SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

INT Workshop INT-14-55W
Studies of 3D Structure of Nucleon

Seattle, USA: February 24-28, 2014.

MODELING QUARK FRAGMENTATION INCLUDING POLARIZATION

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THE UNIVERSITY of ADELAIDE

SUB CRUCE LUMEN

OUTLOOK

- **Introduction and Motivation.**

- **Monte Carlo Approach within the *NJL-jet formalism*.**
  - ***Transverse Momentum Dependent FFs and PDFs.***
  - ***Unpolarised Dihadron Fragmentations.***
  - ***One and Two Hadron Fragmentations of a Transversely Polarized Quark.***

- **Conclusions.**

Unfavored FFs NOT well known!

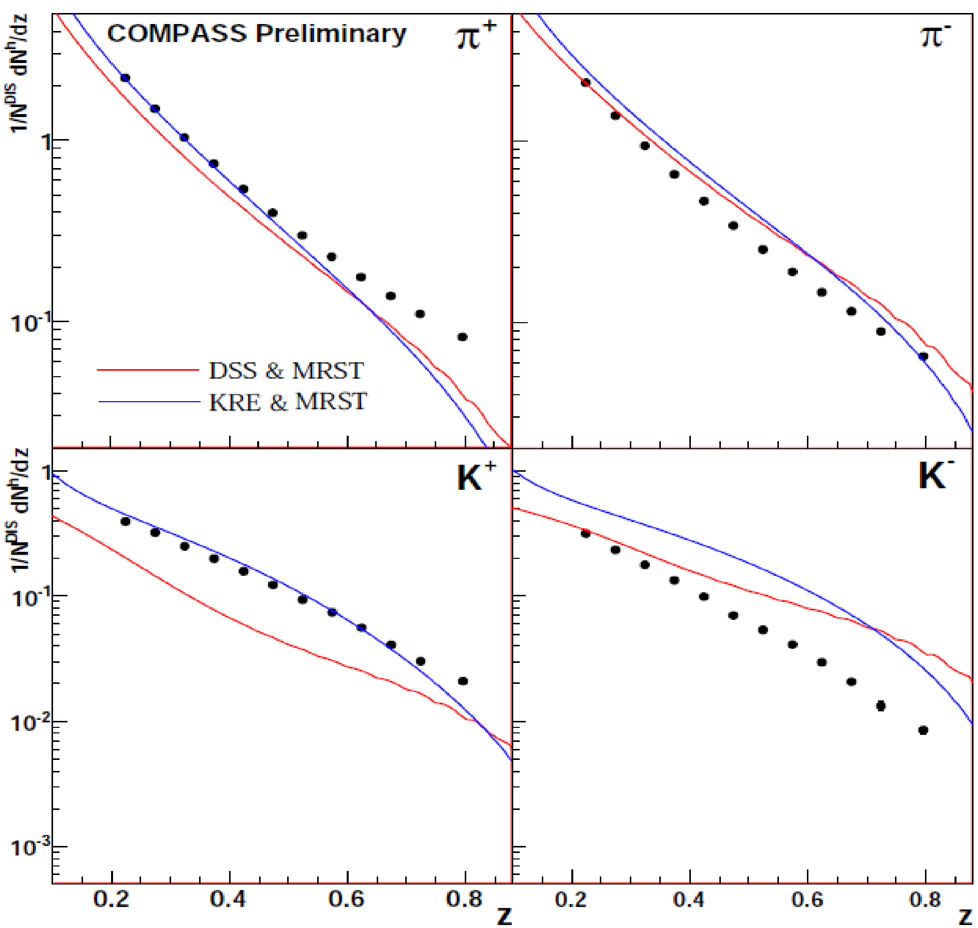
Hadron Multiplicities

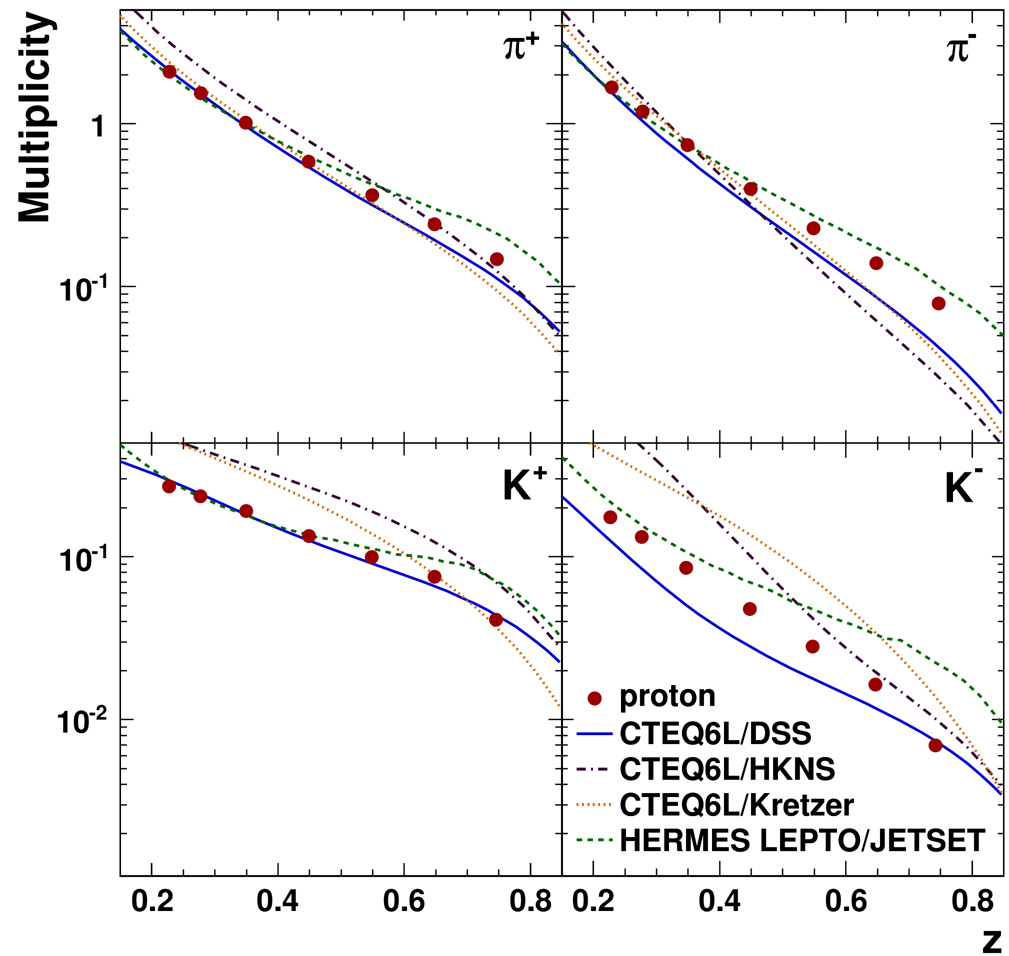
- Preliminary from COMPASS

Talk by C.Franco at CIPANP 2012.

- Also results from HERMES

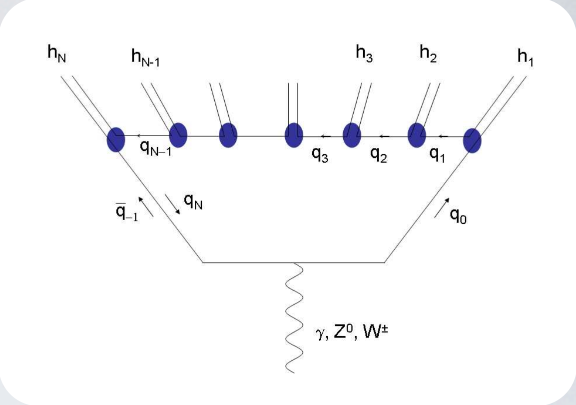
Phys. Rev. D 87, 074029 (2013)



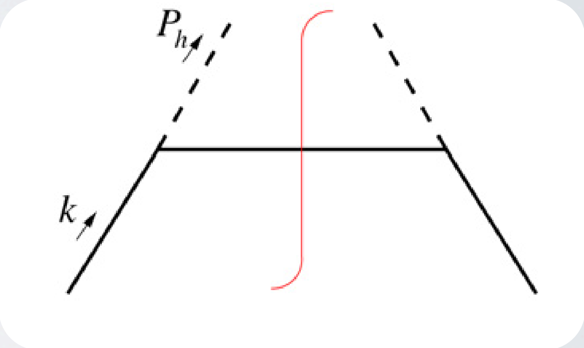


MODELS FOR FRAGMENTATION

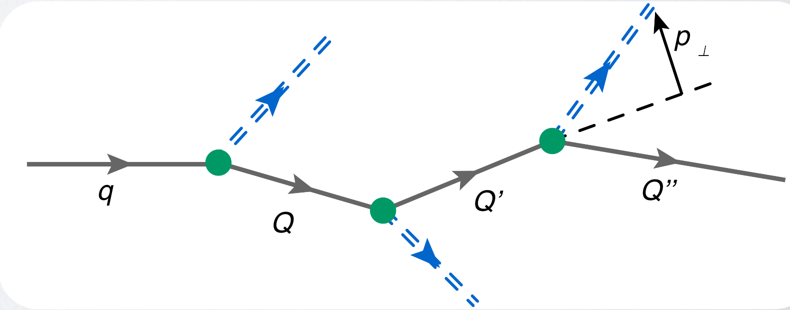
- *Lund String Model*
  - *Very Successful* implementation in *JETSET, PYTHIA*.
  - Highly Tunable - Limited Predictive Power.
  - No Spin Effects - Formal developments by X. Artru et al but no quantitative results!



- *Spectator Model*
  - Quark model calculations with empirical form factors.
  - No unfavored fragmentations.
  - Need to tune parameters for small z dependence.



- *NJL-jet Model*
  - *Multi-hadron* emission framework with effective quark model input.
  - *Monte-Carlo framework* allows flexibility in including the transverse momentum, spin effects, two-hadron correlations, etc.



MOTIVATION

- A *robust* and *expandable* Monte Carlo framework for describing both *Favored and Unfavored* fragmentation functions in multi-hadron emission process using microscopic quark models as input.

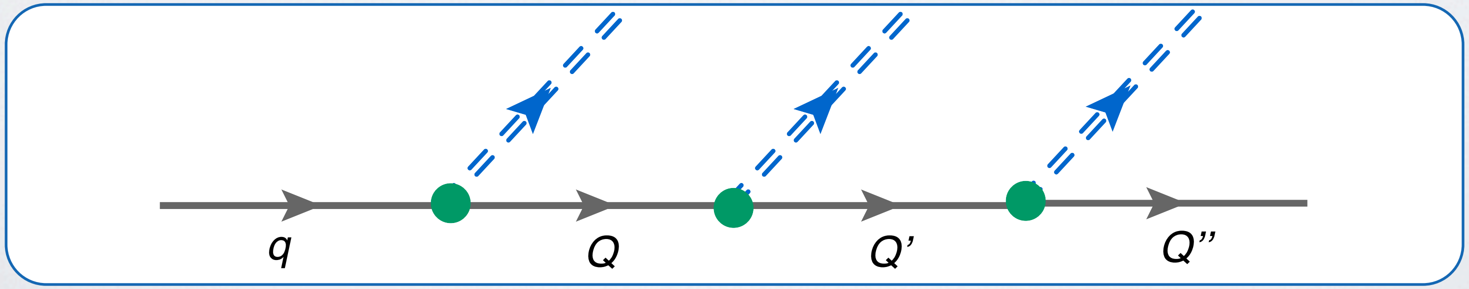
- NO model parameters fitted to fragmentation data!

- Momentum and quark flavor conservation is imposed.

- Extensions to TMD, Polarized Quark Fragmentation, Dihadron Fragmentations.

MONTE-CARLO (MC) APPROACH

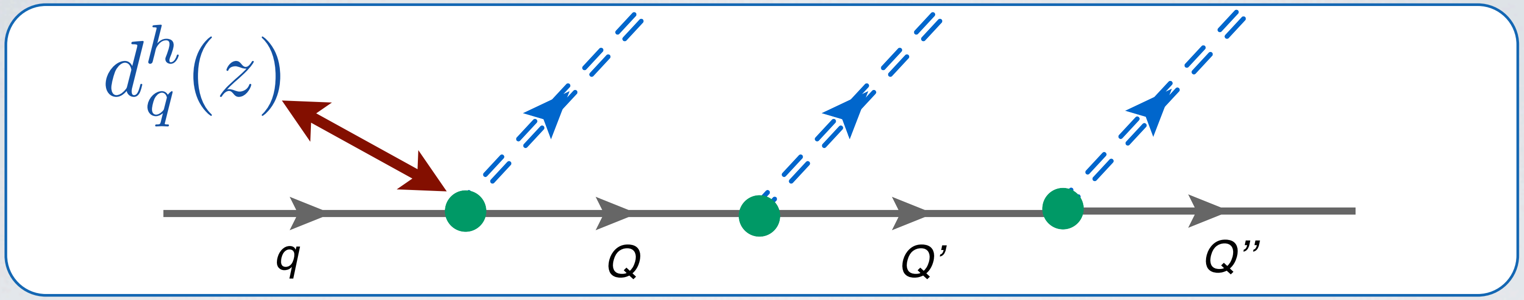
✦ Using the ***probabilistic*** interpretation of fragmentation funcs. to include the effect of ***multiple*** hadron emissions.



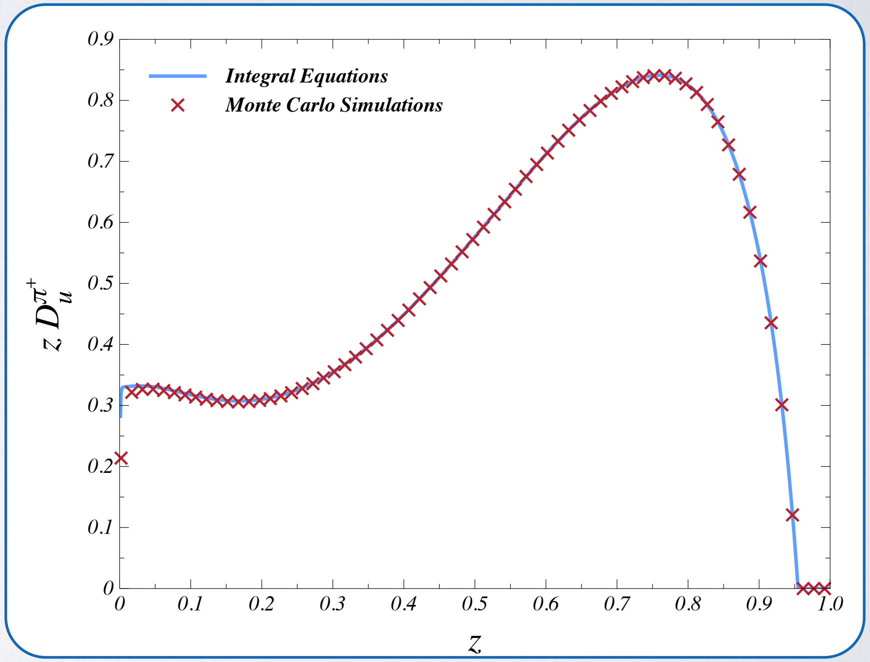
INTEGRATED FRAGMENTATIONS FROM MC

H.M., Thomas, Bentz, PRD. 83:07400; PRD.83:114010, 2011.

