Sivers Function in the Quasi-Classical Approximation

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(based on arXiv:1310.5028 [hep-ph] with Matt Sievert)

Outline

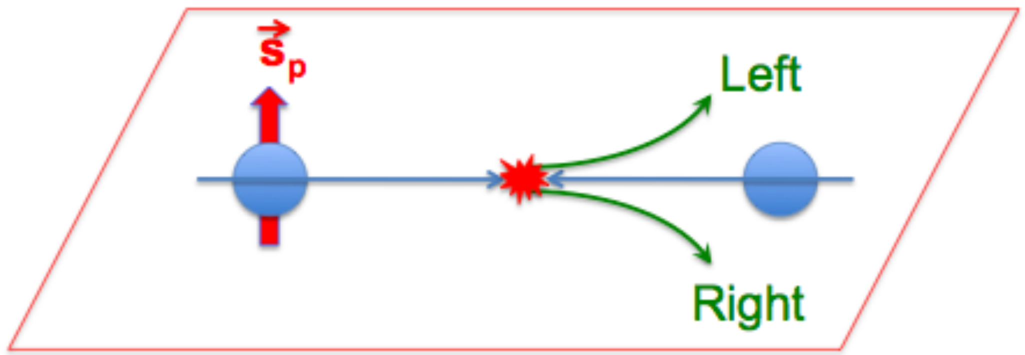
- Preview/Summary
- STSA sign-flip in SIDIS vs DY: diagrammatic interpretation?
- Quasi-classical picture of STSA generation:
  - Quasi-classical STSA: the OAM and transversity channels
  - Space-time picture and physical/diagrammatic interpretation of sign-flip
- Outlook: can calculate any TMD in the quasi-classical picture.

Preview/Summary

Single Transverse Spin Asymmetry

- Consider transversely polarized proton scattering on an unpolarized proton or nucleus.

$$p(\vec{s}_\perp) + p \to h(\pi^\pm, \pi^0, \ldots) + X$$

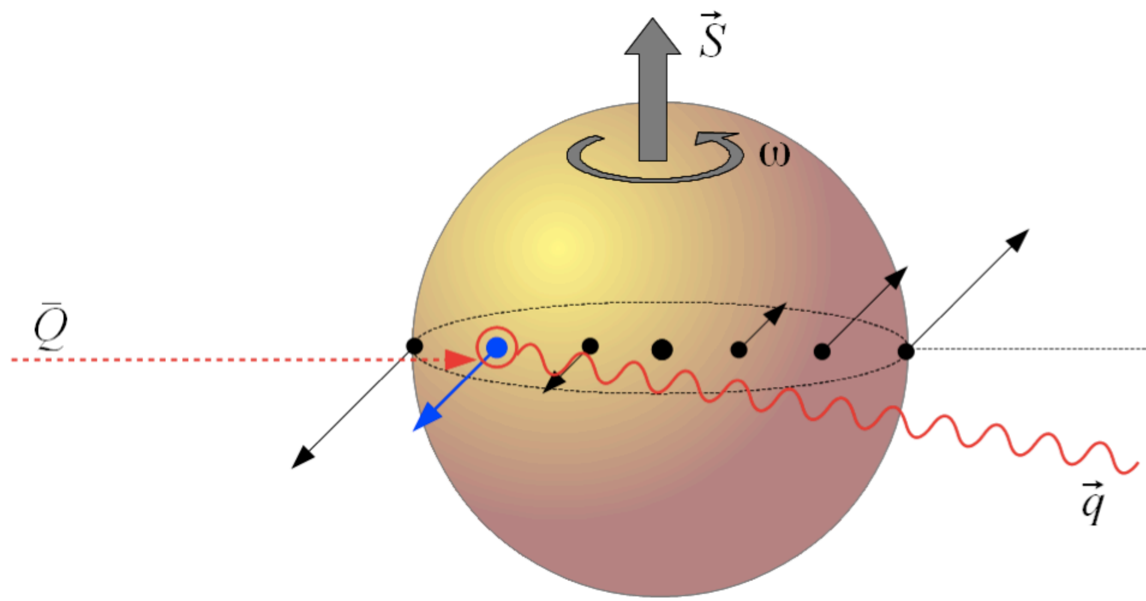


- Single Transverse Spin Asymmetry (STSA) is defined by

$$A_N(\mathbf{k}) \equiv \frac{\frac{d\sigma^\uparrow}{d^2k\,dy} - \frac{d\sigma^\downarrow}{d^2k\,dy}}{\frac{d\sigma^\uparrow}{d^2k\,dy} + \frac{d\sigma^\downarrow}{d^2k\,dy}} = \frac{\frac{d\sigma^\uparrow}{d^2k\,dy}(\mathbf{k}) - \frac{\mathbf{d}\sigma^\uparrow}{\mathbf{d^2k\,dy}}(-\mathbf{k})}{\frac{d\sigma^\uparrow}{d^2k\,dy}(\mathbf{k}) + \frac{\mathbf{d}\sigma^\uparrow}{\mathbf{d^2k\,dy}}(-\mathbf{k})} \equiv \frac{d(\Delta\sigma)}{2\,d\sigma_{unp}}$$

