bar{D}_{2,0}^\mu & S_-^\mu & D_{-1,0}^\mu & D_{-1,-1}^\mu & D_{-1,1}^\mu \\ \bar{D}_{1,0}^\mu & \bar{D}_{-1,0}^\mu & S_0^\mu & D_{0,-1}^\mu & D_{0,1}^\mu \\ \bar{D}_{1,-1}^\mu & \bar{D}_{-1,-1}^\mu & \bar{D}_{0,-1}^\mu & S_B^\mu & D_{0,2}^\mu \\ \bar{D}_{1,1}^\mu & \bar{D}_{-1,1}^\mu & \bar{D}_{0,1}^\mu & \bar{D}_{0,2}^\mu & S_{\bar{B}}^\mu \end{pmatrix} \quad \longrightarrow \quad \begin{aligned} \mathbf{D}_1^\mu &= \text{Tr}(\mathbf{D}^\mu) , \\ \mathbf{D}_{10}^\mu &= \frac{i}{2} (\mathbf{D}^\mu - \mathbf{D}^{\mu T}) , \\ \mathbf{D}_{14}^\mu &= \frac{1}{2} (\mathbf{D}^\mu + \mathbf{D}^{\mu T}) - \frac{1}{5} \text{Tr}(\mathbf{D}^\mu) \mathbf{1}_5 \end{aligned}$$

- Higher body systems: $J=0, 1$, flavour = $\underbrace{\square \square \square \cdots \square \square}_n$, $n=2, \dots, 8$

Simulations

- Wilson gauge and fermion actions
- HMC using modified **chroma**
- 4 lattice spacings (β), 6 masses
 - Isospin symmetric
- 3 or 4 volumes per choice (β, m_0)
- Long streams of configurations

Label	β	m_0	$L^3 \times T$	N_{traj}
<i>A</i>	1.8	-1.0890	$12^3 \times 72$	5,000
			$16^3 \times 72$	4,120
			$20^3 \times 72$	3,250
<i>B</i>	2.0	-0.9490	$12^3 \times 48$	10,000
			$16^3 \times 48$	4,000
			$20^3 \times 48$	3,840
			$24^3 \times 48$	2,930
<i>C</i>	2.0	-0.9200	$12^3 \times 48$	10,000
			$16^3 \times 48$	9,780
			$20^3 \times 48$	10,000
<i>D</i>	2.0	-0.8500	$12^3 \times 48$	9,990
			$16^3 \times 48$	5,040
			$16^3 \times 72$	5,000
			$20^3 \times 48$	5,000
			$24^3 \times 48$	5,050
<i>E</i>	2.1	-0.7700	$12^3 \times 72$	5,000
			$16^3 \times 72$	5,000
			$20^3 \times 72$	4,300
<i>F</i>	2.2	-0.6000	$12^3 \times 72$	5,000
			$16^3 \times 72$	5,000
			$20^3 \times 72$	5,000
			$24^3 \times 72$	5,070

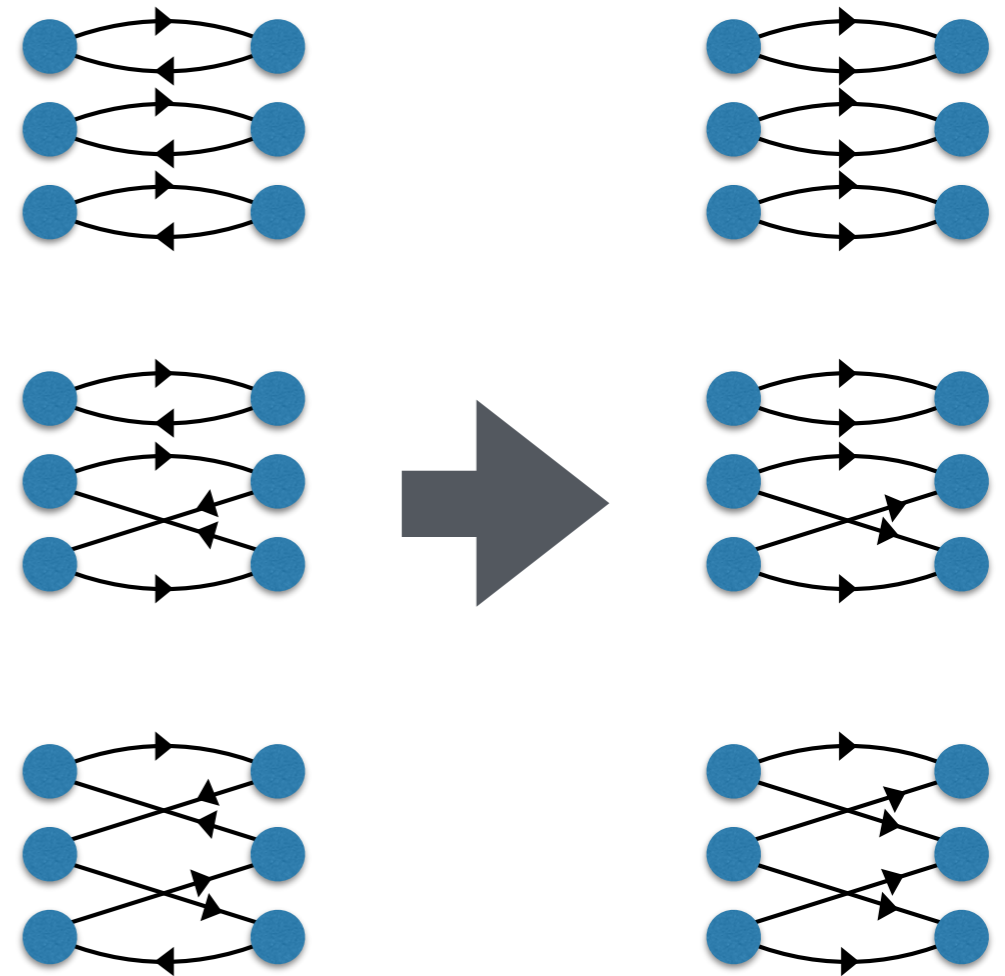
Multi-baryon contractions

- SU(2) multi-baryon contractions equivalent to maximal isospin multi-meson contractions
- Clear from degeneracies but explicitly