- Input: One hadron emission probability



- Sample the emitted hadron type and z according to input splitting.
- ***CONSERVE***: Momentum and Quark Flavor in each step.
- Repeat for decay chains with the same initial quark.



$$D_q^h(z)\Delta z = \langle N_q^h(z, z+\Delta z)\rangle \equiv \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z)}{N_{Sims}}$$

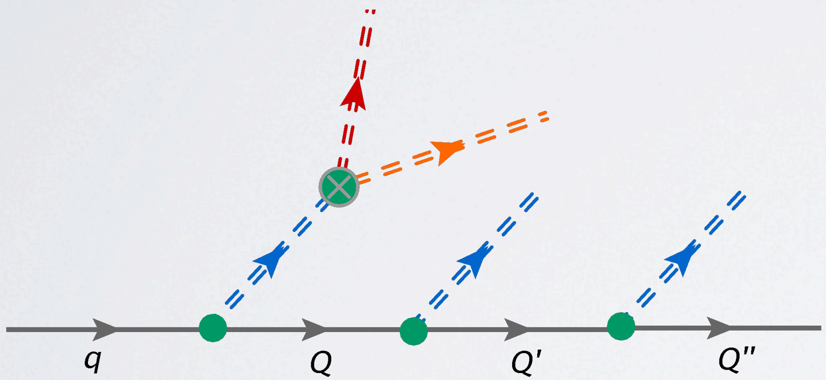
MORE CHANNELS

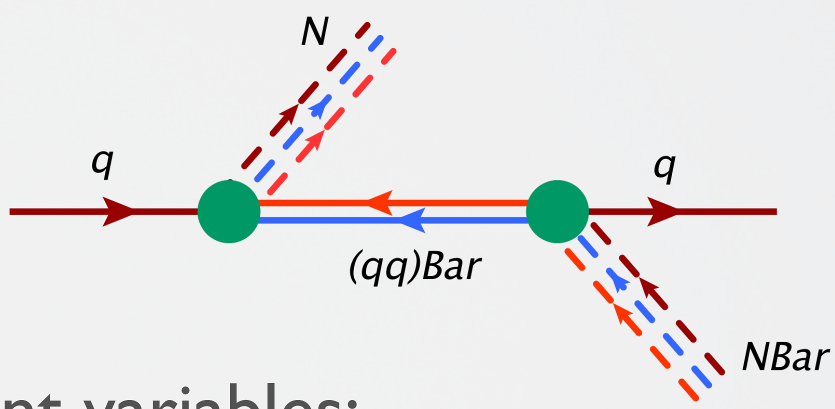
H.M., Thomas, Bentz, PRD. 83:074003, 2011

- Calculate quark splittings to vector mesons, Nucleon Anti-Nucleon: $d_q^h(z)$

$$h = \rho^0, \rho^\pm, K^{*0}, \overline{K}^{*0}, K^{*\pm}, \phi, N, \bar{N}$$

- Add the decay of the resonances:



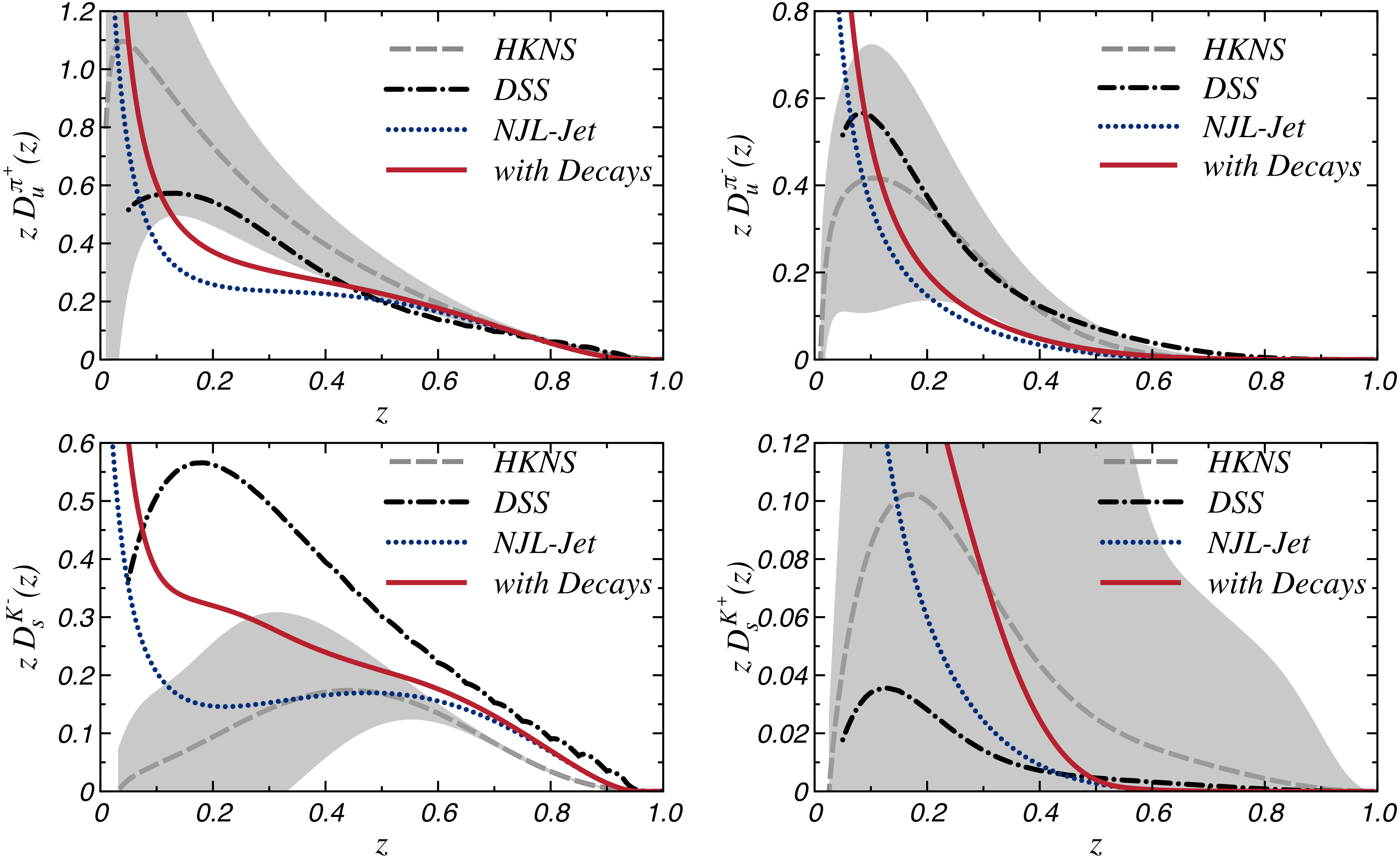


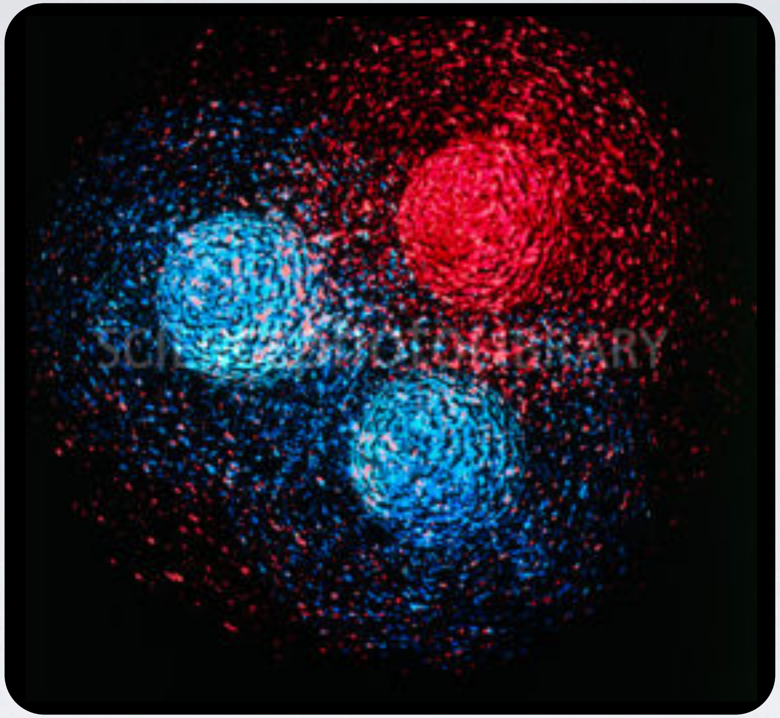
- Decay cross-section in light-front variables:

$$dP^{h\to h_1,h_2}(z_1) = \begin{cases} \frac{C_h^{h_1 h_2}}{8\pi}\, dz_1 & \text{if } z_1 z_2\, m_h^2 - z_2 m_{h1}^2 - z_1 m_{h2}^2 \geq 0;\ z_1 + z_2 = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Results with VM decays: $Q^2 = 4\ \mathrm{GeV}^2$

Favored | Unfavored



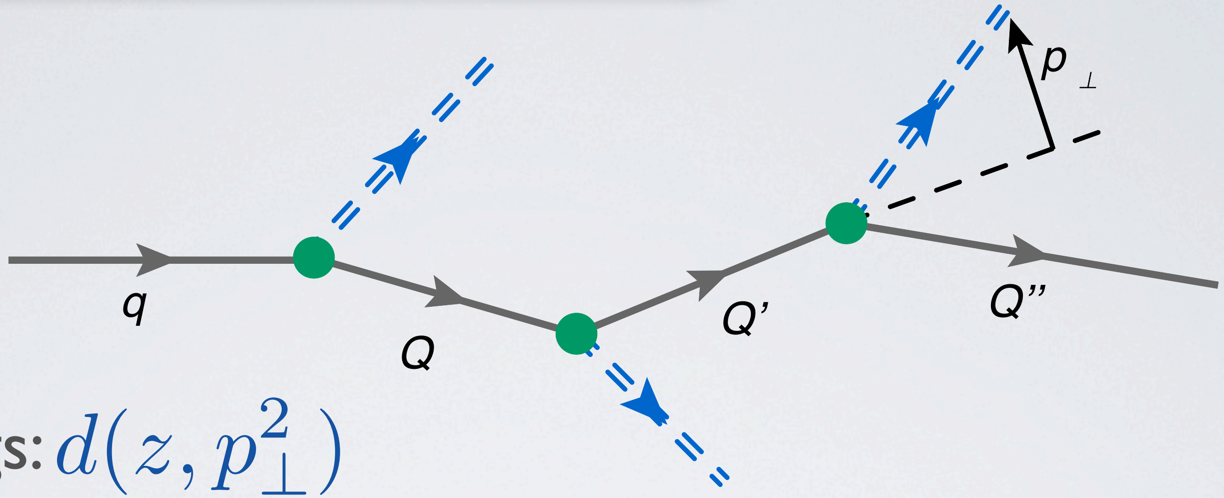


TRANSVERSE MOMENTUM DEPENDENCE:

ACCESSING 3 DIMENSIONAL PICTURE OF NUCLEON FROM SIDIS.

INCLUDING THE TRANSVERSE MOMENTUM

H.M.,Bentz, Cloet, Thomas, PRD.85:014021, 2012

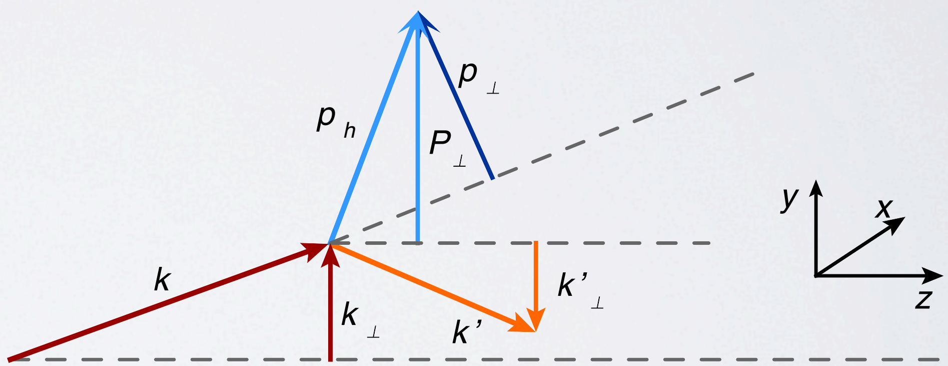


- TMD splittings: $d(z, p_\perp^2)$

- Conserve transverse momenta at each link.

$$\mathbf{P}_\perp = \mathbf{p}_\perp + z^h \mathbf{k}_\perp$$

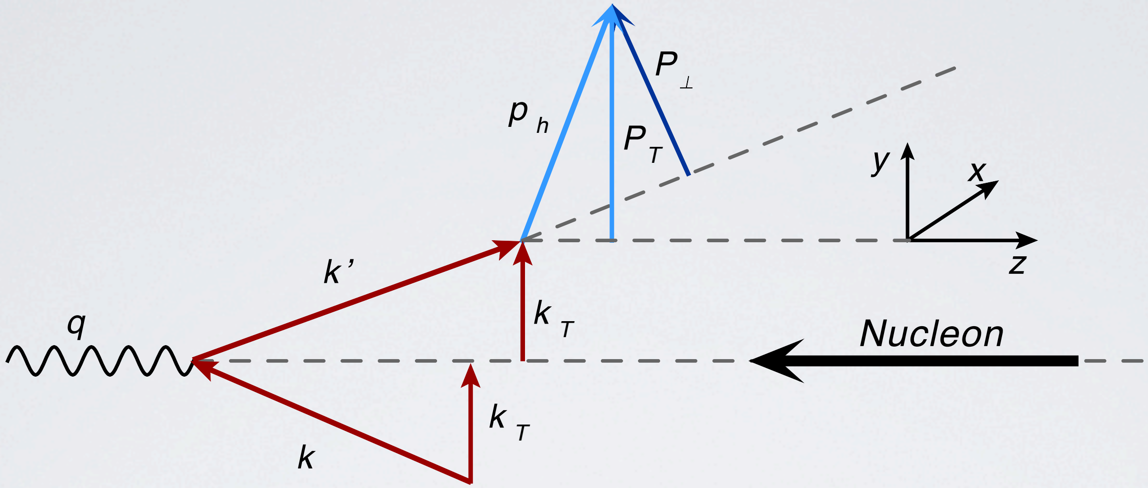
$$\mathbf{k}_\perp = \mathbf{P}_\perp + \mathbf{k}'_\perp$$



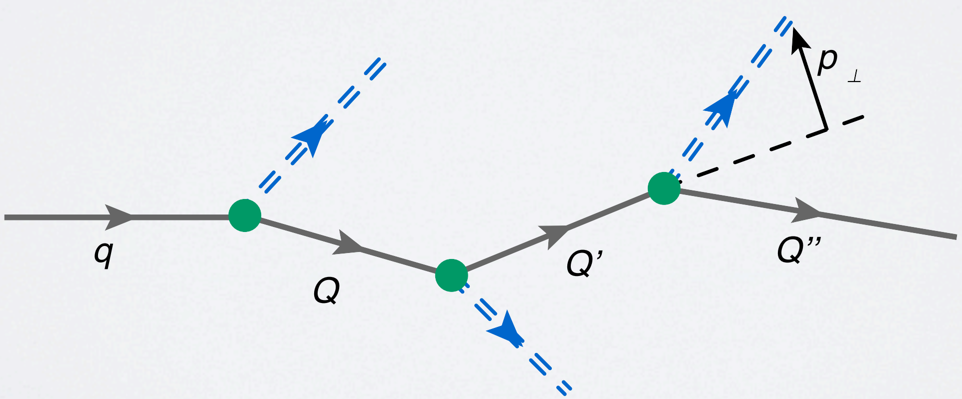
- Calculate the Number Density

$$D_q^h(z, P_\perp^2)\Delta z\ \pi\Delta P_\perp^2 = \frac{\sum_{N_{Sims}} N_q^h(z, z+\Delta z, P_\perp^2, P_\perp^2+\Delta P_\perp^2)}{N_{Sims}}.$$

THE TRANSVERSE MOMENTA OF HADRONS IN SIDIS



- Use TMD quark distribution functions from the NJL model .
- Use NJL-Jet hadronization model.



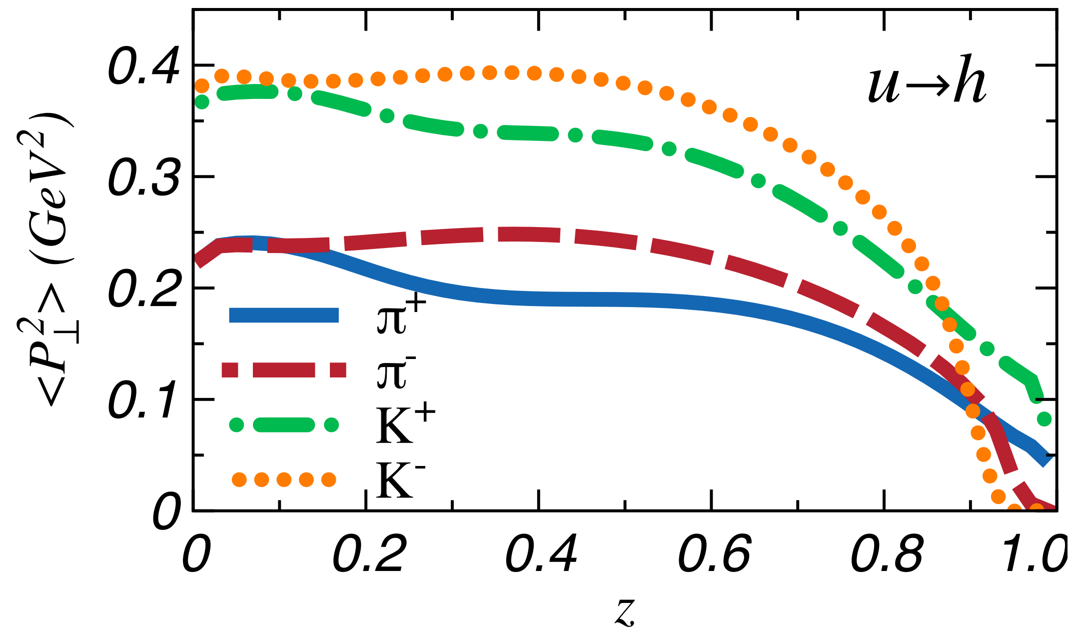
- Evaluate the cross-section using MC simulation.

