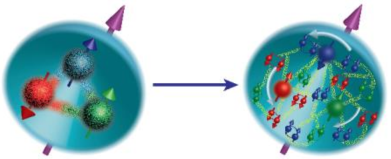
INSTITUTE FOR NUCLEAR THEORY

INT Workshop INT-14-55W
Studies of 3D Structure of Nucleon
February 24-28, 2014



Orbital angular momentum and Wigner distributions

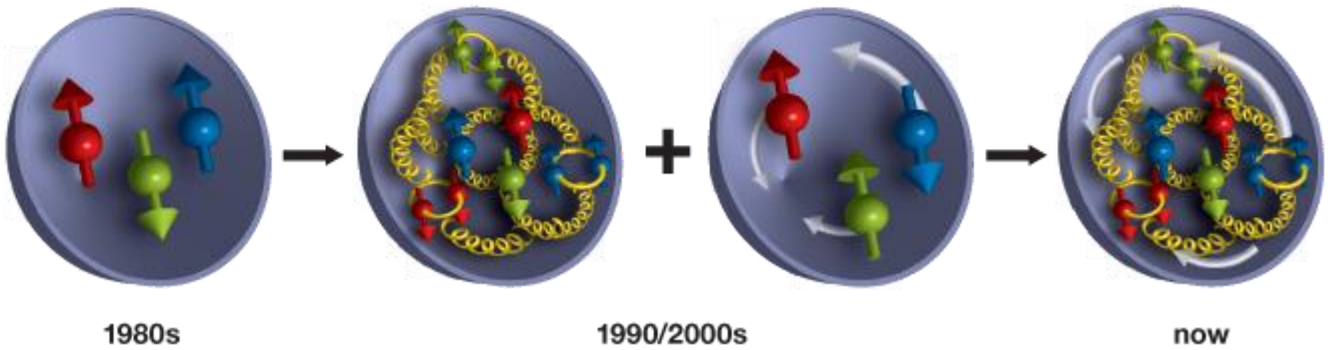
Cédric Lorcé

IPN Orsay - IFPA Liège

February 28th 2014, UW, Seattle, USA

Introduction and motivations

Our picture/understanding of the proton evolves !

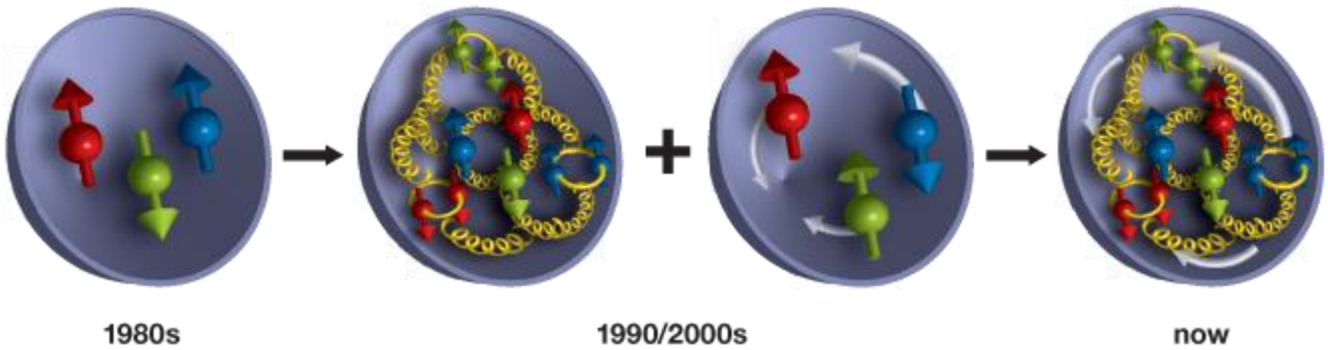


But many questions remain unanswered …

- **What is the proton size ?**
- **Why are quarks and gluons confined ?**
- **How are constituent quarks related to QCD ?**
- **How are quarks and gluons distributed inside the nucleon ?**
- **Where does the proton spin come from ?**
- **…**

Introduction and motivations

Our picture/understanding of the proton evolves !



