TMD evolution: matching SIDIS to Drell-Yan

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LBNL

in collaboration with C. P. Yuan and F. Yuan

Outline

- Goal
  - Global analysis of Sivers (Collins/Transversity) at the next-to-leading logarithmic level
- Framework
  - CSS resummation in b_* prescription

QCD k_T resummation

- Consider the production process $h_1 h_2 \to Z+X$

$$\frac{d\sigma}{dQ_T^2} \sim \frac{1}{Q_T^2}\{ \alpha_S(L+1) + \alpha_S^2(L^3+L^2) + \alpha_S^3(L^5+L^4) + \alpha_S^4(L^7+L^6) + \ldots$$
$$+\alpha_S^2(L+1) + \alpha_S^3(L^3+L^2) + \alpha_S^4(L^5+L^4) + \ldots$$
$$+\alpha_S^3(L+1) + \alpha_S^4(L^3+L^2) + \ldots \}$$

$A^{(1)}, B^{(1)}$

$A^{(2)}, B^{(2)}$

$A^{(3)}, B^{(3)}$

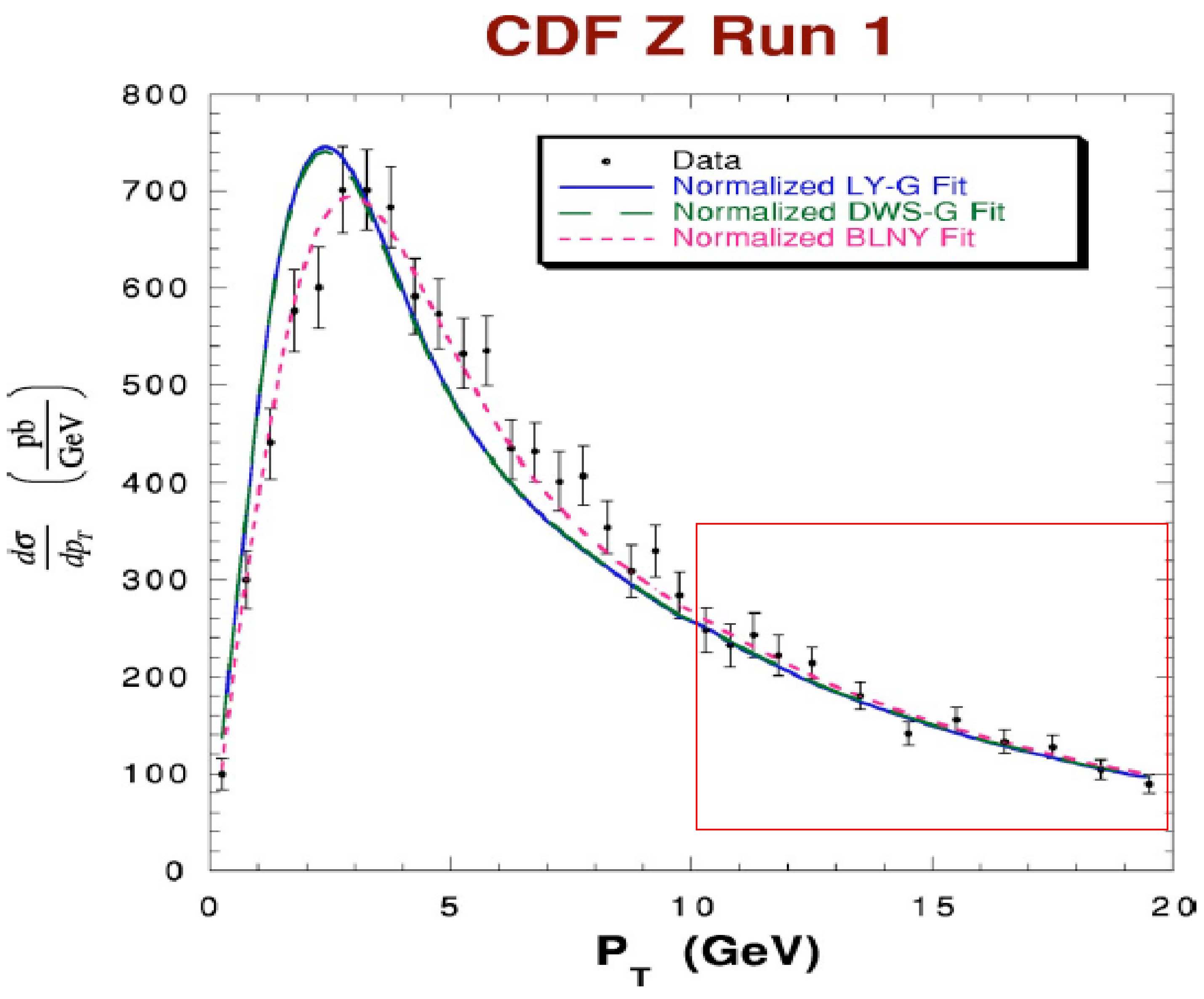
Where Q_T is the transverse momentum, and Q the mass of Z, and $L = \mathrm{Log}[Q^2 / Q_T^2]$.

- We have to resum these large logs to make reliable predictions

$$\frac{d^3\sigma(M^2, P_\perp, y)}{d^2P_\perp dy} = \sigma_0 \int \frac{d^2\vec{b}}{(2\pi)^2} e^{-iP_\perp \cdot b_\perp} W(x_1, x_2, b, M^2)$$

$$A = \sum_i (\alpha_s/\pi)^i A^{(i)}$$

$$W(Q,b) = e^{-\int_{1/b}^{Q} \frac{d\mu}{\mu}\left(\ln\frac{Q}{\mu}A + B\right)} C \otimes f_1 C \otimes f_2$$



C. P. Yuan et al PRD 67, 073016

TMD Evolution

At the small transverse momentum limitation

$$W = H(M, \mu)\, f_1(x_1, b, M, \mu)\, f_2(x_2, b, M, \mu)\, S(\lambda_T, \mu)$$

Fourier transformation

factorization scale

k_t

W satisfies CSS evolution equation

$$\frac{\partial W(x_i, b, M^2)}{\partial \ln M^2} = (K + G')W(x_i, b, M^2)$$

At one-loop order for Drell-Yan process

$$K(b,\mu) = -\frac{\alpha_s C_F}{\pi} \ln \frac{b^2\mu^2}{c_0^2} \qquad G(Q,\mu) = -\frac{\alpha_s C_F}{\pi}\left(\ln\frac{Q^2}{\mu^2} - \frac{3}{2}\right)$$

Substituting the above result into the evolution equation, and taking into account the running effects in K(b, μ)

CSS ⇒ Exp[$\int_{c_0/b}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \left[A \ln \frac{Q^2}{\bar{\mu}^2} + B \right]$]

$$\widetilde{W}_{UU}(Q;b) = e^{-S_{pert}(Q^2,b_*) - S_{NP}(Q,b)} \times \Sigma_{i,j} C_{qi}^{(DY)} \otimes f_{i/A}(z_1, \mu = c_0/b_*) C_{\bar{q}j}^{(DY)} \otimes f_{j/B}(z_2, \mu = c_0/b_*)$$

$C_0 = 2e^{-\gamma} \approx 1$

$e^{-S_{pert}(Q^2,b_*) - S_{NP}(Q,b)}$ → NP part

perturbative part:

$$S_{pert}(Q, b_*) = \int_{c_0/b_*}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \left[A \ln \frac{Q^2}{\bar{\mu}^2} + B \right]$$

S_{pert} is universal

where $A^1 = C_F \times \alpha_s(\bar{\mu})/\pi$, $B^1 = 3/2 \times \alpha_s(\bar{\mu})/\pi$

b_* prescription (CSS, 85) in S_{pert}

$$b \Rightarrow b_* = b/\sqrt{1 + b^2/b_{max}^2} \ , \quad b_{max} < 1/\Lambda_{QCD}$$

Sudakov factor

- There are two parts in the Sudakov factor

$$S_{sud} \Rightarrow S_{pert}(Q;b_*) + S_{NP}(Q;b)$$

- the nonperturbative part

$$S_{NP}(Q,b) = g_2(b)\ln Q + g_1(b;z_1,z_2)$$
$$S_{NP}^T(Q,b) = g_2(b)\ln Q + g_1^T(b;z_1,z_2)$$

This term is from

$$K(b,\mu) = -\frac{\alpha_s C_F}{\pi}\ln\frac{b^2\mu^2}{c_0^2}$$

$\mu = c_0/b^*$

Q dependence always satisfies CSS equation.

$g_2(b)$ is universal in Drell-Yan, SIDIS, and $e^+e^- \rightarrow hh$

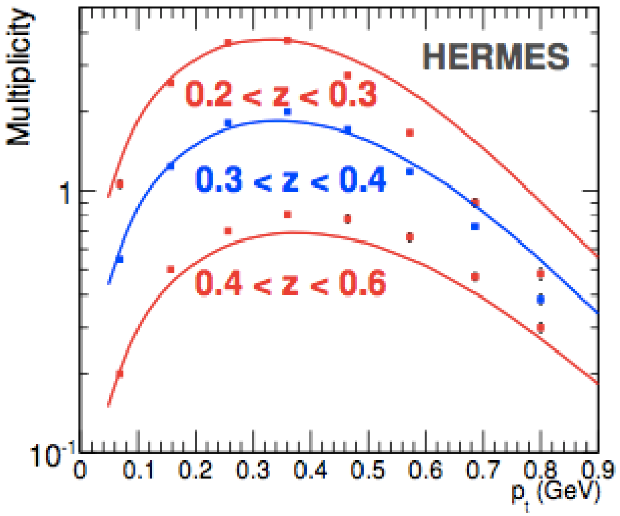
- Gaussian assumptions are usually made for $g_1(b)$ and $g_2(b)$ (BLNY 2002):

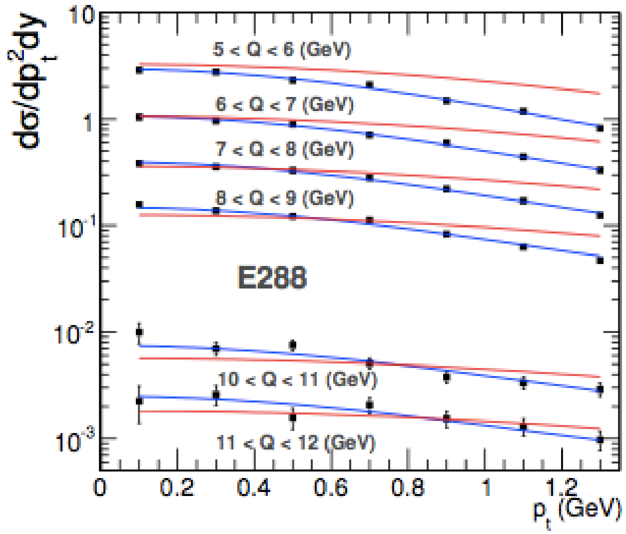
$$S_{NP} = g_1 b^2 + g_2 b^2 \ln(Q/3.2) + g_1 g_3 b^2 \ln(100 x_1 x_2)$$

$$g_1 = 0.21, \quad g_2 = 0.68, \quad g_1 g_3 = -0.2, \quad \text{with} \quad b_{max} = 0.5 \text{GeV}^{-1}$$

- However, these assumptions do not work for SIDIS and Drell-Yan simultaneously in the range of Q^2~(3—100)GeV^2

Sun,Yuan,1308.5003





CT10 and DSS are used here, so are our other fittings.

