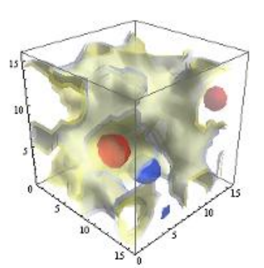
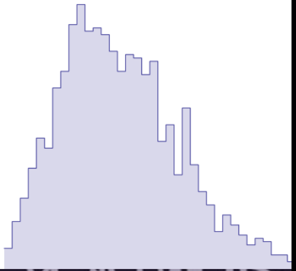
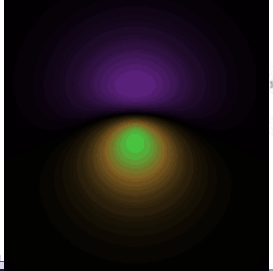
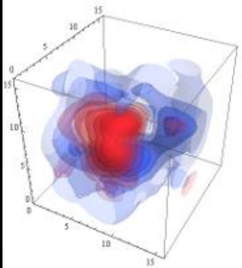
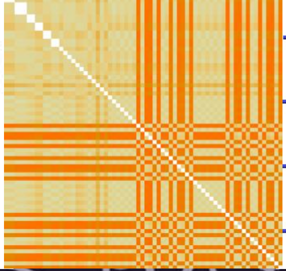
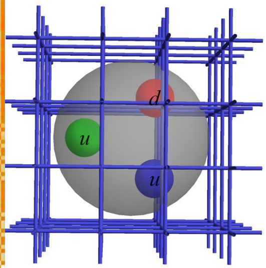


From C to Parton Sea

PDFs on the Lattice

Huey-Wen Lin
University of Washington

This talk is based on the work

"Flavor Structure of the Nucleon Sea from Lattice QCD", 1402.1462 [hep-ph]

performed in collaboration with

Jiunn-Wei Chen
(NTU)

Saul Cohen
(UW)

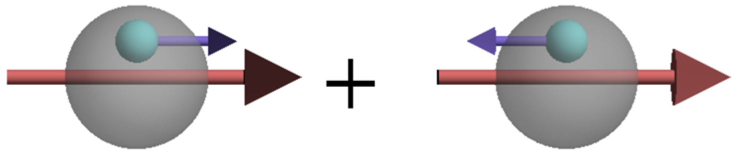
Xiangdong Ji
(UMD/SJTU/
INPAC)

Parton Distribution Functions

most well known

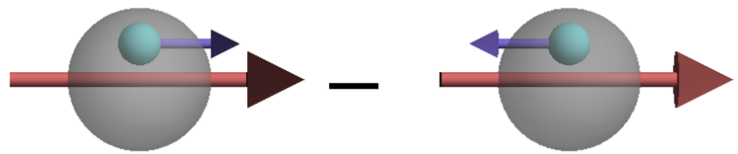
§ Quark distribution

spin-averaged
unpolarized



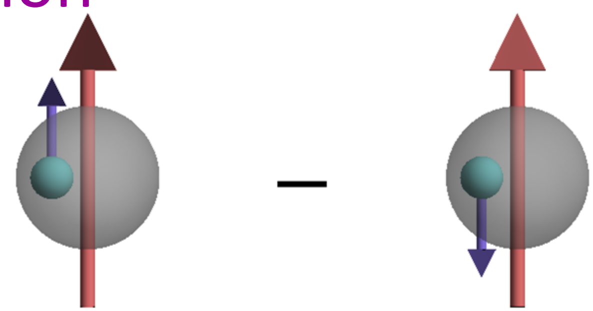
§ Helicity distribution

spin-dependent
longitudinally polarized



§ Transversity distribution

spin-dependent
transversely polarized



very poorly known

What can LQCD Help?

§ Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories

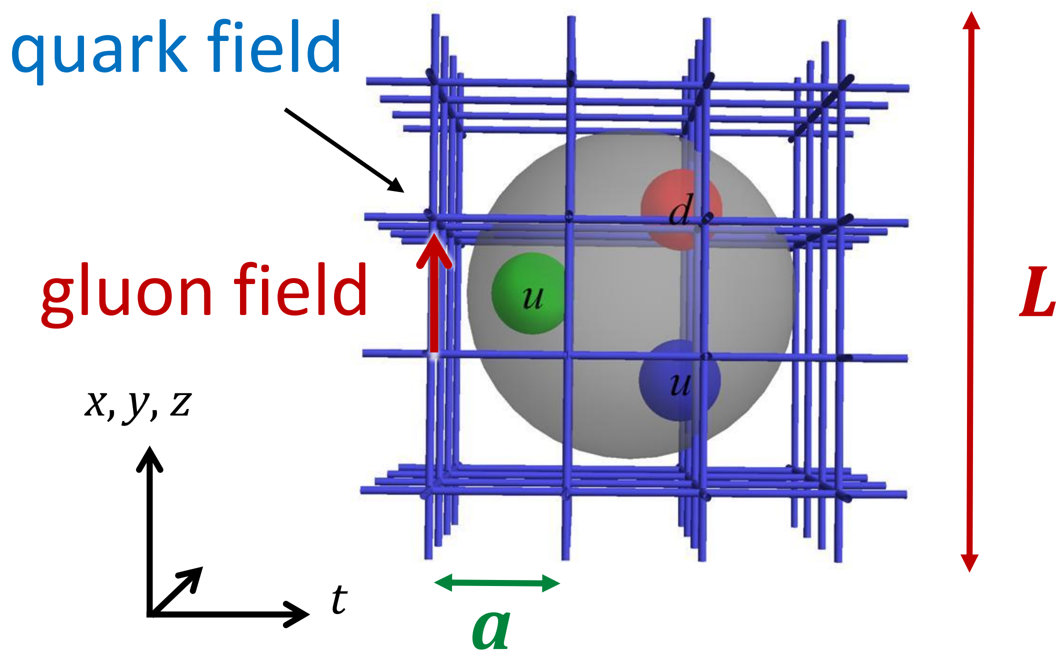
- Ideal tool for studying nonperturbative hadron structure

§ Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}O(\bar{\psi},\psi,A)$$

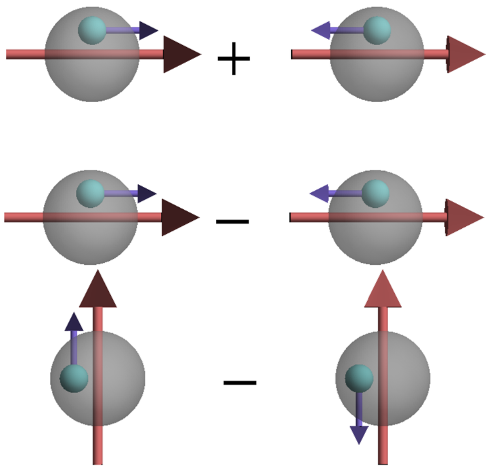
§ Lattice QCD

- Impose a UV cutoff
  discretize spacetime
- Impose an Infrared cutoff
  finite volume



PDFs on the Lattice

§ Many lattice calculations of the moments of the PDFs



$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

- Limited to the lowest few moments
- Might provide constraints on models or tests of experiment

§ Also applies to GPDs: limited to 3rd moment

§ Most progress made in quark contributions

- Very costly to obtain useful gluon signal
- Limited by available computational resources

$\langle x^n \rangle$ Moments

§ Leading moment $\langle x \rangle$, hypercubic decomposition

- $4_1 \otimes 4_1 = \quad 1_1 \oplus 3_1 \oplus 6_1 \oplus \quad 6_2$:

  O … ires $p \neq 0$)

- Both ope…

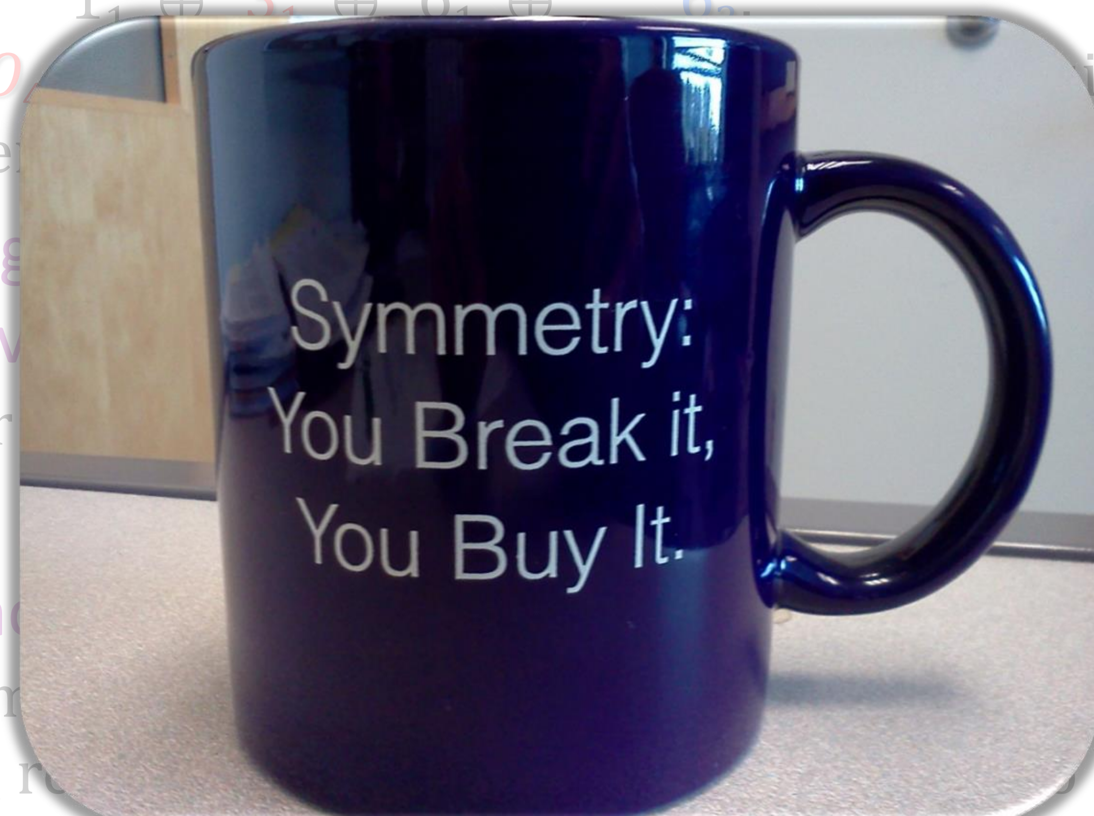
§ No mixing … mension

§ To improv…

- Consider … try:

  … $c_1 \, \overline{q} \, \sigma_{\mu\rho} \overleftrightarrow{D}_{[\nu} \overleftrightarrow{D}_{\rho]} q$

  … $\partial_\rho \left(\overline{q} \, \sigma_{\mu\rho} \overleftrightarrow{D}_\nu q \right)$



§ Higher mo…

- 4_1: O_{111} m…
- 4_2: $O_{\{123\}}$ r… be nonzero
- 8_1: $O_{\{441\}} - (O_{\{221\}} + O_{\{331\}})/2$ mixes under renormalization

§ For higher spin, all ops mix with lower-dimension ops

$\langle x^n \rangle$ Moments

§ Easiest case: first moment $\langle x \rangle$, hypercubic decomposition

- $4_1 \otimes 4_1 = 1_1 \oplus 3_1 \oplus 6_1 \oplus 6_3$:
  $O_{44} - (O_{11}+O_{22}+O_{33})/3$ $\quad O_{14}+O_{41}$, (requires $p \neq 0$)
- Both operators go to same continuum limit
- No mixing with operators of same or lower dimension

§ To improve to $O(a)$

- Consider all irrelevant operators of same symmetry:

$$O_{\{\mu\nu\}} \longrightarrow (1 + a\, m_q\, c_0)\, O_{\{\mu\nu\}} + i\, a\, c_1\, \overline{q}\, \sigma_{\mu\rho}\, \overleftrightarrow{D}_{[\nu}\, \overleftrightarrow{D}_{\rho]}\, q + a\, c_2\, \overline{q}\, \overleftrightarrow{D}_{[\mu}\, \overleftrightarrow{D}_{\nu]}\, q + i\, a\, c_3\, \partial_\rho \left(\overline{q}\, \sigma_{\mu\rho}\, \overleftrightarrow{D}_\nu\, q \right)$$

§ Higher moments $\langle x^2 \rangle$

- 4_1: O_{111} mixes with $\overline{q}\gamma_1 q$ with coefficient $\sim 1/a^2$
- 4_2: $O_{\{123\}}$ requires all momentum components to be nonzero
- 8_1: $O_{\{441\}} - (O_{\{221\}}+O_{\{331\}})/2$ mixes under renormalization

§ For higher spin, all ops mix with lower-dimension ops

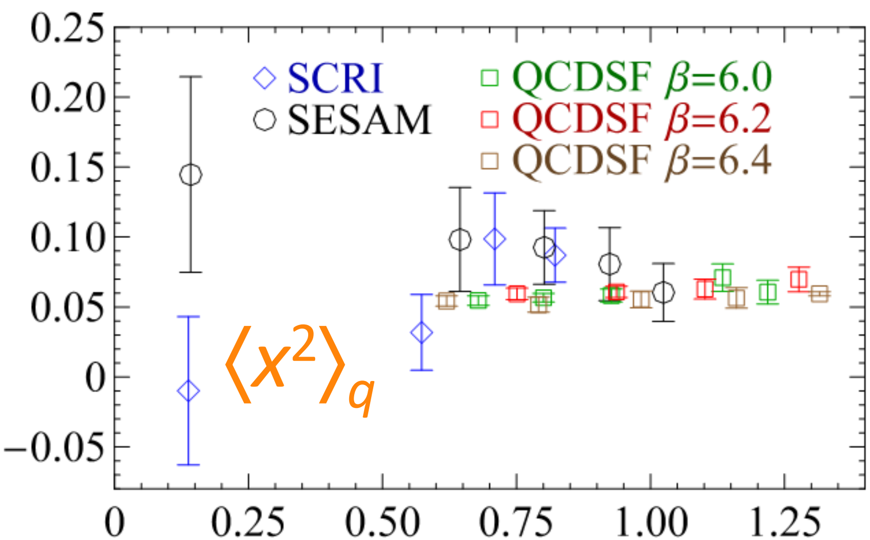
$\langle x^n \rangle$ Moments

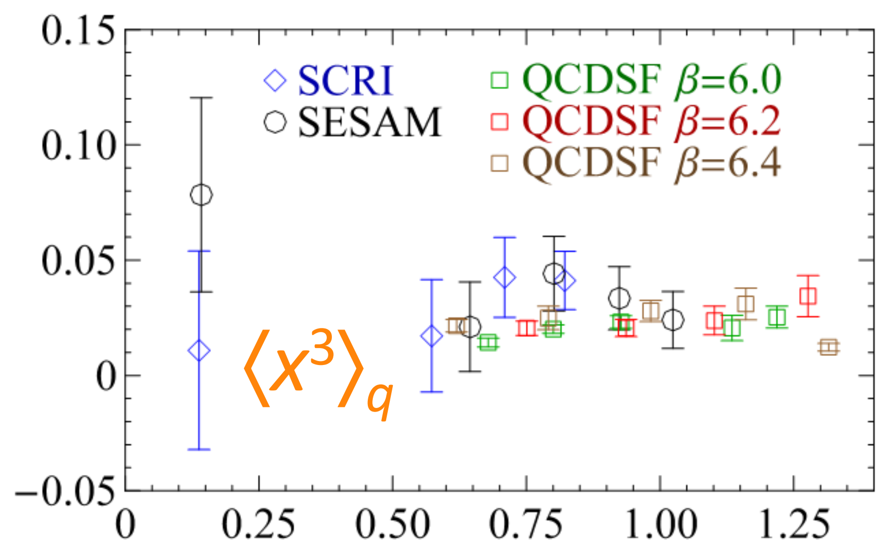
- § For higher spin, all ops mix with lower-dimension ops
  - Tricks: subtraction to remove divergent terms, heavy fields, four-point functions… **None is practical enough**
- § Relative error grows in higher moments
  - Calculation would be costly and difficult

LHPC (SCRI, SESAM): 2f, Wilson and clover

QCDSF: 0f

Dolgov et al. PRD66, 034506 (2002)
Göckeler et al. PRD71, 114511 (2005)



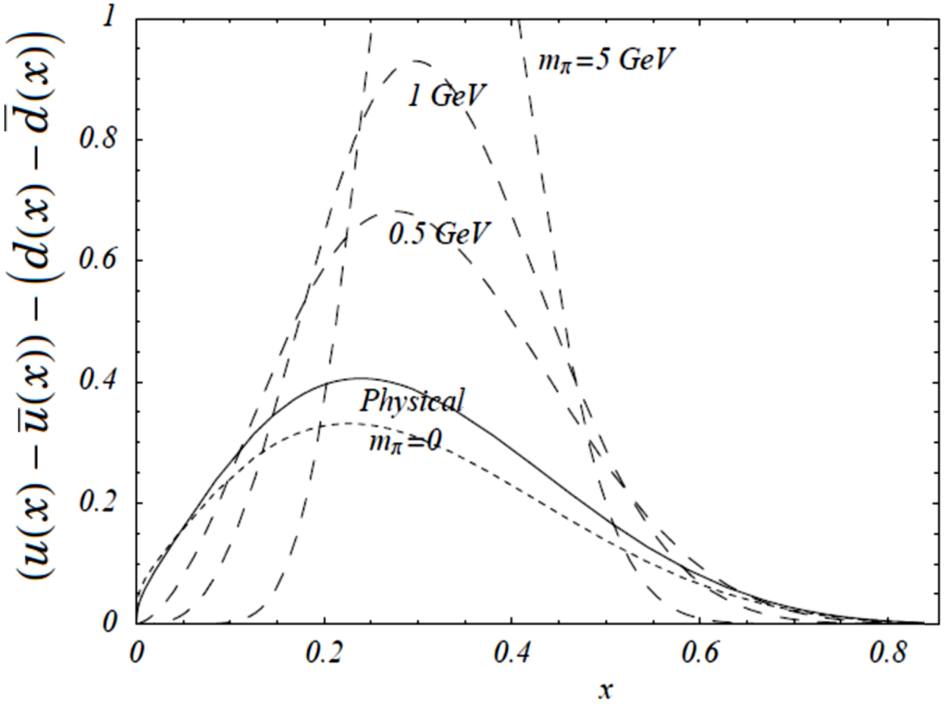


Limited Access

§ What can we learn about the *x*-distribution?

- Make an ansatz of some smooth form for the distribution and fix the parameters by matching to the lattice moments

$$xq(x) = ax^b(1-x)^c(1+\epsilon\sqrt{x}+\gamma x)$$



Cannot separate valence-quark contribution from sea

New idea needed to access the sea!

