TMD evolution of Sivers asymmetry

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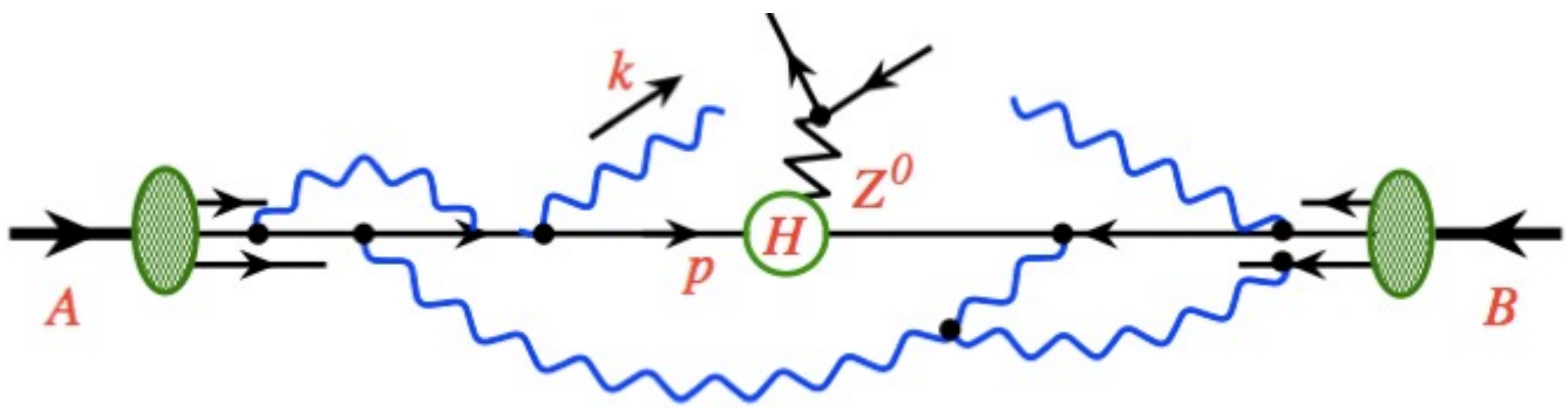
Outline

- Introduction and discussion
- QCD evolution of unpolarized TMDs
- QCD evolution of Sivers asymmetry
- Summary

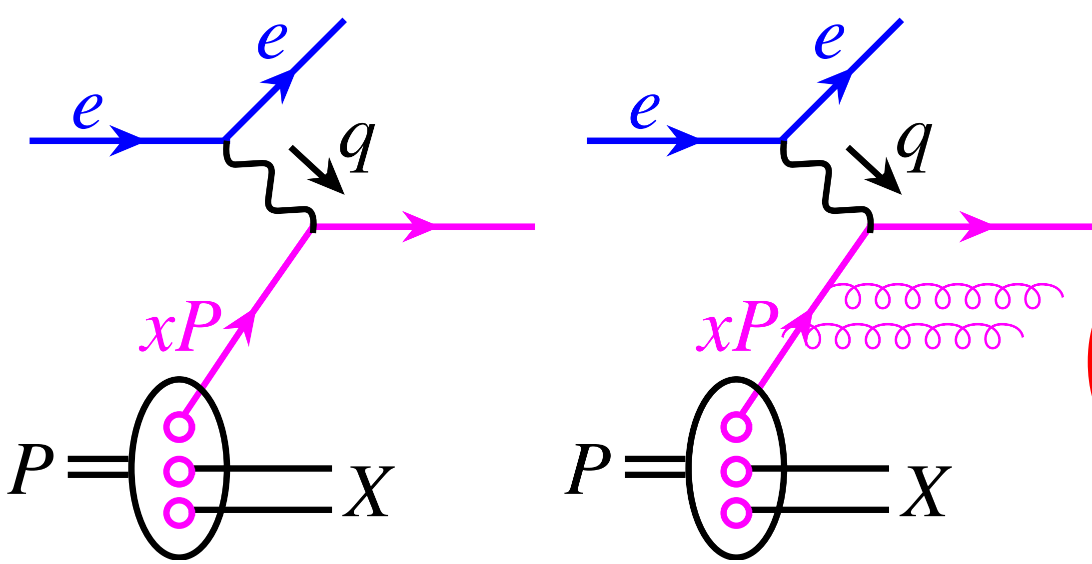
Why QCD evolution is needed

- Experiments are operated in different energy and kinematic regions, to make reliable predictions, one has to take into account these differences
  - Q is different: Q ~ 1 - 3 GeV in SIDIS, Q ~ 10 GeV at e+e-, Q ~ 4 - 90 GeV for DY, W/Z
  - Also $\sqrt{s}$ dependence is important   Qiu-Zhang 1999, ResBos
- We use the energy evolution equation for the relevant parton distribution functions (PDFs) or fragmentation function (FFs) to account for the kinematic differences

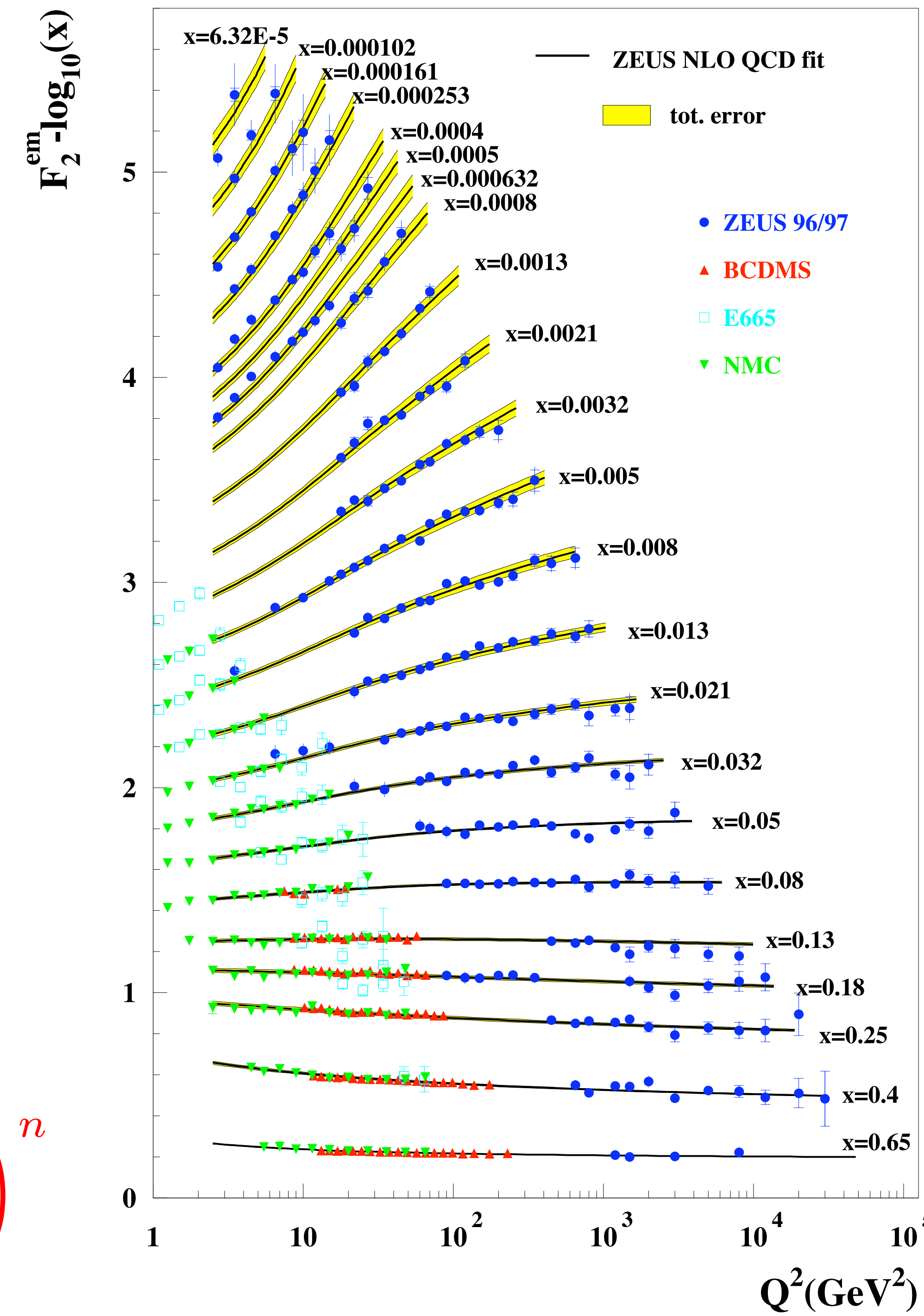


QCD evolution: meaning

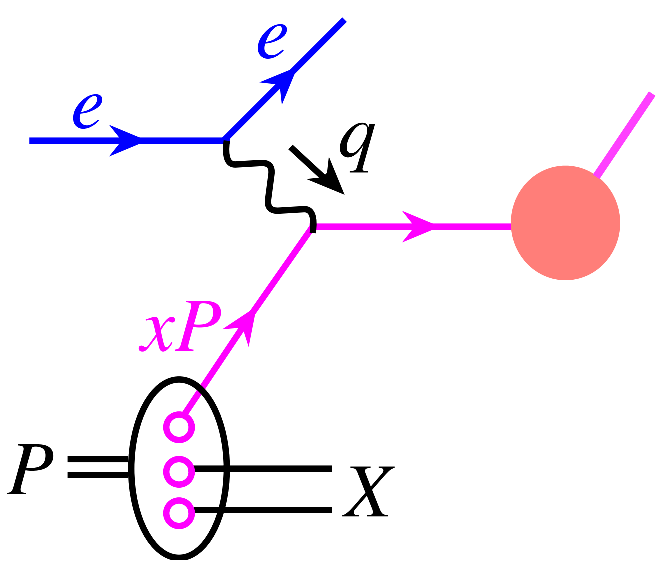
- What is QCD evolution of TMDs anyway?
  - Evolution = include important perturbative corrections
  - One of the well-known examples is the DGLAP evolution of collinear PDFs, which lead to the scaling violation observed in inclusive DIS process
  - What it does is to resum the so-called single logarithms in the higher order perturbative calculations

