Understanding final state interactions & quark-gluon correlations

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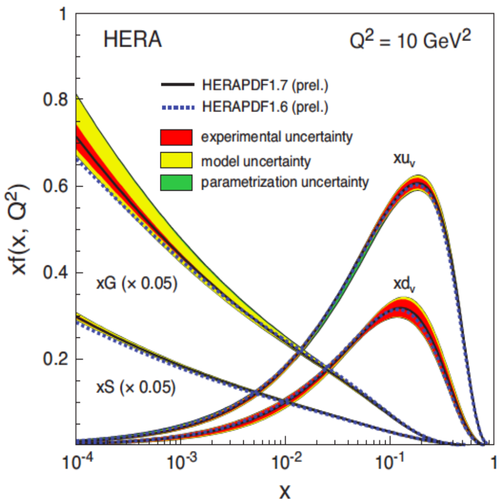
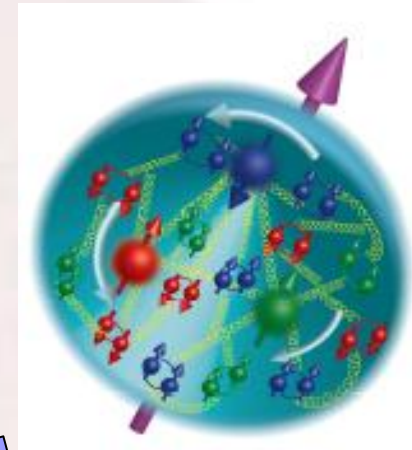
TMDs: center piece of nucleon structure

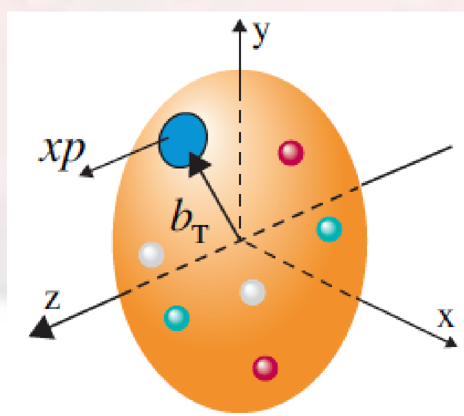


Long. Momentum distributions

QCD:
Factorization,
Universality,
Evolution,
Lattice, …

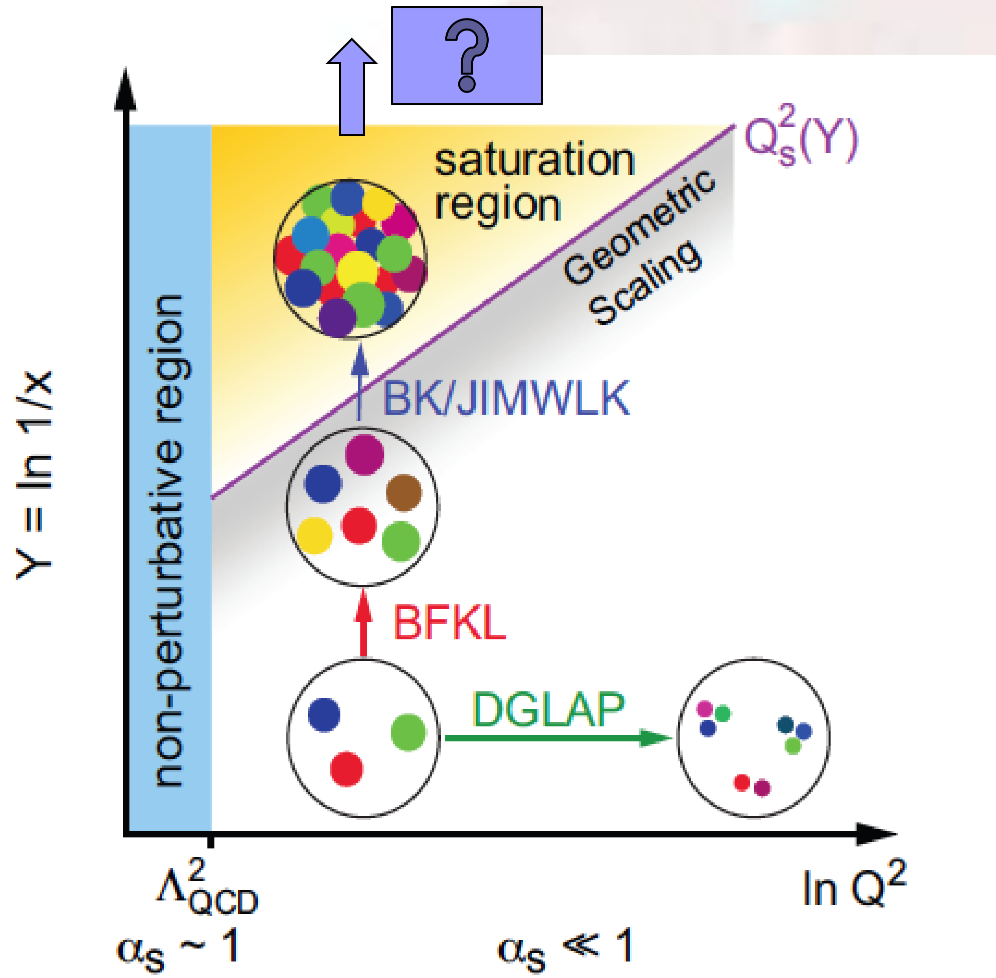


Nucleon Spin

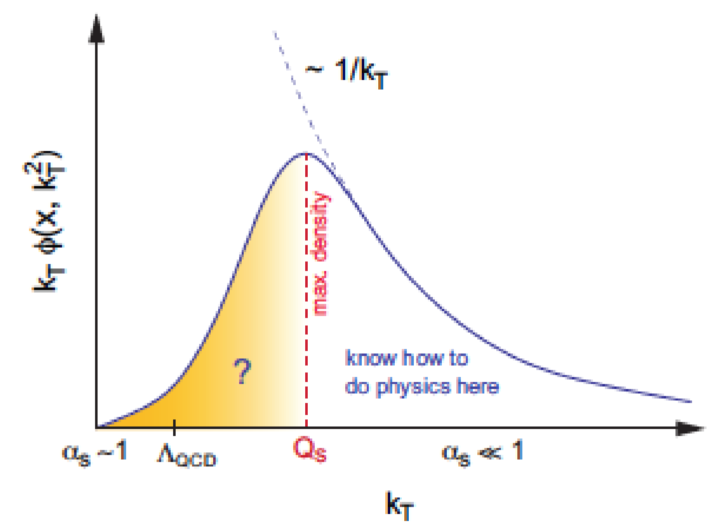


3D imaging
Transverse-momentum-dependent and Generalized PDFs

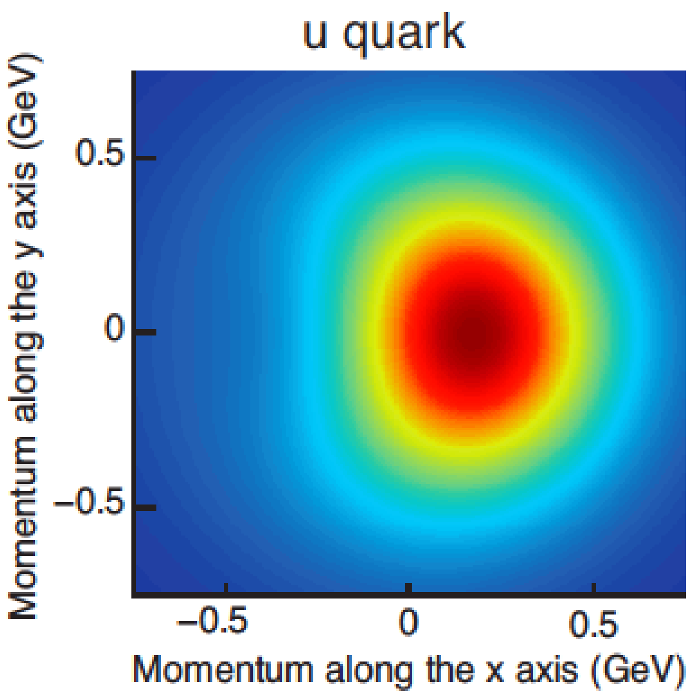
TMDs at small-x

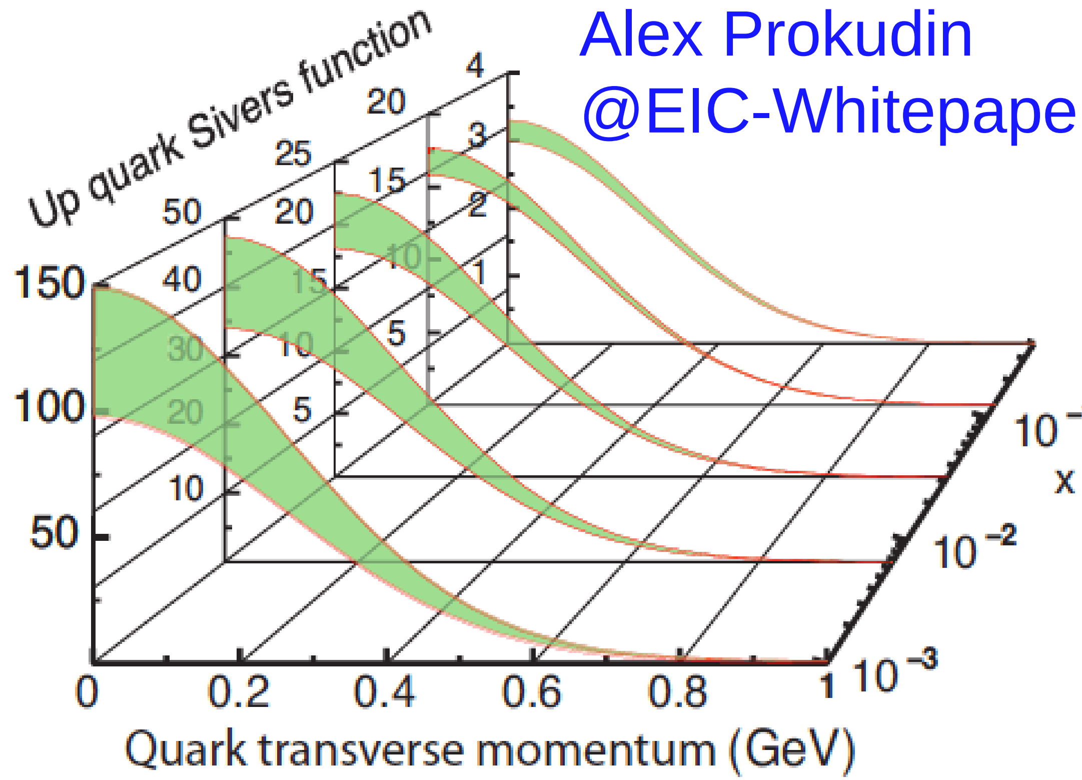


- kt-dependence crucial to the saturation



TMDs in valence region





Alex Prokudin
@EIC-Whitepaper

Quark Sivers function leads to an azimuthal asymmetric distribution of quark in the transverse plane

