

Flavor and spin dependence of transverse densities

Gerald A. Miller, UW

- **Personal view of subject based on my recent papers**
- **Transverse charge densities**, *Ann.Rev.Nucl.Part.Sci.* 60 (2010)
- **Flavor separation**
- **Transverse magnetization density**
- **Spin dependent density**
- Lattice calculation shows proton is not round
- **Experiment (TMD): TMD is momentum-space probability- sdd given by pretzelosity**

Model independent transverse charge density

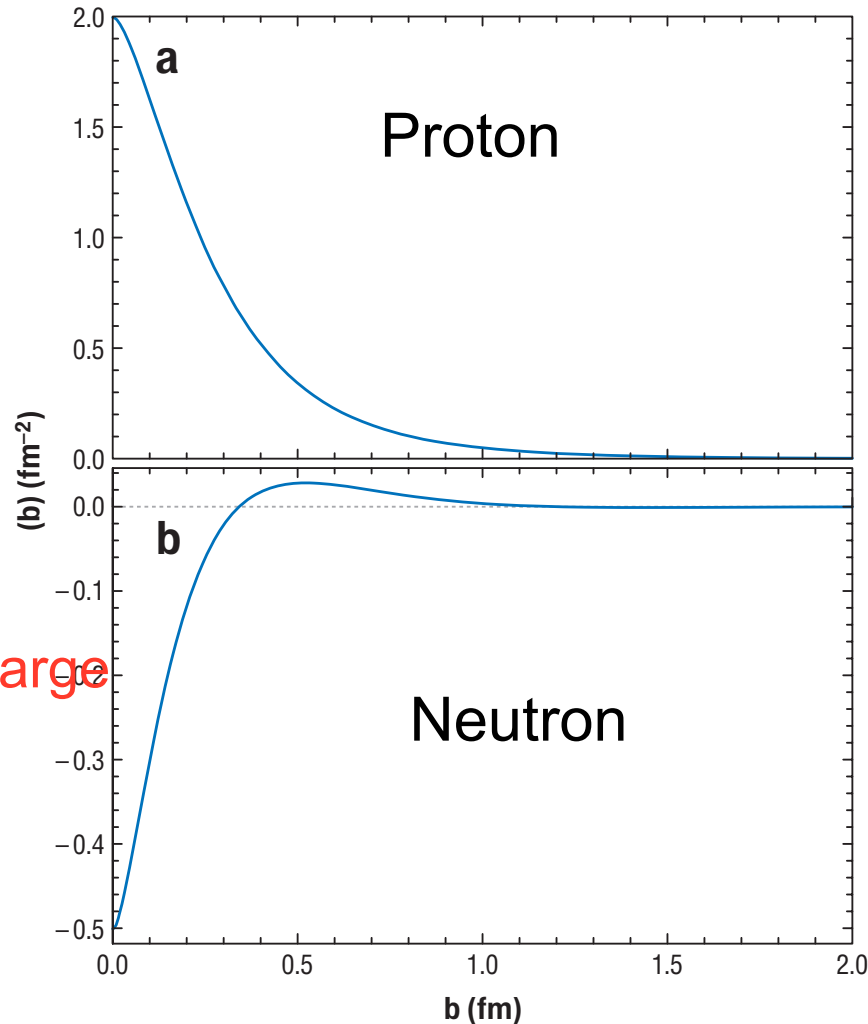
$$J^+(x^-, \mathbf{b}) = \sum_q e_q q_+^\dagger(x^-, b) q_+(x^-, b) \quad \text{Charge Density operator IMF}$$

$$\rho_\infty(x^-, \mathbf{b}) = \langle p^+, \mathbf{R} = \mathbf{0}, \lambda | \sum_q e_q q_+^\dagger(x^-, b) q_+(x^-, b) | p^+, \mathbf{R} = \mathbf{0}, \lambda \rangle$$

$$F_1 = \langle p^+, \mathbf{p}', \lambda | J^+(0) | p^+, \mathbf{p}, \lambda \rangle$$

$$\rho(b) \equiv \int dx^- \rho_\infty(x^-, \mathbf{b}) = \int \frac{Q dQ}{2\pi} F_1(Q^2) J_0(Qb)$$

Transverse charge densities from parameterizations (Alberico)