Sun,Yuan, 1304.5037, 1308.5003

- Sun-Yuan (1308.5003) has shown that direct integration of the evolution kernel from low to high Q can describe both Drell-Yan and SIDIS data
  - This suggests that Log(b) maybe a good choice for $g_2(b)$.

$$\frac{d\sigma}{dy\,dQ^2\,d^2q_\perp} = \sigma_0^{(\mathrm{DY})}\left[\int \frac{d^2b}{(2\pi)^2} e^{i\vec{q}_\perp\cdot\vec{b}}\,\widetilde{W}_{UU}(Q;b) + Y_{UU}(Q;q_\perp)\right]$$

The leading power of $q_\perp/Q$

The higher order correction of $q_\perp/Q$

So, Y piece should be important in small Q region

Fitting Drell-Yan processes

$$S_{NP} = g_1 b^2 + g_2 \ln(b/b_*) \ln(Q/Q_0) + g_3 b^2 \left((x_0/x_1)^\lambda + (x_0/x_2)^\lambda \right)$$

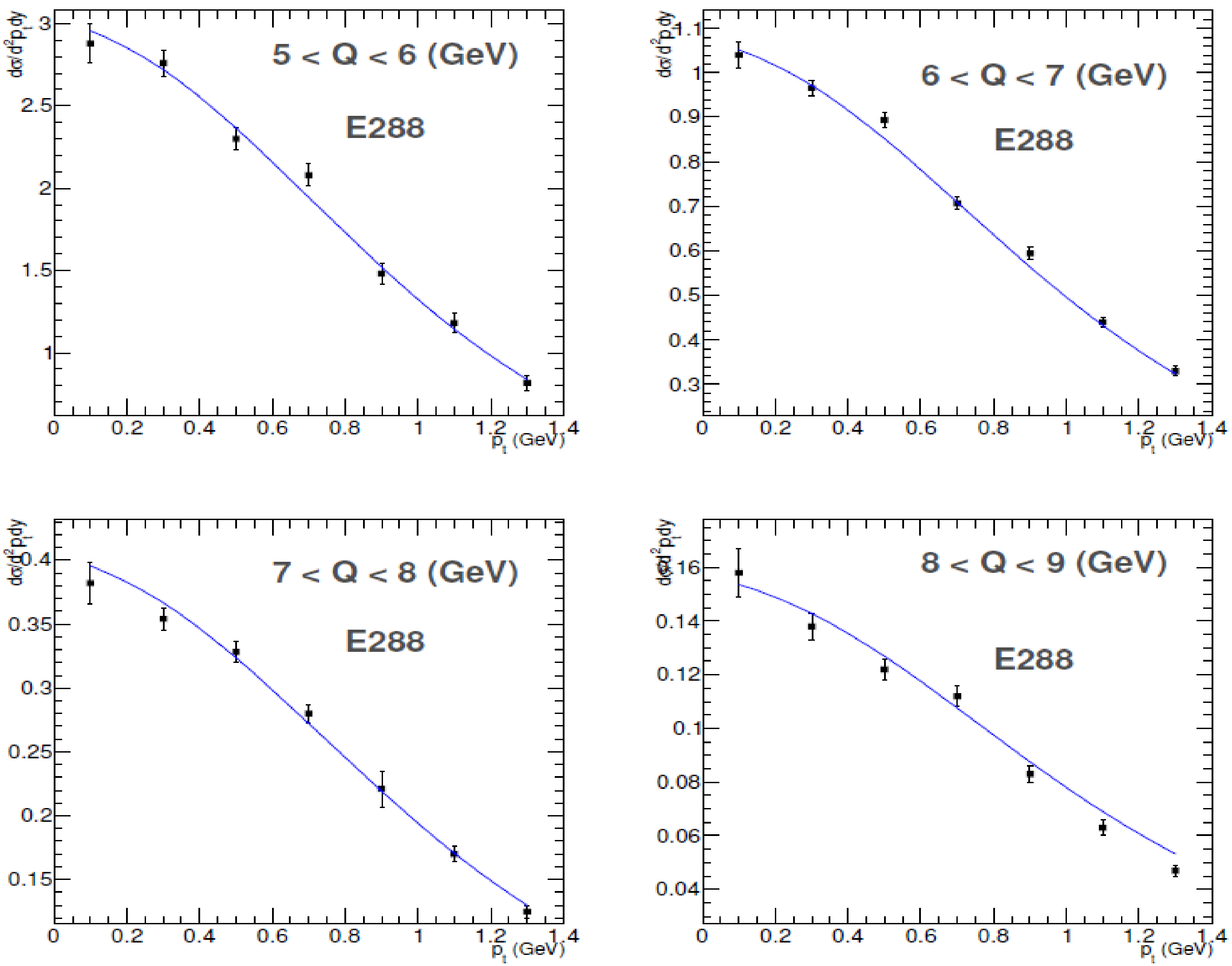
Parameter	SYY fit
g_1	0.140±0.013
g_2	0.925±0.018
g_3	0.032±0.007
E288 (28 points)	$N_{fit} = 0.97$ $\chi^2 = 44$
E605 (35 points)	$N_{fit} = 1.02$ $\chi^2 = 62$
Tevatron (30 points)	$N_{fit} = 1.02$ $\chi^2 = 32$
χ^2	138
χ^2/DOF	1.60

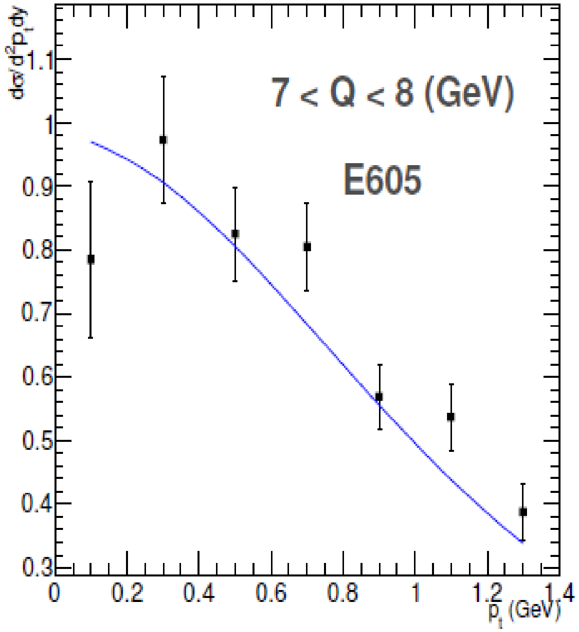
A new non-perturabtive Sudakov factor is used.
Where x_0=0.01, Q_0^2=2.4GeV^2, and λ =0.2

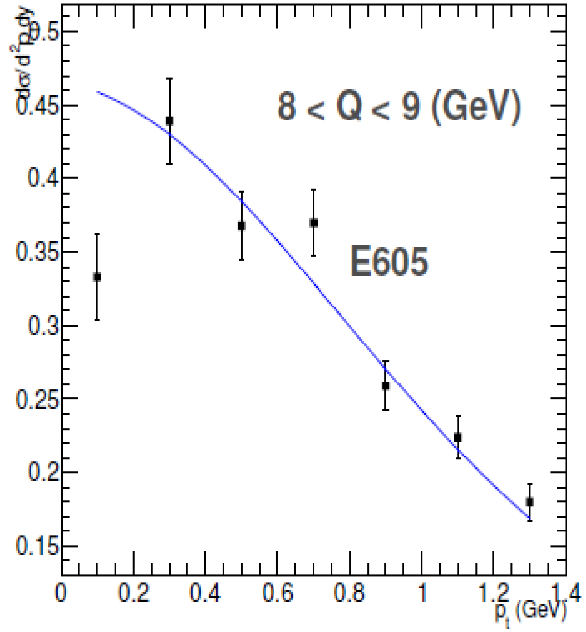
g_1 , g_2 and g_3 are free parameters

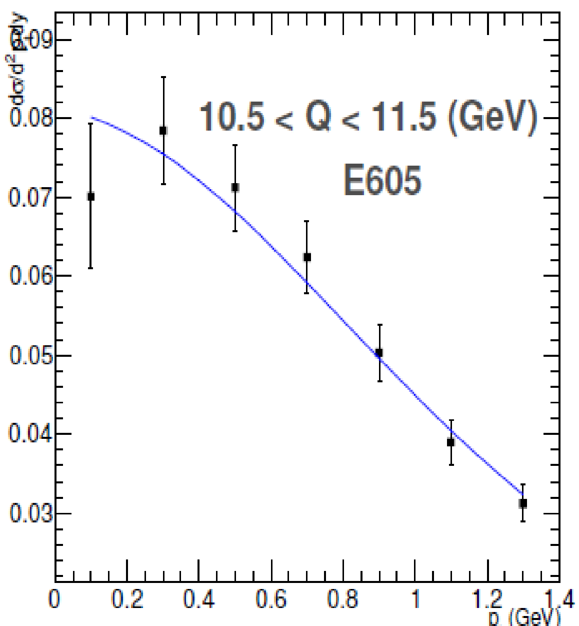
In our fit, we choose b_{max}=1.5GeV^{-1}

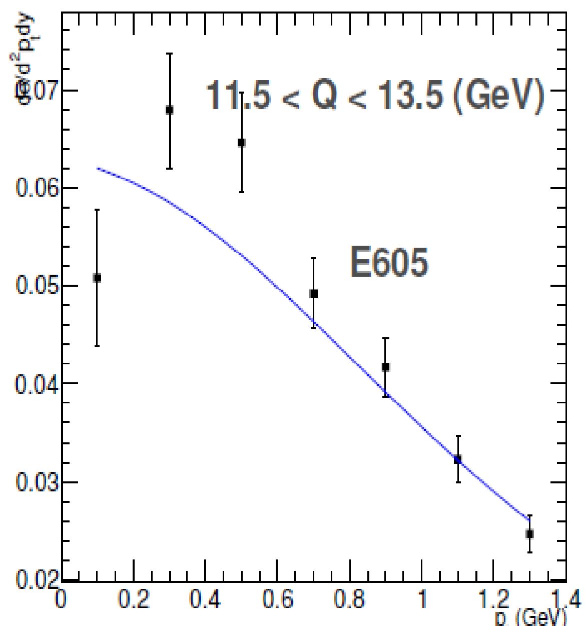
Y piece is included

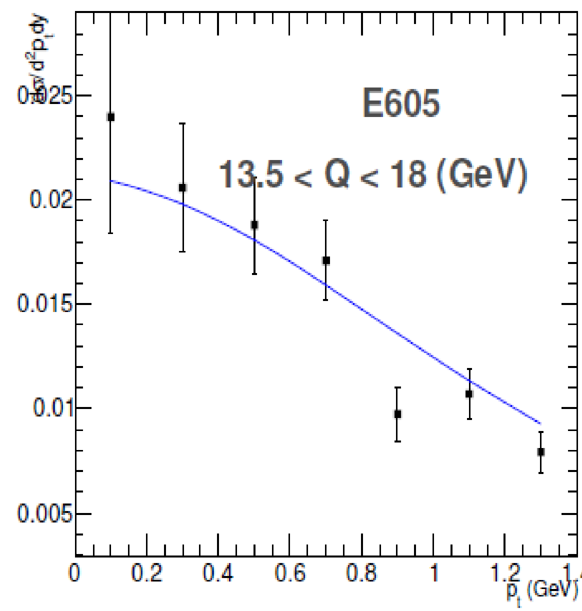

5 < Q < 6 (GeV)
E288
6 < Q < 7 (GeV)
E288
7 < Q < 8 (GeV)
E288
8 < Q < 9 (GeV)
E288
dσ/d²p_t dy
p_t (GeV)

