# Nuclear matrix elements from QCD

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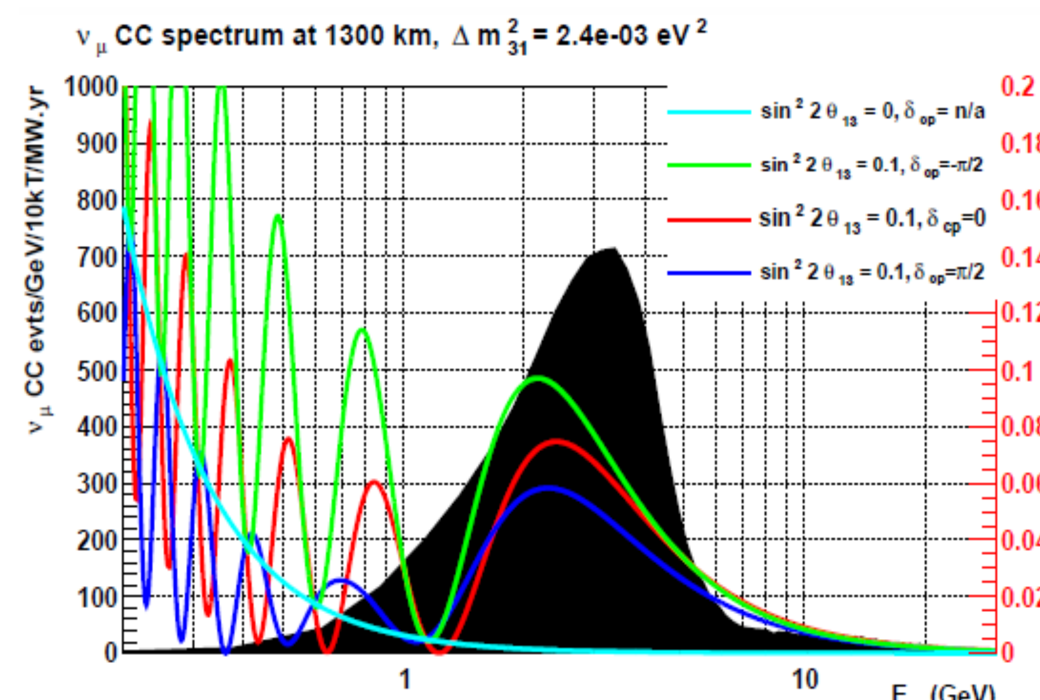
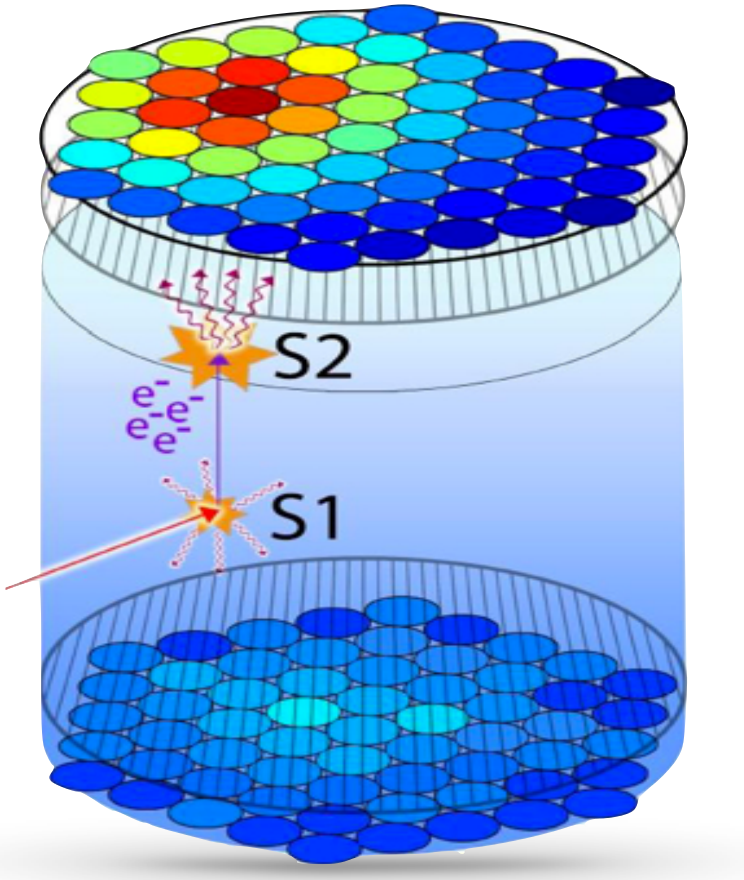
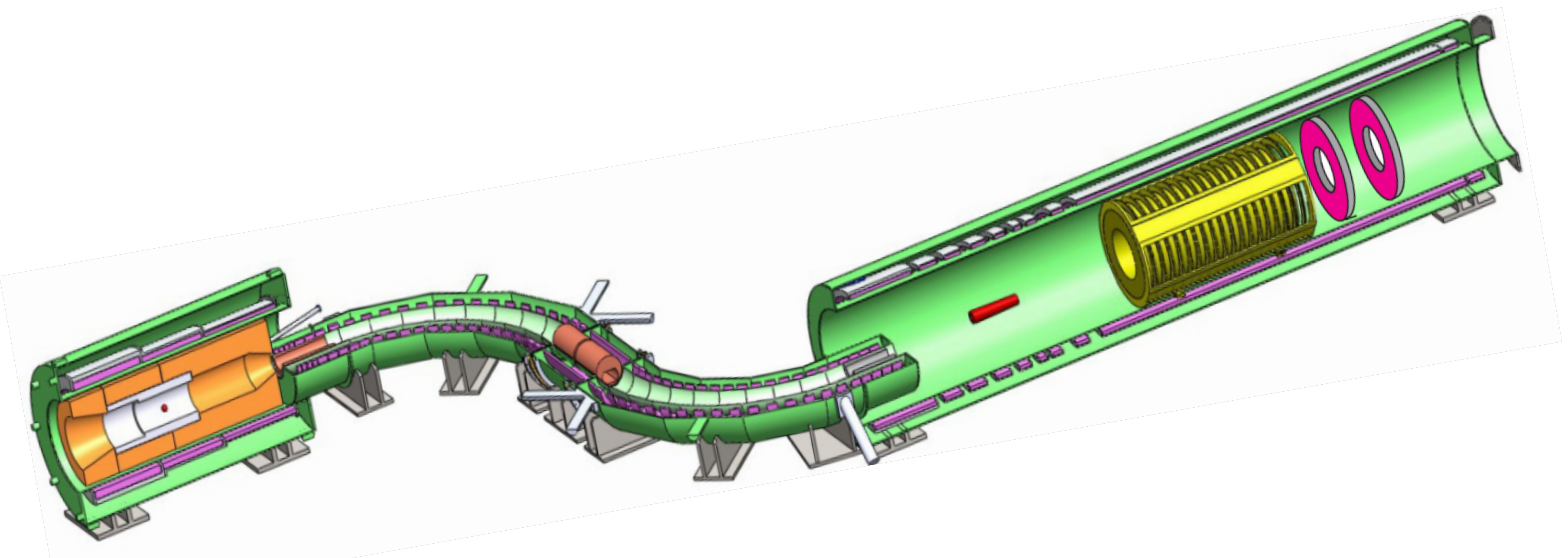


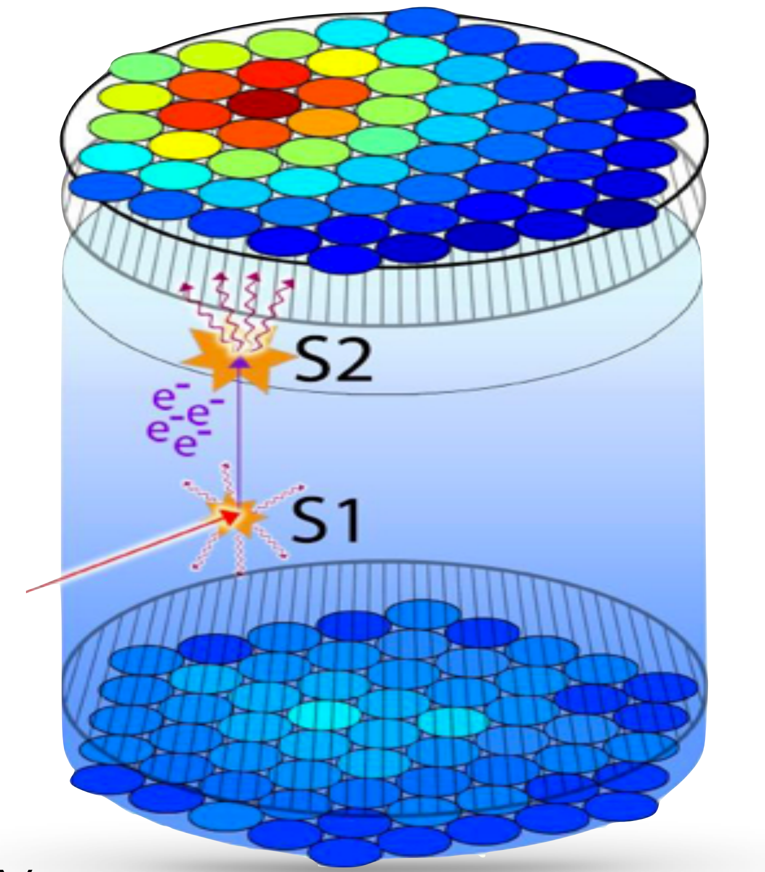
[U Mosel FSNu town meeting]




http://www.hep.ucl.ac.uk/darkMatter/



- Laboratory searches for new physics
  - Dark matter detection: nuclear recoils as signal
    Nuclear matrix elements of exchange current
  - $\mu$2e conversion expt: similar requirements
  - If(when) we detect dark matter or $\mu \to e$, we will need precise nuclear matrix elements with fully quantified uncertainties to discern what it is

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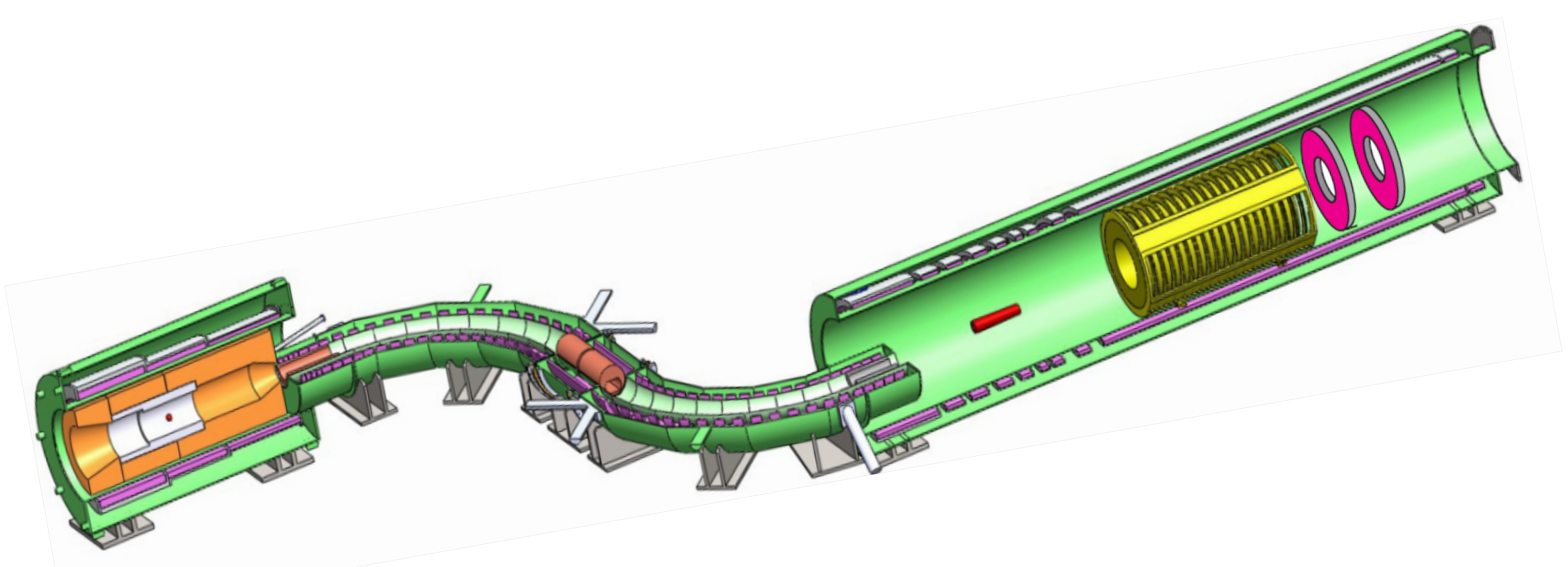

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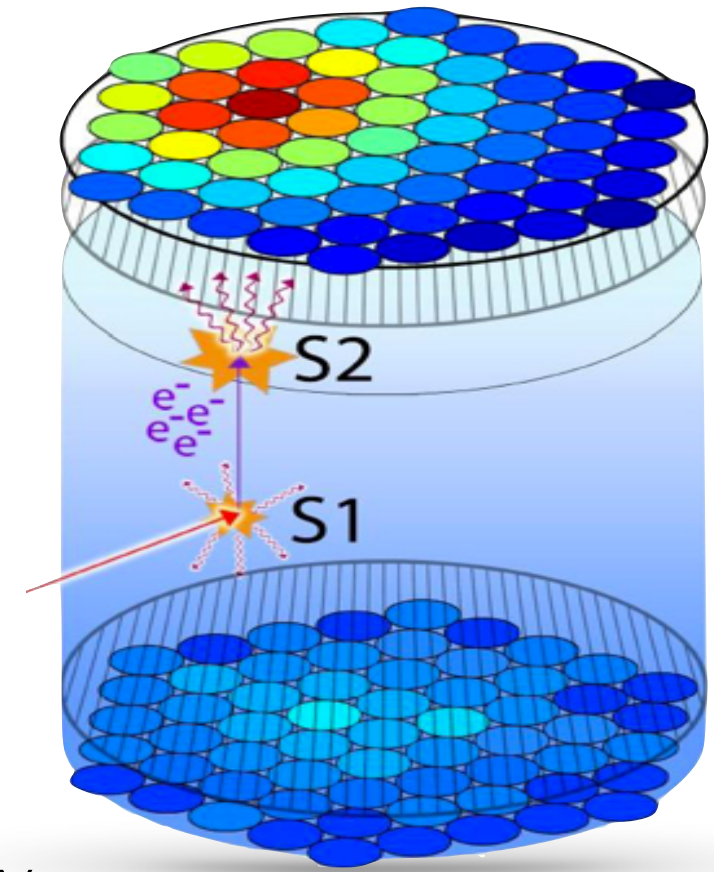
- ***Nuclear physics is the new flavour physics!***
  - Must be based on the Standard Model

