TMD analysis at HERMES using MC

INT Workshop 14-55w

Studies of 3D Structure of Nucleon

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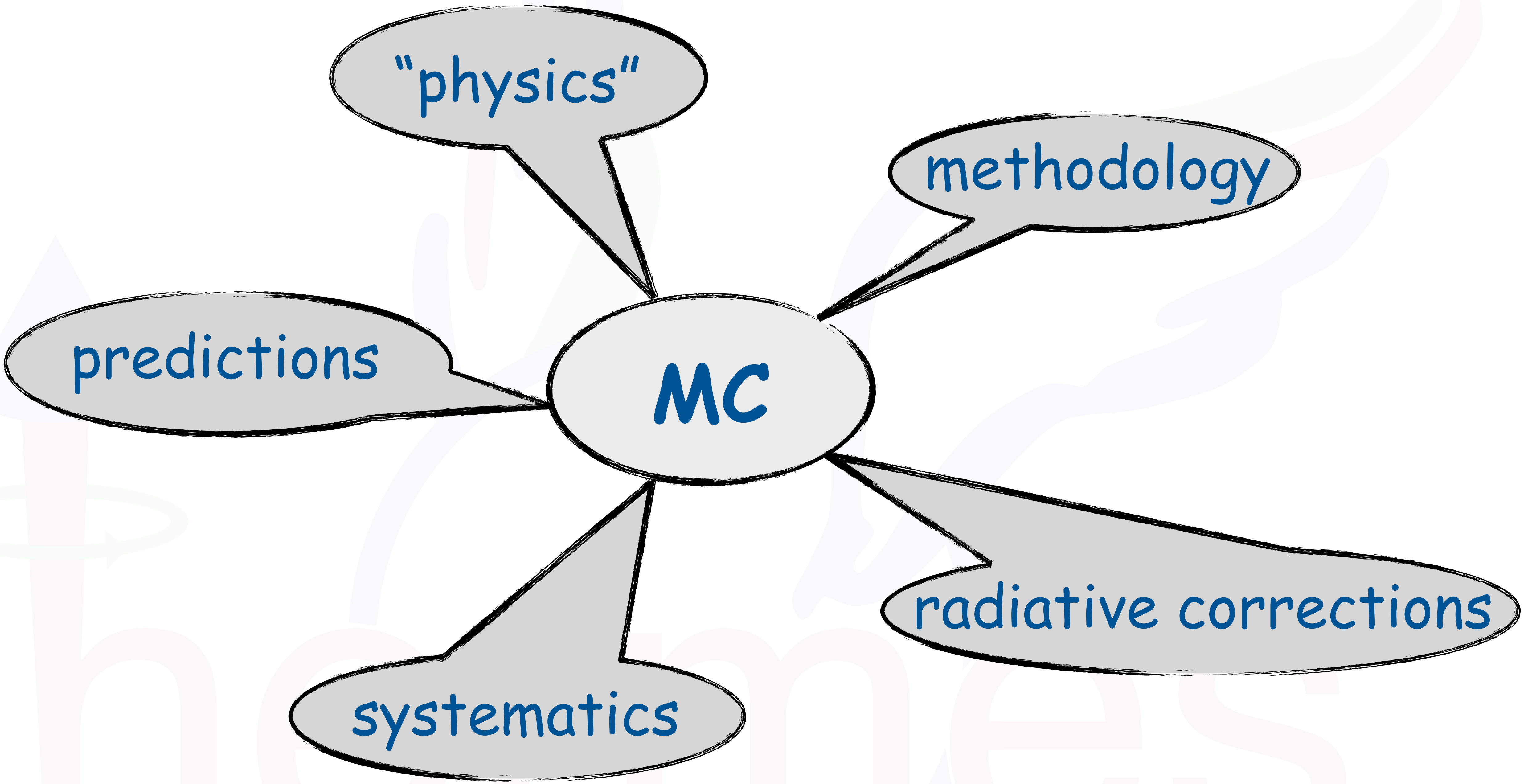
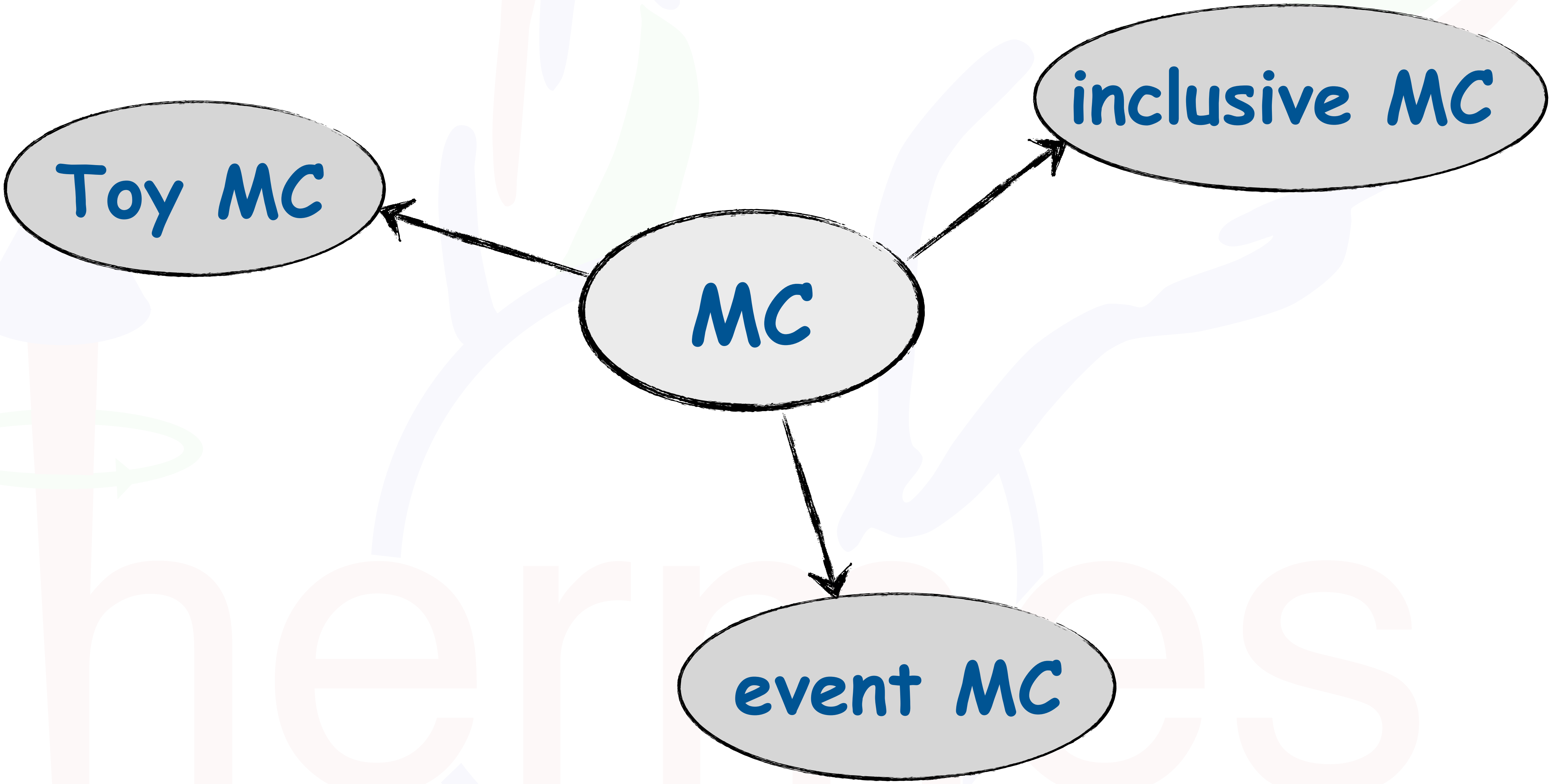
Disclaimers

- focus rather on "MC in TMD analyses at HERMES"
- contains a number of trivial, but hopefully still useful, statements
- can not offer a general recipe, though hopefully some guidance

Some usages for Monte Carlo



Some types of Monte Carlo



Prelude: role of acceptance in experiments

An unfortunate Lemma

- "No particle-physics experiment has a perfect acceptance!"
  - obvious for detectors with gaps/holes
  - but also for "4π", especially when looking at complicated final states

An unfortunate Lemma

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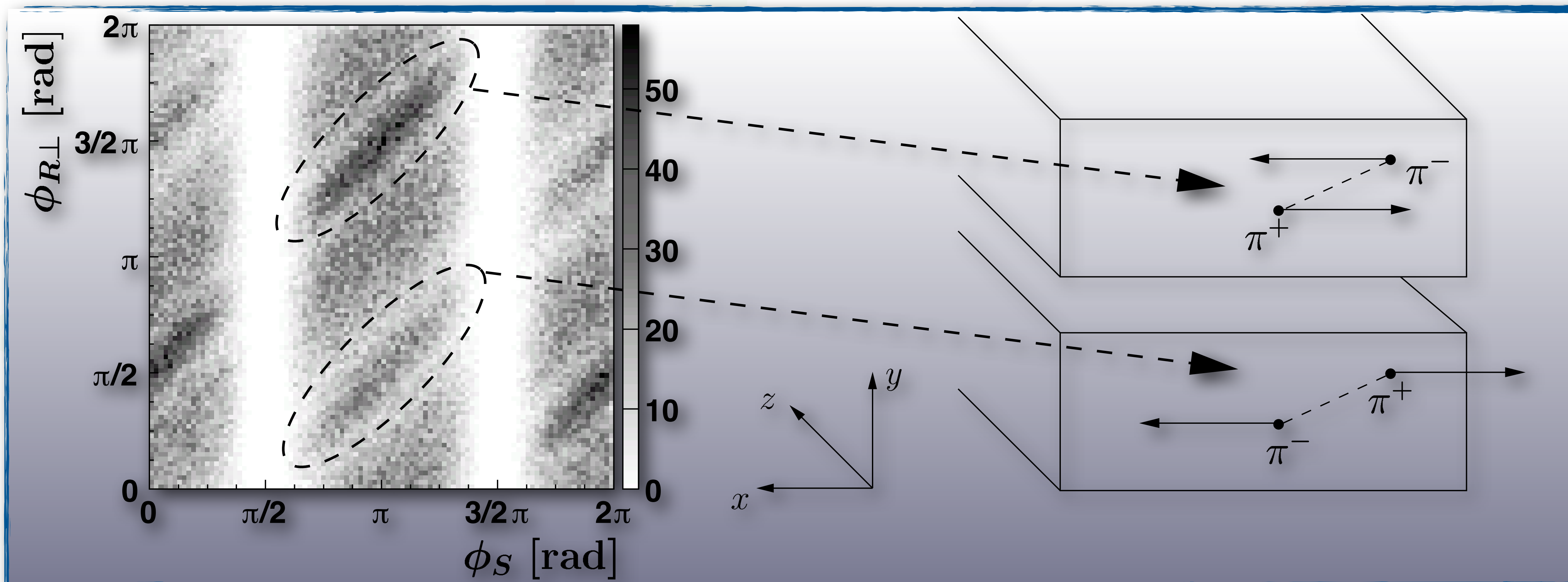
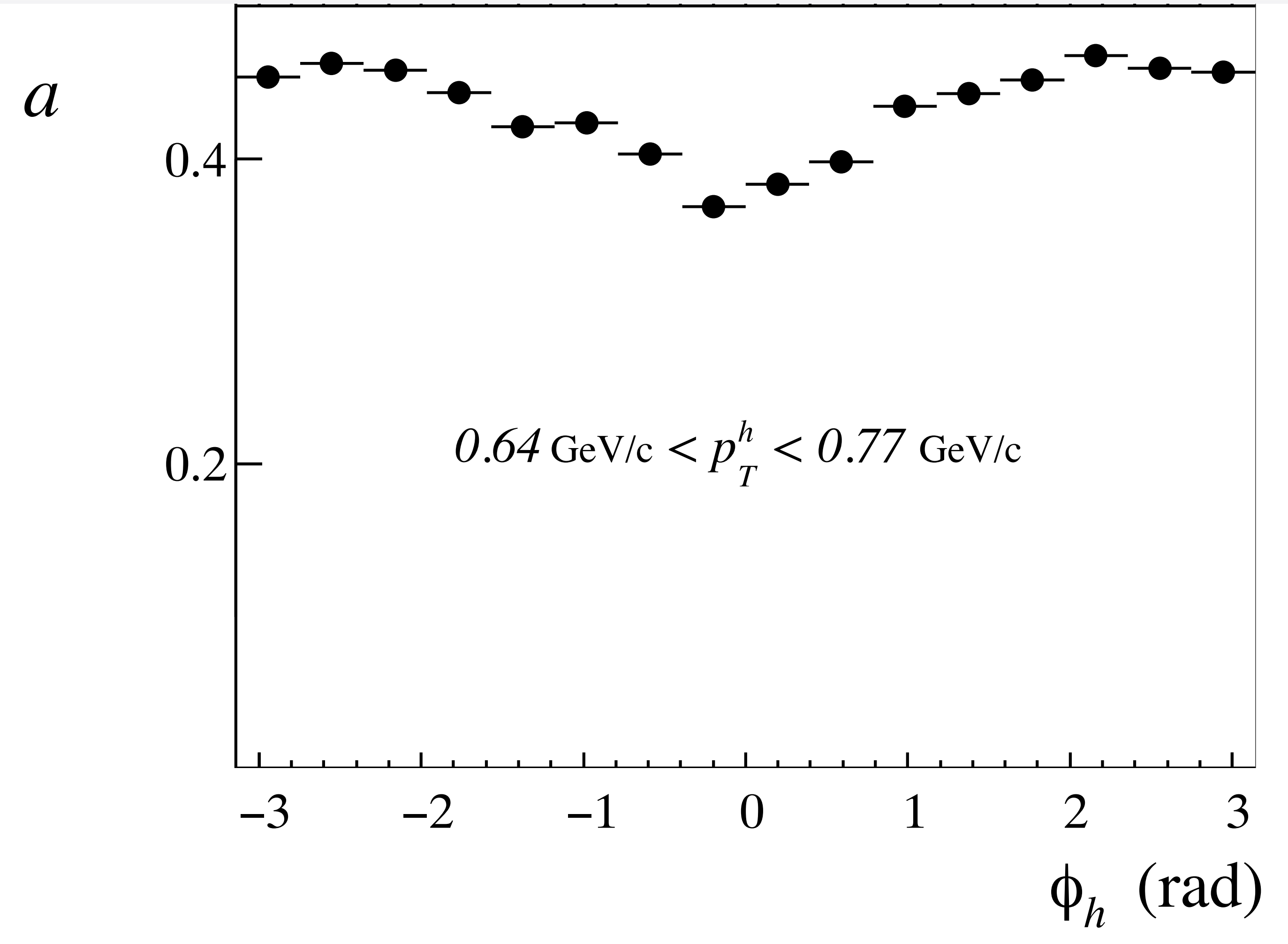


HERMES azimuthal acceptance for 2-hadron production

[P. van der Nat, Ph.D. thesis, Vrije Universiteit (2007)]

An unfortunate Lemma

- "No particle-physics experiment has a perfect acceptance!"



[C. Adolph, arXiv:1401.6284]

maybe "2π" around beam axis, but not around virtual-photon axis because of lower limit on θ

[see also A. Bianconi et al., Eur.Phys.J. A49 (2013) 42]

An unfortunate Lemma

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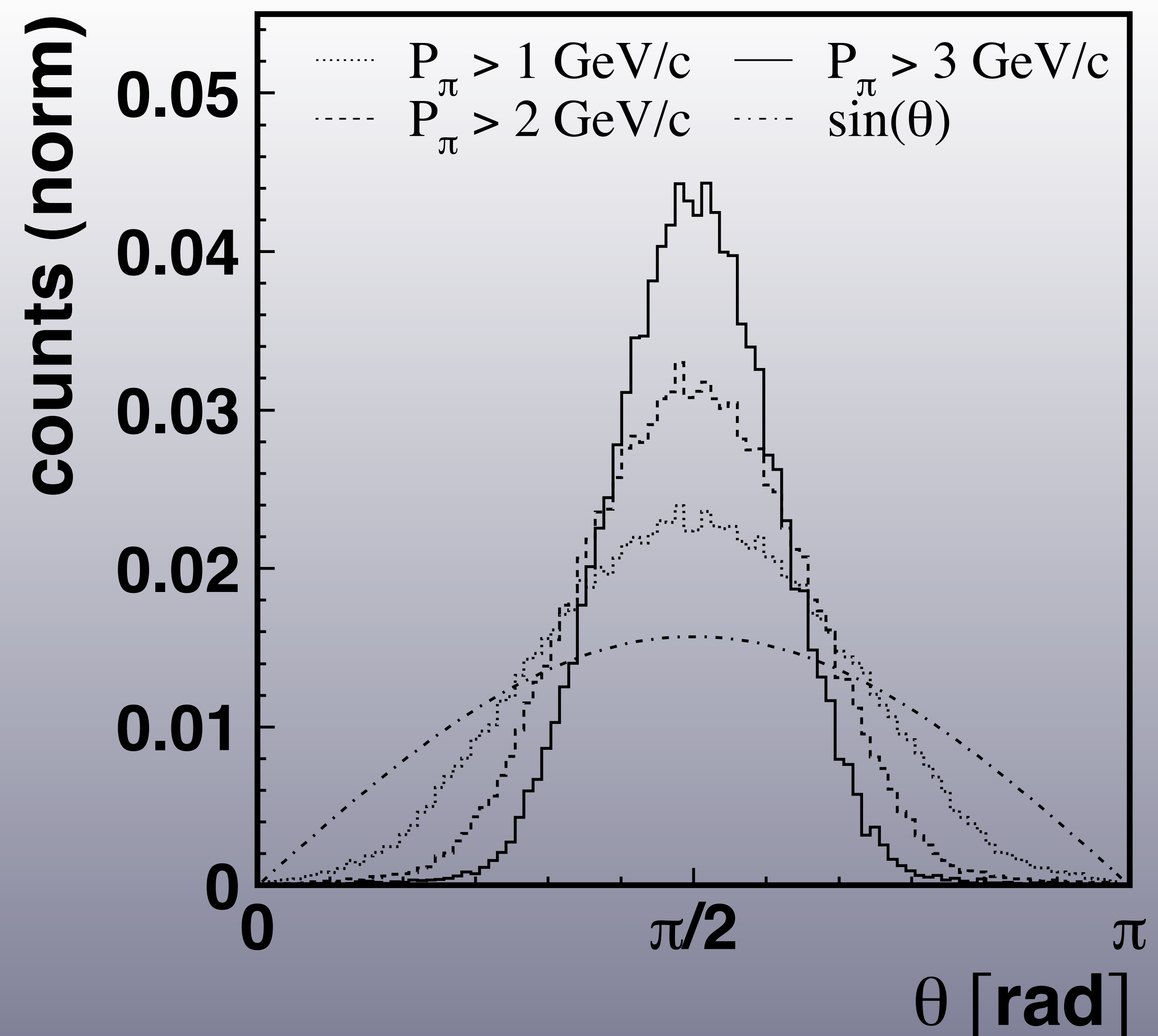

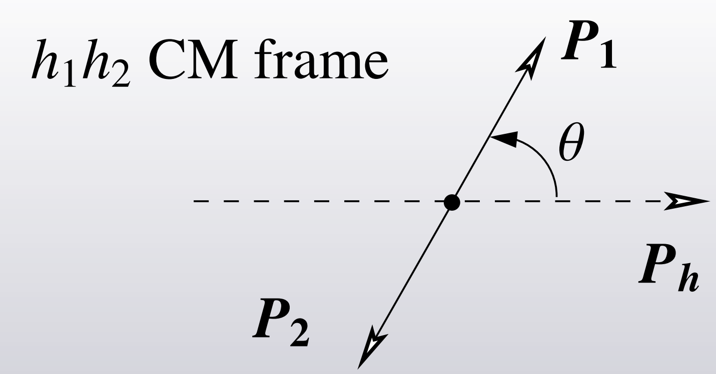

momentum cuts strongly distort kinematic distributions even for "4π" acceptance

[P. van der Nat, Ph.D. thesis, Vrije Universiteit (2007)]

An unfortunate Lemma

- "No particle-physics experiment has a perfect acceptance!"
    - obvious for detectors with gaps/holes
    - but also for "4π", especially when looking at complicated final states
- How acceptance effects are handled is one of the essential questions in experiments!

some acceptance effects

- acceptance in kinematic variable studied, e.g., azimuthal coverage in extraction of azimuthal moments

- acceptance in kinematic variables integrated over, e.g., due to limited statistics not being able to do fully differential analysis

a common misconception

- "acceptance cancels in asymmetries"

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$$A_{UT}(\phi,\Omega) = \frac{\sigma_{UT}(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)} \qquad \Omega = x, y, z, \ldots$$

a common misconception

- "acceptance cancels in asymmetries"

$$A_{UT}(\phi,\Omega) = \frac{\sigma_{UT}(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)} = \frac{\sigma_{UT}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)}$$

$\Omega = x, y, z, \ldots$

ϵ : **detection efficiency**

a common misconception

- "acceptance cancels in asymmetries"

$$A_{UT}(\phi,\Omega) = \frac{\sigma_{UT}(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)} \qquad \Omega = x, y, z, \ldots$$

$$= \frac{\sigma_{UT}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)} \qquad \epsilon : \textbf{detection efficiency}$$

$$\neq \frac{\int \mathrm{d}\Omega\,\sigma_{UT}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\int \mathrm{d}\Omega\,\sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)}$$

a common misconception

- "acceptance cancels in asymmetries"

$$
\begin{aligned}
A_{UT}(\phi,\Omega) &= \frac{\sigma_{UT}(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)} \qquad \Omega = x, y, z, \ldots \\
&= \frac{\sigma_{UT}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)} \qquad \epsilon : \textbf{detection efficiency} \\
&\neq \frac{\int \mathrm{d}\Omega\,\sigma_{UT}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\int \mathrm{d}\Omega\,\sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)} \neq \frac{\int \mathrm{d}\Omega\,\sigma_{UT}(\phi,\Omega)}{\int \mathrm{d}\Omega\,\sigma_{UU}(\phi,\Omega)} \equiv A_{UT}(\phi)
\end{aligned}
$$

a common misconception

- "acceptance cancels in asymmetries"

$$A_{UT}(\phi,\Omega) = \frac{\sigma_{UT}(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)} \qquad \Omega = x, y, z, \ldots$$

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$$\neq \frac{\int \mathrm{d}\Omega\,\sigma_{UT}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\int \mathrm{d}\Omega\,\sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)} \neq \frac{\int \mathrm{d}\Omega\,\sigma_{UT}(\phi,\Omega)}{\int \mathrm{d}\Omega\,\sigma_{UU}(\phi,\Omega)} \equiv A_{UT}(\phi)$$

Acceptance does **not cancel** in general when **integrating** numerator and denominator over (large) ranges in kinematic variables!

… geometric acceptance …

extract acceptance from Monte Carlo simulation?

$$\epsilon(\phi,\Omega) = \frac{\epsilon(\phi,\Omega)\sigma_{UU}(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)} \qquad \Omega = x, y, z, ..$$

simulated acceptance

simulated cross section

… geometric acceptance …

extract acceptance from Monte Carlo simulation?

$$\epsilon(\phi,\Omega) = \frac{\epsilon(\phi,\Omega)\sigma_{UU}(\phi,\Omega)}{\sigma_{UU}(\phi,\Omega)}$$

$$\neq \frac{\int d\Omega\, \sigma_{UU}(\phi,\Omega)\,\epsilon(\phi,\Omega)}{\int d\Omega\, \sigma_{UU}(\phi,\Omega)}$$

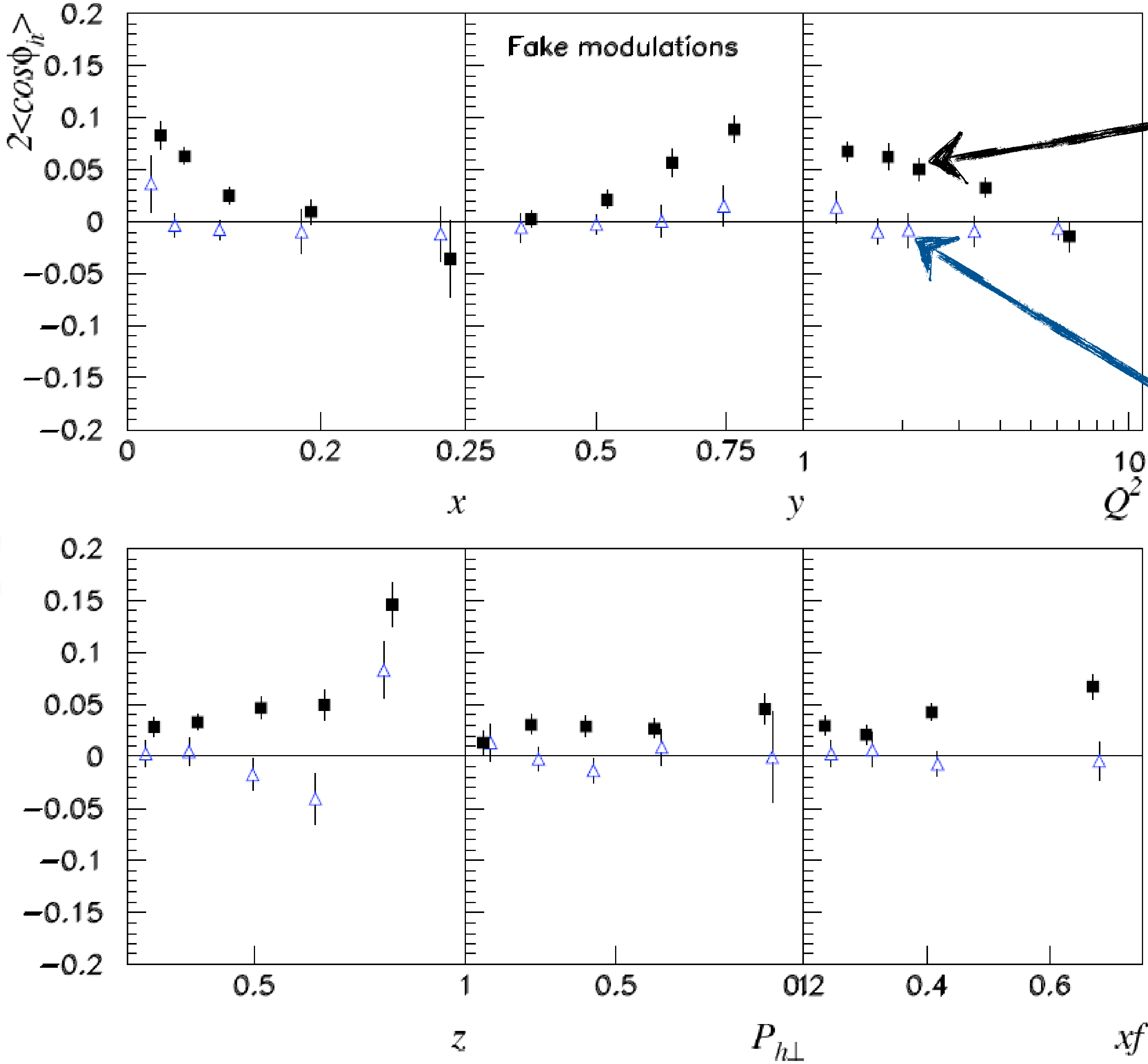
$$\neq \int d\Omega\, \epsilon(\phi,\Omega) \equiv \epsilon(\phi)$$

$$\Omega = x, y, z, \ldots$$

"Aus Differenzen und Summen kürzen nur die Dummen."

Cross-section model does NOT CANCEL in general when integrating numerator and denominator over (large) ranges in kinematic variables!

