ResBos family of programs for TMD factorization

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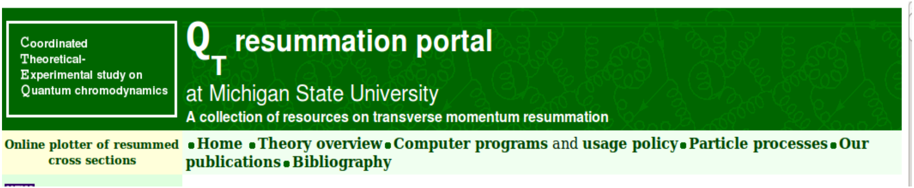
ResBos programs for various applications

ResBos programs have been developed during 20 years to predict a variety of observables using Collins-Soper-Sterman formalism. They incorporate many effects important at the latest precision level.

The computer codes, input files, and online plotter of $d\sigma/(dQ^2\,dy\,dq_T^2)$ are available at the "Q_T resummation portal", *http://hep.pa.msu.edu/resum/*.

Work in progress: produce a streamlined ResBos version and user-friendly interface for resummation at NNLO accuracy.



ResBos, RhicBos, ResBos for DIS

1. **Legacy+ResBos:** TMD factorization for unpolarized Drell-Yan-like processes at approx. NNLL/NNLO

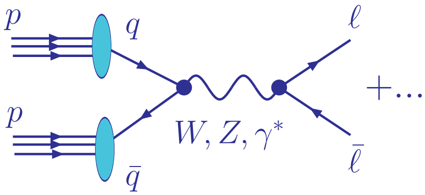
- Production+leptonic decays of γ^*, W, Z bosons *(Ladinsky, Yuan, 1993; Balazs, Qiu, Yuan, 1995; Balazs, Yuan, 1997; Brock, Landry, P.N., Yuan, 2002; ..., Guzzi, P.N., Wang, 2013)*
  - **ResBos-A:** NNLO QCD + final-state NLO QED *(Cao, Yuan, 2004)*
- Higgs bosons in SM and MSSM *(Balazs, Mrenna, Yuan; Balazs, Yuan, 2000; Belyaev, P.N., Yuan, 2006; Cao, Chen, Schmidt, Yuan, 2009; Wang et al., 2012)*
- Photon pair production *(Balazs, Berger, Mrenna, Yuan, 1997; Balazs, P.N., Schmidt, Yuan, 1998; Balazs, Berger, P.N., Yuan, 2007-08)*

2. **ResBos for semi-inclusive DIS** at NLL/NLO: light quarks *(Nadolsky, Stump, Yuan, 2001)* and massive quarks *(Nadolsky, Kidonakis, Olness, Yuan, 2002)*
3. **RhicBos** for polarized W, Z production at NLL/NLO *(Nadolsky, Yuan, 2003)*

ResBos predictions are available for...

- Tevatron and LHC
  - Precision measurement of W boson mass
  - Higgs boson searches
- Fixed-target Drell-Yan pair production
  - measurement of universal power-suppressed resummed contributions
- SIDIS at HERA, HERMES, COMPASS, JLab
  - energy flow and particle multiplicities in the current fragmentation region ($Q > 3$ GeV); heavy-flavor production
- RHIC
  - measurement of polarized parton densities in single-spin W production

TMD factorization on the example of $AB \to (Z \to \ell\bar{\ell})X$



NNLL TMD factorization in ResBos

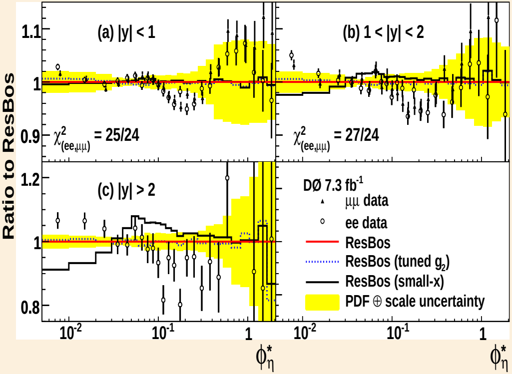
ResBos is an exact QCD calculation that includes dominant NNLL/NNLO perturbative and nonperturbative contributions.

In several comparisons, it describes the Q_T and ϕ_η^* data better than other available codes.