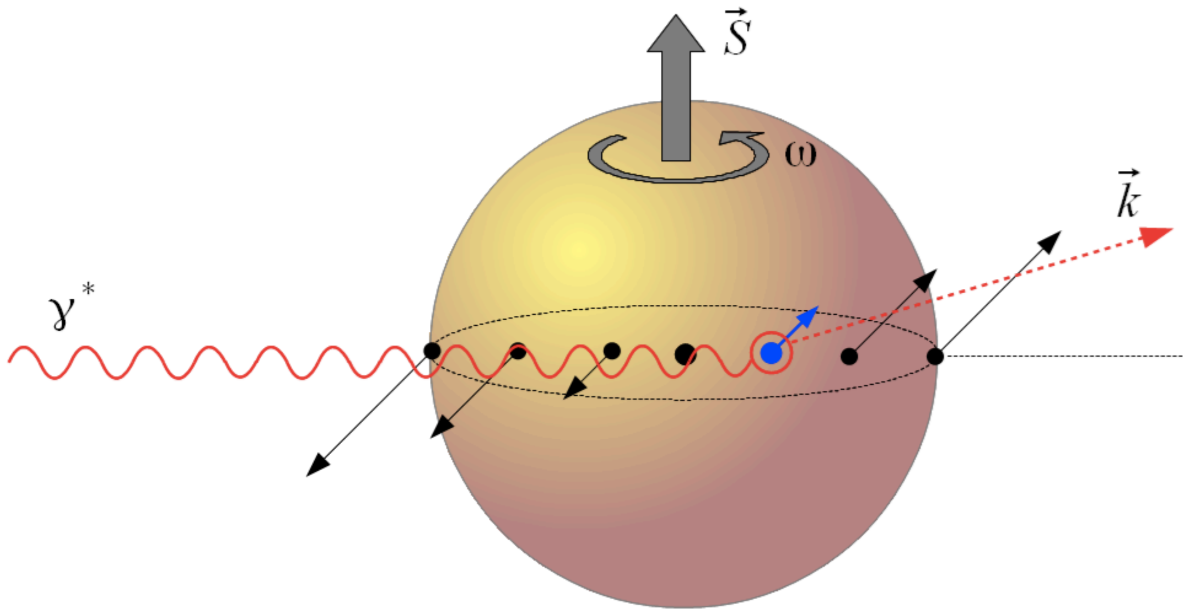
Classical picture of STSA in Drell-Yan

- Think of a transversely polarized proton as a rotating disk with the axis perpendicular to the collision axis
- The proton is not transparent: it has some amount of screening/shadowing (e.g. gray disk, black disk, etc.)
- Incoming anti-quark (in DY) is more likely to interact near the "front" of the proton: hence, due to the rotation, the outgoing virtual photon is more likely to be produced **left-of-beam**, thus generating STSA.



Classical picture of STSA in SIDIS

- Ditto for SIDIS: except now the incoming virtual photon is more likely to interact near the "back" of the proton, in order for the produced quark to be able to escape out of the proton remnants.
- Owing to the same rotation, the outgoing quark is more likely to be produced **right-of-beam**, thus generating STSA in SIDIS with the **opposite sign** compared to STSA in DY!



Summary

- It appears that STSA can be easily interpreted as a combination of OAM and some amount of shadowing.
- Sign-flip between STSA and DY has a very simple interpretation in this framework too.

In search of a diagrammatic interpretation of STSA sign-flip

Definitions

- TMDs are defined through the correlator

$$\Phi_{ij}(x, \underline{k}; P, S) \equiv \int \frac{dx^- \, d^2 x_\perp}{2(2\,\pi)^3} \, e^{i\left(\frac{1}{2}\, x\, P^+\, x^- - \underline{x}\cdot\underline{k}\right)} \, \langle P, S | \bar{\psi}_j(0)\, \mathcal{U}\, \psi_i(x^+ = 0, x^-, \underline{x}) | P, S \rangle$$

with the gauge links

$$\mathcal{U}^{SIDIS} = V_{\underline{0}}^\dagger[+\infty\,,\,0]\, V_{\underline{x}}[+\infty\,,\,x^-]$$
$$\mathcal{U}^{DY} = V_{\underline{0}}[0\,,\,-\infty]\, V_{\underline{x}}^\dagger[x^-\,,\,-\infty]$$

- The correlator is decomposed in terms of quark TMDs as

$$\Phi_{ij}(x, \underline{k}; P, S) = \frac{M}{2\,P^+} \left[f_1(x, k_T)\, \frac{P\cdot\gamma}{M} + \frac{1}{M^2}\, f_{1T}^\perp(x, k_T)\, \epsilon_{\mu\nu\rho\sigma}\, \gamma^\mu\, P^\nu\, k_\perp^\rho\, S_\perp^\sigma - \frac{1}{M}\, g_{1s}(x, \underline{k})\, P\cdot\gamma\,\gamma^5 \right.$$
$$\left. - \frac{1}{M}\, h_{1T}(x, k_T)\, i\,\sigma_{\mu\nu}\,\gamma^5\, S_\perp^\mu\, P^\nu - \frac{1}{M^2}\, h_{1s}^\perp(x, \underline{k})\, i\,\sigma_{\mu\nu}\,\gamma^5\, k_\perp^\mu\, P^\nu + h_1^\perp(x, k_T)\,\sigma_{\mu\nu}\, \frac{k_\perp^\mu\, P^\nu}{M^2} \right]_{ij}$$

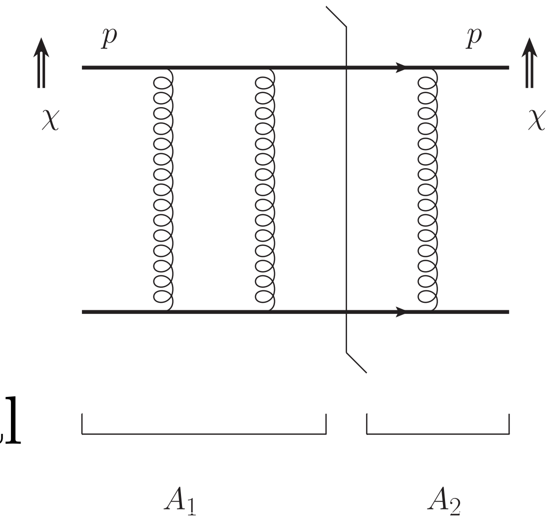
STSA spin-flip between SIDIS and DY

- Operatorially everything is clear (Collins 2002): under T-reversal Wilson lines go from future-pointing (SIDIS) to past-pointing (DY), while the Sivers function, being T-odd, changes sign, such that

$$f_{1T}^{\perp A}(x,k_T)\Big|_{SIDIS} = -f_{1T}^{\perp A}(x,k_T)\Big|_{DY}$$

- Diagrammatically things are not so simple…

What generates STSA



Efremov, Teryaev '82

- To obtain STSA need
  - transverse polarization (χ) dependence (comes with a factor of "i")

$$\bar{u}_\chi(p)\,\Gamma\,u_\chi(p) \sim a + i\,\chi\,b, \qquad a, b \text{ real}$$