FRAGMENTATION

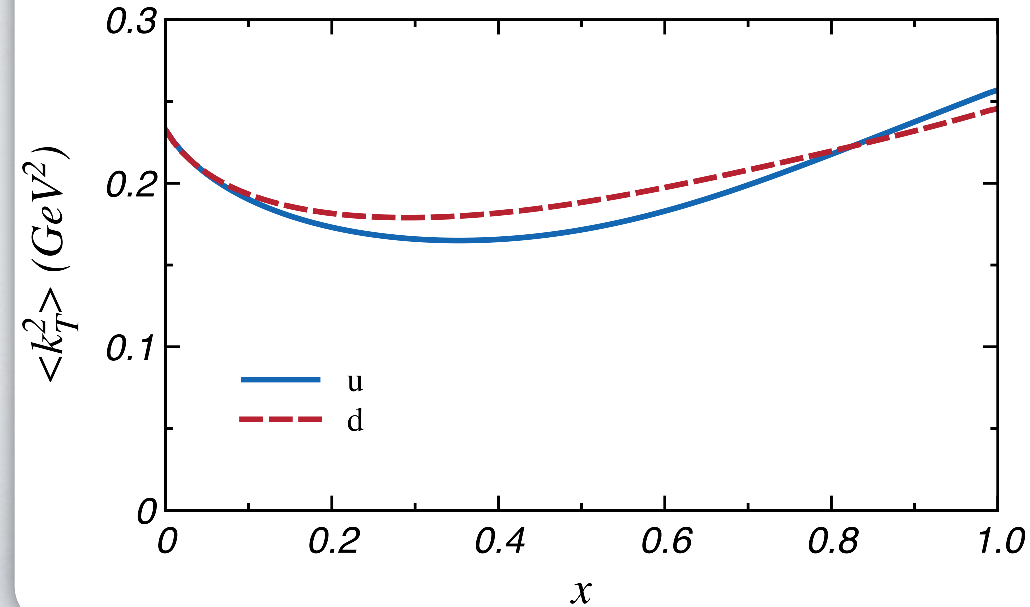
$$\langle P_\perp^2 \rangle_{unf} > \langle P_\perp^2 \rangle_f$$

✦ Indications from HERMES data:

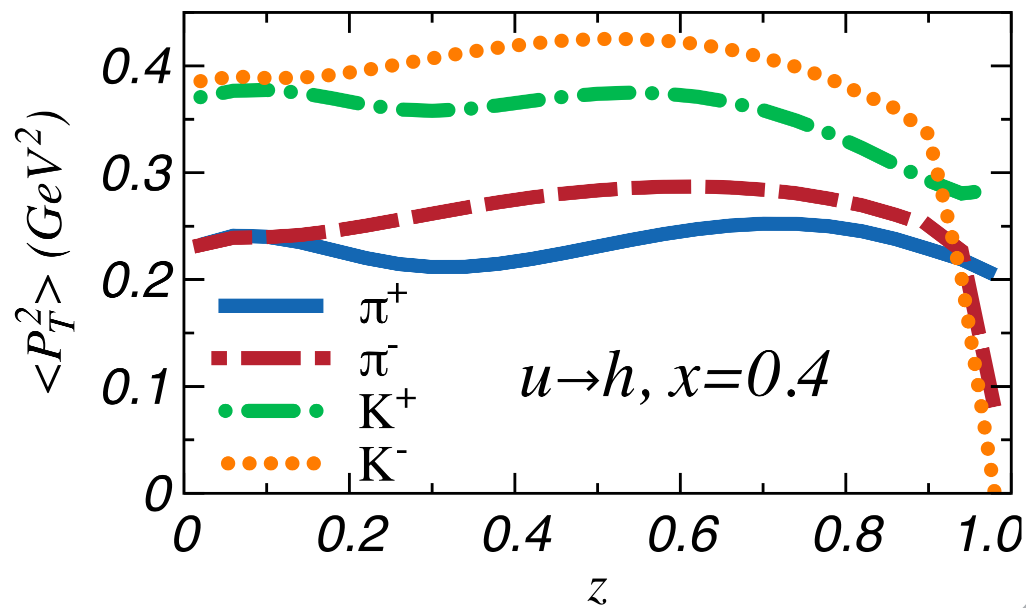
A. Signori, et al: JHEP 1311, 194 (2013)

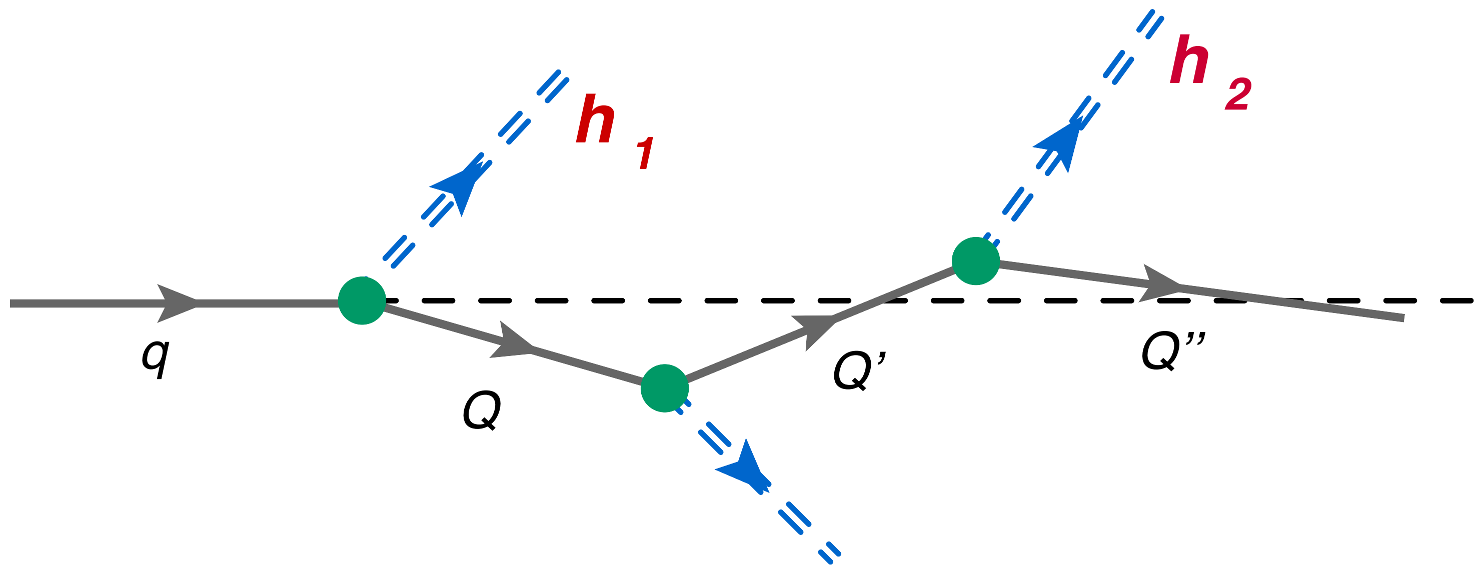


PDF



SIDIS



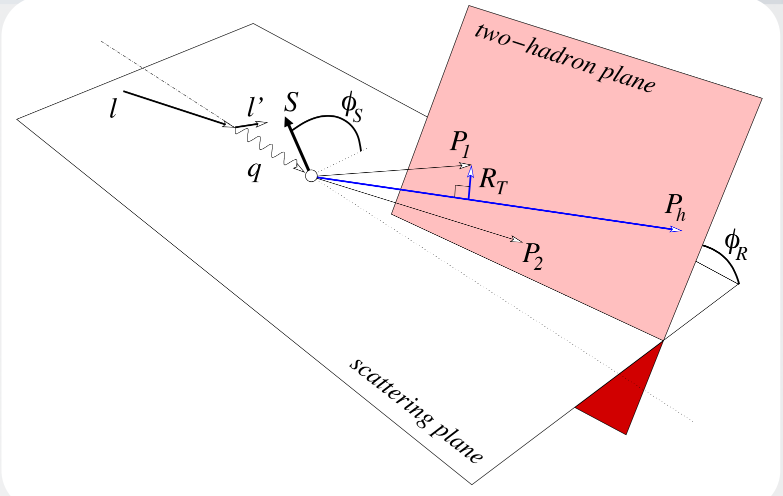


TWO HADRON CORRELATIONS: DIHADRON FRAGMENTATION FUNCTIONS

ACCESS TO TRANSVERSITY PDF FROM DFF

M. Radici, et al: PRD 65, 074031 (2002).

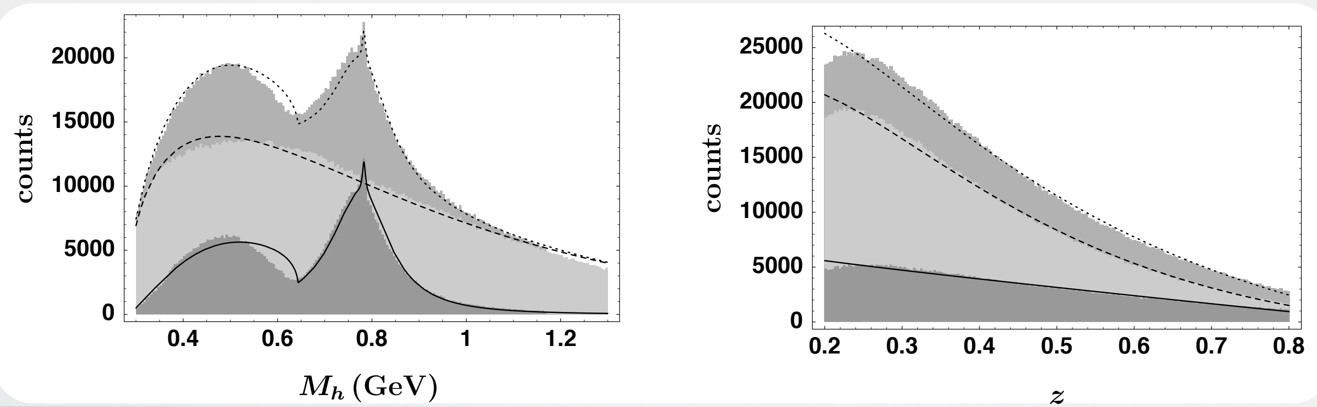
- In two hadron production from polarized target the cross section factorizes *collinearly* - no TMD!
- Allows clean access to *transversity*.
- *Unpolarized* and *Interference* Dihadron FFs are needed!



$$\frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \propto \sin(\phi_R + \phi_S) \frac{\sum_q e_q^2 \; h_1^q(x)/x \; H_1^{\sphericalangle q}(z, M_h^2)}{\sum_q e_q^2 \; f_1^q(x)/x \; D_1^q(z, M_h^2)}$$

- Empirical Model for D_1^q have been fitted to PYTHIA simulations.

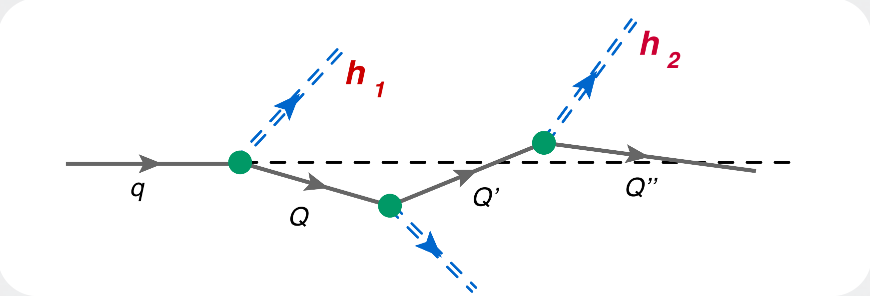
A. Bacchetta and M. Radici, PRD 74, 114007 (2006).



Experiments: BELLE, HERMES, COMPASS.

UNPOLARIZED DIHADRON FRAGMENTATIONS

H.M. Thomas, Bentz, PRD.88:094022, 2013.



- The probability density for observing two hadrons:

$$P_1 = (z_1 k^-, P_1^+, \boldsymbol{P}_{1,\perp}), \;\; P_1^2 = M_{h1}^2$$

$$P_2 = (z_2 k^-, P_2^+, \boldsymbol{P}_{2,\perp}), \;\; P_2^2 = M_{h2}^2$$

- The corresponding number density:

$$D_q^{h_1 h_2}(z, M_h^2)\ \Delta z\ \Delta M_h^2 = \langle N_q^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2 + \Delta M_h^2)\rangle$$

$$z = z_1 + z_2 \qquad M_h^2 = (P_1 + P_2)^2$$

- Kinematic Constraint.

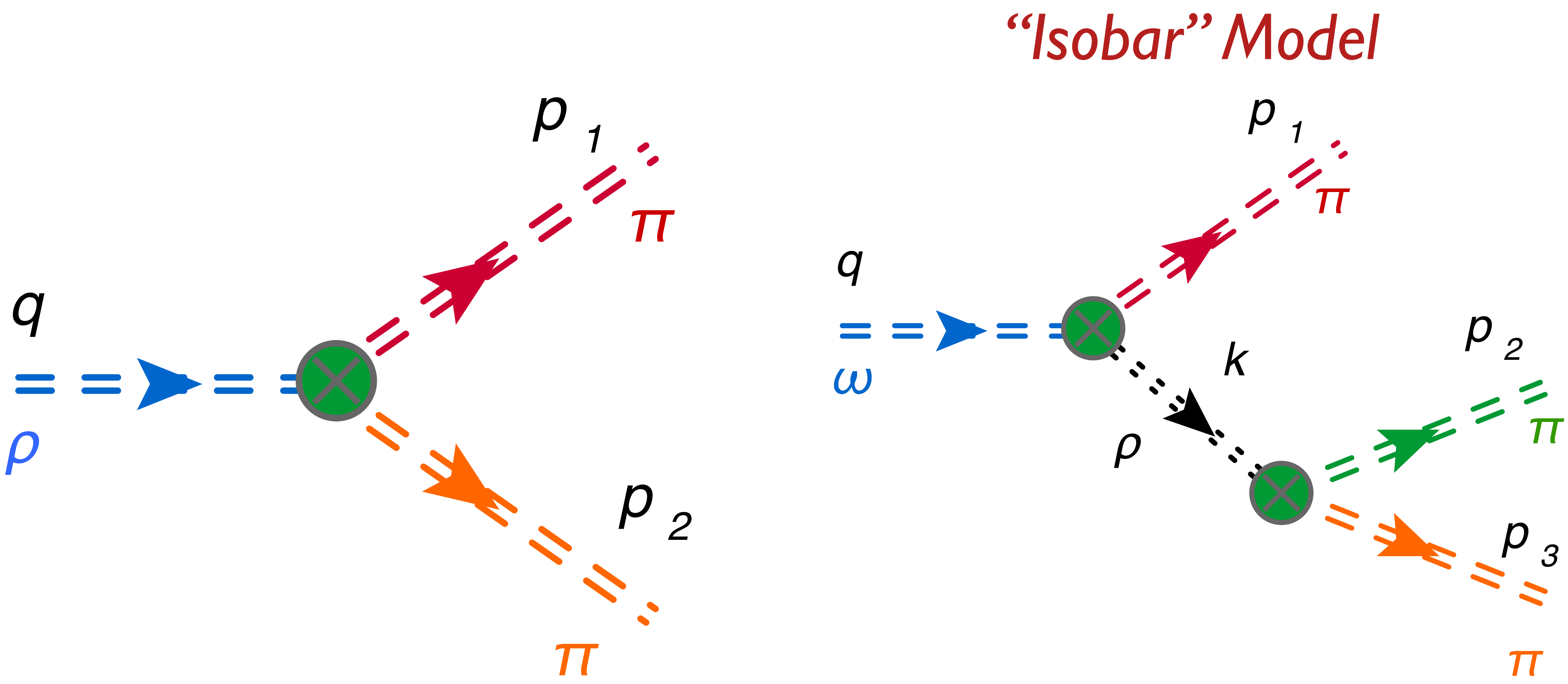
$$z_1 z_2 M_h^2 - (z_1 + z_2)(z_2 M_{h1}^2 + z_1 M_{h2}^2) \geq 0$$

- In MC simulations record all the pairs in every decay chain.

2- AND 3-BODY DECAYS

The M_h^2 spectrum of pseudoscalars is strongly affected by VM decays.

- We include only the 2-body decays ρ, K^*.
- Both 2- and 3-body decays of ω, ϕ .



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- We include only the 2-body decays ρ, K^*.
- Both 2- and 3-body decays of ω, ϕ .

Achasov et al. (SND), PRD 68, 052006, (2003).

- 2-body decay amplitude:

$$M(p_1, p_2) = \frac{g_V^{h_1 h_2} \epsilon^\mu (p_{2\mu} - p_{1\mu})}{D_V(q^2)}$$

- Resonance propagator:

$$D_V(s) = m_V^2 - s - i\sqrt{s}\Gamma_V(s)$$

$$\Gamma_V(s) = \frac{m_V^2}{s}\Gamma_V \left(\frac{q(s)}{q(m_V^2)}\right)^3$$

Relative Momentum of daughters in their CM frame.

- 3-body decay amplitude (ignore small width):

$$M(p_1, p_2, p_3) = \varepsilon_{\mu\alpha\beta\gamma}\epsilon^\mu p_1^\alpha p_2^\beta p_3^\gamma \sum_{i=0,\pm} \frac{g_{V\rho_i\pi}\ g_{\rho_i\pi\pi}}{D_{\rho_i}(v_i^2)}$$

- Simulate 2- and 3-body phase space in LC.

THE TREATMENT OF VM DECAYS: COMPARISON TO PYTHIA.

