Theory of hard exclusive processes and GPD fits

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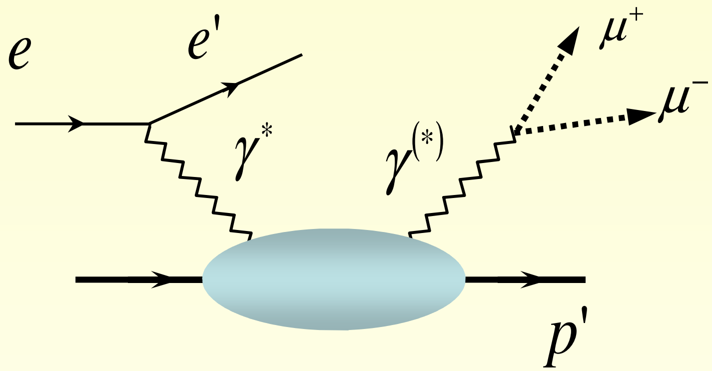
- *A little bit theory*
- *GPD parametrizations*
- *Data description/predicitions*
- *Conclusions*

K. Kumerički (KK), E. Aschenauer, S. Firzo, M. Murray
K. Passek-Kumerički (KP-K), T. Lautenschlager, A. Schäfer; M. Meskauskas
A. Belitsky, Y. Ji; V. Braun, A. Manashov, B. Pirnay
D.S. Hwang

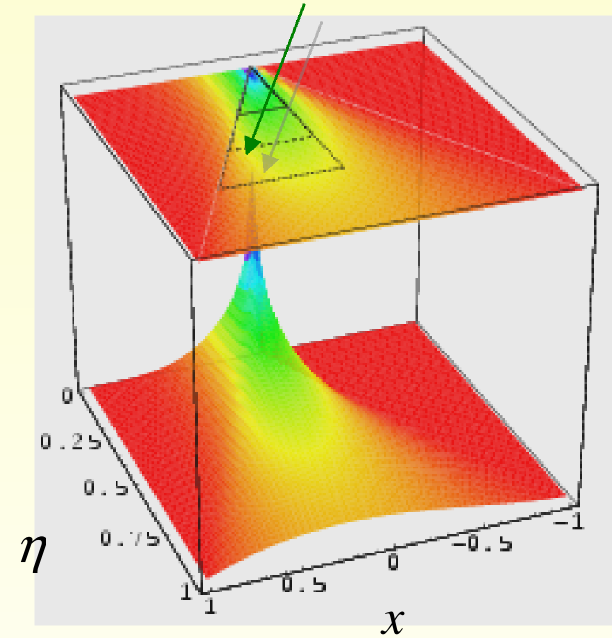
GPD related hard exclusive processes

- Deeply virtual Compton scattering (clean probe)

$$ep \to e'p'\gamma$$
$$ep \to e'p'\mu^+\mu^-$$
$$\gamma p \to p'e^-e^+$$



scanned area of the surface as a functions of lepton energy

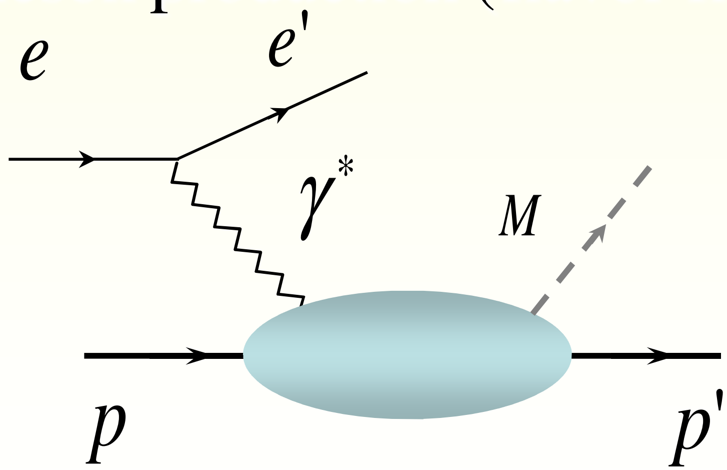


$ep \to e'\,p'\,\mu^+\mu^-$

factorization proof for transversal cross sections
[Collins Freund (99)]

- Deeply virtual meson production (flavor filter)

$$ep \to e'p'\pi$$
$$ep \to e'p'\rho$$
$$ep \to e'n\pi^+$$
$$ep \to e'n\rho^+$$



<u>twist-two observables:</u>

longitudinal cross sections

transverse target spin asymmetries

- etc.

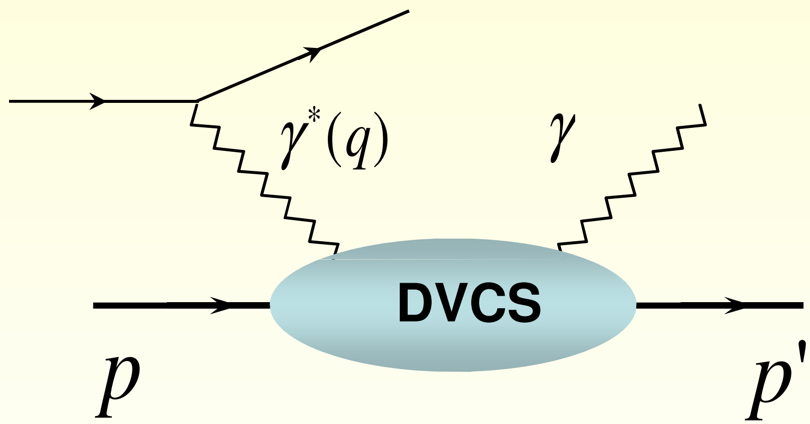
factorization proof for longitudinal cross sections
[Collins, Frankfurt, Strikman (96)]

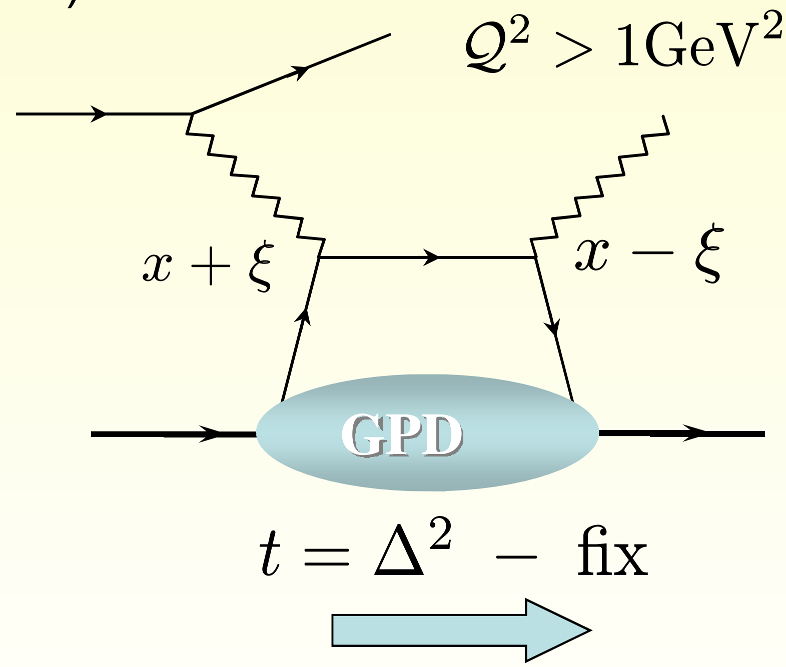
GPDs embed non-perturbative physics

GPDs appear in various hard exclusive processes,

[DM et. al (91/94)
Radyushkin (96)
Ji (96)]

e.g., hard electroproduction of photons (DVCS)





$$\mathcal{F}(\xi,\mathcal{Q}^2,t) = \int_{-1}^{1} dx\, C(x,\xi,\alpha_s(\mu),\mathcal{Q}/\mu) F(x,\xi,t,\mu) + O(\tfrac{1}{\mathcal{Q}^2})$$

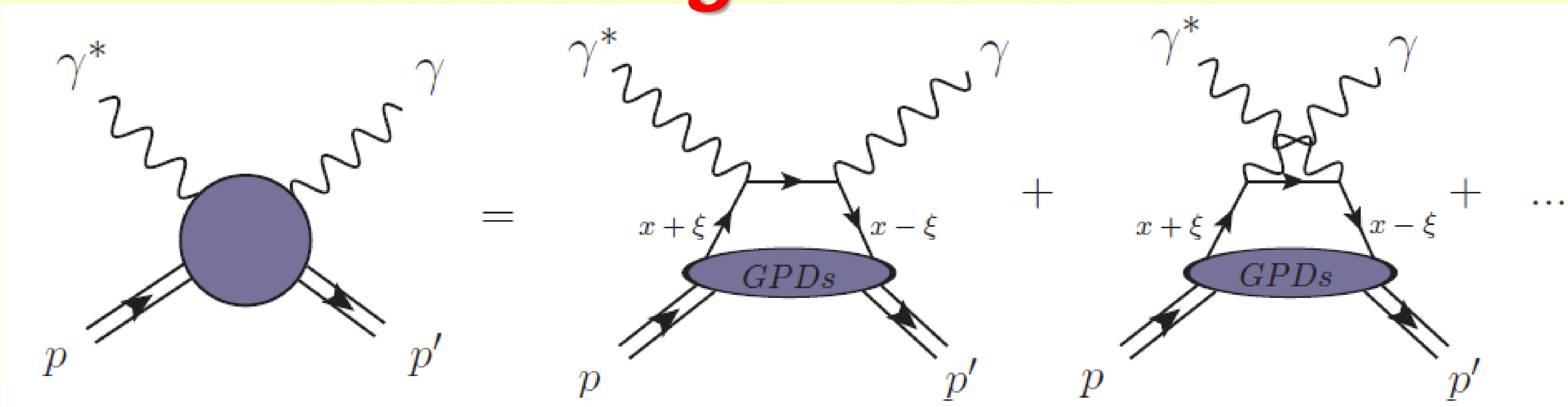
CFF
Compton form factor
observable

hard scattering part
perturbation theory
(our conventions/microscope)

GPD
universal
(conventional)

higher twist
depends on
approximation

Calculating DVCS tensor



$$T_{\mu\nu} = i \int d^4x \, \mathrm{e}^{\frac{i}{2}(q_1+q_2)\cdot x} \langle p_2 | T\left\{ j_\mu(x/2) j_\nu(-x/2) \right\} | p_1 \rangle$$

- collinear factorization approach (calculating Feynman diagrams on partonic level)
- operator product expansion (in terms of light-ray operators)

$$T j_\mu(x/2) j_\nu(-x/2) \stackrel{\mathrm{LO}}{=} \frac{S_{\mu\nu\alpha\beta} i x^\alpha}{(x^2 - i\epsilon)^2} \left[\overline{\psi}(x/2)\gamma^\beta \psi(-x/2) - \overline{\psi}(-x/2)\gamma^\beta \psi(x/2) \right]$$
$$+ \frac{i\epsilon_{\mu\nu\alpha\beta} i x^\alpha}{(x^2 - i\epsilon)^2} \left[\overline{\psi}(x/2)\gamma^\beta\gamma^5 \psi(-x/2) + \overline{\psi}(-x/2)\gamma^\beta\gamma^5 \psi(x/2) \right]$$

- expansion in leading $1/x^2$ singularities is easily done by projection on the light cone $n_\mu \sim q_\mu + \ldots$ and $n^*_\mu \sim P_\mu + \ldots$ or $n_\mu = q_{2\mu}$ and $n^*_\mu = q_{1\mu} + \ldots q_{2\mu}$

  with $q_\mu = (q_{1\mu} + q_{2\mu})/2$ and $P_\mu = p_{1\mu} + p_{2\mu}$

$$T_{\mu\nu} \stackrel{\mathrm{LO}}{=} -g^{\perp}_{\mu\nu} \sum_q \int_{-1}^{1} dx \left[\frac{e_q^2}{\xi - x - i\epsilon} - \frac{e_q^2}{\xi + x - i\epsilon} \right] q(x, \xi, t, \mathcal{Q}^2 | s_1, s_2)$$

$$-i\epsilon^{\perp}_{\mu\nu} \sum_q \int_{-1}^{1} dx \left[\frac{e_q^2}{\xi - x - i\epsilon} + \frac{e_q^2}{\xi + x - i\epsilon} \right] \widetilde{q}(x, \xi, t, \mathcal{Q}^2 | s_1, s_2)$$

GPD nomenclature

$$q(\cdots | s_1, s_2) = \overline{u}(p_2, s_2) \left[n \cdot \gamma H(\cdots) + \frac{i n^{\alpha} \sigma_{\alpha\beta\Delta^{\beta}}}{2M} E(\cdots) \right] u(p_1, s_1)$$

$$\widetilde{q}(\cdots | s_1, s_2) = \overline{u}(p_2, s_2) \left[n \cdot \gamma \gamma^5 \widetilde{H}(\cdots) + \frac{n \cdot \Delta}{2M} \gamma^5 \widetilde{E}(\cdots) \right] u(p_1, s_1)$$

consequences of *1/Q* truncation and restriction to leading order in pQCD

- DVCS tensor structure depends on the choice of *n*
- scaling variable $\xi \sim x_B/(2-x_B)$ depends on the choice of *n*
- gauge invariance holds only to leading power accuracy
- DVCS tensor structure is not complete

to overcome these problems one should go

- to twist-3 accuracy, yields 4 other GPDs (LT photon helicity flips)
- to NLO, yields 4 gluon transversity GPDs (TT photon helicity flips)
- twist-4 accuracy pushes ambiguity to the $1/Q^4$ level **[Braun, Manashov 12]** but yields new parton correlation functions, however, no new structures

Status of theory

[Belitsky, DM (97); Mankiewicz et. al (97); Ji,Osborne (97/98); Pire, Szymanowski, Wagner (11); DM, Pire, Szymanowski, Wagner 11]

✓ ***twist-two*** DVCS coefficients at ***next-to-leading*** order

✓ twist-two DVMP coefficients at ***next-to-leading*** order

NLO effects are well understood generically [Belitsky, DM (01); Ivanov, Szymanowski,Krasnikov (04)]

large-ξ: logarithmical enhancement
valence region: weak evolution implies moderate effects
small-ξ: model dependence

DM, T. Lautschlager, K. Passek-Kumericki. A. Schaefer (13)

✓ anomalous dimensions and evolution kernels at ***next-to-leading*** order

evolution effects can be called moderate, except for H/E at small- ξ
NLO analyses have to include NLO evolution

[Belitsky, DM (98) + Freund (01)]

✓ gluon transversity at ***next-to-leading*** order [Belitsky, DM (00)]

[DM (06); KMP-K, Schaefer 06]

✓ ***next-to-next-to-leading*** DVCS order in a specific conformal subtraction scheme

NLO → NNLO corrections can be called moderate w.r.t. LO → NLO

[Anikin,Teryaev, Pire (00); Polyakov et. al (00), Belitsky DM (00); Kivel et. al, Weiss, Radyushkin (00)]

✓ ***twist-three*** including quark-gluon-quark correlation at LO

✓ partially, ***twist-three*** sector at ***next-to-leading*** order [Kivel, Mankiewicz (03)]

? `target mass corrections' (not understood) [Belitsky DM (01)]

✓ **kinematical twist-four** corrections [Braun, Manashov (11)]

Field theoretical GPD definition

GPDs are defined as matrix elements of **renormalized** ***light-ray*** operators:

DM, Robaschik, Geyer, Dittes, Hořejši (94)

$$F(x,\eta,\Delta^2,\mu^2)=\int_{-\infty}^{\infty}d\kappa\; e^{i\kappa\, x\, n\cdot P}\langle P_2|\mathcal{R}T:\phi(-\kappa n)[(-\kappa n),(\kappa n)]\phi(\kappa n):|P_1\rangle,\;\; n^2=0$$

momentum fraction x , skewness $\eta=\frac{n\cdot\Delta}{n\cdot P}$ $\Delta=P_2-P_1$ $P=P_1+P_2$ $\Delta^2\equiv t$

For a nucleon target we have four chiral even twist-two GPDs:

$$\bar\psi_i\gamma_+\psi_i \quad\Rightarrow\quad {}^i q^V = \bar U(P_2,S_2)\gamma_+ U(P_1,S_1)H_i+\bar U(P_2,S_2)\frac{i\sigma_{+\nu}\Delta^\nu}{2M}U(P_1,S_1)E_i$$

$$\bar\psi_i\gamma_+\gamma_5\psi_i \quad\Rightarrow\quad {}^i q^A = \bar U(P_2,S_2)\gamma_+\gamma_5 U(P_1,S_1)\widetilde H_i+\bar U(P_2,S_2)\frac{\gamma_5\Delta_+}{2M}U(P_1,S_1)\widetilde E_i$$

shorthands:

chiral even GPDs: $F=\{H,E,\widetilde H,\widetilde E\}$ & CFFs: $\mathcal{F}=\{\mathcal{H},\mathcal{E},\widetilde{\mathcal{H}},\widetilde{\mathcal{E}}\}$

chiral odd GPDs: $F_T=\{H_T,E_T,\widetilde H_T,\widetilde E_T\}$ $\mathcal{F}_T=\{\mathcal{H}_T,\mathcal{E}_T,\widetilde{\mathcal{H}}_T,\widetilde{\mathcal{E}}_T\}$

GPD properties (from definition)

- *polynomiality arises from Lorentz covariance*
  (but GPDs are not Lorentz invariant or covariant)

$$\int_{-1}^{1} dx\, x^n F(x,\eta,t) = \text{polynom of order } n \text{ or } n+1 \text{ in } \eta$$

- ***symmetric in η*** *(time reversal invariance+hermiticity)*
- *satisfied within double distribution representation (**GPD duality**)*

$$F(x,\eta,t) = \int_{-1}^{1} dy \int_{-1+|y|}^{1-|y|} dz\, \delta(x-y-z\eta)\,[f(y,z,t) + x\Delta f(y,z,t)]$$

- *lowest moment: partonic form factor – related to observables*
- *first moment: expectation value of energy-momentum tensor*
- *reduction to parton densities (PDFs)*

$$q(x) = \lim_{\Delta\to 0} H(x,\eta,t), \quad \Delta q(x) = \lim_{\Delta\to 0} \widetilde{H}(x,\eta,t)$$

- *positivity constraints (requirement on GPD and scheme)* ***[Pobylitsa(00,02)]***
  *are **only** automatically satisfied in the LCWF **overlap representation***

GPD representations

- *x*-(momentum fraction) representation (mostly indirectly used)
- double distribution representation (used in models: GPV, BMK, GK,...)
- conformal partial wave expansion, starting point for
  smearing **[Radyushkin (97); Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)]**
  Shuvaev transformation **[A. Shuvaev (99), J. Noritzsch (00)]**
  \`dual' param. **[M. Polyakov, A. Shuvaev (02); M. Polyakov (07), Semenov-Tian-Shansky]**
  Mellin-Barnes representation **[DM, Schaefer (05); Kirch, Manashov, Schäfer (05); ...]**
- LC-wave function overlap representation (not used in phenomenology)

toy GPD:

$$F(x,\eta) = \theta\left(\frac{\eta+x}{1-x}\right)\frac{7(1+\eta)}{8\eta^2}\left(\frac{x+\eta}{1+\eta}\right)^{\frac{3}{2}}\left[\frac{1}{2}\frac{1-x}{1+\eta}+1-\frac{x}{\eta}\right]+\{\eta\to-\eta\}$$

$$F(x,0) = \frac{35}{32}\frac{(1-x)^3}{\sqrt{x}}$$

$$F(\xi,\xi) = \frac{7}{4(1+\xi)}\sqrt{\frac{1+\xi}{2\xi}\frac{1-\xi}{1+\xi}} \quad \Leftrightarrow \quad \frac{7}{4}\frac{1-X}{\sqrt{X}}, \quad X=\frac{2\xi}{1+\xi}$$

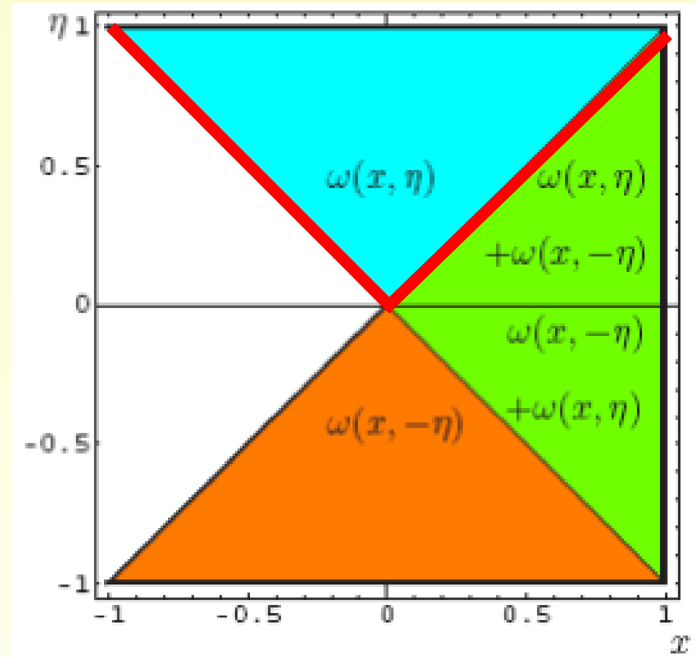
A partonic duality interpretation

quark GPD (anti-quark $x \to -x$):