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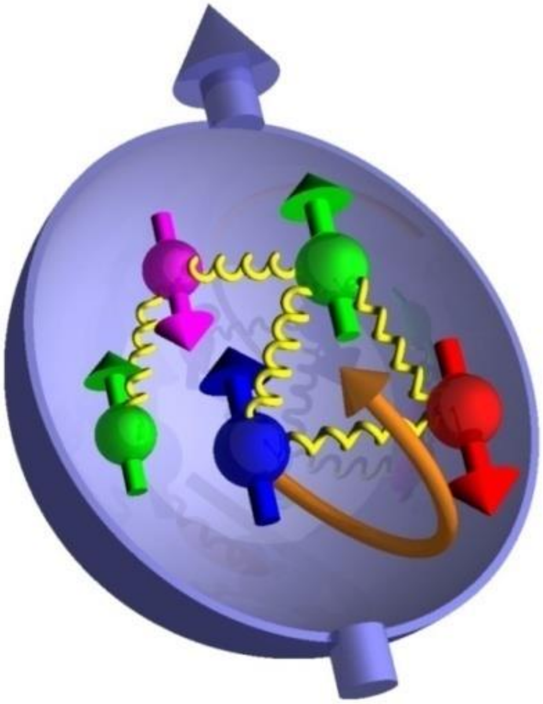
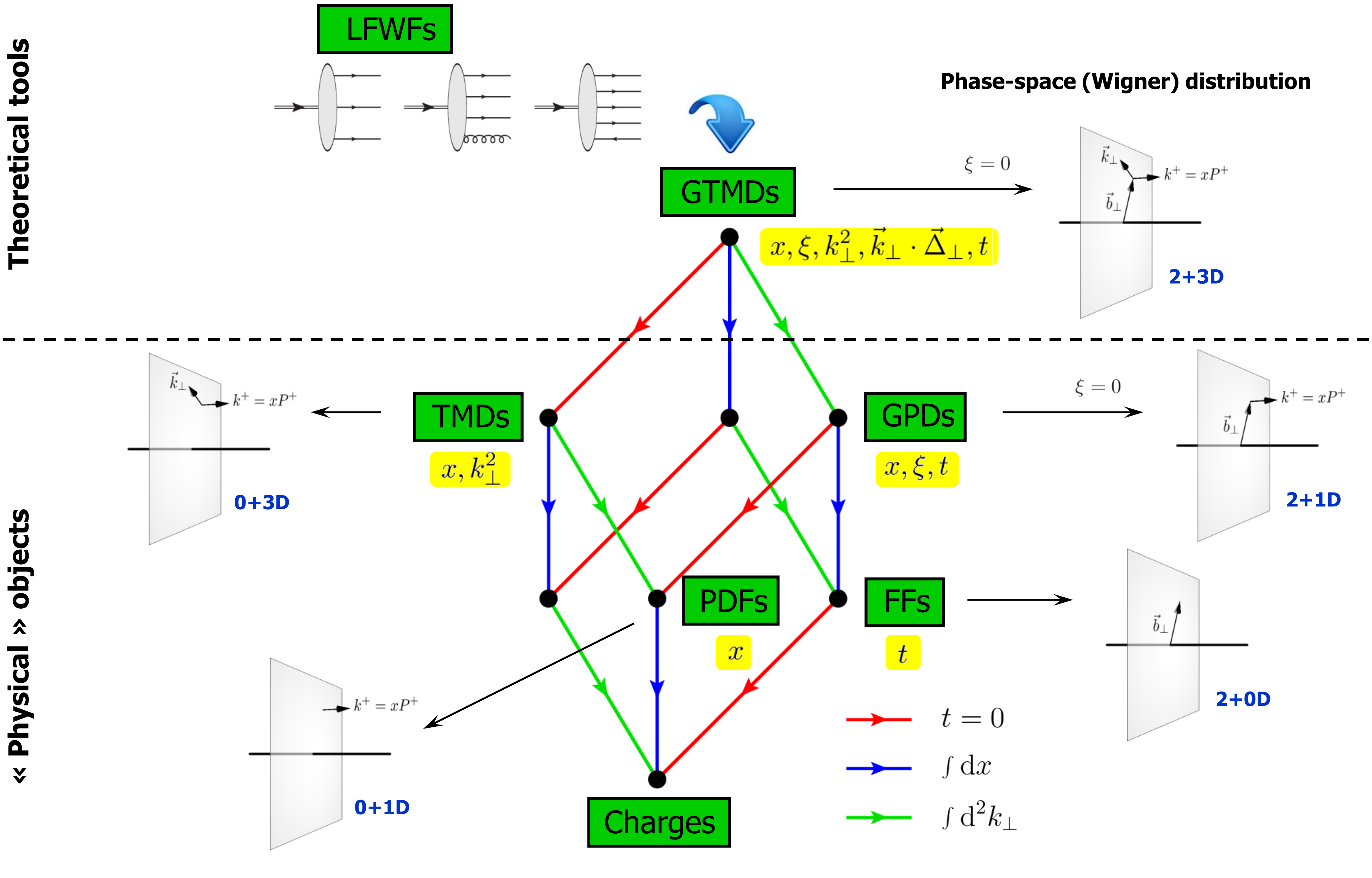
Outline



- Phase-space distributions and OAM
- Does F_{14} exist ?
- Spin-orbit correlations
- Conclusions

Parton distributions (naive)



[C.L., Pasquini, Vanderhaeghen (2011)]

Phase-space distributions

Quark Wigner operator

$$\widehat{W}^{[\Gamma]}(r,k^+,\vec{k}_\perp) = \int \frac{\mathrm{d}^4 z}{(2\pi)^4}\, e^{ik\cdot z}\, \overline{\psi}(r-\tfrac{z}{2})\Gamma\mathcal{W}\psi(r+\tfrac{z}{2})\,\delta(z^+)\,2\pi$$

Non-relativistic Wigner distribution

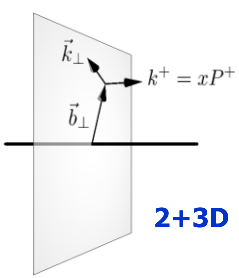
[Ji (2003)]
[Belitsky, Ji, Yuan (2004)]

$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{r},k^+,\vec{k}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}}\,\langle \tfrac{\vec{\Delta}}{2},\Lambda'|\widehat{W}^{[\Gamma]}(0,k^+,\vec{k}_\perp)|-\tfrac{\vec{\Delta}}{2},\Lambda\rangle$$

3+3D

Relativistic Wigner distribution

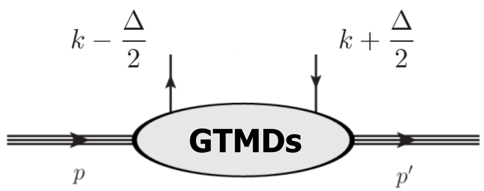
[C.L., Pasquini (2011)]
[C.L., Pasquini, Xiong, Yuan (2012)]



2+3D

$$\rho^{[\Gamma]}_{\Lambda'\Lambda}(\vec{b}_\perp,k^+,\vec{k}_\perp) = \frac{1}{2}\int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\,\underbrace{\langle p^+,\tfrac{\vec{\Delta}_\perp}{2},\Lambda'|\widehat{W}^{[\Gamma]}(0,k^+,\vec{k}_\perp)|p^+,-\tfrac{\vec{\Delta}_\perp}{2},\Lambda\rangle}_{\boxed{\text{GTMDs}}}$$

Light-front overlap representation



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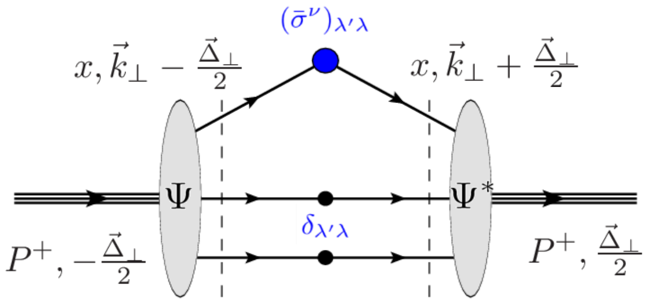


$$\boxed{W^{[\Gamma]}_{\Lambda'\Lambda}(x,\xi,\vec{k}_\perp,\vec{\Delta}_\perp) = \frac{1}{\sqrt{1-\xi^2}} \sum_{\beta',\beta} \int [\mathrm{d}x]_3 [\mathrm{d}^2k_\perp]_3\, \bar{\delta}(\tilde{k})\, \psi^*_{\Lambda'\beta'}(r')\, \psi_{\Lambda\beta}(r)\, M^{[\Gamma]\beta'\beta}}$$