However, cross section has to be real

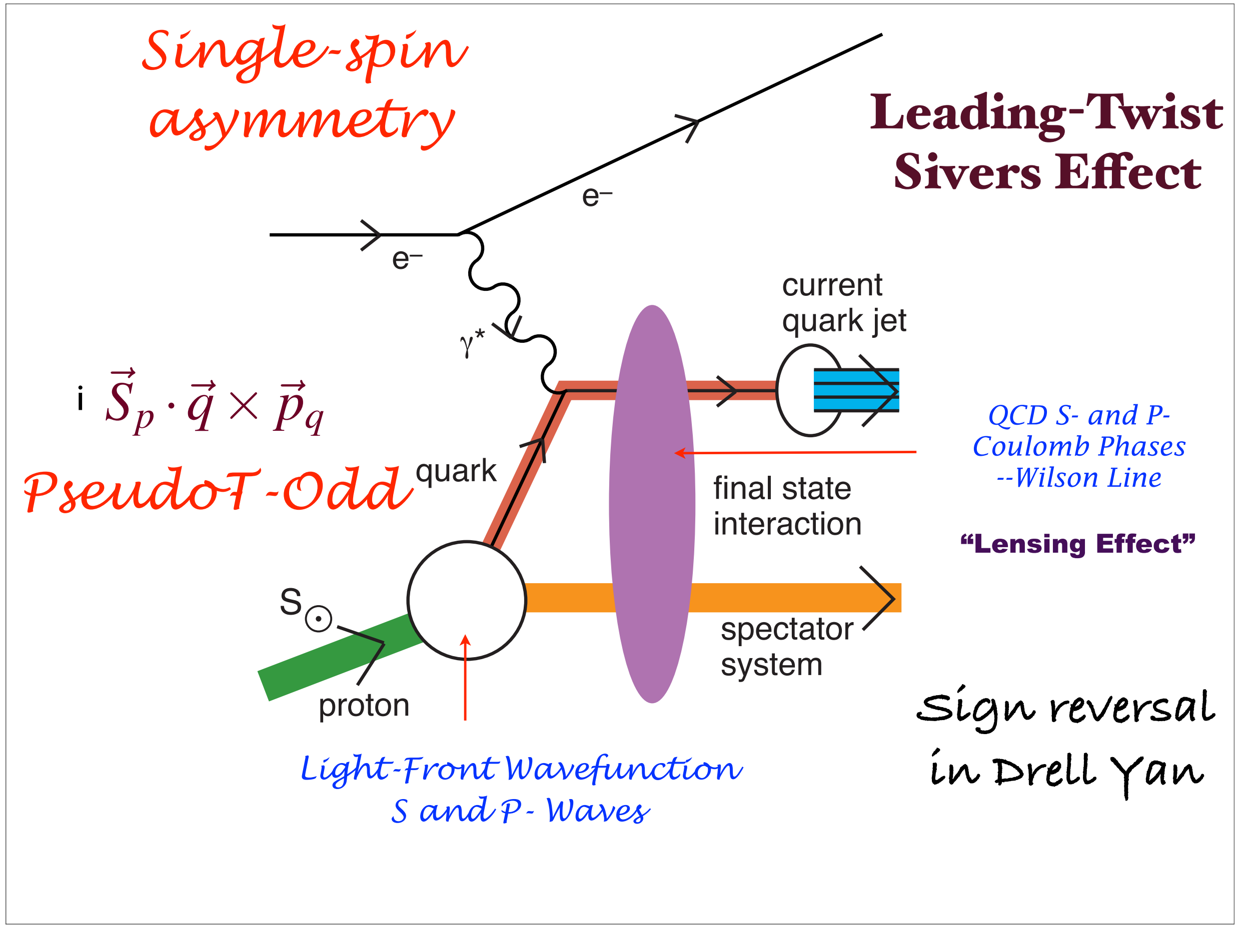
$$\sigma \sim \bar{u}_\chi(p)\,\Gamma\,u_\chi(p)\,A_1\,A_2^* + \text{c.c} = (a + i\,\chi\,b)\,A_1\,A_2^* + (a - i\,\chi\,b)\,A_1^*\,A_2$$
$$= (\chi - \text{independent}) + i\,\chi\,b\,(A_1\,A_2^* - A_1^*\,A_2)$$

such that we also need

  - a complex phase difference between the amplitude (A_1) and the cc amplitude (A_2) to cancel the "i" from χ-dependence

(from Qiu and Sterman, early 90's)

STSA IN SIDIS

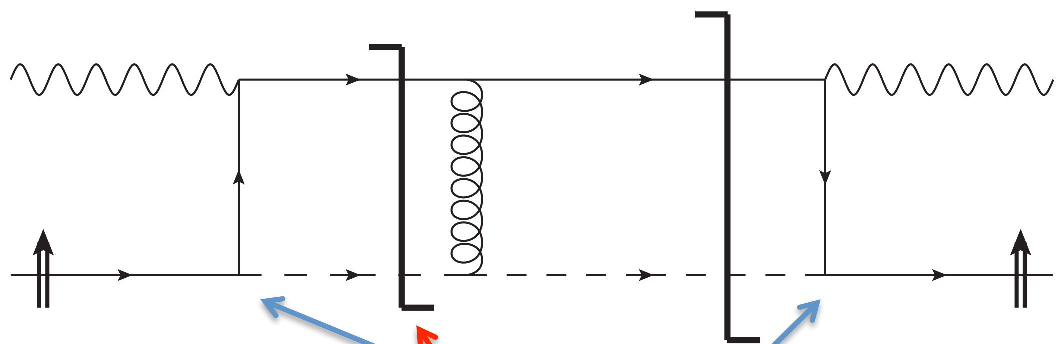


Brodsky, Hwang, Schmidt '02; Collins '02

- To generate STSA need a final state interaction (the blob above).
- In TMD factorization this is usually absorbed into the polarized proton TMD and is referred to as the initial-state effect, and hence identified with the Sivers effect.

STSA in SIDIS

- STSA arises from the interference diagrams between Born-level and the one-rescattering graphs:



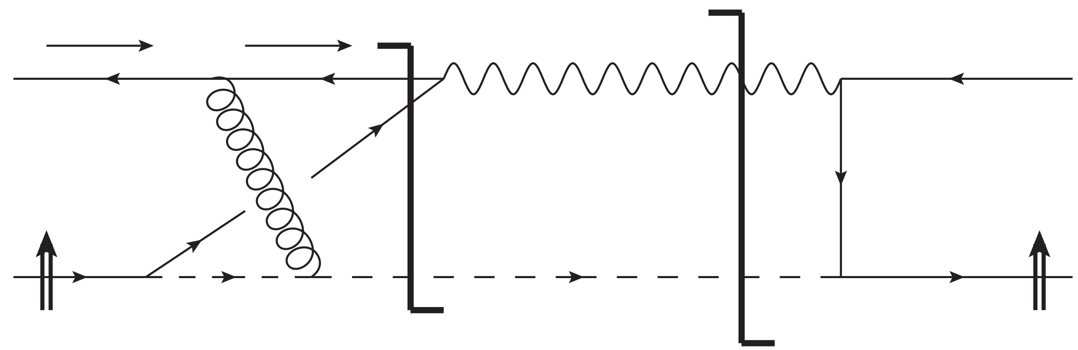
- Spin-dependence comes from the vertex.

