Neutron Matter from non–local Chiral forces and Quantum Monte Carlo

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The r–process: status and challanges

Outline & Acknowledgements

- Quantum Monte Carlo methods
- Low–density Pure Neutron matter with χ-EFT
  - Equation of State
  - Nucleon chemical potential (self–energies)
- Impurities in neutron–matter and constraints for DFT functionals

Collaborators:

- Francesco Pederiva - UNITN
- Lorenzo Contessi - UNITN
- Abhishek Mukherjee - ECT*

Quantum Monte Carlo methods

Central objects are random walks :

$$X^{k+1} = T[X^k] \quad \xrightarrow{\text{choose basis}} \quad X_j^{k+1} = \sum_i T_{ji} X_i^k$$

Under general conditions the asymptotic distribution depends on T in a predictable way

- if you want to sample from some known $F(X) \longrightarrow$ recipe for T

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To sample ground–state $|\Phi_0\rangle$ of Schroedinger equation you can use the propagator in imaginary–time $(it \to \tau)$

$$e^{-\tau\hat{H}}|\Psi_k\rangle = |\Psi_{k+1}\rangle \qquad \lim_{n\to\infty} e^{-n\tau\hat{H}}|\Psi_{in}\rangle \propto |\Phi_0\rangle$$

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Under genera … n T in a predictable wa…

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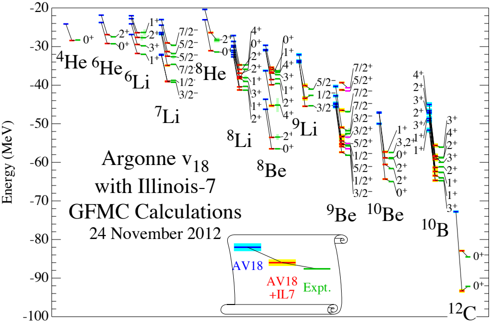
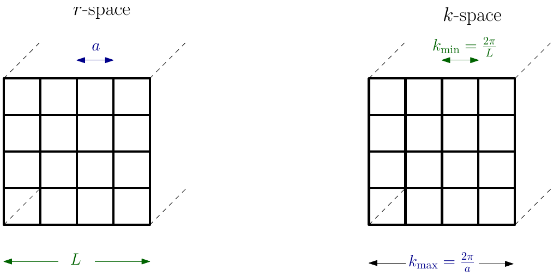


Figure: S. Pieper, R. Wiringa et. al (ANL)

Monte Carlo in Slater–Determinant space

Our basis are A–body slater determinants constructed from a single particle space $\mathcal{S} \longrightarrow$ we can use χ-EFT in this basis!!



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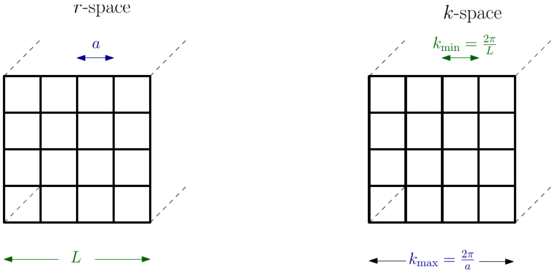

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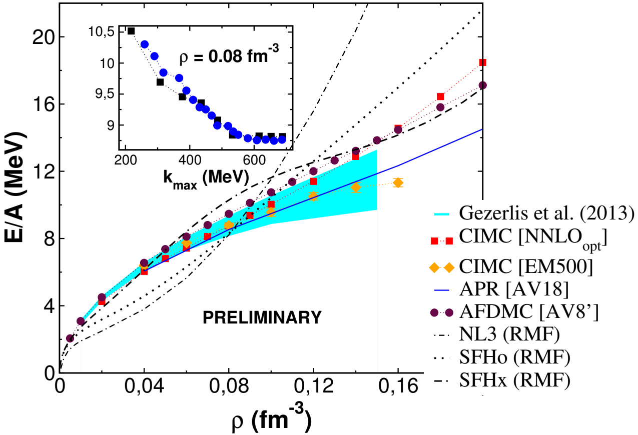
Coulomb gas $\longrightarrow$ good agreement with R–space QMC calculations

[A. R., A. Mukherjee and F. Pederiva, Phys. Rev. B 88,115138 (2013)]

