Extracting TMDs from data and Monte Carlo

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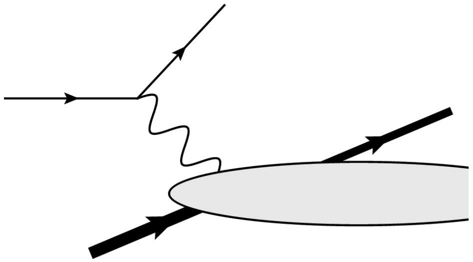
Complementarity of SIDIS, e+e- and Drell-Yan

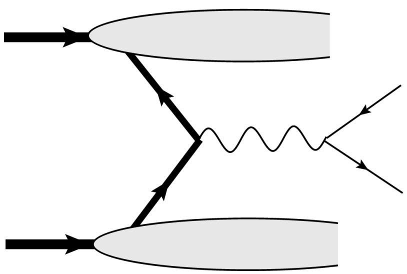
Various processes allow study and test of evolution, universality and extractions of distribution and fragmentation functions. We need information from all of them



$$f(x) \otimes D(z)$$

Semi Inclusive DIS – convolution of distribution functions and fragmentation functions

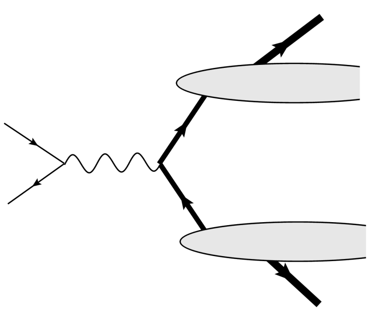
$$\ell + P \rightarrow \ell' + h + X$$



$$f(x_1) \otimes f(x_2)$$

Drell-Yan – convolution of distribution functions

$$P_1 + P_2 \rightarrow \bar{\ell}\ell + X$$



$$D(z_1) \otimes D(z_2)$$

e+ e- annihilation – convolution of fragmentation functions

$$\bar{\ell} + \ell \rightarrow h_1 + h_2 + X$$

Combining measurements from all above is important

Distributions

Unpolarised distributions

$$f_1(x) \propto \langle P, S_P|\bar{\psi}(0)\frac{\gamma^+}{2}\psi(\xi)|P, S_P\rangle$$

Distribution of unpolarised quarks inside unpolarised nucleon

Polarised distributions

$$g_1(x) \propto \langle P, S_L|\bar{\psi}(0)\frac{\gamma^+\gamma_5}{2}\psi(\xi)|P, S_L\rangle$$

Helicity distribution – distribution of longitudinally polarised quarks inside longitudinally polarised nucleon

$$h_1(x) \propto \langle P, S_{T1}|\bar{\psi}(0)\frac{\gamma^+\gamma^1\gamma_5}{2}\psi(\xi)|P, S_{T1}\rangle$$

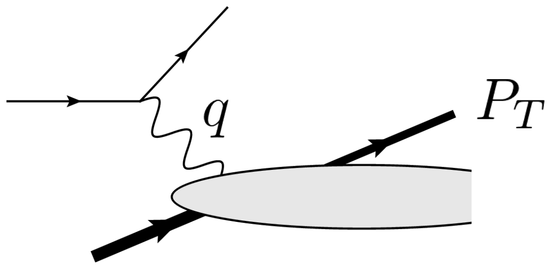
Transversity distribution – distribution of transversely polarised quarks inside transversely polarised nucleon

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

Two scale problem in QCD

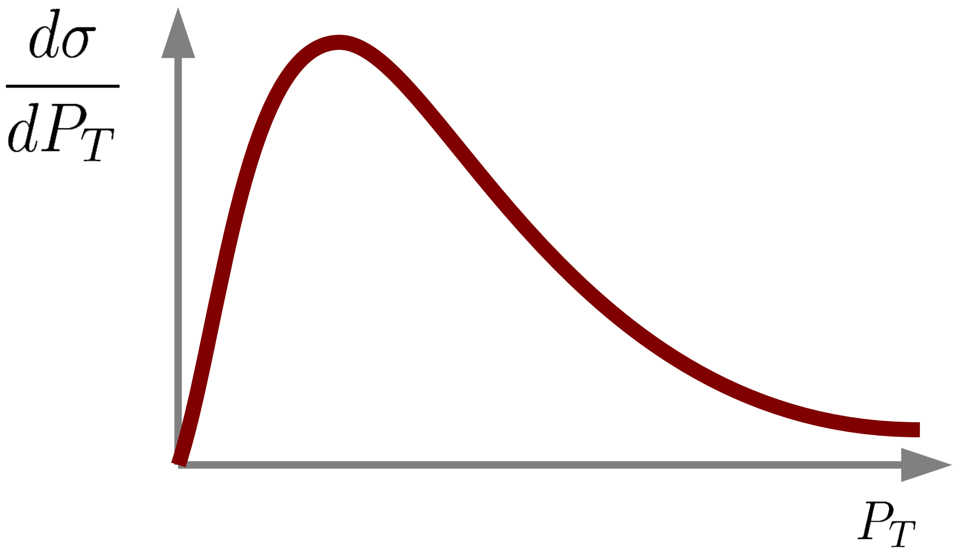
Consider Semi Inclusive Deep Inelastic Scattering:

There are two measured scales P_T, Q



$$\ell + P \to \ell' + h + X$$

Talk by Feng Yuan



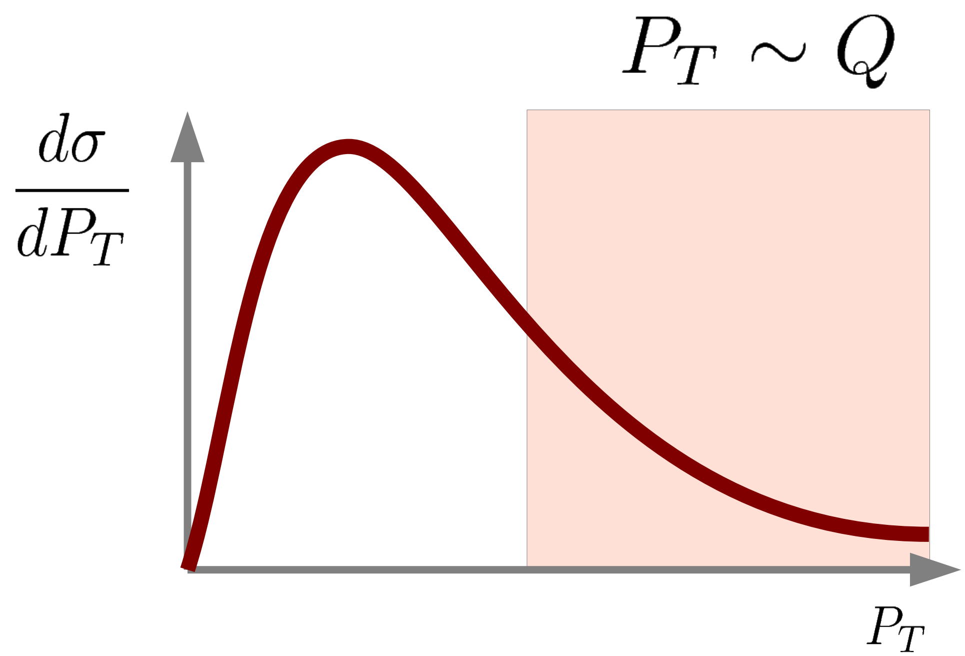
Two scale problem in QCD

Consider Semi Inclusive Deep Inelastic Scattering:

There are two measured scales P_T, Q

One scale problem, $P_T, Q \gg \Lambda_{QCD}$

The treatment is referred to as “collinear”.
One neglects transverse motion of partons.
Observed momentum is generated by gluon recoil.



$f(x;Q^2)$

Beyond leading twist:
Twist-3 matrix elements – dominate asymmetries
Efremov Teryaev (1982), Qiu, Sterman (1991)

Two scale problem in QCD

Consider Semi Inclusive Deep Inelastic Scattering:

There are two measured scales P_T, Q

Two scale problem, $P_T \gg \Lambda_{QCD}$

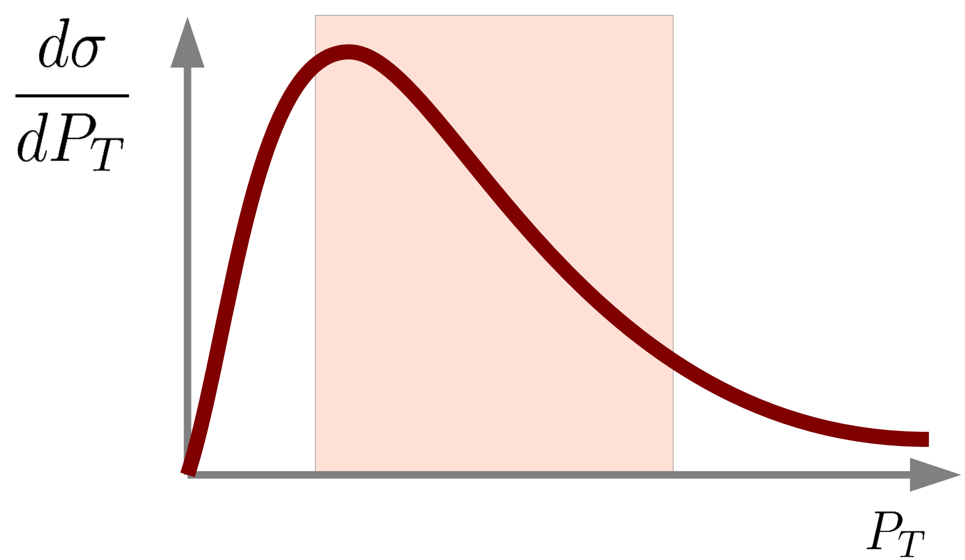
Collinear treatment is still possible,
large logs should be resummed to all orders

$$\alpha_s^n \ln^{2n}\left(Q^2/P_T^2\right)$$

So-called Collins-Soper-Sterman resummation

Collins, Soper, Sterman 1985

$$P_T \ll Q$$



Talk by Pavel Nadolsky

Two scale problem in QCD

Consider Semi Inclusive Deep Inelastic Scattering:

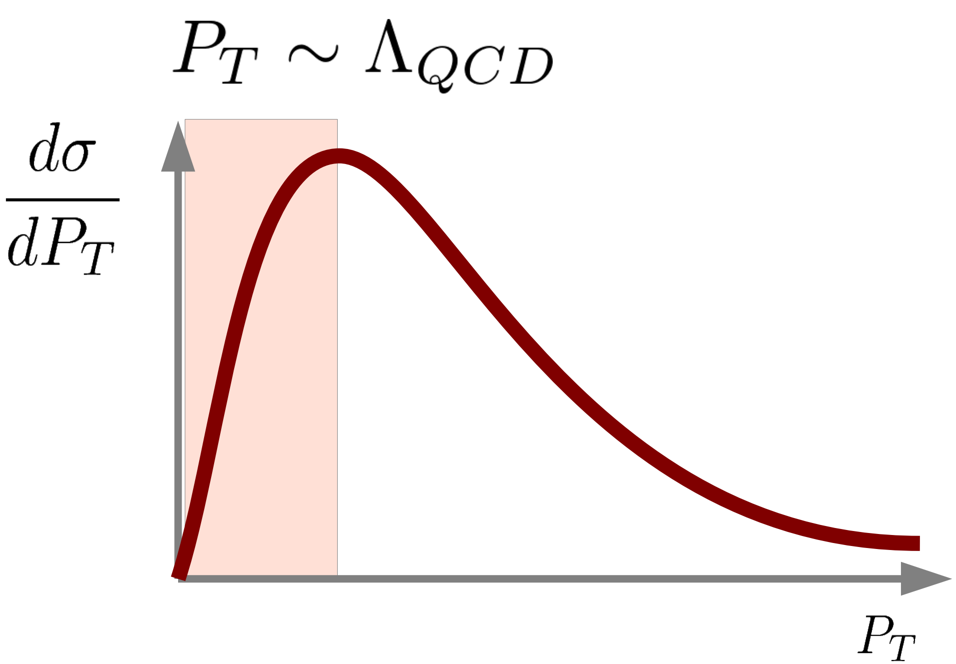
There are two measured scales P_T, Q

Two scale problem, $P_T \sim \Lambda_{QCD}$

One cannot neglect transverse motion of partons

Large logs should be resummed to all orders

$$\alpha_s^n \ln^{2n}\left(Q^2/P_T^2\right)$$



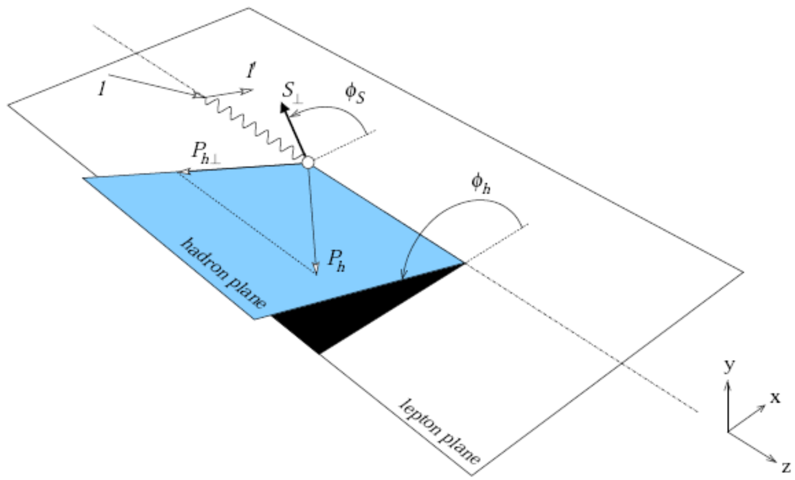
So-called TMD factorization
Mulders, Tangerman 1995
Boer, Mulders 1998,
Ji, Ma, Yuan 2004,
Collins 2011

$$f(x, k_\perp; Q^2)$$

TMD functions

Jefferson Lab kinematics is dominated by this region

Semi Inclusive Deep Inelastic scattering



One can rewrite cross-section in terms of **18** structure functions

Each structure function encodes parton dynamics via convolutions of TMDs

Mulders, Tangerman (1995),
Boer, Mulders (1998)
Bacchetta et al (2007)

$$F_{UU,T} = x \sum_q e_q^2 \int \mathrm{d}^2 k_\perp \mathrm{d}^2 p_\perp \delta^{(2)}(\mathbf{P}_{h\perp} - z\mathbf{k}_\perp - \mathbf{p}_\perp) f^q(x, k_\perp^2) D_q(z, p_\perp^2)$$

$$\frac{d\sigma}{dx\, dy\, d\psi\, dz\, d\phi_h\, dP_{h\perp}^2} =$$

$$\frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \Big\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, F_{UU}^{\cos\phi_h}$$

$$+ \varepsilon \cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, F_{LU}^{\sin\phi_h} + \dots$$

One can write cross-section as product of leptonic and hadronic tensors:

$$\sigma \propto L_{\mu\nu} W^{\mu\nu}$$

Hadronic tensor contains information on the structure, for SIDIS:

Ji, Ma, Yuan, 2004
Collins, 2011

$$W^{\mu\nu} \propto H^{\mu\nu}(\mu, Q) \int \underline{\mathrm{d}^2 k_\perp \mathrm{d}^2 p_\perp \delta^{(2)}(\mathbf{P}_{h\perp} - z\mathbf{k}_\perp - \mathbf{p}_\perp) f(x, k_\perp^2; \mu, \zeta) D(z, p_\perp^2; \mu, \zeta)}$$

$$+(\text{other terms}) + \mathcal{O}((P_{h\perp}/Q)^a)$$

TMD contribution

Applicable in the region $P_{h\perp} \ll Q$

TMDs

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$$+(\text{other terms}) + \underline{Y^{\mu\nu}_{SIDIS}(P_{h\perp}, Q)}$$