The agreement can be further improved both at the Tevatron and LHC by tuning QCD scales and the nonperturbative function

(Guzzi, Nadolsky, Wang, arXiv:1309.1393)

Tevatron



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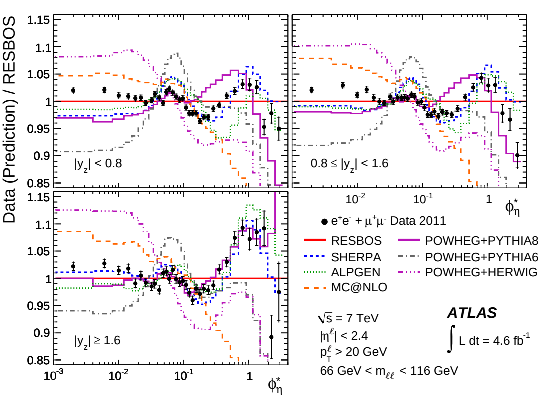
LHC



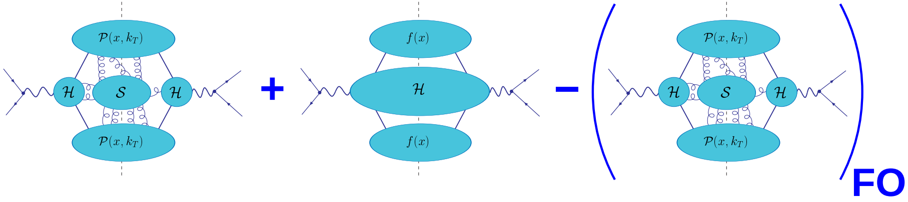
QCD factorization as a function of q_T

(according to Collins, Soper, and Sterman approach)

Small-q_T term $\Lambda^2_{QCD} \ll q_T^2 \ll Q^2$

Large-q_T term $\Lambda^2_{QCD} \ll q_T^2 \sim Q^2$

Overlap term



- k_T-dependent PDFs
$\mathcal{P}(x, \vec{k}_T)$
- Sudakov function
$\mathcal{S}(x, \vec{k}_T)$

▷ actually, their impact parameter (b) space transforms

- Collinear PDFs
$f_a(x, \mu)$
- hard matrix elements
$\mathcal{H}$ of order N

- Truncated perturbative expansion

$$\sum_{k=0}^{N} \alpha_s^k \sum_{m=0}^{2k-1} c_{km} \ln^m \left(\frac{q_T^2}{Q^2}\right)$$

Q_T distribution for $AB \to VX$

$$\frac{d\sigma_{AB\to VX}}{dQ^2 dy dq_T^2} = \sum_{a,b=g,\overset{(-)}{u},\overset{(-)}{d},\ldots} \int \frac{d^2b}{(2\pi)^2} e^{-i\vec{q}_T\cdot\vec{b}} \widetilde{W}_{ab}(b,Q,x_A,x_B) + Y(q_T,Q,x_A,x_B)$$

$$\widetilde{W}_{ab}(b,Q,x_A,x_B) = |\mathcal{H}_{ab}|^2 \; e^{-\mathcal{S}(b,Q)} \overline{\mathcal{P}}_a(x_A,b) \overline{\mathcal{P}}_b(x_B,b)$$

$\mathcal{S}$ is the soft (Sudakov) function:

$$\mathcal{S}(b,Q) = \int_{b_0^2/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\mathcal{A}(\alpha_s(\bar{\mu})) \ln \frac{\bar{\mu}^2}{Q^2} + \mathcal{B}(\alpha_s(\bar{\mu})) \right], \quad b_0 = 2e^{-\gamma_E} \approx 1.12$$

$\overline{\mathcal{P}}_a(x,b)$ are b-dependent PDF's; if $b^2 \ll Q^{-2}$,

$$\overline{\mathcal{P}}_a(x,b) = \sum_c \left[\mathcal{C}_{a/c} \otimes f_c \right] (x, b, \mu_F = \frac{b_0}{b})$$

Y is the difference of the finite-order and overlap (asymptotic) terms

TMD factorization vs. resummation

By tradition, ResBos is described as a Q_T resummation program. But it includes full logarithmic structure according to the TMD factorization.

At $q_T \to 0$, one can formally write *(Bozzi, Catani, de Florian, Grazzini,...)*

$$\frac{d\sigma}{dQ^2\, dy\, dq_T^2} = \int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}} \times$$
$$\sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1, \mu_F)\, f_{b/h_2}(x_2, \mu_F)\, \widetilde{W}_{ab}(\vec{b}, Q, x_1, x_2; \alpha_s(\mu_R), \mu_F),$$

where

$$\widetilde{W}_{ab}(b, Q; \alpha_s(\mu_R), \mu_R, \mu_F) = \mathcal{H}_{ab}^F\left(Q, \alpha_s(\mu_R); Q^2/\mu_R^2, Q^2/\mu_F^2; C_1, C_2\right)$$
$$\times \exp\{\mathcal{G}_{ab}(\alpha_s(\mu_R), C_1/b, C_2 Q)\},$$

and I choose $\mu_R \sim \mu_F \sim Q$ (the boson's virtuality); $C_1,\ C_2 \sim 1$.