W. Detmold et al, Eur.Phys.J.direct C3 (2001) 1–15

A New Kid in Town

The Idea

§ Finite-momentum quark distribution

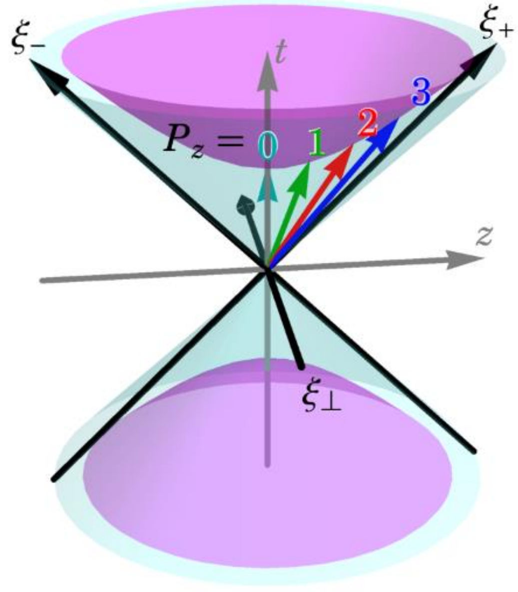
$$q(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z)\gamma_z \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0)\right| P\right\rangle + \mathcal{O}\left(\Lambda_{\rm QCD}^2/P_z^2, M_N^2/P_z^2\right)$$

$x = k_z/P_z$

Lattice z coordinate

Nucleon momentum $P_\mu = \{P_0, 0, 0, P_z\}$

Product of lattice gauge links



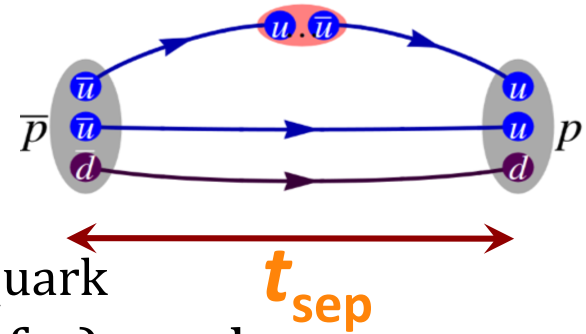
- In $P_z \longrightarrow \infty$ limit, parton distribution is recovered
- For finite P_z, corrections are needed

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013); this workshop

Some Lattice Details

§ Exploratory study

MILC Collaboration, Phys. Rev. D87, 054505 (2013)



- N_f = 2+1+1 clover/HISQ lattices (MILC)
  $M_\pi \approx$ **310 MeV**, $a \approx$ **0.12 fm** ($L \approx 2.88$ fm)
- Isovector only ("disconnected" suppressed)
  gives us flavor asymmetry between up and down quark
- 2 source-sink separation ($t_{sep} \approx 0.96$ and 1.2 fm) used

§ Properties known on these lattices

- Lattice Z_Γ for bilinear operator ~ 1
  (with HYP-smearing)
- $M_\pi L \approx 4.6$ large enough to avoid finite-volume effects

Hyak @ UW

§ Feasible with today's computational resources!

- 8/16 nodes on UW Hyak cluster

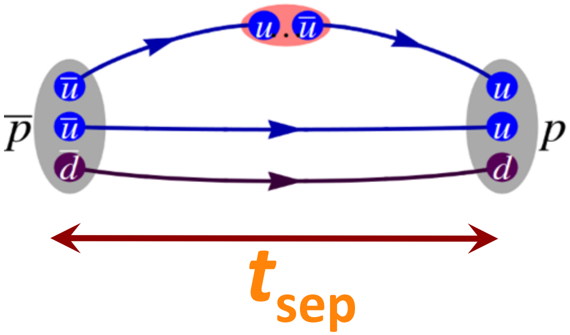
HWL et al, 1402.1463 (submitted to Phys. Rev. Lett.)

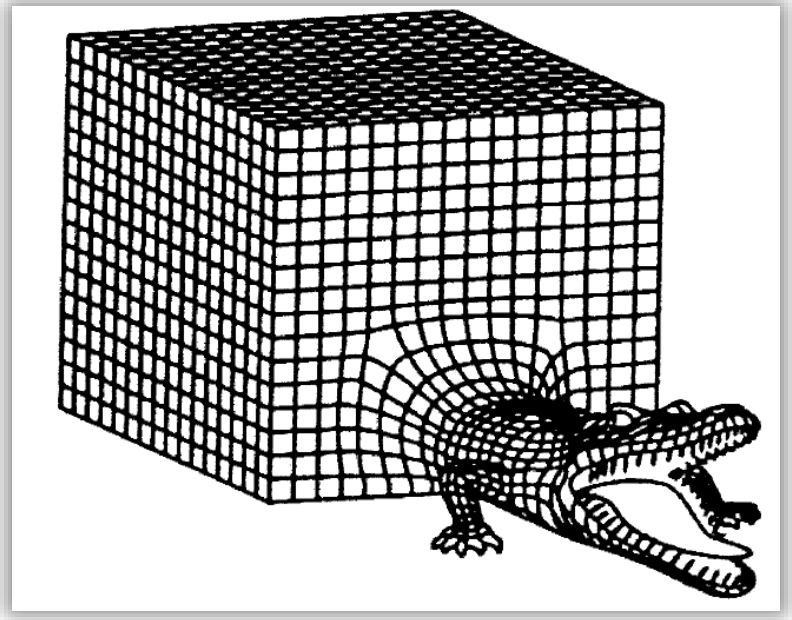
Warning!

MILC Collaboration, Phys. Rev. D87, 054505 (2013)

§ Exploratory study

- N_f = 2+1+1 clover/HISQ lattices (MILC)
  $M_\pi \approx$ **310 MeV**, $a \approx 0.12$ fm ($M_\pi L \approx 4.5$)





NO SYSTEMATICS YET!

§ Demonstration that the method works!

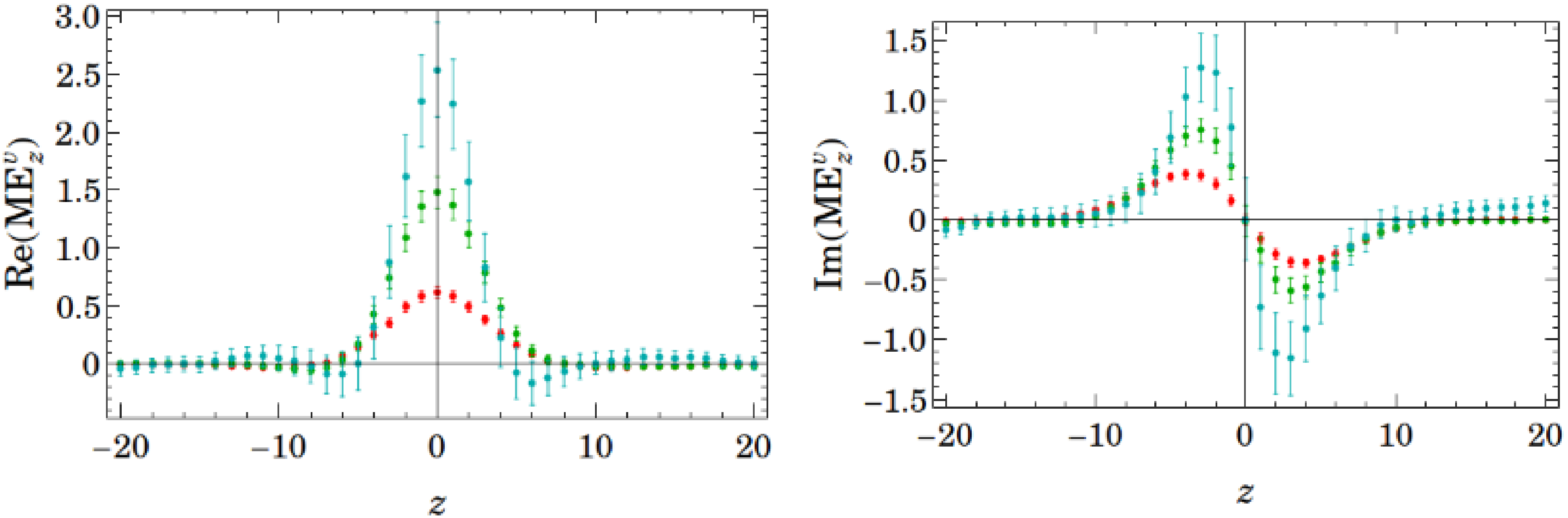
- Intend to motivate future LQCD work on many quantities

Quark Distribution

§ Exploratory study

$$\left\langle P \,\middle|\, \overline{\psi}(z)\, \gamma_z \exp\!\left(-i g \int_0^z dz'\, A_z(z')\right) \psi(0) \,\middle|\, P \right\rangle$$

- How many links are needed?



