An Avenue for Extracting Generalized Parton Distributions from Experiment

Simonetta Liuti
University of Virginia

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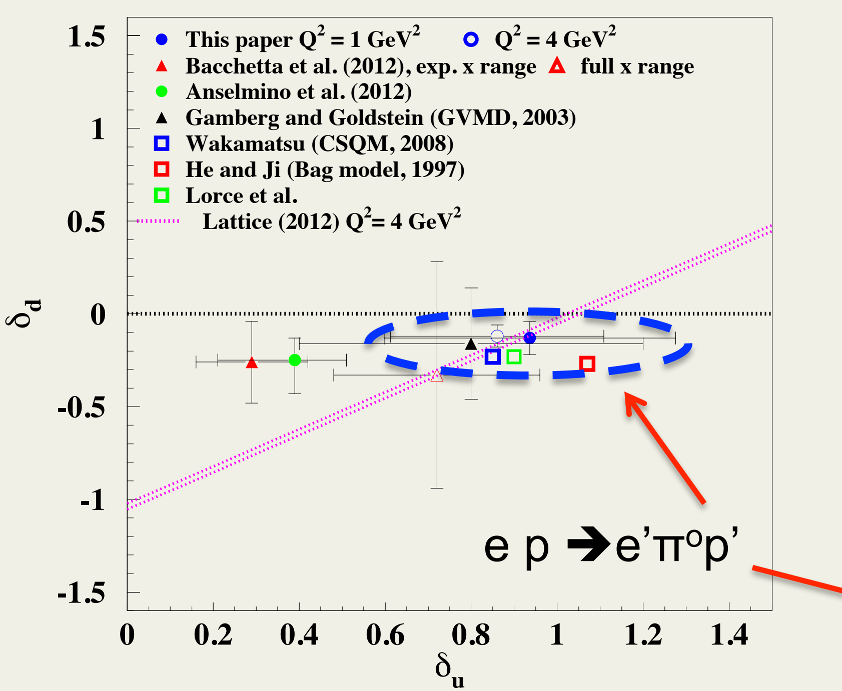
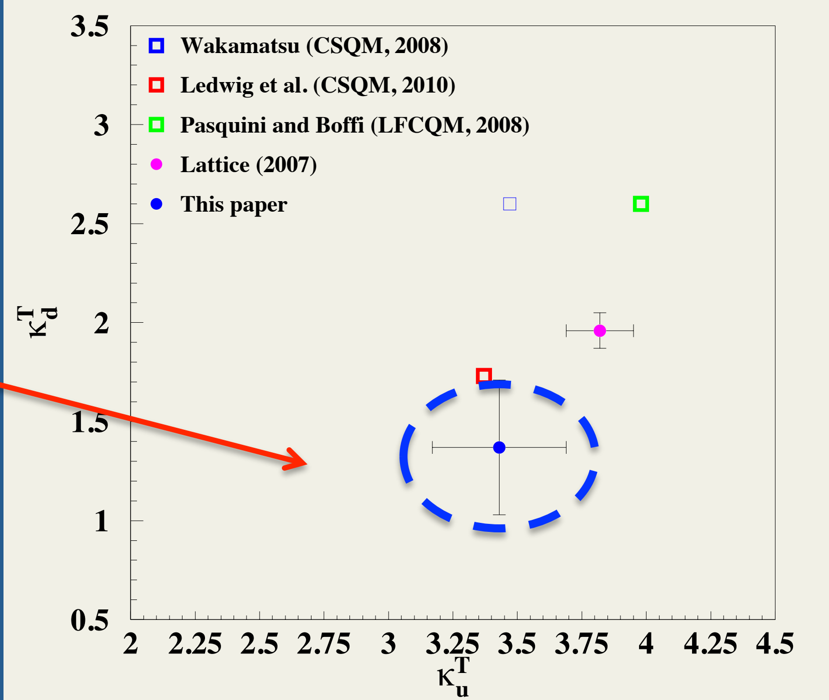
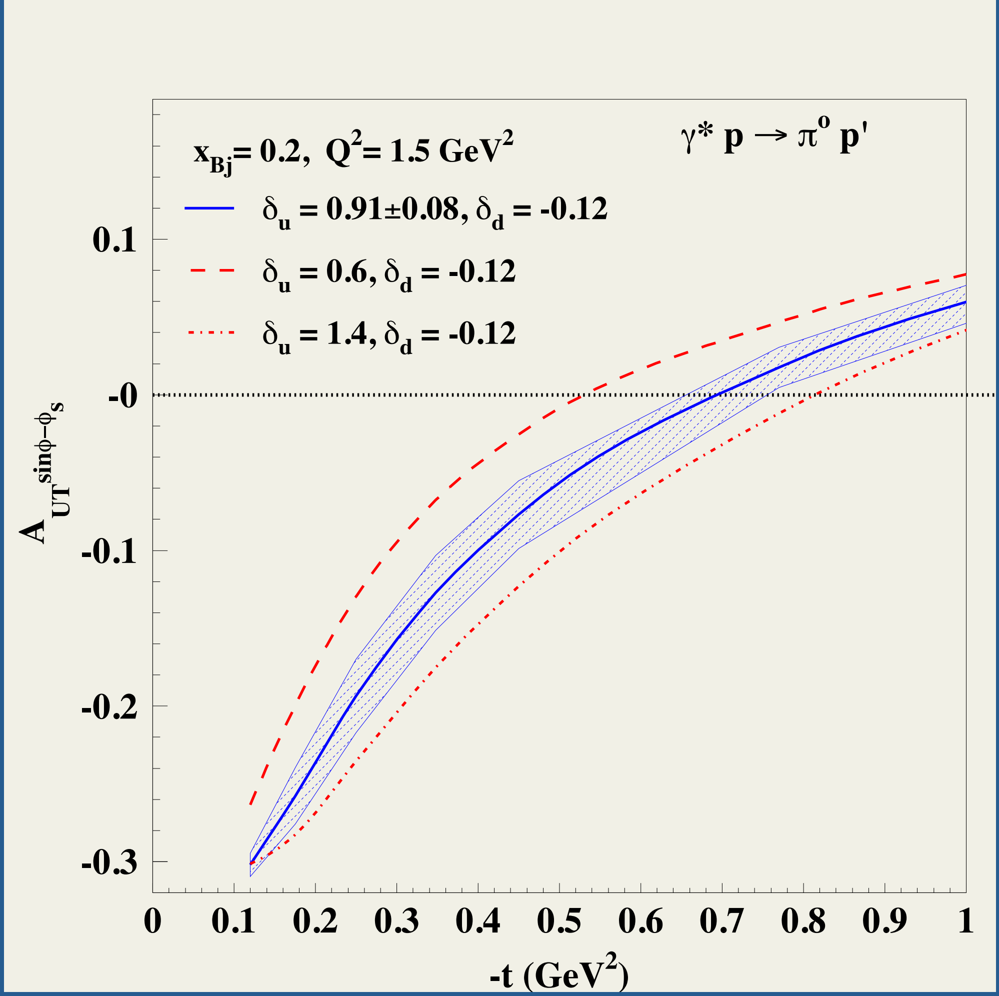
Motivation I

Tensor charge



Transv. anomalous magnetic moment

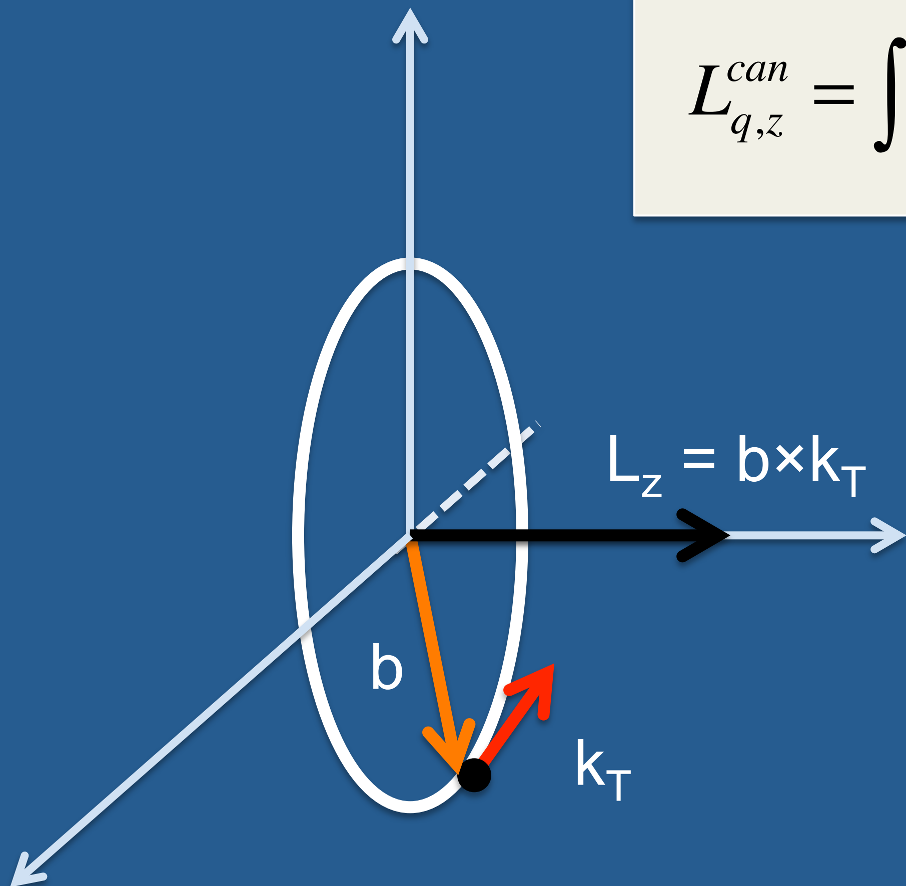




x_{Bj}= 0.2, Q^2= 1.5 GeV^2
$\gamma^* p \rightarrow \pi^o p'$
δ_u = 0.91±0.08, δ_d = -0.12
δ_u = 0.6, δ_d = -0.12
δ_u = 1.4, δ_d = -0.12
$A_{UT}^{\sin\phi-\phi_s}$
0.1
-0
-0.1
-0.2
-0.3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
-t (GeV^2)

Motivation II

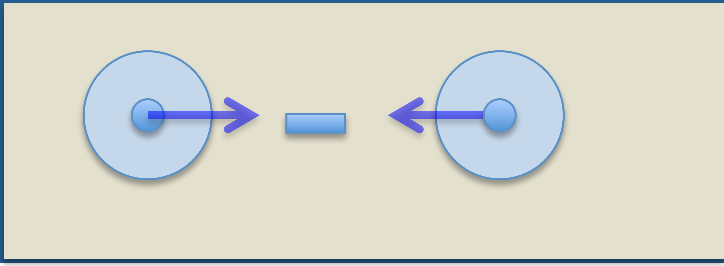
OAM: The "naive" picture



$$L_{q,z}^{can} = \int d^2b\, d^2k_T \left(\vec{b} \times \vec{k}_T\right)_z W_{LC}(x, \vec{b}, \vec{k}_T)$$

Leading twist Wigner distribution with gauge link in LC direction

In arXiv:1310.5157 (PLB 2/2014) we asked what is the spin configuration corresponding to quark OAM in QCD?



Analysis done using twist 3 GTMDs

$$-\frac{4}{P^+}\left[\frac{\bar{\mathbf{k}}_T\cdot\mathbf{\Delta}_T}{\Delta_T}F_{27}+\Delta_T F_{28}-\left(\frac{\bar{\mathbf{k}}_T\cdot\mathbf{\Delta}_T}{\Delta_T}G_{27}+\Delta_T G_{28}\right)\right] = A^{tw3}_{++,++}+A^{tw3}_{+-,+-}-A^{tw3}_{-+,-+}-A^{tw3}_{--,--}$$

$\downarrow G_2$ $\qquad$ $\downarrow \tilde{G}_2$

$G_2 \rightarrow$ $\sigma_{ij}\Delta^j \Rightarrow \vec{S}_L \times \vec{\Delta}$

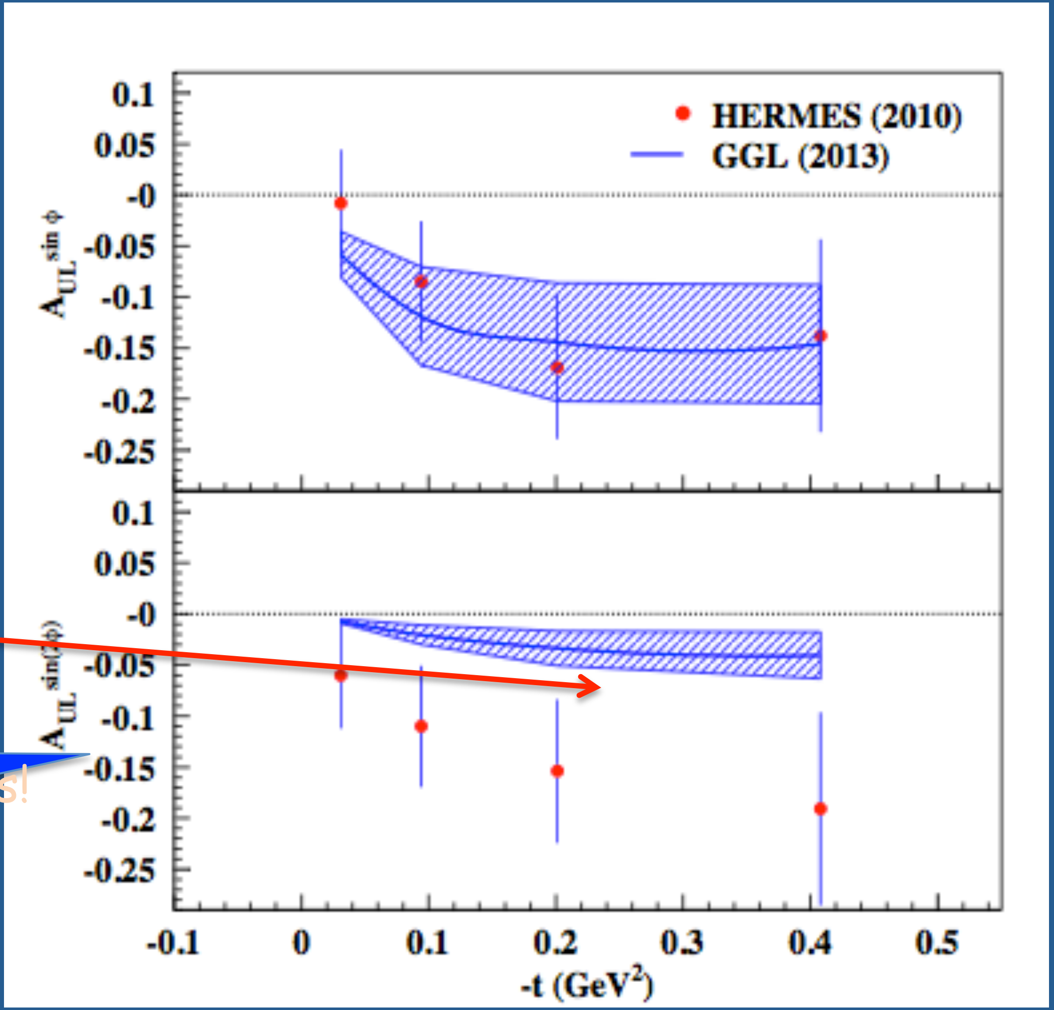
This is no longer Parity odd but it has a transverse component, OAM is associated with a transverse spin component in the proton

DVCS on a longitudinally polarized target

$$A_{UL} = \frac{\sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\epsilon \sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T}}$$

sin 2φ term is tw 3!

