

Nuclear matrix elements from QCD

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"Nuclear Aspects of Dark Matter Searches", INT, Seattle, Dec 8-12 2014

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http://www.hep.ucl.ac.uk/darkMatter/

- Laboratory searches for new physics
	- Dark matter detection: nuclear recoils as signal Nuclear matrix elements of exchange current
	- µ2e conversion expt: similar requirements
	- detect dark matter or $\mu \rightarrow e$, we

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	acertainties to discern what it is If(when) we detect dark matter or $\mu \rightarrow e$, we will need precise nuclear matrix elements with fully quantified uncertainties to discern what it is
- *Nuclear physics is the new flavour physics!*
	- Must be based on the Standard Model

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We need to develop the tools for precision predictions $\mathcal{L}_{\mathcal{A}}$

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Z

- Exploit effective degrees of freedom
- Establish quantitative control through linkages between different methods
	- QCD forms a foundation determines few body interactions & matrix elements
	- Match existing EFT and $\mathcal{L}_{\mathcal{A}}$ many body techniques onto QCD

Quantum Chromodynamics

- Lattice QCD: tool to deal with quarks and gluons
	- Formulate problem as functional integral $\mathcal{L}_{\mathcal{A}}$ over quark and gluon d.o.f. on R4

$$
\langle \mathcal{O} \rangle = \int dA_\mu dq d\bar{q} \, \mathcal{O}[q,\bar{q},A] e^{-S_{QCD}[q,\bar{q},A]} \label{eq:correlation}
$$

- Discretise and compactify system
	- Finite but large number of d.o.f (10¹⁰)
- Integrate via importance sampling (average over important configurations)
- Undo the harm done in previous steps

Spectroscopy

- How do we measure the proton mass? $\mathcal{L}_{\mathcal{A}}$
- Create three quarks at a source: and annihilate the three quarks \mathbb{R}^n at sink far from source
- QCD adds all the quark anti-quark pairs and gluons \mathbb{R}^n automatically: only eigenstates with correct q#'s propagate

Spectroscopy

Correlation decays exponentially with distance

at late times $C(t) = \sum Z_n \exp(-E_n t)$ *n* all eigenstates with q#'s of proton

 $\rightarrow Z_0 \exp(-E_0 t)$

Ground state mass revealed m. through "effective mass plot"

$$
M(t) = \ln \left[\frac{C(t)}{C(t+1)} \right] \stackrel{t \to \infty}{\longrightarrow} E_0
$$

QCD spectrum

Ground state hadron spectrum reproduced

QCD spectrum

QCD spectrum

Precise isospin mass splittings in QCD+QED

Nuclear Spectra

QCD for Nuclear Physics

- QCD (+EW) describes nuclear physics
	- Can compute the mass of lead nucleus ... in principle
- In practice: a hard problem
- At least two exponentially \mathbb{R}^n difficult challenges
	- Noise: probabilistic method $\mathcal{L}_{\mathcal{A}}$ so statistical uncertainty grows exponentially with A
	- Contraction complexity grows factorially

QCD for Nuclear Physics

- Quarks need to be tied together in all possible ways i.
	- $N_{\text{contractions}} = N_u/N_d/N_s$

- Managed using algorithmic trickery [WD & Savage, WD & Orginos; Doi & Endres] \mathbb{R}^n
	- Study up to N=72 pion systems, A=5 nuclei

Light nuclei

Light hypernuclear spectrum @ 800 MeV a.

[NPLQCD Phys.Rev. D87 (2013), 034506]

Heavy quark universe

[Barnea et al. 1311.4966]

- Combining LQCD and nuclear EFT (pionless EFT) T.
- For heavy quarks, even spectroscopy requires QCD matching:

Equally important for matrix elements

Nuclear Structure

- Current-nucleus interaction $\mathcal{L}_{\mathcal{A}}$
	- Born approximation interacts with a single $\mathcal{L}_{\mathcal{A}}$ nucleon

 $\sigma \sim |A~\langle N|J|N\rangle|^2$

N

Born approximation – interacts with a single $\left\{\begin{array}{ccc} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{array}\right\}$ $\mathcal{L}_{\mathcal{A}}$ nucleon

$$
\sigma \sim |A \langle N|J|N\rangle|^2
$$
\nknown from `expt/LQCD`

N

Born approximation – interacts with a single $\left\{\begin{array}{ccc} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{array}\right\}$ p. nucleon

$$
\sigma \sim |A \langle N|J|N\rangle|^2
$$
 known from $\exp t/\text{LQCD}$

Interact non-trivially with multiple nucleons

 $\sigma \sim |A\ \langle N|J|N\rangle + \alpha\ \langle NN|J|NN\rangle + \ldots |^2$

Born approximation – interacts with a single \blacksquare nucleon and, as such, are not expected to be enhanced in WIMP-nucleus interactions. To determine

$$
\sigma \sim |A \langle N|J|N\rangle|^2
$$
 known from $\exp t/\text{LQCD}$

$$
\overline{a} \rightarrow \overline{a}
$$

N

which is two orders lower than the impulse approximation from the impulse approximation. This term is the impulse approximation from the impulse approximation. This term is the impulse approximation. This term is the impul is the origin of the enhancement suggested in Ref. [1]. The isoscalar interactions with the strange and heavier quarks do not contribute to the non-derivative interaction with pions

order contribution to the single-nucleon -term in PT. The middle (pion-exchange) and right

Ideally, one would simply determine the matrix element of the Lagrange density in Eq. (2)

in the ground state of a given nucleus, at the relevant momentum transfer, without performing the intermediate matchings in Eq. (3) and in Eq. (3) and in Eq. (5). This would sum the contributions in Eq. (3) and in Eq. (3). This would sum the contributions in Eq. (5). This would sum the contributions in Eq. (5).