TMD factorization vs. resummation

$$\widetilde{W}_{ab}(b,Q;\alpha_s(\mu_R),\mu_R,\mu_F) = \mathcal{H}^F_{ab}\left(Q,\alpha_s(\mu_R);Q^2/\mu_R^2,Q^2/\mu_F^2;C_1,C_2\right) \times \exp\{\mathcal{G}_{ab}(\alpha_s(\mu_R),C_1/b,C_2 Q)\}$$

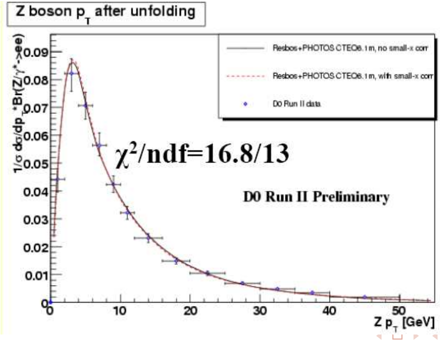
In genuine resummation (e.g., in HRes program), $\mathcal{G}_{ab}$ is evaluated as an expansion in α_s up to order N_{order} and in leading logarithms,

$$\mathcal{G}_{ab} = \sum_{n=1}^{N_{order}} \left(\frac{\alpha_s(\mu_R)}{\pi}\right)^n \sum_{k=1}^{2n-3} g_{nk}(C_1,C_2)\ln^k\left(\frac{C_2 Q\, b}{C_1}\right).$$

The CSS representation does not perform this expansion, includes logarithmic terms to all orders in α_s in the solutions for RG and gauge invariance.

What is in the latest ResBos?

- Resummation module for W and Z production and NNLO contributions – slow
  (**Legacy** — *Ladinsky, Yuan, 1993; Brock, Landry, P. N., Yuan, 2002)*
- Monte-Carlo integration module for W and Z decay and matching of small-q_T and large-q_T terms – fast
  (**ResBos** — *Balazs, Yuan, 1997;..., Guzzi, Nadolsky, Wang, 2013)*



What is in the latest ResBos?

★ Close approximation to the full NNLL/NNLO computation at the lepton level

★ Sufficient for describing the current Z data, will continue to advance to include remaining small NNLO terms.

- Small Q_T: Exact coefficients $\mathcal{A}^{(3)}, \mathcal{B}^{(2)}$; the $\mathcal{C}^{(2)}$ coefficient found numerically using CANDIA *(Guzzi, Cafarella, Corianò, 2006)*

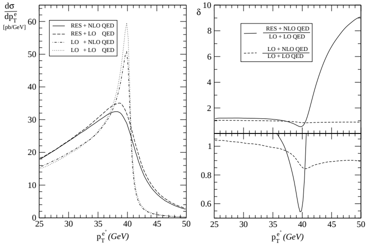
- Large Q_T: The $Y = Y_{NLO} K_{NNLO}$ piece is computed up to $O(\alpha_s^2)$ by *Arnold and Reno Nucl.Phys. B319 (1989); Arnold and Kauffman Nucl.Phys. B349 (1991), for the dominant structure function.*

- Complete scale dependence at NNLL/NNLO; reduced scale uncertainty compared to NLL/NLO

What is in the latest ResBos?

Electroweak contributions at all Q_T

- W, Z width in effective Born approximation
- full $\gamma^* - Z$ interference
- ResBos-A: + final-state QED radiation in W and Z production *(Cao, Yuan)*
  - both W term (2004) and Y term

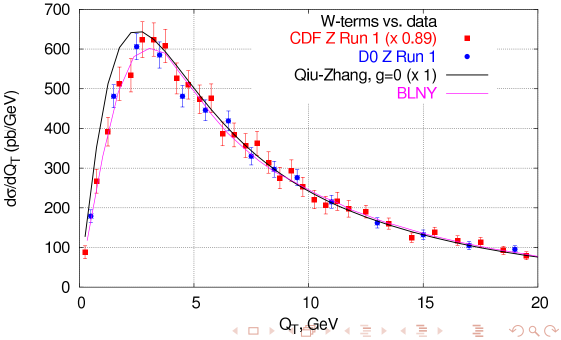


What is in the latest ResBos?