WW, small ξ



Jlab data analysis in progress!
Avakian, Pisano

Outline

1) State of the art of global fit for GPDs

2) Angular Momentum and OAM

3) Chiral odd sector

4) Self Organizing Maps as a future tool for GPDs/TMDs analyses

Collaborations

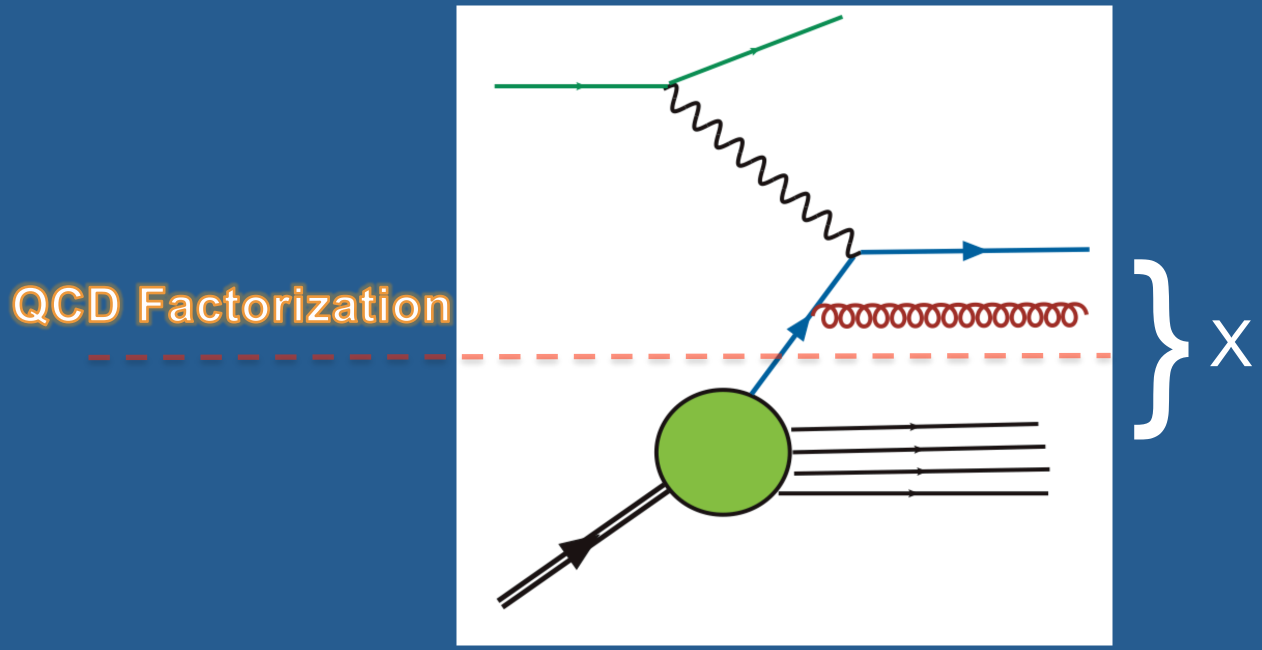
GPDs Fit
Aurore Courtoy, Gary Goldstein, Osvaldo Gonzalez Hernandez, S.L., Silvia Pisano, Jon Poage, Abha Rajan

Angular Momentum/OAM
Aurore Courtoy, Gary Goldstein, Osvaldo Gonzalez Hernandez, S.L., Abha Rajan

Extension to Chiral Odd Sector
Gary Goldstein, Osvaldo Gonzalez Hernandez, S.L.

Self Organizing Maps Fit
Evan Askanazi, Katherine Holcomb, S.L.

DIS



Observable

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha}{2xQ^4}\left[\left(1+(1-y)^2\right)F_2(x,Q^2) - y^2 F_L(x,Q^2)\right]$$

PDFs

$$F_2(x,Q^2) = x\sum_q e_q^2\, q(x,Q^2)$$

Global Analyses Basic Points

- Select experimental data sets
- Factorization theorems: choose a parametric form for PDFs at an initial Q_o^2

$$q(x,Q_o^2) = A_q\, x^{\alpha_q} (1-x)^{\beta_q} F(x, c_q, d_q, \ldots)$$

- $q(x,Q_o^2)$ is the input for QCD evolution equations (choose the factorization scheme), solve and obtain $q(x,Q_{exp}^2)$
- Construct observable

$$F_2(x,Q_{\exp}^2) = x \sum_q e_q^2\, q(x,Q_{\exp}^2)$$

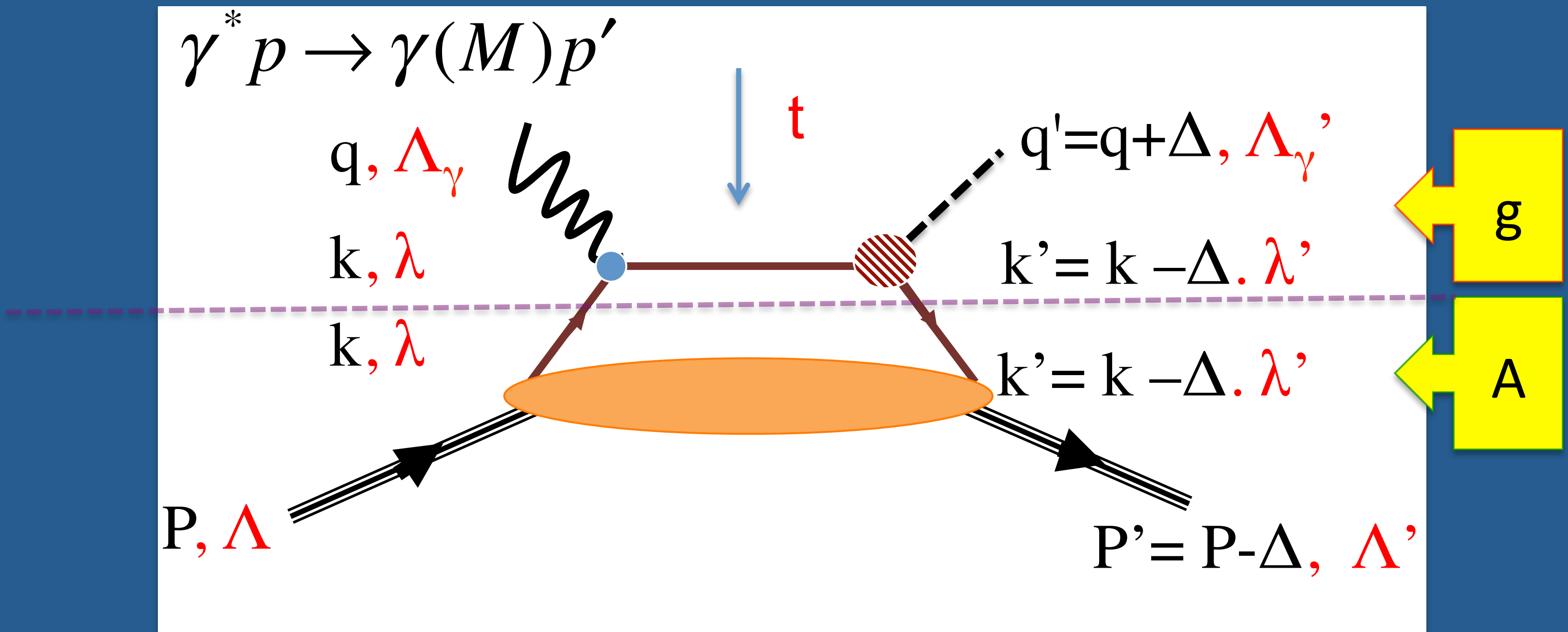
- Calculate χ^2
- Select an algorithm to minimize χ^2 (this fixes the initial parameters)

Deeply Virtual Exclusive Processes

↓

GPDs

Factorization in exclusive processes (DVCS, DVMP…)



Convolution of "hard part" with quark-proton amplitudes

$$f_{\Lambda_\gamma,\Lambda;\Lambda'_\gamma,\Lambda'} = \sum_{\lambda,\lambda'} g_{\lambda,\lambda'}^{\Lambda_\gamma,\Lambda'_{\gamma(M)}}(x,k_T,\zeta,t;Q^2) \otimes A_{\Lambda',\lambda';\Lambda,\lambda}(x,k_T,\zeta,t),$$

Observables

Chiral Even

$$A_{\Lambda'\pm,\Lambda\pm} \Leftrightarrow H, E, \tilde{H}, \tilde{E}$$

Chiral Odd

$$A_{\Lambda'\pm,\Lambda\mp} \Leftrightarrow H_T, E_T, \tilde{H}_T, \tilde{E}_T$$

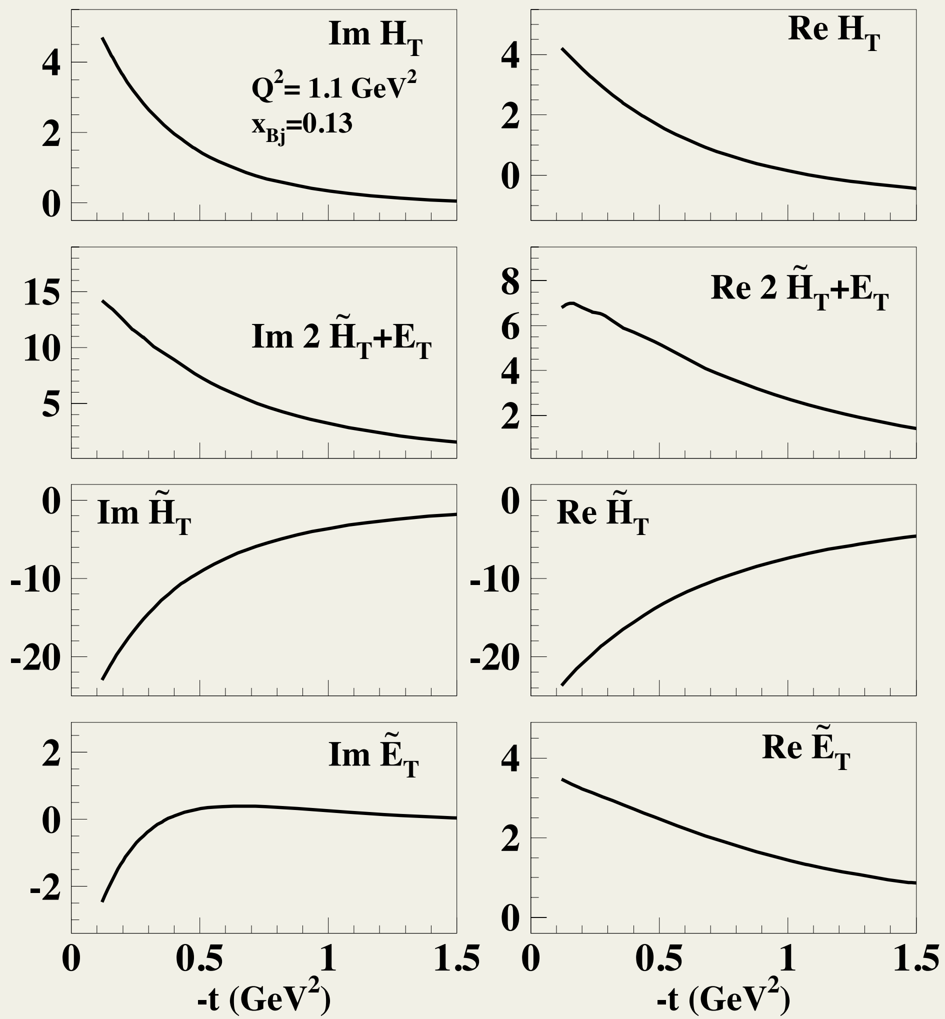
Compton Form Factors

$$\mathcal{H}(\xi,t;Q^2) = \int dx \left[\frac{1}{x-\xi-i\varepsilon} \mp \frac{1}{x+\xi-i\varepsilon} \right] H(x,\xi,t;Q^2)$$