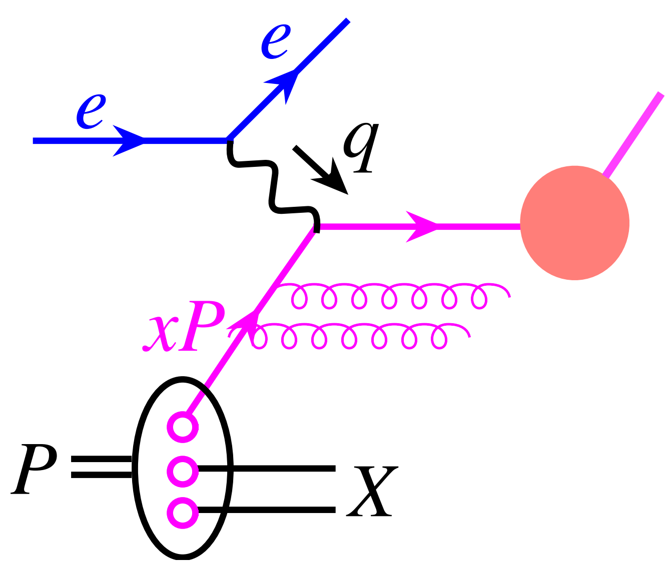


$$\left(\alpha_s \ln \frac{Q^2}{\mu^2}\right)^n$$



- TMD factorization works in the situation where there are two observed momenta in the process, such as SIDIS, DY, W/Z production and in the kinematic region where $Q>>q_T$
- Evolution again = include important perturbative corrections
- What it does is to resum the so-called double logarithms in the higher order perturbative corrections
- For SIDIS: q_T is the transverse momentum of the final-state hadron





$$\left(\alpha_s \ln^2 \frac{Q^2}{q_T^2}\right)^n$$

Many approaches for TMD evolution

- Collins-Soper-Sterman (CSS) resummation framework

  Collins-Soper-Sterman 1985
  ResBos: C.P. Yuan, P. Nadolsky
  Qiu-Zhang 1999, Vogelsang ...
  Kang-Xiao-Yuan 2011, Sun-Yuan 2013,
  Echevarria-Idilbi-Kang-Vitev 2014

- New Collins approach

  Aybat-Rogers 2011,
  Aybat-Collins-Rogers-Qiu, 2012
  Aybat-Prokudin-Rogers 2012

- Soft Collinear Effective Theory (SCET)

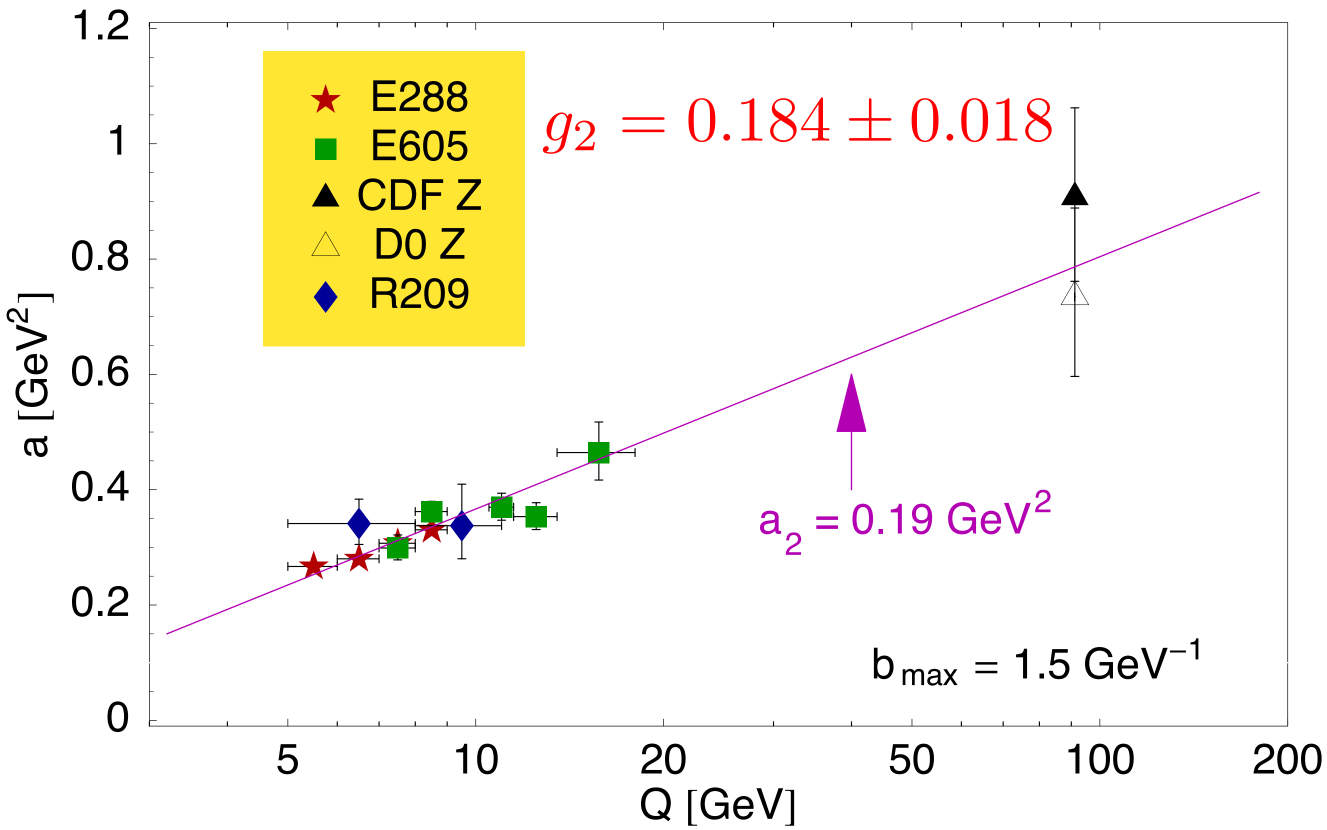
  Echevarria-Idilbi-Schafer-Scimemi 2012

They are all consistent with each other perturbatively
However, they could have very different phenomenological predictions

Kinematic dependence of TMD evolution

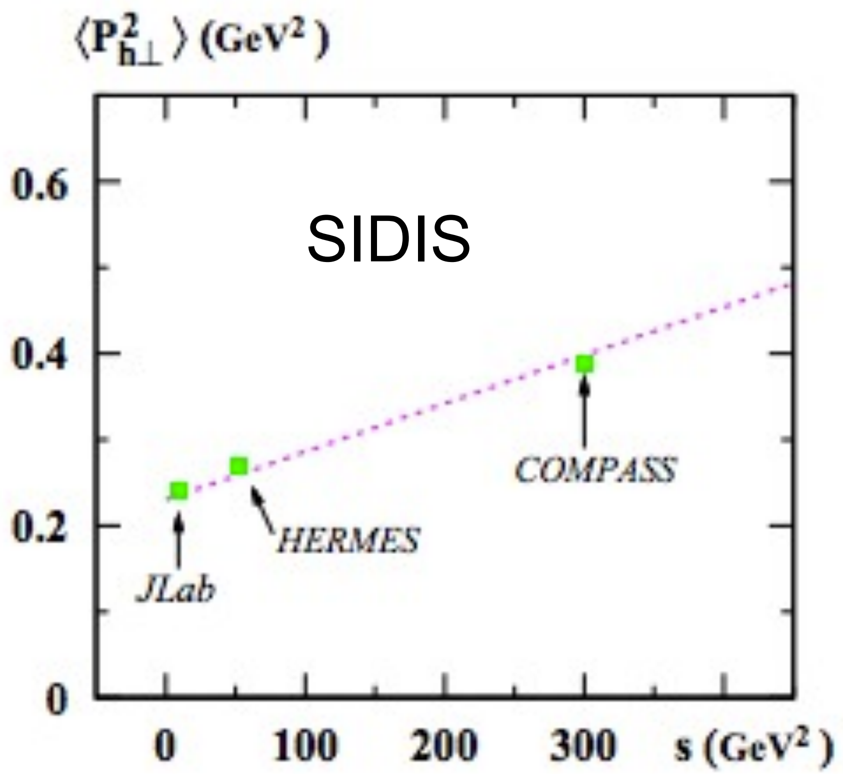
- Any correct TMD evolution should contain
  - Q dependence
  - $\sqrt{s}$ dependence
- These are supported by the data
  - Q dependence seems to be generally accepted