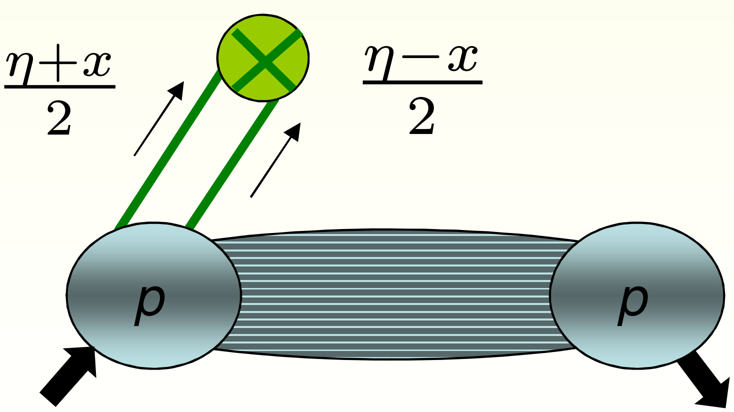
$$F(x,\eta,t) \quad =$$

$$\theta(-\eta \le x \le 1)\,\omega(x,\eta,t) + \theta(\eta \le x \le 1)\,\omega(x,-\eta,t)$$

$$\omega\,(x,\eta,t) = \frac{1}{\eta}\int_0^{\frac{x+\eta}{1+\eta}} dy\,(a+bx)\,f(y,(x-y)/\eta,t)$$

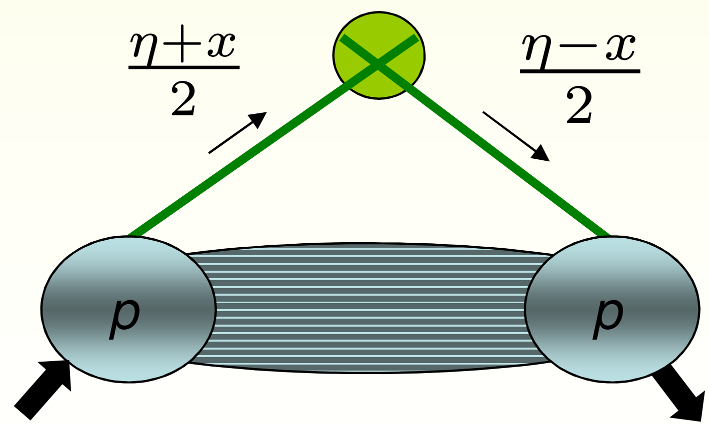


dual interpretation on partonic level:



support extension is unique [DM et al. 92]

ambiguous (*D*-term)
[DM, A. Schäfer (05)
KMP-K (07)]



central region - $\eta < x < \eta$
mesonic exchange in *t*-channel

outer region $\eta < x$
partonic exchange in *s*-channel

Double distribution (DD) representations

- general DD representation might be quoted as

$$F(x,\eta,t) = \int_0^1 dy \int_{-1+y}^{1-y} dz\, \delta(x-y-z\eta)\,[f(y,z,t) + x\Delta f(y,z,t)]$$

$$\Delta f(y,z,t) = 0 \quad \text{for signature-odd and} \quad F \in \{H+E, \widetilde{H}\}$$

DD can be converted into *f+Δf + η z Δf with f,Δf symmetric in z*

- if representation is fixed, DDs are obtained by Radon transform
- alternatively one may use, e.g., **[Polyakov, Weiss]** *D*-term **[Belitsky, DM et al.; Teryaev (01)]**

$$H^{q^{(+)}}(x,\eta,t) = \int_0^1 dy \int_{-1+y}^{1-y} dz\, \delta(x-y-z\eta) f{'}'(y,z,t) + \theta(|x| \leq |\eta|)\text{sign}(\eta) D(x/\eta)$$

$$\text{with} \quad f' = f + w \otimes \Delta f \quad \text{and} \quad D(x) = \lim_{\eta\to\infty} H^{q^{(+)}}(x\eta,\eta,t)$$

- ambiguity: DDs are generalized functions, i.e., it is allowed to add $\delta^{(n)}(y)$ [or D-, F-, ...-terms], one might end up with double counting
- Is *D*-term an integral part of a GPD or not? (answer depends on believes)

Uses of DDs in phenomenology

Radyushkin`s double distribution ansatz (RDDA) is employed
(original DD + D-term for *H,E* + *π*-pole for *Ẽ*)

GPV (VGG code), BMK, GK:

$$f(y,z,t) = F_f(t) y^{-\alpha' t} \frac{q_f(y)}{1-y} \frac{\Gamma\left(\frac{3}{2}+b\right)\left(1-\frac{z^2}{(1-y)^2}\right)^b}{\sqrt{\pi}\,\Gamma(1+b)}$$

in form factor modeling:

$$\int_{-1+x}^{1-x} dz\, f(x,z,t) = q_f(x) \exp\{t g_f(x)\}$$

<u>NOTE:</u>

- GPV & BMK (I give it up 2005) α'=0, quark angular momentum mostly fixed
- VGG code now (form factor sum rule can be violated, J_u/J_d issue)
- GK uses not Diehl-Kroll ansatz from form factor fits; only J_{sea} is a free parameter
  profile parameter *b* is fixed (integer value)
  NLO PDFs are refitted with integer *β*, evolution is **not** GPD evolution
- RDDA is so ***rigid*** that it is a ***holographic*** model **[Kumericki, DM (10)]**
  (*F(x,x,t)* and *F(x,0,t)* allow to restore the whole GPD)

large-*x* & small-*x* behavior are tied:

$$\frac{F(\xi,\xi,t)}{F(\xi,0,t)} \overset{\xi\to 1}{=} \frac{2^b\Gamma\left(\frac{3}{2}+b\right)\Gamma(1+b-\alpha(t))\Gamma(\beta-b)}{\sqrt{\pi}\,\Gamma(1+b)\Gamma(1-\alpha(t)+\beta)} \frac{(1-\xi)^b}{(1-\xi)^\beta}$$

$$\frac{F(\xi,\xi,t)}{F(\xi,0,t)} \overset{\xi\to 0}{=} \frac{\Gamma\left(\frac{3}{2}+b\right)\Gamma(1+b-\alpha(t))}{\Gamma\left(1+b-\frac{\alpha(t)}{2}\right)\Gamma\left(\frac{3}{2}+b-\frac{\alpha(t)}{2}\right)}$$

Conformal partial wave expansion of GPDs

- a GPD can be expanded with respect to conformal partial waves of the collinear conformal group SO(2,1) (similar to SO(3) expansion)
  - expansion in terms of discrete conformal spin j+2 for $\eta > 1, \; |x/\eta| \leq 1$

$$F(x,\eta,t) = \sum_{j=0}^{\infty} (-1)^j p_j(x,\eta) F_j(\eta,t)$$

$z = x/\eta \longleftrightarrow j+2$

  - conformal moments (partial wave amplitudes) are polynomials:

$$F_j(x,\eta) = \frac{\Gamma(3/2)\Gamma(1+j)}{2^j\Gamma(3/2+j)} \int_{-1}^{1} dx\, \eta^{j+1} C_j^{3/2}\left(\frac{x}{\eta}\right) F(x,\eta,t)$$

  - conformal partial waves ensure the polynomiality condition:

$$p_j(x,\eta) = \frac{\Gamma(5/2+j)}{j!\Gamma(1/2)\Gamma(2+j)} \frac{d^j}{dx^j} \int_{-1}^{1} du (1-u^2)^{j+1} \delta(x-u\eta)$$

✓ ***crossing symmetry*** allows for a more convenient representation (technicality, e.g., Sommerfeld-Watson transform, numerous failures in the literature)

✓ PWs evolve autonomously ⇒ trivial implementation of LO evolution
NLO done by perturbative expansion

Implementing constraints

- form factor and PDF constraints can be trivially implemented, e.g,
  (also Lattice constraints could be treated in this way, if they are considered as reliable)

$$F_j(\eta=0,t) = q_{F,j} F_F(t|M_j)\,, \quad q_{F,j} = \int_0^1 dx\, x^j q_F(x)\,,\; F_F(t|M_{j=0}) = F_F(t)$$

- flexible skewness dependence can be implemented, e.g., by SO(3) PW expansion (Wigner matrices) [`dual' model]

[Polyakov (99), Lebed, Ji (00), Diehl(03),...]

$$F_j(t,\eta) = \sum_J^{j\,(+1)} f_j^J(t)\, \eta^{j\,(+1)-J}\, \hat{d}_J^F(\eta)\,, \quad \hat{d}_J^F(\eta=0)=1$$

- all PWs contribute in the small-ξ approximation of CFFs
- taking leading PW yields the Shuvaev claim (tying GPDs to PDFs at small- ξ)
- two PWs can be used to mimic the RDDA
- three PWs can be used to control normalization and evolution flow at small-ξ
- to have flexibility at large-ξ one must resum, i.e., in fitting one should replace Wigner matrices by some effective functions

GPD ansatz from t-channel view

- at short distance a quark/anti-quark state is produced, labeled by ***conformal spin*** *j+2*
- they form an intermediate mesonic state with total angular momentum ***J*** strength of ***coupling*** is

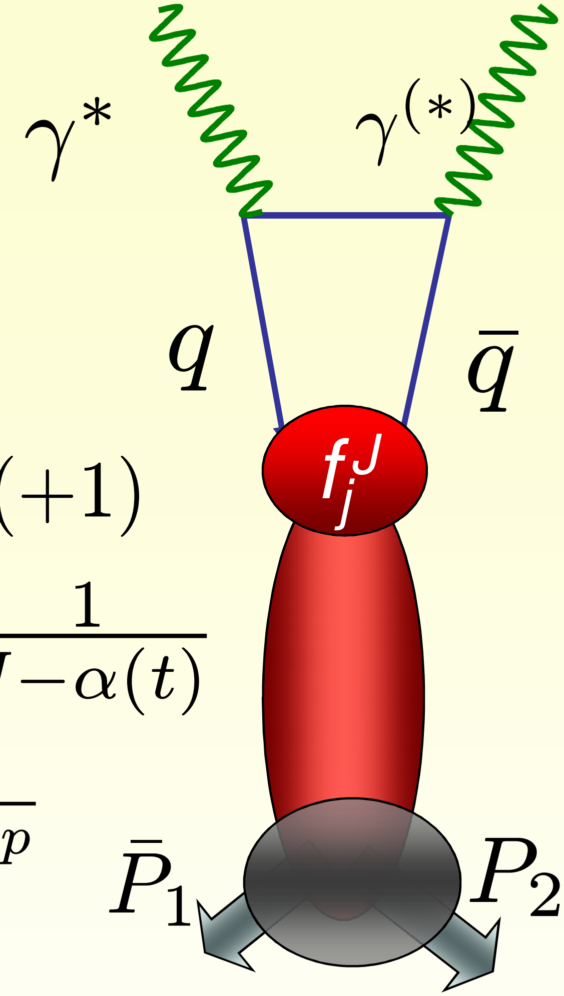
$$f_j^J,\ J \leq j\,(+1)$$

- mesons propagate with

$$\frac{1}{m^2(J)-t} \propto \frac{1}{J-\alpha(t)}$$

- decaying into nucleon anti-nucleon pair with given angular momentum *J*, described by an ***impact form factor***

$$\frac{1}{(1-\frac{t}{M^2(J)})^p}$$



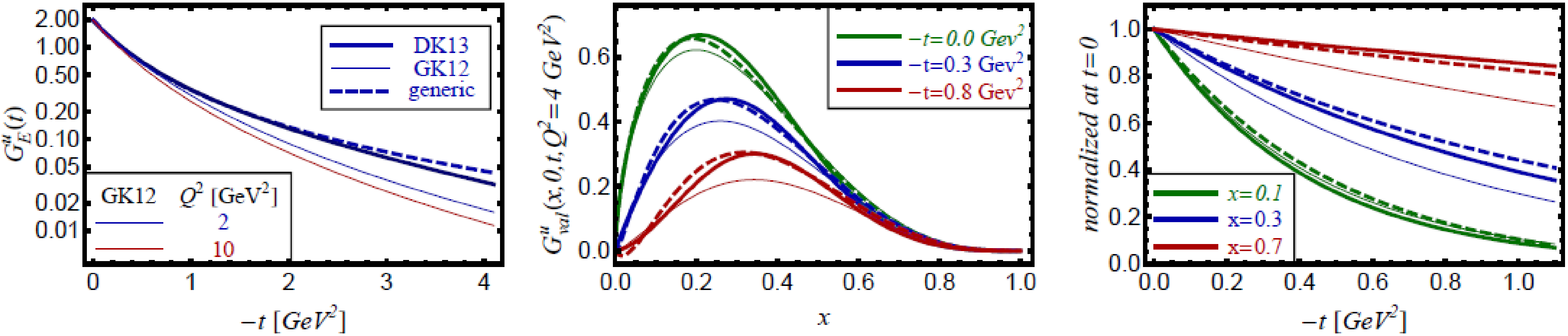
➢ (conformal) GPD moments expanded in Wigner`s rotation matrices

$$F_j(t,\eta) = \sum_J^{j\,(+1)} \frac{f_j^J}{J-\alpha(t)} \frac{1}{(1-\frac{t}{M^2(J)})^p}\, \eta^{j\,(+1)-J}\, \hat{d}_J^F(\eta)\,, \quad \hat{d}_J^F(\eta=0)=1$$

[Polyakov (99), Lebed, Ji (00), Diehl(03),...]

- labeling by *t*-channel quantum numbers J^{PC}
- so-called D-term arises from *0⁺⁺, (f⁰ or σ) 2⁺⁺, 4⁺⁺, ...,* has even *J=j+1 (or j = -1 in DR)* pole (*J (=0)* has multiple meanings **[KMP-K(07&08)]**)
- usable for large *x* (employing effective rotation matrices)

Comments on skewless GPD modeling



- GK12 uses as in VGG a Regge inspired ansatz for valence quarks
  - not the ansatz as in Guidal et. al or Diehl-Kroll form factor fits
  - induces only a slight violation of form form factor sum rules
  - $J_u^{(-)}/J_d^{(-)}$ values fixed (strong J_u/J_d variations are senseless)!
- generic modeling agrees with DK13, having a different functional form

$$F_j^{q^{(-)}}(x,t) = q_F(x) e^{tB(x)} \quad G_j^{u^{(-)}}(t) = \frac{2}{\left(1-\frac{t}{M_j^2}\right)^2} \frac{\Gamma(1+j-\alpha)\Gamma(2-\alpha+\beta)}{\Gamma(1-\alpha)\Gamma(2+j-\alpha+\beta)}, \; M_j^2 = \frac{\alpha'}{1+j-\alpha}$$

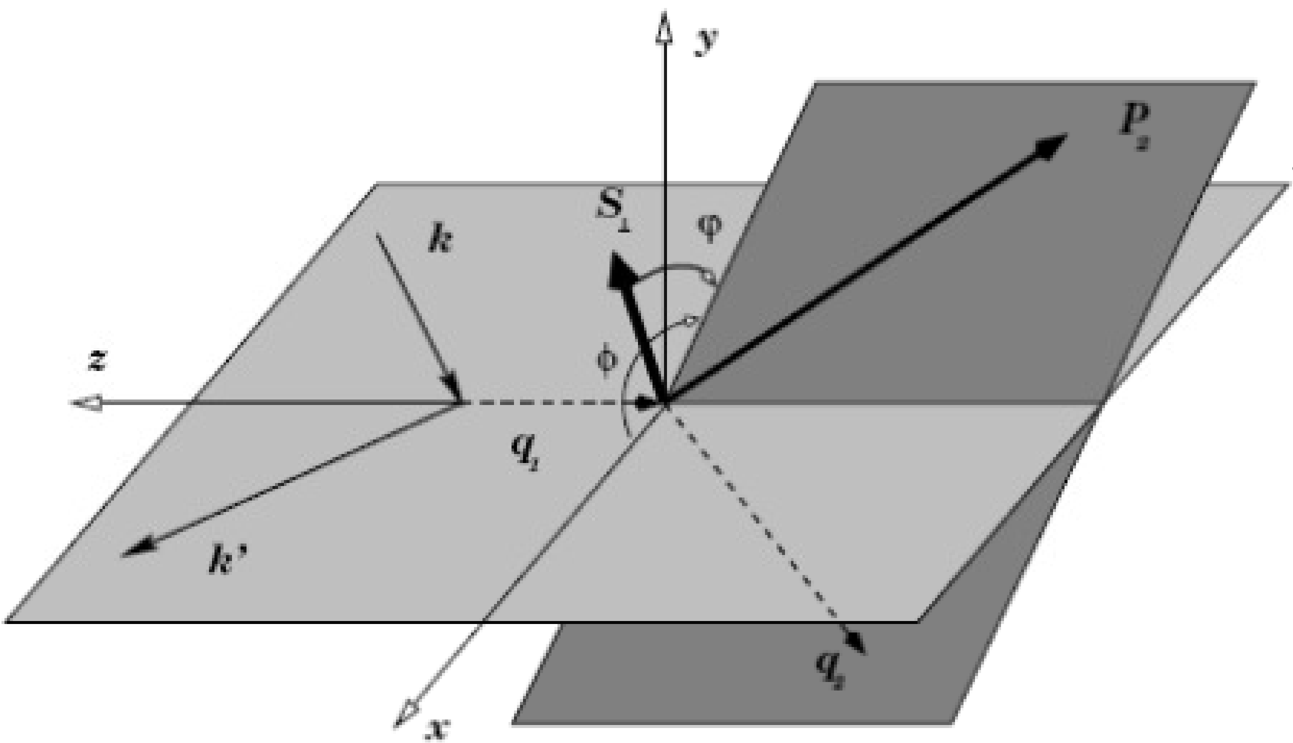
- How strongly influences the PDF parameterization the *t*-dependence?
- Do *η*- and *t*-dependencies factorize (as commonly assumed)? (properly not)

Photon leptoproduction $e^{\pm}N \to e^{\pm}N\gamma$

measured by ***H1, ZEUS, HERMES, CLAS, HALL A*** collaborations

planed at ***COMPASS, JLAB@12GeV***, perhaps at ? EIC, ?? LHeC

$$\frac{d\sigma}{dx_{\rm Bj}dyd|\Delta^2|d\phi d\varphi} = \frac{\alpha^3 x_{\rm Bj} y}{16\,\pi^2\,\mathcal{Q}^2}\left(1+\frac{4M^2x_{Bj}^2}{\mathcal{Q}^2}\right)^{-1/2}\left|\frac{\mathcal{T}}{e^3}\right|^2,$$



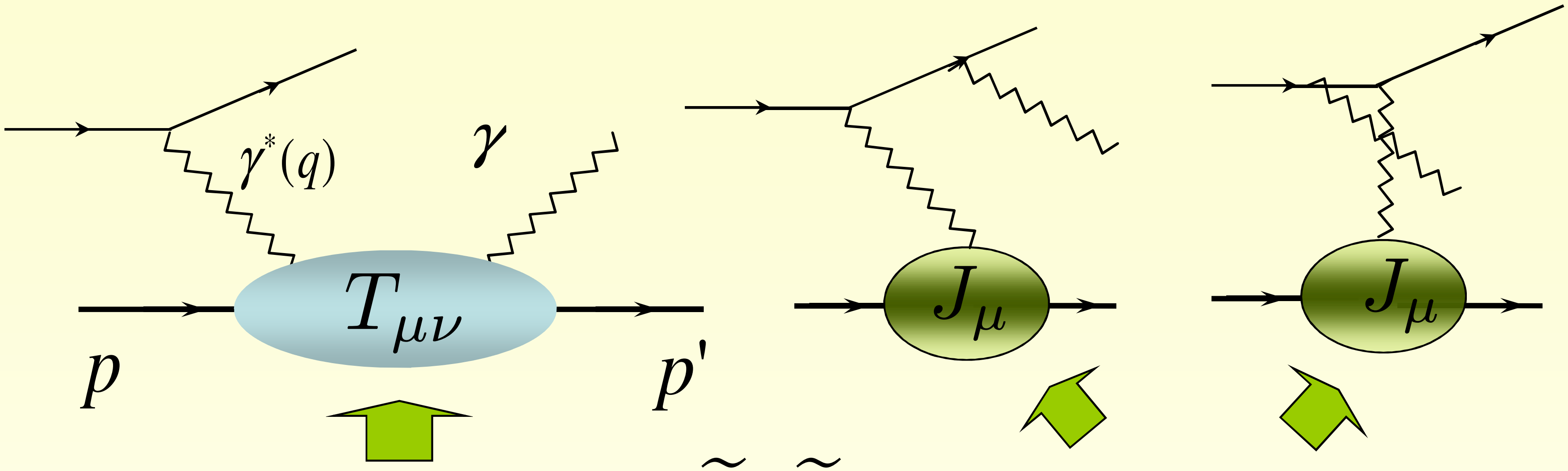
$$x_{\rm Bj} = \frac{\mathcal{Q}^2}{2P_1\cdot q_1} \approx \frac{2\xi}{1+\xi},$$

$$y = \frac{P_1\cdot q_1}{P_1\cdot k},$$

$$\Delta^2 = t \quad (\text{fixed, small}),$$

$$\mathcal{Q}^2 = -q_1^2 \quad (> 1\text{GeV}^2),$$

interference of *DVCS* and *Bethe-Heitler* processes



12 Compton form factors $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}, \cdots$ elastic form factors F_1, F_2
(helicity amplitudes)

$$|\mathcal{T}_{\mathrm{BH}}|^2 = \frac{e^6(1+\epsilon^2)^{-2}}{x_{\mathrm{Bj}}^2 y^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathrm{BH}} + \sum_{n=1}^{2} c_n^{\mathrm{BH}} \cos(n\phi) \right\},$$

exactly known
(LO, QED)

$$|\mathcal{T}_{\mathrm{DVCS}}|^2 = \frac{e^6}{y^2 \mathcal{Q}^2} \left\{ c_0^{\mathrm{DVCS}} + \sum_{n=1}^{2} \left[c_n^{\mathrm{DVCS}} \cos(n\phi) + s_n^{\mathrm{DVCS}} \sin(n\phi) \right] \right\},$$

harmonics
1:1
helicity ampl.