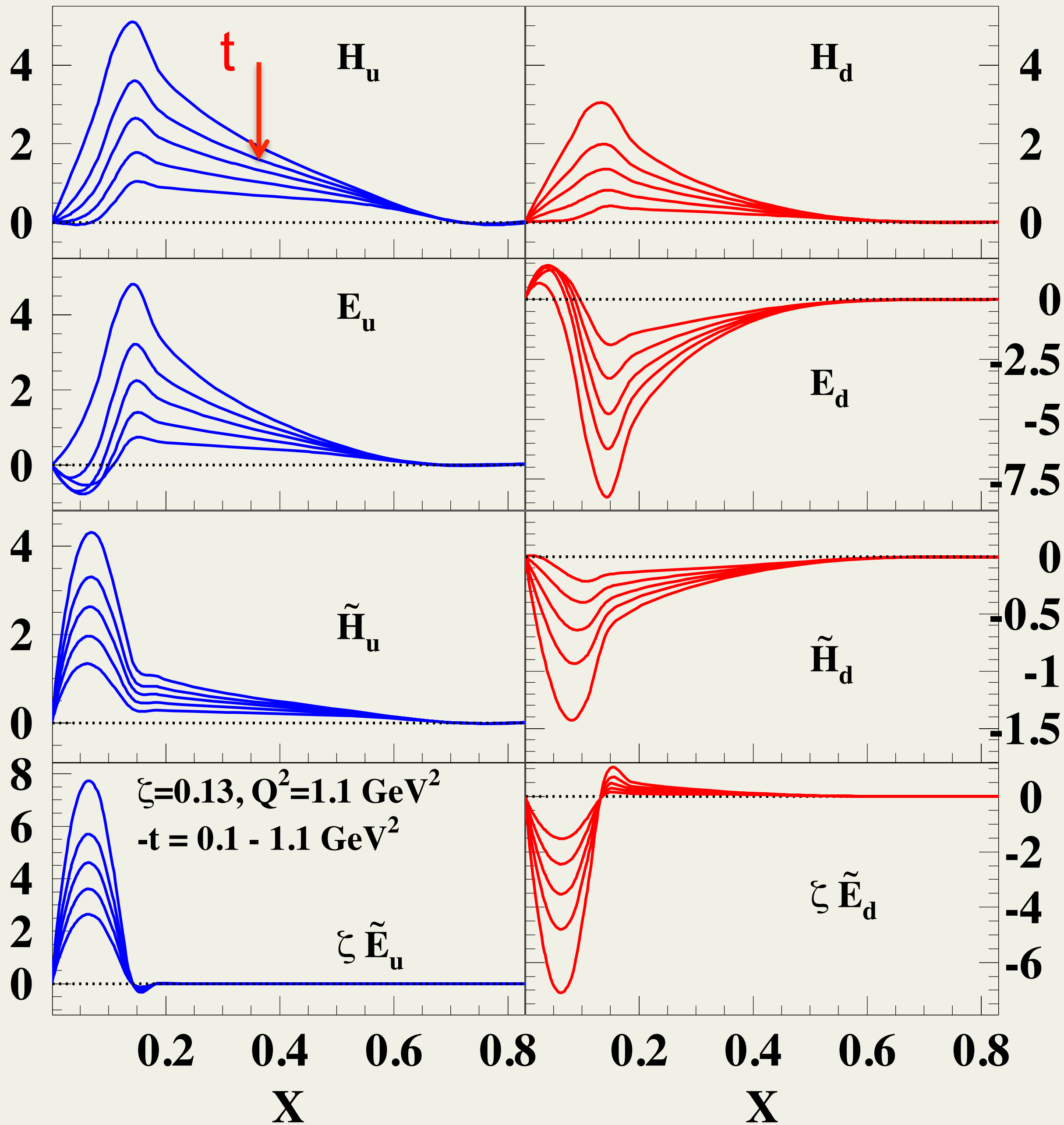
$$\rightarrow \left(P.V. \int dx \frac{H(x,\xi,t;Q^2)}{x-\xi} + i\pi H(\xi,\xi,t;Q^2) \right) \mp (symm.term)$$

Re $\mathcal{H}$ Im $\mathcal{H}$

Compton form factors



Chiral even GPDs (u and d valence only)



$$\frac{d^4\sigma}{dx_{Bj}dyd\phi dt} = \Gamma\Big\{\Big[F_{UU,T} + \epsilon F_{UU,L} + \epsilon\cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{2\epsilon(\epsilon+1)}\cos\phi F_{UU}^{\cos\phi} + h\sqrt{2\epsilon(1-\epsilon)}\,\sin\phi F_{LU}^{\sin\phi}\Big]$$

$$+\ S_{||}\Big[\sqrt{2\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi} + \epsilon\sin 2\phi F_{UL}^{\sin 2\phi} + h\Big(\sqrt{1-\epsilon^2}\,F_{LL} + \sqrt{2\epsilon(1-\epsilon)}\,\cos\phi\,F_{LL}^{\cos\phi}\Big)\Big]$$

$$+\ S_{\perp}\Big[\sin(\phi-\phi_S)\Big(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)}\Big) + \epsilon\Big(\sin(\phi+\phi_S)F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi-\phi_S)F_{UT}^{\sin(3\phi-\phi_S)}\Big)$$

$$+\ \sqrt{2\epsilon(1+\epsilon)}\Big(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\Big)\Big]$$

$$+\ S_{\perp}h\Big[\sqrt{1-\epsilon^2}\cos(\phi-\phi_S)F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)}\Big(\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S)F_{LT}^{\cos(2\phi-\phi_S)}\Big)\Big]\Big\}$$

Example: e p → e'π^{o}p'

GPDs
in helicity
amplitudes

$$F_{UU,T} = \mathcal{N}\,[|\,f_{10}^{++}\,|^2 + |\,f_{10}^{+-}\,|^2 + |\,f_{10}^{-+}\,|^2 + |\,f_{10}^{--}\,|^2]$$

$$F_{UU,L} = \mathcal{N}\,[|\,f_{00}^{++}\,|^2 + |\,f_{00}^{+-}\,|^2]$$

$$F_{UU}^{\cos 2\phi} = -\mathcal{N}\,2\Re e\,[(f_{10}^{++})^*(f_{10}^{--}) - (f_{10}^{+-})^*(f_{10}^{-+})]$$

$$F_{UU}^{\cos\phi} = -\mathcal{N}\Re e[(f_{00}^{+-})^*(f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} - f_{10}^{--})]$$

$$F_{LU}^{\sin\phi} = \mathcal{N}\Im m[(f_{00}^{+-})^*(f_{10}^{+-} + f_{10}^{-+}) + (f_{00}^{++})^*(f_{10}^{++} - f_{10}^{--})]$$

How do we perform a global fit

-- given the enhanced complexity –

how do we choose the “initial parametrization”?

Our method: Recursive fit

Advantage: control over the number of parameters to be fitted at different stages so that it can be optimized

Functional form:

From DIS

$$q(x,Q_o^2) = A_q\, x^{-\alpha_q}(1-x)^{\beta_q} F(x,c_q,d_q,...)$$

to DVCS, DVMP

$$H_q(x,\xi,t;Q_o^2) = N_q\, x^{-\left[\alpha_q+\alpha'_q(1-x)^p t\right]} G^{a_1 a_2 a_3 ..}(x,\xi,t)$$

$$a_1 = m_q,\ a_2 = M_X^q,\ a_3 = M_\Lambda^q, ...$$

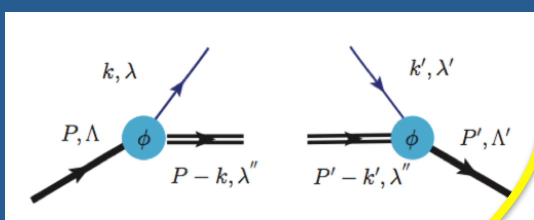
"Flexible" parameterization based on the reggeized quark-diquark model.

First step: DIS cross section

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha}{2xQ^4}\left[\left(1+(1-y)^2\right)F_2(x,Q^2) - y^2F_L(x,Q^2)\right]$$

PDFs from same helicity amps. based analysis with reggeized diquark model

$data\ F_2^{p,d},\ G_1,\ldots$



Q^2 evolution

LC functions → Helicity amps. → Structure fcns. →

$\phi_{\Lambda\lambda}^{a_1,a_2,a_3,\ldots}(k,P)$ → $A_{\Lambda\pm\Lambda\pm}^{S=0,S=1}$ → $f_1^{S=0,S=1},\ g_1^{S=0,S=1}$ → $f_1^{u,d},\ g_1^{u,d}$

SU(4) → flavor

Now GPDs...

✓ $H_q(x,0,0;Q^2) = f_1^q(x,Q^2)$, …

Total number of parameters = N

fix $n_1 < N$ parameters

DIS data

$F_2^{p,d}, G_1, \ldots$

Q^2 evol

$t=\xi=0$

$\phi_{\Lambda\lambda}^{a_1,a_2,a_3,\ldots}(k,P)$ → $A_{\Lambda\pm\Lambda\pm}^{S=0,S=1}$ → $f_1^{S=0,S=1}, g_1^{S=0,S=1}$ → $f_1^{u,d}, g_1^{u,d}$