$$S(y, x) = C^\dagger (-i\sigma_2)^\dagger S(x, y)^T (-i\sigma_2) C$$

$$S(y, x) = \gamma_5 S^\dagger(x, y) \gamma_5$$

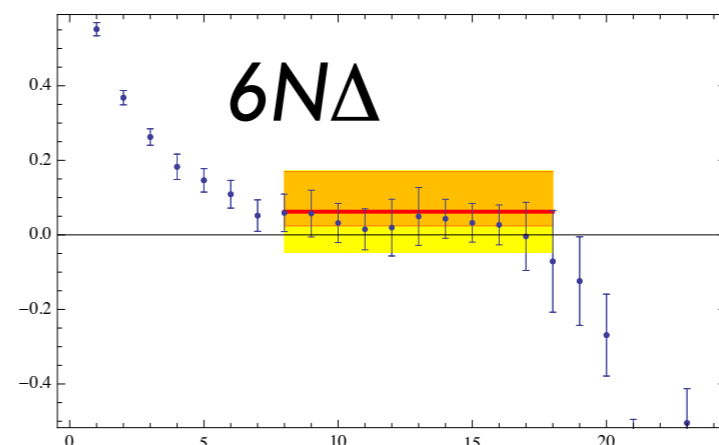
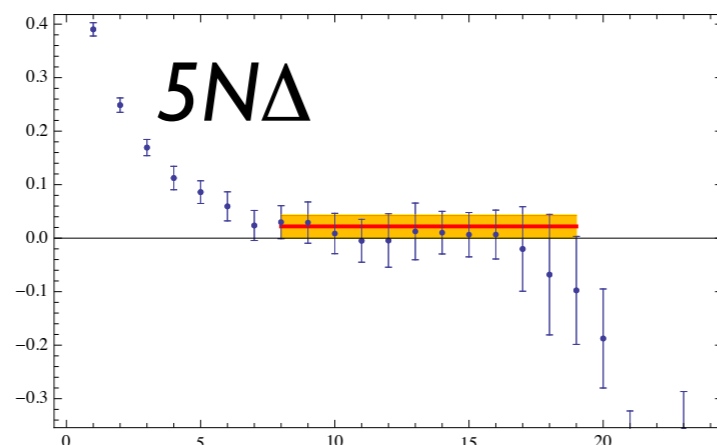
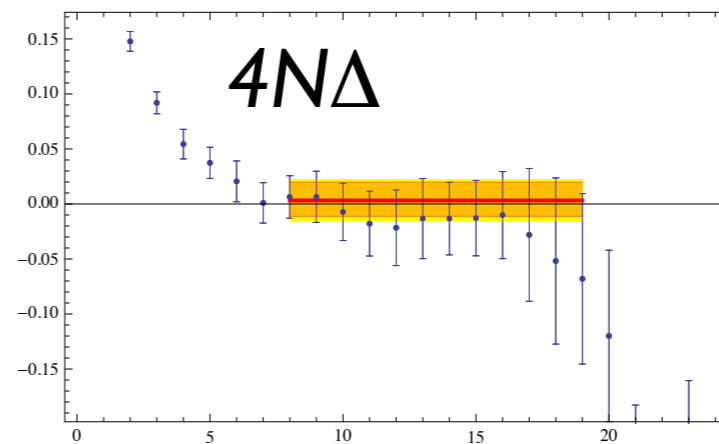
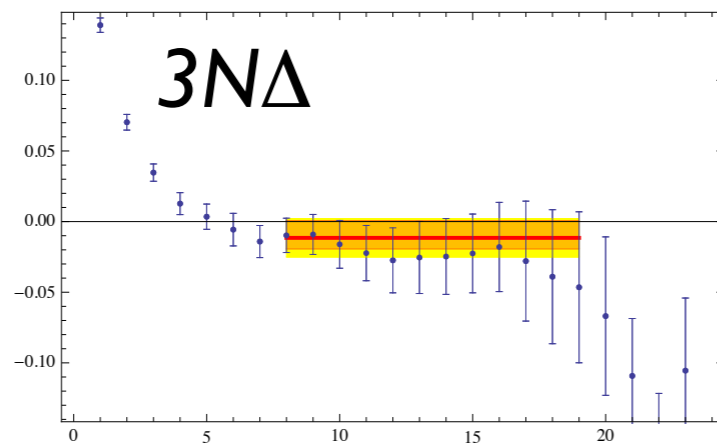
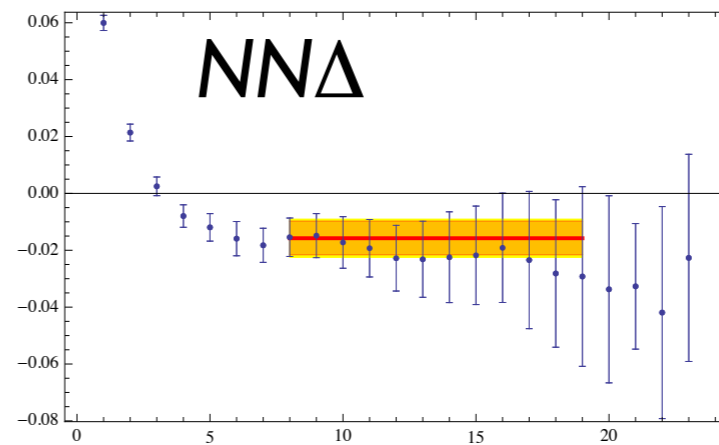
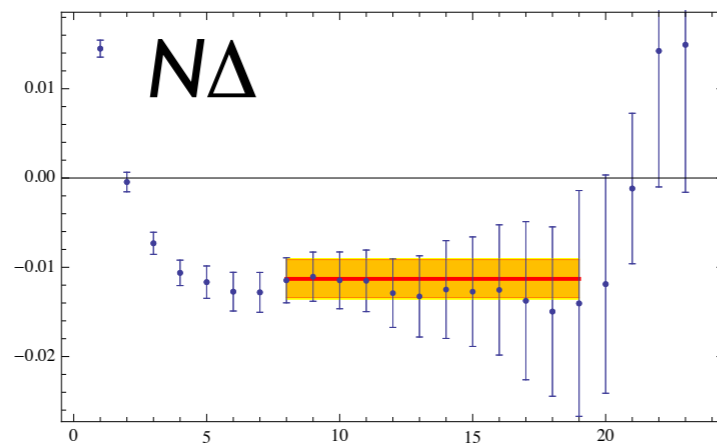
first relation specific to $N_c=2$