Negative central charge density

Negative central density - GAM

Phys.Rev.Lett. 99 (2007) 112001

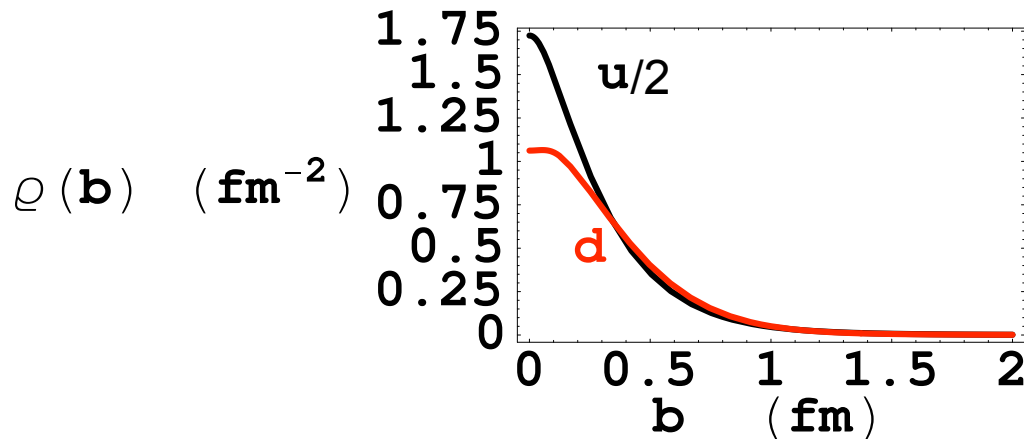
Flavor separation

- Charge symmetry (u in proton is d in neutron)

$$F^p = \frac{2}{3}F^u - \frac{1}{3}F^d, \quad F^u = \langle p | \bar{u}\gamma u | p \rangle, \quad \text{etc.}$$

$$F^n = \frac{2}{3}F^d - \frac{1}{3}F^u$$

- Neglect $s\bar{s}$ pairs

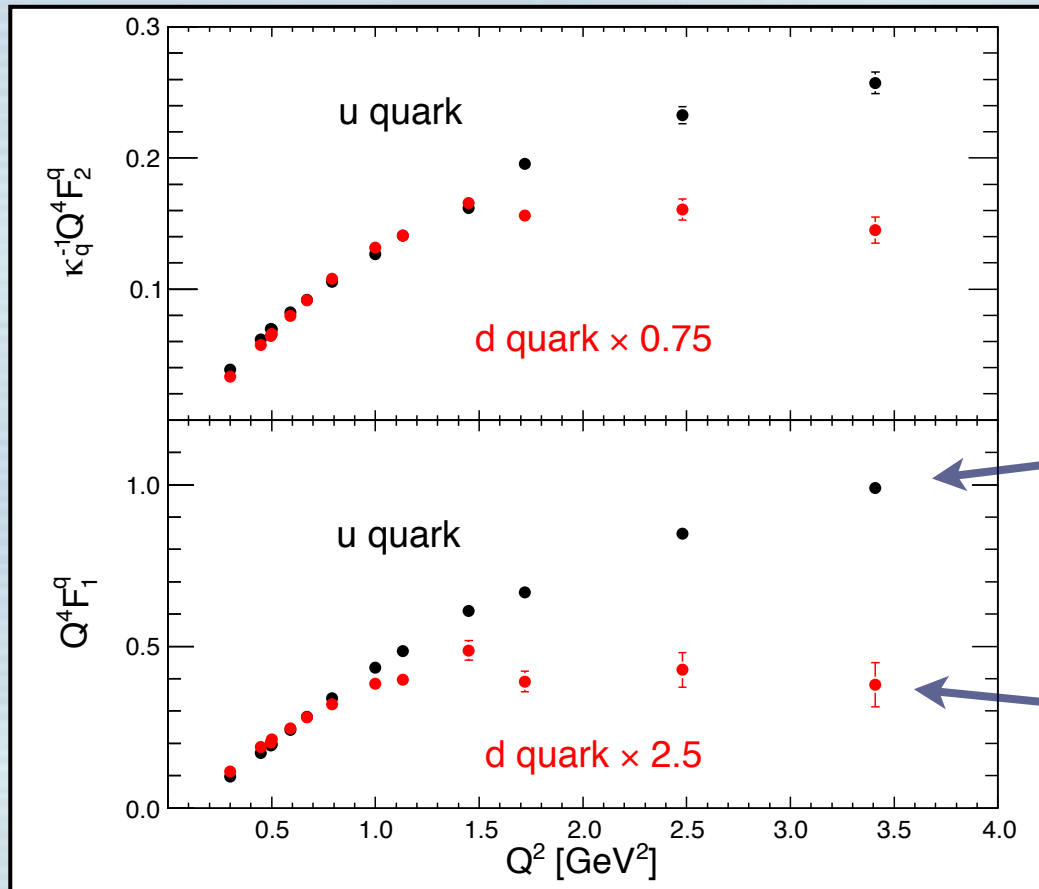


Form factors are more interesting

Next three slides from G Cates

The flavor separated form factors for the up and down quarks have very different Q^2 behavior above 1 GeV^2

Cates, de Jager, Riordan and Wojtsekhowski, PRL vol. 106, pg 252003 (2011)



F^u seems to scale more like $1/Q^2$ (if at all).