Nonperturbative model at $b \gtrsim 1$ GeV^{-1}:

- revised "b_*" approximation + a power-suppressed term $\propto b^2$ *(Collins, Soper, Sterman, 1985; Konychev, P. N., 2005)*
- replaces BLNY model *(Brock, Landry, P.N., Yuan)* used in Tevatron Run-2 M_W measurements

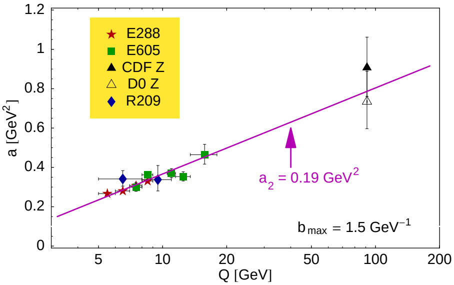
- can approximate a variety of nonperturbative models *(BLNY, Qiu, Zhang; Kulesza, Sterman, Vogelsang;...)*



What is in the latest ResBos?

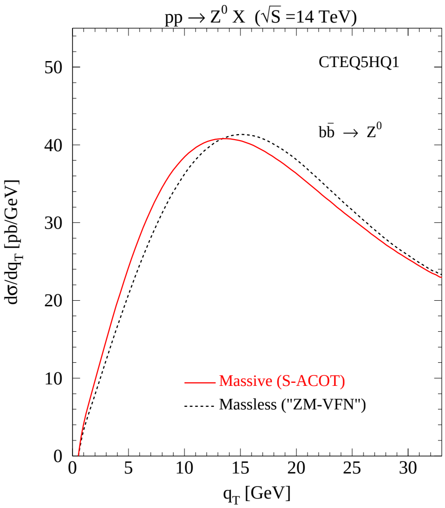
Gaussian $\mathcal{F}_{NP}(b,Q) = b^2\left[0.20 + 0.19\ln(Q/3.2) - 0.026\ln(100x_A x_B)\right]$

- linear $\ln Q$ dependence, in **quantitative** agreement with SIDIS q_T fit and infrared renormalon estimates *(Tafat)*
- small $\sqrt{s}$ dependence
- no tangible flavor dependence
- supports dominance of soft contributions in $\mathcal{F}_{NP}(b,Q)$
- **applies at** $x \gtrsim 10^{-2}$



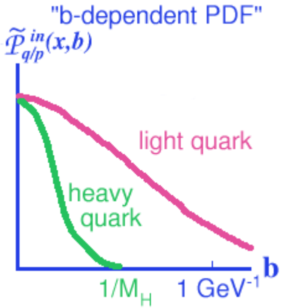
m_b dependence in $b\bar{b} \to Z^0$

(Nadolsky, Kidonakis, Olness, Yuan, 2002; Berge, Nadolsky, Olness, 2005)



The convolutions $[\mathcal{C}_{j/g} \otimes f_g](x, b, \mu_F)$ with c, b quarks are evaluated with $m_H \neq 0$ in the S-ACOT scheme

The Sudakov exponential is kept massless



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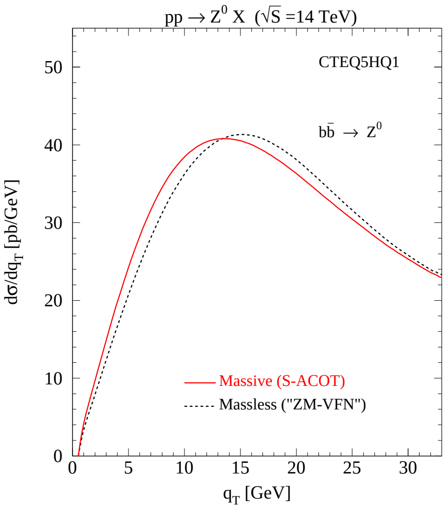
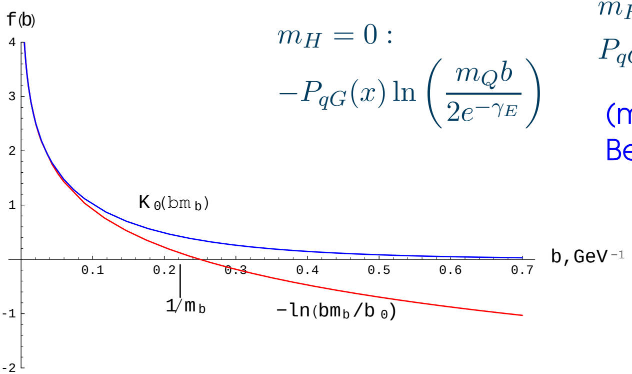


- The shape of "massless" $d\sigma/dQ_T$ varies considerably depending on the assumed continuation to $b > 1/m_b$
- With full m_b dependence, $d\sigma/dQ_T$ is well-defined; low sensitivity to nonperturbative scattering contributions
- 5 MeV effects on M_W at the LHC

m_Q suppresses contributions from $1/b \lesssim m_H$



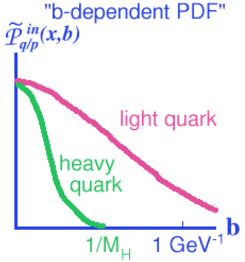
$m_H = 0:$

$$-P_{qG}(x)\ln\left(\frac{m_Q b}{2e^{-\gamma_E}}\right)$$

$m_H \neq 0:$

$P_{qG}(x)K_0\left(m_Q b\right)$

(modified Bessel function)



For $m_Q^2 \gg \Lambda_{QCD}^2$, the resummed cross section can be calculated without the nonperturbative input from $b \sim \Lambda_{QCD}$!

