Studies of TMD resummation and evolution

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Outline:

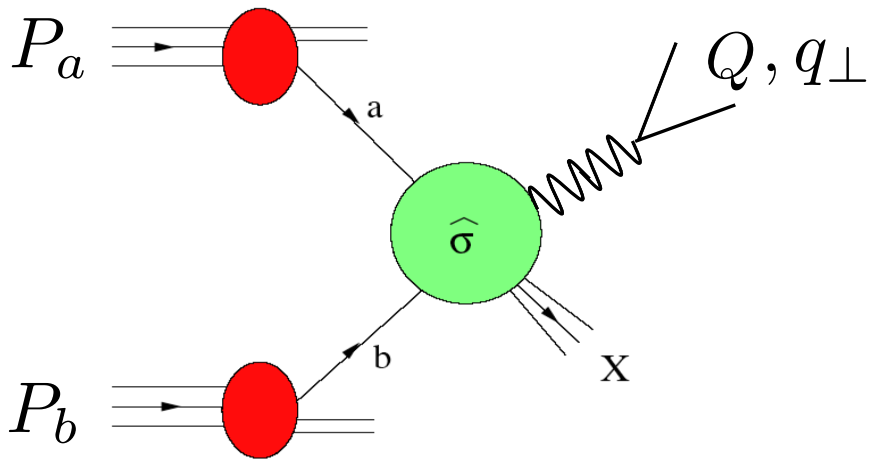
- Resummation for color-singlet processes
- Contact with TMD evolution
- Phenomenology
- Conclusions

Earlier work with A. Kulesza, E. Laenen, G. Sterman;
J. Nagashima, Y. Koike

Work in progress with M. Lambertsen and M. Schlegel

Resummation for color-singlet processes

Collinear factorization:
e.g. Drell-Yan



$$d\sigma \approx \sum_{ab} \int dx_a \int dx_b\, f_a(x_a,\mu)\; f_b(x_b,\mu)\; d\hat{\sigma}_{ab}\left(x_a P_a, x_b P_b, \alpha_s(\mu), \mu, Q, q_\perp, \ldots\right)$$

- especially $\dfrac{d\sigma}{dQ^2}, \; \dfrac{d\sigma}{dQ^2 d^2 q_\perp}$

- partonic cross sections: pQCD

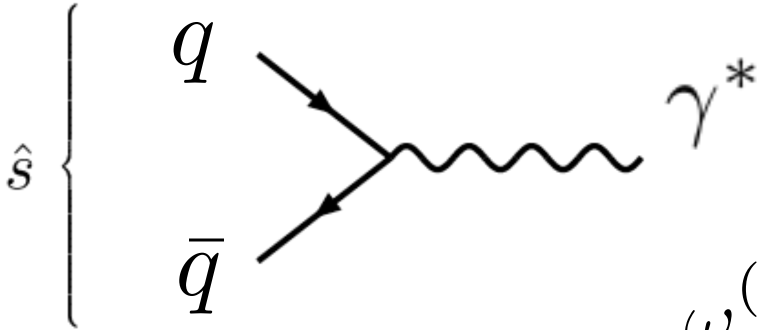
$$d\hat{\sigma}_{ab} = d\hat{\sigma}_{ab}^{(0)} + \frac{\alpha_s}{2\pi} d\hat{\sigma}_{ab}^{(1)} + \ldots$$

- sometimes, large (double-)logarithmic corrections to $d\hat{\sigma}_{ab}^{(k)}$

- first example:

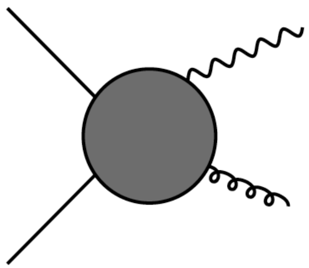
$$Q^4 \frac{d\sigma}{dQ^2} = \sum_{ab} \int dx_a dx_b \, f_a(x_a, \mu) \, f_b(x_b, \mu) \, \omega_{ab} \left(z = \frac{Q^2}{\hat{s}}, \alpha_s(\mu), \frac{Q}{\mu} \right) + \ldots$$

- LO :



$$\omega^{(0)}_{q\bar{q}} \propto \delta(1-z)$$

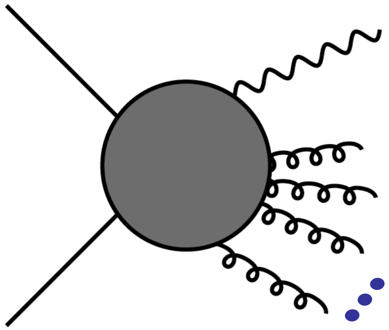
- NLO correction:



$$z \longrightarrow 1 :$$

$$\omega^{(1)}_{q\bar{q}} \propto \alpha_s \left(\frac{\log(1-z)}{1-z} \right)_+ + \ldots$$

- yet higher orders:



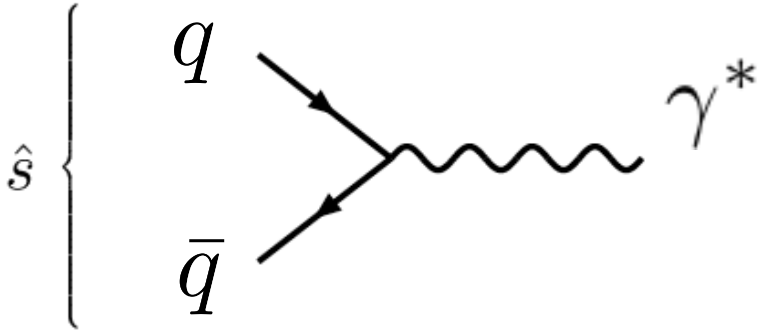
$$\omega_{q\bar{q}}^{(k)} \propto \alpha_s^k \left(\frac{\log^{2k-1}(1-z)}{1-z} \right)_+ + \ldots$$

“threshold logarithms”

- for z → 1 real radiation inhibited / exclusive boundary

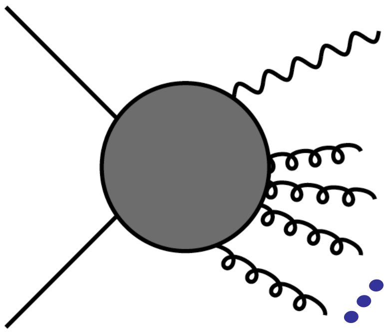
- second example: $\dfrac{d\sigma}{dQ^2\, d^2 q_\perp}$

  - LO :



$$\frac{d\hat{\sigma}^{(0)}_{q\bar{q}}}{d^2 q_\perp} \propto \delta^{(2)}(\vec{q}_\perp)$$

- higher orders:



$$\frac{d\hat{\sigma}^{(k)}_{q\bar{q}}}{d^2 q_\perp} \propto \alpha_s^k \left(\frac{\log^{2k-1}(q_\perp^2/Q^2)}{q_\perp^2} \right)_+$$

"q_T logarithms"

- close correspondence with TMD evolution

Large logs can be resummed to all orders directly from perturbative diagrams

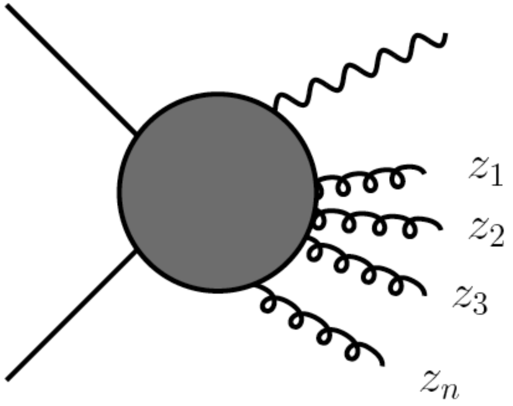
- originate from soft / collinear gluon emission
- QCD matrix elements simplify, particularly so for color-singlet processes
- near threshold, exponentiation of eikonal diagrams