$$g_2 \ln(Q/Q_0)$$

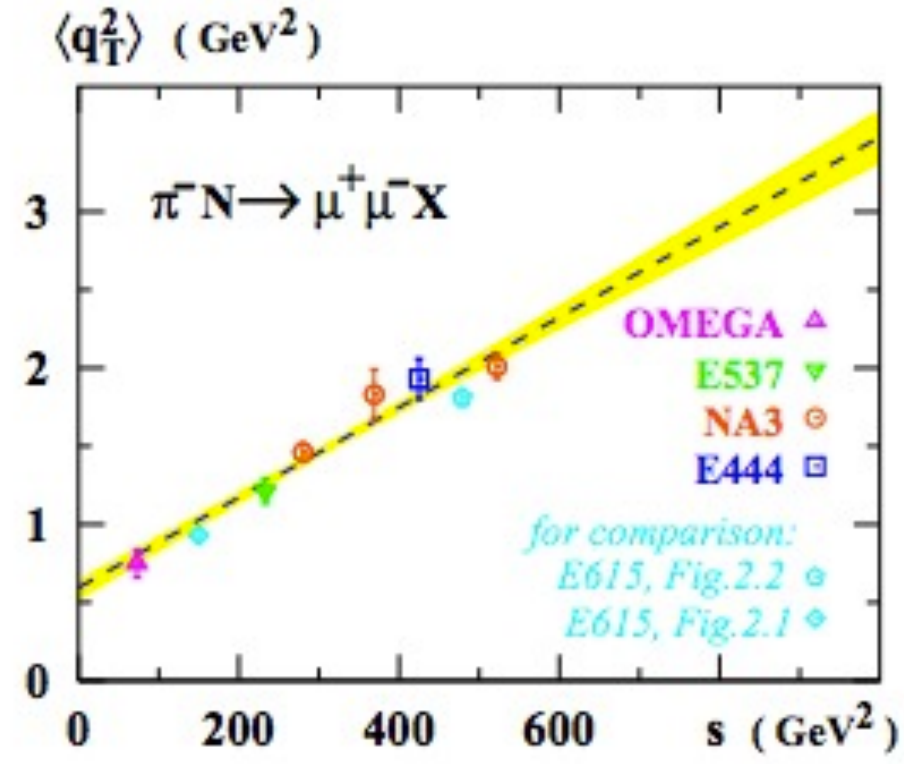


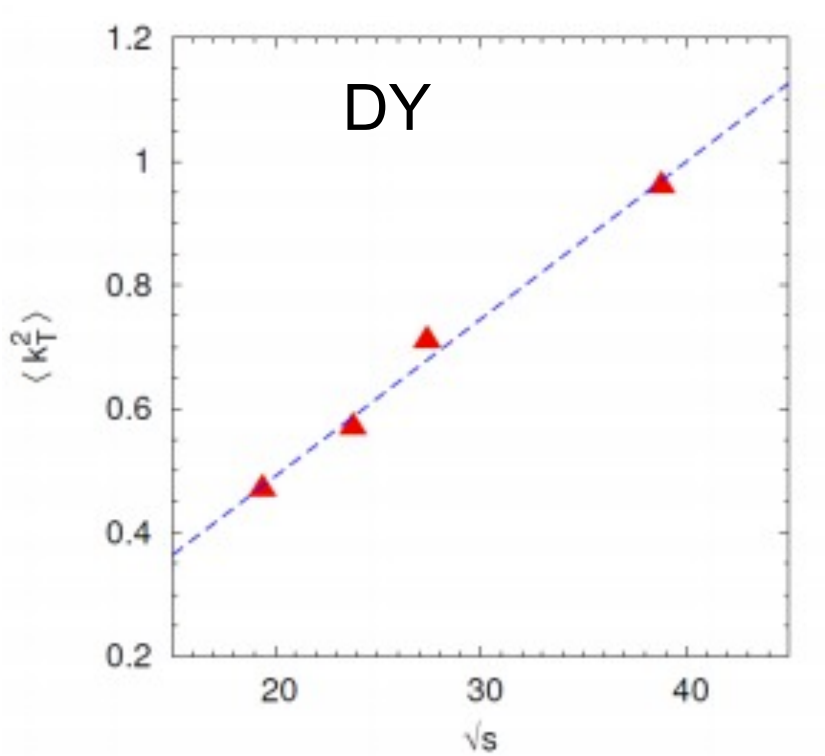
CM energy $\sqrt{s}$ dependence is also important

- Phenomenological studies



Schweitzer-Teckentrup-Metz, 2010





Boglione, QCD evolution 2013

What the evolution looks like?

- We have a TMD distribution $F(x, k_\perp; Q)$ (take $\zeta = \mu = Q$) measured at a scale Q
  - It is easy to deal in the Fourier transformed space

$$F(x, b; Q) = \int d^2 k_\perp e^{-ik_\perp \cdot b} F(x, k_\perp; Q)$$

  - Standard CSS formalism tells us it evolves from an initial scale

$$\mu_b = c/b$$

$$c = 2e^{-\gamma_E} \sim O(1)$$

$$F(x, b; Q) = F(x, b; c/b) \exp\left\{ -\int_{c/b}^{Q} \frac{d\mu}{\mu} \left(A \ln \frac{Q^2}{\mu^2} + B \right) \right\}$$

$$A = \sum_{n=1} A^{(n)} \left(\frac{\alpha_s}{\pi}\right)^n, \qquad B = \sum_{n=1} B^{(n)} \left(\frac{\alpha_s}{\pi}\right)^n$$

$$A^{(1)} = C_F$$

$$A^{(2)} = \frac{C_F}{2}\left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} T_R n_f \right]$$

$$B^{(1)} = -\frac{3}{2} C_F$$

Connection to other approaches: new Collins

- Derive new Collins evolution from CSS

$$F(x,b;Q_f) = F(x,b;c/b)\exp\left\{-\int_{c/b}^{Q_f}\frac{d\mu}{\mu}\left(A\ln\frac{Q_f^2}{\mu^2}+B\right)\right\}$$

$$F(x,b;Q_i) = F(x,b;c/b)\exp\left\{-\int_{c/b}^{Q_i}\frac{d\mu}{\mu}\left(A\ln\frac{Q_i^2}{\mu^2}+B\right)\right\}$$

$$F(x,b;Q_f) = F(x,b;Q_i)\exp\left\{-\int_{Q_i}^{Q_f}\frac{d\mu}{\mu}\left(A\ln\frac{Q_f^2}{\mu^2}+B\right)\right\}\left(\frac{Q_f^2}{Q_i^2}\right)^{-\int_{c/b}^{Q_i}\frac{d\mu}{\mu}A}$$

- This is the same as in SCET

What's the complication in QCD evolution?

- So far the evolution kernel is calculated in perturbation theory, so valid only for small b region:

$$F(x,b;Q_f) = F(x,b;Q_i)\exp\left\{-\int_{Q_i}^{Q_f}\frac{d\mu}{\mu}\left(A\ln\frac{Q_f^2}{\mu^2}+B\right)\right\}\left(\frac{Q_f^2}{Q_i^2}\right)^{-\int_{c/b}^{Q_i}\frac{d\mu}{\mu}A}$$

- Fourier transform back to the momentum space, one needs the whole b region (also large b): need some non-perturbative extrapolation

$$F(x,k_\perp;Q) = \frac{1}{(2\pi)^2}\int d^2b e^{ik_\perp\cdot b}F(x,b;Q)$$
$$= \frac{1}{2\pi}\int_0^\infty db\, bJ_0(k_\perp b)F(x,b;Q)$$

- Widely used prescription (CSS):

$$F(x,b;Q_f) = F(x,b;Q_i)R^{\rm pert}(b_*,Q_i,Q_f)\times R^{NP}(b,Q_i,Q_f)$$

$$b_* = b/\sqrt{1+(b/b_{\rm max})^2}$$

Use conventional CSS formalism

- In the conventional CSS formalism, one further calculate TMD at c/b scale in terms of collinear PDFs

$$F(x,b;Q) = F(x,b;c/b)\exp\left\{-\int_{c/b}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\}$$

  - Expand our initial TMD $F(x,b;c/b)$ in terms of the corresponding the collinear PDF

$$F_{i/A}(x,b;\mu=c/b) = \sum_a\int_x^1 C_{i/a}\left(\frac{x}{\xi},\mu=c/b\right)f_{a/A}(\xi,\mu=c/b)$$

$$C_{i/a} = \sum_n C_{i/a}^{(n)}(\alpha_s/\pi)^n \qquad C_{i/a}^{(0)} = \delta_{ia}\delta(x-1)$$