What is in the latest ResBos?

PDF reweighting and ROOT ntuple output

If the central PDF cross section σ_0 and PDF uncertainty $\Delta\sigma^2$ are estimated by generating $\overline{N}$ Monte-Carlo integrator events for each error PDF $f^{(i)}(x,\mu)$ ($i = 0{,}2N$), their MC estimates are

$$\overline{\sigma}_0 \sim \sigma_0 + \frac{c}{\overline{N}^{1/2}} \text{ and}$$

$$\overline{\Delta\sigma^2} \sim \Delta\sigma^2 + \frac{c' N}{\overline{N}^{1/2}}$$

- a large factor of $N \sim 22$ in the MC error for $\overline{\Delta\sigma^2}$ due to randomness of event generation for each PDF!
- need N^2 more MC events to evaluate σ^2

What is in the latest ResBos?

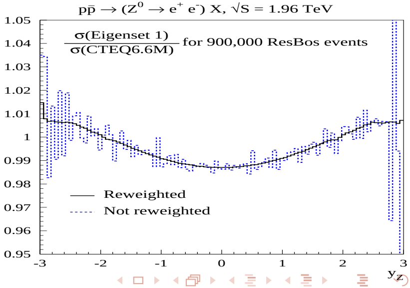
PDF reweighting and ROOT ntuple output

- PDF reweighting generates the same sequence of events to compute each of $2N$ cross sections
  - $\overline{\Delta\sigma^2} \approx \Delta\sigma^2 + \mathcal{O}(\overline{N}^{-1})$
- In multi-loop calculations, PDF reweighting saves CPU time drastically by reducing slow computations of hard-scattering

```
// These are the C functions accessible from Fortran.


extern "C" {
  //Initialization of the ROOT file
  void initrootnt_(const char *title, const char *access, int ltitle, int laccess);
  void reinitrootnt_(const char *access, int laccess);
  void addntbranch_(float *element, const char *chtag, int ltag);
  void fillntbranch_(const char *chtag, int ltag);
  int getnumbranches_();
  void rootntoutp_();
  void printnt_();
  void teststr_(const char *str, int lstr);

}//extern "C"
```



QCD scale dependence and matching

The full small-b form factor in ResBos is

$$\widetilde{W}^{pert}_{\alpha,j} = \sum_{j=u,d,s\ldots} |H_{\alpha,j}(Q,\Omega,C_2Q)|^2$$

$$\times \exp\left[-\int_{C_1^2/b^2}^{C_2^2Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\mathcal{A}(\bar{\mu};C_1)\,\ln\left(\frac{C_2^2Q^2}{\bar{\mu}^2}\right) + \mathcal{B}(\bar{\mu};C_1,C_2)\right]$$

$$\times \sum_{a=g,q,\bar{q}} [\mathcal{C}_{ja}\otimes f_{a/h_1}]\left(\chi_1,\frac{C_1}{C_2},\frac{C_3}{b}\right)\sum_{b=g,q,\bar{q}} [\mathcal{C}_{\bar{j}b}\otimes f_{b/h_2}]\left(\chi_2,\frac{C_1}{C_2},\frac{C_3}{b}\right).$$

1. The scales $C_1 = b\bar{\mu}$ and $C_2 = \bar{\mu}/Q$ provide lower and upper integration limits; $\mu_F = C_3/b$ is the factorization scale in collinear PDFs.

2. $\chi_{1,2}$ are longitudinal variables that improve matching at $Q_T \sim Q$. They reduce to $x_{1,2}$ at q_T^2/Q^2 and suppress $W - ASY$ at $q_T^2 \sim Q^2$.