Gatherall; Franklin,Taylor; Sterman; …

- for symmetric multi-gluon phase space
- in the following, Drell-Yan as example. Easily extended to SIDIS, e^+e^-

Sterman, WV

- total Drell-Yan cross section:

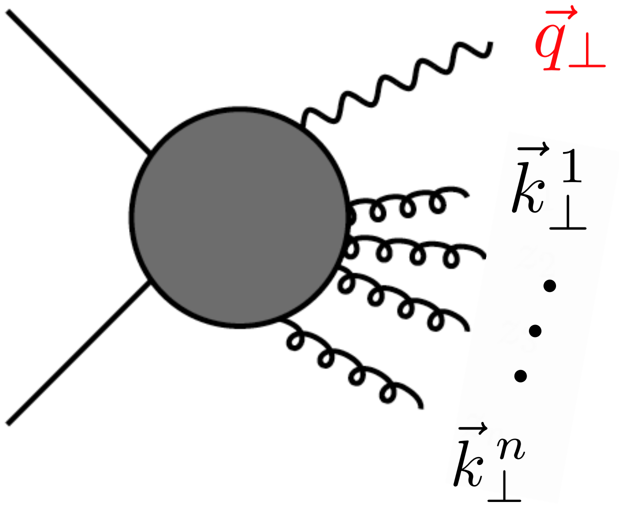


$$\delta\Big(1 - z - \sum_j z_j\Big) = \frac{1}{2\pi i}\int_C dN\, \mathrm{e}^{N(1-z-\sum_j z_j)}$$

$$z_i = \frac{2E_i}{\sqrt{\hat{s}}}$$

$$\left(\frac{\log^{2k-1}(1-z)}{1-z}\right)_+ \leftrightarrow \log^{2k}(N) + \ldots$$

- q_T-differential cross section:



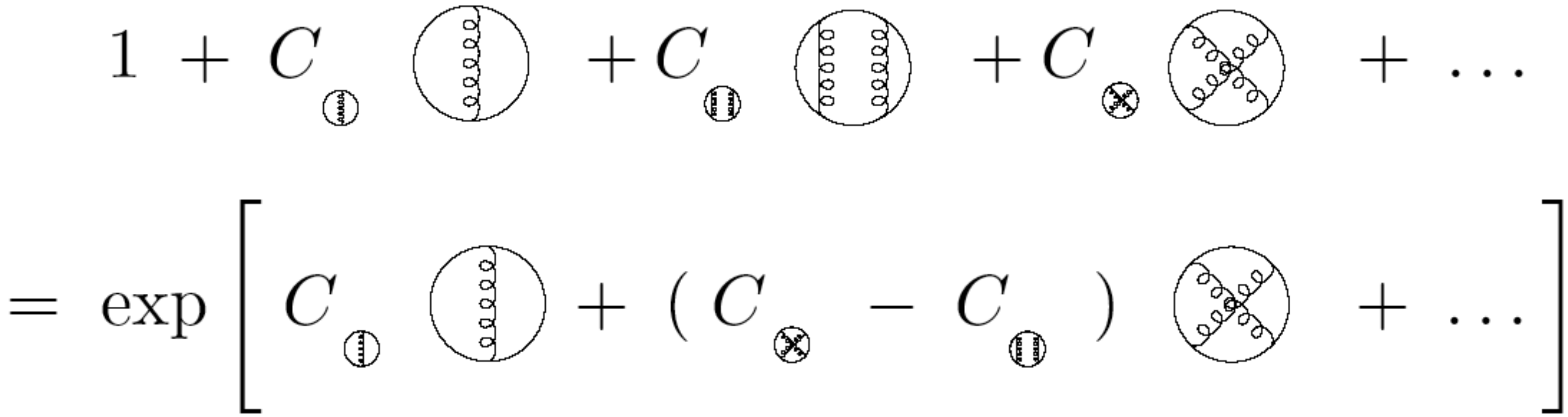
$$\delta\Big(\vec{q}_\perp + \sum_j \vec{k}_\perp^{\,j}\Big) = \frac{1}{(2\pi)^2}\int d^2b\, \mathrm{e}^{-i\vec{b}\cdot(\vec{q}_\perp + \sum_j \vec{k}_\perp^{\,j})}$$

$$\left(\frac{\log^{2k-1}(q_\perp^2/Q^2)}{q_\perp^2}\right)_+ \leftrightarrow \log^{2k}(bQ) + \ldots$$

- both transforms can be taken simultaneously Laenen,Sterman,WV

- non-abelian exponentiation:

Gatherall; Franklin, Taylor; Sterman
Berger, Sterman



$$1 + \overbrace{\alpha_s L^2 + \alpha_s^2 L^4}^{\alpha_s^k L^{2k}} + \ldots + \overbrace{\alpha_s L + \alpha_s^2 L^3}^{\alpha_s^k L^{2k-1}} + \ldots$$

$$\leftrightarrow \; \exp\Big[\underbrace{\alpha_s L^2 + \alpha_s^2 L^3}_{\alpha_s^k L^{k+1}} + \ldots + \underbrace{\alpha_s L + \alpha_s^2 L^2}_{\alpha_s^k L^k} + \ldots\Big]$$

$$\sigma^{\mathrm{eik}}(N,b) \;=\; \exp\left[\mathcal{E}^{\mathrm{eik}}(N,b,\varepsilon)\right]$$

$$\mathcal{E}^{\mathrm{eik}}(N,b,\varepsilon) \;\propto\; \int^{Q} d^{4-2\varepsilon}k \;\; \mathcal{W}(k^2,(k\cdot\beta)(k\cdot\beta'),\mu^2,\alpha_s(\mu),\varepsilon)$$

"web function"

$$\times\;\left(\mathrm{e}^{-N k^0/Q - i\,\vec{b}\cdot\vec{k}_\perp} - 1\right)$$

$$\mathcal{W} \;=\; \frac{2C_F\alpha_s}{\pi}\,\frac{1}{k_\perp^2}\,\delta(k^2) \;+\; \mathcal{O}(\alpha_s^2)$$

- after performing k⁺ integral and subtraction of collinear div.

$$\sigma^{\mathrm{eik}}(N,b) = \exp\left[2\int_{k_\perp^2<Q^2}\frac{d^{2-2\epsilon}k_\perp}{[2\pi^{1-\varepsilon}/\Gamma(1-\varepsilon)]}\left\{\int_0^{Q^2-k_\perp^2}dk^2\,\mathcal{W}\left(k^2,k_\perp^2+k^2,\mu^2,\alpha_s,\varepsilon\right)\right.\right.$$

$$\left.\left.\times\left[\mathrm{e}^{-i\,\vec{b}\cdot\vec{k}_\perp}K_0\left(2N\sqrt{\frac{k_\perp^2+k^2}{Q^2}}\right)-\ln\sqrt{\frac{Q^2}{k_\perp^2+k^2}}\right]+\frac{2}{(k_\perp^2)^{1-\varepsilon}}\ln\bar{N}\,A_q\left(\alpha_s(k_\perp)\right)\right\}\right]$$

$$(\bar{N} = N\mathrm{e}^{\gamma_E})$$

where

$$A_q(\alpha_s) = C_F\left\{\frac{\alpha_s}{\pi}+\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{C_A}{2}\left(\frac{67}{18}-\zeta(2)\right)-\frac{5}{9}T_R n_f\right]+\ldots\right\}$$