- Lattice momenta discretized by finite size of volume

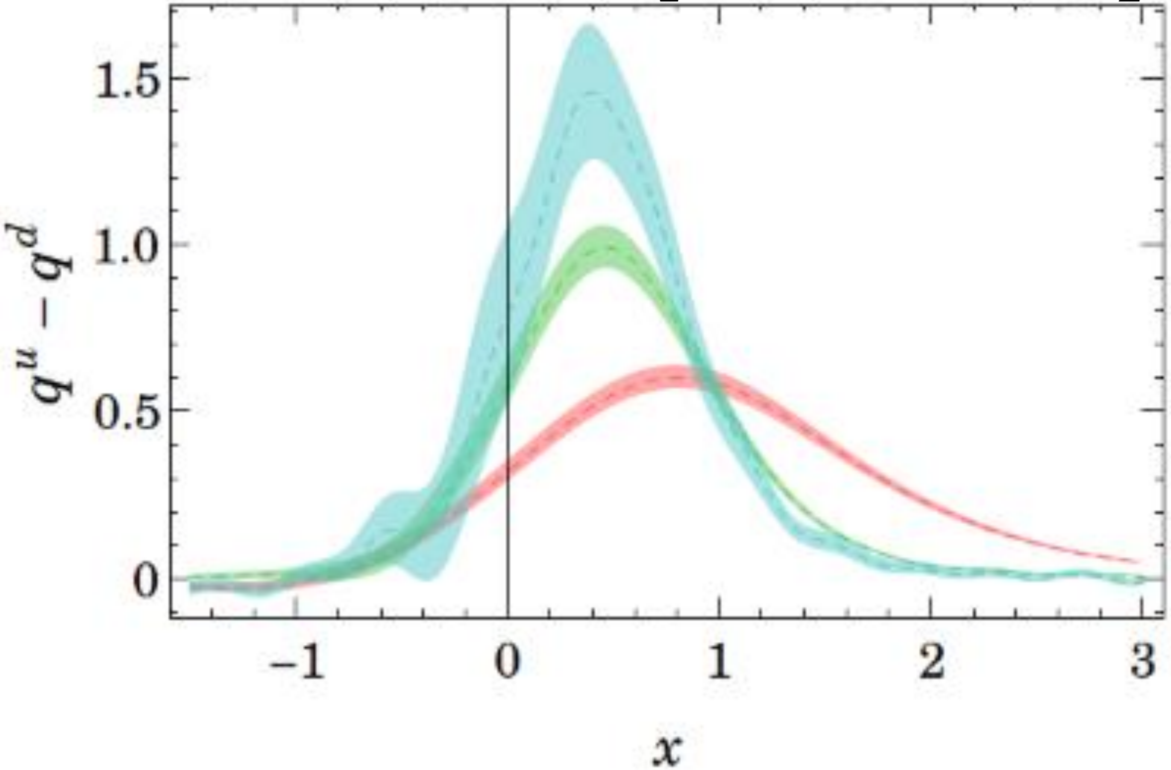
$$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_{L}$$

Quark Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi}\, e^{-izk_z} \left\langle P \,\middle|\, \overline{\psi}(z)\,\gamma_z \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right)\psi(0) \,\middle|\, P \right\rangle$$

$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_{L}$



Uncorrected bare lattice results

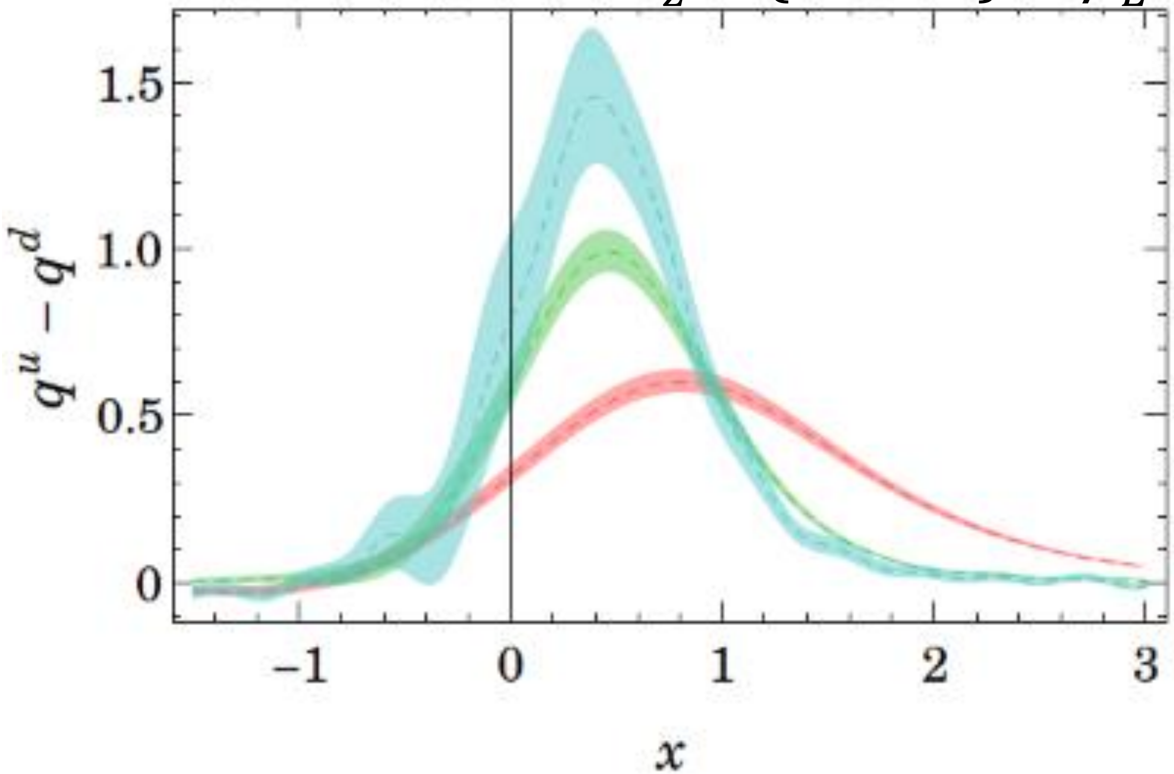
$x = k_z / P_z$

Quark Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi}\, e^{-izk_z} \left\langle P \middle| \overline{\psi}(z)\, \gamma_z \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$

$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_{L}$



Distribution gets sharper as P_z increases

Artifacts due to finite P_z on the lattice

Improvement?

Work out leading-P_z corrections

Quark Distribution

§ Back to the continuum

Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

$$q(x,\mu) = q_{\rm FP}(x,\mu,P_z) + O\left(\Lambda^2_{\rm QCD}/P_z^2\right) + O\left(M_N^2/P_z^2\right) + O(\alpha_s)$$

What we want

What we calculate on the lattice
$P_z \in \{1, 2, 3\}\ ^{2\pi}/_L$

Smaller P_z correction but complicated higher-twist operator (extrapolate it away)
J.-H. Zhang, Y. Zhao, J.-W. Chen et al. (in preparation)

Dominant correction (for nucleon); known scaling form
J.-W. Chen et al. (in preparation)

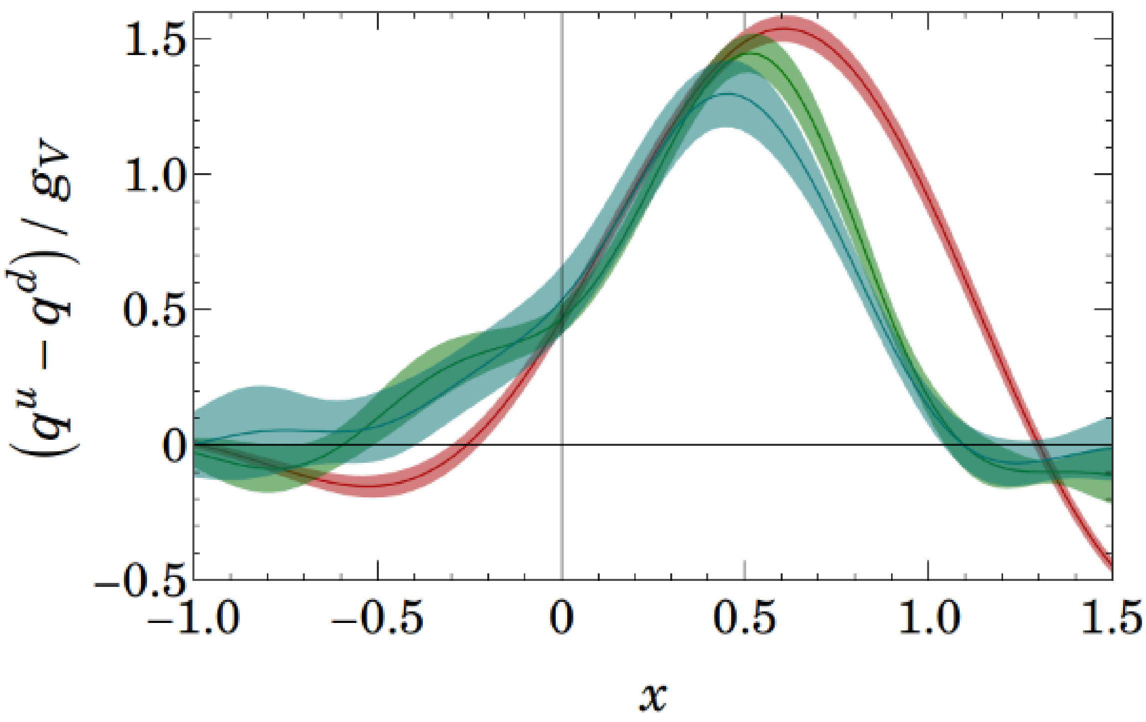
Finite $P_z \to \infty$
Estimate $O(20\%)$ effect
X. Xiong, X. Ji, J. Zhang, 1310.7471 [hep-ph]; this wokshop

Quark Distribution

§ Exploratory study

- Take ratios (partially cancel statistical and systematic errors)

$$q_{\mathrm{norm}}(x, \mu, P_z) = \frac{q(x, \mu, P_z)}{\int dx\, q(x, \mu, P_z)}$$