- The phase is generated by an extra rescattering, which gives the amplitude an Im part represented by the second “cut”.

Brodsky, Hwang, Schmidt ’02;
(see also Brodsky, Hwang, YK, Schmidt, Sievert ’13)

STSA in Drell-Yan

- Here we also need interference between the Born level amplitude and a one-rescattering correction to it.



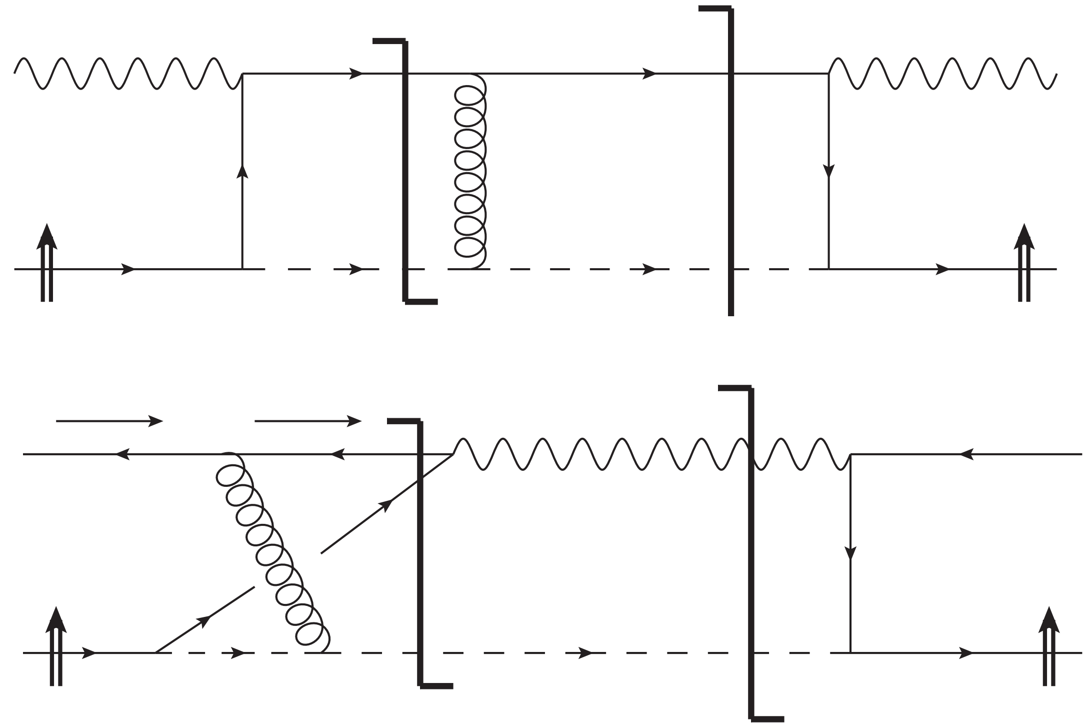
- The DY STSA is also caused by two essential ingredients: (i) spin dependence from the quark-proton vertex and (ii) phase due to the extra cut (intermediate state) in the amplitude.
- Note that the 2nd cuts in SIDIS and DY are different! Still at large Q^2 the two STSAs are equal up to a sign reversal:

$$A_N^{DY} = -A_N^{SIDIS}$$

(Collins ’02; Brodsky, Hwang, Schmidt ’02; same+ YK, Sievert ‘13).

Side-by-side: SIDIS and DY

- The second cuts in the SIDIS and DY STSA diagrams look very different: in SIDIS we cut the s-channel quark and the di-quark, while in DY one has to cut the s-channel anti-quark and the exchanged quark.



- Diagrammatically the sign-flip is not obvious. (Math works, but can we see why diagrammatically?)

Quasi-Classics

Rules of the game

- We want to evaluate STSA in the Glauber-Mueller (GM)/ McLerran-Venugopalan (MV) quasi-classical approximation.
- This approximation is valid for a large nucleus or a "dense" proton.
- I will talk about the nucleus simply because the nuclear atomic number A allows for a controlled approximation: quasi-classical physics resums powers of

$$\alpha_s^2 \, A^{1/3}$$

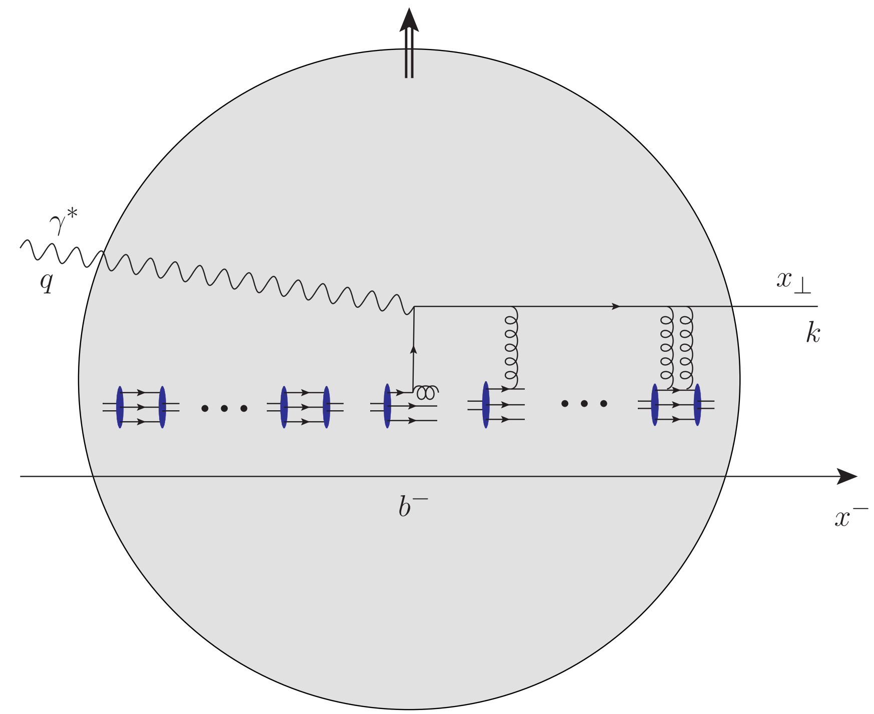
- Note that the GM/MV quasi-classics is not just a model, but is a valid QCD asymptotics in the high-energy & large-A limit.

Quark Production

- Start with inclusive classical quark production cross section in SIDIS.