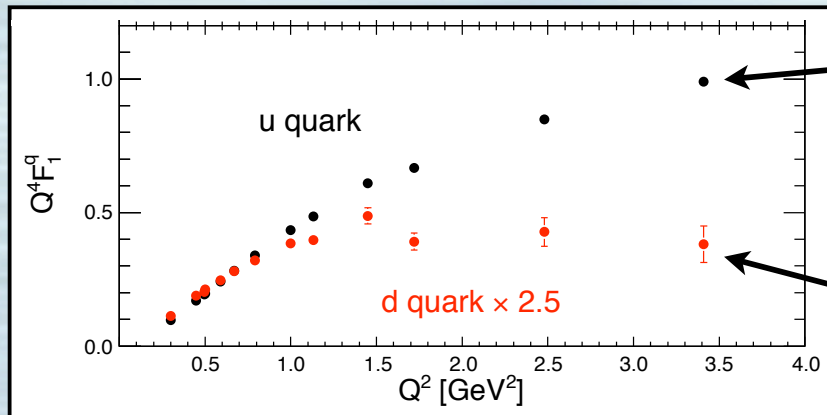
F^d seems to scale roughly like $1/Q^4$

Cates slide

What is the significance of these different behaviors?

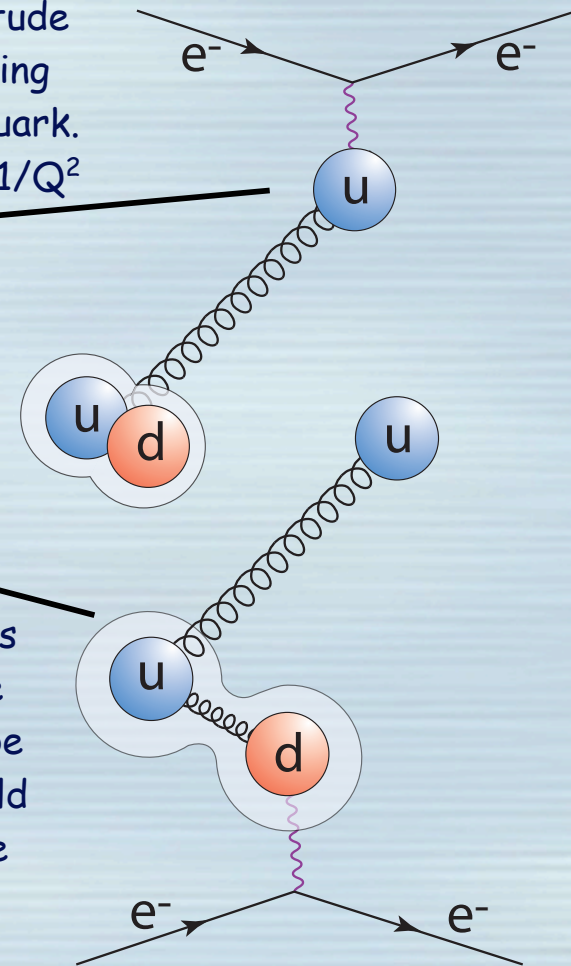
Jerry Miller's suggestion explaining the different scaling by using diquarks

u-quark scattering amplitude is dominated by scattering from the lone "outside" quark. Two constituents implies $1/Q^2$



Cates slide

d-quark scattering amplitude is necessarily probing inside the diquark. Two gluons need to be exchanged (or the diquark would fall apart), so scaling goes like $1/Q^4$



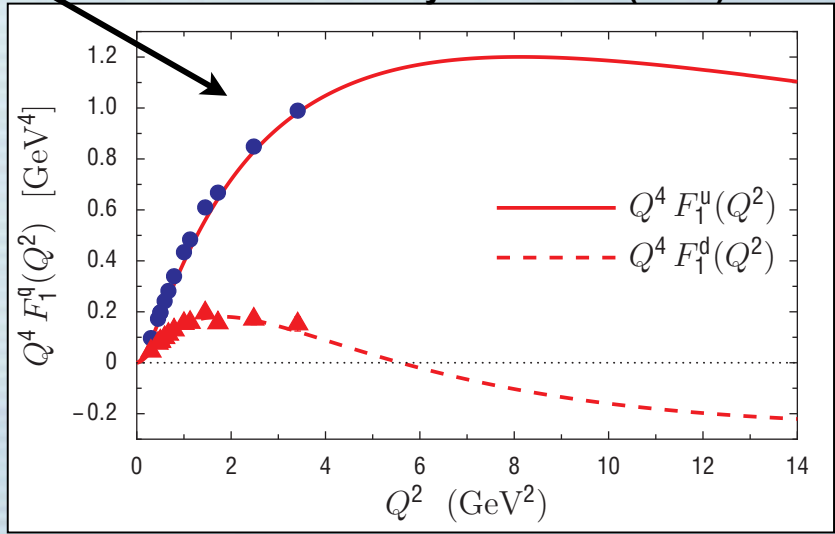
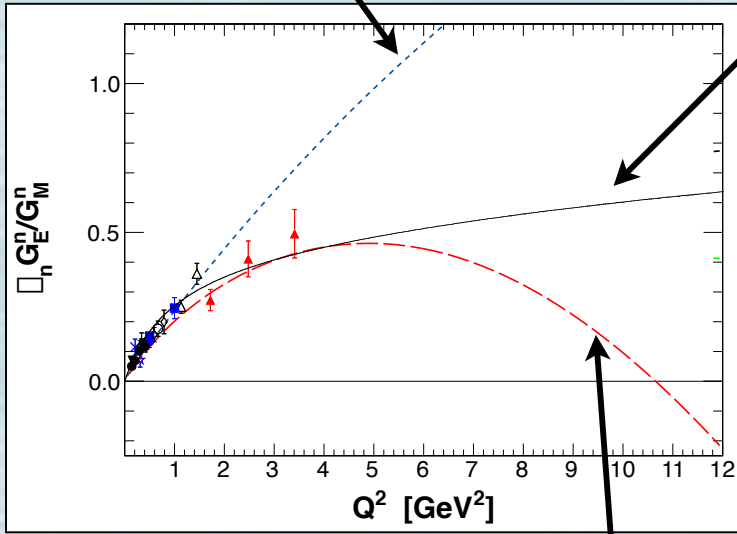
While at present this idea is at the conceptual stage, it is an intriguingly simple interpretation for the very different behaviors.