- 2-body decay amplitude: non-relativistic Breit-Wigner:

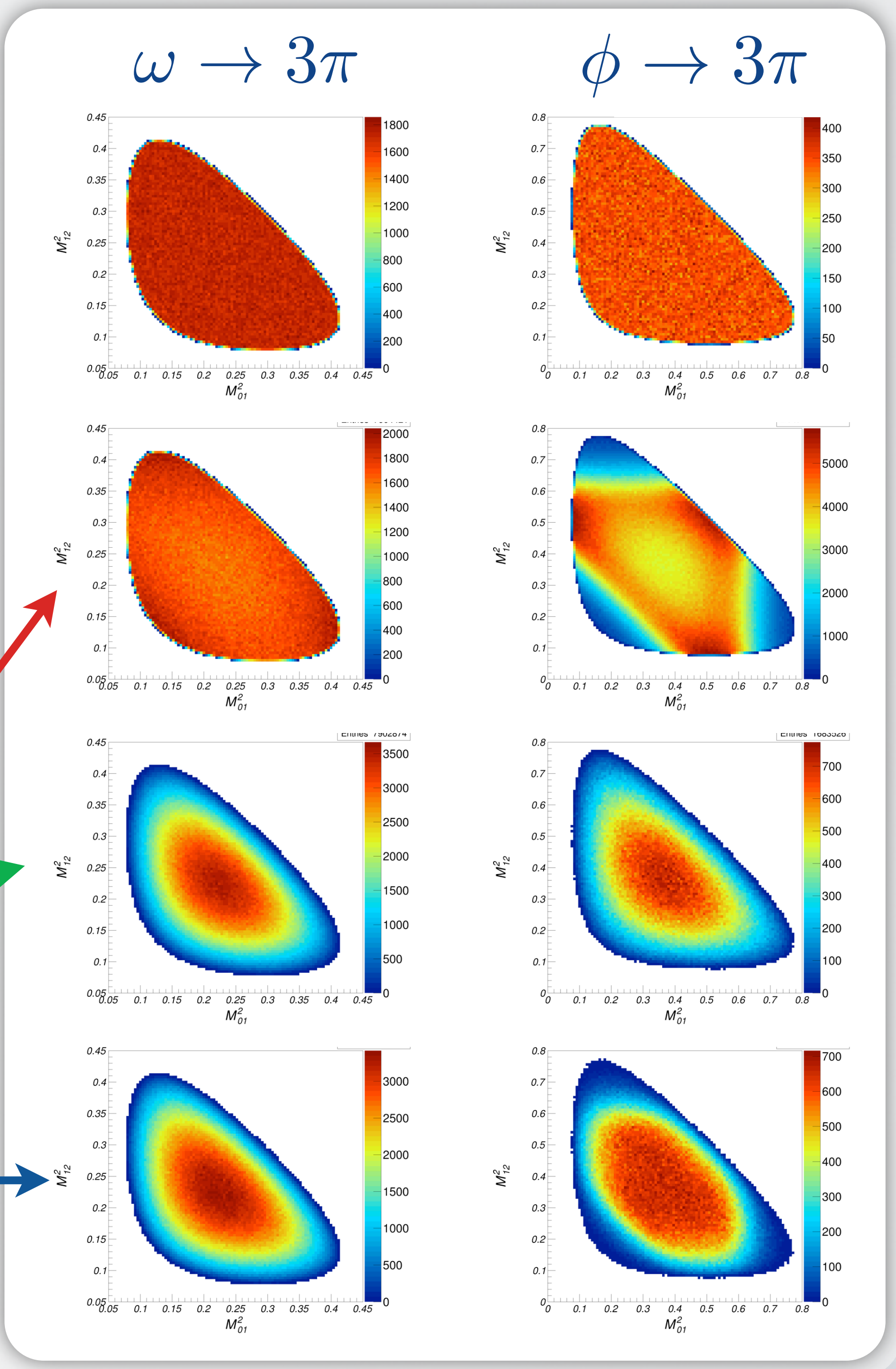
$$\mathcal{P}(m)dm \propto \frac{1}{(m-m_0)^2+\Gamma^2/4}dm$$

- Constant decay width of VM.

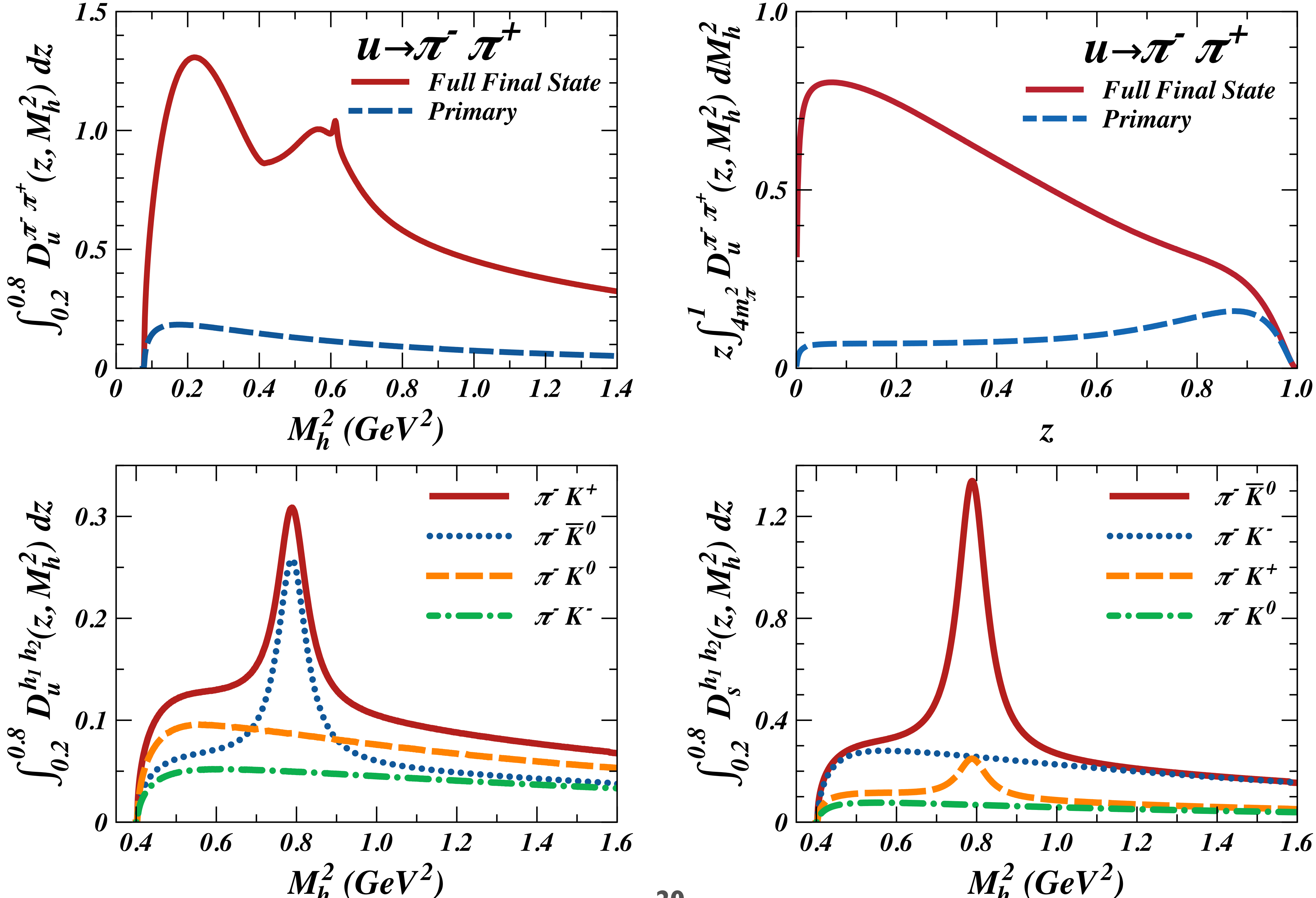
$$\Gamma_V(s)=\frac{m_V^2}{s}\Gamma_V\left(\frac{q(s)}{q(m_V^2)}\right)^3$$

- 3-body decay amplitude:
  - Point-like coupling (PYTHIA).
  - "Isobar" model (HERWIG, NJL-jet).

$$M=\varepsilon_{\mu\alpha\beta\gamma}\epsilon^{\mu}p_1^{\alpha}p_2^{\beta}p_3^{\gamma}\sum_{i=0,\pm}\frac{g_{V\rho_i\pi}\,g_{\rho_i\pi\pi}}{D_{\rho_i}(v_i^2)}$$



RESULTS FOR DFFS $N_{Links} = 8$



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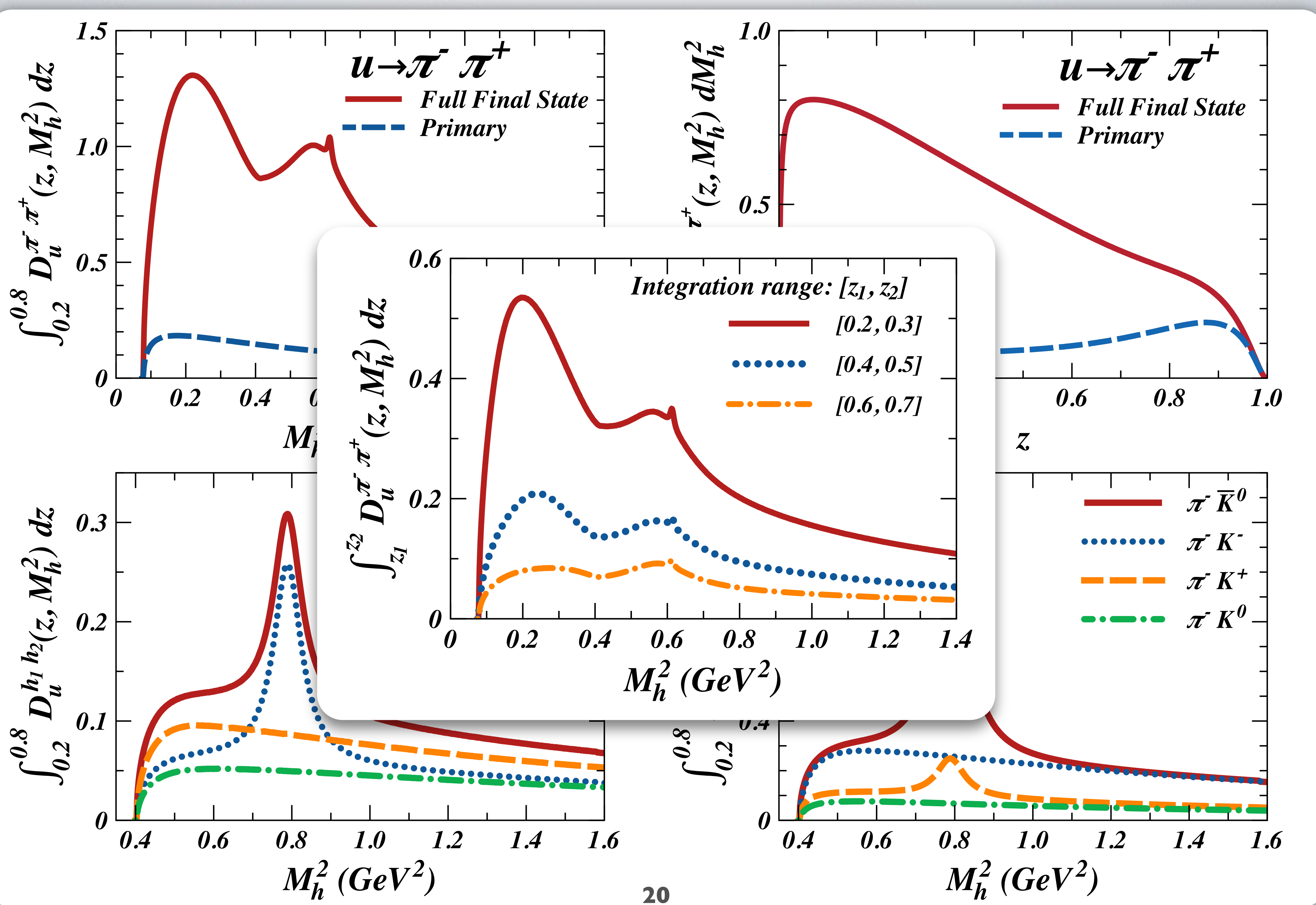
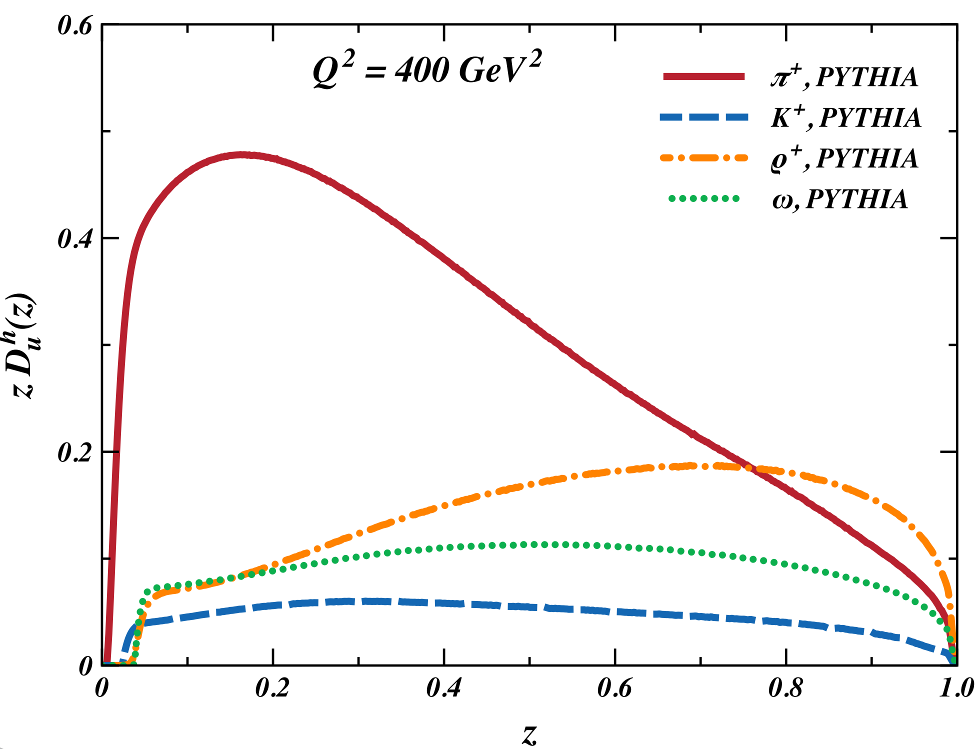


PYTHIA SIMULATIONS

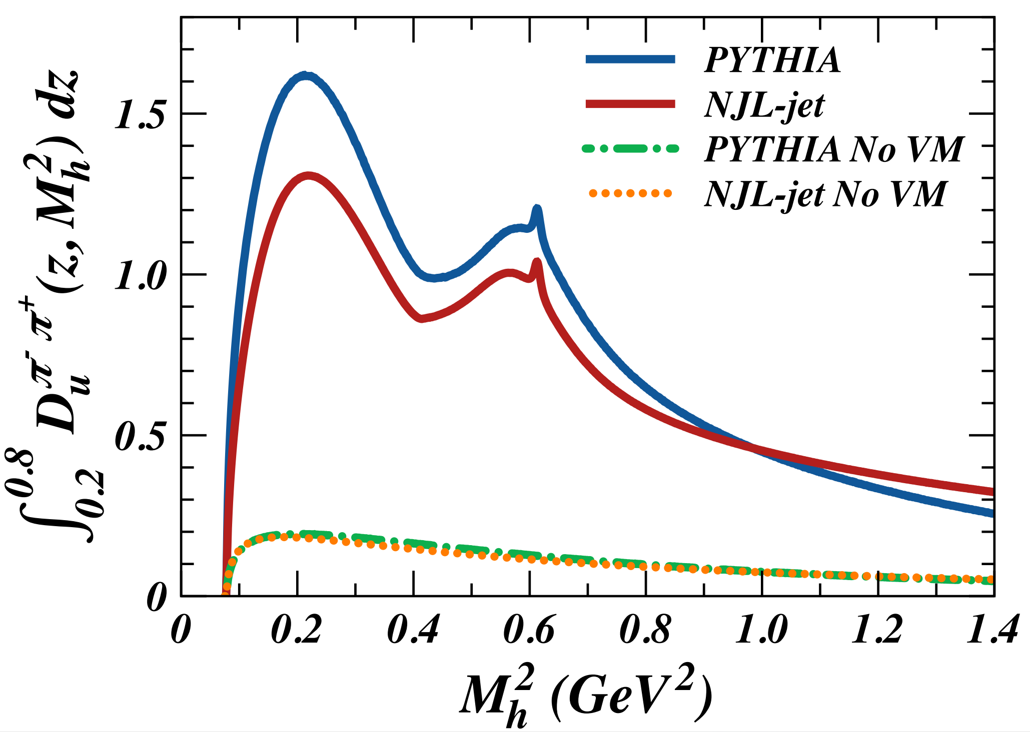
- Setup hard process with back to back $q\ \bar{q}$ along **z** axis.
- ***Only Hadronize.*** Allow the same resonance decays as NJL.
- Assign hadrons with positive p_z to q fragmentation.