Outlines

- General Remarks
- Applying to single spin asymmetries
- Quark-gluon correlations and TMD observables

Collinear vs TMD factorization

- TMD factorization is an extension and simplification to the collinear factorization
- Extends to the region where collinear fails
- Simplifies the kinematics
  - Power counting, correction 1/Q neglected

$$\sigma(P_T,Q)=H(Q)\ f_1(k_{1T},Q)\ f_2(k_{2T},\ Q)\ S(\lambda_T)$$

DGLAP vs CSS

- DGLAP for integrated parton distributions
  - One hard scale

$$\sigma(Q)=H(Q/\mu)\ f_1(\mu)\ldots$$

- Collins-Soper-Sterman for TMDs
  - Two scales, large double logs

$$\frac{d\sigma}{dQ_1^2} = \frac{1}{Q_1^2} f_1 \otimes f_2 \otimes \sum_i \alpha_s^i \ln^{2i-1} \frac{Q^2}{Q_1^2} + \cdots$$

Evolution vs resummation

- Any evolution is to resum large logarithms
- DGLPA resum single large logarithms
- CSS evolution resum double logarithms

Sudakov Large Double Logarithms

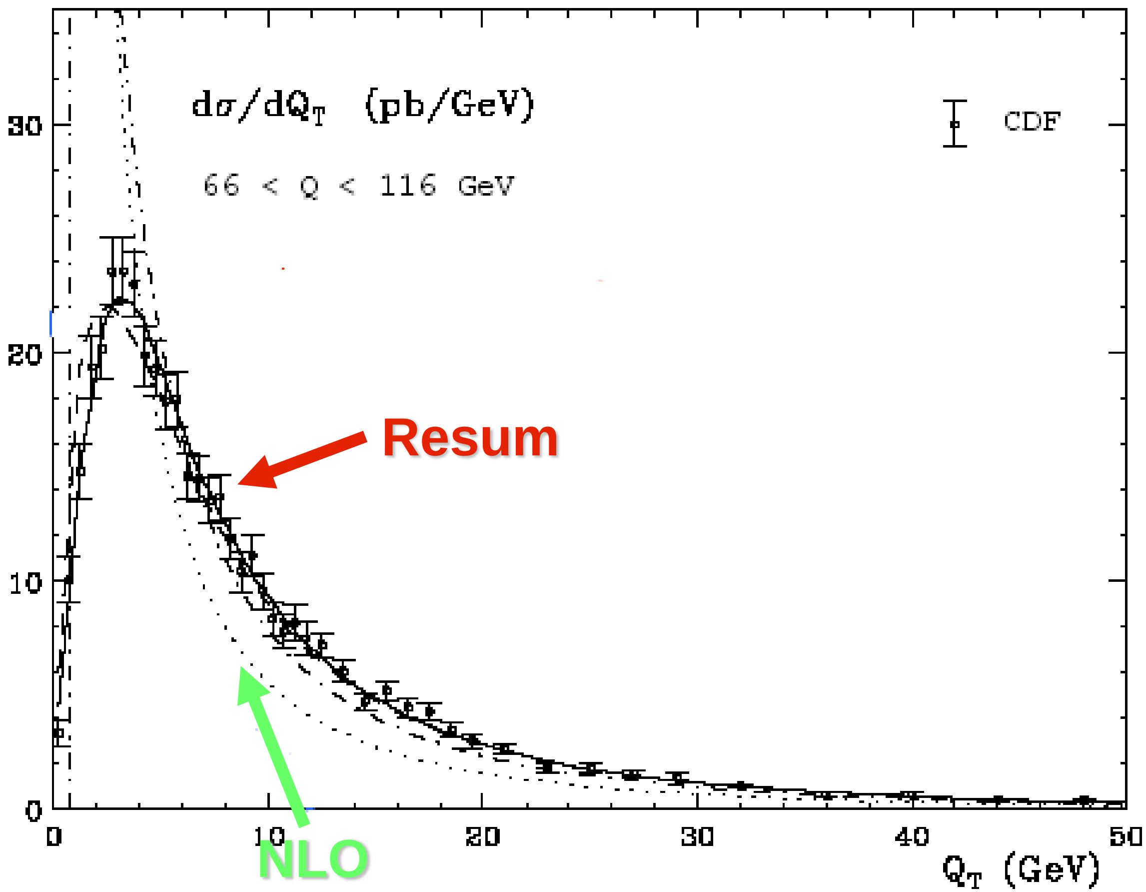
Sudakov, 1956

- Differential cross section depends on Q_1, where $Q^2 \gg Q_1^2 \gg \Lambda^2_{QCD}$

$$\frac{d\sigma}{dQ_1^2} = \frac{1}{Q_1^2} f_1 \otimes f_2 \otimes \sum_i \alpha_s^i \ln^{2i-1} \frac{Q^2}{Q_1^2} + \cdots$$

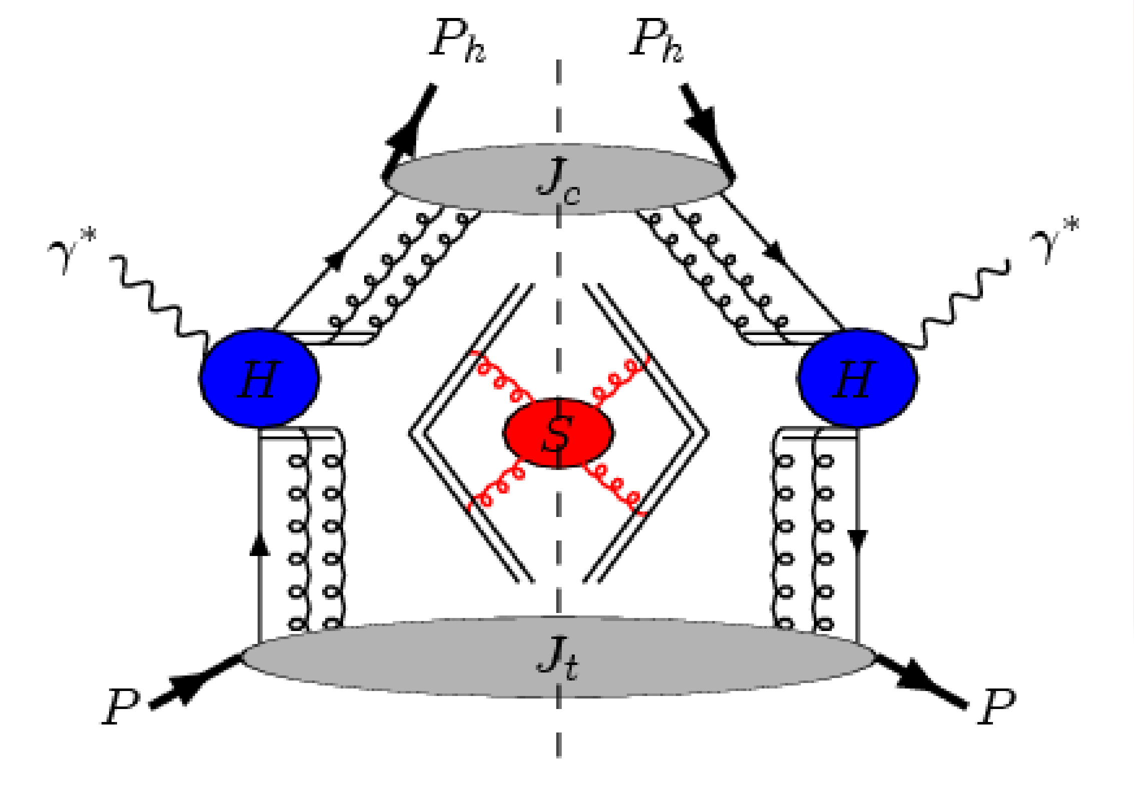
- We have to resum these large logs to make reliable predictions
  - Q_T: Dokshitzer, Diakonov, Troian, 78; Parisi Petronzio, 79; Collins, Soper, Sterman, 85
  - Threshold: Sterman 87; Catani and Trentadue 89

How Large of the Resummation effects



Kulesza, Sterman, Vogelsang, 02

TMD factorization: a nutshell



Collins-Soper 81

- Axial gauge was used

$$\mathcal{P}_{j/A}(x, k_T) = \frac{1}{2(2\pi)^3}\int dy^- d^2y_T\, e^{-i(xP^+y^- - k_T\cdot y_T)}\langle P|\bar{\psi}_j(0, y^-, y_T)\gamma^+\psi_j(0)|P\rangle$$

$$\zeta \equiv (2P\cdot n)^2/|n^2|$$

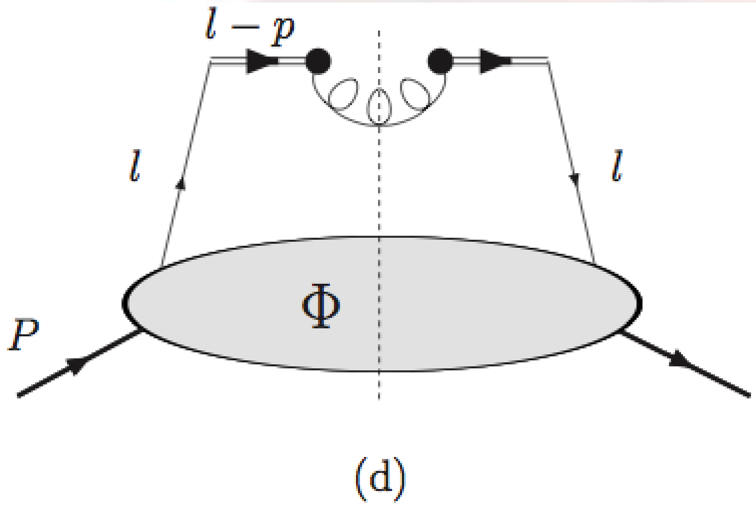
- a gauge link along n shall be included
  - Bacchetta-Boer-Diehl-Mulders, JHEP, 2008

$$F_{UU,T} = \left|H\left(x\zeta^{1/2}, z^{-1}\zeta_h^{1/2}\right)\right|^2 \sum_a x e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T$$

$$\times\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T\right) f_1^a(x, p_T^2; \zeta)\, D_1^a(z, k_T^2; \zeta_h)\, U(l_T^2)$$

Subtlety: pinch pole singularity?

