TMD Evolution Overview

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- High Energy Collisions and Transverse Momentum
- Transverse Momentum Dependent (TMD) Functions and Intrinsic, non-perturbative Transverse Momentum
- Phenomenology

INT Workshop – February 27, 2014

Motivation

- HEP (high energy QCD, BSM, etc…)

Theme of talk.

Transverse Momentum Dependent (TMD) Factorization

- Hadronic structure studies that use pQCD.

Quark and Gluon
Degrees of Freedom

Talk Strategy

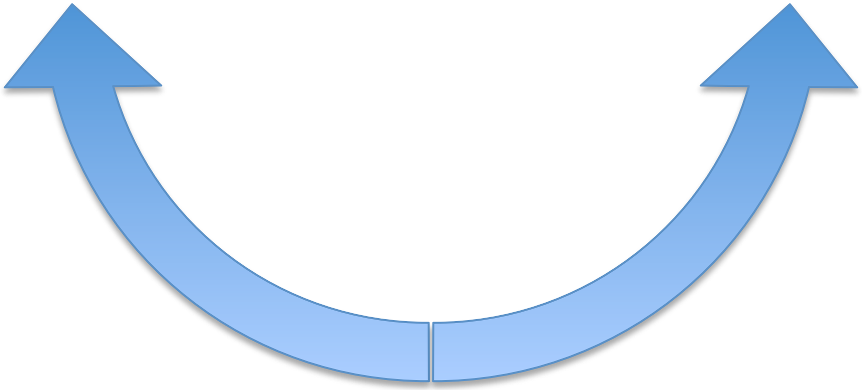
- Parton Model Intuition

- Real QCD

Talk Strategy

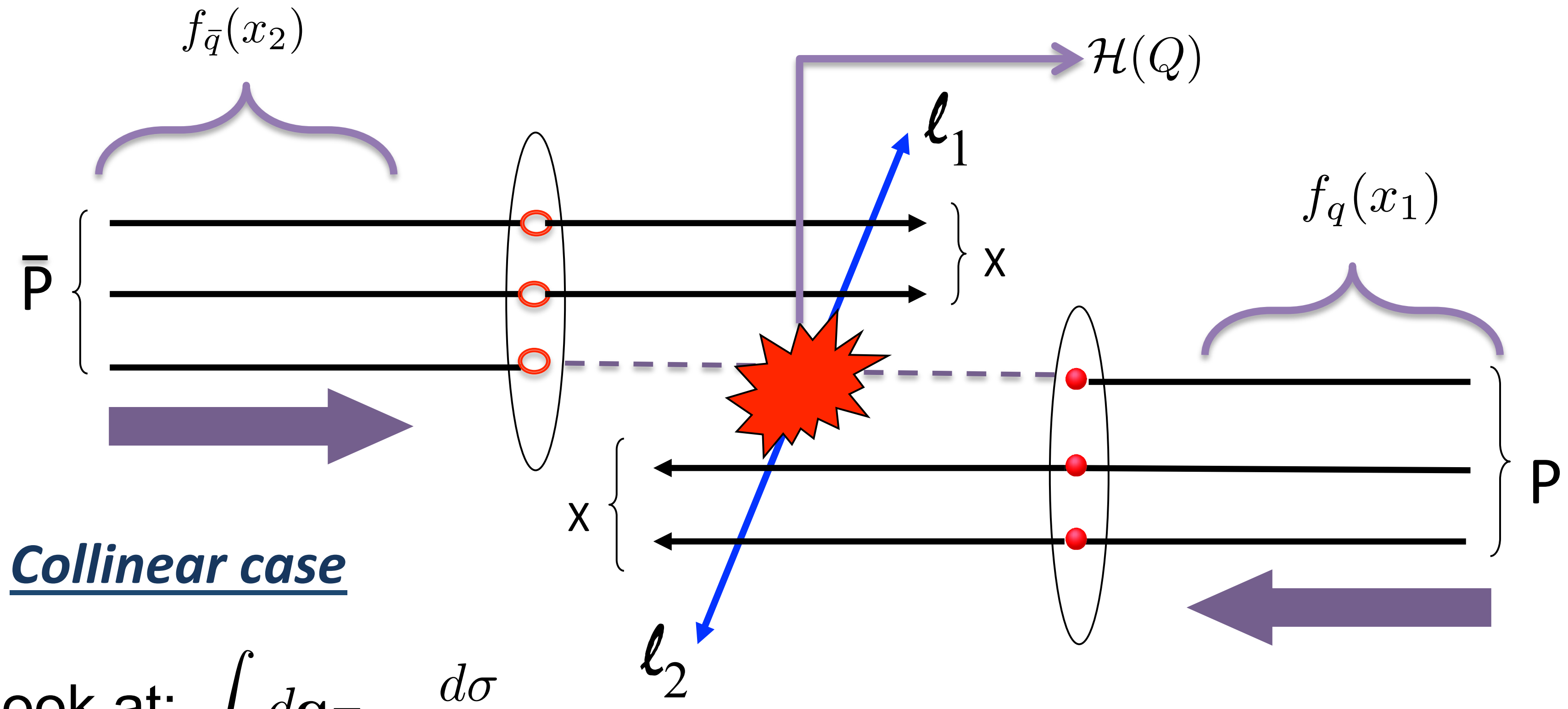
- Collinear Factorization
  - Collinear PDF, FFs
  - Scale Evolution

- TMD Factorization
  - TMD PDF, FFs
  - Scale Evolution



<u>Analogies / Broken Analogies</u>

Collinear parton model



Collinear case

Look at: $\int d\mathbf{q}_T \, \frac{d\sigma}{d\mathbf{q}_T \, \cdots}$

$$q^2 = (l_1 + l_2)^2 = Q^2 \gg \Lambda^2_{\rm QCD}$$

$\sim 1/Q$

Small Scales

Collinear Drell-Yan

- Factorization theorem

$$\sigma \sim \int \underbrace{\mathcal{H}(\mu/Q, \alpha_s(\mu))} \otimes f_{q/P}(x_1; \mu) \otimes f_{\bar{q}/\bar{P}}(x_2; \mu)$$

Small Coupling:
Perturbation Theory

$$C_0 + C_1 \alpha_s(\mu) + C_2 \alpha_s(\mu)^2 + C_3 \alpha_s(\mu)^3 + \cdots$$

Defined in terms of elementary fields

$$f_{j/p}(\xi) = \int \frac{dw^-}{(2\pi)} e^{-i\xi P^+ w^-} \langle P | \bar{\psi}_j(0, w^-, \mathbf{0}_t) \frac{\gamma^+}{2} \psi_j(0, 0, \mathbf{0}_t) | P \rangle$$

Collinear (Standard) Case

- Perturbative QCD factorization theorem:

$$\sigma \sim \int \underbrace{\mathcal{H}(\mu/Q, \alpha_s(\mu))}_{} \otimes f_{q/P}(x_1;\mu) \otimes f_{\bar{q}/\bar{P}}(x_2;\mu)$$

Auxiliary parameter: Arbitrary

- DGLAP evolution

(Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)

$$\frac{d}{d\ln\mu} f_{j/P}(x;\mu) = 2\int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

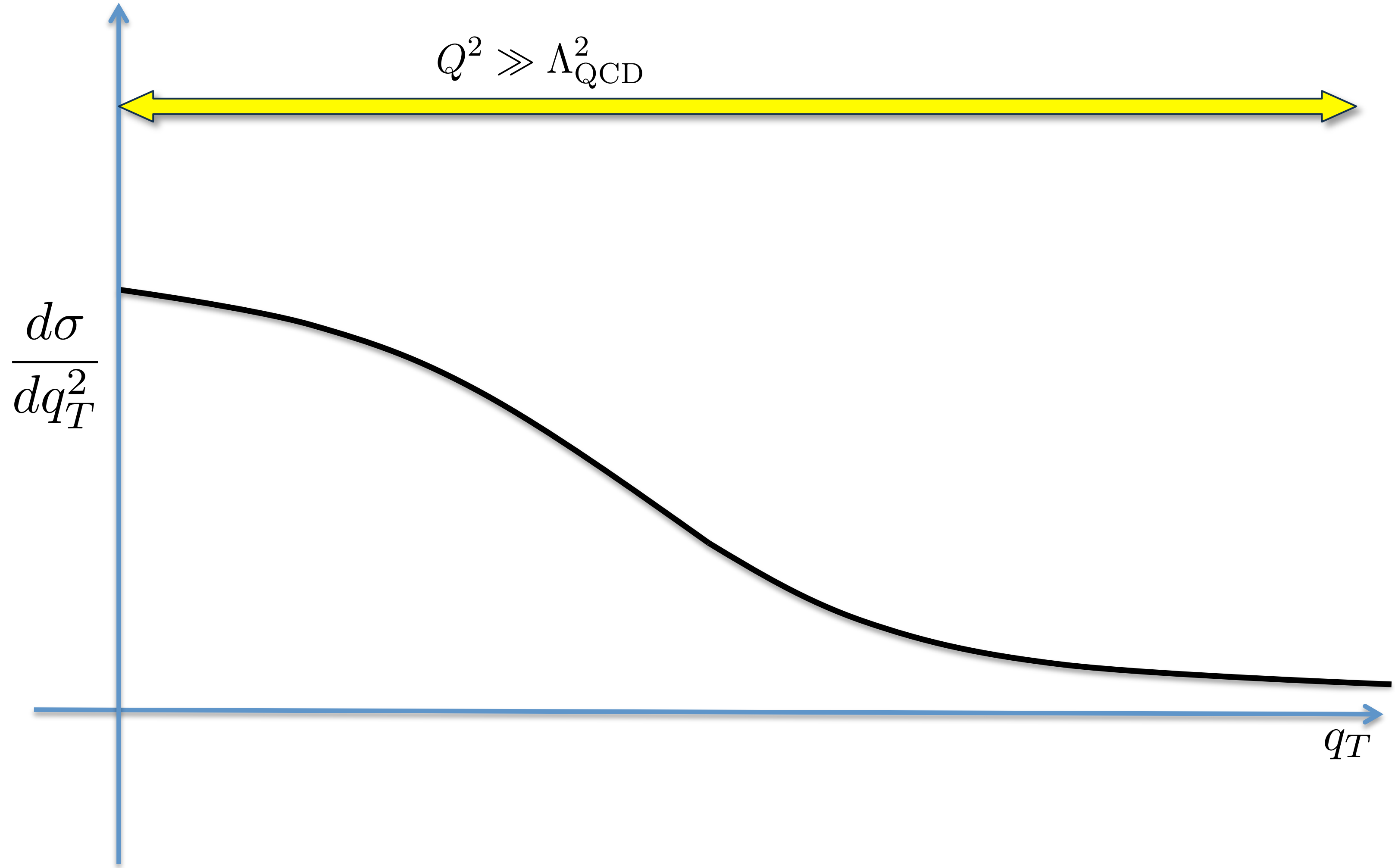
- *Factorization* + *Evolution*: Universal PDFs

“Portable”

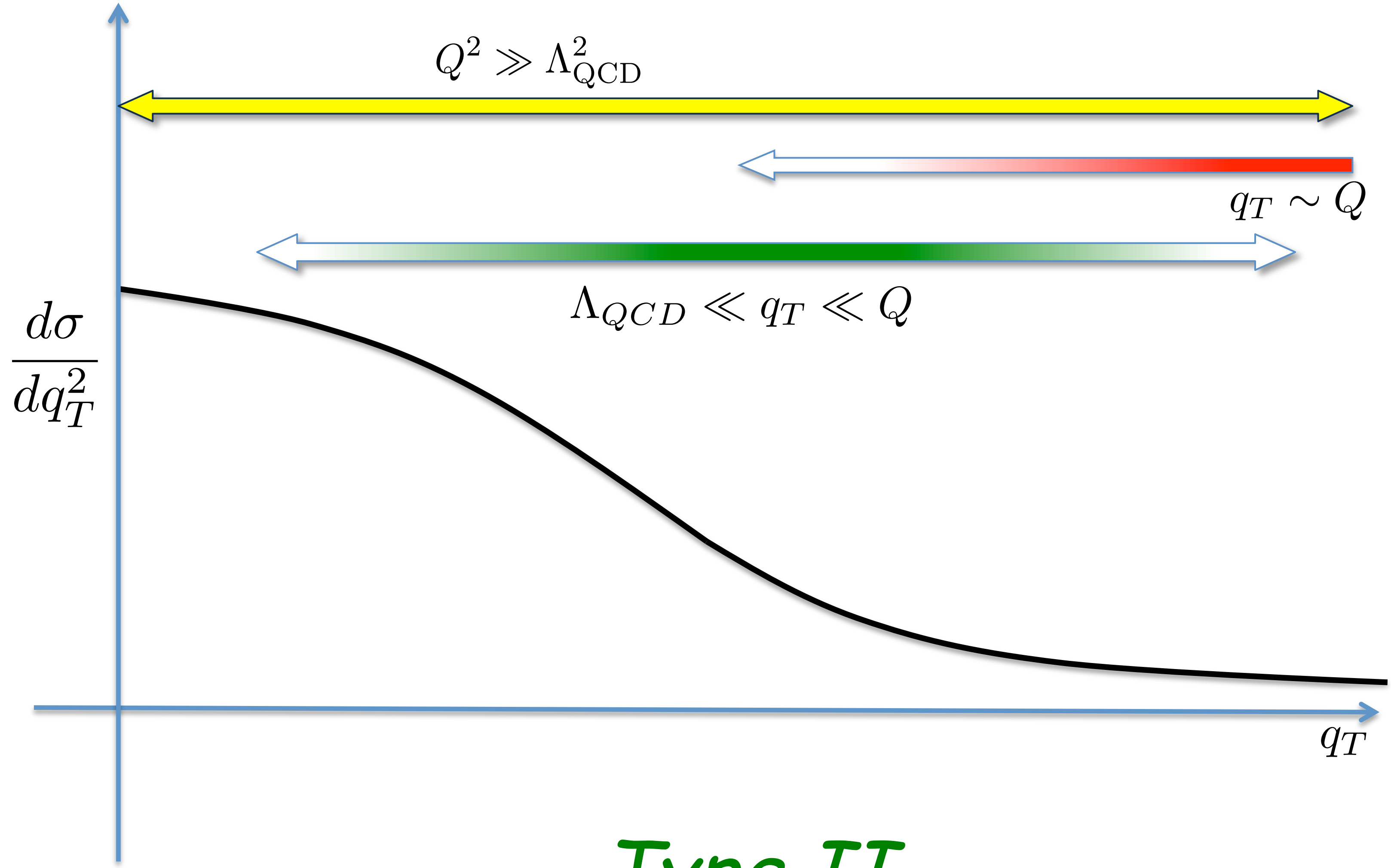
High Energy Collisions
& Transverse Momentum

(Less Inclusive)

Transverse Momentum:

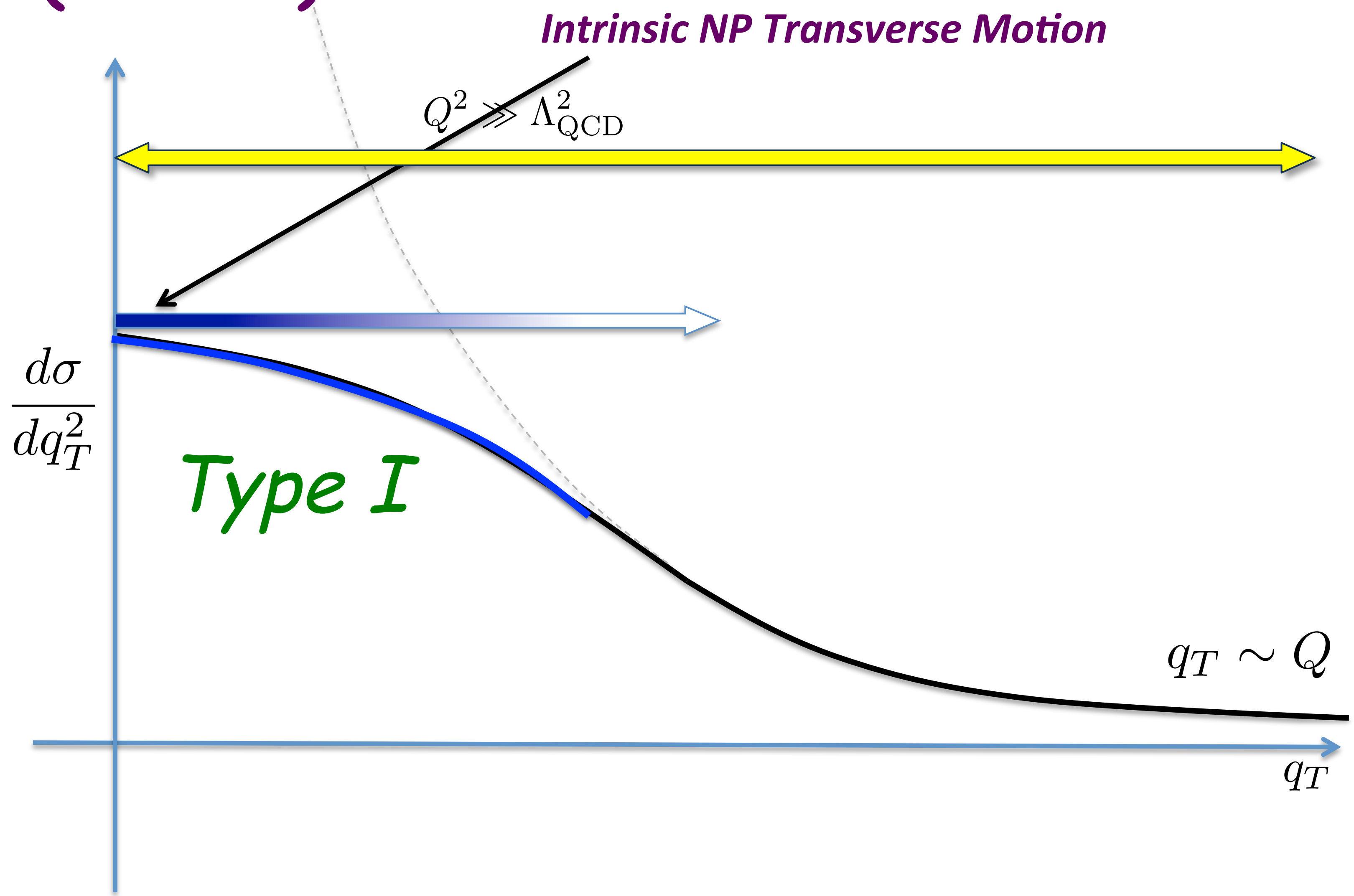


(Large) Transverse Momentum:



Type II

(Small) Transverse Momentum:



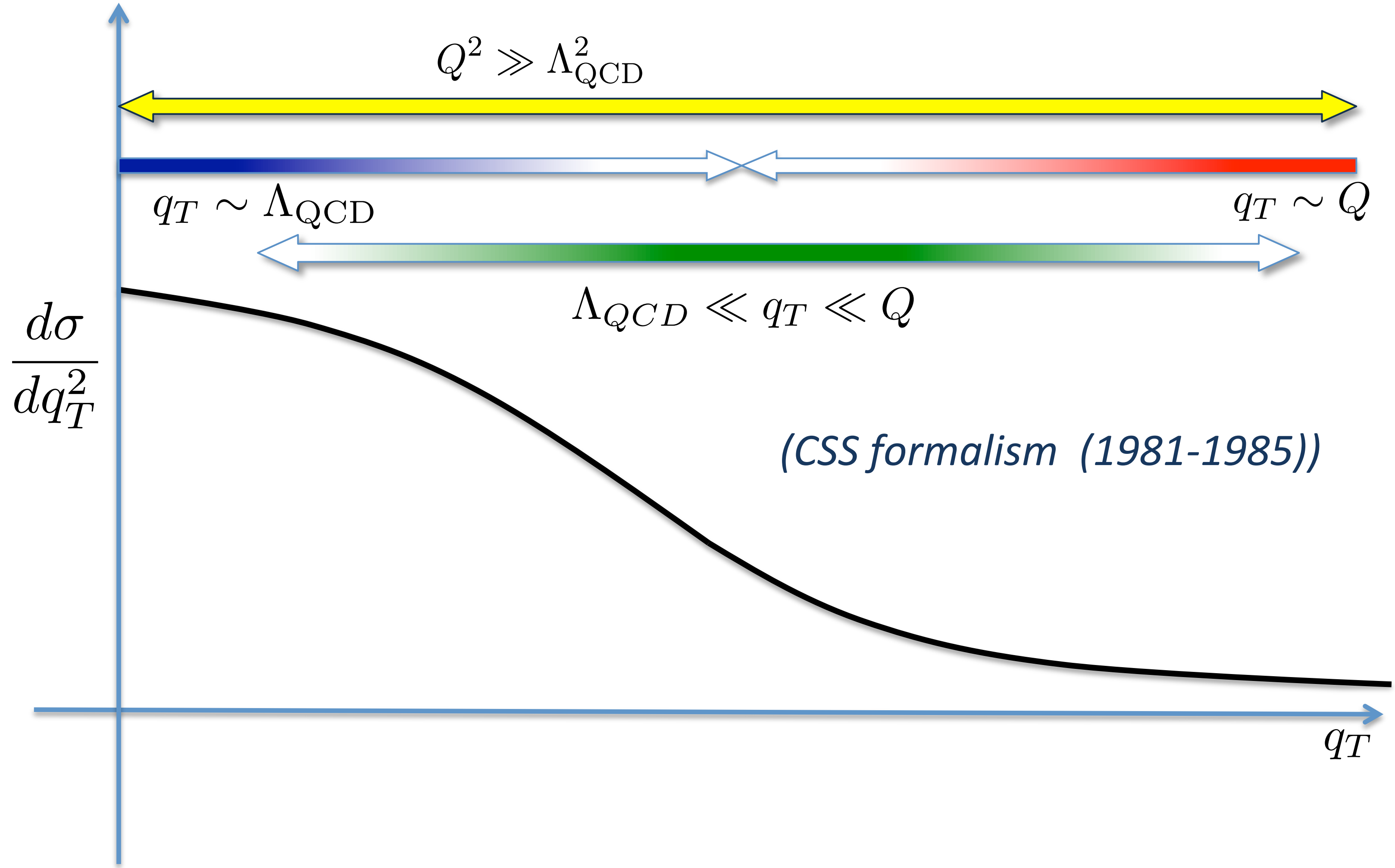
TMD Parton Model

$$\frac{d\sigma}{d^2\mathbf{q}_t} \sim \int \underbrace{\mathcal{H}(Q)}_{\text{Elementary Collision}} \otimes \underbrace{F_{q/P}(x_1, \mathbf{k}_{1T})} \otimes \underbrace{F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T})}$$