- Note: above coefficient functions are different from the C-functions in CSS formalism
  - Take into account the hard-part function -> C-function in CSS

$$\frac{d\sigma}{dQ^2d^2q_T} \propto F_1(x_1,k_{1T};Q)\otimes F_2(x_2,k_{2T}\otimes H(Q,\mu)$$

$$H(Q,\mu)_{\rm SIDIS,DY} = 1 + C_F\frac{\alpha_s}{2\pi}\left[3\ln\frac{Q^2}{\mu^2} - \ln^2\frac{Q^2}{\mu^2} - 8+\pi^2\right]$$

Non-perturbative Sudakov factor

- Still have to choose non-perturbative Sudakov function

$$F(x,b;Q) = F(x,c/b_*)R^{\rm pert}(Q,b_*)R^{\rm NP}(Q,b)$$

$$R^{\rm NP}(Q,b) = \exp(-S^{\rm NP})$$

- Typical simplest form for unpolarized PDF and FF

$$S^{\rm NP}_{pdf} = b^2\left[g_1^{pdf} + \frac{g_2}{2}\ln(Q/Q_0)\right]$$

$$S^{\rm NP}_{ff} = b^2\left[g_1^{ff}/z^2 + \frac{g_2}{2}\ln(Q/Q_0)\right]$$

- This way still okay to obtain your TMD in momentum space, thus to perform 3D structure as usual

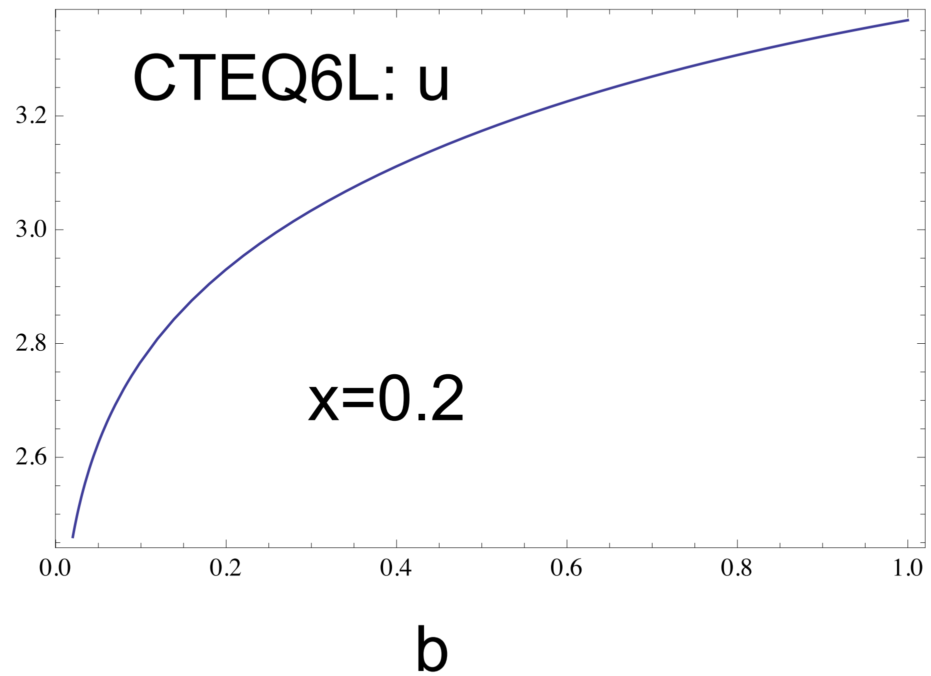
$$F(x,k_\perp;Q) = \frac{1}{(2\pi)^2}\int d^2b e^{ik_\perp\cdot b}F(x,b;Q)$$
$$= \frac{1}{2\pi}\int_0^\infty db\, b J_0(k_\perp b)F(x,b;Q)$$

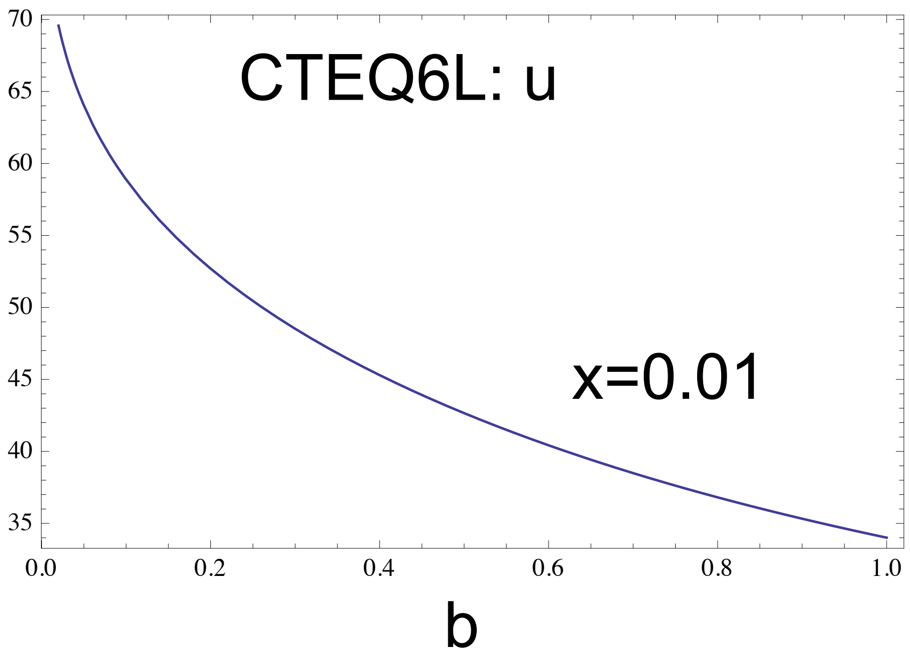
Interesting thing: not a universal Gaussian

- Full expression in b-space

$$F(x,b;Q) = F(x,c/b_*)\exp\left\{-\int_{c/b_*}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\}\exp\left\{-b^2\left[g_1^{pdf}+\frac{g_2}{2}\ln(Q/Q_0)\right]\right\}$$

  - At large b region, b* → b_max, thus the above form will be closer to the usual Gaussian: thus might be okay to think g_1 as the "starting/intrinsic" kt at Q0
  - At intermediate or small b, apparently they do not look Gaussian at all; because F(x, c/b) has different behavior as a function of b for different x, it is very complicated functional form





  - In the usual Gaussian ansatz: same b-dependence at all x

$$F(x,b;Q) = F(x,Q)\exp\left\{-\frac{b^2\langle k_\perp^2\rangle}{4}\right\}$$

F(x, c/b) builds into the $\sqrt{s}$ dependence

- At cross section level for DY

$$\frac{d\sigma}{dQ^2 dy dq_\perp^2} = \frac{1}{(2\pi)^2}\int d^2b e^{iq_\perp\cdot b} W(b, Q, x_a, x_b)$$

$$\sim \frac{1}{2\pi}\int_0^\infty db\, J_0(q_\perp b)\left[b e^{-S(b,Q)} \sum_{i,j} f_{i/A}(x_a, c/b) f_{j/B}(x_b, c/b)\right]$$

- Perform the saddle-point approximation, which will reflect the peak in the b-space

$$\frac{d}{db}\ln\left[b e^{-S(b,Q)} \sum_{i,j} f_{i/A}(x_a, c/b) f_{j/B}(x_b, c/b)\right]_{b=b_{\rm sp}} = 0$$

$$b_{\rm sp} = \frac{c}{\Lambda_{\rm QCD}}\left(\frac{Q}{\Lambda_{\rm QCD}}\right)^{-\frac{A^{(1)}}{A^{(1)}+\beta 1\left[1-F(\sqrt{s}, y, c/b_{\rm sp})\right]}}$$

$$F(\sqrt{s}, y, \mu = c/b) = \frac{d}{d\ln\mu^2}\sum_{i,j} f_{i/A}(x_a, \mu) f_{j/B}(x_b, \mu)$$