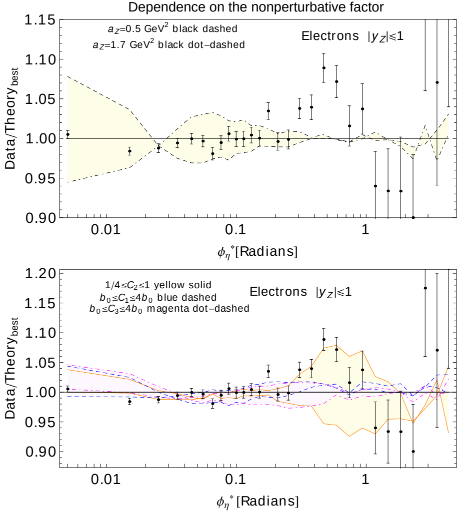
Separating dependence on QCD scales and nonperturbative Q_T smearing

The nonpert. function $\mathcal{F}_{NP}$ found from the experiment depends on the assumed QCD scales.

Variations in $\mathcal{F}_{NP}$ affect only the region $Q_T^2 \ll Q^2$

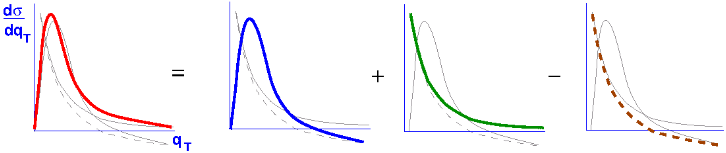
Variations of QCD scales affect a wide range of Q_T

This difference is used to separate $\mathcal{F}_{NP}$ dependence from scale dependence *(⇒Guzzi)*

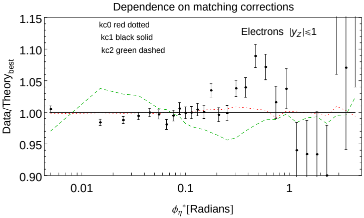


Data from D0 Run-2, 1010.0262(hep-ex)

Matching of W and Y terms



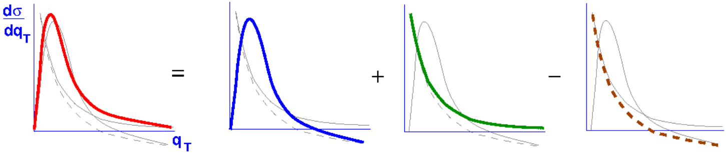
Away from the $Q_T \ll Q$ region, there is an additional uncertainty associated with matching of the W and Y terms.



In ResBos, the matching uncertainty is evaluated explicitly by choosing different scaling variables, e.g.,

$$\chi_{1,2} = \underbrace{\frac{Q}{\sqrt{s}} e^{\pm y}}_{x_{1,2}} \cdot \left(1 + \frac{q_T^2}{Q^2}\right).$$

Matching of W and Y terms



In other calculations, this systematic uncertainty is hidden, a fixed matching prescription is used.

For instance, matching in the resummation calculation by Bozzi et al. (hep-ph/0508068) is based on replacement

$$L = \log\left(\frac{C_2 Q\, b}{2e^{-\gamma_E}}\right) \to L' = \log\left(\frac{C_2 Q\, b}{2e^{-\gamma_E}} + 1\right).$$

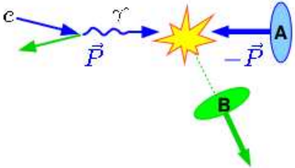
It provides one possible way to match the $W + Y$ and FO contributions

TMD factorization for semi-inclusive DIS

(ResBos for DIS)

P. Nadolsky, D. Stump, C.-P. Yuan, hep-ph/9906280; hep-ph/0012261; hep-ph/0012262

Semi-inclusive DIS in $\gamma^* p$ c.m. frame

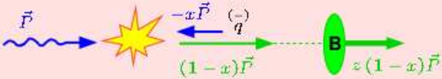


Incoming particles

Outgoing particles

$$0 \le x \le 1$$
$$0 \le z \le 1$$

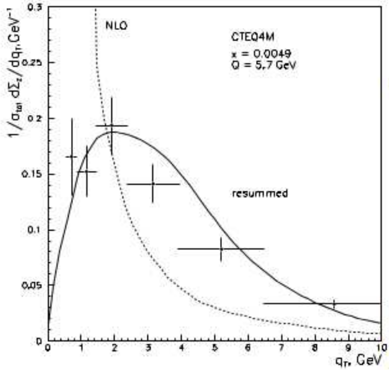
No QCD radiation



QCD radiation

Soft and collinear radiation in SIDIS

CSS refactorization formalism can be applied to resum large logarithmic terms in the hadronic energy flow (R. Meng, F. Olness, D. Soper, 1996) and particle cross sections (P. N., D. Stump, C.-P. Yuan, 1999-2000)