Parton Model

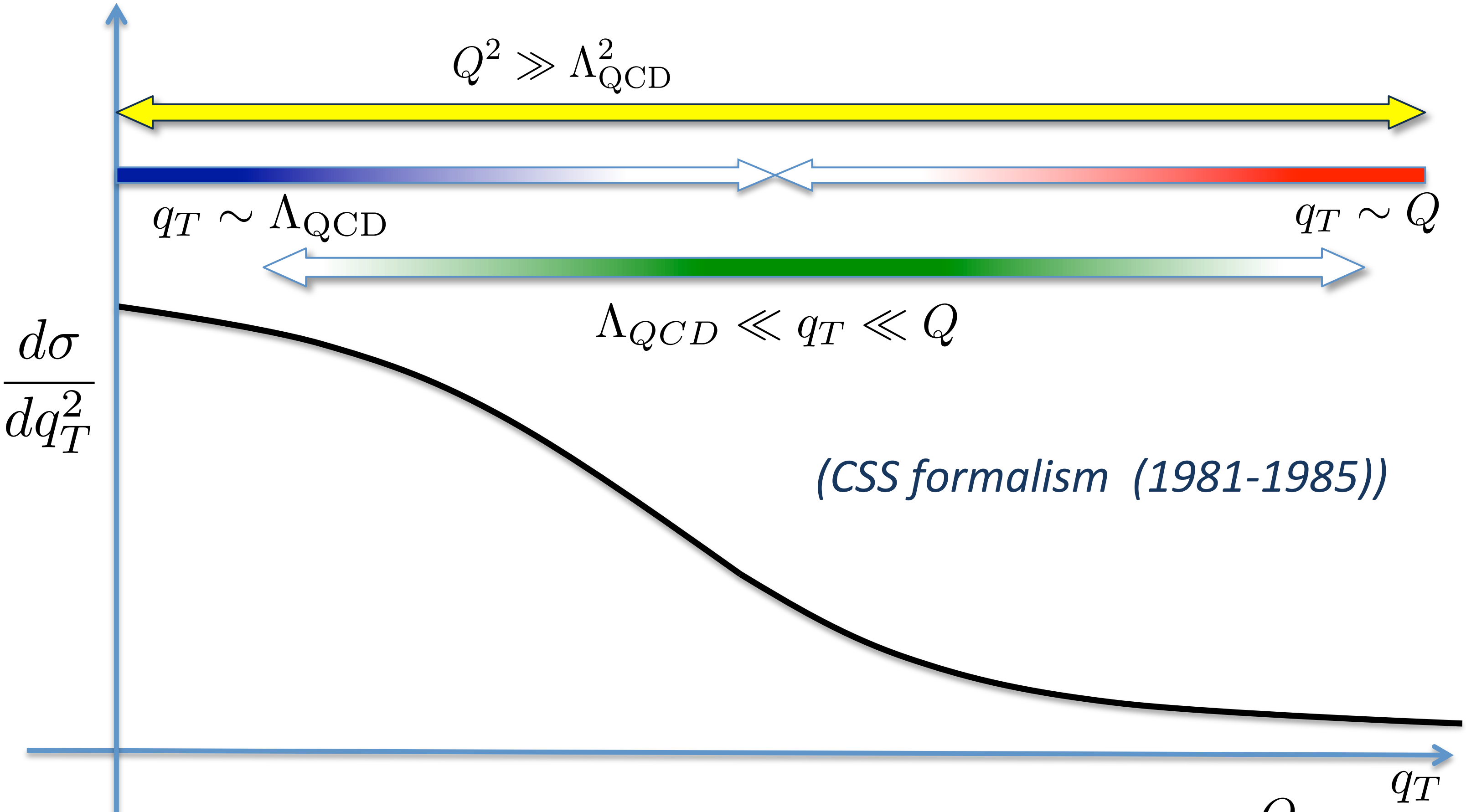
Number densities

Unified: Transverse Momentum:



(CSS formalism (1981-1985))

Unified: Transverse Momentum:



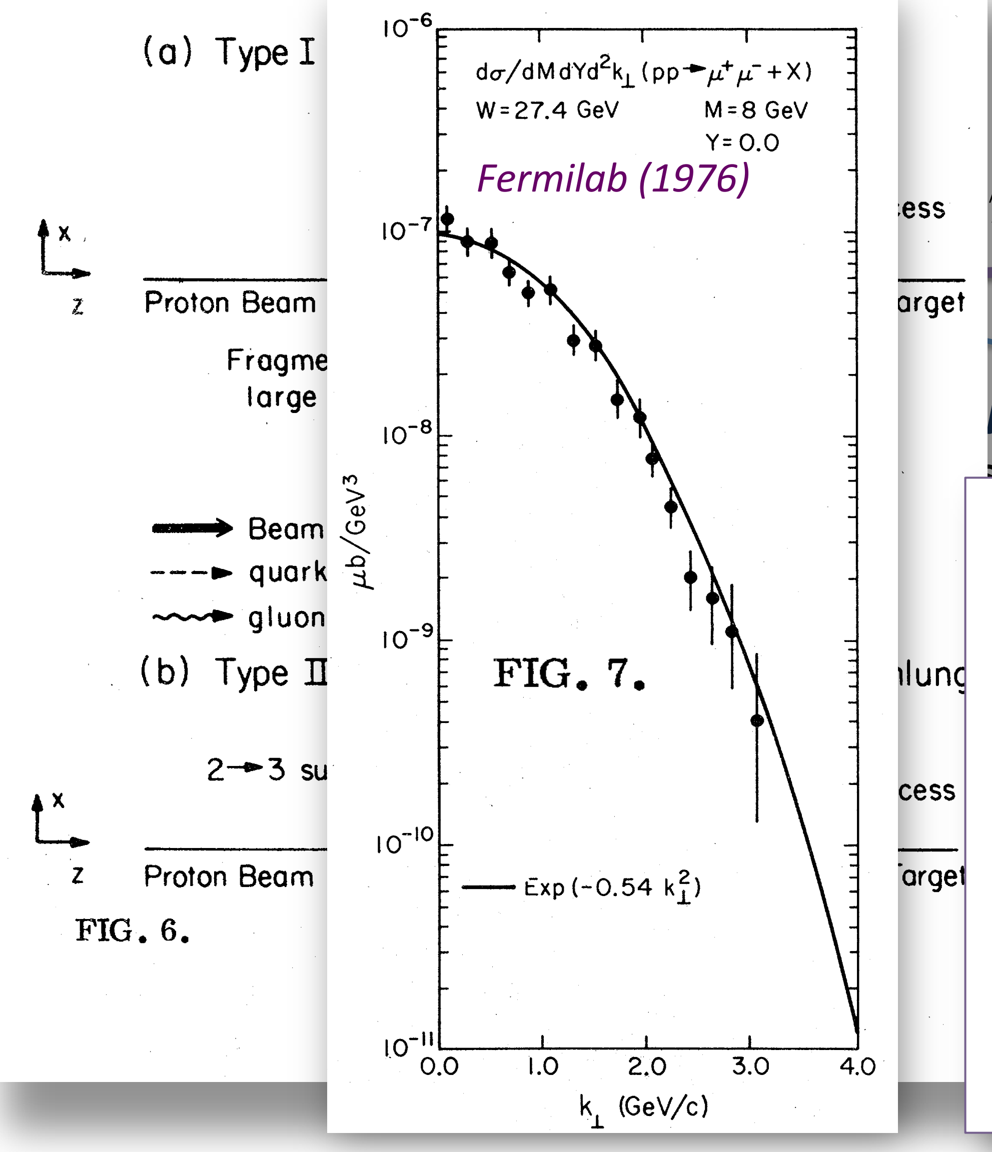
(CSS formalism (1981-1985))

Non-perturbative q_T dependence vanishes in literal limit of $\frac{Q}{\Lambda} \to \infty$

(Parisi, Petronzio (1979))

Type II Dominance

TMD Parton Model



Fermilab (1976)

FIG. 6.

FIG. 7.

$$\ldots_T) \otimes \underbrace{F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T})}$$

Number densities

e Momentum Dependent

> *"There has been much speculation about how much of the dimuon k_T spectra shown in Fig.7 is due to the wave function **(Type I)** and how much is explained by QCD perturbation calculations **(Type II)**."*
>
> \- R. Feynman, R. Field, G. Fox
> Phys.Rev. D18 **(1978)** 3320

All Transverse Momenta

Basic Structure

$$\frac{d\sigma}{dq_t^2} = \underset{\substack{\uparrow \\ q_T \ll Q}}{\mathrm{W}} \quad + \quad \underset{\substack{\uparrow \\ q_T \sim Q}}{\mathrm{Y}} \quad + \quad 1/\mathrm{Q} \quad \text{corrections}$$

(Collins, Soper, Sterman (CSS) formalism (1981-1985)… (many similar formalisms))

All Transverse Momenta

W term

$$\frac{d\sigma}{dq_t^2} \sim \int d^2\mathbf{b}_T \; e^{-i\mathbf{b}_T\cdot\mathbf{q}_T} \times$$

Collinear OPE

$\alpha_s(\sim 1/b_T)$

$$\left\{ \begin{array}{l} \times \displaystyle\int_{x_1}^{1} \frac{d\hat{x}_1}{\hat{x}_1} \tilde{C}_{f/j}(x_1/\hat{x}_1, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}_1, \mu_b) \times \\ \\ \times \displaystyle\int_{x_2}^{1} \frac{d\hat{x}_2}{\hat{x}_2} \tilde{C}_{\bar{f}/j'}(x_2/\hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j'/\bar{P}}(\hat{x}_2, \mu_b) \times \end{array} \right.$$

Ex: Matching Prescription:

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_\mathrm{max}^2}}$$

$$\mu_b \equiv C_1/|\mathbf{b}_*(b_T)|$$

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Perturbative Logs

$$\times \exp\left\{ \int_{\mu_b}^{Q} \frac{d\mu'^{\,2}}{\mu'^{\,2}} \left[\mathcal{B}(g(\mu')) + \ln \frac{Q^2}{\mu'^{\,2}} \mathcal{A}(g(\mu')) \right] \right\} \times$$

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Non-perturbative Large b_T

$$\times \exp\left\{ \underbrace{-g_1(x_1, b_T)}_{\textit{Hadron 1 Intrinsic}} \underbrace{-g_2(x_2, b_T)}_{\textit{Hadron 2 Intrinsic}} \underbrace{-2g_K(b_T)}_{\textit{NP soft Evolution}} \ln\frac{Q}{Q_0} \right\}$$

Ex: Matching Prescription:

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_{\mathrm{max}}^2}}$$

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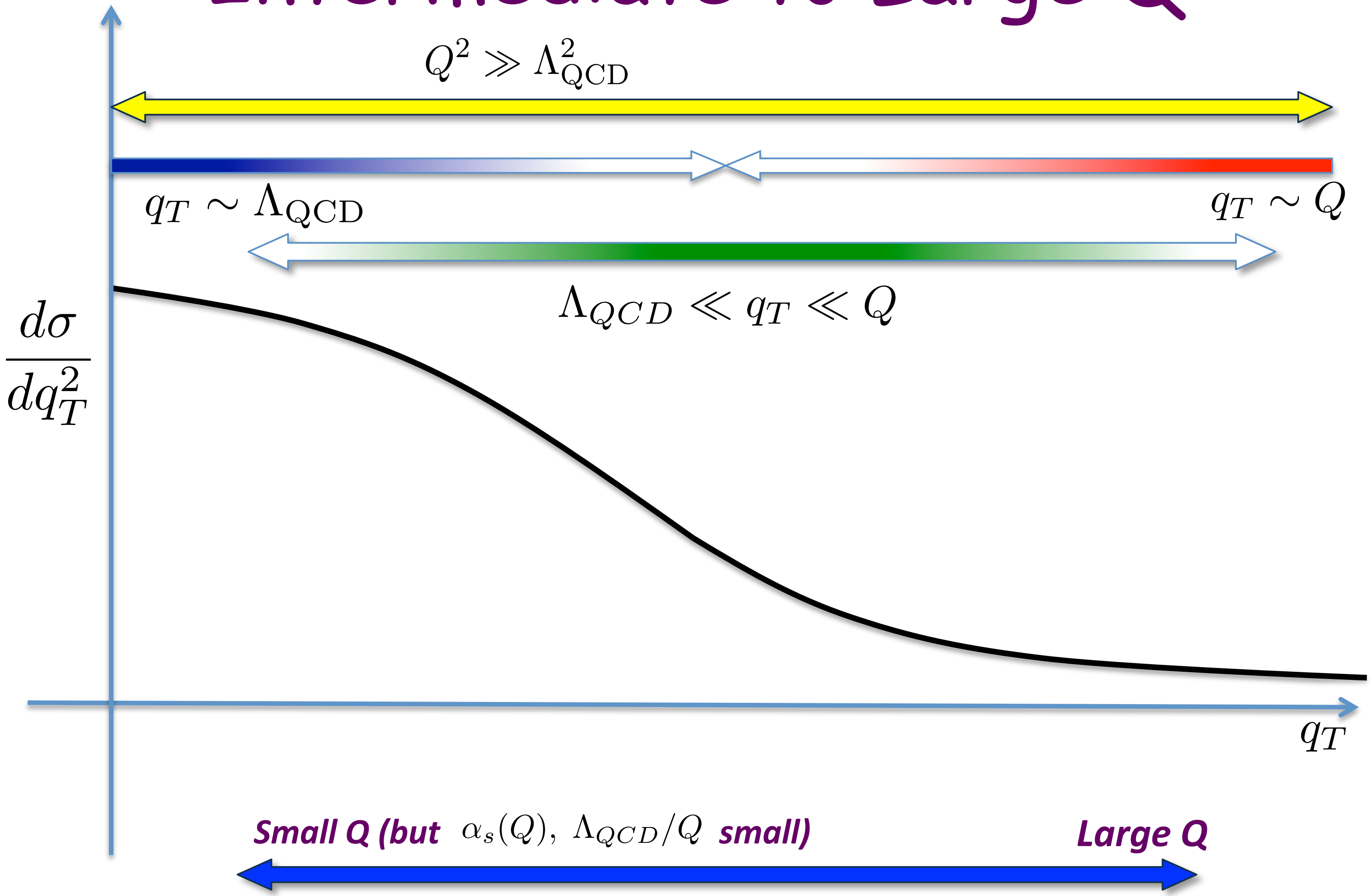
(Collins, Soper, Sterman (CSS) formalism (1981-1985)… (many similar formalisms))

Unified: All Transverse Momentum, Intermediate to Large Q:



(Collins, Soper, Sterman (CSS) formalism (1981-1985)… (many similar formalisms))

Motivation I
High Energy Physics
& Transverse Momentum

Small Transverse Momentum, Motivation Ex:

- Constraining SM parameters.
  - Example: W, Z masses and widths

> *"While significant effort has been put into the study of **W(b)** at large **b** [36, 42, 43, 44], none … adequately describe the observed **Z** boson distribution without introducing free parameters."*

- P. Nadolsky, (2004) *Theory of W and Z Production*, pg. 9

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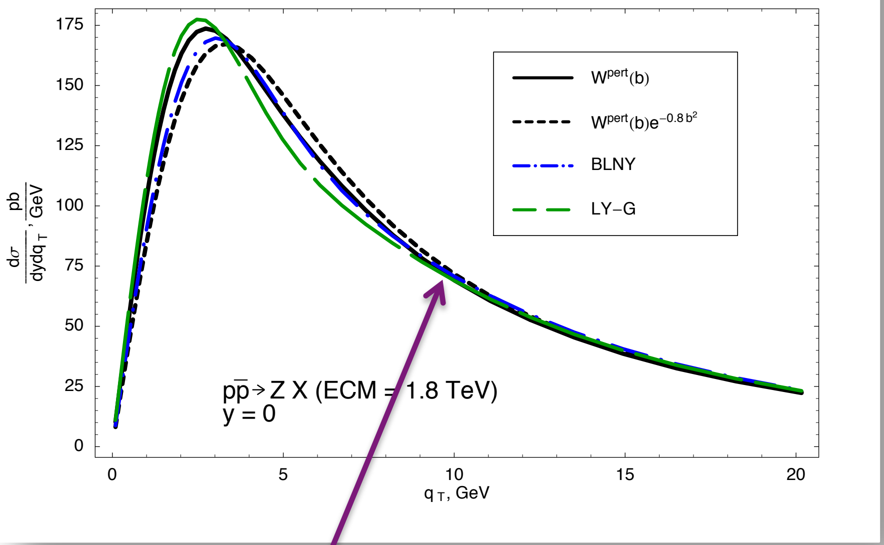
- P. Nadolsky, (2004) *Theo*

P. Nadolsky, (2004) Theory of W and Z Production



Type II - like

All Transverse Momenta

W term

$$\frac{d\sigma}{dq_t^2} \sim \int d^2\mathbf{b}_T\; e^{-i\mathbf{b}_T\cdot\mathbf{q}_T} \times$$

Collinear OPE

$\alpha_s(\sim 1/b_T)$

$$\times \int_{x_1}^{1} \frac{d\hat{x}_1}{\hat{x}_1} \tilde{C}_{f/j}(x_1/\hat{x}_1, b_*; \mu_b^2, \mu_b, g(\mu_b))\, f_{j/P}(\hat{x}_1, \mu_b) \times$$

$$\times \int_{x_2}^{1} \frac{d\hat{x}_2}{\hat{x}_2} \tilde{C}_{\bar{f}/j'}(x_2/\hat{x}_2, b_*; \mu_b^2, \mu_b, g(\mu_b))\, f_{j'/\bar{P}}(\hat{x}_2, \mu_b) \times$$

Perturbative Logs

$$\times \exp\left\{ \int_{\mu_b}^{Q} \frac{d\mu'^2}{\mu'^2} \left[\mathcal{B}(g(\mu')) + \right.\right.$$

Ansatz:

$$-g_K(b_T)\ln\left(\frac{Q}{Q_0}\right) = -g_2\frac{1}{2}b_T^2\ln\left(\frac{Q}{Q_0}\right)$$

Non-perturbative Large b_T

$$\times \exp\left\{ \underbrace{-g_1(x_1, b_T)}_{\text{Hadron 1 Intrinsic}} \underbrace{-g_2(x_2, b_T)}_{\text{Hadron 2 Intrinsic}} \underbrace{-2g_K(b_T)}_{\text{NP soft Evolution}} \ln\frac{Q}{Q_0} \right\}$$

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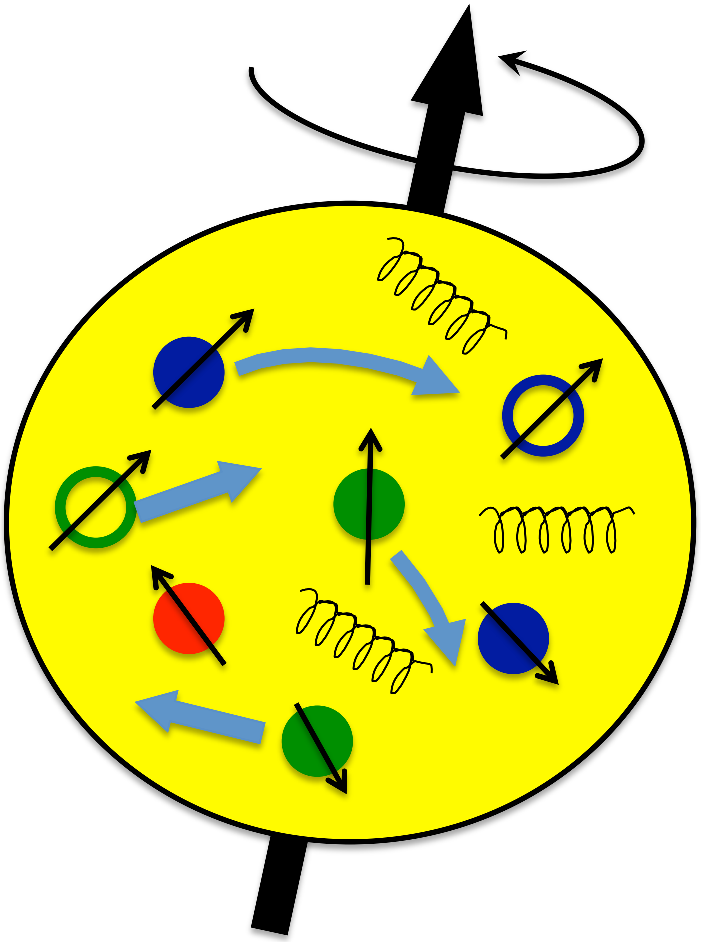
> *"The observed boson q_T spectrum in this measurement is mostly sensitive to g_2 and has very limited sensitivity to the other non-perturbative parameters..."*

(See Talk of M. Guzzi)

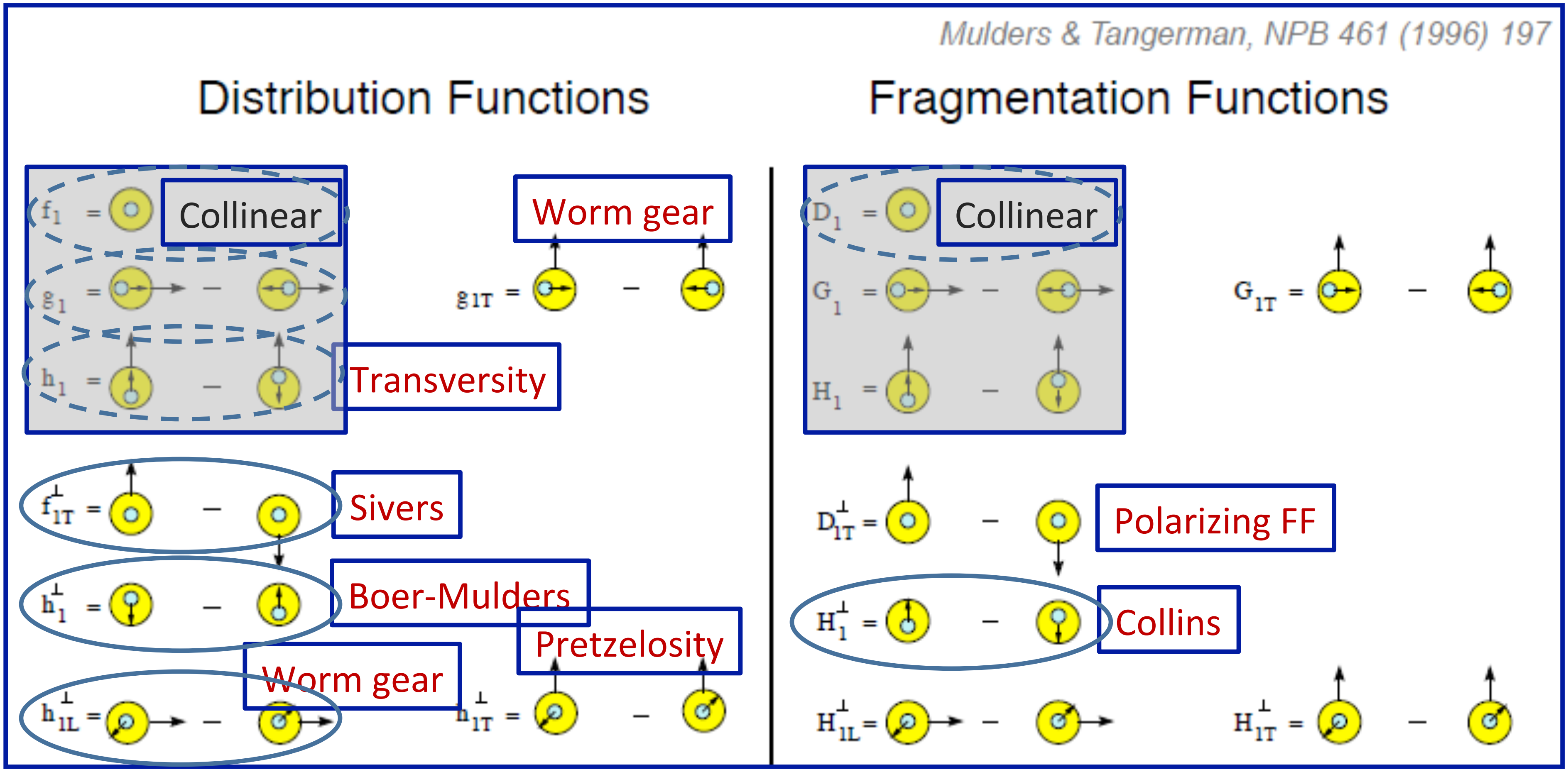
o Lopes de Sá, (2013) ,
urement of the W boson Mass with the D0 Detector, pg. 57, Ph.D. Thesis, Stony Brook University

Motivation II
Hadron Structure and Transverse Momentum

TMD vs. Collinear



TMD Taxonomy



(P. Mulders, R. Tangerman (1996))

(Gaussian Parametrizations)

Recall Collinear Case:

- Parton Model

$$\sigma \sim \int \underbrace{\mathcal{H}(Q)} \otimes \underbrace{f_{q/P}(x_1)} \otimes \underbrace{f_{\bar{q}/\bar{P}}(x_2)}$$

Elementary collision
Short distance scales

Hadron Structure: large distance scales

Real QCD?

TMD Parton Model

$$\frac{d\sigma}{d^2\mathbf{q}_t} \sim \int \underbrace{\mathcal{H}(Q)}_{} \otimes \underbrace{F_{q/P}(x_1, \mathbf{k}_{1T})}_{} \otimes \underbrace{F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T})}_{}$$

Parton Model

Elementary collision

Short distance scales

$\sim 1/Q$

Number densities

“Transverse Momentum Dependent Parton Distribution Functions” (TMD PDFs)

⬇

Parton model-like picture in QCD?

TMD PDF Definitions

- Exact, gauge invariant operator definitions needed to address questions of hadronic structure.

(See Collins, POS (2003) for list of complications)

TMD PDF Definitions

- Exact, gauge invariant operator definitions needed to address questions of hadronic structure.