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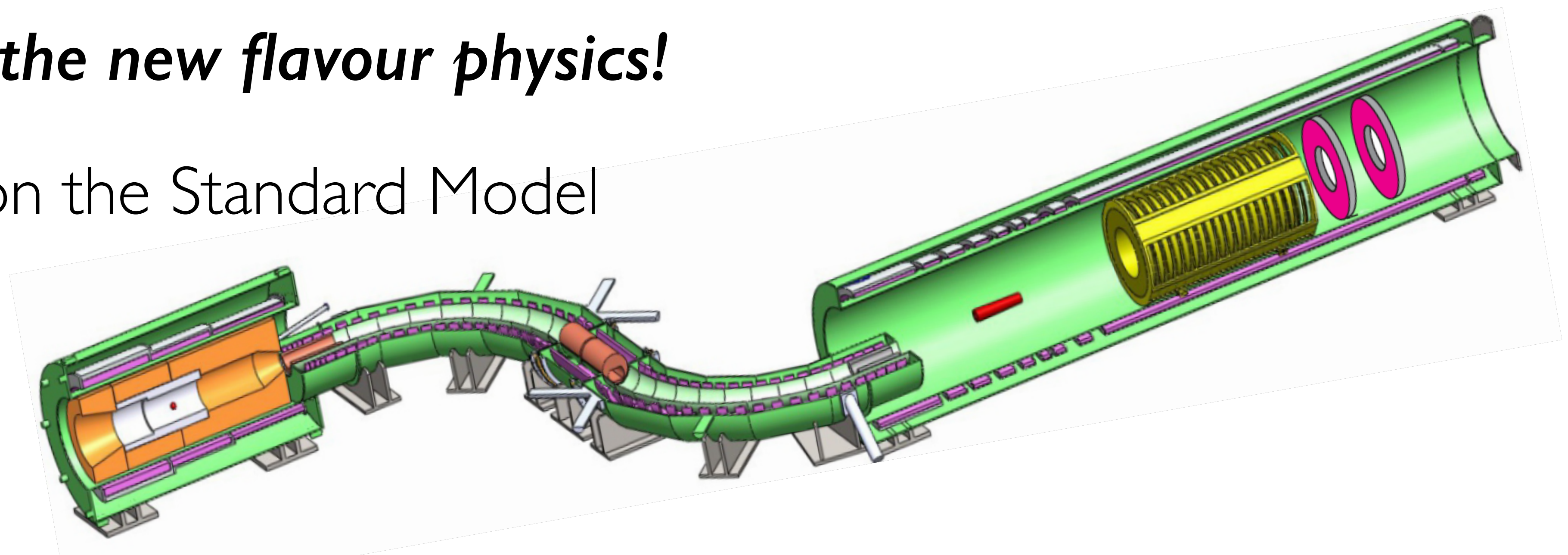

# Precision nuclear physics

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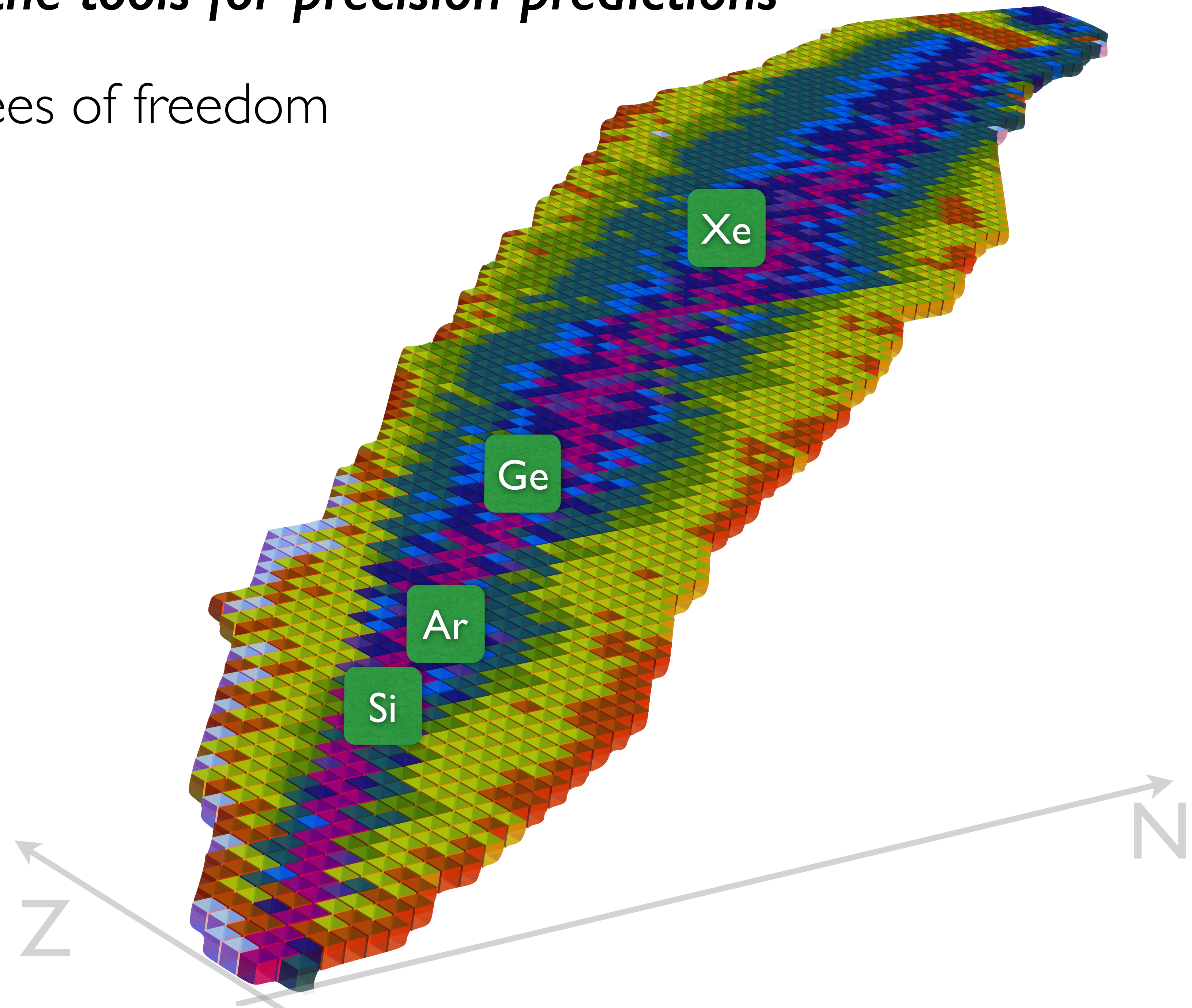
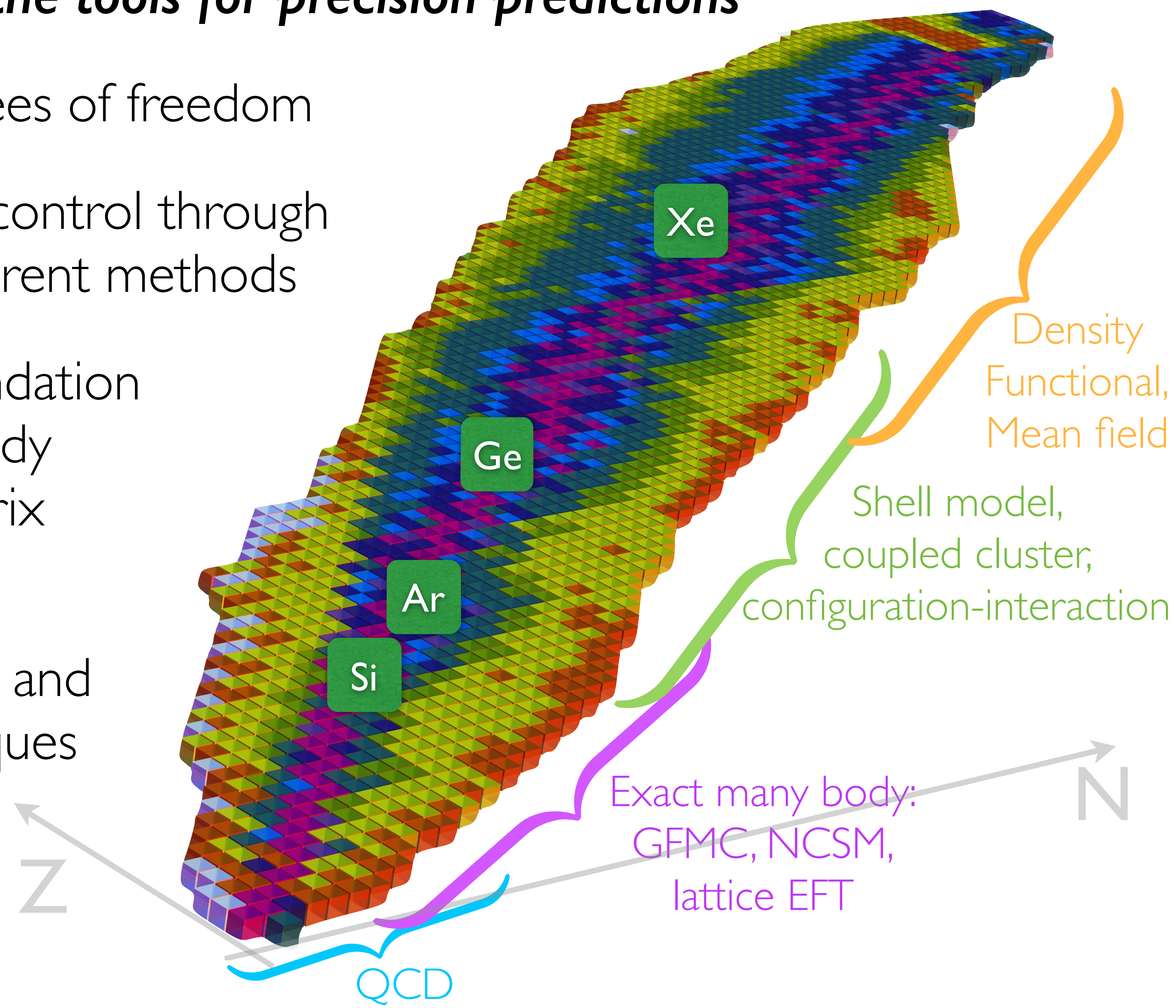


# Precision nuclear physics

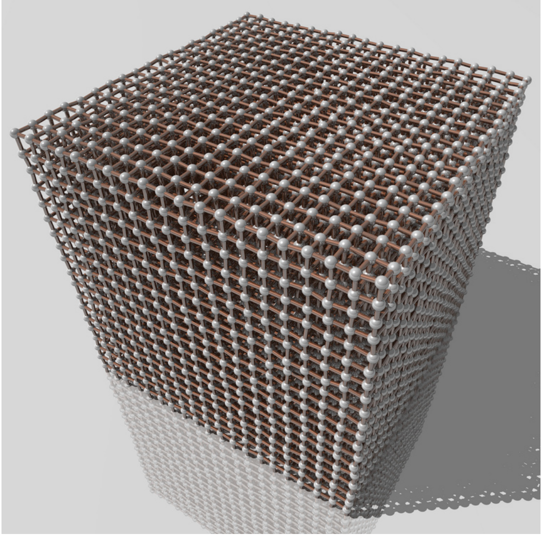
- ***We need to develop the tools for precision predictions***
- Exploit effective degrees of freedom
- Establish quantitative control through linkages between different methods
  - QCD forms a foundation determines few body interactions & matrix elements
  - Match existing EFT and many body techniques onto QCD

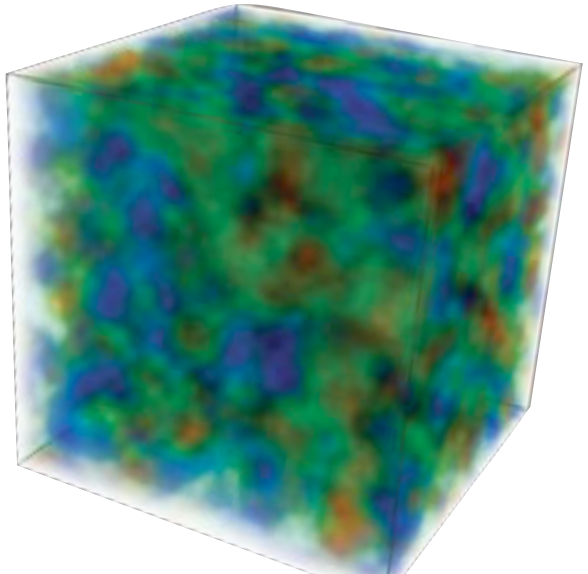


- Lattice QCD: tool to deal with quarks and gluons
  - Formulate problem as functional integral over quark and gluon d.o.f. on $R_4$

$$\langle \mathcal{O} \rangle = \int dA_\mu dq d\bar{q}\, \mathcal{O}[q, \bar{q}, A] e^{-S_{QCD}[q, \bar{q}, A]}$$

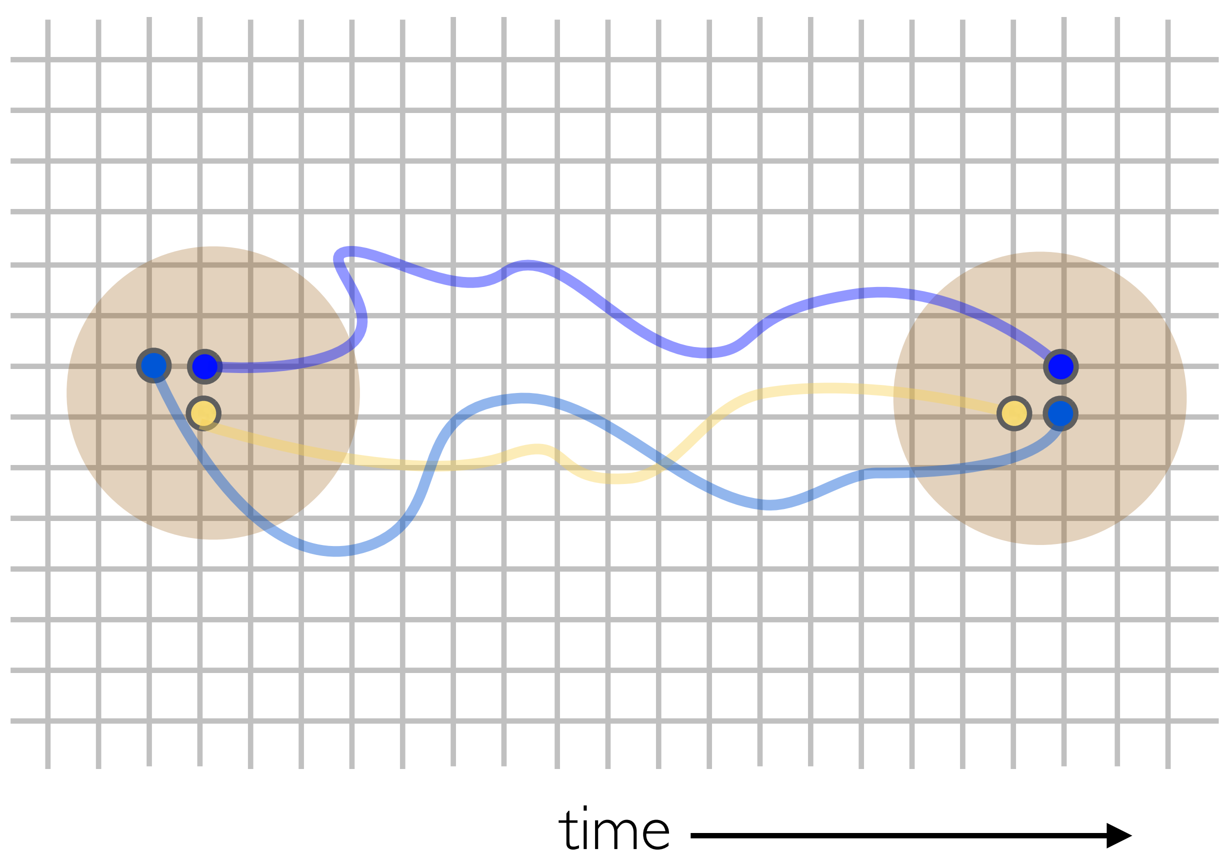
  - Discretise and compactify system
    - Finite but large number of d.o.f $(10^{10})$
  - Integrate via importance sampling (average over important configurations)
  - Undo the harm done in previous steps