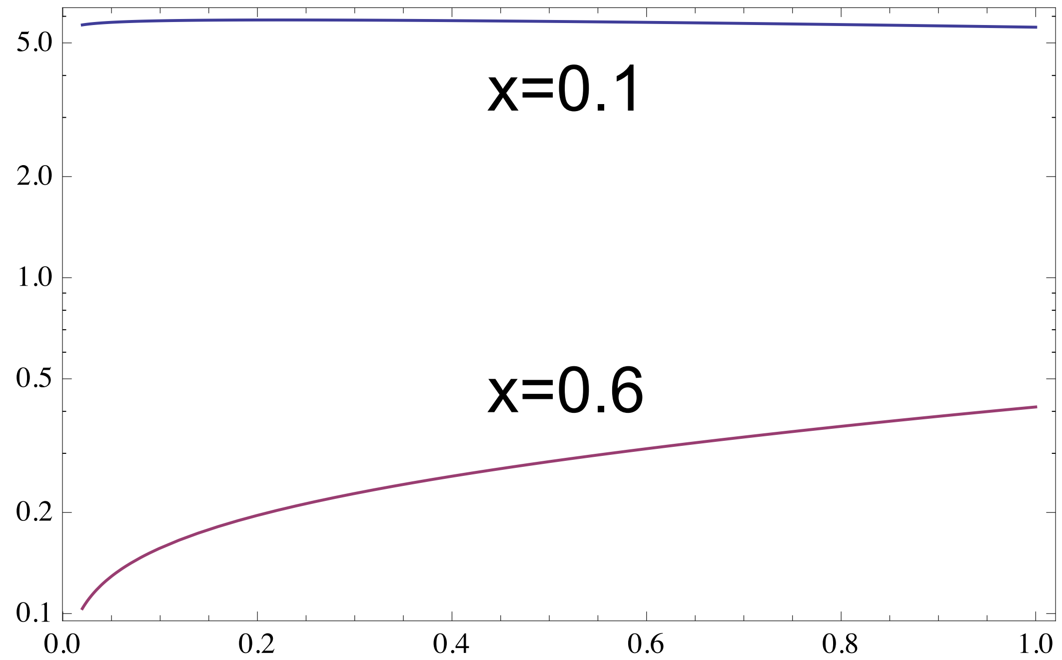
Saddle point b_{sp}

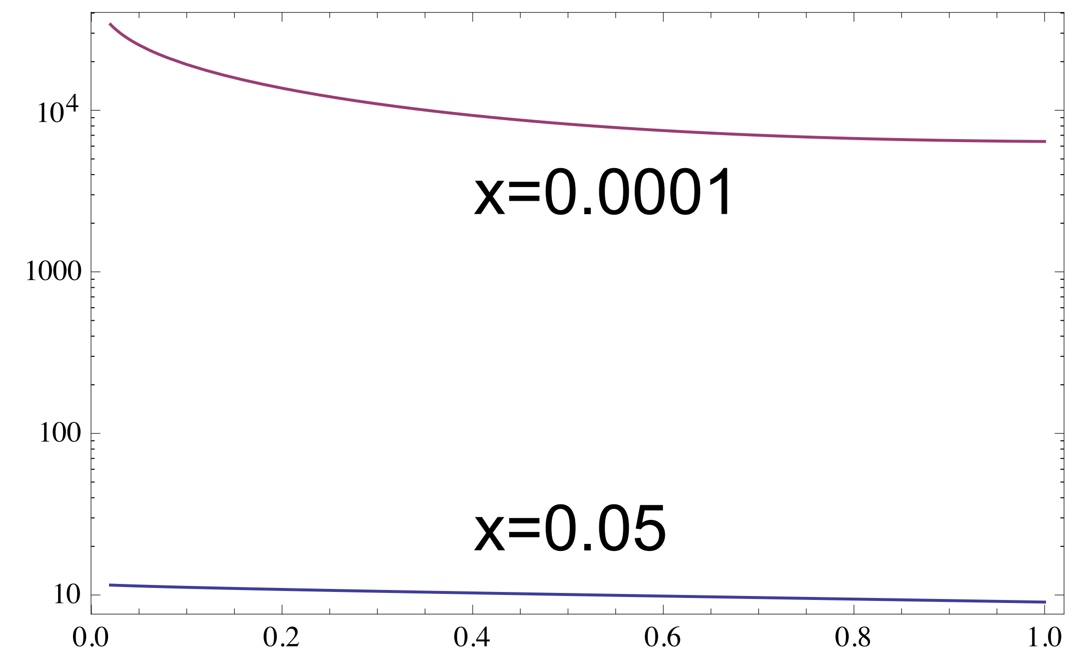
- Generic features

$$b_{sp} = \frac{c}{\Lambda_{QCD}} \left(\frac{Q}{\Lambda_{QCD}} \right)^{-\frac{A^{(1)}}{A^{(1)} + \beta 1 \left[1 - F(\sqrt{s}, y, c/b_{sp})\right]}}$$

$$F(\sqrt{s}, y, \mu = c/b) = \frac{d}{d \ln \mu^2} \sum_{i,j} f_{i/A}(x_a, \mu) f_{j/B}(x_b, \mu) \qquad x_{a,b} = \frac{Q}{\sqrt{s}} e^{\pm y}$$

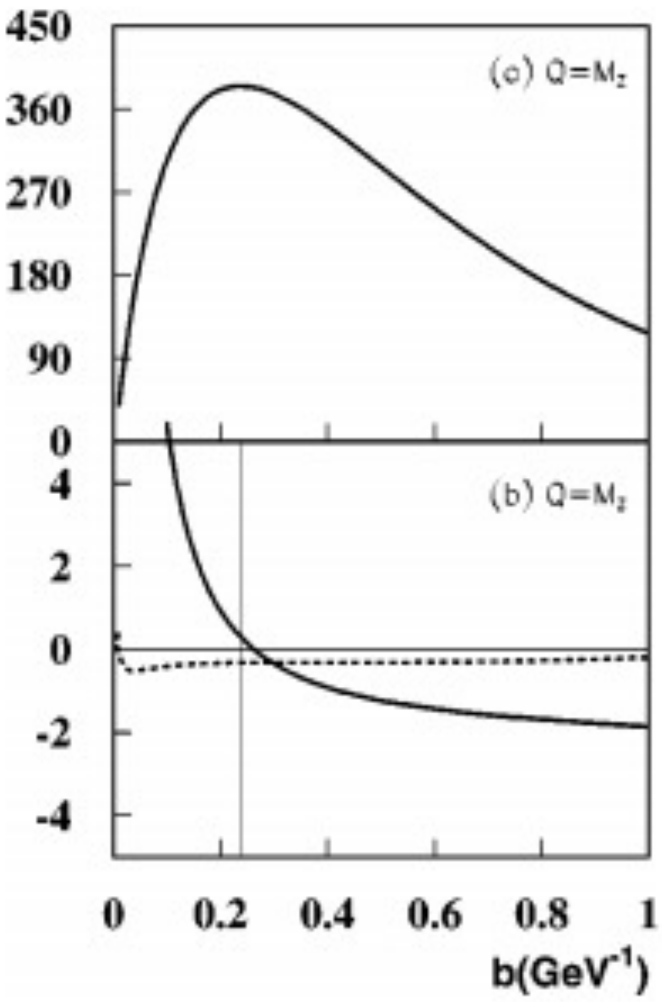
  - If Q is large, b_{sp} is small, so the integrand is more dominant by the small b region when performing Fourier transformation
  - roots enters through the derivative of the PDFs: high roots -> smaller b_{sp}