Relativistic Constituent Quark Models (RCQMs) that emphasize diquark features fit the data well

Light-front cloudy bag model Jerry Miller (PRC v66, pg032201, 2002).

Updated RCQM model emphasizing quark-diquark structure: Ian Cloët and Jerry Miller

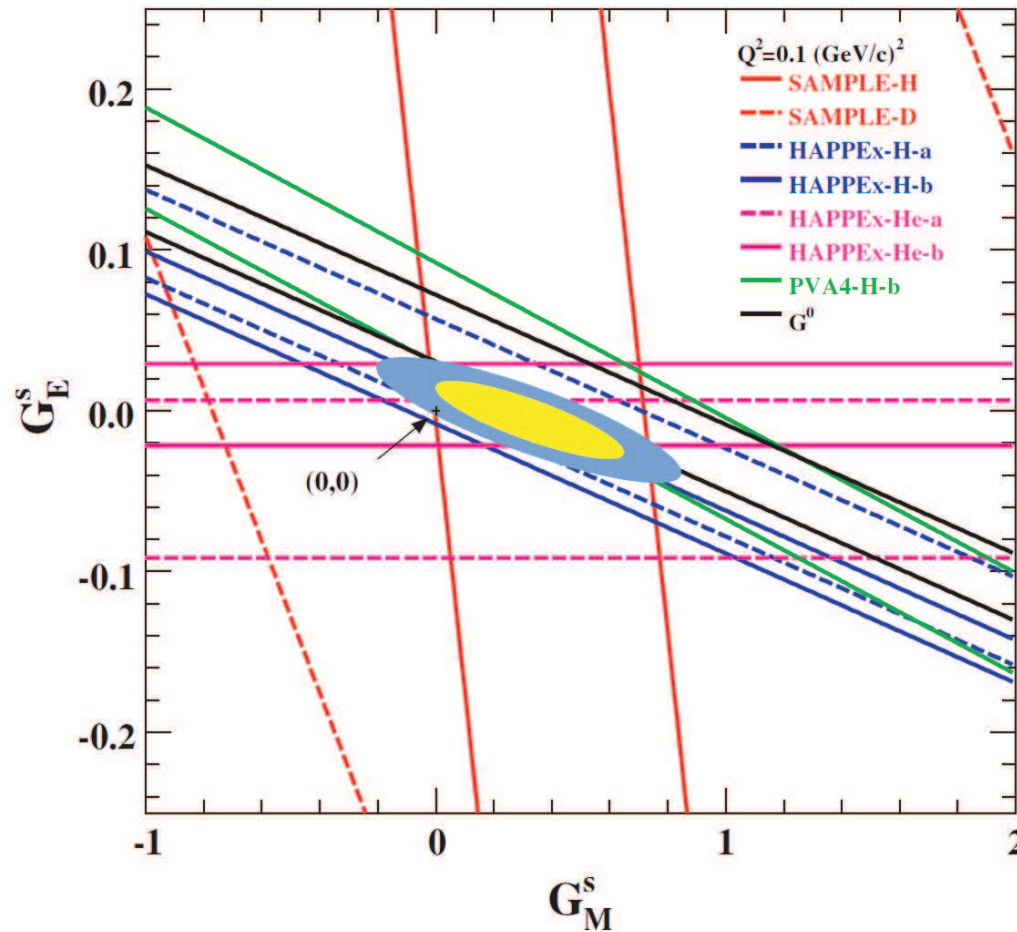
Phys.Rev. C86 (2012) 015208



The QCD DSE model of Cloët, Roberts et al. in which the constituent quark mass is dynamically generated and diquark degrees of freedom are incorporated.
(Few Body Systems v46, no1, 2009)

It appears that it is important to include terms related to diquarks in RCQMs in order to fit the behavior of the flavor decomposed form factors.