$$E_q = 10 \text{ GeV}$$

Single Hadron



Dihadron



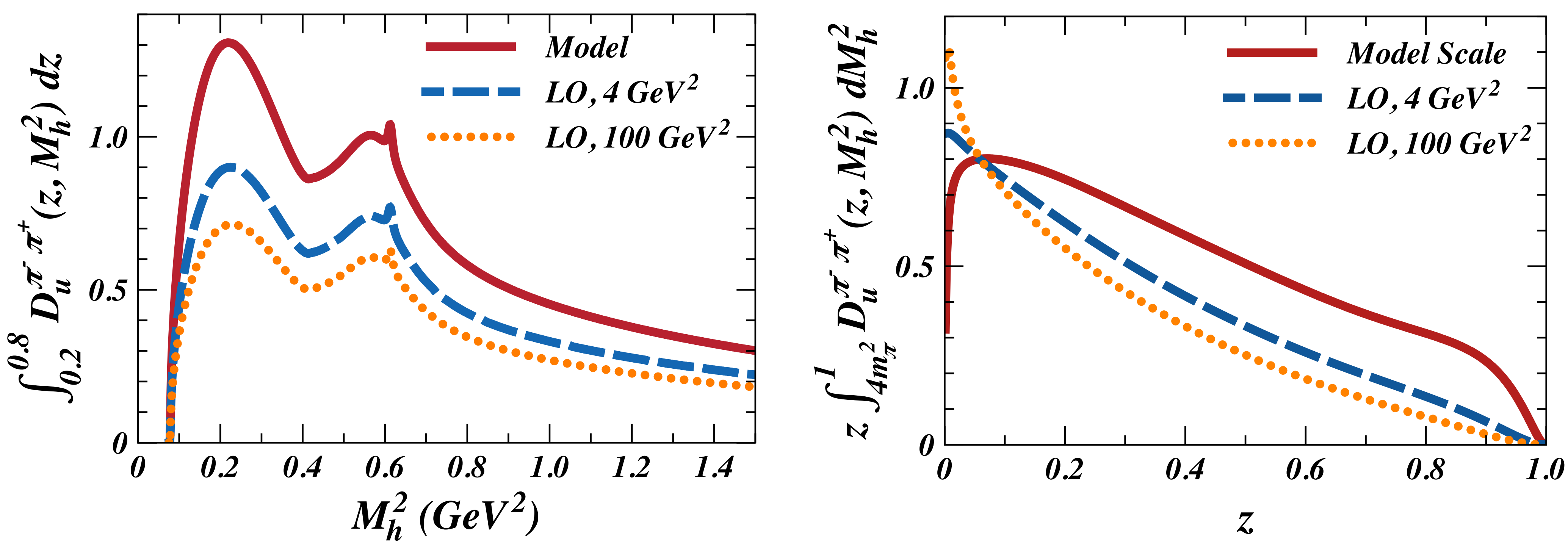
EVOLUTION OF DFF

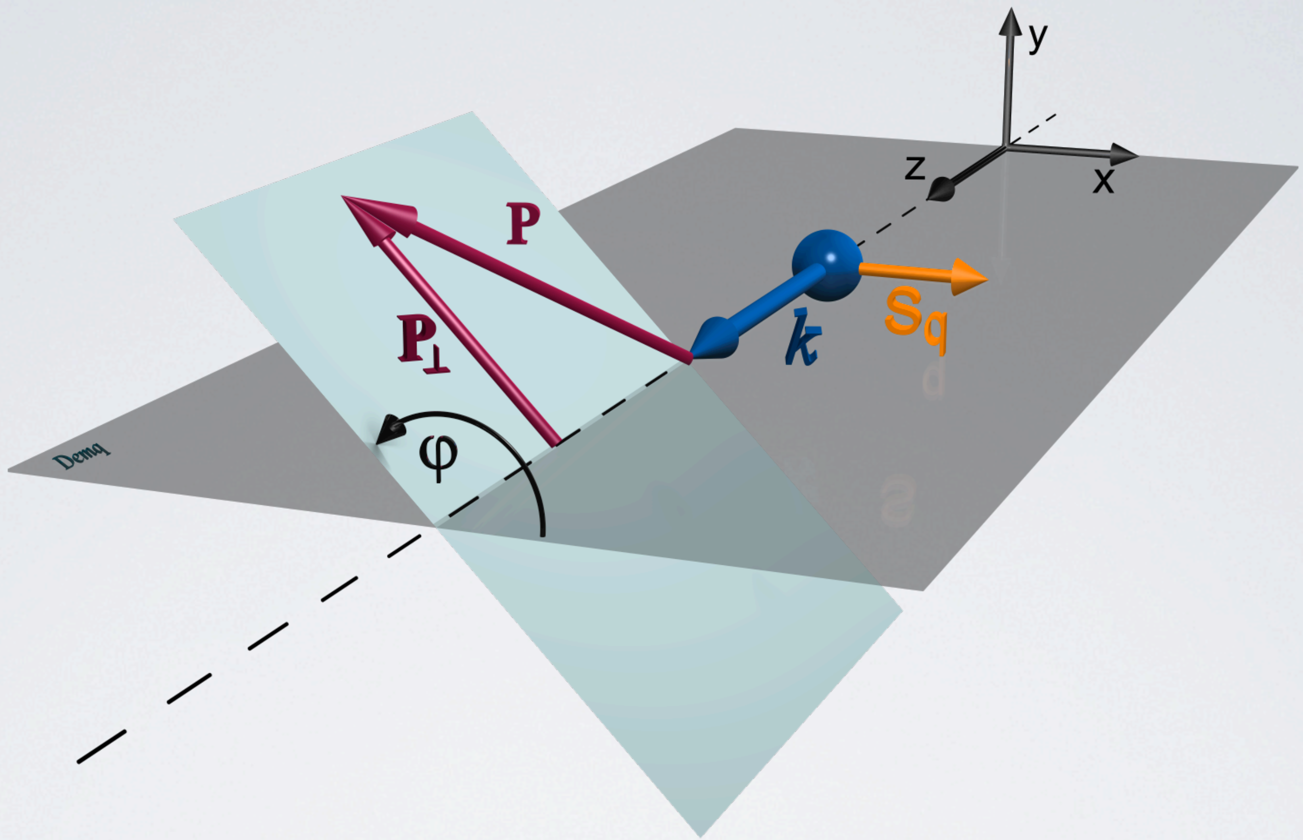
Bacchetta et. al., Phys.Rev. D79, 034029 (2009).

At leading order:

$$\frac{d}{d\log Q^2} D_{1,q}(z, M_h^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{du}{u} D_{1,q'}\left(\frac{z}{u}, M_h^2, Q^2\right) P_{q'q}(u)$$

$$u \to \pi^- \pi^+$$



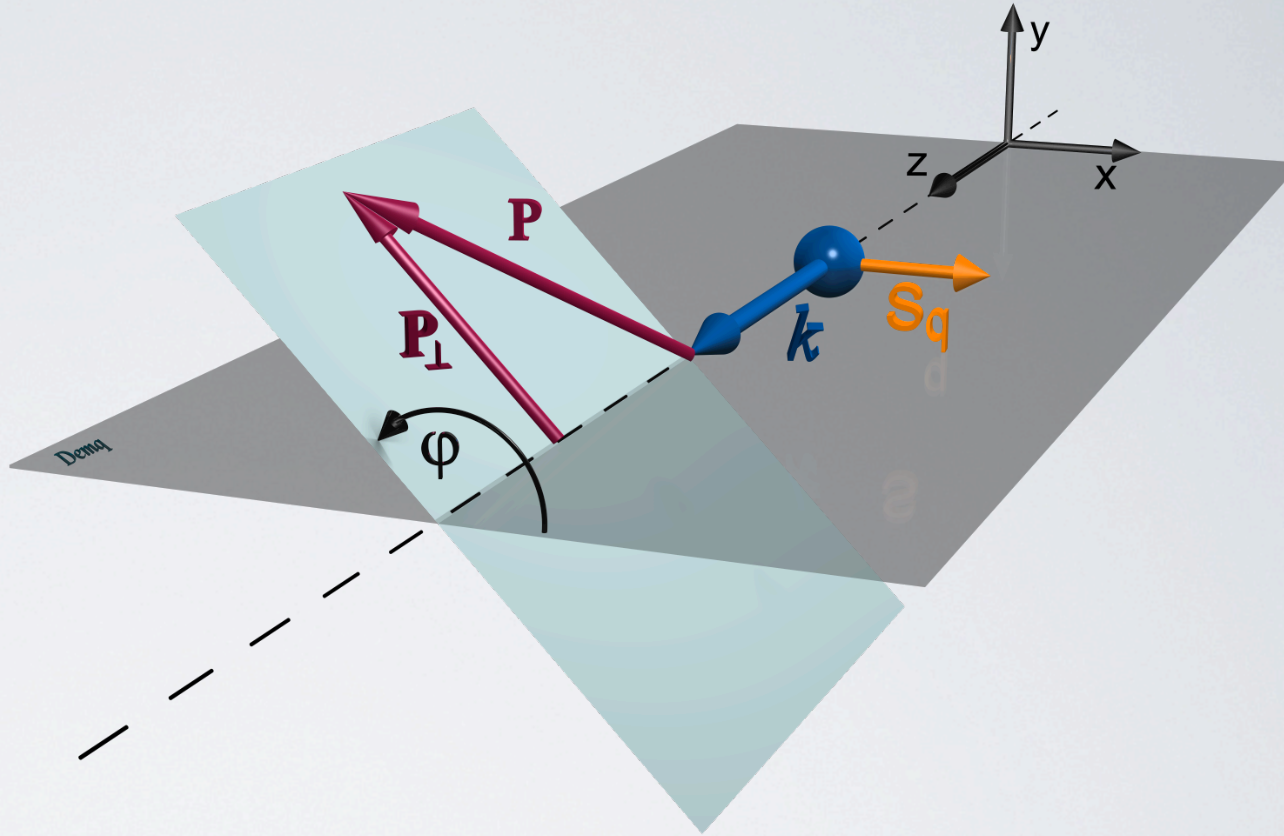


TRANSVERSELY POLARIZED QUARK FRAGMENTATION:
COLLINS EFFECT AND TWO-HADRON CORRELATIONS

COLLINS FRAGMENTATION FUNCTION

- **Collins Effect:**

  Azimuthal Modulation of Transversely Polarized Quark' Fragmentation Function.



Unpolarized

$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi) = D_1^{h/q}(z, P_\perp^2) - H_1^{\perp h/q}(z, P_\perp^2)\frac{P_\perp S_q}{z m_h}\sin(\varphi)$$

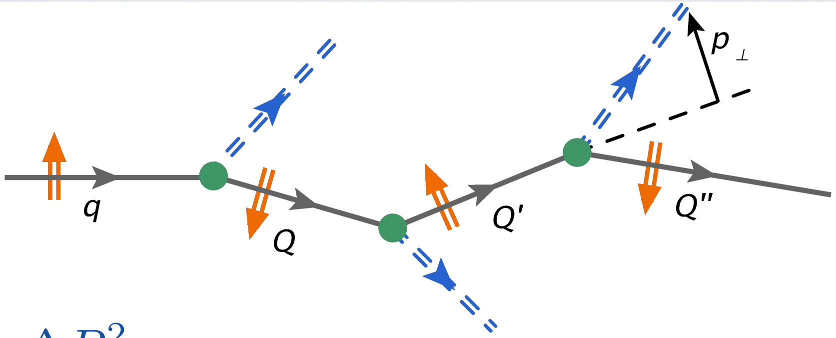
Collins

- **Chiral-ODD: Needs to be coupled with another chiral-odd quantity to be observed.**

COLLINS FRAGMENTATION FUNCTION FROM NJL-JET

H.M.,Bentz, Thomas, PRD.86:034025, 2012.

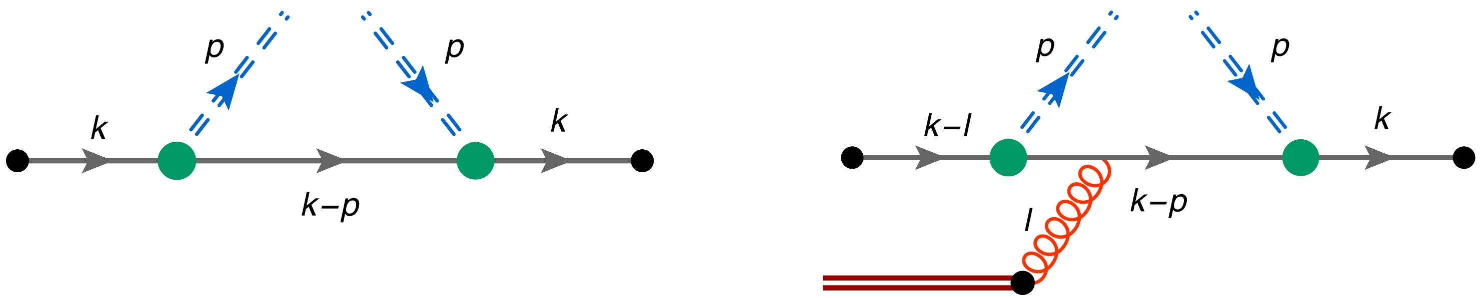
- **Extend the NJL-jet Model to Include the Quark's Spins.**



$$D_{h/q^\uparrow}(z, P_\perp^2, \varphi)\, \Delta z\, \frac{\Delta P_\perp^2}{2}\, \Delta\varphi = \left\langle N_{q^\uparrow}^h(z, z+\Delta z; P_\perp^2, P_\perp^2 + \Delta P^2; \varphi, \varphi + \Delta\varphi) \right\rangle$$

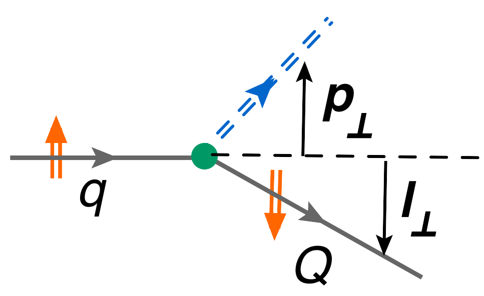
- **Model Calculated Elementary Collins Function as Input**

A. Bacchetta et. al., PLB659, 234 (2008).



- ***Spin flip probability:*** $\mathcal{P}_{SF}$

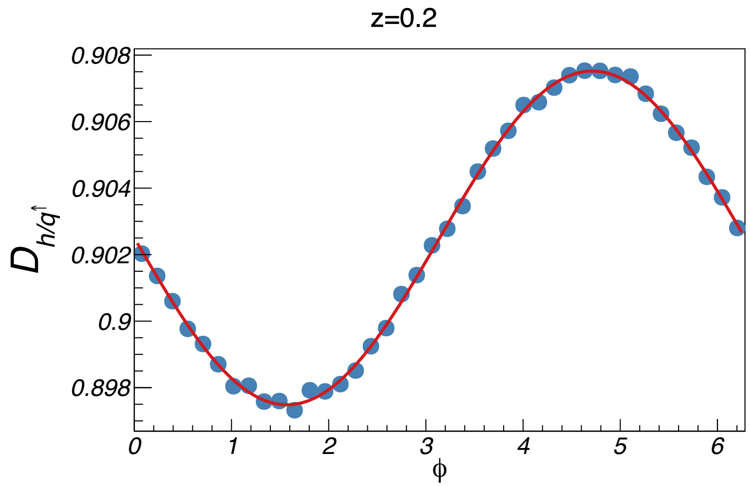
$$\mathcal{P}_{SF} = \mathcal{C}\,\frac{l_y^2 + (M_2 - (1-z)M_1)^2}{l_\perp^2 + (M_2 - (1-z)M_1)^2}$$

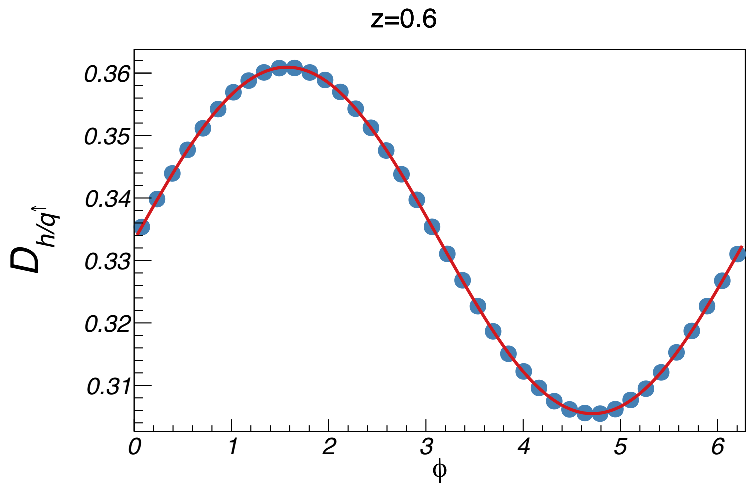


INTEGRATED POLARIZED FRAGMENTATIONS

- Integrate Polarized Fragmentations over $P_\perp^2$

$$D_{h/q^\uparrow}(z,\varphi) \equiv \int_0^\infty dP_\perp^2 \; D_{h/q^\uparrow}(z,P_\perp^2,\varphi) = \frac{1}{2\pi}\left[D_1^{h/q}(z) \; - 2H_{1(h/q)}^{\perp(1/2)}(z) S_q \sin(\varphi)\right]$$