Momentum **Polarization**

Light-front quark models

$$\psi_{\Lambda\beta}(r) = \mathcal{N}\, \Psi(r) \sum_{\sigma_i} \Phi^{\sigma_1\sigma_2\sigma_3}_{\Lambda} \prod_{i=1}^{3} D_{\lambda_i\sigma_i}(\tilde{k}_i)$$

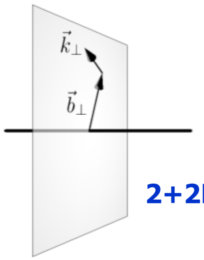
Wigner rotation

$$D(\tilde{k}) = \frac{1}{|\vec{K}|} \begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}, \qquad K_{R,L} = K_x \pm i K_y$$

Model	$\Psi(r)$	K_z	$\vec{K}_\perp$	κ_z
LFCQM	$\tilde{\psi}(r)$	$m + y\mathcal{M}_0$	$\vec{\kappa}_\perp$	$y\mathcal{M}_0 - \omega$
LFχQSM	$\prod_{i=1}^{3} \lvert\vec{K}_i\rvert$	$f_{/\!/}(y,\kappa_\perp)$	$\vec{\kappa}_\perp\, f_\perp(y,\kappa_\perp)$	$y\mathcal{M}_N - E_{\text{lev}}$

[C.L., Pasquini, Vanderhaeghen (2011)]

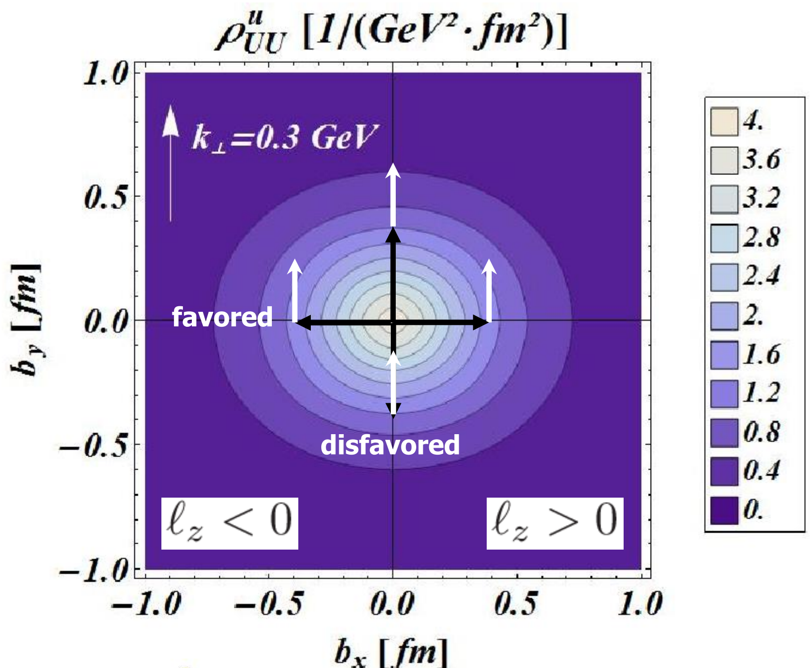
Model results

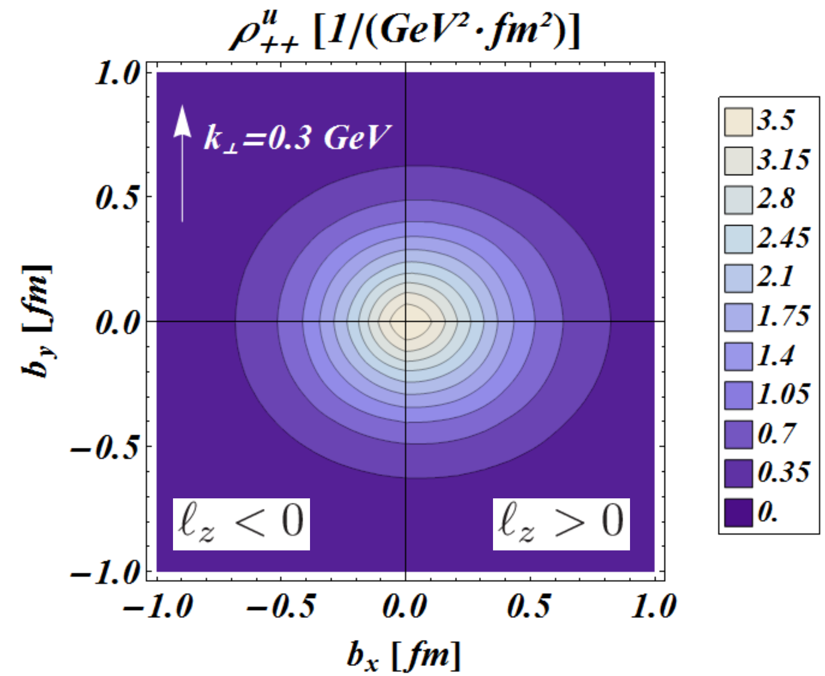


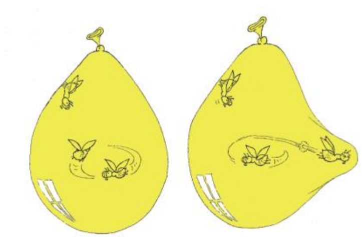
2+2D

$$\rho_{UU} = \frac{1}{2}\left(\rho_{++}^{[\gamma^+]} + \rho_{--}^{[\gamma^+]}\right) \propto F_{11}$$