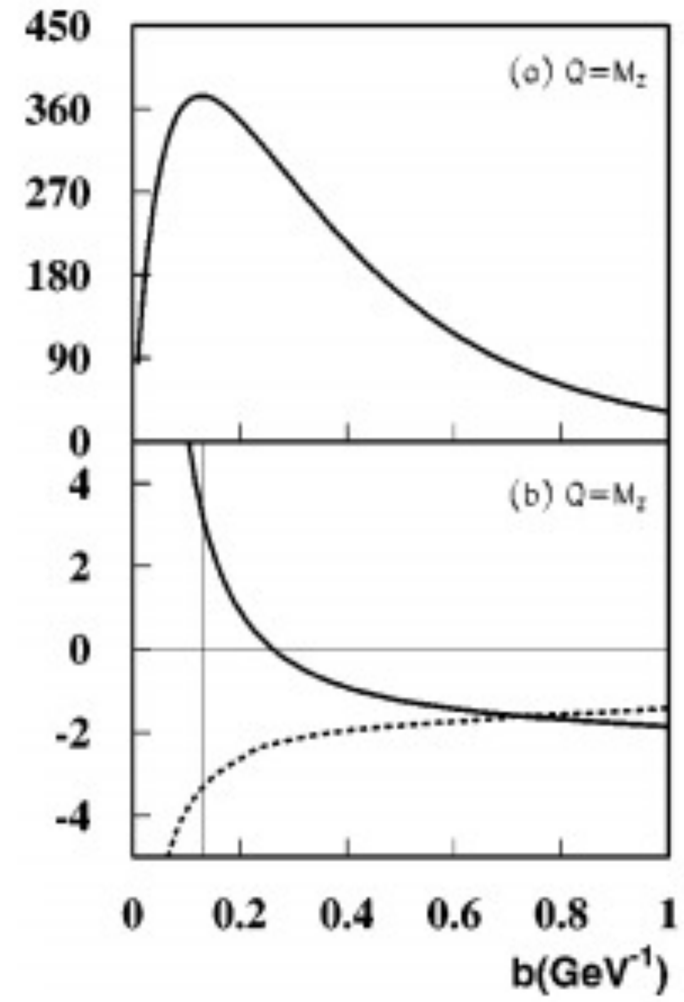




Example illustration

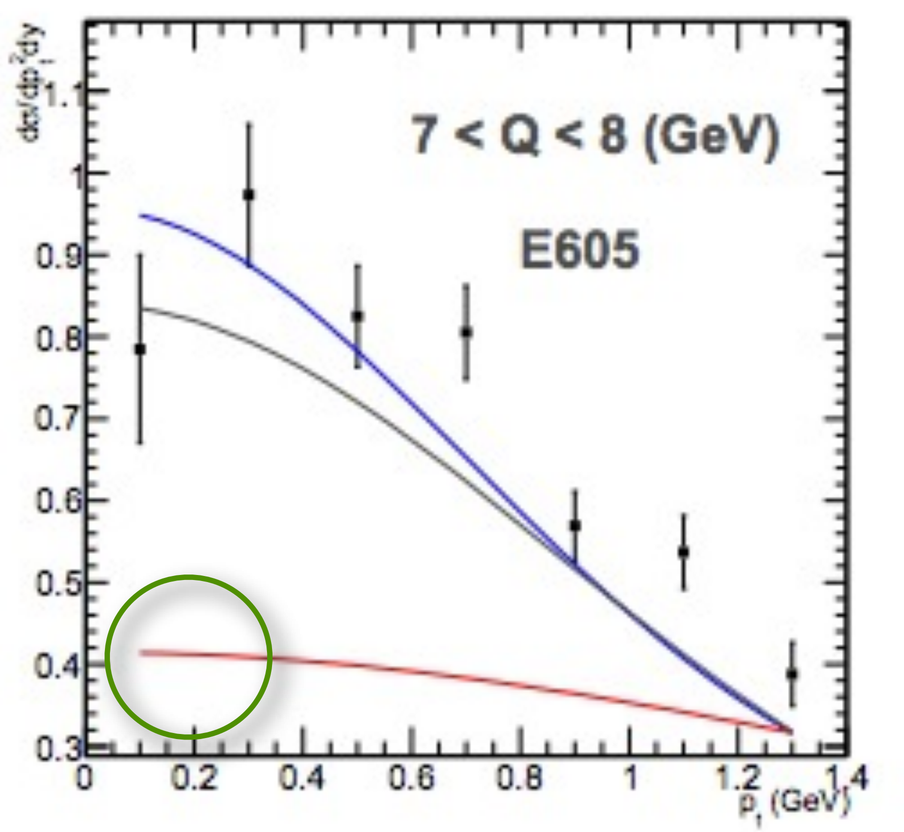
- At 1.8 TeV, left; at 14 TeV, right   Qiu-Zhang 1999



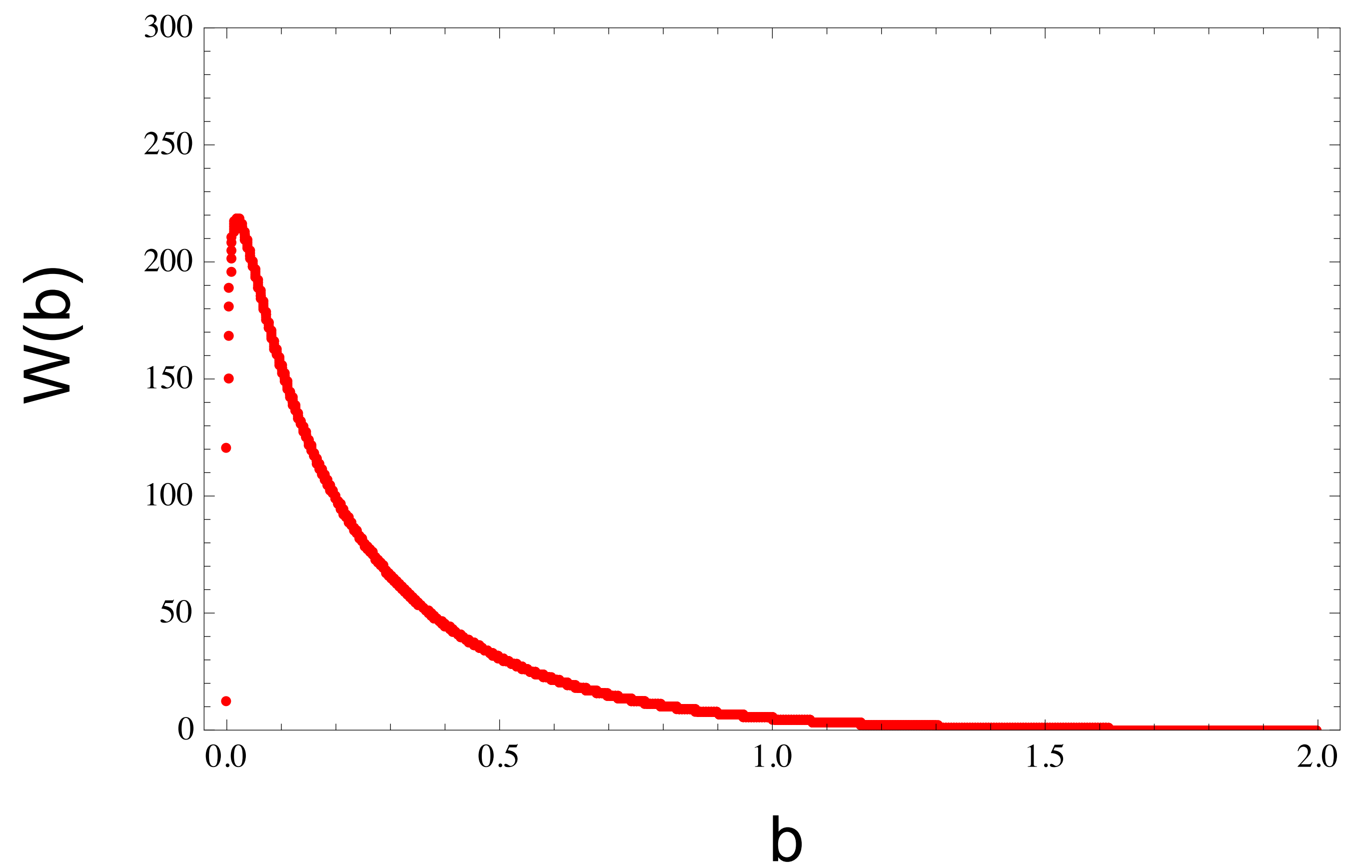


New Collins evolution + Gaussian ansatz

- Choose some Gaussian form for TMDs at initial scale Q0, then evolve to W/Z scale, to see if it describes the pt distribution
  - It does not (use a reasonable b_{max}). It always leads to a rather flat pt distribution: the integrand in b-space is almost a delta-function concentrated at b=0
  - It will then lead to a rather flat pt distribution: curvature much smaller than data



See, e.g., Sun-Yuan arXiv:1308.5003



Intuitive meaning of these parameters

- Let us understand these parameters

$$S_{pdf}^{\rm NP} = b^2\left[g_1^{pdf} + \frac{g_2}{2}\ln(Q/Q_0)\right] \qquad S_{ff}^{\rm NP} = b^2\left[g_1^{ff}/z^2 + \frac{g_2}{2}\ln(Q/Q_0)\right]$$

$g_1^{\rm pdf} = \langle k_\perp^2\rangle/4$ intrinsic transverse momentum width for PDFs at scale Q0

$g_1^{\rm ff} = \langle p_\perp^2\rangle/4$ intrinsic transverse momentum width for FFs at scale Q0

g_2 mimic the increase in the width observed by the experiments
large Q leads to more shower

- Sivers asymmetry is very sensitive to g2 (though the Drell-Yan unpolarized cross section is not)
  - Choose a wrong g2 leads to very different result