- Use algorithms from $N_c=3$ QCD
[WD & Savage 2011; WD, Orginos, Shi 2012]
- $(n-1)N\Delta \sim$ mixed pion-kaon contractions
[WD & Smigielski 2011]



Ex.: three types of contractions for $I=3$ $\pi\pi\pi$ and NNN

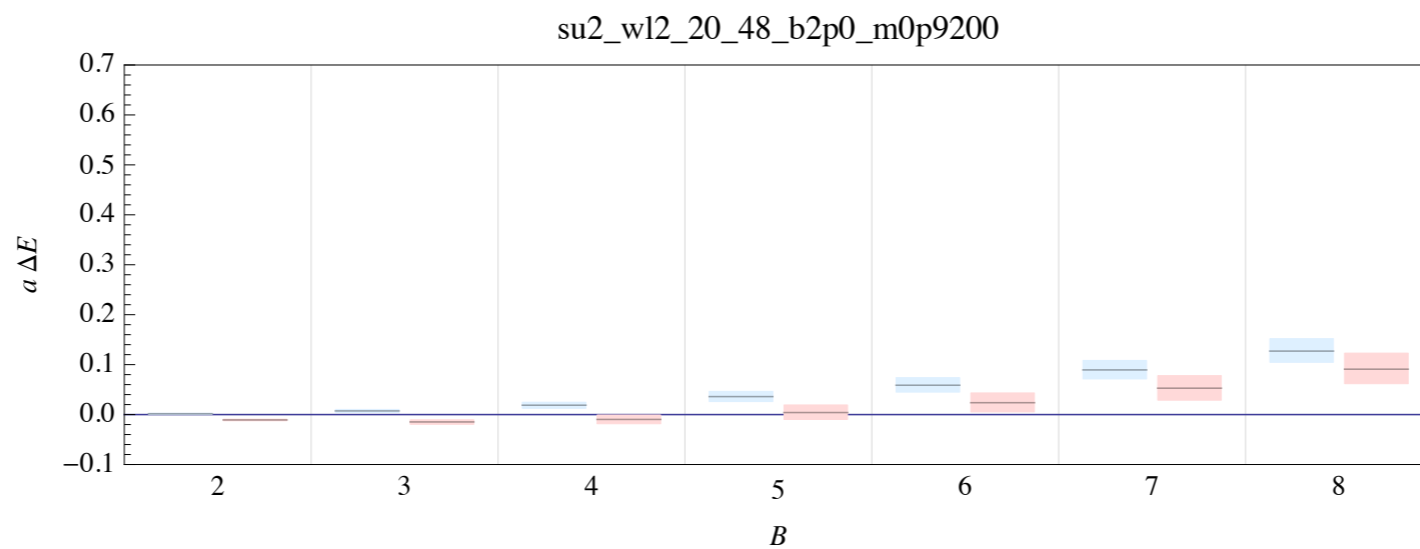
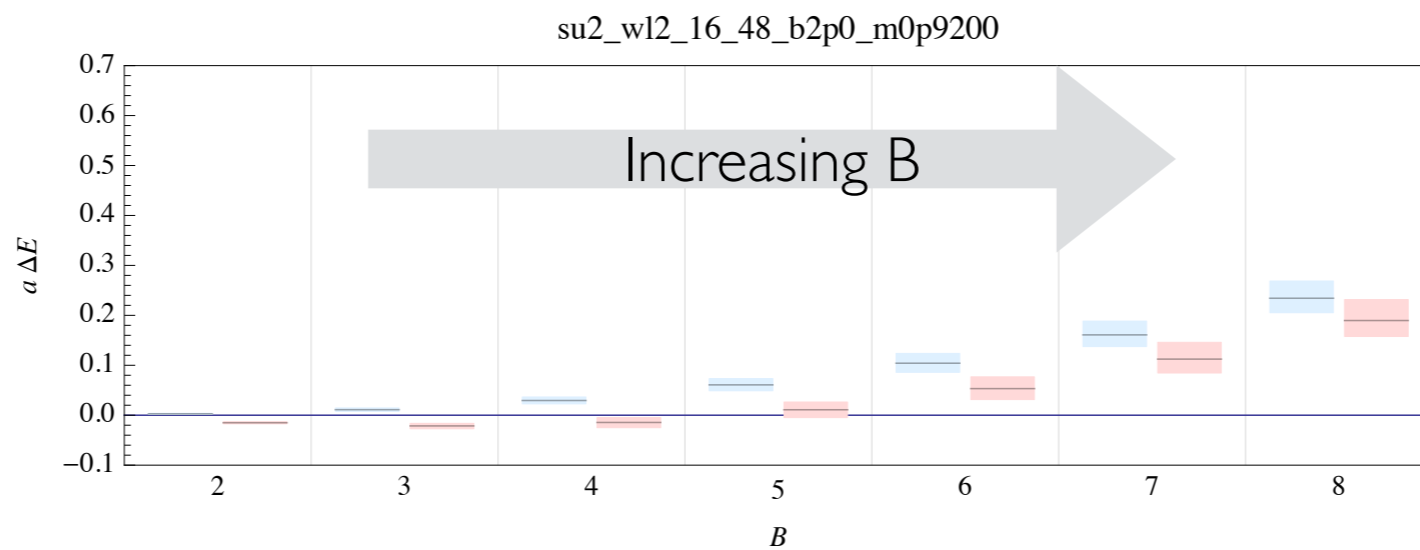
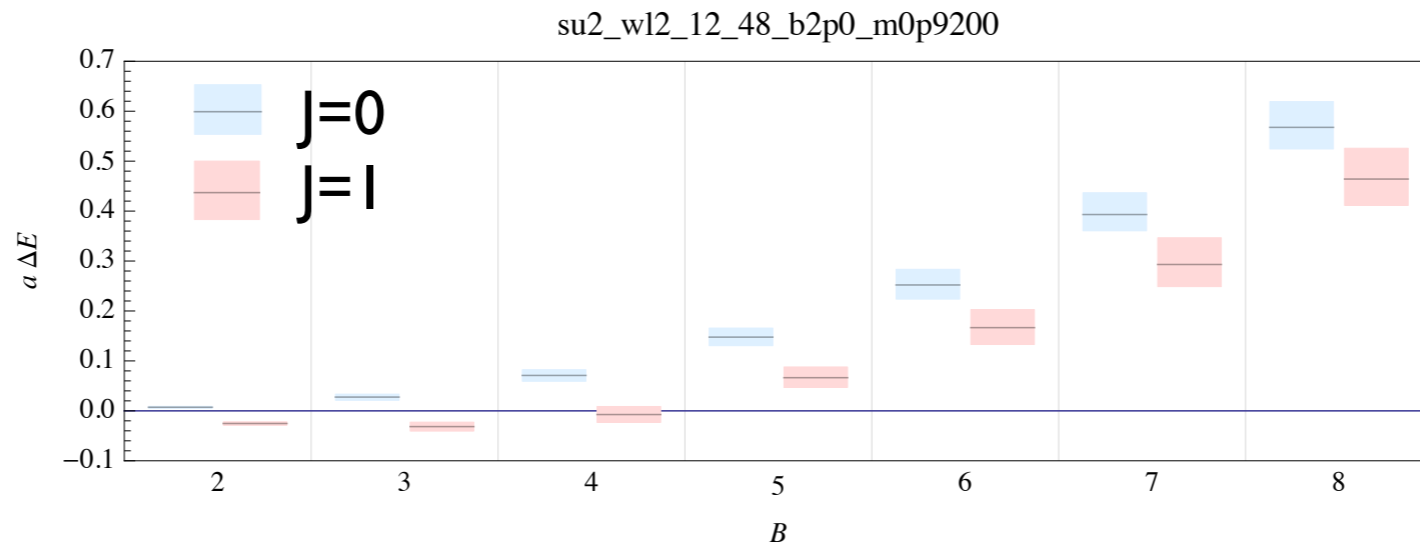
Example effective mass shift plots