$$\mathcal{I} = \frac{\pm e^6}{x_{\mathrm{Bj}} y^3 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\}.$$

harmonics
1:1
helicity ampl.

all harmonics are given by twist-2 and -3 GPDs: **[Diehl et. al (97) Belitsky, DM, Kirchner (01)]**

$$\begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta}{\mathcal{Q}}\ \text{tw-2(GPDs)} + O(1/\mathcal{Q}^3), \qquad c_0^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2}\ \text{tw-2(GPDs)} + O(1/\mathcal{Q}^4),$$

$$\begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2}\ \text{tw-3(GPDs)} + O(1/\mathcal{Q}^4), \qquad \begin{Bmatrix} c_3 \\ s_3 \end{Bmatrix}^{\mathcal{I}} \propto \frac{\Delta\alpha_s}{\mathcal{Q}} (\text{tw-2})^{\text{T}} + O(1/\mathcal{Q}^3),$$

$$c_0^{\text{CS}} \propto\ (\text{tw-2})^2, \qquad \begin{Bmatrix} c_1 \\ s_1 \end{Bmatrix}^{\text{CS}} \propto \frac{\Delta}{Q}\ (\text{tw-2})\ (\text{tw-3}), \qquad \begin{Bmatrix} c_2 \\ s_2 \end{Bmatrix}^{\text{CS}} \propto \alpha_s (\text{tw-2})(\text{tw-2})^{\text{GT}}$$

e.g., n=1 odd harmonic is approximately given by `CFF' combination

$$\begin{Bmatrix} c^{\mathcal{I}}_{1,\text{unp}} \\ s^{\mathcal{I}}_{1,\text{unp}} \end{Bmatrix} = 8K \begin{Bmatrix} -(2-2y+y^2) \\ \lambda y(2-y) \end{Bmatrix} \begin{Bmatrix} \Re\text{e} \\ \Im\text{m} \end{Bmatrix} \mathcal{C}^{\mathcal{I}}_{\text{unp}}(\mathcal{F})\ , \mathcal{C}^{\mathcal{I}}_{\text{unp}} = F_1\mathcal{H} + \frac{x_{\text{B}}}{2-x_{\text{B}}}(F_1+F_2)\widetilde{\mathcal{H}} - \frac{\Delta^2}{4M^2}F_2\mathcal{E}$$

relations among **harmonics** and (helicity dependent) **CFFs** are not more based on a $1/Q$ expansion: **[Belitsky, DM (10) -- Belitsky, DM, Ji (12)]**

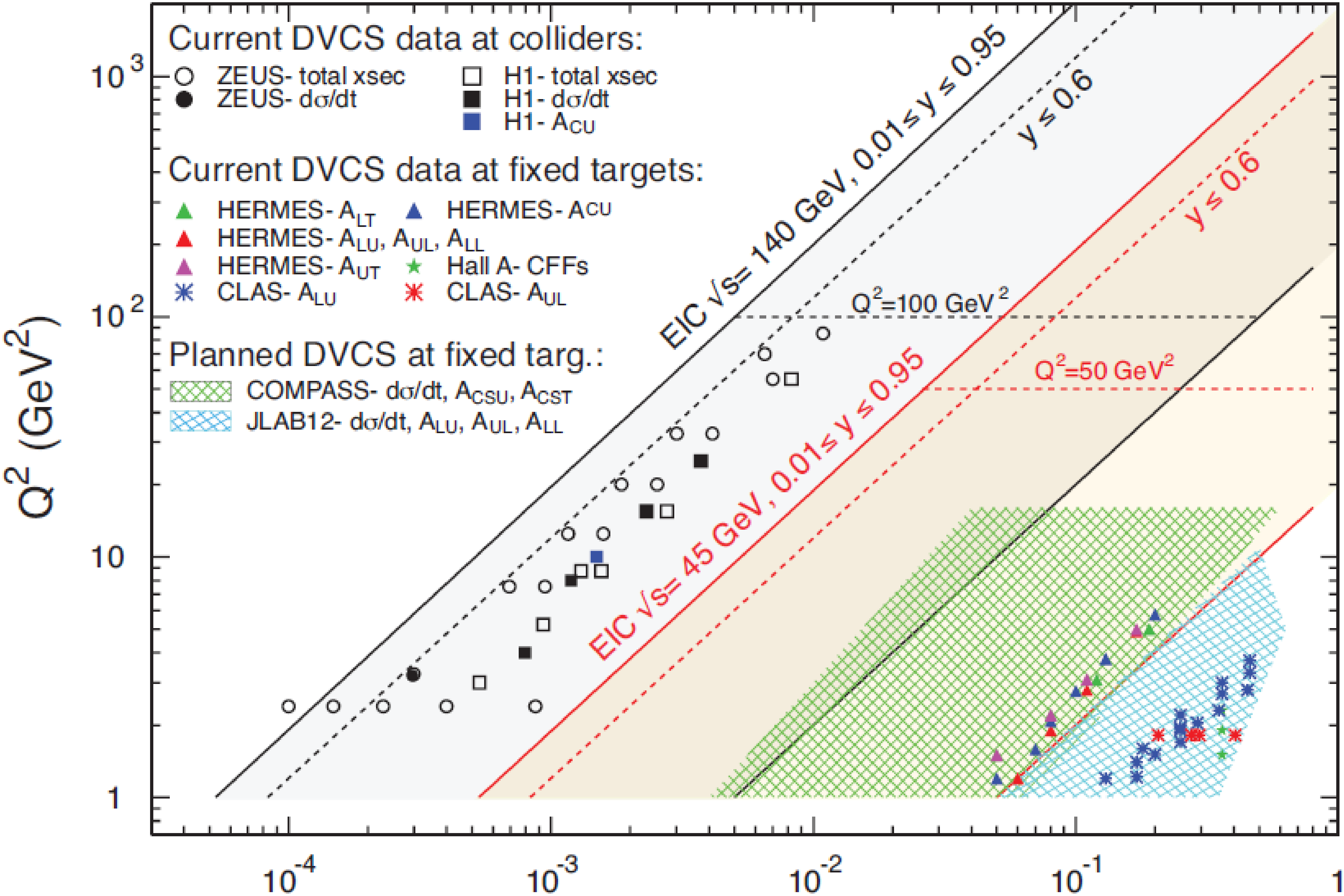
$$s^{\mathcal{I}}_{1,\text{unp}} = \frac{8\widetilde{K}\lambda\sqrt{1-y-\frac{y^2\gamma^2}{4}}(2-y)y}{Q(1+\gamma^2)} \Im\text{m}\Big\{ \mathcal{C}^{\mathcal{I}}_{\text{unp}}\Big(\Big[1-\frac{\varkappa}{2Q^2}\frac{Q^2+t}{\sqrt{1+\gamma^2}}\Big]\mathcal{F}_{++} + \Big[1-\frac{2+\varkappa}{2Q^2}\frac{Q^2+t}{\sqrt{1+\gamma^2}}\Big]\mathcal{F}_{-+} + \frac{(Q^2+t)\varkappa_0}{Q^2\sqrt{1+\gamma^2}}\mathcal{F}_{0+}\Big)$$

$$+ \frac{-t(Q^2+t)}{\sqrt{1+\gamma^2}Q^4}\Delta\mathcal{C}^{\mathcal{I}}_{\text{unp}}\left(\mathcal{F}_{-+} + \frac{\varkappa}{2}[\mathcal{F}_{++}+\mathcal{F}_{-+}] - \varkappa_0\mathcal{F}_{0+}\right)\Big\}, \qquad (70)$$

new improved C coefficients ensure the cancellation of kinematical singularities

relations among CFFs and GPDs are always based on a $1/Q$ expansion

DVCS world data set



Can one `measure' GPDs?

- ***CFF*** given as ***GPD*** **convolution**:

$$\mathcal{F}(\xi, t, \mathcal{Q}^2) \stackrel{\rm LO}{=} \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} \mp \frac{1}{\xi + x - i\epsilon}\right) F(x, \eta = \xi, t, \mathcal{Q}^2)$$

$$\stackrel{\rm LO}{=} i\pi F^{\pm}(x = \xi, \eta = \xi, t, \mathcal{Q}^2) + \mathrm{PV} \int_0^1 dx \frac{2x}{\xi^2 - x^2} F^{\pm}(x, \eta = \xi, t, \mathcal{Q}^2)$$

- $F(x,x,t,\mathcal{Q}^2)$ viewed as **"spectral function"** (s-channel cut):

$$F^{\pm}(x, x, t, Q^2) \equiv F(x, x, t, Q^2) \mp F(-x, x, t, Q^2) \stackrel{\rm LO}{=} \frac{1}{\pi} \Im\mathrm{m} \mathcal{F}(\xi = x, t, Q^2)$$

[Frankfurt et al (97)
Chen (97)
Terayev (05)
KMP-K (07)
Diehl, Ivanov (07)]

- ***CFFs*** satisfy `**dispersion relations**'
(not the physical ones, threshold ξ_0 set to 1)

$$\Re\mathrm{e}\mathcal{F}(\xi, t, Q^2) = \frac{1}{\pi} \mathrm{PV} \int_0^1 d\xi' \left(\frac{1}{\xi - \xi'} \mp \frac{1}{\xi + \xi'}\right) \Im\mathrm{m}\mathcal{F}(\xi', t, Q^2) + \mathcal{C}(t, Q^2)$$

[Terayev (05)]

access to the ***GPD*** on the **cross-over line** $\eta = x$ (at LO)

access to the subtraction constant (for H,E related to `D-term')

Strategies to analyze DVCS data

(ad hoc) modeling: **VGG** code [**Goeke et. al (01)** based on **Radyushkin's** DDA]
BMK model [**Belitsky, DM , Kirchner (01)** based on RDDA]
`aligned jet' model [**Freund, McDermott, Strikman (02)**]
Goloskokov/Kroll (05) based on RDDA (pinned down by DVMP)
`dual' model [**Polyakov,Shuvaev 02;Guzey,Teckentrup 06;Polyakov 07**]
" -- " [**KMP-K (07)** in MBs-representation]
polynomials **[Belitsky et al. (98), Liuti et. al (07), Moutarde (09)]**

dynamical models: not applied **[Radyushkin et.al (02); Tiburzi et.al (04); Hwang DM (07)]…**
(respecting Lorentz symmetry)

flexible models: any representation by including ***unconstrained*** degrees of freedom
(for fits) **KMP-K (07/08)** for H1/ZEUS in MBs-integral-representation

CFFs (real and imaginary parts) and GPD fits/predictions

i. CFF extraction with formulae (local) **[BMK (01), HALL-A (06)]** and **[KK,DM, Murray]**
least square fits (local) **[Guidal, Moutarde (08...)]**
neural networks – a start up **[KMS (11)]**
ii. `dispersion integral' fits **[KMP-K (08),KM (08...)]**
iii. flexible GPD modeling **[KM (08...)]**
vi. model comparisons **VGG code, however also BMK01 (up to 2005)**
& predictions **Goloskokov/Kroll (07) model** based on RDDA

Asking for CFFs (physics case)

- CFFs are defined for the whole kinematical region [Belitsky, DM, Ji (12)]
- contain (generalized) polarizabilities
- their access requires a complete measurement

toy example DVCS off a scalar target [KK, DM, Murray (13)]

- for the first step we use *s*-channel helicity conservation hypothesis (neglecting twist-three and transversity associated CFFs)

• linearized set of equations (approximately valid)

$$A_{\mathrm{LU,I}}^{\sin(1\phi)} \approx N c_{\Im\mathfrak{m}}^{-1} \mathcal{H}^{\Im\mathfrak{m}} \quad \text{and} \quad A_{\mathrm{C}}^{\cos(1\phi)} \approx N c_{\Re\mathfrak{e}}^{-1} \mathcal{H}^{\Re\mathfrak{e}}$$

• normalization *N* is bilinear in CFFs

$$0 \lesssim N(\boldsymbol{A}) \approx \frac{1}{1+\frac{k}{4}|\mathcal{H}|^2} \approx \frac{\int_{-\pi}^{\pi} d\phi\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi) d\sigma_{\mathrm{BH}}(\phi)}{\int_{-\pi}^{\pi} d\phi\, \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)\left[d\sigma_{\mathrm{BH}}(\phi)+d\sigma_{\mathrm{DVCS}}(\phi)\right]} \lesssim 1$$

• cubic equation for *N* with two non-trivial solutions

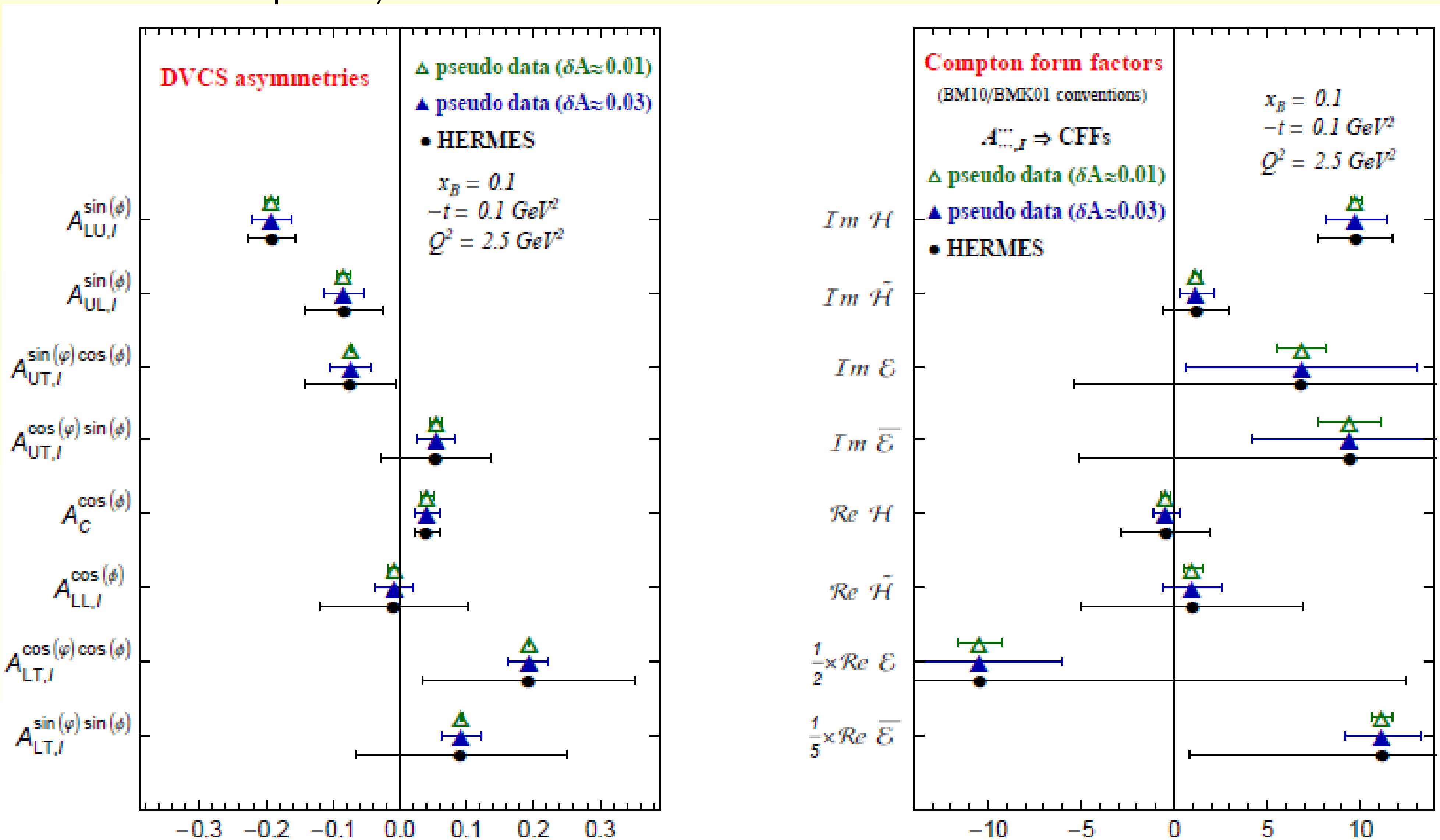
$$N(\boldsymbol{A}) \approx \frac{1}{2}\left(1 \pm \sqrt{1 - k\, c_{\Im\mathfrak{m}}^2 \left(A_{\mathrm{LU,I}}^{\sin(1\phi)}\right)^2 - k\, c_{\Re\mathfrak{e}}^2 \left(A_{\mathrm{C}}^{\cos(1\phi)}\right)^2}\right)$$

+ BH regime
- DVCS regime

• standard error propagation

NOTE: there is no need to **linearize**, we do mapping numerically

- a complete measurement allows in principle to pin down all CFFs **KK, DM, Murray (13)**
- missing information in incomplete measurements can be filled with noise
  (Guidal`s philosophy: use noise together with hypotheses and model constraints, our results are compatible)



- larger statistics:
  some CFF *E* constraint might have been obtained by HERMES

A simple valence quarks GPD model

- model of GPD *H(x,x,t) within DD motivated ansatz at Q^2=2* GeV^2

fixed: *PDF normalization* *eff. Reage pole* *large t-counting rules*

$$H(x,x,t) = \frac{n\, r\, 2^{\alpha}}{1+x}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}.$$

free parameters: r-ratio at small x *large x-behavior* *p-pole mass*

- unpolarized valence quarks : asking for *r, b, M* parameters

$$n = 1.0, \quad \alpha(t) = 0.43 + 0.85t/\mathrm{GeV}^2, \quad p = 1$$

- flexible parameterization of subtraction constant (so-called D-term convoluted with hard amplitude)

$$\mathcal{D}(t) = \frac{-C}{(1-t/M_c^2)^2}$$

- analogous ansatz for porlarized quark GPD + pion-pole contribution
- no E(x,x,t) nor Ê(x,x,t) is set up
- KM...> 2010 hybrid models GPD evolution for sea /gluon + DR for valence

KM10 fits to DVCS off unpolarized proton

- a hybrid model: three effective SO(3) PWs for sea quarks/gluons
  dispersion relations for valence
  still E GPD is neglected (only D-term)
  still $\hat{E}$ GPD only flexible pion pole contribution
- asking for GPD H and `*D-term*' *($\hat{H}$ is considered as effective d.o.f.)*

leading order, including evolution for sea quarks/ gluons
quark twist-two dominance hypothesis within CFF convention **[BM10]**

- data selection (taking moments of azimuthal angle harmonics)

KM10a: neglecting HALL-A data
KM10b: forming ratios of moments
KM10: original HALL-A data
neglecting large $-t$ BSA CLAS data

15 parameter fit, e.g.,
including all HALL-A data

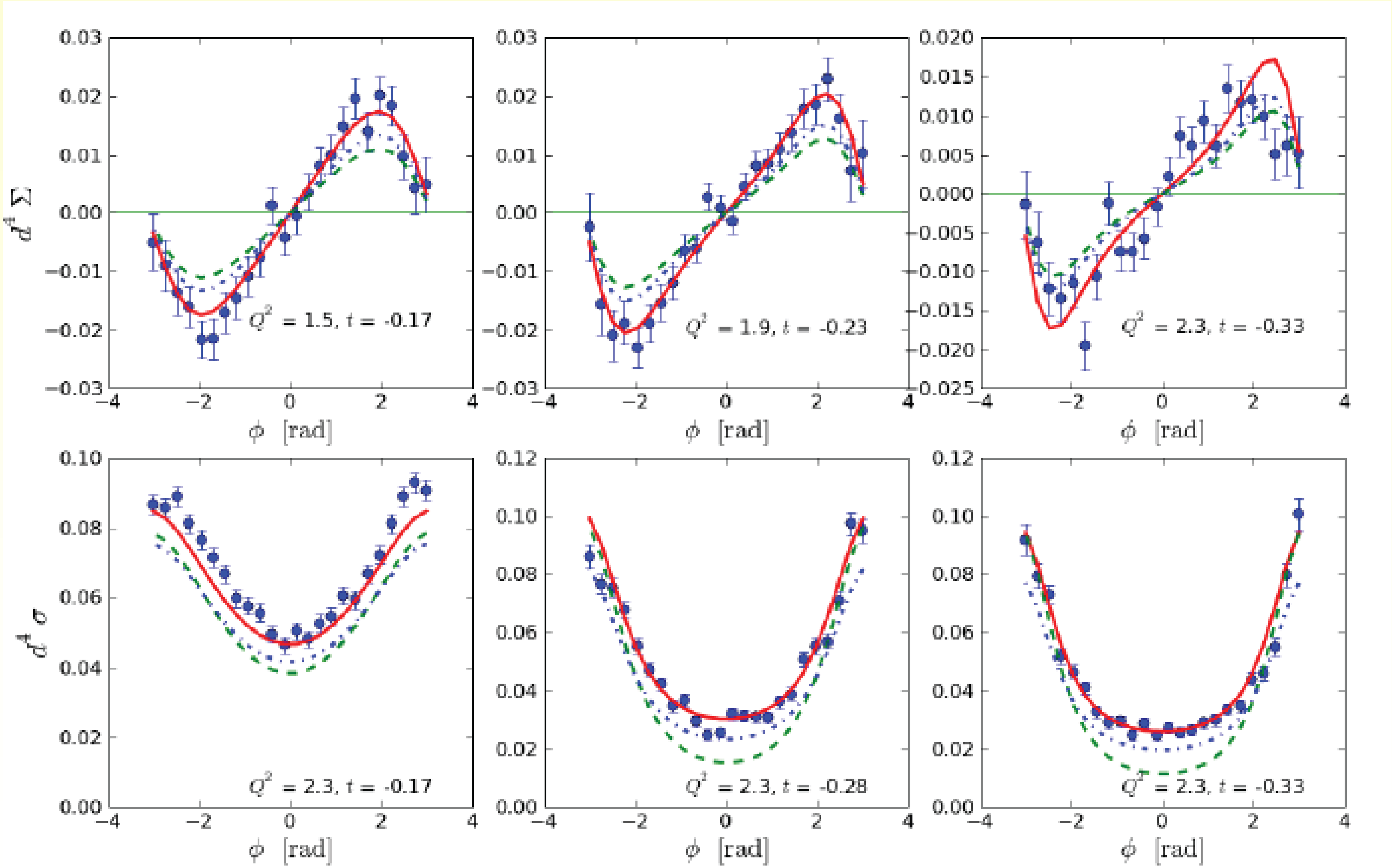
175 data points
$\chi^2/d.o.f. = 132/165$

```
------------------------------
M02S  =  0.51  +-  0.02
SECS  =  0.28  +-  0.02
SECG  = -2.79  +-  0.12
THIS  = -0.13  +-  0.01
THIG  =  0.90  +-  0.05
  Mv  =  4.00  +-  3.33  (edge)
  rv  =  0.62  +-  0.06
  bv  =  0.40  +-  0.67
   C  =  8.78  +-  0.98
  MC  =  0.97  +-  0.11
 tMv  =  0.88  +-  0.24
 trv  =  7.76  +-  1.39
 tbv  =  2.05  +-  0.40
 rpi  =  3.54  +-  1.77
 Mpi  =  0.73  +-  0.37
------------------------------
```