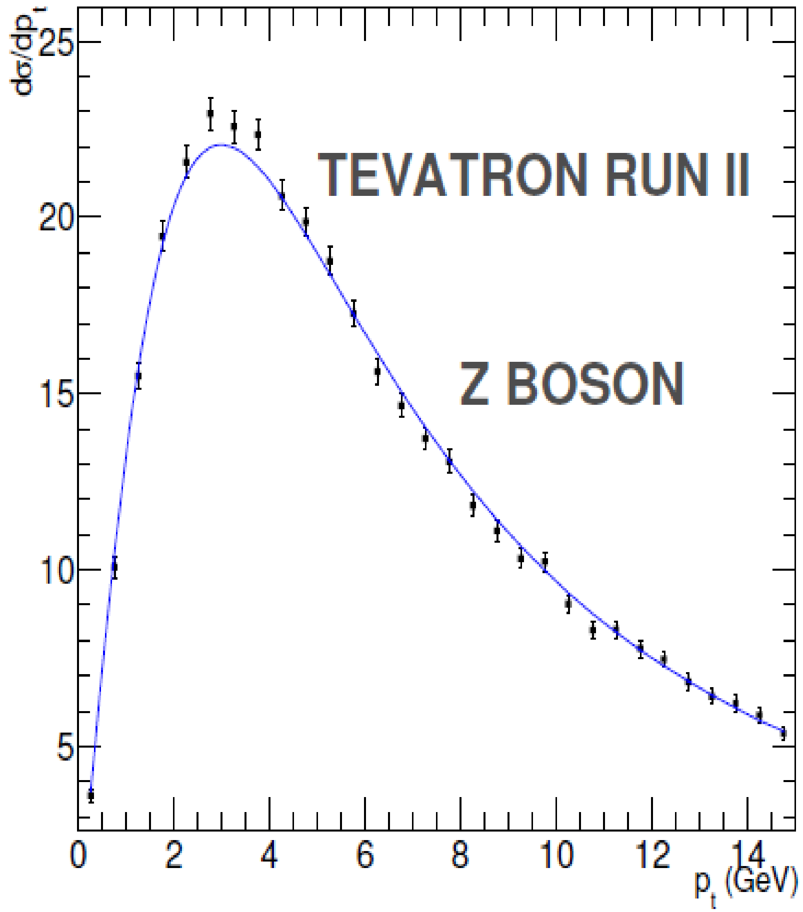

7 < Q < 8 (GeV)
E605
$d\sigma/d^2p_t dy$
0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1
0 0.2 0.4 0.6 0.8 1 1.2 1.4
p_t (GeV)

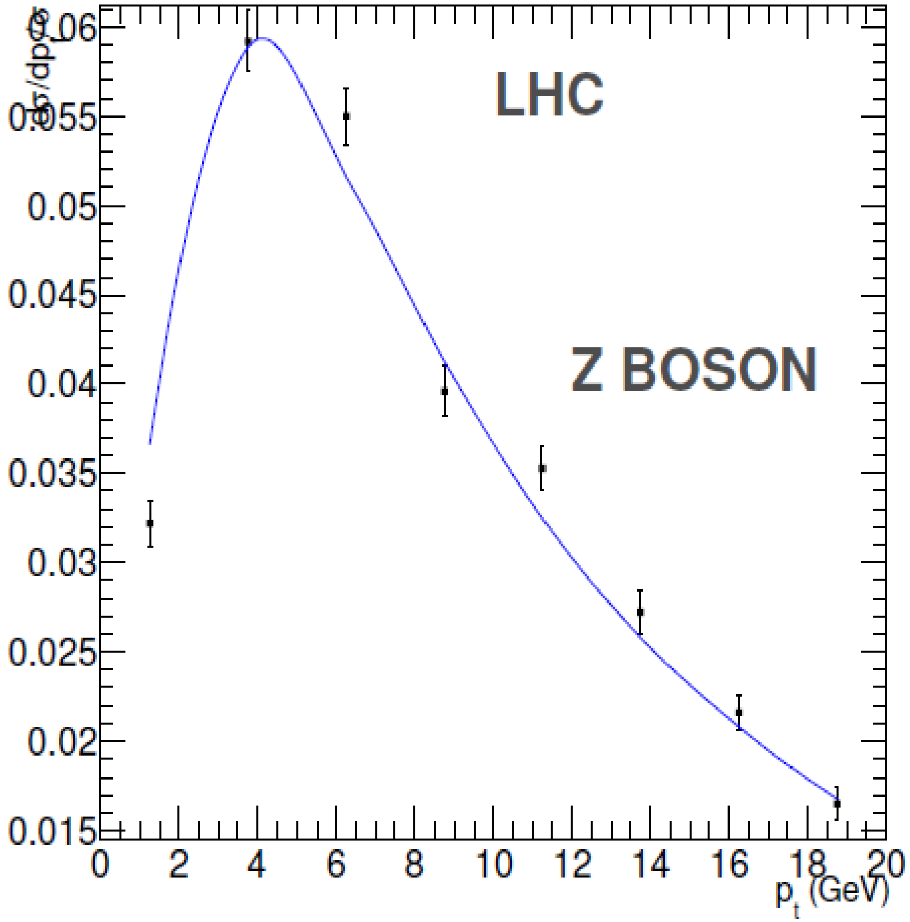

8 < Q < 9 (GeV)
E605
$d\sigma/d^2p_t dy$
0.15 0.2 0.25 0.3 0.35 0.4 0.45 0.5
0 0.2 0.4 0.6 0.8 1 1.2 1.4
p_t (GeV)


10.5 < Q < 11.5 (GeV)
E605
$d\sigma/d^2p_t dy$
0.03 0.04 0.05 0.06 0.07 0.08 0.09
0 0.2 0.4 0.6 0.8 1 1.2 1.4
p_t (GeV)


11.5 < Q < 13.5 (GeV)
E605
$d\sigma/d^2p_t dy$
0.02 0.03 0.04 0.05 0.06 0.07
0 0.2 0.4 0.6 0.8 1 1.2 1.4
p_t (GeV)

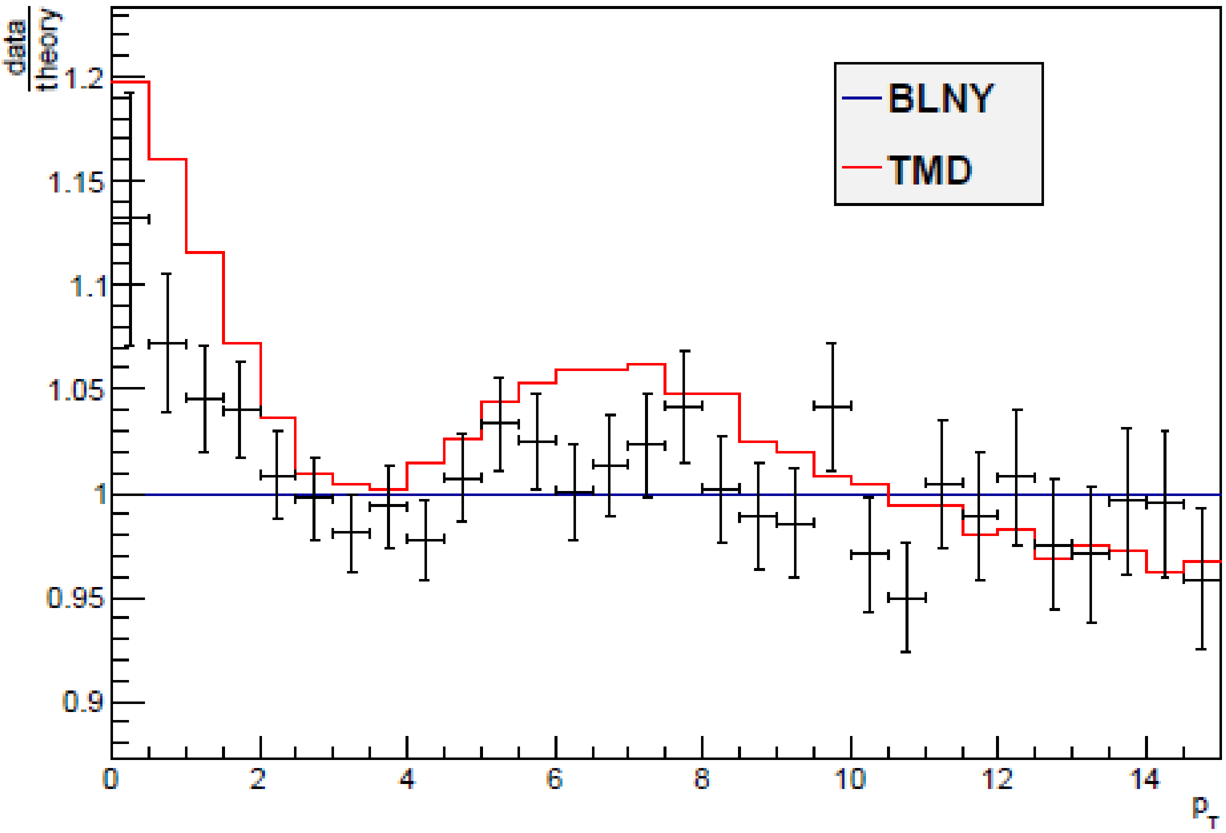

E605
13.5 < Q < 18 (GeV)
$d\sigma/d^2p_t dy$
0.005 0.01 0.015 0.02 0.025
0 0.2 0.4 0.6 0.8 1 1.2 1.4
p_t (GeV)