$$R_{nN}(t) \equiv \frac{C_{nN}(t)}{[C_{1N}(t)]^n}, \quad R_{nN,\Delta}(t) \equiv \frac{\sum_i C_{nN,\Delta}^{(i,i)}(t)}{[C_{1N}(t)]^n \sum_i C_{0N,\Delta}^{(i,i)}(t)}$$



Energy shifts for different volumes

Increasing volume



- Bound/scattering state hypotheses (Lüscher):

$$H_1 : \quad \Delta E_{\text{bound}}(L) = -\Delta E_{\infty} \left[1 + C \frac{e^{-\kappa L}}{L} \right],$$

$$H_2 : \quad \Delta E_{\text{scatter}}(L) = \frac{2\pi A}{\mu L^3} \binom{n}{2} \left[1 - \left(\frac{A}{\pi L} \right) \mathcal{I} + \left(\frac{A}{\pi L} \right)^2 [\mathcal{I}^2 + (2n - 5)\mathcal{J}] \right] + \frac{B}{L^6}$$

- Assess support for each hypothesis using the Bayes factor

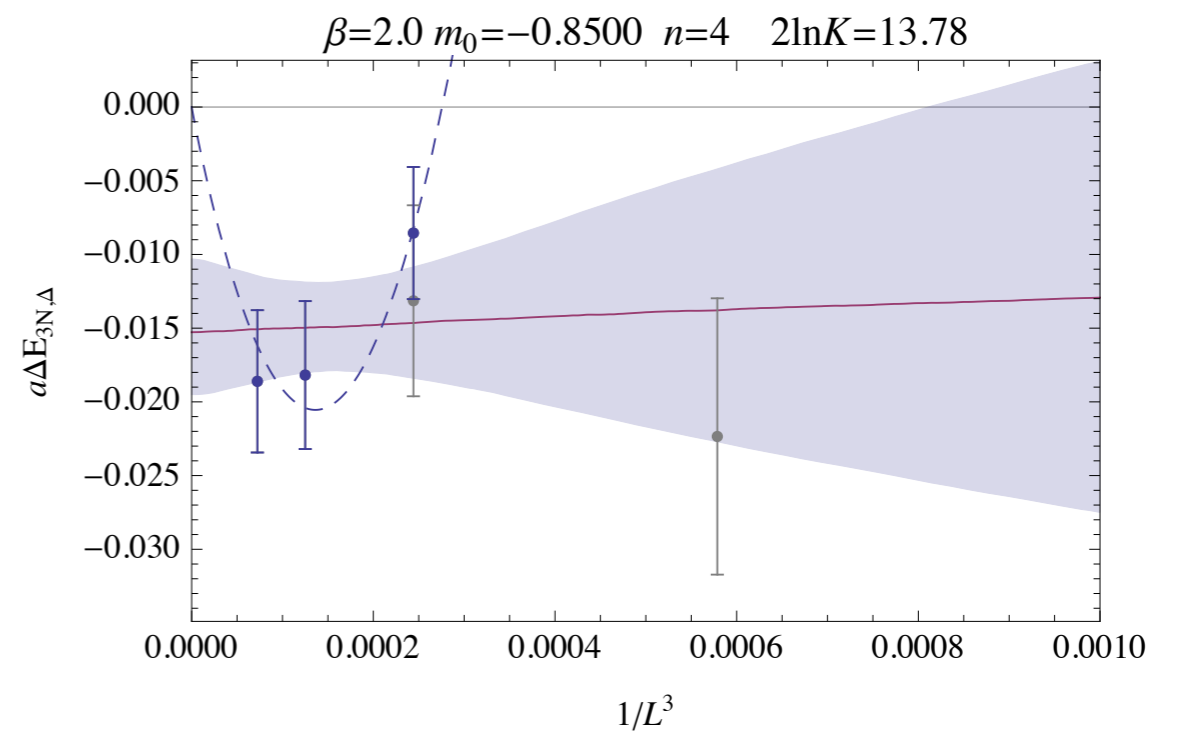
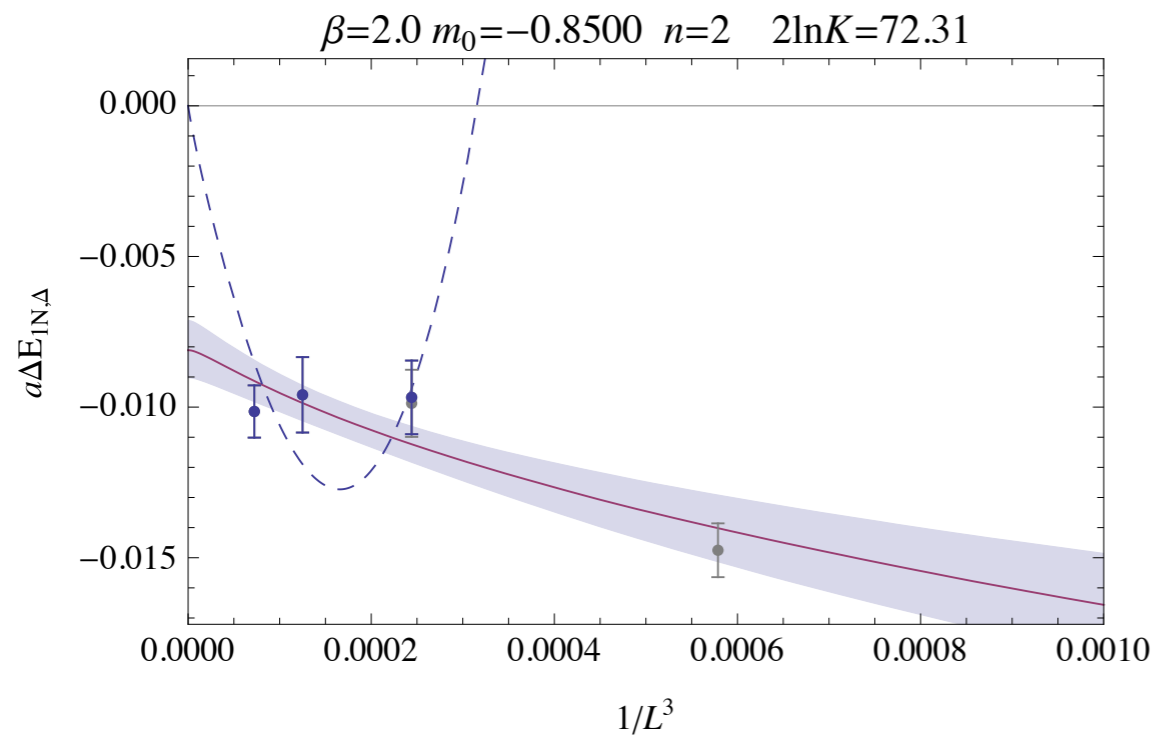
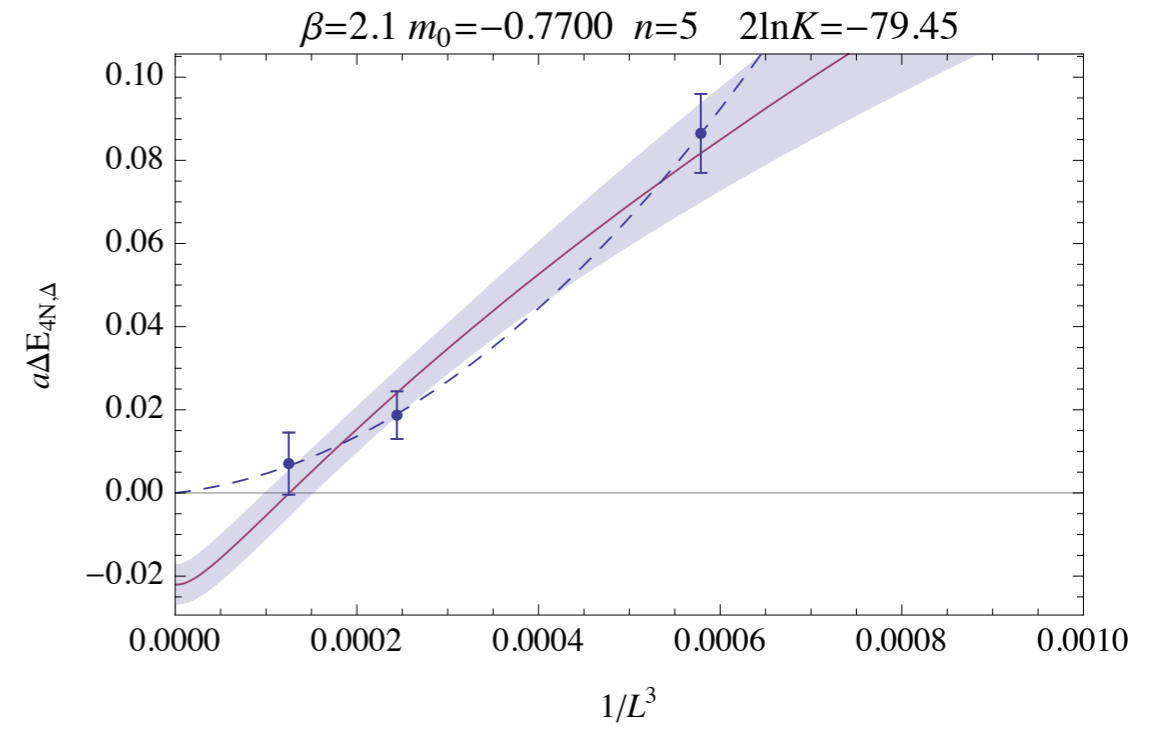
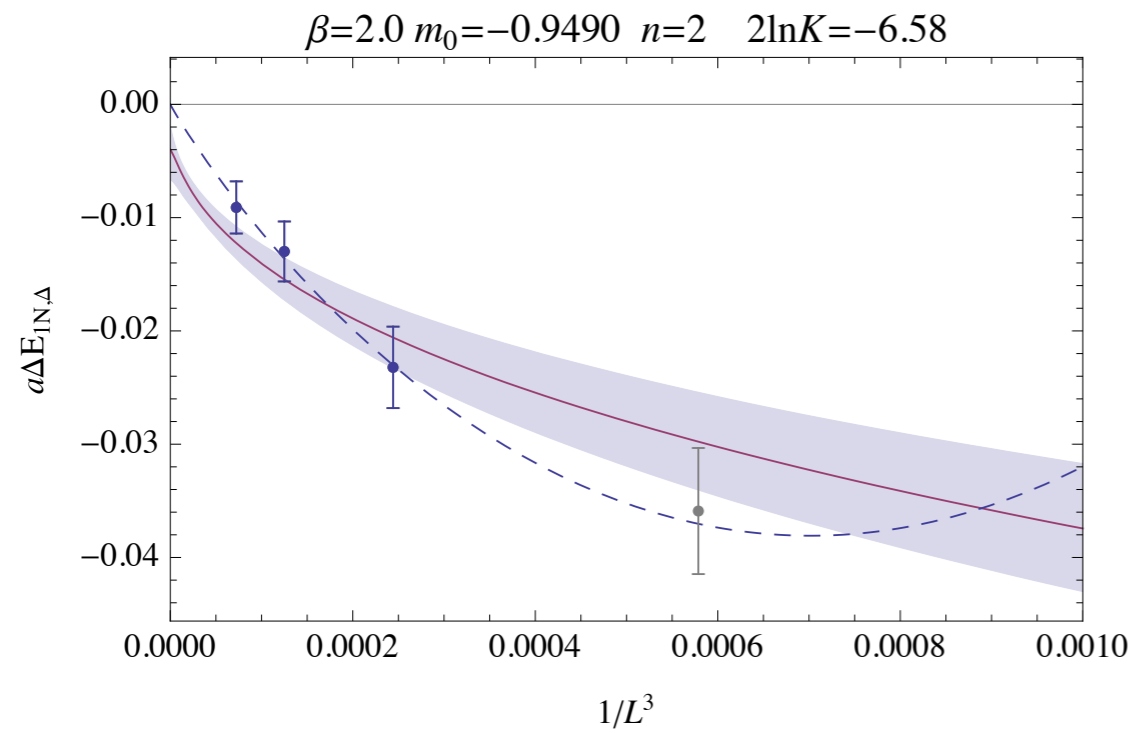
$$K = \frac{P(D|H_1)}{P(D|H_2)} = \frac{\int P(D|H_1, p_1)P(p_1|H_1)dp_1}{\int P(D|H_2, p_2)P(p_2|H_2)dp_2}$$

where $\log P(D|H_i, p_i) = -\frac{1}{2} \sum_{j=1}^N \frac{[d_j - H_i(x_j; p_i)]^2}{\sigma_j^2}$

and $P(p_i|H_i)$ are broad prior distributions for convergence

- If $2 \ln[K] > 6$: “strong evidence” of preference for H_1 over H_2
then ask what are the bounds on the binding energy