Tune the parameters to describe all data

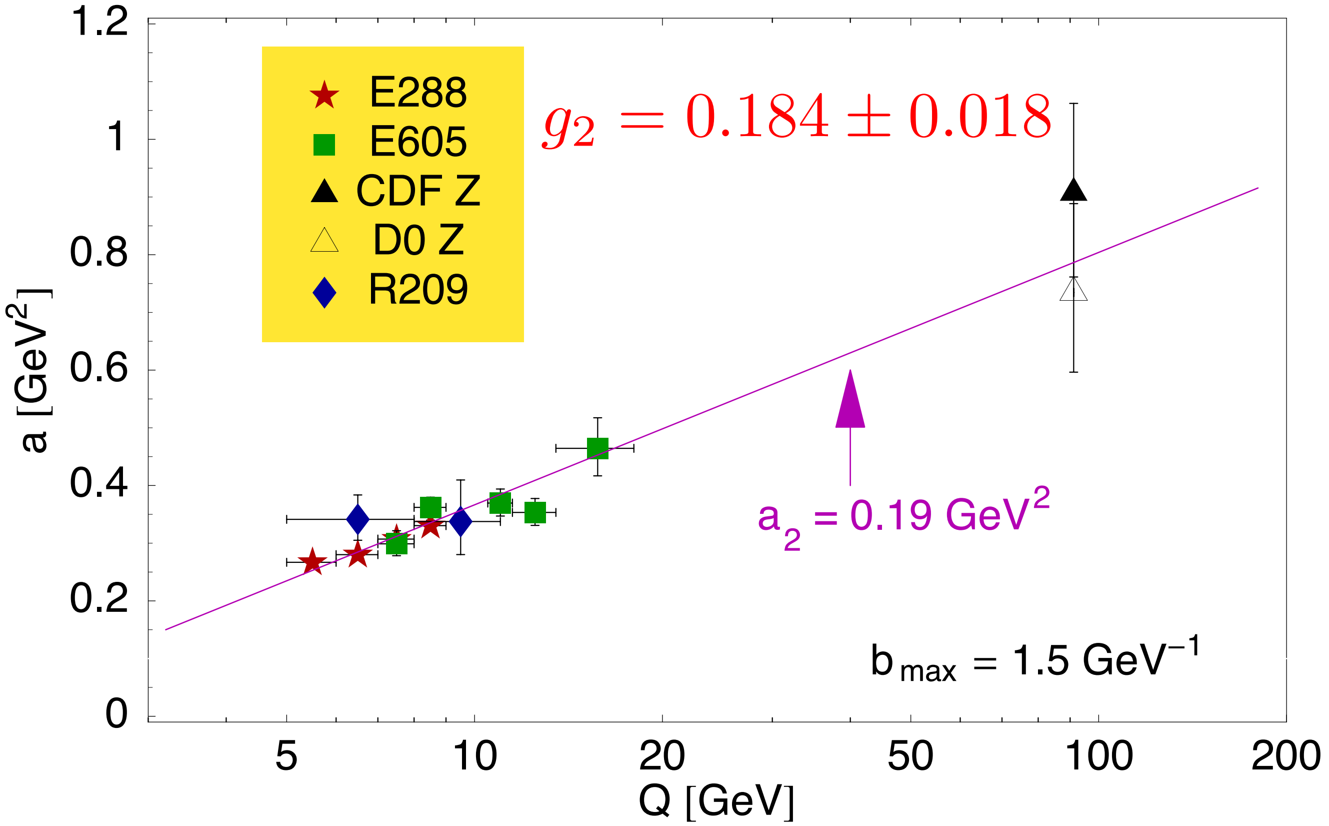
- Now we will try to tune these parameters to describe all the world data for pt distribution for SIDIS, DY, W/Z at all energies
  - Let us choose $Q_0^2 = 2.4$ GeV^2, i.e., the HERMES scale
  - At this scale, the intrinsic transverse momentum width is already extracted by different group: there are some freedom

arXiv:1003.2190 & Torino

$$\langle k_\perp^2 \rangle = 0.25 - 0.44 \text{ GeV}^2 \qquad \langle p_T^2 \rangle = 0.15 - 0.2 \text{ GeV}^2$$

Finding a way to describe both SIDIS and DY/WZ

- Study unpolarized cross section, and pin-down g2
  - Slightly adjust g2 (within their fitted uncertainty) such that non-perturbative Sudakov can predict $\langle k_\perp^2 \rangle$ at HERMES
  - Once this is fixed, adjust $\langle p_T^2 \rangle$ such that it gives a good description of SIDIS



hep-ph/0506225

$$g_2 \approx 0.16$$

$$g_1^{\rm pdf} = \langle k_\perp^2 \rangle / 4$$

$$g_1^{\rm ff} = \langle p_\perp^2 \rangle / 4$$

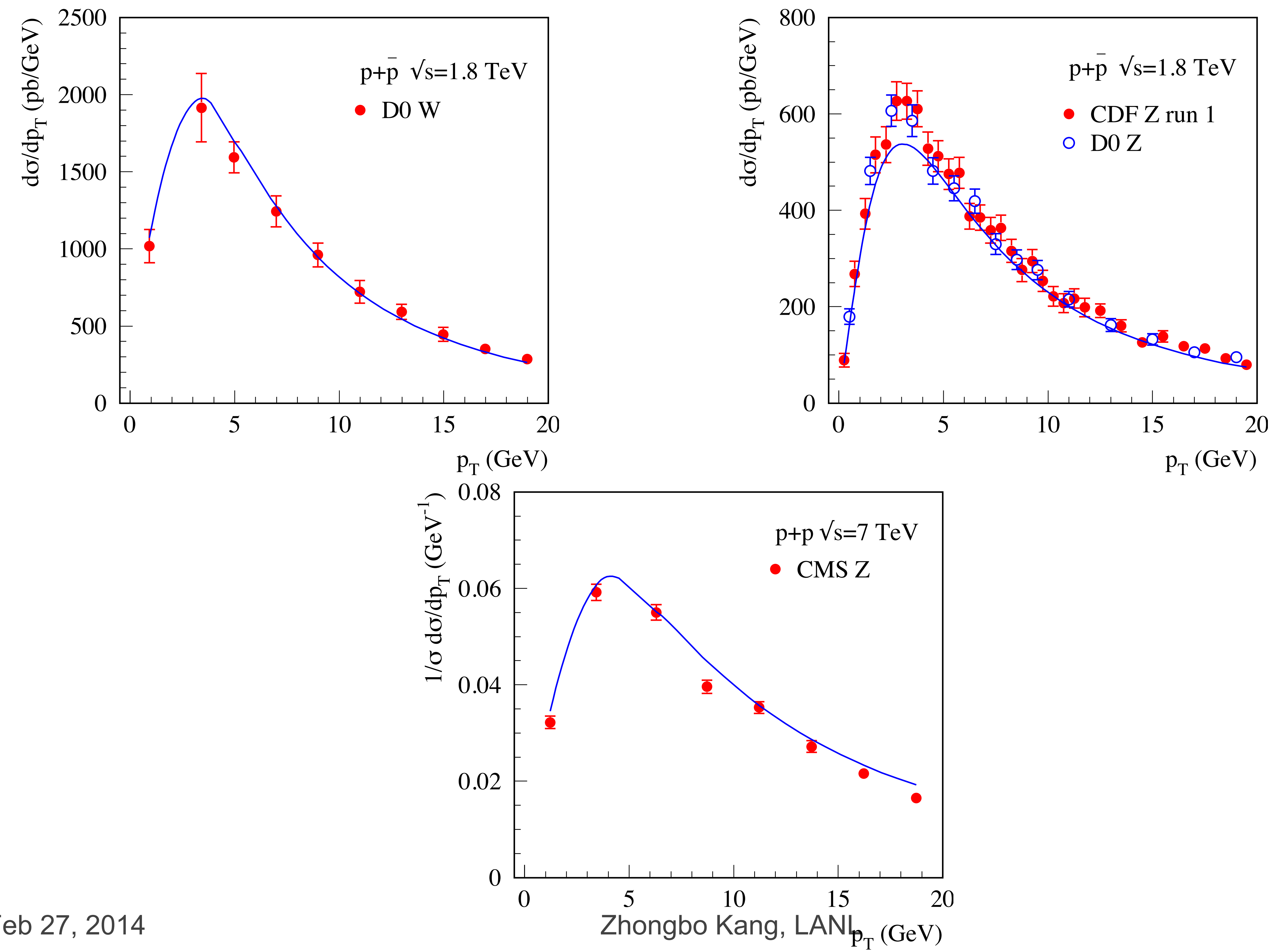
$$\langle k_\perp^2 \rangle = 0.38 {\rm GeV}^2$$