Removing $O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$

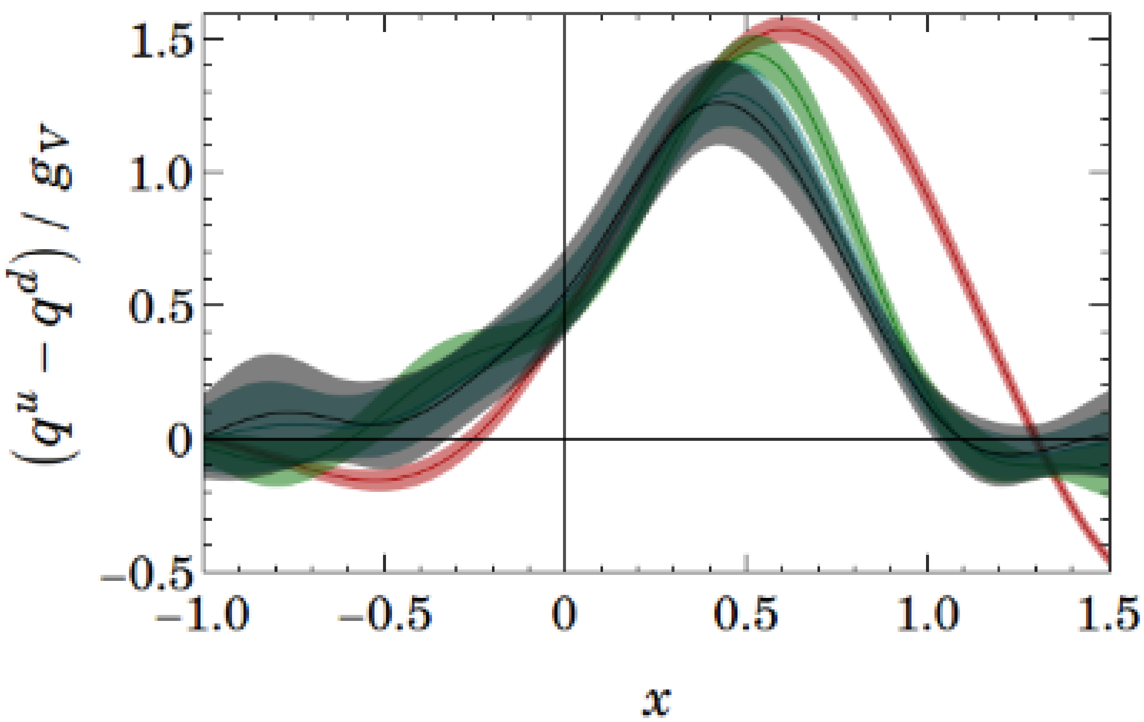
No significant finite-momentum effect seen for P_z>1

Quark Distribution

§ Exploratory study

- Take ratios (partially cancel statistical and systematic errors)

$$q_{\text{norm}}(x,\mu,P_z)=\frac{q(x,\mu,P_z)}{\int dx\, q(x,\mu,P_z)}$$



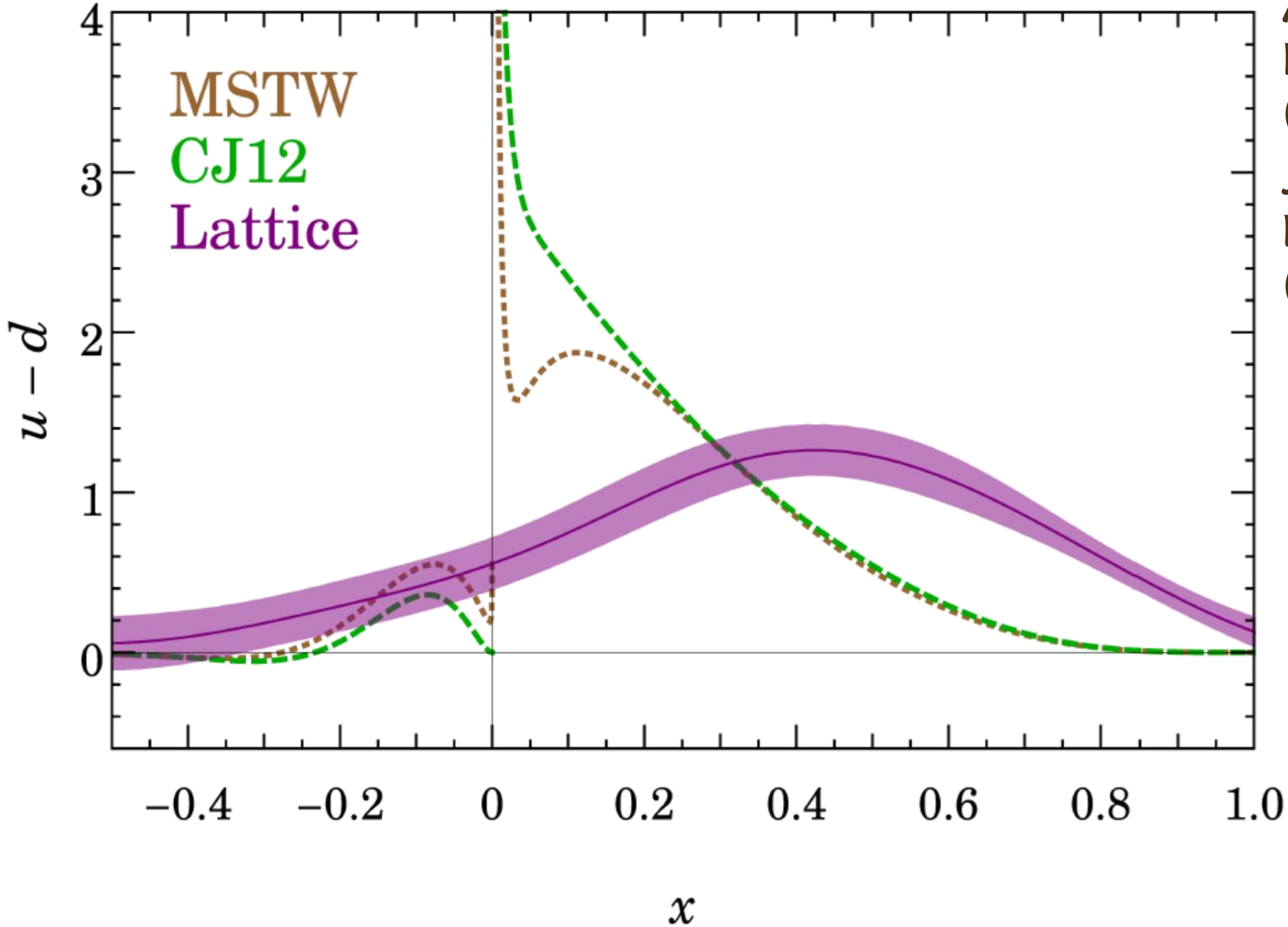
Removing $O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$

No significant finite-momentum effect seen for $P_z>1$

Further removing $O(\Lambda_{\text{QCD}}^2/P_z^2)$ errors

Quark Distribution

§ Exploratory study



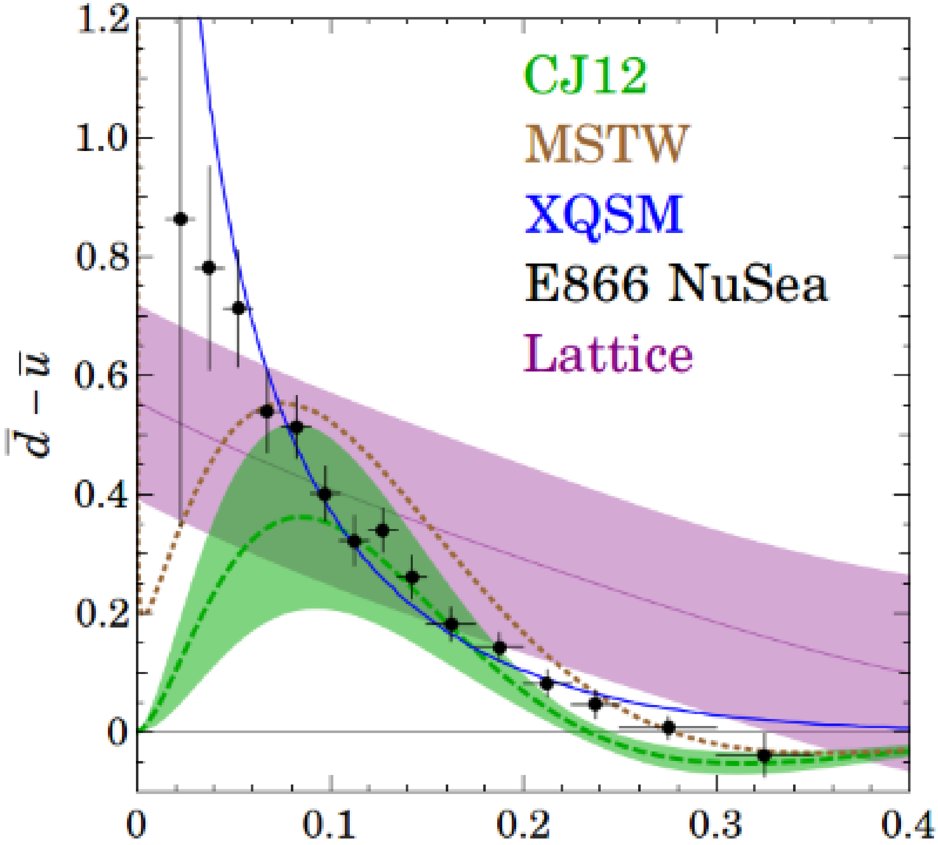
A.D. Martin et al. Eur.Phys.J. C63, 189 (2009)

J.F. Owens et al. PRD 87, 094012 (2012)

Quark Distribution

§ Compare with experiments

$$\bar{q}(x) = -q(-x)$$



R.$_x$Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)

Compared with E866

Too good to be true?