"Classique" Example: $\langle \cos\phi \rangle_{UU}$



1D correction

(input: MC without azimuthal modulation)

5D correction

[F. Giordano, Transversity 2008, Ferrara]

… averaging …

often enough one has to average observables over available phase space:

properly normalized for simplicity

$$\langle A(\Omega)\rangle_\epsilon \equiv \int \mathrm{d}\Omega\, A(\Omega)\epsilon(\Omega)$$

... averaging ...

often enough one has to average observables over available phase space:

$$\langle A(\Omega) \rangle_\epsilon \equiv \int \mathrm{d}\Omega \, A(\Omega) \epsilon(\Omega)$$

$$\neq \int \mathrm{d}\Omega \, A(\Omega) \equiv \langle A(\Omega) \rangle_{\text{``}4\pi\text{''}}$$

life (of the experimentalist) simplifies if asymmetries are weakly (i.e. not more than linearly) dependent on kinematics:

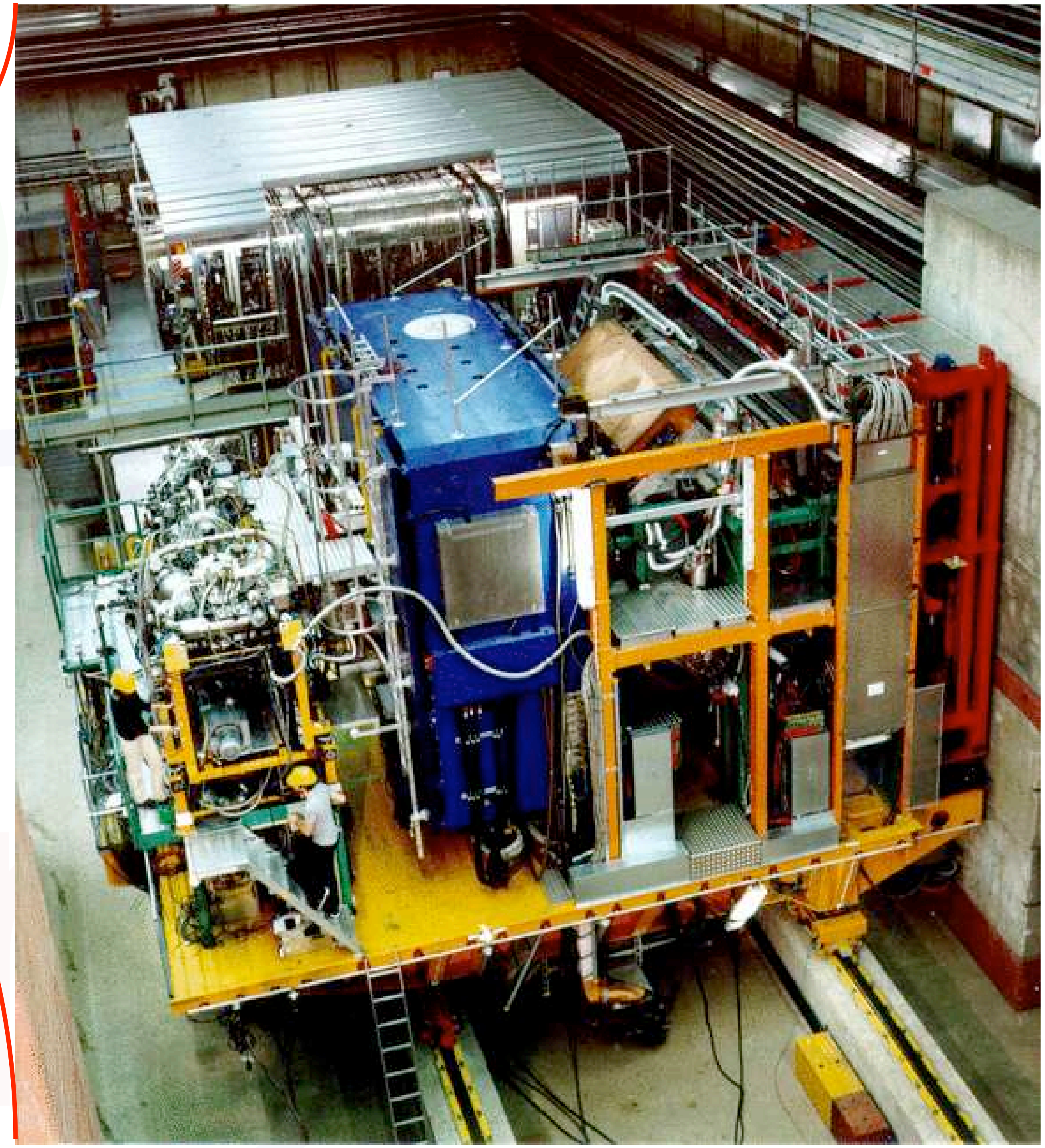
$$\langle A(\Omega) \rangle_\epsilon = A(\langle \Omega \rangle_\epsilon) \quad \text{for} \quad A(\Omega) = A_0 + A_1 \Omega$$

The HERMES experiment

(HERMES = HERA Measurement of Spin)

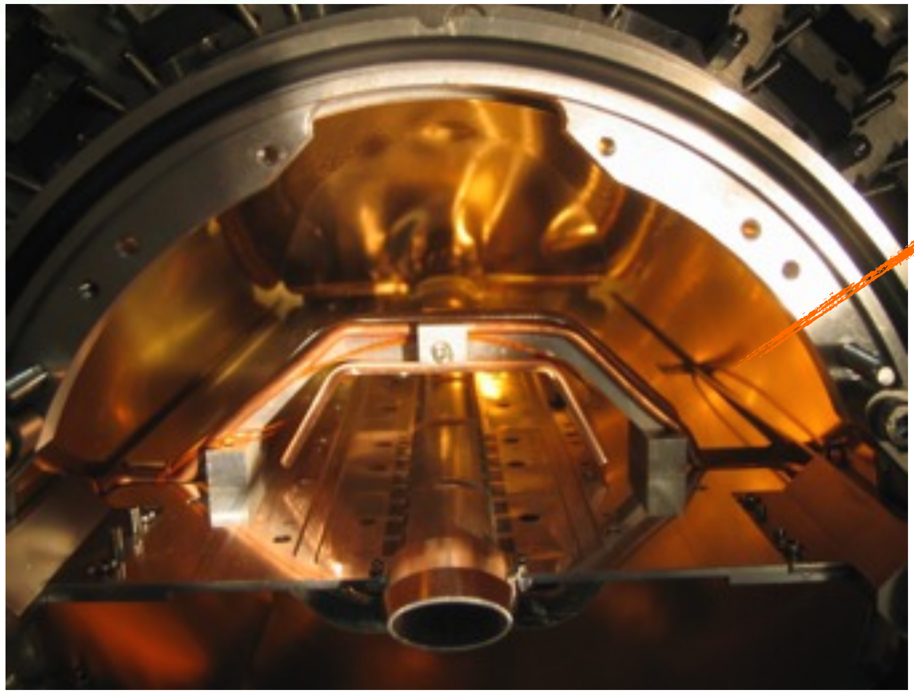
The HERMES Experiment

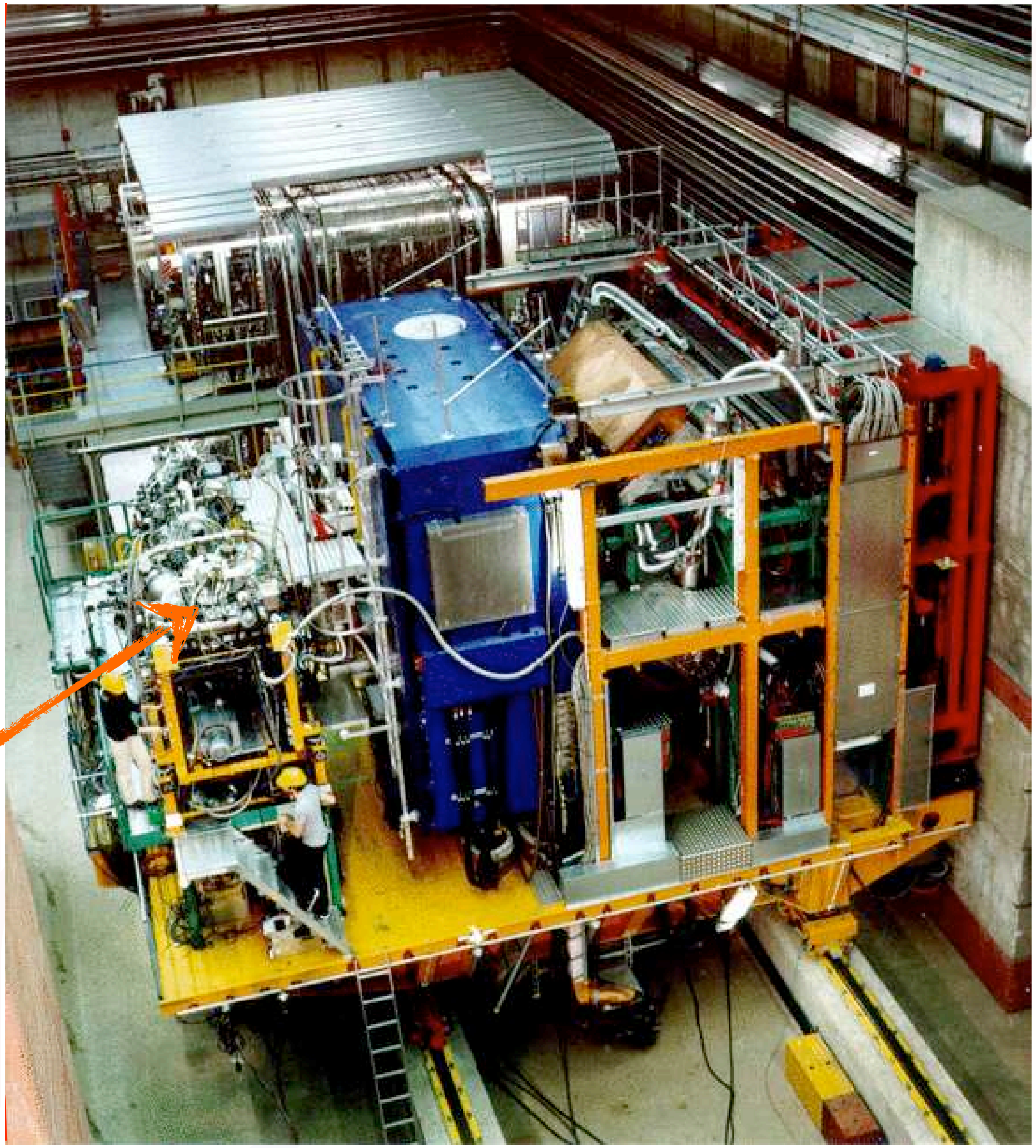
27.5 GeV e^+/e^- beam of HERA



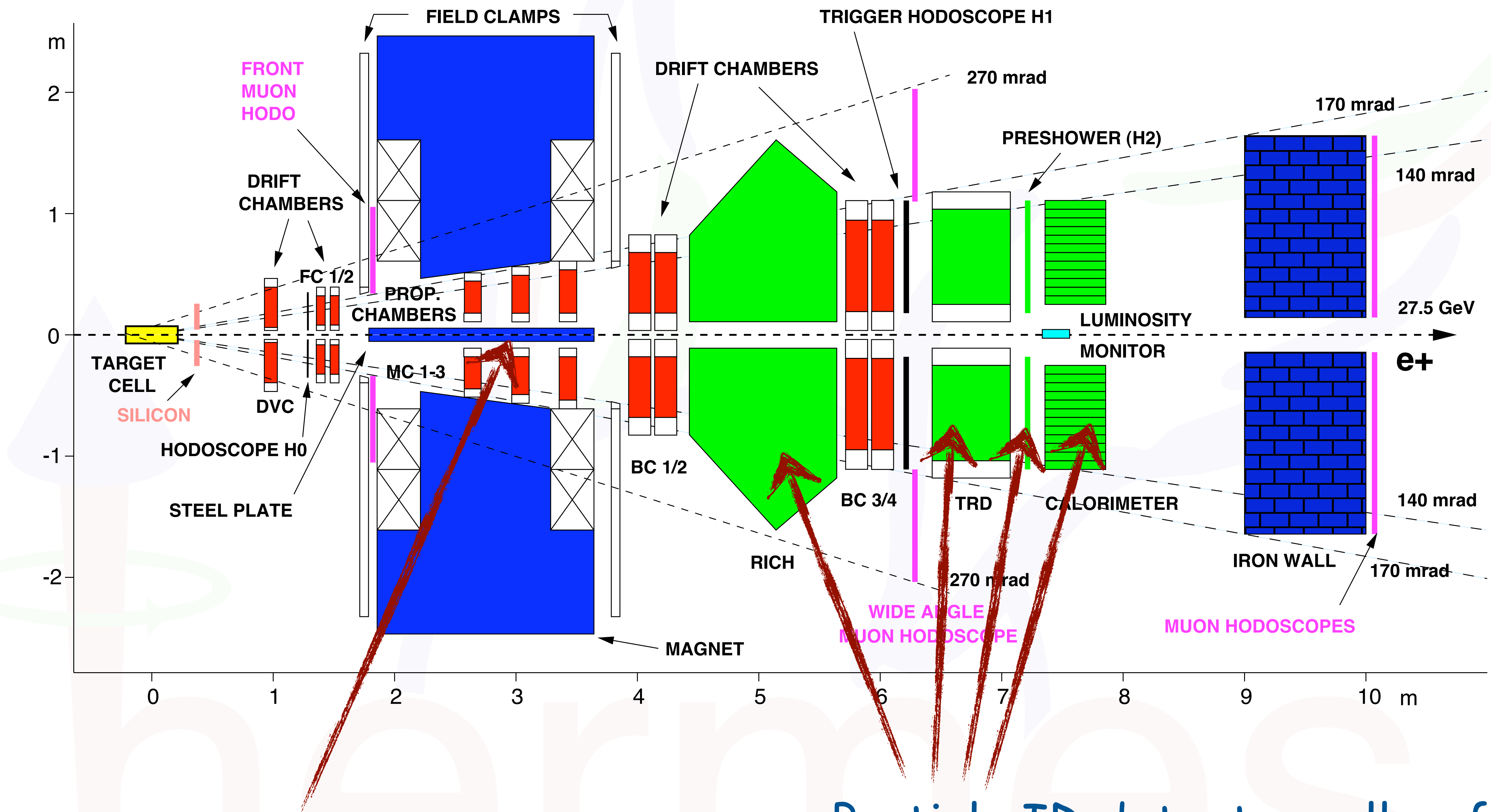
The HERMES Experiment

- pure gas targets
- internal to lepton ring
- unpolarized (^{1}H … Xe)
- long. polarized: ^{1}H, ^{2}H, ^{3}He
- transversely polarized: ^{1}H





HERMES (1998-2005) schematically



two (mirror-symmetric) halves -> no homogenous azimuthal coverage

Particle ID detectors allow for
- lepton/hadron separation
- RICH: pion/kaon/proton discrimination 2GeV<p<15GeV

1-Hadron production (ep→ehX)

$$d\sigma = d\sigma_{UU}^{0} + \cos 2\phi \, d\sigma_{UU}^{1} + \frac{1}{Q} \cos\phi \, d\sigma_{UU}^{2} + \lambda_e \frac{1}{Q} \sin\phi \, d\sigma_{LU}^{3}$$

$$+ S_L \left\{ \sin 2\phi \, d\sigma_{UL}^{4} + \frac{1}{Q} \sin\phi \, d\sigma_{UL}^{5} + \lambda_e \left[d\sigma_{LL}^{6} + \frac{1}{Q} \cos\phi \, d\sigma_{LL}^{7} \right] \right\}$$

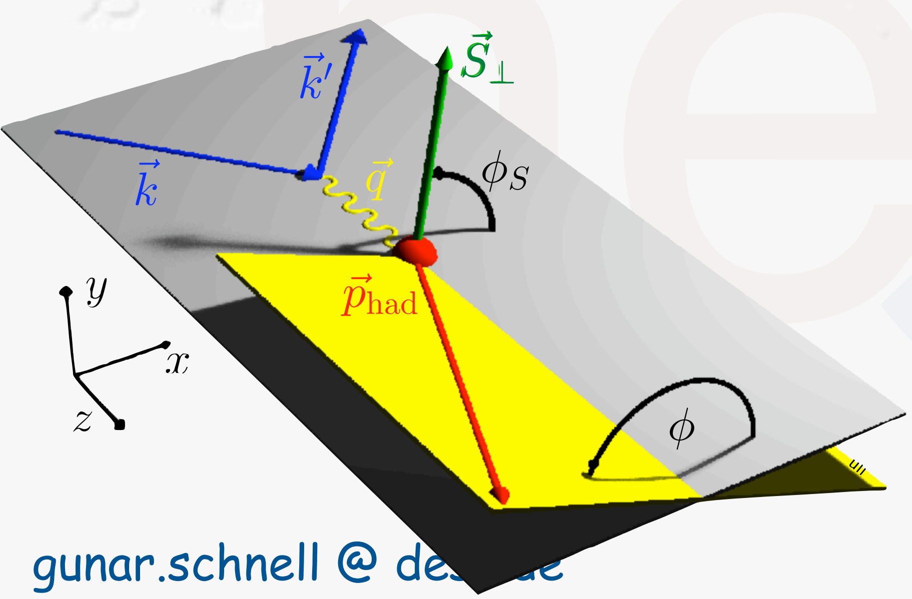
$$+ S_T \left\{ \sin(\phi - \phi_S) \, d\sigma_{UT}^{8} + \sin(\phi + \phi_S) \, d\sigma_{UT}^{9} + \sin(3\phi - \phi_S) \, d\sigma_{UT}^{10} \right.$$

$$+ \frac{1}{Q} \left(\sin(2\phi - \phi_S) \, d\sigma_{UT}^{11} + \sin\phi_S \, d\sigma_{UT}^{12} \right)$$

$$\left. + \lambda_e \left[\cos(\phi - \phi_S) \, d\sigma_{LT}^{13} + \frac{1}{Q} \left(\cos\phi_S \, d\sigma_{LT}^{14} + \cos(2\phi - \phi_S) \, d\sigma_{LT}^{15} \right) \right] \right\}$$

σ_{XY}

Beam **Target**
Polarization



Mulders and Tangermann, Nucl. Phys. B 461 (1996) 197

Boer and Mulders, Phys. Rev. D 57 (1998) 5780

Bacchetta et al., Phys. Lett. B 595 (2004) 309

Bacchetta et al., JHEP 0702 (2007) 093

"Trento Conventions", Phys. Rev. D 70 (2004) 117504

1-Hadron production (ep→ehX)

$$d\sigma = d\sigma^0_{UU} + \cos 2\phi\, d\sigma^1_{UU} + \frac{1}{Q}\cos\phi\, d\sigma^2_{UU} + \lambda_e \frac{1}{Q}\sin\phi\, d\sigma^3_{LU}$$

$$+S_L\left\{\sin 2\phi\, d\sigma^4_{UL} + \frac{1}{Q}\sin\phi\, d\sigma^5_{UL} + \lambda_e\left[d\sigma^6_{LL} + \frac{1}{Q}\cos\phi\, d\sigma^7_{LL}\right]\right\}$$

$$+S_T\left\{\sin(\phi-\phi_S)\, d\sigma^8_{UT} + \sin(\phi+\phi_S)\, d\sigma^9_{UT} + \sin(3\phi-\phi_S)\, d\sigma^{10}_{UT}\right.$$

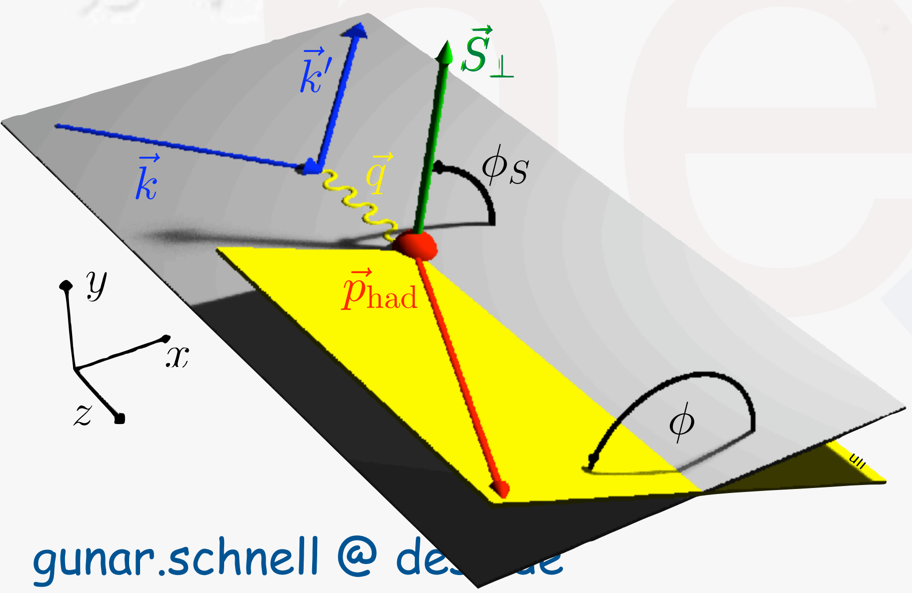
$$+\frac{1}{Q}\left(\sin(2\phi-\phi_S)\, d\sigma^{11}_{UT} + \sin\phi_S\, d\sigma^{12}_{UT}\right)$$

$$\left.+\lambda_e\left[\cos(\phi-\phi_S)\, d\sigma^{13}_{LT} + \frac{1}{Q}\left(\cos\phi_S\, d\sigma^{14}_{LT} + \cos(2\phi-\phi_S)\, d\sigma^{15}_{LT}\right)\right]\right\}$$

σ_{XY}

Beam **Target**

Polarization



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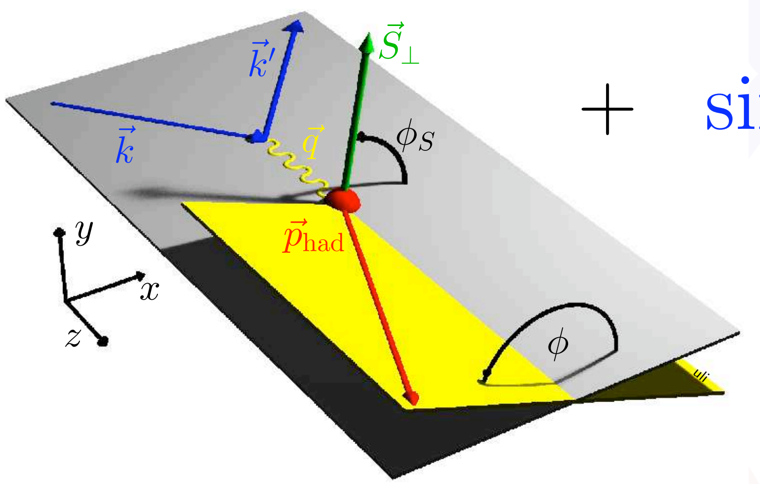
Measuring azimuthal SSA

$$A_{UT}(\phi,\phi_S) = \frac{1}{\langle|S_\perp|\rangle}\frac{N_h^\uparrow(\phi,\phi_S)-N_h^\downarrow(\phi,\phi_S)}{N_h^\uparrow(\phi,\phi_S)+N_h^\downarrow(\phi,\phi_S)}$$

$$\sim \sin(\phi+\phi_S)\sum_q e_q^2\,\mathcal{I}\left[\frac{\boldsymbol{k_T}\hat{\boldsymbol{P}}_{\boldsymbol{h}\perp}}{M_h}h_1^q(x,p_T^2)H_1^{\perp,q}(z,k_T^2)\right]$$

$$+ \sin(\phi-\phi_S)\sum_q e_q^2\,\mathcal{I}\left[\frac{\boldsymbol{p_T}\hat{\boldsymbol{P}}_{\boldsymbol{h}\perp}}{M}f_{1T}^{\perp,q}(x,p_T^2)D_1^q(z,k_T^2)\right]$$

$$+ \quad \cdots$$



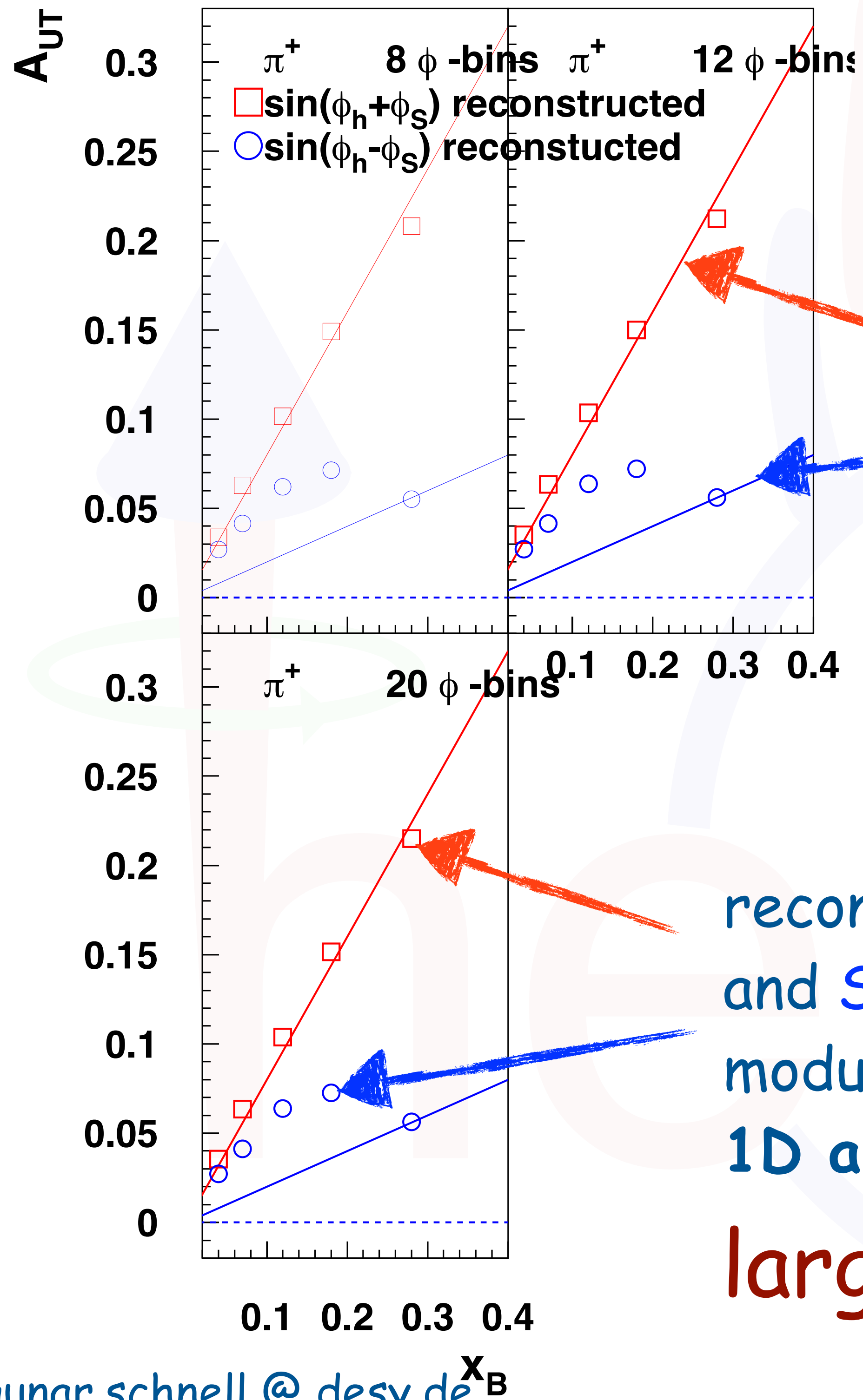
$\mathcal{I}[\ldots]$: **convolution integral over initial (p_T) and final (k_T) quark transverse momenta**

⇒ 2D Max.Likelihd. fit of to get Collins and Sivers amplitudes:

$$PDF(2\langle\sin(\phi\pm\phi_S)\rangle_{UT},\ldots,\phi,\phi_S) = \tfrac{1}{2}\{1+P_T(2\langle\sin(\phi\pm\phi_S)\rangle_{UT}\sin(\phi\pm\phi_s)+\ldots)\}$$