Bacchetta-Boer-Diehl-Mulders, 2008



$$\frac{v^2}{\left[(\bar{l}-\bar{p})\cdot v+i\epsilon\right]\left[(\bar{l}-\bar{p})\cdot v-i\epsilon\right]}$$

- Cured by subtraction method, a la Collins 2011

$$q^{sub.}(x,b_\perp)=q^{unsub.}(x,b_\perp)\sqrt{\frac{S^{\bar{n},n}(b_\perp)}{S^{\bar{n},v}(b_\perp)S^{v,n}(b_\perp)}}$$

Ji-Ma-Yuan 2004

- Two off-light-cone vectors: v_1, v_2
  - $v_1^+ >> v_1^-$, $v_2^- >> v_2^+$, $v_1^2 > 0$
- Soft factor depends on $v_1.v_2$

$$F(x_B, z_h, P_{h\perp}, Q^2) = \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp$$
$$\times q\left(x_B, k_\perp, \mu^2, x_B\zeta, \rho\right) \hat{q}_T\left(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho\right) S(\vec{\ell}_\perp, \mu^2, \rho)$$
$$\times H\left(Q^2, \mu^2, \rho\right) \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp}) ,$$

Collins 2011

- Retain light-cone correlators in the unsubtracted TMDs
- Light-cone singularity is cancelled by the subtraction term

$$A_{\text{JCC, unren}}(\zeta_A) = \lim_{\substack{y'_1 \to +\infty \\ y'_2 \to -\infty}} A_{\text{JCC},0}(y'_2) \sqrt{\frac{S_{\text{JCC},0}(y'_1 - y_n)}{S_{\text{JCC},0}(y'_1 - y'_2)\, S_{\text{JCC},0}(y_n - y'_2)}}$$

$$= A_{\text{JCC},0}(-\infty) \sqrt{\frac{S_{\text{JCC},0}(+\infty - y_n)}{S_{\text{JCC},0}(+\infty - (-\infty))\, S_{\text{JCC},0}(y_n - (-\infty))}},$$

Collins-Soper-Sterman Resummation

- $\sigma(P_T,Q)=H(Q)\ f_1(k_{1T},Q)\ f_2(k_{2T},Q)\ S(\lambda_T)$
- Large Logs are resummed by solving the energy evolution equation of the TMDs

$$\frac{\partial}{\partial \ln Q} f(k_\perp,Q) = (K(q_\perp,\mu)+G(Q,\mu))\otimes f(k_\perp,Q)$$

- K and G obey the renormalization group eq.

$$\frac{\partial}{\partial \ln \mu} K = -\gamma_K = \frac{\partial}{\partial \ln \mu} G$$

(Collins-Soper 81, Collins-Soper-Sterman 85)

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CSS Formalism (II)

- The large logs will be resummed into the exponential form factor

$$W(Q,b) = e^{-\int_{1/b}^{Q}\frac{d\mu}{\mu}\left(\ln\frac{Q}{\mu}A+B\right)} C\otimes f_1 C\otimes f_2$$

  - A,B,C functions are perturbative calculable
  - f_1,f_2 are integrated PDFs
  - all are scheme-independent
  - Collins 2011 is slightly different, but final results are the same

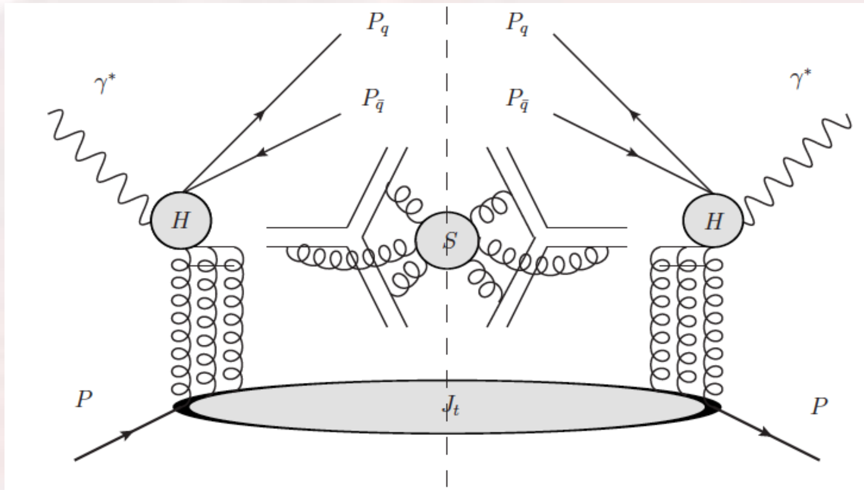
(Collins-Soper-Sterman 85)

Low Transverse Momentum Heavy Quark Pair Production to Probe Gluon Tomography

Ruilin Zhu,[1,2] Peng Sun,[2] and Feng Yuan[2]

PLB, 2013

all order factorization



$$W(x, b_\perp, \widetilde{Q}^2) = g(x, b_\perp, \widetilde{Q}_0, \widetilde{Q}_0)\overline{S}(b_\perp, \widetilde{Q}_0)H(\widetilde{Q}, \widetilde{Q})e^{-S_{sud}(\widetilde{Q}, \widetilde{Q}_0)}$$

$$S_{sud} = -\int_{\widetilde{Q}_0}^{\widetilde{Q}} \frac{d\mu}{\mu}\left(\ln\frac{\widetilde{Q}}{\mu}\gamma_K(\mu) - \gamma_S(\mu, 1) + \frac{\alpha_s C_A}{\pi}(1 - 2\beta_0 - \ln\frac{\widetilde{Q}_0^2 b_\perp^2}{c_0^2})\right)$$

Take both Ji-Ma-Yuan and Collins 11 formalisms, Obtain the consistent result

Where are the TMDs?

- Non-perturbative TMDs are in the initial scale Q_0,$1/b_*$
- Can we have a factorization in terms of the TMDs calculable on lattice?
  - Musch et al, 2011

TMDs in Euclidean Space

Ji, 2013

- Large momentum effective theory

$$q(x_z, k_\perp) = \frac{1}{2}\int \frac{d^3z}{(2\pi)^3} e^{ik\cdot z} \langle PS|\overline{\psi}(0)\mathcal{L}^\dagger_{n_z(0,-\infty)}\gamma^z \mathcal{L}_{n_z(z,-\infty)}\psi(z)|PS\rangle$$

- To fulfill the TMD factorization, a subtraction is necessary

$$q^{sub.}(x_z, b_\perp) = q^{unsub.}(x_z, b_\perp)\frac{1}{S(b_\perp)}$$

- Collins-Soper evolution (respect to P_z)
  - Lattice simulation will be very useful in constraining the evolution, in particular, at large b

TMDs at large b

- Phenomenogical applications of the QCD resummation to the P_T spectrum of EW bosons production have been very successful

  Yuan, Nadolsky, Ladinsky, Landry,

  Qiu, Zhang, Berger, Li,

  Laenen, Sterman, Vogelsang, Kulesza,

  Bozzi, Catani, deFlorian,

  Kulesza, Stirling, and many others, …

  working even at NNLL level for some

b_* prescription

$$W(Q,b) = e^{-\int_{1/b}^{Q}\frac{d\mu}{\mu}\left(\ln\frac{Q}{\mu}A+B\right)} C\otimes f_1 C\otimes f_2$$

- b_* always in perturbative region

$$b \Rightarrow b_* = b/\sqrt{1+b^2/b_{max}^2}\ , \quad b_{max} < 1/\Lambda_{QCD}\ ,$$

- This will introduce a non-perturbative form factors $\mathcal{S}_{sud} \Rightarrow \mathcal{S}_{pert}(Q;b_*) + S_{NP}(Q;b)$
- Generic behavior

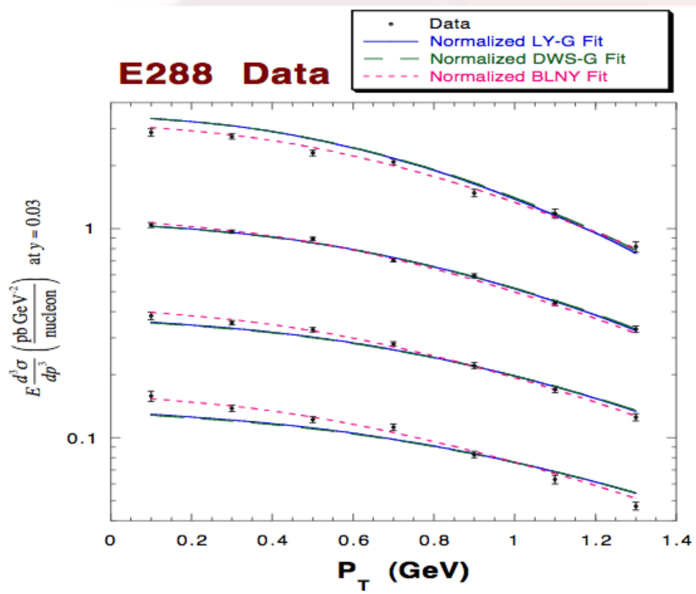
$$S_{NP} = g_2(b)\ln Q + g_1(b)$$

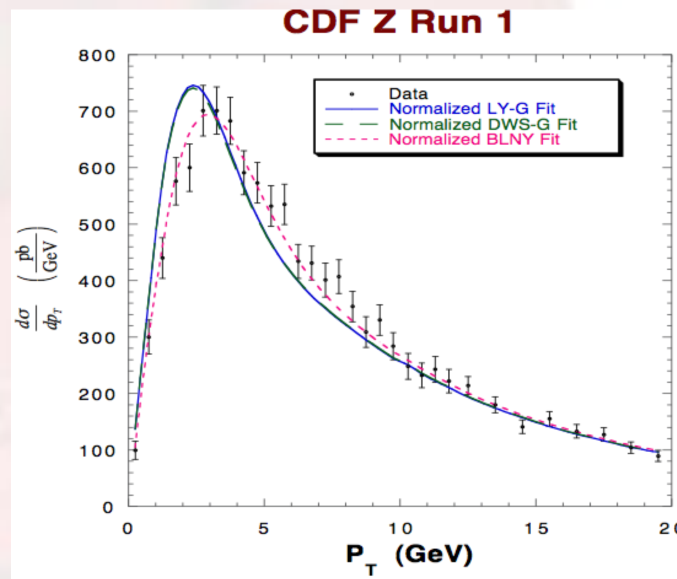
Collins-Soper-Sterman 85

BLNY form factors Phys. Rev. D **67**, 073016 (2003)