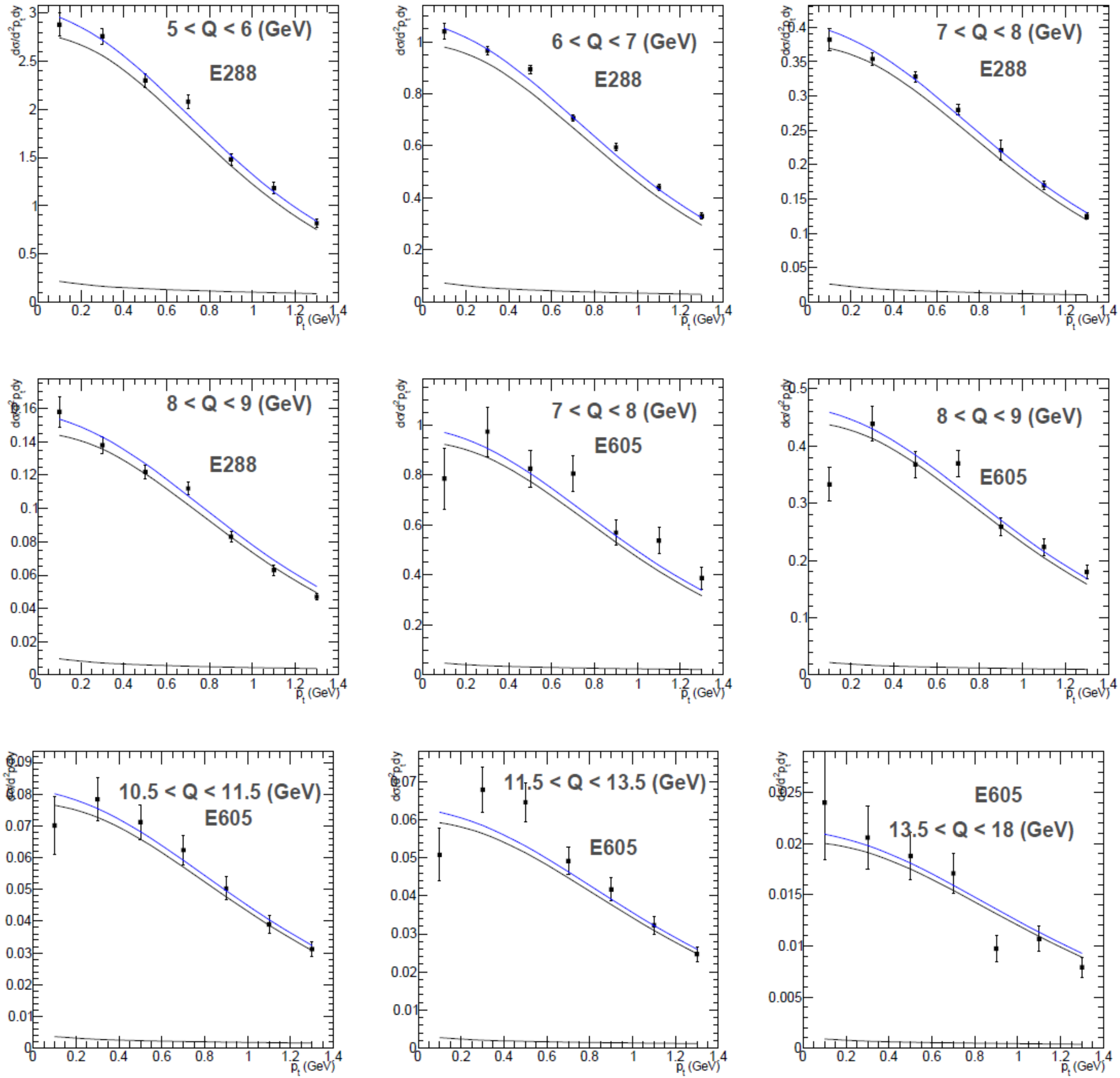


For Z boson production $A^{(2)}$ is very important

Compare to ResBos



Drawn by Joshual Isaacson

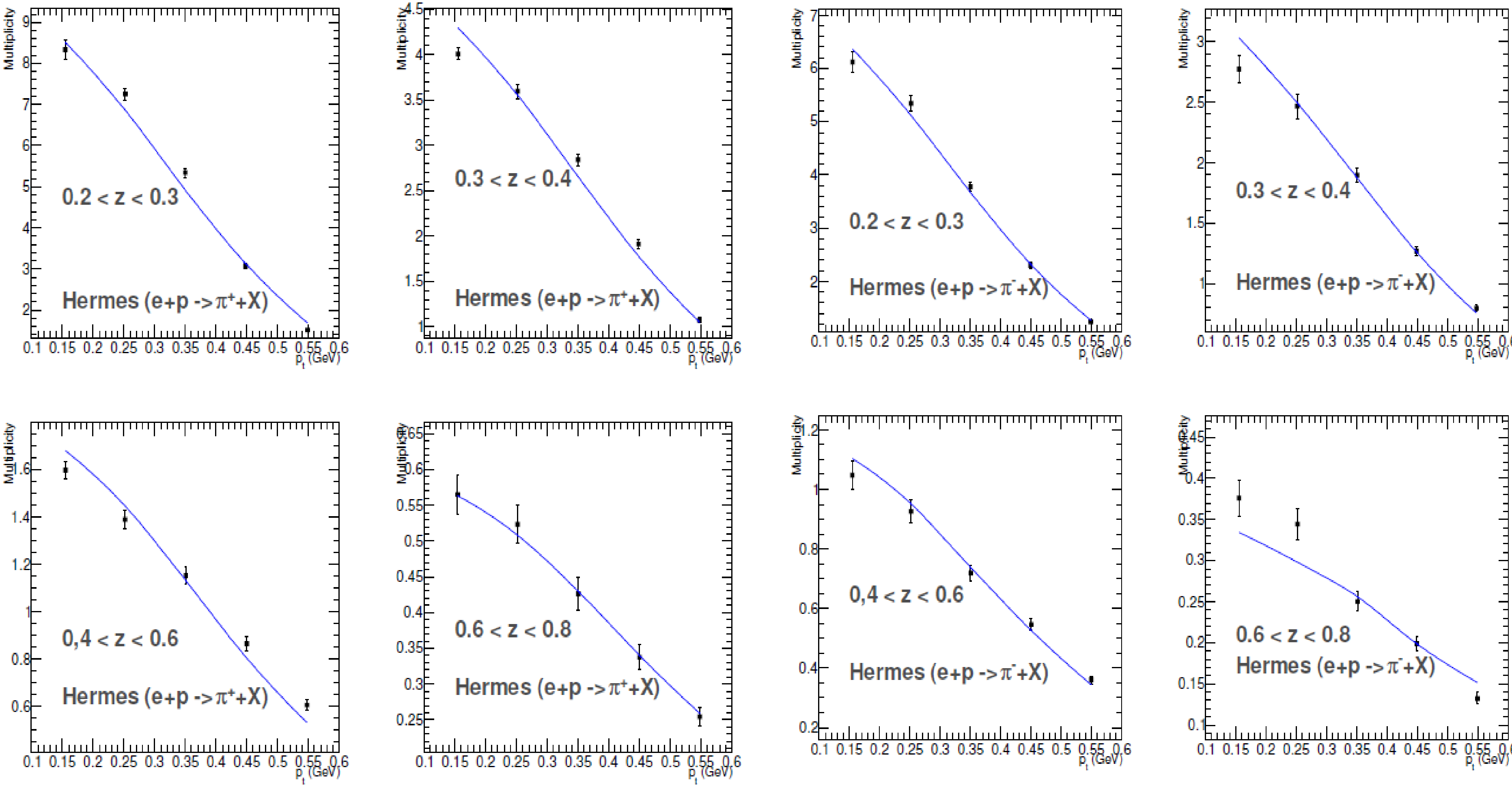


5 < Q < 6 (GeV)
E288
6 < Q < 7 (GeV)
E288
7 < Q < 8 (GeV)
E288
8 < Q < 9 (GeV)
E288
7 < Q < 8 (GeV)
E605
8 < Q < 9 (GeV)
E605
10.5 < Q < 11.5 (GeV)
E605
11.5 < Q < 13.5 (GeV)
E605
E605
13.5 < Q < 18 (GeV)
dσ/d²p_t dy
p_t (GeV)

- In this fit, we take into account the A^1,A^2, B^1,B^2,C^1,Y^1.
- For these Drell-Yan process, the Y piece is very small, which is about few percent of resummation part (W function).
- We have resolved the problem of S_{NP}, then we can matching SIDIS to Drell-Yan.

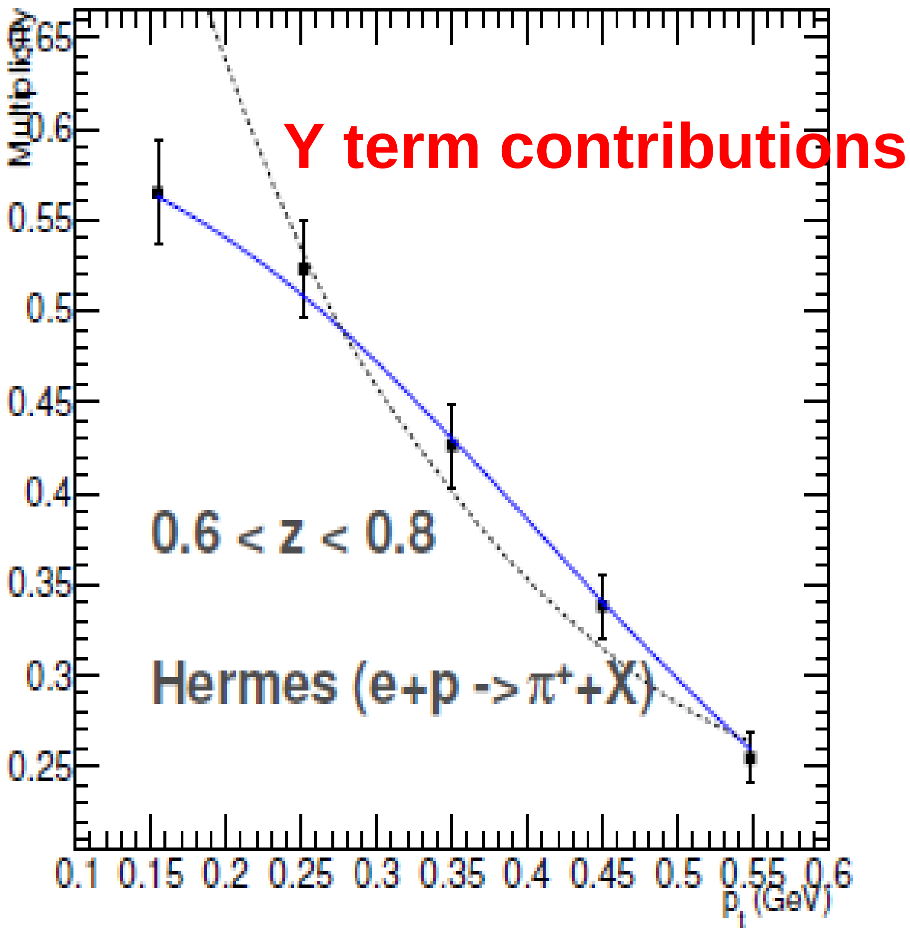
SIDIS at HERMES

$Q^2 = 3GeV^2$



Where these curves do not include the Y piece

Where are the Y terms??