TMD contribution

Matching to large momenta $P_{h\perp} \sim Q$

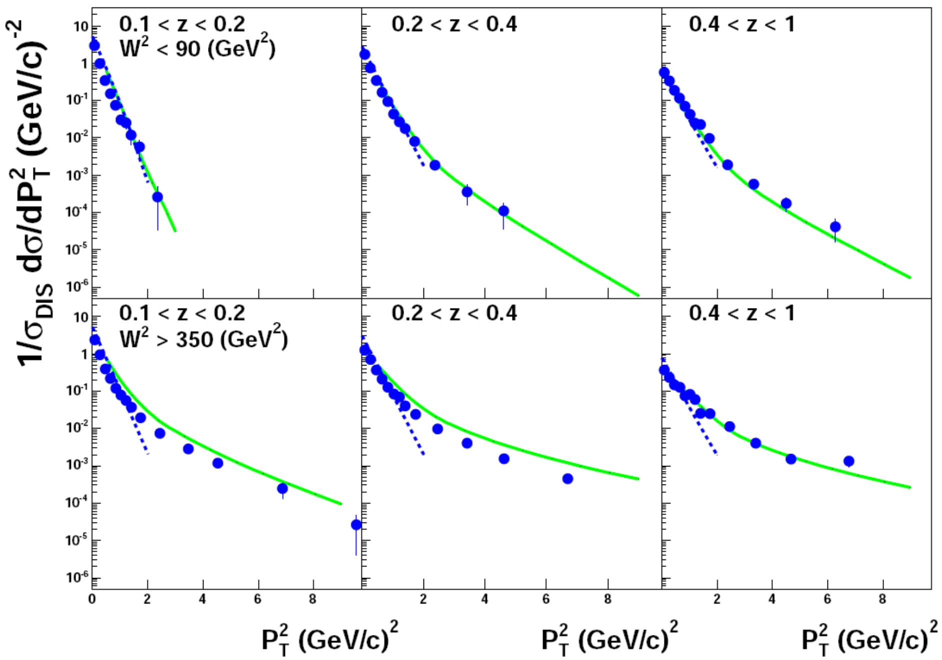
Talk by Feng Yuan,
Pavel Nadolsky

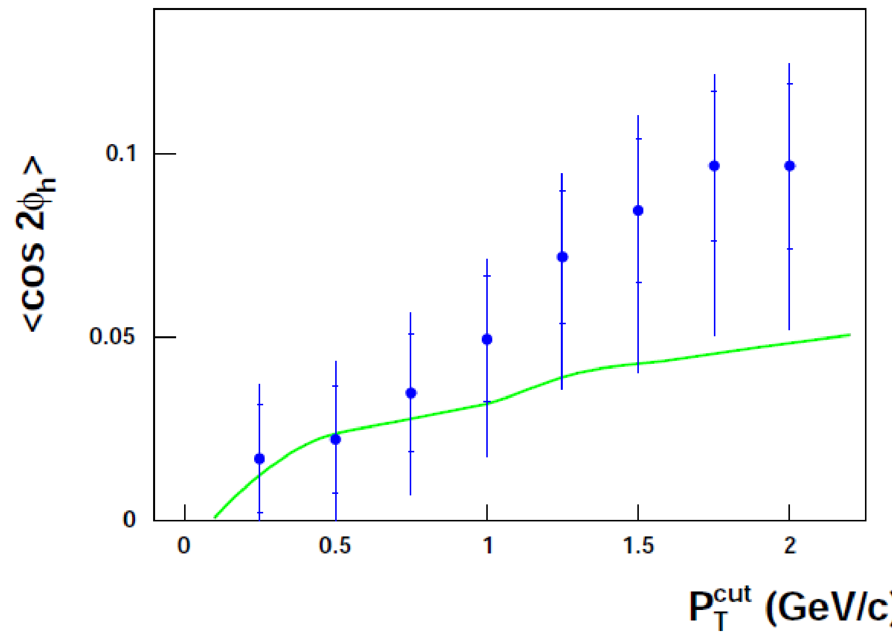
Applicable in the whole range of $P_{h\perp}$

Very naïve method

$$W^{\mu\nu} \propto W^{\mu\nu}_{TMD}\theta(P^0_{h\perp} - P_{h\perp}) + W^{\mu\nu}_{FO}\theta(P_{h\perp} - P^0_{h\perp})$$

was implemented Anselmino et al (2006)





Data EMC: Ashman et al 1991

Data ZEUS: Breitweg et al 2000

TMDs

Aidala, Field, Gamberg, Rogers 2014

SIDIS kinematics at fixed target experiments

$$P_{h\perp} \sim 0.5(\text{GeV}) \quad Q \sim 1(\text{GeV}) \qquad q_T \simeq \frac{P_{h\perp}}{z}$$

Separation to TMD and collinear regions become problematic

Drell-Yan on the other hand allows clear separation

$$q_T < 1(\text{GeV}) \qquad Q > 4(\text{GeV})$$

TMD PDF and TMD FF evolve according to Collins-Soper evolution equations

Collins, Soper, Sterman 1985
Ji, Ma, Yuan, 2004
Idilbi, Ji, Ma, Yuan 2004
Collins, 2011

Maximizing "perturbative" content, we can write

$$\tilde{F}(x, b_T; \mu, \zeta) = \left(C(b_*) \otimes f(x)\right)(\mu_b) e^{S(b_*)} e^{S_{NP}(b_T)} \tilde{F}_{NP}(x, b_T)$$

$$\mu_b = C_1/b_*$$

$$b_* \in [0, b_{max})$$

Perturbative — Non perturbative

Other methods of including non-perturbative inputs are also known

Qiu, Zhang (2001)
Sterman, Vogelsang (2000)
Many others

Evolution

Non perturbative part of TMD PDF

$$e^{S_{NP}(b_T)}\tilde{F}_{NP}(b_T) = e^{-g(x,b_T)-\frac{1}{2}g_K(b_T)\ln(Q/Q_0)}$$

is to be ***extracted*** from comparison to experimental data

“Standard” assumption $g(x, b_T) = \dfrac{b_T^2\langle k_\perp^2\rangle}{4}$ leads to gaussian type behaviour

to be tested with experimental data. Other functional forms are readily available, however “gaussian” form is very successful phenomenologically

Anselmino et al (2006),
Schweitzer, Teckentrup, Metz (2010)

Non perturbative part of TMD PDF

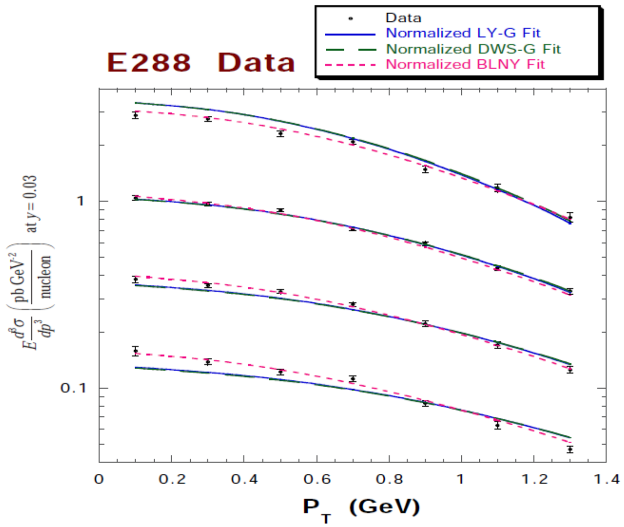
$$e^{S_{NP}(b_T)}\tilde{F}_{NP}(b_T) = e^{-g(x,b_T)-\frac{1}{2}g_K(b_T)\ln(Q/Q_0)}$$

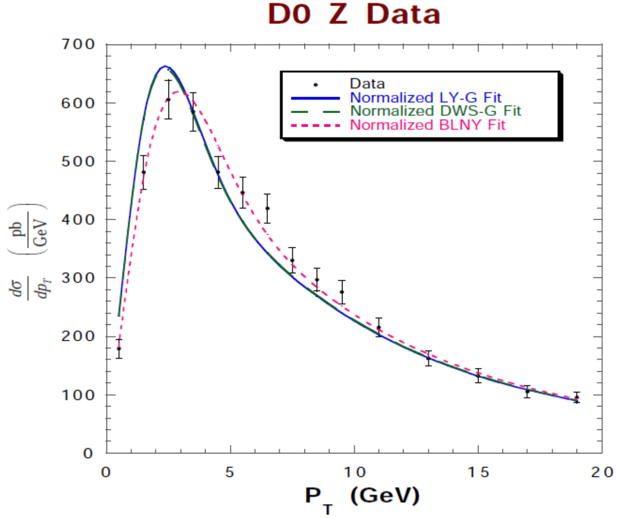
is to be ***extracted*** from comparison to experimental data

"Standard" assumption $g_K(b_T) \sim g_2 b_T^2 + \ldots$

Konychev, Nadolsky, (2005)
Landry,Brock, Nadolsky, Yuan (2002)

leads to a very successful description of DY, Z, W data





Evolution

Including HERMES, COMPASS data and matching low energy and high energy data becomes highly non trivial.
Several proposals for non-perturbative contributions:

Talks by Ted Rogers, Peng Sun, Zhongbo Kang

Collins $g_K(b_T) \sim const$ at large b_T — Collins (2013)

$$g(x, b_T) \sim g b_T$$

Sun-Yuan — Sun, Yuan (2013)

$$g_K(b_T) \sim g_1 \ln(b/b_*) + g_2 b_T^2$$

$$g_K(b_T) \sim g_1/2 \ln(1 + b^2/b_{max}^2) + g_2 b_T^2$$

Aidala-Field-Gamberg-Rogers — Aidala, Field, Gamberg, Rogers (2014)

$$g_K(b_T) \sim g_2 b_{NP}^2 \ln(1 + b^2/b_{NP}^2)$$

Echevarria-Idilbi-Kang-Vitev — Echevarria, Idilbi, Kang, Vitev (2014)

$$g_K(b_T) \sim g_2 b_T^2$$

$$g(x, b_T) \sim g b_T^2$$

Experiments

Many possible solutions. The challenge is to find an optimal one.
Most probably we will need to explore a more complicated non-perturbative input

Fit SIDIS data (JLab at plab = 6 GeV, HERMES at plab = 27.5 GeV, COMPASS at plab = 160 GeV)

Targets: H and D targets. COMPASS uses LiD, NH3 target

Relatively low Q, difficult to separate into regions

Drell-Yan data, E288 at three energies plab= 200, 300, 400 GeV

Target: CU

Q > 4 GeV, easy to separate into regions

No substantial nuclear corrections found – Pavel Nadolsky

Example

Many possible solutions. Challenge is to find out an optimal one.
Let's consider an example:

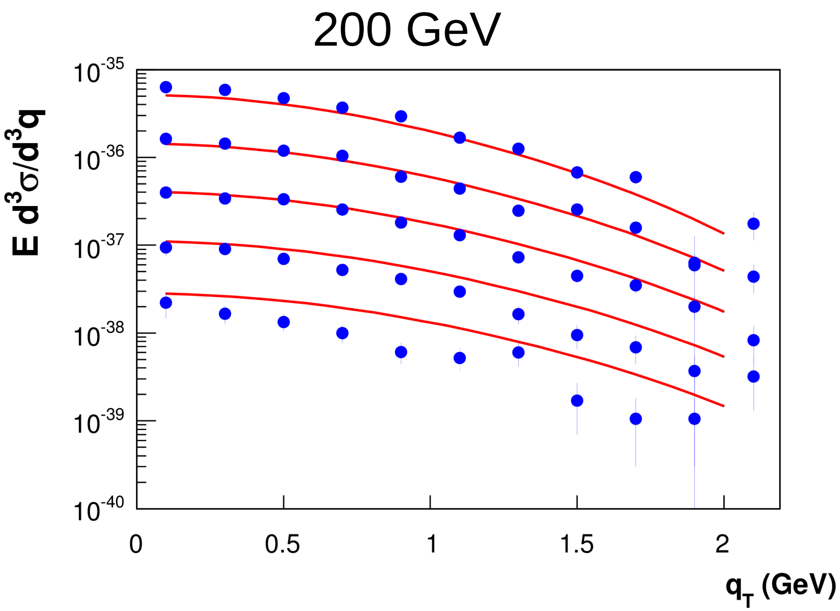
$$b_{max} = 1(\mathrm{GeV}^{-1})$$

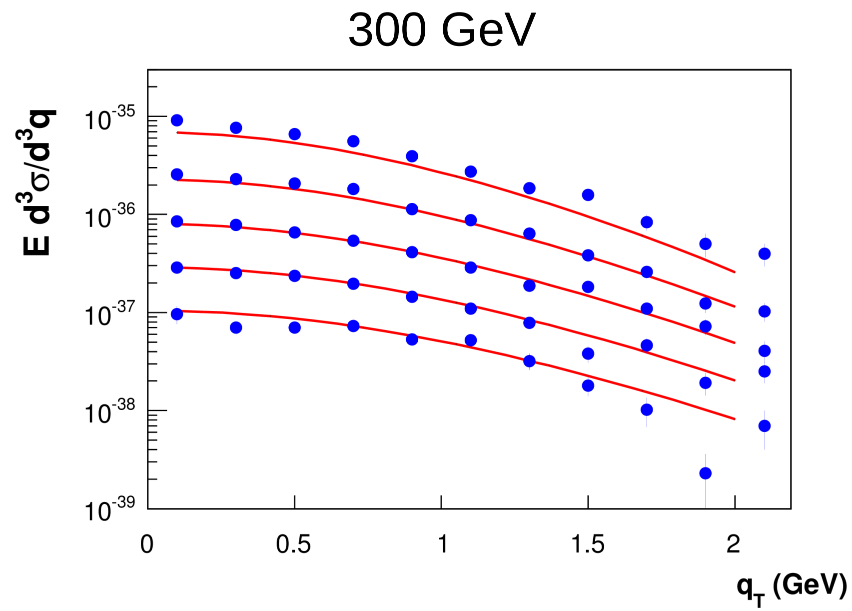
$$g_K(b_T) \sim g_2 b_T^2 + g_1 b_T$$ linear in bT

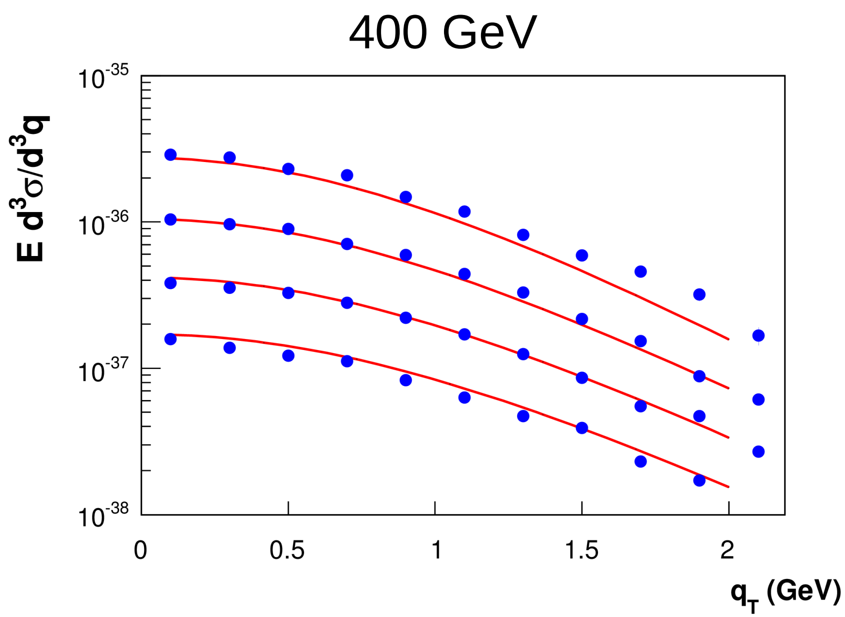
$$g(x, b_T) \sim g b_T^2$$ Gaussian input