✓ Switch on t: $H_q(x,0,t;Q^2)$

fix n_2 parameters $n_1 + n_2 < N$

form factor data

$F_1(t), F_2(t), G_A, G_P$

Q^2 evol

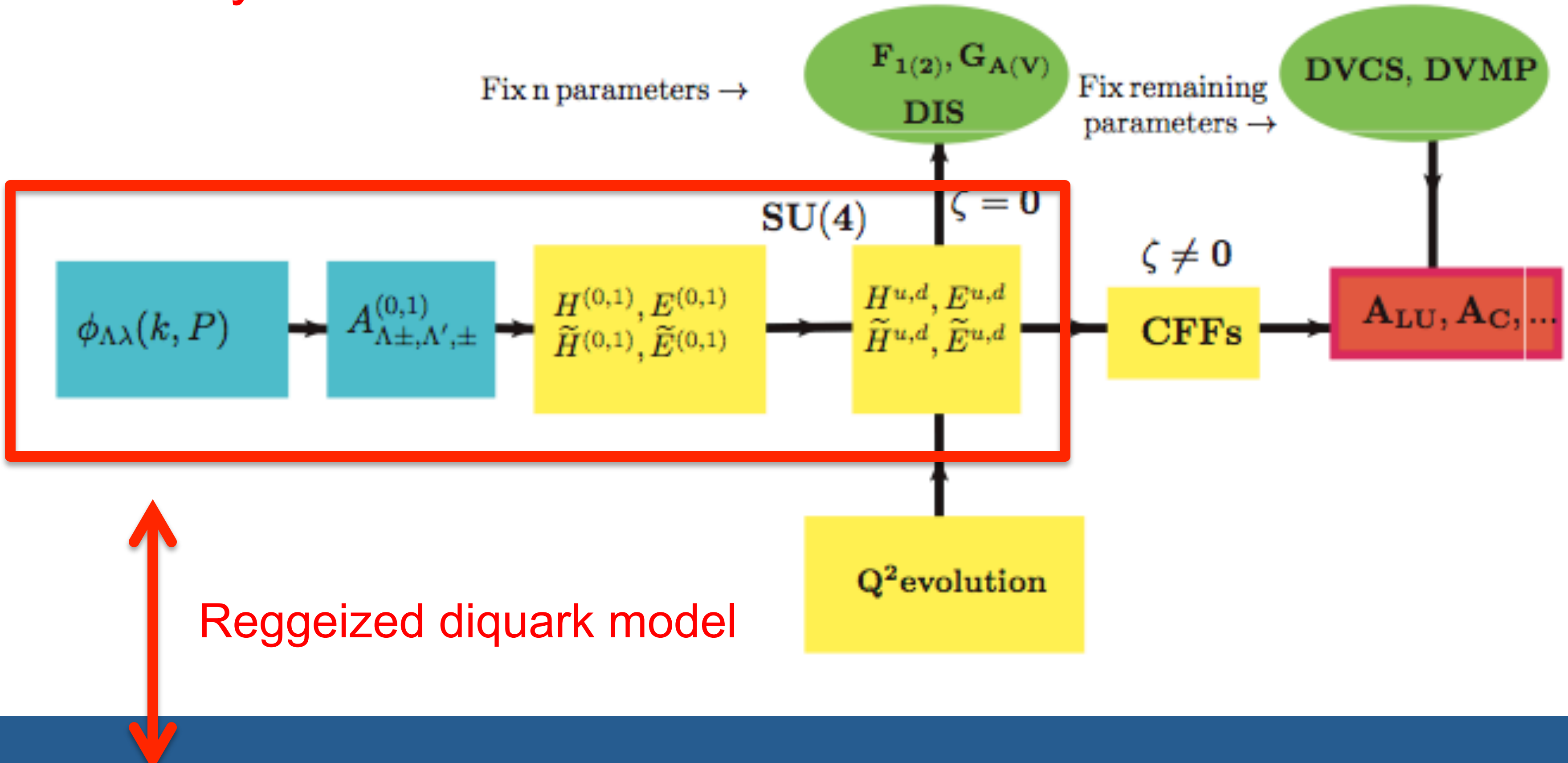
✓ Switch on ξ: $H_q(x,\xi,t;Q^2)$

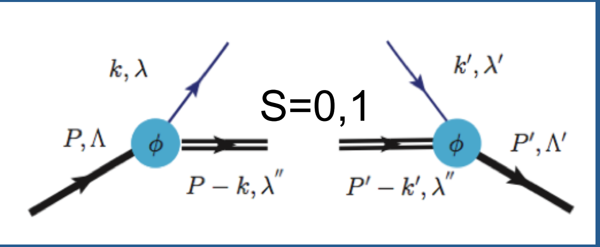
fix remaining $N-(n_1+n_2)$ parameters

DVCS data

$A_{UL}(\xi,t), A_{LU}(\xi,t), A_{LL}(\xi,t), \ldots$

Summary so far

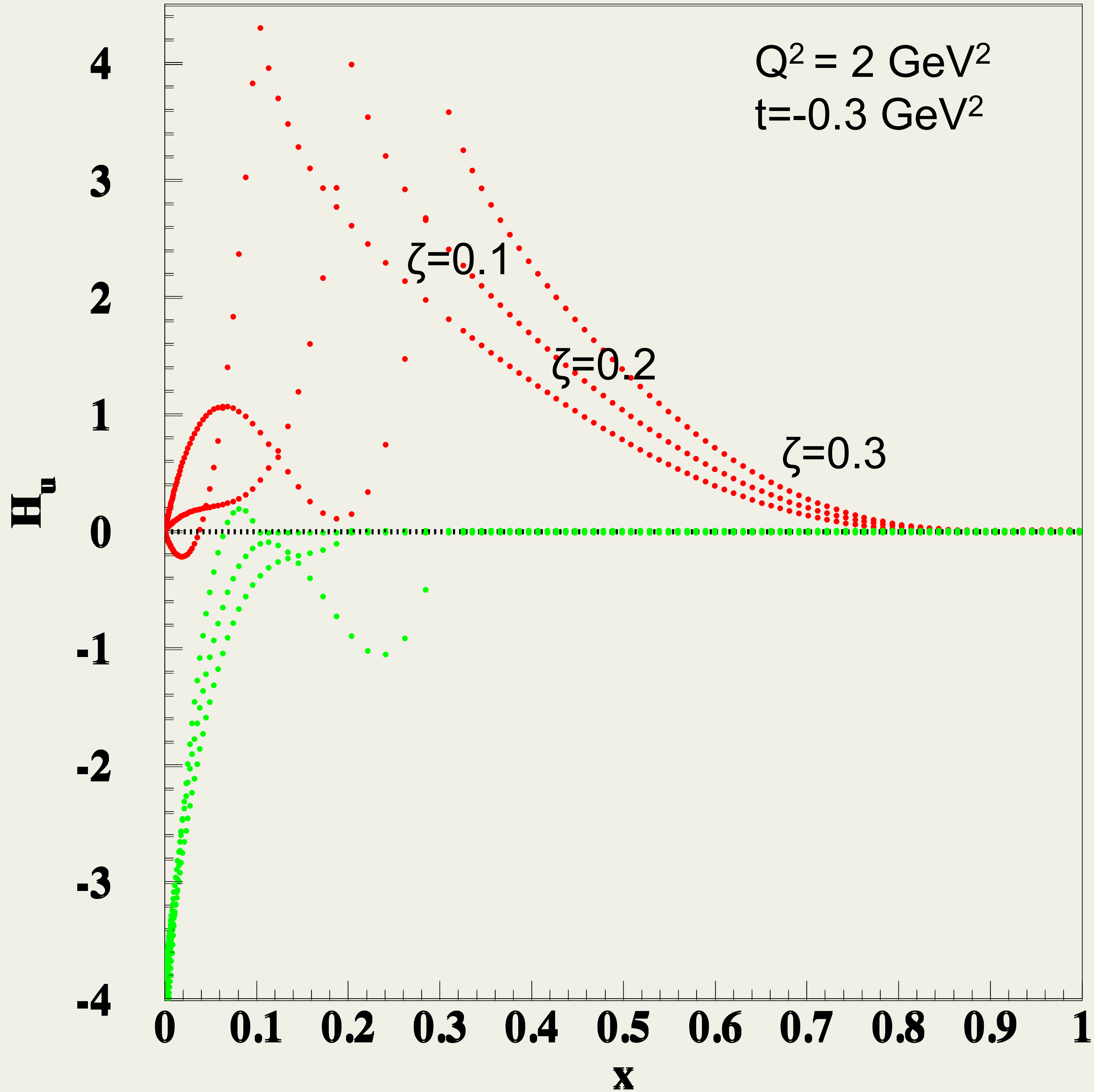




We asked the question: "What is the minimal number of parameters necessary to fit X, ξ and t ?" It can be addressed with Recursive Fit

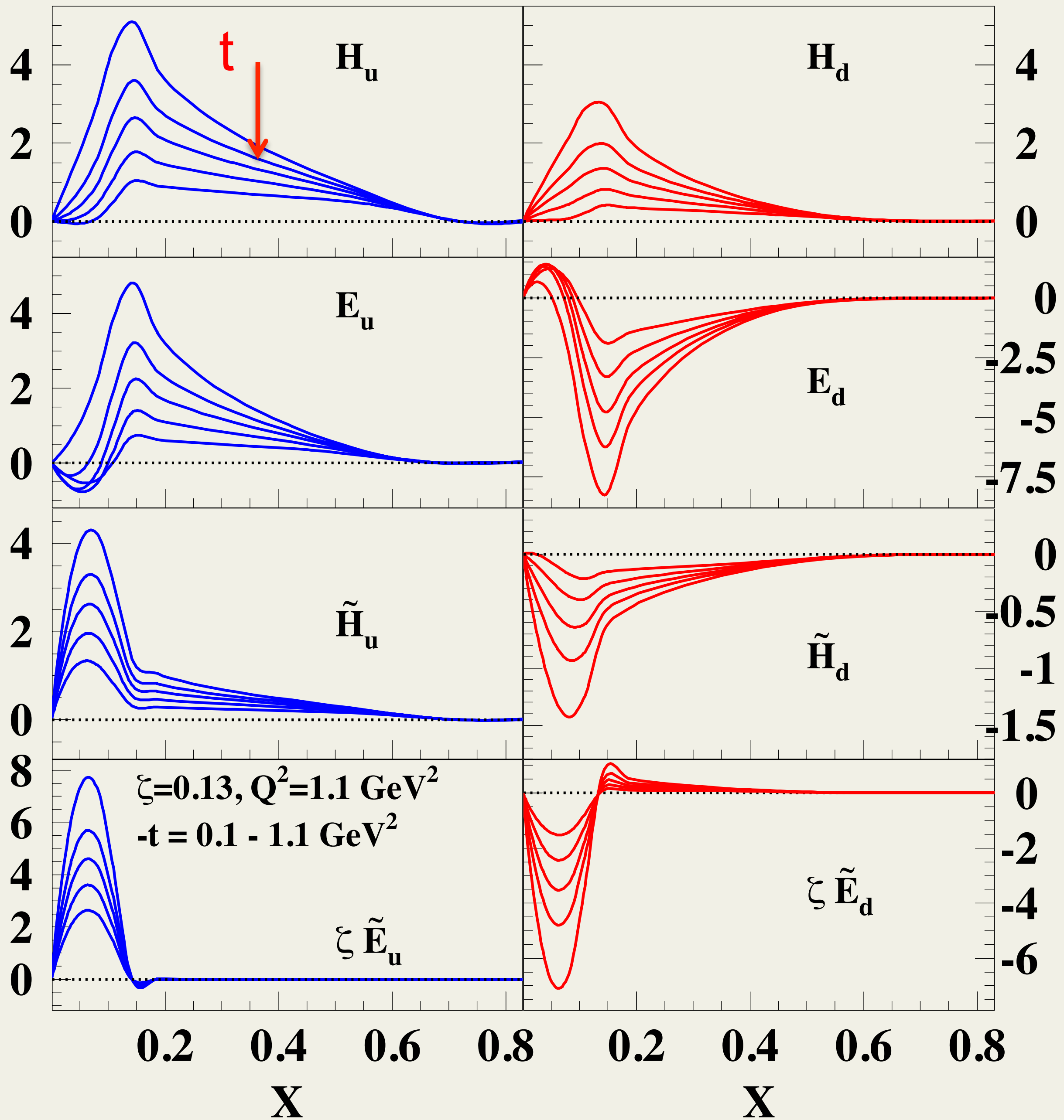
Parameters	H	E	$\widetilde{H}$	$\widetilde{E}$
m_u (GeV)	0.420	0.420	2.624	2.624
M_X^u (GeV)	0.604	0.604	0.474	0.474
M_Λ^u (GeV)	1.018	1.018	0.971	0.971
α_u	0.210	0.210	0.219	0.219
α'_u	1.814 ± 0.022	2.835 ± 0.051	1.543 ± 0.296	5.130 ± 0.101
p_u	0.449 ± 0.017	0.969 ± 0.031	0.346 ± 0.248	3.507 ± 0.054
$\mathcal{N}_u$	2.043	1.803	0.0504	1.074
χ^2	0.5	3.2	0.12	2.0
m_d (GeV)	0.275	0.275	2.603	2.603
M_X^d (GeV)	0.913	0.913	0.704	0.704
M_Λ^d (GeV)	0.860	0.860	0.878	0.878
α_d	0.0317	0.0317	0.0348	0.0348
α'_d	1.139 ± 0.056	1.281 ± 0.031	1.298 ± 0.245	3.385 ± 0.145
p_d	-0.113 ± 0.104	0.726 ± 0.0631	0.974 ± 0.358	2.326 ± 0.137
$\mathcal{N}_d$	1.570	-2.800	-0.0262	-0.966
χ^2	0.9	4.8	0.11	1.0

New set of parameters using flavor separated data of Cates et al. (2012)

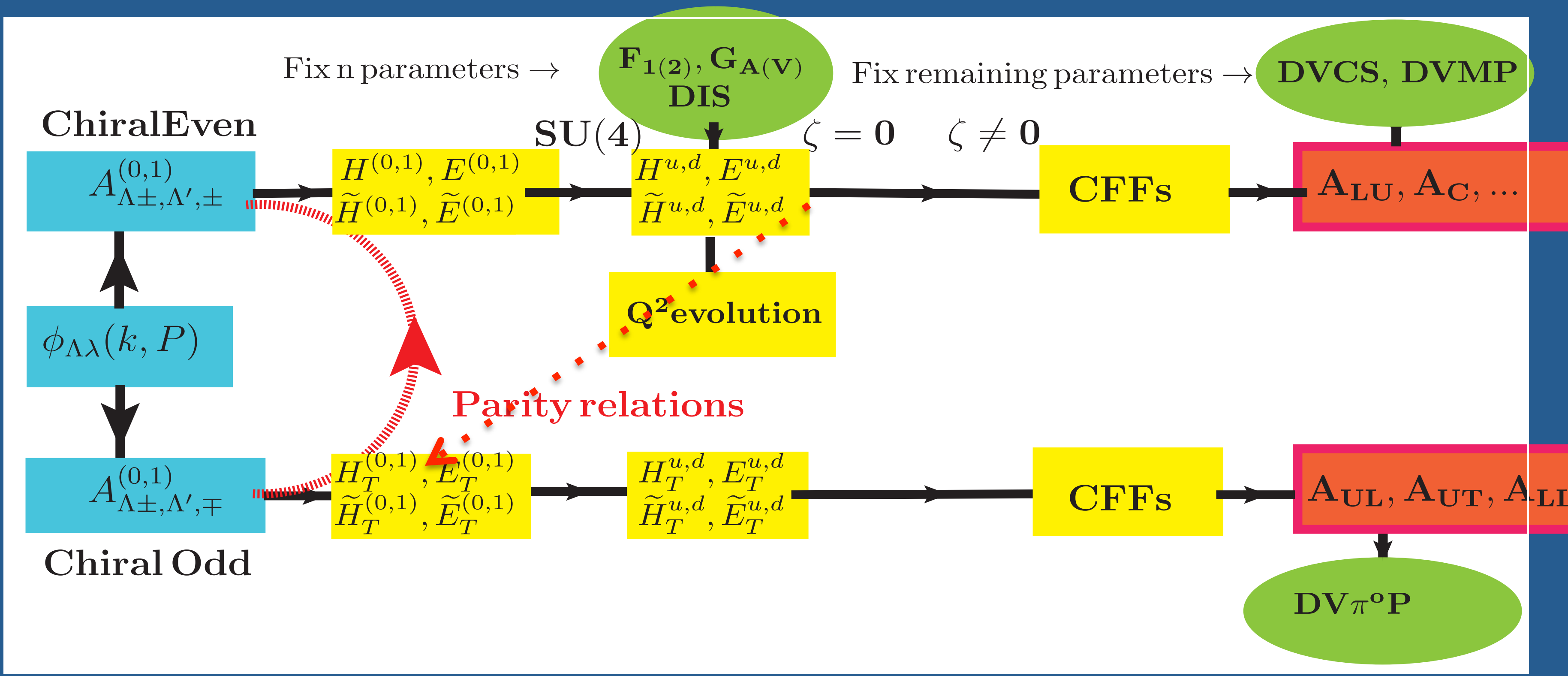


Q^2 = 2 GeV^2
t=-0.3 GeV^2
ζ=0.1
ζ=0.2
ζ=0.3
H_u
x
4
3
2
1
0
-1
-2
-3
-4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

RESULT: This is how we determined the chiral even GPDs



Chiral Odd Sector


Fix n parameters $\rightarrow$
$\mathbf{F_{1(2)}, G_{A(V)}}$
DIS
Fix remaining parameters $\rightarrow$
DVCS, DVMP
ChiralEven
SU(4)
$\zeta = 0$
$\zeta \neq 0$
$A^{(0,1)}_{\Lambda\pm,\Lambda',\pm}$
$H^{(0,1)}, E^{(0,1)}$
$\widetilde{H}^{(0,1)}, \widetilde{E}^{(0,1)}$
$H^{u,d}, E^{u,d}$
$\widetilde{H}^{u,d}, \widetilde{E}^{u,d}$
CFFs
$\mathbf{A_{LU}, A_{C}, \ldots}$
$\phi_{\Lambda\lambda}(k, P)$
$\mathbf{Q^2}$evolution
Parity relations
$A^{(0,1)}_{\Lambda\pm,\Lambda',\mp}$
$H_T^{(0,1)}, E_T^{(0,1)}$
$\widetilde{H}_T^{(0,1)}, \widetilde{E}_T^{(0,1)}$
$H_T^{u,d}, E_T^{u,d}$
$\widetilde{H}_T^{u,d}, \widetilde{E}_T^{u,d}$
CFFs
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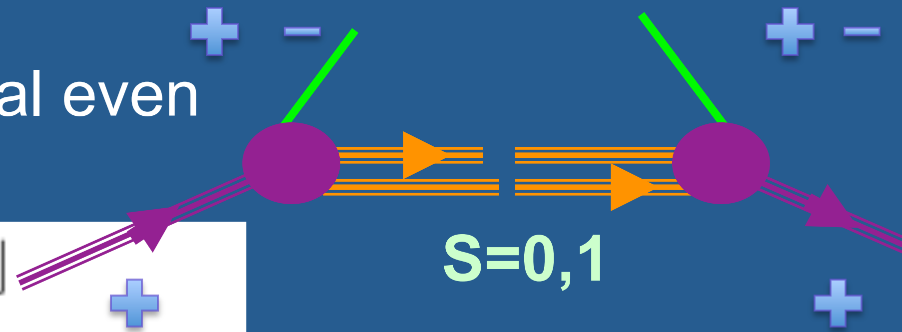
Chiral Odd
DVπ^oP