1D vs. 2D fitting

- limited acceptance introduces correlations to originally orthogonal azimuthal Fourier amplitudes



input models for Collins- and Sivers-type modulations

reconstructed Collins- and Sivers-type modulations

1D analysis

large false asymmetries

1D vs. 2D fitting

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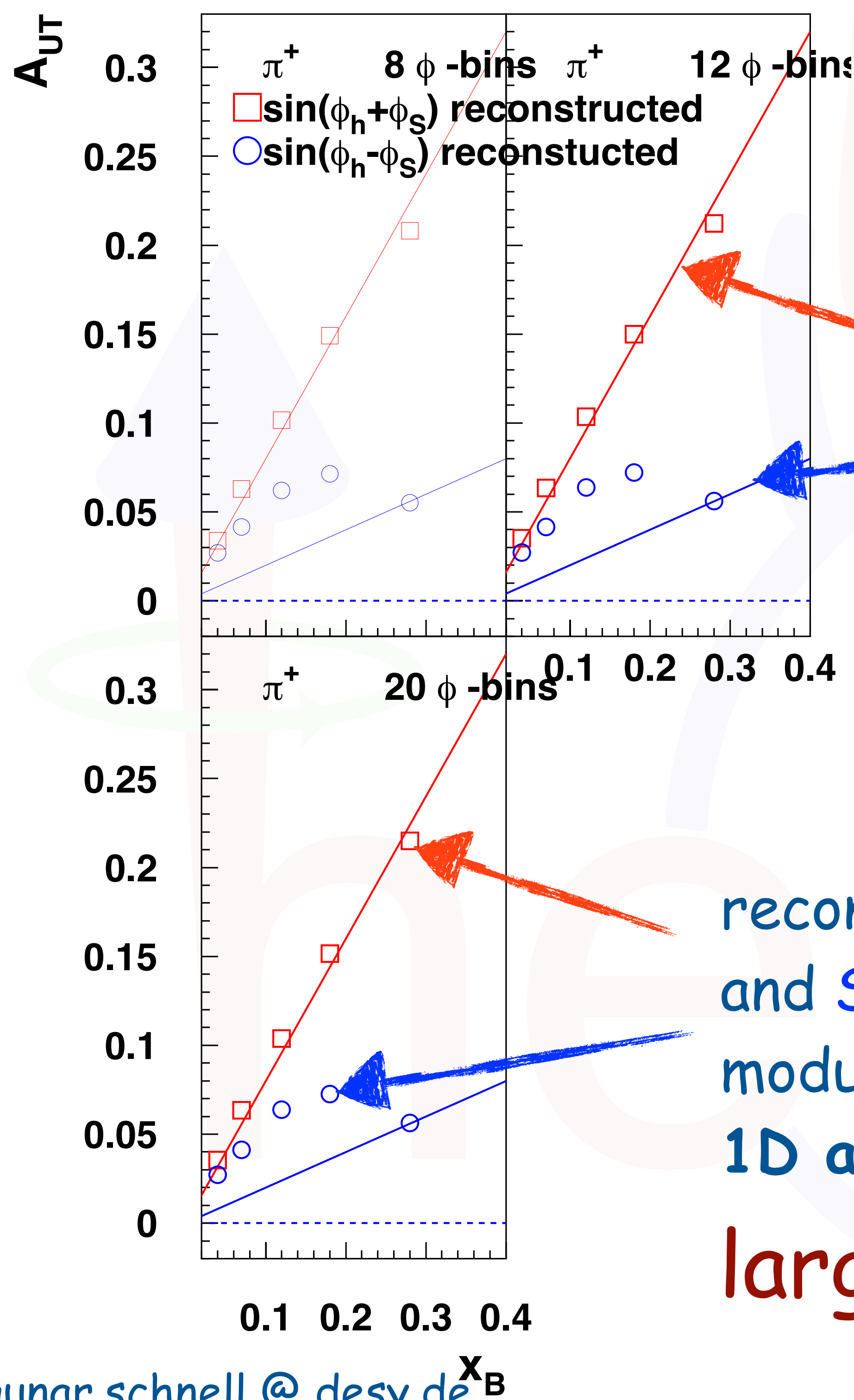
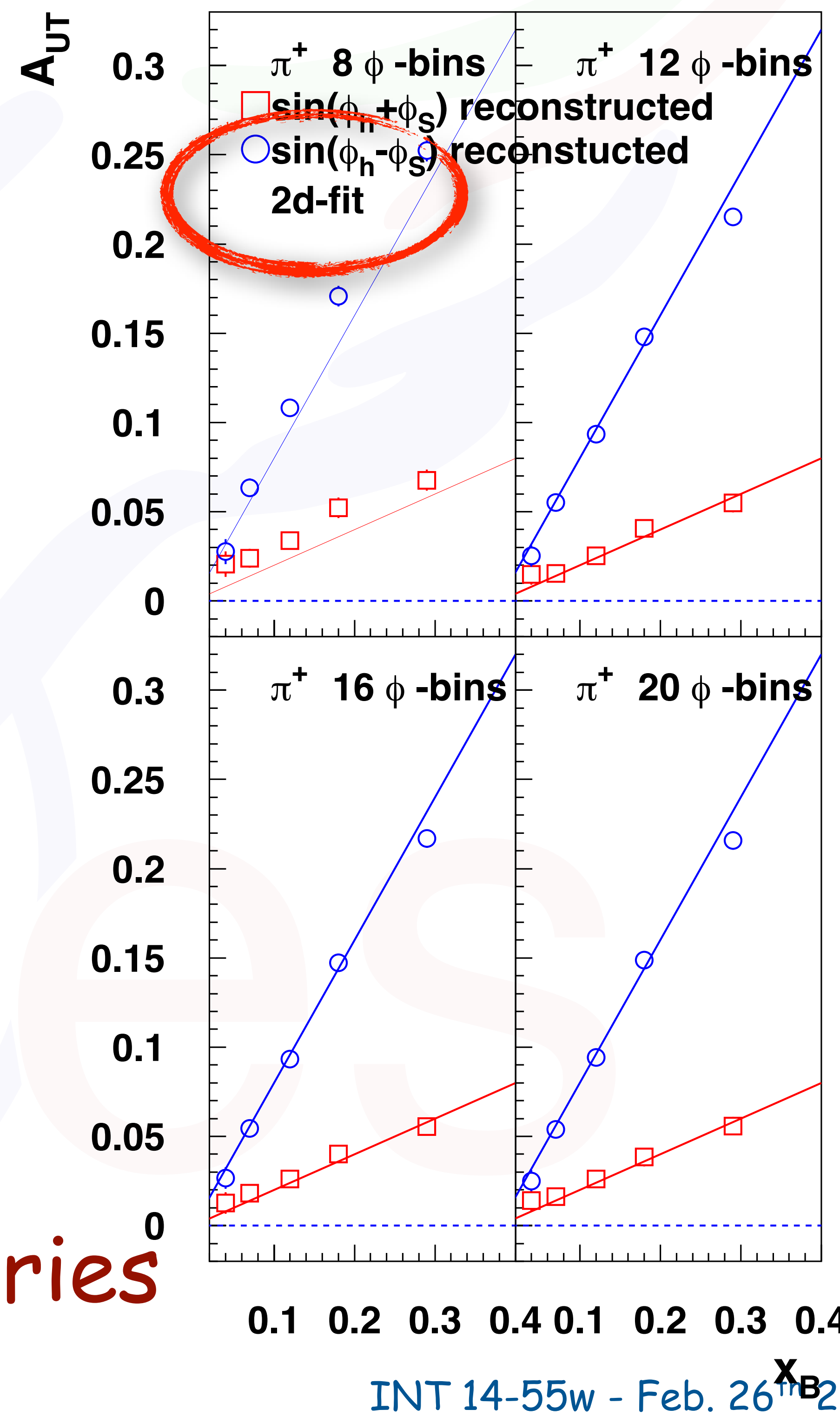


input models for Collins- and Sivers-type modulations

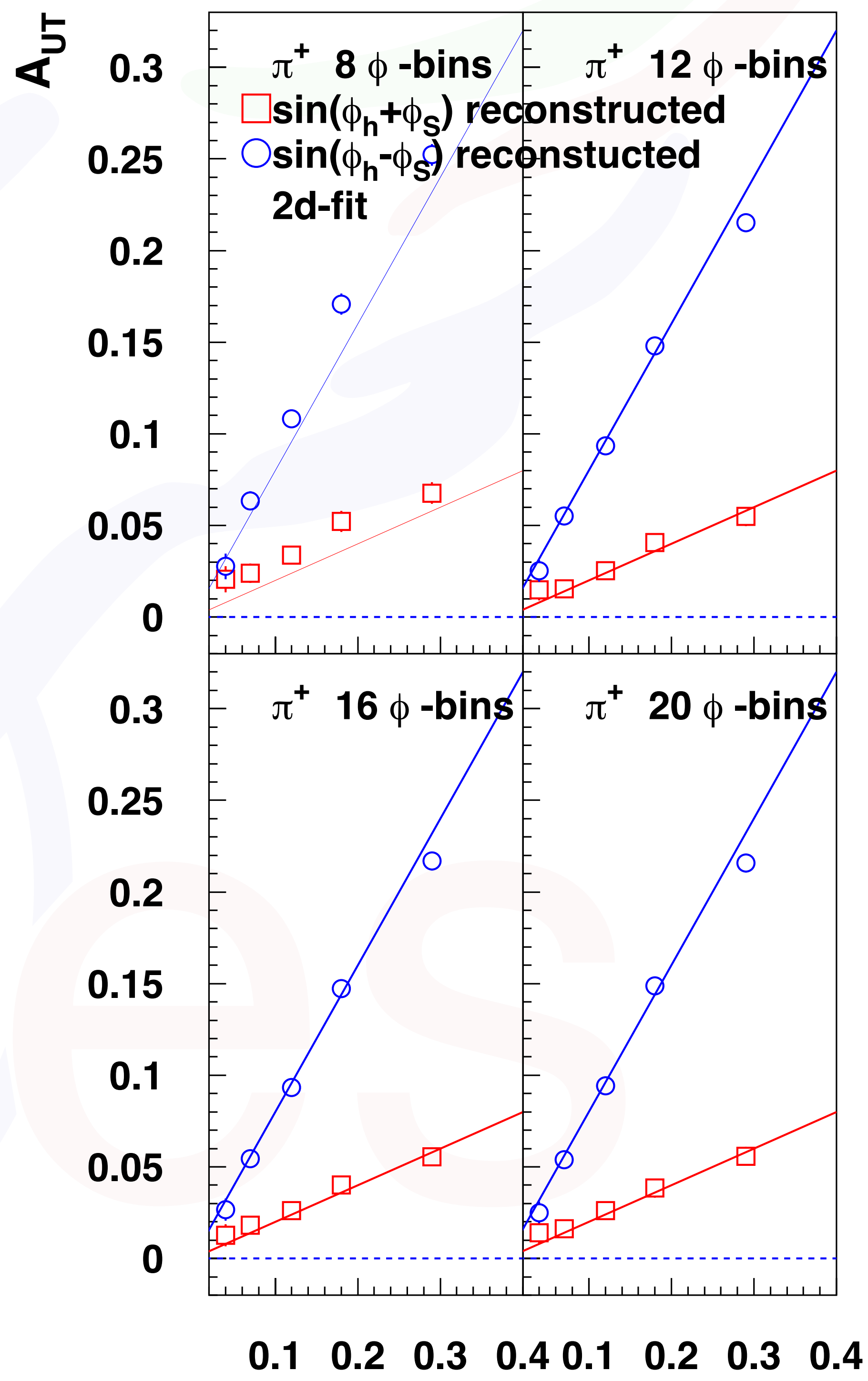
reconstructed Collins- and Sivers-type modulations
1D analysis



large false asymmetries

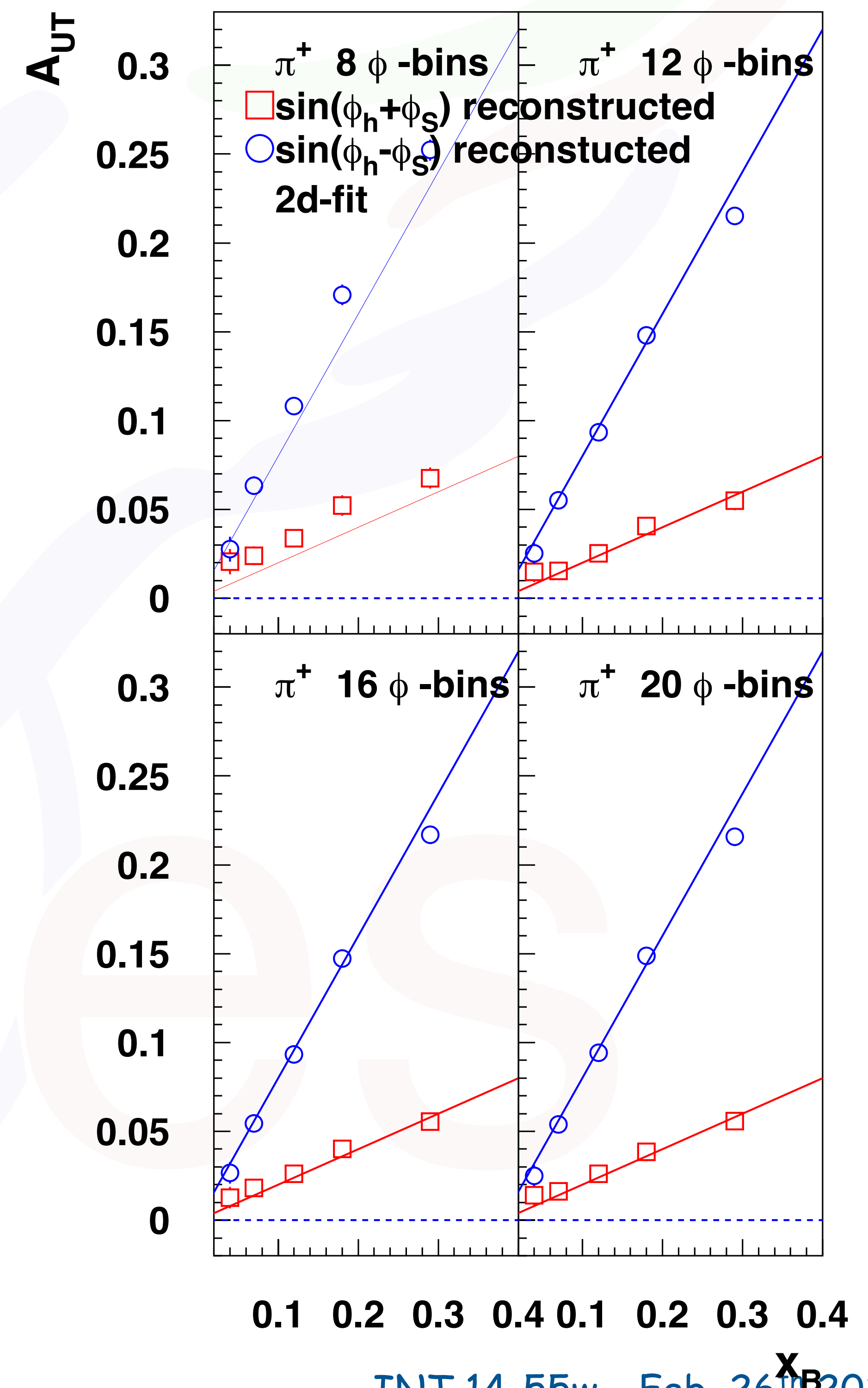
choice of models

- linear dependence kind of trivial to reproduce (see earlier slide)



choice of models

- linear dependence kind of trivial to reproduce (see earlier slide)
- need more realistic model



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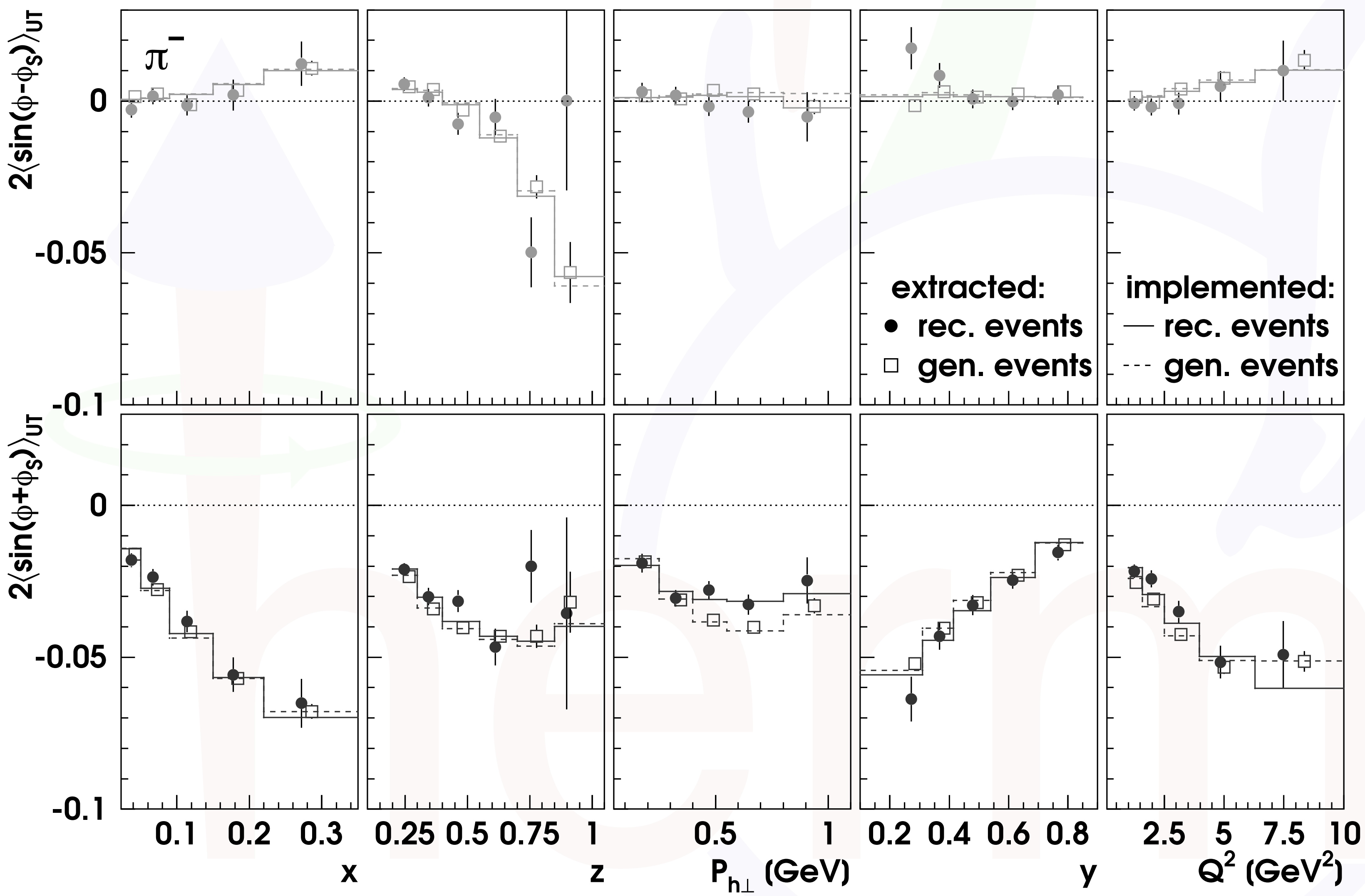
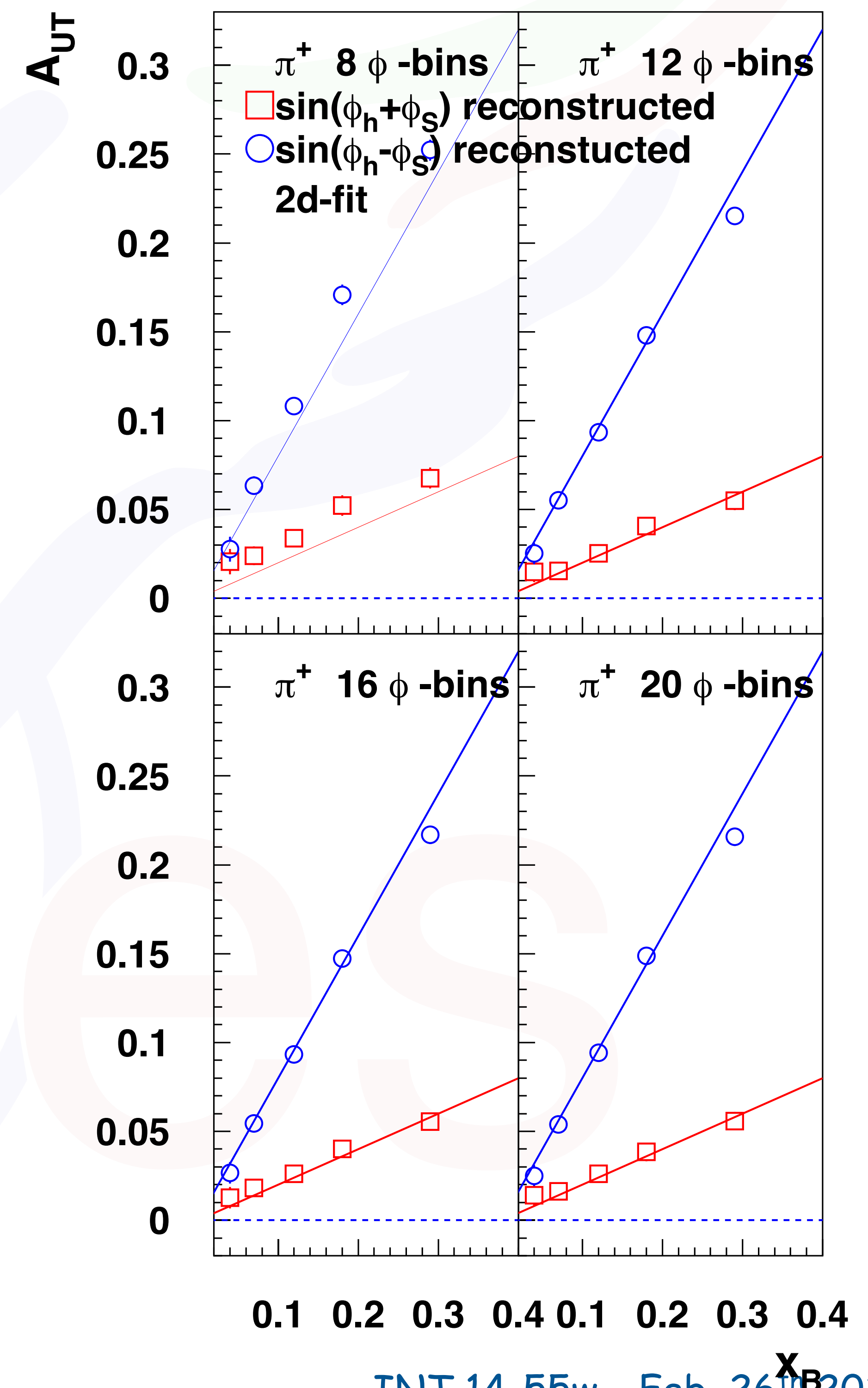


GMCtrans



GMC$_{\text{TRANS}}$ ingredients

Initial goals

- physics generator for SIDIS pion production
- include transverse-momentum dependence, in particular simulate Collins and Sivers effects
- be fast
- allow comparison of input model and reconstructed amplitudes
- to be used with standard HERMES Monte Carlo
- be extendable (e.g., open for new models)

Basic workings

- use cross section that can be calculated analytically
- do not simulate full event
- start from 1-hadron SIDIS expressions of Mulders & Tangerman (Nucl.Phys.B461:197-237,1996) and others
- use Gaussian Ansatz for all transverse-momentum dependences of DFs and FFs
- unpolarized DFs (as well as helicity distribution) and FFs from fits/parametrizations (e.g., Kretzer FFs etc.)
- "polarized" DFs and FFs either related to unpolarized ones (e.g., saturation of Soffer bound for transversity)or some parametrizations used

SIDIS Cross Section incl. TMDs

$$d\sigma_{UT} \equiv d\sigma_{UT}^{\text{Collins}} \cdot \sin(\phi + \phi_S) + d\sigma_{UT}^{\text{Sivers}} \cdot \sin(\phi - \phi_S)$$

$$d\sigma_{UT}^{\text{Collins}}(x, y, z, \phi_S, P_{h\perp}) \equiv -\frac{2\alpha^2}{sxy^2} B(y) \sum_q e_q^2 \, \mathcal{I}\left[\left(\frac{k_T \cdot \hat{P}_{h\perp}}{M_h}\right) \cdot h_1^q H_1^{\perp q}\right]$$

$$d\sigma_{UT}^{\text{Sivers}}(x, y, z, \phi_S, P_{h\perp}) \equiv -\frac{2\alpha^2}{sxy^2} A(y) \sum_q e_q^2 \, \mathcal{I}\left[\left(\frac{p_T \cdot \hat{P}_{h\perp}}{M_N}\right) \cdot f_{1T}^{\perp q} D_1^q\right]$$

$$d\sigma_{UU}(x, y, z, \phi_S, P_{h\perp}) \equiv \frac{2\alpha^2}{sxy^2} A(y) \sum_q e_q^2 \, \mathcal{I}\left[f_1^q D_1^q\right]$$

where

$$\mathcal{I}\left[\mathcal{W}\, f\, D\right] \equiv \int d^2p_T d^2k_T \, \delta^{(2)}\left(p_T - \frac{P_{h\perp}}{z} - k_T\right)\left[\mathcal{W}\, f(x, p_T)\, D(z, k_T)\right]$$

Gaussian Ansatz

- want to deconvolve convolution integral over transverse momenta
- easy Ansatz: Gaussian dependences of DFs and FFs on intrinsic (quark) transverse momentum:

$$\mathcal{I}[f_1(x,\boldsymbol{p_T^2})D_1(z,z^2\boldsymbol{k_T^2})] = f_1(x)\cdot D_1(z)\cdot \frac{R^2}{\pi z^2}\cdot e^{-R^2\frac{P_{h\perp}^2}{z^2}}$$

with $\boldsymbol{f_1(x,p_T^2) \;=\; f_1(x)\frac{1}{\pi\langle p_T^2\rangle}e^{-\frac{p_T^2}{\langle p_T^2\rangle}}}$ $\qquad \frac{1}{R^2} \equiv \langle k_T^2\rangle + \langle p_T^2\rangle = \frac{\langle P_{h\perp}^2\rangle}{z^2}$

(similar: $D_1(z,z^2\boldsymbol{k_T^2})$)

Caution: different notations for intrinsic transverse momenta exist! (Here: "Amsterdam notation")

Positivity Constraints

- DFs (FFs) have to fulfill various positivity constraints (resulting cross section has to be positive!)
- based on probability considerations one can derive positivity limits for leading-twist functions: Bacchetta et al., Phys. Rev. Lett. 85 (2000) 712-715

➡ transversity: e.g., Soffer bound

➡ Sivers and Collins functions: e.g., loose bounds:

$$\frac{|p_T|}{2M_N} f_{1T}^{\perp}(x, p_T^2) \equiv f_{1T}^{\perp(1/2)}(x, p_T^2) \leq \frac{1}{2} f_1(x, p_T^2)$$

$$\frac{|k_T|}{2M_h} H_1^{\perp}(z, z^2 k_T^2) \equiv H_1^{\perp(1/2)}(z, z^2 k_T^2) \leq \frac{1}{2} D_1(z, z^2 k_T^2)$$

Positivity and the Gaussian Ansatz

$$\frac{|\boldsymbol{p_T}|}{2M_N} f_{1T}^{\perp}(x, \boldsymbol{p_T^2}) \leq \frac{1}{2} f_1(x, \boldsymbol{p_T^2})$$

with
$$\boldsymbol{f_1(x, p_T^2) = f_1(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}}$$

$$\boldsymbol{f_{1T}^{\perp}(x, p_T^2) = f_{1T}^{\perp}(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}}$$

$$\Rightarrow \quad \boldsymbol{|p_T| f_{1T}^{\perp}(x) \leq M_N f_1(x)}$$

Positivity and the Gaussian Ansatz

$$\frac{|\boldsymbol{p_T}|}{2M_N} f_{1T}^{\perp}(x, \boldsymbol{p_T^2}) \leq \frac{1}{2} f_1(x, \boldsymbol{p_T^2})$$

with

$$f_1(x, p_T^2) = f_1(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}$$

$$f_{1T}^{\perp}(x, p_T^2) = f_{1T}^{\perp}(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-\frac{p_T^2}{\langle p_T^2 \rangle}}$$

$$\rightarrow \quad |p_T| f_{1T}^{\perp}(x) \leq M_N f_1(x)$$

No (useful) solution for non-zero Sivers function!