Lost resolution in small-*x* region

Future improvement to have larger lattice volume

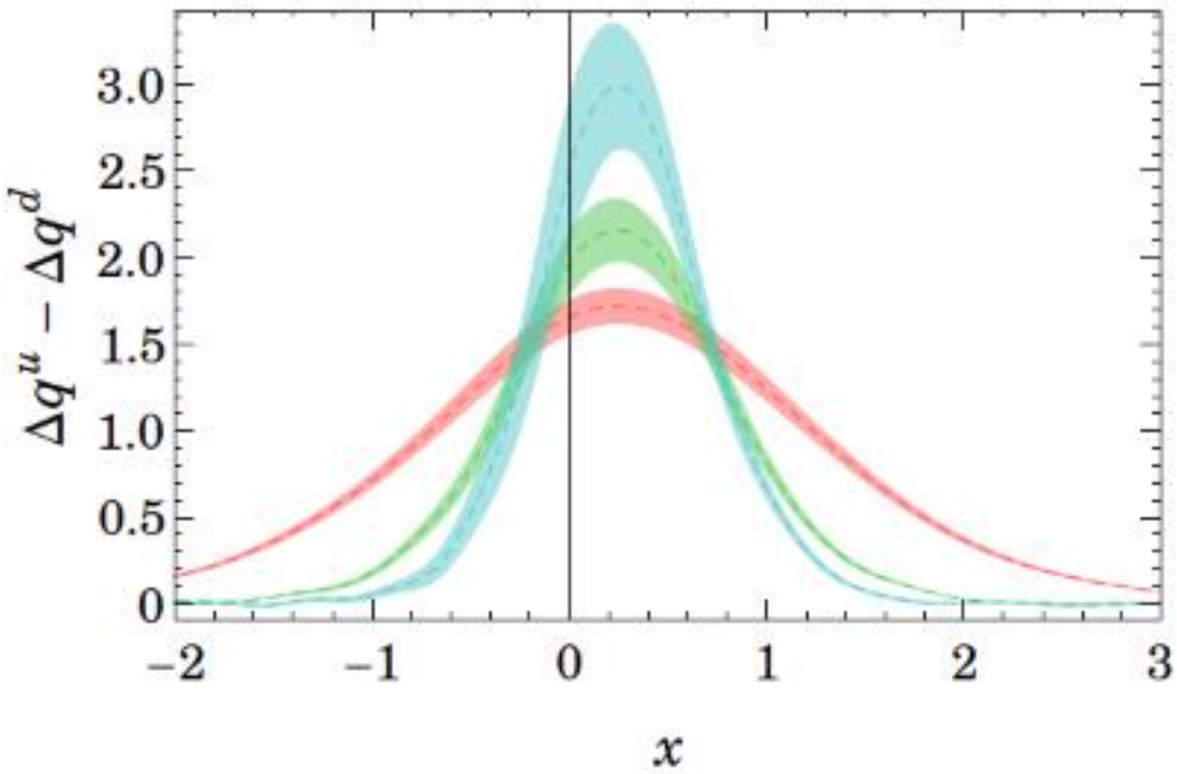
$$\int dx\,(\bar{u}(x) - \bar{d}(x)) \approx -0.16(7)$$

Experiment	x range	$\int_0^1[\overline{d}(x)-\overline{u}(x)]dx$
E866	$0.015<x<0.35$	0.118 ± 0.012
NMC	$0.004<x<0.80$	0.148 ± 0.039
HERMES	$0.020<x<0.30$	0.16 ± 0.03

Helicity Distribution

§ Exploratory study

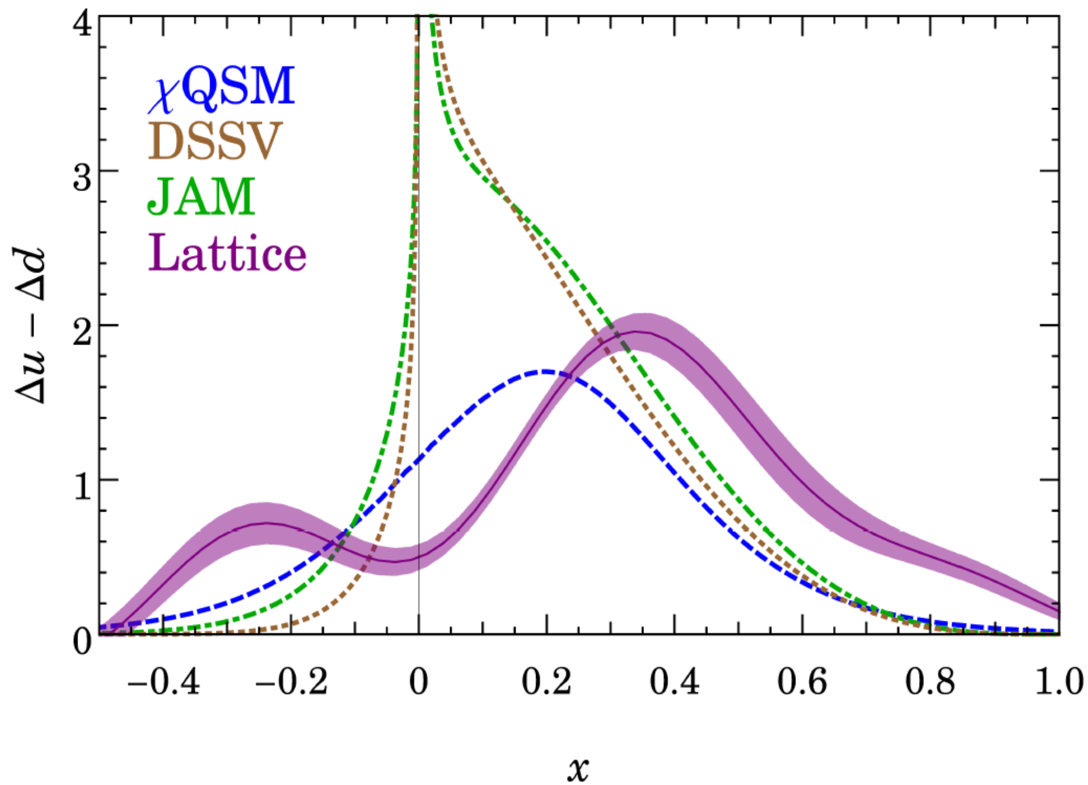
$$\int \frac{dz}{4\pi}\, e^{-izk_z} \left\langle P \middle| \bar{\psi}(z)\,\gamma_z\,\gamma_5 \exp\!\left(-ig\int_0^z dz'\,A_z(z')\right)\psi(0) \middle| P \right\rangle$$



Uncorrected bare lattice results

Helicity Distribution

§ Exploratory study



Removing $O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$

D. de Florian et al. PRD 80, 034030 (2009)

P. Jimenez-Delgado et al. arXiv:1310.3734 (2013)

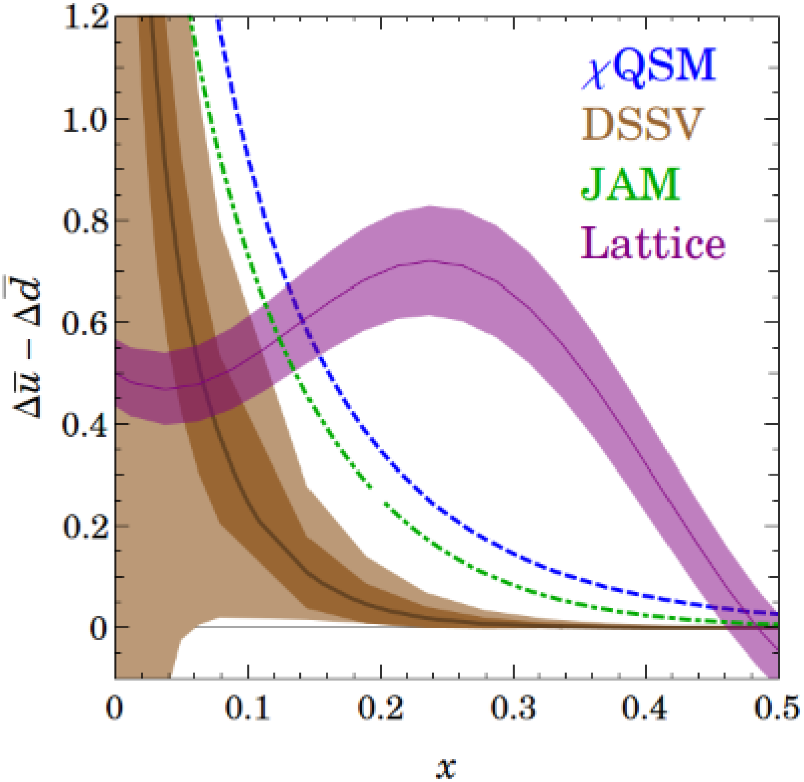
- We found $\Delta\bar{u} > \Delta\bar{d}$ with large sea asymmetry

$$\int dx\,\left(\Delta\bar{u}(x) - \Delta\bar{d}(x)\right) \approx 0.24(6)$$

Helicity Distribution

- Model: large-N_c predicts larger polarized antiquark asymmetry
  chiral quark-soliton model $\int dx \left(\Delta\bar{u}(x) - \Delta\bar{d}(x)\right) \approx 0.31$
- Experimental comparison