Fit SIDIS data (HERMES at plab = 27.5 GeV) and Drell-Yan data (E288 at three energies plab= 200, 300, 400 GeV)

Drell-Yan, E288



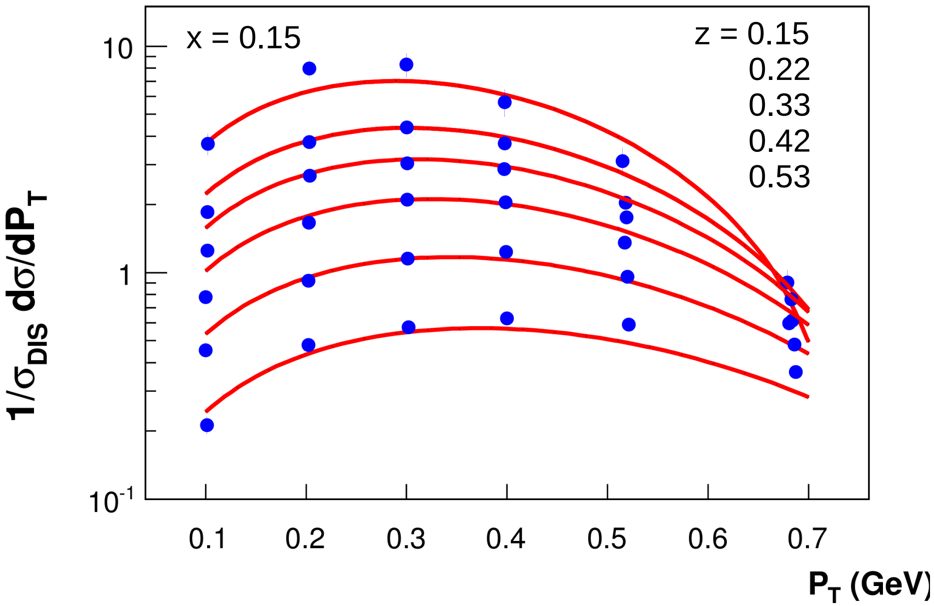




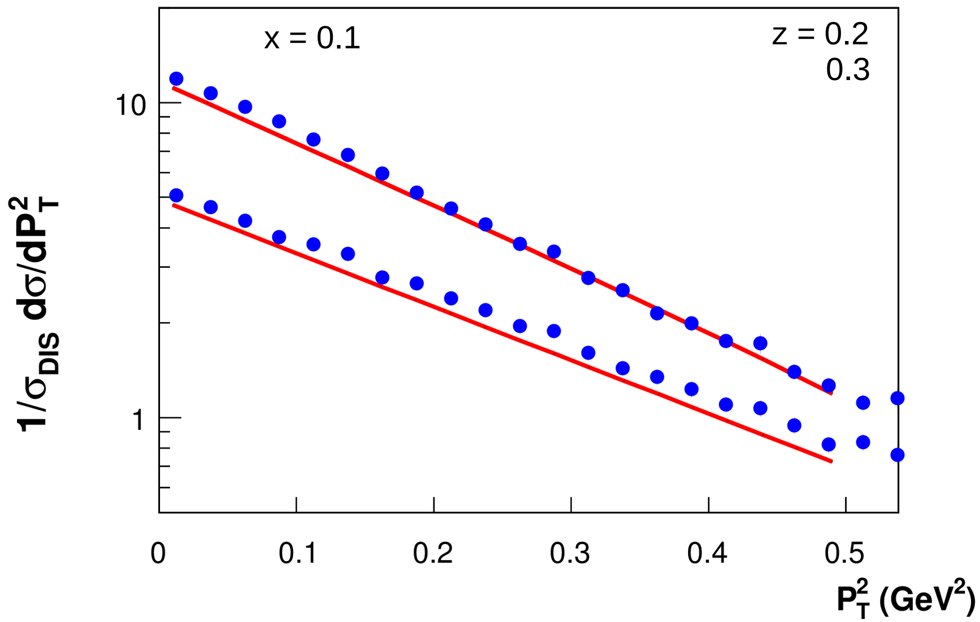
$$g_1 \sim 0.5(\text{GeV})$$

SIDIS

HERMES 27.5 GeV



COMPASS 160 GeV



$$\langle k_\perp^2 \rangle \sim 0.4(\mathrm{GeV}^2)$$

$\langle p_\perp^2 \rangle \sim 0.2(\mathrm{GeV}^2)$ close to tree level result

$$\chi^2/d.o.f \sim 2.5$$

Similar results to
Echevarria, Idilbi, Kang, Vitev (2014)

Evolution

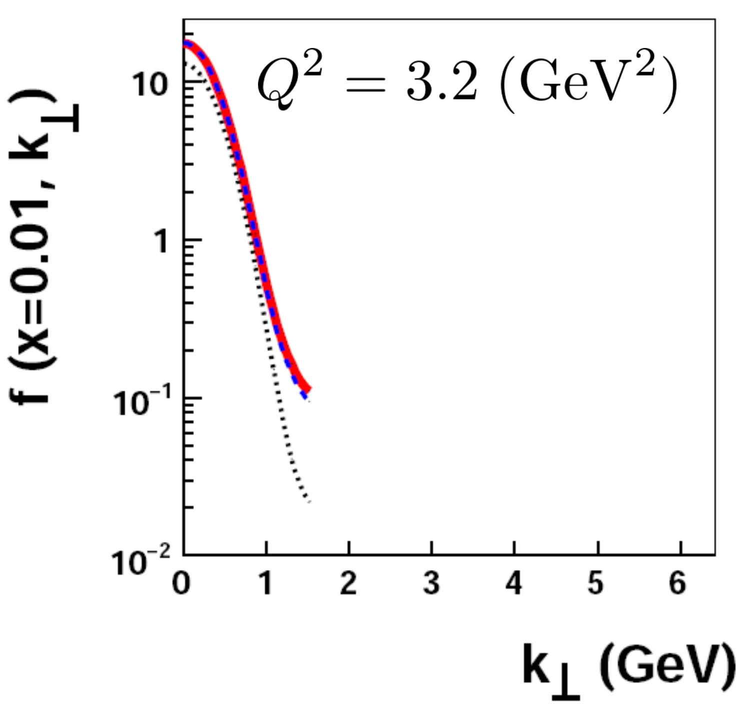
At fixed scale the form resembles very much usual “gaussian” form

$$\tilde{F}(x, b_T; Q_0, Q_0^2) \sim f(x, \mu_b)\tilde{F}_{NP}(b_T) \qquad \tilde{F}_{NP}(b_T) = e^{-\frac{b_T^2 \langle k_\perp^2 \rangle}{4}}$$