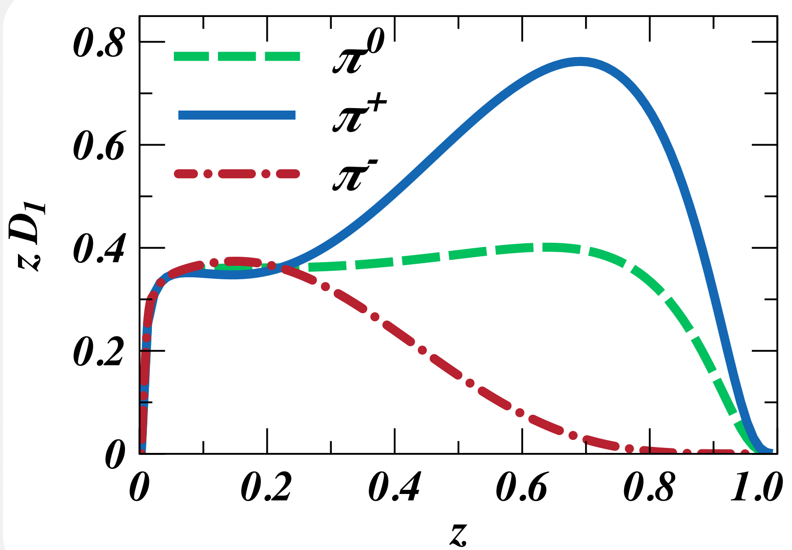




$$D_1^{h/q}(z) \equiv \pi \int_0^\infty dP_\perp^2 \; D_1^{h/q}(z,P_\perp^2)$$

$$H_{1(h/q)}^{\perp(1/2)}(z) \equiv \pi \int_0^\infty dP_\perp^2 \frac{P_\perp}{2zm_h} H_1^{\perp h/q}(z,P_\perp^2)$$

- Fit with form: $F(c_0,c_1) = c_0 - c_1 \sin(\varphi)$



COLLINS EFFECT - MK2

MK2 Model Assumptions:

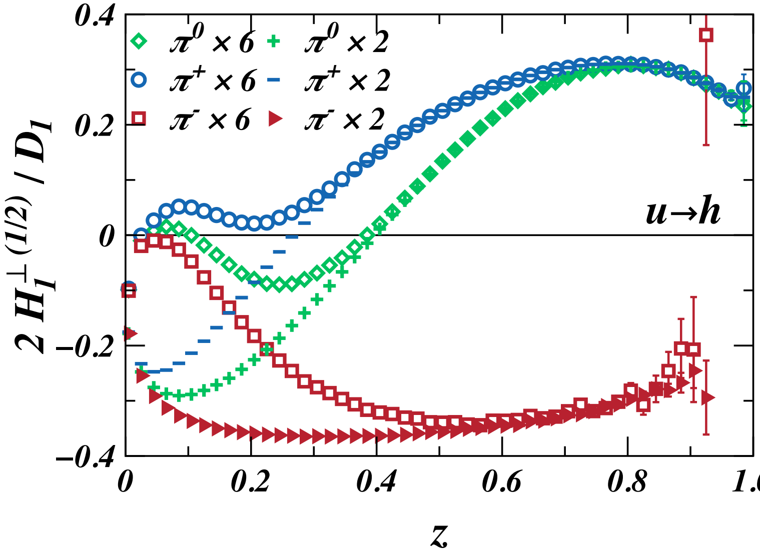
H.M., Kotzinian, Thomas, arXiv:1312.4556 (2013).

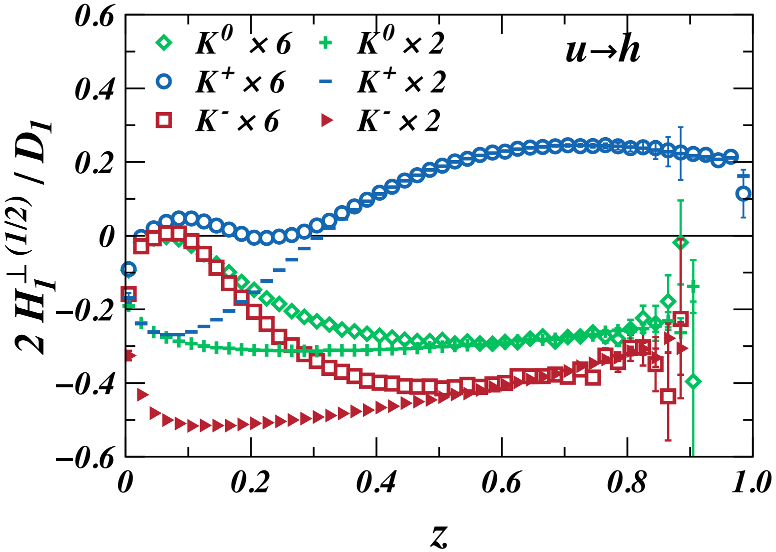
1. Allow for Collins Effect only in a SINGLE emission vertex - N_L^{-1} scaling of the resulting Collins function.
2. Use constant values for $\mathcal{P}_{SF}$.

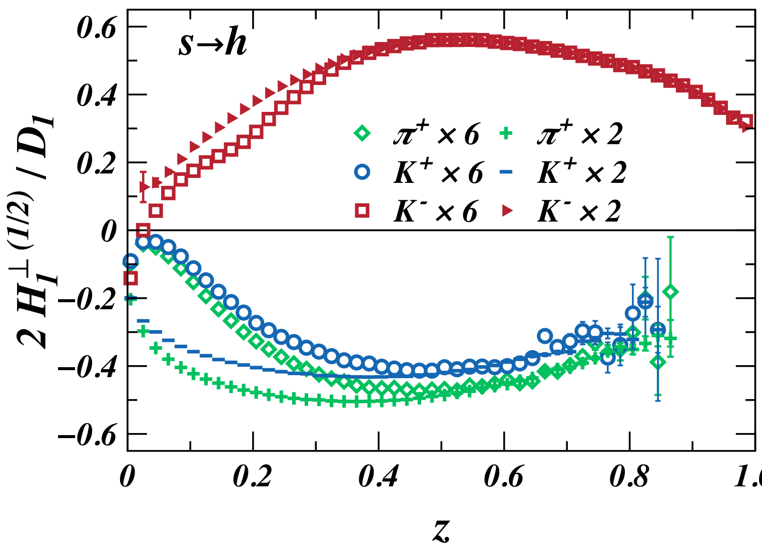
✦ The results for N_L=2 and N_L=6, scaled up by a factor N_L.

$$\mathcal{P}_{SF} = 1$$

$$F(c_0, c_1) = c_0 - c_1 \sin(\varphi)$$

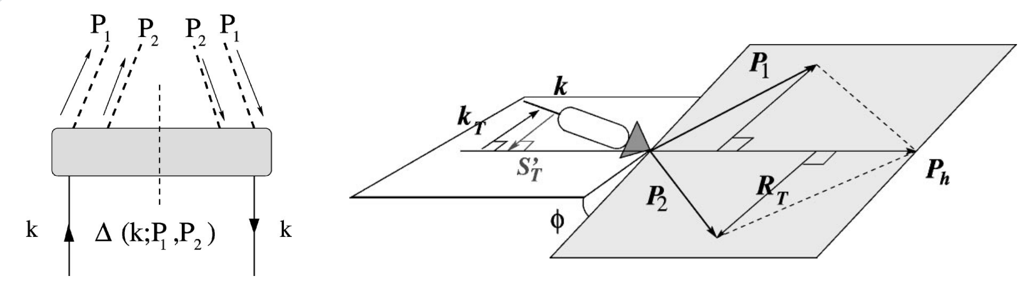






TWO-HADRON FRAGMENTATION

A. Bianconi, et al: PRD 62, 034008 (2000). M. Radici, et al: PRD 65, 074031 (2002).



- Kinematic Variables:

$$P_1=\left[\xi P_h^-, \frac{M_1^2+\vec{R}_T^2}{2\,\xi P_h^-}, \vec{R}_T\right], \qquad k=\left[\frac{P_h^-}{z}, z\frac{k^2+\vec{k}_T^2}{2P_h^-}, \vec{k}_T\right] \qquad z \equiv z_h = z_1 + z_2$$

$$P_2=\left[(1-\xi)P_h^-, \frac{M_2^2+\vec{R}_T^2}{2(1-\xi)P_h^-}, -\vec{R}_T\right] \qquad \mathbf{R} = \frac{\mathbf{P}_1 - \mathbf{P}_2}{2} \qquad \xi = \frac{z_1}{z_1+z_2}$$

- The relevant terms of the quark correlator at leading order for a **Transversely Polarized Quark:**

Unpolarized

$$\Delta^{[\gamma^-]} = D_1(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T\cdot\mathbf{R}_T)$$

Interference

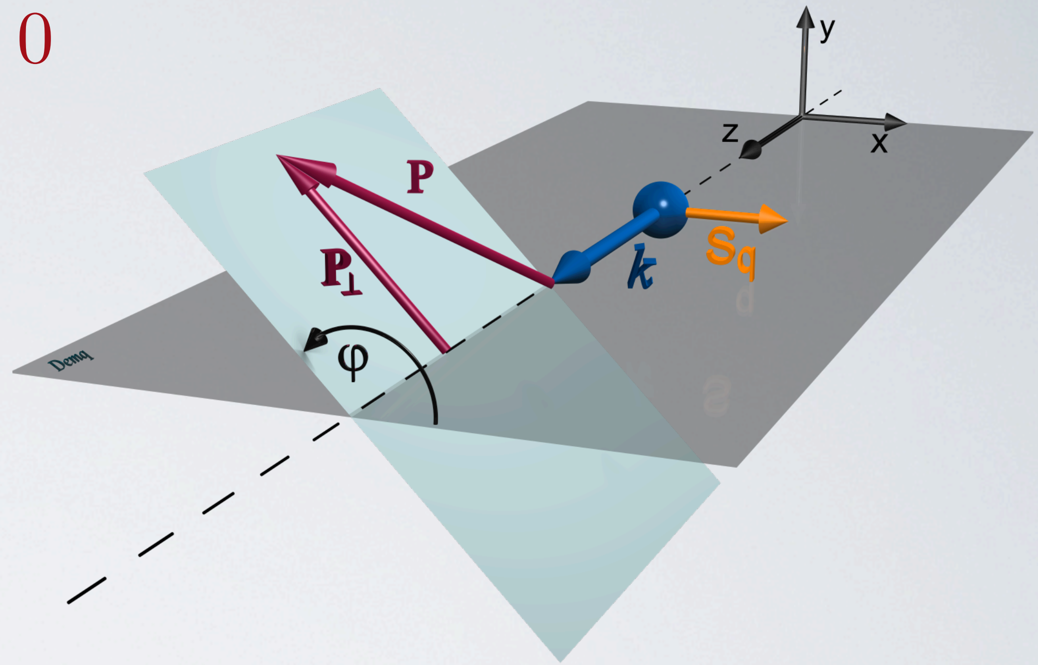
$$\Delta^{[i\sigma^{i-}\gamma_5]} = \frac{\epsilon_T^{ij} R_{Tj}}{M_1+M_2} H_1^{\sphericalangle}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T\cdot\mathbf{R}_T) + \frac{\epsilon_T^{ij} k_{Tj}}{M_1+M_2} H_1^{\perp}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T\cdot\mathbf{R}_T)$$

- **IFFS are Chiral-ODD: Need to be coupled with another chiral-odd quantity to be observed (e.g. transversity).**

TWO-HADRON FRAGMENTATION

✦ **Transformation to frame $\mathbf{k}_T = 0$**

$$k = (k^-, k^+, \mathbf{0})$$
$$\mathbf{k}_T = -\mathbf{P}_T / z_h$$



$$\mathbf{P}_T = \mathbf{P}^{\perp}_{h_1} + \mathbf{P}^{\perp}_{h_2}$$
$$\mathbf{R} = (\mathbf{P}^{\perp}_{h_1} - \mathbf{P}^{\perp}_{h_2})/2$$

✦ **Integrate over one or other momentum:**

$$D^{h_1 h_2}_{q^\uparrow}(\varphi_R) = D^{h_1 h_2}_{1,q} + \sin(\varphi_R - \varphi_S)\mathcal{F}[H_1^{\sphericalangle}, H_1^{\perp}]$$
$$D^{h_1 h_2}_{q^\uparrow}(\varphi_T) = D^{h_1 h_2}_{1,q} + \sin(\varphi_T - \varphi_S)\mathcal{F}'[H_1^{\sphericalangle}, H_1^{\perp}]$$

✦ **The IFF surviving after $\mathbf{k}_T$ integration is redefined as**

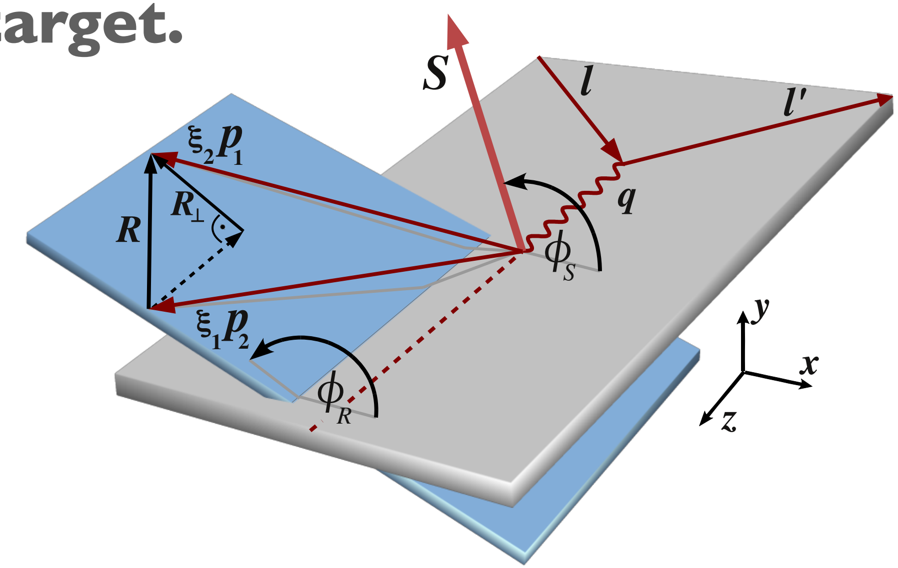
A. Bacchetta, M. Radici: PRD 69, 074026 (2004).

$$H_1^{\sphericalangle}(z_h, \xi, M_h^2) \equiv \int d^2\mathbf{k}_T \left[H_1^{\sphericalangle' e}(z_h, \xi, M_h^2, k_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) + \frac{k_T^2}{2M_h^2} H_1^{\perp o}(z_h, \xi, k_T^2, R_T^2, \mathbf{k}_T \cdot \mathbf{R}_T) \right]$$

RECENT COMPASS RESULTS

COMPASS Collaboration, arXiv:1401.7873 (2014).

✦ **SIDIS with transversely polarized target.**



✦ **Collins single spin asymmetry:**

$$A_{Coll} = \frac{\sum_q e_q^2\ \Delta_T q \otimes H_1^{\perp h/q}}{\sum_q e_q^2\ q \otimes D_1^{h/q}}$$

✦ **Two hadron single spin asymmetry:**

$$A_{UT}^{\sin\phi_{RS}} = \frac{|\boldsymbol{p}_1 - \boldsymbol{p}_2|}{2M_{h^+h^-}} \frac{\sum_q e_q^2 \cdot h_1^q(x) \cdot H_{1,q}^{\sphericalangle}(z,\, M_{h^+h^-}^2,\, \cos\theta)}{\sum_q e_q^2 \cdot f_1^q(x) \cdot D_{1,q}(z,\, M_{h^+h^-}^2,\, \cos\theta)}$$

✦ **Note the choice of the vector**

$$\boldsymbol{R}_{Artru} = \frac{z_2 \boldsymbol{P}_1 - z_1 \boldsymbol{P}_2}{z_1 + z_2}$$

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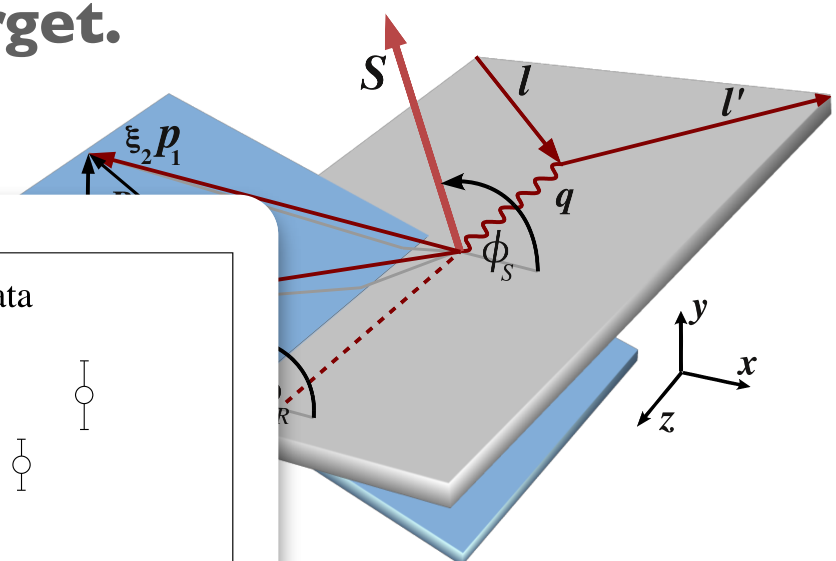
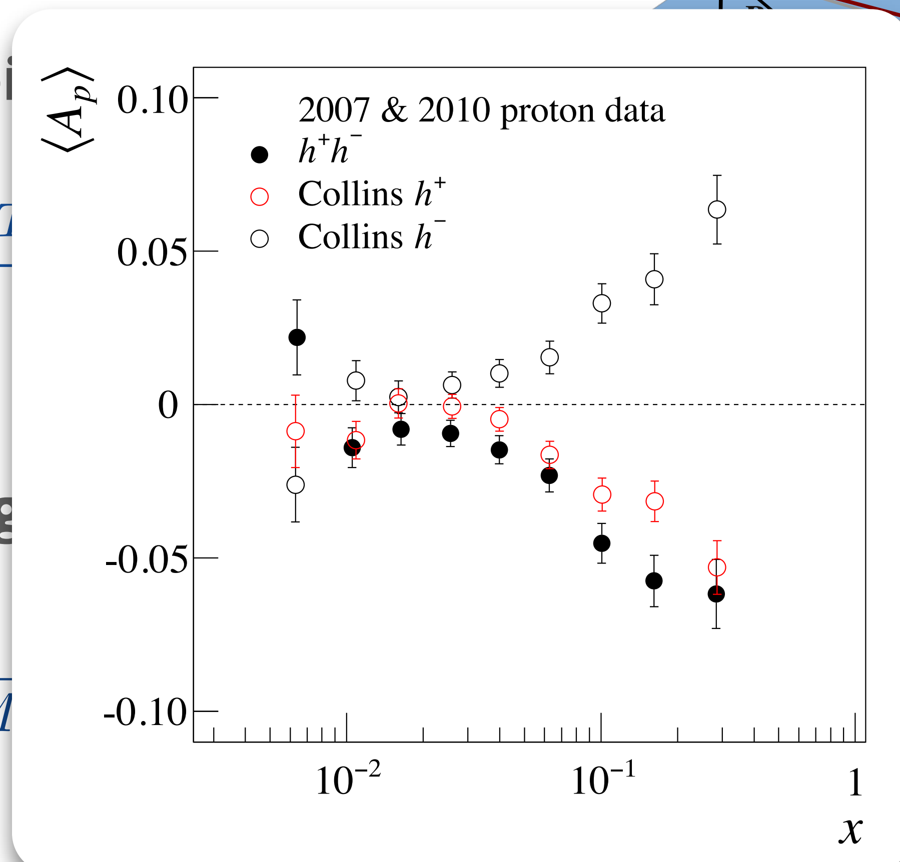