- finally, to NLL

$$\sigma^{\mathrm{eik}}(N,b) = \exp\left[2\int_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\,A_q(\alpha_s(k_\perp))\left[J_0(bk_\perp)\,K_0\left(\frac{2Nk_\perp}{Q}\right)+\ln\left(\frac{\bar{N}k_\perp}{Q}\right)\right]\right]$$

$$\sigma^{\mathrm{eik}}(N,b) \;=\; \exp\left[2\int_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\,A_q(\alpha_s(k_\perp))\left[J_0(bk_\perp)\,K_0\left(\frac{2Nk_\perp}{Q}\right)+\ln\left(\frac{\bar{N}k_\perp}{Q}\right)\right]\right]$$

- "jointly resummed" cross section: Laenen,Sterman,WV

$N \gg bQ:$ threshold logs (e.g. b=0)

$bQ \gg N:$ q_T logs

- for the latter case:

$$\sigma^{\mathrm{eik}}(N,b) \;\approx\; \exp\left[-2\int_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\,A_q(\alpha_s(k_\perp))\left(J_0(bk_\perp)-1\right)\,\ln\left(\frac{\bar{N}k_\perp}{Q}\right)\right]$$

$$-2\int_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\,A_q(\alpha_s(k_\perp))\left(J_0(bk_\perp)-1\right)\,\ln\left(\frac{\bar{N}k_\perp}{Q}\right)$$

- vanishes at b=0

- $\left(J_0(bk_\perp)-1\right)$ cuts off integral at $k_\perp \sim \dfrac{2\mathrm{e}^{-\gamma_E}}{b}$

- write exponent as

$$2\int_{Q^2/\eta^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\,A_q(\alpha_s(k_\perp))\,\ln\left(\frac{\bar{N}k_\perp}{Q}\right)$$

$$\eta^2 \equiv \left(\frac{bQ}{2\mathrm{e}^{-\gamma_E}}\right)^2 + 1$$

Bozzi, Catani, de Florian, Grazzini; Laenen,Sterman,WV

$$2\int_{Q^2/\eta^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2}\, A_q(\alpha_s(k_\perp))\, \ln\left(\frac{\bar{N}k_\perp}{Q}\right)$$

- write as

$$-\int_{Q^2/\eta^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \left[A_q(\alpha_s(k_\perp))\, \ln\left(\frac{Q^2}{k_\perp^2}\right) + B_q(\alpha_s(k_\perp))\right]$$
$$+\int_{Q^2/\eta^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \left[2A_q(\alpha_s(k_\perp))\, \ln\bar{N} + B_q(\alpha_s(k_\perp))\right]$$

where $B_q(\alpha_s) = -\frac{3}{2}C_F\,\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)$

$$- \underbrace{\int_{Q^2/\eta^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \left[A_q(\alpha_s(k_\perp)) \ln\left(\frac{Q^2}{k_\perp^2}\right) + B_q(\alpha_s(k_\perp)) \right]}_{\text{“standard” Sudakov exponent}}$$

$$+ \int_{Q^2/\eta^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \underbrace{\left[2A_q(\alpha_s(k_\perp)) \ln \bar{N} + B_q(\alpha_s(k_\perp)) \right]}_{\approx\ -\frac{\alpha_s}{\pi} P_{qq}^N}$$

→ DGLAP evolution of PDFs from μ = Q to Q/η

- matches standard CSS result

$$\frac{d\sigma}{dQ^2\, d^2q_\perp} \sim \int \frac{dN}{2\pi i}\, \tau^{-N} \int \frac{d^2b}{(2\pi)^2}\, e^{i\vec{q}_\perp\cdot\vec{b}}\ \ f_q^N(\mu = Q/\eta)\, \mathrm{e}^{-\mathcal{S}(b,Q)}\, f_{\bar{q}}^N(\mu = Q/\eta)$$

- can be systematically extended (Y-term, qg contribution,…)

- emphasize: exponent vanishes at b=0

- nonperturbative contributions?

$$-2\int_0^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\,A_q(\alpha_s(k_\perp))\left(J_0(bk_\perp)-1\right)\,\ln\left(\frac{\bar{N}k_\perp}{Q}\right)$$

$$\longrightarrow\quad -b^2\,\frac{C_F}{2\pi}\int_0 dk_\perp^2\,\alpha_s(k_\perp)\,\ln\left(\frac{Q}{\bar{N}k_\perp}\right)\;+\;\mathcal{O}(b^4)$$

- suggests form

universal

$$\mathcal{S}^{\rm NP}\;=\;-\left[g_1+g_2\log\left(\frac{Q}{M}\right)\right]b^2\;+\;\mathcal{O}(b^4)$$

- for "joint" resummation:

$$\left(-b^2+\frac{4N^2}{Q^2}\right)\frac{C_F}{2\pi}\int_0 dk_\perp^2\,\alpha_s(k_\perp)\,\ln\left(\frac{Q}{\bar{N}k_\perp}\right)\;+\;\mathcal{O}(b^4)$$

Contact with TMD evolution

Ji, Ma, Yuan; Collins; Mert Aybat, Rogers,…

hard coefficient

$$\frac{d\sigma^{\mathrm{TMD}}}{dQ^2\, d^2 q_\perp} = \sum_{q,\bar{q}} \mathcal{H}_{q\bar{q}} \int_{C_N} \frac{dN}{2\pi i}\, \tau^{-N} \int \frac{d^2 b}{(2\pi)^2}\, e^{i\vec{q}_\perp \cdot \vec{b}}\, f_q(N,b,Q)\, f_{\bar{q}}(N,b,Q)$$

- comparison to resummation formula yields

$$\begin{aligned} f_q(N,b,Q) &= \exp\left\{ -\frac{1}{2} \int_{Q^2/\eta^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \left[A_q(\alpha_s(k_\perp)) \ln\left(\frac{Q^2}{k_\perp^2}\right) + B_q(\alpha_s(k_\perp)) \right] \right\} \\ &\times \exp\left\{ -\frac{1}{2} \left[g_1 + g_2 \log\left(\frac{Q}{M}\right) \right] b^2 \right\} \\ &\times \exp\left\{ \int_{\mu_F^2}^{Q^2/\eta^2} \frac{dk_\perp^2}{k_\perp^2} \frac{\alpha_s(k_\perp)}{2\pi} P_{qq}^N \right\} f_q^N(\mu_F) \end{aligned}$$

- probably best for use in phenomenology

Sun, Yuan; Echeverria et al.

Mert Aybat, Prokudin, Rogers
Anselmino et al.