In the diquark model the chiral odd helicity amps are related to the chiral even ones through Parity



$S=0$	$S=1$
$\phi^{*}_{\Lambda'\lambda'}\phi_{\Lambda\lambda}$	$\phi^{\mu}_{\Lambda'\lambda'}\left(\sum_{\lambda''}\epsilon^{*\lambda''}_{\mu}\epsilon^{\lambda''}_{\nu}\right)\phi^{\nu}_{\Lambda\lambda}$

In order to explain the working of Parity transformations, we write the LHS and RHS of Fig. 1

	RHS	LHS
$S=0$	$\phi^{*}_{\Lambda'\lambda'}$	$\phi_{\Lambda\lambda}$
$S=1$	$\phi^{\mu}_{\Lambda'\lambda'}\epsilon^{*\lambda''}_{\mu}$	$\epsilon^{\lambda''}_{\nu}\phi^{\nu}_{\Lambda\lambda}$

S=1

Odd Even

$$A^{(1)}_{++,--} = -\frac{X+X'}{1+XX'}\,A^{(1)}_{++,++}$$

$$A^{(1)}_{+-,-+} = 0$$

$$A^{(1)}_{++,+-} = -\sqrt{\frac{\langle\tilde{k}_\perp^2\rangle}{X'^2+\langle\tilde{k}_\perp^2\rangle/P^{+2}}}\,A^{(1)}_{++,-+}$$

$$A^{(1)}_{+-,++} = -\sqrt{\frac{\langle k_\perp^2\rangle}{X^2+\langle k_\perp^2\rangle/P^{+2}}}\,A^{(1)}_{-+,++},$$

S=0

Odd Even

$$A^{(0)}_{++,--} = A^{(0)}_{++,++}$$

$$A^{(0)}_{++,+-} = -A^{(0)}_{++,-+}$$

$$A^{(0)}_{+-,++} = -A^{(0)}_{-+,++},$$

$$A^{(0)}_{+-,-+} = \frac{t_0-t}{4M}\,\frac{1}{\sqrt{1-\zeta}}\,\frac{1}{(1-\zeta/2)}\,\frac{\tilde{X}}{m+MX'}\left[E-(\zeta/2)\tilde{E}\right]$$

In terms of GPDs

Odd Even

$S = 0$

$$\widetilde{H}_T^{(0)} = -\frac{M(1-x)}{m+Mx} E^{(0)} \quad (27a)$$

$$E_T^{(0)} = 2\left(1 + \frac{M(1-x)}{m+Mx}\right) E^{(0)} \quad (27b)$$

$$\widetilde{E}_T^{(0)} = 0 \quad (27c)$$

$$H_T^{(0)} = \frac{H^{(0)} + \widetilde{H}^{(0)}}{2} - \frac{t_0 - t}{4M^2} \frac{M(1-x)}{m+Mx} E^{(0)} \quad (27d)$$

$S = 1$

$$\widetilde{H}_T^{(1)} = 0 \quad (28a)$$

$$E_T^{(1)} = 2E^{(1)} \quad (28b)$$

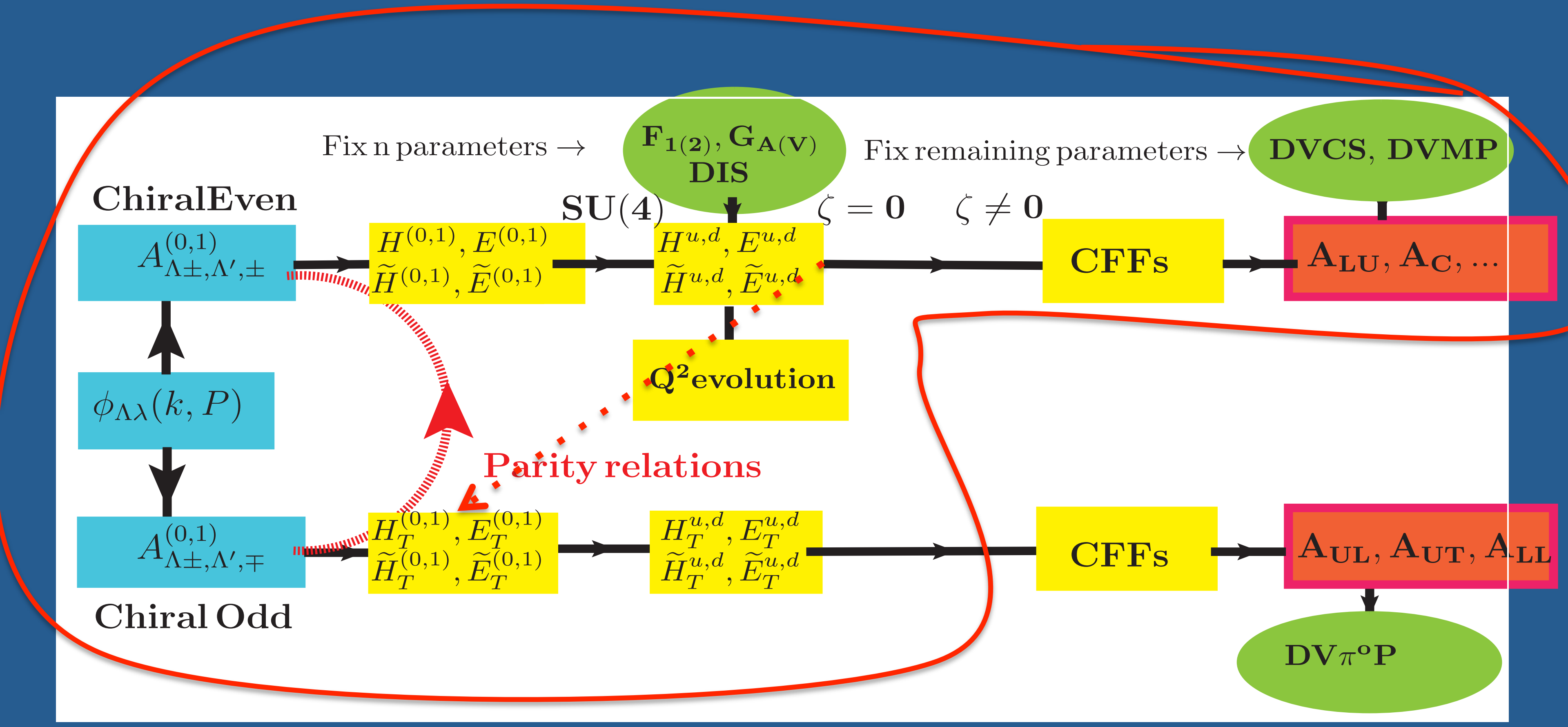
$$\widetilde{E}_T^{(1)} = 0 \quad (28c)$$

$$H_T^{(1)} = -\frac{2x}{1+x^2} \frac{H^{(1)} + \widetilde{H}^{(1)}}{2} \quad (28d)$$

Final step: use SU(4)

$$H_T^u = \frac{3}{2} H_T^{S=0} - \frac{1}{6} H_T^{S=1}$$

$$H_T^d = -\frac{1}{3} H_T^{S=1}$$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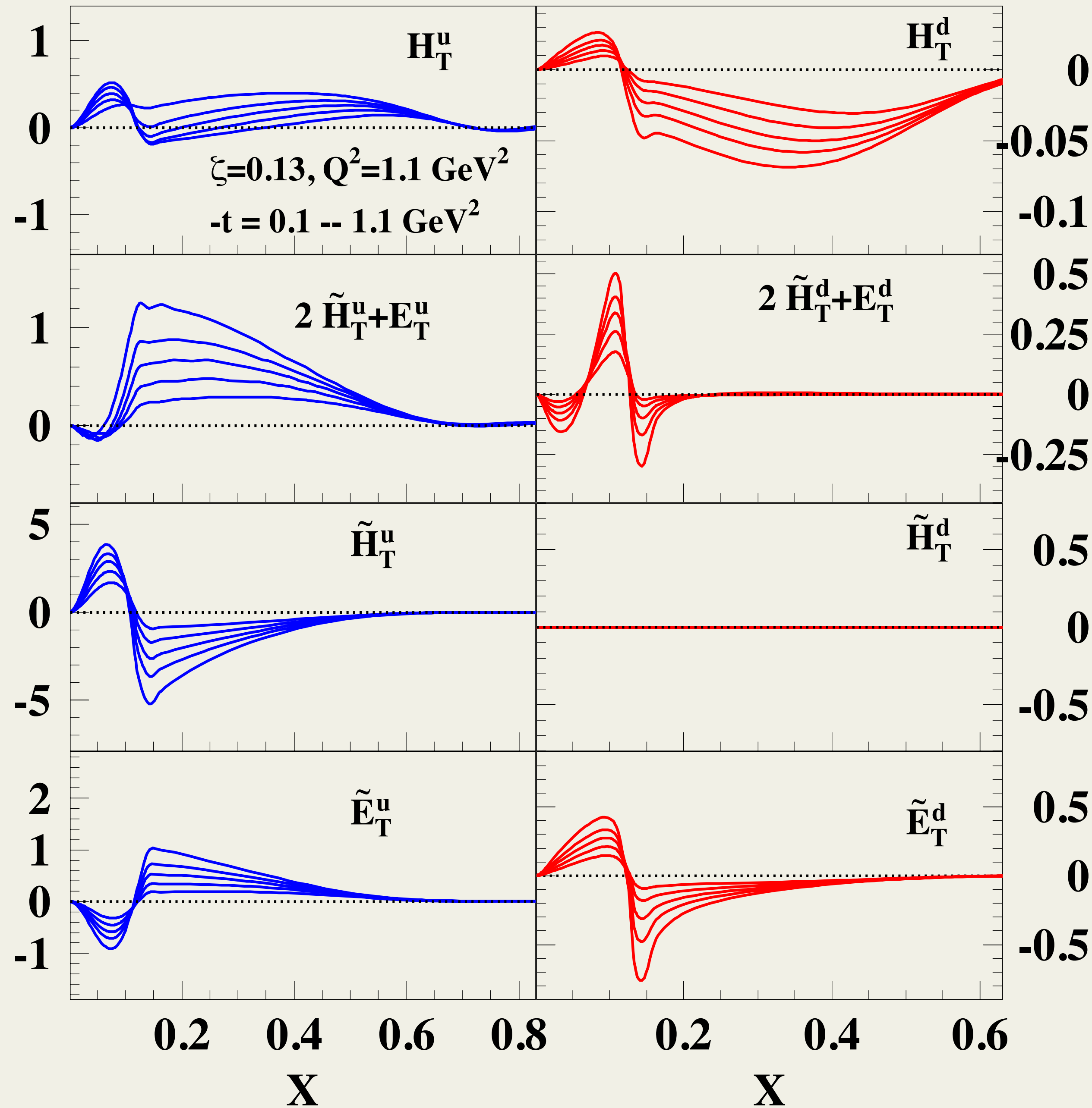
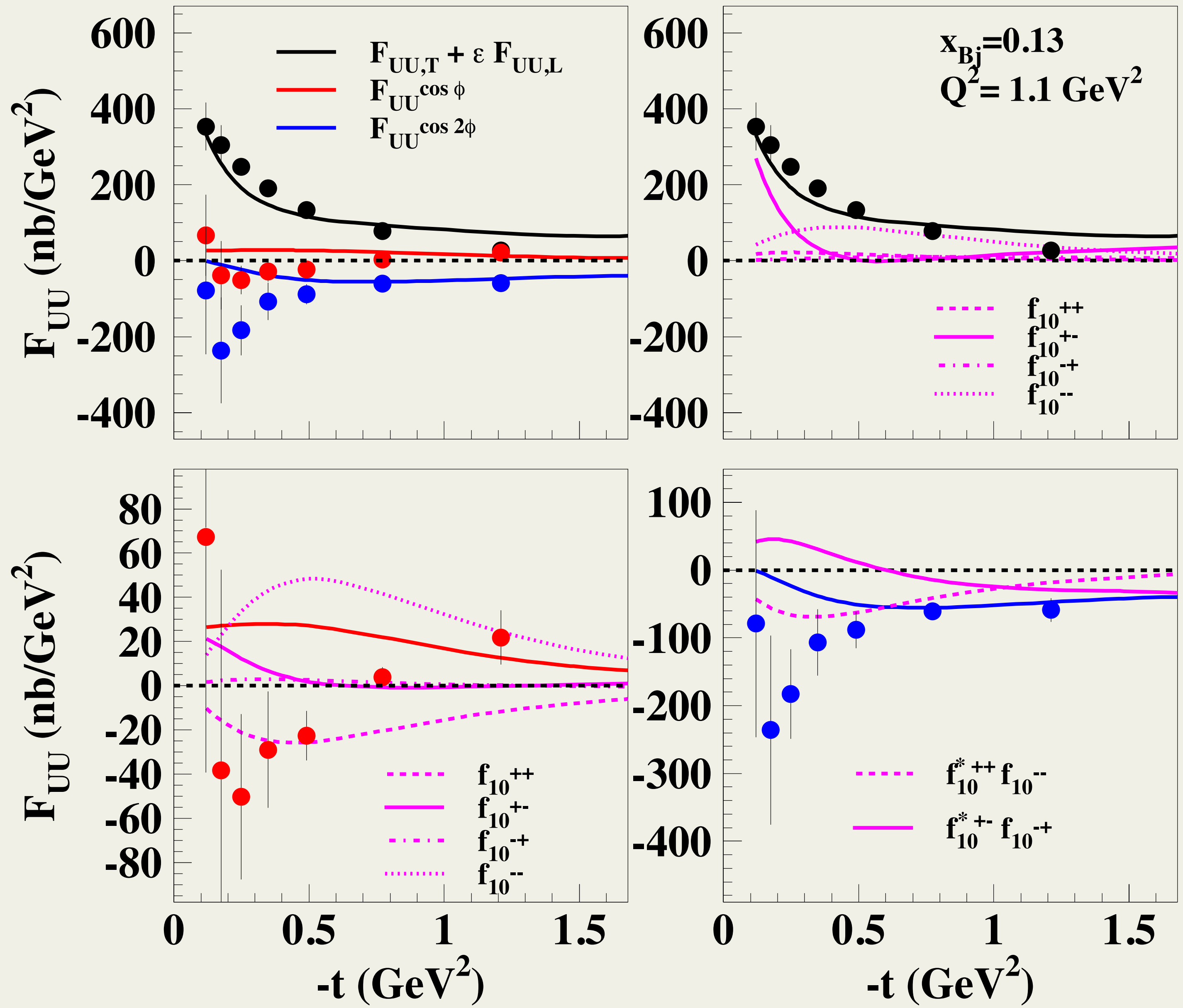
A_{UL}, A_{UT}, A_{LL}
Chiral Odd
DVπ^oP