Validity of flavor separation $s\bar{s}$

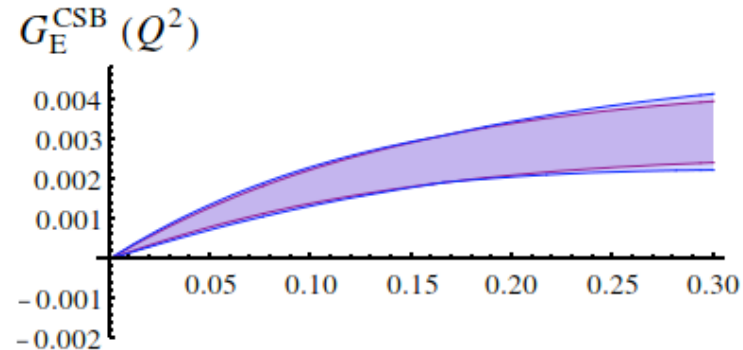
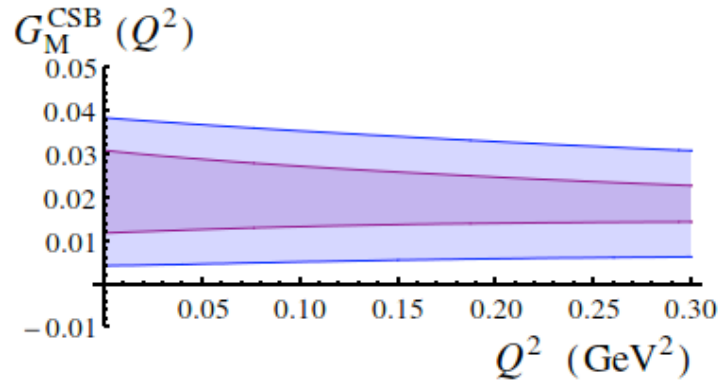


$$Q^2 = 0.1 \text{ GeV}^2$$

This is not zero

Validity of flavor separation CSB

$$\text{CSB} \ll s\bar{s}$$



Wagman & Miller 2014

arXiv:1402.7169

CSB effects at least 10 times smaller than current error bar

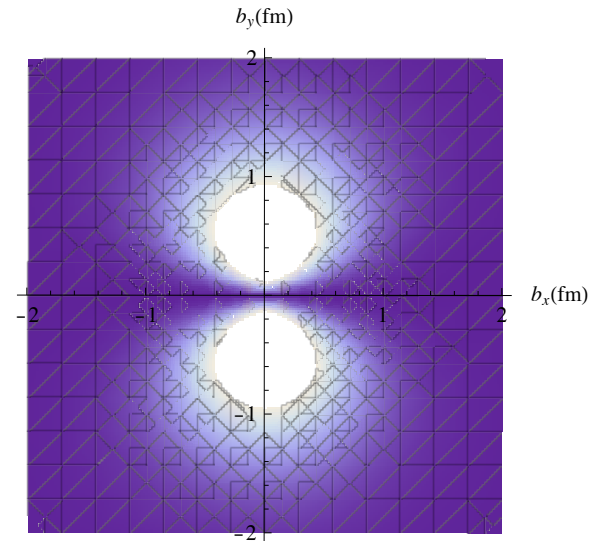
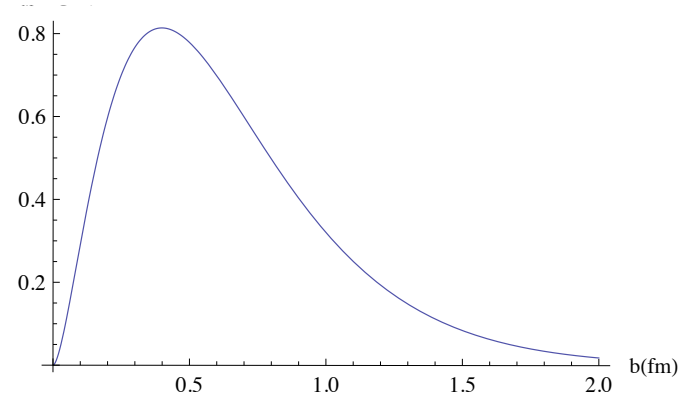
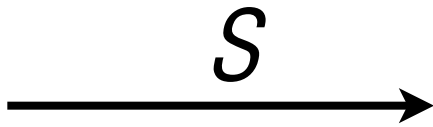
Spin effects: Magnetization density