- alternatively, relate to $f_q(N,b,Q_0)$:

$$f_q(N,b,Q) = f_q(N,b,Q_0)\,\exp\left\{-\frac{1}{2}\int_{Q^2/\eta^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\left[A_q(\alpha_s(k_\perp))\,\ln\left(\frac{Q^2}{k_\perp^2}\right)+B_q(\alpha_s(k_\perp))\right]\right\}$$

$$\times\,\exp\left\{+\frac{1}{2}\int_{Q_0^2/\eta_0^2}^{Q_0^2}\frac{dk_\perp^2}{k_\perp^2}\left[A_q(\alpha_s(k_\perp))\,\ln\left(\frac{Q_0^2}{k_\perp^2}\right)+B_q(\alpha_s(k_\perp))\right]\right\}$$

$$\times\,\exp\left\{-\frac{1}{2}\,g_2\log\left(\frac{Q}{Q_0}\right)b^2\right\}\,\exp\left\{\int_{Q_0^2/\eta_0^2}^{Q^2/\eta^2}\frac{dk_\perp^2}{k_\perp^2}\,\frac{\alpha_s(k_\perp)}{2\pi}\,P_{qq}^N\right\}$$

$$= f_q(N,b,Q_0)\,\exp\left\{-\mathcal{S}(b,Q)+\mathcal{S}(b,Q_0)-\frac{1}{2}\,g_2\log\left(\frac{Q}{Q_0}\right)b^2\right\}$$

$$\times\,\exp\left\{\int_{Q_0^2/\eta_0^2}^{Q^2/\eta^2}\frac{dk_\perp^2}{k_\perp^2}\,\frac{\alpha_s(k_\perp)}{2\pi}\,P_{qq}^N\right\}$$

- coincides with standard result, except that

$$\eta^2 \equiv \left(\frac{bQ}{2\mathrm{e}^{-\gamma_E}}\right)^2 + 1 \qquad\qquad \eta_0^2 \equiv \left(\frac{bQ_0}{2\mathrm{e}^{-\gamma_E}}\right)^2 + 1$$

$$
\begin{aligned}
f_q(N,b,Q) &= f_q(N,b,Q_0)\ \exp\left\{-\mathcal{S}(b,Q)+\mathcal{S}(b,Q_0)-\frac{1}{2}\,g_2\log\left(\frac{Q}{Q_0}\right)b^2\right\} \\
&\quad\times\exp\left\{\int_{Q_0^2/\eta_0^2}^{Q^2/\eta^2}\frac{dk_\perp^2}{k_\perp^2}\,\frac{\alpha_s(k_\perp)}{2\pi}\,P_{qq}^N\right\}
\end{aligned}
$$

- at b=0:

$$
f_q(N,0,Q) = f_q(N,0,Q_0)\ \exp\left\{\int_{Q_0^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\,\frac{\alpha_s(k_\perp)}{2\pi}\,P_{qq}^N\right\}
$$

DGLAP evolution for $k_\perp$- integrated PDF !

Phenomenology

- NLL expansion of perturbative exponent:

$$-\frac{1}{2}\int_{Q^2/\eta^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\left[A_q(\alpha_s(k_\perp))\,\ln\left(\frac{Q^2}{k_\perp^2}\right)+B_q(\alpha_s(k_\perp))\right] = \frac{1}{\alpha_s(\mu)}h^{(0)}(\beta)+h^{(1)}(\beta)$$

$$\beta = b_0\,\alpha_s(Q)\ln\left(\eta^2\right) = b_0\,\alpha_s(Q)\ln\left(\left(\frac{bQ}{2\mathrm{e}^{-\gamma_E}}\right)^2+1\right)$$

$$\begin{aligned}
h^{(0)}(\beta) &= \frac{A_q^{(1)}}{2\pi b_0^2}\left[\beta+\ln(1-\beta)\right]\\
h^{(1)}(\beta) &= \frac{A_q^{(1)}b_1}{2\pi b_0^3}\left[\frac{1}{2}\ln^2(1-\beta)+\frac{\beta+\ln(1-\beta)}{1-\beta}\right]+\frac{B_q^{(1)}}{2\pi b_0}\ln(1-\beta)\\
&- \frac{A_q^{(2)}}{2\pi^2 b_0^2}\left[\frac{\beta}{1-\beta}+\ln(1-\beta)\right]
\end{aligned}$$

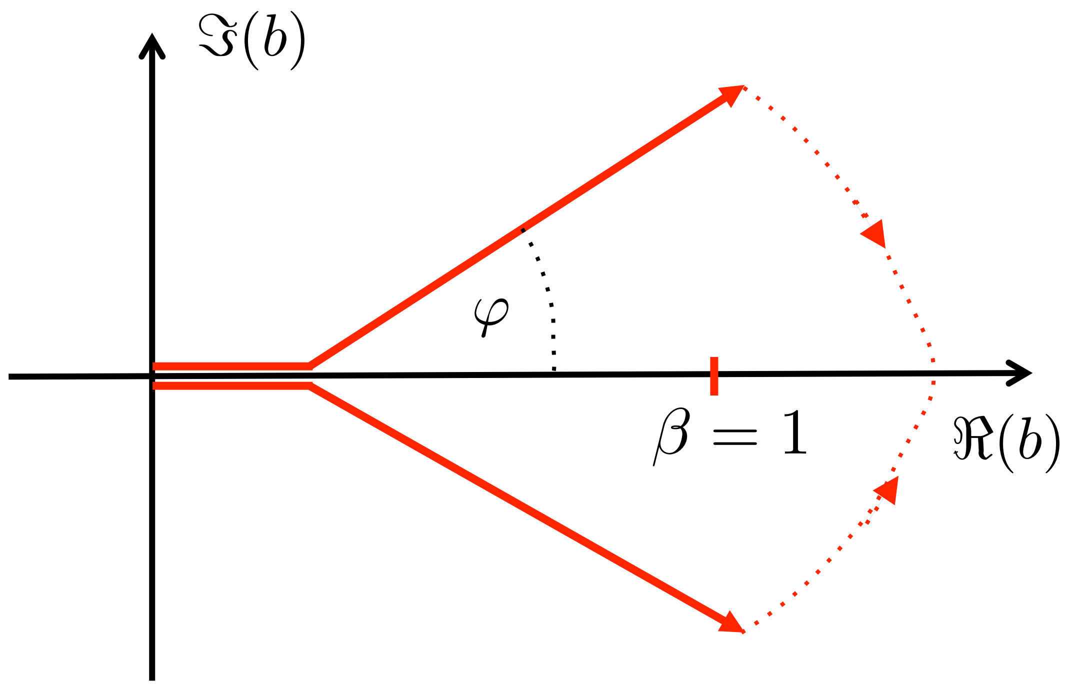
Treatment of large-b region:

- b* prescription

Collins, Soper, Sterman; …

- “contour method”

Laenen, Sterman, WV



$$2\pi \int_0^\infty db\, b\; J_0(bq_T)\, f(b) = \pi \int_0^\infty db\, b\, \left[\, h_1(bq_T, v) + h_2(bq_T, v)\,\right] f(b)$$

(h_i Hankel functions)

- “parameter free” (can be used even w/o Gaussian)

In the following, investigate:

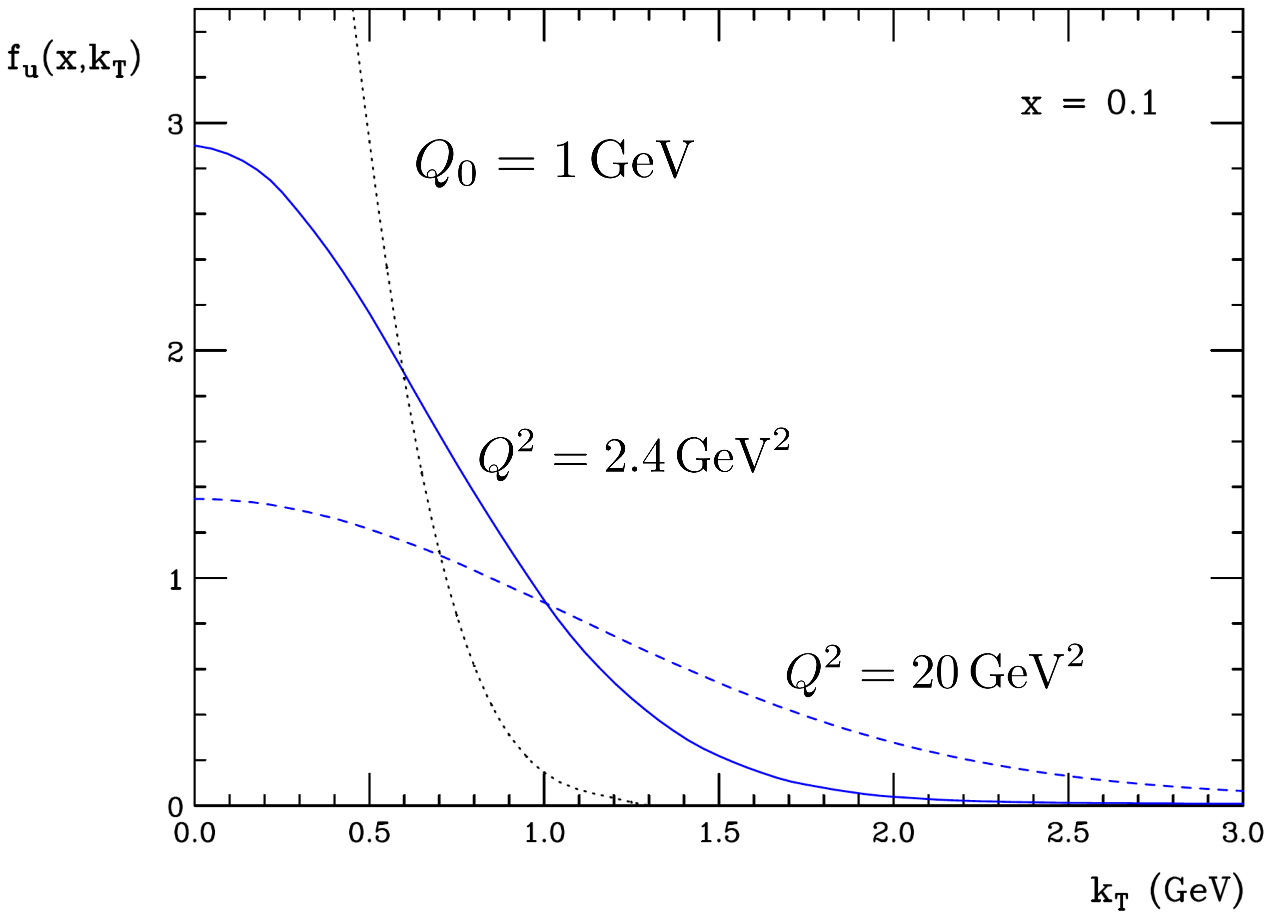
- complex-b method vs b*
- role of “boundary condition” at b=0
- choice for $f_q(N, b, Q_0)$:
  Gaussian vs “standard resummation”

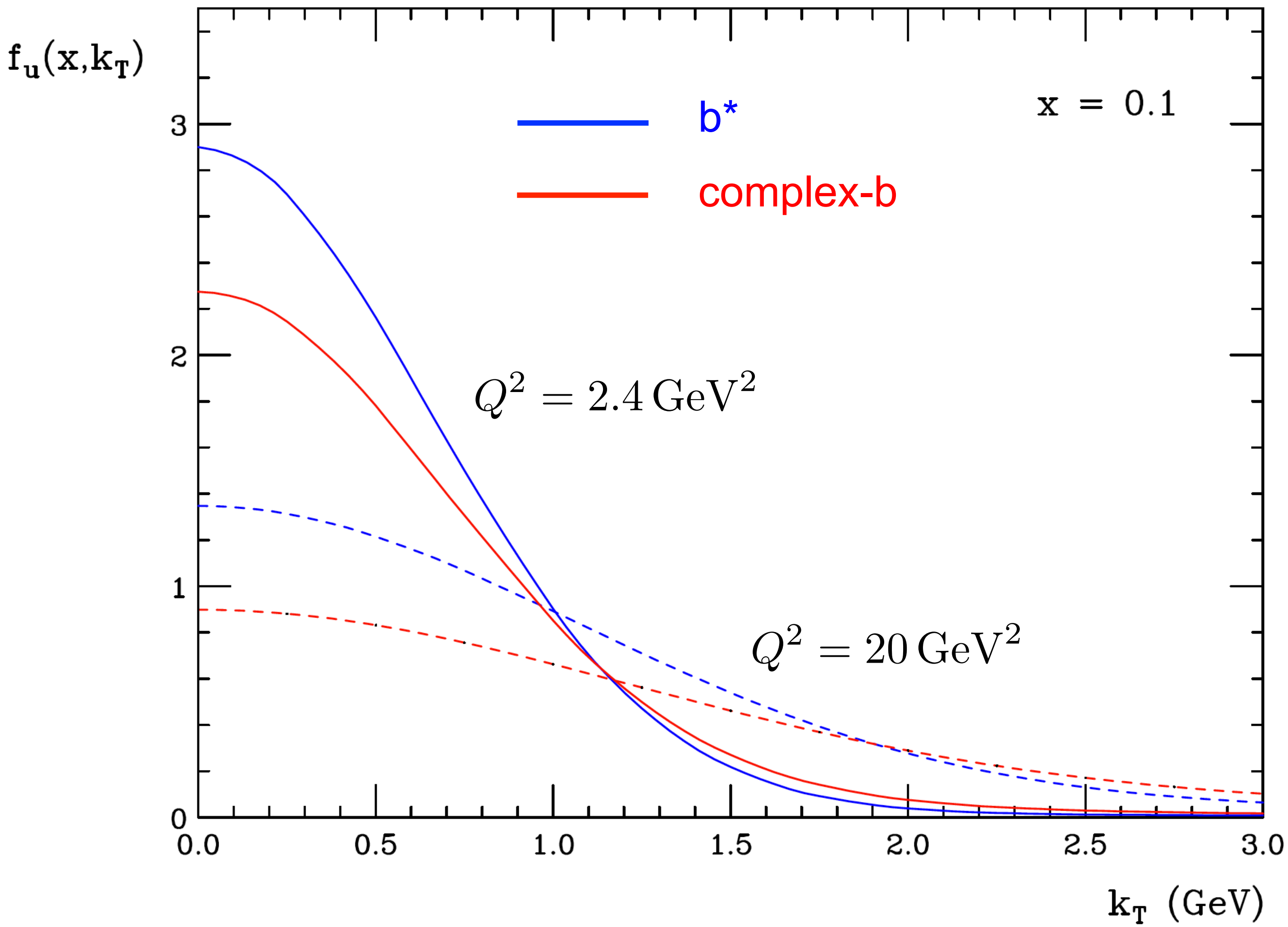
$$f_u(x, k_\perp, Q_0) \propto \exp^{-k_\perp^2/\langle k_\perp^2 \rangle} f_u^{\mathrm{CTEQ}}(x, Q_0)$$

Anselmino et al.

$\langle k_\perp^2 \rangle = 0.25\,\mathrm{GeV}^2$ $\quad g_2 = 0.68\,\mathrm{GeV}^2$ $\quad Q_0 = 1\,\mathrm{GeV}$