- The kinematics is standard:

$$s \sim Q^2 \gg \perp^2$$



- The result is

Wigner distribution

$$\frac{d\sigma^{\gamma^* + A \to q + X}}{d^2 k \, dy} = A \int \frac{dp^+ \, d^2 p \, db^-}{2(2\pi)^3} \int d^2 x \, d^2 y \; W\left(p, b^-, \frac{\underline{x} + \underline{y}}{2}\right)$$
$$\times \int \frac{d^2 k'}{(2\pi)^2} \, e^{-i\,(\underline{k} - \underline{k}') \cdot (\underline{x} - \underline{y})} \, \frac{d\hat{\sigma}^{\gamma^* + N \to q + X}}{d^2 k' \, dy} (p, q) \; D_{\underline{x}\,\underline{y}}[+\infty, b^-]$$

LO cross sect **Wilson lines**

Wilson lines

- Here

$$D_{\underline{x}\,\underline{y}}[+\infty, b^-] = \left\langle \frac{1}{N_c} \,\mathrm{Tr} \left[V_{\underline{x}}[+\infty, b^-] \, V_{\underline{y}}^\dagger[+\infty, b^-] \right] \right\rangle$$

is the quark dipole scattering S-matrix with

$$V_{\underline{x}}[b^- \, , \, a^-] \equiv \mathcal{P} \, \exp \left[\frac{ig}{2} \int\limits_{a^-}^{b^-} dx^- A^+(x^+ = 0, x^-, \underline{x}) \right]$$

denoting Wilson lines.

- In the quasi-classical approximation D_{xy} is real!

Dipole scattering in the GM/MV limit

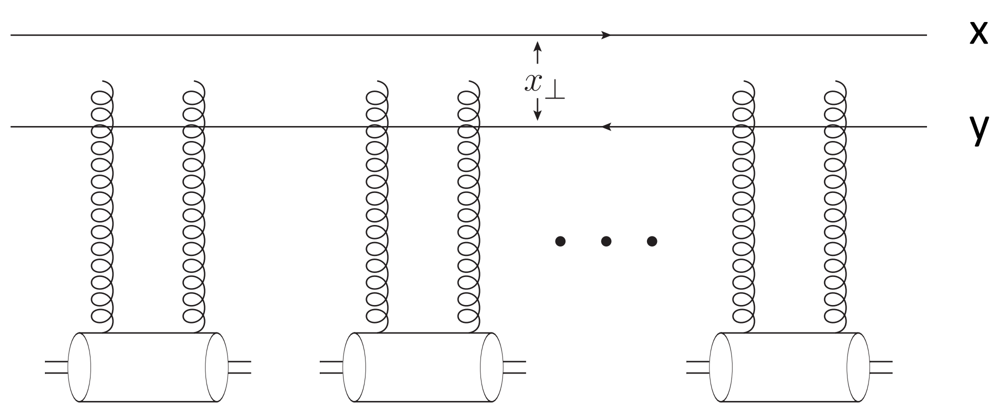
- The dipole S-matrix $$D_{\underline{x}\,\underline{y}}[+\infty, b^-] = \left\langle \frac{1}{N_c} \, \text{Tr} \left[V_{\underline{x}}[+\infty, b^-] \, V_{\underline{y}}^\dagger[+\infty, b^-] \right] \right\rangle$$

  taken in the symmetrized form $$S_{\underline{x}\underline{y}} = \frac{D_{\underline{x}\underline{y}} + D_{\underline{y}\underline{x}}}{2}$$

  in the GM/MV approximation is (note – all real)

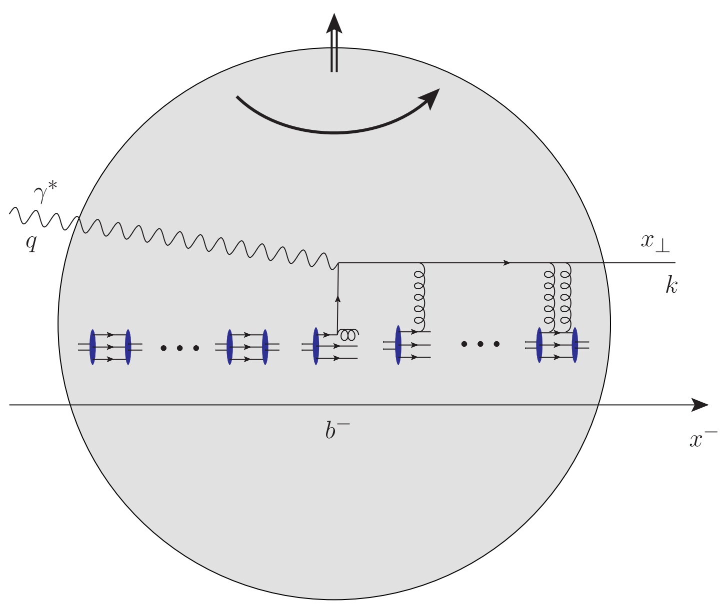
$$S_{\underline{x}\underline{y}}[+\infty, b^-] = \exp\left[-\frac{1}{4} |\underline{x} - \underline{y}|^2 \, Q_s^2 \left(\frac{\underline{x} + \underline{y}}{2} \right) \left(\frac{R^-(\underline{b}) - b^-}{2R^-(\underline{b})} \right) \ln \frac{1}{|\underline{x} - \underline{y}|\Lambda} \right]$$

- This corresponds to multiple rescatterings as shown here:

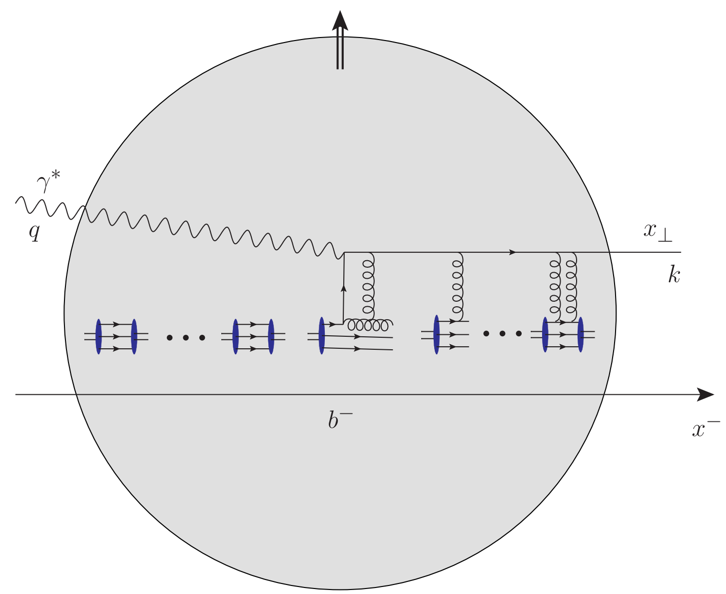


Quasi-Classical STSA in SIDIS

- To generate STSA we need spin-dependence and a complex phase. They may come from three sources:
  - Sivers functions of the (polarized) nucleons: transversity channel
  - Orbital rotation due to OAM (just gives real OAM-dependence)
  - Going beyond classical approximation in D_{xy} by including extra rescatterings per nucleon (the odderon): this gives a phase, but is A-suppressed. Hence we will drop it.