G A Miller

Ann.Rev.Nucl.Part.Sci. 60 (2010) 1-25

$$\mu_a = \frac{1}{2M} \int d^2b \tilde{\rho}_M(\mathbf{b})$$

$$\tilde{\rho}_M(\mathbf{b}) = \sin^2 \phi b \int_0^\infty \frac{q^2 dq}{2\pi} J_1(qb) F_2(q^2),$$



Spin effects- spin-dependent density

- Probability that quark is in a given **location** and has a spin in a given direction GPD
OR
- Probability that quark has a given **momentum** and has a spin in a given direction TMD
- Condensed matter physicists measure former using neutrons
First step - non relativistic example

I: Non-Rel. $p_{1/2}$ proton outside 0^+ core

$$\langle \mathbf{r}_p | \psi_{1,1/2s} \rangle = R(r_p) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}}_p |s\rangle \quad \text{Binding pot'l} \\ \text{rotationally invariant}$$

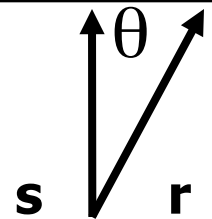
$$\rho(r) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p) | \psi_{1,1/2s} \rangle = R^2(r)$$

probability proton at \mathbf{r} & spin direction \mathbf{n} :

$$\rho(\mathbf{r}, \mathbf{n}) = \langle \psi_{1,1/2s} | \delta(\mathbf{r} - \mathbf{r}_p) \frac{(1 + \boldsymbol{\sigma} \cdot \mathbf{n})}{2} | \psi_{1,1/2s} \rangle$$

$$= \frac{R^2(r)}{2} \langle s | \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} (1 + \boldsymbol{\sigma} \cdot \mathbf{n}) \boldsymbol{\sigma} \cdot \hat{\mathbf{r}} | s \rangle$$

Three directions spin, \mathbf{r} , \mathbf{n}



$$\mathbf{n} \parallel \hat{\mathbf{s}} : \quad \rho(\mathbf{r}, \mathbf{n} = \hat{\mathbf{s}}) = R^2(r) \cos^2 \theta$$

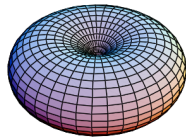
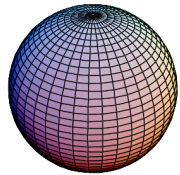
$$\mathbf{n} \parallel -\hat{\mathbf{s}} : \quad \rho(\mathbf{r}, \mathbf{n} = -\hat{\mathbf{s}}) = R^2(r) \sin^2 \theta$$

non-spherical shape depends on spin direction

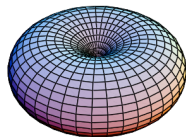
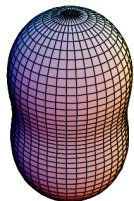
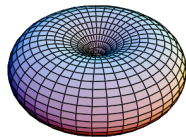
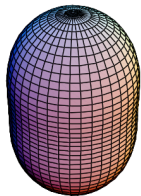
Shapes of the proton

Phys.Rev. C68 (2003) 022201

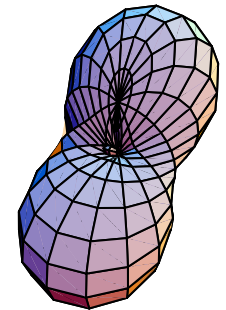
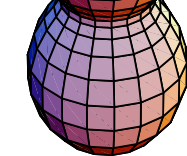
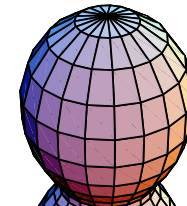
Momentum space



vectors \mathbf{n} , \mathbf{K} , \mathbf{S}



Coordinate space



Pretzelosity

How to measure?-Lattice and/or experiment

Relation between coordinate and momentum space densities? Model independent technique needed.

Generalized Coordinate Space Densities

$$\rho^\Gamma(\mathbf{b}) = \sum_q e_q \int dx^- q_+^\dagger(x^-, \mathbf{b}) \gamma^+ \Gamma q_+(x^-, \mathbf{b})$$

Diehl Hagler 2005

$\Gamma = \frac{1}{2} (1 + \hat{\mathbf{s}} \cdot \boldsymbol{\gamma} \gamma_5)$ gives spin-dependent density

PRL **98**, 222001 (2007)

PHYSICAL REVIEW LETTERS

week ending
1 JUNE 2007

Transverse Spin Structure of the Nucleon from Lattice-QCD Simulations

M. Göckeler,¹ Ph. Hägler,^{2,*} R. Horsley,³ Y. Nakamura,⁴ D. Pleiter,⁴ P.E.L. Rakow,⁵ A. Schäfer,¹ G. Schierholz,^{6,4}
H. Stüben,⁷ and J.M. Zanotti³

$$\begin{aligned} \rho^n(b_\perp, s_\perp, S_\perp) &= \int_{-1}^1 dx x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp) = \\ &\frac{1}{2} \left\{ A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{Tn0}(b_\perp^2) - \frac{1}{4m^2} \Delta_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2) \right) \right. \\ &+ \frac{b_\perp^j \epsilon^{ji}}{m} \left(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \bar{B}'_{Tn0}(b_\perp^2) \right) \\ &\left. + s_\perp^i (2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}) S_\perp^j \frac{1}{m^2} \tilde{A}''_{Tn0}(b_\perp^2) \right\}, \end{aligned} \quad (1)$$