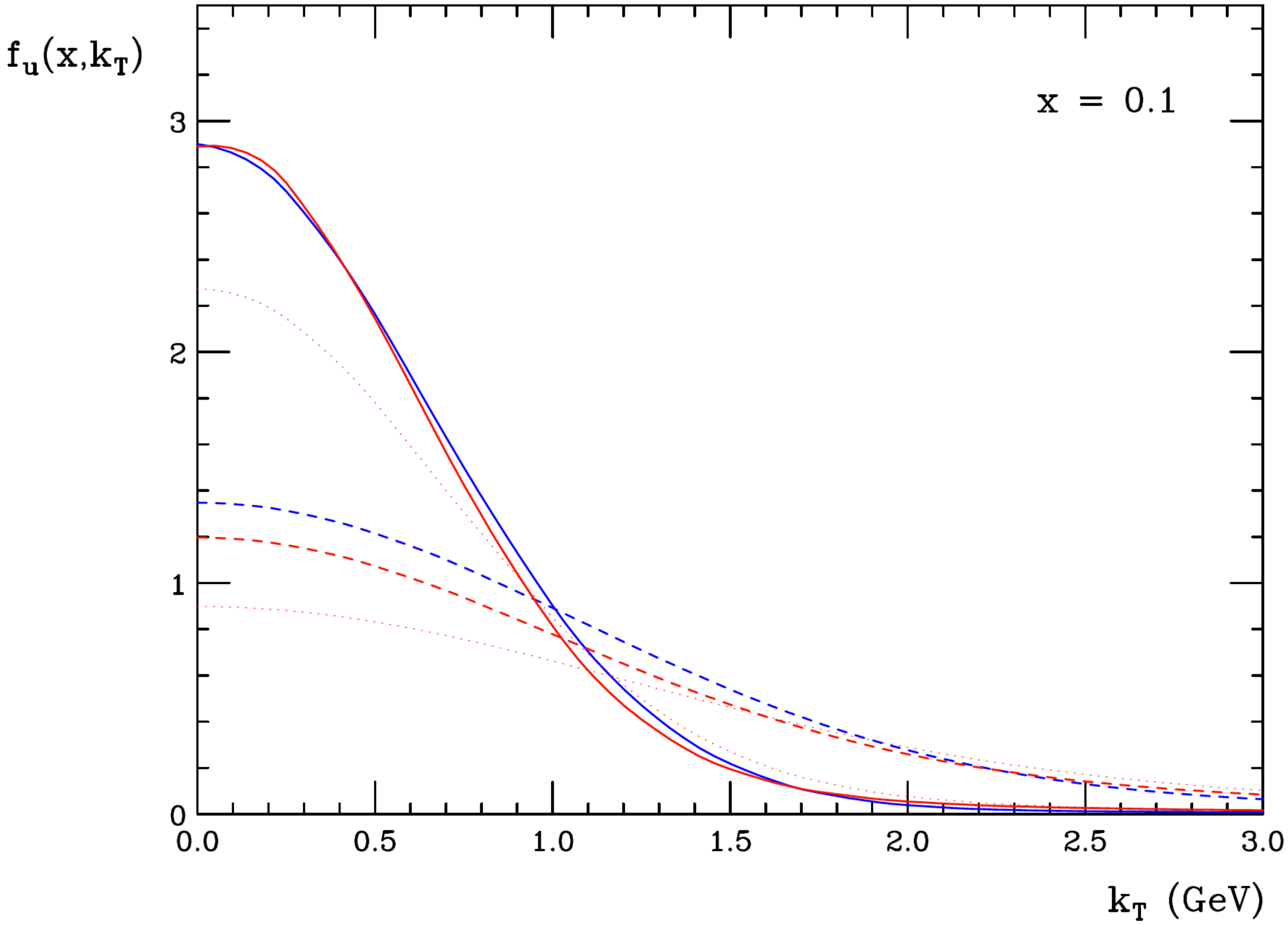
b* prescription with b_{max}=0.5 GeV^{-1} , no boundary condition



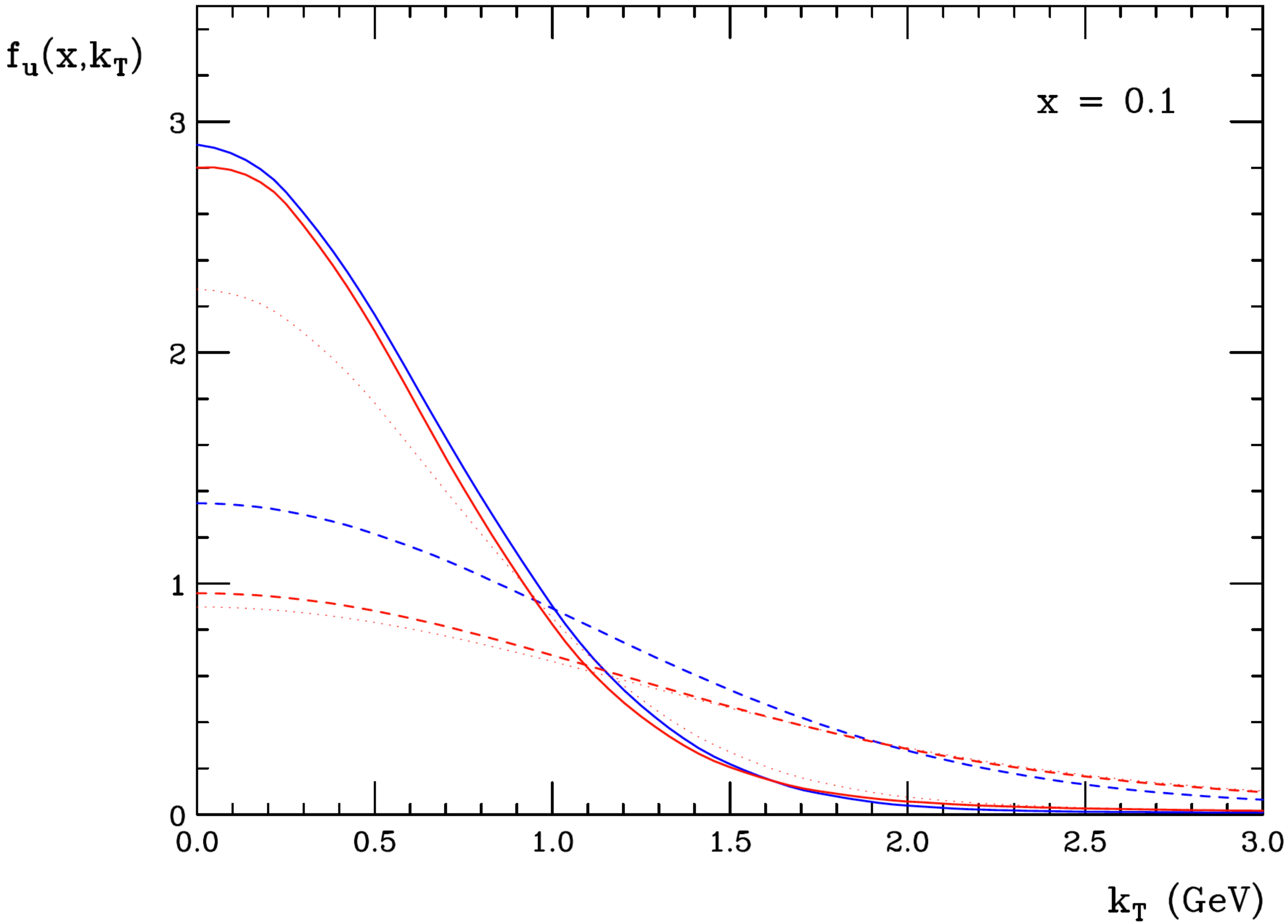


$f_u(x,k_T)$
b*
complex-b
x = 0.1
$Q^2 = 2.4\,\mathrm{GeV}^2$
$Q^2 = 20\,\mathrm{GeV}^2$
0
1
2
3
0.0
0.5
1.0
1.5
2.0
2.5
3.0
k_T (GeV)