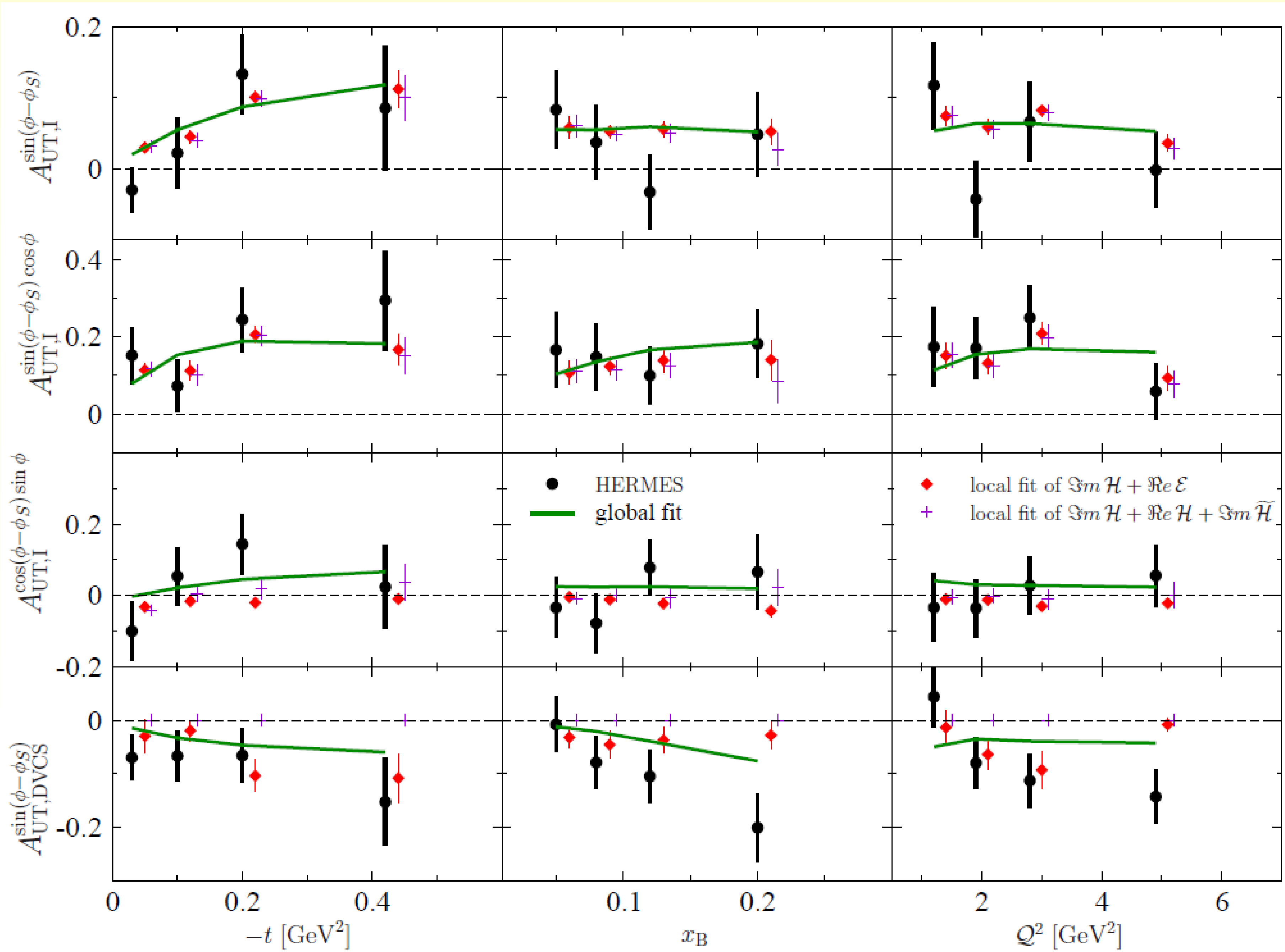

- results are given as **xs.exe** on ***http://calculon.phy.hr/gpd/***

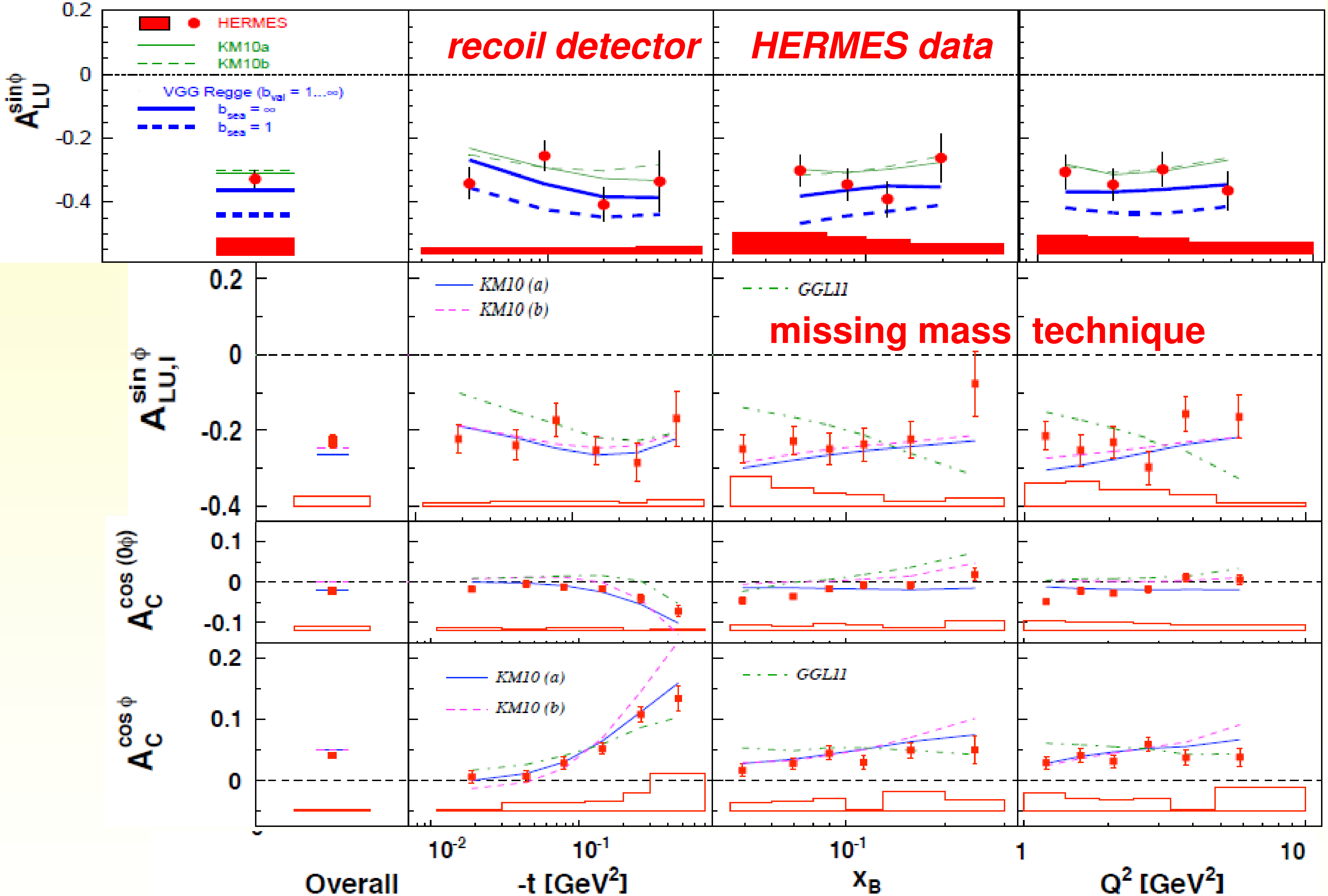
HALL A φ-dependence

- φ-dependence is described (if we fit to it)



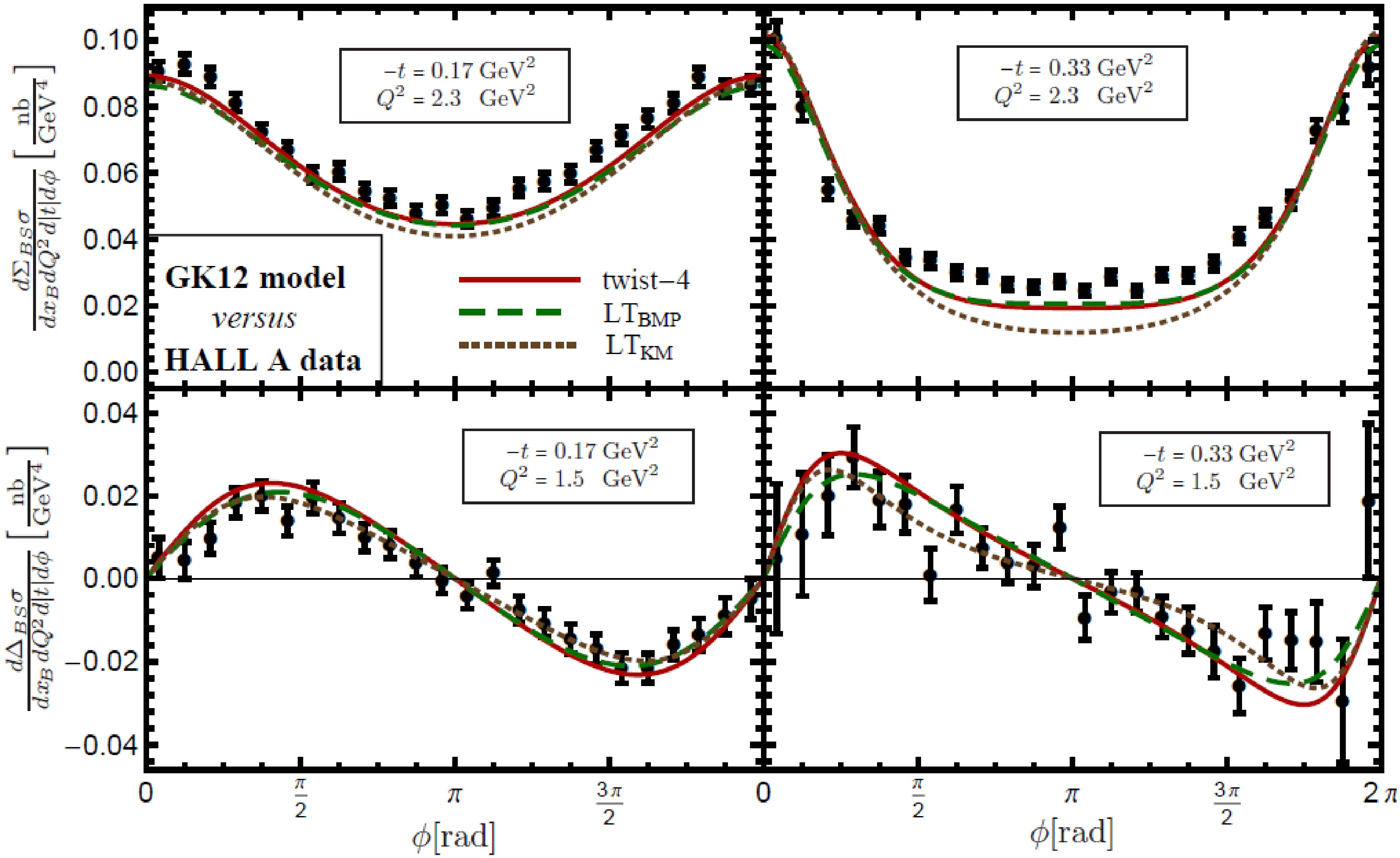
- KMM12 (KM10 type model) includes polarized target DVCS data (global fit to most of data , $\chi^2/d.o.f \approx 1.6$ - best what is there at present e.g., transverse polarized HERMES asymmetries looks as)





- recoil detector data are compatible with missing mass technique ones
- fit procedure: curves were data are scattered around
- recoil data: RDDA is not so much disfavored as it was before the case

How to understand Hall A data?



[Braun, Manashov, Pirnay, DM (14)]

GK12 model evaluated with KM and BMP prescription

including kinematical corrections

- standard models can not explain HALL A data
- wrong understanding on CFF hierarchy? – inclusion of higher-twist?
- exclusivity issue in all other fixed target data?
- Is (QED) correction procedure understood?
- naive understanding of `power corrections' **[VGG (99)]** is misleading

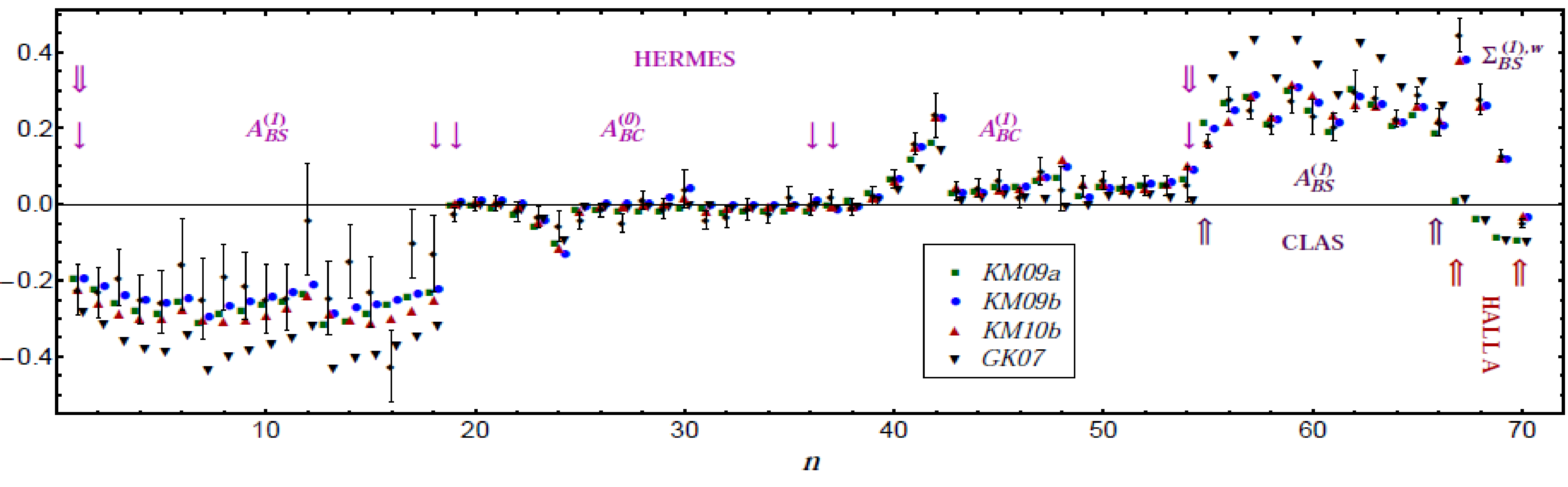
Fixed target DVCS data

- HERMES(02-12) 12x34 asymmetries (+few bins) $0.05 \le \langle x_B \rangle \le 0.2$, $\langle |t| \rangle \le 0.6$ GeV^2
  [$\sin(\varphi)$, ..., $\cos(3\varphi)$, $\langle Q^2 \rangle \approx 2.5$ GeV^2
  two kinds of electrons, all polarization options]

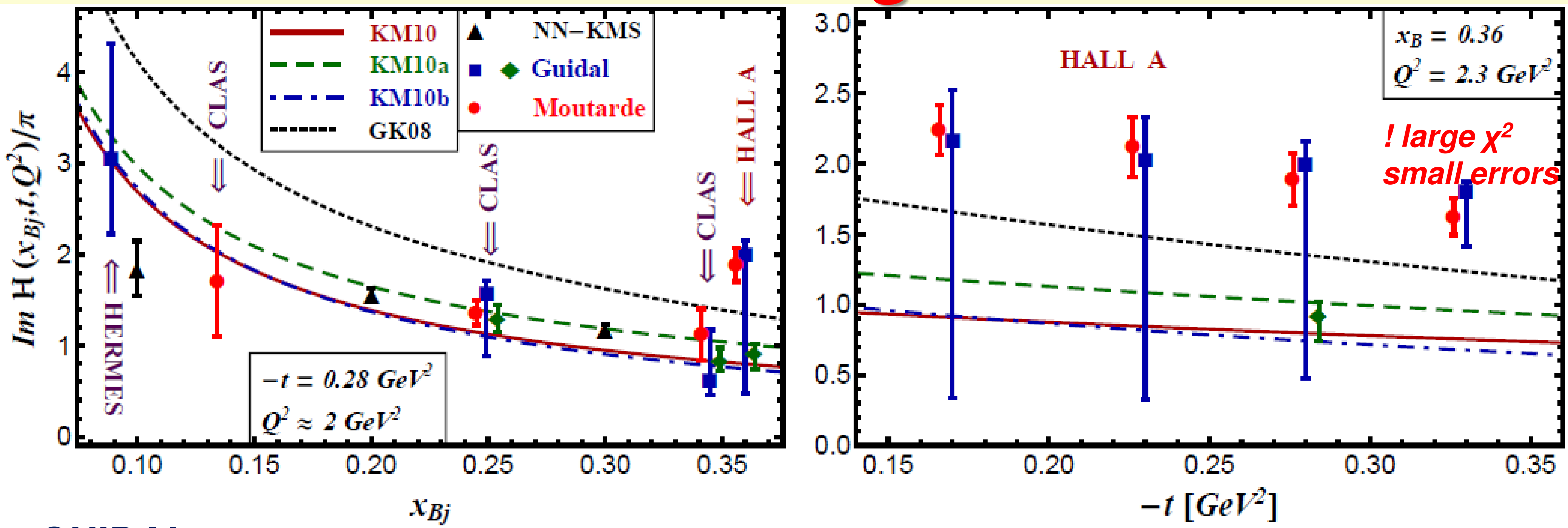
- HERMES(12) A_{LU} with recoil detector
  (compatible with old data, differences in GPD interpretation)

- CLAS(07) 12x12 [$A_{LU}(\varphi)$] $0.14 \le \langle x_B \rangle \le 0.35$, $\langle |t| \rangle \le 0.3$ GeV^2
  40x12 [$A_{LU}(\varphi)$] (large |t| or bad sta.) $\langle Q^2 \rangle \approx 1.8$ GeV^2
  (06,08) A_{UL} and A_{LU}

- HALL A(06) 12x24 [$\Delta\sigma(\varphi)$] $\langle x_B \rangle = 0.36$, $\langle |t| \rangle \le 0.33$ GeV^2
  3x24 [$\sigma(\varphi)$] $\langle Q^2 \rangle \approx 1.8$ GeV^2