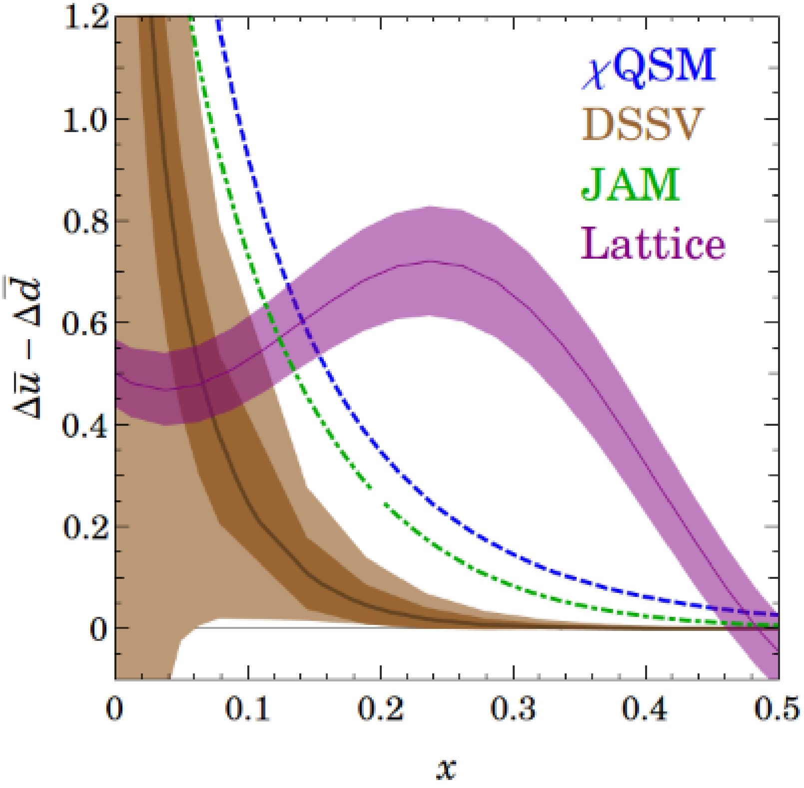
B. Dressler et al, hep-ph/9809487



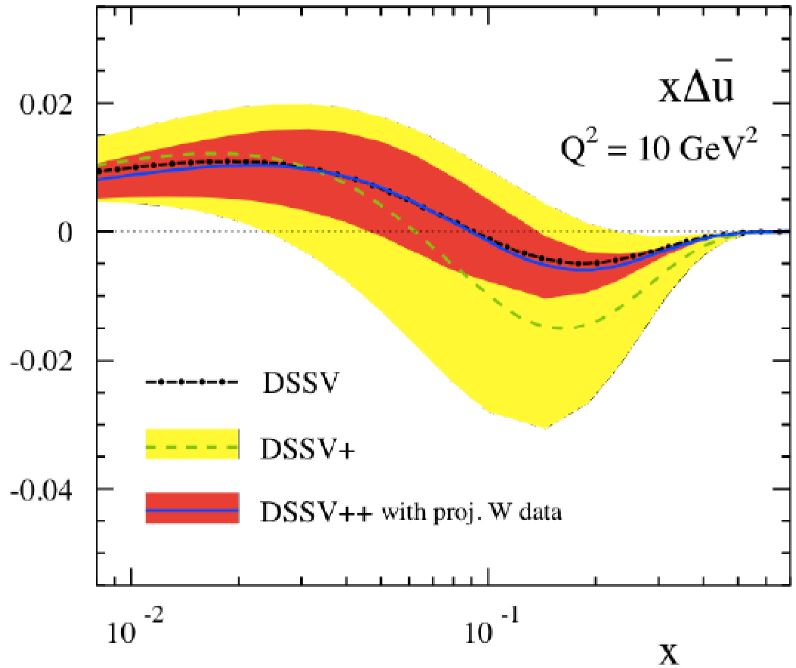
D. de Florian et al. PRD 80, 034030 (2009); P. Jimenez-Delgado et al. arXiv:1310.3734

Helicity Distribution

- § Model: large-N_c predicts larger polarized antiquark asymmetry
  chiral quark-soliton model $\int dx\,(\Delta\bar{u}(x) - \Delta\bar{d}(x)) \approx 0.31$
  B. Dressler et al, hep-ph/9809487
- § Experimental comparison



Improved fits with preliminary 2012STAR data



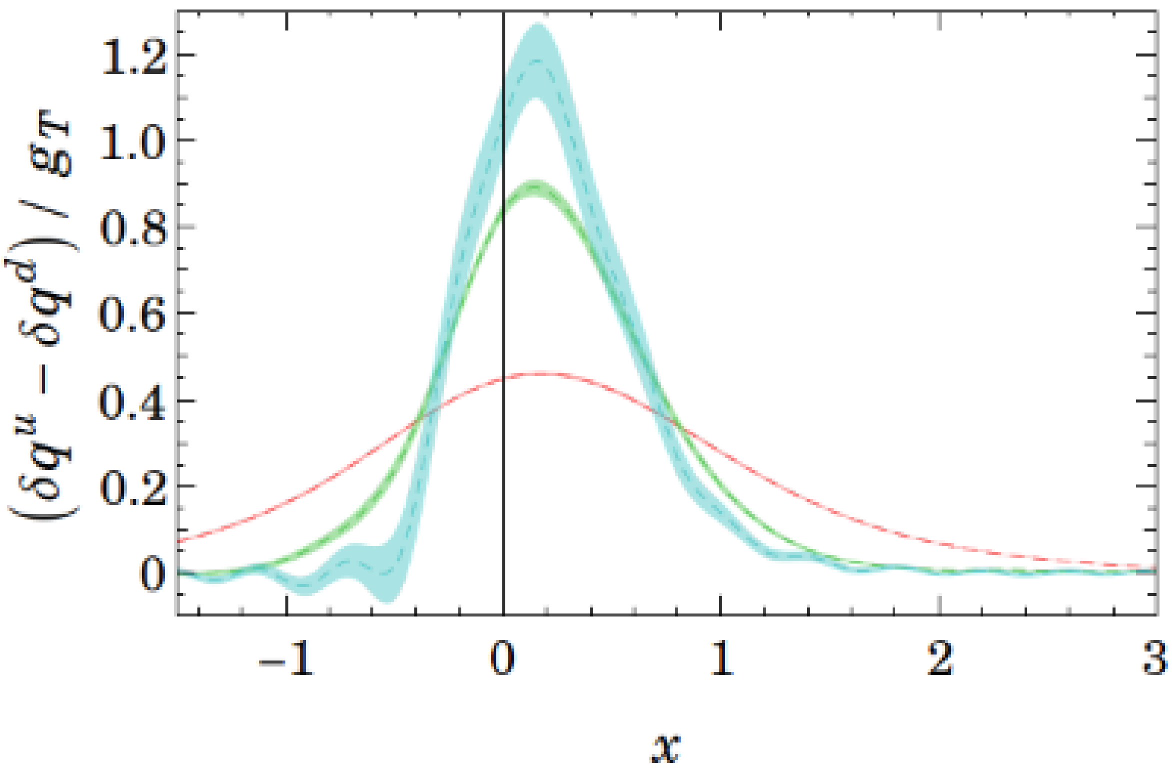
E. Aschenauer et al. (2013), 1304.0079

- § Future full data analysis from RHIC will reveal more info

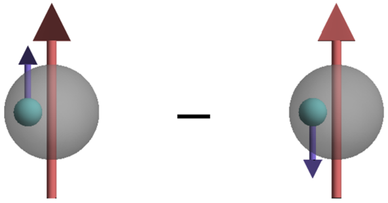
Transversity Distribution

§ Exploratory study

$$\int \frac{dz}{4\pi}\, e^{-izk_z} \left\langle P \,\middle|\, \overline{\psi}(z)\, \sigma_{xy} \exp\!\left(-ig\int_0^z dz'\, A_z(z')\right) \psi(0) \,\middle|\, P \right\rangle$$



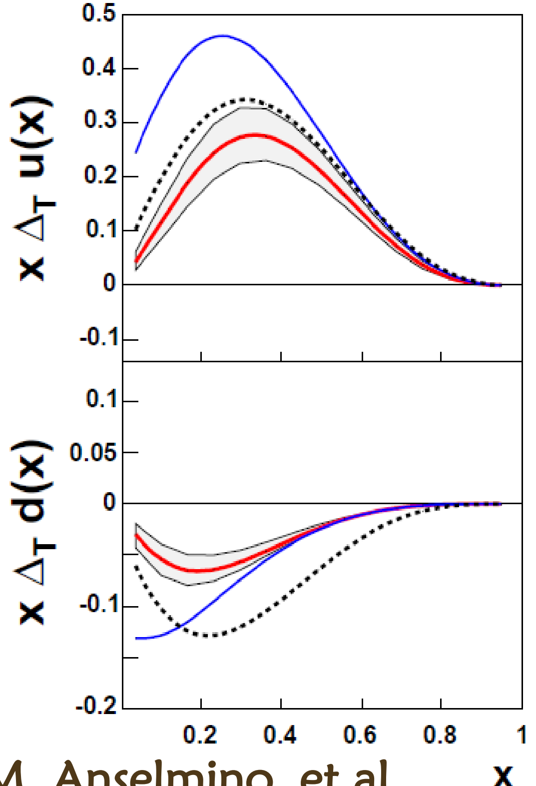
Uncorrected bare lattice results



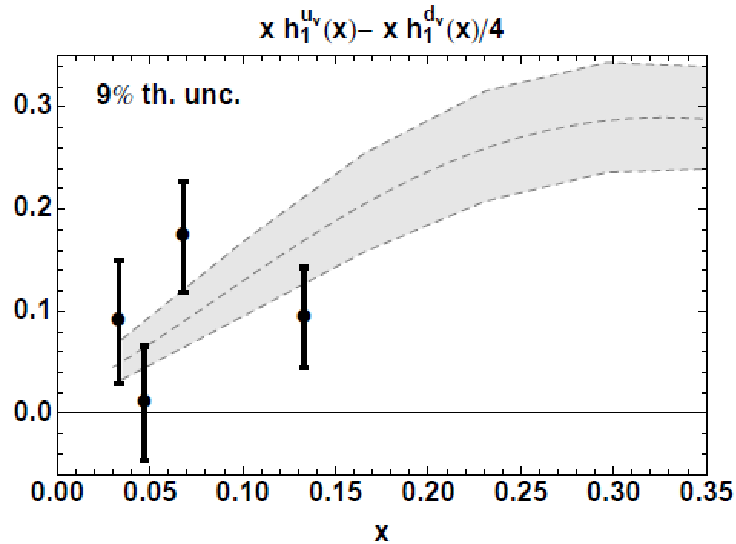
Transversity

§ There have only been 2 attempts (still very preliminary)