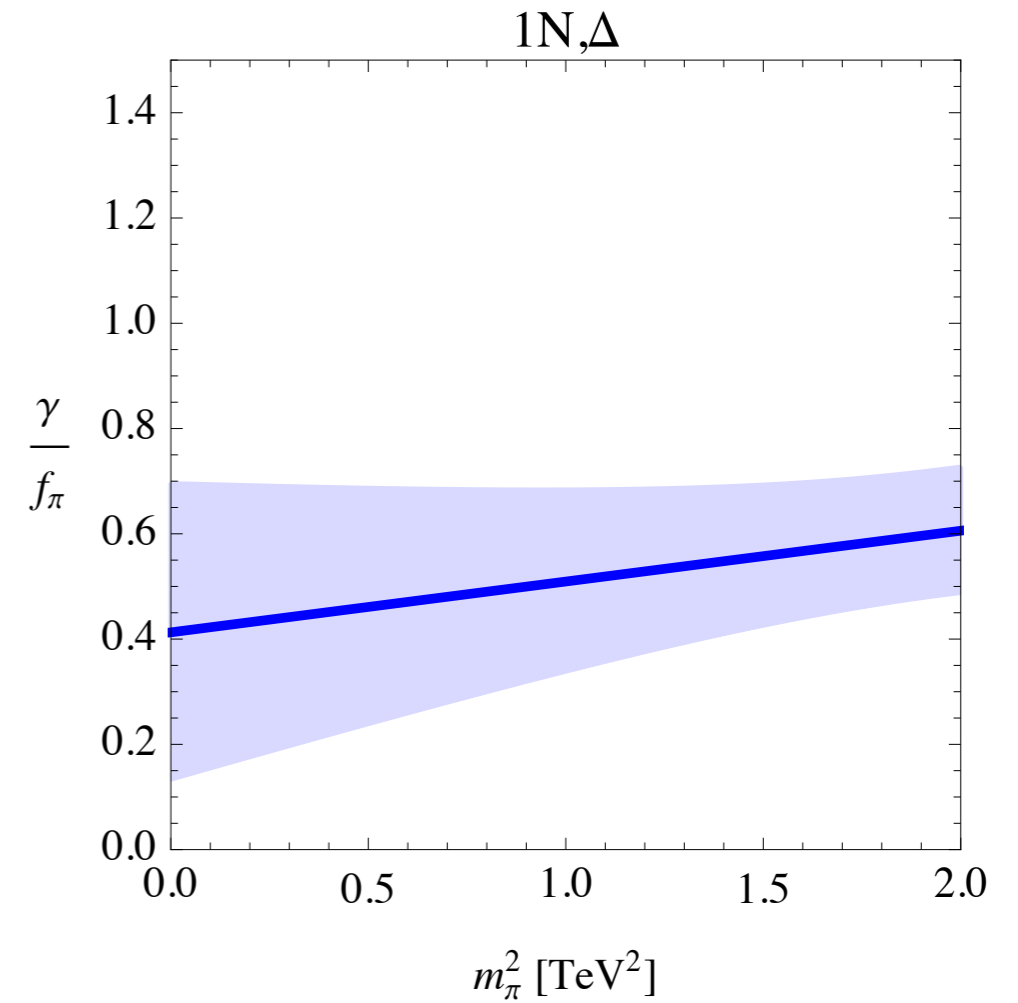
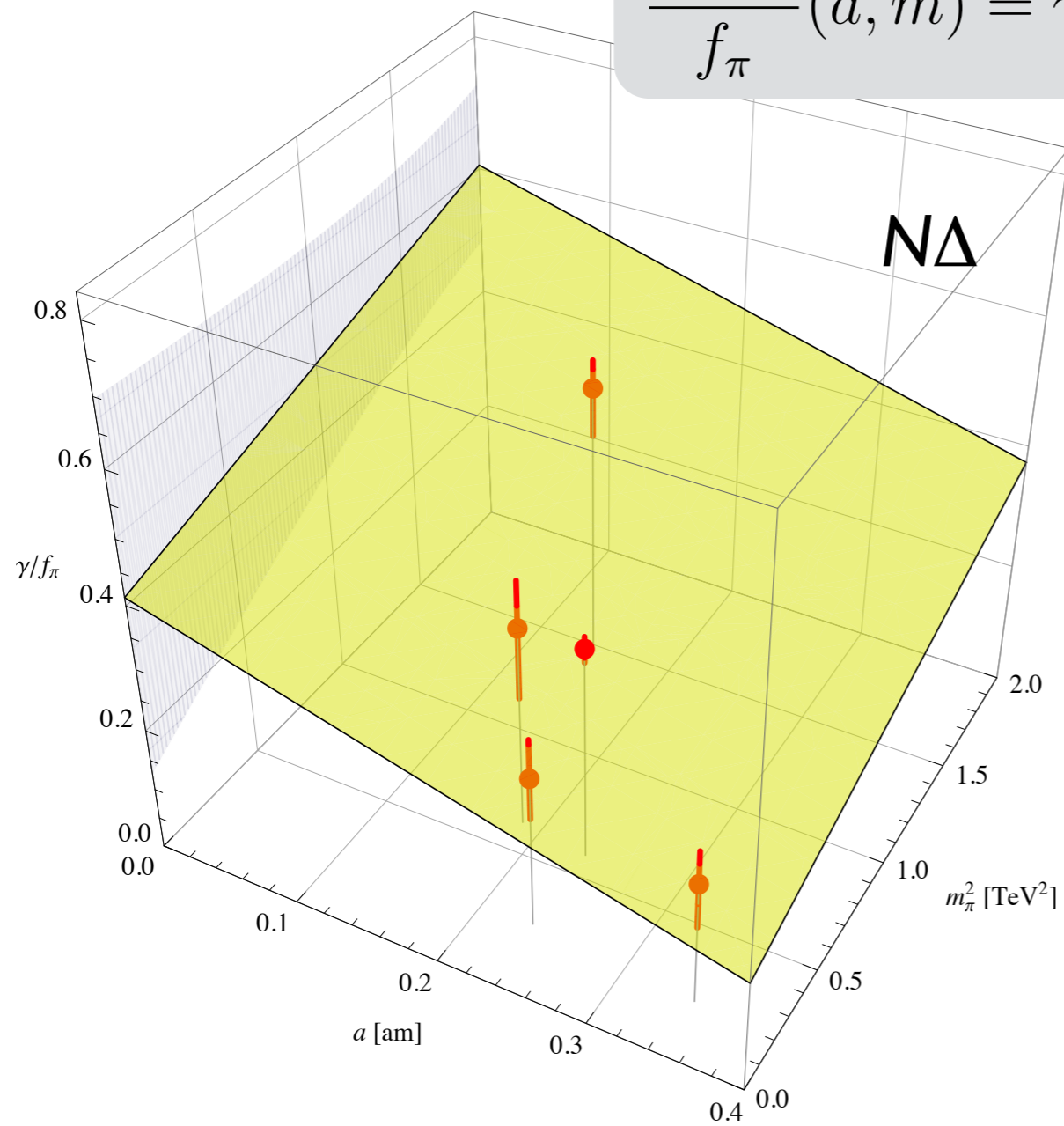
Infinite volume extrapolations



Continuum extrapolations

- Simple continuum limit extrapolation of binding momentum, γ

$$\frac{\gamma_{nN,\Delta}}{f_\pi}(a, m) = \gamma_{nN,\Delta}^{(0)} + a \delta_n^{(a)} + m_\pi^2 \delta_n^{(m)}$$