Modify Gaussian width

$$f_{1T}^{\perp}(x,p_T^2) = f_{1T}^{\perp}(x)\,\frac{1}{(1-C)\pi\langle p_T^2\rangle}\,e^{-\frac{p_T^2}{(1-C)\langle p_T^2\rangle}}$$

➡ **positivity limit:**

$$f_{1T}^{\perp}(x)\,\frac{|p_T|}{2M_N}\,\frac{1}{\pi(1-C)\langle p_T^2\rangle}\,e^{-\frac{p_T^2}{(1-C)\langle p_T^2\rangle}} \leq 1/2\, f_1(x)\,\frac{1}{\pi\langle p_T^2\rangle}\,e^{-\frac{p_T^2}{\langle p_T^2\rangle}}$$

➡ $$\frac{|p_T|}{1-C}\,e^{-\frac{C}{1-C}\frac{p_T^2}{\langle p_T^2\rangle}} \leq M_N\,\frac{f_1(x)}{f_{1T}^{\perp}(x)}$$

SIDIS Cross Section incl. TMDs

$$\sum_q \frac{e_q^2}{4\pi} \frac{\alpha^2}{(MExyz)^2} \left[X_{UU} + |\mathbf{S}_T| X_{SIV} \sin(\phi_h - \phi_s) + |\mathbf{S}_T| X_{COL} \sin(\phi_h + \phi_s) \right]$$

using Gaussian Ansatz for transverse-momentum dependence of DFs and FFs:

$$X_{UU} = R^2 e^{-R^2 P_{h\perp}^2/z^2} \left(1 - y + \frac{y^2}{2}\right) f_1(x) \cdot D_1(z)$$

$$X_{COL} = + \frac{|P_{h\perp}|}{M_\pi z} \frac{(1-C)\langle k_T^2 \rangle}{\left[\langle p_T^2 \rangle + (1-C)\langle k_T^2 \rangle\right]^2} \exp\left[- \frac{P_{h\perp}^2/z^2}{\langle p_T^2 \rangle + (1-C)\langle k_T^2 \rangle} \right] \times (1-y) \cdot h_1(x) \cdot H_1^\perp(z)$$

$$X_{SIV} = - \frac{|P_{h\perp}|}{M_p z} \frac{(1-C')\langle p_T^2 \rangle}{\left[\langle k_T^2 \rangle + (1-C')\langle p_T^2 \rangle\right]^2} \exp\left[- \frac{P_{h\perp}^2/z^2}{\langle k_T^2 \rangle + (1-C')\langle p_T^2 \rangle} \right] \times \left(1 - y + \frac{y^2}{2}\right) f_{1T}^\perp(x) \cdot D_1(z)$$

Example: Sivers (azimuthal) moments

use cross section expressions to evaluate azimuthal moments:

$$-\langle \sin(\phi - \phi_s) \rangle_{UT} = \frac{\sqrt{(1-C)\langle p_T^2 \rangle}}{\sqrt{(1-C)\langle p_T^2 \rangle + \langle k_T^2 \rangle}} \frac{A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp(1/2)}(x) D_1(z)}{A(y) \frac{1}{xy^2} \sum e_q^2 f_1(x) D_1(z)}$$

$$-\langle \sin(\phi - \phi_s) \rangle_{UT} = \frac{M_N \sqrt{\pi}}{2\sqrt{(1-C)\langle p_T^2 \rangle + \langle k_T^2 \rangle}} \frac{A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp(1)}(x) D_1(z)}{A(y) \frac{1}{xy^2} \sum e_q^2 f_1(x) D_1(z)}$$

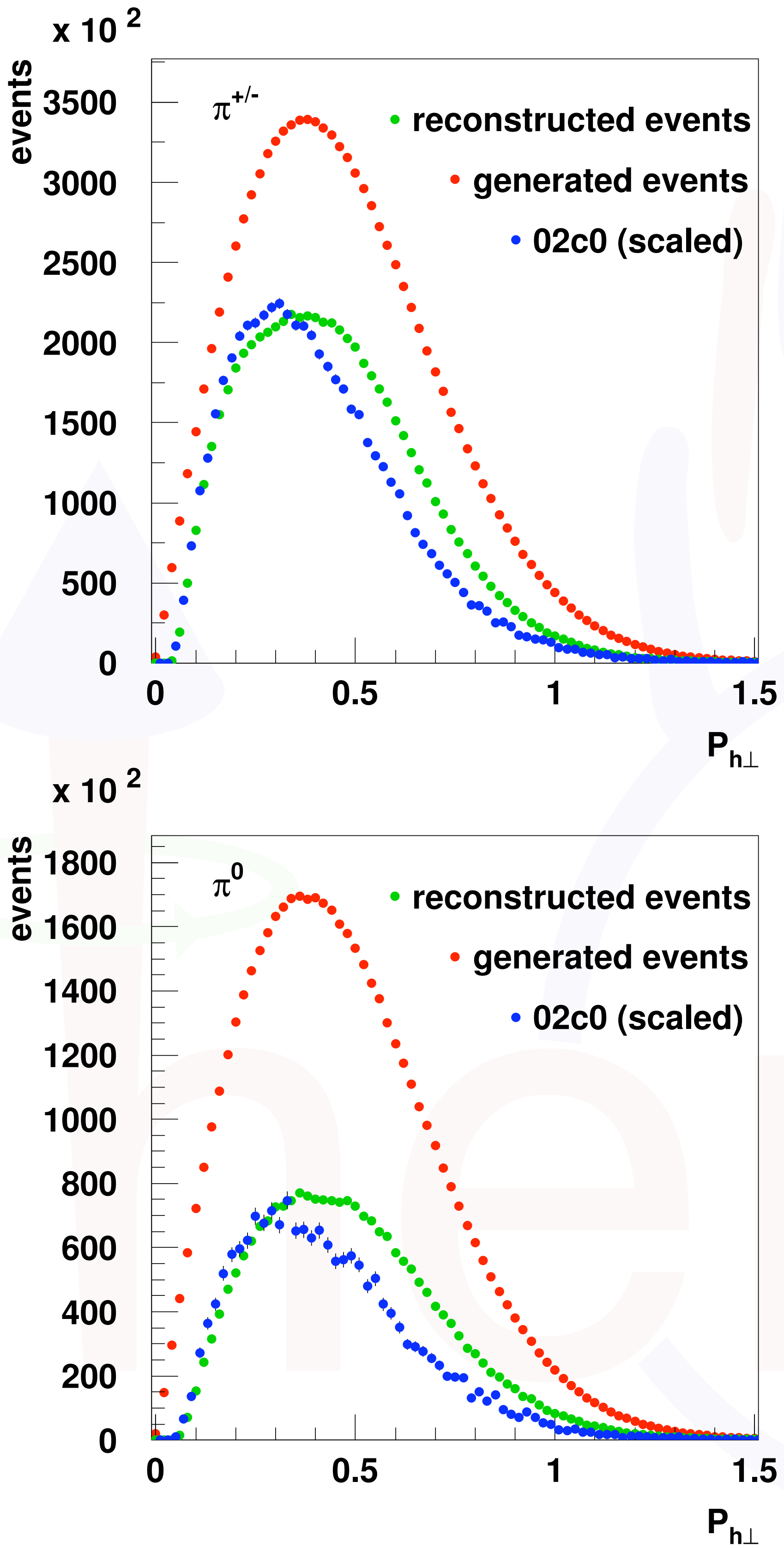
$$-\langle \frac{|P_{h\perp}|}{zM_N} \sin(\phi - \phi_s) \rangle_{UT} = \frac{2\sqrt{(1-C)\langle p_T^2 \rangle}}{M_N \sqrt{\pi}} \frac{A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp(1/2)}(x) D_1(z)}{A(y) \frac{1}{xy^2} \sum e_q^2 f_1(x) D_1(z)}$$

$$-\langle \frac{|P_{h\perp}|}{zM_N} \sin(\phi - \phi_s) \rangle_{UT} = \frac{A(y) \frac{1}{xy^2} \sum e_q^2 f_{1T}^{\perp(1)}(x) D_1(z)}{A(y) \frac{1}{xy^2} \sum e_q^2 f_1(x) D_1(z)}$$

model-dependence on transverse momenta "swallowed" by p_T^2 - moment of Sivers fct.: $f_{1T}^{\perp(1)}$

Selected Results

Tuning the Gaussians in gmc_trans

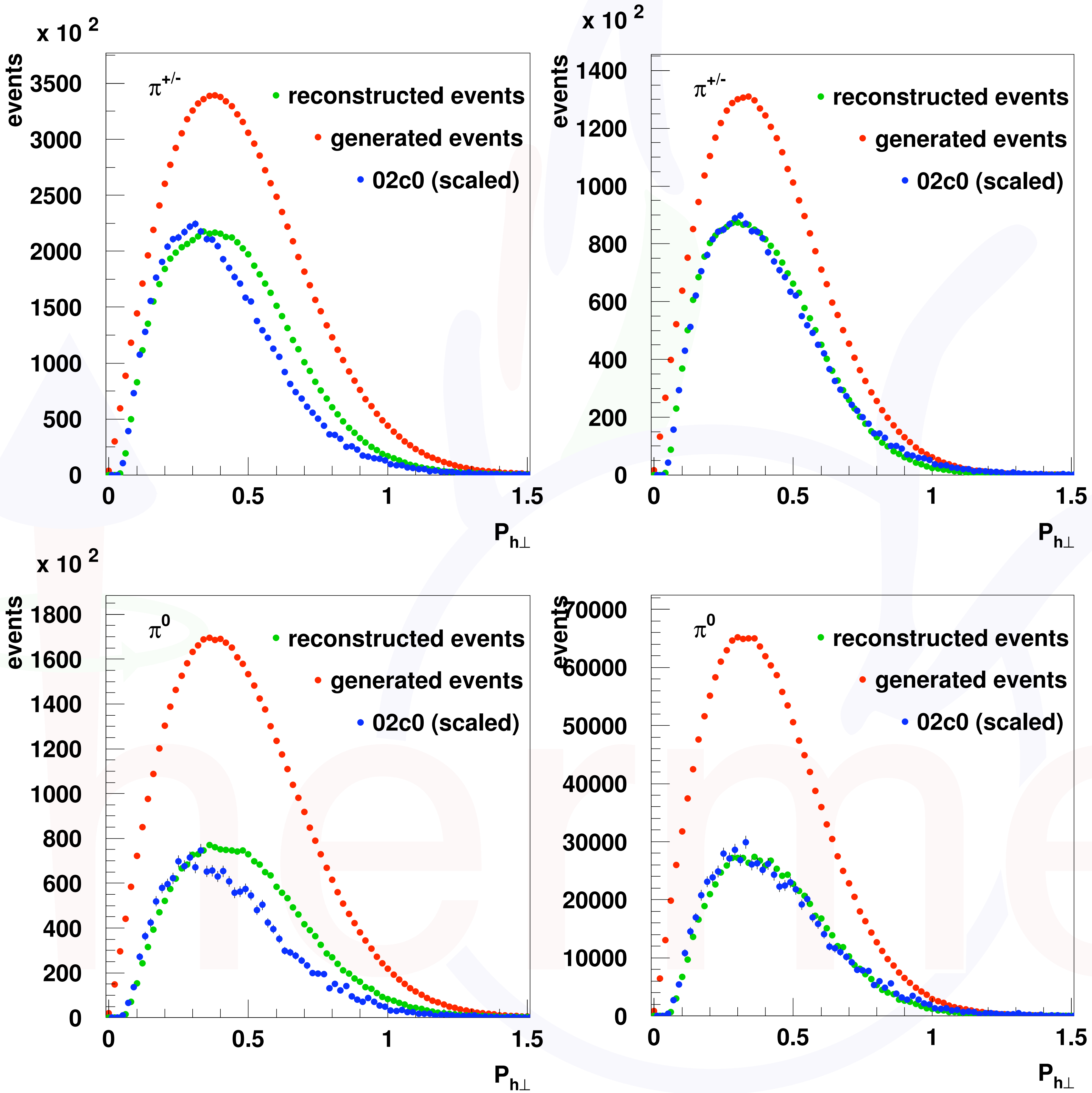


- constant Gaussian widths, i.e., no dependence on x or z:

$$\langle p_T \rangle = 0.44$$
$$\langle K_T \rangle = 0.44$$

- tune to data integrated over whole kinematic range

Tuning the Gaussians in gmc_trans



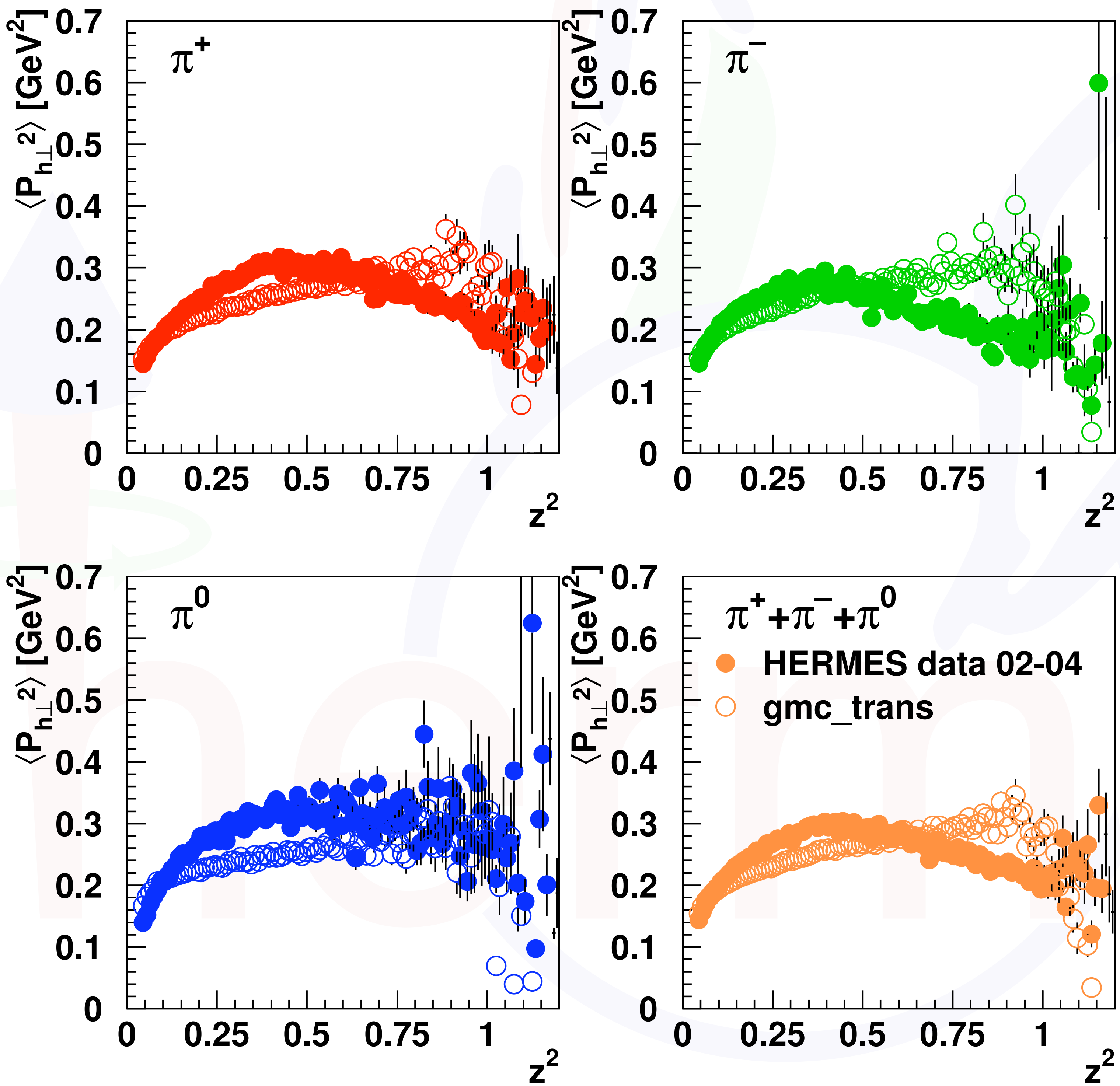
Better:

$$\langle p_T \rangle = 0.38$$

$$\langle K_T \rangle = 0.38$$

Tuning the Gaussians in gmc_trans

so far: $\langle P_{h\perp}^2(z)\rangle = z^2 \langle p_T^2\rangle + \langle K_T^2\rangle$



$\langle p_T\rangle = 0.38$

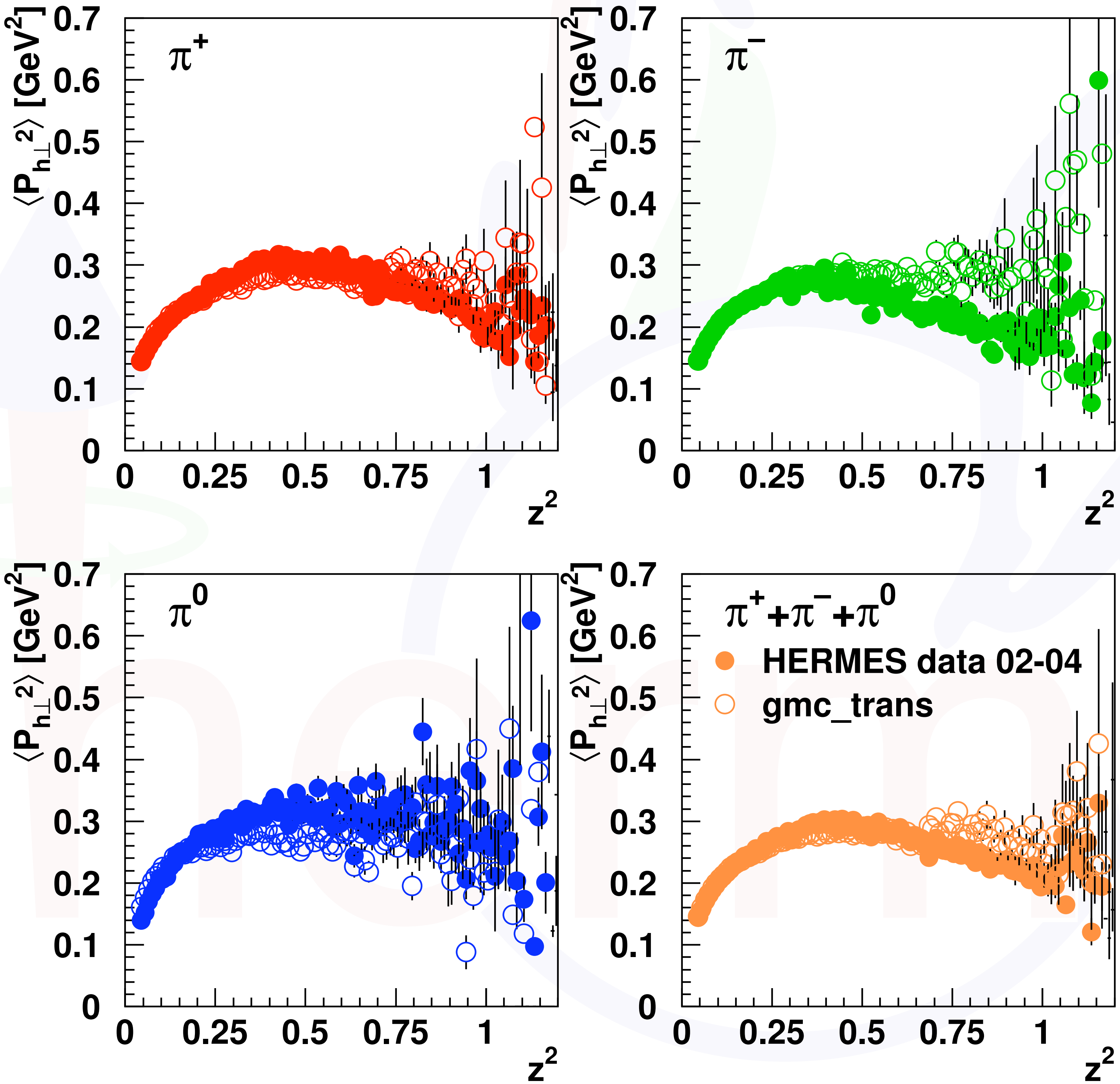
$\langle K_T\rangle = 0.38$

$\langle p_T^2\rangle \simeq 0.185$

$\langle K_T^2\rangle \simeq 0.185$

Tuning the Gaussians in gmc_trans

$$\langle P_{h\perp}^2(z)\rangle = z^2 \langle p_T^2\rangle + \langle K_T^2(z)\rangle$$

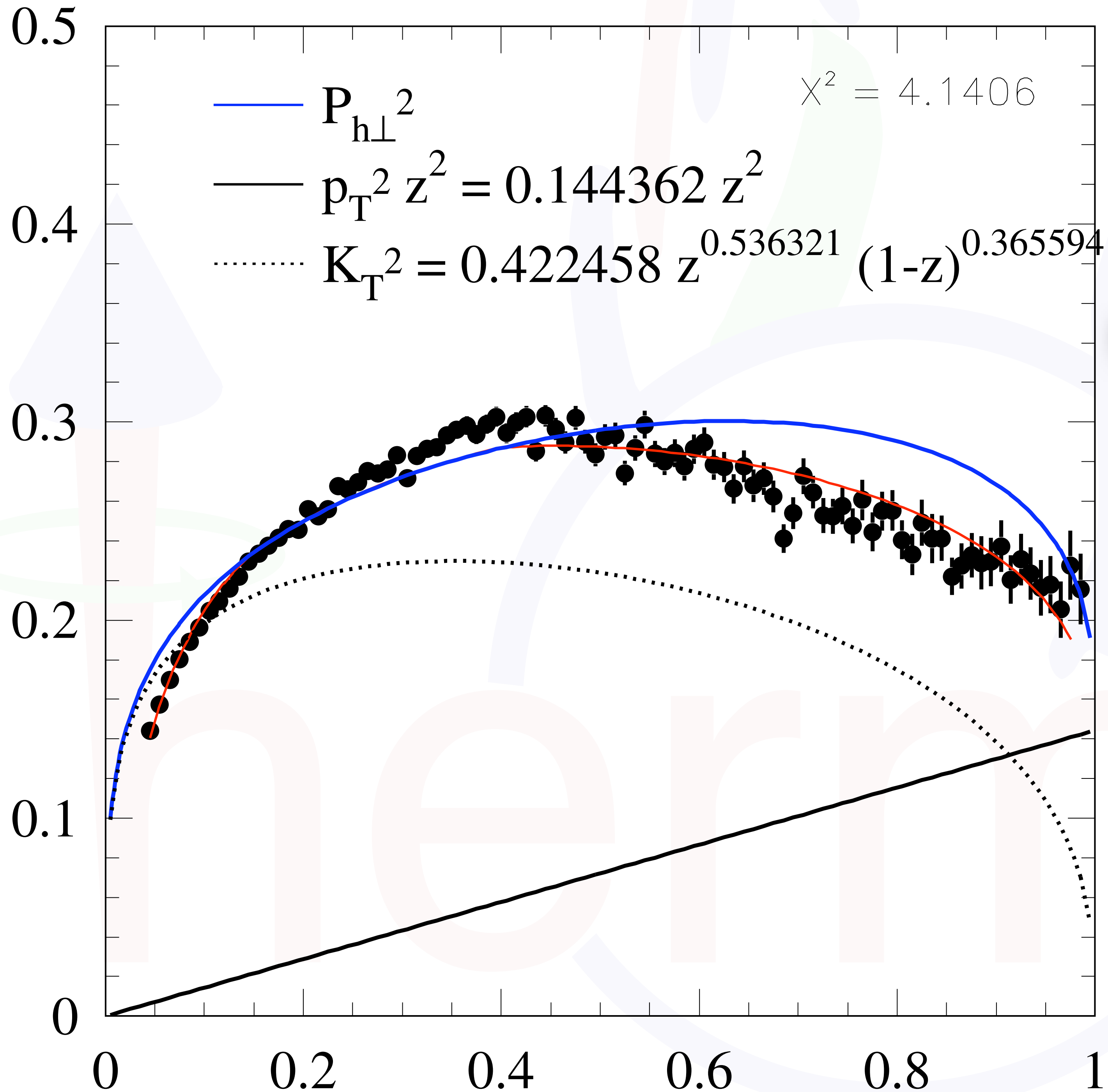


z-dependent!

"Hashi set"

Tuning the Gaussians in gmc_trans

now: $\langle P_{h\perp}^2(z)\rangle = z^2 \langle p_T^2\rangle + \langle K_T^2(z)\rangle$



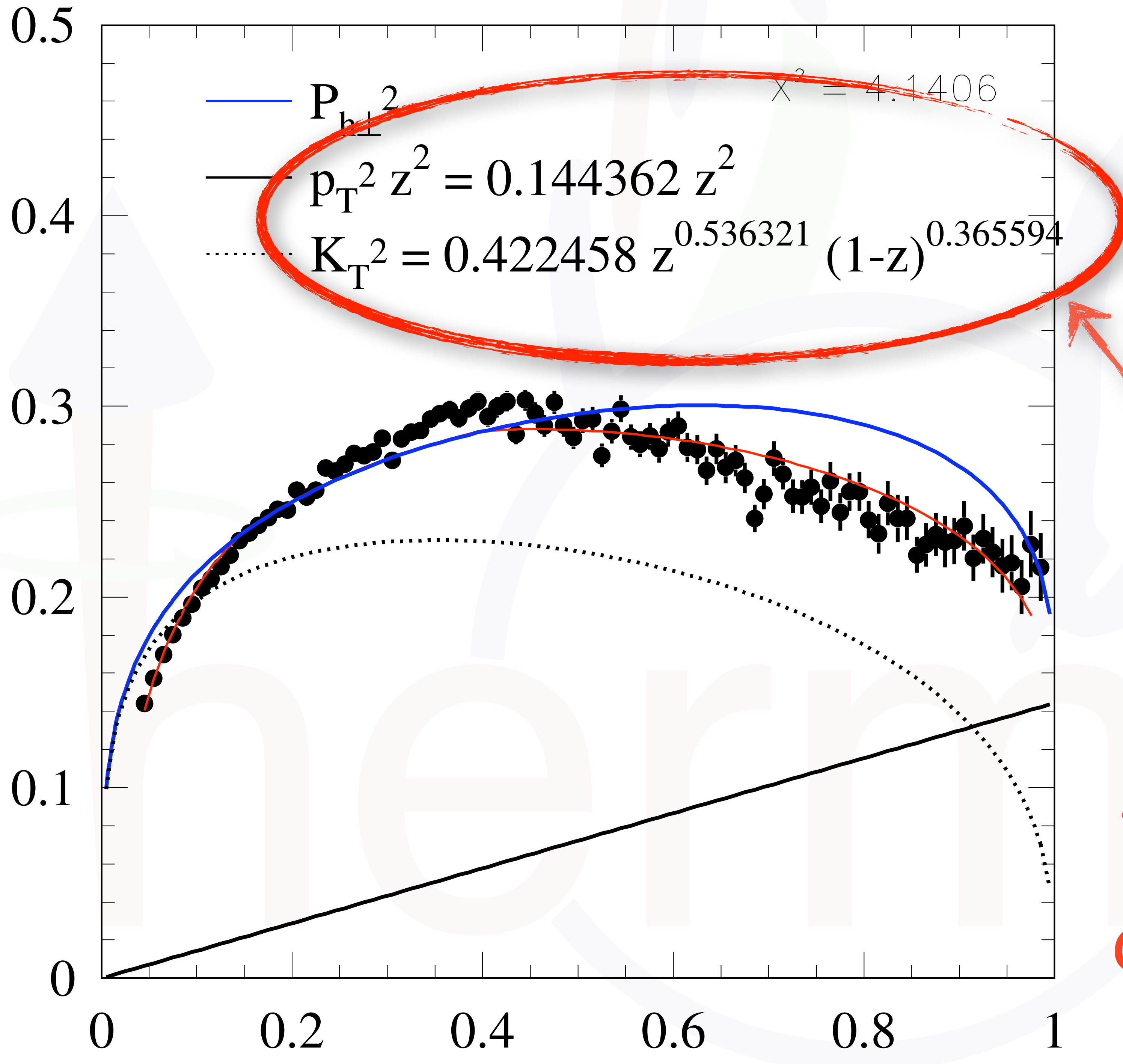
z-dependent!

"Hashi set"

tuned to HERMES data in acceptance

Tuning the Gaussians in gmc_trans

now: $\langle P_{h\perp}^2(z)\rangle = z^2 \langle p_T^2\rangle + \langle K_T^2(z)\rangle$



z-dependent!