  (See Collins, POS (2003) for list of complications)

- Universality / Modified Universality.
  - Sivers Function: Non-zero, reverses sign in Drell-Yan vs. SIDIS

    (Brodsky, Hwang, Schmidt (2002)), (Collins, (2002))

TMD Parton Model

$$\frac{d\sigma}{d^2\mathbf{q}_t} \sim \int \underbrace{\mathcal{H}(Q)} \otimes \underbrace{F_{q/P}(x_1, \mathbf{k}_{1T})} \otimes \underbrace{F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T})}$$

Parton Model

Elementary collision

Short distance scales

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"Transverse Momentum Dependent Parton Distribution Functions" (TMD PDFs)

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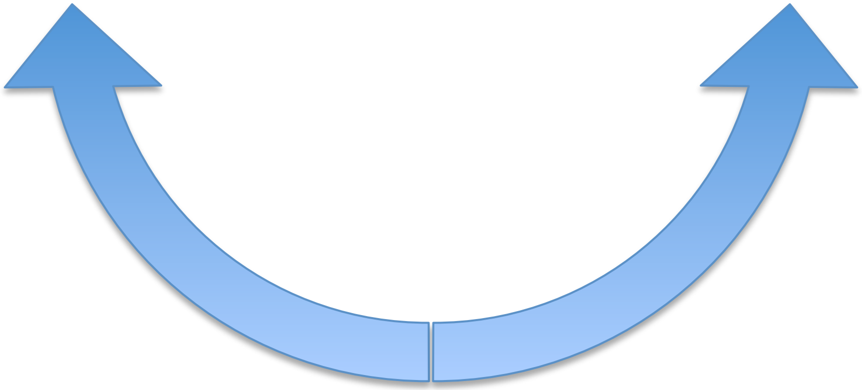
Parton model-like picture in QCD?

Talk Strategy

- Collinear Factorization
  - Collinear PDF, FFs
  - Scale Evolution

- TMD Factorization
  - TMD PDF, FFs
  - Scale Evolution



<u>Analogies / Broken Analogies</u>

TMD-Factorization

- Unified Formalism

$$\frac{d\sigma}{d^2\mathbf{q}_t} \sim \int \underbrace{\mathcal{H}(\mu/Q, \alpha_s(\mu))} \otimes F_{q/P}(x_1, \mathbf{k}_{1T}, \mu, \zeta_1) \otimes F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T}, \mu, \zeta_2)$$

Pert. QCD

Small Coupling:
Perturbation Theory

$$C_0 + C_1\alpha_s(\mu) + C_2\alpha_s(\mu)^2 + C_3\alpha_s(\mu)^3 + \cdots$$

+ Y-term

2 Auxiliary parameters: Arbitrary

$$\zeta_1\zeta_2 \sim Q^4$$

(J.C. Collins Extension of CSS formalism: (Book, 2011), Chapts. 10,13,14)

(See also SCET language: Echevarria, Idilbi, Scimemi (2011-2014))

Talk Strategy

- Collinear Factorization
  - Collinear PDF, FFs
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- TMD Factorization
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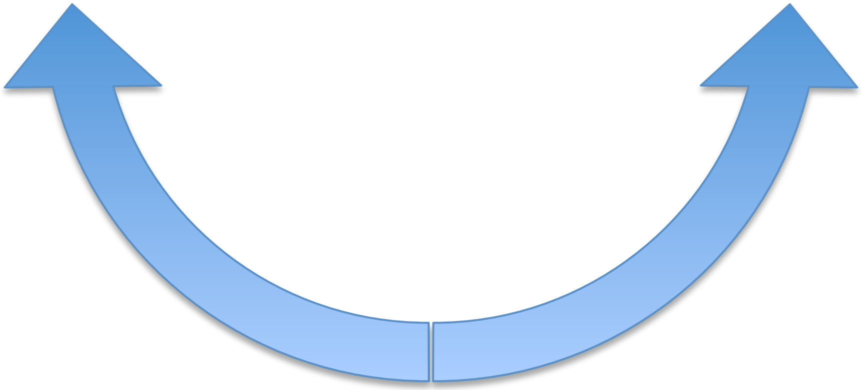


Analogies / Broken Analogies

TMD-Evolution

- Recall Collinear / DGLAP:

$$\frac{d}{d\ln\mu} f_{j/P}(x;\mu) = 2\int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

TMD-Evolution

- Recall Collinear / DGLAP:

$$\frac{d}{d\ln\mu} f_{j/P}(x;\mu) = 2\int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

- TMD Case:

$$\frac{\partial \ln \tilde{F}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T;\mu)$$

Large b_T:
Non-perturbative

Small b_T:
perturbative

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d\ln \tilde{F}(x, b_T; \mu, \zeta)}{d\ln\mu} = \gamma_F(g(\mu); \zeta/\mu^2)$$

(Collins Extension: (2011), Chapts. 10,13,14)

Solution: One TMD PDF

One physical scale for evolution

$$\mu \sim \sqrt{\zeta_1} \sim \sqrt{\zeta_2} \sim Q$$

$$\zeta_1 \zeta_2 \sim Q^4$$

Ex: Matching Prescription:

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_{\mathrm{max}}^2}}$$

$$\mu_b \equiv C_1/|\mathbf{b}_*(b_T)|$$

$$\tilde{F}_{f/P}(x, \mathbf{b}_\mathrm{T}; Q, Q^2) =$$

Collinear PDFs

$$\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times$$

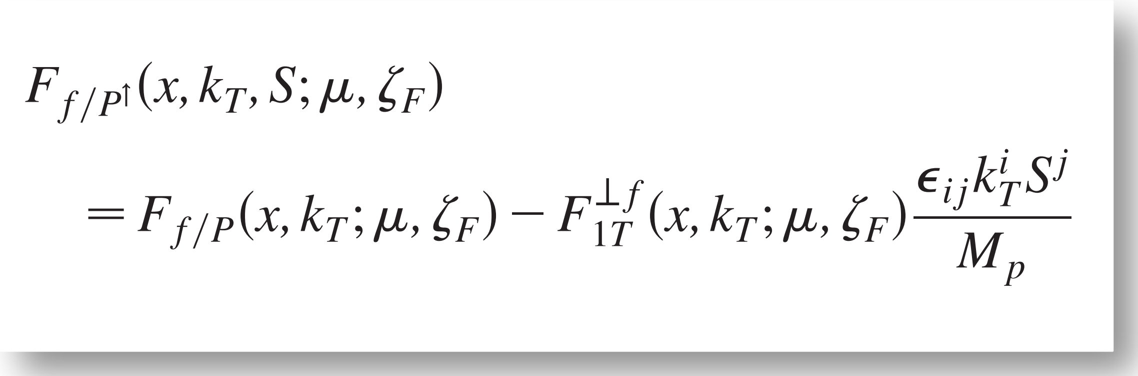
$$\times \exp\left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times$$

$$\times \exp\left\{ -\underline{g_{f/P}(x, b_T)} - \underline{g_K(b_T)} \ln \frac{Q}{Q_0} \right\}$$

Nonperturbative large b_T

Polarized TMD PDFs:

- Same definition, same evolution equations



$$F_{f/P^{\uparrow}}(x, k_T, S; \mu, \zeta_F)$$

$$= F_{f/P}(x, k_T; \mu, \zeta_F) - F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F)\frac{\epsilon_{ij} k_T^i S^j}{M_p}$$

Solution: One TMD PDF

One physical scale for evolution

$$\mu \sim \sqrt{\zeta_1} \sim \sqrt{\zeta_2} \sim Q$$

$$\zeta_1 \zeta_2 \sim Q^4$$

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Collinear PDFs

$$\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times$$

$$\times \exp\left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times$$

$$\times \exp\left\{ -\underline{g_{f/P}(x, b_T)} - \underline{g_K(b_T)} \ln \frac{Q}{Q_0} \right\}$$

Nonperturbative large b_T

Polarized TMD PDFs:

One physical scale for evolution

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$$\tilde{F}'_{1T,\,f/P}(x,\mathbf{b}_\mathrm{T};Q,Q^2) =$$

Qiu-Sterman Collinear Function

$$\sum_j \frac{M_p b_T}{2} \int_x^1 \frac{d\hat{x}_1}{\hat{x}_1} \frac{d\hat{x}_2}{\hat{x}_2} \tilde{C}^{\text{Sivers}}_{f/j}(\hat{x}_1,\hat{x}_2,b_*;\mu_b^2,\mu_b,g(\mu_b)) T_{\mathrm{F},\,j/P}(\hat{x}_1,\hat{x}_2,\mu_b) \times$$

$$\times \exp\left\{ \ln\frac{Q}{\mu_b}\tilde{K}(b_*;\mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(g(\mu'))\right]\right\} \times$$

$$\times \exp\left\{ \underline{-g^{\text{Sivers}}_{f/P}(x,b_T)} \underline{-g_K(b_T)} \ln\frac{Q}{Q_0} \right\}$$

Same as for unpolarized!

Nonperturbative large b_T behavior

(Aybat, Collins, Qiu, TCR (2012))

TMD Factorization

- Incorporate all processes.
  - SIDIS, DY, e^+e^-, different targets….
  - Unpolarized cross sections, spin asymmetries…

$$\begin{aligned}
d\sigma_{\mathrm{SIDIS}} &= \sum_f \mathcal{H}_{f,\mathrm{SIDIS}}(Q) \otimes F_{f/H_1}(x, k_{1T}, Q) \otimes D_{H_2/f}(z, k_{2T}, Q) &&+ Y_{\mathrm{SIDIS}} \\
d\sigma_{\mathrm{DY}} &= \sum_f \mathcal{H}_{f,\mathrm{DY}}(Q) \otimes F_{f/H_1}(x_1, k_{1T}, Q) \otimes F_{\bar{f}/H_2}(x_2, k_{2T}, Q) &&+ Y_{\mathrm{Drell-Yan}} \\
d\sigma_{\mathrm{e^+e^-}} &= \sum_f \mathcal{H}_{f,\mathrm{e^+e^-}}(Q) \otimes D_{H_1/\bar{f}}(z_1, k_{1T}, Q) \otimes D_{H_2/f}(z_2, k_{2T}, Q) &&+ Y_{\mathrm{e^+e^-}}
\end{aligned}$$

Phenomenology

Constraining Non-Perturbative Parts

- Incorporate all processes.
  - SIDIS, DY, e⁺e⁻, different targets….
  - Unpolarized cross sections, spin asymmetries…

$$
\begin{aligned}
d\sigma_{\rm SIDIS} &= \sum_f \mathcal{H}_{f,\rm SIDIS}(Q) \otimes F_{f/H_1}(x, k_{1T}, Q) \otimes D_{H_2/f}(z, k_{2T}, Q) &&+ Y_{\rm SIDIS} \\
d\sigma_{\rm DY} &= \sum_f \mathcal{H}_{f,\rm DY}(Q) \otimes F_{f/H_1}(x_1, k_{1T}, Q) \otimes F_{\bar{f}/H_2}(x_2, k_{2T}, Q) &&+ Y_{\rm Drell-Yan} \\
d\sigma_{\rm e^+e^-} &= \sum_f \mathcal{H}_{f,\rm e^+e^-}(Q) \otimes D_{H_1/\bar{f}}(z_1, k_{1T}, Q) \otimes D_{H_2/f}(z_2, k_{2T}, Q) &&+ Y_{\rm e^+e^-}
\end{aligned}
$$

Solution: One TMD PDF

One physical scale for evolution

$$\mu \sim \sqrt{\zeta_1} \sim \sqrt{\zeta_2} \sim Q$$

$$\zeta_1 \zeta_2 \sim Q^4$$

Ex: Matching Prescription:

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_\mathrm{max}^2}}$$

$$\mu_b \equiv C_1/|\mathbf{b}_*(b_T)|$$

$$\tilde{F}_{f/P}(x, \mathbf{b}_\mathrm{T}; Q, Q^2) =$$

Collinear PDFs

$$\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times$$

$$\times \exp\left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times$$

$$\times \exp\left\{ -\underline{g_{f/P}(x, b_T)} - \underline{g_K(b_T)} \ln \frac{Q}{Q_0} \right\}$$

Depends on Hadron

Nonperturbative large b_T

TMD Factorization

- Incorporate all processes.
  - SIDIS, DY, e^+e^-, different targets….
  - Unpolarized cross sections, spin asymmetries…

$$\begin{aligned}
d\sigma_{\mathrm{SIDIS}} &= \sum_f \mathcal{H}_{f,\mathrm{SIDIS}}(Q) \otimes F_{f/H_1}(x, k_{1T}, Q) \otimes D_{H_2/f}(z, k_{2T}, Q) && \cancel{+\, Y_{\mathrm{SIDIS}}} \\
d\sigma_{\mathrm{DY}} &= \sum_f \mathcal{H}_{f,\mathrm{DY}}(Q) \otimes F_{f/H_1}(x_1, k_{1T}, Q) \otimes F_{\bar{f}/H_2}(x_2, k_{2T}, Q) && \cancel{+\, Y_{\mathrm{Drell-Yan}}} \\
d\sigma_{\mathrm{e^+e^-}} &= \sum_f \mathcal{H}_{f,\mathrm{e^+e^-}}(Q) \otimes D_{H_1/\bar{f}}(z_1, k_{1T}, Q) \otimes D_{H_2/f}(z_2, k_{2T}, Q) && \cancel{+\, Y_{\mathrm{e^+e^-}}}
\end{aligned}$$

Type I - like

Approximation

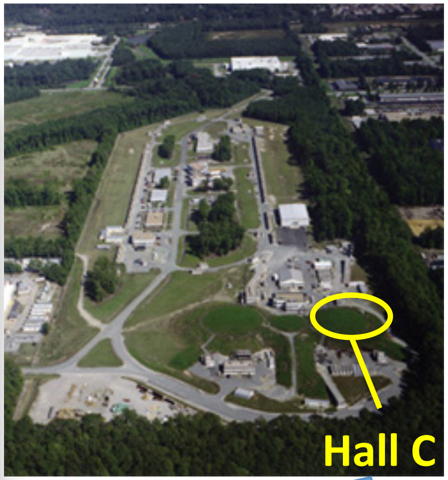
Extractions of TMD PDFs

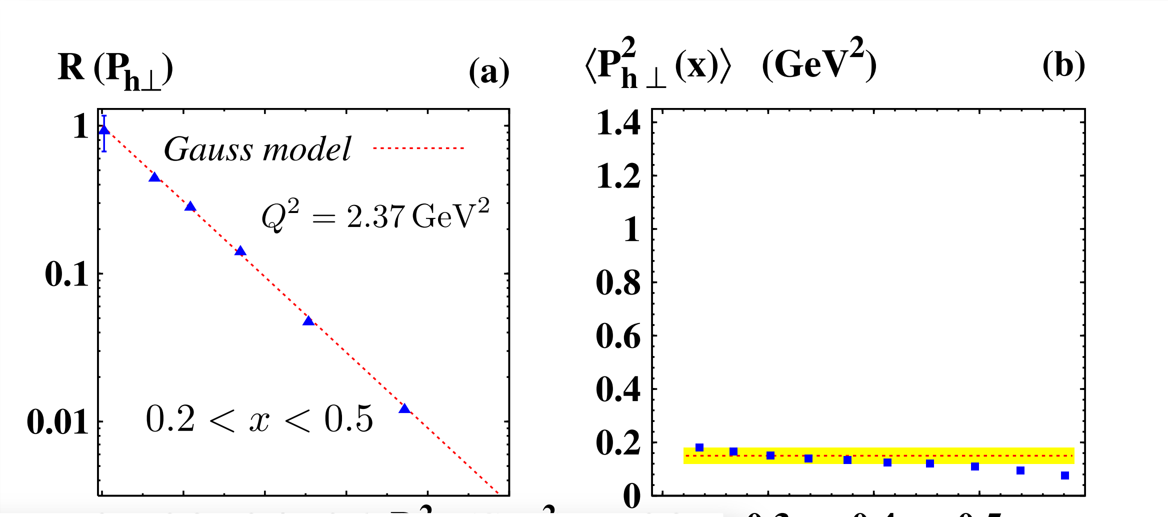
- Fixed Scale Fits.

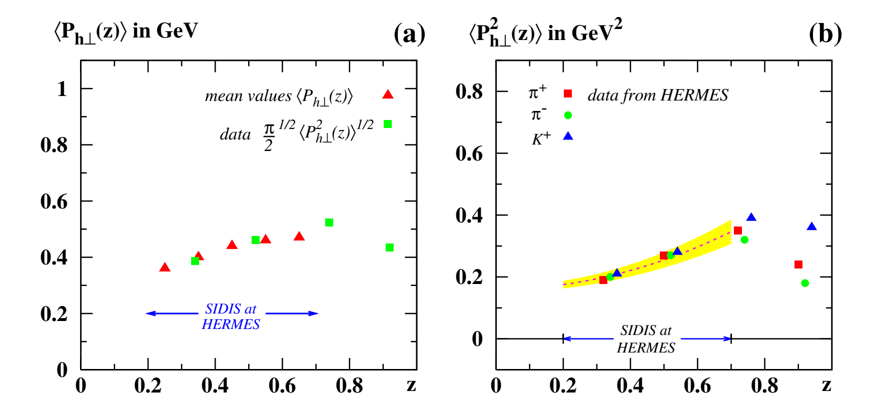
(Ex: Schweitzer, Teckentrup, Metz (2010))

Type I - like

JLab







Good evidence for
Gaussian shape at small Q, P_T

$$\langle x\rangle = 0.09 \quad Q^2 = 2.4\,\mathrm{GeV}^2$$

HERMES $\langle p_T^2\rangle = (0.38 \pm 0.06)\ \mathrm{GeV}^2$

$$\langle K_T^2\rangle = (0.16 \pm 0.01)\ \mathrm{GeV}^2$$

Extractions of TMD PDFs

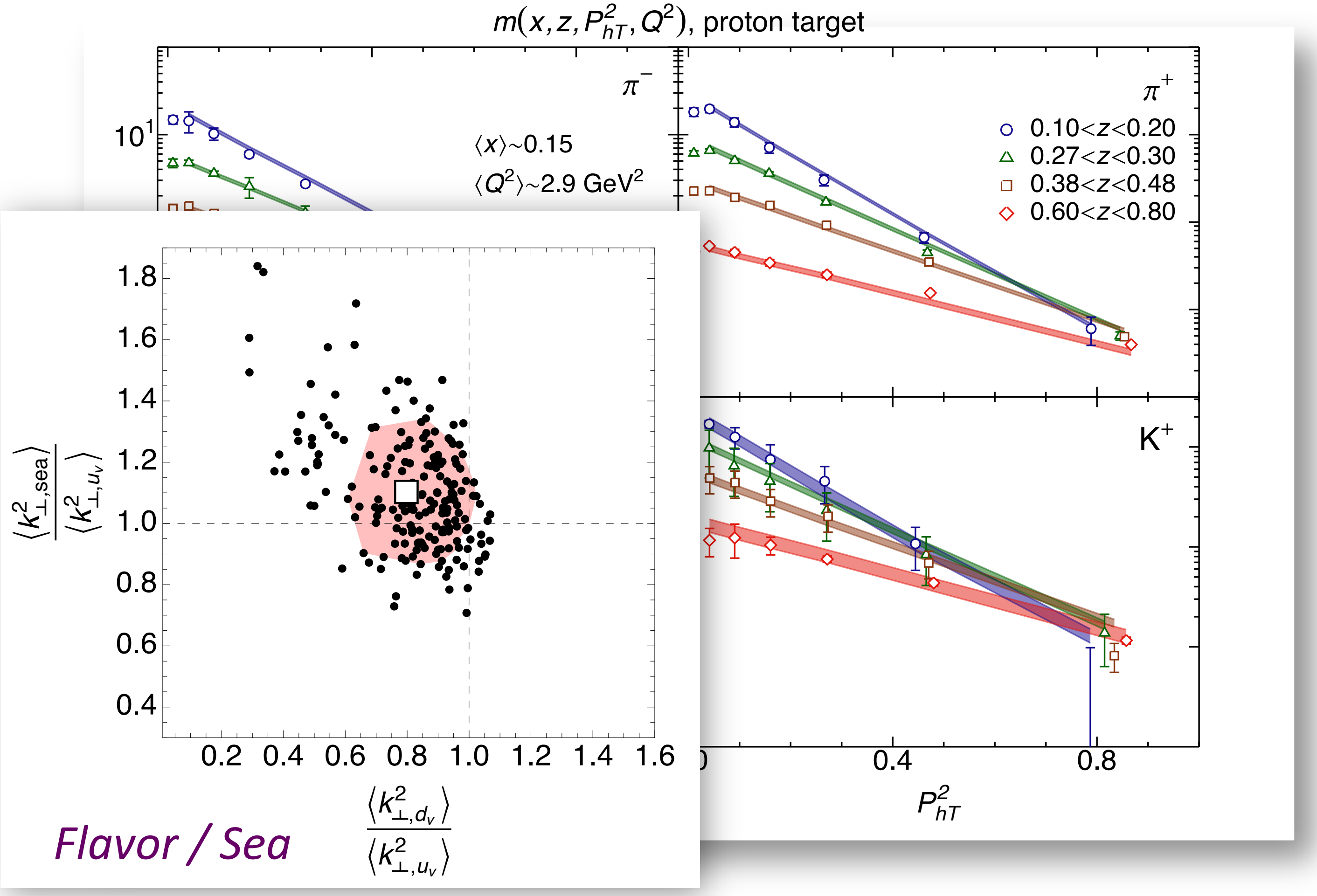
- Fixed Scale Fits.

(Signori, Bacchetta, Radici, Schnell (2013))

Type I - like

Gaussian shape

See also Bacchetta talk



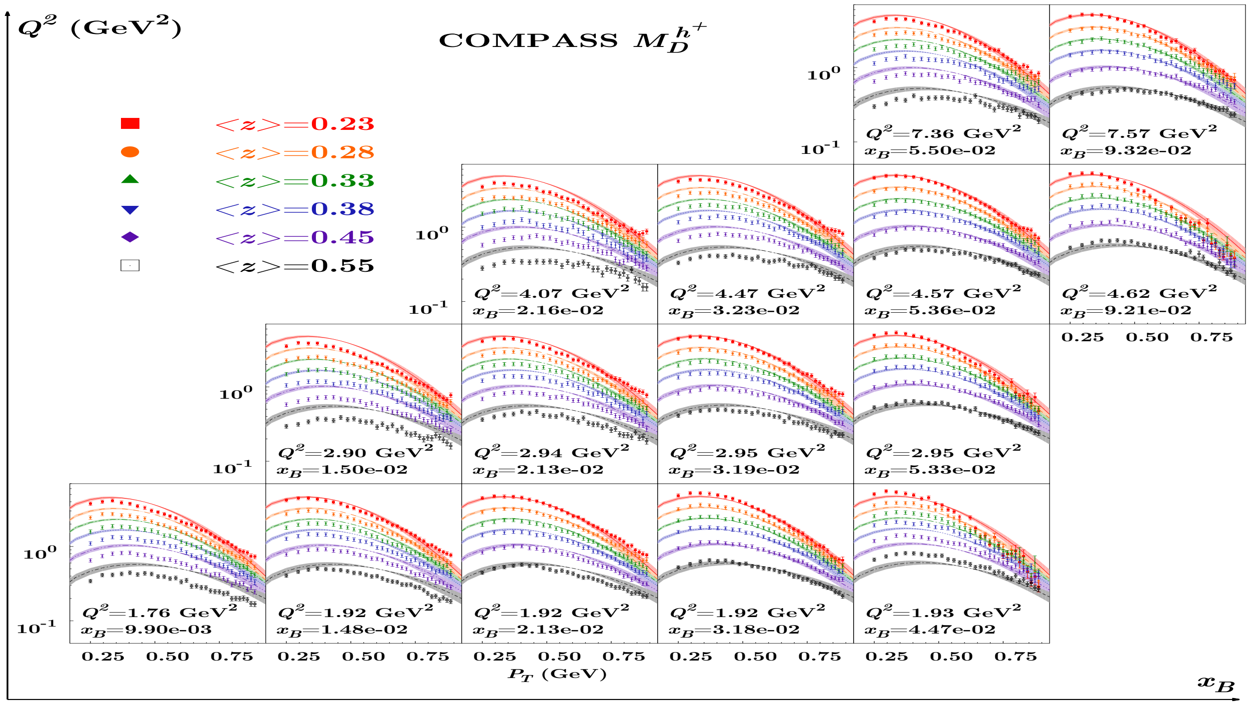
Flavor / Sea

Extractions of TMD PDFs

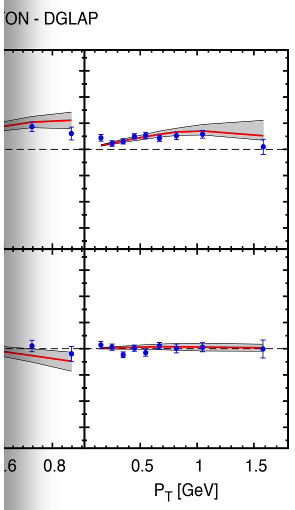
- Fixed Scale Fits.