- Fit to Drell-Yan and W/Z boson production

$$S_{NP} = g_1 b^2 + g_2 b^2 \ln (Q/3.2) + g_1 g_3 b^2 \ln(100 x_1 x_2)$$



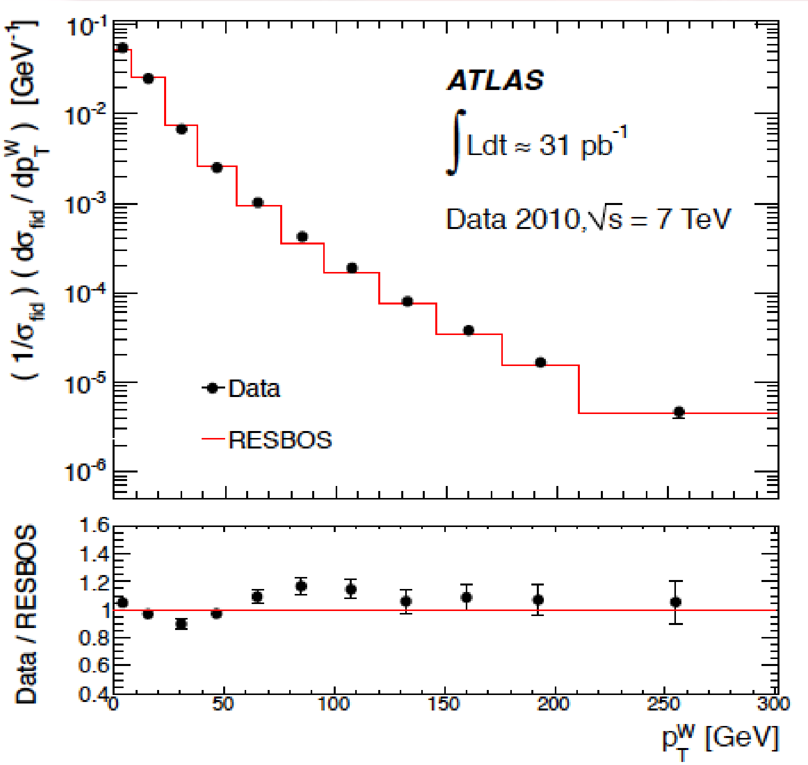


$$g_1 = 0.21^{+0.01}_{-0.01}\ \mathrm{GeV}^2\ ,\ g_2 = 0.68^{+0.01}_{-0.02}\ \mathrm{GeV}^2\ ,\ g_3 = -0.6^{+0.05}_{-0.04}.$$

b_{max}=0.5GeV^{-1}

Very successful phenomenology

- Most quoted comparisons at the LHC for W/Z production



ResBos: Nadolsky, et al., PRD 2003
CSS resummation built in

Top quark pair production

$$\frac{d^4\sigma}{dq_T^2\,dy\,dM\,d\cos\theta} = \frac{\beta_t\alpha_s^3}{4sMq_T^2}\sum_i \frac{1}{d_i}$$

$$\times\left\{ f_{i/N_1}(\xi_1)\, f_{\bar{i}/N_2}(\xi_2)\,\mathrm{Tr}\left[\boldsymbol{H}^{(0)}_{i\bar{i}}\left(\boldsymbol{A}_{i\bar{i}}\ln\frac{M^2}{q_T^2}+\boldsymbol{B}_{i\bar{i}}\right)\right]\right.$$

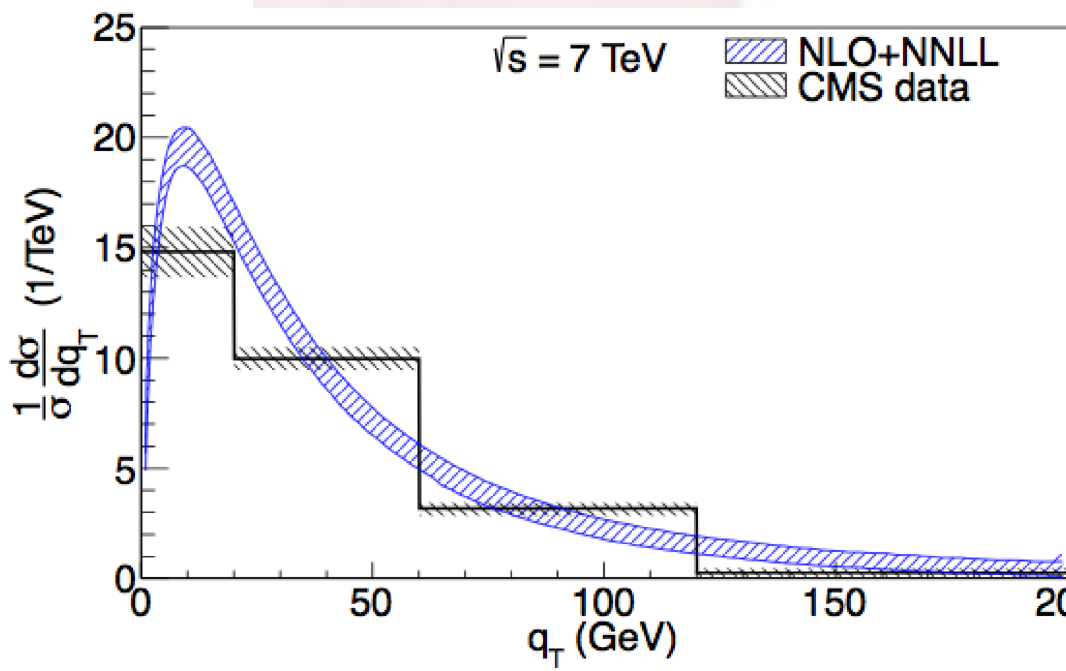
$$+\,\mathrm{Tr}\left[\boldsymbol{H}^{(0)}_{i\bar{i}}\,\boldsymbol{S}^{(0)}_{i\bar{i}}\right]\left[\sum_a \left[P^{(1)}_{ia}\otimes f_{a/N_1}\right](\xi_1)\, f_{\bar{i}/N_2}(\xi_2)\right.$$

$$+\sum_b f_{i/N_1}(\xi_1)\left[P^{(1)}_{\bar{i}b}\otimes f_{b/N_2}\right.$$

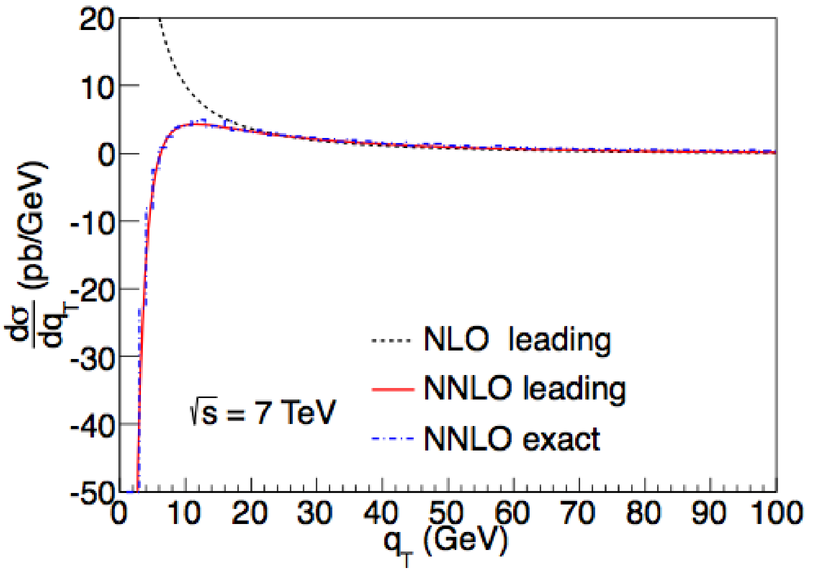
Zhu,H.X., et al, PRL 2012
Derived in SCET

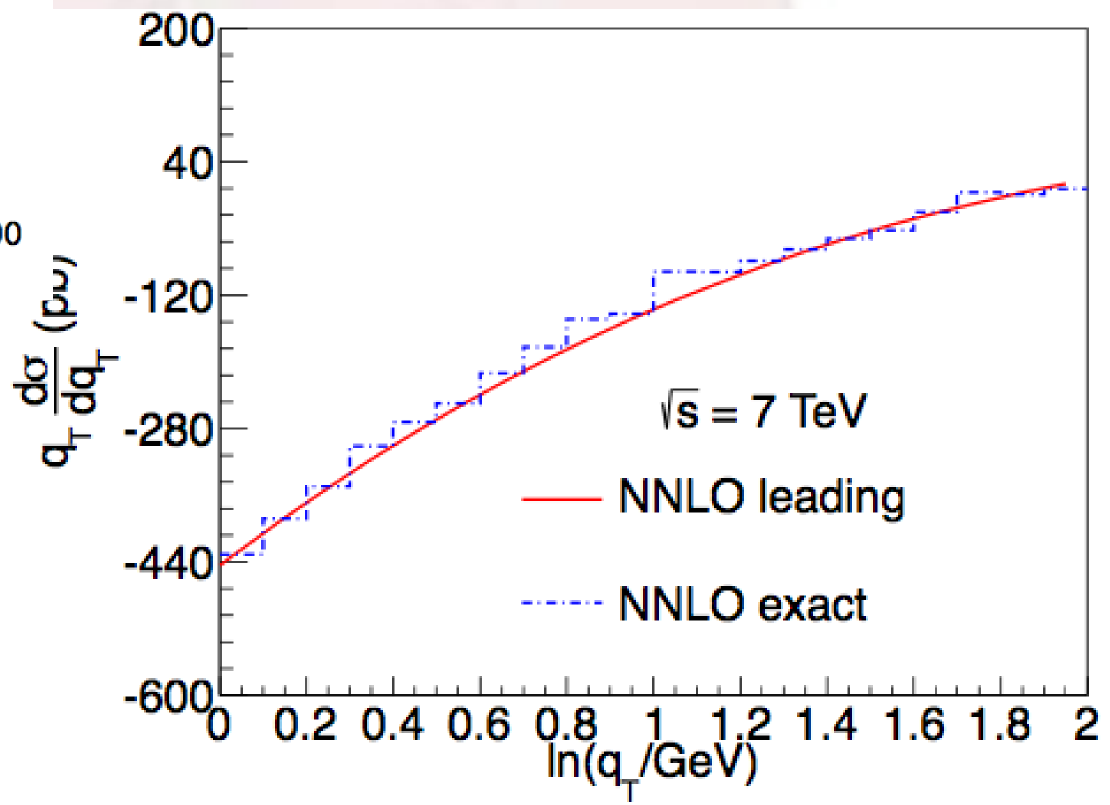
the consistent resummation formula can be derived following Ji-Ma-Yuan,