"Hashi set"

tuned to HERMES data in acceptance

Some rather simple models for Transversity & friends

$$\delta u(x) = 0.7 \cdot \Delta u(x) \qquad f_{1T}^{\perp u}(x) = -0.3 \cdot u(x)$$

$$\delta d(x) = 0.7 \cdot \Delta d(x) \qquad f_{1T}^{\perp d}(x) = 0.9 \cdot d(x)$$

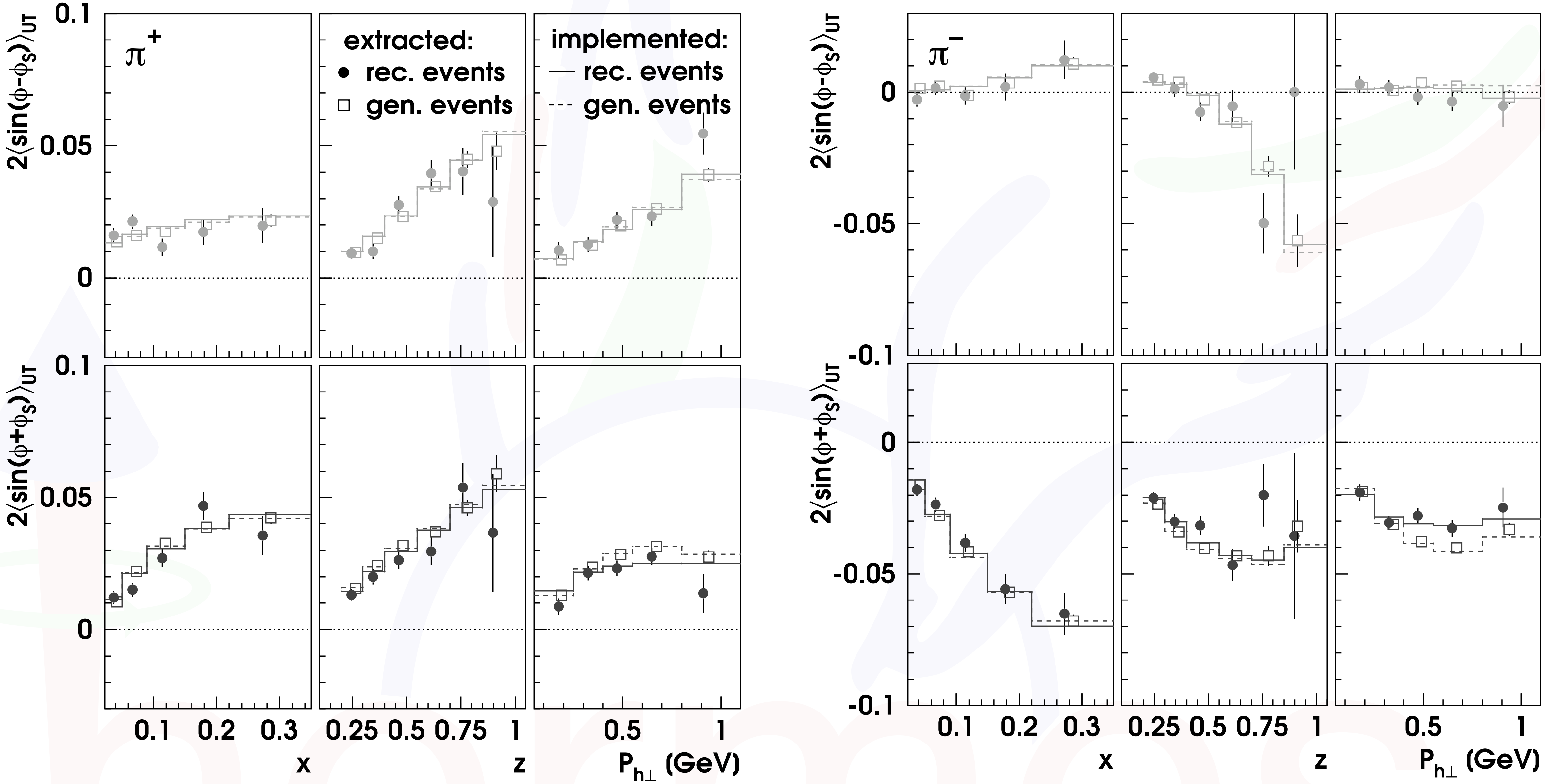
$$\delta q(x) = 0.3 \cdot \Delta q(x) \qquad f_{1T}^{\perp q}(x) = 0.0 \qquad q = \bar{u}, d, s, \bar{s}$$

$$H_{1,\mathrm{fav}}^{\perp(1)}(z) = 0.65 \cdot D_{1,\mathrm{fav}}(z)$$

$$H_{1,\mathrm{dis}}^{\perp(1)}(z) = -1.30 \cdot D_{1,\mathrm{dis}}(z)$$

GRSV for PDFs and Kretzer FF for D_1

Generated vs. extracted amplitudes



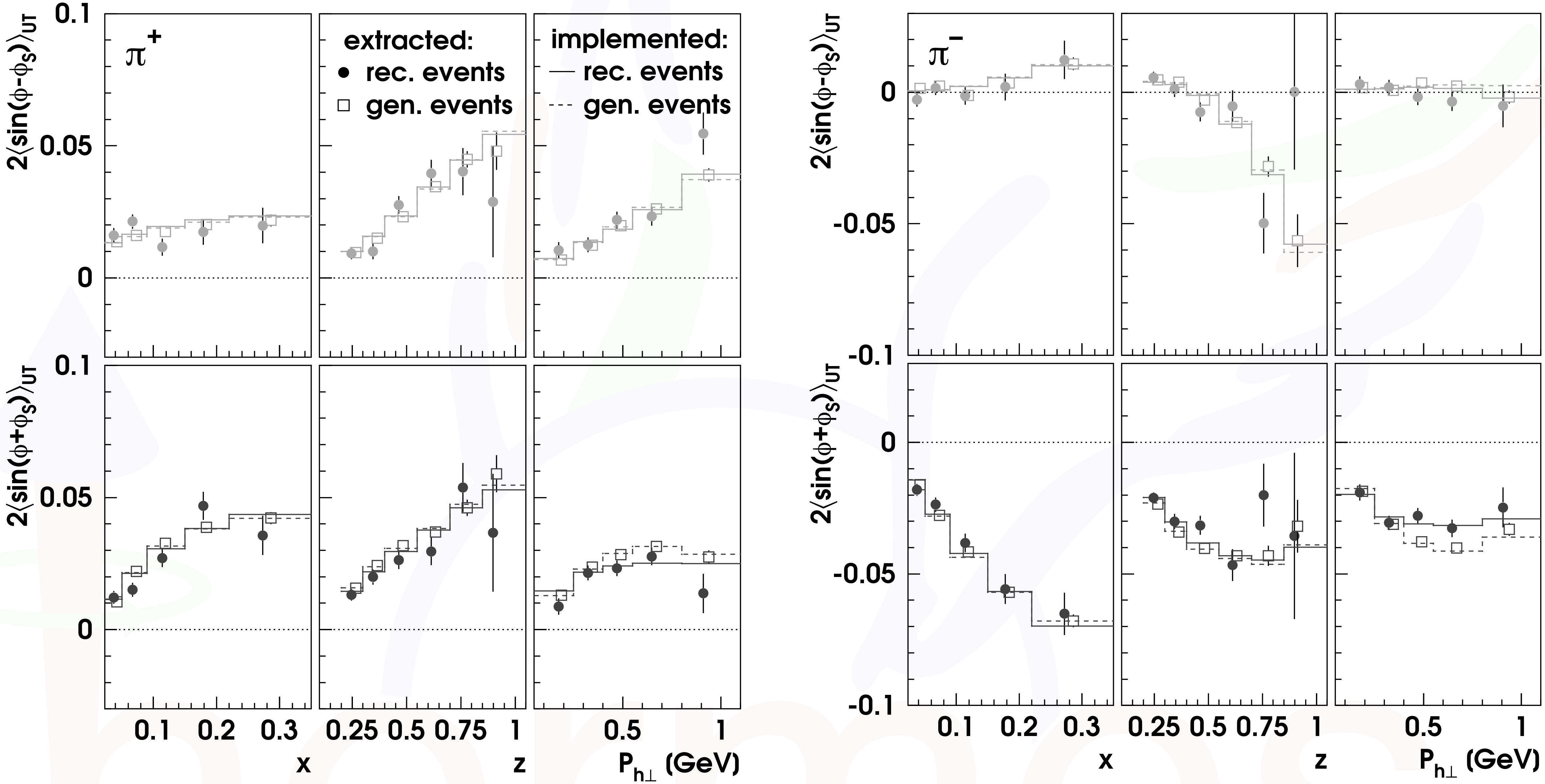
$$\delta u(x) = 0.7 \cdot \Delta u(x) \quad f_{1T}^{\perp u}(x) = -0.3 \cdot u(x) \quad H_{1,\mathrm{fav}}^{\perp(1)}(z) = 0.65 \cdot D_{1,\mathrm{fav}}(z)$$

$$\delta d(x) = 0.7 \cdot \Delta d(x) \quad f_{1T}^{\perp d}(x) = 0.9 \cdot d(x) \quad H_{1,\mathrm{dis}}^{\perp(1)}(z) = -1.30 \cdot D_{1,\mathrm{dis}}(z)$$

$$\delta q(x) = 0.3 \cdot \Delta q(x) \quad f_{1T}^{\perp q}(x) = 0.0 \quad q = \bar{u}, \bar{d}, s, \bar{s}$$

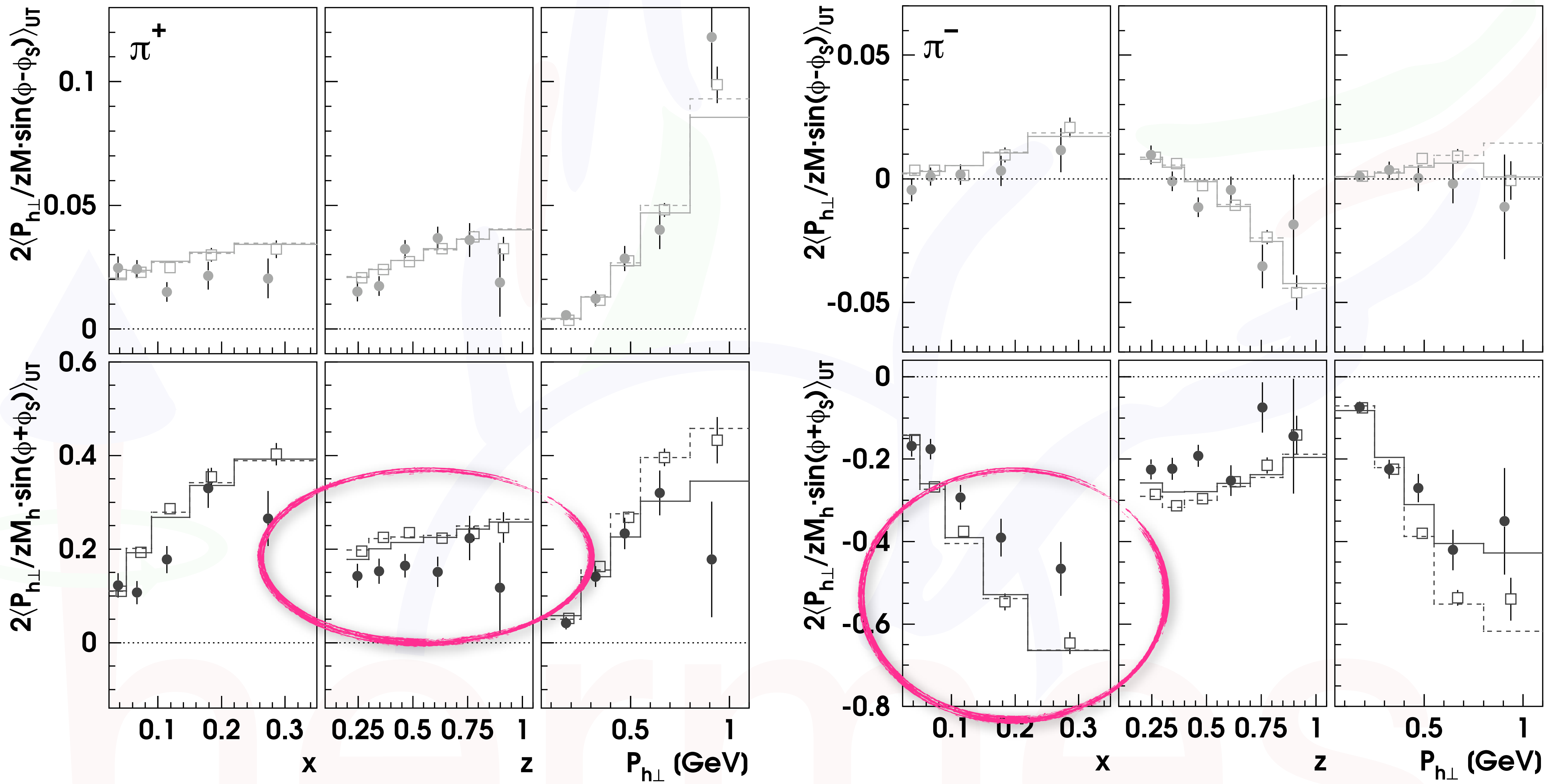
$$C_S = C_C = 0.25$$

Generated vs. extracted amplitudes



Extraction method works well!

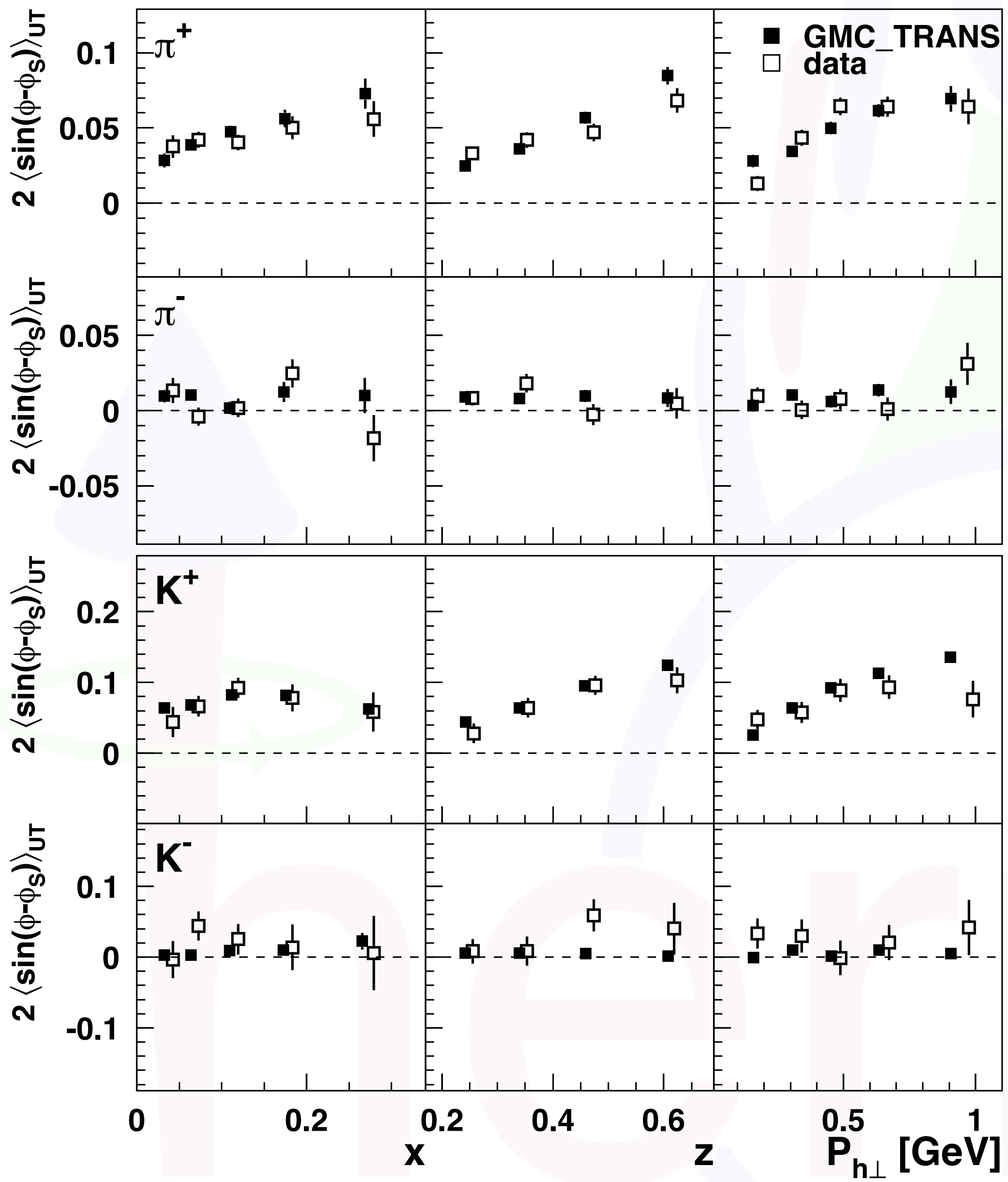
Extraction of weighted moments



Not so good news for weighted moments!

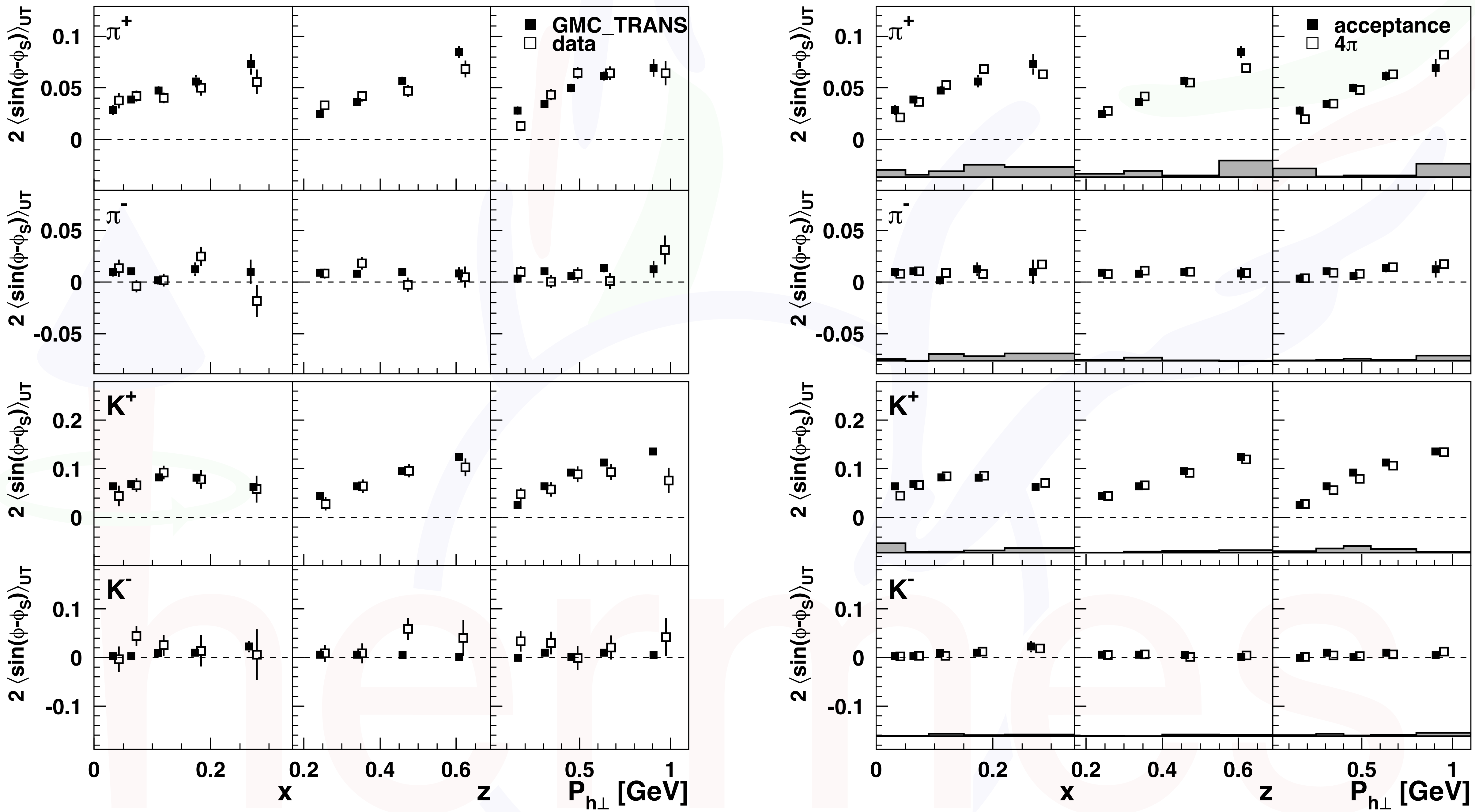
further improvement of the models

- DSS FFs and modified Anselmino et al. Sivers fit:



further improvement of the models

- DSS FFs and modified Anselmino et al. Sivers fit:



- could in principle be used for systematics, but ...

missing items in gmc_trans

- not so good model for transversity & Collins FF
- missing models for other single- and double-spin asymmetries
- no azimuthal modulations of unpolarized cross section
- no radiative corrections
- no full event generation (missing track multiplicities and correlations etc.)

"reshuffling" PYTHIA events

- use model for azimuthal distribution to introduce spin dependence in PYTHIA

- throw random number ρ and assign spin state up if, e.g.,

$$\rho < \frac{1}{2}\left(1 + \sin\left(\phi - \phi_S\right)\Xi_{11}^{\sin(\phi-\phi_S),h} + \sin\left(\phi + \phi_S\right)\Xi_{11}^{\sin(\phi+\phi_S),h} + \sin\left(\phi_S\right)\Xi_{11}^{\sin(\phi_S),h}\right)$$

parametrization of azimuthal dependences
(extracted, e.g., from real data)

Parametrization of azimuthal dependence

- fully differential model extracted in M.L. fit to data with PDF

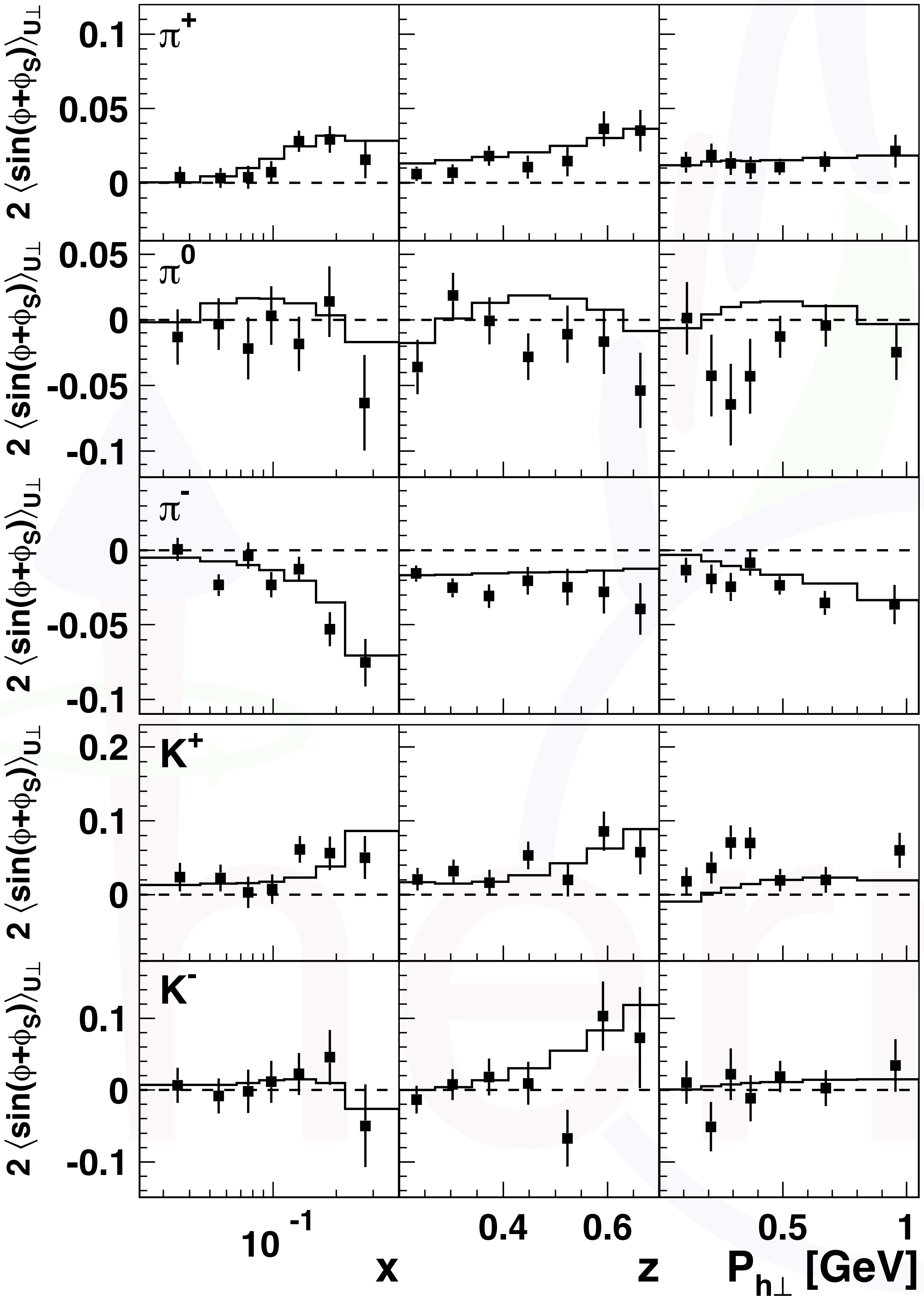
$$P\left(x, Q^2, z, |\mathbf{P}_{h\perp}|, \phi, \phi_S; \Xi_{22}^{\sin(\phi-\phi_S),h}, \Xi_{22}^{\sin(\phi+\phi_S),h}\right)$$

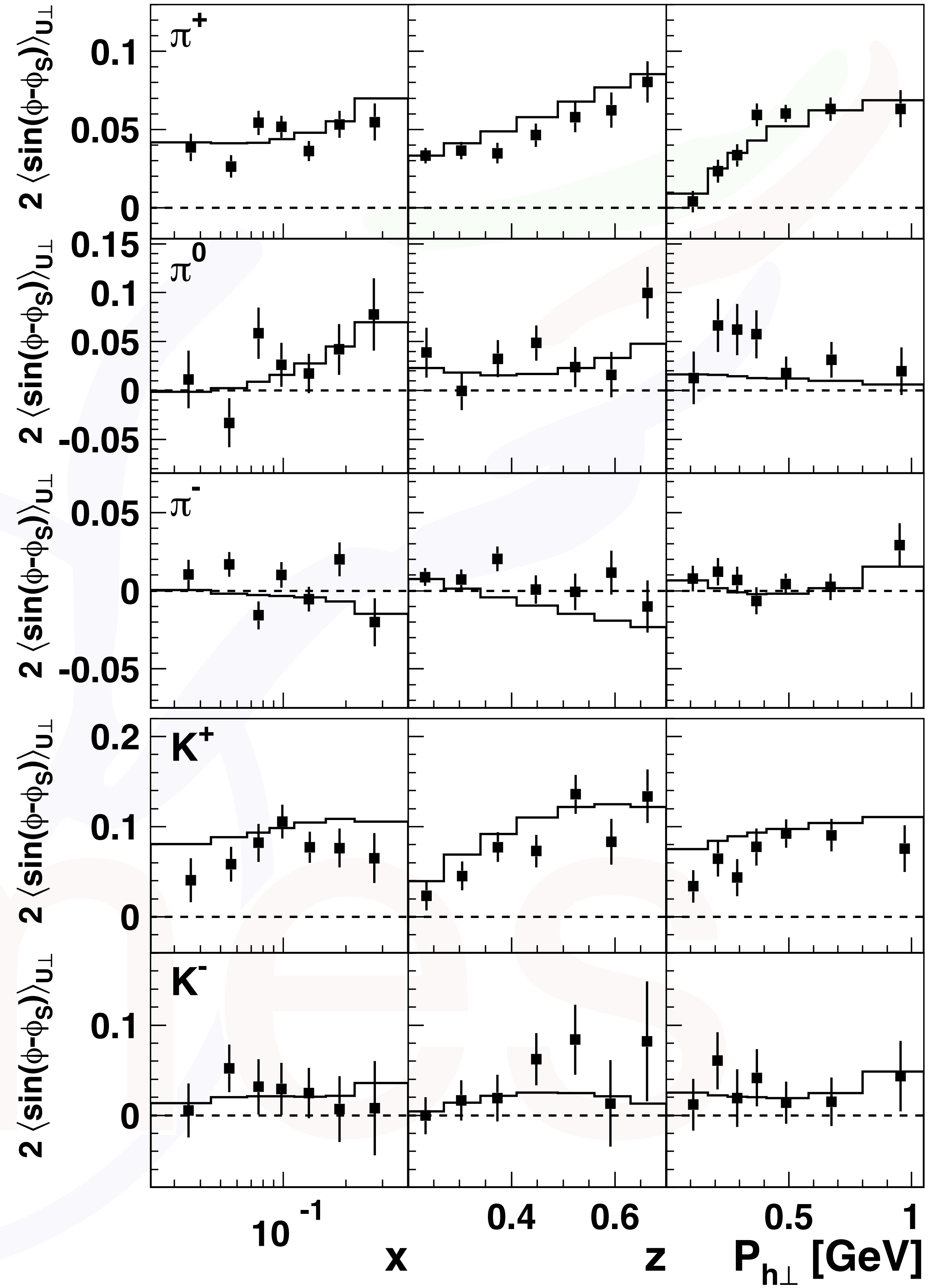
$$= 1 + S_\perp \left(\sin(\phi - \phi_S)\, \Xi_{22}^{\sin(\phi-\phi_S),h} + \sin(\phi + \phi_S)\, \Xi_{22}^{\sin(\phi+\phi_S),h}\right)$$

$$\begin{aligned}
\Xi_{22}^{\sin(\phi\pm\phi_S),h} = \; & \Xi_{22,1}^{\sin(\phi\pm\phi_S),h} && + \Xi_{22,2}^{\sin(\phi\pm\phi_S),h}\, x' && + \\
& \Xi_{22,3}^{\sin(\phi\pm\phi_S),h}\, Q^{2\prime} && + \Xi_{22,4}^{\sin(\phi\pm\phi_S),h}\, z' && + \\
& \Xi_{22,5}^{\sin(\phi\pm\phi_S),h}\, |\mathbf{P}_{h\perp}|' && + \Xi_{22,6}^{\sin(\phi\pm\phi_S),h}\, x'^2 && + \\
& \Xi_{22,7}^{\sin(\phi\pm\phi_S),h}\, z'^2 && + \Xi_{22,8}^{\sin(\phi\pm\phi_S),h}\, |\mathbf{P}_{h\perp}|'^2 && + \\
& \Xi_{22,9}^{\sin(\phi\pm\phi_S),h}\, x'z' && + \Xi_{22,10}^{\sin(\phi\pm\phi_S),h}\, x'\,|\mathbf{P}_{h\perp}|' && + \\
& \Xi_{22,11}^{\sin(\phi\pm\phi_S),h}\, z'\,|\mathbf{P}_{h\perp}|' && + \Xi_{22,12}^{\sin(\phi\pm\phi_S),h}\, x'^3 && + \\
& \Xi_{22,13}^{\sin(\phi\pm\phi_S),h}\, x'z'^2 && + \Xi_{22,14}^{\sin(\phi\pm\phi_S),h}\, x'^2 z' && + \\
& \Xi_{22,15}^{\sin(\phi\pm\phi_S),h}\, x'^2\,|\mathbf{P}_{h\perp}|' && + \Xi_{22,16}^{\sin(\phi\pm\phi_S),h}\, x'|\mathbf{P}_{h\perp}|'^2 && + \\
& \Xi_{22,17}^{\sin(\phi\pm\phi_S),h}\, z'^2\,|\mathbf{P}_{h\perp}|' && + \Xi_{22,18}^{\sin(\phi\pm\phi_S),h}\, z'|\mathbf{P}_{h\perp}|'^2 && + \\
& \Xi_{22,19}^{\sin(\phi\pm\phi_S),h}\, x'^2|\mathbf{P}_{h\perp}|'^2 && + \Xi_{22,20}^{\sin(\phi\pm\phi_S),h}\, z'^2|\mathbf{P}_{h\perp}|'^2 && + \\
& \Xi_{22,21}^{\sin(\phi\pm\phi_S),h}\, x'z'\,|\mathbf{P}_{h\perp}|' && + \Xi_{22,22}^{\sin(\phi\pm\phi_S),h}\, x'^2 z'\,|\mathbf{P}_{h\perp}|' && .
\end{aligned}$$