$$q_T = W e^{-\eta_{c.m.}},$$

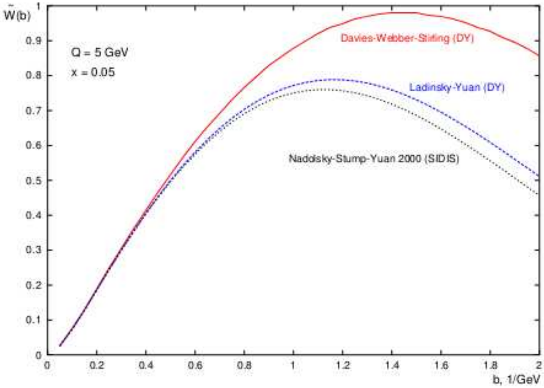
where

$$W^2 = Q^2\left(\frac{1}{x} - 1\right)$$

$$\lim_{\theta\to 0} q_T = \frac{W}{2}\left(\theta + \frac{\theta^3}{12} + \dots\right)$$

Σ_z is a rescaled transverse energy flow in the $\gamma^* p$ c.m. frame

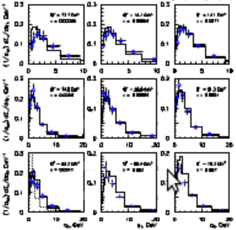
Comparison of b-space form-factors in Drell-Yan process and SIDIS energy flow



$x > 10^{-2}$: $\mathcal{F}_{NP}$ is SIDIS is similar to $\mathcal{F}_{NP}$ in DY process at comparable Q

q_T dependence of E_T flow at small x

Data from *H1 Collaboration*



$13.1 < \langle Q^2 \rangle < 70.2$ GeV^2,

$8 \times 10^{-5} < \langle x \rangle < 7 \times 10^{-3}$

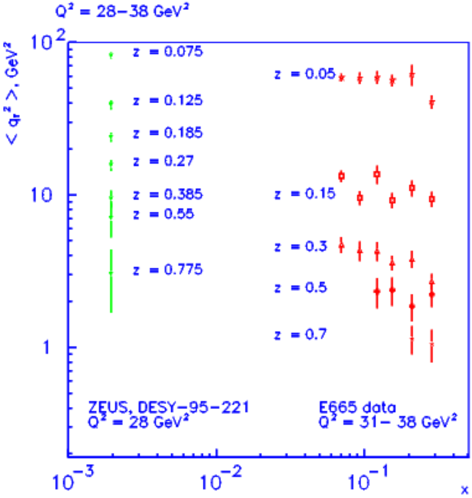
Resummed z-flow: CTEQ5M1 PDFs,

$$S_z^{NP} = b^2 \left\{ 0.013 \frac{(1-x)^3}{x} + 0.19 \ln\left(\frac{Q}{2\ \text{GeV}}\right) \right\}$$

Possible interpretation:
rapid increase of "intrinsic" k_T when x decreases (first BFKL signs???)
No mechanism for such increase in the $\mathcal{O}(\alpha_s)$ part of the CSS formula

$x < 10^{-2}$: Strong x dependence in $\mathcal{F}_{NP}$ of SIDIS, corresponding to broader $d\sigma/dq_T$

$\langle q_T^2 \rangle$ vs. x and z in charged particle production



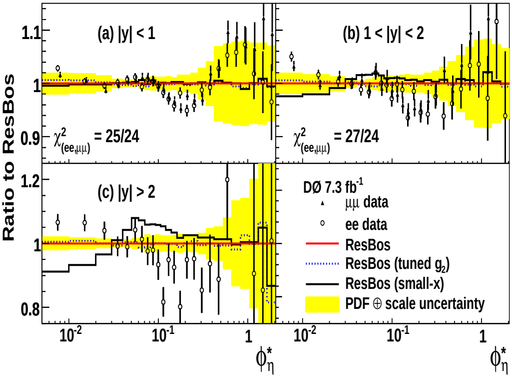
Similar increase of $\langle q_T^2 \rangle$ as $x \to 0$ as in the E_T flow data

Z production at the Tevatron and LHC: strong small-x broadening is disfavored

No rapidity dependence is observed in $\mathcal{F}_{NP}(b, Q, x_1, x_2)$ at $Q \approx M_Z$

Compatibility with low x SIDIS data?

Cancellation of large x and small x dependence in $\mathcal{F}_{NP}$?