$$\rho_{++}^{[\gamma^+]} \propto F_{11},\, F_{14}$$







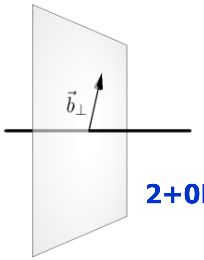
favored **disfavored**

$\boxed{\ell_z^{\rm tot} = 0}$

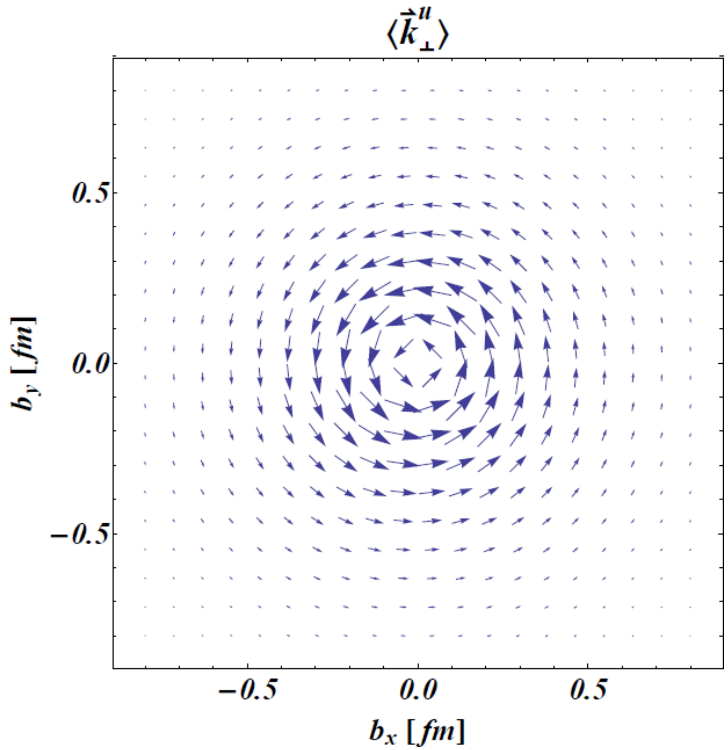
$\boxed{\ell_z^{\rm tot} > 0}$

[C.L., Pasquini (2011)]

Orbital angular momentum



2+0D



Model result

Average quark momentum

$$\langle \vec{k}_\perp \rangle (\vec{b}_\perp) = \int \mathrm{d}x\, \mathrm{d}^2 k_\perp\, \vec{k}_\perp\, \rho^{[\gamma^+]}_{++}(x, \vec{k}_\perp, \vec{b}_\perp)$$

Phase-space density

Average quark OAM

$$\begin{aligned} \ell_z &= \int \mathrm{d}^2 b_\perp\, \vec{b}_\perp \times \langle \vec{k}_\perp \rangle (\vec{b}_\perp) \\ &= -\int \mathrm{d}x\, \mathrm{d}^2 k_\perp\, \frac{\vec{k}^2_\perp}{M^2}\, F_{14}(x, 0, \vec{k}_\perp, \vec{0}_\perp) \end{aligned}$$

« Vorticity »

[C.L., Pasquini (2011)]
[Hatta (2012)]
[C.L., Pasquini, Xiong, Yuan (2012)]

Path dependence

Quarks Wigner operator

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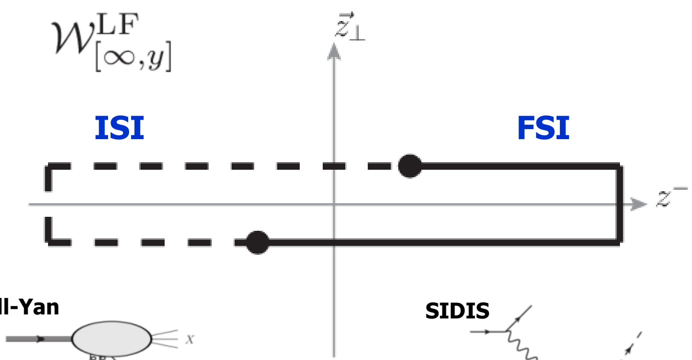
Canonical OAM

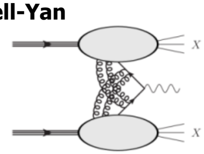
[Jaffe, Manohar (1990)]

$\boxed{\vec{r}\times i\vec{D}_{\text{pure}}}$ $\quad D^{\text{pure}}_\mu \overset{A^+=0}{=} \partial_\mu$

$\mathcal{W}^{\text{LF}}_{[\infty,y]}$

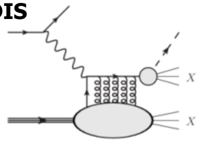


Drell-Yan



$\ell_z^{\text{DY}}=\ell_z^{\text{SIDIS}}$

SIDIS



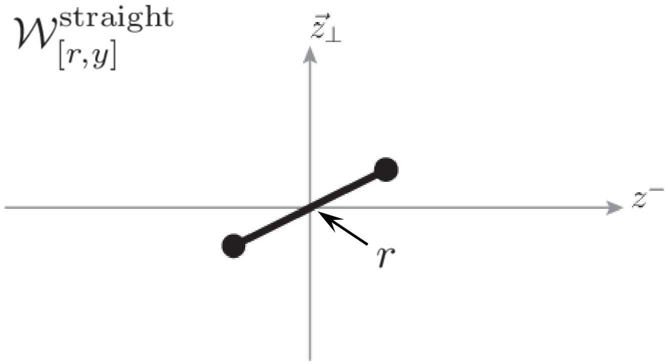
[Hatta (2012)]
[C.L., Pasquini, Xiong, Yuan (2012)]

Kinetic OAM

[Ji (1997)]

$\boxed{\vec{r}\times i\vec{D}}$

$\mathcal{W}^{\text{straight}}_{[r,y]}$



[Ji, Xiong, Yuan (2012)]
[C.L. (2013)]

Leading-twist parametrization

GTMDs $\xi = 0$

Quark polarization

Nucleon polarization

	U
U	F_{11}
T_x	$\frac{i}{M}\left[k_y F_{12} + \Delta_y(F_{13} - \frac{1}{2}F_{11})\right]$
T_y	$-\frac{i}{M}\left[k_x F_{12} + \Delta_x(F_{13} - \frac{1}{2}F_{11})\right]$
L	$\frac{i(\vec{k}_\perp \times \vec{\Delta}_\perp)_z}{M^2} F_{14}$

$t = 0$ $\int \mathrm{d}^2 k_\perp$

TMDs

	U
U	f_1
T_x	$\frac{k_y}{M} f_{1T}^\perp$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$
L	

GPDs $\xi = 0$

	U
U	H
T_x	$i\,\frac{\Delta_y}{2M}\,E$
T_y	$-i\,\frac{\Delta_x}{2M}\,E$
L	

Vector quark GTMDs

$$W^{[\gamma^+]}_{\Lambda'\Lambda} = \frac{1}{2M}\,\overline{u}'\left[F_{11} - \frac{i\sigma^{k_\perp +}}{P^+}F_{12} - \frac{i\sigma^{\Delta_\perp +}}{P^+}F_{13} + \frac{i\sigma^{k_\perp \Delta_\perp}}{M^2}F_{14}\right]u$$

[Meissner, Metz, Schlegel (2009)]

Similarly for gluons **[C.L., Pasquini (2013)]**

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But …

Recent claim that F_{14} should not appear !