Description of data

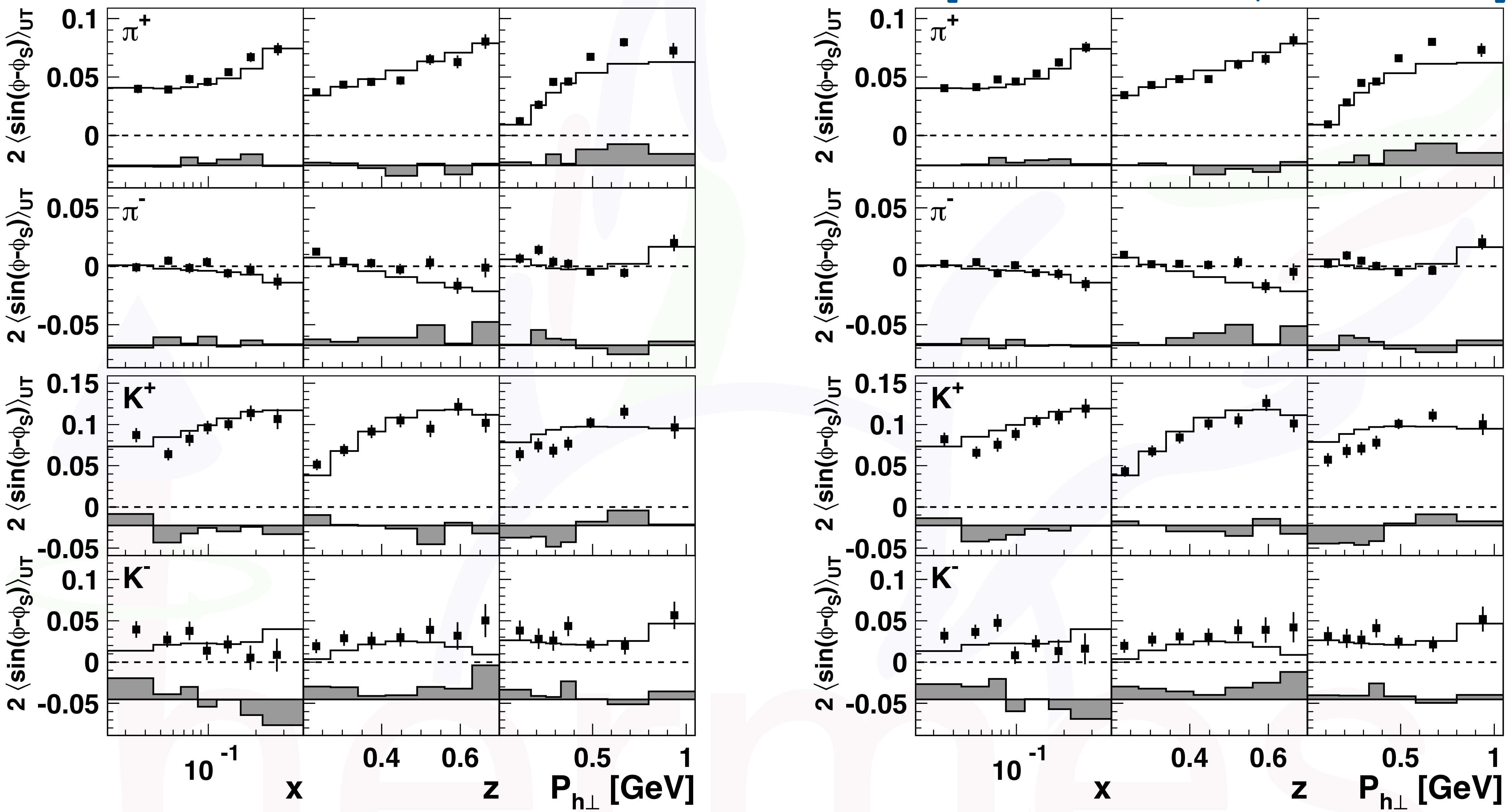
[M. Diefenthaler, Ph.D. thesis]





Evaluation of detector effects

[M. Diefenthaler, Ph.D. thesis]



- differences include effects from internal and external radiative effects, acceptance, PID, (mis)alignment etc.
- in further step "smoothed" to reduce statistical fluctuations

some Pro&Cons of "reshuffling"

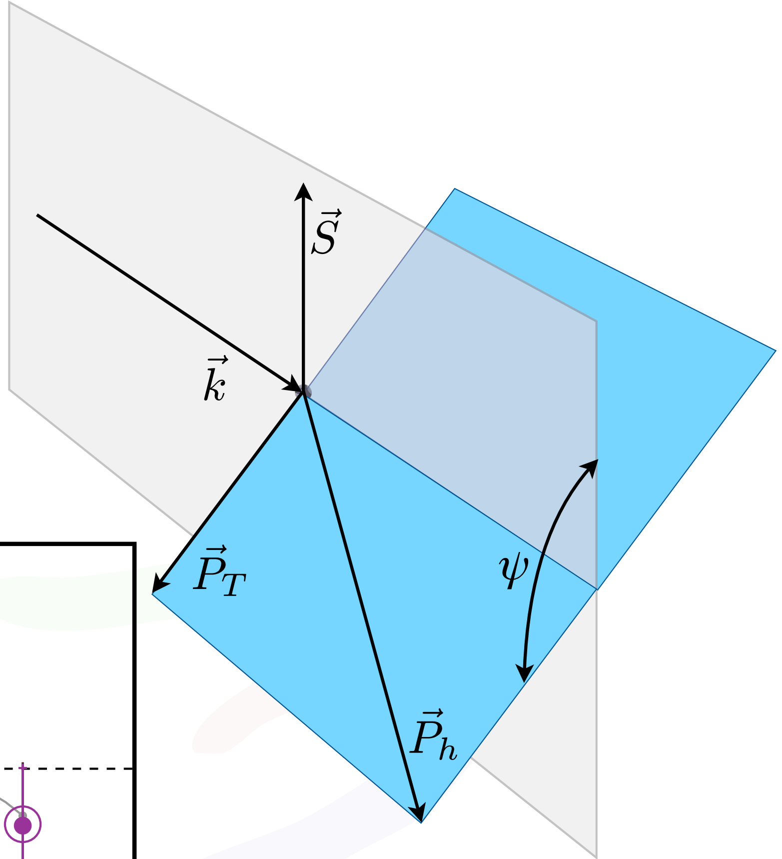
- whole event topology and correlations available
- flexible
- applicable also to cases where no guidance from theory available on shape/magnitude of modulations

some Pro&Cons of "reshuffling"

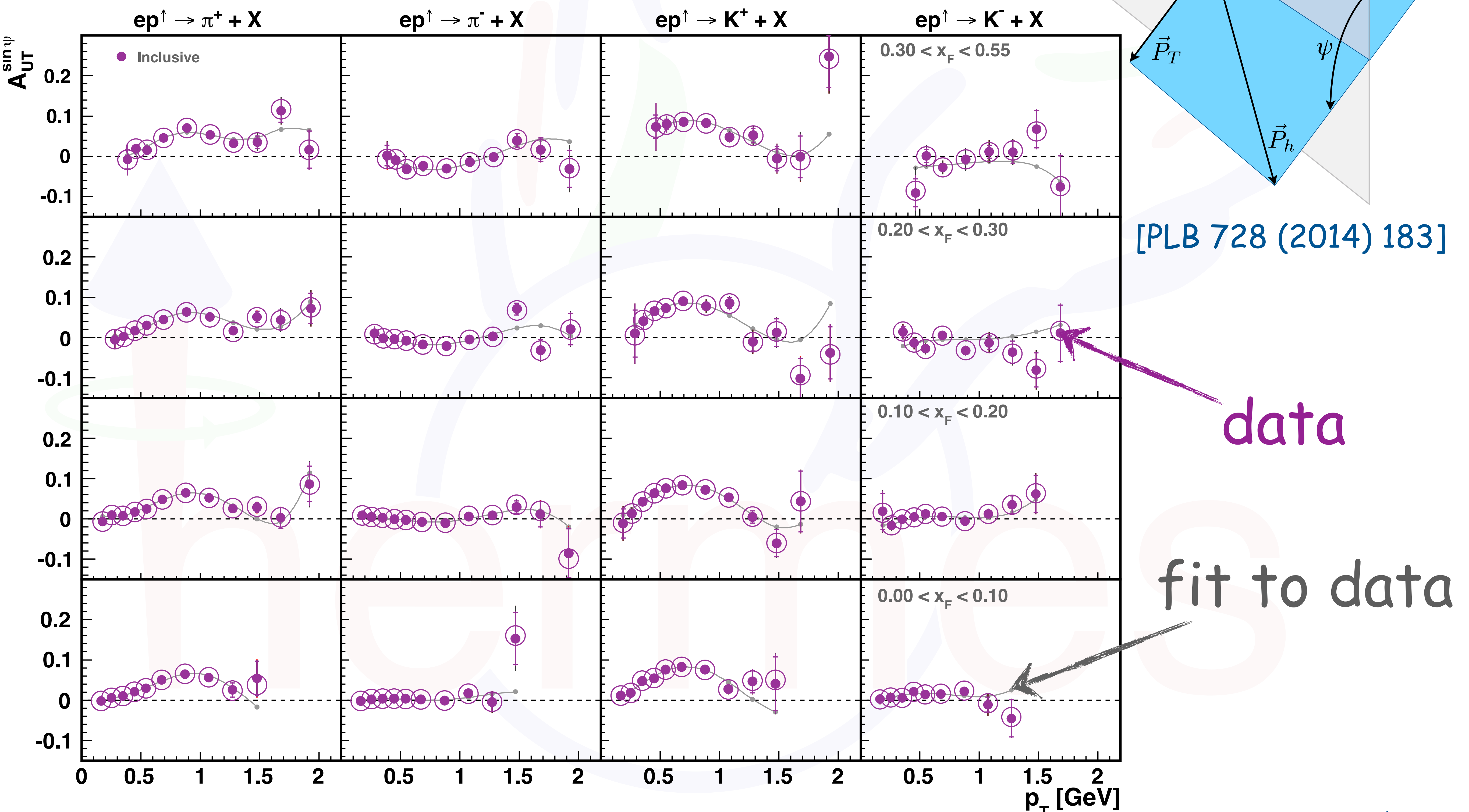
- whole event topology and correlations available
- flexible
- applicable also to cases where no guidance from theory available on shape/magnitude of modulations

- need parametrization
  if from real data, where to stop Taylor (or other) expansion?
- large uncertainties on (some) parameters can introduce large spurious effects in systematics calculation
- relies on good description of unpolarized cross section

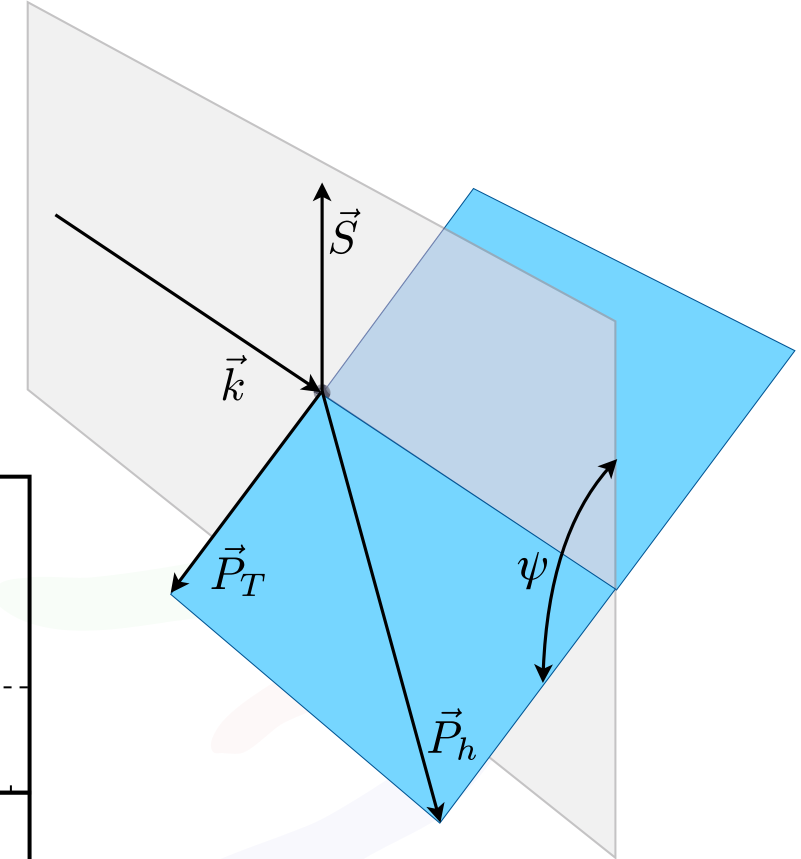
Another example: A_{UT} in inclusive hadron production

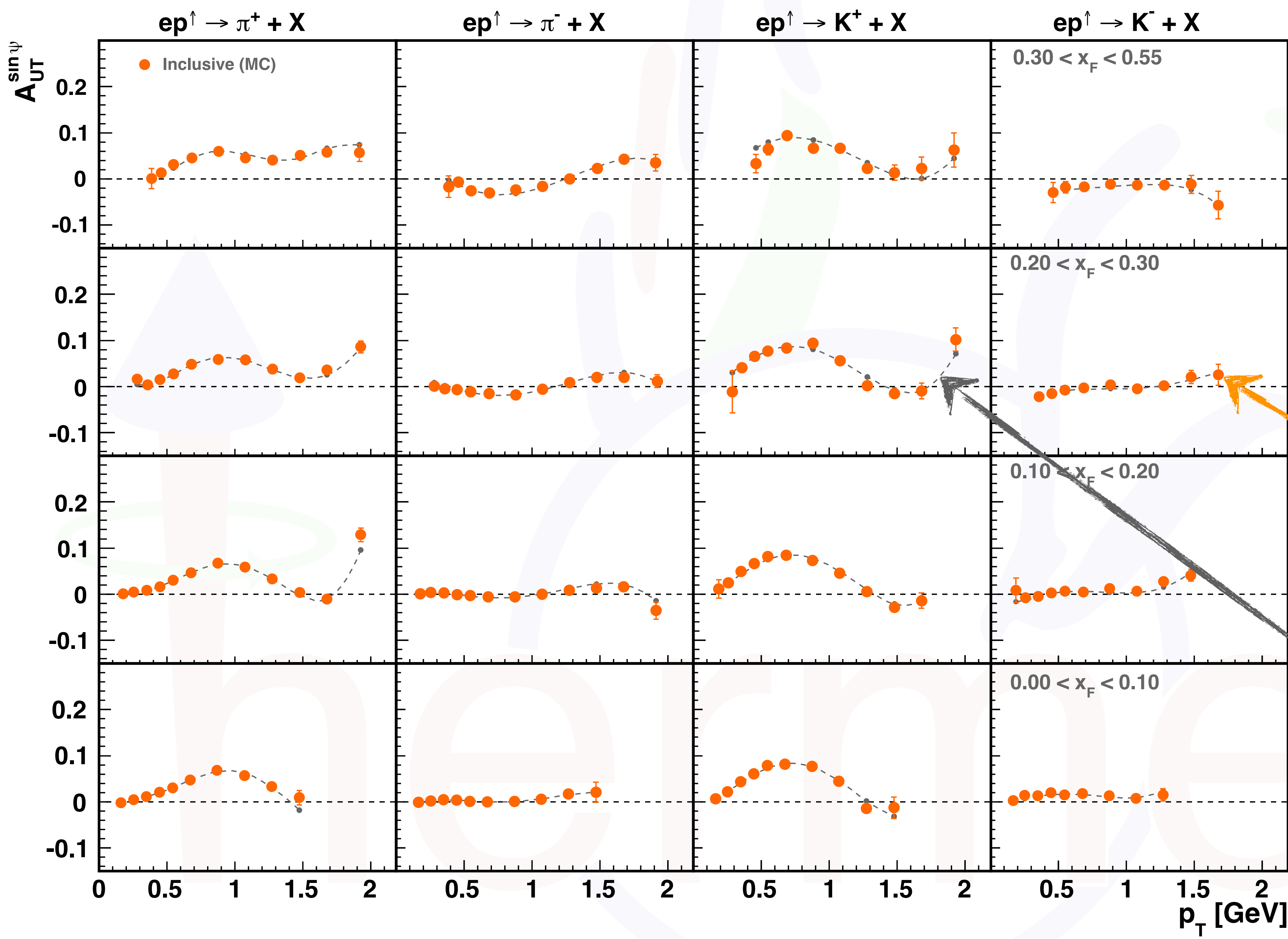


[PLB 728 (2014) 183]



Another example: A_{UT} in inclusive hadron production

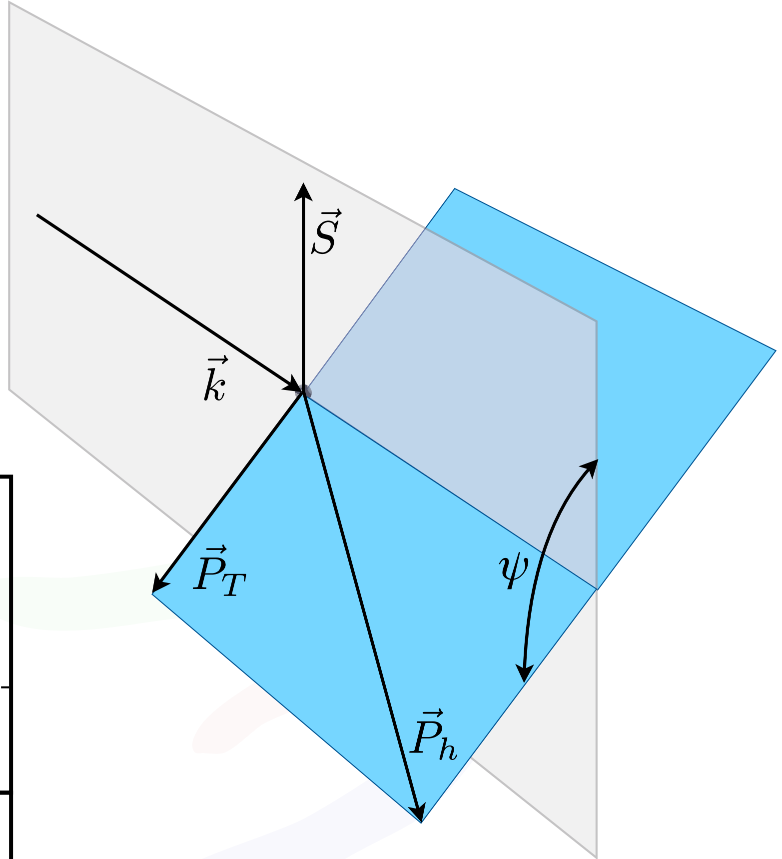


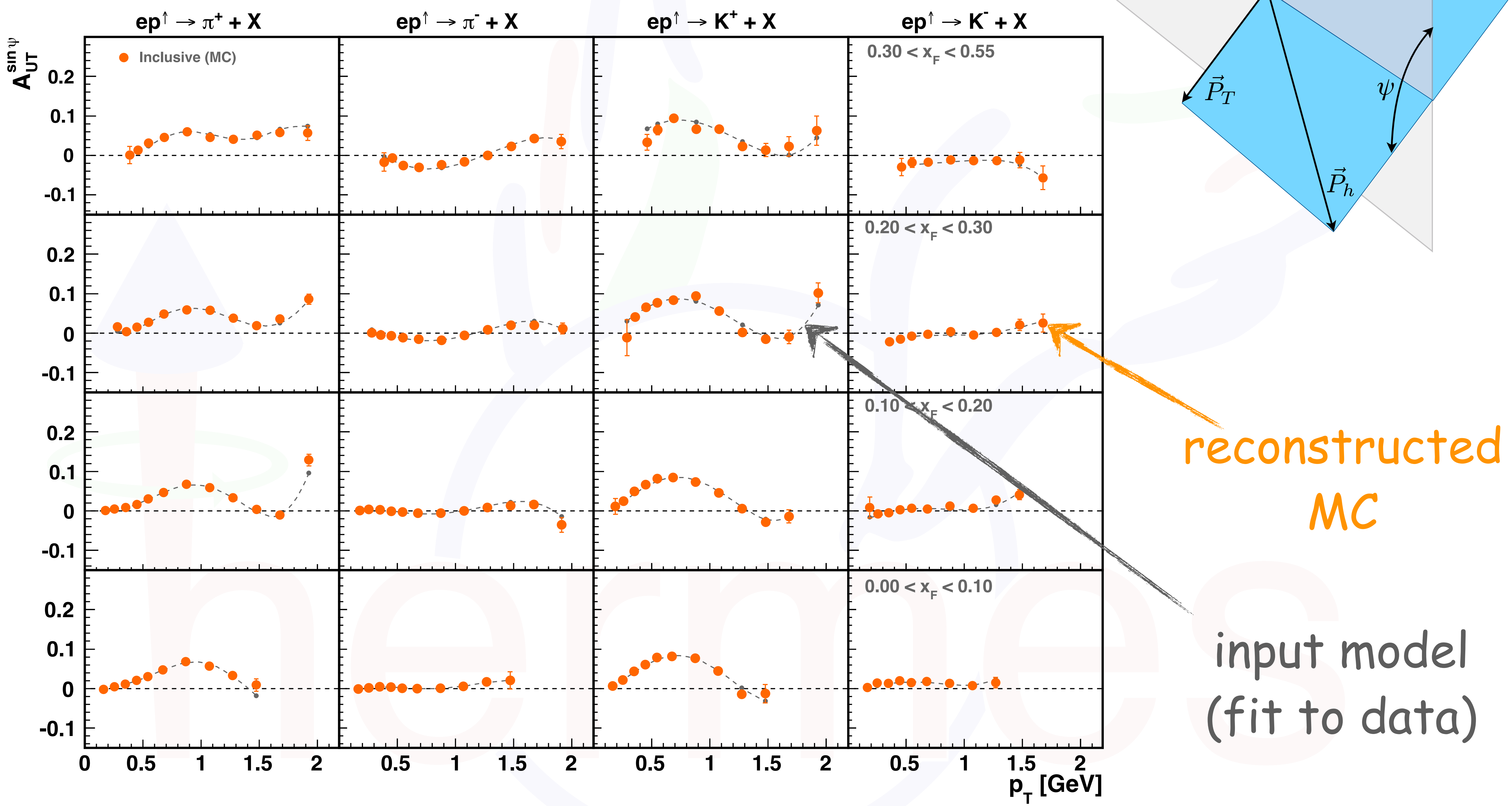


reconstructed MC

input model (fit to data)

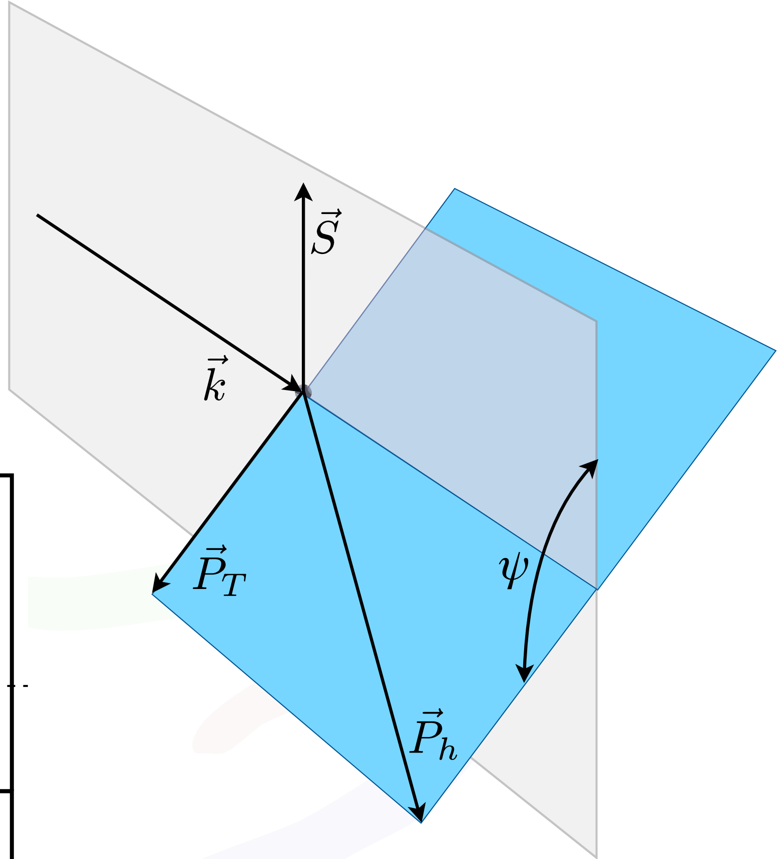
Another example: A_{UT} in inclusive hadron production