Issues with the Y terms in SIDIS

- Leading behavior

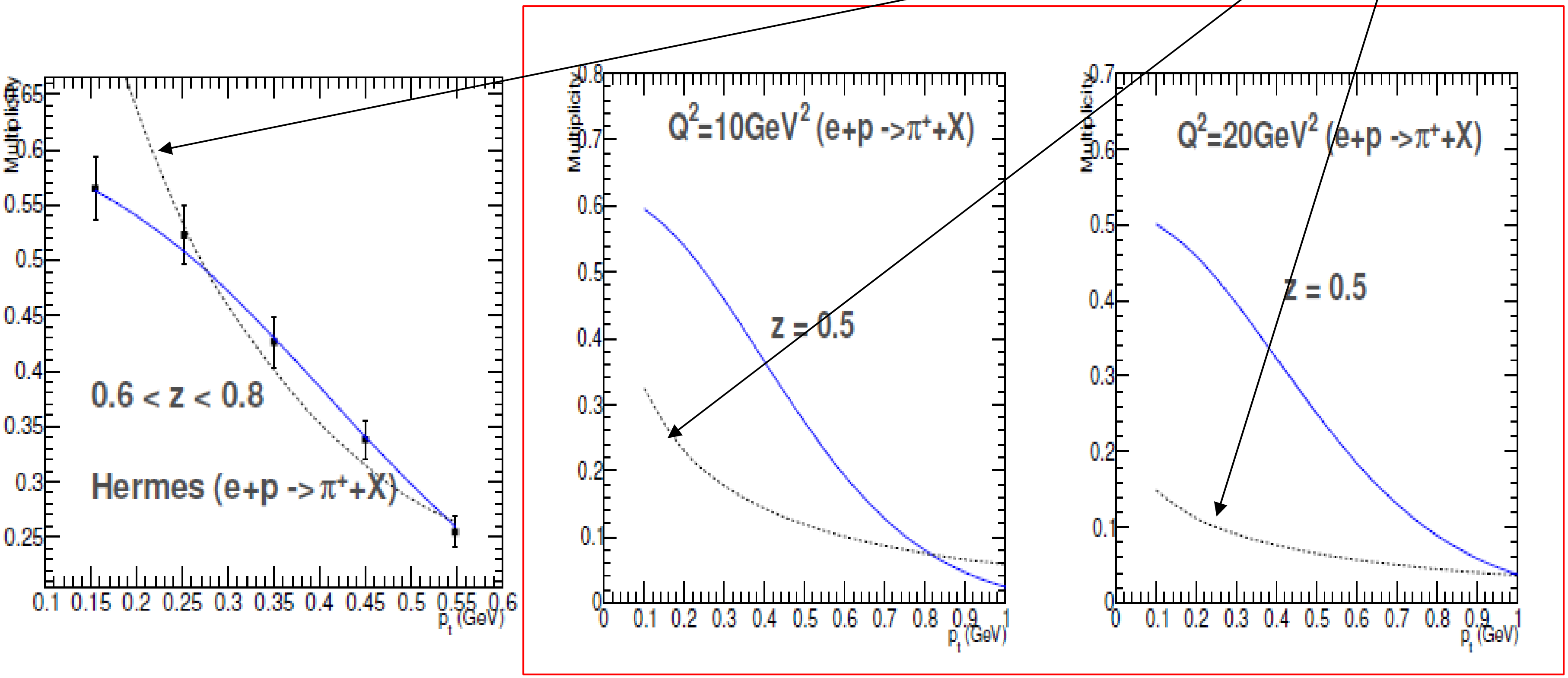
$$\frac{d\sigma}{d^2P_{h\perp}} \sim \frac{1}{P_{h\perp}^2}\left[\frac{1+\xi^2}{(1-\xi)_+}+\frac{1+\hat{\xi}^2}{(1-\hat{\xi})_+}+\delta(1-\xi)\delta(1-\hat{\xi})\ln\frac{z_h^2Q^2}{P_{h\perp}^2}\right]$$

- Subleading Y terms

$$Y \sim \frac{1}{Q^2}\left[\frac{1+\xi}{(1-\xi)_+}+\frac{1+\hat{\xi}}{(1-\hat{\xi})_+}+\delta(1-\xi)\delta(1-\hat{\xi})\ln\frac{z_h^2Q^2}{P_{h\perp}^2}\right]$$

- Similar terms arise in the azimuthal angular asymmetries in Drell-Yan and SIDIS
  - Boer-Vogelsang 2006, Qiu et al. 2007
  - Bacchetta-Boer-Diehl-Mulders, 2008
- Resummation or not?
- Fortunately, increasing Q^2 can help

$$\frac{d\sigma}{dx_B dy dz_h d^2\vec{P}_{h\perp}} = \sigma_0^{(\mathrm{DIS})}\left[\frac{1}{z_h^2}\int \frac{d^2b}{(2\pi)^2} e^{i\vec{P}_{h\perp}\cdot\vec{b}/z_h}\widetilde{F}_{UU}(Q;b) + Y_{UU}(Q;P_{h\perp})\right]$$



The Q^2 is larger, and the Y piece is less and less important.

Outlook

- Assuming the TMD applications in SIDIS experiments, global analysis of Sivers and Collins functions with new parameterizations of the TMDs
  - Work in progress
- Y terms in SIDIS need more detailed studies
  - Experiments with high Q^2 at EIC is a must

Soft Gluon Resummations in Dijet Azimuthal Angular Correlations at the Collider

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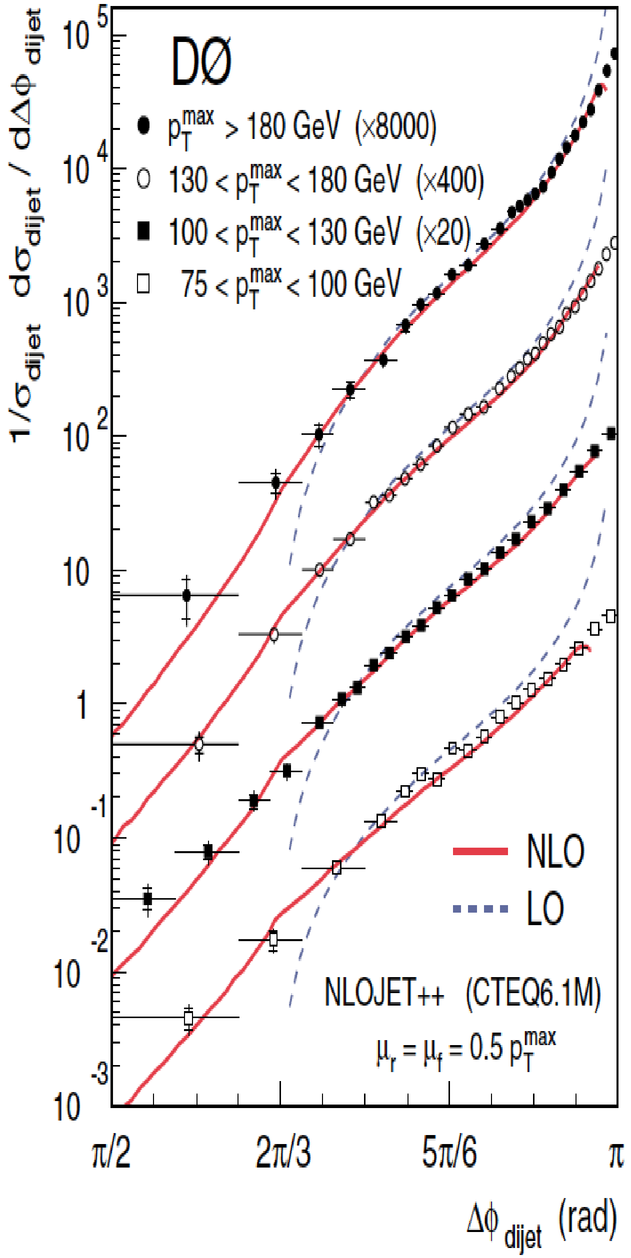
Motivations:

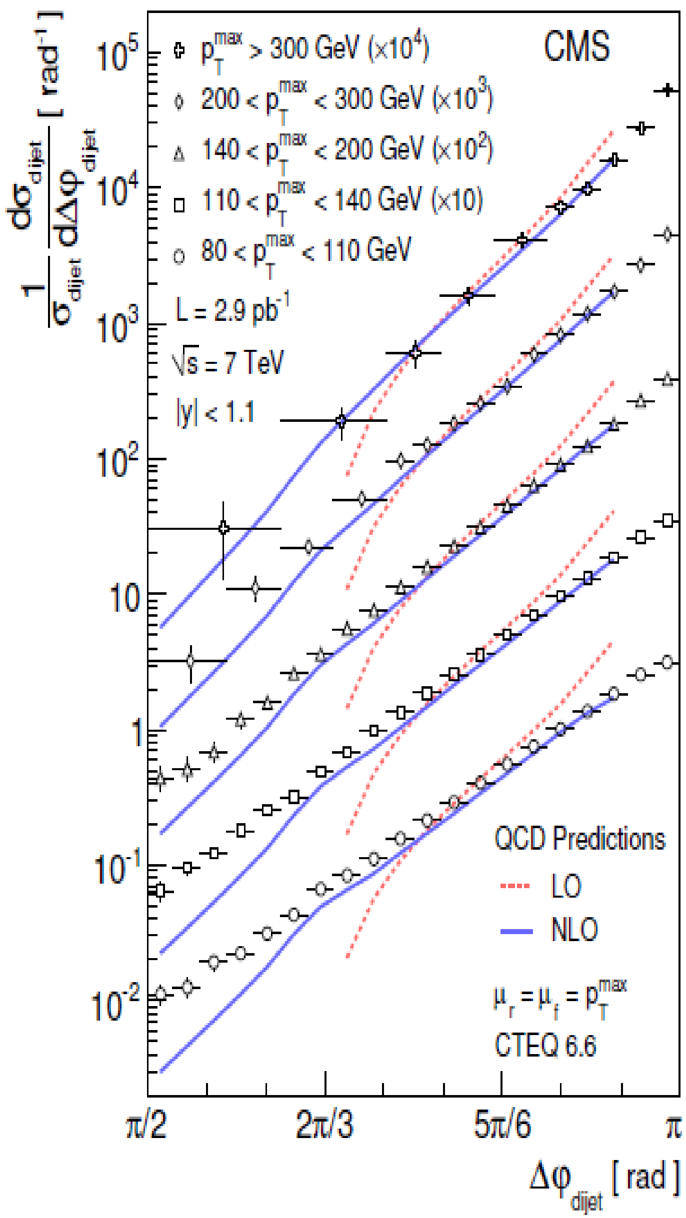
reummation of large logs in dijet production

factorization breaking effects

Collins-Qiu, 2007; Vogelsang-Yuan, 2007;