Zhu-Qiao-Sun-Yuan, to appear



How good the factorization



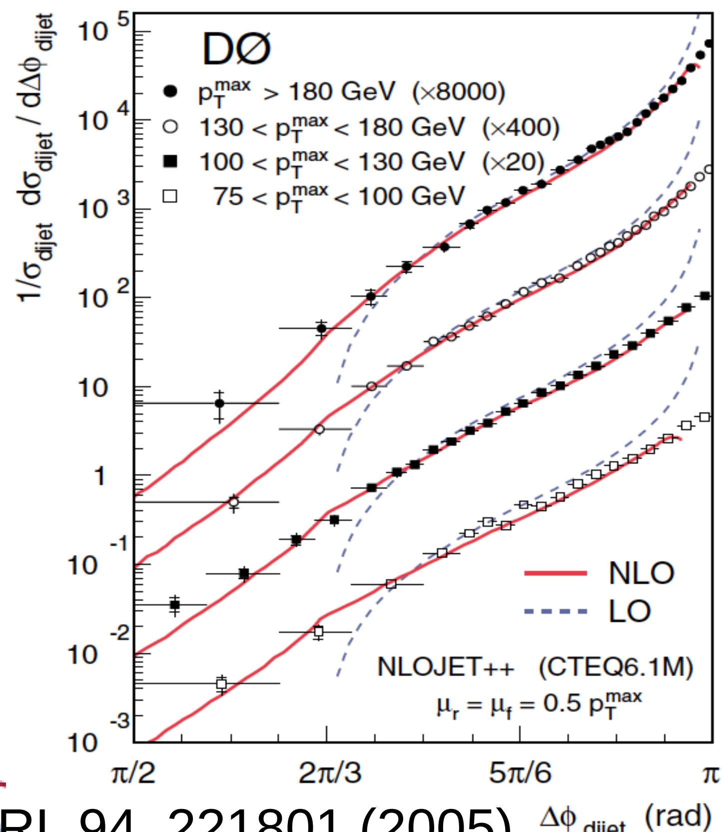


Zhu H.X., PRD 2013

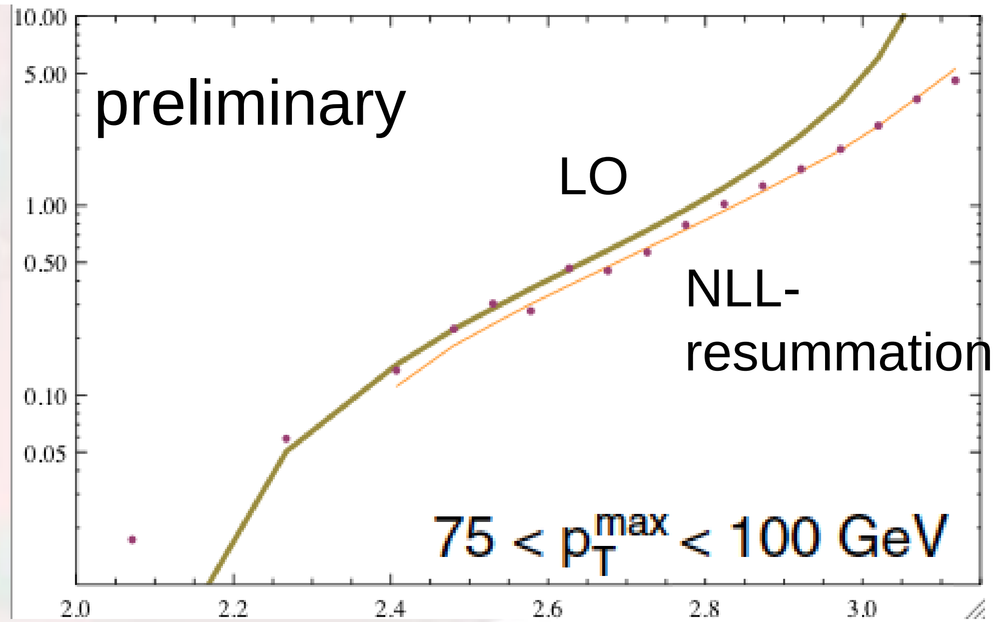
Dijet azimuthal correlation at colliders

$$x_1 f_a(x_1, \mu = c_0/b_\perp) x_2 f_b(x_2, \mu = c_0/b_\perp) e^{-S_{Sud}(Q^2, b_\perp)}$$

$$\mathrm{Tr}\left[\mathbf{H}_{ab\to cd} \exp[-\int_{c_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s\dagger}] \mathbf{S}_{ab\to cd} \exp[-\int_{c_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s}]\right]$$



PRL 94, 221801 (2005)



Leading logarithms consistent with Mueller-Xiao-Yuan, PRD 2013;
Nontrivial results at NNL;
Peng Sun, et al., to appear

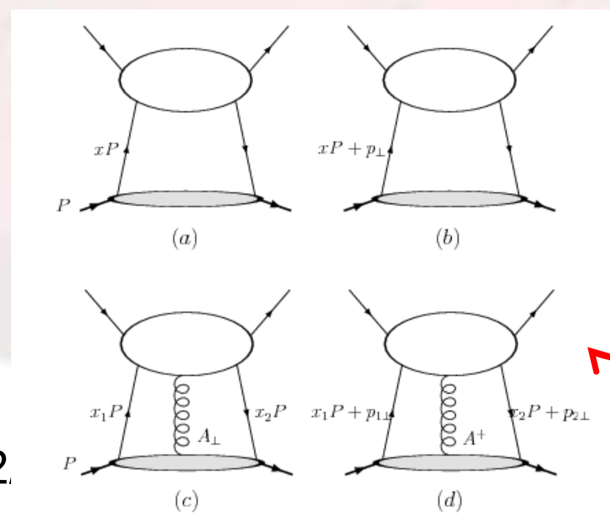
Perturbative tail is calculable

- Transverse momentum dependence

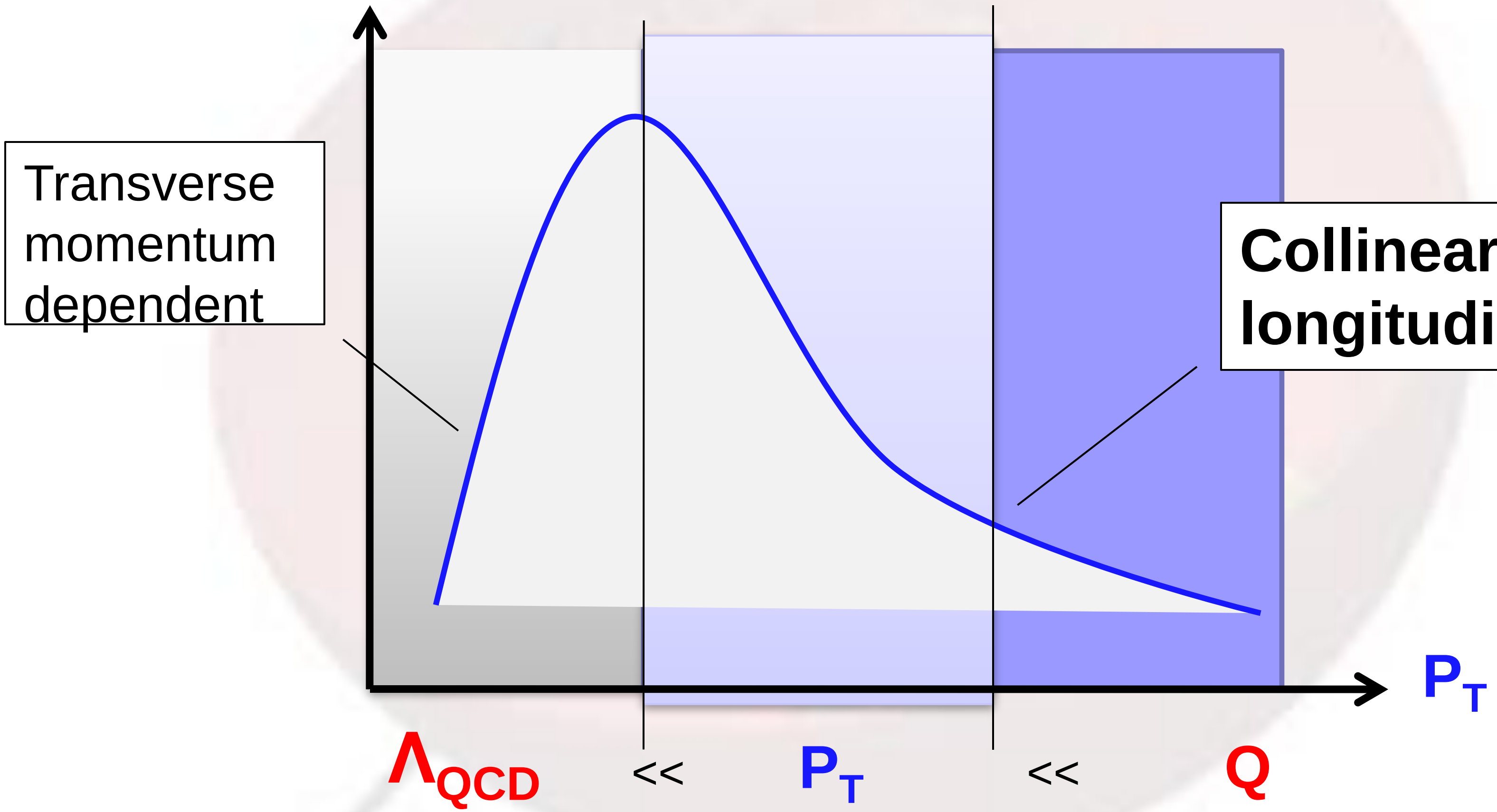
$$q(x,k_\perp)|_{k_\perp \gg \Lambda_{QCD}} = \frac{1}{(k_\perp^2)^n} \int \frac{dx'}{x'} f_i(x') \times \mathcal{H}_{q/i}(x;x')$$