KM... versus CFF fits & large-x "model" fit



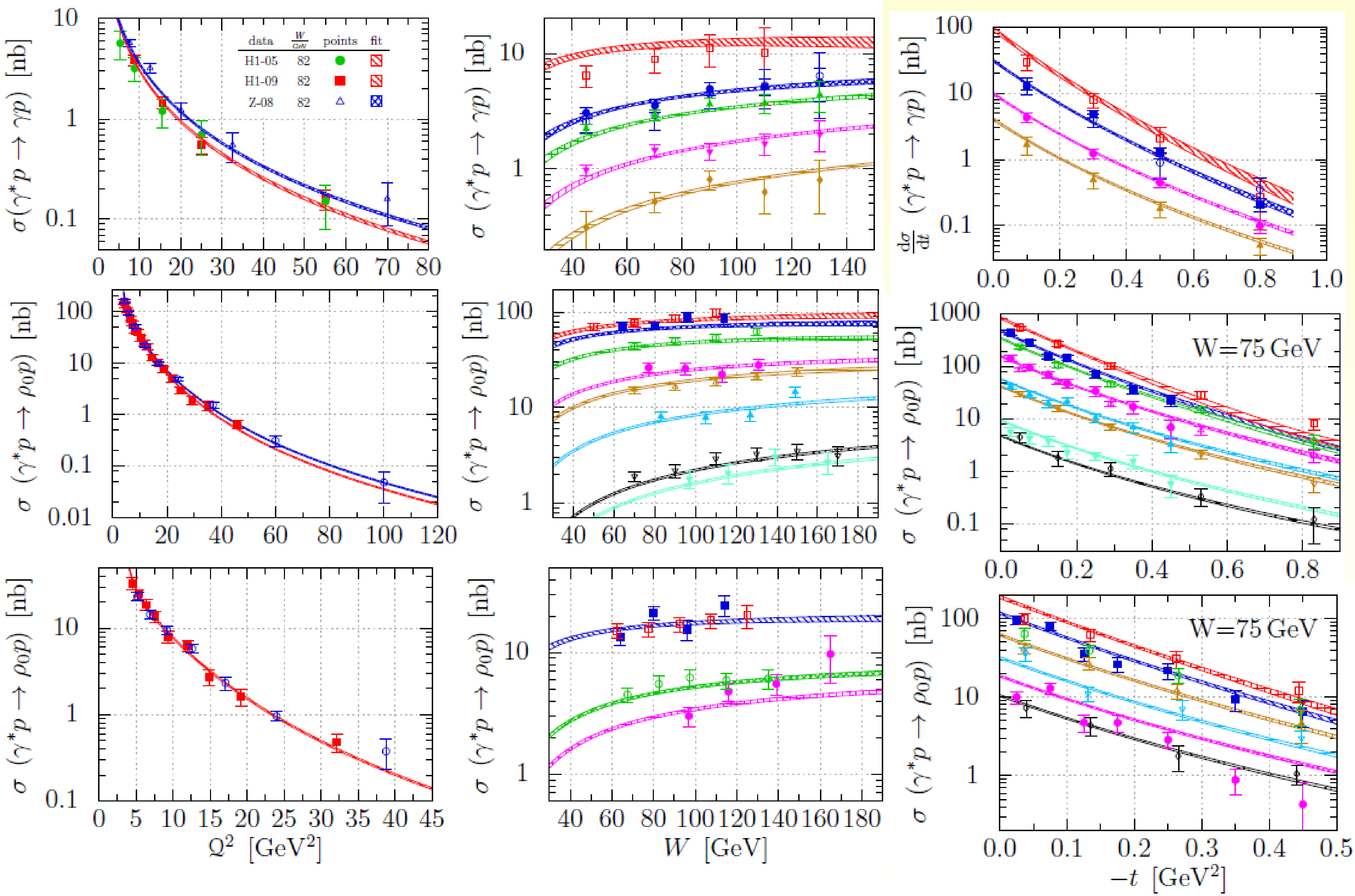
GUIDAL	twist-two dominance hypothesis 7 parameter fit to all harmonics of unpolarized cross section propagated errors + "theoretical" error estimate
GUIDAL	same + longitudinal TSA
Moutarde	H dominance hypothesis within a smeared polynomial expansion propagated errors + "theoretical" error estimate
NN	neural network within H dominance hypothesis green (blue) [red] curves (KM10...) without (with) HALL A data (ratios)
GK08	black curve GPDs (based on RDDA) obtained from handbag approach to DVMP

- reasonable agreement for HERMES and CLAS kinematics
- large *x*-region and real part remains unsettled

DIS+DVCS+DVMP phenomenology at small-x_B (H1,ZEUS)

works somehow without DIS at LO **[T. Lautenschlager, DM, A. Schäfer (13)]**

works at NLO ($Q^2 > 4\ GeV^2$), done with Bayes theorem (probability distribution function)

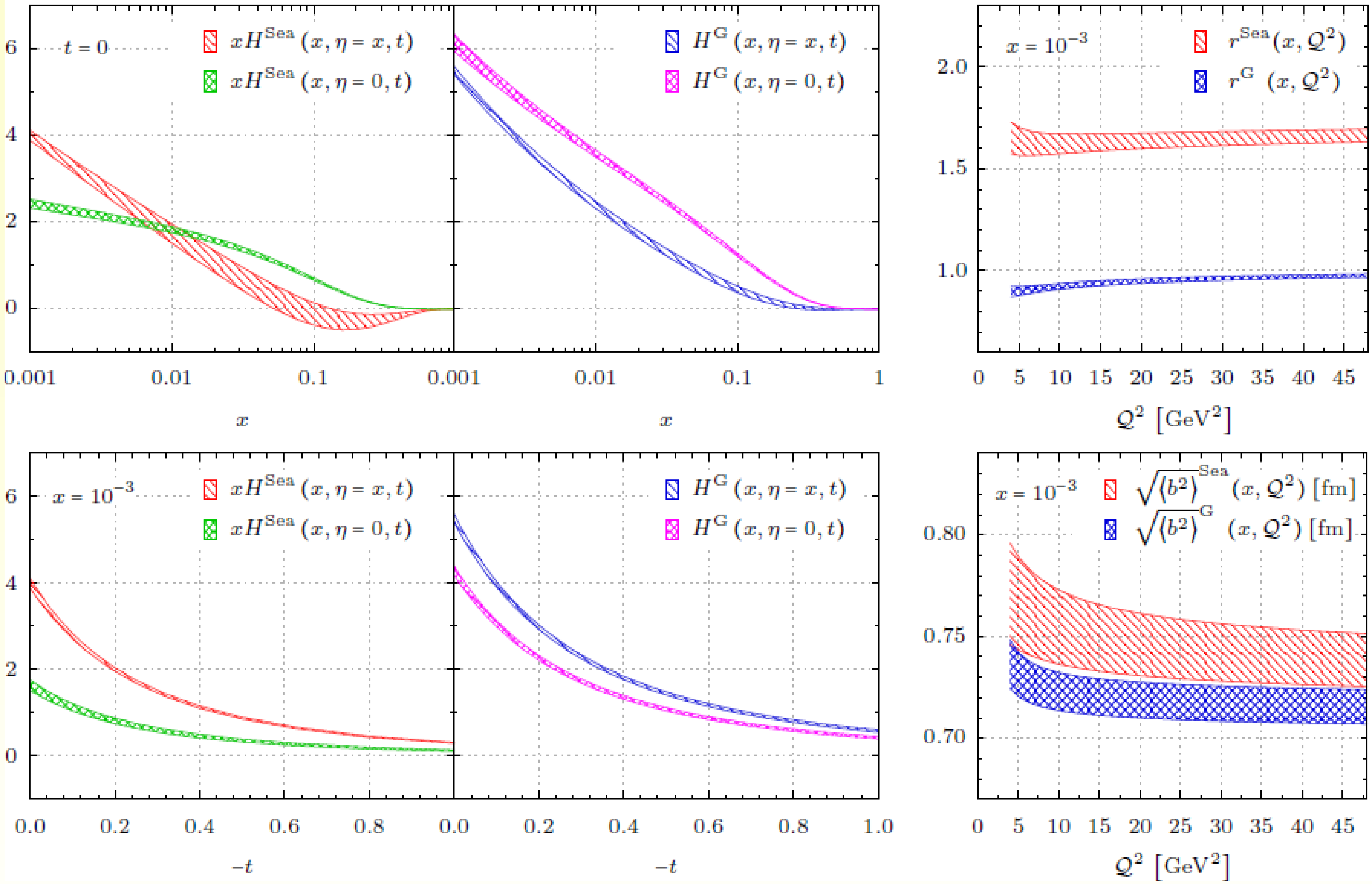


fixed:

meson DA flavor content

errors might be perhaps larger

entirely model dependency for x> 10^{-2}



- going from LO to NLO increases the skewness ratios (known since `ever', **[KMP-K(07)]**)
- gluons are more centralized as sea quarks (expected from DVCS & *J/ψ* interpretation)
- cross-talk of skewness and *t*-dependency has been addressed by pdf
- NLO GPDs look rather compatible to Goloskokov/Kroll and Martin et. al finding
- there is also DVCS beam charge and perhaps **beam spin** data are coming up

GPD phenomenology lessons: first decade

- qualitatively GPD formalism works in DVCS (from the start up)
- first look: no serious problems in DVMP (apart from ? about very large x_B data)
  also supported by hand-bag model description of Goloskokov/Kroll
- description of present DVCS data is reached/feasible with flexible models
  for unpolarized target– but GPD understanding induces tension among data
  large unidentified contribution called Ĥ is disfavored by polarized target data
- many uncertainties: exclusivity, correction procedure, assumptions
- HERMES gave proof of principle that on can go for a complete measurement

partonic interpretation:

- RDDA (GVP01,BMK01, VGG code in its many versions, GK07, ...)
  a bit disfavored at LO can not reach a $\chi^2/dof \sim 1...1.6$ (its like χ^2/nop ~5...10)
  should work at NLO **[Freund, McDermott (02)]**
- GPD H is dominant (? 15% accuracy), tomography at small-x_B
- GPD $\hat{H}$ is constrained
- no access to GPD E from present data, pion pole model for $\hat{E}$ is disfavored
- D-term related subtraction constant comes out negative (& sizable)
  Goke et. al model prediction (perhaps fit result might be not stable)

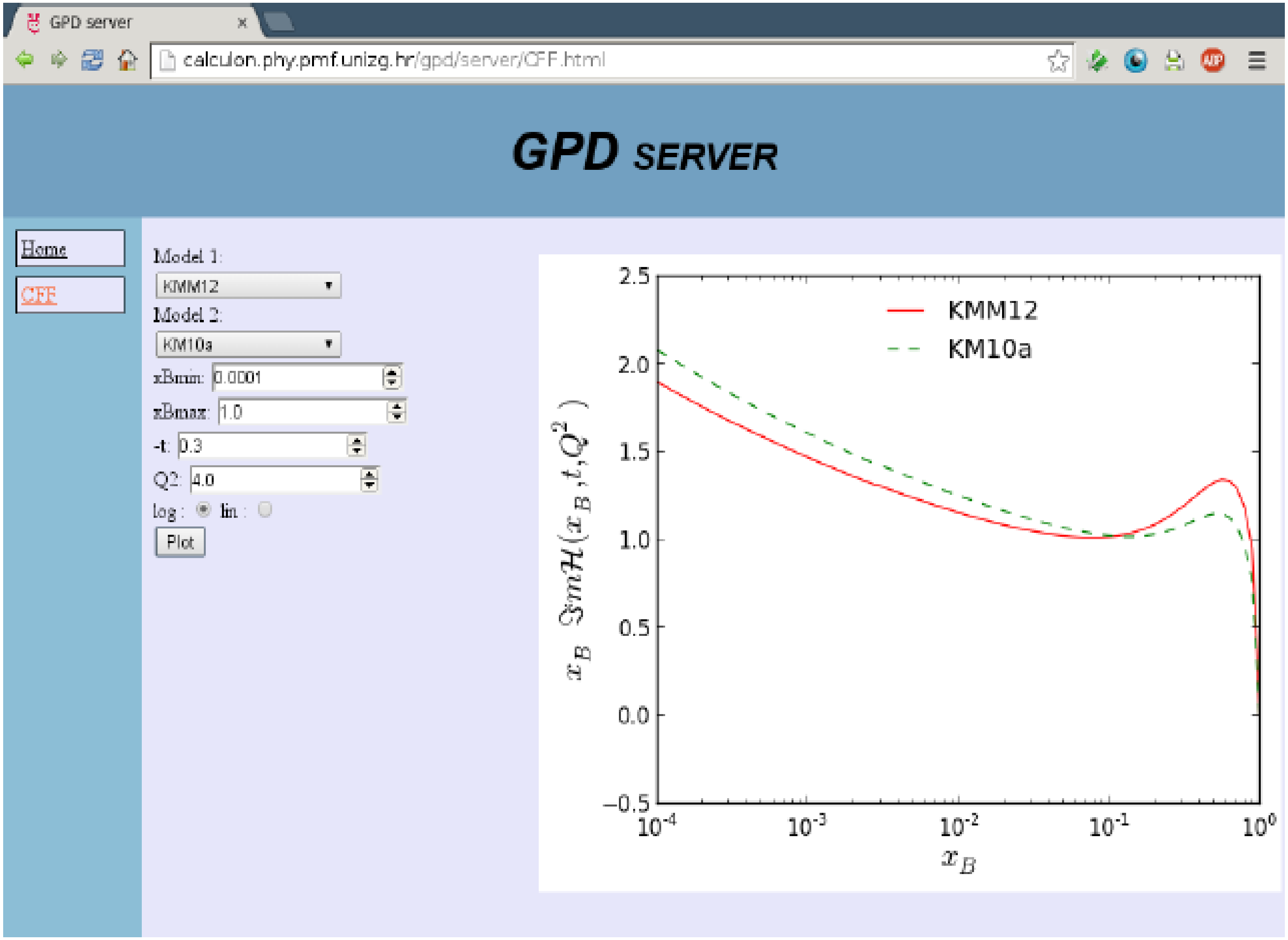
KM models are available at WWW

- http://calculon.phy.hr/gpd/ — binary code for cross sections

```
% xs.exe

  xs.exe  ModelID  Charge  Polarization  Ee  Ep  xB  Q2  t  phi

returns cross section (in nb) for scattering of lepton of energy Ee
on unpolarized proton of energy Ep. Charge=-1 is for electron.

ModelID is one of
   0 debug, always returns 42,
   1 KM09a - arXiv:0904.0458 fit without Hall A,
   2 KM09b - arXiv:0904.0458 fit with Hall A,
   3 KM10  - preliminary hybrid fit with LO sea evolution, from Trento presentation,
   4 KM10a - preliminary hybrid fit with LO sea evolution, without Hall A data
   5 KM10b - preliminary hybrid fit with LO sea evolution, with Hall A data

xB Q2 t phi  -- usual kinematics (phi is in Trento convention)

% xs.exe 1 -1 1 27.6 0.938  0.111 3. -0.3 0

  0.18584386497251
```

GPD page and server

- Durham-like CFF/GPD server page

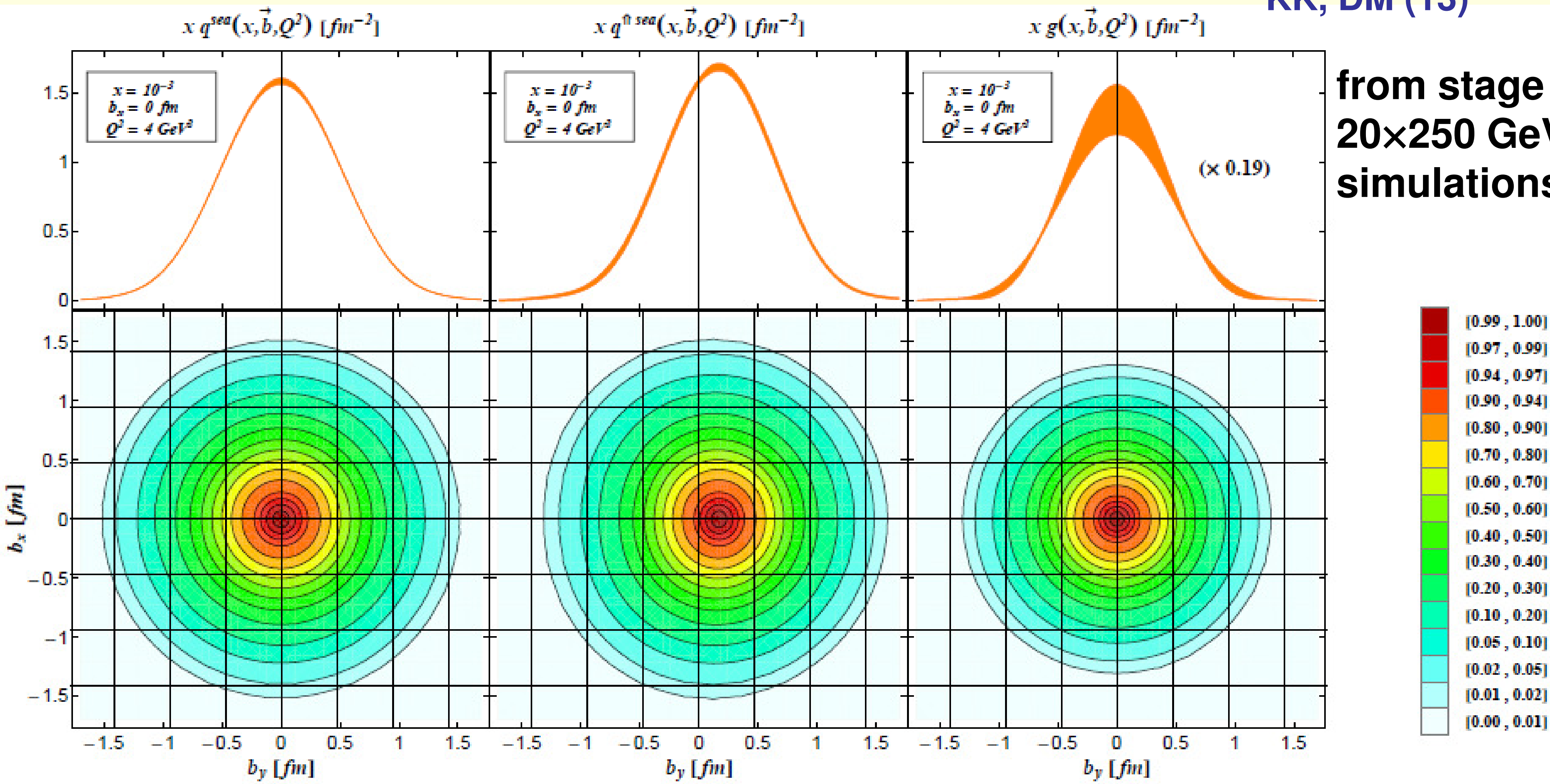


- Do we need "Les Houches Accord" CFF/GPD interface?

The Future

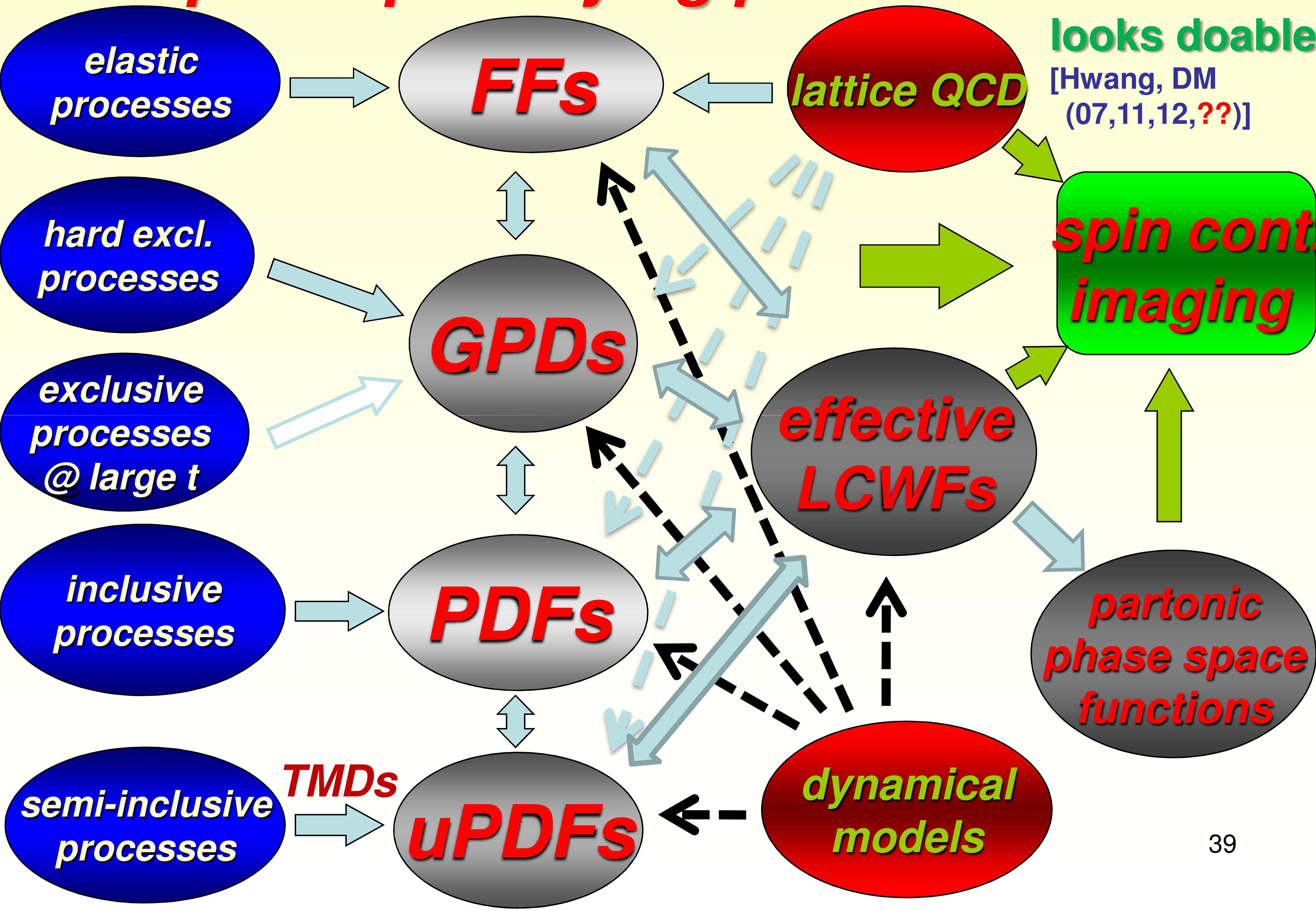
- ✓ COMPASS II
- ✓ JLAB@12 GeV
- ? ENC@GSI
- ? LHeC@CERN
- ? EIC@BNL or EIC@JLAB