OAM Channel



Transversity Channel

Quasi-Classical STSA in SIDIS

- When the dust settles one gets

$$\hat{z}\cdot(\underline{J} \times \underline{k})\, f_{1T}^{\perp A}(\bar{x}, k_T) = M_A \int \frac{dp^+\, d^2p\, db^-}{2(2\pi)^3}\, d^2x\, d^2y\, \frac{d^2k'}{(2\pi)^2}\, e^{-i\,(\underline{k}-\underline{k}')\cdot(\underline{x}-\underline{y})}$$

$$\times \left\{ i\, x\, \underline{p} \cdot (\underline{x} - \underline{y})\, A\, W_{unp}^{OAM} \left(p^+, \underline{p}, b^-, \frac{\underline{x}+\underline{y}}{2} \right) f_1^N(x, k'_T) \right.$$

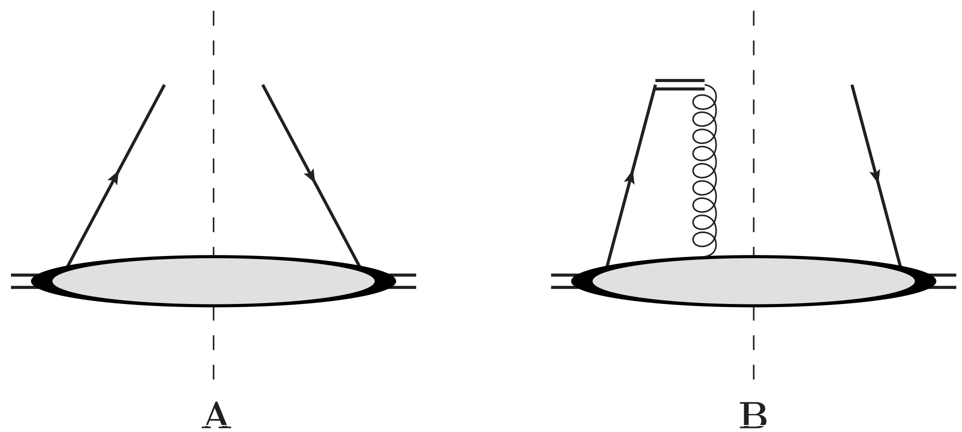
$$\left. + \frac{1}{m_N}\, \hat{z} \cdot \left(\underline{S} \times \underline{k}'\right)\, W_{trans}^{symm} \left(p^+, \underline{p}, b^-, \frac{\underline{x}+\underline{y}}{2} \right) f_{1T}^{\perp N}(x, k'_T) \right\} S_{\underline{x}\underline{y}}[+\infty, b^-]$$

- J = L+S = total spin of the nucleus (analogue of proton spin)
  L = net OAM of the nucleons (analogue of quark and gluon OAM)
  S = net spin of all nucleons (analogue of net spin of quarks and gluons)
- S_{xy} = dipole rescattering S-matrix
- W = Wigner distributions of nucleons, unp = unpolarized (may have <S>=0), trans = transversity, with

$$W^{\binom{symm}{OAM}}(p, b) \equiv \tfrac{1}{2}\left[W(p, b) \pm (\underline{p} \to -\underline{p}) \right]$$

LO TMDs

- f_1^N and $f_{1T}^{\perp N}$ are the LO unpolarized quark TMD (in a nucleon) and the "nucleon" Sivers function:



- They can be arbitrary non-perturbative objects: we can always "run" them through the quasi-classical "grinder". ;)
- Resulting quasi-classical Sivers function can be used as initial condition for evolution equations (e.g. CSS for s ~Q^2 or small-x evolution if s>>Q^2).

OAM and Transversity channels

- The final formula is a sum of the contributions of the OAM and Transversity (Sivers function density) channels:

$$\hat{z}\cdot(\underline{J}\times\underline{k})\, f_{1T}^{\perp A}(\bar{x},k_T) = M_A \int \frac{dp^+\, d^2p\, db^-}{2(2\pi)^3}\, d^2x\, d^2y\, \frac{d^2k'}{(2\pi)^2}\, e^{-i\,(\underline{k}-\underline{k}')\cdot(\underline{x}-\underline{y})}$$

$$\times \left\{ i\, x\, \underline{p}\cdot(\underline{x}-\underline{y})\, A\, W_{unp}^{OAM}\left(p^+,\underline{p},b^-,\frac{\underline{x}+\underline{y}}{2}\right) f_1^N(x,k_T') \right.$$

OAM channel

$$\left. + \frac{1}{m_N}\,\hat{z}\cdot\left(\underline{S}\times\underline{k}'\right) W_{trans}^{symm}\left(p^+,\underline{p},b^-,\frac{\underline{x}+\underline{y}}{2}\right) f_{1T}^{\perp N}(x,k_T') \right\} S_{\underline{xy}}[+\infty,b^-]$$

Transversity channel

- The non-perurbative input comes though the Wigner functions and LO TMDs: the dipole scattering is perturbative due to $Q_s >> \Lambda_{QCD}$.

Rigid Rotator Model

- To get the feel for what the result is like, our main formula can be evaluated using a rigid-rotator nucleus with simple (powers of k_T) models of LO TMDs.