Power counting,
Brodsky-Farrar, 1973

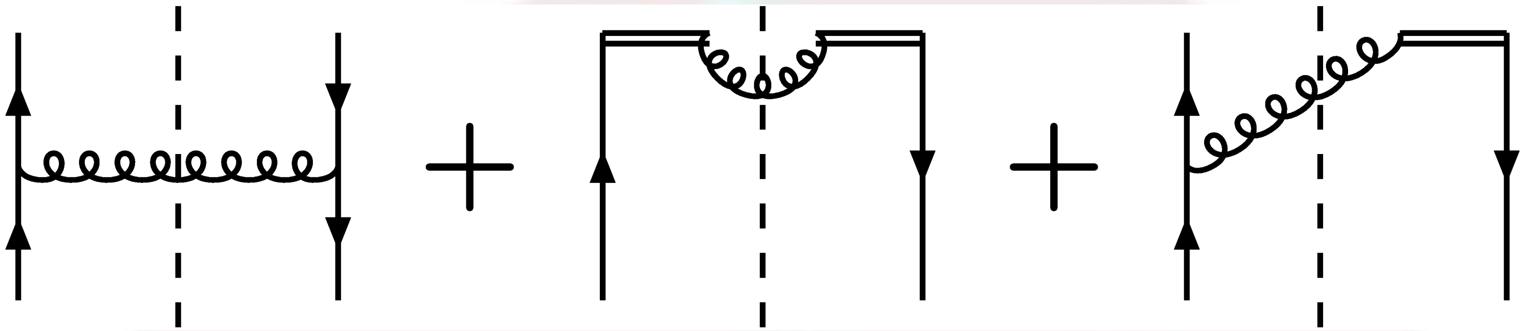
Integrated Parton Distributions
Twist-three functions



A unified picture (leading pt/Q)



Simple example



- Ji-Ma-Yuan

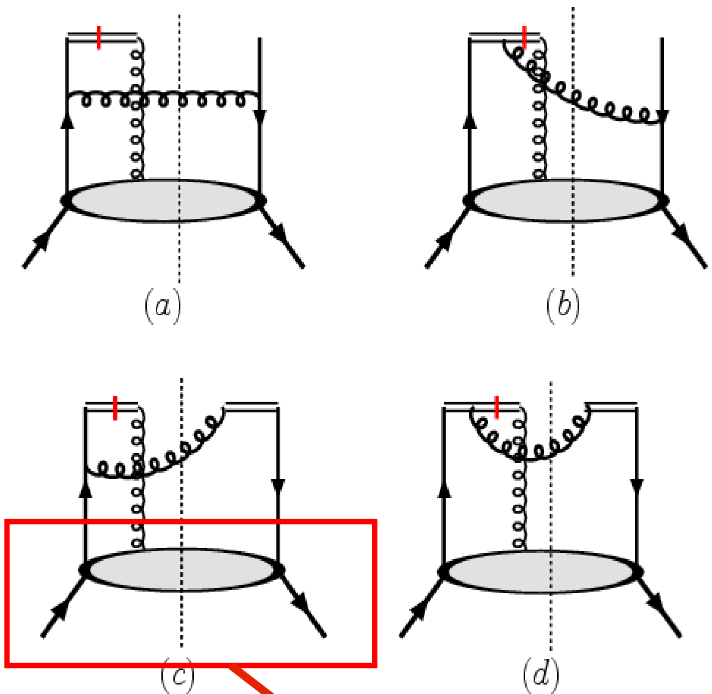
$$\frac{\alpha_s}{2\pi^2}\frac{1}{k_\perp^2}C_F\int\frac{dx}{x}q(x)\left\{\frac{1+\xi^2}{(1-\xi)_+}+\frac{D-2}{2}(1-\xi)+\delta(1-\xi)\left(\ln\frac{z^2\zeta^2}{k_\perp^2}-1\right)\right\}$$

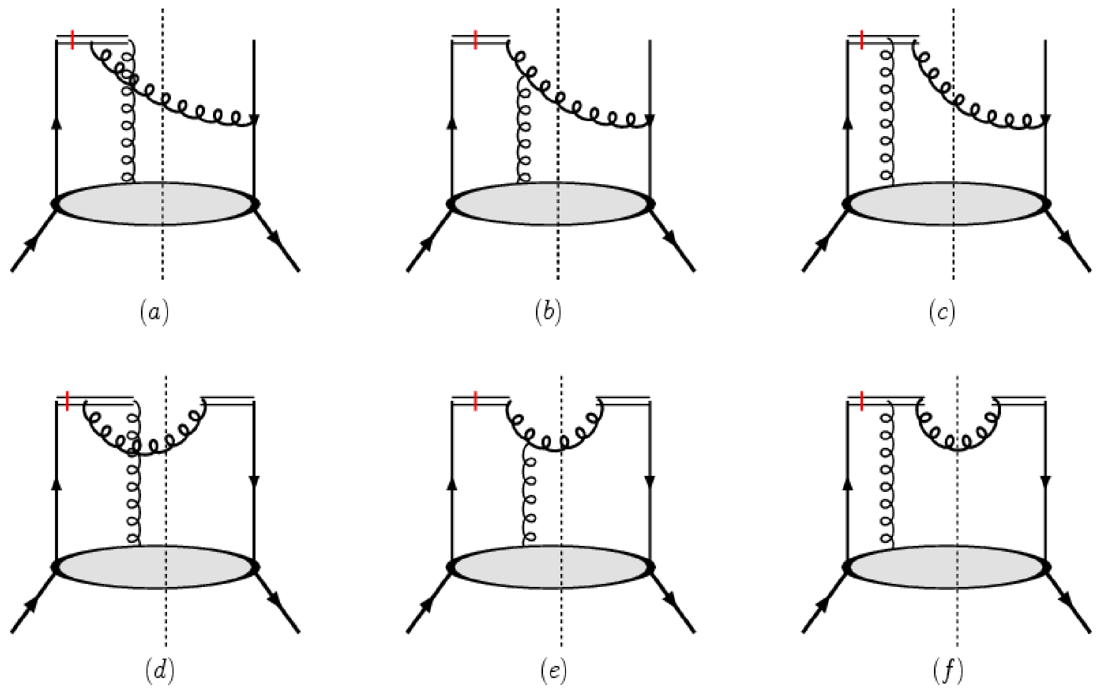
- Collins 11

$$\frac{\alpha_s}{2\pi^2}\frac{1}{k_\perp^2}C_F\int\frac{dx}{x}q(x)\left\{\frac{1+\xi^2}{(1-\xi)_+}+\frac{D-2}{2}(1-\xi)+\delta(1-\xi)\left(\ln\frac{\zeta_c^2}{k_\perp^2}\right)\right\},$$

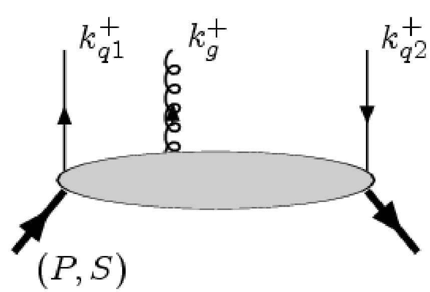
- Although the integral does not reproduce the integrated PDFs, the infrared behavior in the integral reproduces the infrared behavior for the integrated PDFs
- That’s why we have small b factorization

Sivers Function at large k_T



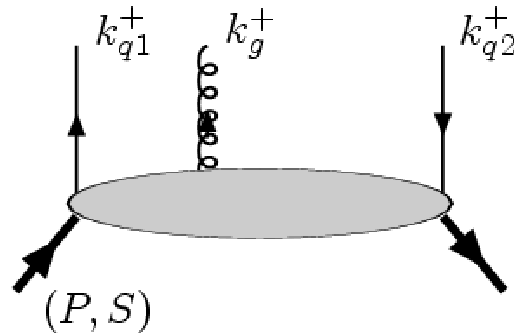


Quark-gluon Correlation



Qiu, Sterman, 91,99

Qiu-Sterman matrix element



$$T_{a,F}(x_1,x_2) = \int \frac{dy_1^- dy_2^-}{4\pi} e^{ix_1P^+y_1^- + i(x_2-x_1)P^+y_2^-}$$
$$\times \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ \left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right] \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

Sivers Function at Large k_T

Ji-Qiu-Vogelsang-Yuan, 06
Sun-Yuan, 13

$$
\begin{aligned}
f_{1T}^{\perp}(z,k_\perp) = \frac{\alpha_s}{2\pi^2}\frac{M}{(k_\perp^2)^2}\int\frac{dx}{x}\Bigg\{ & \frac{C_A}{2}T_F(x,z)\frac{1+\xi}{(1-\xi)_+} + T_F(x,x)\frac{-1}{2N_c}\frac{D-2}{2}(1-\xi \\
& + \frac{1}{2N_C}\left[\left(x\frac{\partial}{\partial x}T_F(x,x)\right)(1+\xi^2) + T_F(x,x)\frac{(1-\xi)^2(2\xi+1)-2}{(1-\xi)_+}\right] \\
& + T_F(x,x)\delta(1-\xi)C_F\left(\ln\frac{x^2\zeta^2}{k_\perp^2}-2\right)\Bigg\} .
\end{aligned}
$$

- $1/k_T^4$ follows a power counting
- Drell-Yan Sivers function has opposite sign
- Plugging this into the factorization formula, we indeed reproduce the polarized cross section calculated from twist-3 correlation