- Requires more theory input and experimental data
- More assumptions are made to extract the distribution



M. Anselmino, et al., Nucl.Phys.Proc.Suppl. 191, 98–107 (2009)



A. Bacchetta, A. Courtoy, and M. Radici, Phys.Rev.Lett. 107, 012001 (2011)

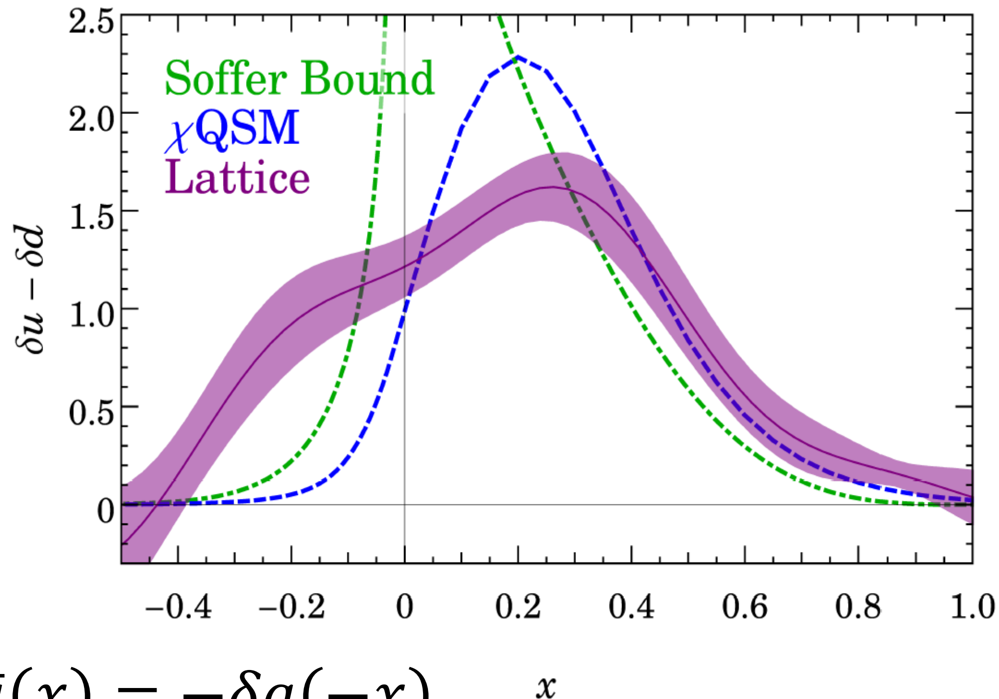
Transversity Distribution

§ Exploratory study

- We found $\delta\bar{u} < \delta\bar{d}$ with large sea asymmetry
- Chiral quark-soliton model

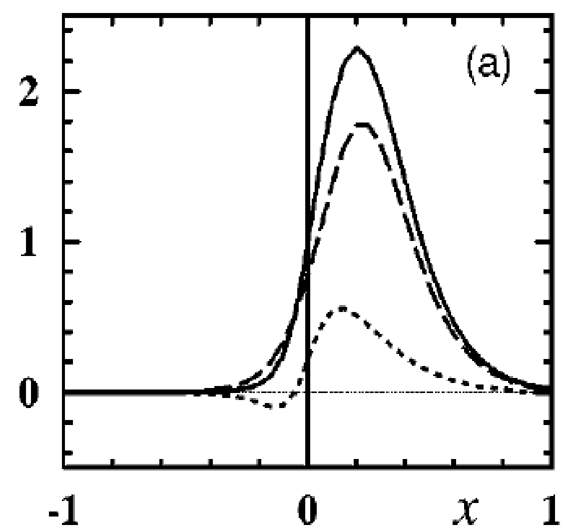
$$\int dx\,\left(\delta\bar{u}(x) - \delta\bar{d}(x)\right) \approx -0.26(10)$$

$$\int dx\,\left(\delta\bar{u}(x) - \delta\bar{d}(x)\right) \approx -0.082$$



$$\delta\bar{q}(x) = -\delta q(-x)$$

CQS model



P. Schweitzer et al. PRD 64, 034013 (2001)

A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful

Summary and Outlook

Exciting time for hadron structure on the lattice

§ Overcoming longstanding obstacle to full *x*-distribution

- Demonstrates Ji proposal can be used for practical calculations

§ First ab-initio approach to study sea asymmetry

- Promising results on unpolarized and polarized sea asymmetry compared with experiments, even at non-physical pion mass
- Prediction of transversity sea asymmetry

§ Caveats

- Not a precision calculation *yet* , better statistics, improve large-momentum signal, proper renormalization,…

§ Opens doors for future lattice-QCD work

- Wide variety of light-cone quantities can be computed

Outlook

Lattice Parton Physics Project (LP3) Workshop

https://sites.google.com/a/lbl.gov/lp3dc/

Maryland Center for
Fundamental Physics
University of Maryland
College Park, MD
March 31–April 2

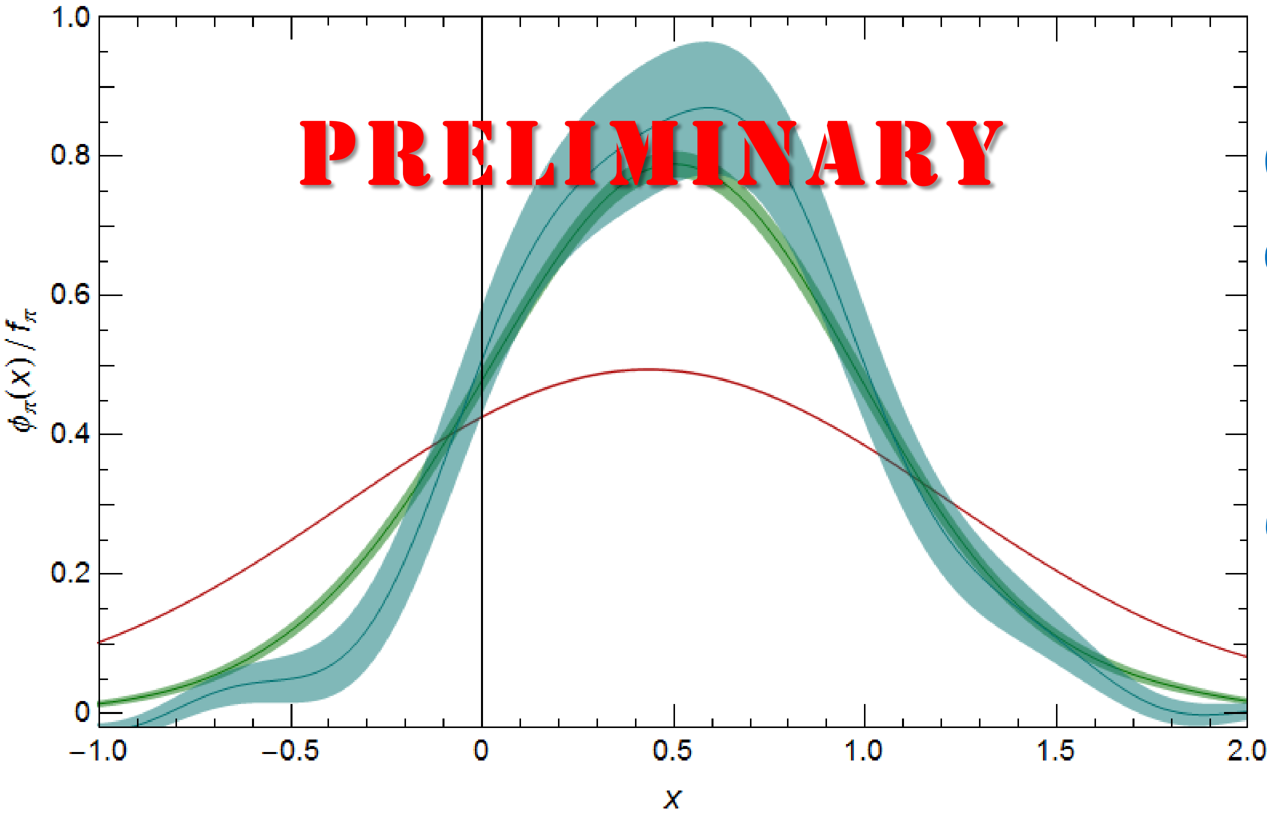
Organizers:
Xiangdong Ji (Maryland/Shanghai Jiaotong)
Huey-Wen Lin (University of Washington)
Kostas Orginos (William and Mary/JLab)
Jianwei Qiu (Brookhaven)
Christian Weiss (JLab)
Feng Yuan (LBNL)

Backup Slides

Pion Distribution Amplitude

§ Exploratory study

$$\int \frac{dz}{2\pi} e^{-izk_z} \left\langle 0 \left| \overline{d}(z)\, \gamma_z\, \gamma_5 \exp\left(-ig \int_0^z dz'\, A_z(z')\right) u(0) \right| \pi^+(P) \right\rangle$$



Only leading mass correction applied

Dominated by $O(\Lambda_{QCD}^2/P_z^2)$ errors

$P_z \in \{1, 2, 3\}\, {}^{2\pi}/_L$

Outlook

Exciting time for hadron structure on the lattice

§ Improved calculations planned

- Obtained time at NERSC for physical pion mass ensemble

2018	HP14 (new)	Extract accurate information on spin-dependent and spin-averaged valence quark distributions to momentum fractions x above 60% of the full nucleon momentum

- Achievable with supercomputing

§ More: strange and heavy-quark distributions, gluons, TMD…

- Charm: future EIC (eRHIC?), LHC $pp \to \gamma c X$, …
- Polarized gluon: polarized pp collisions at RHIC, …