(Torino Group 1999…)

Type I - like



(Anselmino, Boglione, Gonzalez, Melis, Prokudin (2013))



(Anselmino, Boglione, D'Alesio, Melis, Murgia, Prokudin (2013))

Recall: One TMD PDF

One physical scale for evolution

$$\mu \sim \sqrt{\zeta_1} \sim \sqrt{\zeta_2} \sim Q$$

$$\zeta_1 \zeta_2 \sim Q^4$$

Ex: Matching Prescription:

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

$$\mu_b \equiv C_1/|\mathbf{b}_*(b_T)|$$

<u>Collinear PDFs</u>

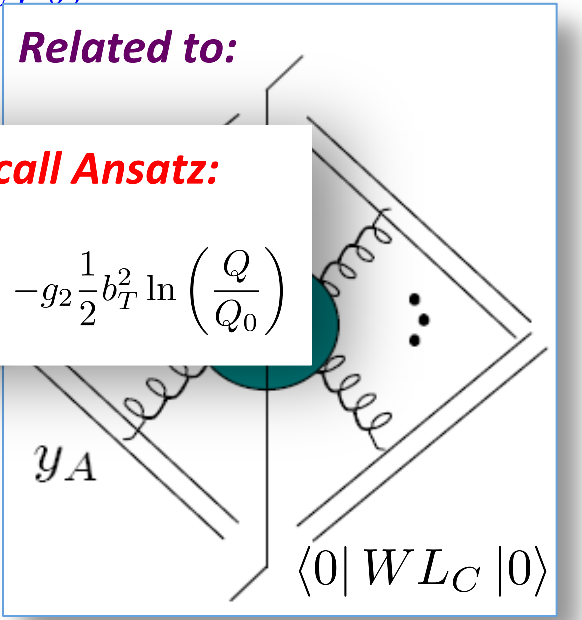
$$\tilde{F}_{f/P}(x, \mathbf{b}_\mathrm{T}; Q, Q^2) =$$

$$\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times$$

$$\times \exp\left\{ \ln\frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int\right.$$

$$\times \exp\left\{ \underline{g_{f/P}(x, b_T)} + \underline{g_K(b_T)} \ln\frac{Q}{Q_0} \right\}$$

Related to:

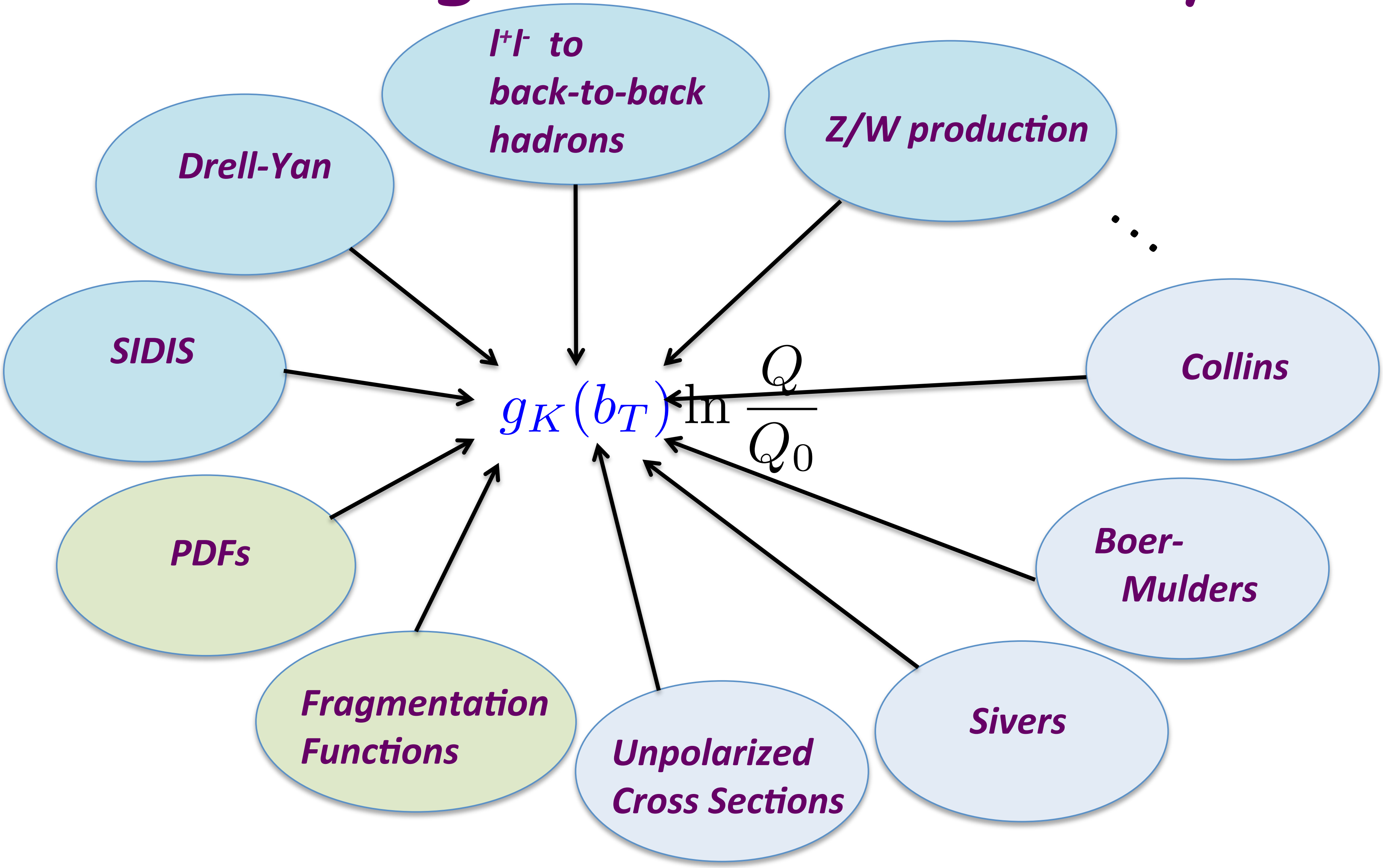


Recall Ansatz:

$$g_K(b_T) \ln\left(\frac{Q}{Q_0}\right) = -g_2 \frac{1}{2} b_T^2 \ln\left(\frac{Q}{Q_0}\right)$$

Nonperturbative Soft Evolution

Strong Soft Universality



l^+l^- to back-to-back hadrons

Drell-Yan

Z/W production

…

SIDIS

$$g_K(b_T)\ln\frac{Q}{Q_0}$$

Collins

PDFs

Boer-Mulders

Fragmentation Functions

Unpolarized Cross Sections

Sivers

Extractions of TMD PDFs

- Ex: ResBos: CSS formalism

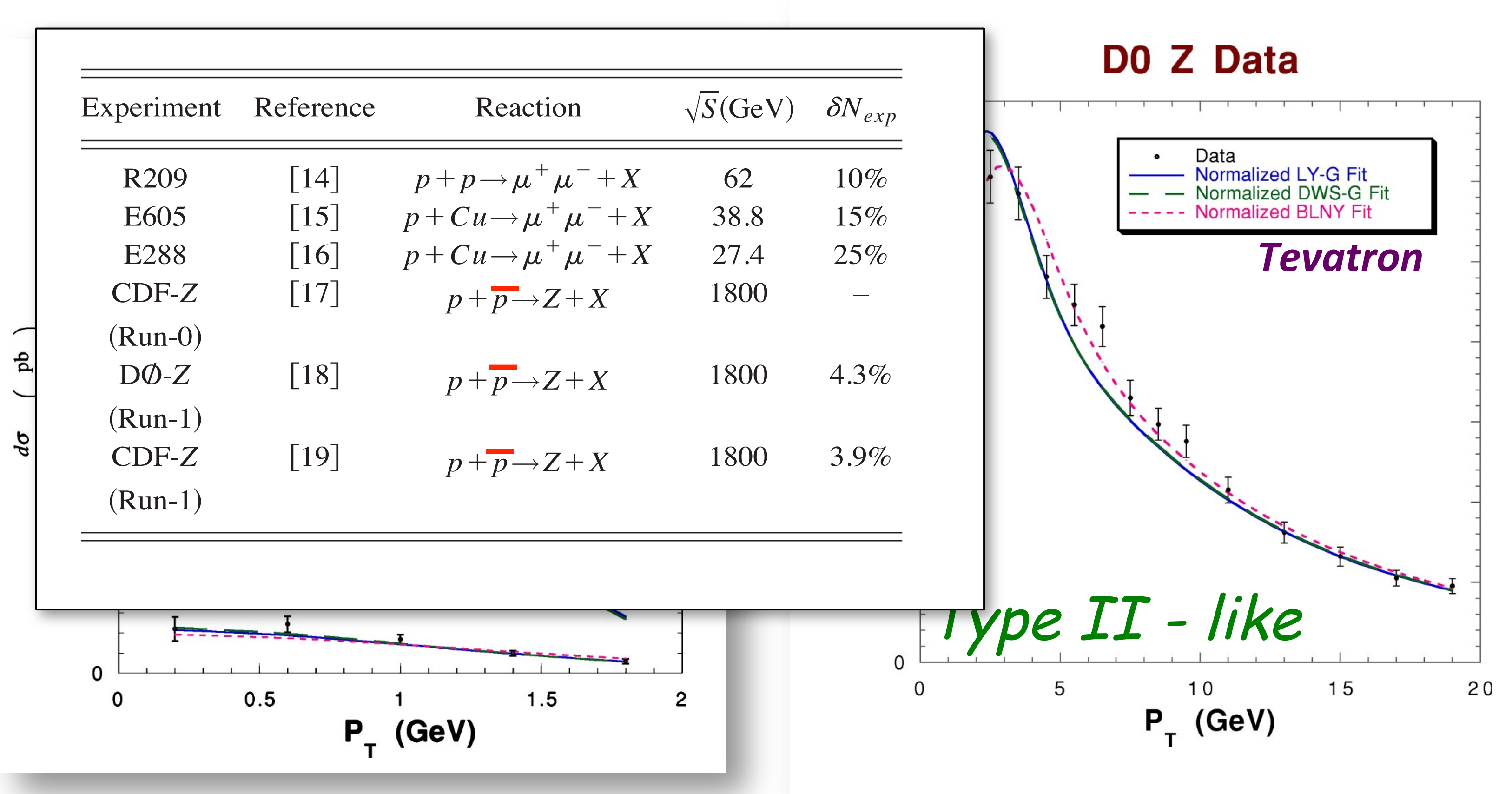
http://hep.pa.msu.edu/resum/

(Landry, Brock, Nadolsky, Yuan, (2003))

$$g_K(b_T)\ln\left(\frac{Q}{Q_0}\right) = -g_2\frac{1}{2}b_T^2\ln\left(\frac{Q}{Q_0}\right)$$

Gaussian ansatz

$g_2 = .68\ \mathrm{GeV}^2$ $\quad b_{\max} = .5\ \mathrm{GeV}^{-1}$



Experiment	Reference	Reaction	$\sqrt{S}$(GeV)	δN_{exp}
R209	[14]	$p+p\to\mu^+\mu^-+X$	62	10%
E605	[15]	$p+Cu\to\mu^+\mu^-+X$	38.8	15%
E288	[16]	$p+Cu\to\mu^+\mu^-+X$	27.4	25%
CDF-Z (Run-0)	[17]	$p+\bar{p}\to Z+X$	1800	–
DØ-Z (Run-1)	[18]	$p+\bar{p}\to Z+X$	1800	4.3%
CDF-Z (Run-1)	[19]	$p+\bar{p}\to Z+X$	1800	3.9%

Tevatron

Type II - like

TMD Factorization

- Incorporate all processes.
  - SIDIS, DY, e$^+$e$^-$, different targets….
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$$
\begin{aligned}
d\sigma_{\mathrm{SIDIS}} &= \sum_f \mathcal{H}_{f,\mathrm{SIDIS}}(Q) \otimes F_{\cancel{f/H_1}}(x, k_{1T}, Q) \otimes D_{\cancel{H_2/f}}(z, k_{2T}, Q) \quad + Y_{\mathrm{SIDIS}} \\
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\end{aligned}
$$

Type II - like

(Approximations)

New Stage in Fitting

Strategy: “Apples-to-apples“

- Incorporate all processes.
  - SIDIS, DY, e^+e^-, different targets….
  - Unpolarized cross sections, spin asymmetries…

$$
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\end{aligned}
$$

Strategy: “Apples-to-apples“

- Evolution: Fix x,z, process/hadron species. *(As much as possible)*
  - Vary Q to determine $g_K(b_T;b_{max})$

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- Determine x, z dependence

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- Dependence on hadron species, TMD PDF vs. TMD fragmentation function, flavor

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- Hadron structure

Example:

- Sea quark TMDs vs. valence quark TMDs:
  - $\bar{p}A \rightarrow l^+ + l^- + X$
    - $\left. F_{\bar{q}/\bar{p}} \otimes F_{q/A} \right\} \longrightarrow$ **Valence Quarks**
  - $pA \rightarrow l^+ + l^- + X$
    - $F_{q/p} \otimes \boxed{F_{\bar{q}/A}} \longrightarrow$ **Sea Quarks**

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- Sea quark TMDs vs. valence quark TMDs:
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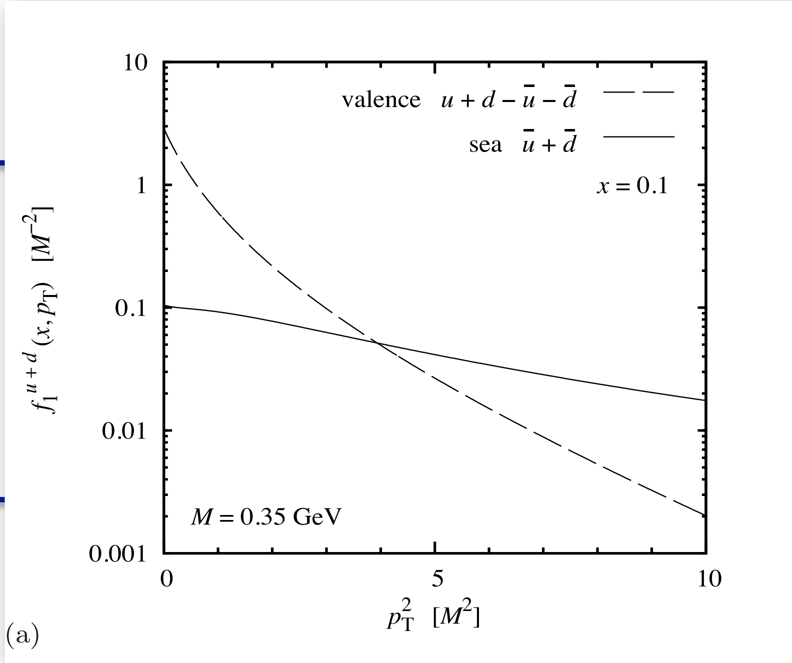
    - $F_{\bar{q}/\bar{p}} \otimes F_{q/A}$
  - $pA \to l^{+} + l^{-} + X$
    - $F_{q/p} \otimes \boxed{F_{\bar{q}/A}}$

(Schweitzer, Strikman, Weiss, (2013))



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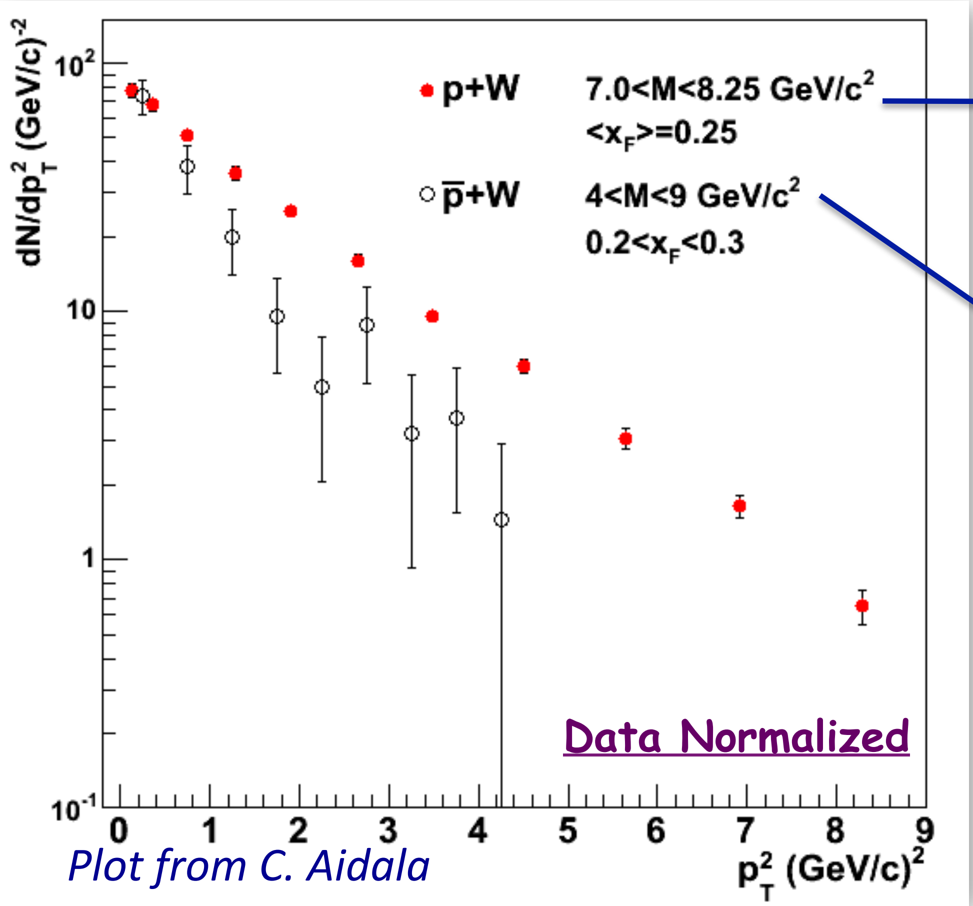

Data Normalized

Plot from C. Aidala

Fermilab, Oliver et al.,
AIP Conf.Proc. 45 (1978) 93-102
400 GeV proton

Anassontzis et al.,
Phys.Rev. D38 (1988) 1377
125 GeV antiprotons

Is there a difference?

Need better control of evolution for moderate Q

Solution: One TMD PDF

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<u>Collinear PDFs</u>

$$\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times$$

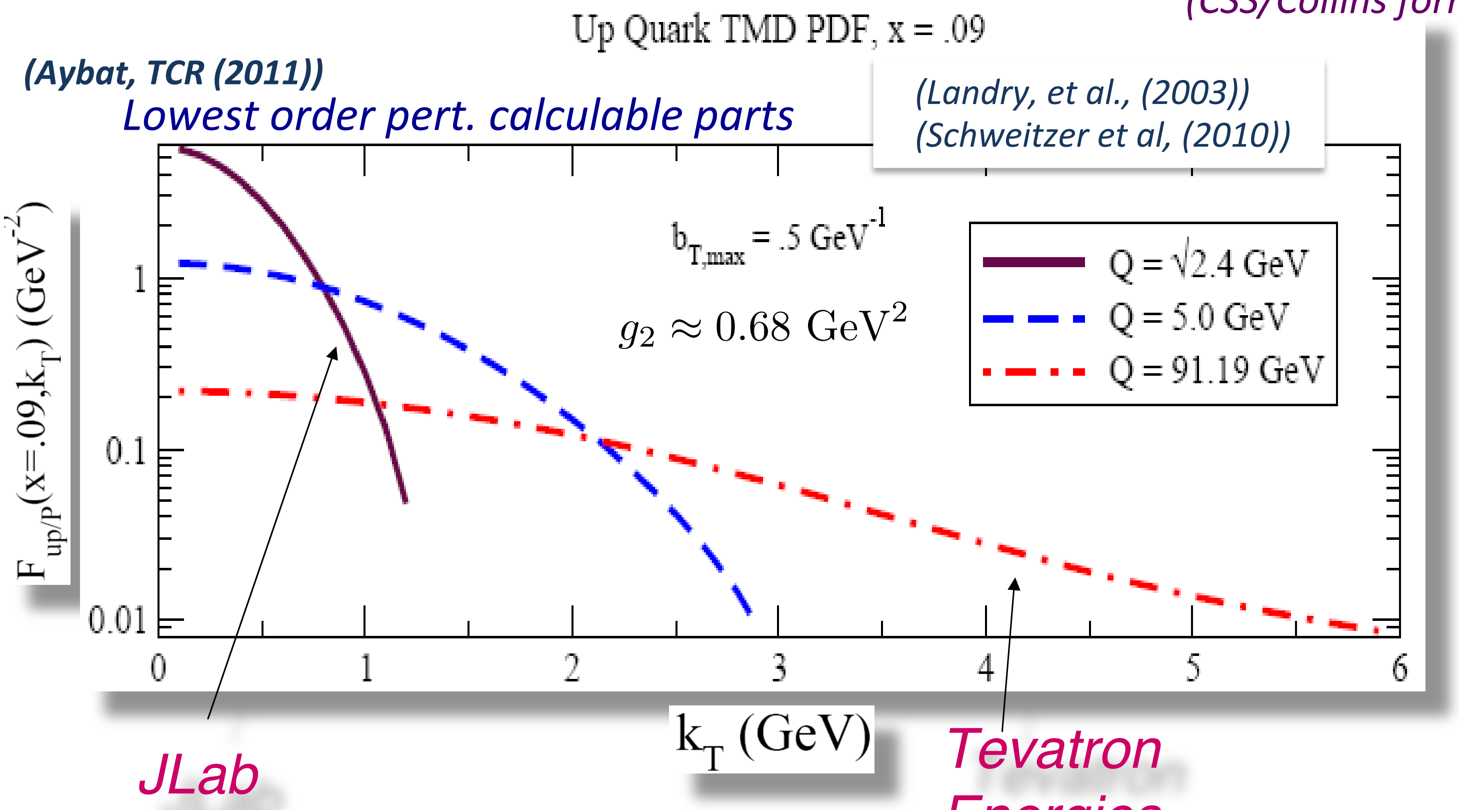
$$\times \exp\left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times$$

$$\times \exp\left\{ -\underline{g_{f/P}(x, b_T)} - \underline{g_K(b_T)} \ln \frac{Q}{Q_0} \right\}$$

Nonperturbative large b_T

Evolved TMD PDFs: constructed from old fits

(CSS/Collins formalism.)