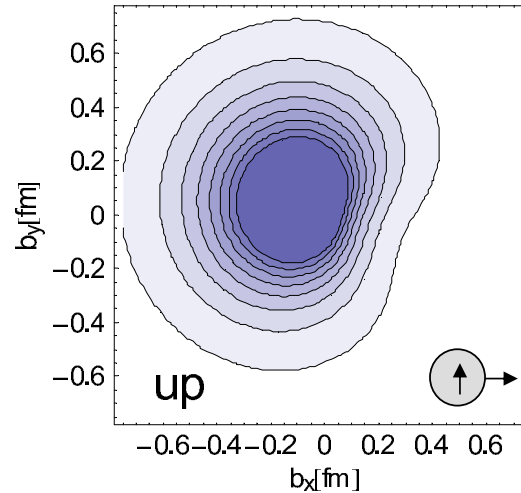
spin-dependent density
-depends on direction
of \mathbf{b} : proton is not round

A,B: $f(b_\perp^2) \equiv \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-ib_\perp \cdot \Delta_\perp} f(t = -\Delta_\perp^2),$

$$\tilde{A}_{T10}''(b^2)$$

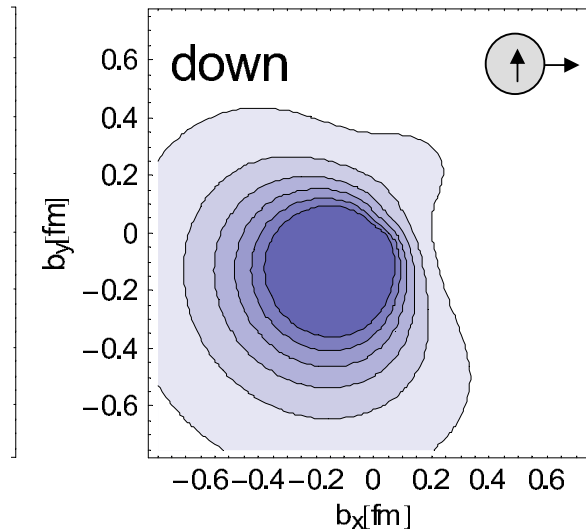
Sent by
Schierholz

This is not symmetric.
Proton is not round!
effect is small



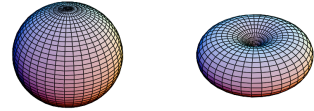
n=1 moment

nucleon spin:
sideways
quark spin: up



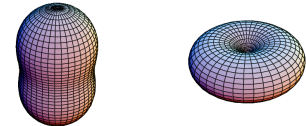
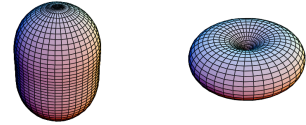
Shapes of the proton

Relate spin dependent density to experiment



Phys.Rev.C76:065209,2007

Field-theoretic spin dependent
momentum density is related to the
transverse momentum distribution h_{1T}^\perp