Interact non-trivially with multiple nucleons \mathbb{R}^n FIG. 1: Some of the diagrams contributing to nuclear -terms. The left panel shows the leading

$$
\sigma \sim |A \langle N|J|N\rangle + \alpha \langle NN|J|NN\rangle + \dots|^2
$$

unknown/poorly known! Second term may be significant which is two orders lower than the contribution from the impulse approximation. I. is the origin of the enhancement suggested in Ref. [1]. The isoscalar interactions with the

- May shift cross sections and, and, and are not expected to be enhanced in WIMP-nucleus in WIMP-nucleus in WIMPstrange and heavier quarks do not contribute to the non-derivative interaction with pions
- May scale differently with Z and A
- Leads to significant uncertainty

Particlear uncertainties
Particles, Centre Scientifique, University Pasteur, Bosnie 28, F–67037 Strasbourg, Bosnie 28, F–67037 Strasbourg, Bo

- Gamow-Teller transitions in nuclei 64 decays of nuclei in the mass range A = 41–50. In all the are a stark example of problems $0.8\begin{bmatrix} -0.77 \ 0.77 \end{bmatrix}$
- Well measured
- Best nuclear structure calculations are systematically off by 20–30% $\begin{array}{ccc} 0.2 \end{array}$
- Large range of nuclei (30<A<60) where spectrum is well described \Box Large range of nuclei artion
are ff t
del
del artic
del artic
- **Example 28 CRPA, shell-model,...** \Box \Box is \Box plane, with the \Box
- **Correct for it by "quenching"** axial charge in nuclei ... \overline{a}

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(August 12, 2013)

$$
T(GT)\sim\sqrt{\sum_f\langle\boldsymbol{\sigma}\boldsymbol{\cdot}\boldsymbol{\tau}\rangle_{i\rightarrow f}}
$$

1 Correct for it by "quenching"
$$
\langle \sigma \tau \rangle = \frac{\langle f \mid \sum_{k} \sigma^{k} t_{\pm}^{k} \mid i \rangle}{\sqrt{2J_{i}+1}}
$$

Nuclear matrix elements

For deeply bound nuclei, use the techniques as for single hadron matrix elements

- At large time separations gives ground-state matrix element of $\mathcal{L}_{\mathcal{A}}$ current
- For near threshold states, need to be careful with volume effects
- Calculations of matrix elements of currents in light nuclei just \mathbb{R}^n beginning for A<5

production in a Background field method with ¹, ¹, ¹, 2^{, 1}, 2, ¹, 2, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1,

Hadron/nuclear two-point functions are modified in presence of fixed eternal fields t charge. Therefore, the magnetic moments presented momen here considerations are completed complete complete complete calculations are Γ

of gauge-field configurations are *^e|*B*[|]* ⇠ ⁰*.*⁰⁴⁶ *[|]n*˜*[|]* GeV².

To optimize the re-use of light-quark propagators in the

to be such that in this initial initial intervals in the such that in the such that in the such that is not th
In the such that is in the such that is not the

- Eg: fixed B field: modified exponential behaviour $E(\mathbf{B}) = M + \frac{|Q e \mathbf{B}|}{2 M}$ $\frac{\mathbf{z} \cdot \mathbf{z}}{2M}$ - $\mu \cdot \mathbf{B}$ $- 2\pi \beta_{M0} |\mathbf{B}|^2 - 2\pi \beta_{M2} T_{ij} B_i B_j + ...$
	- QCD spectroscopy with multiple fields enable extraction of coefficients of response extraction or coemercing or response
	- Eg: magnetic moments, polarisabilities, … Γ the second is resonance mass, the second is the second is the second is the energy of the Ly. Haylicus Hollicus, polarisau liucs, ...
	- Not restricted to simple EM fields (axial, twist-2,...) tion of its magnetic moment, \mathbf{r} , and fifther fourth and fifther fourth and fifther first and fifther first and final field and fifth and first and terms are from its scalar areas in the from the from the from the from the front α