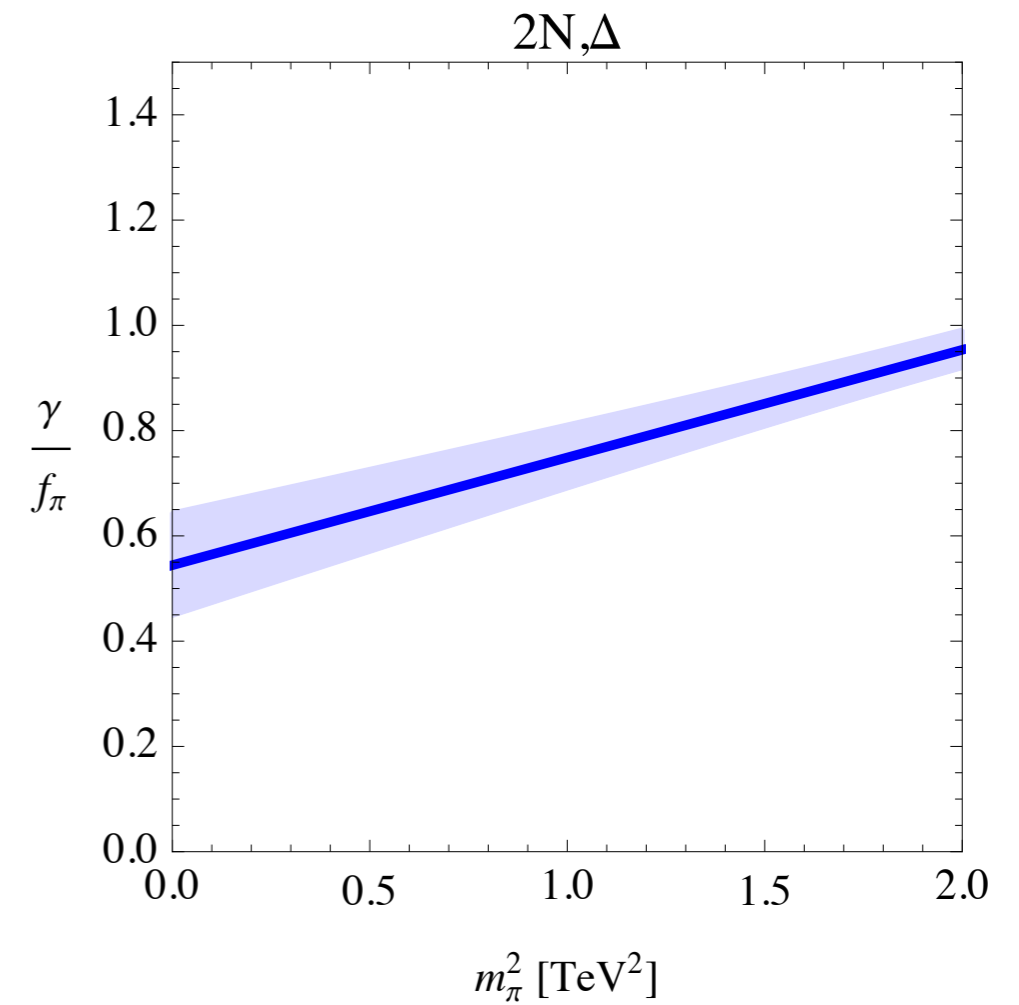
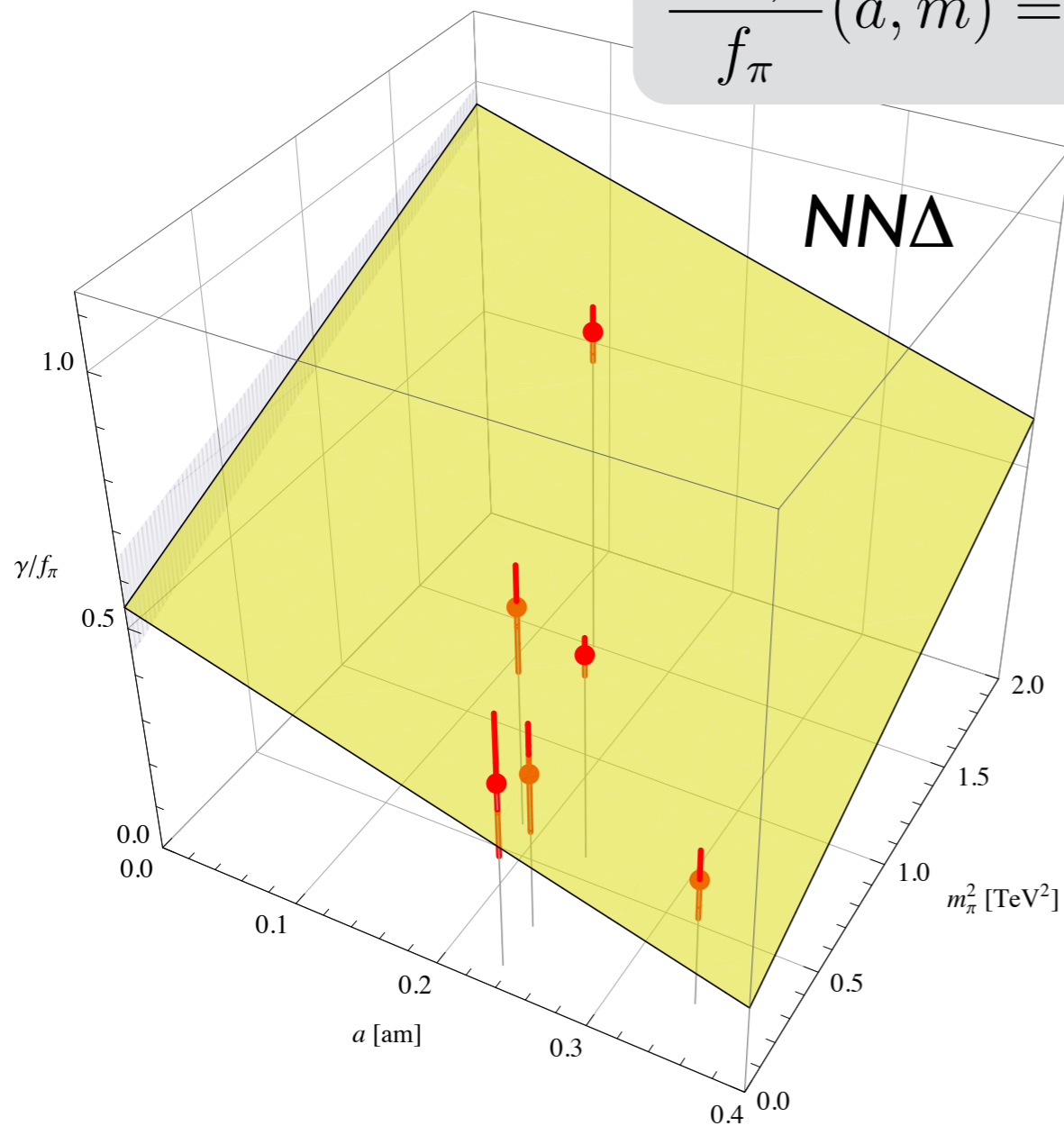


NB: physical scale set by demanding $f_\pi=246$ GeV (arbitrary)

Continuum extrapolations

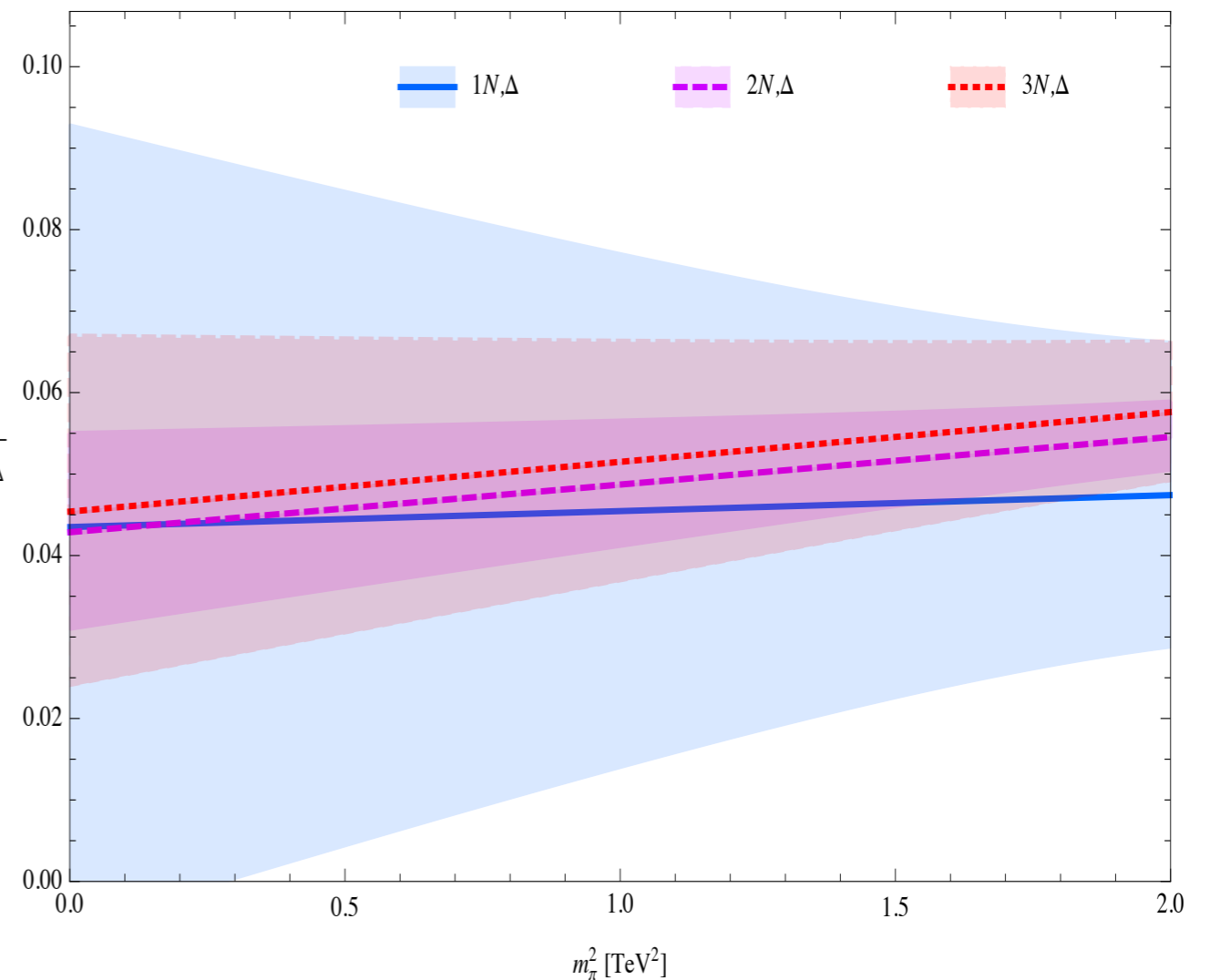
- Simple continuum limit extrapolation of binding momentum, γ

$$\frac{\gamma_{nN,\Delta}}{f_\pi}(a, m) = \gamma_{nN,\Delta}^{(0)} + a \delta_n^{(a)} + m_\pi^2 \delta_n^{(m)}$$