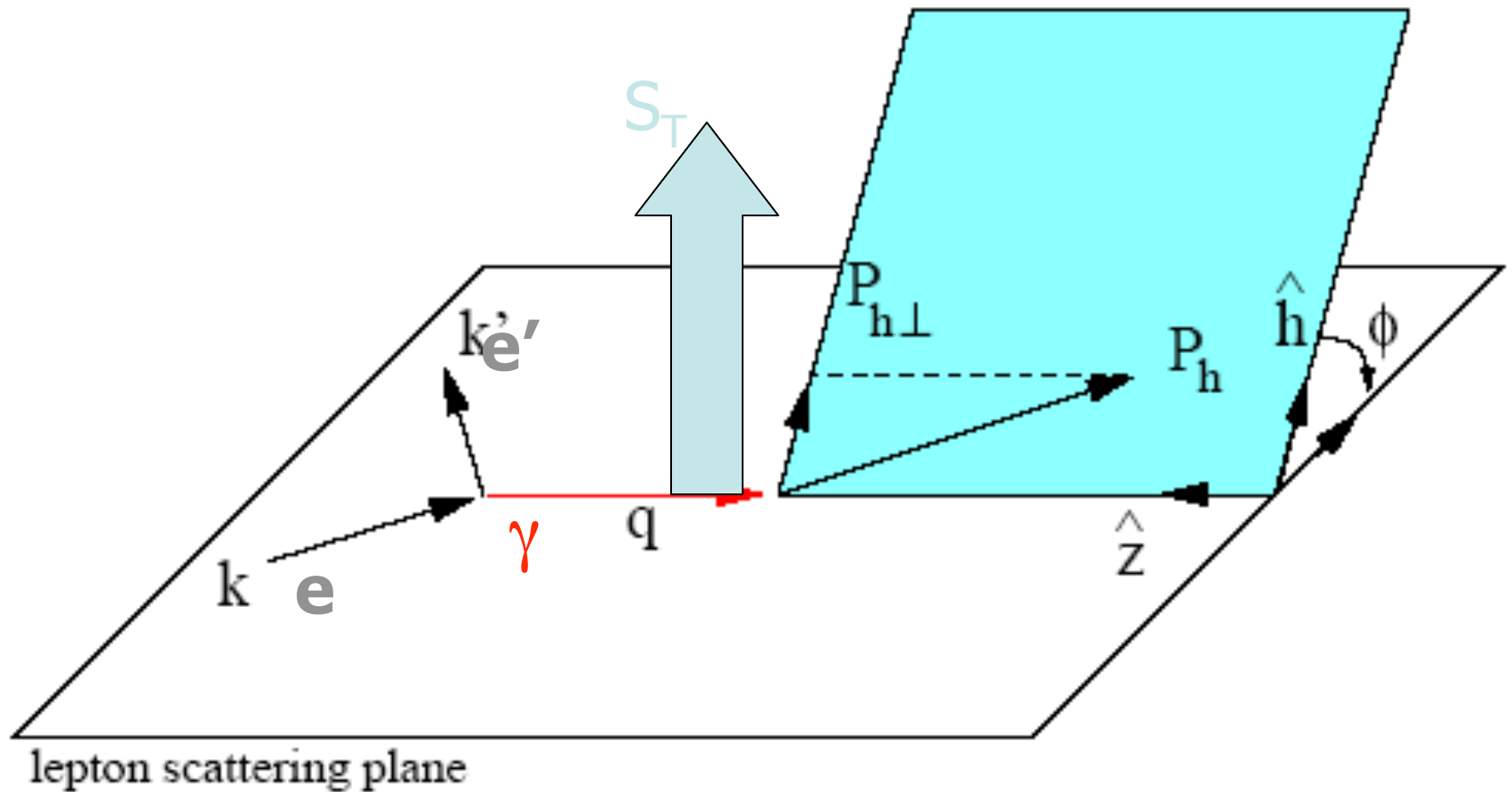


$$\Phi^{[\Gamma]}(x, \mathbf{K}_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{iK \cdot \xi} \langle P, S | \bar{\psi}(0) \Gamma \mathcal{L}(0, \xi; n_-) \psi(\xi) | P, S \rangle \Big|_{\xi^+ = 0}$$

Mulders Tangerman'96

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x, \mathbf{K}_T) = S_T^i h_1(x, K_T^2) + \frac{(K_T^i K_T^j - \frac{1}{2} K_T^2 \delta_{ij}) S_T^j}{M^2} h_{1T}^\perp(x, K_T^2)$$

Measure h_{1T}^\perp : $\mathbf{e}, \uparrow \mathbf{p} \rightarrow \mathbf{e}', \pi \mathbf{X}$



Cross section has term proportional to $\cos 3\phi$
 Boer Mulders '98 there are other ways to see h_{1T}^\perp

Measurement of pretzelosity asymmetry of charged pion production in Semi-Inclusive Deep Inelastic Scattering on a polarized ^3He target

Y. Zhang,^{1,*} X. Qian,^{2,3} K. Allada,^{4,5} C. Dutta,⁴ J. Huang,^{5,6} J. Katich,⁷ Y. Wang,⁸ K. Aniol,⁹ J.R.M. Annand,¹⁰

.....

arXiv:1312.3047

An experiment to measure single-spin asymmetries in semi-inclusive production of charged pions in deep-inelastic scattering on a transversely polarized ^3He target was performed at Jefferson Lab in the kinematic region of $0.16 < x < 0.35$ and $1.4 < Q^2 < 2.7 \text{ GeV}^2$. The pretzelosity asymmetries on ^3He , which can be expressed as the convolution of the h_{1T}^\perp transverse momentum dependent distribution functions and the Collins fragmentation functions in the leading order, were measured for the first time. Using the effective polarization approximation, we extracted the corresponding neutron asymmetries from the measured ^3He asymmetries and cross-section ratios between the proton and ^3He . Our results show that for both π^\pm on ^3He and on the neutron the pretzelosity asymmetries are consistent with zero within experimental uncertainties.

Summary

- Form factors, **GPDs**, **TMDs**, understood from unified light-front formulation, **GPD-coordinate space density**, **TMD momentum space density**
- Neutron central transverse density is negative-
- Proton is not round- lattice QCD spin-dependent-density in coordinate space is **not** zero
- **Experiment can whether or not proton is round by measuring**

$$h_{1T}^{\perp}$$



The Proton