Neutron Matter with χ-EFT interactions

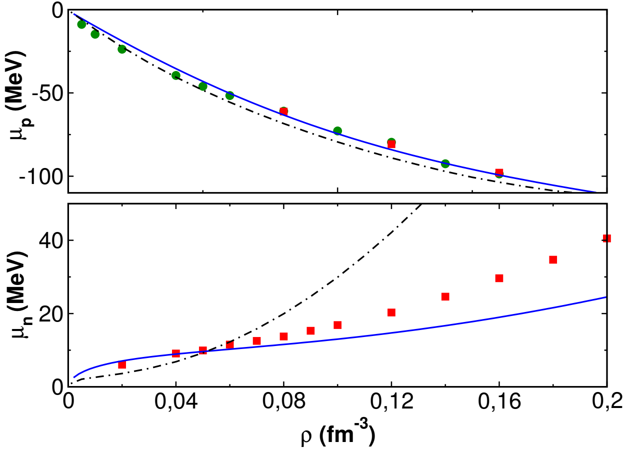
Equation of State (NN–only)



[A. R., A. Mukherjee and F. Pederiva, PRL 112, 221103 (2014)]

Neutron Matter with χ-EFT interactions

Nucleon chemical potential ($\sim$ self–energy at zero momentum)



[A. R., A. Mukherjee and F. Pederiva, PRL 112, 221103 (2014)]

Constrining Nuclear Energy Density Functionals

Energy density functional for uniform matter:

$$\mathcal{E} = \mathcal{E}_{\text{kin}} + \sum_{t=0,1} \left(C_t^{\rho} \rho_t^2 + C_t^{\tau} \rho_t \tau_t + C_t^{s} s_t^2 + C_t^{T} s_t T_t \right).$$

- contributions from both time–even and time–odd components.
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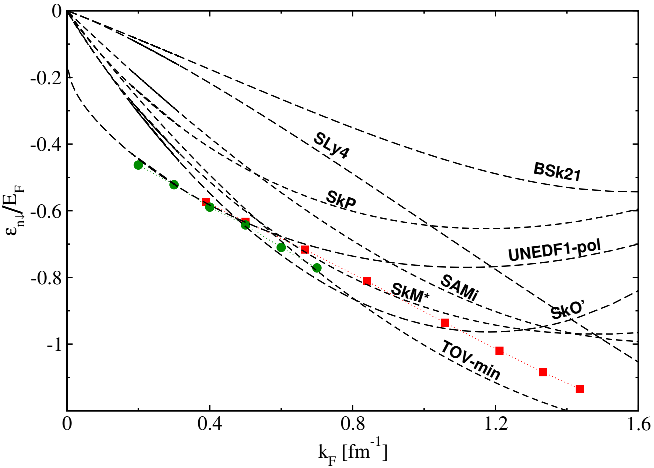
Idea: [M. M. Forbes et al. PRC 89, 041301(R) (2014)]

Calculate binding energy of an impurity in polarized neutron matter

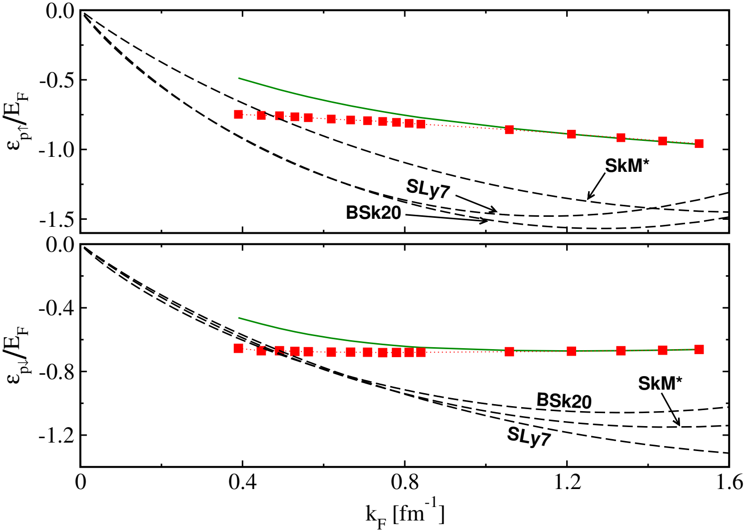
$$\varepsilon_{\tau\sigma} = \left.\frac{\partial \mathcal{E}}{\partial \rho_{\tau\sigma}}\right|_{\rho_{\tau\sigma}\to 0} \quad \rightarrow \quad \text{eg} \quad \varepsilon_{n\downarrow} \propto (C_0^s + C_1^s), (C_0^T + C_1^T)$$