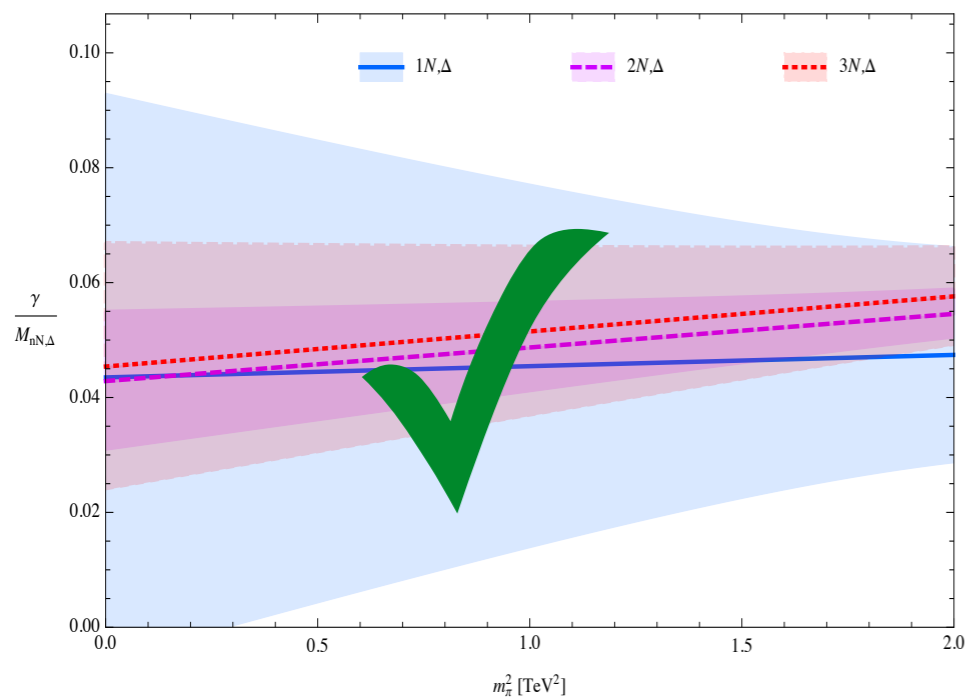
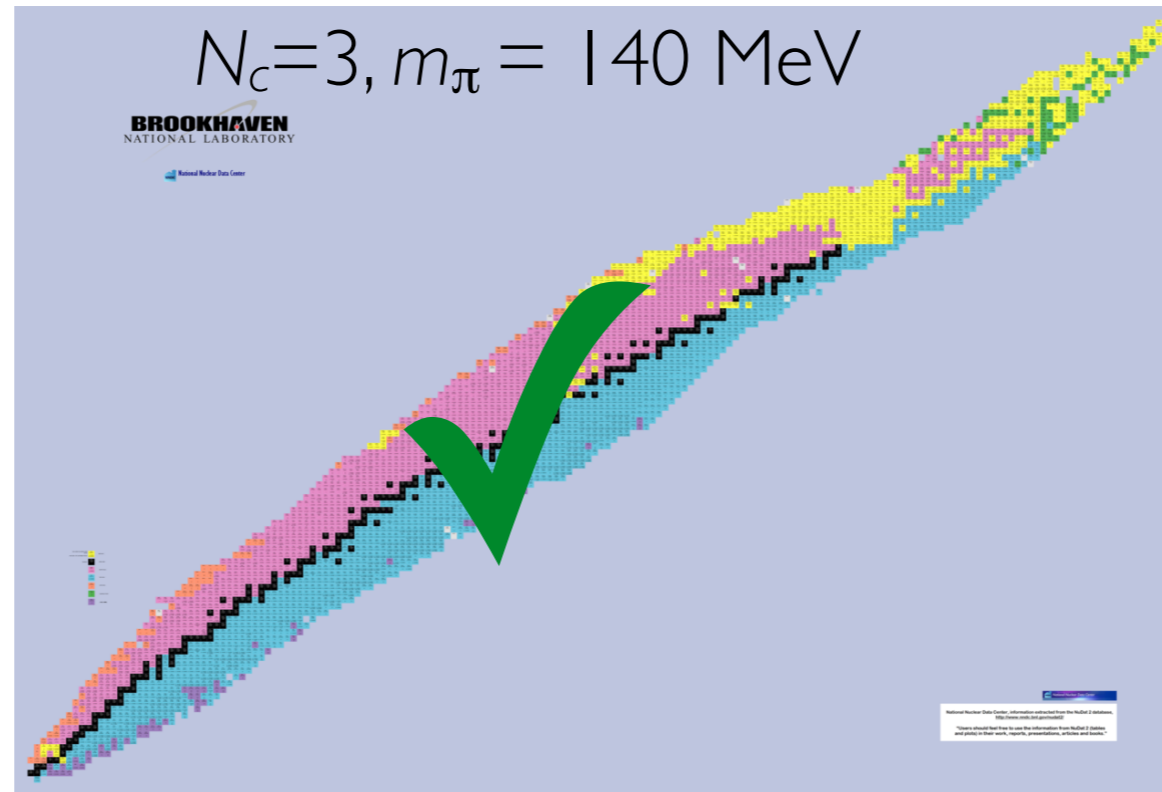


Dark nuclei

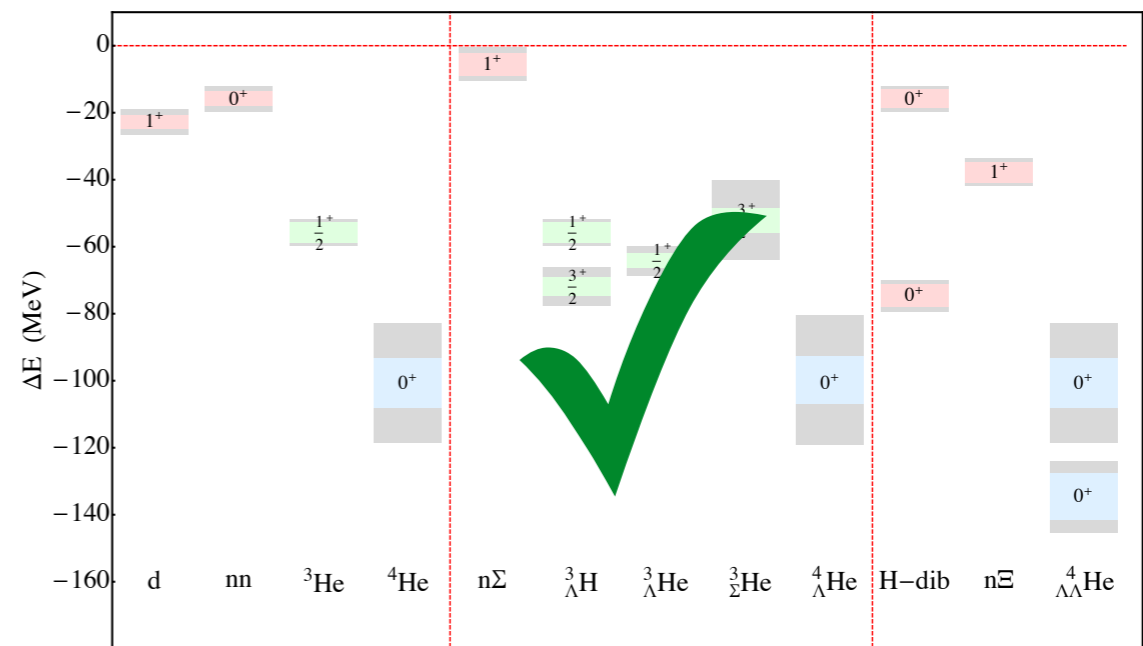
- $J=0$ nuclei: very likely unbound (all positively shifted)
- **$J=1$, strong evidence for bound states for $B=2,3,4(?)$**
 $B=5,\dots,8$ seem unbound
- Bindings decrease with quark mass and increase towards continuum
- Binding is few % w.r.t. mass $\frac{\gamma}{M_{hN\Delta}}$
- Nuclear states with other quantum #s may also be bound



The ubiquity of nuclei?



$N_c=2, m_\pi = 1 \text{ TeV}$



$N_c=3, m_\pi = 400\text{--}800 \text{ MeV}$
NPLQCD/PACS-CS

The ubiquity of nuclei?

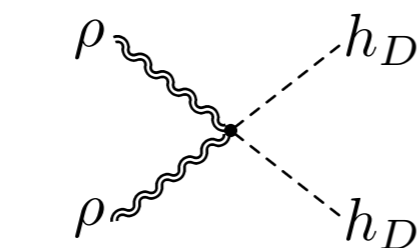
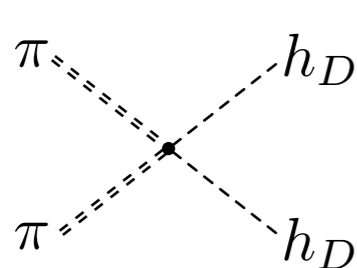
- So far appears that nuclei are rather generic and not an accident of parameters
- What are nuclei? e.g. shell-model like states vs quark blobs
 - More detailed studies necessary (eg magnetic moments)
- How generic are layers of effective degrees of freedom?
 - nucleons \rightarrow alpha clusters \rightarrow nuclei