Rogers-Mulders 2010





$$q_i\bar{q}_i \to q_i\bar{q}_i$$

$$q_i\bar{q}_j \to q_i\bar{q}_j$$

$$q_iq_j \to q_iq_j$$

$$q_iq_i \to q_iq_i$$

$$gq_i(\bar{q}_i) \to gq_i(\bar{q}_i)$$

$$gg \to q_i\bar{q}_i$$

$$q_i\bar{q}_i \to gg$$

$$gg \to gg$$

$$\frac{d^4\sigma}{dy_1 dy_2 dP_J^2 d^2q_\perp} = \sum_{ab} \sigma_0 \left[\int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-iq_\perp \cdot b_\perp} W_{ab\to cd}(x_1, x_2, b_\perp) + Y_{ab\to cd} \right]$$

$$W_{ab\to cd}(x_i, b) = x_1 f_a(x_1, b, \zeta^2, \mu^2, \rho) x_2 f_b(x_2, b, \bar{\zeta}^2, \mu^2, \rho) \mathrm{Tr}\left[\mathbf{H}_{ab\to cd}(Q^2, \mu^2, \rho) \mathbf{S}_{ab\to cd}(b, \mu^2, \rho)\right]$$

$$S_{IJ} = \int_0^\pi \frac{d\phi_0}{\pi} C_{Iii'}^{bb'} C_{Jll'}^{aa'} \langle 0 | \mathcal{L}_{vcb'}^\dagger(b_\perp) \mathcal{L}_{vbc}(b_\perp) \mathcal{L}_{\bar{v}ca'}^\dagger(0) \mathcal{L}_{\bar{v}ac}(0) \mathcal{L}_{nji}^\dagger(b_\perp) \mathcal{L}_{\bar{n}i'k}(b_\perp) \mathcal{L}_{\bar{n}kl}^\dagger(0) \mathcal{L}_{nl'j}(0) | 0 \rangle$$

For qq → gg

$$c_1 = \delta^{a_1 a_2} \delta_{a_3 a_4}, \qquad c_2 = i f^{a_1 a_2 c} t^c_{a_3 a_4}, \qquad c_3 = d^{a_1 a_2 c} t^c_{a_3 a_4}$$

For gg → gg

$$c_1 = f^{a_1 a_2 c_1} f_{a_3 a_4 c_1}, \qquad c_2 = f^{a_1 a_3 c_1} f_{a_2 a_4 c_1} + f^{a_1 a_4 c_1} f_{a_2 a_3 c_1}, \qquad c_3 = d^{a_1 a_2 c_1} f_{a_3 a_4 c_1},$$

$$c_4 = f^{a_1 a_2 c_1} d_{a_3 a_4 c_1}, \qquad c_5 = d^{a_1 a_4 c_1} f_{a_2 a_3 c_1}, \qquad c_6 = \delta^{a_1 a_2} \delta^{a_3 a_4}, \qquad c_7 = \delta^{a_1 a_3} \delta^{a_2 a_4}, \qquad c_8 = \delta^{a_1 a_4} \delta^{a_2 a_3}$$

After solving the evolution equation

$$W_{ab\to cd}(x_1,x_2,b) = x_1 f_a(x_1,\mu=c_0/b_\perp) x_2 f_b(x_2,\mu=c_0/b_\perp) e^{-S_{Sud}(Q^2,b_\perp)} \times \mathrm{Tr}\left[\mathbf{H}_{ab\to cd}\exp[-\int_{c_0/b_\perp}^{Q}\frac{d\mu}{\mu}\gamma^{s\dagger}]\mathbf{S}_{ab\to cd}\exp[-\int_{c_0/b_\perp}^{Q}\frac{d\mu}{\mu}\gamma^{s}]\right]$$

where

$$S_{Sud}(Q^2,b_\perp,C_1,C_2) = \int_{C_1^2/b_\perp^2}^{C_2^2 Q^2}\frac{d\mu^2}{\mu^2}\left[\ln\left(\frac{Q^2}{\mu^2}\right)A + B + D_1\ln\frac{2}{R_1} + D_2\ln\frac{2}{R_2}\right]$$

For gg → jj $\quad A = C_A\frac{\alpha_s}{\pi},\ B = -2C_A\beta_0\frac{\alpha_s}{\pi}$

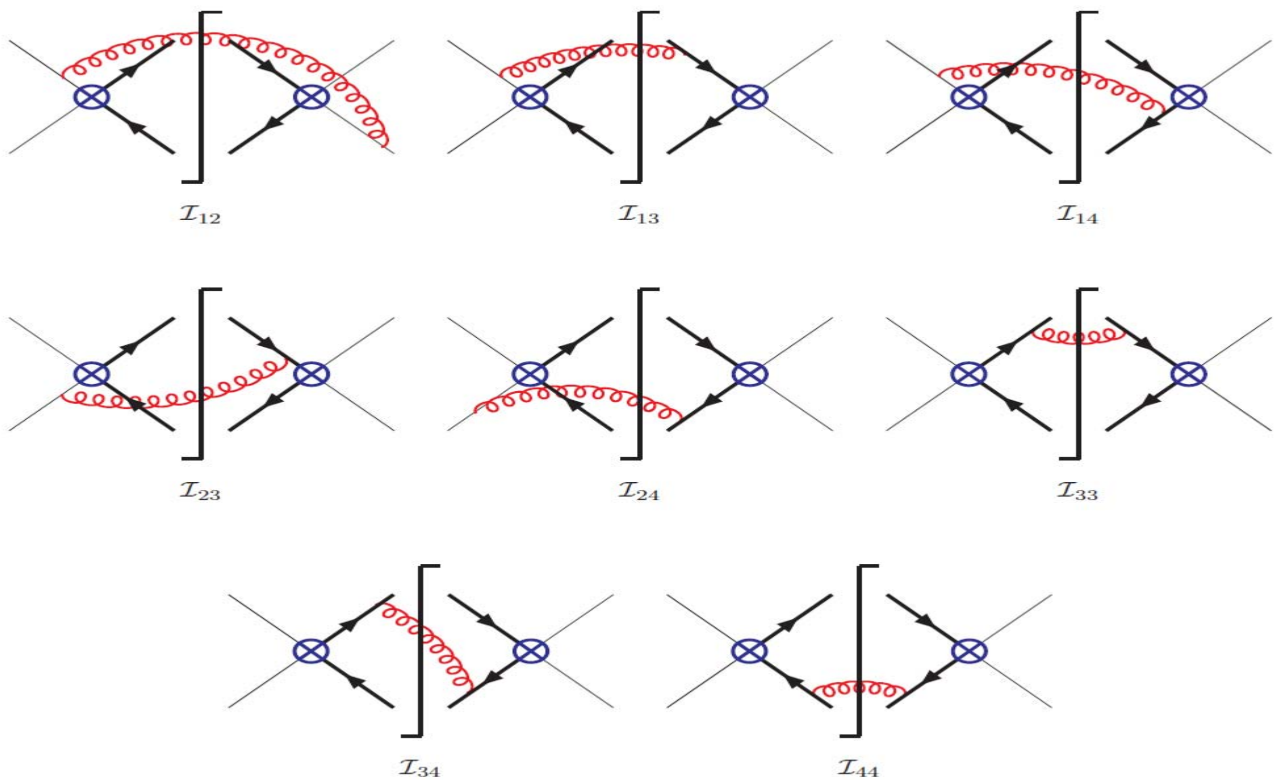
For qq → jj $\quad A = C_F\frac{\alpha_s}{\pi},\ B = \frac{-3C_F}{2}\frac{\alpha_s}{\pi}$

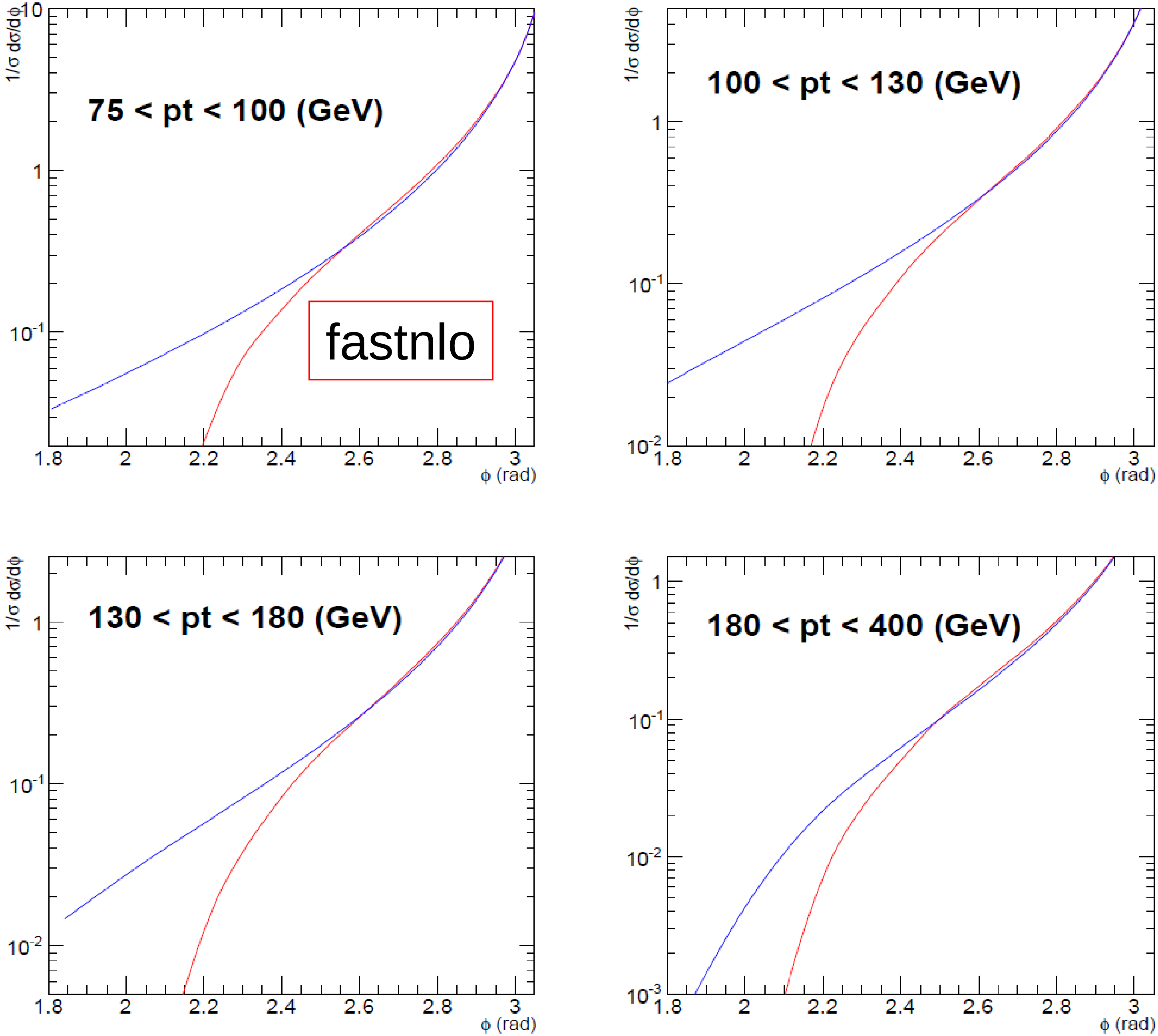
For qg → jj $\quad A = \frac{(C_F+C_A)}{2}\frac{\alpha_s}{\pi} \quad B = \left(\frac{-3C_F}{4}+C_A\beta_0\right)\frac{\alpha_s}{\pi}$