# Spectroscopy

- How do we measure the proton mass?
- Create three quarks at a source: and annihilate the three quarks at sink far from source
- QCD adds all the quark anti-quark pairs and gluons automatically: only eigenstates with correct q#'s propagate



# Spectroscopy

- Correlation decays exponentially with distance

$$C(t) = \sum_n Z_n \exp(-E_n t)$$

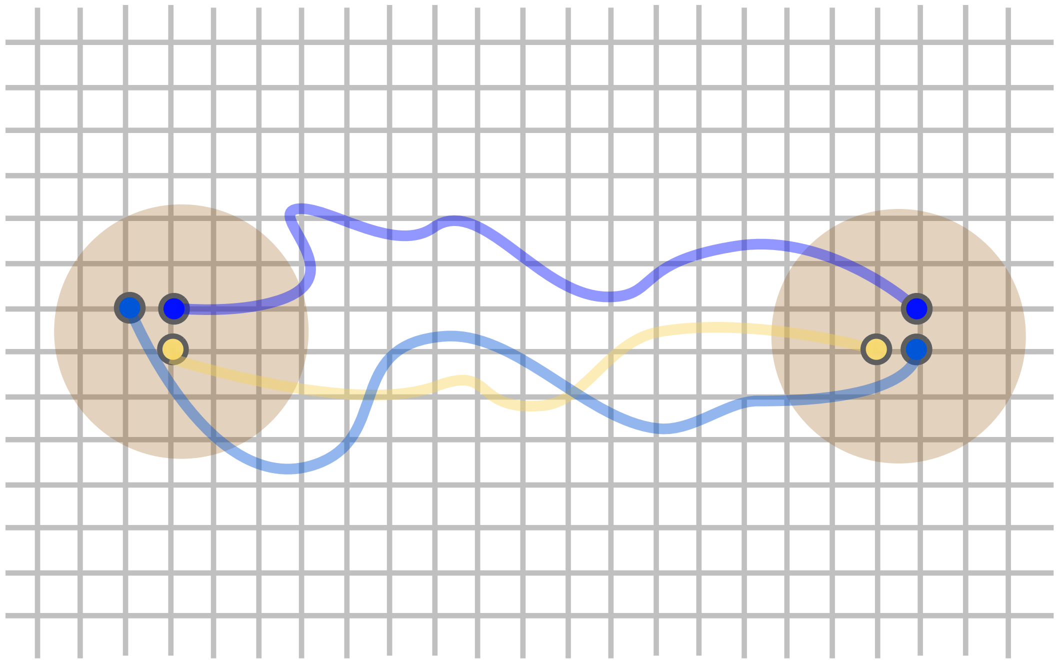
$n$ ← all eigenstates with q#'s of proton

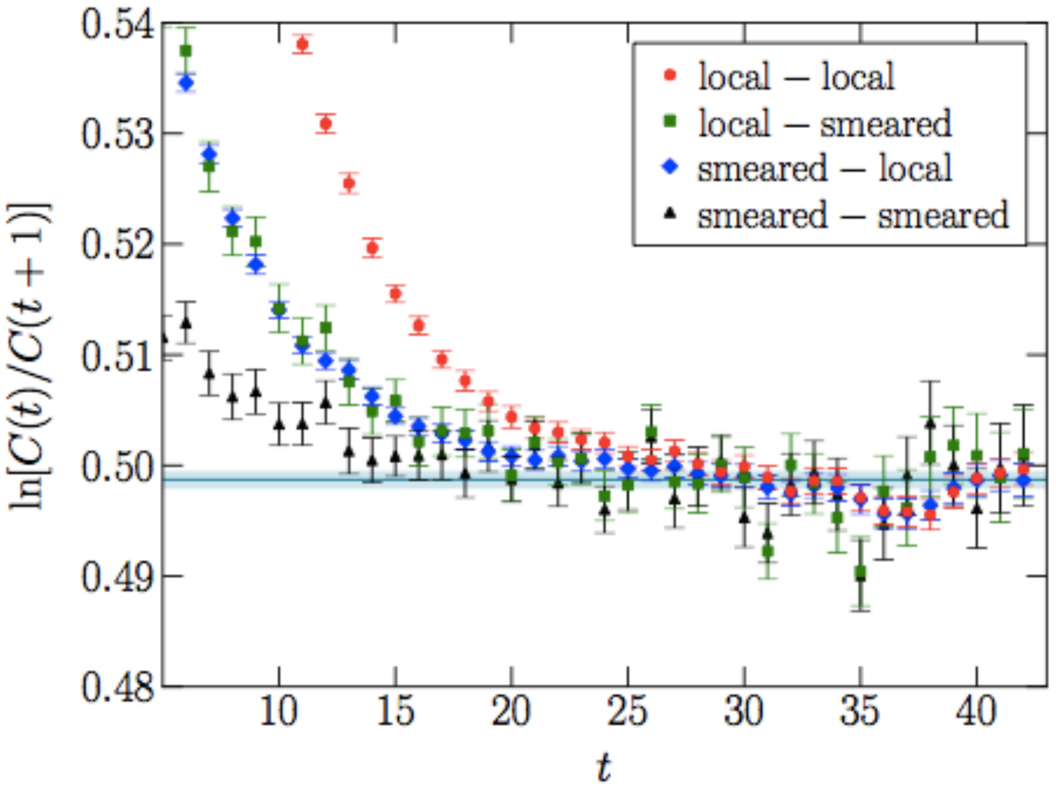
at late times

$$\rightarrow Z_0 \exp(-E_0 t)$$

- Ground state mass revealed through "effective mass plot"

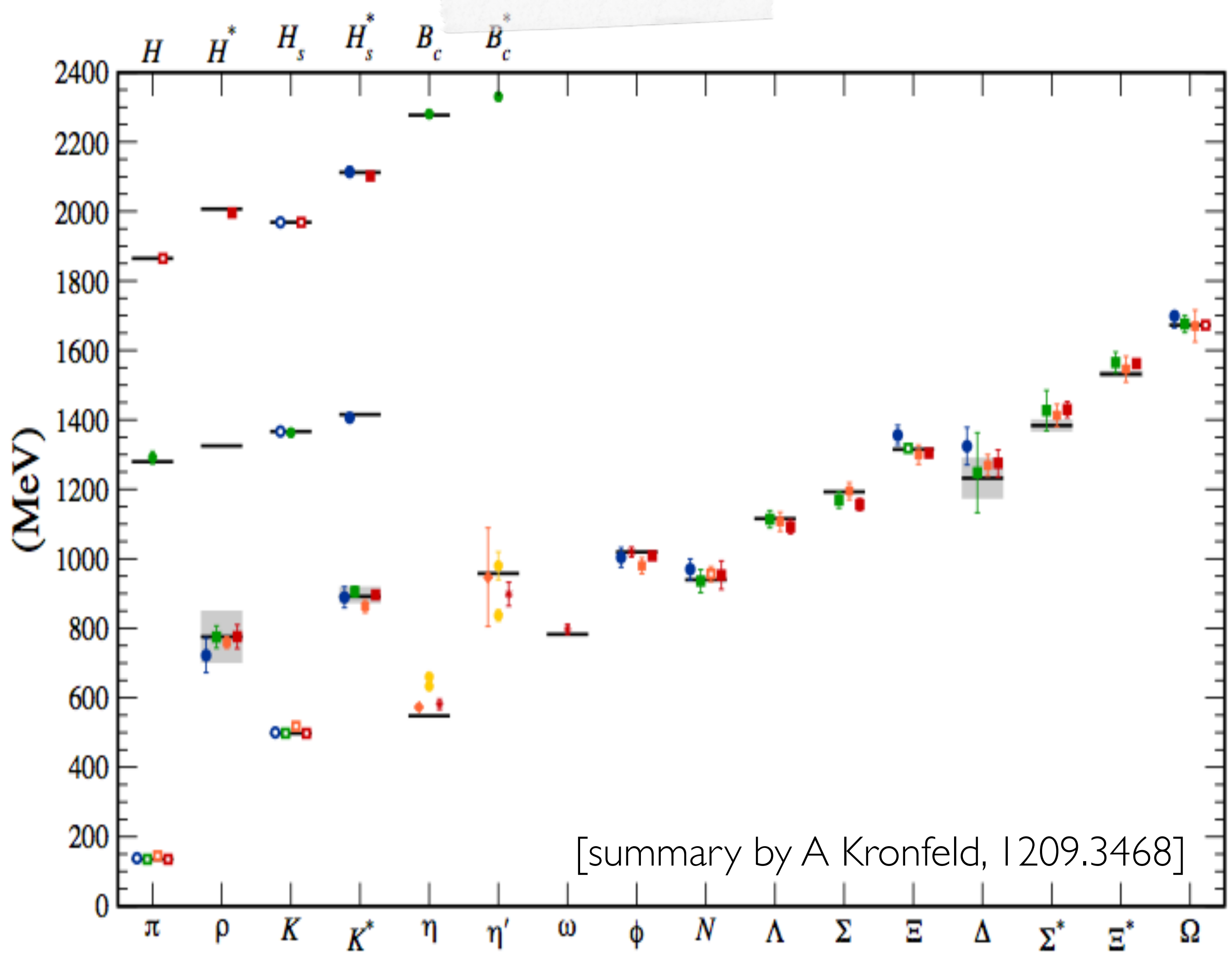
$$M(t) = \ln\left[\frac{C(t)}{C(t+1)}\right] \xrightarrow{t\to\infty} E_0$$





# QCD spectrum

- After 30 years of developments
- 😍 Ground state hadron spectrum reproduced



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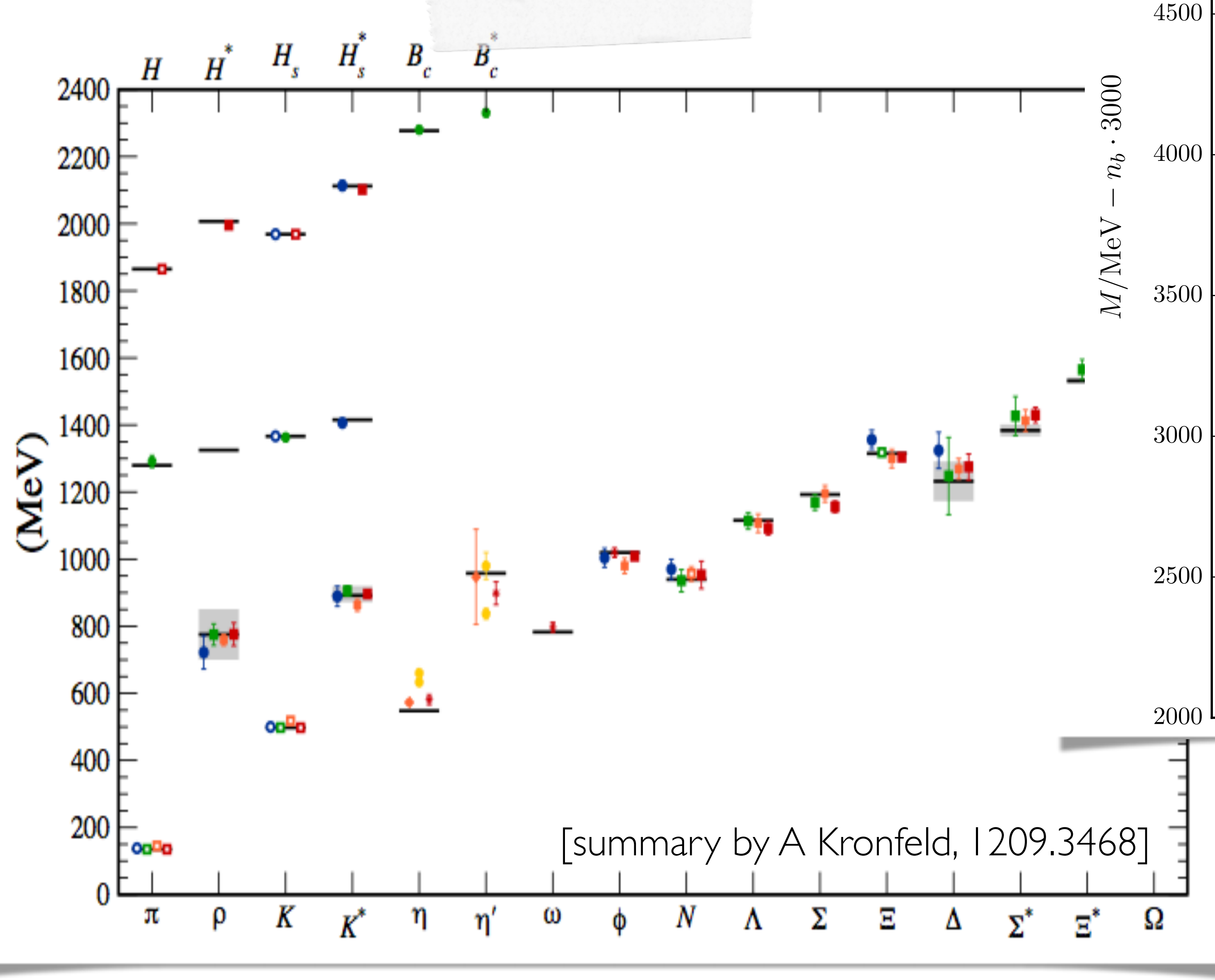
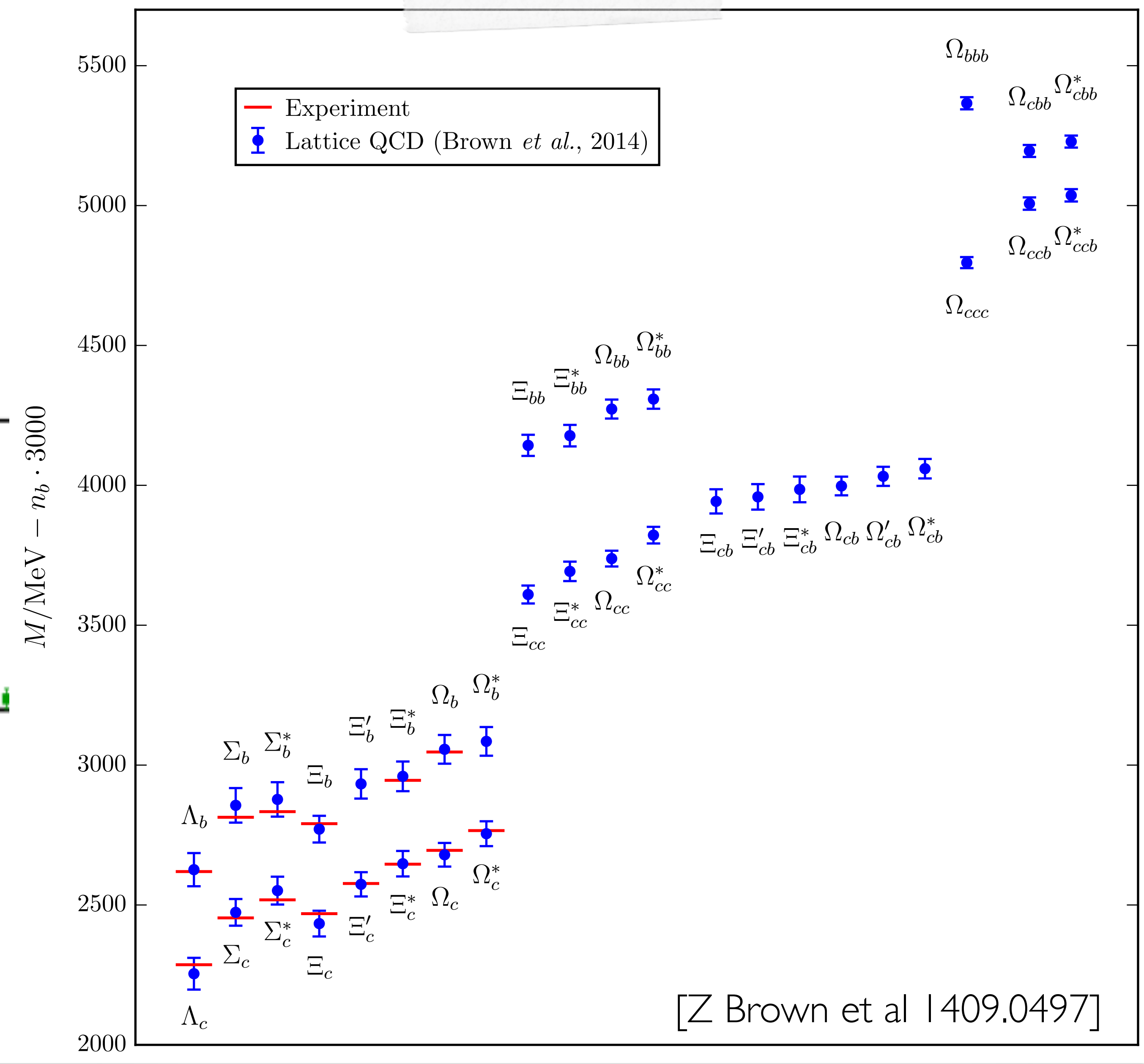
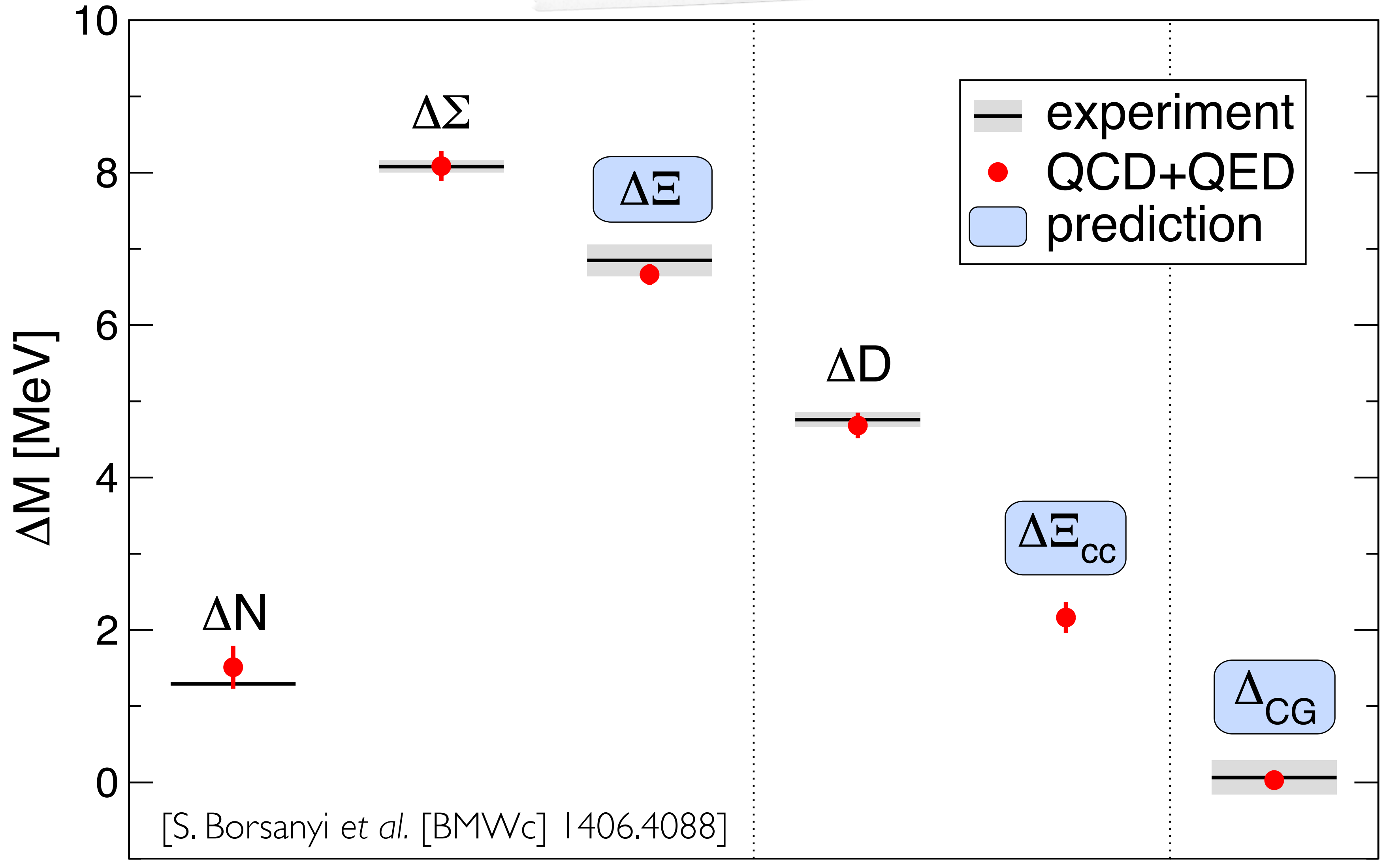