(Aybat, TCR (2011))

Lowest order pert. calculable parts

(Landry, et al., (2003))
(Schweitzer et al, (2010))

JLab Energies

Tevatron Energies

https://projects.hepforge.org/tmd/

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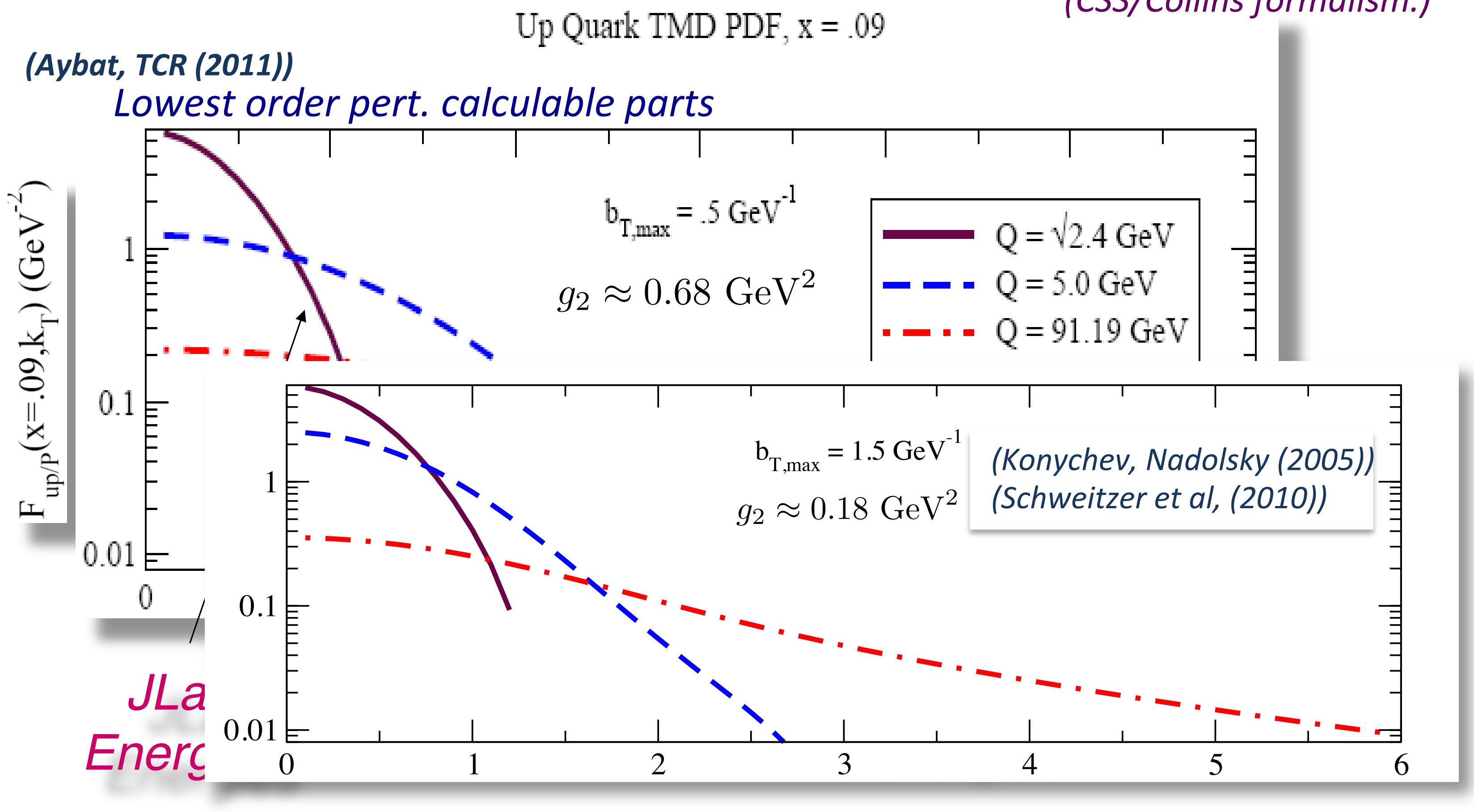

(Aybat, TCR (2011))

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(Konychev, Nadolsky (2005))
(Schweitzer et al, (2010))

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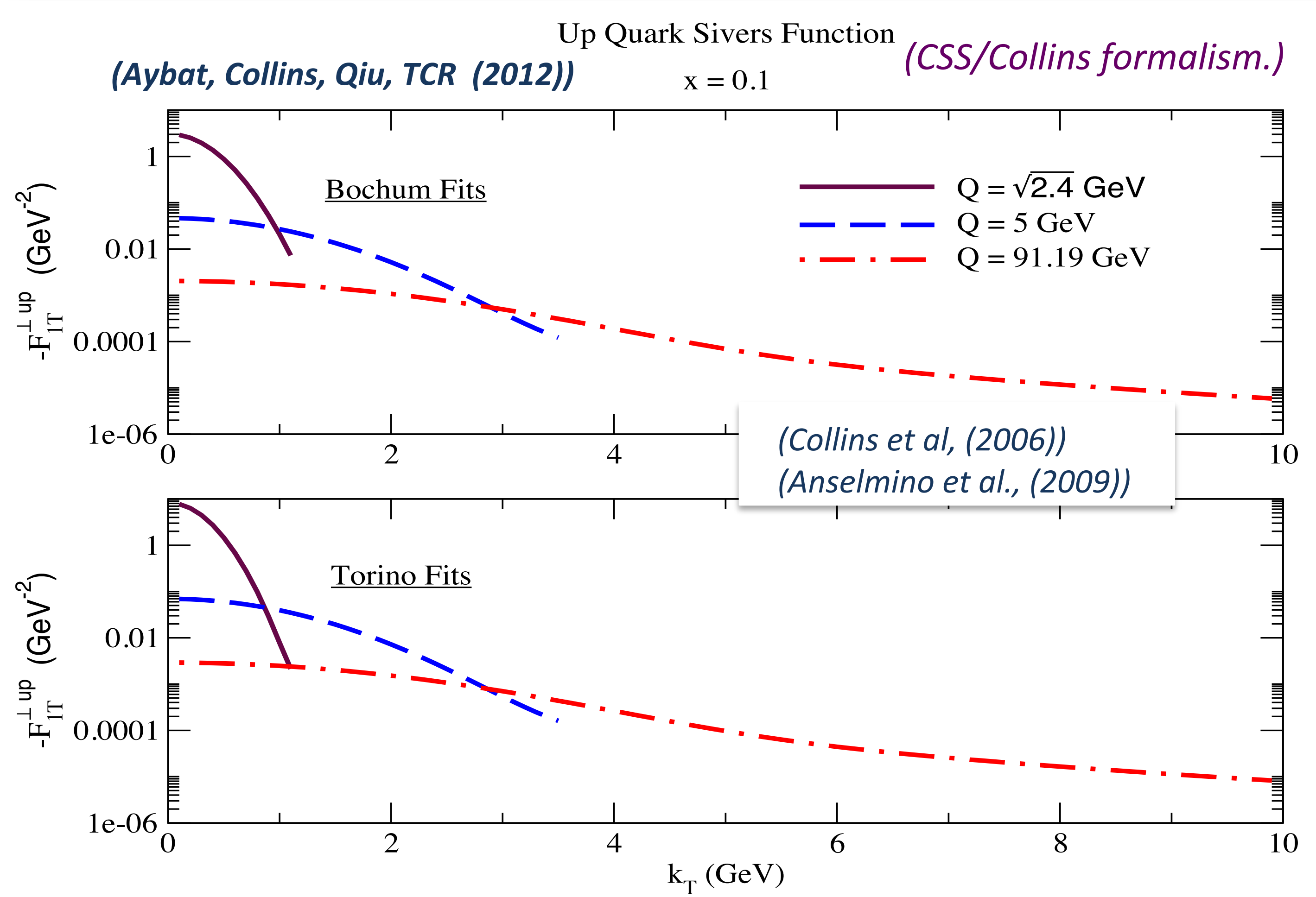


(Aybat, Collins, Qiu, TCR (2012))

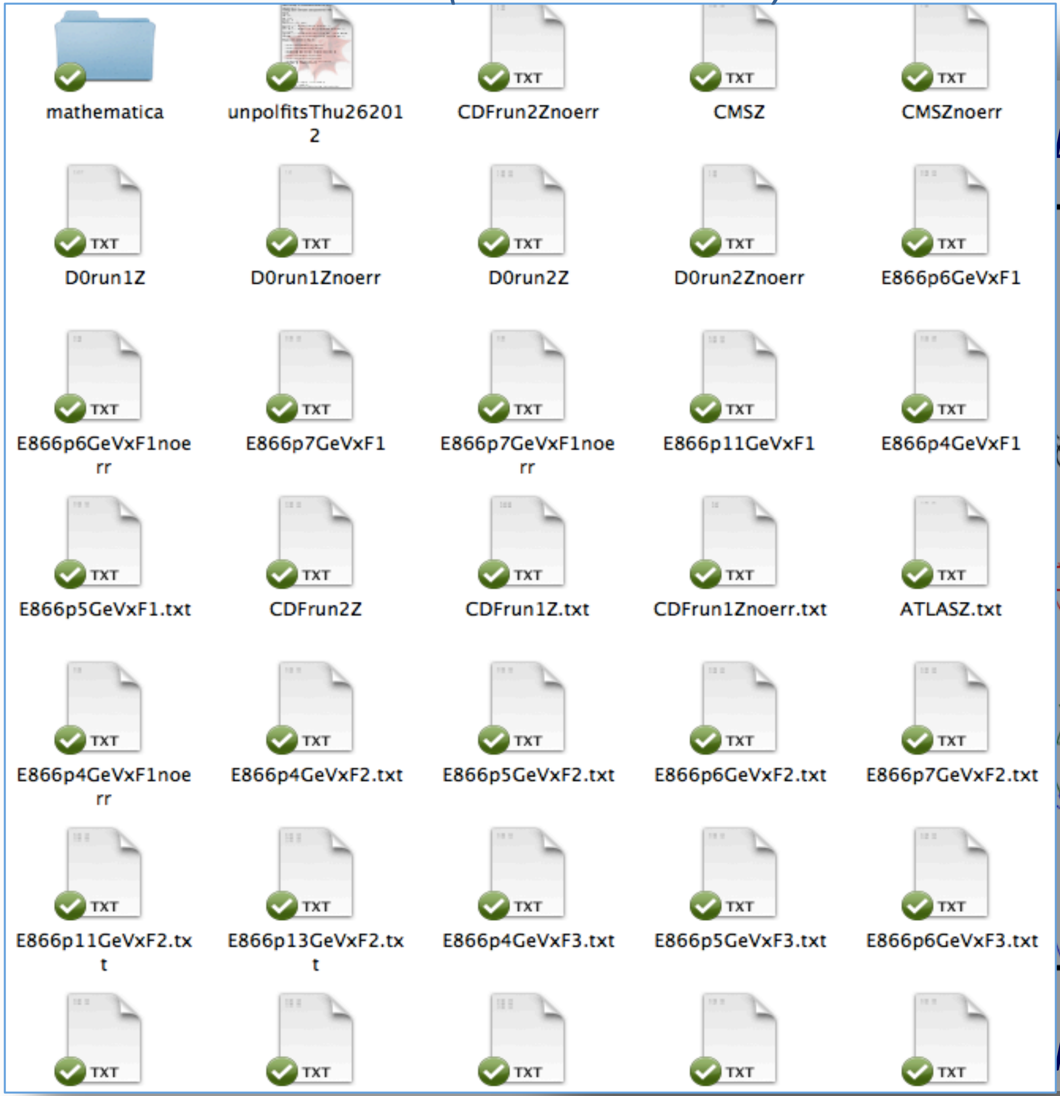
(CSS/Collins formalism.)

(Collins et al, (2006))
(Anselmino et al., (2009))

Sign flip for Drell-Yan!

Unpolarized Fitting

(With C. Aidala…)



mathematica
unpolfitsThu262012
CDFrun2Znoerr
CMSZ
CMSZnoerr
D0run1Z
D0run1Znoerr
D0run2Z
D0run2Znoerr
E866p6GeVxF1
E866p6GeVxF1noerr
E866p7GeVxF1
E866p7GeVxF1noerr
E866p11GeVxF1
E866p4GeVxF1
E866p5GeVxF1.txt
CDFrun2Z
CDFrun1Z.txt
CDFrun1Znoerr.txt
ATLASZ.txt
E866p4GeVxF1noerr
E866p4GeVxF2.txt
E866p5GeVxF2.txt
E866p6GeVxF2.txt
E866p7GeVxF2.txt
E866p11GeVxF2.txt
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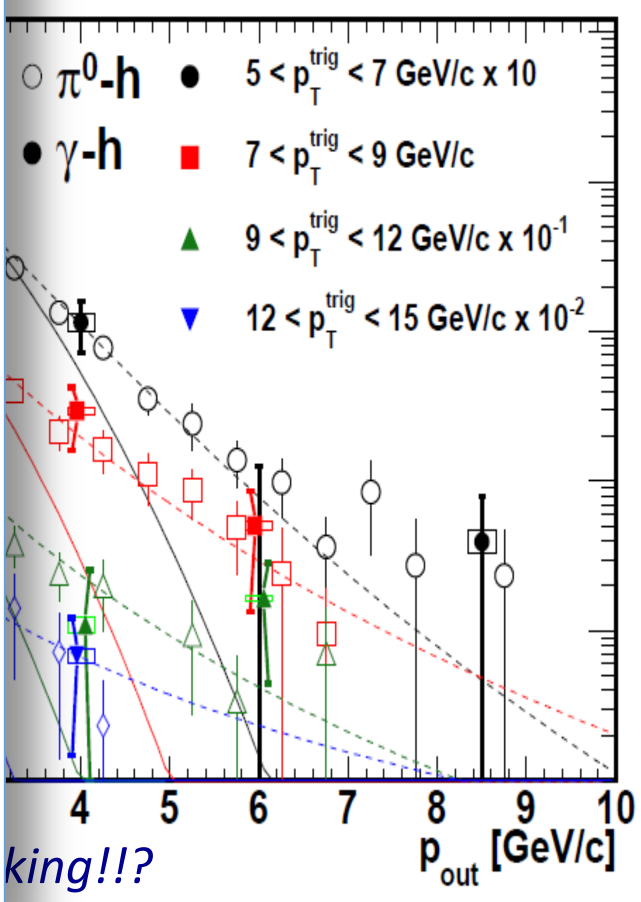
E866p5GeVxF3.txt
E866p6GeVxF3.txt

D82, 072001 (2010)



king!!?

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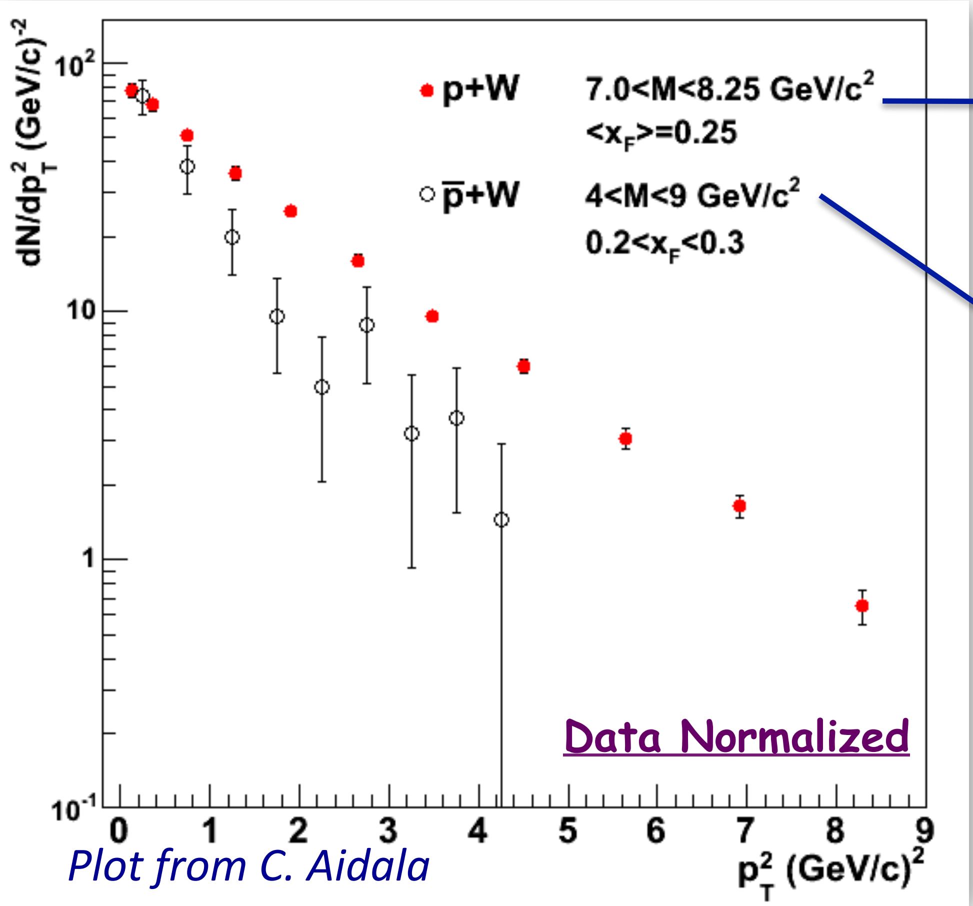

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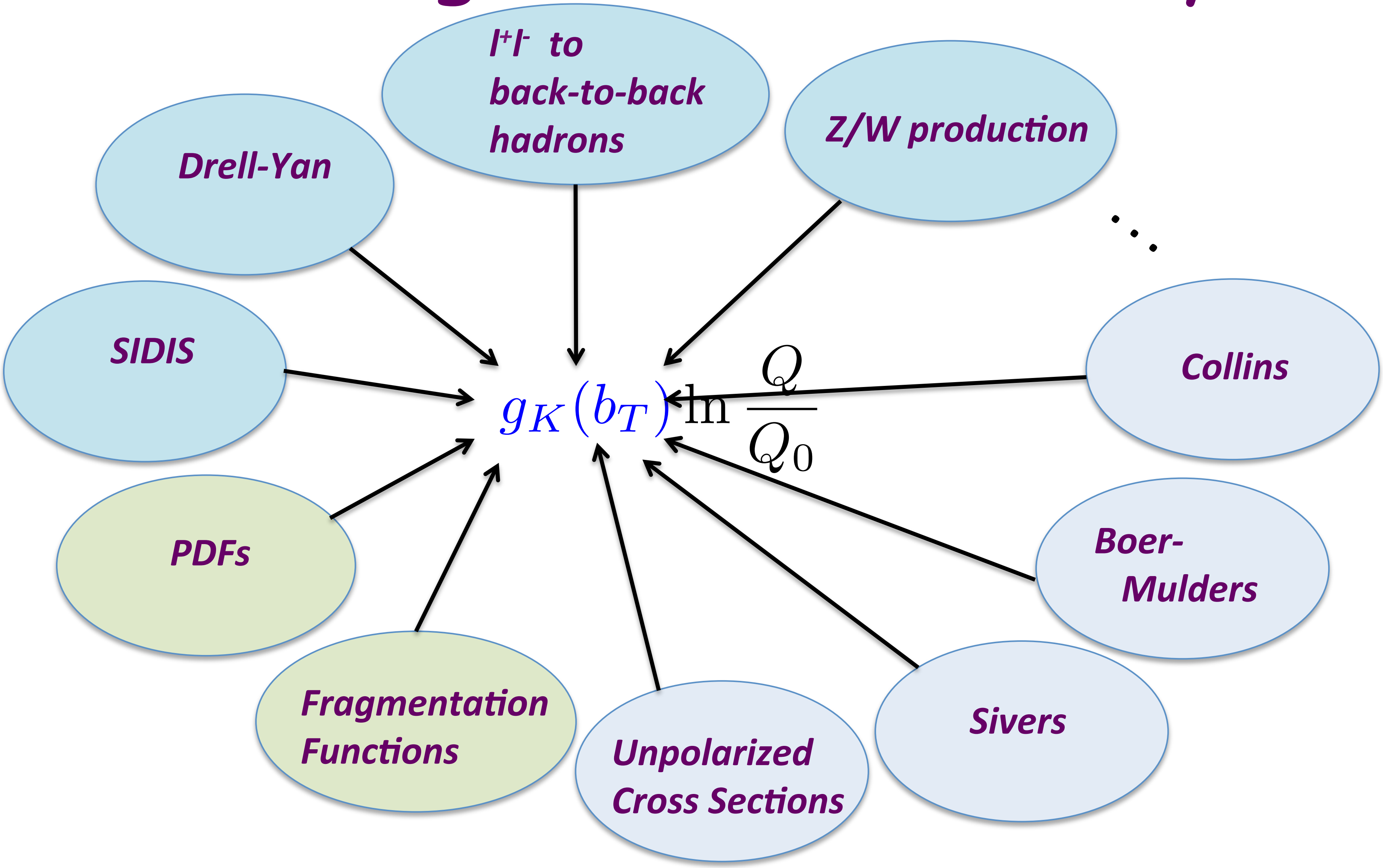

$$g_K(b_T)\ln\frac{Q}{Q_0}$$

Drell-Yan

l^+l^- to back-to-back hadrons

Z/W production

…

SIDIS

Collins

PDFs

Boer-Mulders

Fragmentation Functions

Unpolarized Cross Sections

Sivers

Non-Perturbative Evolution

- High Q fits extrapolated to low Q (≈ 1 GeV) gives extremely rapid evolution.
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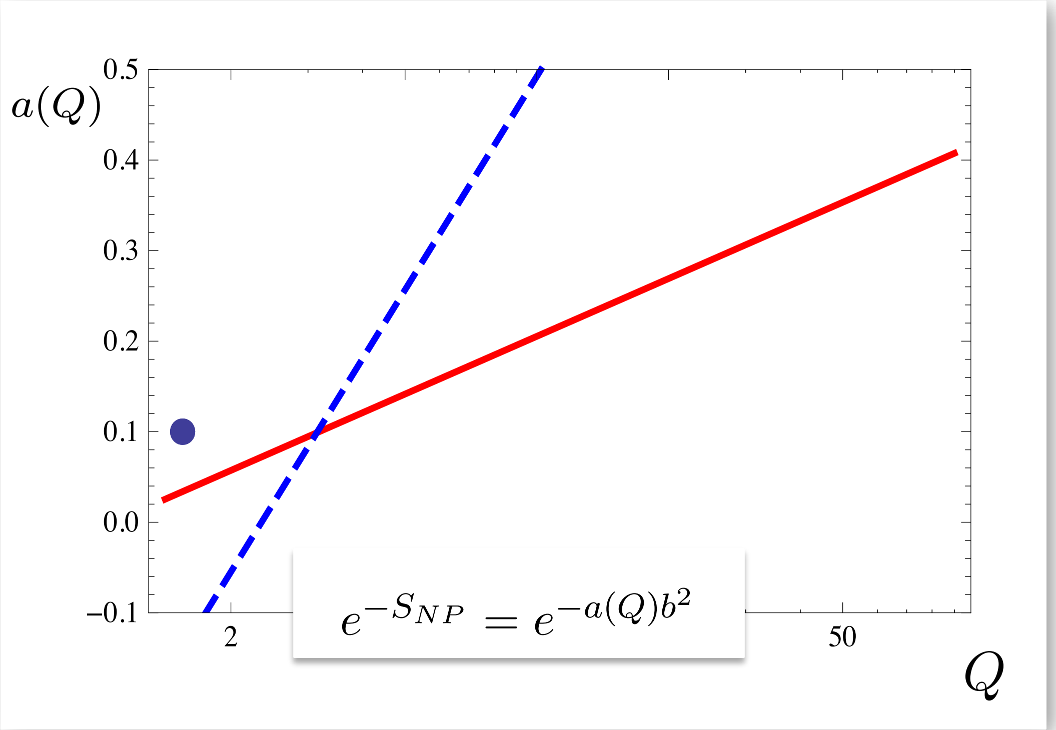
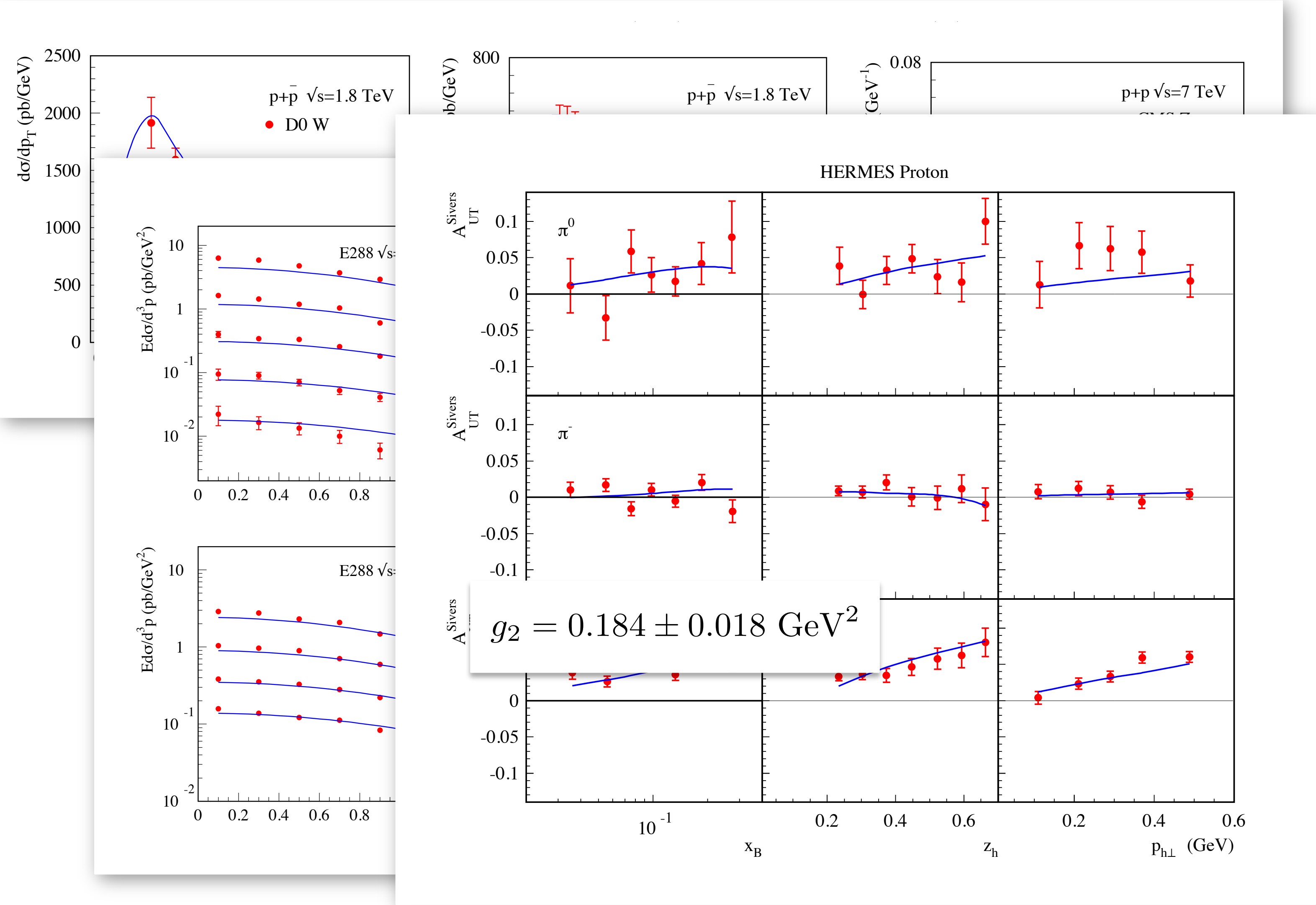


$$e^{-S_{NP}} = e^{-a(Q)b^2}$$

Fits in TMD formalism

(Echevarria, Idilbi, Kang, Vitev (2014))



$$g_2 = 0.184 \pm 0.018\ \mathrm{GeV}^2$$

Extractions of TMD PDFs

- Ex: ResBos: CSS formalism

http://hep.pa.msu.edu/resum/

(Landry, Brock, Nadolsky, Yuan, (2003))