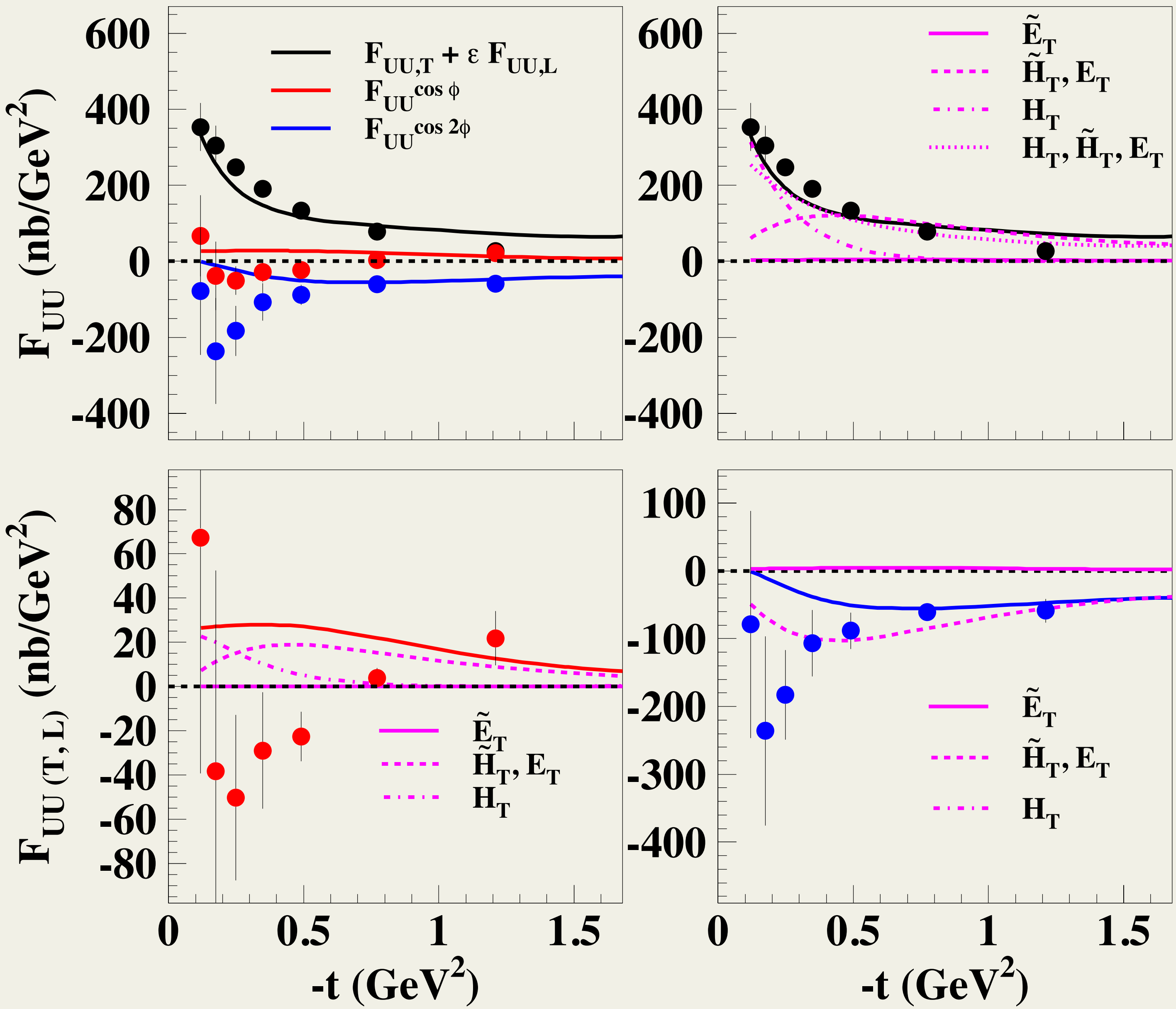
RESULT: Chiral odd GPDs (but we have not fixed the ξ dependence)