In a restricted region tree level extractions are justified

Can be improved step-by-step

“systematically improvable approximation”



Aybat, Rogers, 2011
Bacchetta, AP, 2013

Unpolarised structure function becomes

$$F_{UU,T} = x \sum_q e_q^2 f^q(x, k_\perp^2) D_q(z, p_\perp^2) \frac{e^{-P_{h\perp}^2/\langle P_{h\perp}^2 \rangle}}{\pi \langle P_{h\perp}^2 \rangle}$$

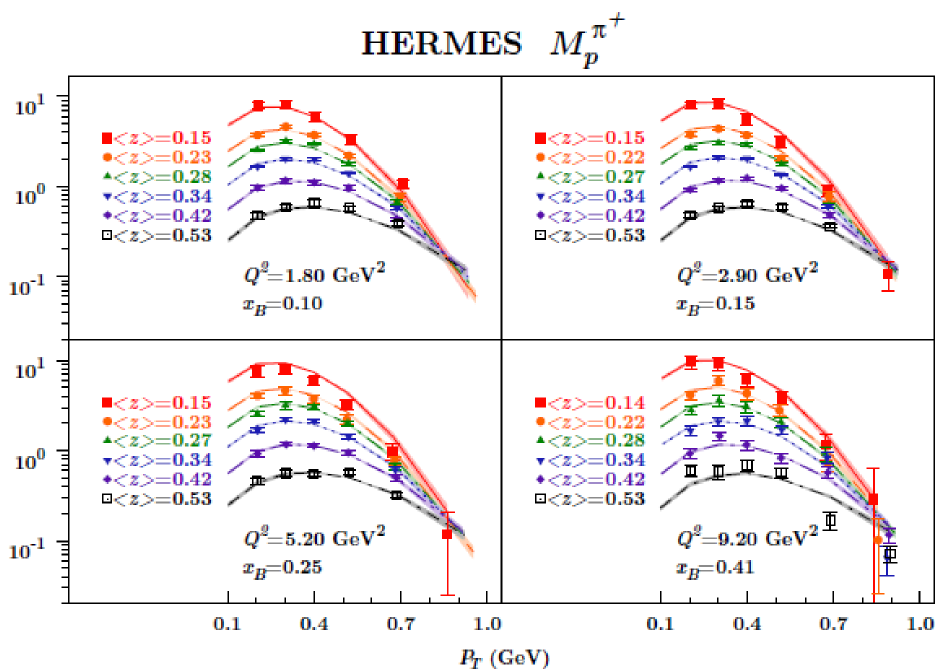
Anselmino et al 2006, 2013

$$\langle P_{h\perp}^2 \rangle = z^2 \langle k_\perp^2 \rangle + \langle p_\perp^2 \rangle$$

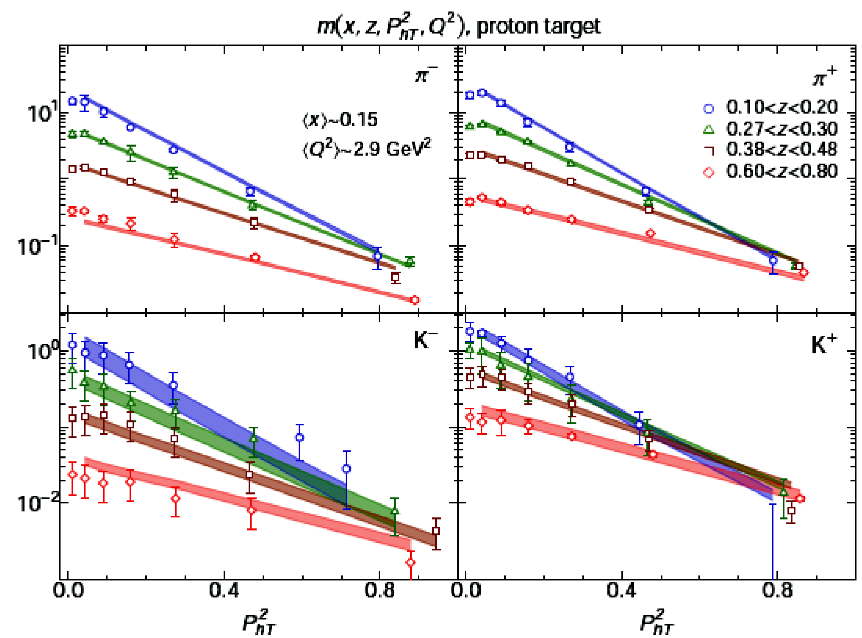
This simple model works incredibly well with HERMES data