Aschenauer, Firzo KK, DM (13)



from stage II
20×250 GeV^2
simulations

Prospect: quantifying partonic content



Summary

GPDs are intricate and (thus) a promising tool

- to reveal the transverse distribution of partons (to some extend done at small x_B)
- to address the spin content of the nucleon (not possible at present in pheno.)
- providing a bridge to LCWFs & non-perturbative methods (e.g., lattice)
- modeling in terms of effective LCWFs is doable (require efforts)

first decade of hard exclusive leptoproduction measurements

- CFFs have their own interest, bridging low and high virtuality regimes
- should be straightforward to improve global (flexible) model fits to DVCS
- DVCS and DVMP data are describable in global fits at small x
- moving on: to NLO, kinematical twist, full GPD models, DVCS+DVMP+...
- covering the kinematical region between HERA (COMPASS) experiments within a high luminosity machine and dedicated detectors is needed to quantify exclusive and inclusive QCD phenomena: handle on GPD E & 3D

need :

tools/technology for global NLO QCD fits (inclusive + exclusive)

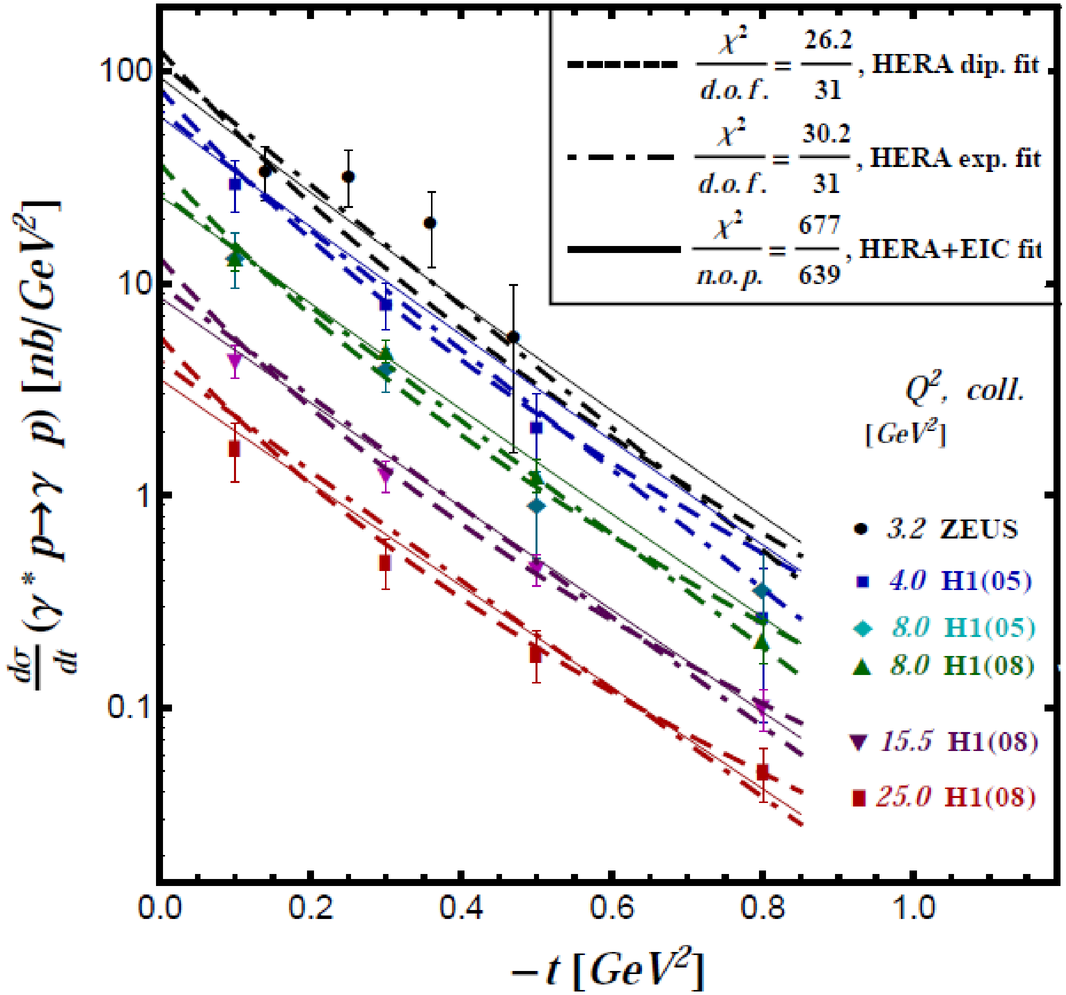
theory development ***(desired but not urgent needed for phenomenology)***

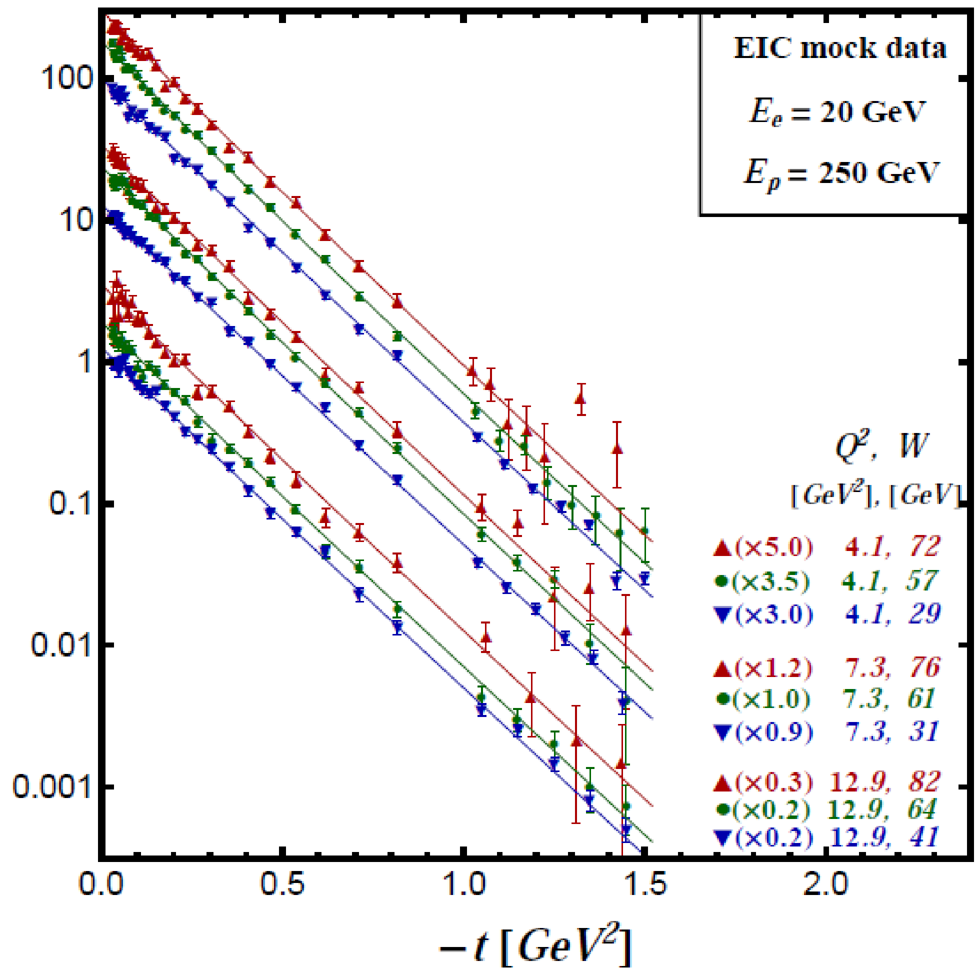
back ups

Impact of EIC data to extract GPD H

two simulations from S. Fazio for DVCS cross section ~ 650 data points
$-t <$ ~0.8 GeV^2 for ~ 10/fb
1 GeV^2 $< -t < 2$ GeV^2 for ~ 100/fb (cut: $-t < 1.5$ GeV^2, 4 GeV^2 $< Q2$ to ensure $-t < Q^2$)

pseudo data are re-generated with GeParD
statistical errors rescaled
5% systematical errors added in quadrature, 3% Bethe-Heitler uncertainty





Imaging (probabilistic interpretation)

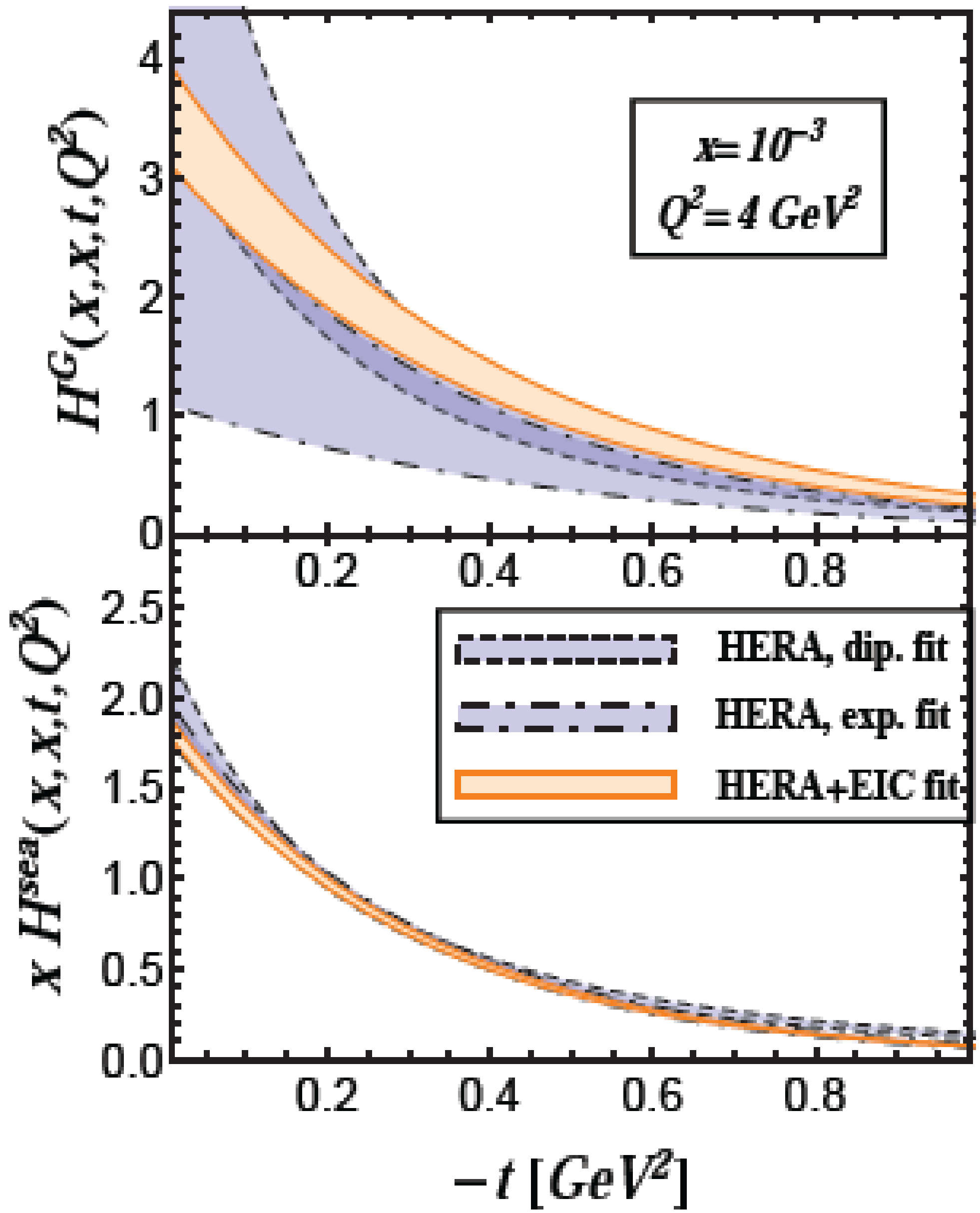
$$q(x, \vec{b}, \mu^2) = \frac{1}{4\pi} \int_0^\infty d|t|\, J_0(|\vec{b}|\sqrt{|t|}) H(x, \eta = 0, t, \mu^2)$$

skewness effect vanishes (s_2 , $s_4 \rightarrow 0$)

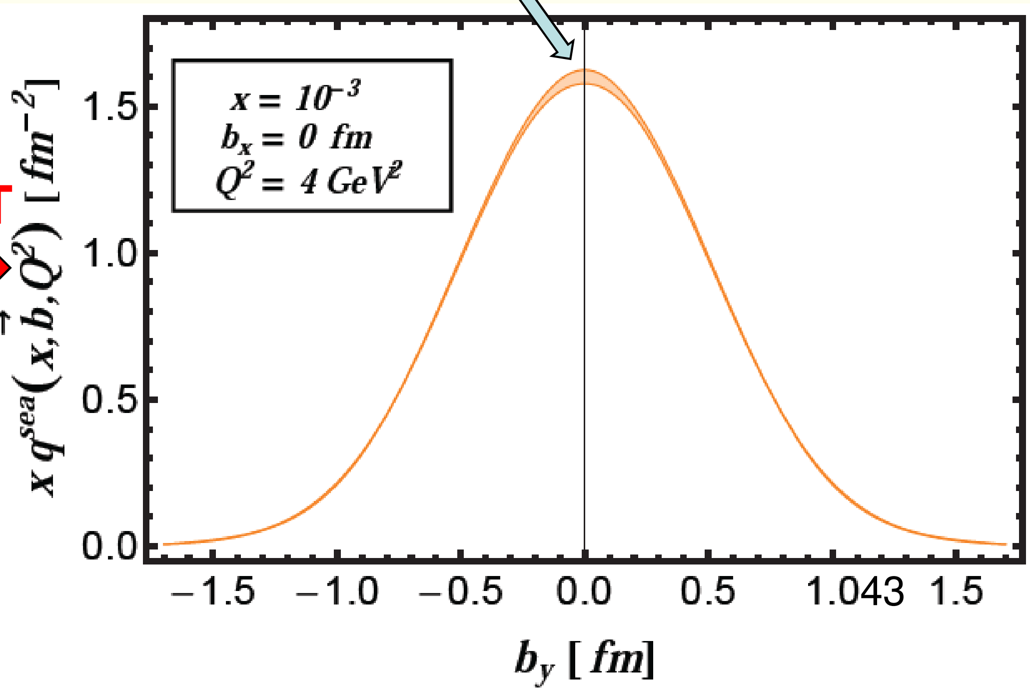
- reduce fit uncertainties
- increase model uncertainties

extrapolation errors for $-t \rightarrow 0$
(large b uncertainties – small effect)

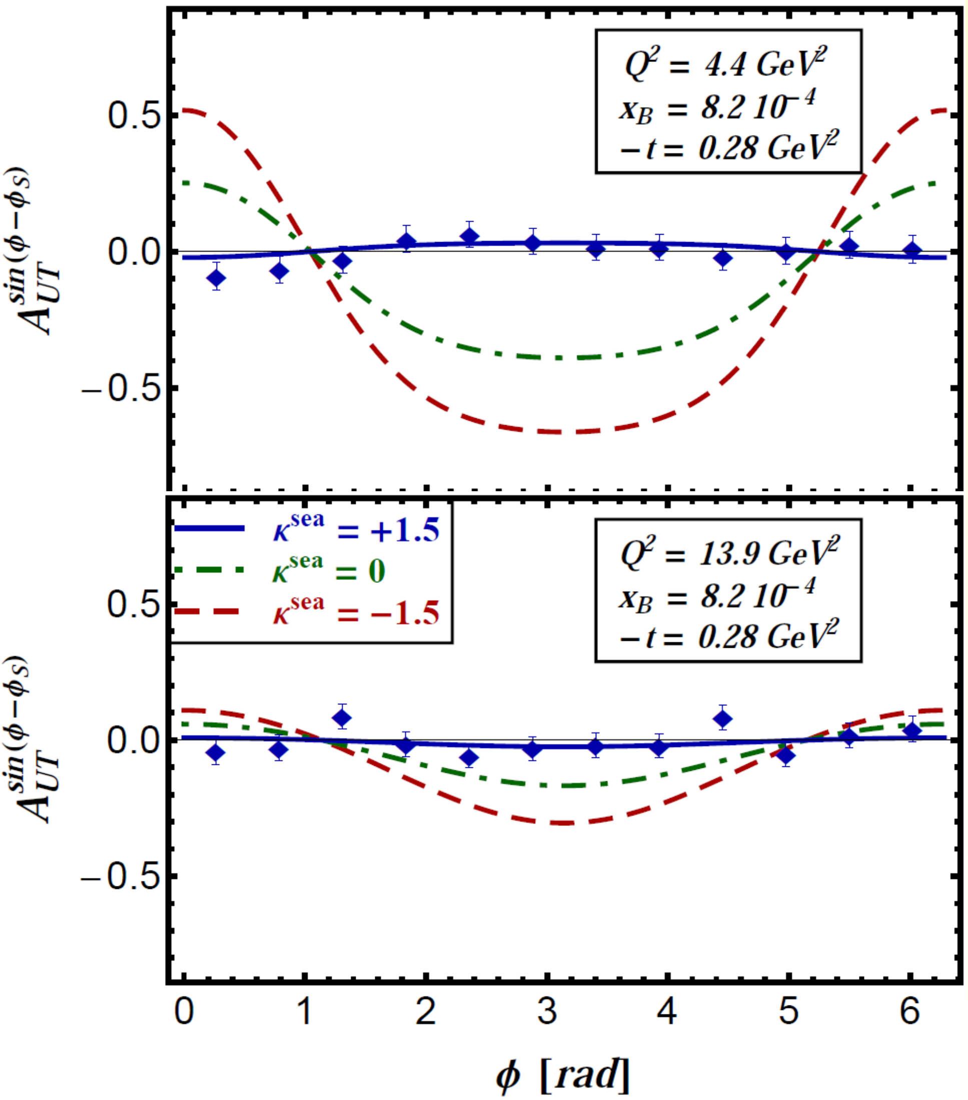
extrapolation errors into $-t > 1.5$ GeV^2
(small b uncertainties)



FT



Single transverse target spin asymmetry

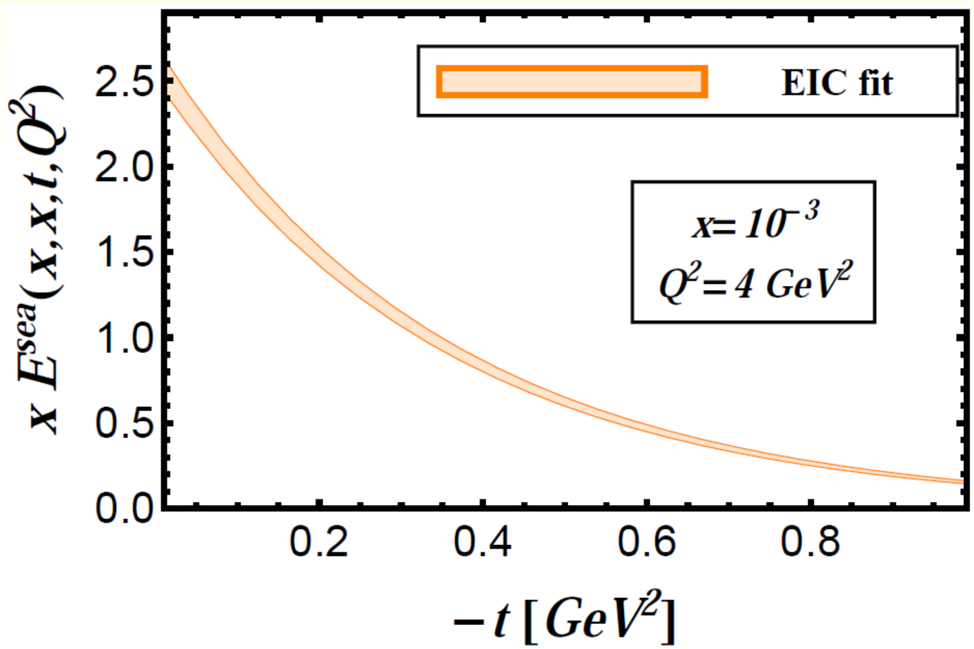


20x250 2x5/fb mock data
(~1200 data points with statistical errors
+ 5% systematics at cross section level)

flexible GPD model for E^{sea} and E^G

normalization (and t-dependency) of E^{sea} is reasonable constraint

E^G is essentially unconstraint

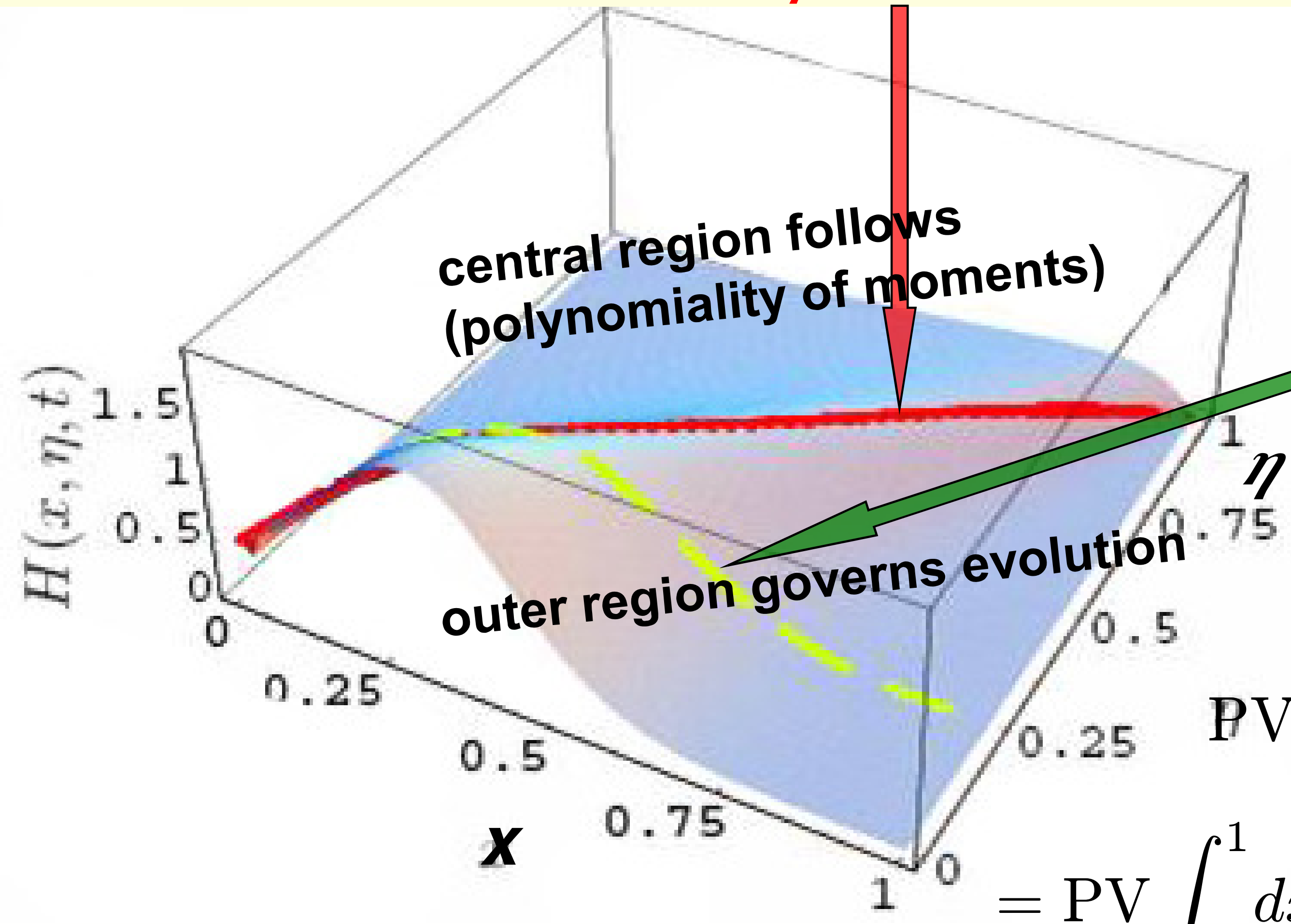


Modeling & Evolution

outer region governs the evolution at the cross-over trajectory

$$\mu^2 \frac{d}{d\mu^2} F(x,x,t,\mu^2) = \int_x^1 \frac{dy}{x} V(1, x/y, \alpha_s(\mu)) F(y,x,\mu^2)$$