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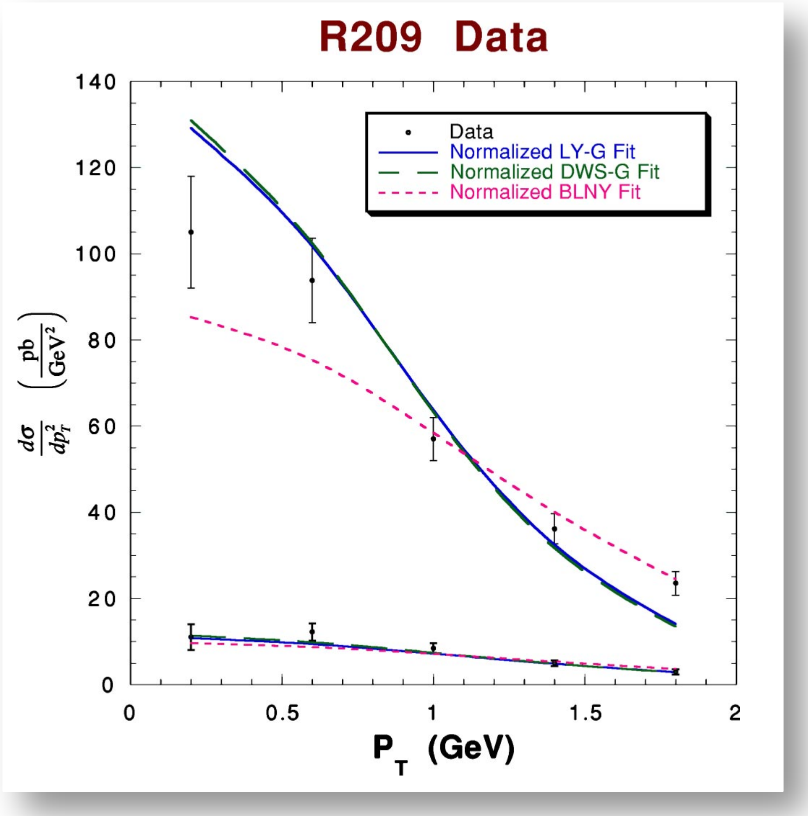
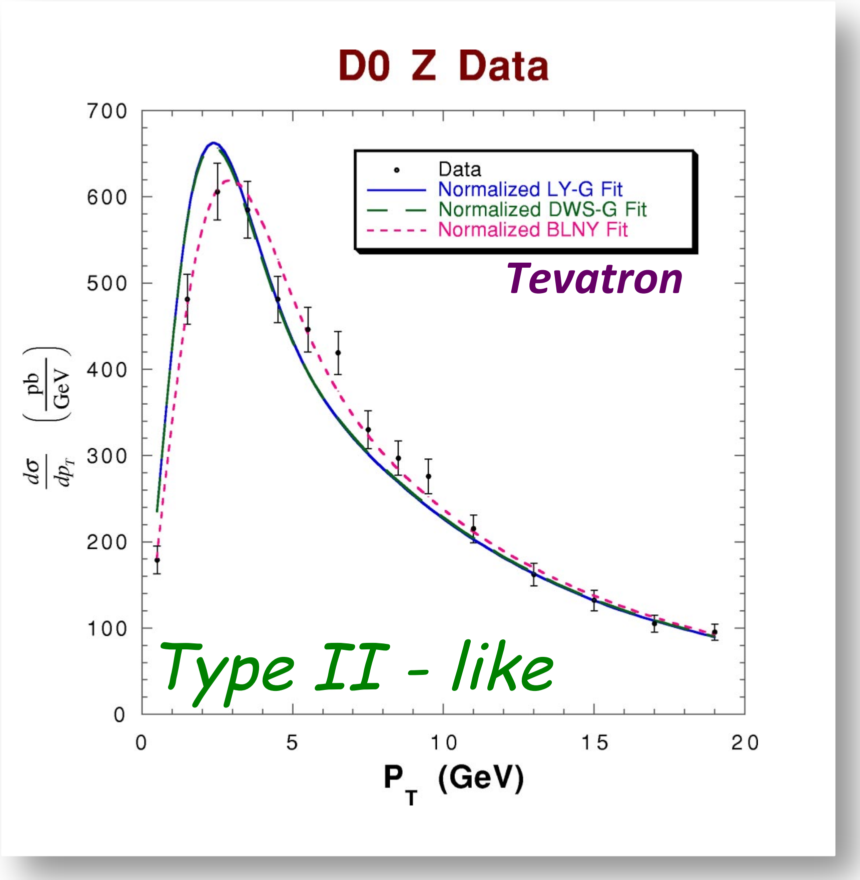
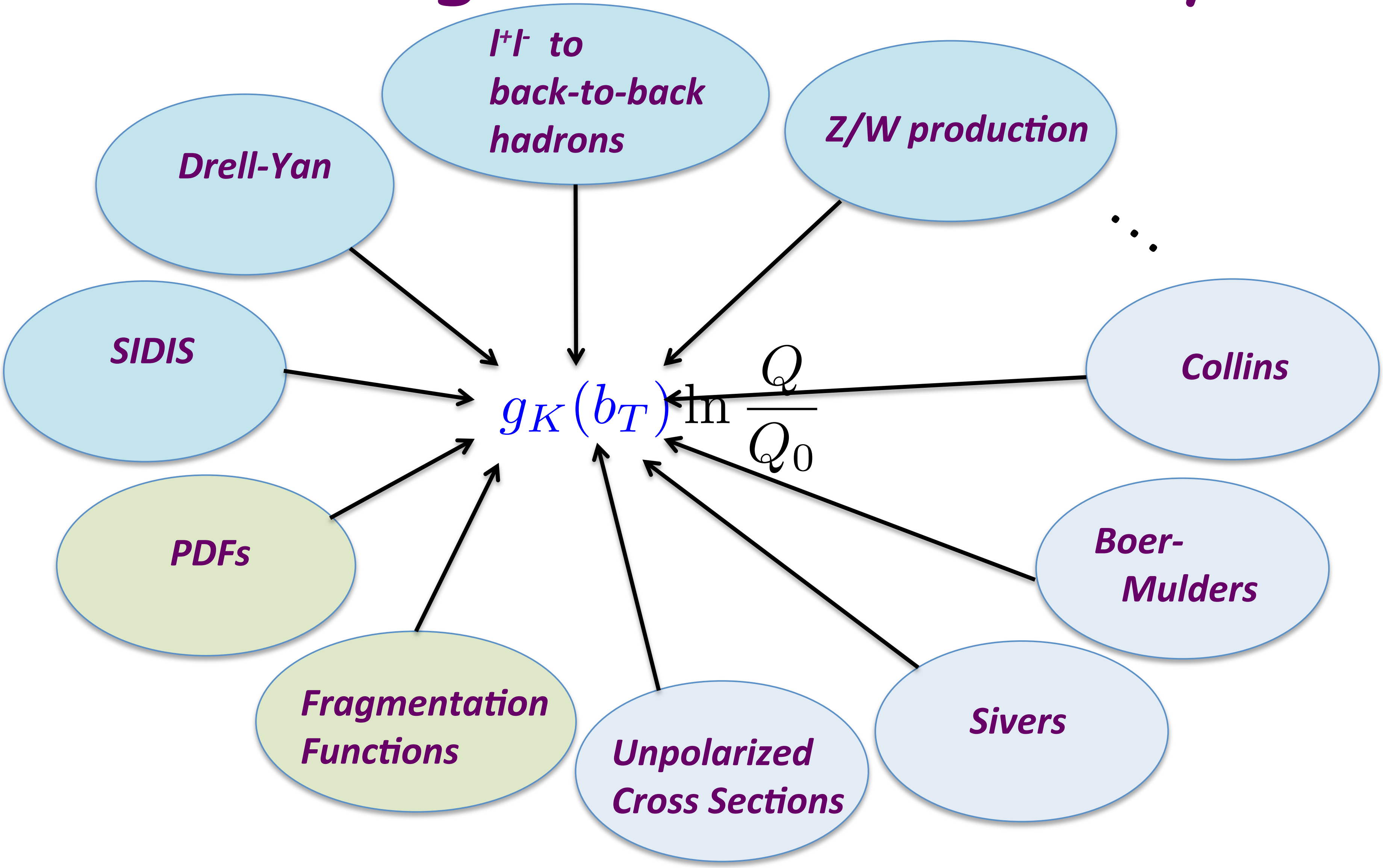




Tevatron

Type II - like

Strong Soft Universality



l^+l^- to back-to-back hadrons

Drell-Yan

Z/W production

. . .

SIDIS

$$g_K(b_T) \ln \frac{Q}{Q_0}$$

Collins

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    (Sun,Yuan (2013))

- Importance of non-perturbative *(Type II)* TM dependence?
  - Contribution should vanish at Q = Infinity *(Parisi, Petronzio (1979))*

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  - Global fits find small but important important non-perturbative evolution of k_T-dependence at Q ≈ 90 GeV.

    (Recent: (Guzzi, Nadolksy, Wang (2013))

Non-Perturbative Evolution

- High Q fits extrapolated to low Q (≈ 1 GeV) gives extremely rapid evolution.
  - Too Rapid!

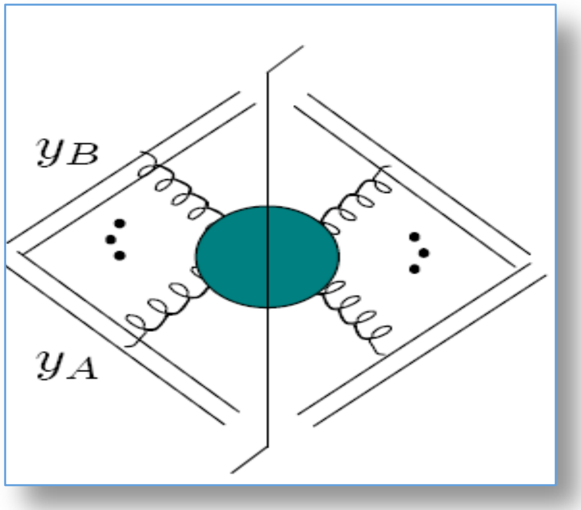
    (Sun,Yuan (2013))
- Importance of non-perturbative *(Type II)* TM dependence?
  - Contribution should vanish at Q = Infinity *(Parisi, Petronzio (1979))*
  - *Collins, Soper, Sterman (1985)*: non-perturbative k_T-dependence becomes negligible at approximately Q ≈ 10^8 GeV.
  - Global fits find small but important important non-perturbative evolution of k_T-dependence at Q ≈ 90 GeV.

    (Recent: (Guzzi, Nadolksy, Wang (2013))
  - Only perturbative evolution down to Q ≈ 1 GeV ??

    (Sun,Yuan (2013)) (Echevarria, Idilbi, Schafer, Scimemi (2012))

Non-Perturbative Evolution

- Lattice



$$g_K(b_T; b_{\max})$$

- Renormalons / Power corrections
  - Korchemsky, Sterman (1995), Tafat (2001) $g_2 \approx 0.16 \text{ GeV}^2$
  - Laenen, Sterman, Vogelsang (2000,2001)

$$g_K(b_T; b_{\max}) \propto b_T^2 \qquad \lim b_T \to 0$$

TMD-Evolution

- Recall Collinear / DGLAP:

$$\frac{d}{d\ln\mu} f_{j/P}(x;\mu) = 2\int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

- TMD Case:

$$\frac{\partial \ln \tilde{F}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T;\mu)$$

Large b_T:
Non-perturbative

Small b_T:
perturbative

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d\ln \tilde{F}(x, b_T; \mu, \zeta)}{d\ln\mu} = \gamma_F(g(\mu);\zeta/\mu^2)$$

(Collins Extension: (2011), Chapts. 10,13,14)

Test non-perturbative evolution in unpolarized SIDIS

Recent: arXiv:1401.2654 C. Aidala, B. Field, and L. Gamberg, TCR

***COMPASS,** C. Adolph et al., arXiv:1305.7317*

"Apples-to-Apples"

$$Q = 1 \sim 2 \ \ \mathrm{GeV}$$

Approx. fixed
x,z bins

Fit:

$$\frac{d\sigma}{dP_T^2} \propto \exp\left\{-\frac{P_T^2}{\langle P_T^2\rangle}\right\}$$

W term

Assume
Type I
behavior
dominates

$$\left.\frac{d\ln\tilde{\sigma}}{d\ln Q^2}\right|_{\mathrm{b_T\ dep}} = \left.\tilde{K}(b_T;\mu_0)\right|_{\mathrm{b_T\ dep}}$$

Universal,
Perturbative
and Non-Perturbative

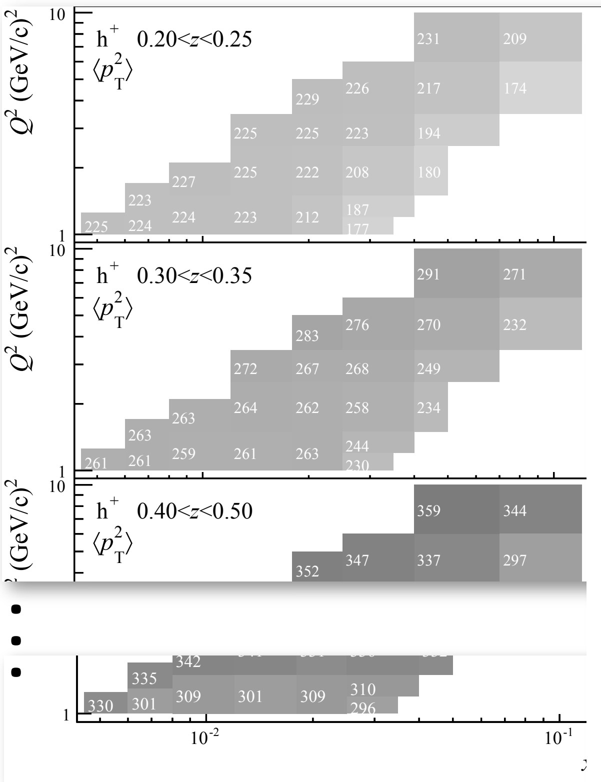
(Collins, Soper, Sterman (1985), Eq.(3.3))

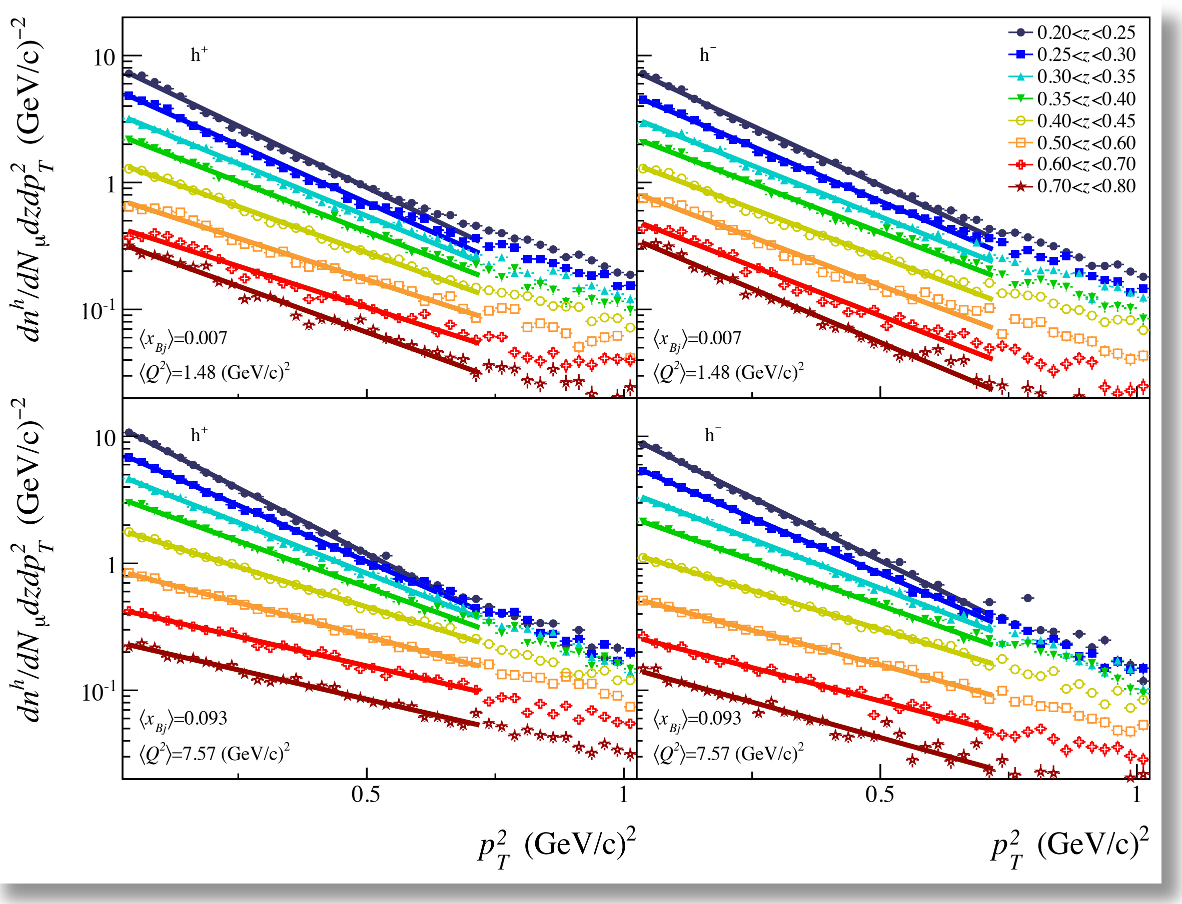
Maintain
Gaussian
Form

$$\frac{d\sigma}{dP_T^2} \propto \mathrm{F.T.}\,\exp\left\{-\frac{b_T^2}{4}\left(\langle P_T^2\rangle_0 + 4C_{\mathrm{evol}}\ln\left(\frac{Q_2}{Q_1}\right)\right)\right\}$$

Test non-perturbative evolution in unpolarized SIDIS

***From COMPASS,** C. Adolph et al., arXiv:1305.7317*





Type I - like

Test non-perturbative evolution in unpolarized SIDIS

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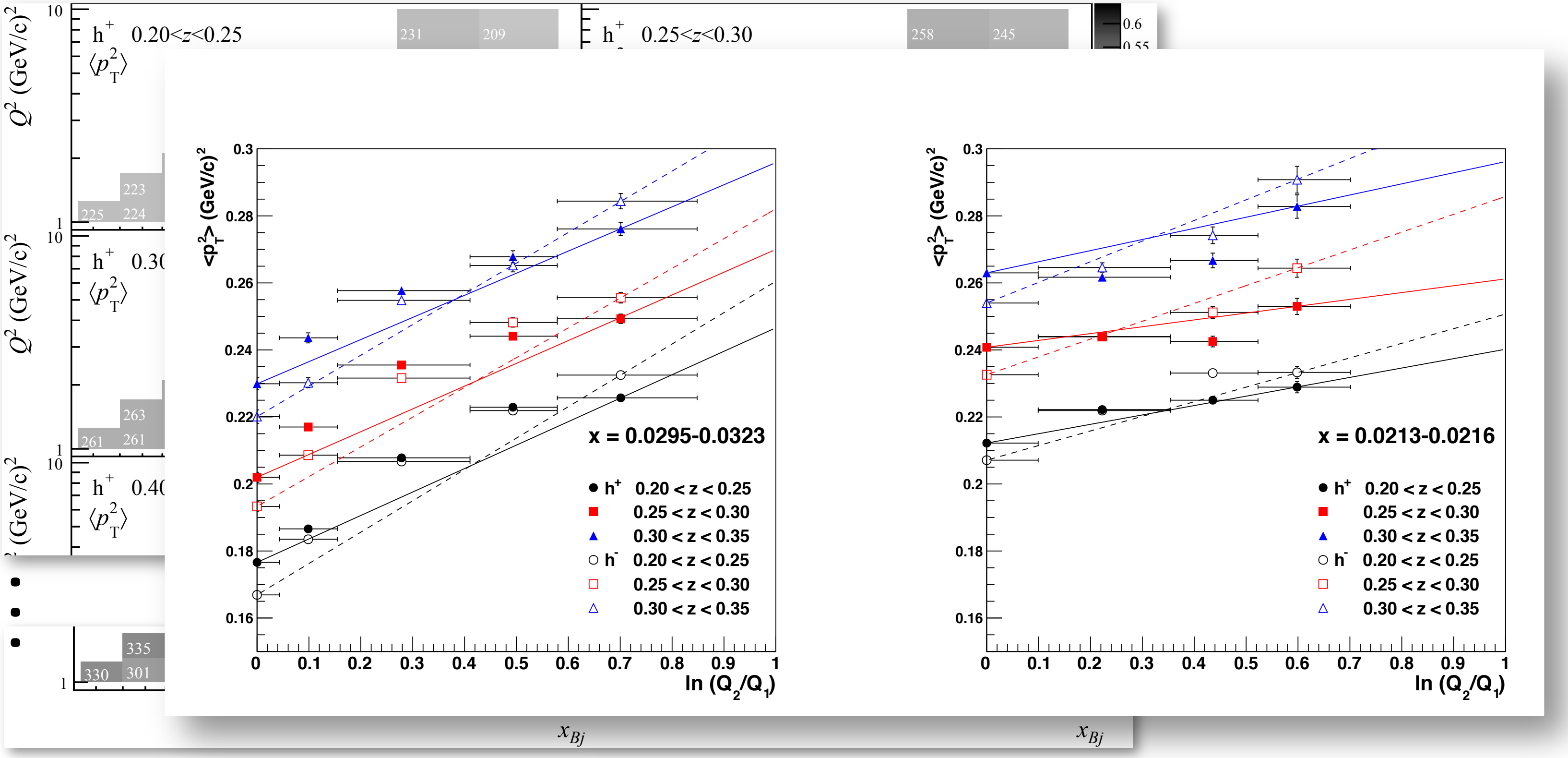
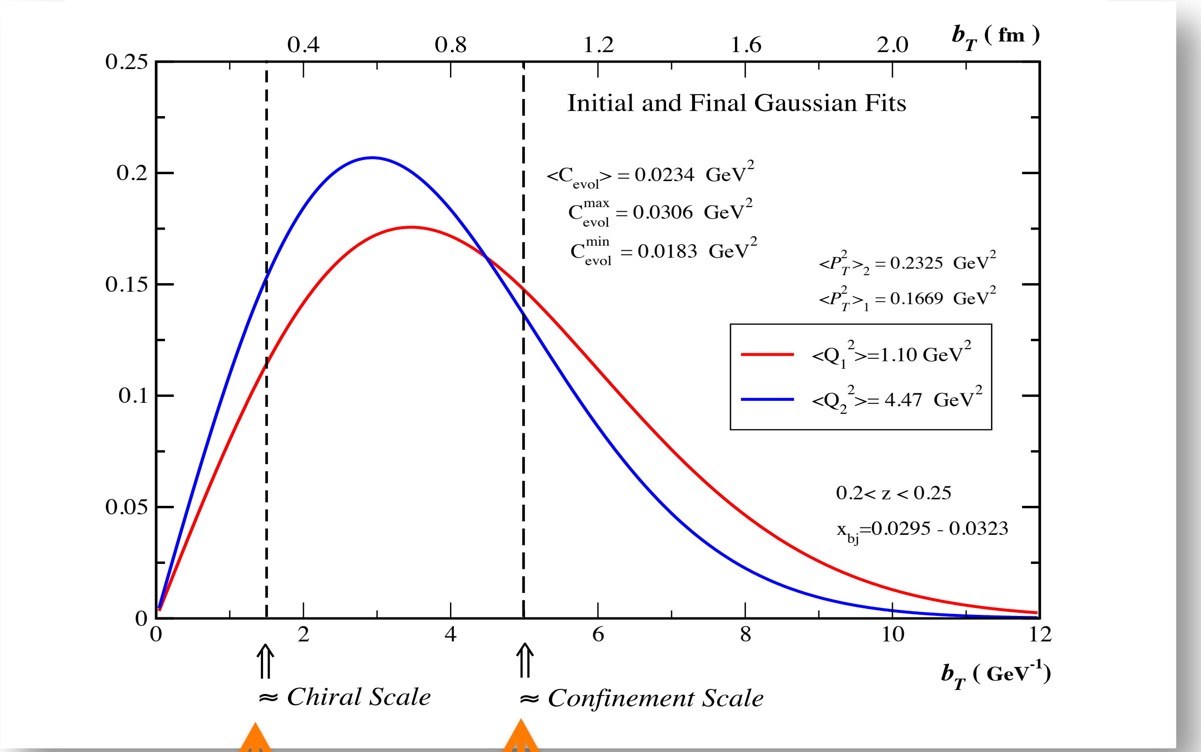