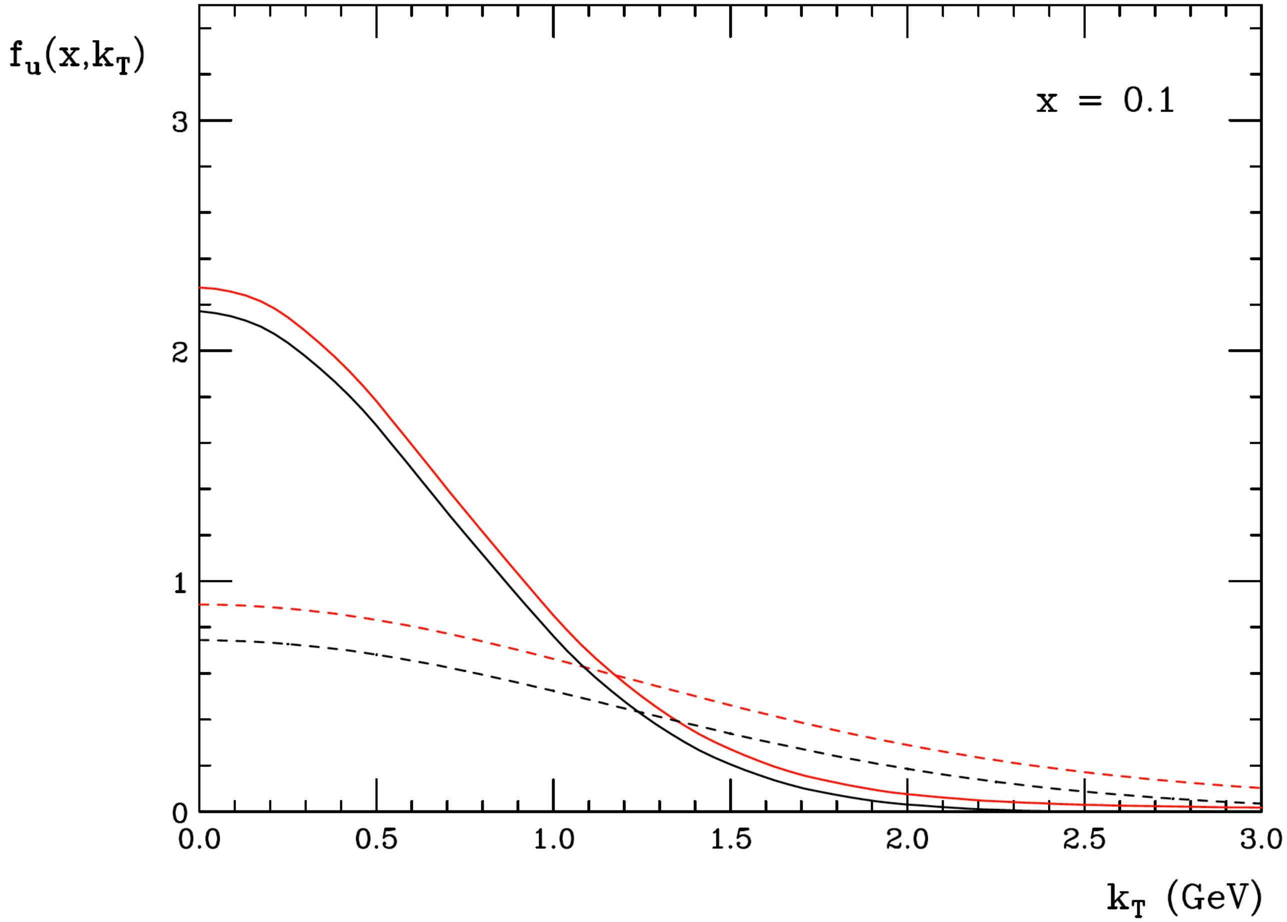
complex-b method w/ g_2=0.42 GeV^2



complex-b method with $\langle k_\perp^2 \rangle = 0.05\,\mathrm{GeV}^2$

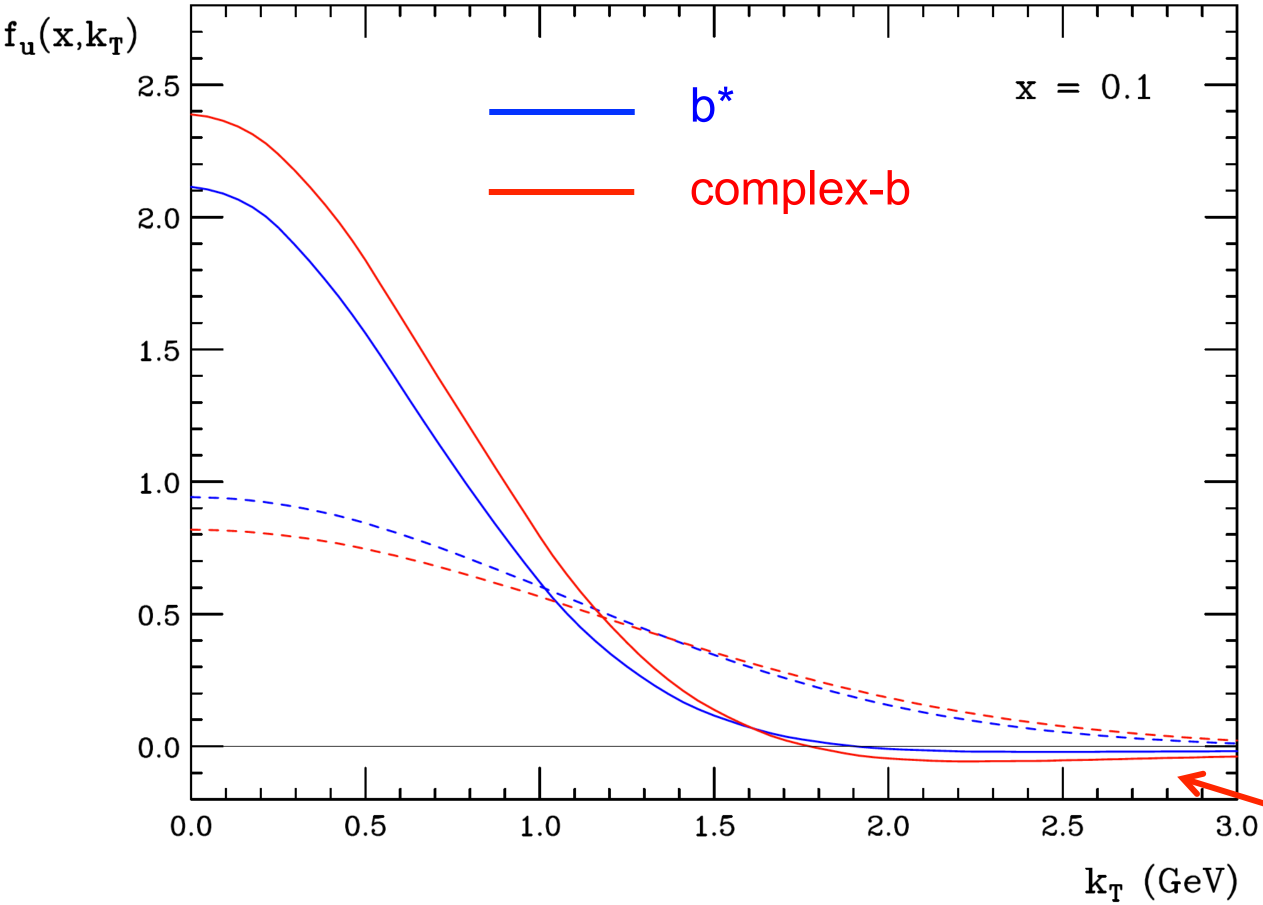


effect of “boundary condition” at b=0



“standard resummation”:

$$
\begin{aligned}
f_q(N,b,Q) &= \exp\left\{-\frac{1}{2}\int_{Q^2/\eta^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\left[A_q(\alpha_s(k_\perp))\,\ln\left(\frac{Q^2}{k_\perp^2}\right)+B_q(\alpha_s(k_\perp))\right]\right\}\\
&\times\ \exp\left\{-\frac{1}{2}\left[g_1+g_2\log\left(\frac{Q}{M}\right)\right]b^2\right\}\\
&\times\ \exp\left\{\int_{\mu_F^2}^{Q^2/\eta^2}\frac{dk_\perp^2}{k_\perp^2}\,\frac{\alpha_s(k_\perp)}{2\pi}\,P_{qq}^N\right\}\ f_q^N(\mu_F)
\end{aligned}
$$