Small b factorization

Kang-Xiao-Yuan, 11
Sun-Yuan, 13

$$q(z,b) = \frac{\alpha_s}{2\pi}C_F\int\frac{dx}{x}q(x)\left\{\left(-\frac{1}{\epsilon}+\ln\frac{c_0^2}{b^2\bar{\mu}^2}\right)\mathcal{P}_{q\to q}(\xi)-\delta(1-\xi)\ln\frac{c_0^2}{b^2\mu^2}+(1-\xi)\right.$$
$$\left.+\delta(1-\xi)\left[\frac{3}{2}\ln\frac{b^2\mu^2}{c_0^2}+\ln\frac{z^2\zeta^2}{\mu^2}-\frac{1}{2}\left(\ln\frac{z^2\zeta^2b^2}{c_0^2}\right)^2-2-\frac{\pi^2}{2}\right]\right\}.$$

$$\tilde{f}_{1T}^{\alpha}(z,b) = \frac{\alpha_s}{2\pi}\left(\frac{-ib^\alpha}{2}\right)\int\frac{dx}{x}\left\{\left(-\frac{1}{\epsilon}+\ln\frac{c_0^2}{b^2\mu^2}\right)\mathcal{P}^T_{qg\to qg}\otimes T_F(z,z)\right.$$
$$-\delta(1-\xi)T_F(x,x)C_F\ln\frac{c_0^2}{b^2\mu^2}-\frac{1}{2N_c}T_F(x,x)(1-\xi)$$
$$\left.+\delta(1-\xi)T_F(x,x)C_F\left[\frac{3}{2}\ln\frac{b^2\mu^2}{c_0^2}+\ln\frac{z^2\zeta^2}{\mu^2}-\frac{1}{2}\left(\ln\frac{z^2\zeta^2b^2}{c_0^2}\right)^2-2-\frac{\pi^2}{2}\right]\right\}$$

Reproduce the DGLAP evolution for the Qiu-Sterman matrix element

Braun et al, 2008; Vogelsang 2008
Zhou-Schafer, 2012
Kang-Qiu, 2012

Single Transverse Spin Asymmetry

$$\frac{d\Delta\sigma(S_\perp)}{dydQ^2d^2q_\perp} = \sigma_0\epsilon^{\alpha\beta}S_\perp^\alpha W_{UT}^\beta(Q;q_\perp)$$

- Separate the singular and regular parts

$$W_{UT}^\alpha(Q;q_\perp) = \int \frac{d^2b}{(2\pi)^2} e^{i\vec{q}_\perp\cdot\vec{b}}\widetilde{W}_{UT}^\alpha(Q;b) + Y_{UT}^\alpha(Q;q_\perp)$$

- TMD factorization in b-space

$$\widetilde{W}_{UT}^\alpha(Q;b) = \tilde{f}_{1T}^{(\perp\alpha)}(z_1,b,\zeta_1)\bar{q}(z_2,b,\zeta_2) \times H_{UT}(Q)\left(S(b,\rho)\right)^{-1} ,$$

Kang, Xiao, Yuan, PRL 11;
Rogers et al., PRD, 2012

Evolution equations

Idilbi-Ji-Ma-Yuan, PRD04

$$\zeta \frac{\partial}{\partial \zeta} \partial_b^i q_T(x, b, \mu, x\zeta, \rho) = [K(b, \mu, \rho) + G(x\zeta, \mu, \rho)] \times \partial_b^i q_T(x, b, \mu, x\zeta, \rho), \quad (30)$$

$$Q^2 \frac{\partial}{\partial Q^2} \mathcal{F}_{UT}(x_B, z_h, b, Q^2) = \{K[b\mu, g(\mu), \rho] + G'_{UT}[Q/\mu, g(\mu), \rho]\} \times \mathcal{F}_{UT}(x_B, z_h, b, Q^2),$$

$$\mathcal{F}_{UT}(x_B, z_h, b, Q^2) = \mathcal{F}_{UT}(x_B, z_h, b, \mu_1^2/C_2^2) e^{-S(Q^2, b, C_2)}, \quad (42)$$

Boer, NPB, 2002

Final resum form

$$\widetilde{W}_{UT}^{\alpha}(Q;b) = e^{-\mathcal{S}_{UT}(Q^2,b)}\widetilde{W}_{UT}^{\alpha}(C_1/b,b)$$
$$= (-ib_\perp^\alpha/2)\, e^{-\mathcal{S}_{UT}(Q^2,b)}\Sigma_{i,j}$$
$$\times \Delta C_{qi}^T \otimes f_{i/A}^{(3)}(z_1) C_{\bar{q}j} \otimes f_{j/B}(z_2)$$

- Sudakov the same

$$\mathcal{S}_{UT}(Q^2,b) = \int_{C_1^2/b^2}^{C_2^2Q^2} \frac{d\mu^2}{\mu^2}\left[\ln\left(\frac{C_2^2Q^2}{\mu^2}\right) A_{UT}(C_1;g(\mu)) + B_{UT}(C_1,C_2;g(\mu))\right] ,$$

Coefficients at one-loop order

$$A_{UT}^{(1)} = C_F,\ B_{UT}^{(1)} = -3/2C_F,\ \Delta C_{qq}^{T(0)} = \delta(1-x)\ ,$$

$$\Delta C_{qq}^{T(1)} = -\frac{1}{4N_c}(1-x) + \frac{C_F}{2}\delta(x-1)\left[\frac{\pi^2}{2} - 4\right]\ ,$$

- Coefficients are (TMD) scheme independent
- Can be done consistently with the unpolarized cross sections

TMDs in the resummation

- b_* prescription

$$\widetilde{W}_{UU}(Q;b) = e^{-\mathcal{S}_{pert}(Q^2,b_*)-S_{NP}(Q,b)}\Sigma_{i,j}C_{qi}^{(DY)} \otimes f_{i/A}(z_1)C_{\bar{q}j}^{(DY)} \otimes f_{j/B}(z_2) \ ,$$

$$\widetilde{W}_{UT}^{\alpha}(Q;b) = \left(\frac{-ib^\alpha}{2}\right) e^{-\mathcal{S}_{pert}(Q^2,b_*)-S_{NP}^T(Q,b)}\Sigma_{i,j}\Delta C_{qi}^{T(DY)} \otimes f_{i/A}^{(3)}(z_1,z_1)C_{\bar{q}j}^{(DY)} \otimes f_{j/B}(z_2),$$

- TMDs at initial scale embedded in the non-perturbative form factors

$$S_{NP}(Q,b) = g_2(b)\ln Q + g_1(b;z_1,z_2)$$
$$S_{NP}^T(Q,b) = g_2(b)\ln Q + g_1^T(b;z_1,z_2)$$

Polarized TMD Quark Distributions

Quark \ Nucleon	Unpol.	Long.	Trans.
Unpol.	$f_1(x,k_\perp)$		$f_{1T}^\perp(x,k_\perp)$
Long.		$g_1(x,k_\perp)$	$g_{1T}(x,k_\perp)$
Trans.	$h_1^\perp(x,k_\perp)$	$h_{1L}(x,k_\perp)$	$h_1(x,k_\perp)$ $h_{1T}^\perp(x,k_\perp)$

Boer, Mulders, Tangerman (96&98)

TMDs and Quark-gluon Correlations (twist-3)

- Kt-odd distribution

$f_{1T}^{\perp}(x,k_\perp)$ ⟺ $G_D(x_1,x_2\ \tilde{G}_D(x_1,x_2)$

$g_{1T}(x,k_\perp)$ ⟺ $[T_F(x_1,x_2)\tilde{T}_F(x_1,x_2)]$

$h_1^{\perp}(x,k_\perp)$ ⟺ $T_F^{(\sigma)}(x_1,x_2)$

$h_{1L}(x,k_\perp)$ ⟺ $H_D(x_1,x_2)$

$$T_F(x,x) = -\int \frac{d^2\vec{k}_\perp}{2\pi}\frac{\vec{k}_\perp^2}{M^2} f_{1T}^{\perp}|_{\rm DIS}(x,k_\perp)$$

$$T_F^{(\sigma)}(x,x) = -\int \frac{d^2\vec{k}_\perp}{2\pi}\frac{\vec{k}_\perp^2}{M^2} h_1^{\perp}|_{\rm DIS}(x,k_\perp)$$

$$\tilde{g}(x) = \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{2M^2} g_{1T}(x,k_\perp)$$

$$\tilde{h}(x) = \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{2M^2} h_{1L}(x,k_\perp)$$

Boer-Mulders-Pijlman, 2003

Quark-gluon correlations (twist-three)