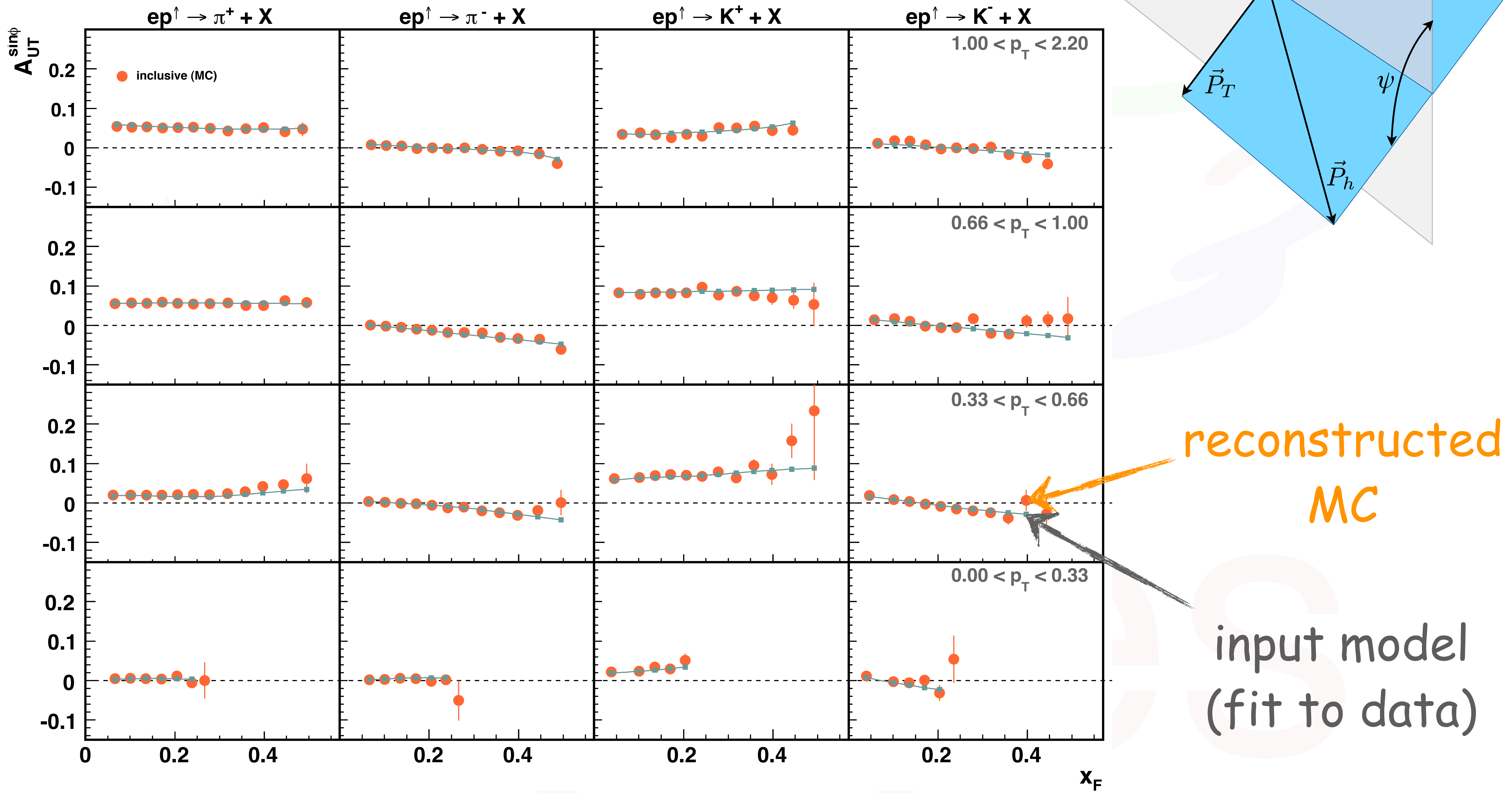




small detector effects in fully differential analysis

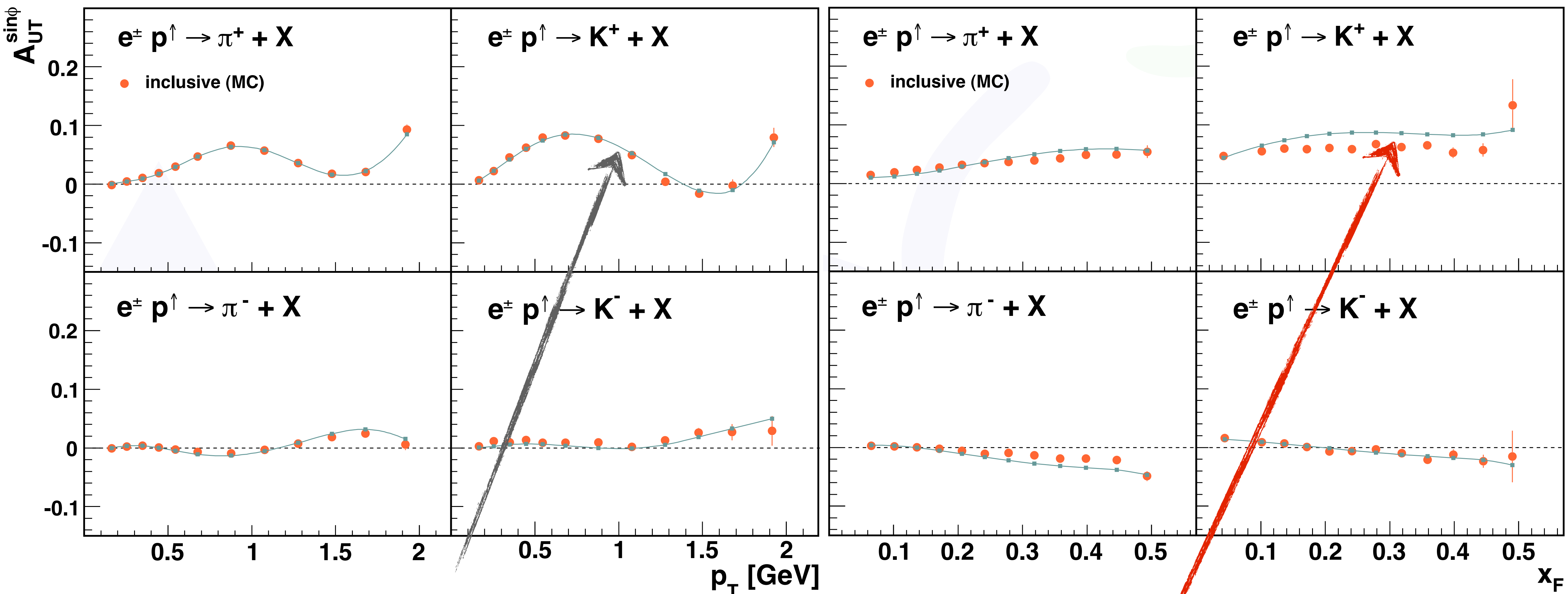
Another example: A_{UT} in inclusive hadron production





small detector effects in fully differential analysis

Another example: A_{UT} in inclusive hadron production



strong kinematic dependence can lead to large systematic effects if integrated over

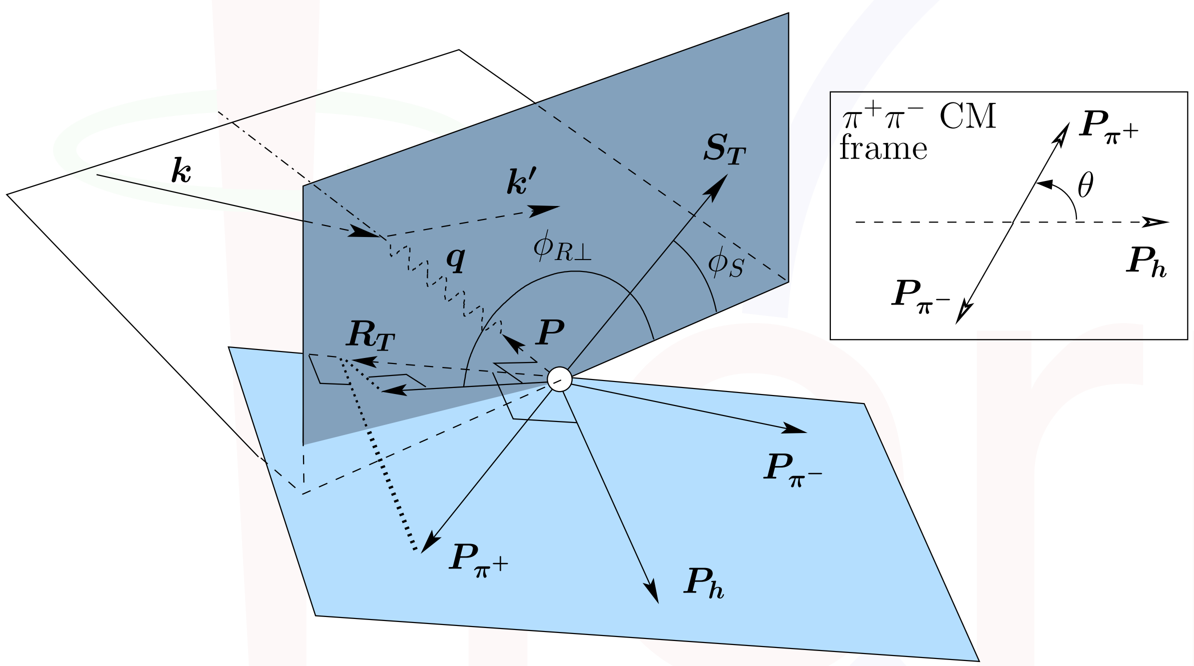
not so small detector effects in 1D analysis

similar problematics: di-hadron A_{UT}

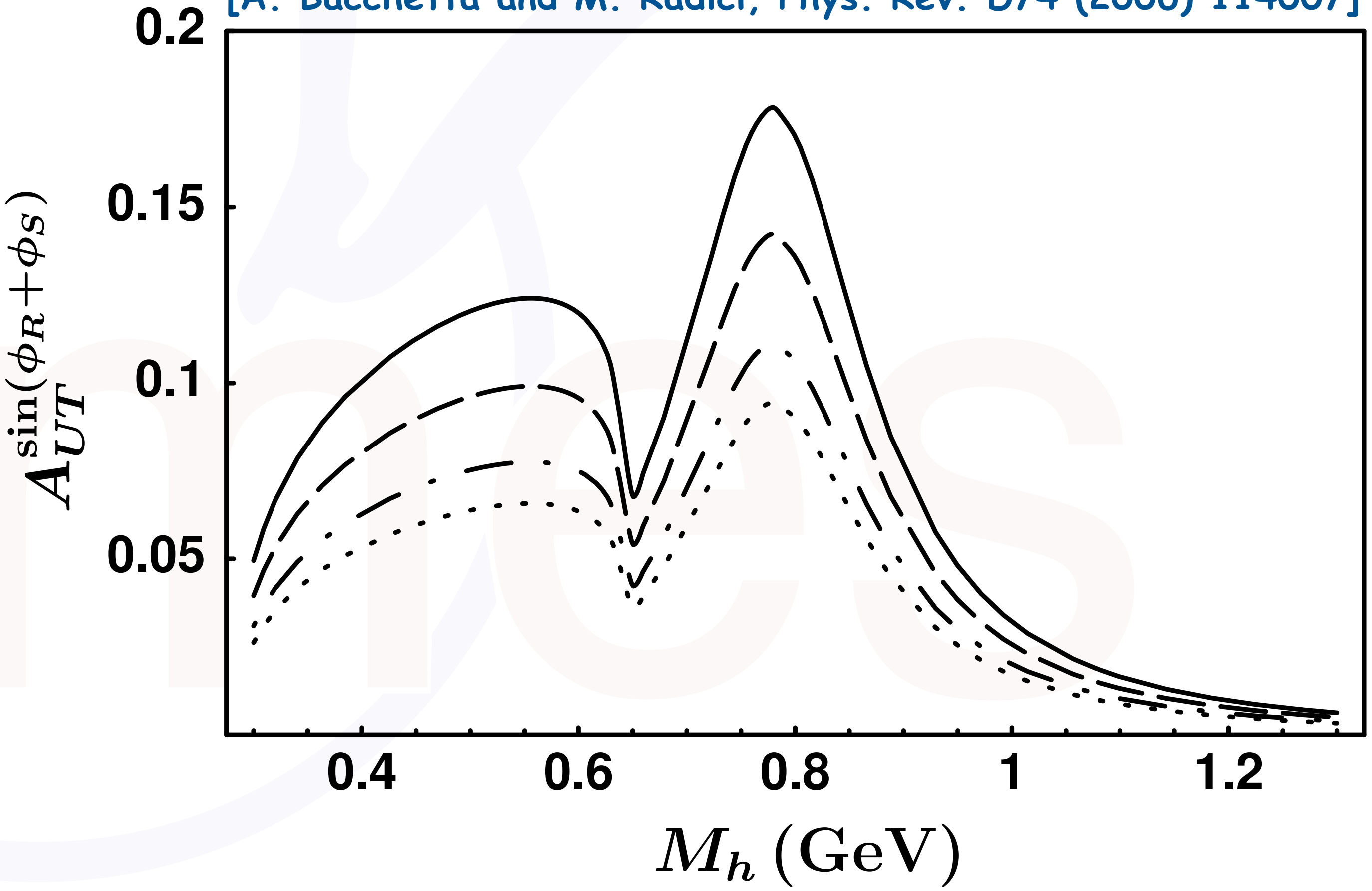
- many kinematic variables needed to describe process

$$N^{\uparrow(\downarrow)}(\phi_{R\perp}, \phi_S, \theta, M_{\pi\pi}) \propto \int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}^2\boldsymbol{P_{h\perp}}\,\epsilon(x, y, z, \boldsymbol{P_{h\perp}}, \phi_{R\perp}, \phi_S, \theta, M_{\pi\pi}) \times$$
$$\times\ \sigma_{U\uparrow(\downarrow)}(x, y, z, \boldsymbol{P_{h\perp}}, \phi_{R\perp}, \phi_S, \theta, M_{\pi\pi}),$$

- at least for one of them strong dependence expected:



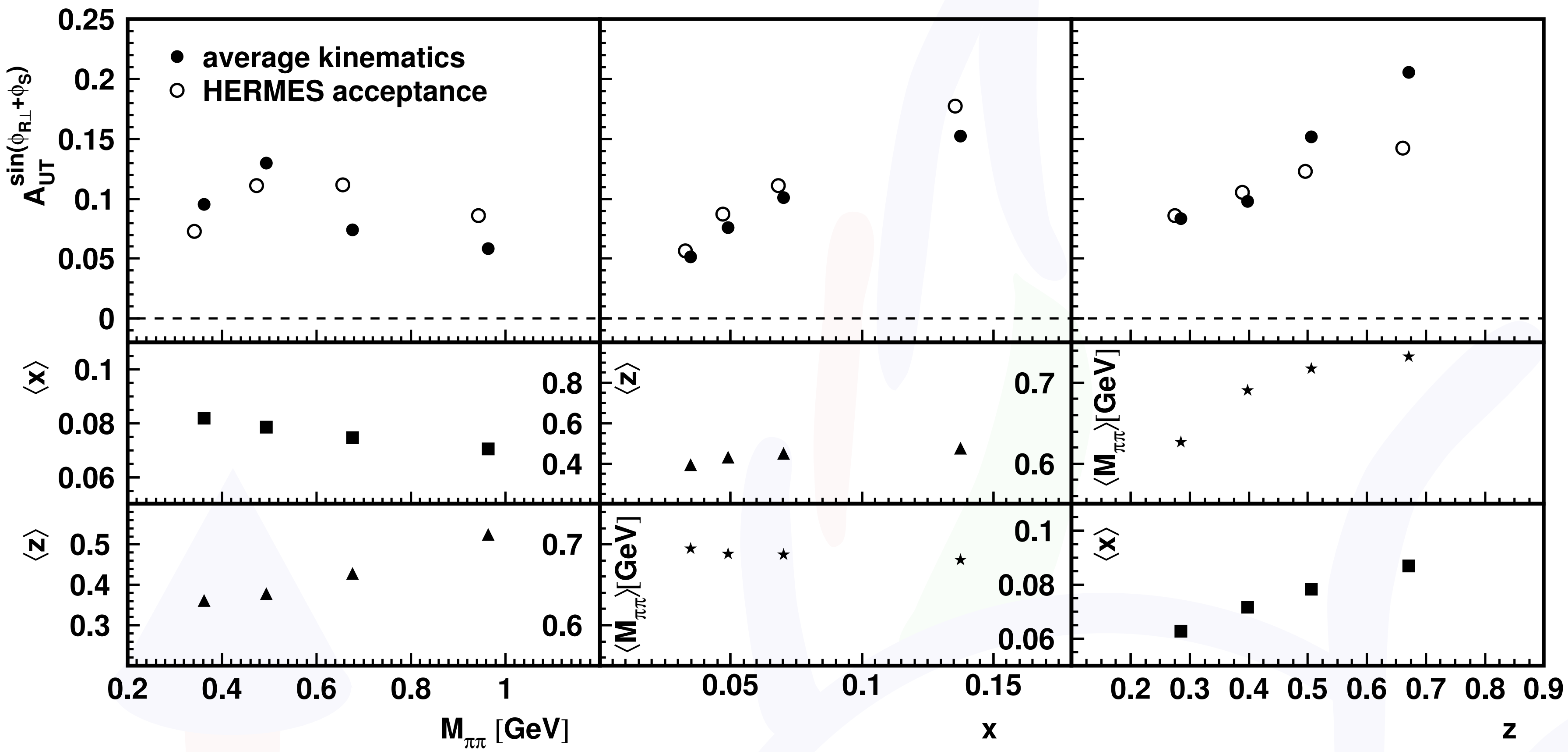
[A. Bacchetta and M. Radici, Phys. Rev. D74 (2006) 114007]



define your measurement wisely

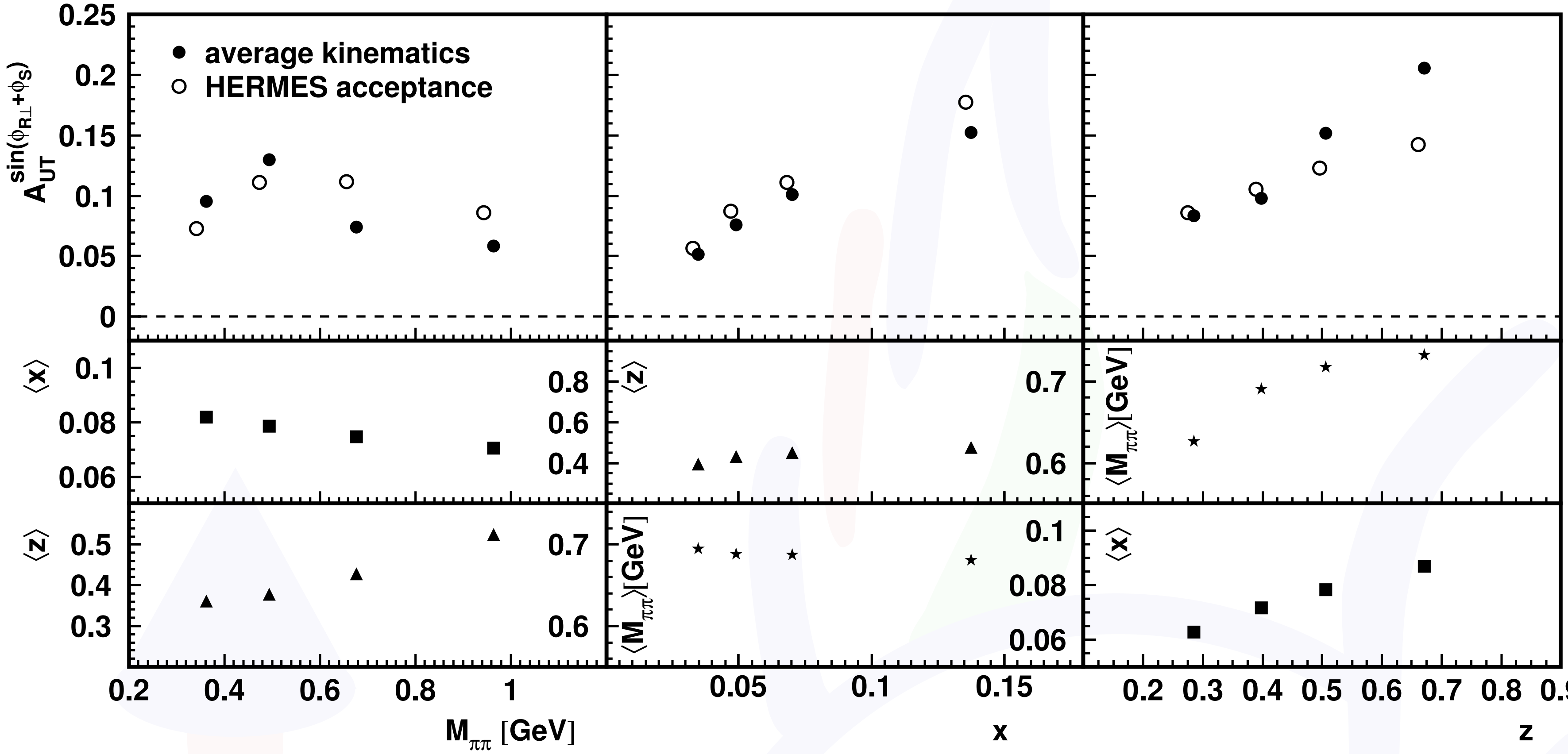
- data point interpreted as asymmetry
  - at the average kinematics given
  - integrated over kinematic ranges
- results in different systematics -> select the one with smallest systematics?

back to di-hadron production

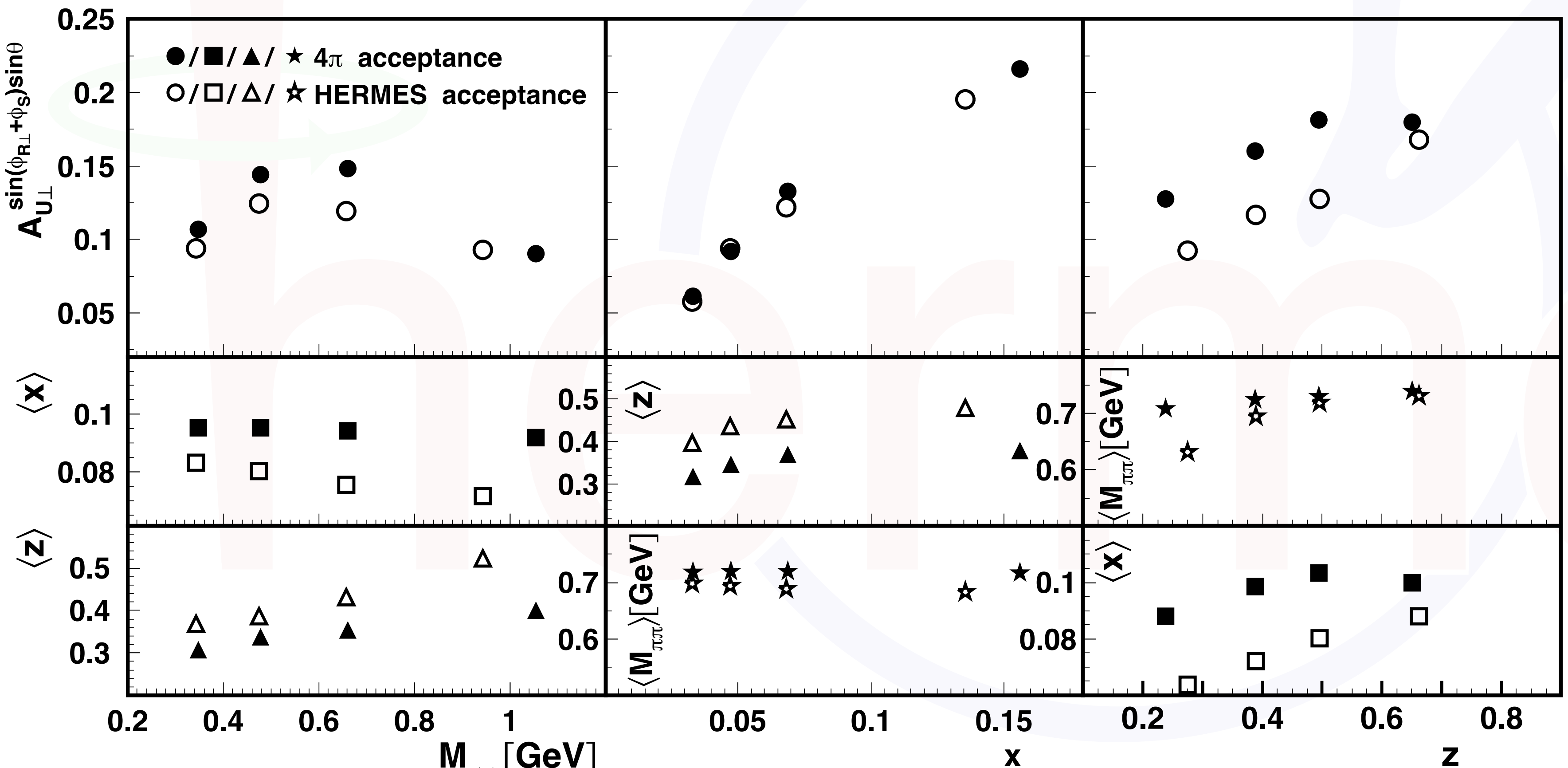


- asymmetries at average kinematics -> large effects with strong model dependence

back to di-hadron production



- asymmetries at average kinematics -> large effects with strong model dependence



- integrated over kinematic range -> still large effects but less model dependent

Unpolarized SIDIS

SIDIS cross section

$$\frac{d^5\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{h\perp}^2} \propto \left(1+\frac{\gamma^2}{2x}\right)\{F_{UU,T}+\epsilon F_{UU,L}$$

$$+\sqrt{2\epsilon(1-\epsilon)}\,F_{UU}^{\cos\phi_h}\cos\phi_h+\epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h\}$$

SIDIS cross section

hadron multiplicity:
normalize to inclusive DIS cross section

$$\frac{d^2\sigma^{\text{incl.DIS}}}{dx\,dy} \propto F_T + \epsilon F_L$$

$$\frac{d^4\mathcal{M}^h(x,y,z,P_{h\perp}^2)}{dx\,dy\,dz\,dP_{h\perp}^2} \propto \left(1+\frac{\gamma^2}{2x}\right)\frac{F_{UU,T}+\epsilon F_{UU,L}}{F_T+\epsilon F_L}$$

$$\approx \frac{\sum_q e_q^2\, f_1^q(x,p_T^2)\otimes D_1^{q\to h}(z,K_T^2)}{\sum_q e_q^2\, f_1^q(x)}$$

$$\frac{d^5\sigma}{dx\,dy\,dz\,d\phi_h\,dP_{h\perp}^2} \propto \left(1+\frac{\gamma^2}{2x}\right)\{F_{UU,T}+\epsilon F_{UU,L}$$

$$+\sqrt{2\epsilon(1-\epsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h+\epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h\}$$

SIDIS cross section

hadron multiplicity:
normalize to inclusive DIS cross section

$$\frac{d^2\sigma^{\rm incl.DIS}}{dxdy} \propto F_T + \epsilon F_L$$

$$\frac{d^4\mathcal{M}^h(x,y,z,P_{h\perp}^2)}{dxdydzdP_{h\perp}^2} \propto \left(1+\frac{\gamma^2}{2x}\right)\frac{F_{UU,T}+\epsilon F_{UU,L}}{F_T+\epsilon F_L}$$

$$\approx \frac{\sum_q e_q^2\, f_1^q(x,p_T^2)\otimes D_1^{q\to h}(z,K_T^2)}{\sum_q e_q^2\, f_1^q(x)}$$

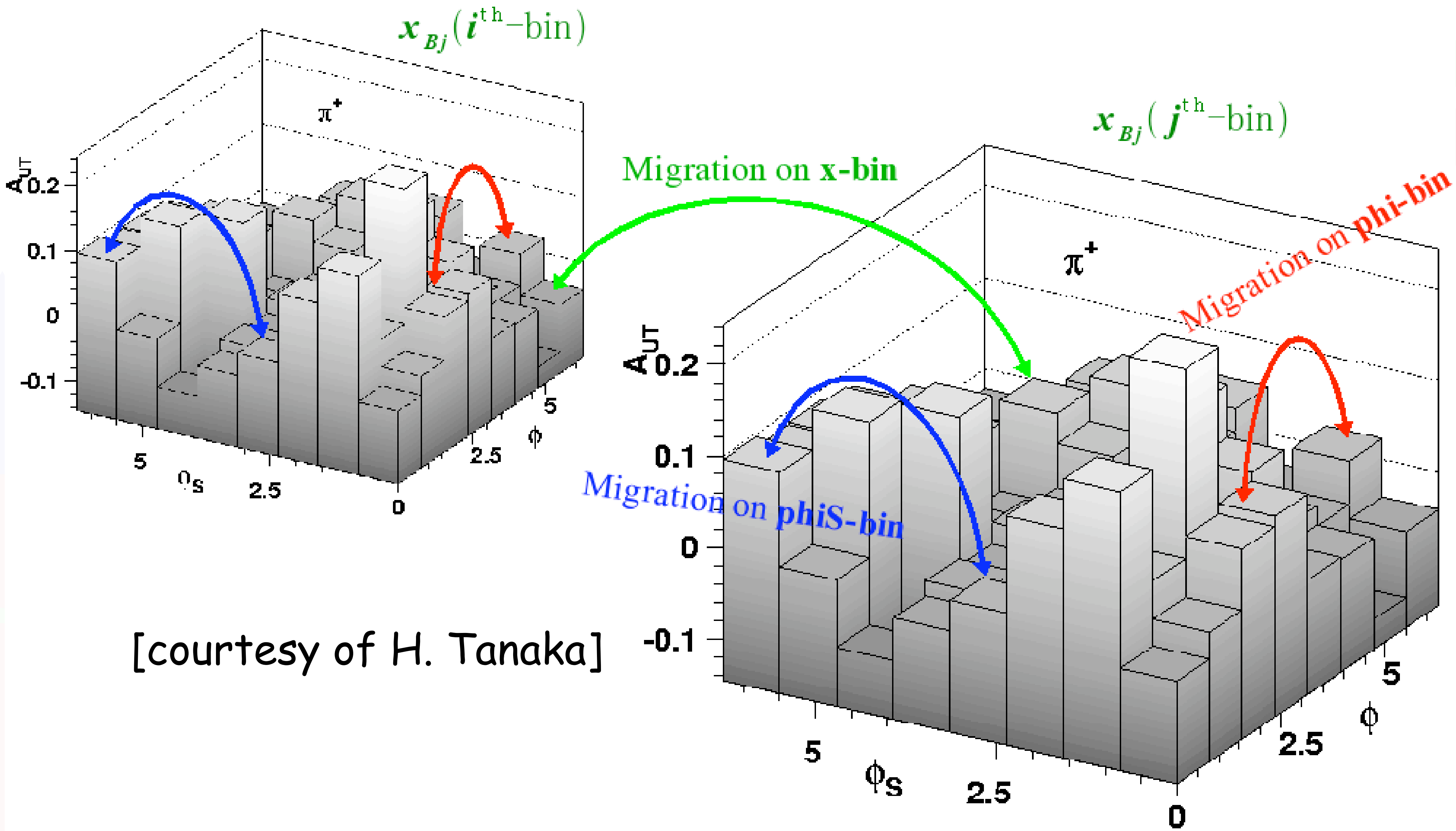
$$\frac{d^5\sigma}{dxdydzd\phi_h dP_{h\perp}^2} \propto \left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\epsilon F_{UU,L} + \sqrt{2\epsilon(1-\epsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h + \epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h\Big\}$$

$$2\langle\cos 2\phi\rangle_{UU} \equiv 2\frac{\int d\phi_h \cos 2\phi\, d\sigma}{\int d\phi_h d\sigma} = \frac{\epsilon F_{UU}^{\cos 2\phi}}{F_{UU,T}+\epsilon F_{UU,L}}$$

$$\approx \epsilon\frac{\sum_q e_q^2\; h_1^{\perp,q}(x,p_T^2)\otimes_{\rm BM} H_1^{\perp,q\to h}(z,K_T^2)}{\sum_q e_q^2\; f_1^q(x,p_T^2)\otimes D_1^{q\to h}(z,K_T^2)}$$