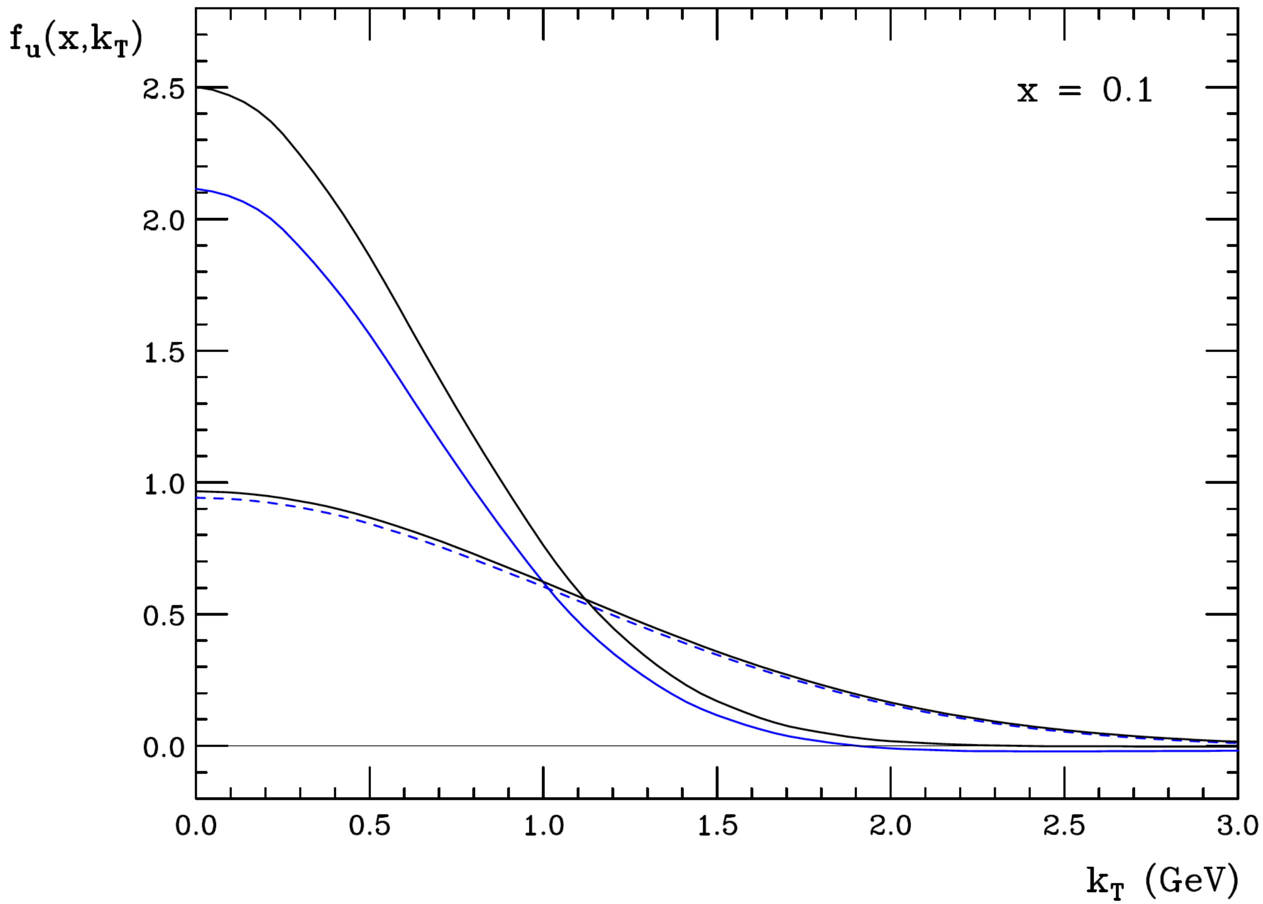
$$g_1 = 0.0625\,\mathrm{GeV}^2$$

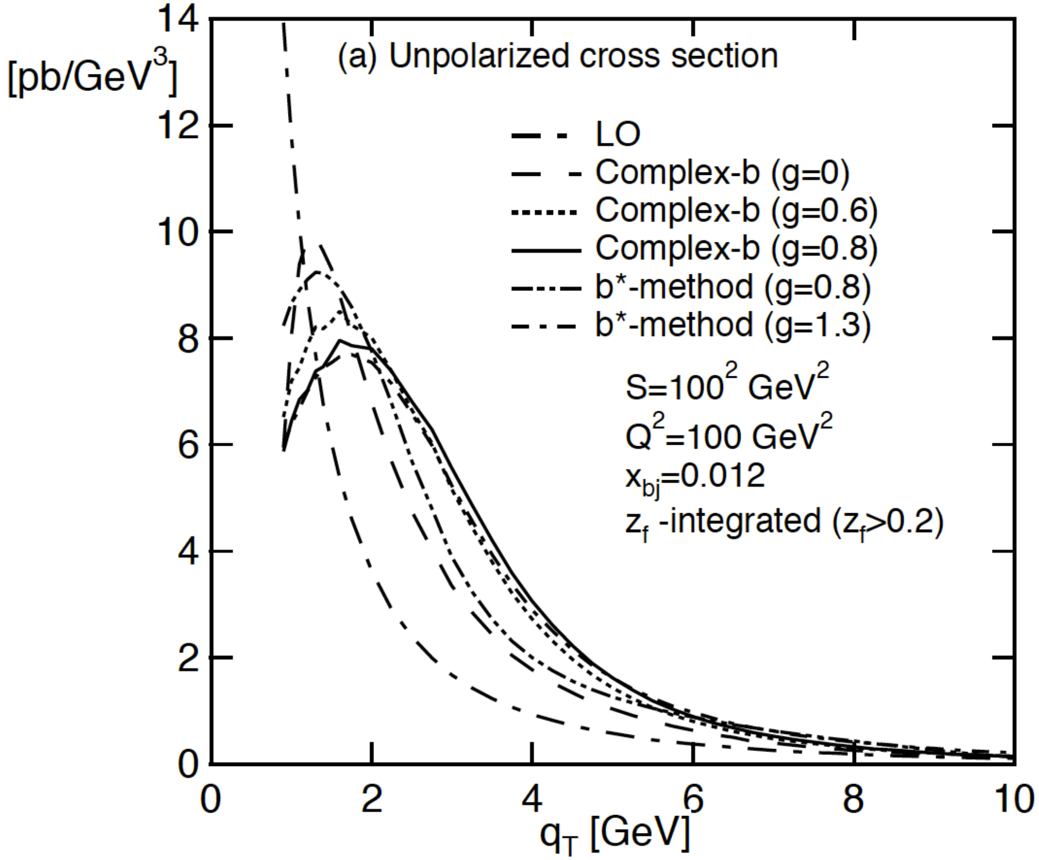
$$g_2 = 0.68\,\mathrm{GeV}^2$$

$$M = 1\,\mathrm{GeV}$$

“matching” region

effect of “boundary condition” at b=0





Koike, Nagashima, WV

Conclusions:

- complex-b method is an alternative to b*, parameter-free. Will need more detailed studies.

- role of subleading effects

- ansatz for $f_q(N, b, Q_0)$: probably "standard" resummation more useful

- "joint" resummation could be relevant in presently relevant kinematic regimes