[Liuti *et al.* (2013)]
[Courtoy *et al.* (2013)]

Leading-twist parametrization

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Quark polarization / **Nucleon polarization**

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$t = 0$ → TMDs

$\int \mathrm{d}^2 k_\perp$ → GPDs $\xi = 0$

TMDs

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GPDs

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[Liuti *et al.* (2013)]
[Courtoy *et al.* (2013)]

But …

Arguments are not valid !

[Kanazawa, C.L., Metz, Pasquini, Schlegel (in preparation)]

Argument 1 : Parity

Lorentz structure associated with *F*$_{14}$

$$\overline{u}'\, i\sigma^{k_\perp \Delta_\perp} u \propto \vec{S}_L \cdot (\vec{k}_\perp \times \vec{\Delta}_\perp)$$
$$\propto \Lambda\, i(\vec{k}_\perp \times \vec{\Delta}_\perp)_z$$

P-odd

[Liuti *et al.* (2013)]
[Courtoy *et al.* (2013)]

P-even

[Meissner, Metz, Schlegel (2009)]
[Hatta (2012)]
[C.L., Pasquini (2013)]
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P-even

[Meissner, Metz, Schlegel (2009)]
[Hatta (2012)]
[C.L., Pasquini (2013)]
[Kanazawa *et al.* (in preparation)]

1) $\overline{u}'\, i\sigma^{k_\perp \Delta_\perp} u$ **has no** $\epsilon_{\mu\nu\rho\sigma}$ **or** γ_5 $\Longrightarrow$ **naturally P-even !**

2) $\boxed{\boxed{\vec{S}_L} \cdot \boxed{(\vec{k}_\perp \times \vec{\Delta}_\perp)}}$ **P-odd (pseudo-vector)** **P-even (scalar)**

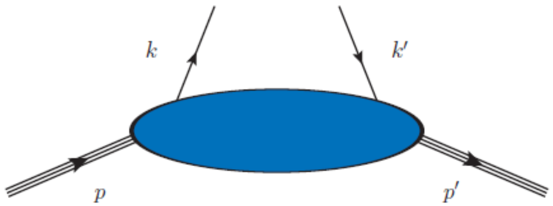
3) **Light-front parity :** $\Lambda \overset{\mathsf{P_{LF}}}{\mapsto} -\Lambda$ **&** $i(\vec{k}_\perp \times \vec{\Delta}_\perp)_z = \frac{1}{2}(k_L\Delta_R - k_R\Delta_L)$

$a_{R,L} \overset{\mathsf{P_{LF}}}{\mapsto} -a_{L,R}$ $\qquad \overset{\mathsf{P_{LF}}}{\mapsto} -i(\vec{k}_\perp \times \vec{\Delta}_\perp)_z$

Argument 2 : Two-body scattering picture

Quark DGLAP region

$x > \xi \geq 0$



GTMD correlator

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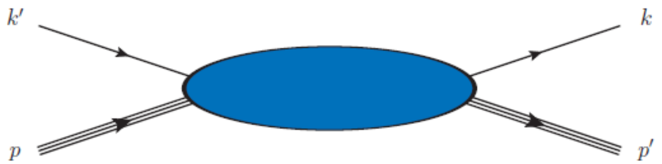

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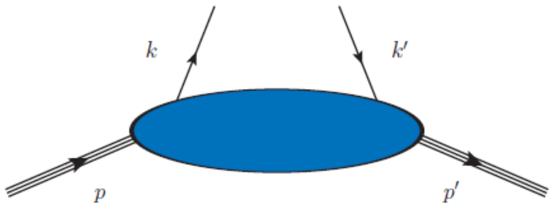

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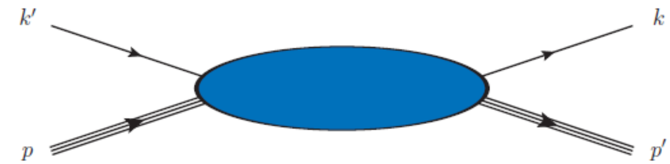
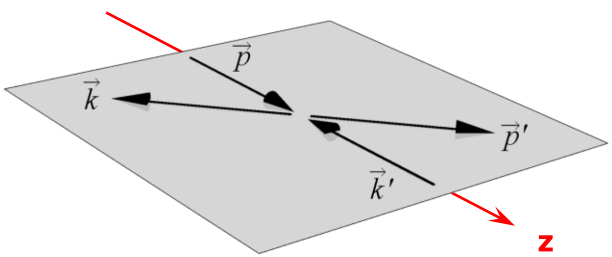


Two-body scattering

Two-body CM frame



$$i(\vec{k}_\perp \times \vec{\Delta}_\perp)_z = 0$$

Missing independent direction
provided by higher twist (extra gluon)

[Liuti *et al.* (2013)]
[Courtoy *et al.* (2013)]

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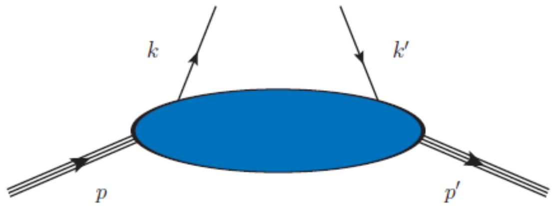