- The result is (for k_T not much larger than Q_s)

$$f_{1T}^{\perp A}(\bar{x}, k_T) = \frac{m_N \, N_c}{2\pi \, \alpha_s \, C_F} \, \frac{1}{\beta + \frac{8}{5} \, p_{max} \, R} \, \frac{1}{k_T^2}$$

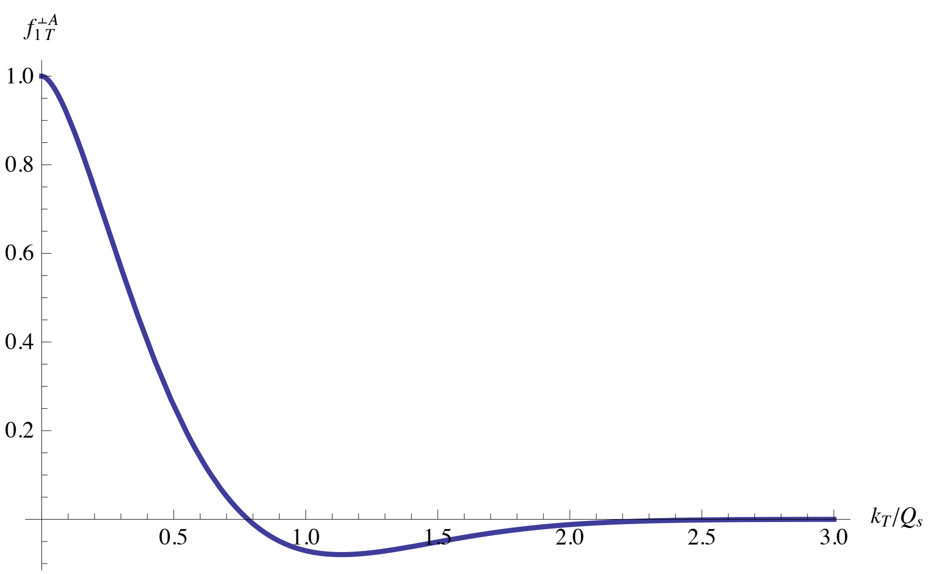
$$\times \int d^2 b \left\{ 4 \, \bar{x} \, p_{max}(\underline{b}) \, C_1 \, \left[e^{-k_T^2/Q_s^2(\underline{b})} + 2 \, \frac{k_T^2}{Q_s^2(\underline{b})} \, Ei \left(- \frac{k_T^2}{Q_s^2(\underline{b})} \right) \right] + \alpha_s \beta \, m_N \, C_2 \, e^{-k_T^2/Q_s^2(\underline{b})} \right\}$$

OAM channel **Transversity channel**

- Note a “new” functional form for Sivers function due to the OAM channel (not a Gaussian).

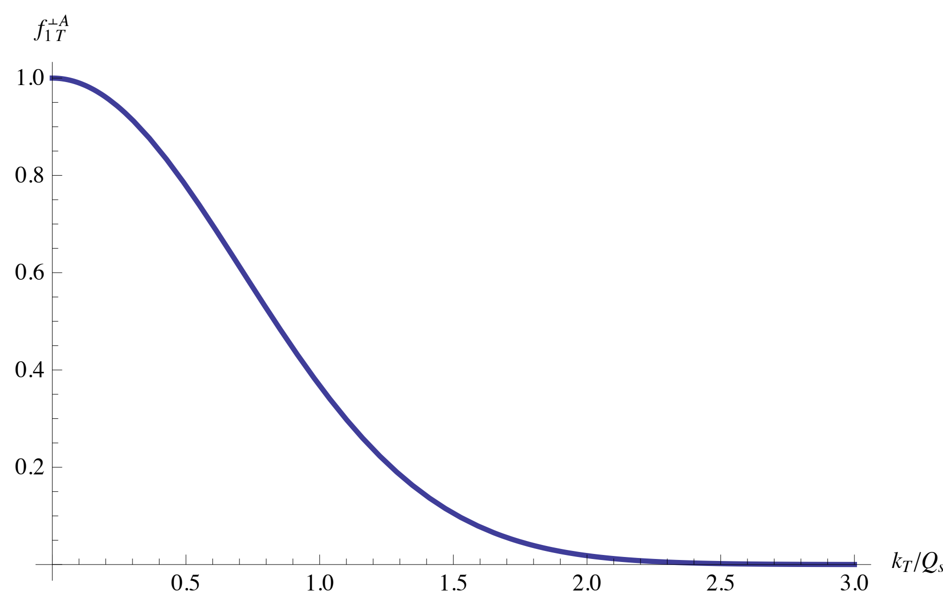
Rigid Rotator Model

- The OAM contribution in this model (!) calculation is shown below (arbitrary units). It changes sign as a function of k_T.



Rigid Rotator Model

- The transversity (Sivers density) contribution comes out to be a simple Gaussian (in this calculation); units are again arbitrary:



STSA at Large-k_T

- For $k_T >> Q_s$ we get

$$f_{1T}^{\perp A}(\bar{x}, k_T)\big|_{k_T \gg Q_s} = \frac{S}{J}\left[-\frac{4\,\alpha_s\,m_N\,\bar{x}\,C_1}{3\beta\,k_T^6}\,\ln\frac{k_T^2}{\Lambda^2}\int d^2b\,T(\underline{b})\,p_{max}(\underline{b})\,Q_s^2(\underline{b}) + A\,f_{1T}^{\perp N}(\bar{x}, k_T)\right]$$

$$= \frac{\beta}{\beta + \frac{8}{5}\,p_{max}\,R}\left[-\frac{4\,\alpha_s\,m_N\,\bar{x}\,C_1}{3\beta\,k_T^6}\,\ln\frac{k_T^2}{\Lambda^2}\int d^2b\;T(\underline{b})\,p_{max}(\underline{b})\,Q_s^2(\underline{b}) + \frac{A\,\alpha_s^2\,m_N^2\,C_2}{k_T^4}\,\ln\frac{k_T^2}{\Lambda^2}\right]$$

OAM channel **Transversity channel**

- At high-k_T the transversity (Sivers density) channel dominates over the OAM one.

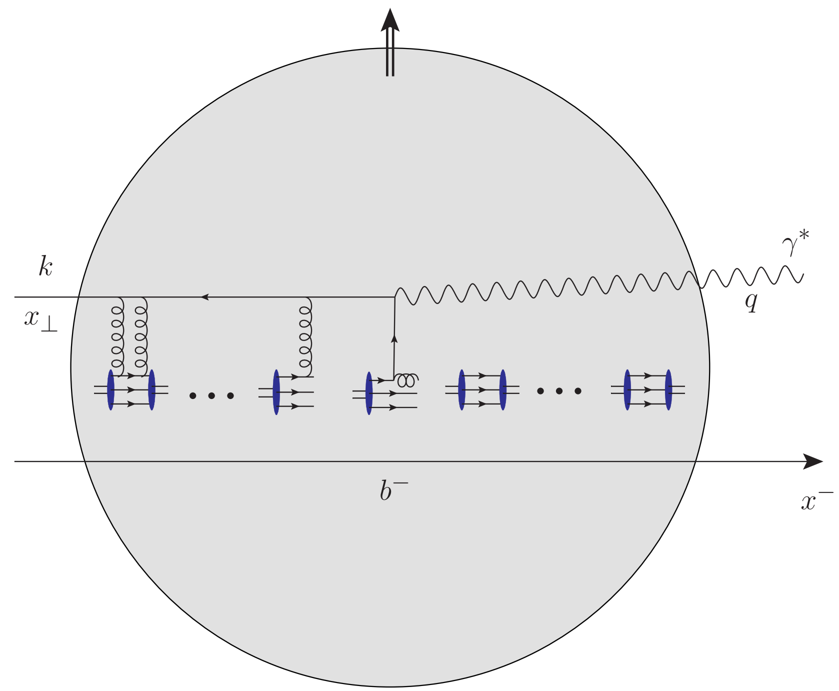
- However, the OAM channel dominates over a fairly broad range

$$k_T < \frac{Q_s}{\sqrt{\alpha_s}}$$

(if $p_T \sim m_N$).