- **Collins single spi**

$$A_{Coll} = \frac{\sum_q e_q^2\ \Delta_T \cdots}{\sum_q e_q^2 \cdots}$$



- **Two hadron sing**

$$A_{UT}^{\sin\phi_{RS}} = \frac{|\boldsymbol{p}_1 \cdots}{2M \cdots} \cdots \frac{\cdots \cos\theta)}{\cdots \cos\theta)}$$

- **Note the choice of the vector**

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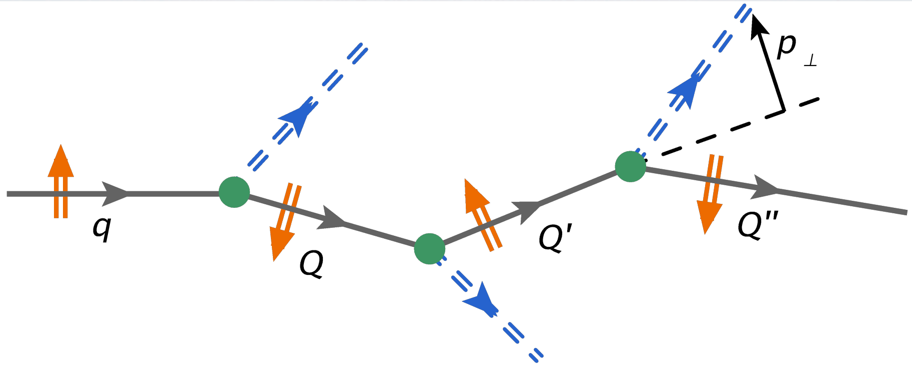
POLARIZED QUARK DIFF IN QUARK-JET.

H.M., Kotzinian, Thomas, arXiv:1312.4556 (2013).

- **Use the NJL-jet Model including Collins effect (Mk 2) to study DiFFs.**



$$D_{q^\uparrow}^{h_1 h_2}(z, M_h^2, \varphi_R)\ \Delta z\ \Delta M_h^2\ \Delta\varphi_R = \left\langle N_{q^\uparrow}^{h_1 h_2}(z, z+\Delta z; M_h^2, M_h^2+\Delta M_h^2; \varphi_R, \varphi_R+\Delta\varphi_R) \right\rangle .$$

- ***Choose a constant Spin flip probability:*** $\mathcal{P}_{SF}$

- ***Simple model to start with:***

  Only pions and extreme ansatz for the Collins term in elementary function.

$$d_{h/q^\uparrow}(z, \mathbf{p}_\perp) = d_1^{h/q}(z, p_\perp^2)(1 - 0.9\sin\varphi)$$

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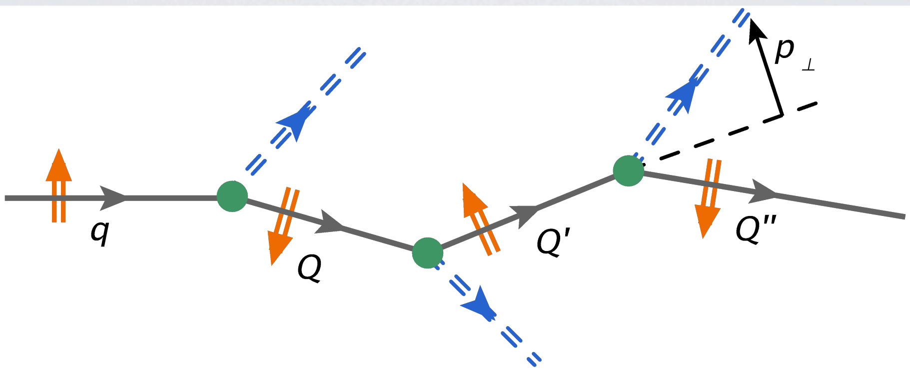

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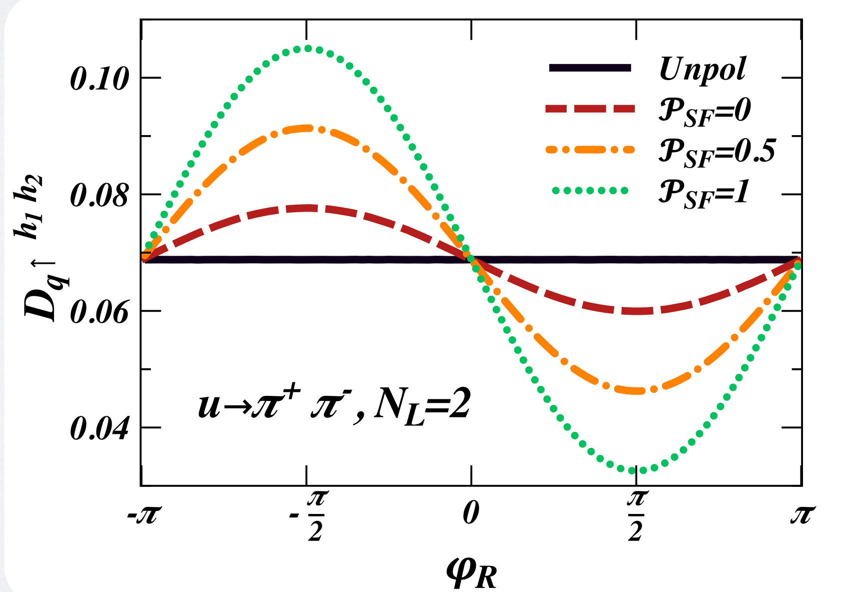
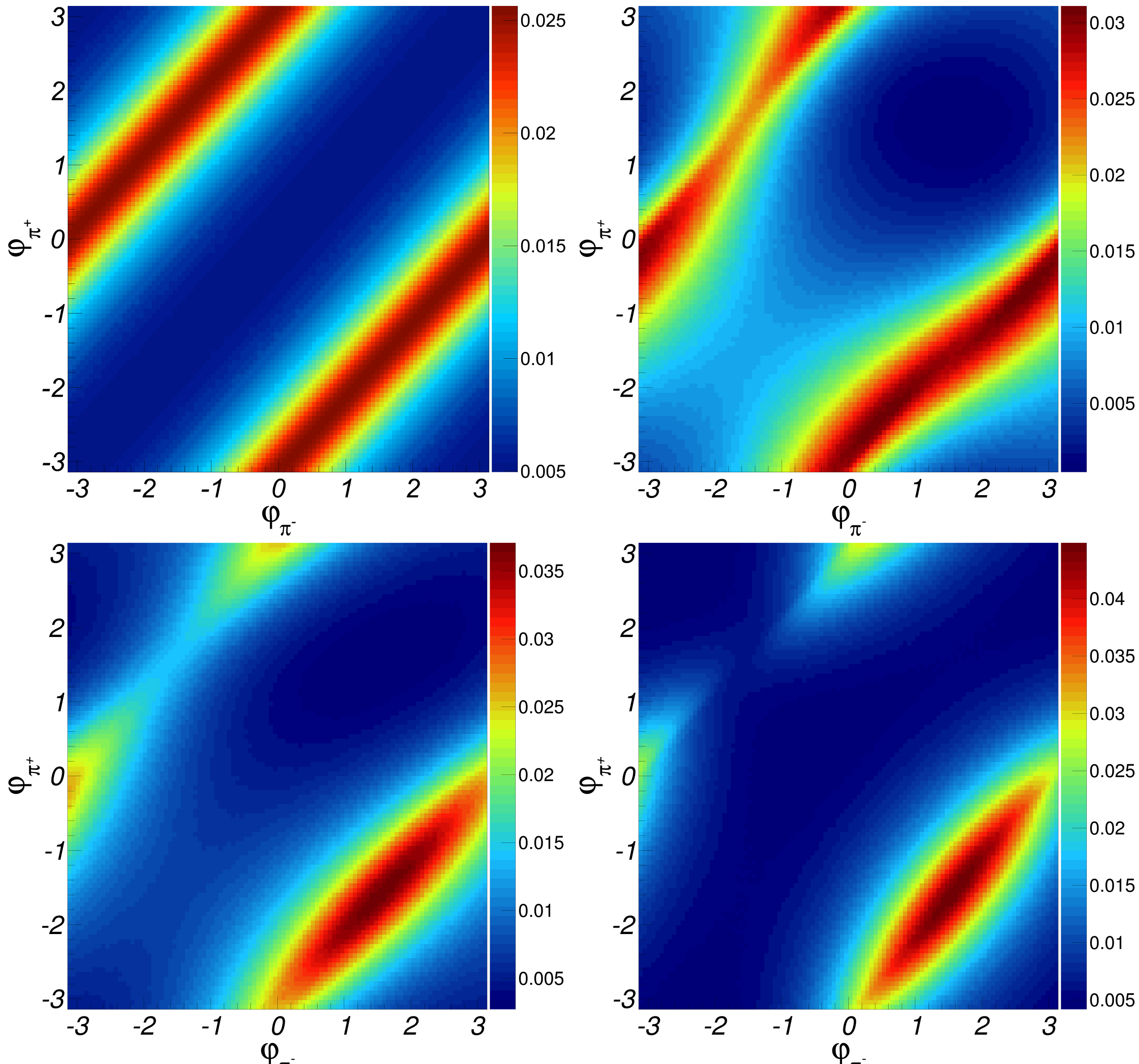


ANGULAR CORRELATIONS: $u \to \pi^+\pi^-$

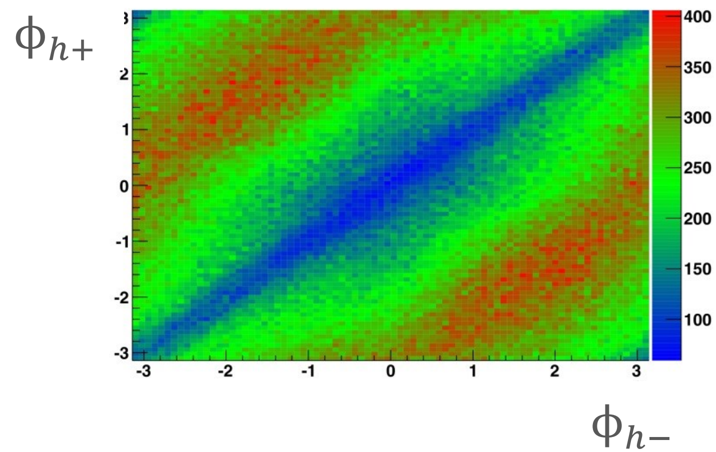
Unpolarized $\mathcal{P}_{SF} = 0$



$\mathcal{P}_{SF} = 0.5$ $\mathcal{P}_{SF} = 1$

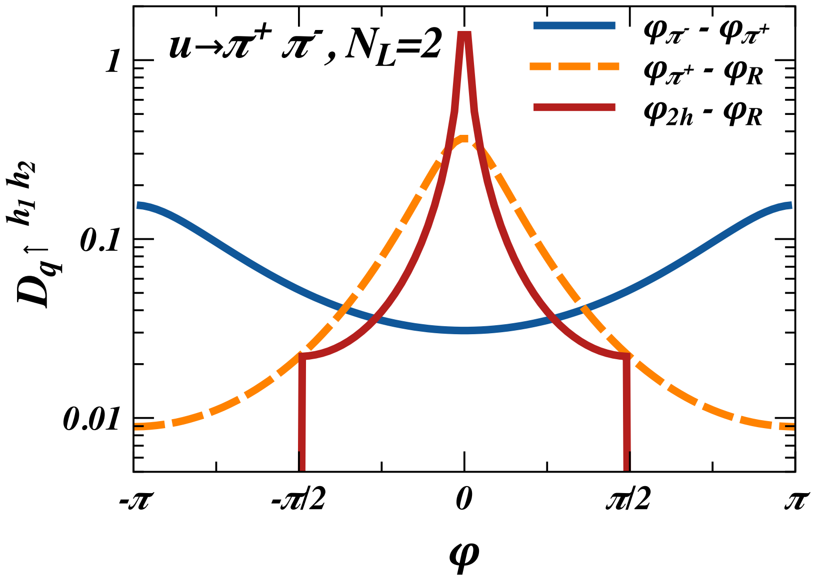
COMPASS Preliminary:

F. Bradamante - COMO 2013.



ANGULAR CORRELATIONS: $u \to \pi^+\pi^-$

Quark-Jet



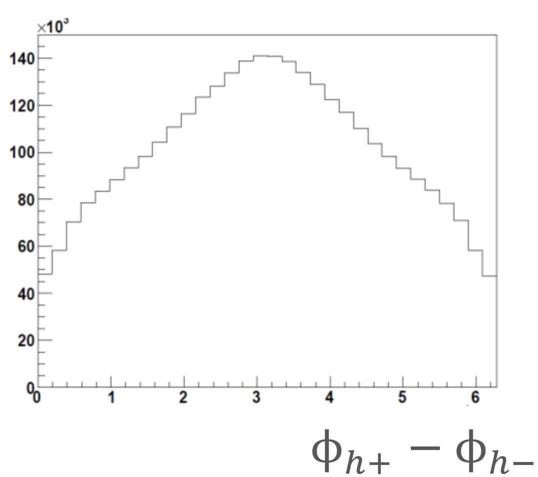
- We define:

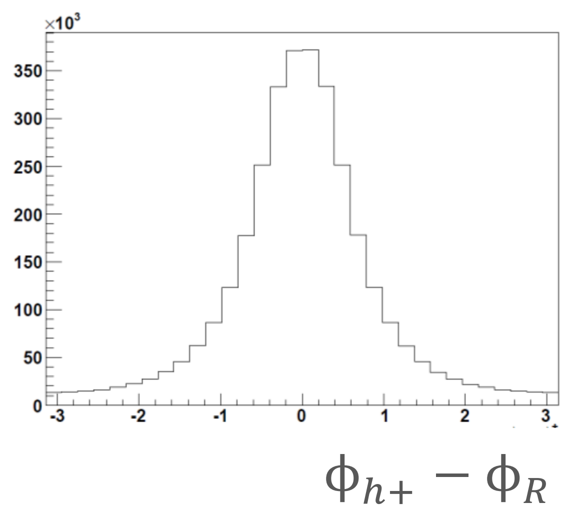
$$\mathbf{P}_{2h} = \frac{\widehat{\mathbf{P}}_1 - \widehat{\mathbf{P}}_2}{2}$$

- No Spin Dependence!