New studies of x dependence of $\mathcal{F}_{NP}(b, Q, x, z)$ in SIDIS will be very interesting

SM Higgs and $\gamma\gamma$ production in ResBos

Balazs, Yuan, hep-ph/0001103; Cao, Chen, Schmidt, Yuan, arXiv:0909.2305;

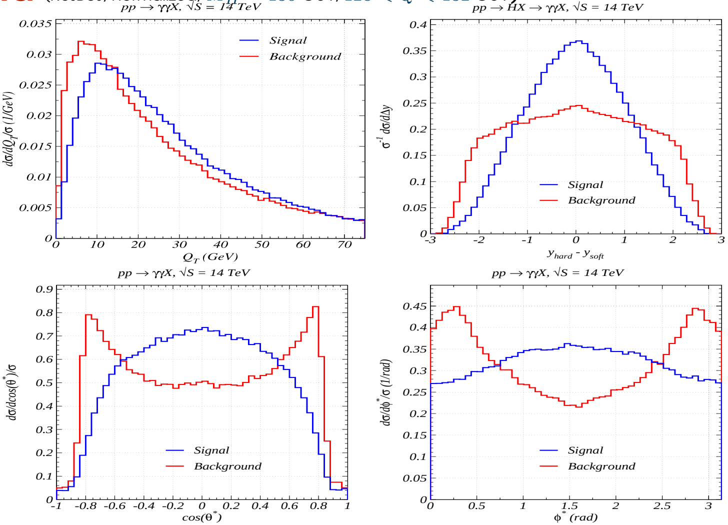
Balazs, Berger, Mrenna, Yuan, hep-ph/9712471; Balazs, Nadolsky, Schmidt, Yuan, hep-ph/9905551; Nadolsky, Schmidt, hep-ph/0211398; Balazs, Berger, Nadolsky, Yuan, hep-ph/0603037; hep-ph/0702003; arXiv:0704.0001

NNLL/NLO distributions for Higgs $\to \gamma\gamma$ signal and background (ResBos, normalized; $M_H = 130$ GeV, $128 < Q < 132$ GeV)

Q_T and $y_{\gamma_1} - y_{\gamma_2}$ in the lab frame

Decay angles θ_*, φ_* in the $\gamma\gamma$ rest frame

no singularities, in contrast to the fixed-order rate



Spin-flip collinear term in the $g_1 g_2 \to \gamma_3 \gamma_4 g_5$ amplitude

(PRD 76, 013008 (2007); see also Bern, Dixon, Schmidt, hep-ph/0206194)

- includes a novel $1/q_T^2$ term proportional to interference between $2 \to 2$ matrix elements with opposite spins of gluon 1 and a universal spin-flip splitting function $P'_{g/g}(x)$
- is also present in the Catani-Seymour dipole formalism
- arises because of incomplete factorization of helicity dependence in TMD distributions of linearly polarized gluons
- affects dependence on the azimuthal (φ_*) and polar (θ_*) angles of photons in the Collins-Soper $\gamma\gamma$ rest frame
- The full formalism for resummation of the spin-flip term is developed by *Catani & Grazzini, arXiv:1011.3918*
- Resummation of the spin-flip term is now implemented in ResBos (Zhao Li, C.-P. Yuan)

Spin-flip collinear term in the $g_1 g_2 \to \gamma_3 \gamma_4 g_5$ amplitude

(PRD 76, 013008 (2007); see also Bern, Dixon, Schmidt, hep-ph/0206194)

$$|\mathcal{M}_5(1,2,3,4,5)|^2 \xrightarrow{5\|1} \frac{\sigma_g^{(1)}}{2\widehat{x}_1 p_1 \cdot p_5} \left\{ P_{g/g}(\widehat{x}_1) L_g(\theta_\star) + P'_{g/g}(\widehat{x}_1)\, L'_g(\theta_\star)\; \cos 2\varphi_\star \right\}$$

$P_{g/g} L_g$ is the usual collinear term, with

$$, \qquad P_{g/g} = 2C_A \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \beta_0 \delta(1-x),$$

$$L_g(\theta_\star) \equiv \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4=\pm 1} |\mathcal{M}_4(\lambda_1,\lambda_2,\lambda_3,\lambda_4)|^2$$

$P'_{g/g} L'_g \cos 2\varphi_\star$ is the interference (spin-flip) term, with

$$P'_{g/g}(x) = 2C_A(1-x)/x$$

$$L'_g(\theta_\star)\cos 2\varphi_\star = \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4=\pm 1} \mathcal{M}_4^*(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\mathcal{M}_4(-\lambda_1,\lambda_2,\lambda_3,\lambda_4)$$

Conclusions

- ResBos for Drell-Yan-like processes continues to develop to include various effects relevant in precision tests of TMD factorization
- ResBos for SIDIS at NLO needs an upgrade to be confronted with the low-Q SIDIS data
    - Measurements of hadronic energy flow, particle multiplicities in unpolarized SIDIS at different x, $q_T = p_T/z$, $\eta_{c.m.}$ are very instructive tests of TMD factorization
- See talks by Marco Guzzi and Ted Rogers about the latest applications based on ResBos programs