GTMD correlator

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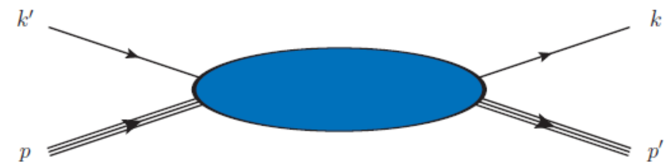
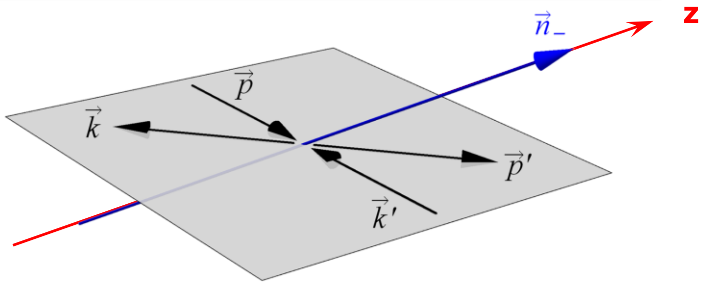


Two-body scattering

Two-parton correlator

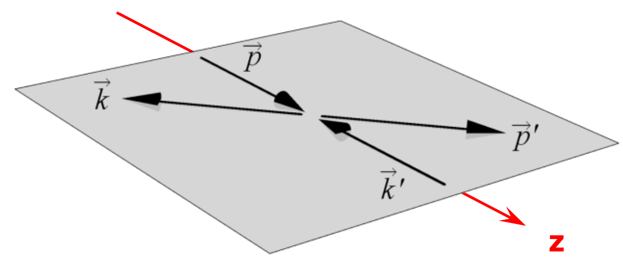


$$i(\vec{k}_\perp \times \vec{\Delta}_\perp)_z \neq 0$$

Independent direction provided by light-front vector (canonical twist)

[Kanazawa *et al.* (in preparation)]

Two-body CM frame



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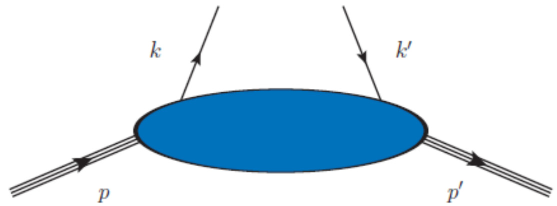
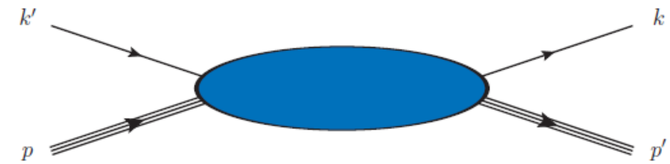


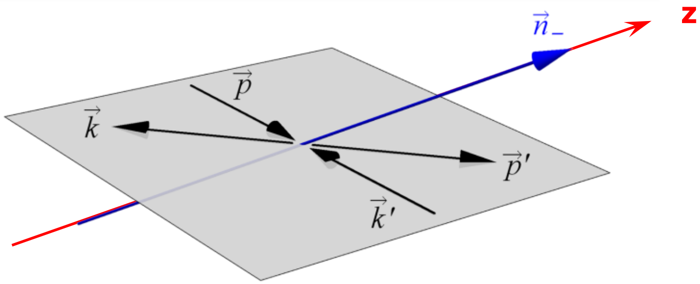
GTMD correlator

⚠ ~



Two-body scattering

Two-parton correlator



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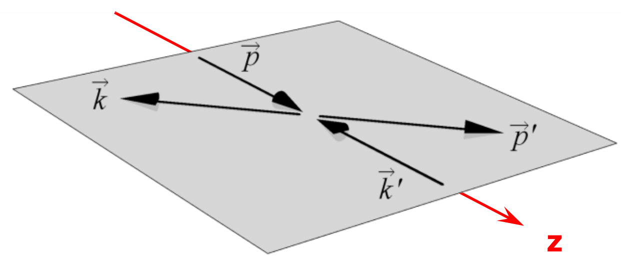
➡ **4 chiral-odd GPDs** **[Diehl (2001)]** **[Diehl (2003)]**

Two-body CM frame



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[Liuti *et al.* (2013)]
[Courtoy *et al.* (2013)]

➡ **2 chiral-odd GPDs** **[Hoodbhoy, Ji (1998)]**

Argument 3 : Explicit model results

F_{14} non-zero in SDM, LFCQM and LFχQSM

[Meissner, Metz, Schelgel (2009)]
[C.L., Pasquini (2011)]
[C.L., Pasquini, Xiong, Yuan (2012)]

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But …

Claim : "These non-zero results are coming about from the kinematics or from effective higher twist components arising from quarks' confinement"

[Liuti *et al.* (2013)]

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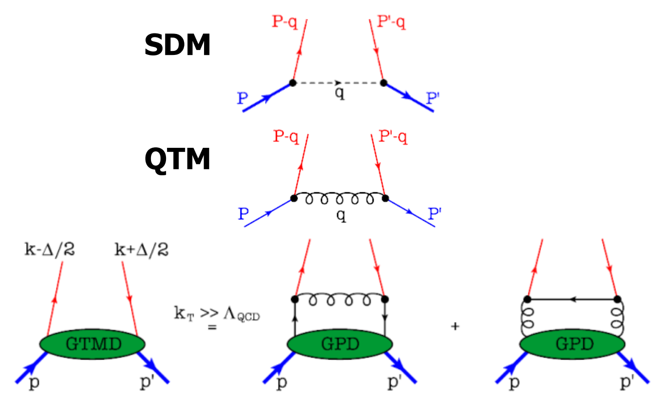
***Kinematics* ?** **GTMDs depend on intrinsic variables only !**

***Confinement* ?** **F_{14} non-zero in perturbative models !**

$$
\begin{aligned}
F^q_{14} &= -\frac{\lambda^2}{2(2\pi)^3}\,\frac{(1-x)^2M^2}{[\vec k'^2_\perp+\mathcal{M}^2(x)][\vec k^2_\perp+\mathcal{M}^2(x)]}\\
&= \frac{C_F\alpha_S}{2\pi^2}\,\frac{(1-x^2)m_q^2}{[\vec k'^2_\perp+(1-x)^2m_q^2][\vec k^2_\perp+(1-x)^2m_q^2]}\\
&= \frac{\alpha_S}{2\pi^2}\int_x^1\frac{\mathrm{d}z}{z}\,\frac{C_F\tilde H^q(\frac{x}{z},0,\vec\Delta^2_\perp)-T_R(1-z)^2\tilde H^g(\frac{x}{z},0,\vec\Delta^2_\perp)}{[\vec k'^2_\perp+z(1-z)\,\frac{\vec\Delta^2_\perp}{4}][\vec k^2_\perp+z(1-z)\,\frac{\vec\Delta^2_\perp}{4}]}
\end{aligned}
$$



[Kanazawa *et al.* (in preparation)]

Link with OAM

Active « parton » OAM

$$\boxed{\ell_z = -\int \mathrm{d}x\,\mathrm{d}^2k_\perp\,\frac{\vec{k}_\perp^2}{M^2}\,F_{14}(x,0,\vec{k}_\perp,\vec{0}_\perp)}$$

Model-independent relation !