Test non-perturbative evolution in unpolarized SIDIS

Recent: arXiv:1401.2654 C. Aidala, B. Field, and L. Gamberg, TCR

- Largest “apples-to-apples” evolution (in b_T-space):



≈ *Chiral Sym. Breaking Scale*

≈ *Proton Charge Radius*

$0.2 < z < 0.25$

$x_{bj} = 0.0295 - 0.0323$

Recall: One TMD PDF

One p

$$-g_{PDF,f}(x,b_T) \equiv -g_{f/P}(x,b_T) + \ln\left(\tilde{F}_{f/P}(x,b_*;\,\mu_b,\mu_b^2)\right)$$

tching Prescription:

$$) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1+b_T^2/b_\mathrm{max}^2}}$$

$$\mu_b \equiv C_1/|\mathbf{b}_*(b_T)|$$

$$\tilde{F}_{f/P}(x,\mathbf{b}_\mathrm{T};Q,Q^2) =$$

$$\tilde{F}_{f/P}(x,b_*;\,\mu_b,\mu_b^2)\times$$

$$\times \exp\left\{\ln\frac{Q}{\mu_b}\tilde{K}(b_*;\mu_b) + \int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_F(g(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(g(\mu'))\right]\right\}\times$$

$$\times \exp\left\{\underline{-g_{f/P}(x,b_T)}\underline{-g_K(b_T)}\ln\frac{Q}{Q_0}\right\}$$

Nonperturbative large b_T behavior

Recall: One TMD PDF

One physical scale for evolution

$$\mu \sim \sqrt{\zeta_1} \sim \sqrt{\zeta_2} \sim Q$$

$$\zeta_1 \zeta_2 \sim Q^4$$

Ex: Matching Prescription:

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_\mathrm{max}^2}}$$

$$\mu_b \equiv C_1/|\mathbf{b}_*(b_T)|$$

$$\tilde{F}_{f/P}(x, \mathbf{b}_\mathrm{T}; Q, Q^2) =$$

$$\exp\left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times$$

$$\times \exp\left\{ -g_{PDF,f}(x, b_T) - g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

↑

Not vanishing at small b_T

TMD Factorization: W-term

$$\frac{d\sigma}{dP_T^2} \propto \mathcal{H}(\alpha_s(Q)) \int d^2 b_T e^{i b_T \cdot P_T} \; \tilde{F}_{H_1}(x, b_T; Q, Q^2) \, \tilde{D}_{H_2}(z, b_T; Q, Q^2) \; + \; \cancel{Y_{\mathrm{SIDIS}}}$$

TMD Factorization: W-term

$$\frac{d\sigma}{dP_T^2} \propto \mathcal{H}(\alpha_s(Q)) \int d^2 b_T e^{ib_T \cdot P_T}\ \tilde{F}_{H_1}(x, b_T; Q, Q^2)\, \tilde{D}_{H_2}(z, b_T; Q, Q^2) \ + \ \cancel{Y_{\text{SIDIS}}}$$

$$\frac{d\sigma}{dP_T^2} \propto \text{F.T.} \exp\left\{ -g_{\text{PDF}}(x, b_T; b_{\text{max}}) - g_{\text{FF}}(z, b_T; b_{\text{max}}) - 2 g_K(b_T; b_{\text{max}}) \ln\left(\frac{Q}{Q_0}\right) + \right.$$

$$\left. + 2 \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_{\text{PDF}}(\alpha_s(\mu'); 1) + \gamma_{\text{FF}}(\alpha_s(\mu'); 1) - 2 \ln\left(\frac{Q}{\mu'}\right) \gamma_K(\alpha_s(\mu')) \right] \right\}$$

$$g_K(b_T) \ln\left(\frac{Q}{Q_0}\right) = -g_2 \frac{1}{2} b_T^2 \ln\left(\frac{Q}{Q_0}\right)$$

Ansatz!

$$-g_{\text{PDF}}(x, b_T; b_{\text{max}}) - g_{\text{FF}}(z, b_T; b_{\text{max}})$$

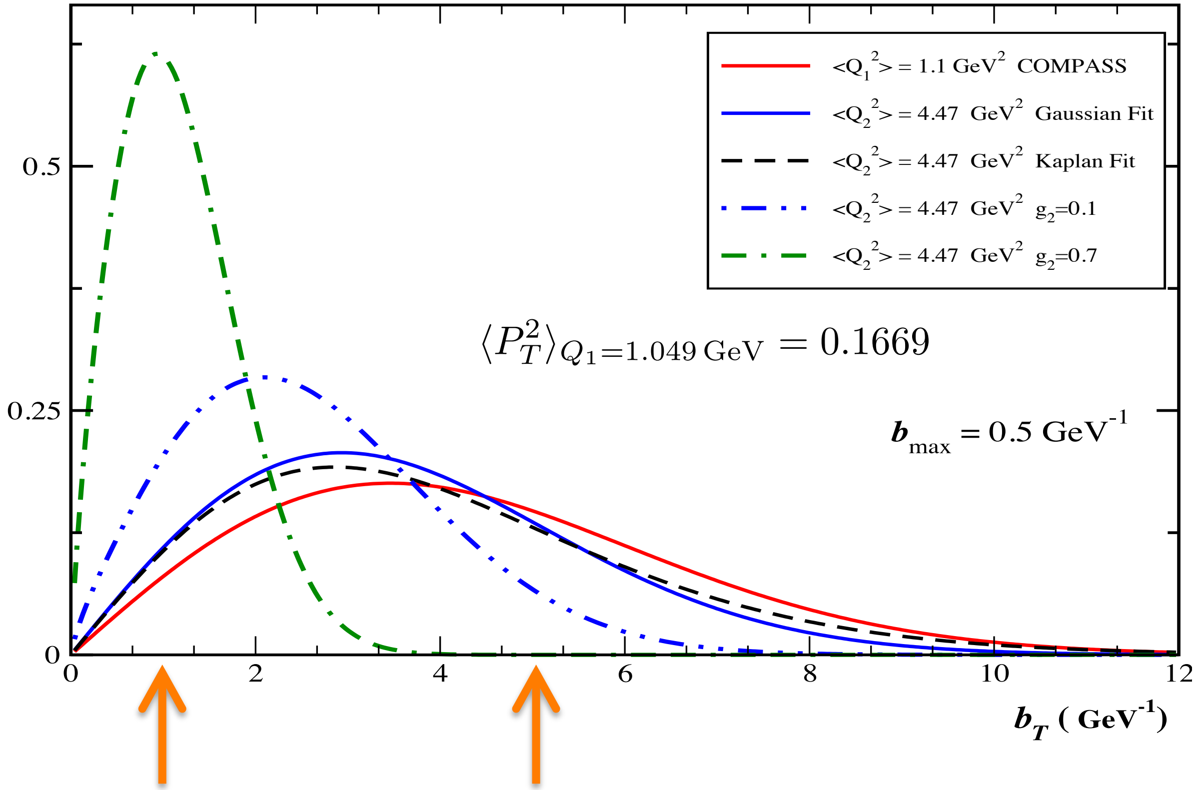
$$= -\frac{b_T^2 \langle P_T^2 \rangle_{Q_0}}{4} - 2 \ln\left(\frac{Q_0}{\mu_b}\right) \tilde{K}(b_*; \mu_b) - \int_{\mu_b}^{Q_0} \frac{d\mu'}{\mu'} \left[\gamma_{\text{PDF}}(\alpha_s(\mu'); 1) + \gamma_{\text{FF}}(\alpha_s(\mu'); 1) - 2 \ln\left(\frac{Q_0}{\mu'}\right) \gamma_K(\alpha_s(\mu')) \right]$$

Test non-perturbative evolution in unpolarized SIDIS

Recent: arXiv:1401.2654 C. Aidala, B. Field, and L. Gamberg, TCR

- Gaussian $g_K(b_T)$

$$g_K(b_T)\ln\left(\frac{Q}{Q_0}\right) = -g_2\frac{1}{2}b_T^2\ln\left(\frac{Q}{Q_0}\right)$$



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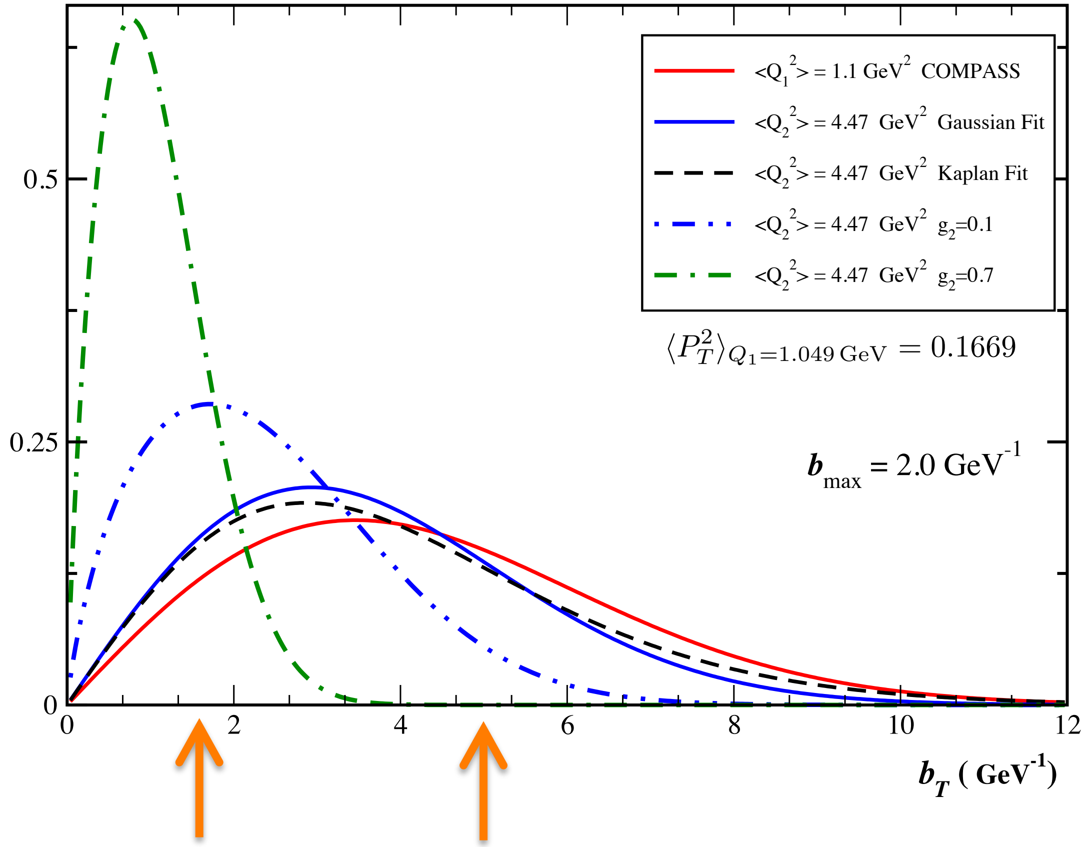

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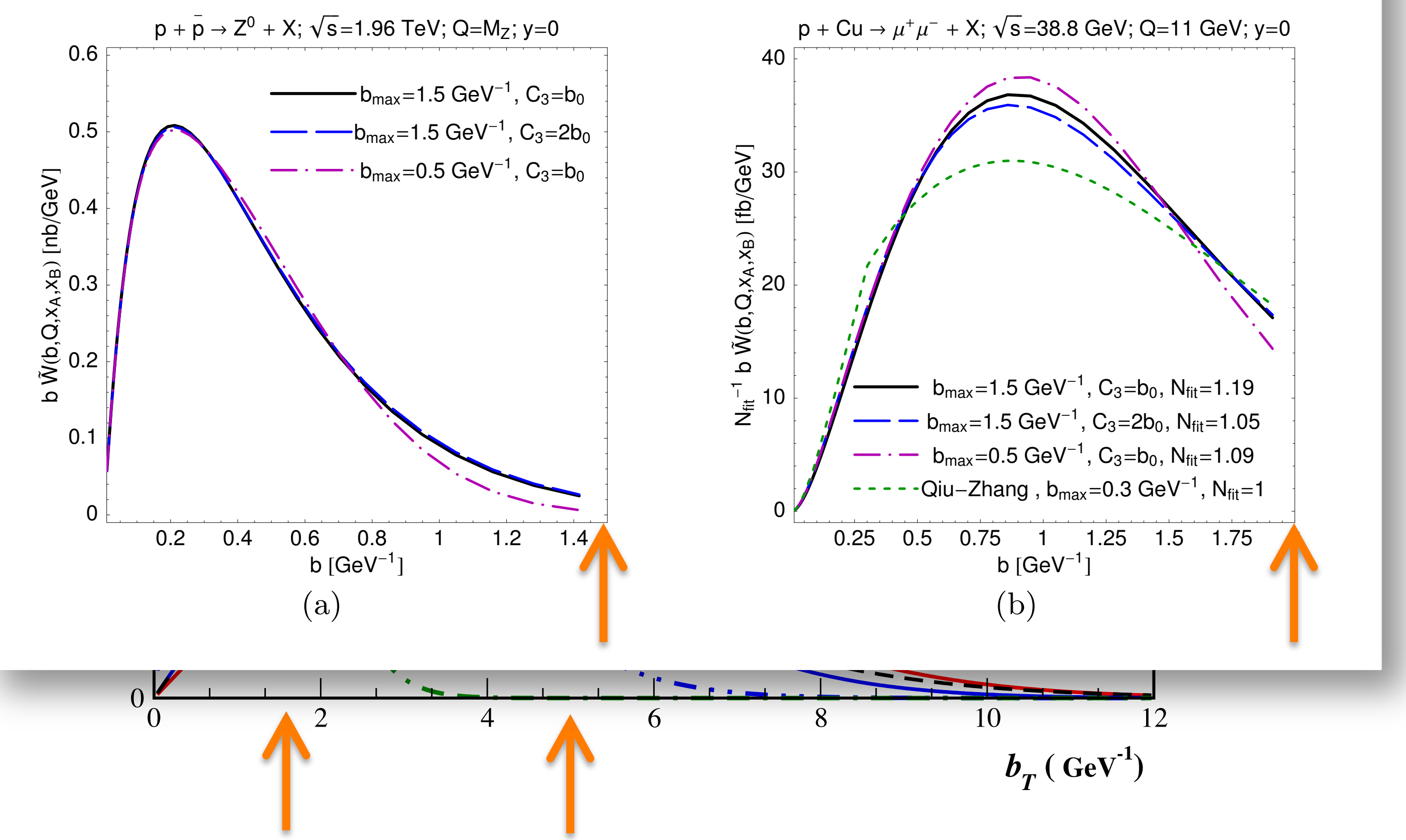
(Konychev, Nadolsky (2006))



To Do:

- New global fits to semi-inclusive deep inelastic scattering over wide range of large and small Q.

  (Meng, Soper, Yuan (1995): CSS Factorization for SIDIS)

  (Nadolsky, Stump, Yuan (2000,2001): ResBos SIDIS)

- Recast in terms of Collins 2011 TMD factorization.

  (with P. Nadolsky, in progress….)

- Constrain non-perturbative evolution.
  - Different non-perturbative forms.
  - Purely non-perturbative considerations.

- Fix x, z, hadron species as much as possible (or account for variations).

To Do:

- Try non-power-law form.
  - Requirements:
    - Quadratic *(or power law)* at small b_T

$$g_K(b_T; b_{\max}) = a_1 \left(\frac{b_T^2}{b_{\mathrm{NP}}^2} \right) + a_2 \left(\frac{b_T^4}{b_{\mathrm{NP}}^4} \right) + \cdots$$

  - Constant at very large b_T

(Schweitzer, Strikman, Weiss, (2013))

(with J. Collins, in progress….)

- $a_1/\mathrm{b}_{\mathrm{NP}}^2 \sim 0.1 \ \mathrm{GeV}^2$

- $b_{\mathrm{NP}} \gtrsim 1.0 \ \mathrm{GeV}^2$

Example:

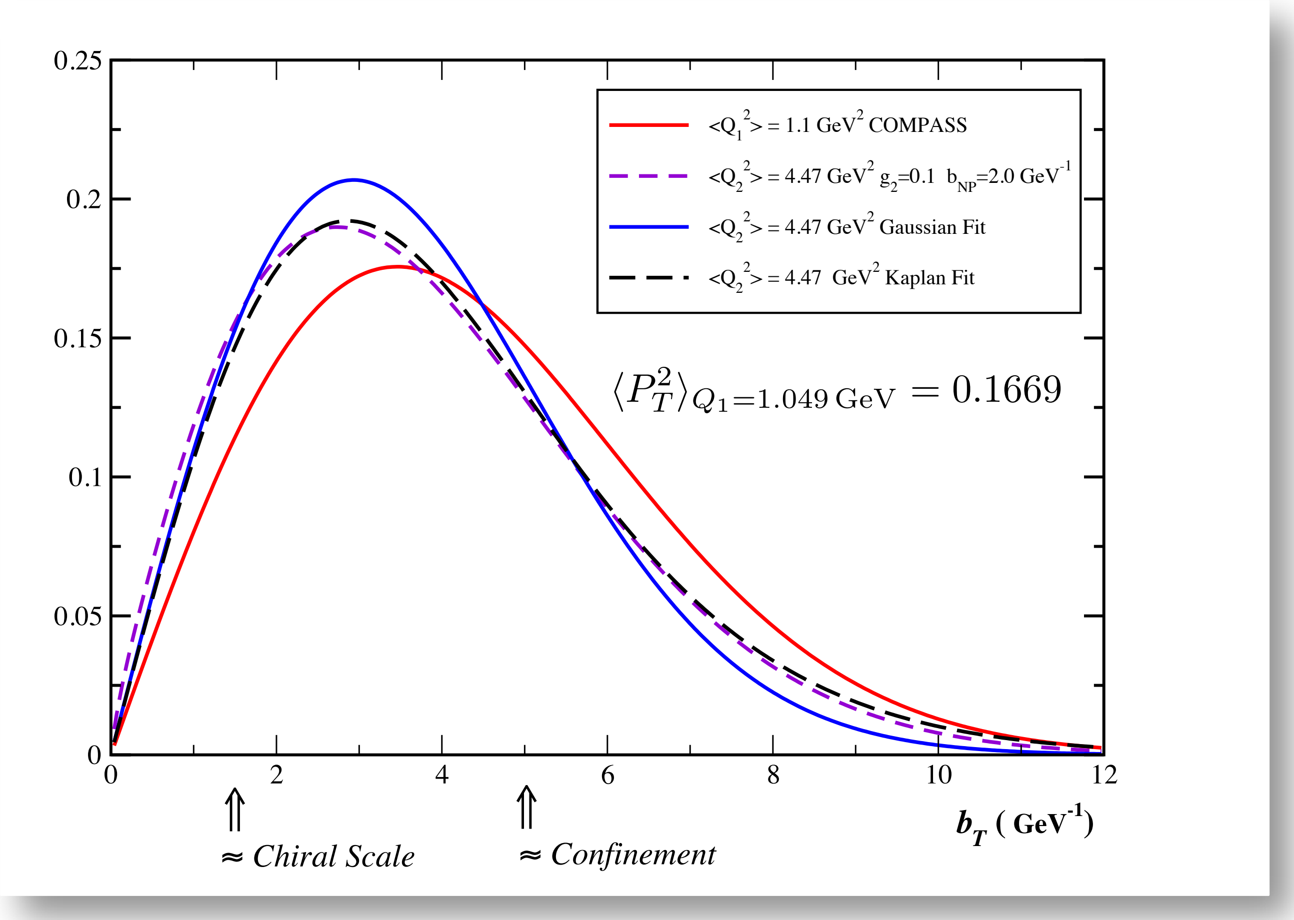
- Try non-power-law forms.
  - Ex: $g_K(b_T; b_{\max}) = \frac{g_2(b_{\max}) b_{\mathrm{NP}}^2}{2} \ln\left(1 + \frac{b_T^2}{b_{\mathrm{NP}}^2}\right)$
    - $g_2 \gtrsim 0.1\ \mathrm{GeV}^2$
    - $b_{\mathrm{NP}} \gtrsim 1.0\ \mathrm{GeV}^2$

Dependence on b_{max}

Example:

Recent: arXiv:1401.2654 C. Aidala, B. Field, and L. Gamberg, TCR

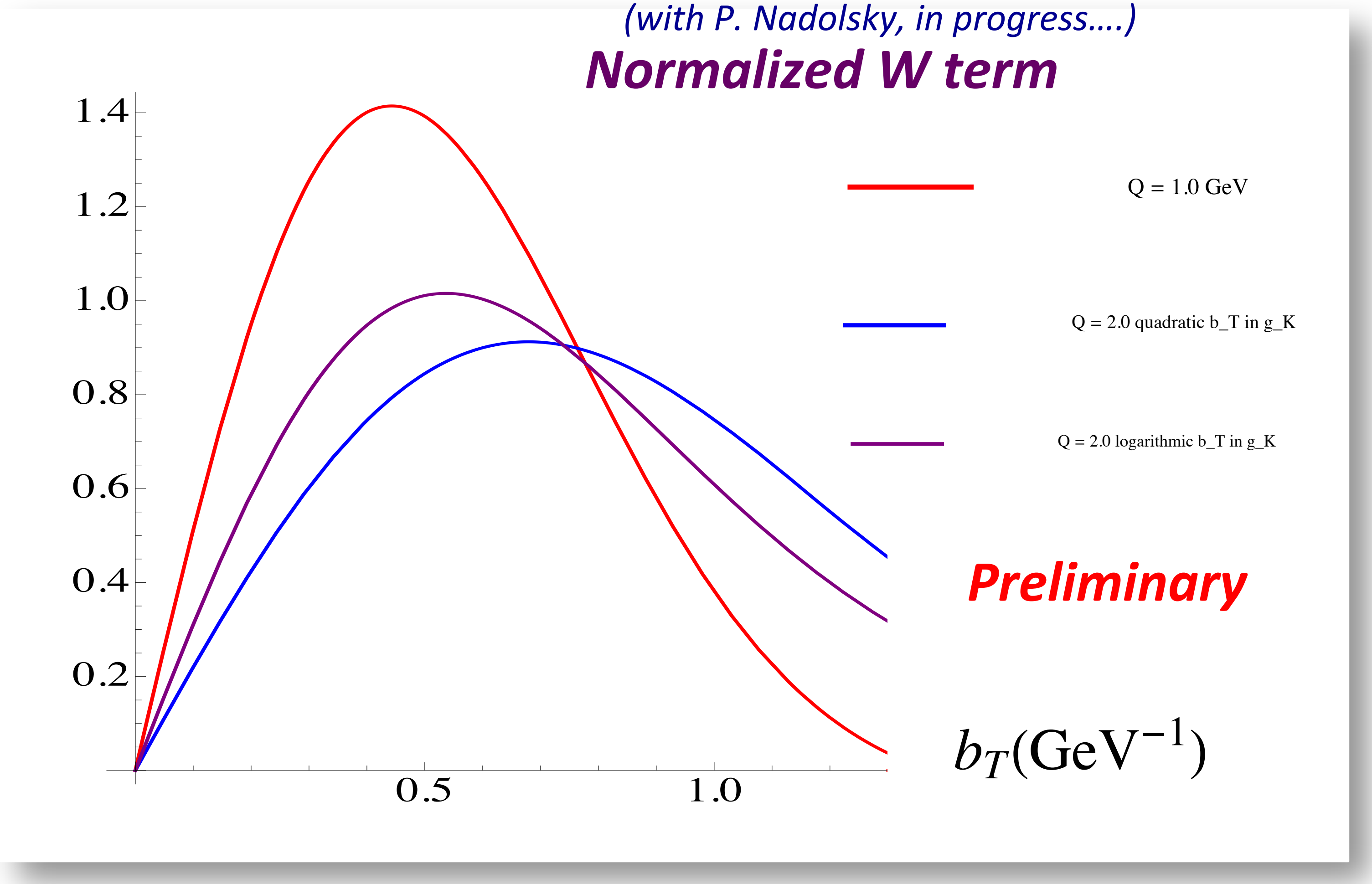
- Try non-power-law form.