Quasi-Classical STSA in DY

- The DY process in the quasi-classical approximation looks as follows:



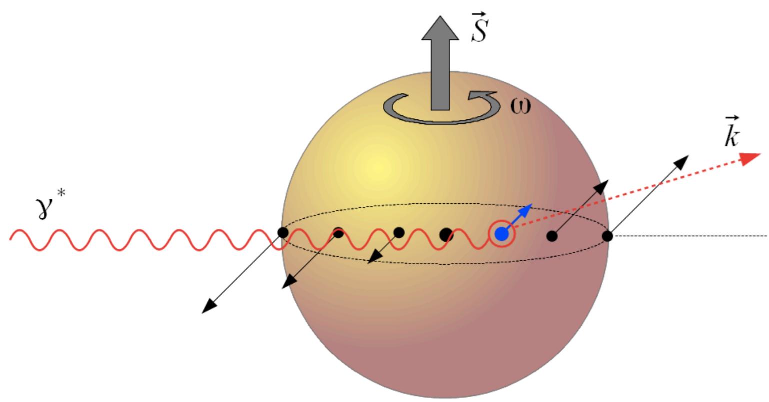
- Note that ordering interactions along the x⁻ direction makes the reversal of the Wilson line direction between SIDIS and DY explicit.

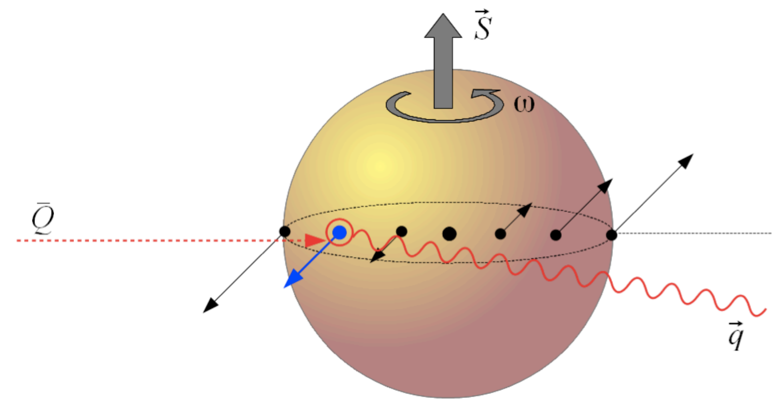
Origin of Sign Reversal

$$f_{1T}^{\perp A}(x,k_T)\Big|_{SIDIS} = -f_{1T}^{\perp A}(x,k_T)\Big|_{DY}$$

- In the transversity (Sivers density) channel the origin of the sign reversal is simple: the (LO) Sivers function of the nucleon changes sign, and multiple rescatterings do not affect this.

- In the OAM channel the reversal ultimately happens for the simple reasons described in the beginning:



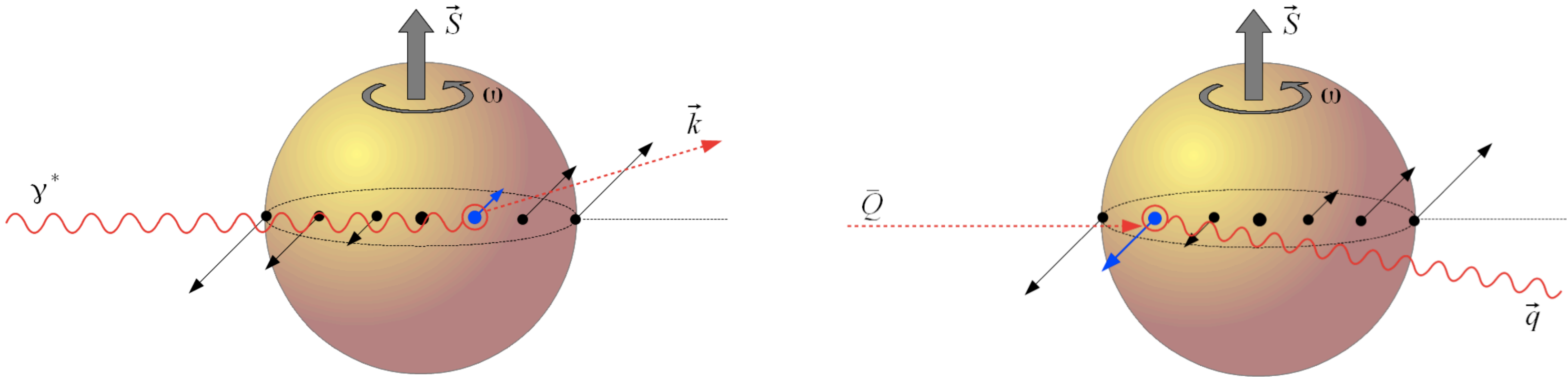


Summary

- We have calculated Sivers function in the quasi-classical approximation. Valid for a nucleus or a "dense" proton.
- The result can serve as an initial condition for QCD evolution (CSS for $s \sim Q^2$ or small-x evolution for $s \gg Q^2$). New functional form(s) of initial conditions could be used in phenomenology and may perhaps help build MCs.
- In general, given a guess for proton Wigner distribution one can construct new initial conditions for Sivers function evolution.
- The asymmetry is generated through two channels: Sivers density (conventional, BHS-type) and OAM channel.
- In the OAM channel the asymmetry is due to a combination of OAM and shadowing: could this be a general trend?

Summary

- OAM channel allows for a simple classical-physics interpretation of the asymmetry and of the sign reversal between STSA in SIDIS and in DY!



- Our technique applies to other TMDs: all of them can be (and are being) calculated in the quasi-classical limit.