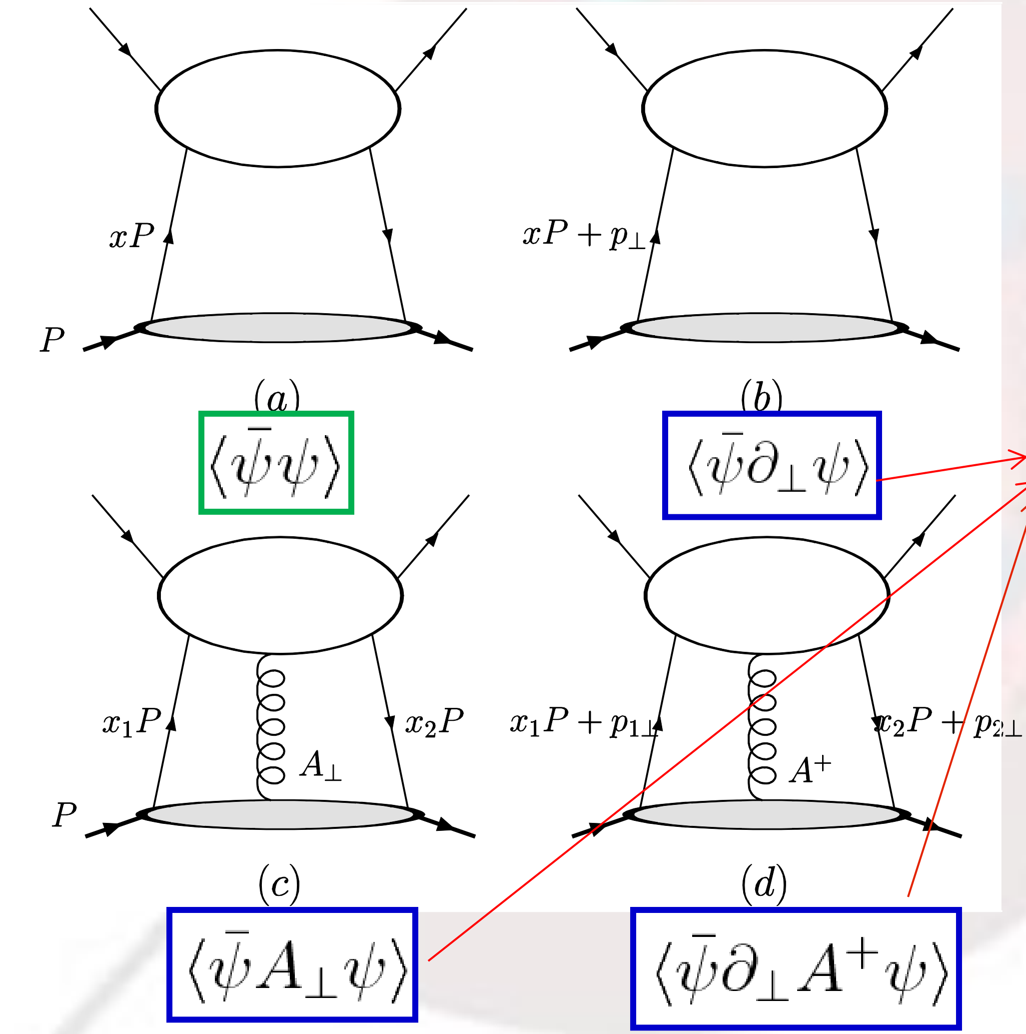
- Have long been studied,

$$D^i_\Gamma(y_1, y_2, s) = \langle P, s|\ \bar\psi(0)\ \Gamma\ D^i(y_2)\ \psi(y_1)\ |P, s\rangle$$
$$F^i_\Gamma(y_1, y_2, s) = \langle P, s|\ \bar\psi(0)\ \Gamma\ n_\mu F^{i\mu}(y_2)\ \psi(y_1)\ |P, s\rangle$$

- F-type and D-type are related to each other,
Ellis-Furmanski-Petronzio 82, Eguchi-Koike-Tanaka 06

$$G_D(x, x_1) = P\frac{1}{x - x_1}T_F(x, x_1),$$
$$\tilde G_D(x, x_1) = P\frac{1}{x - x_1}\tilde T_F(x, x_1) + \delta(x - x_1)\tilde g(x),$$
$$E_D(x, x_1) = P\frac{1}{x - x_1}T_F^{(\sigma)}(x, x_1),$$
$$H_D(x, x_1) = P\frac{1}{x - x_1}\tilde T_F^{(\sigma)}(x, x_1) + \delta(x - x_1)\tilde h(x)$$

twist and collinear expansion



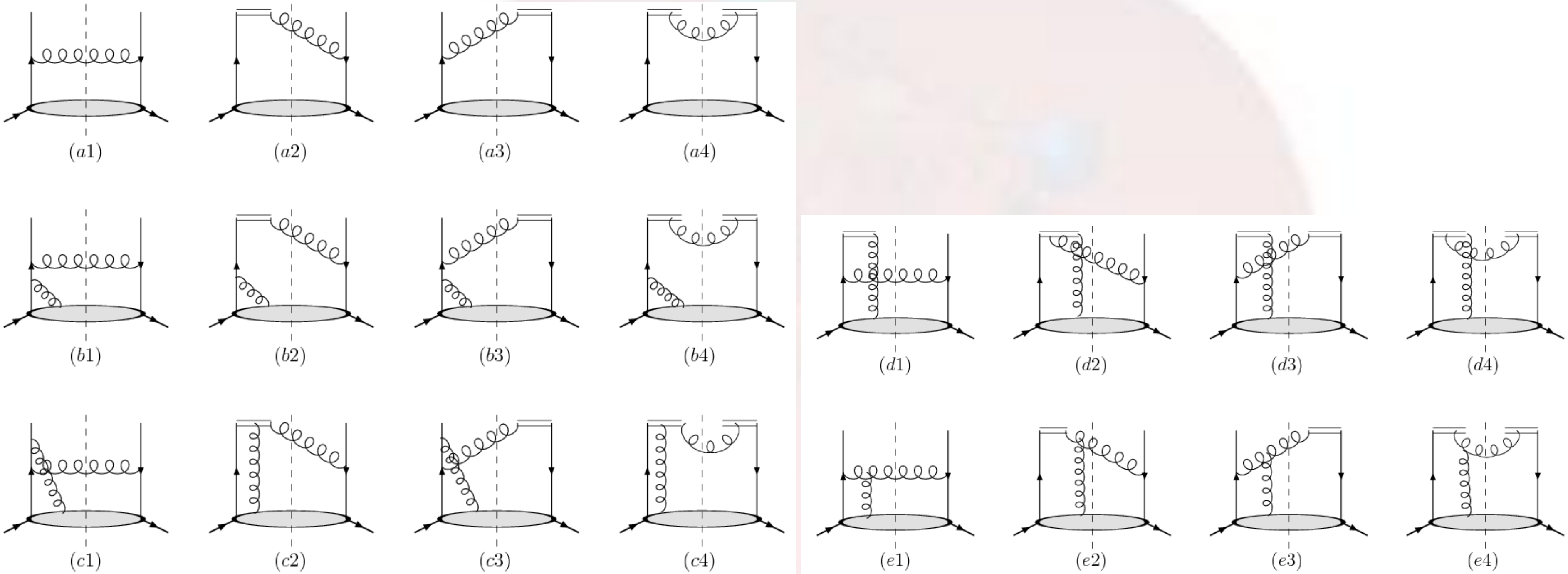
R.K. Ellis et al., 82;
Qiu-Sterman, 90

Twist-three matrix

Gauge invariant twist-3 Quark-gluon correlatio functions: D- or F-type

Large kt TMDs

Zhou-Yuan-Liang 09



- Color factors,
  C_F: a1-4,b1-4,c2,c4
  $1/2N_c$: c1,c3, d1-4
  $C_A/2$: e1-4

- $\langle \psi \partial_\perp \psi \rangle$ a1-4
- $\langle \bar{\psi} A_\perp \psi \rangle$ b1-4,c1,c3,e1-4
- $\langle \bar{\psi} \partial_\perp A^+ \psi \rangle$ b1-4,c1-4,d1-4,e1-4

Generic results

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- Kt-even TMDs

$$f_1(x_B,k_\perp) = \frac{\alpha_s}{2\pi^2}\frac{1}{\vec{k}_\perp^2}C_F\int\frac{dx}{x}f_1(x)\left[\frac{1+\xi^2}{(1-\xi)_+}+\delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2}-1\right)\right]$$

$$g_{1L}(x_B,k_\perp) = \frac{\alpha_s}{2\pi^2}\frac{1}{\vec{k}_\perp^2}C_F\int\frac{dx}{x}g_{1L}(x)\left[\frac{1+\xi^2}{(1-\xi)_+}+\delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2}-1\right)\right]$$

$$h_1(x_B,k_\perp) = \frac{\alpha_s}{2\pi^2}\frac{1}{\vec{k}_\perp^2}C_F\int\frac{dx}{x}f_1(x)\left[\frac{2\xi}{(1-\xi)_+}+\delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2}-1\right)\right]$$

Splitting kernel

Large logs

Sivers and Boer-Mulders

$$f_{1T}^{\perp}|_{\rm DY}(x_B,k_\perp) = \frac{\alpha_s}{\pi}\frac{M^2}{(\vec{k}_\perp^2)^2}\int\frac{dx}{x}\left[A_{f_{1T}^\perp} + C_F T_F(x,x)\delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2}-1\right)\right]$$

$$h_1^{\perp}|_{\rm DY}(x_B,k_\perp) = \frac{\alpha_s}{\pi}\frac{M^2}{(\vec{k}_\perp^2)^2}\int\frac{dx}{x}\left[A_{h_1^\perp} + C_F T_F^{(\sigma)}(x,x)\delta(1-\xi)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2}-1\right)\right]$$

$$A_{f_{1T}^\perp} = -\frac{1}{2N_c}T_F(x,x)\frac{1+\xi^2}{(1-\xi)_+} + \frac{C_A}{2}T_F(x,x_B)\frac{1+\xi}{(1-\xi)_+} + \frac{C_A}{2}\tilde{T}_F(x_B,x)$$

$$A_{h_1^\perp} = -\frac{1}{2N_c}T_F^{(\sigma)}(x,x)\frac{2\xi}{(1-\xi)_+} + \frac{C_A}{2}T_F^{(\sigma)}(x,x_B)\frac{2}{(1-\xi)_+}\ .$$

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- g_{1T} and h_{1L}

$$g_{1T}(x_B,k_\perp) = \frac{\alpha_s}{\pi^2}\frac{M^2}{(k_\perp^2)^2}\int\frac{dx}{x}\left\{A_{g_{1T}} + C_F\tilde{g}(x)\delta(\xi-1)\left(\ln\frac{x_B^2\zeta^2}{k_\perp^2}-1\right)\right\}$$

$$h_{1L}(x_B,k_\perp) = \frac{\alpha_s}{\pi^2}\frac{M^2}{(k_\perp^2)^2}\int\frac{dx}{x}\left\{A_{h_{1L}} + C_F\tilde{h}(x)\delta(\xi-1)\left(\ln\frac{x_B^2\zeta^2}{k_\perp^2}-1\right)\right\}$$

$$\begin{aligned} A_{g_{1T}} &= \int dx_1\left\{\frac{1}{2N_C}\tilde{g}(x)\frac{1+\xi^2}{(1-\xi)_+}\delta(x_1-x)\right. \\ &+ \left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{x_1}-\frac{2x_B^2}{x_1 x}-\frac{x_B}{x}-1\right)+\frac{C_A}{2}\frac{(x_B^2+xx_1)(2x_B-x-x_1)}{(x_B-x_1)(x-x_1)x_1}\right]\tilde{G}_D(x,x_1 \\ &+ \left.\left[C_F\left(\frac{x_B^2}{x^2}+\frac{x_B}{x_1}-\frac{x_B}{x}-1\right)+\frac{C_A}{2}\frac{x_B^2-xx_1}{(x_1-x_B)x_1}\right]G_D(x,x_1)\right\} \qquad (\end{aligned}$$

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Conclusions

- TMDs are important tool to investigate the partonic structure of nucleon/nucleus, and the associated QCD dynamics
- Although complicated, the evolution effects have been well understood
  - Provide solid ground for phen. Applications
  - Unique place to study QCD

talks on Thursday