Example:

- <u>Work in progress:</u> ResBos SIDIS including moderate Q

(with P. Nadolsky, in progress….)



Conclusions

- Many types of physics unified in a single TMD formalism

(Types I,II)

(No need for pessimism)

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- Moderate Q ideal for extracting details of large b_T non-perturbative (but universal) physics.

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    - Q too small → Factorization begins to fail.
    - Y - term

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Conclusions

- Many types of physics unified in a single TMD formalism
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- Embrace "apples-to-apples" style for meaningfully extracting hadron structure.

- Embrace and exploit Strong Universality of $g_K(b_T;b_{max})$.

TMD Evolution Overview

T. C. Rogers

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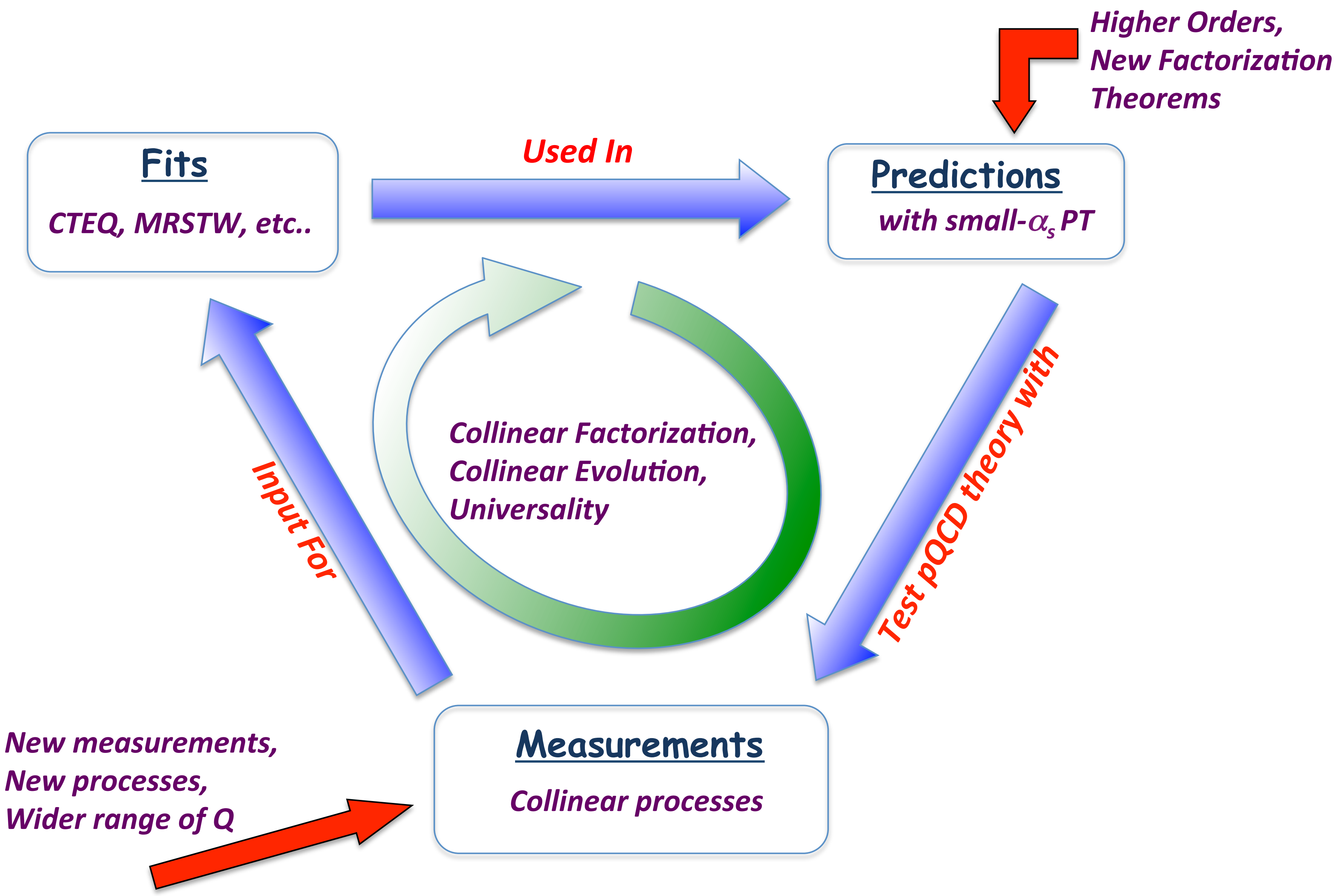
Thank You!

INT Workshop – February 27, 2014

Implementing Collinear Factorization



Higher Orders,
New Factorization
Theorems

Fits

CTEQ, MRSTW, etc..

Used In

Predictions

with small-α_s PT

Test pQCD theory with

Collinear Factorization,
Collinear Evolution,
Universality

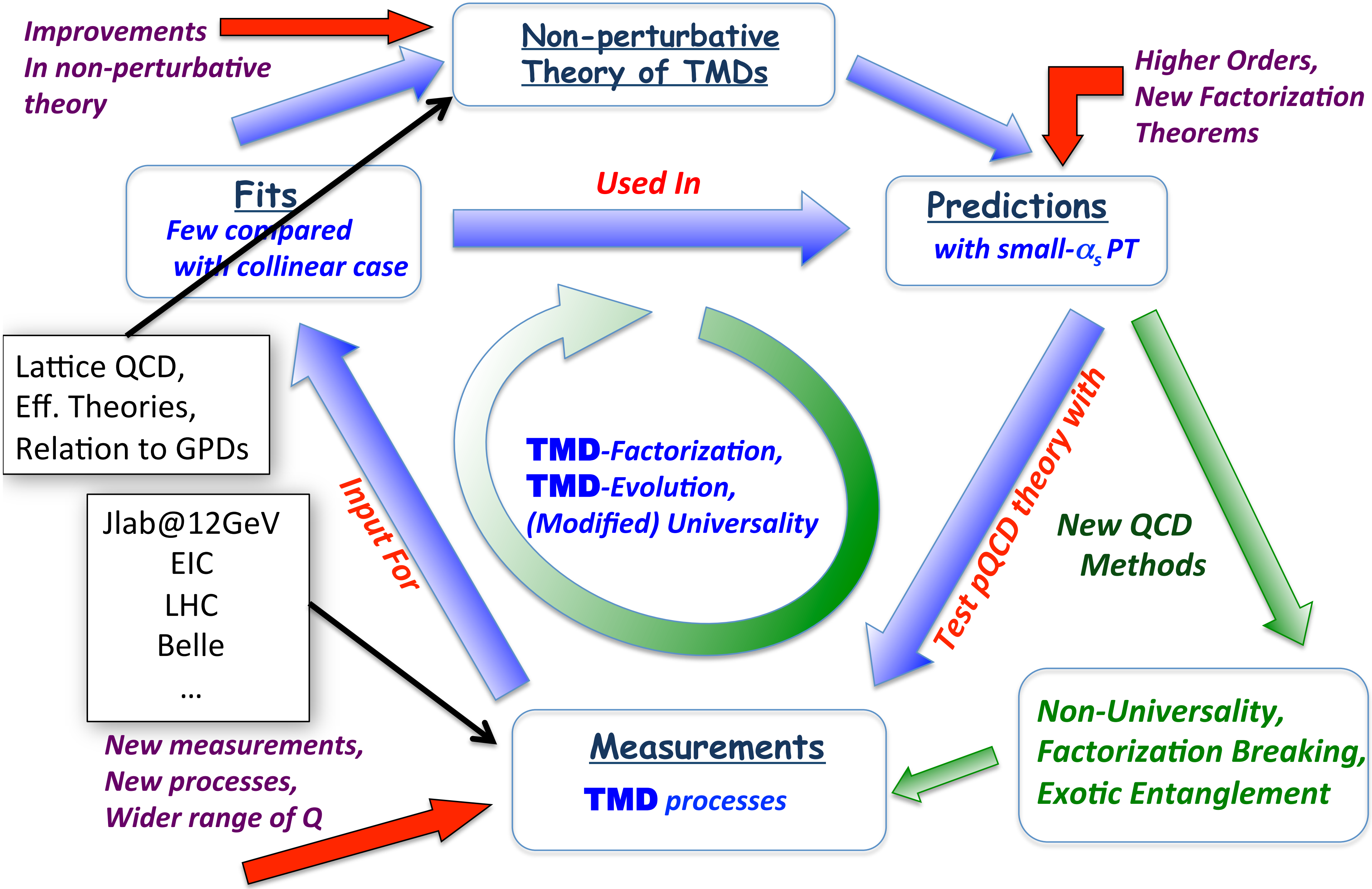
Input For

Measurements

Collinear processes

New measurements,
New processes,
Wider range of Q

Implementing TMD-Factorization



Improvements
In non-perturbative
theory

Non-perturbative Theory of TMDs

Higher Orders,
New Factorization
Theorems

Fits

Few compared with collinear case

Used In

Predictions

with small-α_s PT

Lattice QCD,
Eff. Theories,
Relation to GPDs

TMD-*Factorization,*
TMD-*Evolution,*
(Modified) Universality

Input For

Test pQCD theory with

New QCD Methods

Jlab@12GeV
EIC
LHC
Belle
…

Measurements

TMD *processes*

Non-Universality,
Factorization Breaking,
Exotic Entanglement

New measurements,
New processes,
Wider range of Q

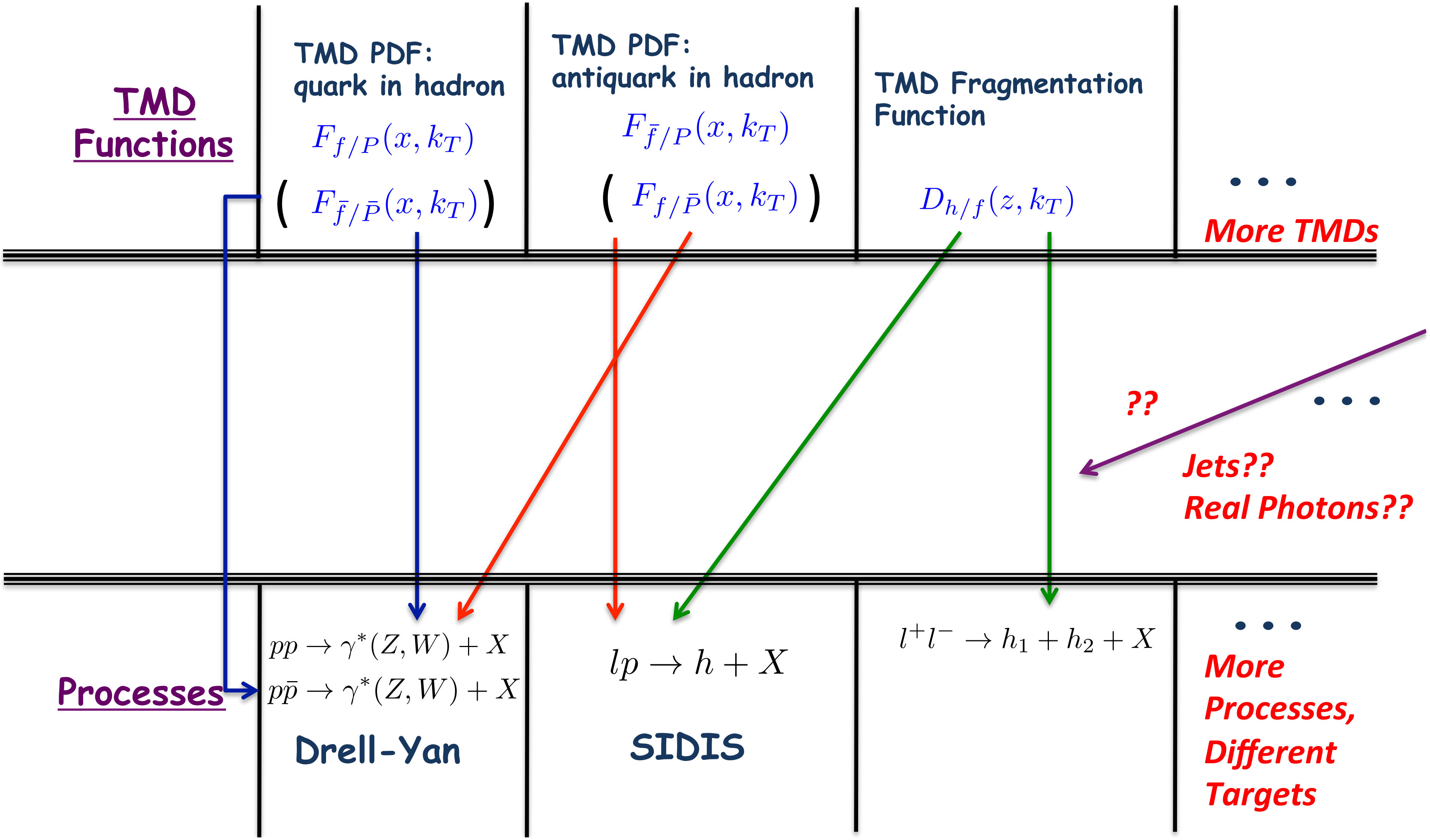
Strategy: "Apples-to-apples"

TMD Functions	**TMD PDF:** **quark in hadron** $F_{f/P}(x, k_T)$ ($F_{\bar{f}/\bar{P}}(x, k_T)$)	**TMD PDF:** **antiquark in hadron** $F_{\bar{f}/P}(x, k_T)$ ($F_{f/\bar{P}}(x, k_T)$)	**TMD Fragmentation Function** $D_{h/f}(z, k_T)$	• • • ***More TMDs***

Strategy: "Apples-to-apples"

TMD Functions	TMD PDF: quark in hadron $F_{f/P}(x, k_T)$ $\left(F_{\bar{f}/\bar{P}}(x, k_T) \right)$	TMD PDF: antiquark in hadron $F_{\bar{f}/P}(x, k_T)$ $\left(F_{f/\bar{P}}(x, k_T) \right)$	TMD Fragmentation Function $D_{h/f}(z, k_T)$	. . . *More TMDs*
Processes	$pp \to \gamma^*(Z, W) + X$ $p\bar{p} \to \gamma^*(Z, W) + X$ Drell-Yan	$lp \to h + X$ SIDIS	$l^+l^- \to h_1 + h_2 + X$	. . . *More Processes, Different Targets*

Strategy: “Apples-to-apples”



TMD Functions

TMD PDF:
quark in hadron

$F_{f/P}(x,k_T)$

$\left(F_{\bar{f}/\bar{P}}(x,k_T) \right)$

TMD PDF:
antiquark in hadron

$F_{\bar{f}/P}(x,k_T)$

$\left(F_{f/\bar{P}}(x,k_T) \right)$

TMD Fragmentation Function

$D_{h/f}(z,k_T)$

• • •

More TMDs

??

• • •

Jets??
Real Photons??

Processes

$pp \to \gamma^*(Z,W) + X$

$p\bar{p} \to \gamma^*(Z,W) + X$

Drell-Yan

$lp \to h + X$

SIDIS

$l^+l^- \to h_1 + h_2 + X$

• • •

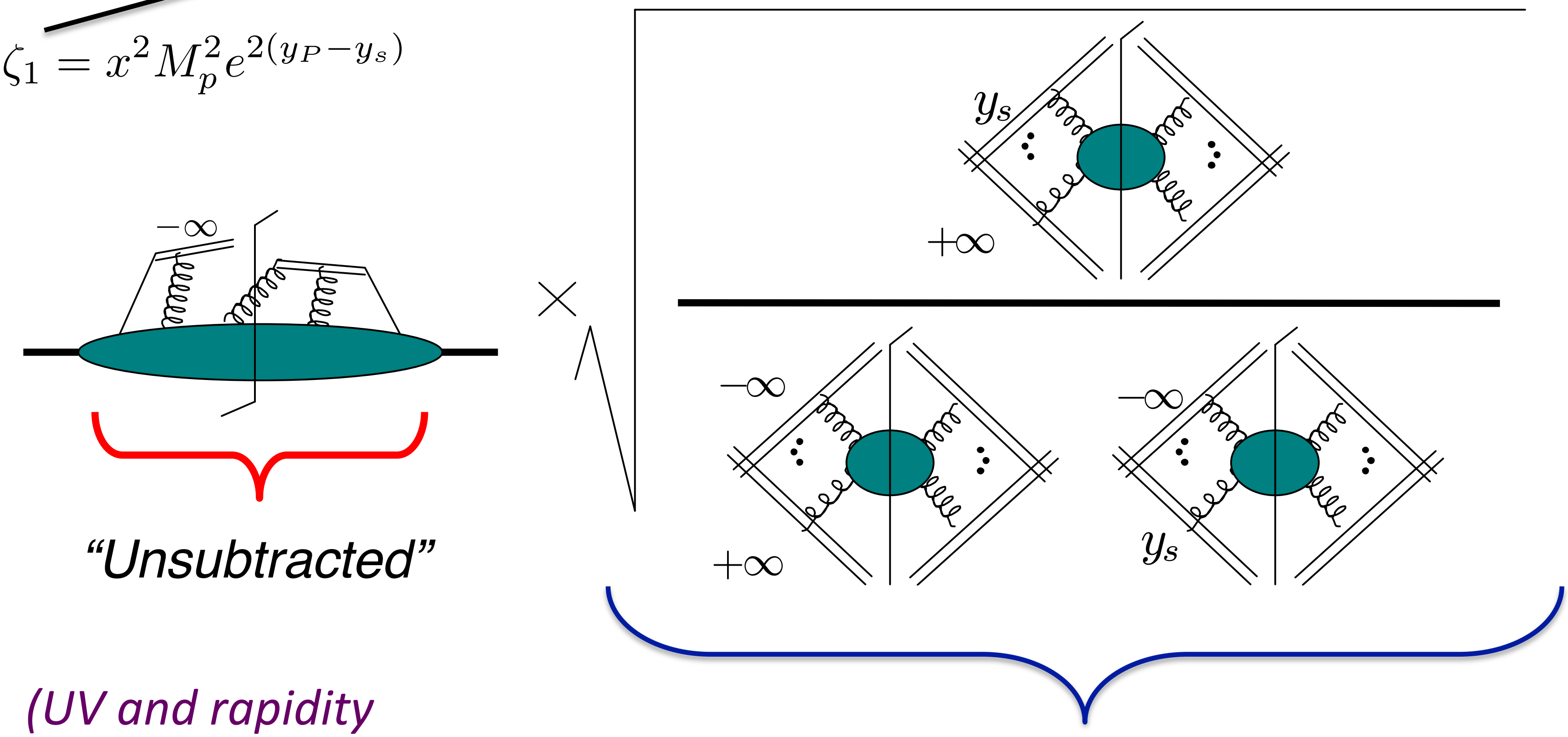
More Processes,
Different Targets

TMD PDF Definition

$\zeta_1 \zeta_2 \sim Q^4$

$F_{q/P}(x_1, \mathbf{k}_{1T}, \mu, \zeta_1)$

$\zeta_1 = x^2 M_p^2 e^{2(y_P - y_s)}$



"Unsubtracted"

(UV and rapidity renormalization needed)

(Collins (2011), chapt. 13)
Generalized Renormalization Factor

TMD PDF Definition

- Collinear PDFs:

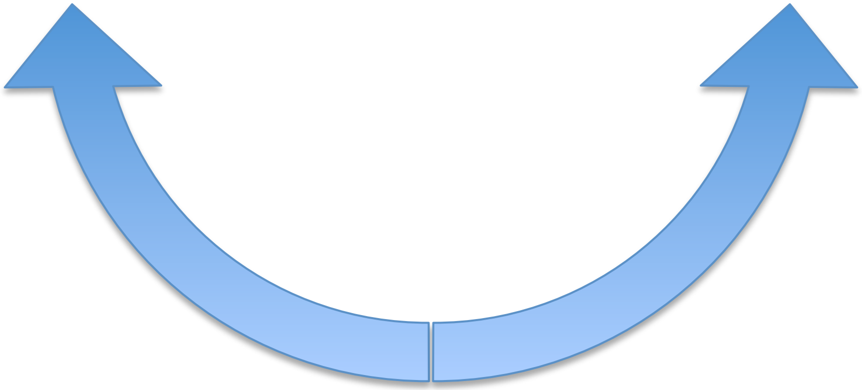
$$f_{j/p}(\xi;\mu) = \sum_i \int \frac{dz}{z}\, Z_{ji}(z, \alpha_s(\mu))\, f_{0,i/p}(\xi/z) \;=\; \overbrace{Z_{ji}}^{\textit{Independent of hadron properties}} \otimes f_{0,i/p}$$

$$f_{0,i/p}(\xi) = \int \frac{dw^-}{(2\pi)}\, e^{-i\xi P^+ w^-} \;\langle P|\, \bar{\psi}_{0,j}(0, w^-, \mathbf{0}_t) U^{[+]}(w^-, 0) \frac{\gamma^+}{2} \psi_{0,j}(0, 0, \mathbf{0}_t)\, |P\rangle$$

Talk Strategy

- Collinear Factorization
  - Collinear PDF, FFs
  - Scale Evolution

- TMD Factorization
  - TMD PDF, FFs
  - Scale Evolution



Analogies / Broken Analogies

TMD PDF Definition

- Collinear PDFs:

$$f_{j/p}(\xi;\mu) = \sum_i \int \frac{dz}{z}\, Z_{ji}(z,\alpha_s(\mu))\, f_{0,i/p}(\xi/z) = \overbrace{Z_{ji}}^{\textit{Independent of hadron properties}} \otimes f_{0,i/p}$$

$$f_{0,i/p}(\xi) = \int \frac{dw^-}{(2\pi)}\, e^{-i\xi P^+ w^-}\, \langle P|\, \bar{\psi}_{0,j}(0,w^-,\mathbf{0}_t) U^{[+]}(w^-,0) \frac{\gamma^+}{2} \psi_{0,j}(0,0,\mathbf{0}_t)\, |P\rangle$$

- TMD PDFs, CS Equation:

$$\tilde{F}^{\text{unsub.}}_{f/P}(x_1,\mathbf{b}_T;\mu,y_s) = \tilde{F}^{\text{unsub.}}_{f/P}(x_1,\mathbf{b}_T;\mu,-\infty) \times Z_{\text{CS}}(\mathbf{b}_T;y_s,+\infty,-\infty)$$

Or

$$\tilde{F}^{\text{unsub.}}_{f/P}(x_1,\mathbf{b}_T;\mu,y_s) = \lim_{\text{WL Raps}\to\infty} \left(\tilde{F}^{\text{unsub.}}_{f/P}(x_1,\mathbf{b}_T;\mu) \times \underbrace{Z_{\text{CS}}(\mathbf{b}_T;y_s)}_{\textit{Independent of hadron properties}} \right)$$