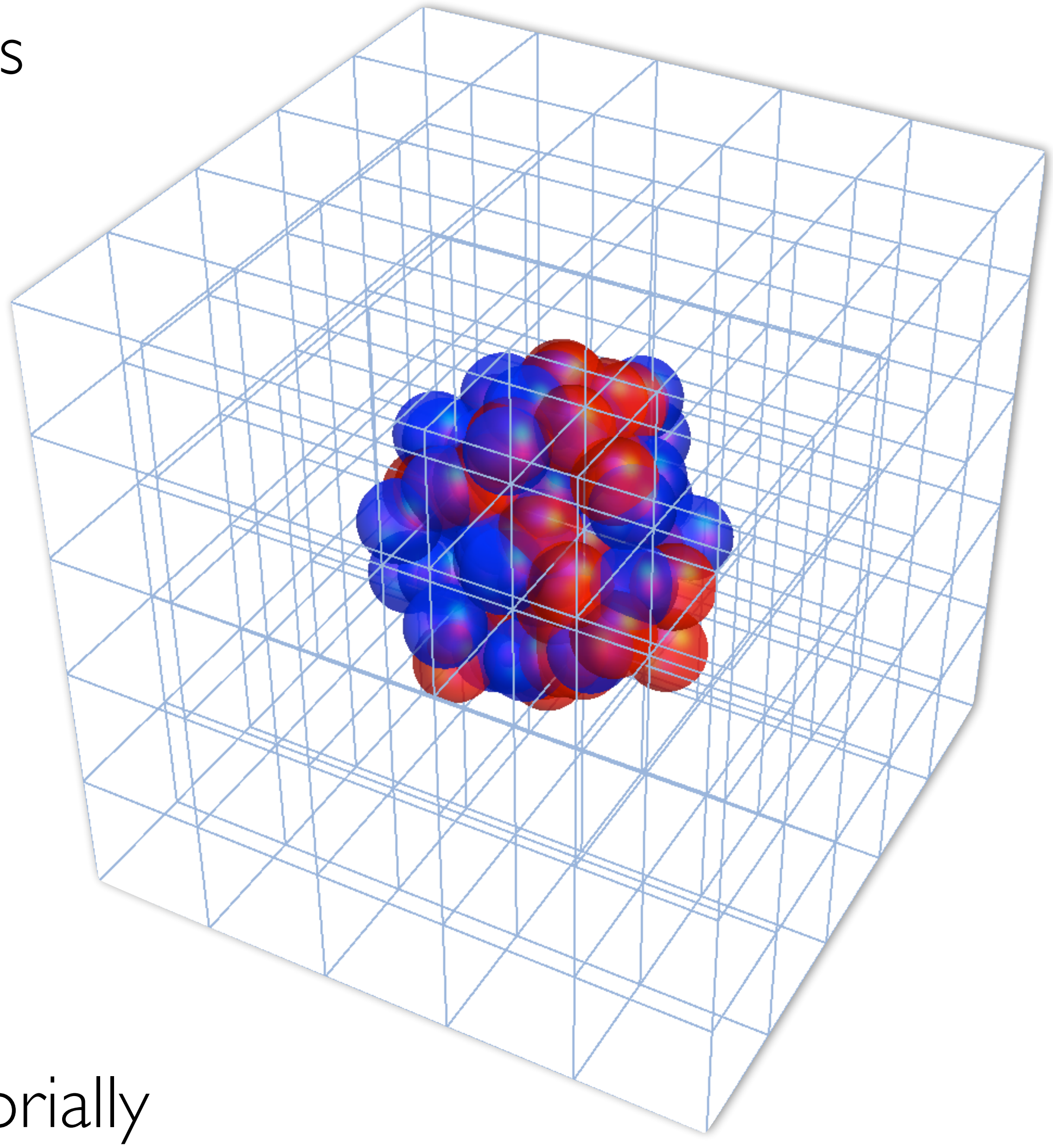
😍 Precise isospin mass splittings in QCD+QED



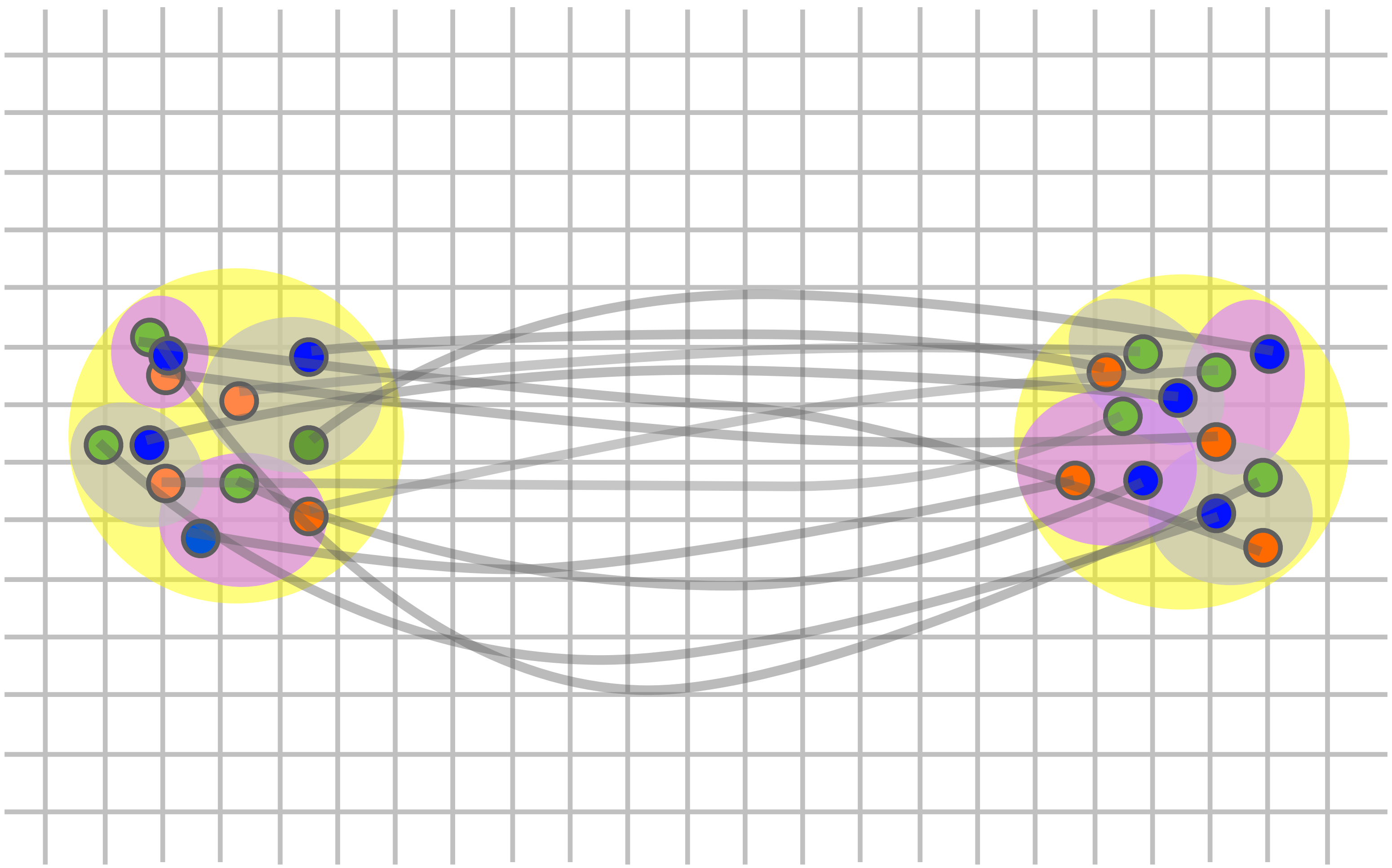
# Nuclear Spectra

# QCD for Nuclear Physics

- QCD (+EW) describes nuclear physics
  - Can compute the mass of lead nucleus ... in principle
- In practice: a hard problem
- At least two exponentially difficult challenges
  - Noise: probabilistic method so statistical uncertainty grows exponentially with A
  - Contraction complexity grows factorially



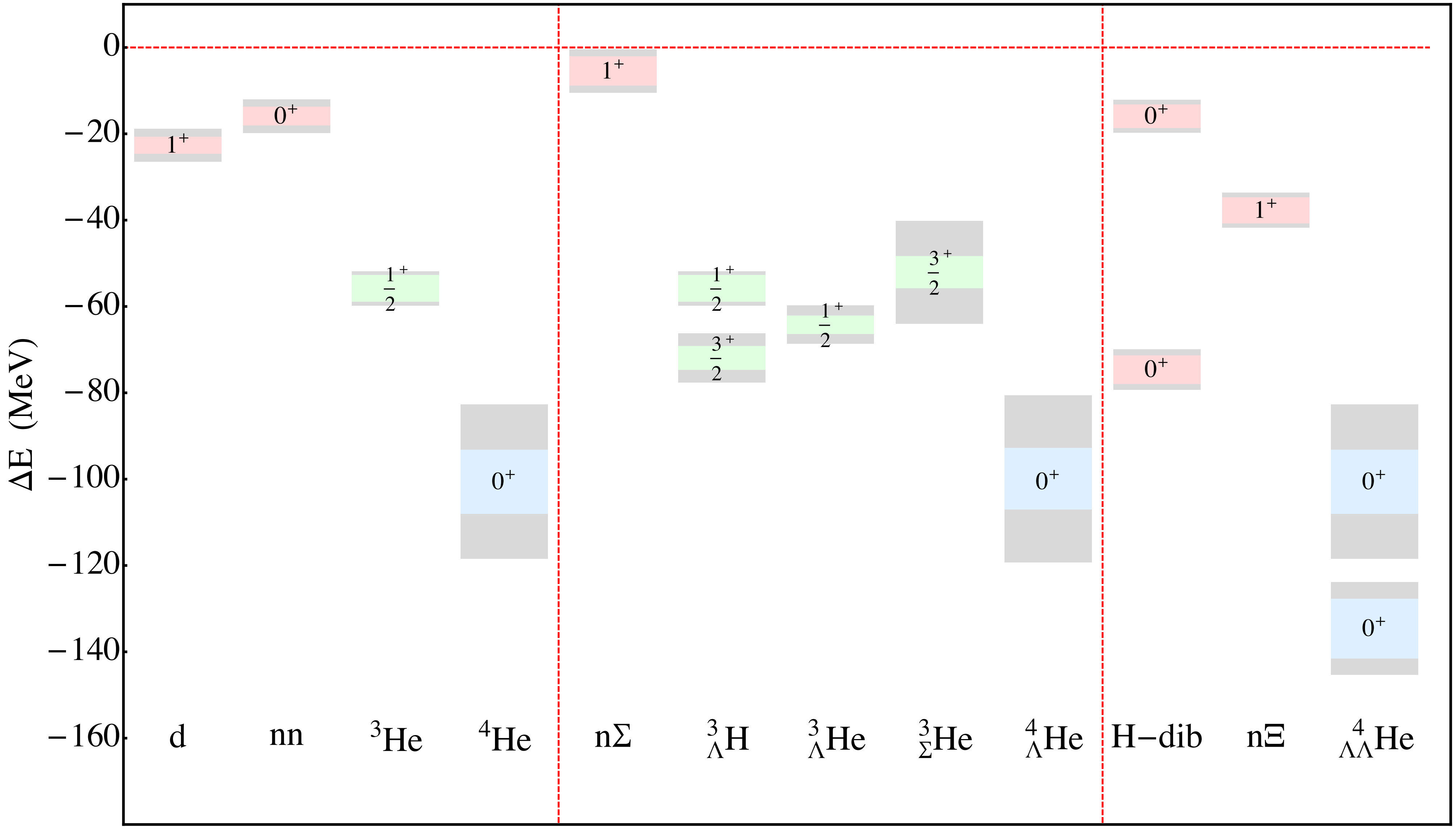
- Quarks need to be tied together in all possible ways
  - $N_{\text{contractions}} = N_u! N_d! N_s!$