The neutron impurity [arXiv:1406.1631]



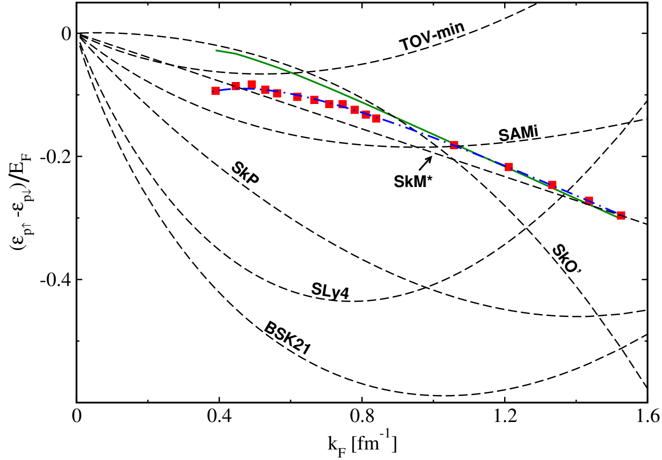
- Green pts from: M. M. Forbes et al. PRC 89, 041301(R) (2014)

The proton impurities I [arXiv:1406.1631]



The proton impurities II [arXiv:1406.1631]

$$\frac{\varepsilon_{p\uparrow} - \varepsilon_{p\downarrow}}{E_F} = \frac{4m(C_0^s - C_1^s)}{3\pi^2\hbar^2} k_F - \frac{2m(C_0^T - C_1^T)}{5\pi^2\hbar^2} k_F^3 .$$



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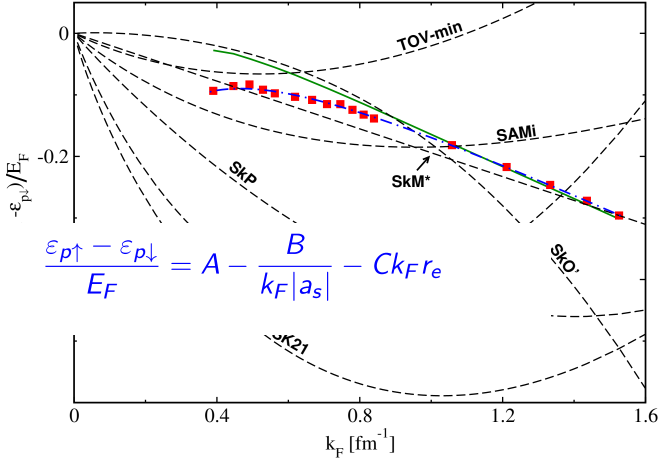


$$\frac{\varepsilon_{p\uparrow}-\varepsilon_{p\downarrow}}{E_F}=A-\frac{B}{k_F|a_s|}-Ck_Fr_e$$

Summary

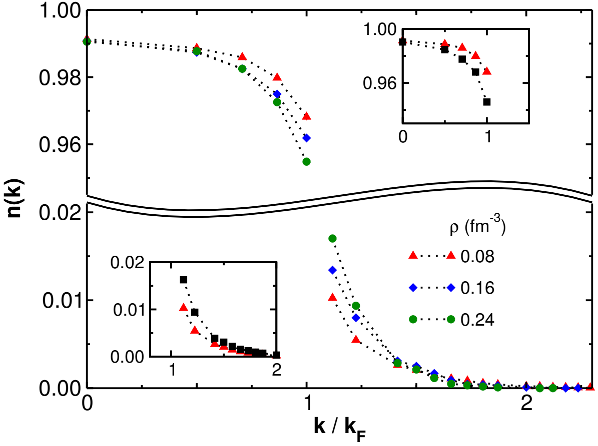
- we have developed a MC method that works for general interactions (non–local too!) providing rigourus upper–bounds on energy
- low density neutron–matter from many–body calculations and realistic forces to constrain Mean–Field theories
- proton impurities in polarized neutron matter as tight constraint on time–odd part of nuclear EDF

Goals and needs for the future:

- extend to three–body forces (coming soon) and finite nuclei
- symmetric matter (also soon)
- estimate uncertaintes coming from interaction (soon)
- Finite-temperature? (not so soon with χ-EFT)

Neutron Matter with χ-EFT interactions

Momentum distribution



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