Dark matter model building

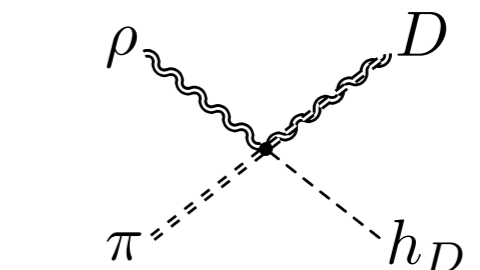
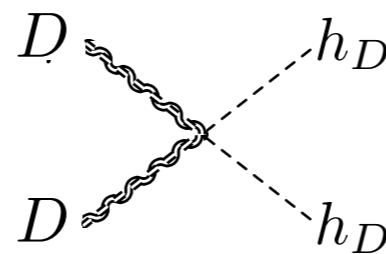
- Extend strongly-interacting dark sector to talk minimally to SM
 - Simple extension: add scalar particle that kinematically mixes with Higgs

$$\mathcal{L} = \mathcal{L}_{\text{strong}} - \frac{\lambda}{4} (v_D - H_D^2)^2 - \left(\kappa H_D (u_R^\dagger u_L + d_L^\dagger d_R) + h.c. \right) + \delta H_D^2 |H|^2$$

- Dark Higgs vev gives quark masses
 - Kinematic mixing controlled by δ : must be small $\sim 10^{-3}$
- Hadronic theory: consider only pions, rhos, “deuterons” (LQCD calculations provide motivation to consider deuterons)
- Interactions



Annihilation



Nucleosynthesis

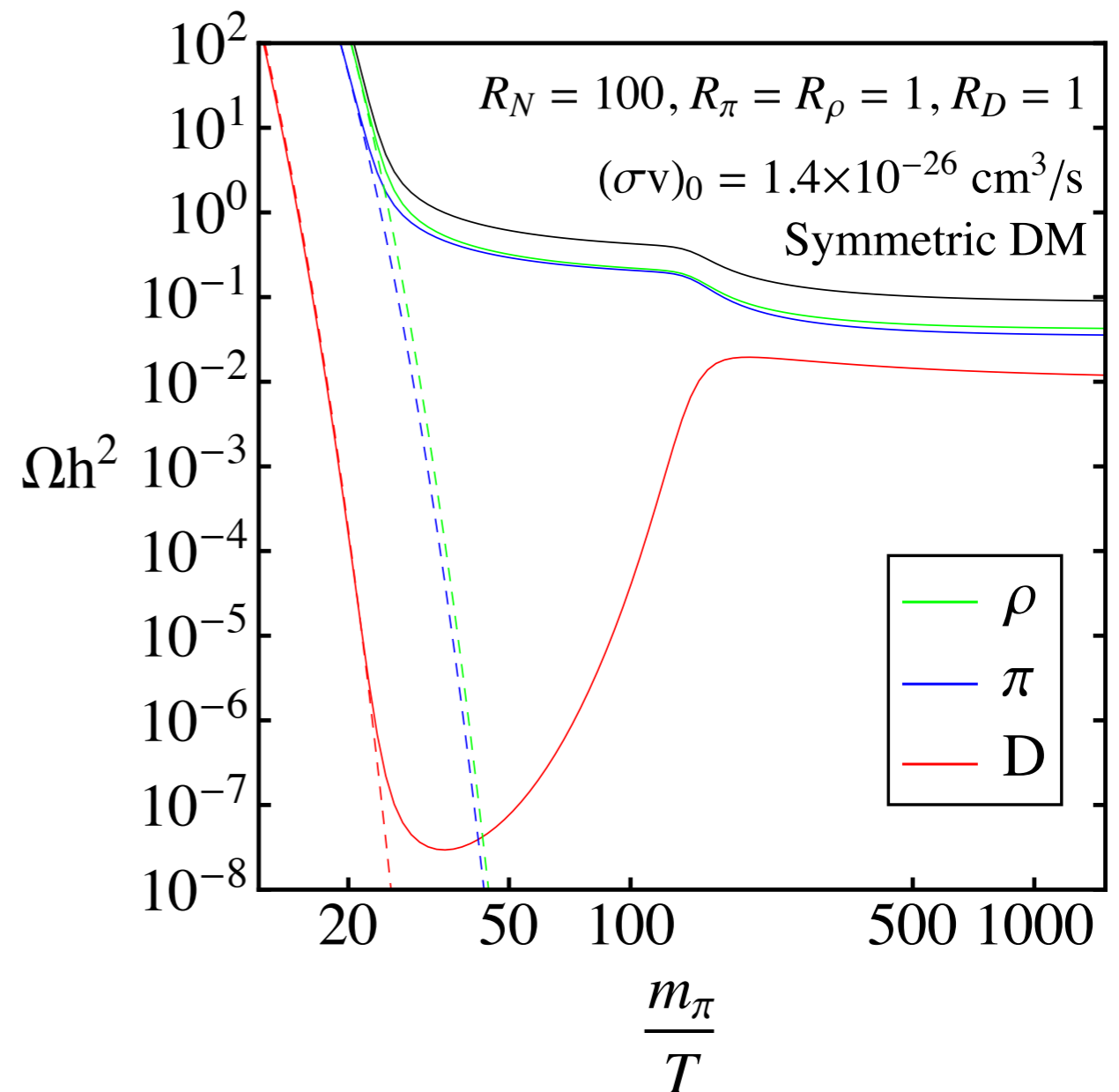
- Dark nucleosynthesis, dark capture processes modify early universe cosmology (both symmetric & asymmetric scenarios)
- Presence of nuclear binding energies: new scale for phenomenology is significantly different than M, Λ_{QC2D}

Signature	Collider	Direct Detection	Annihilation	Nucleosynthesis	Capture
Sym-DM	$M, 2M$	$M, 2M$	$M, 2M$	$B_D \ll M$	M
Asym-DM	$M, 2M$	$M, 2M$	—	$B_D \ll M$	—

- For symmetric DM, additional scale/process may lead to signals at multiple different energy scales with identical spatial morphology
- For asymmetric DM scenarios (only dark baryon number carrying states remain) nucleosynthesis allows indirect detection signals

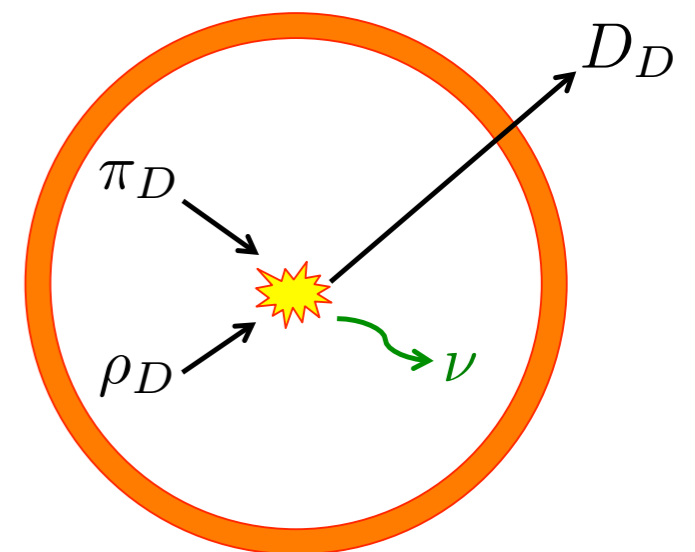
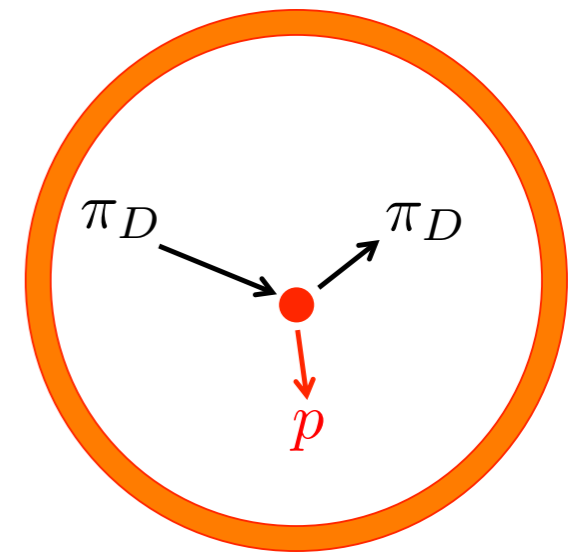
An example of relic densities

- Dark nucleosynthesis beaten out by annihilation at high densities
- Becomes dominant at lower temperatures $B_D/20 < T < M/20$
- Repopulates the nuclear state



Compact Objects

- Significant modifications to physics of astrophysical bodies
 - Dark matter gravitationally captured after scattering on visible matter
 - Helioseismology and neutron star lifetimes strongly modified – strongly constrains asymmetric DM models
- Very rich phenomenology!
 - Liberation of binding energy may allow ejection of dark matter
- Star develops a co-located dark nuclear process site



Summary

- Two-flavour, two-colour QCD has a complex spectrum exhibiting the analogues of nuclei
 - Ubiquity of nuclei?
- Composite dark matter is a natural scenario to consider
 - Composite doesn't just mean simple hadrons
Need to consider “nuclei”
 - Nuclear binding provides a scale for free that is small relative to the QCD scale in a natural way
 - Predicts a range of different phenomenology that beyond what is possible in simpler models

fin