[C.L., Pasquini (2011)]
[Hatta (2012)]
[C.L., Pasquini, Xiong, Yuan (2012)]
[Ji, Xiong, Yuan (2012)]
[C.L. (2013)]
[Kanazawa *et al.* (in preparation)]

Explicitly checked with :

- **LFCQM**
- **LFχQSM**
- **SDM**
- **QTM**
- **LFWF overlap representation**

$$\frac{i(\vec{k}_\perp\times\vec{\Delta}_\perp)_z}{M^2}\,F_{14} = \frac{1}{2(2\pi)^3}\sum_{\Lambda,\lambda,\mu}\Lambda\,\psi'^{\Lambda\dagger}_{\lambda,\mu}\psi^{\Lambda}_{\lambda,\mu}$$

$$l_z = \Lambda - \lambda - \mu$$

$$= -\frac{i(\vec{k}_\perp\times\vec{\Delta}_\perp)_z}{\vec{k}_\perp^2}\underbrace{\boxed{\frac{1}{2(2\pi)^3}\sum_{\Lambda,\lambda,\mu}\Lambda\,l_z\,|\psi^{\Lambda}_{\lambda,\mu}|^2}}_{\ell_z} + \mathcal{O}(\vec{\Delta}_\perp^2)$$

Conclusion :

Vanishing F_{14} ⟺ **Vanishing OAM**

Parity partner

Similarly with extra γ_5

GTMDs $\xi = 0$

$W^{[\gamma^+]}_{\Lambda'\Lambda}$ **Quark polarization** $W^{[\gamma^+\gamma_5]}_{\Lambda'\Lambda}$

Nucleon polarization

	U	L
U	F_{11}	$-\frac{i(\vec{k}_\perp \times \vec{\Delta}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M}\left[k_y F_{12} + \Delta_y(F_{13} - \frac{1}{2}F_{11})\right]$	$\frac{1}{M}\left[k_x G_{12} + \Delta_x G_{13} - \Delta_y \frac{(\vec{k}_\perp \times \vec{\Delta}_\perp)_z}{2M^2} G_{11}\right]$
T_y	$-\frac{i}{M}\left[k_x F_{12} + \Delta_x(F_{13} - \frac{1}{2}F_{11})\right]$	$\frac{1}{M}\left[k_y G_{12} + \Delta_y G_{13} + \Delta_x \frac{(\vec{k}_\perp \times \vec{\Delta}_\perp)_z}{2M^2} G_{11}\right]$
L	$\frac{i(\vec{k}_\perp \times \vec{\Delta}_\perp)_z}{M^2} F_{14}$	G_{14}

$t = 0$ $\int \mathrm{d}^2 k_\perp$

TMDs

	U	L
U	f_1	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		g_{1L}

GPDs $\xi = 0$

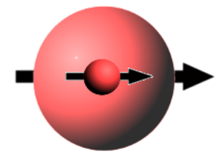
	U	L
U	H	
T_x	$i\,\frac{\Delta_y}{2M} E$	
T_y	$-i\,\frac{\Delta_x}{2M} E$	
L		$\tilde{H}$

Proton spin structure

« Quark spin »

$$\langle\langle S_z^q\rangle\rangle \sim \frac{1}{2(2\pi)^3}\sum_{\Lambda,\lambda,\mu}\Lambda\,\lambda\,|\psi^\Lambda_{\lambda,\mu}|^2 \sim \langle S_z^N S_z^q\rangle$$

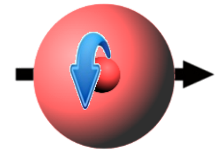
$$\overline{\psi}\gamma^+\gamma_5\psi$$



« Quark OAM »

$$\langle\langle L_z^q\rangle\rangle \sim \frac{1}{2(2\pi)^3}\sum_{\Lambda,\lambda,\mu}\Lambda\,l_z\,|\psi^\Lambda_{\lambda,\mu}|^2 \sim \langle S_z^N L_z^q\rangle$$

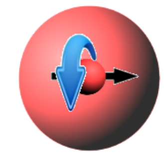
$$\overline{\psi}\gamma^+(\vec{r}_\perp\times i\vec{D}_\perp^{(\mathrm{pure})})_z\psi$$



Quark spin-orbit correlation

$$\langle\langle C_z^q\rangle\rangle \sim \frac{1}{2(2\pi)^3}\sum_{\Lambda,\lambda,\mu}\lambda\,l_z\,|\psi^\Lambda_{\lambda,\mu}|^2 \sim \langle S_z^q L_z^q\rangle$$

$$\overline{\psi}\gamma^+\gamma_5(\vec{r}_\perp\times i\vec{D}_\perp^{(\mathrm{pure})})_z\psi$$



[C.L., Pasquini (2011)]
[C.L. (2014)]

OAM *vs* spin-orbit

OAM

$$L_z = \int \mathrm{d}^3r \left[r^1 \langle\langle T^{+2}\rangle\rangle - r^2 \langle\langle T^{+1}\rangle\rangle\right]$$

Parametrization

$$\begin{aligned} T^{\mu\nu} &= \langle p', \Lambda' | \overline{\psi}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{D}^\nu \psi | p, \Lambda\rangle \\ &= \overline{u}' \left[\frac{P^{\{\mu}\gamma^{\nu\}}}{2} A + \frac{P^{\{\mu} i\sigma^{\nu\}\Delta}}{4M} B \right. \\ &\qquad \left. + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{M} C + M g^{\mu\nu} \bar{C} + \frac{P^{[\mu}\gamma^{\nu]}}{2} D \right] u \end{aligned}$$