For quark jet $D_i=C_F\, a_s/\pi$, and for gluon jet $D_i=C_A\, a_s/\pi$

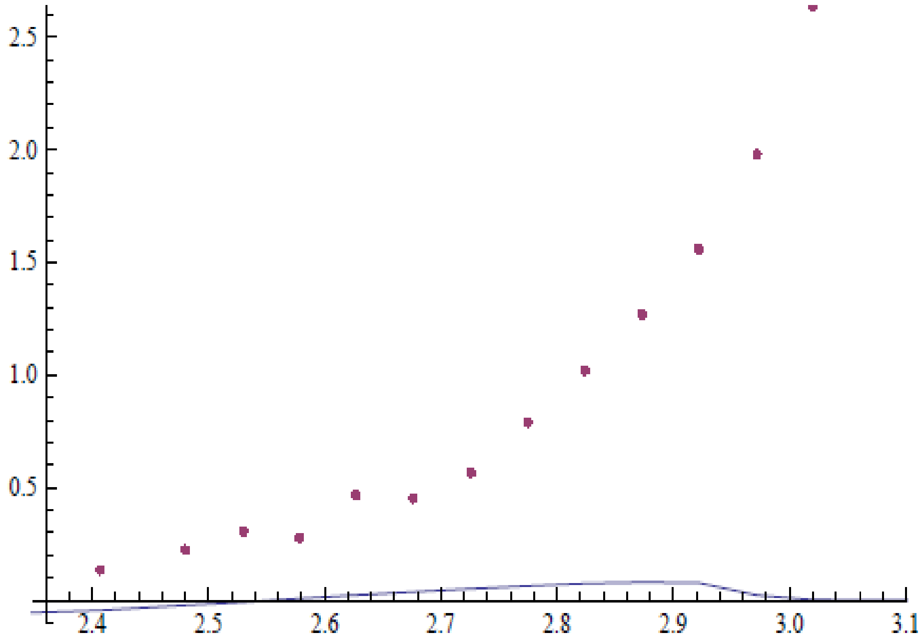
The soft factor satisfies

$$\frac{d}{d\ln\mu} S_{IJ}(\mu) \;=\; -\Gamma^{s\dagger}_{IJ'} S_{J'J}(\mu) - S_{IJ'}(\mu)\Gamma^{s}_{J'J}$$





75 < pt < 100 (GeV)
fastnlo
100 < pt < 130 (GeV)
130 < pt < 180 (GeV)
180 < pt < 400 (GeV)
1/σ dσ/dφ
φ (rad)


2.5
2.0
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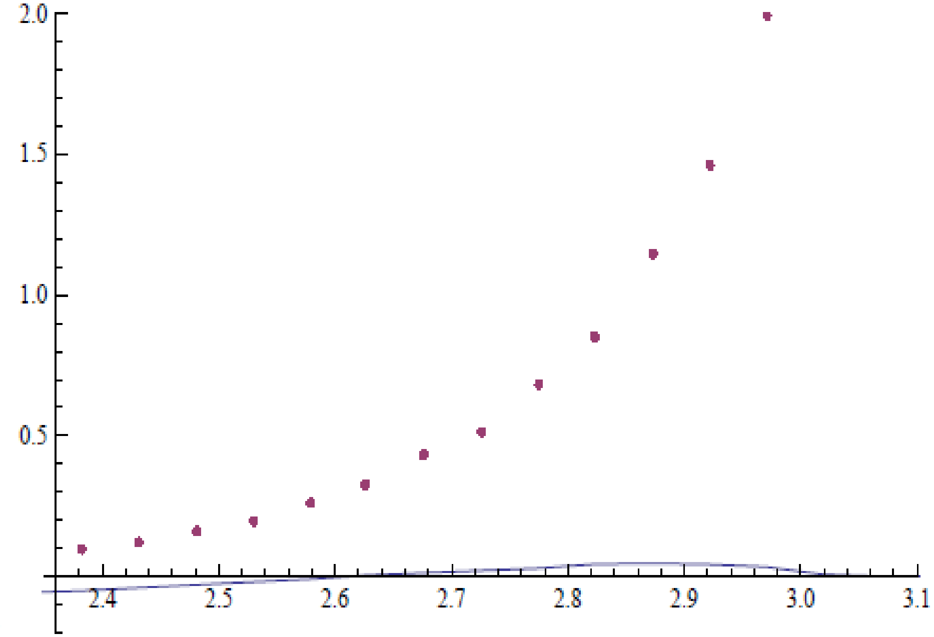
3.1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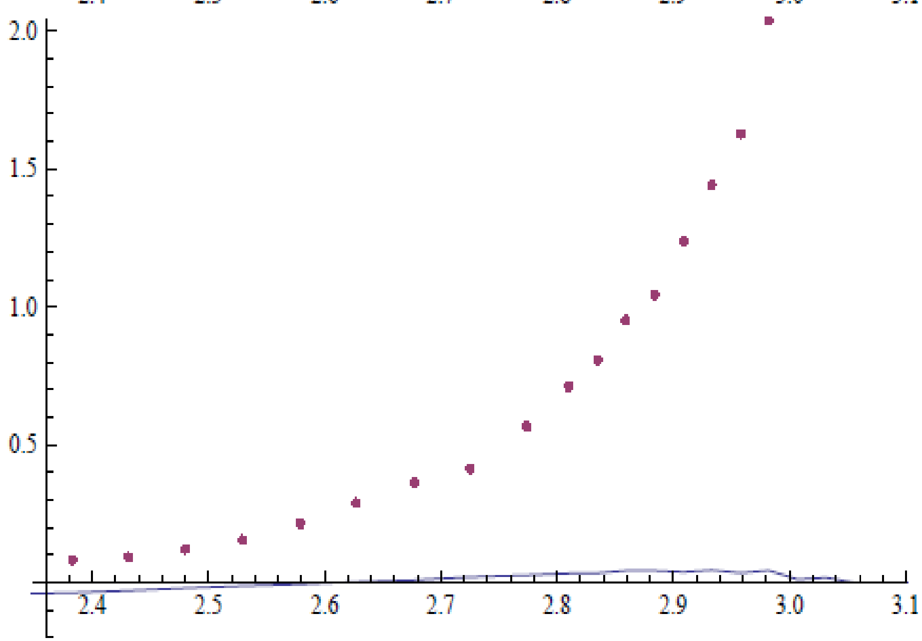
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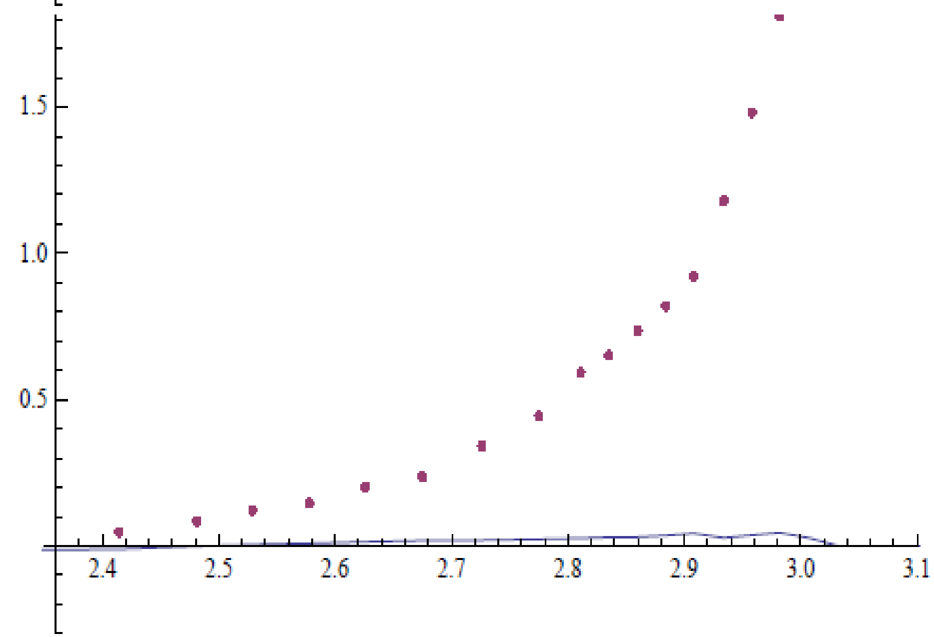
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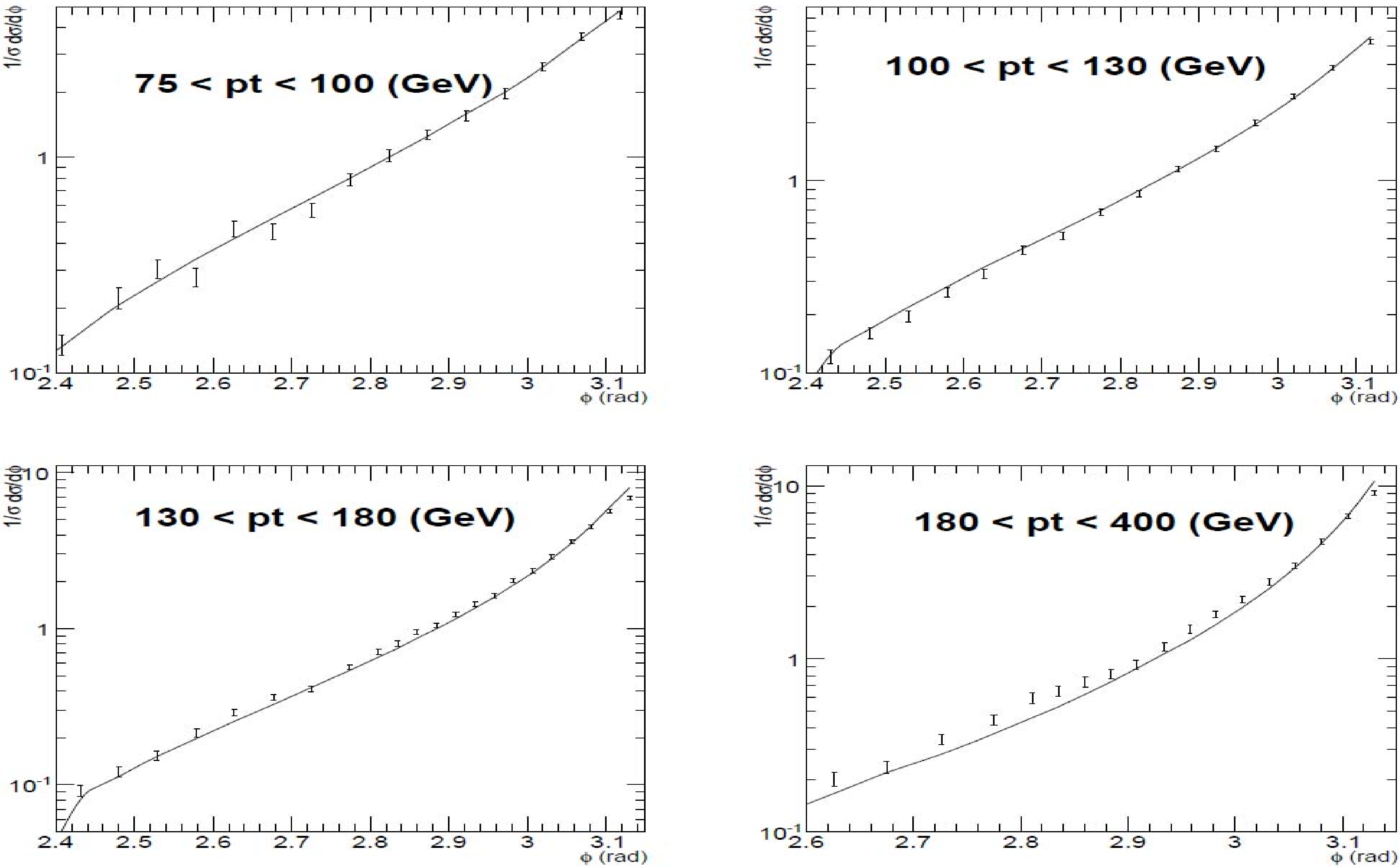

75 < pt < 100 (GeV)
100 < pt < 130 (GeV)
130 < pt < 180 (GeV)
180 < pt < 400 (GeV)
1/σ dσ/dφ
φ (rad)
2.4
2.5
2.6
2.7
2.8
2.9
3
3.1
1
10
10^{-1}

Thank you very much!