GPD at $\eta = x$ is `measurable' (LO)



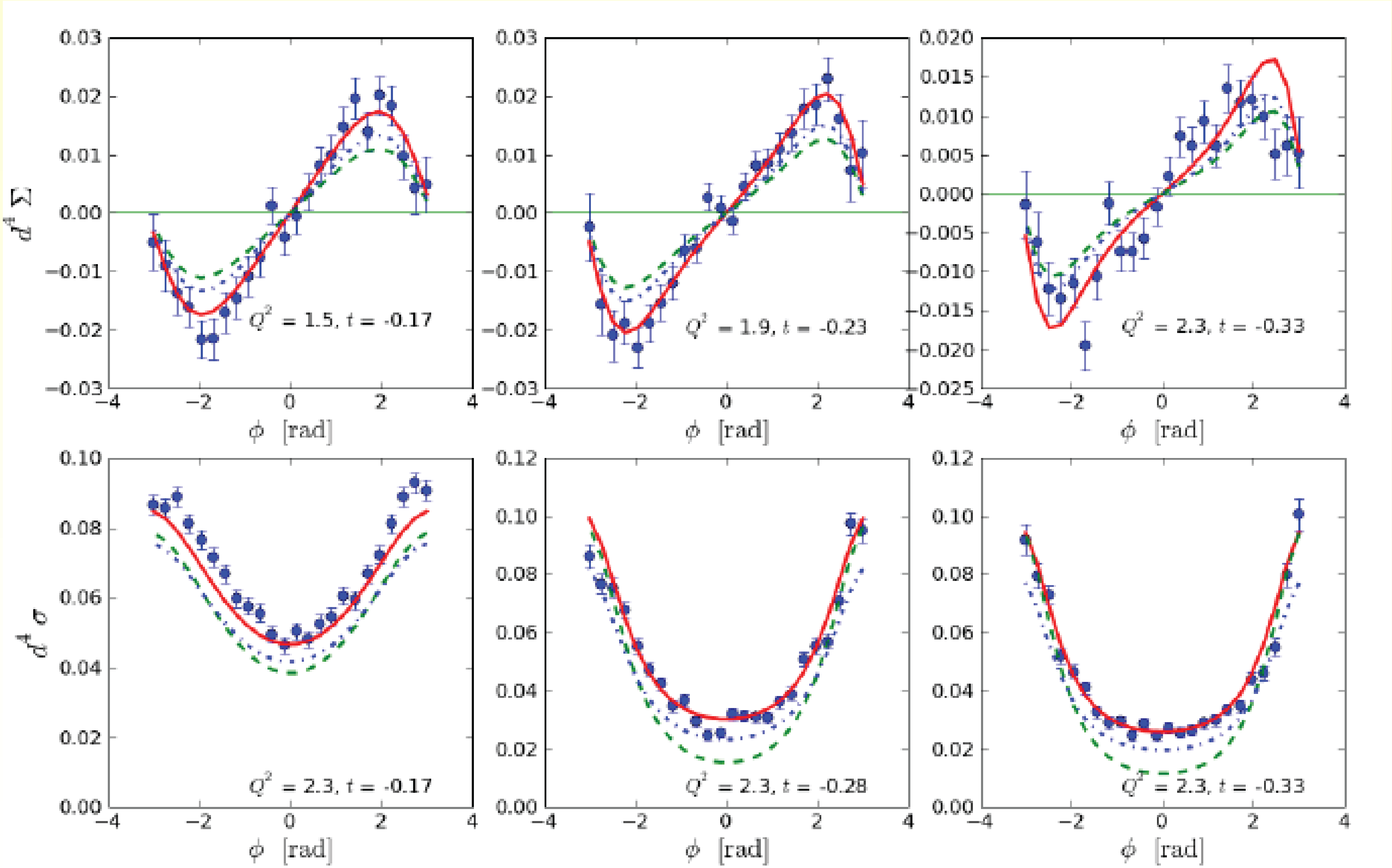
net contribution of outer + central region is governed by a sum rule:

$$\mathrm{PV}\int_0^1 dx\,\frac{2x}{\eta^2 - x^2} F^+(x,\eta,t)$$

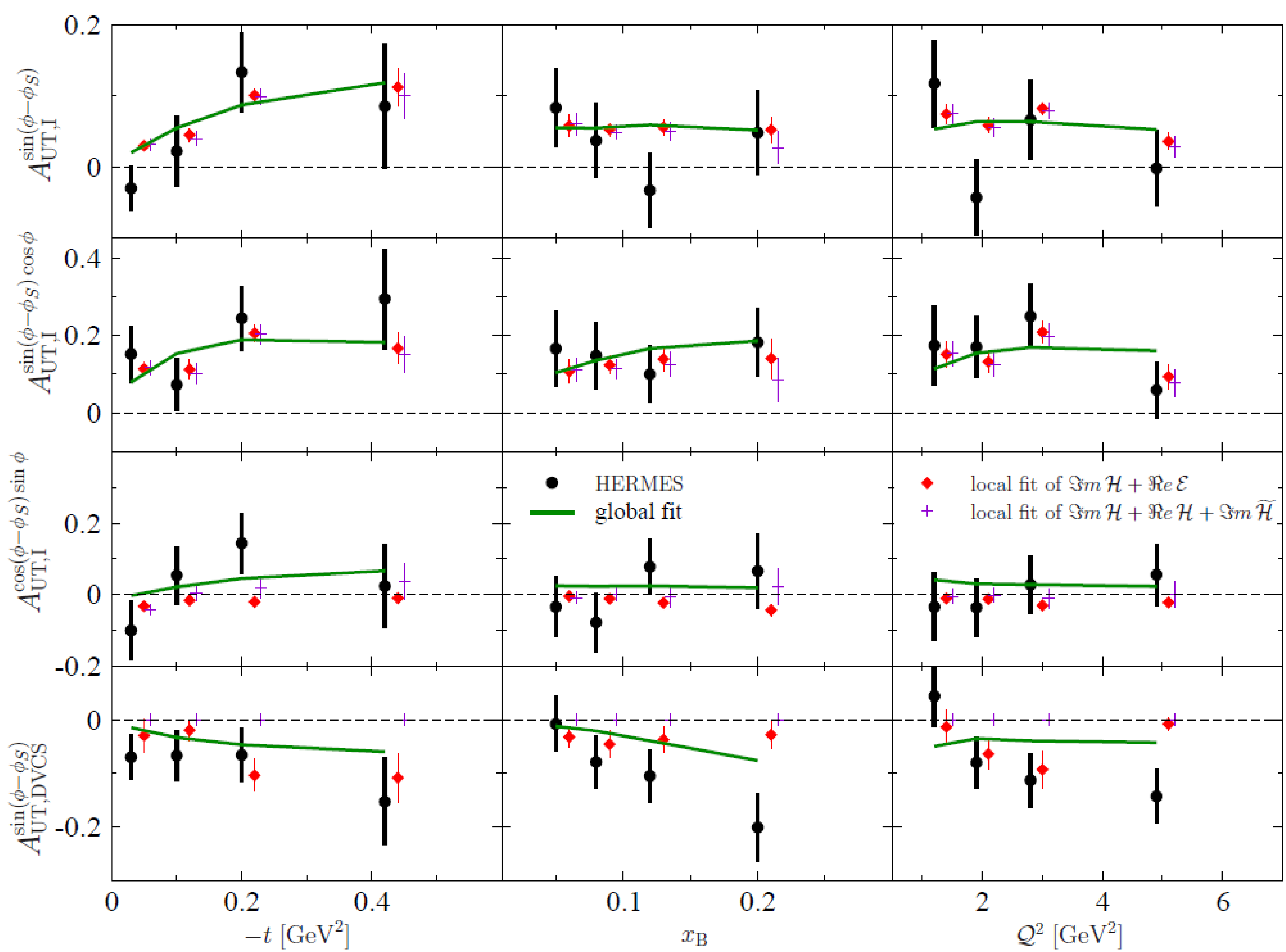
$$= \mathrm{PV}\int_0^1 dx\,\frac{2x}{\eta^2 - x^2} F^+(x,x,t) + C(t)$$

HALL A φ-dependence

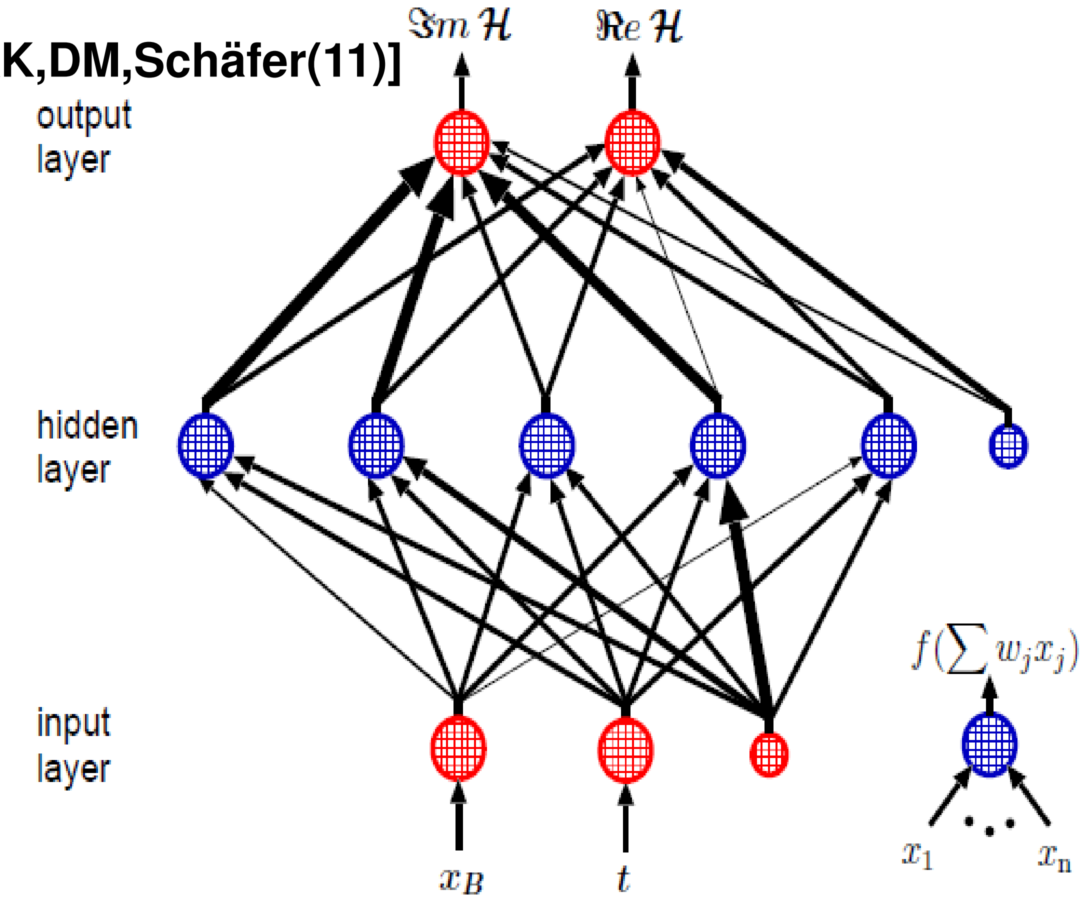
- φ-dependence is described (if we fit to it)



• KMM12 (KM10 type model) includes polarized target DVCS data (global fit to most of data , $\chi^2/d.o.f \approx 1.6$ - best what is there at present e.g., transverse polarized HERMES asymmetries looks as)



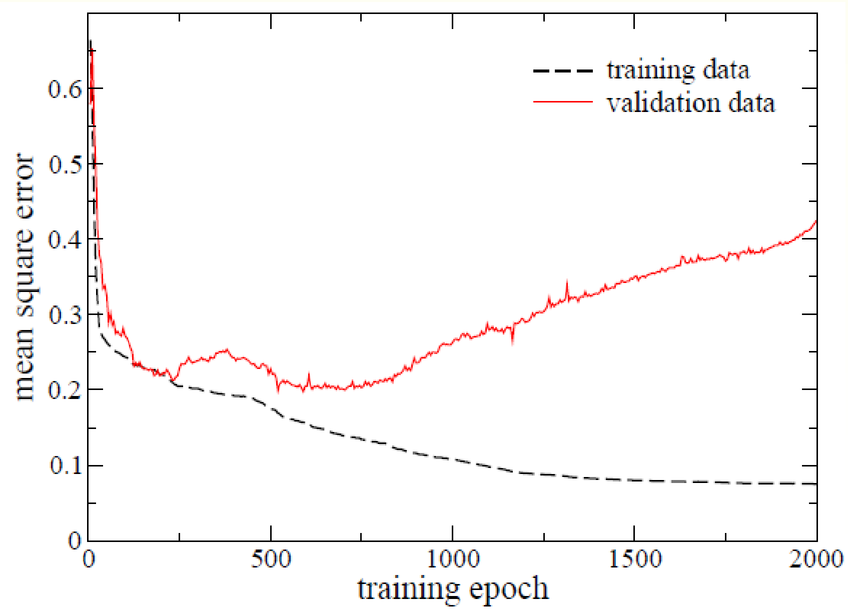
Neural Networks [KK,DM,Schäfer(11)]



- kinematical values are represented by the input layer
- propagated trough the network, where weights are set randomly
- random values for $Im\mathcal{H}$ and $Re\mathcal{H}$
- calculation of χ^2
- backwards propagation (PyBrain)
- adjusting weights so that error decreases
- repeat procedure
- taking next kinematical point

Monte Carlo procedure to propagate errors, i.e., generating a replica data set

avoiding over fitting (fitting to noise), dividing data set, taking a control example if error increases after decreasing – one stops

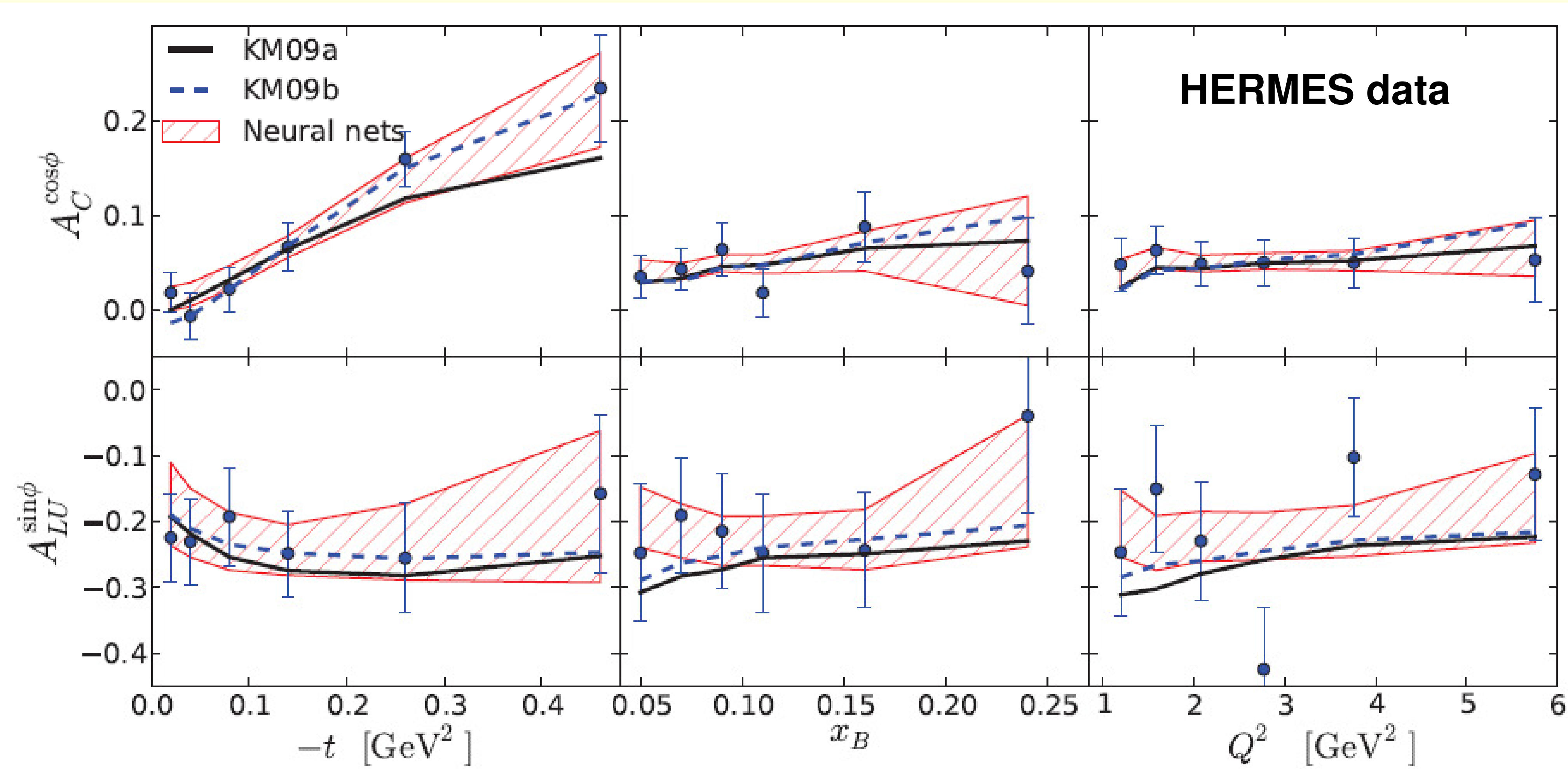


A first use of neural network fits

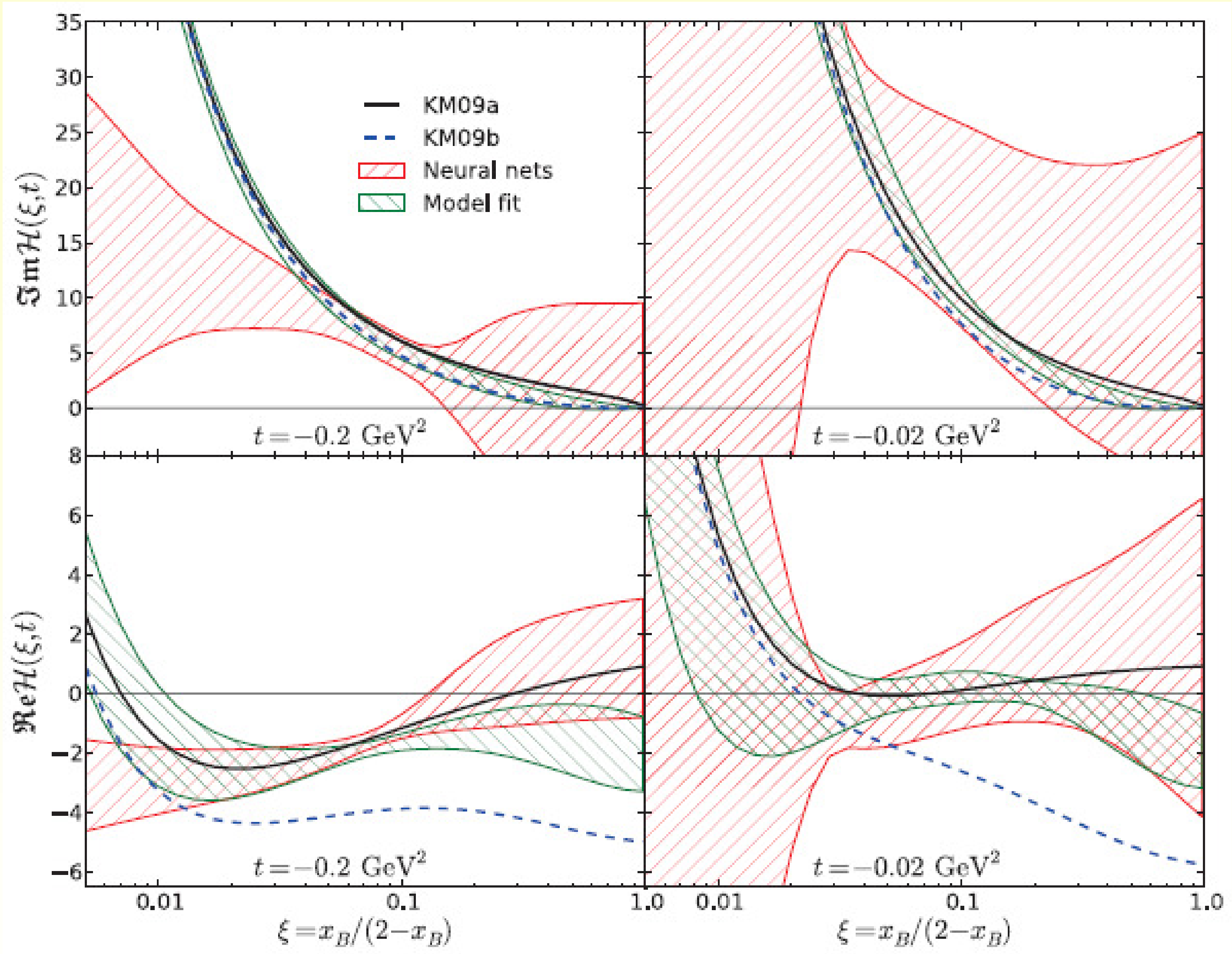
(ideal) tool for error propagation and quantifying model uncertainties

used to access real and imaginary part of $\mathcal{H}$ CFF from HERMES

results are compatible to model, CFF fits, and mapping



Model prediction versus unbiased error propagation



- model fits and neural networks are complimentary
- meaning of error bands should be properly understood
- error propagation is practically an art (full information is not given)