Relations

$$L_z = \tfrac{1}{2}(A + B + D)$$ [Shore, White (2000)]

$$= \int \mathrm{d}x \, \tfrac{1}{2}[x(H+E) - \tilde{H}]$$ [Ji (1997)]

$$= -\int \mathrm{d}x \, x \, G_2$$ [Penttinen *et al.* (2000)]

$$= -\int \mathrm{d}x \, \mathrm{d}^2k_\perp \, \frac{\vec{k}_\perp^2}{M^2} F_{14}$$ [C.L., Pasquini (2011)]
[Hatta (2012)]

OAM *vs* spin-orbit

OAM

$$L_z = \int \mathrm{d}^3r\, \left[r^1\langle\langle T^{+2}\rangle\rangle - r^2\langle\langle T^{+1}\rangle\rangle\right]$$

Parametrization

$$\begin{aligned}T^{\mu\nu} &= \langle p',\Lambda'|\overline{\psi}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{D}^\nu\psi|p,\Lambda\rangle\\ &= \overline{u}'\left[\frac{P^{\{\mu}\gamma^{\nu\}}}{2}A + \frac{P^{\{\mu}i\sigma^{\nu\}\Delta}}{4M}B\right.\\ &\quad\left. + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}C + Mg^{\mu\nu}\bar{C} + \frac{P^{[\mu}\gamma^{\nu]}}{2}D\right]u\end{aligned}$$

Relations

$$L_z = \tfrac{1}{2}(A+B+D)$$ **[Shore, White (2000)]**

$$= \int \mathrm{d}x\, \tfrac{1}{2}[x(H+E) - \tilde{H}]$$ **[Ji (1997)]**

$$= -\int \mathrm{d}x\, x\, G_2$$ **[Penttinen *et al.* (2000)]**

$$= -\int \mathrm{d}x\, \mathrm{d}^2k_\perp\, \frac{\vec{k}_\perp^2}{M^2}\, F_{14}$$ **[C.L., Pasquini (2011)]**
[Hatta (2012)]

Spin-orbit

$$C_z = \int \mathrm{d}^3r\, \left[r^1\langle\langle T_5^{+2}\rangle\rangle - r^2\langle\langle T_5^{+1}\rangle\rangle\right]$$

Parametrization

$$\begin{aligned}T_5^{\mu\nu} &= \langle p',\Lambda'|\overline{\psi}\gamma^\mu\gamma_5 \tfrac{i}{2}\overleftrightarrow{D}^\nu\psi|p,\Lambda\rangle\\ &= \overline{u}'\left[\frac{P^{\{\mu}\gamma^{\nu\}}\gamma_5}{2}\tilde{A} + \frac{P^{\{\mu}\Delta^{\nu\}}\gamma_5}{4M}\tilde{B}\right.\\ &\quad\left. + \frac{P^{[\mu}\gamma^{\nu]}\gamma_5}{2}\tilde{C} + \frac{P^{[\mu}\Delta^{\nu]}\gamma_5}{4M}\tilde{D} + Mi\sigma^{\mu\nu}\gamma_5\tilde{F}\right]u\end{aligned}$$

Relations

$$\left.\begin{aligned}C_z &= \tfrac{1}{2}(\tilde{A}+\tilde{C})\\ &= \int \mathrm{d}x\, \tfrac{1}{2}(x\tilde{H} - H) + \mathcal{O}(\tfrac{m_q}{M})\\ &= -\int \mathrm{d}x\, x(\tilde{G}_2 + 2\tilde{G}_4)\end{aligned}\right\}$$ **[C.L. (2014)]**

$$= \int \mathrm{d}x\, \mathrm{d}^2k_\perp\, \frac{\vec{k}_\perp^2}{M^2}\, G_{11}$$ **[C.L., Pasquini (2011)]**
[C.L. (2014)]

Some figures

Valence number $F_1^u(0) = 2$ $F_1^d(0) = 1$

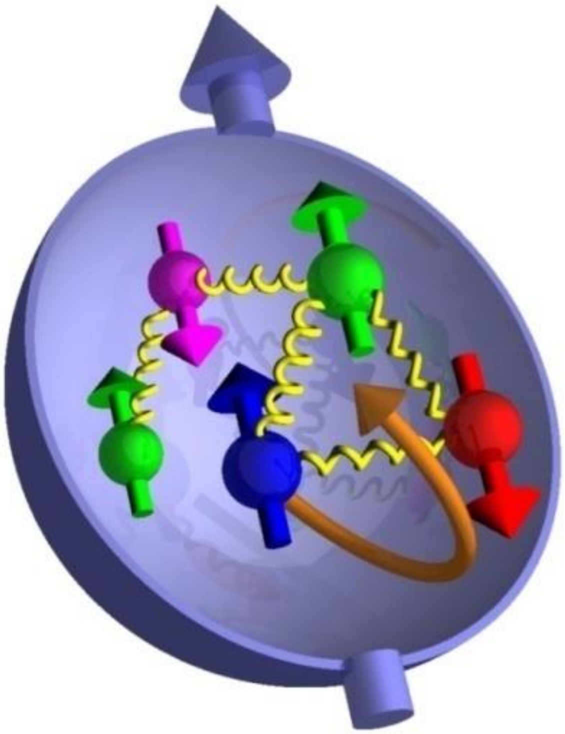
	$\int \mathrm{d}x\, x\tilde{H}^u$	$\int \mathrm{d}x\, x\tilde{H}^d$	C_z^u	C_z^d
NQM	4/9	-1/9	-7/9	-5/9
LFCQM	0.34	-0.09	-0.83	-0.54
LFχQSM	0.39	-0.10	-0.80	-0.55
LSS2010	0.19	-0.06	-0.90	-0.53

Conclusion :

Spin and kinetic OAM of valence quarks are anti-correlated !

[C.L. (2014)]

Conclusions



- Phase-space distributions are intuitive tools (observables ?)
- Contrary to recent claims, F_{14} does appear at (canonical) twist-2
- Valence quark spin and kinetic OAM are anti-correlated !