- Managed using algorithmic trickery [WD & Savage, WD & Orginos; Doi & Endres]
  - Study up to N=72 pion systems, A=5 nuclei

# Light nuclei

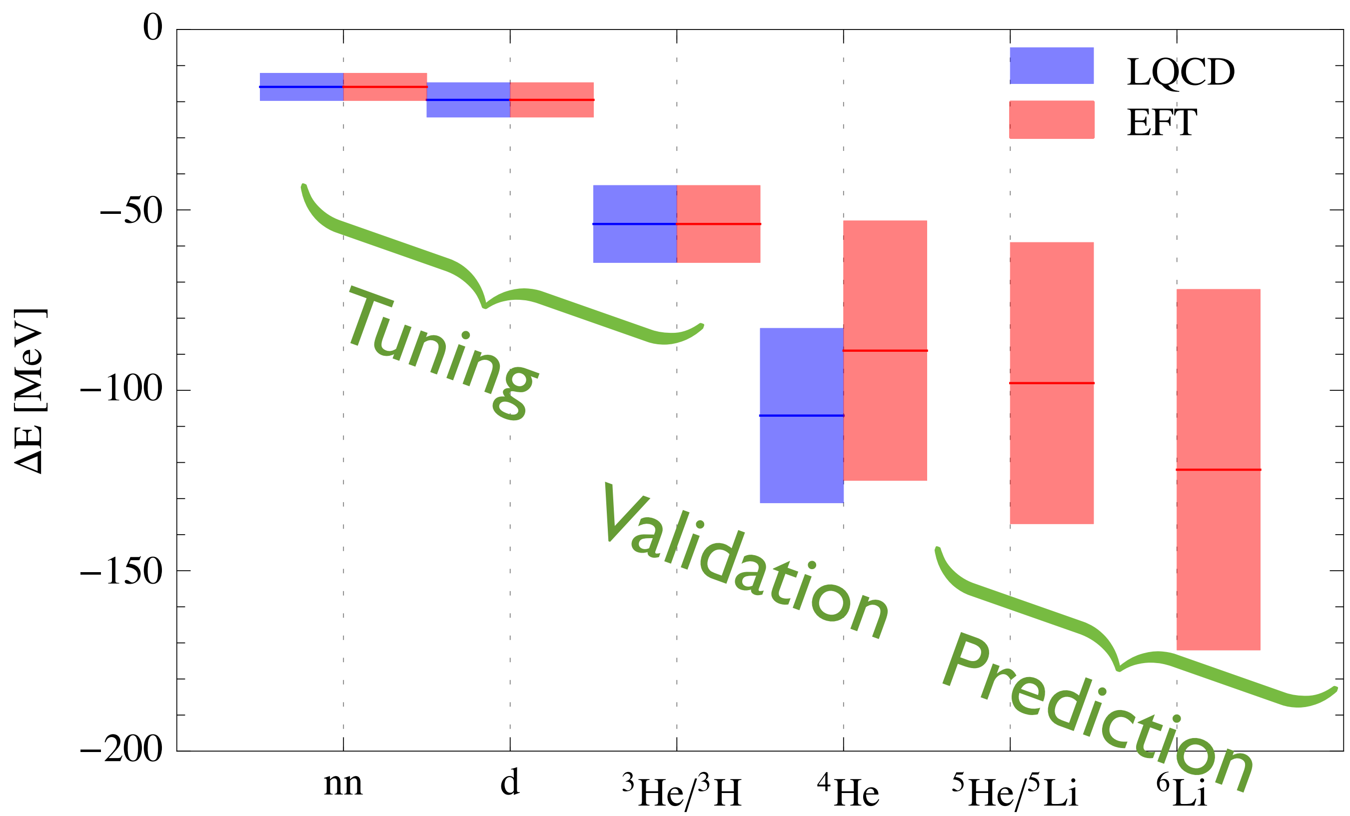
- Light hypernuclear spectrum @ 800 MeV




[NPLQCD Phys.Rev. D87 (2013), 034506 ]

[Barnea et al. 1311.4966]

- Combining LQCD and nuclear EFT (pionless EFT)
- For heavy quarks, even spectroscopy requires QCD matching:




**In a world @ $m_\pi$ = 800 MeV**

- Equally important for matrix elements

# Nuclear Structure

# External currents and nuclei

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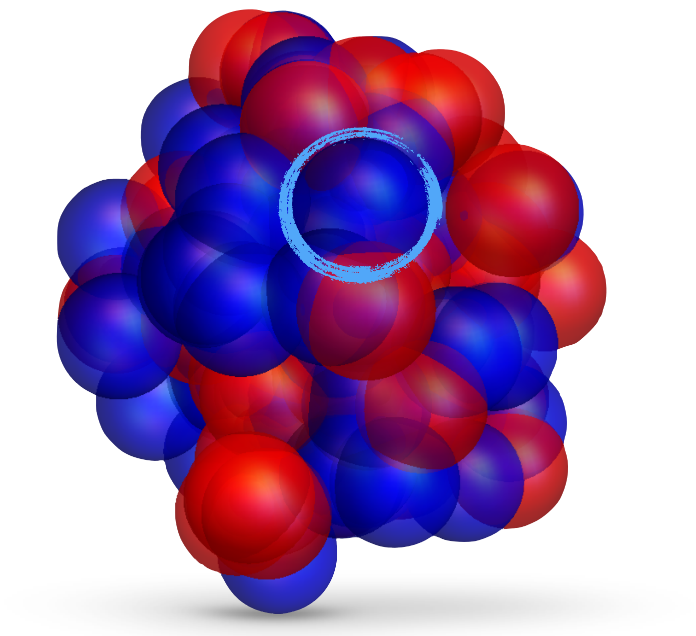
- Current-nucleus interaction
  - Born approximation – interacts with a single nucleon

$$\sigma \sim |A \langle N|J|N\rangle|^2$$



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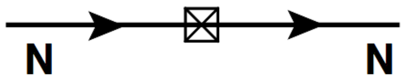


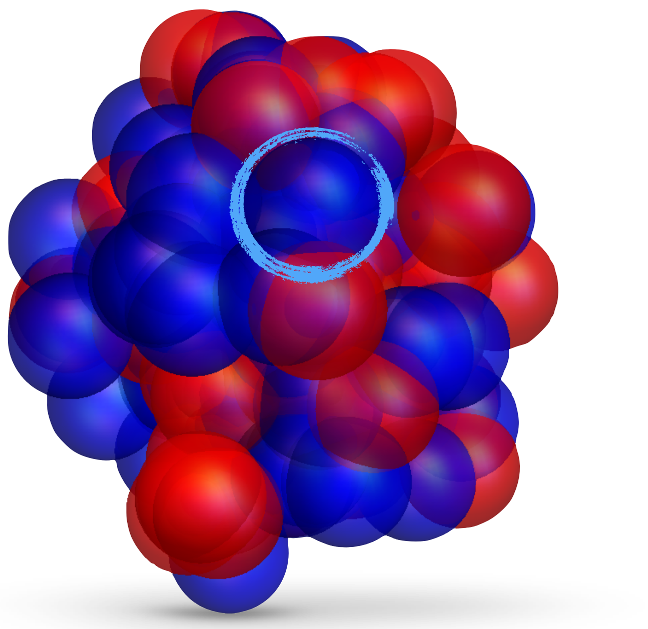

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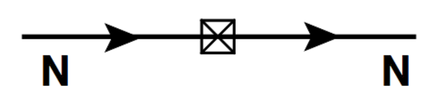
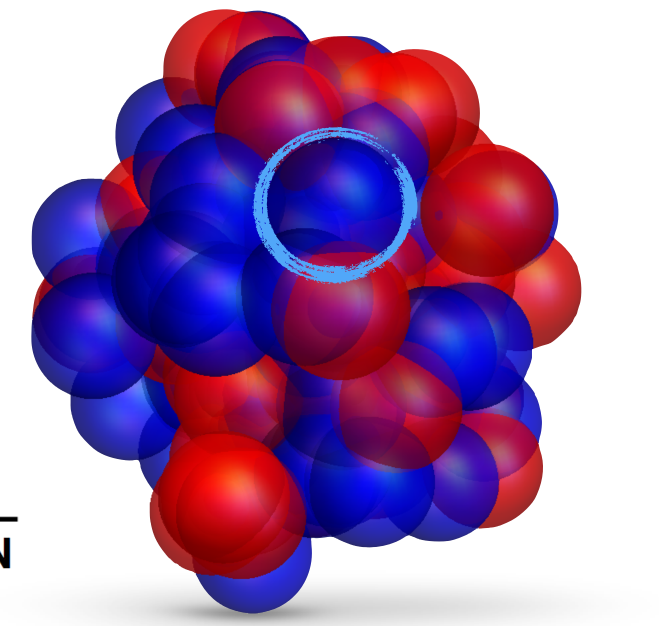
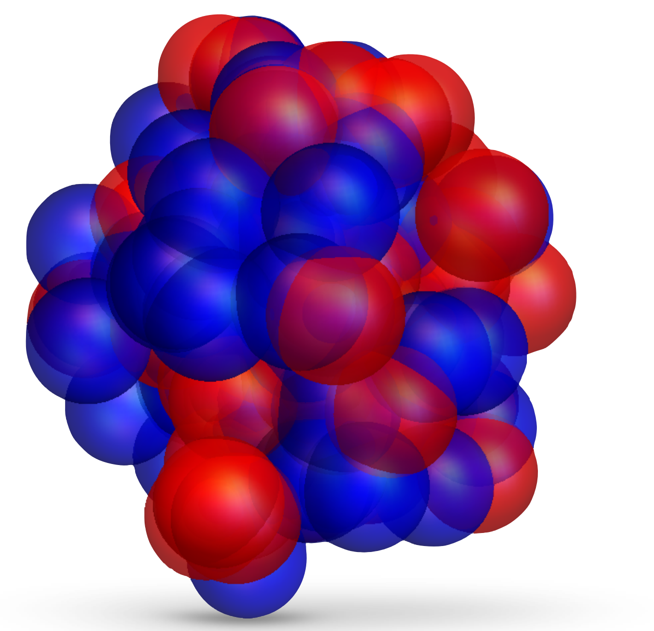


  - Interact non-trivially with multiple nucleons

$$\sigma \sim |A \langle N|J|N\rangle + \alpha \langle NN|J|NN\rangle + \ldots|^2$$



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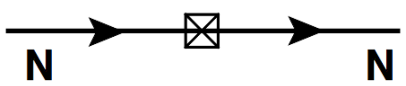


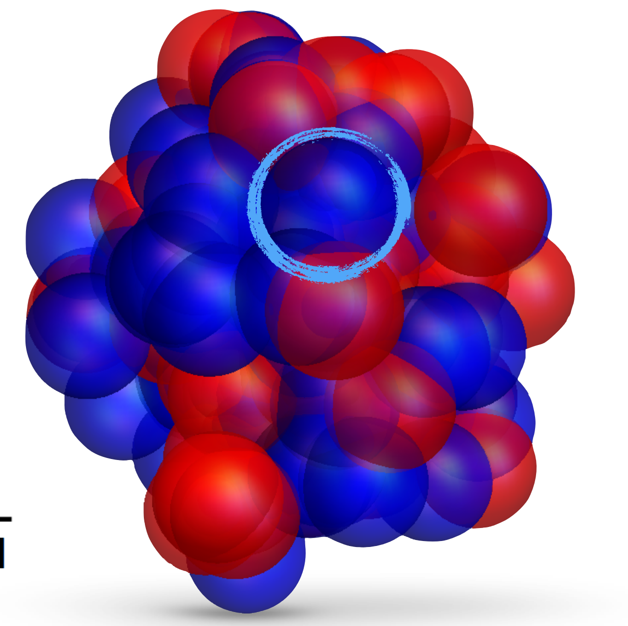

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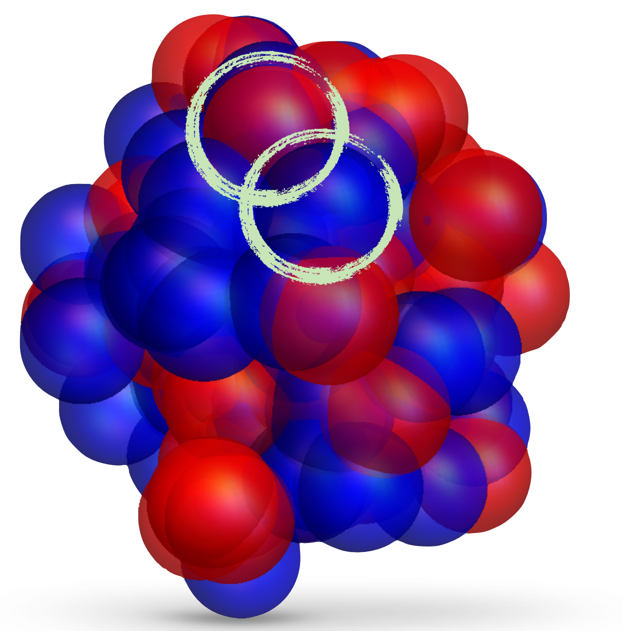

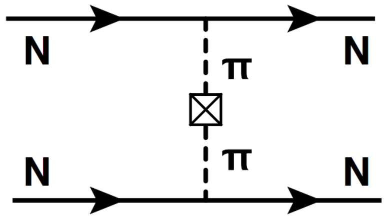


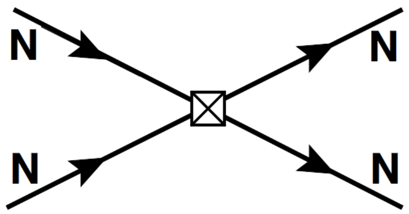

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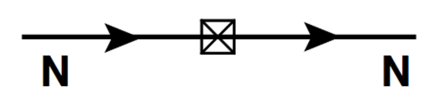