What is used for the (D)VCS tensor? (helicity amplitudes)

A simple electromagnetic form factor parametrization is accepted:

$$\langle p_2, s_2 | j_\rho(0) | p_1, s_1 \rangle \;=\; \overline{u}(p_2, s_2) \left[\gamma_\rho \, F_1(t) + i\sigma_{\rho\sigma} \frac{\Delta^\sigma}{2M} \, F_2(t) \right] u(p_1, s_1)$$

What about (DV)CS-tensor parmetrizations?

$$T_{\mu\nu} = i \int d^4x \, \mathrm{e}^{\frac{i}{2}(q_1+q_2)\cdot x} \langle p_2 | T \left\{ j_\mu(x/2) j_\nu(-x/2) \right\} | p_1 \rangle$$

Prange (1958) [real CS, over-counting amplitudes, Dirac-spinors]

Hearn, Leader (1962) [VCS, Pauli-spinor representation]

Tarrach (1975) [VVCS, kinematical constraints are removed]

DVCS (calculated in terms of GPDs since 1992, various similar parametrizations)

VCS Kroll et al. (1995) in terms of helicity amplitudes [diquark model for nucleon]

VCS Drechsel et al. (1998) [VCS, generalized polarizabilieties]

etc.

unique parametrization for the (D)VCS tensor is desired

Tarrach
(1975)

? minimal

The so-obtained basis of gauge invariant tensors is expected to be minimal and pole free by construction and is the following one :

$$\mathcal{T}_1 = k\cdot k'\, T_1 - T_3$$
$$\mathcal{T}_2 = k^2 k'^2\, T_1 + k\cdot k'\, T_2 - \frac{k^2+k'^2}{2} T_4 + \frac{k^2-k'^2}{2} T_5$$
$$\mathcal{T}_3 = (\ell\cdot K)^2\, T_1 + k\cdot k'\, T_6 - \ell\cdot K\, T_7$$
$$\mathcal{T}_4 = \ell\cdot K\,(k^2+k'^2)\, T_1 - \ell\cdot K\, T_4 - \frac{k^2+k'^2}{2} T_7 + \frac{k^2-k'^2}{2} T_8 + k\cdot k'\, T_9$$
$$\mathcal{T}_5 = -\ell\cdot K\,(k^2-k'^2)\, T_1 + \ell\cdot K\, T_5 + \frac{k^2-k'^2}{2} T_7 - \frac{k^2+k'^2}{2} T_8 + k\cdot k'\, T_{10}$$
$$\mathcal{T}_6 = \ell\cdot K\, T_2 - \frac{k^2+k'^2}{4} T_9 - \frac{k^2-k'^2}{4} T_{10} - M\, T_{12} + M\,\frac{k^2+k'^2}{4} T_{23} - M\,\frac{k^2-k'^2}{4} T_{24}$$
$$\qquad + \frac{k^2-k'^2}{8} T_{29} - \frac{k^2+k'^2}{8} T_{30} - \frac{k^2 k'^2}{4} T_{33}$$
$$\mathcal{T}_7 = 8\, T_{16} - 4\,\ell\cdot K\, T_{21} + \ell\cdot K\, T_{34}$$
$$\mathcal{T}_8 = T_{19} + \frac{k^2-k'^2}{2} T_{22} - \ell\cdot K\, T_{23} + \frac{k^2+k'^2}{8} T_{34}$$
$$\mathcal{T}_9 = T_{20} - \frac{k^2+k'^2}{2} T_{22} + \ell\cdot K\, T_{24} - \frac{k^2-k'^2}{8} T_{34}$$
$$\mathcal{T}_{10} = -8\, k\cdot k'\, T_6 + 4\,\ell\cdot K\, T_7 + 4M\, k\cdot k'\, T_{21} - 4M\,\ell\cdot K\, T_{25} - 2\,\ell\cdot K\, T_{32} - 2\,k\cdot k'\,\ell\cdot K\, T_{33} + M\, k\cdot k'\, T_{34}$$
$$\mathcal{T}_{11} = T_{18} - k\cdot k'\, T_{22} + \ell\cdot K\, T_{26}$$
$$\mathcal{T}_{12} = \ell\cdot K\, T_4 - \frac{k^2-k'^2}{2} T_8 - k\cdot k'\, T_9 - M\, T_{14} + M\, k\cdot k'\, T_{23} - M\,\frac{k^2-k'^2}{2} T_{26} - \frac{k^2+k'^2}{4} T_{32} - k\cdot k'\,\frac{k^2+k'^2}{4} T_{33}$$
$$\mathcal{T}_{13} = \ell\cdot K\, T_5 - \frac{k^2+k'^2}{2} T_8 + k\cdot k'\, T_{10} - M\, T_{15} + M\, k\cdot k'\, T_{24} - M\,\frac{k^2+k'^2}{2} T_{26} - \frac{k^2-k'^2}{4} T_{32} - k\cdot k'\,\frac{k^2-k'^2}{4} T_{33}$$
$$\mathcal{T}_{14} = 2\,\ell\cdot K\, T_8 - 2M\, k\cdot k'\, T_{22} + 2M\,\ell\cdot K\, T_{26} - k\cdot k'\, T_{27} + \ell\cdot K\, T_{31}$$
$$\mathcal{T}_{15} = -(k^2-k'^2)\, T_7 + (k^2+k'^2)\, T_8 - 2\,k\cdot k'\, T_{10} - 2M\, k\cdot k'\, T_{24} + M(k^2-k'^2)\, T_{25} + M(k^2+k'^2)\, T_{26}$$
$$\qquad - k\cdot k'\, T_{29} + \frac{k^2+k'^2}{2} T_{31} + \frac{k^2-k'^2}{2} T_{32}$$
$$\mathcal{T}_{16} = -(k^2+k'^2)\, T_7 + (k^2-k'^2)\, T_8 + 2\,k\cdot k'\, T_9 - 2M\, k\cdot k'\, T_{23} + M(k^2+k'^2)\, T_{25} + M(k^2-k'^2)\, T_{26}$$
$$\qquad - k\cdot k'\, T_{30} + \frac{k^2-k'^2}{2} T_{31} + \frac{k^2+k'^2}{2} T_{32}$$
$$\mathcal{T}_{17} = -4\,\ell\cdot K\, T_1 + 2\, T_7 + 4M\, T_{11} - 2M\, T_{25} + T_{32} + k\cdot k'\, T_{33} \qquad (12)$$
$$\mathcal{T}_{18} = 4\, T_{17} - 4\,\ell\cdot K\, T_{25} + k\cdot k'\, T_{34}$$

Requirements on the parametrization of DVCS tensor

- Lorentz-covariance + gauge invariance + implementing discrete symmetries
- scalar amplitudes (6 for RCS, 12 for (D)VCS, 18 for general parametrization without kinematical constraints
- simplicity (including simple relation to what is already used)

DVCS tensor parametrizations arise from approximate calculations

- $x^\mu \propto n^\mu + \ldots$ different choices for light-like vector $n \propto q + a\, p_1 + b\, p_2$,
  (constructed from in- and out-particle momenta)
- various results differ at twist-2, twist-3, (twist-4) by the order O(1/Q), O(1/Q^2), O(1/Q^3)
- DVCS/GPD results are not exact and suffer from breaking of gauge+Lorentz symmetries
- to relate GPDs to observables a convention is needed (if one likes to compare results)

embed GPD findings in a general DVCS tensor parametrization

[Belitsky, DM, Kirchner (01)]

- this does not solve the ambiguity problem in GPD calculations **(see Volodya`s talk)**
- provides the basis to discuss the physics case of (D)VCS measurements

$$\mathcal{T}_{ab}^{\rm VCS}(\phi) = (-1)^{a-1}\varepsilon_2^{\mu*}(b)T_{\mu\nu}\varepsilon_1^{\nu}(a)$$

[Belitsky, DM, Kirchner (01) -- BM Ji (12)]

$$\mathcal{T}_{ab}^{\rm VCS} = \mathcal{V}(\mathcal{F}_{ab}) - b\,\mathcal{A}(\mathcal{F}_{ab}) \quad \text{for} \quad a\in\{0,+,-\},\ b\in\{+,-\}$$

$$\mathcal{V}(\mathcal{F}_{ab}) = \bar{u}_2\left(\not{m}\mathcal{H}_{ab} + i\sigma_{\alpha\beta}\frac{m^\alpha\Delta^\beta}{2M}\mathcal{E}_{ab}\right)u_1$$

$$\mathcal{A}(\mathcal{F}_{ab}) = \bar{u}_2\left(\not{m}\gamma_5\,\widetilde{\mathcal{H}}_{ab} + \gamma_5\frac{m\cdot\Delta}{2M}\widetilde{\mathcal{E}}_{ab}\right)u_1\,, \qquad m^\mu = \frac{q_1^\mu + q_2^\mu}{(p_1+p_2).(q_1+q_2)}$$

parameterization of (DV)CS helicity amplitudes

$$\begin{aligned} T_{\mu\nu} = & -\tilde{g}_{\mu\nu}\frac{q\cdot V_{\rm T}}{p\cdot q} + i\tilde{\varepsilon}_{\mu\nu}\frac{q\cdot A_{\rm T}}{p\cdot q} + \left(q_{2\,\mu} - \frac{q_2^2}{p\cdot q}p_\mu\right)\left(q_{1\,\nu} - \frac{q_1^2}{p\cdot q}p_\nu\right)\frac{q\cdot V_{\rm L}}{p\cdot q} \\ & + \left(q_{1\,\nu} - \frac{q_1^2}{p\cdot q}p_\nu\right)\left(g_{\mu\rho} - \frac{p_\mu\,q_{2\,\rho}}{p\cdot q}\right)\left[\frac{V_{\rm LT}^\rho}{p\cdot q} + \frac{i\epsilon^\rho{}_{q p\sigma}}{p\cdot q}\frac{A_{\rm LT}^\sigma}{p\cdot q}\right] \\ & + \left(q_{2\,\mu} - \frac{q_2^2}{p\cdot q}p_\mu\right)\left(g_{\nu\rho} - \frac{p_\nu\,q_{1\,\rho}}{p\cdot q}\right)\left[\frac{V_{\rm TL}^\rho}{p\cdot q} + \frac{i\epsilon^\rho{}_{q p\sigma}}{p\cdot q}\frac{A_{\rm TL}^\sigma}{p\cdot q}\right] \\ & + \left(g_\mu{}^\rho - \frac{p_\mu\,q_2^\rho}{p\cdot q}\right)\left(g_\nu{}^\sigma - \frac{p_\nu\,q_1^\sigma}{p\cdot q}\right)\left[\frac{\Delta_\rho\Delta_\sigma + \widetilde{\Delta}_\rho^\perp\widetilde{\Delta}_\sigma^\perp}{2M^2}\frac{q\cdot V_{\rm TT}}{p\cdot q} + \frac{\Delta_\rho\widetilde{\Delta}_\sigma^\perp + \widetilde{\Delta}_\rho^\perp\Delta_\sigma}{2M^2}\frac{q\cdot A_{\rm TT}}{p\cdot q}\right] \end{aligned}$$

(one) parameterization of (DV)CS tensor

equivalent to *Tarrach*'s one

relations of CFFs to helicity dependent CFFs are easily calculated:

$$\begin{aligned} \mathcal{F}_{+b} = & \left[\frac{1+b\sqrt{1+\epsilon^2}}{2\sqrt{1+\epsilon^2}} + \frac{(1-x_{\rm B})x_{\rm B}^2(4M^2-t)\left(1+\frac{t}{\mathcal{Q}^2}\right)}{\mathcal{Q}^2\sqrt{1+\epsilon^2}\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)^2}\right]\mathcal{F}_{\rm T} \\ & + \frac{1-b\sqrt{1+\epsilon^2}}{2\sqrt{1+\epsilon^2}}\frac{\widetilde{K}^2}{M^2\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)^2}\mathcal{F}_{\rm TT} + \frac{2x_{\rm B}\widetilde{K}^2}{\mathcal{Q}^2\sqrt{1+\epsilon^2}\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)^2}\mathcal{F}_{\rm LT} \\ \mathcal{F}_{0+} = & \frac{\sqrt{2}\,\widetilde{K}}{\sqrt{1+\epsilon^2}\mathcal{Q}\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)}\left\{\left[1+\frac{2x_{\rm B}^2\left(4M^2-t\right)}{\mathcal{Q}^2\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)}\right]\mathcal{F}_{\rm LT}\right. \\ & \left. + x_{\rm B}\left[1+\frac{2x_{\rm B}(4M^2-t)}{\mathcal{Q}^2\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)}\right]\mathcal{F}_{\rm T} + x_{\rm B}\left[2-\frac{4M^2-t}{M^2\left(2-x_{\rm B}+\frac{x_{\rm B}t}{\mathcal{Q}^2}\right)}\right]\mathcal{F}_{\rm TT}\right\} \end{aligned}$$

usable for DVCS - RCS, extendable to timelike (D)VCS, double(D)VCS or DIS

this will not be our last suggestion

What is used to connect GPDs to DVCS?

Vanderhaeghen Guidal Guichon *(VGG) code*

- numerical squaring, well defined wrt. convention (includes twist-three, no transversity)
- implemented GPD model is often called VGG – it is GPV
- model is often not specified if confronted with data

[Goeke, Polyakov, Vanderhaeghen (01), based on Radyushkin ansatz]

Guchion (Vanderhaeghen) code used by Moutarde & Sabatie

- unpublished, I am not able to figure out what goes into the code (users can not tell me)
- users claims: the code is `exact' and uniquely separates leptonic and hadronic parts, which is simply wrong

BM(K)J versions

- analytic squaring with (BMK) and without (BMJ) approximtaion
- approximate versions went into `private codes' and Monte Carlos
  (Freund & McDermott (MILOU), Guzey&Teckentrup, HERMES, JLAB, COMPASS)
- we (KM) use (apart from transverse target) BM11 contained in BMJ12
  (will be upgraded to 12 CFFs, needed for kinematical twist-4 corrections)

➢ numerical comparison on precision level is only possible with VGG

➢ practically, results agree often very well since we use similar light-like vector *n*
this has not to be the case, see Volodya`s talk

Summing up conformal PWs

- GPD support is a consequence of Poincaré covariance (polynomiality)

$$H_j(\eta, t, \mu^2) = \int_{-1}^{1} dx\, c_j(x,\eta) H(x,\eta,t,\mu^2)\,, \qquad c_j(x,\eta) = \eta^j C_j^{3/2}(x/\eta)$$

- conformal moments evolve autonomous (to LO and beyond in a special scheme)

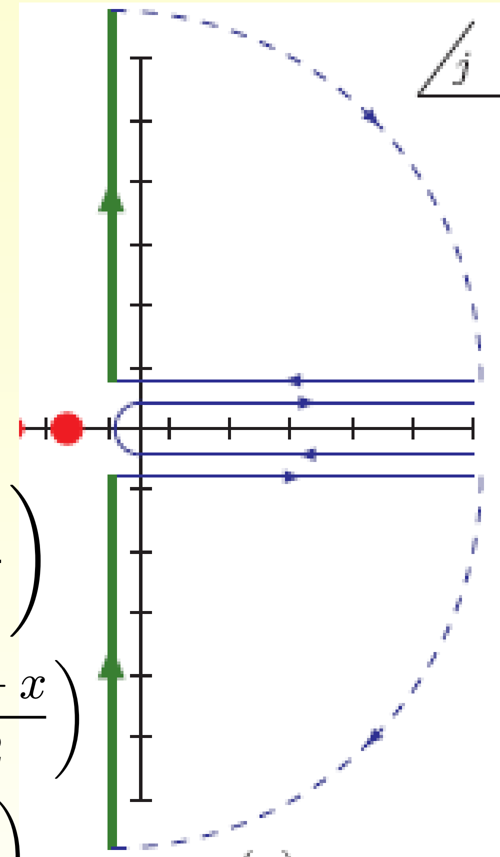
$$\mu \frac{d}{d\mu} H_j(\eta, t, \mu^2) = -\frac{\alpha_s(\mu)}{2\pi} \gamma_j^{(0)} H_j(\eta, t, \mu^2)$$

- inverse relation is given as series of (mathematical) generalized distributions:

$$H(x,\eta,t) = \sum_{j=0}^{\infty} (-1)^j p_j(x,\eta) H_j(\eta,t)\,,\; p_j(x,\eta) \propto \theta(|x| \leq \eta) \frac{\eta^2 - x^2}{\eta^{j+3}} C_j^{3/2}(-x/\eta)$$

- various ways of resummation were proposed:
  - smearing method [Radyushkin (97); Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)]
  - mapping to a kind of forward PDFs [A. Shuvaev (99), J. Noritzsch (00)]
  - dual parameterization [M. Polyakov, A. Shuvaev (02), Polyakov (07), Semenov-Tian-Shansky]
  - based on conformal light-ray operators [Balitsky, Braun (89); Kivel, Mankewicz (99)]
  - ***Mellin-Barnes integral*** [DM, Schäfer (05); A. Manashov, M. Kirch, A. Schäfer (05)]

Sommerfeld-Watson transform



✓ rewrite sum as an integral around the real axis:

$$F(x,\eta,\Delta^2) = \frac{1}{2i}\oint_{(0)}^{(\infty)} dj\, \frac{1}{\sin(\pi j)} p_j(x,\eta)\, F_j(\eta,\Delta^2)$$

✓ find appropriate analytic continuation of p_j and F_j (Carlson's theorem)

$$p_j(x,\eta) = \theta(\eta - |x|)\eta^{-j-1}\mathcal{P}_j\left(\frac{x}{\eta}\right) + \theta(x-\eta)\eta^{-j-1}\mathcal{Q}_j\left(\frac{x}{\eta}\right)$$

$$\mathcal{P}_j(x) = \frac{2^{j+1}\Gamma(5/2+j)}{\Gamma(1/2)\Gamma(1+j)}(1+x)\,{}_2F_1\left(\begin{matrix}-j-1, j+2\\ 2\end{matrix}\Big|\frac{1+x}{2}\right)$$

$$\mathcal{Q}_j(x) = -\frac{\sin(\pi j)}{\pi}\, x^{-j-1}\,{}_2F_1\left(\begin{matrix}(j+1)/2, (j+2)/2\\ 5/2+j\end{matrix}\Big|\frac{1}{x^2}\right)$$

✓ change integration path so that singularities remain on the l.h.s.

$$F(x,\eta,\Delta^2) = \frac{i}{2}\int_{c-i\infty}^{c+i\infty} dj\, \frac{1}{\sin(\pi j)} p_j(x,\eta)\, F_j(\eta,\Delta^2)$$

✓ **NOTE:** continuation of GPD conformal moments has not worked out numerically
RDDA with integer *b, β*, like GK model, can be transformed in Mellin space