0 5 10 15 20

- **1** Magnetic moments from spin splittings will be spin-dependent interactions, and the different interactions, and the different interactions, and the dif- $\delta E^{(B)} \equiv E_{+j}^{(B)} - E_{-j}^{(B)} = -2\mu |\mathbf{B}| + \gamma |\mathbf{B}|^3 + \dots$ $\delta E^{(B)} \equiv E^{(B)}_{+j} - E^{(B)}_{-j} = -2\mu |\mathbf{B}| + \gamma |\mathbf{B}|^3 + \dots$
- Extract splittings from ratios of correlation functions of *E*(*B*) that is linear in B determines *µ* via Eq. (3). $E\times$ under spillungs irom ratios of ϵ

charge matrix, there are no contributions from coupling

$$
R(B) = \frac{C_j^{(B)}(t) C_{-j}^{(0)}(t)}{C_{-j}^{(B)}(t) C_j^{(0)}(t)} \stackrel{t \to \infty}{\longrightarrow} Z e^{-\delta E^{(B)}t}
$$

Careful to be in single exponential region of each correlator qui didi co do ni dirigio di portoriene. κ iderical which we compute using the methods of κ

[NPLQCD 1409.3556, PRL to appear]

Lattice results appear to suggest $\mathcal{L}_{\mathcal{A}}$ shown as the solid bands. The inner bands to inner bands to inner bands. The inner bands to inner bands to in $\overline{}$ heavy quark nuclei are shell-model the statistical uncertainties, which the outer bands corresponds corresponding to the outer bands corresponding $\overline{}$ δ LL 0.01(3)(7) -0.34(2)(9) 0.45(4)(16) like! \mathcal{L}_max , including estimates of the uncertainties from lat-uncertainties from lat-uncertainties from lat-uncertainties from lat-

> Γ P merence memm Difference from NSM expectation [NPLQCD 1409.3556, PRL to appear]

d ³ ³

 $\delta \mu$ 0.01(3)(7) -0.34(2)(9) 0.45(4)(16)

One possible DM interaction is through scalar exchange $\mathcal{L}_{\mathcal{A}}$

$$
\mathcal{L} = \frac{G_F}{2} \sum_q a_S^{(q)}(\overline{\chi}\,\chi)(\overline{q}\,q)
$$

- Accessible via Feynman-Hellman theorem T.
- At hadronic/nuclear level T. $\mathcal{L} \rightarrow G_F \overline{\chi} \chi \left(\frac{1}{4} \langle 0 | \overline{q} q | 0 \rangle \text{ Tr} \left[a_S \Sigma^{\dagger} + a_S^{\dagger} \Sigma \right] +$ 1 $\frac{1}{4} \langle N|\overline{q}q|N\rangle N^{\dagger}N \text{Tr}\left[a_{S}\Sigma^{\dagger}+a_{S}^{\dagger}\Sigma\right]$ $\qquad \qquad - \left. \frac{1}{4} \langle N | \overline{q} \tau^3 q | N \rangle \left(N^\dagger N {\rm Tr} \left[a_S \Sigma^\dagger + a_S^\dagger \Sigma \right] \right. \right. \nonumber \\ \left. - \left. 4 N^\dagger a_{S,\xi} N \right) \right. \nonumber \\ \left. + \right. \left. \ldots \right)$ $\int \frac{1}{4} \langle N|T|T|T|N\rangle \langle N|T|T|T|S^2 + d_S^2 \rangle = 4N^4 \langle S, \xi N \rangle + \dots$ strange and heavier quarks do not contribute to the non-derivative interaction with pions \mathcal{L}

■ Contributions: To describe to be enhanced in WIMP-nucleus interactions. To descriptions. To determine the single-hadron matrix elements of the single-hadron matrix elements of the single-hadron matrix elements of the si

658 *BZ,N* (MeV) 9.1(3.7)(4.6)(0.9) 50.8(8.0)(7.0)(5.1) 75(26)(19)(8)

F Quark mass dependence of nuclear binding energies bounds such contributions. The interactions and outer shaded regions at zero momentum transfer, and outer shaded regions contributions correspond to the inner and outer shaded regions contributions. viations of nuclear hinding energies h de vaarintings appenaente of haerear Binant gener gros b dependence-of nuclear binding energies bounds responds to the corrections due to interactions between the nucleons, including the possibly endhanced contributions from MECs. It is useful to define the ratio of \mathcal{L}

$$
\delta \sigma_{Z,N} = \frac{\langle Z, N(\text{gs}) | \overline{u}u + \overline{d}d | Z, N(\text{gs}) \rangle}{A \langle N | \overline{u}u + \overline{d}d | N \rangle} - 1 = -\frac{1}{A\sigma_N} \frac{m_\pi}{2} \frac{d}{dm_\pi} B_{Z,N}
$$

Lattice calculations + physical point suggest such contributions are $O(10\%)$ or less for light nuclei $(A<4)$ as con divided Lattice calculations prijsiear pontesus subs t ulations t physical point suggest such viations of $\mathcal{P}^{(1)}$ and the impulse approximation, the impulse $\mathcal{P}^{(1)}$ ons are \cup (TU%) or less for light nuclei (AS4) $-$

[NPLQCD PRD to appear] T_{NPI}

QCD for nuclear physics

- Nuclei are under serious study directly from QCD
	- Spectroscopy of light nuclei and exotic nuclei (strange, charmed, …)
	- Nuclear properties/matrix elements
- Prospect of a quantitative connection to QCD makes this a very exciting time for nuclear physics
	- Critical role in current and upcoming intensity frontier experimental program

Learn many interesting things about nuclear physics along the way