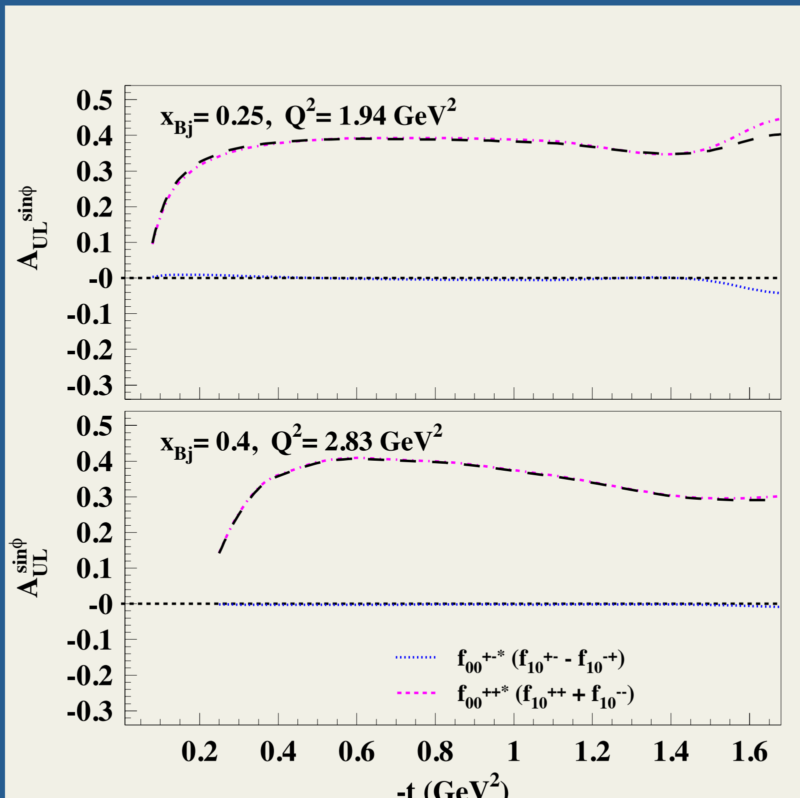
Unpolarized Helicity Amplitudes

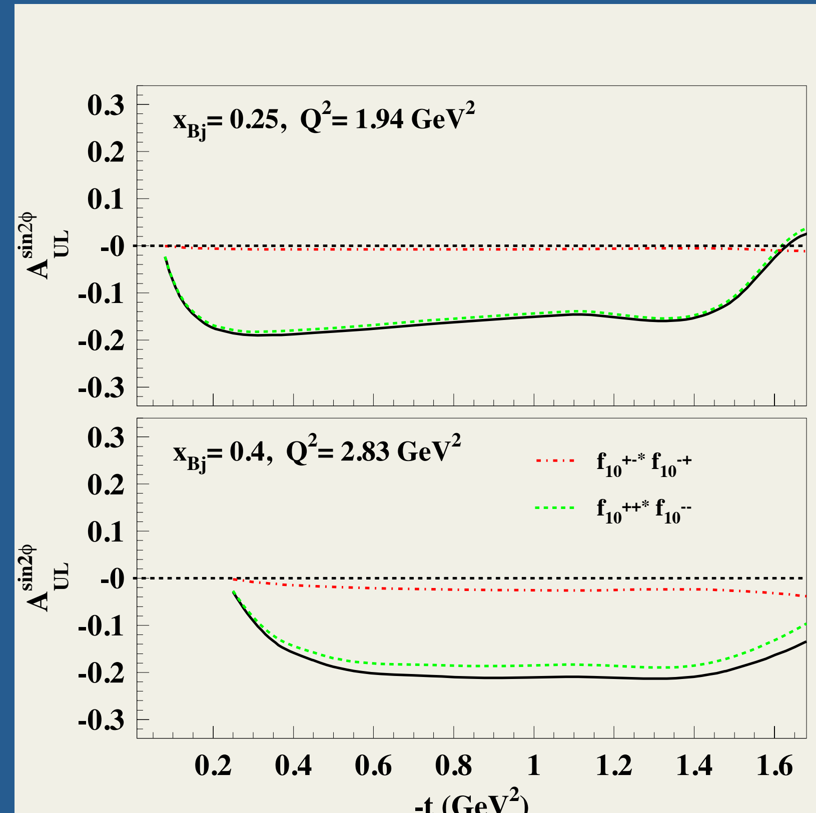


Same, separating the GPDs contribution



Longitudinally polarized target

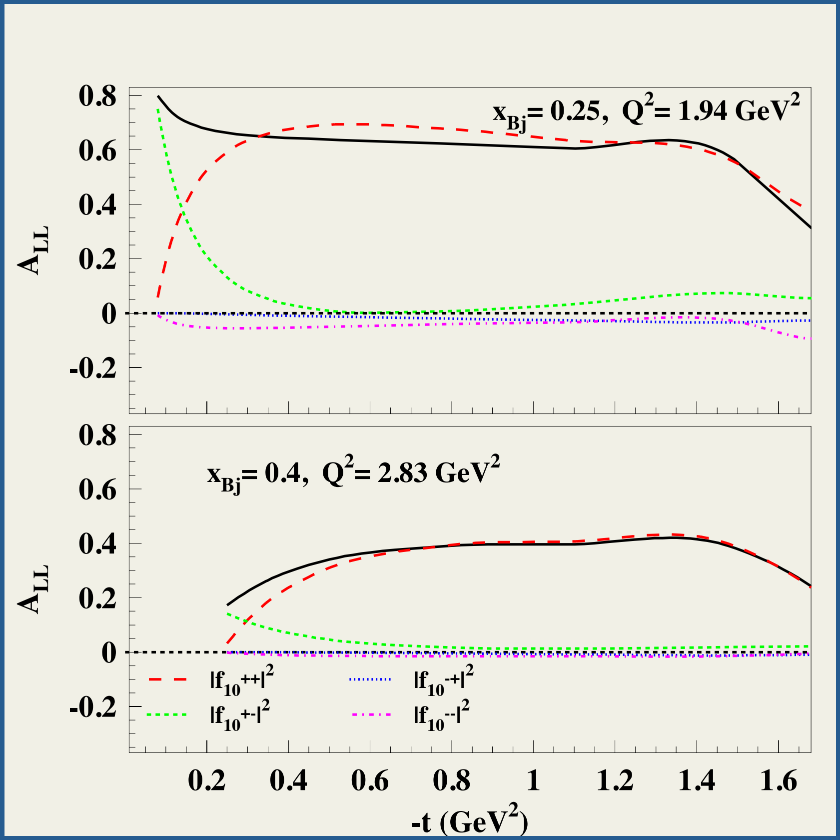


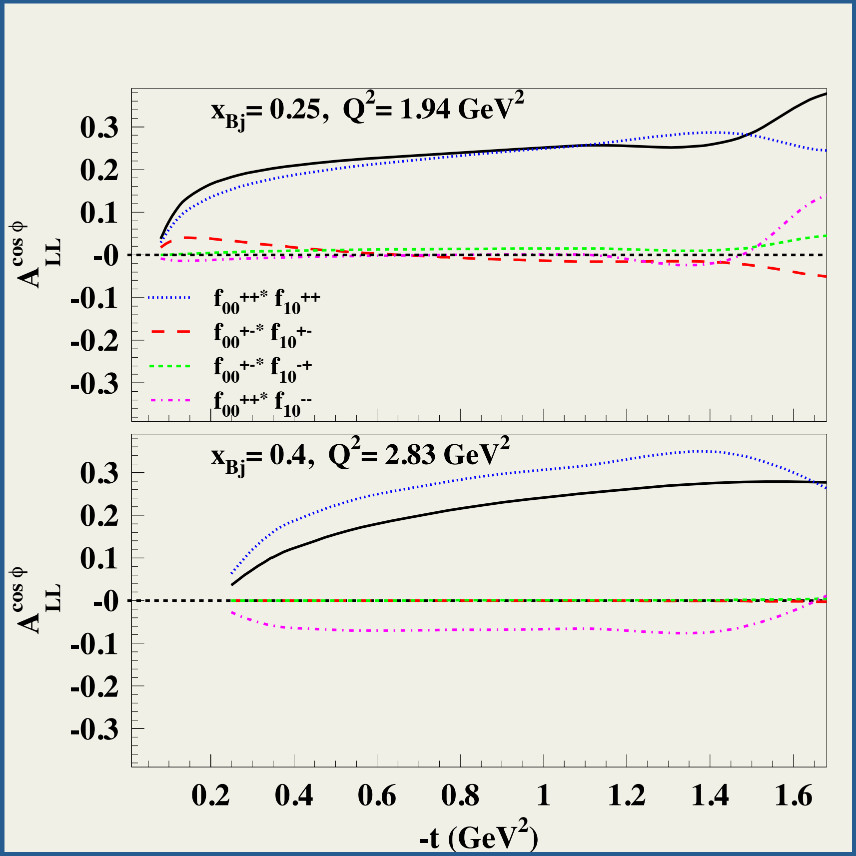


Look for tensor charge in f^{+-}

Tensor Anom. Moment in f^{++},f^{--}

Longitudinally polarized beam and target

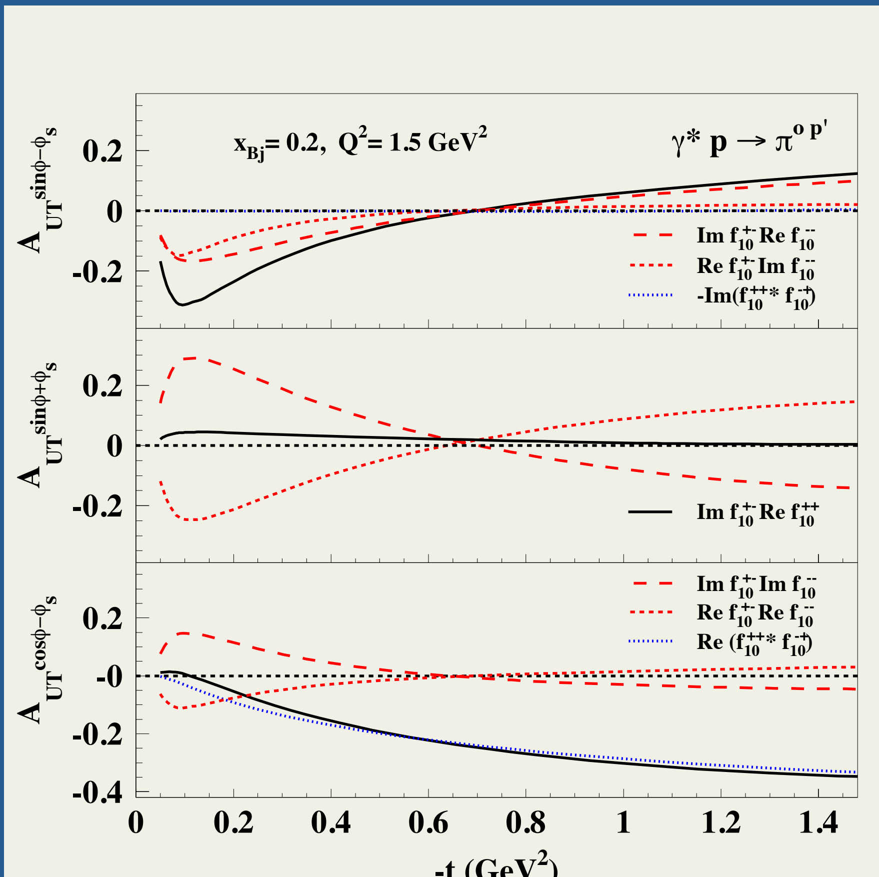


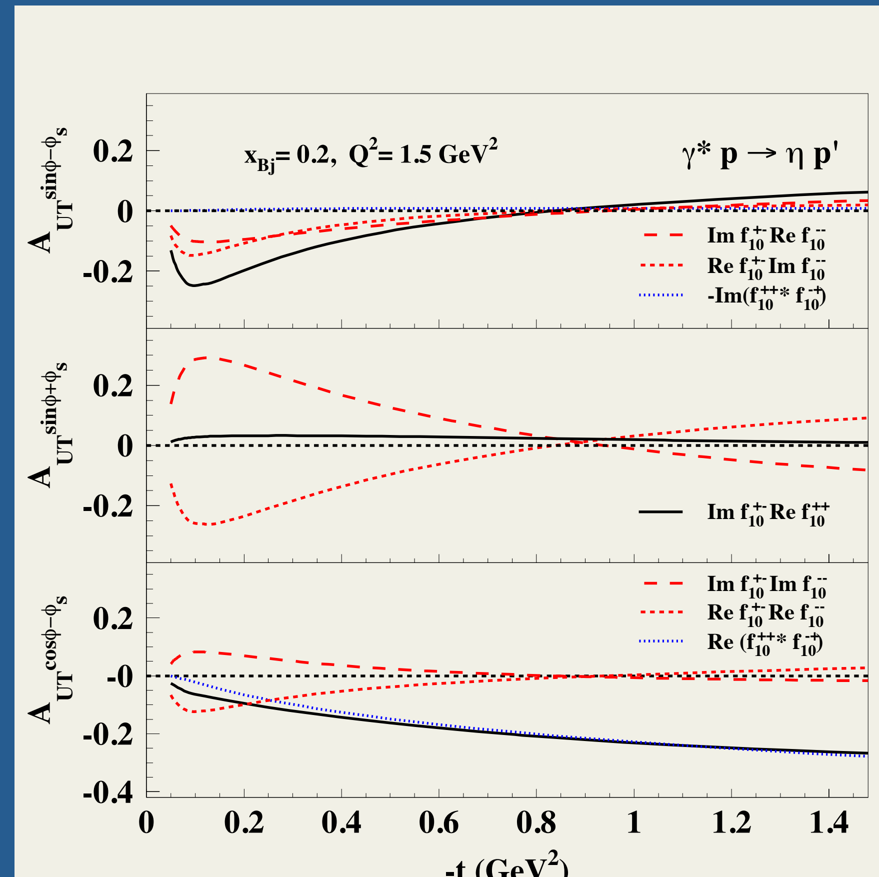


Look for tensor charge in f^{+-}

Tensor Anom. Moment in f^{++},f^{--}

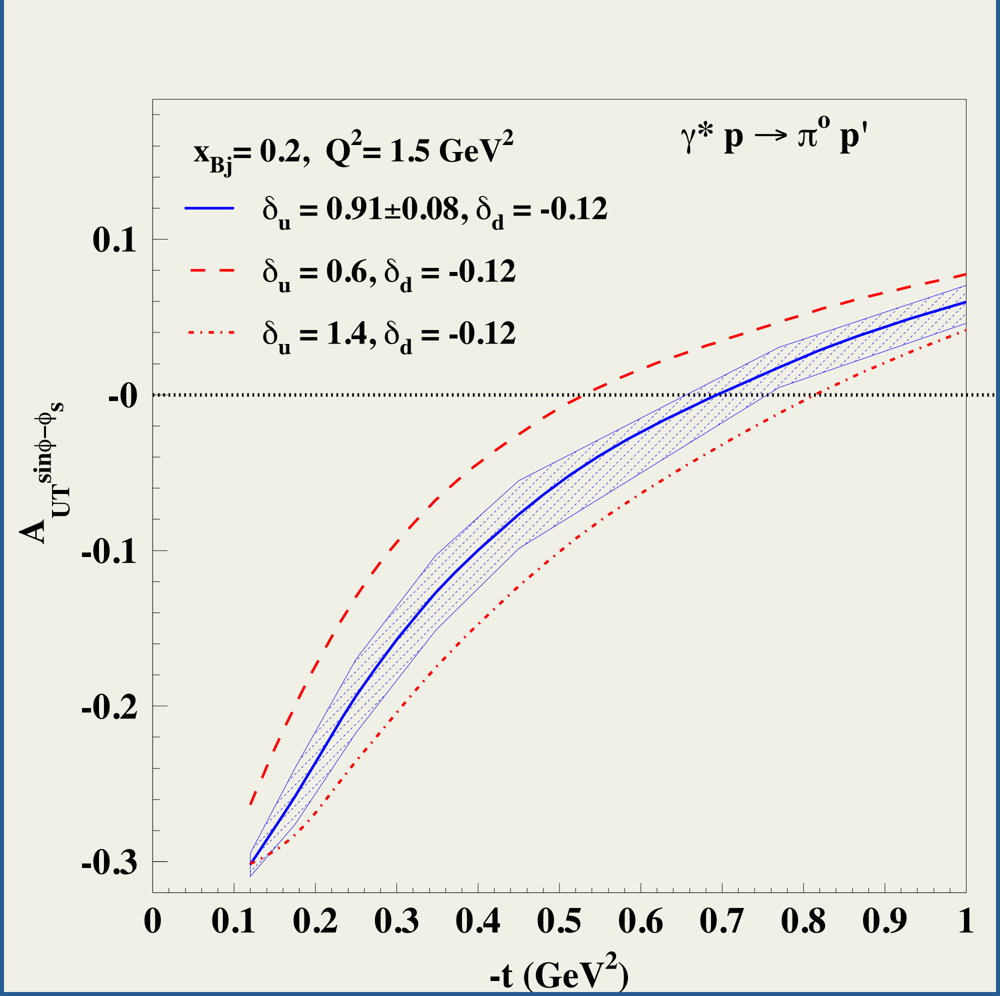
Transverse target





Look for tensor charge in f^{+-}

Tensor Anom. Moment in f^{++},f^{--}



x_{Bj}= 0.2, Q^2= 1.5 GeV^2
$\gamma^* p \rightarrow \pi^o p'$
δ_u = 0.91±0.08, δ_d = -0.12
δ_u = 0.6, δ_d = -0.12
δ_u = 1.4, δ_d = -0.12
$A_{UT}^{\sin\phi-\phi_s}$
0.1
-0
-0.1
-0.2
-0.3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
-t (GeV^2)

State of the art

After these preliminary studies we are now attacking a global fit

A. DVCS

Unpolarized scattering cross section

$$d^4\sigma = F_{UU,T} = c_0 + c_1 \cos\phi + c_2 \cos 2\phi \tag{1}$$

BSA

$$A_{LU} = \sqrt{\epsilon(1-\epsilon)}\,\frac{F_{LU}^{\sin\phi}}{F_{UU,T}} = \frac{a_1 \sin\phi}{c_0 + c_1\cos\phi + c_2\cos 2\phi} \tag{2}$$

TSA

$$\begin{aligned} A_{UL} &= \frac{\sqrt{\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi}}{F_{UU,T} + \epsilon\, F_{UU,L}} + \frac{\epsilon \sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T}} \\ &= \frac{a_2 \sin\phi + a_3 \sin 2\phi}{c_0 + c_1\cos\phi + c_2\cos 2\phi} \end{aligned} \tag{3}$$

Double TSA

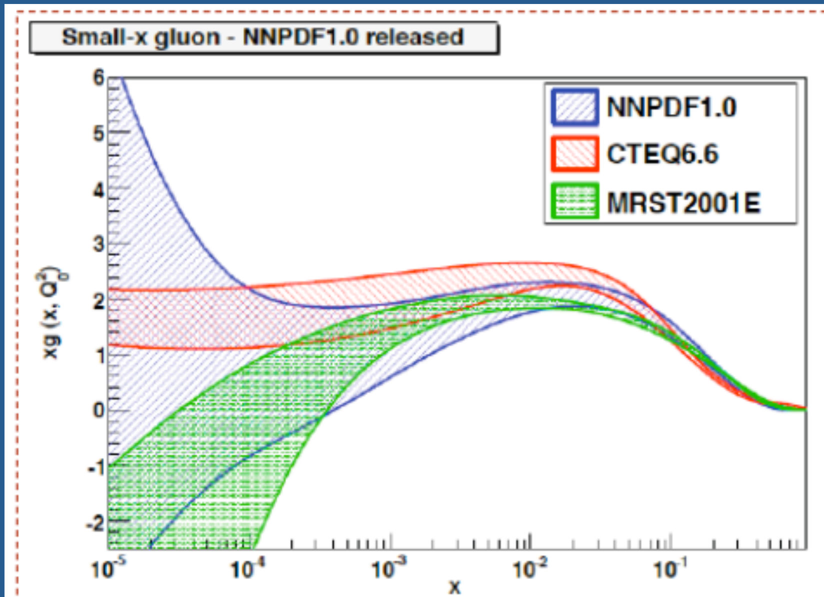
$$\begin{aligned} A_{LL} &= \frac{\sqrt{1-\epsilon^2}\; F_{LL}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\sqrt{\epsilon(1-\epsilon)}\;\cos\phi\, F_{LL}^{\cos\phi}}{F_{UU,T} + \epsilon F_{UU,L}} \\ &= \frac{a_4 + a_5\cos\phi}{c_0 + c_1\cos\phi + c_2\cos 2\phi} \end{aligned} \tag{4}$$

Finally….