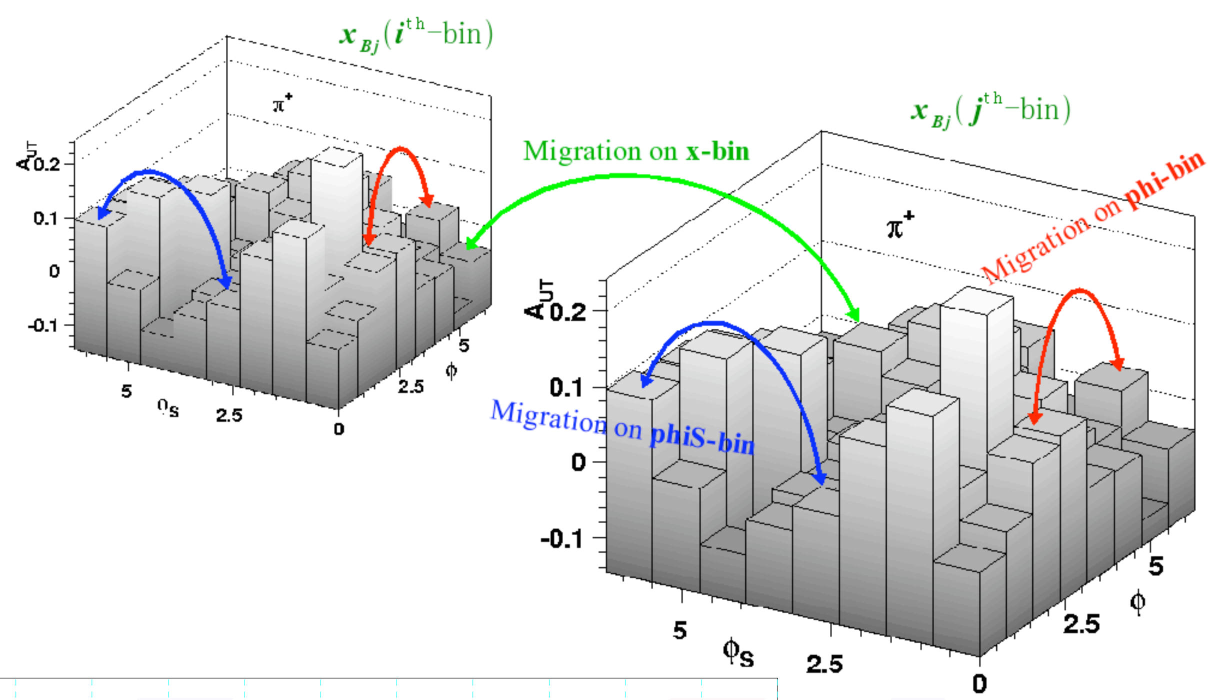
moments:
normalize to azimuth-independent cross-section

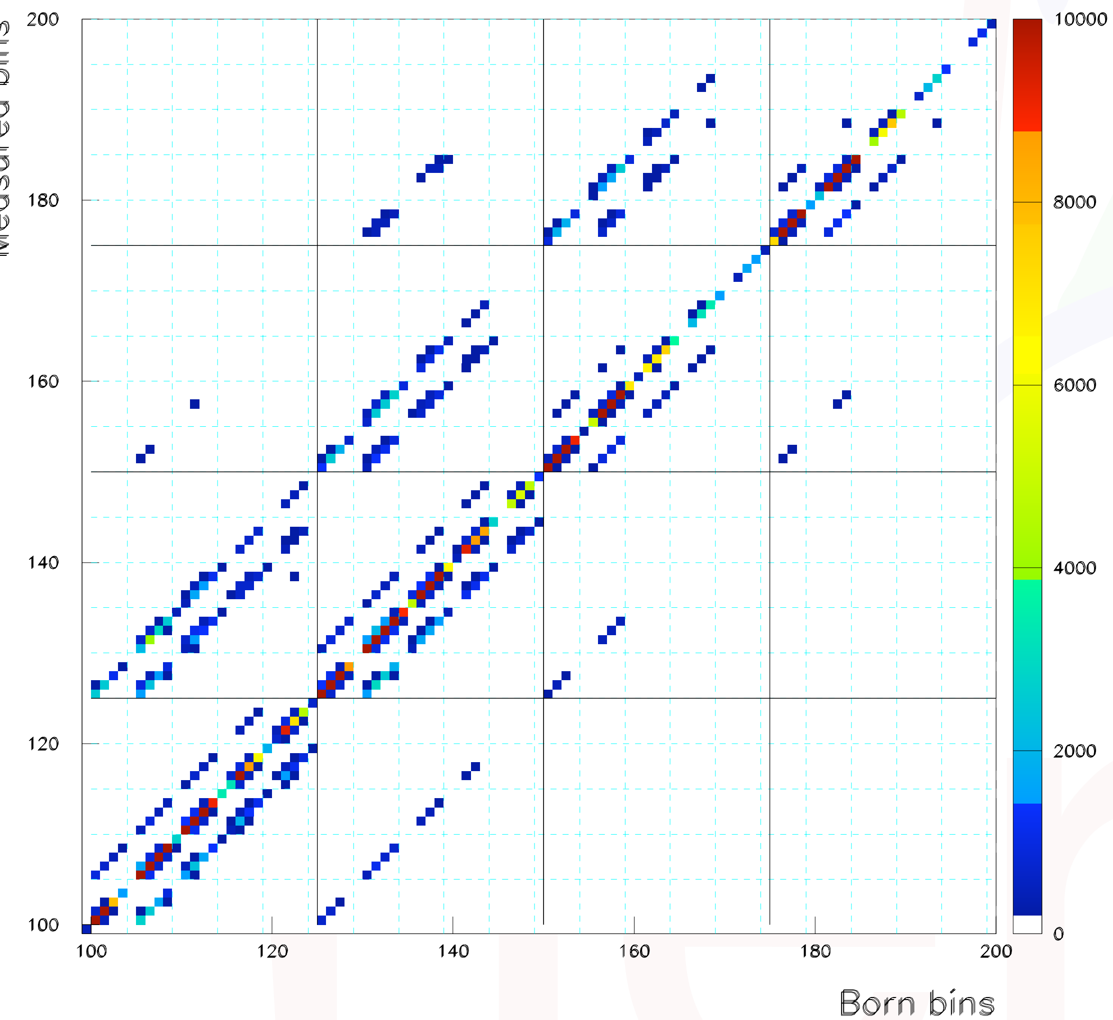
… event migration …

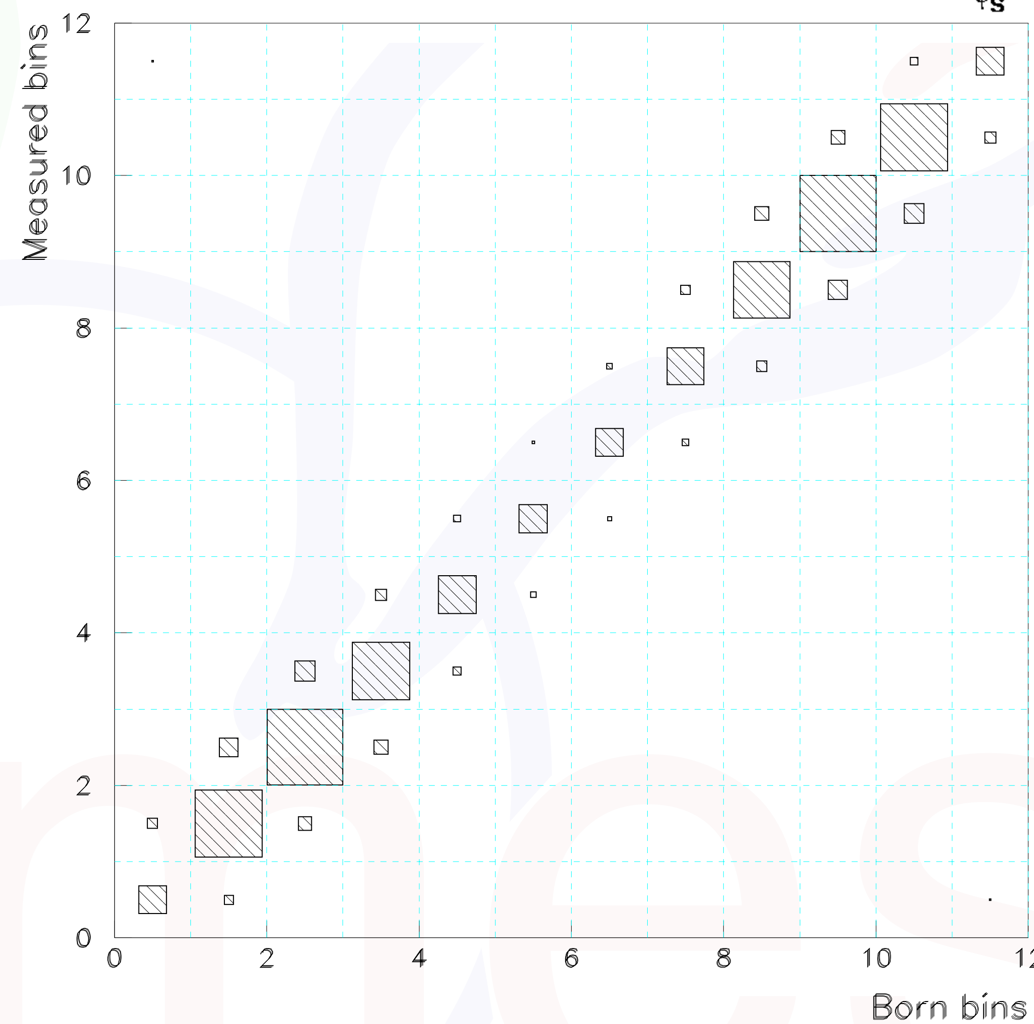


[courtesy of H. Tanaka]

… event migration …







- migration correlates yields in different bins
- can't be corrected properly in bin-by-bin approach

... event migration -> unfolding

$$\mathcal{Y}^{\exp}(\Omega_i) \propto \sum_{j=1}^{N} S_{ij} \int_j d\Omega \, d\sigma(\Omega) + \mathcal{B}(\Omega_i)$$

… event migration -> unfolding

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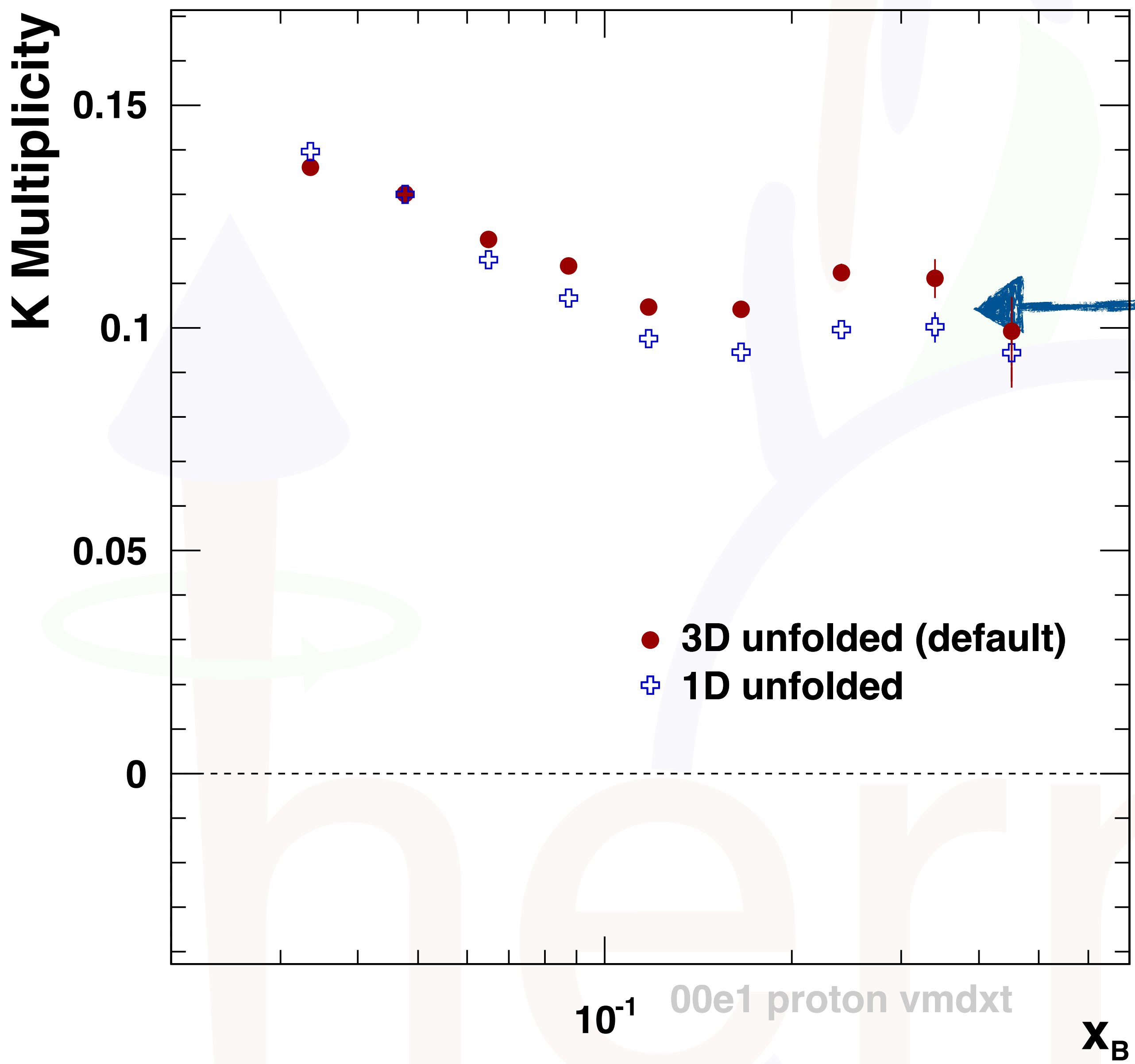
- inversion of relation gives Born cross section from measured yields

Multi-D vs. 1D unfolding at work

[S.J. Joosten, PhD thesis UIUC (2013)]



Neglecting to unfold in z changes x dependence dramatically

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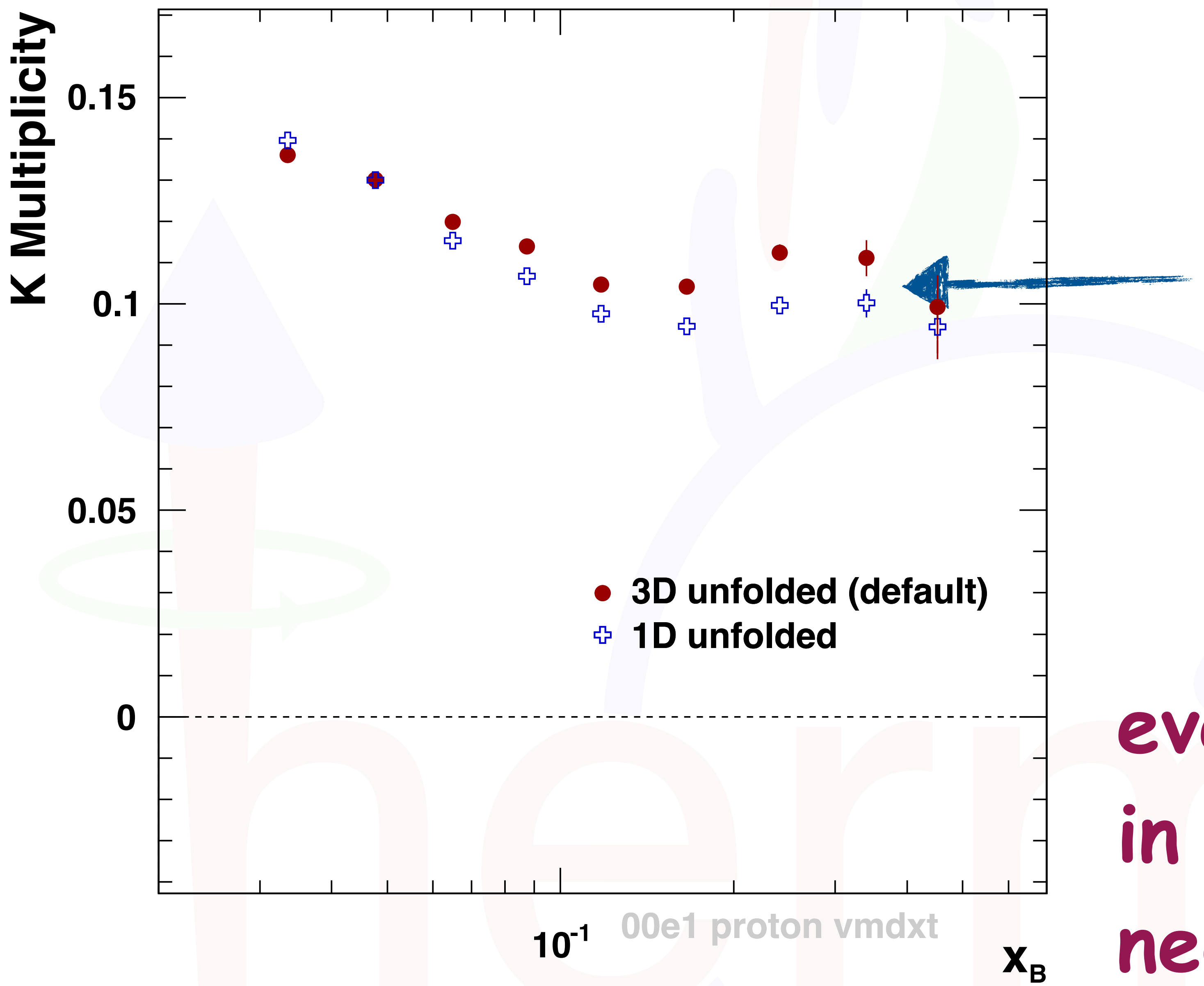

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even though only interested in collinear observable, need to carefully consider transverse d.o.f.

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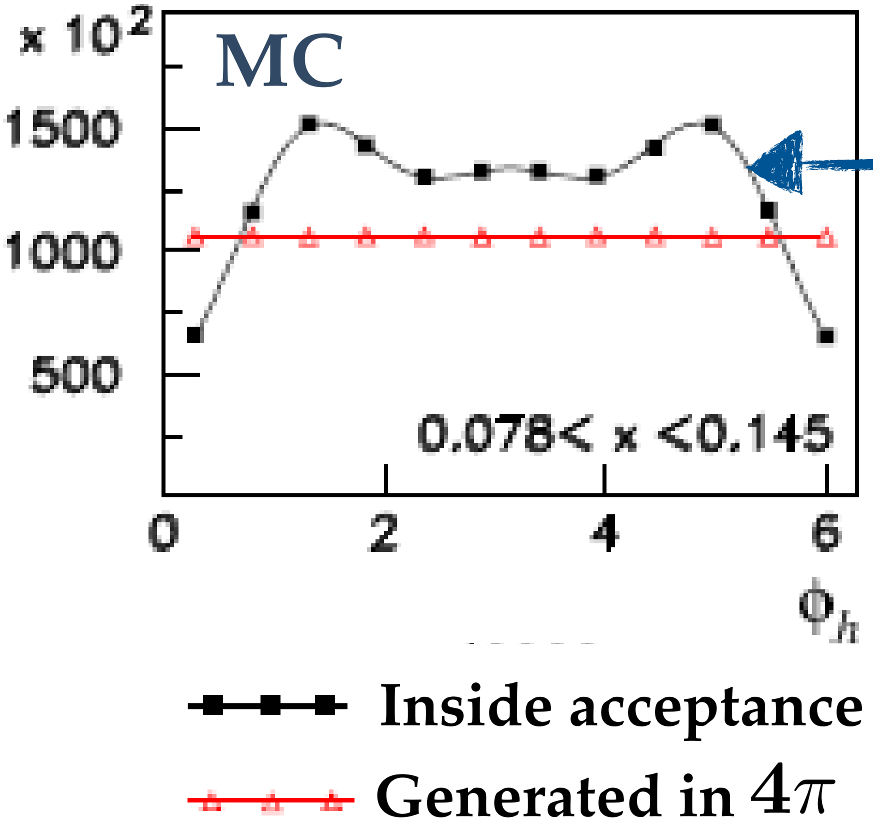

simulated yield with clear cosine modulations from migration and acceptance

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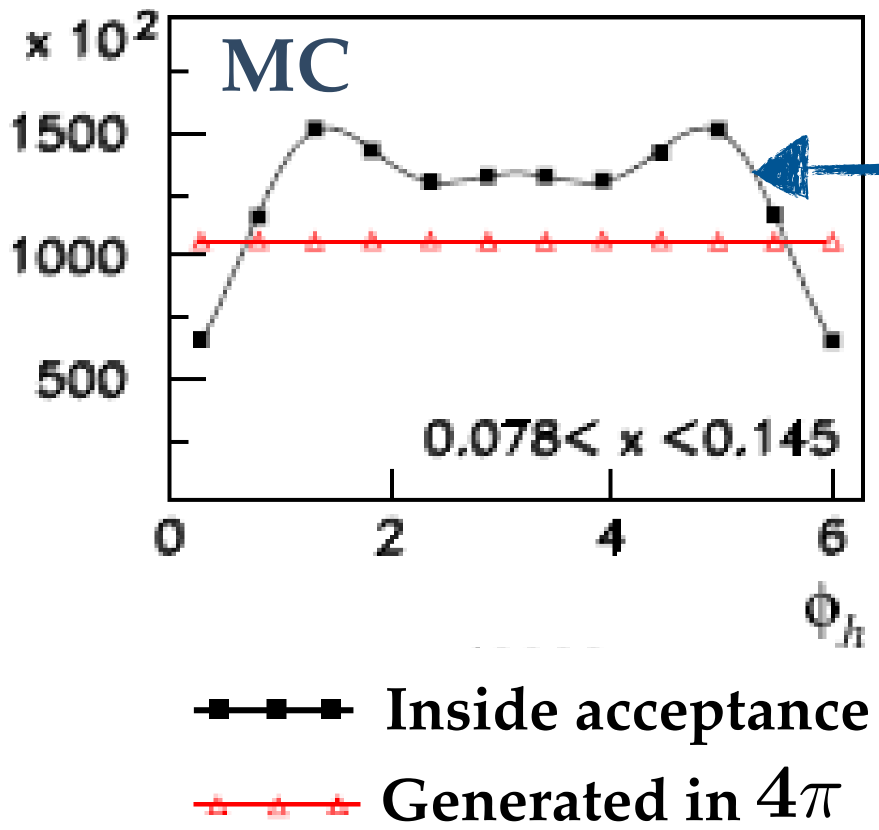

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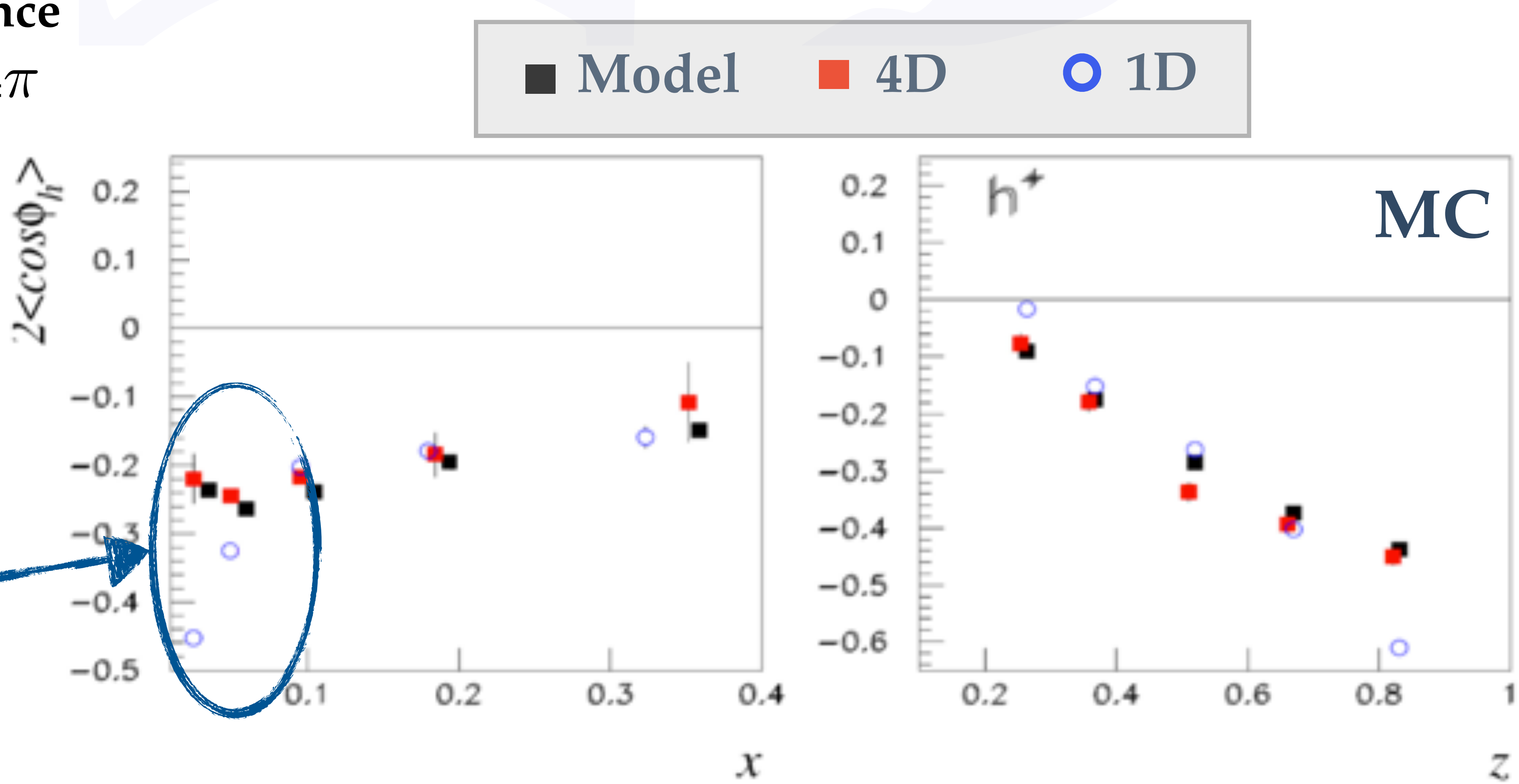

1D clearly not sufficient

summary

- acceptance plays crucial part, especially in analysis of multi-particle final states, and that even for asymmetries
- acceptance studies and/or corrections (e.g., unfolding) require realistic Monte Carlo simulation of underlying physics
- gmc_trans provides reasonably realistic Collins and Sivers amplitudes for pions and kaons based on Gaussian Ansatz
- reshuffling PYTHIA events, guided by, e.g., real data, provides a powerful tool to study systematics
  - still relies on good description of unpolarized cross section
- make a careful choice of how data points are to be interpreted (at average kinematics or average over kinematic range)
  - evaluate systematics accordingly
- fully differential analyses clearly preferred, though more challenging