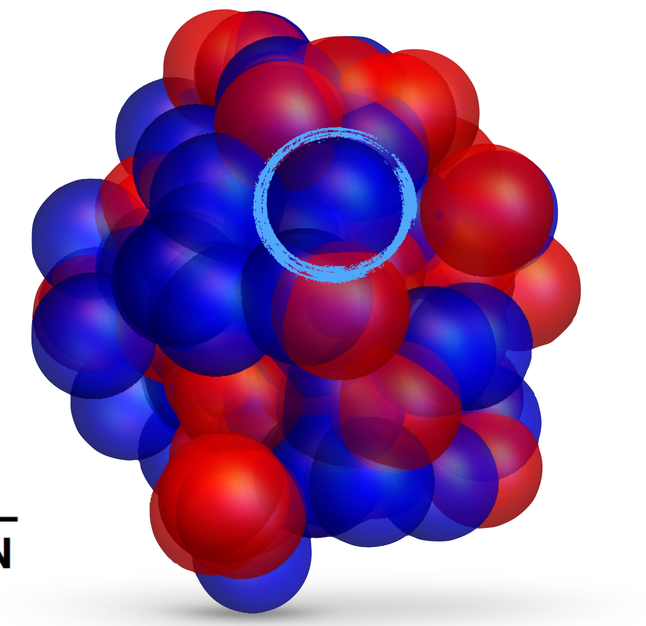

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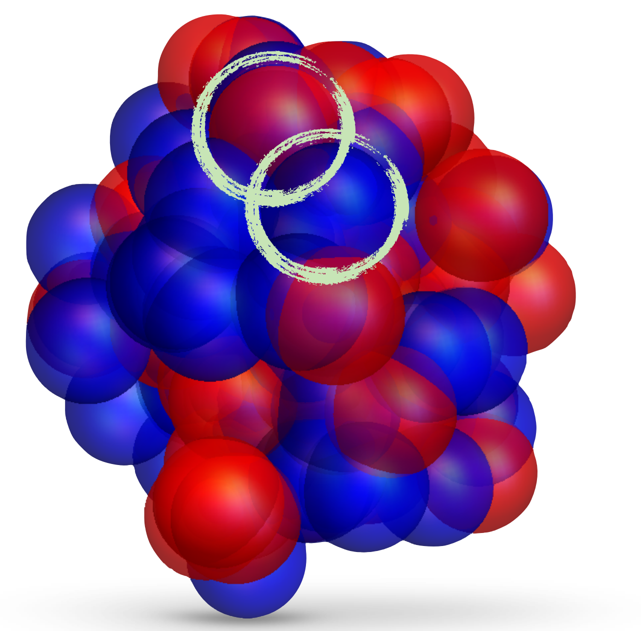
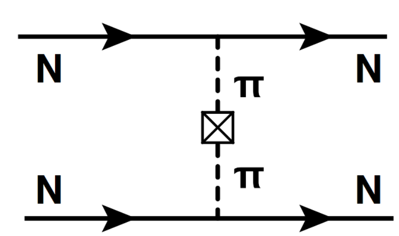
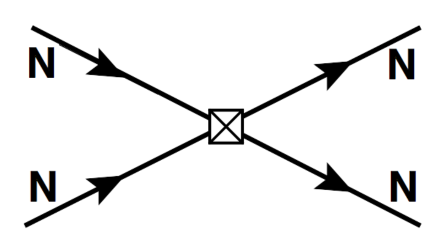


- Second term may be significant
  - May shift cross sections
  - May scale differently with Z and A
  - Leads to significant uncertainty

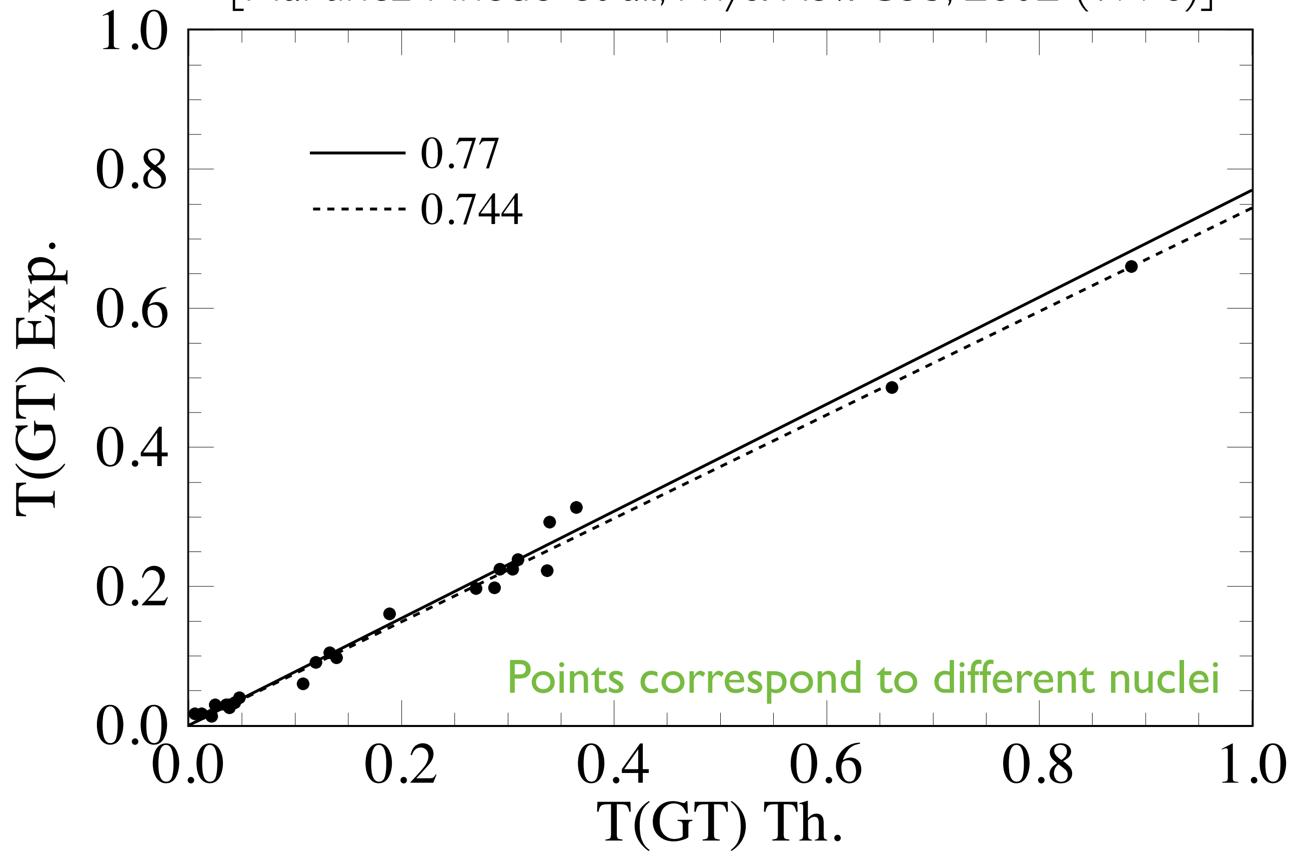


# Nuclear uncertainties

- Gamow-Teller transitions in nuclei are a stark example of problems
- Well measured
- Best nuclear structure calculations are systematically off by 20–30%
  - Large range of nuclei (30<A<60) where spectrum is well described
  - QRPA, shell-model,...
  - Correct for it by "quenching" axial charge in nuclei ...

[Martinez-Pinedo et al., Phys. Rev. C53, 2602 (1996)]

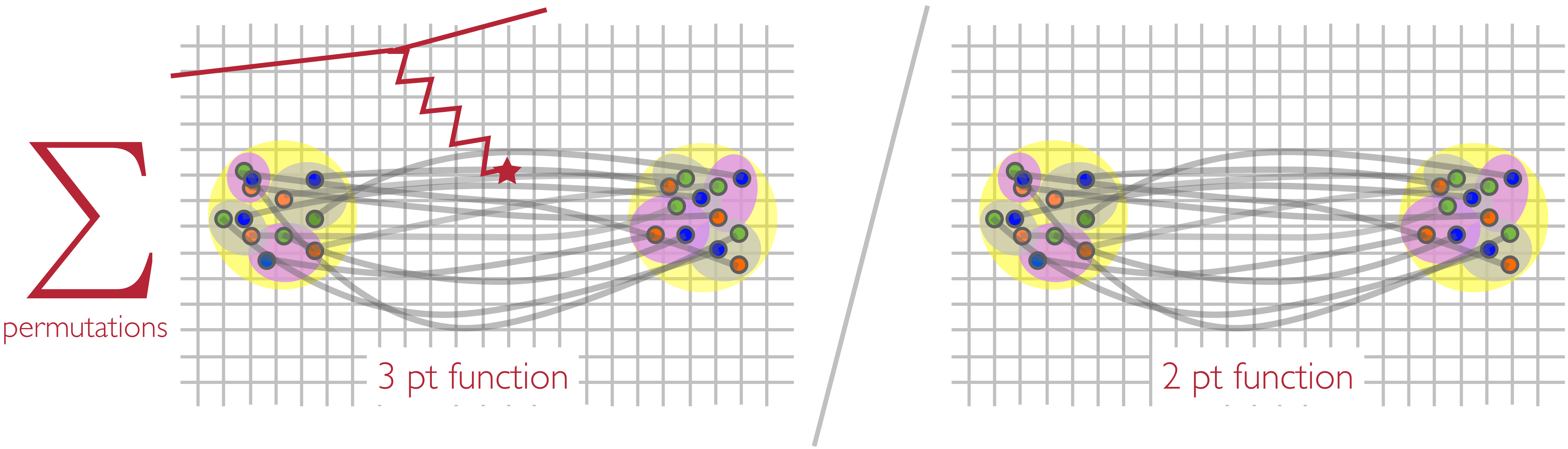



$$T(GT) \sim \sqrt{\sum_f \langle \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \rangle_{i \to f}}$$

$$\langle \boldsymbol{\sigma\tau} \rangle = \frac{\langle f || \sum_k \boldsymbol{\sigma}^k \boldsymbol{t}^k_{\pm} || i \rangle}{\sqrt{2J_i + 1}}$$

# Nuclear matrix elements

- For deeply bound nuclei, use the techniques as for single hadron matrix elements




  - At large time separations gives ground-state matrix element of current

- For near threshold states, need to be careful with volume effects

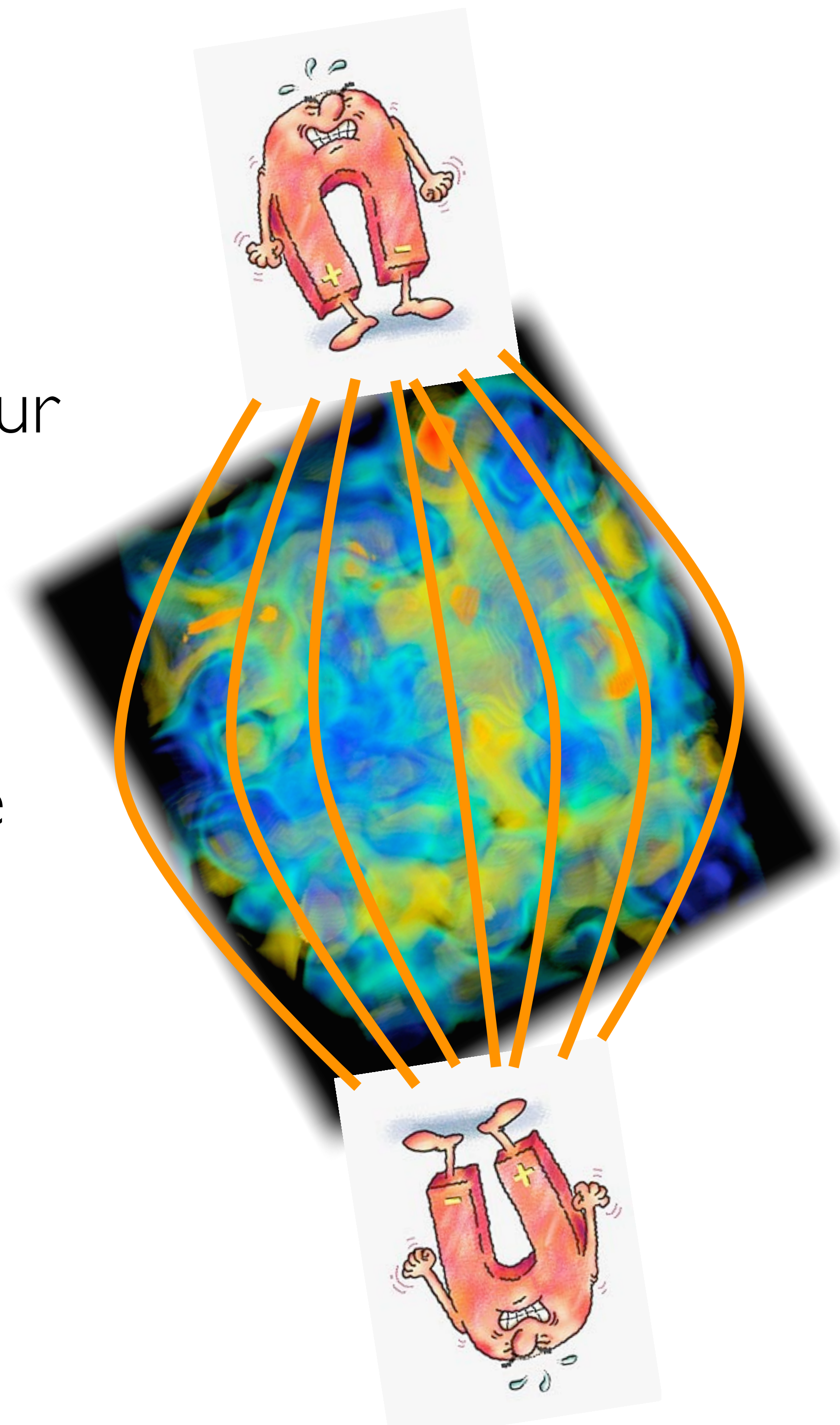
- Calculations of matrix elements of currents in light nuclei just beginning for A<5

# Background field method

- Hadron/nuclear two-point functions are modified in presence of fixed eternal fields
- Eg: fixed B field: modified exponential behaviour

$$E(\mathbf{B}) = M + \frac{|Q\,e\,\mathbf{B}|}{2M} - \boldsymbol{\mu}\cdot\mathbf{B} - 2\pi\beta_{M0}\,|\mathbf{B}|^2 - 2\pi\beta_{M2}T_{ij}B_iB_j + \ldots$$

- QCD spectroscopy with multiple fields enable extraction of coefficients of response
  - Eg: magnetic moments, polarisabilities, …
  - Not restricted to simple EM fields (axial, twist-2,…)