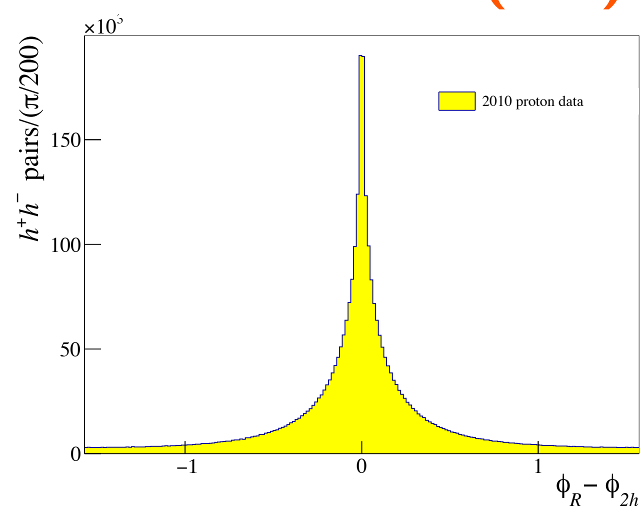
COMPASS Results

F. Bradamante - COMO 2013.

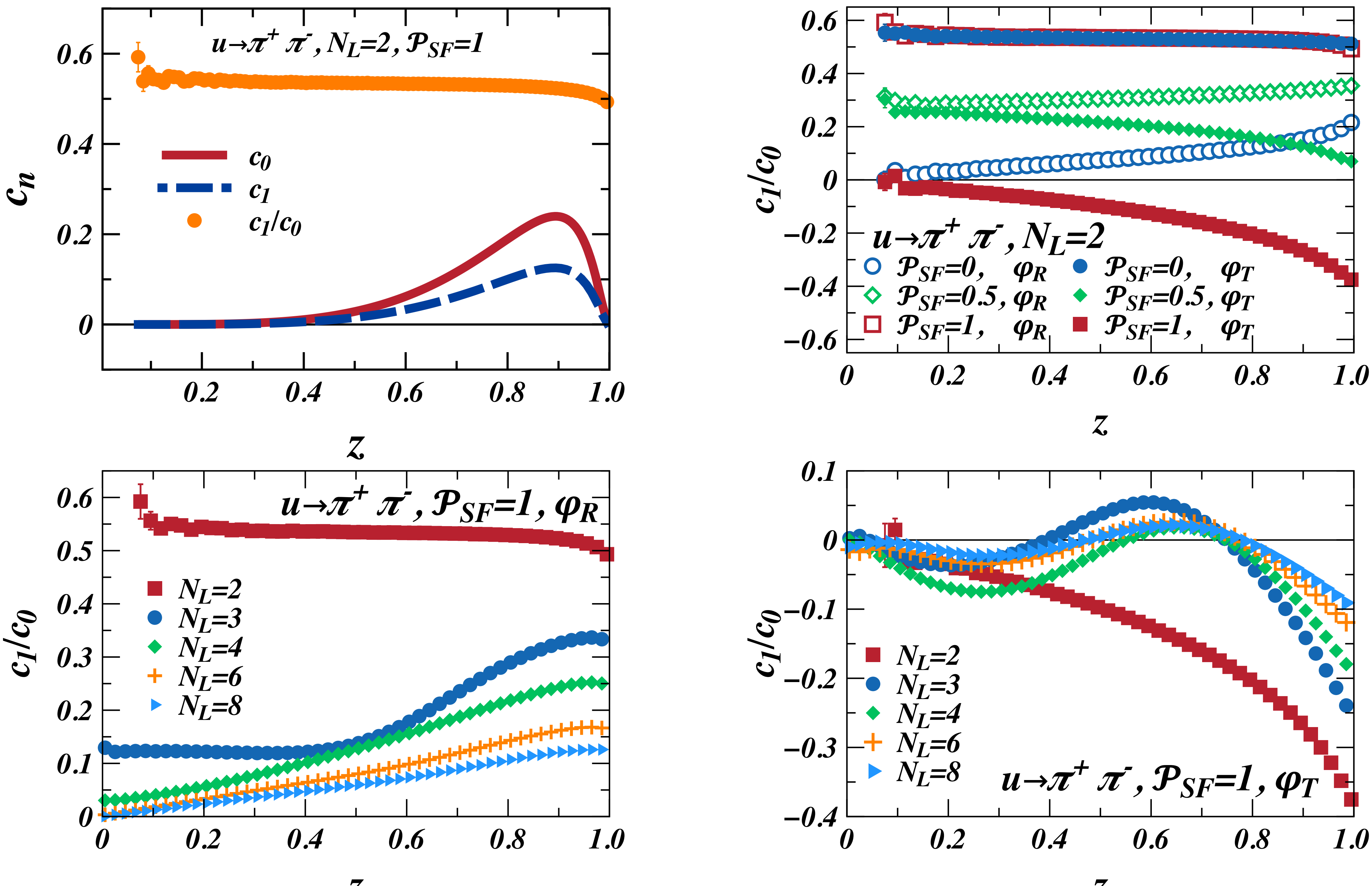




arXiv:1401.7873 (2014).

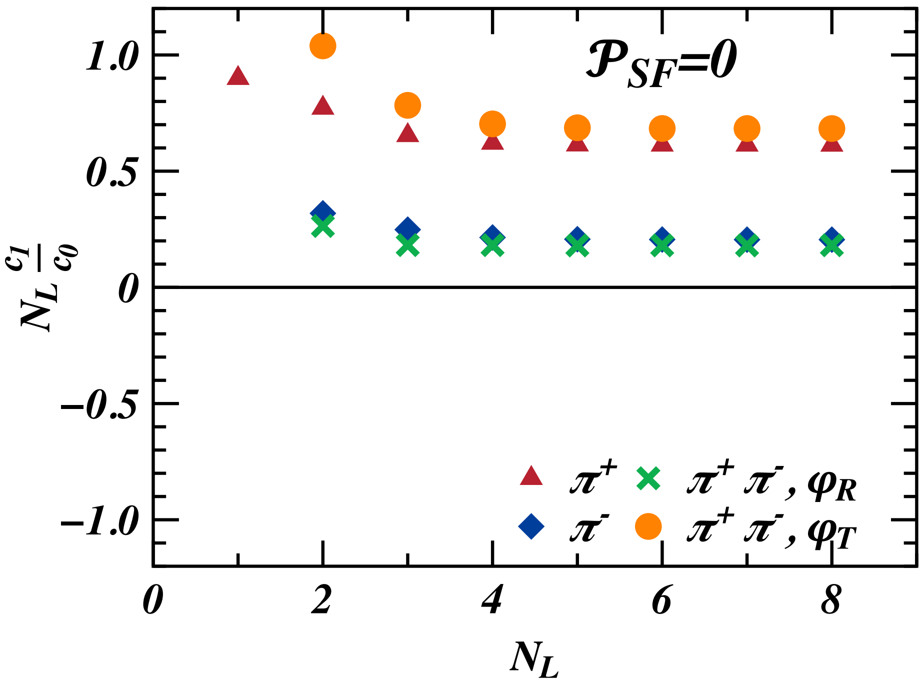


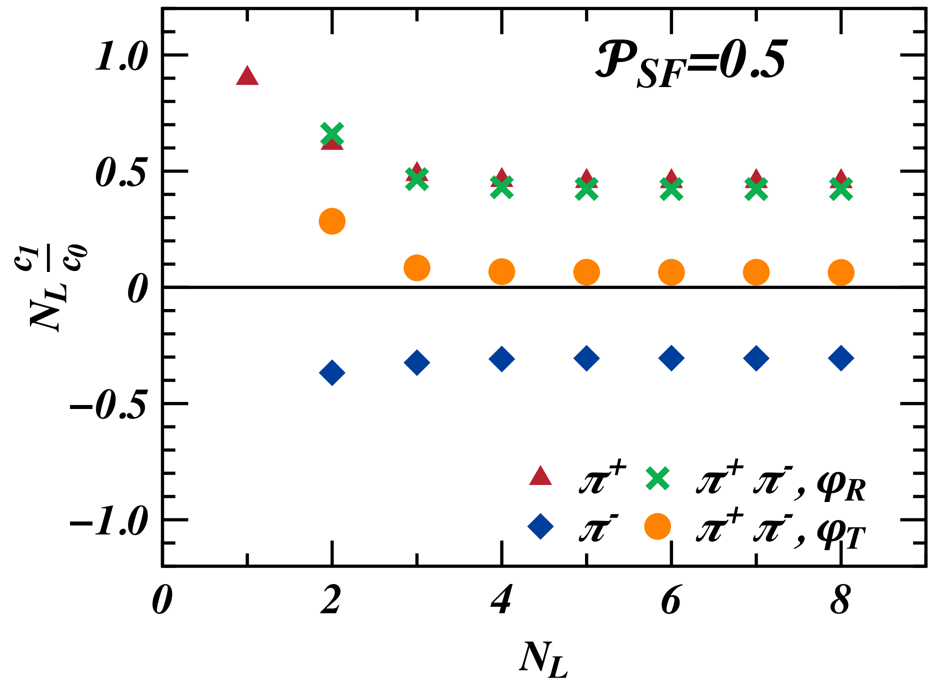
ANALYZING POWERS

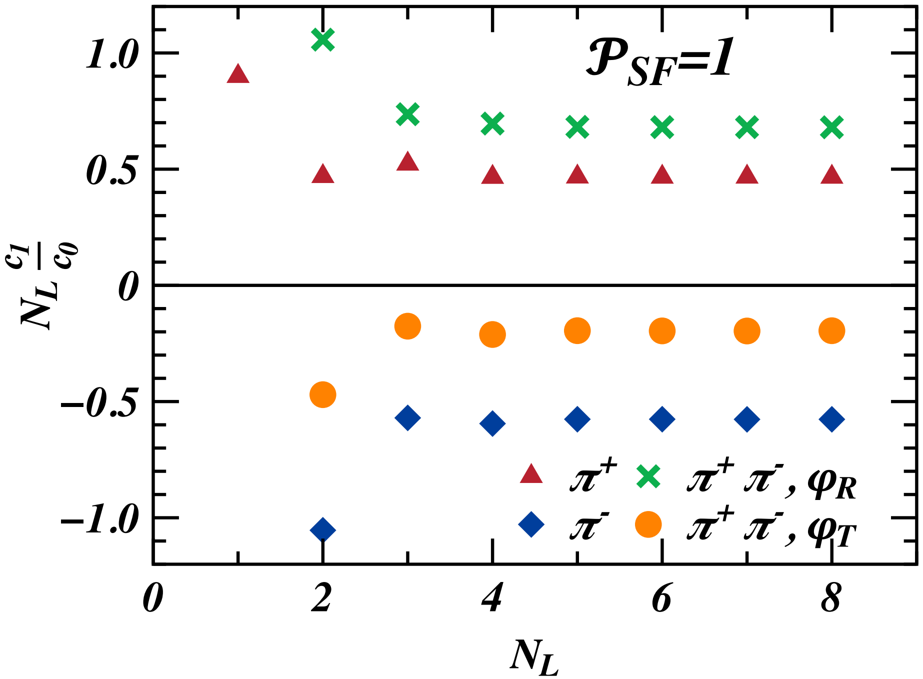


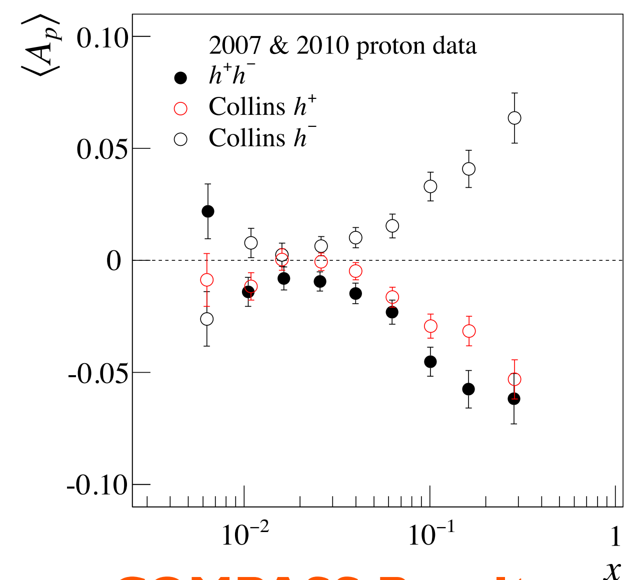
INTEGRATED ANALYZING POWERS

$z_{1,2} > 0.2, z > 0.2$









COMPASS Results
arXiv:1401.7873 (2014).

INTEGRATED ANALYZING POWERS

Single Hadron

$$N_h \propto \sigma_{UU}(1 + \sin(\phi_C) A_C G)$$

$$\phi_C = \phi_h - \phi_{S'}$$
$$= \phi_h + \phi_S - \pi$$

$$A_C = \frac{\sum_q e_q^2 \, \Delta_T q \otimes H_1^{\perp h/q}}{\sum_q e_q^2 \, q \otimes D_1^{h/q}}$$

DiHadron

$$N_{h^+h^-} \propto \sigma_{UU}(1 + \sin(\phi_{RS}) A_{UT}^{\sin\phi_{RS}} F)$$

$$\phi_{RS} = \phi_R - \phi_{S'}$$
$$= \phi_R + \phi_S - \pi$$

$$A_{UT}^{\sin\phi_{RS}} \propto \frac{\sum_q e_q^2 \cdot h_1^q \cdot H_{1,q}^{\sphericalangle}}{\sum_q e_q^2 \cdot f_1^q \cdot D_{1,q}}$$

NJL-Jet Fits

$$D_{q\uparrow} = c_0 - \sin(\phi) c_1$$

arXiv:1401.7873 (2014).

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NJL-Jet Fits

$$D_{q\uparrow} = c_0 - \sin(\phi) c_1$$

arXiv:1401.7873 (2014).

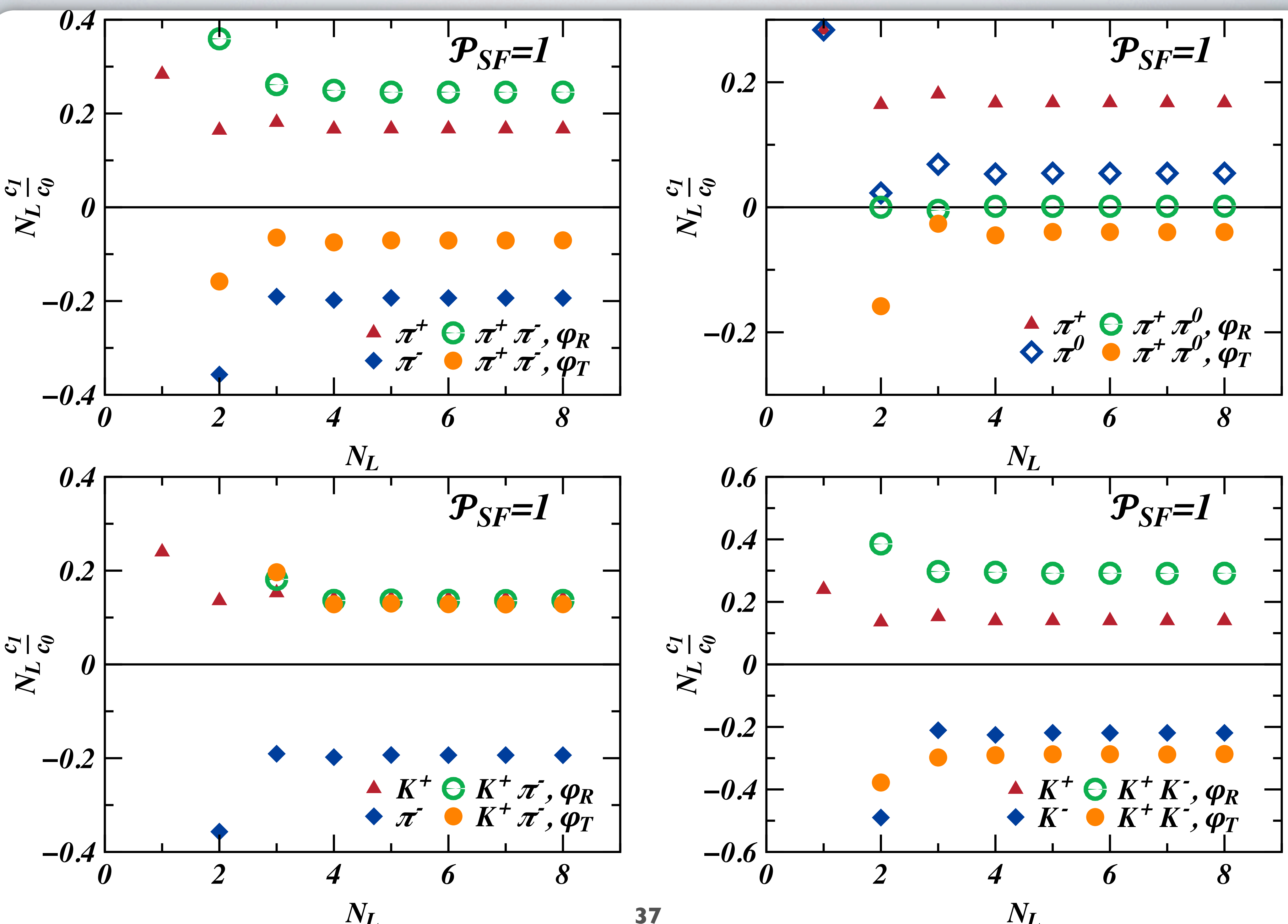
IMPROVED MODEL

- ✦**Use the *spectator model* for Collins function:**

  No singularities at vanishing transverse momentum.

- ✦**Include both *pion and kaon* channels.**

IMPROVED MODEL RESULTS



CONCLUSIONS

- ***Multi-hadron*** emissions are ***essential*** to complete description of both Favored and Unfavored fragmentation functions!

- The ***NJL-Jet*** model provides a ***robust*** and ***extendable*** framework for microscopic description of various fragmentation phenomena using MC simulations: TMD, Collins, DiHadron.

- ***NJL-Jet*** MC helps us to test and understand important aspects of various processes using a specific underlying quark model:
  - **z** dependence of $\langle P_\perp^2 \rangle$.
  - Effect of VM decays on Dihadron FFs.
  - The role of the Collins mechanism in IFFs.

- Further developments of the model are underway:
  - Including ***vector mesons*** in polarized fragmentations.
  - Exploring the ***target fragmentation***.