TMDs

Anselmino et al 2013



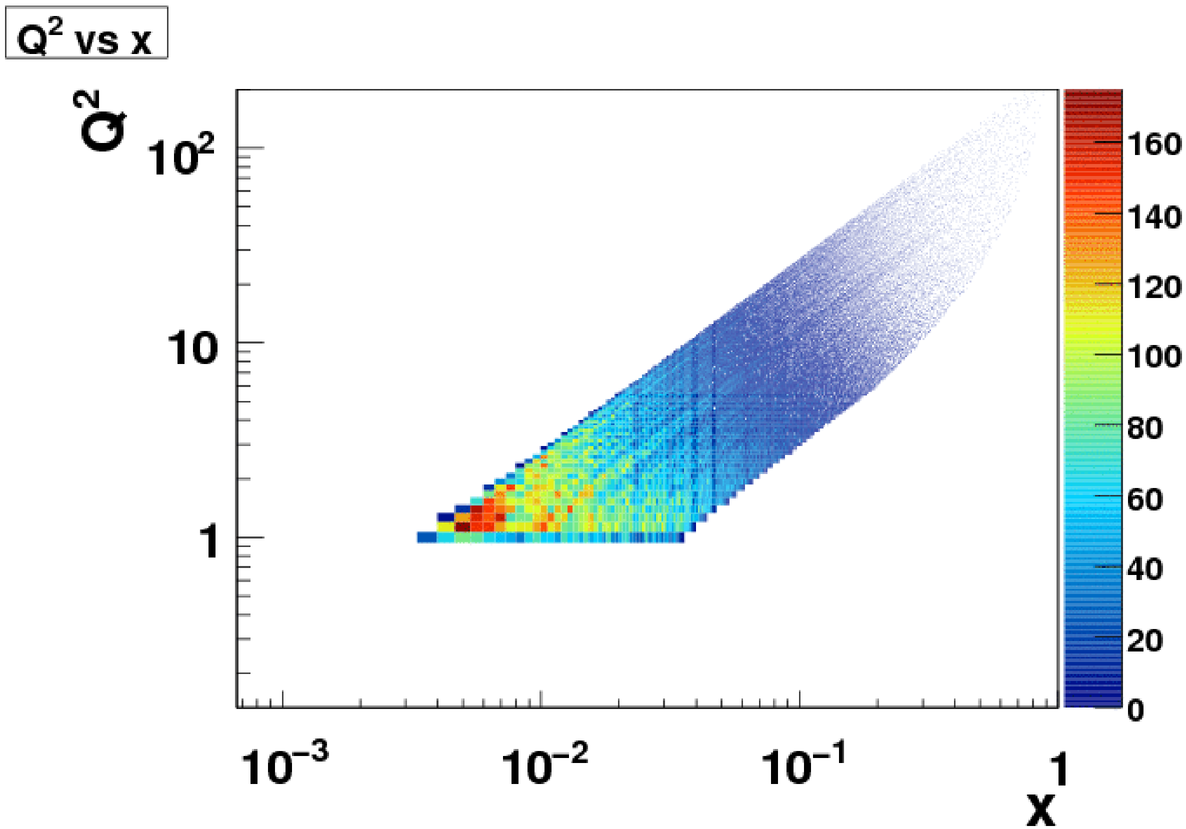
Signori, Bacchetta, Radici, Schnell 2013



See talks by
Alessandro Bacchetta and
Elena Boglione for details

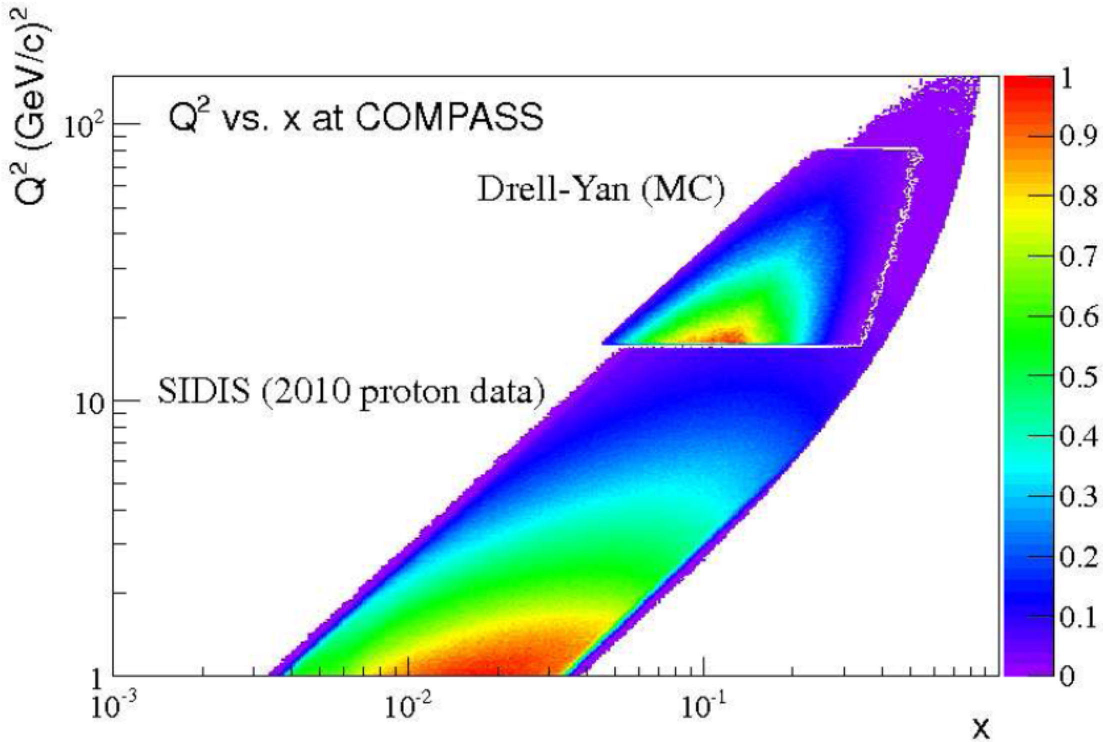
TMDs

This model is not a Monte Carlo, but we use “Monte Carlo” integration at several stages,
So, for instance, we can calculate distributions



Result of $\int \mathrm{d}z \int \mathrm{d}P_{h\perp} \int \mathrm{d}^2 k_\perp \int \mathrm{d}^2 p_\perp$ for COMPASS kinematics

To be compared to actual distribution from COMPASS



Talk by Catarina Quintans

Challenges

Partonic kinematics is simplified

$$\int_{\color{red}{0}}^{\color{red}{\infty}} k_\perp dk_\perp \int_0^{2\pi} d\phi$$

This leads to non physical negative cross-sections at large $P_{h\perp}$

It should be corrected by Y term. In any case gaussian suppresses contributions from large momenta region.

Restrictions on parton momenta might lead to substantial correction in final results

Talk by
Mher

We can provide reliable cross-sections for eventual Monte Carlo simulations

Conclusions

- TMD formalism is quite well developed both theoretically and phenomenologically. Very large amount of data spanning energies from a few GeV up to TeVs is available for analysis
- We have formalism that allows to simultaneously fit various processes, SIDIS, DY, e+e-, and extract corresponding TMD distributions
- TMD Monte Carlo is needed for experiments and should be developed