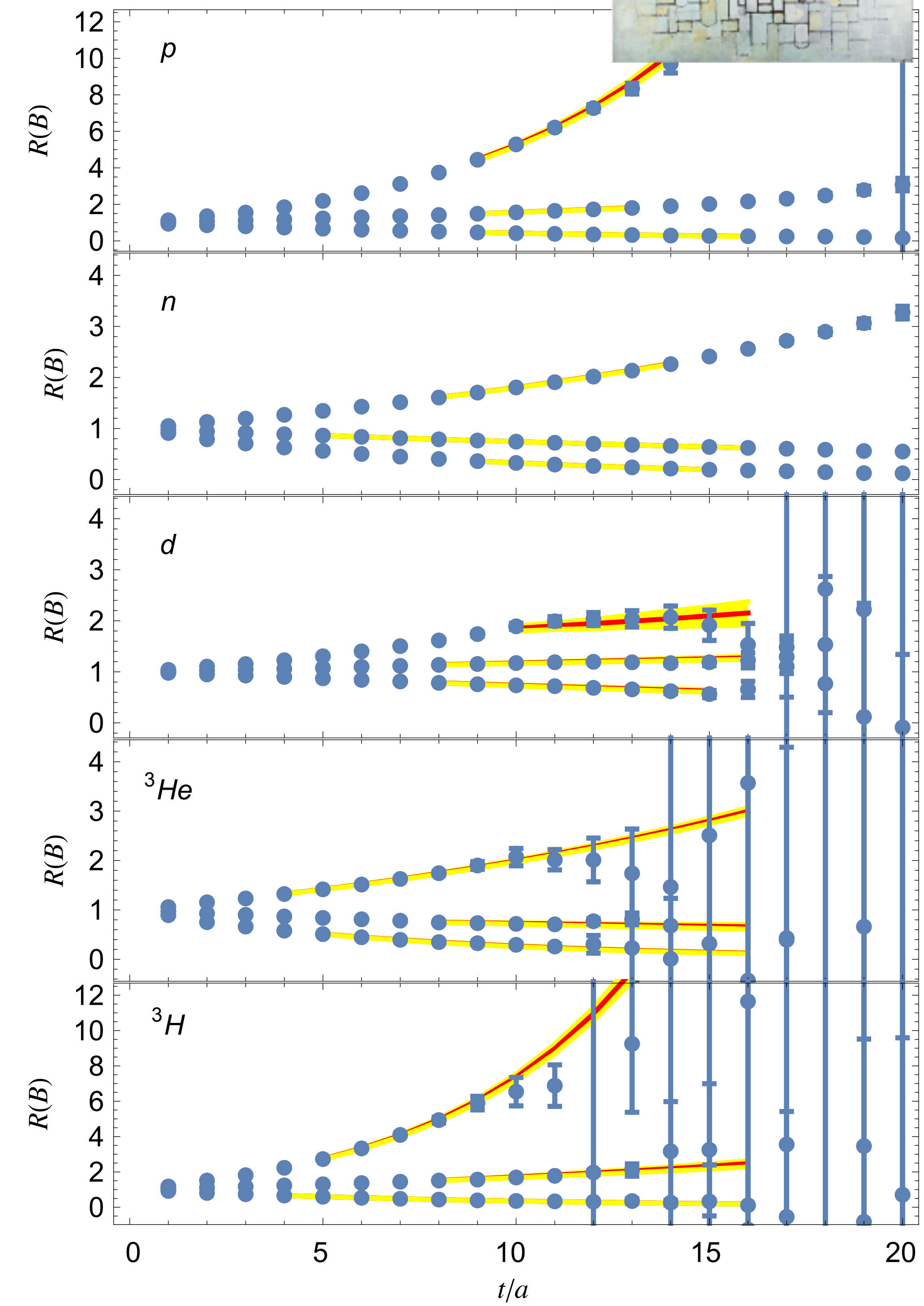
- Magnetic moments from spin splittings

$$\delta E^{(B)} \equiv E_{+j}^{(B)} - E_{-j}^{(B)} = -2\mu|\mathbf{B}| + \gamma|\mathbf{B}|^3 + \ldots$$

- Extract splittings from ratios of correlation functions

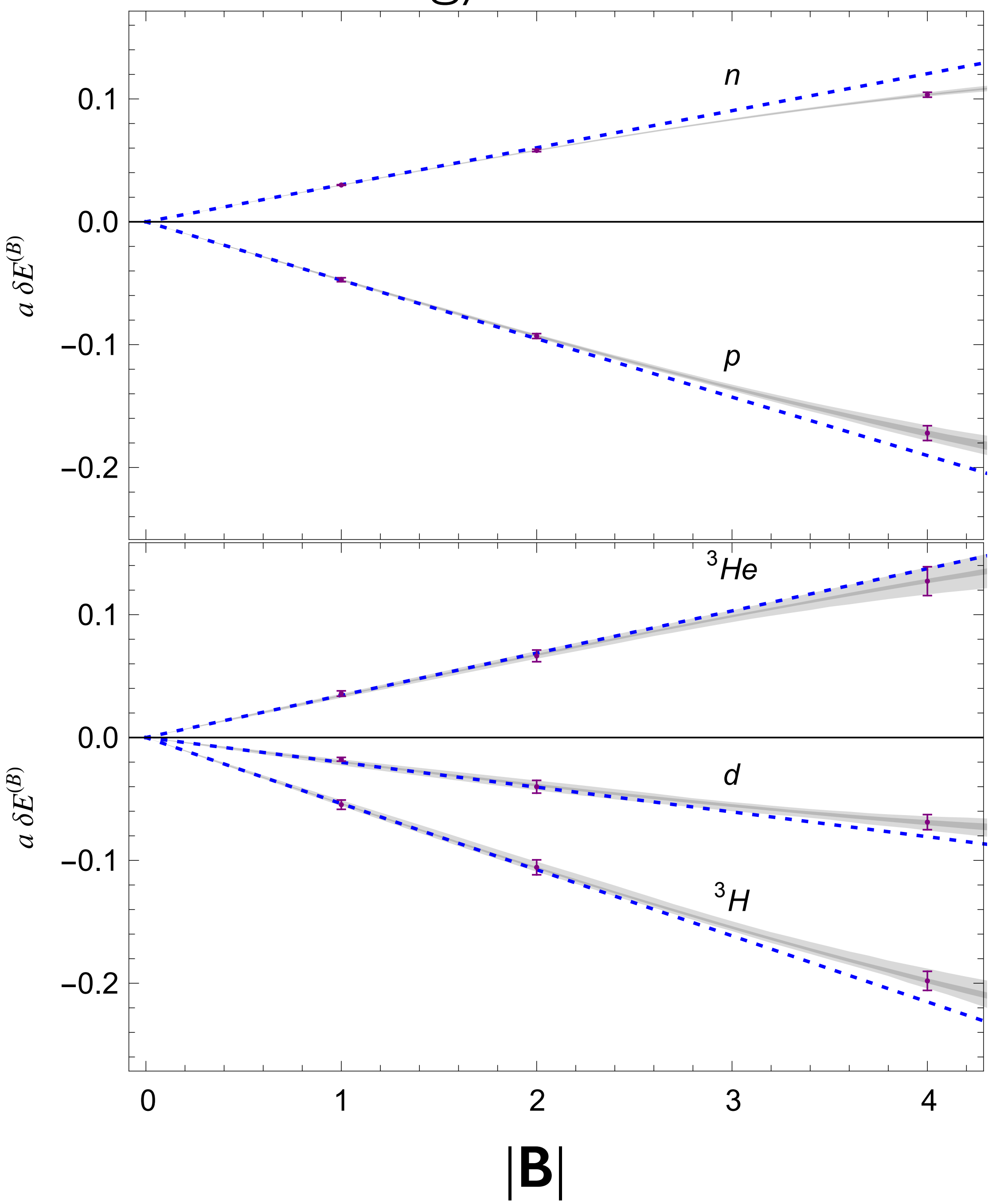
$$R(B) = \frac{C_j^{(B)}(t)\; C_{-j}^{(0)}(t)}{C_{-j}^{(B)}(t)\; C_j^{(0)}(t)} \;\overset{t\to\infty}{\longrightarrow}\; Z e^{-\delta E^{(B)} t}$$

- Careful to be in single exponential region of each correlator




[NPLQCD 1409.3556, PRL to appear]

Energy shift vs B

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# Nuclear magnetic moments

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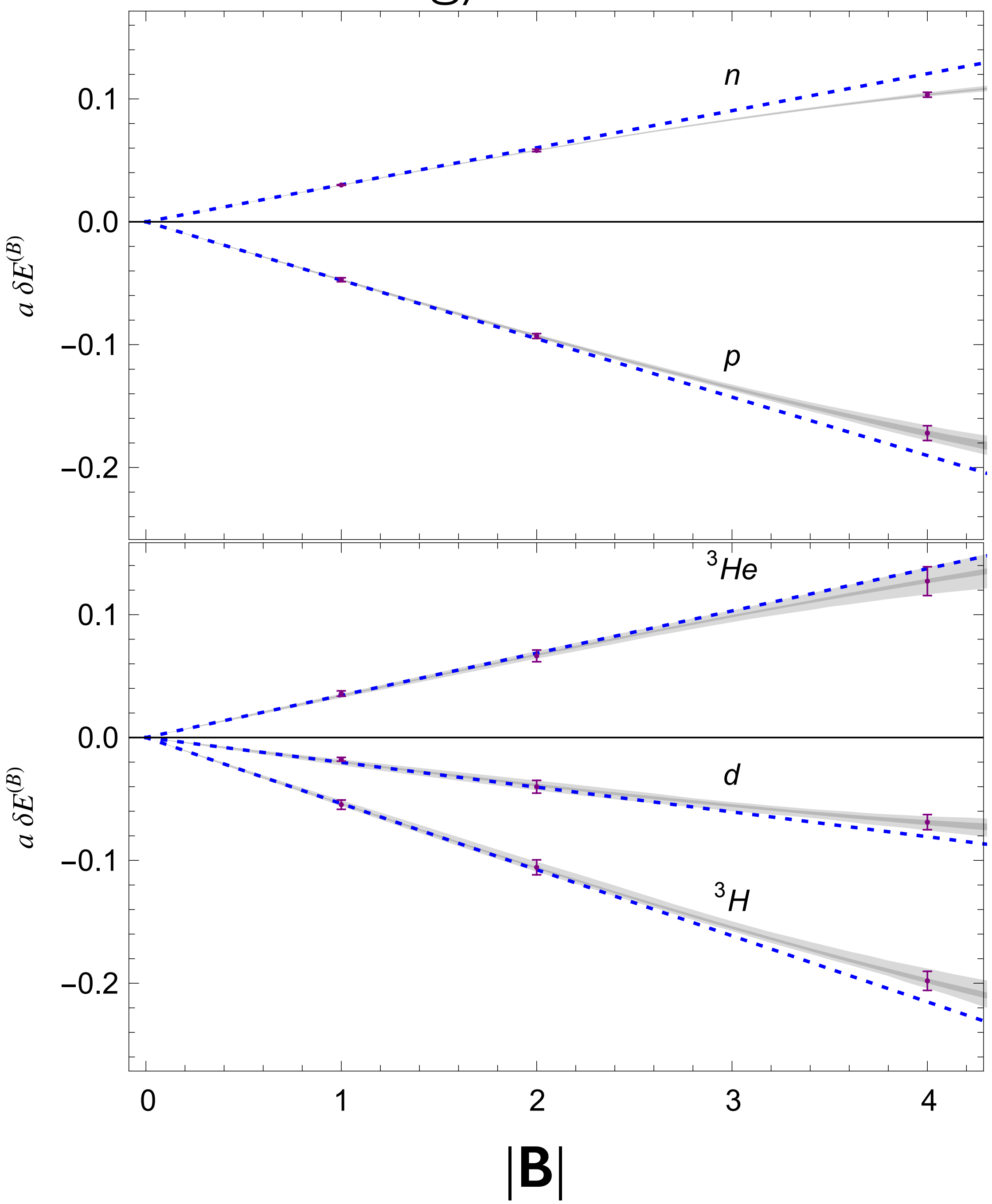
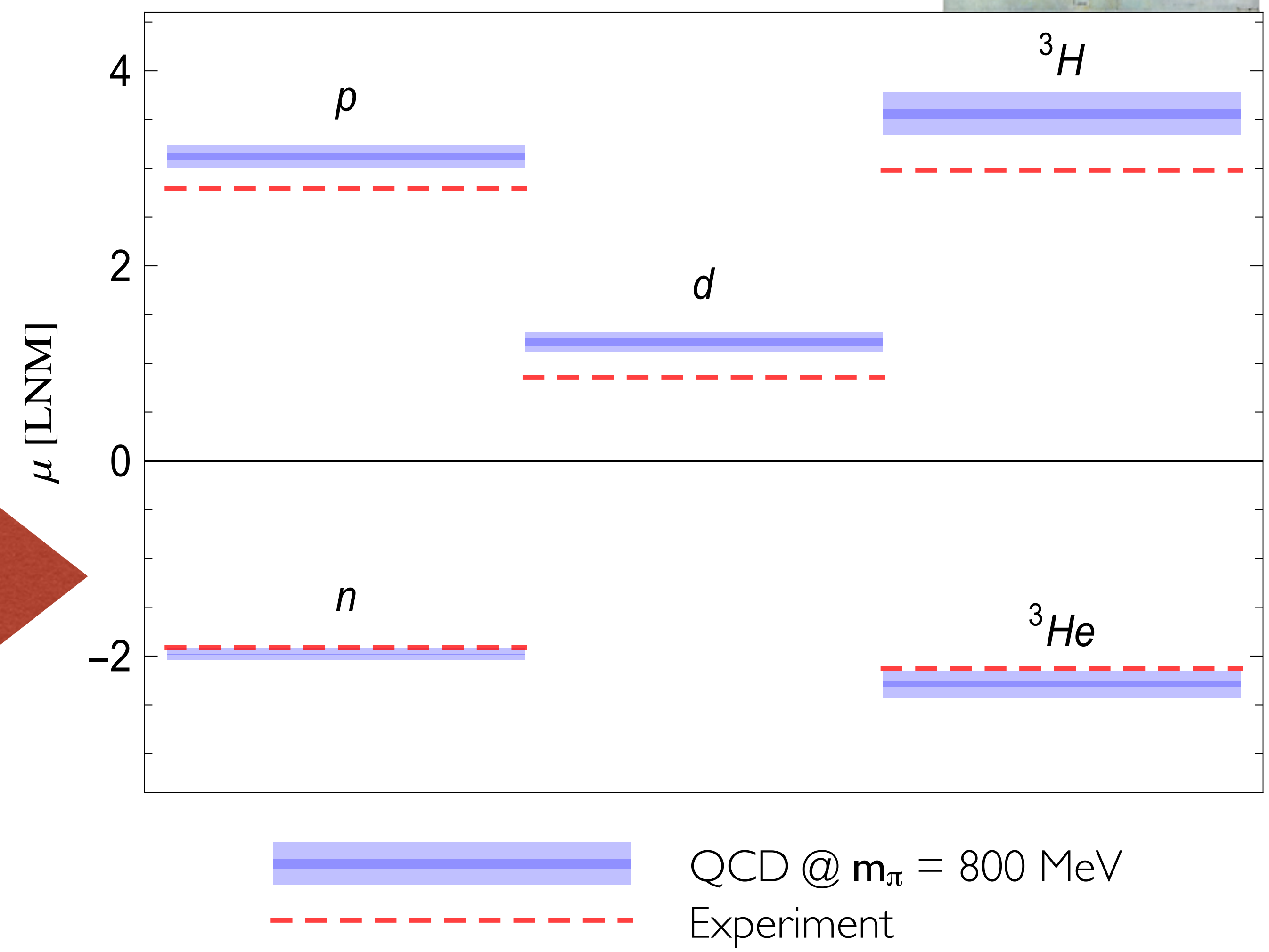