Sudakov factor

- There are two parts in the Sudakov factor

$$\mathcal{S}_{sud} \Rightarrow \mathcal{S}_{pert}(Q; b_*) + S_{NP}(Q; b)$$

For b → ∞:

S_{pert} → constant

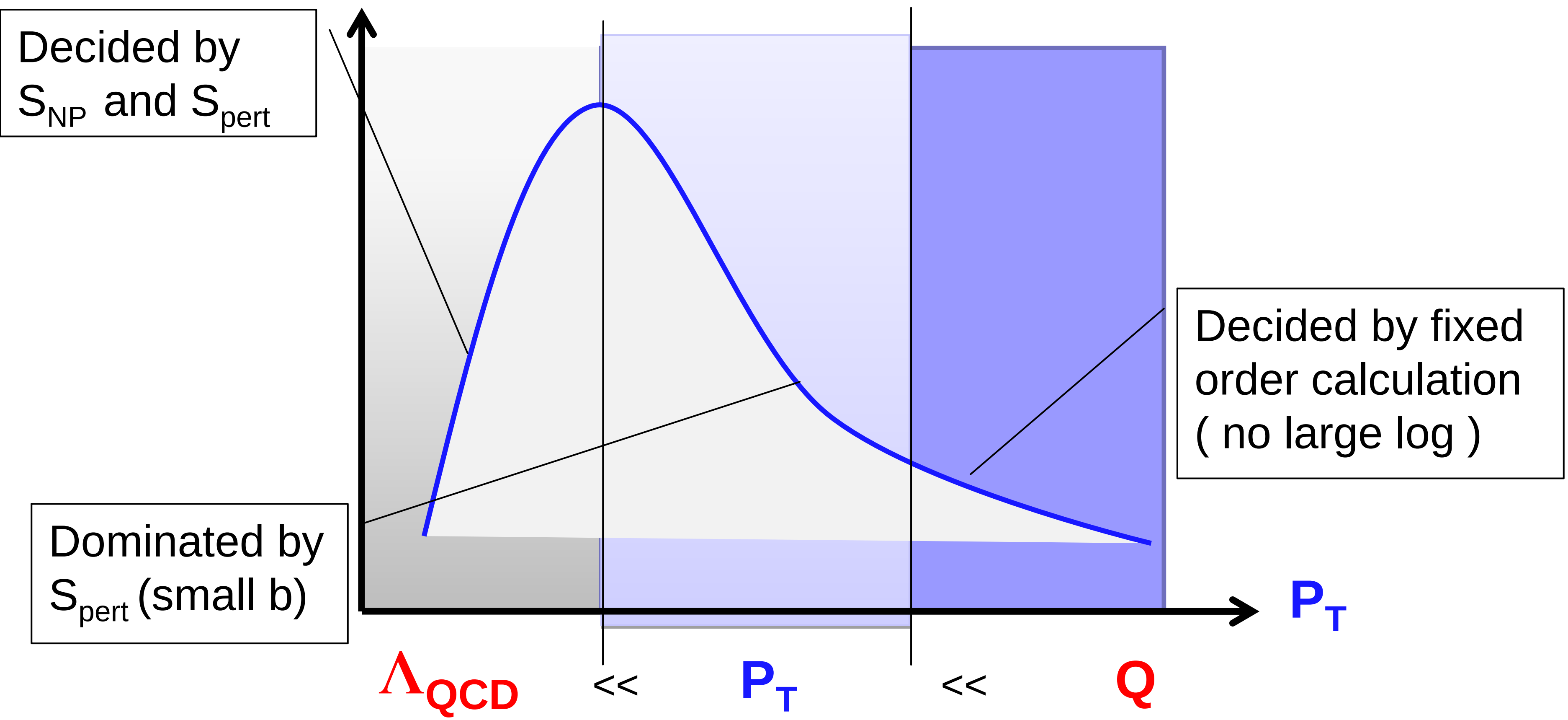
S_{NP} → b^2, b , log(b) (∞)

For b → 0:

$S_{pert} \rightarrow \int_{c_0/b}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \left[A \ln \frac{Q^2}{\bar{\mu}^2} + B \right]$

S_{NP} → 0

A unified picture



$n_j = (1, 0 , 0 , 1) \to n_j^2 = 0$

$n_k = (1, 0 , \sin(x) , \cos(x))$ with $x > R$

$n_j . n_k = 1 - \cos(x) > R^2/2$

$n_j = (1^+, 0\perp, R^2/2) \to n_j^2 = R^2$

Then for any n_k ($n_k^2 = 0$) , we have :

$n_j . n_k > R^2/2$

The difference between these two way is high order of R

$$S_{Sud} = 2C_F \int_{Q_0}^{Q} \frac{d\bar{\mu}}{\bar{\mu}} \frac{\alpha_s(\bar{\mu})}{\pi} \left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) + \ln\frac{Q_0^2 b^2}{c_0^2} - \frac{3}{2} \right]$$

- But, this Sudakov factor does not work in the high k_t region

- This is because the small b behavior is wrong in this Sudakov factor

So in this work, CSS formulism is chosen

Outline

- QCD resummation in TMD factorization

  Sudakov factor (2 parts)

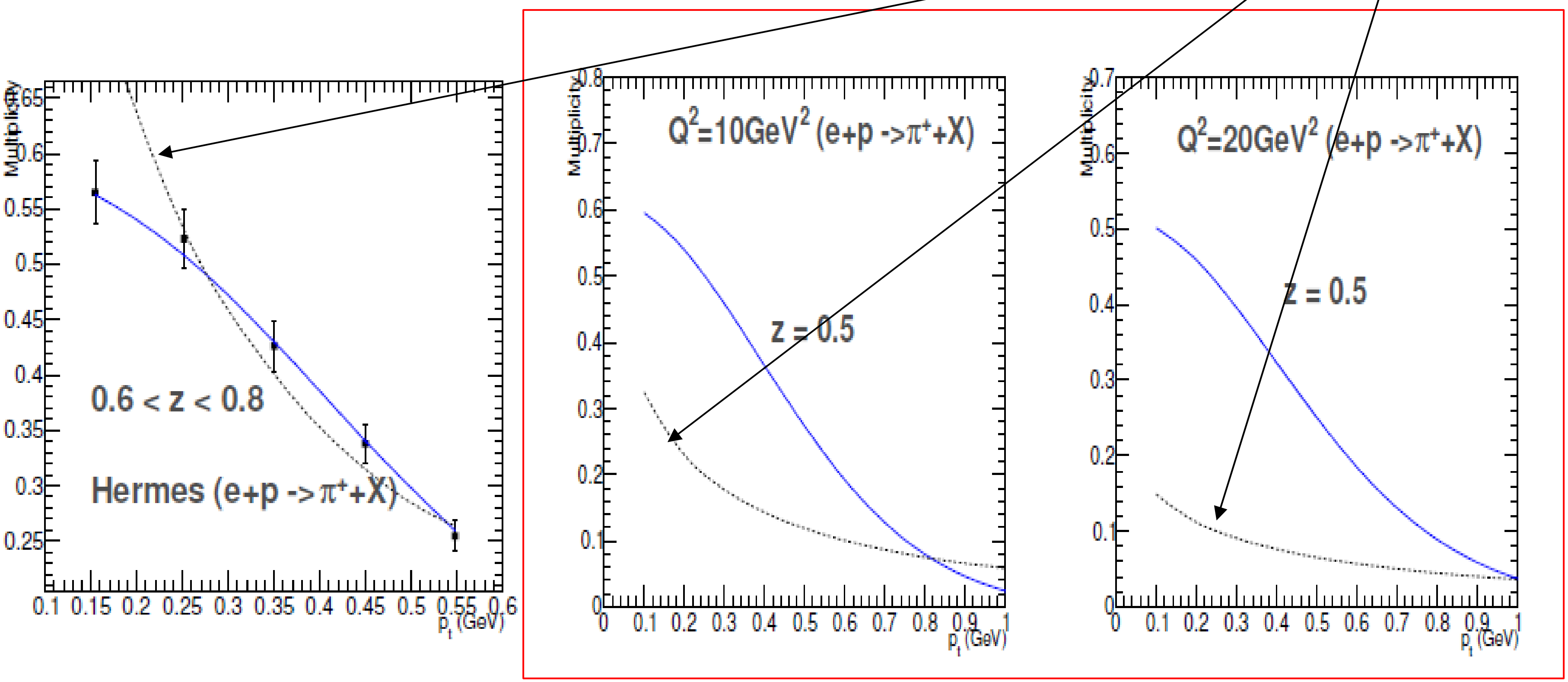
- Nonperturbative Sudakov factor fitting from Drell-Yan processes

- Matching the SIDIS to Drell-Yan

- Summary

- Dijet production at hadron collider

$$\frac{d\sigma}{dx_B dy dz_h d^2\vec{P}_{h\perp}} = \sigma_0^{(\mathrm{DIS})}\left[\frac{1}{z_h^2}\int\frac{d^2b}{(2\pi)^2}e^{i\vec{P}_{h\perp}\cdot\vec{b}/z_h}\widetilde{F}_{UU}(Q;b) + Y_{UU}(Q;P_{h\perp})\right]$$



The Q^2 is smaller, and the Y piece is more important.

Summary

- A new S_{NP} is obtained by fitting the Drell-Yan data
- This S_{NP} can be used in very small Q^2 region
- We apply the universal parameters in this S_{NP} to describe SIDIS processes at Hermes
- For the SIDIS process, at the Hermes energy the Y piece is so large that the data at Hermes can not be described by leading twist resummation