We can attack the problem from a different perspective:

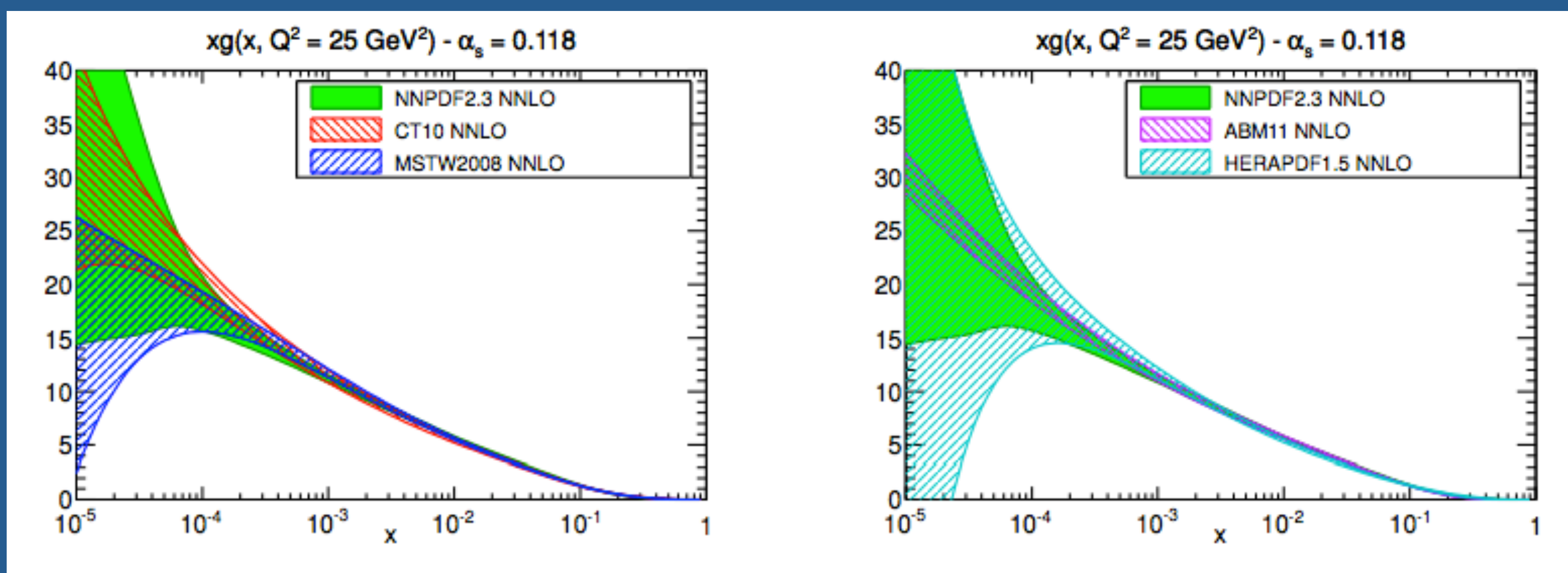
Study the behavior of multi-particle systems as they evolve from a large and varied number of initial conditions

This goal is at reach with HPC



NNPDF before LHC data

NNPDF including LHC data, JHEP(2012)

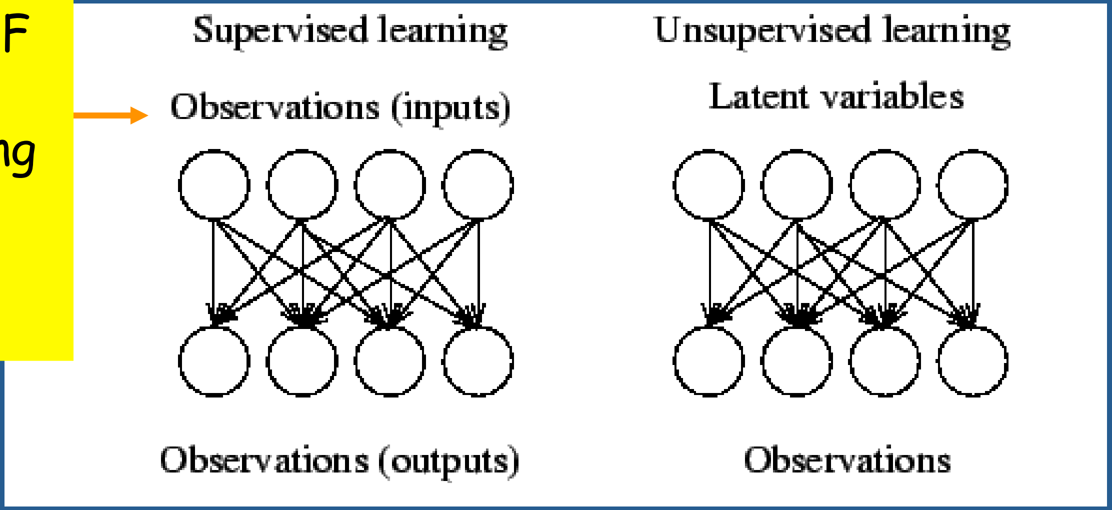


Most NNs (including NNPDFs) learn with supervised learning

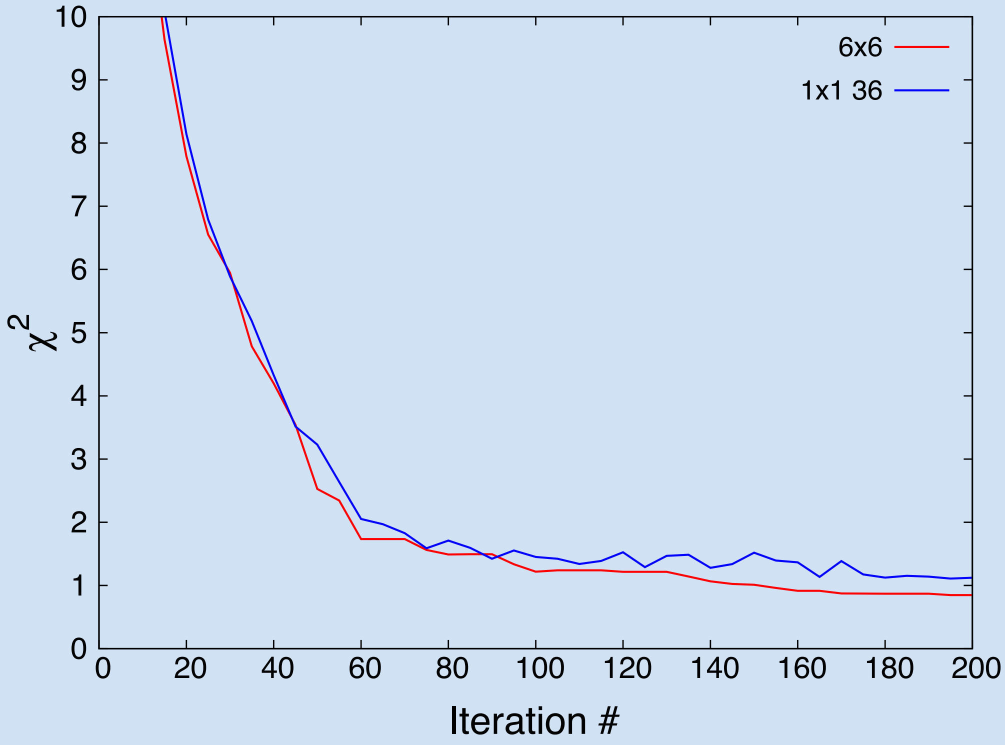
Supervised Learning → A set of examples is given.
The goal is to force the data
To match the examples as closely as possible.
The cost function includes information about the domain

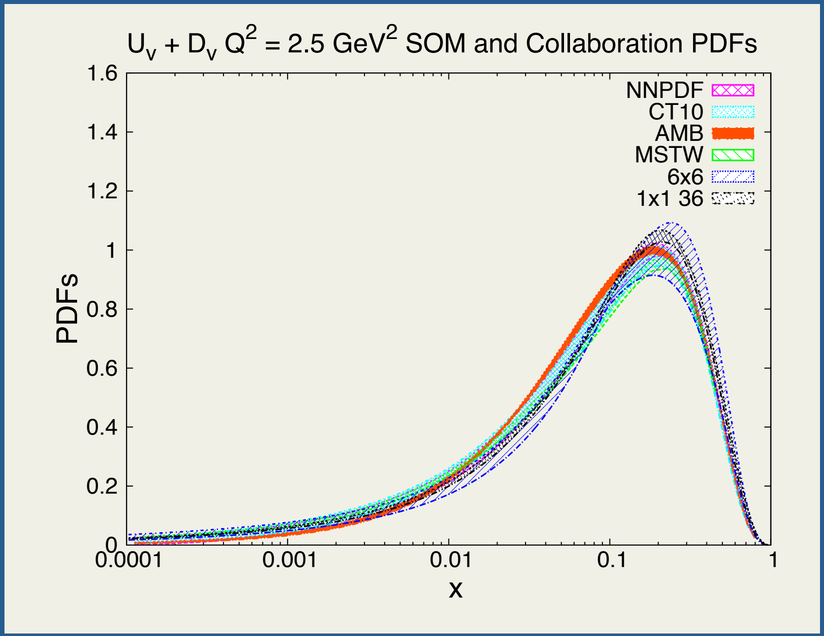
Unsupervised Learning → No a priori examples are given.
The goal is to minimize the cost function by similarity relations, or by finding how the data cluster or self-organize
➔ global optimization problem

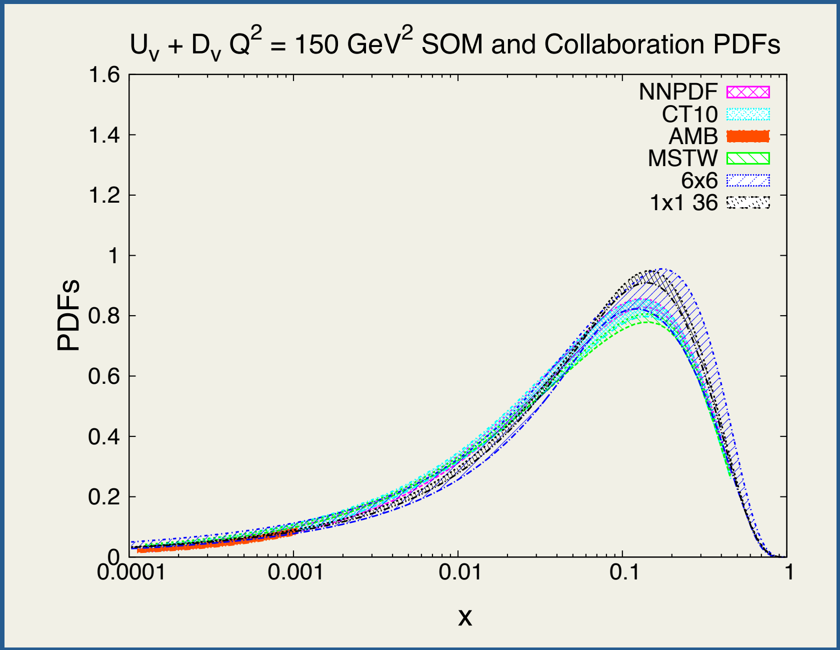
Important for PDF analysis!
If data are missing it is not possible to determine the output!



Minimizing χ^2







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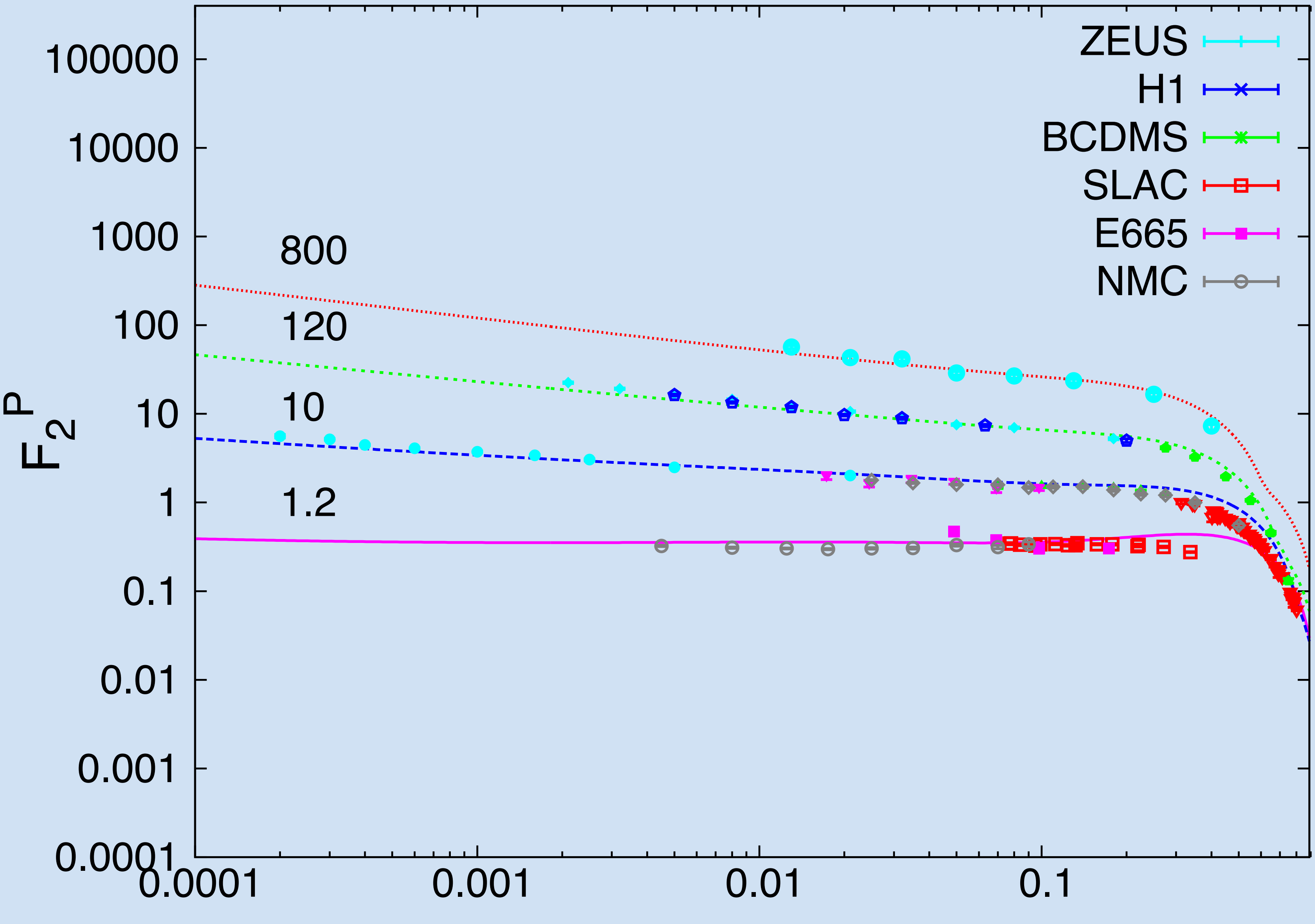
Observable



ZEUS
H1
BCDMS
SLAC
E665
NMC
800
120
10
1.2
100000
10000
1000
100
10
1
0.1
0.01
0.001
0.0001
0.0001
0.001
0.01
0.1
F_2^P
x

Conclusions

We got a usable GPDs parameterization that satisfies all theoretical requirements (polynomiality, positivity, forward limit, …), and that is ‘’flexible”: it is physically motivated (based on reggeized diquark model), but, most importantly, it allows us to monitor the various parameters.

There are several papers on arXiv…

sl4y@virginia.edu

References

- pi0 and eta → arXiv:1401.0438
- GPDs from flavor separated form factors →arXiv:1206.1876
-
- Chiral odd approach →arXiv:1311.0483
- Chiral even approach→arXiv:1012.3776
- Observability of OAM→arXiv:1310.5157
- Angular momentum in spin 1 →arXiv:1101.0581
- Self Organizing Maps parametrization →arXiv:0810.2598, arXiv:1008.2137, arXiv:1309.7085