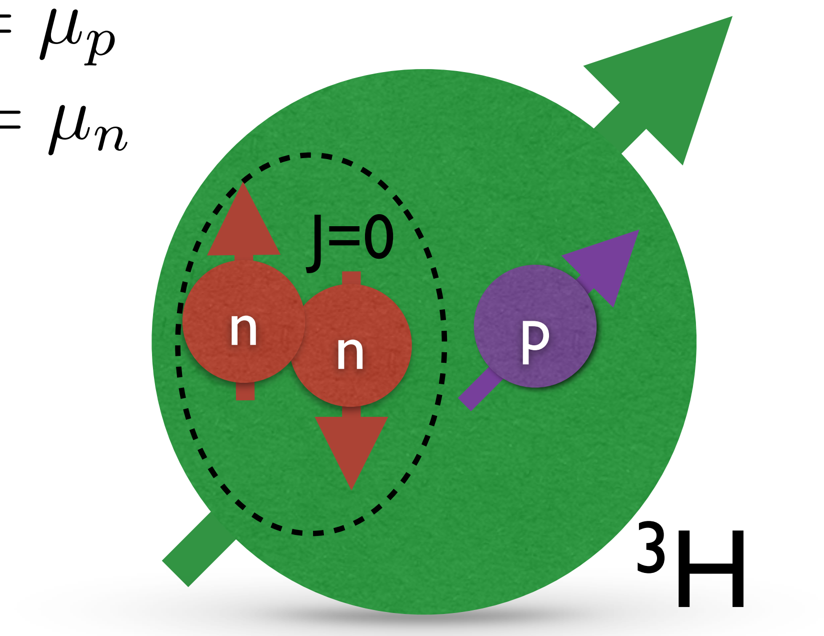

| | n | p | d | 3 | 3 |
|---|---|---|---|---|---|
| μ | -1.98(1)(2) | 3.21(3)(6) | 1.22(4)(9) | -2.29(3)(12) | 3.56(5)(18) |

In units of appropriate nuclear magnetons (heavy $M_N$)

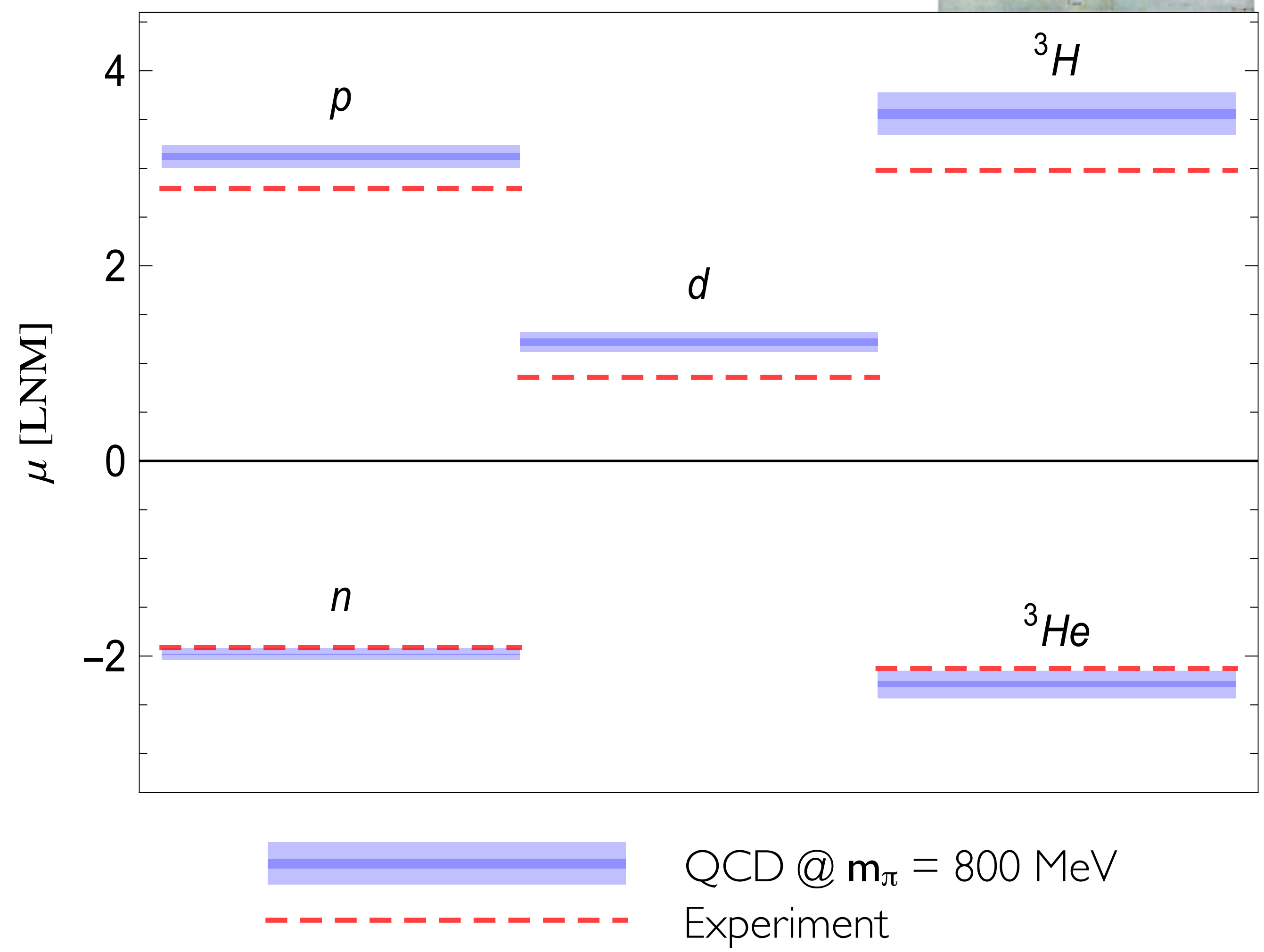
[NPLQCD 1409.3556, PRL to appear]

- Numerical values are surprisingly interesting
- Shell model expectations

$$\mu_d = \mu_p + \mu_n$$
$$\mu_{^3\mathrm{H}} = \mu_p$$
$$\mu_{^3\mathrm{He}} = \mu_n$$




- Lattice results appear to suggest heavy quark nuclei are shell-model like!

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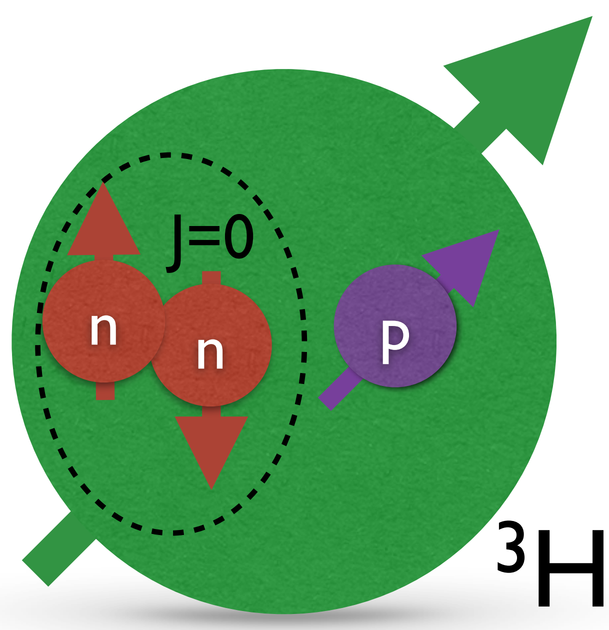


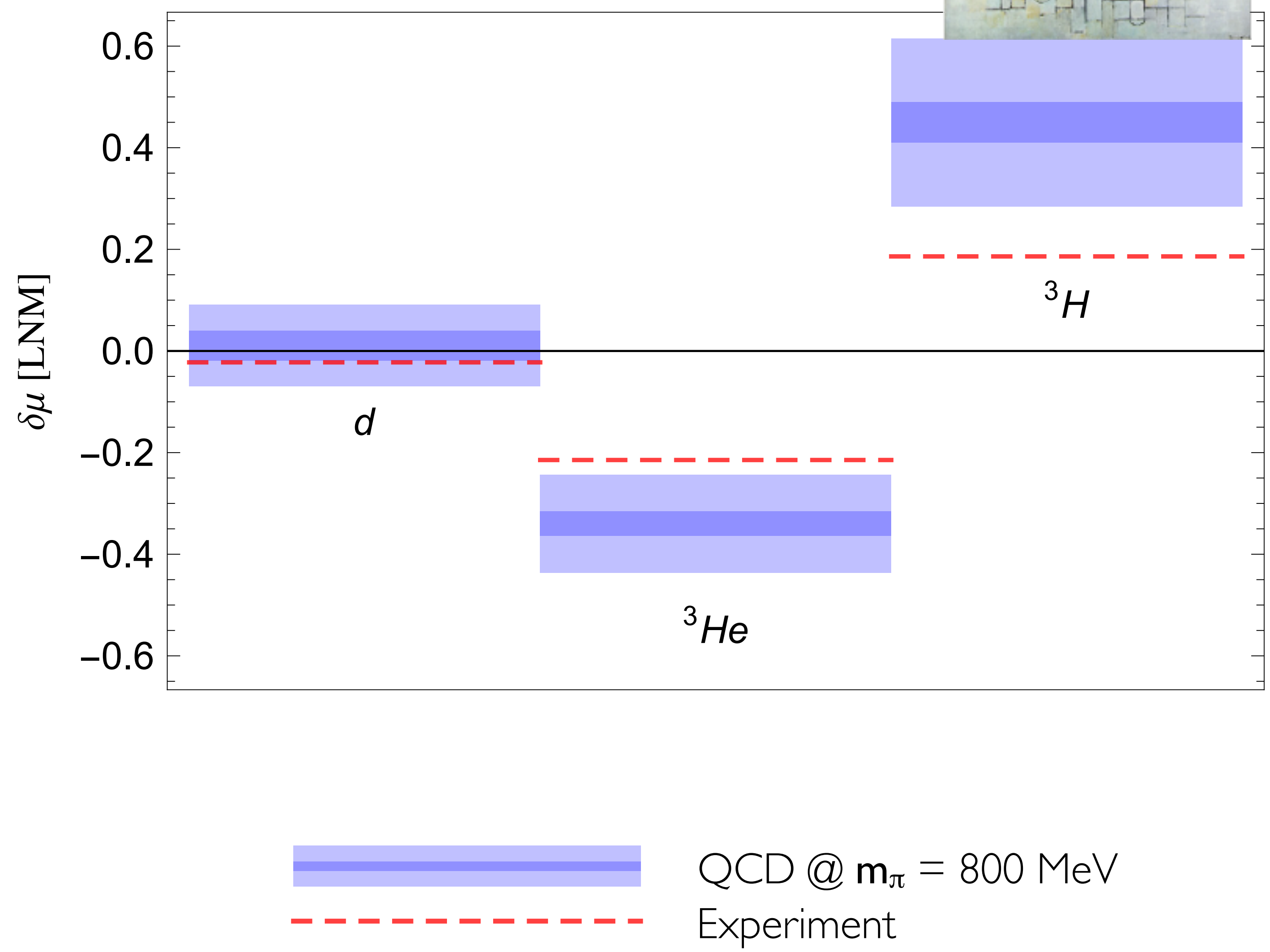


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| | d | 3 | 3 |
|---|---|---|---|
| δμ | 0.01(3)(7) | -0.34(2)(9) | 0.45(4)(16) |

Difference from NSM expectation

[NPLQCD 1409.3556, PRL to appear]

# Nuclear sigma terms

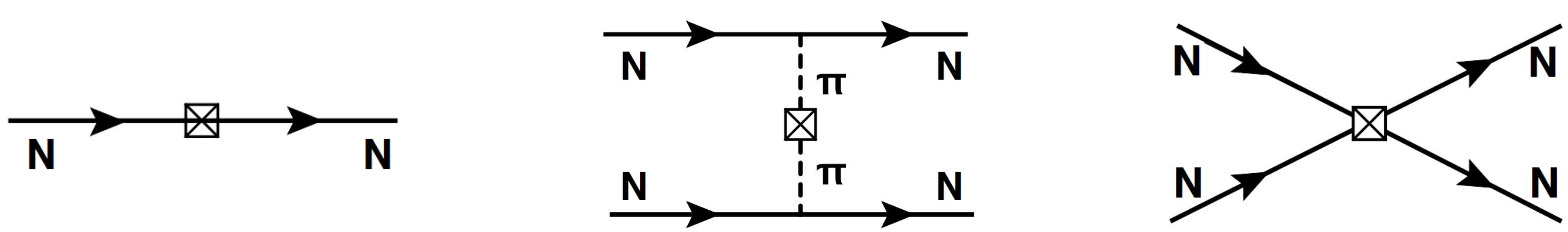
- One possible DM interaction is through scalar exchange

$$\mathcal{L} = \frac{G_F}{2} \sum_q a_S^{(q)} (\overline{\chi}\,\chi)(\overline{q}\,q)$$

- Accessible via Feynman-Hellman theorem
- At hadronic/nuclear level

$$\mathcal{L} \to G_F\,\overline{\chi}\chi \left( \frac{1}{4}\langle 0|\overline{q}q|0\rangle\ \mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] + \frac{1}{4}\langle N|\overline{q}q|N\rangle N^\dagger N \mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] \right.$$
$$\left. - \frac{1}{4}\langle N|\overline{q}\tau^3 q|N\rangle \left(N^\dagger N \mathrm{Tr}\left[a_S\Sigma^\dagger + a_S^\dagger\Sigma\right] - 4N^\dagger a_{S,\xi} N\right) + \ldots \right)$$

  - Contributions:

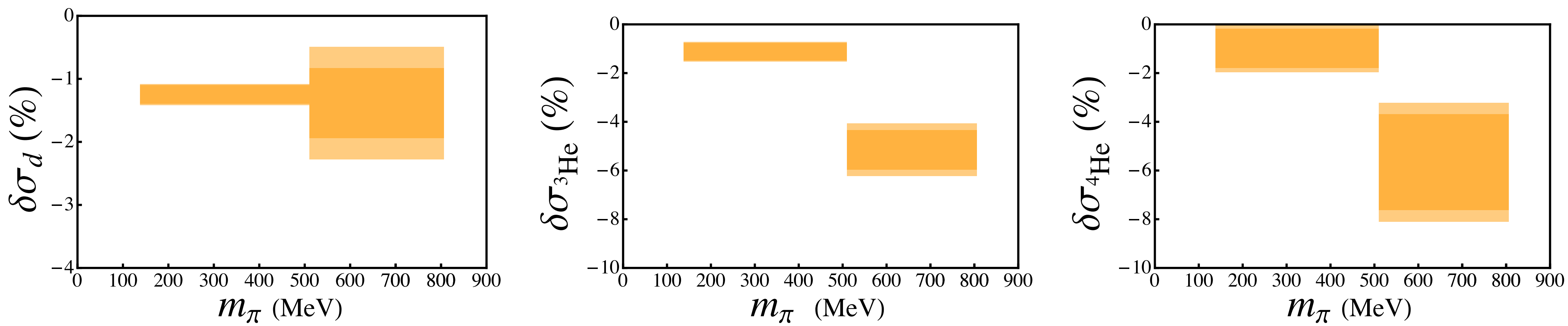


# Nuclear sigma terms

- Previous work suggested scalar dark matter couplings to nuclei have O(50%) uncertainty arising from MECs [Prezeau et al 2003]
- Quark mass dependence of nuclear binding energies bounds such contributions

$$\delta\sigma_{Z,N} = \frac{\langle Z,N(\mathrm{gs})|\ \overline{u}u+\overline{d}d|Z,N(\mathrm{gs})\rangle}{A\ \langle N|\ \overline{u}u+\overline{d}d|N\rangle} - 1 = -\frac{1}{A\sigma_N}\frac{m_\pi}{2}\frac{d}{dm_\pi}B_{Z,N}$$

  - Lattice calculations + physical point suggest such contributions are O(10%) or less for light nuclei (A<4)




[NPLQCD PRD to appear]

# QCD for nuclear physics

- Nuclei are under serious study directly from QCD
  - Spectroscopy of light nuclei and exotic nuclei (strange, charmed, …)
  - Nuclear properties/matrix elements
- Prospect of a quantitative connection to QCD makes this a very exciting time for nuclear physics
  - Critical role in current and upcoming intensity frontier experimental program
- Learn many interesting things about nuclear physics along the way

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