$$\langle p_\perp^2 \rangle = 0.19 {\rm GeV}^2$$

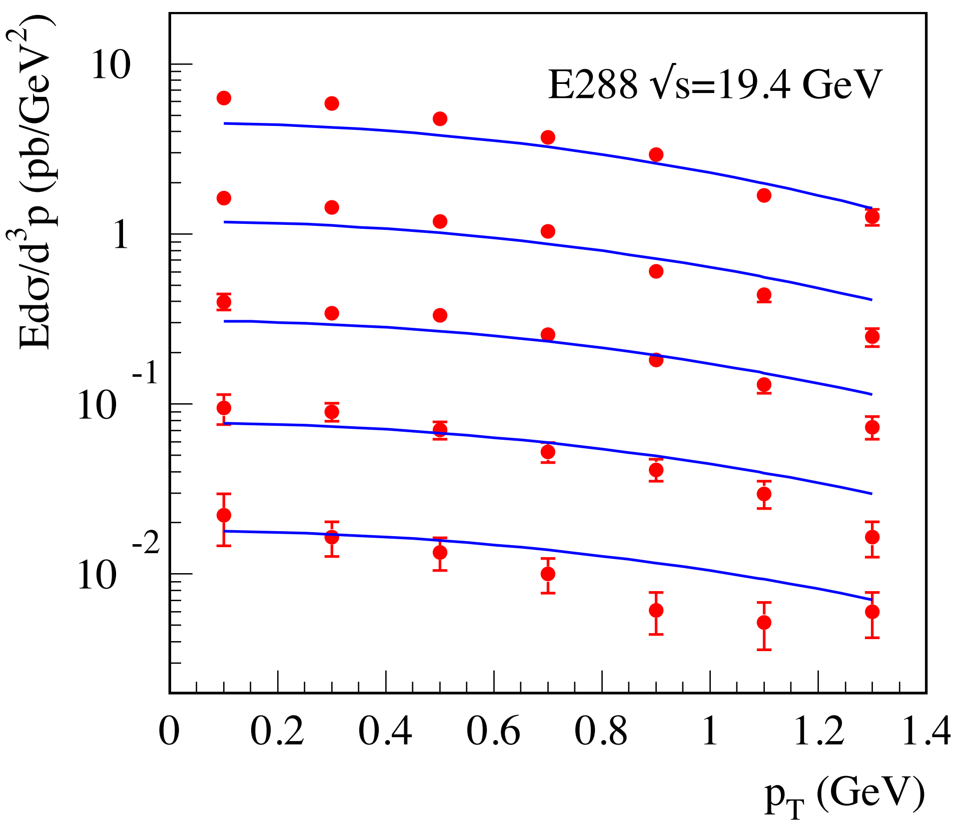
consistent with arXiv:1309.3507

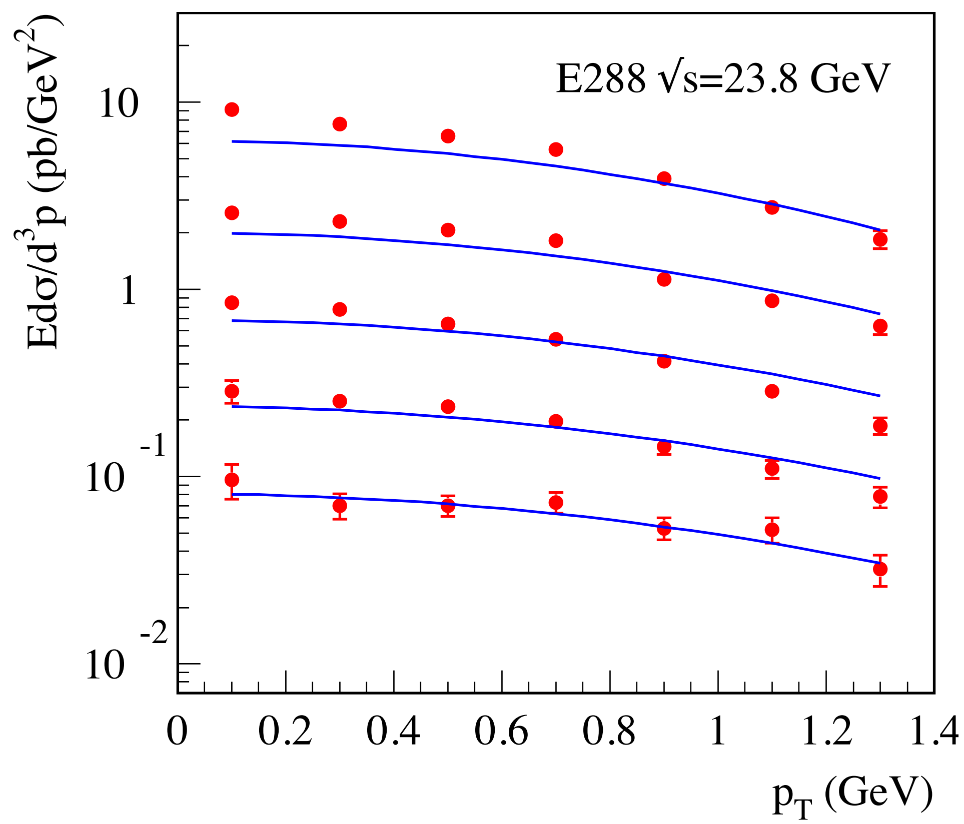
This actually works

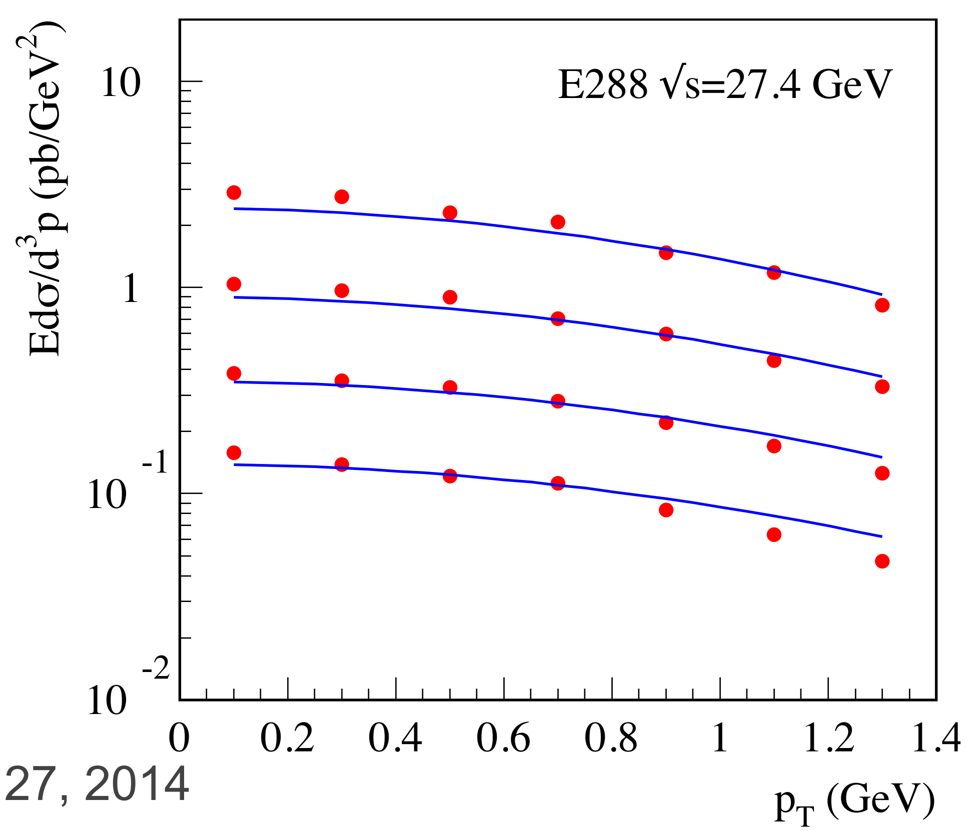
- Description of W/Z data at Tevatron and LHC Echevarria-Idilbi-Kang-Vitev, 1401.5078

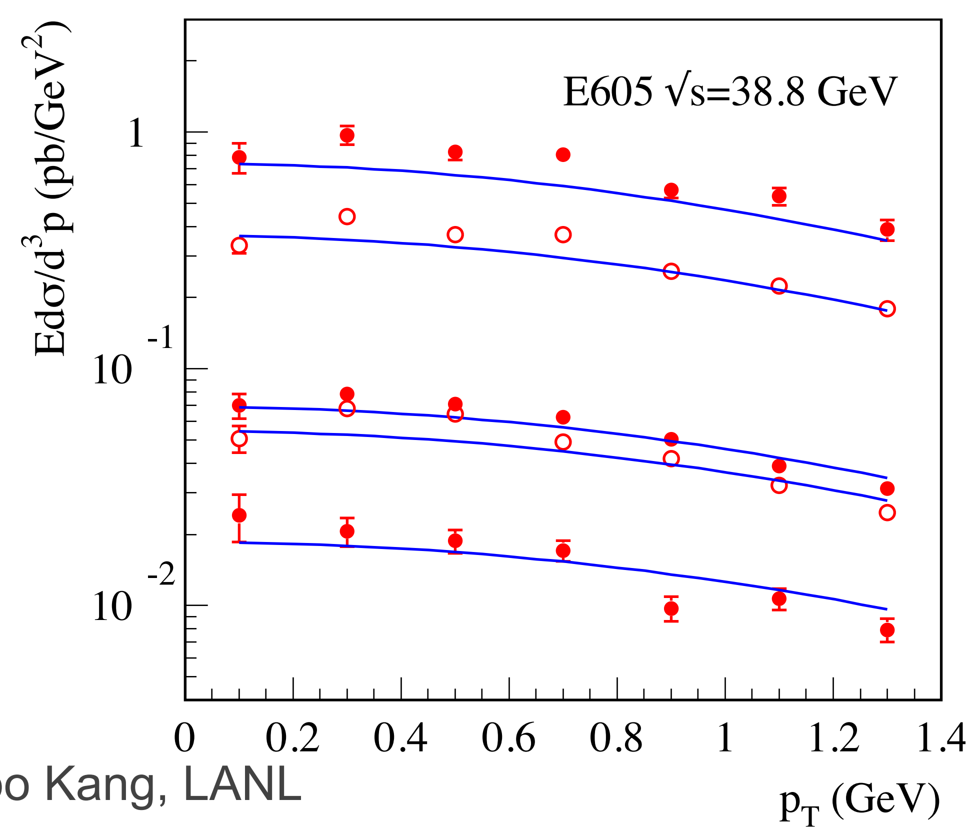


Drell-Yan lepton pair production









Multiplicity distribution in SIDIS 1

- Comparison with COMPASS data

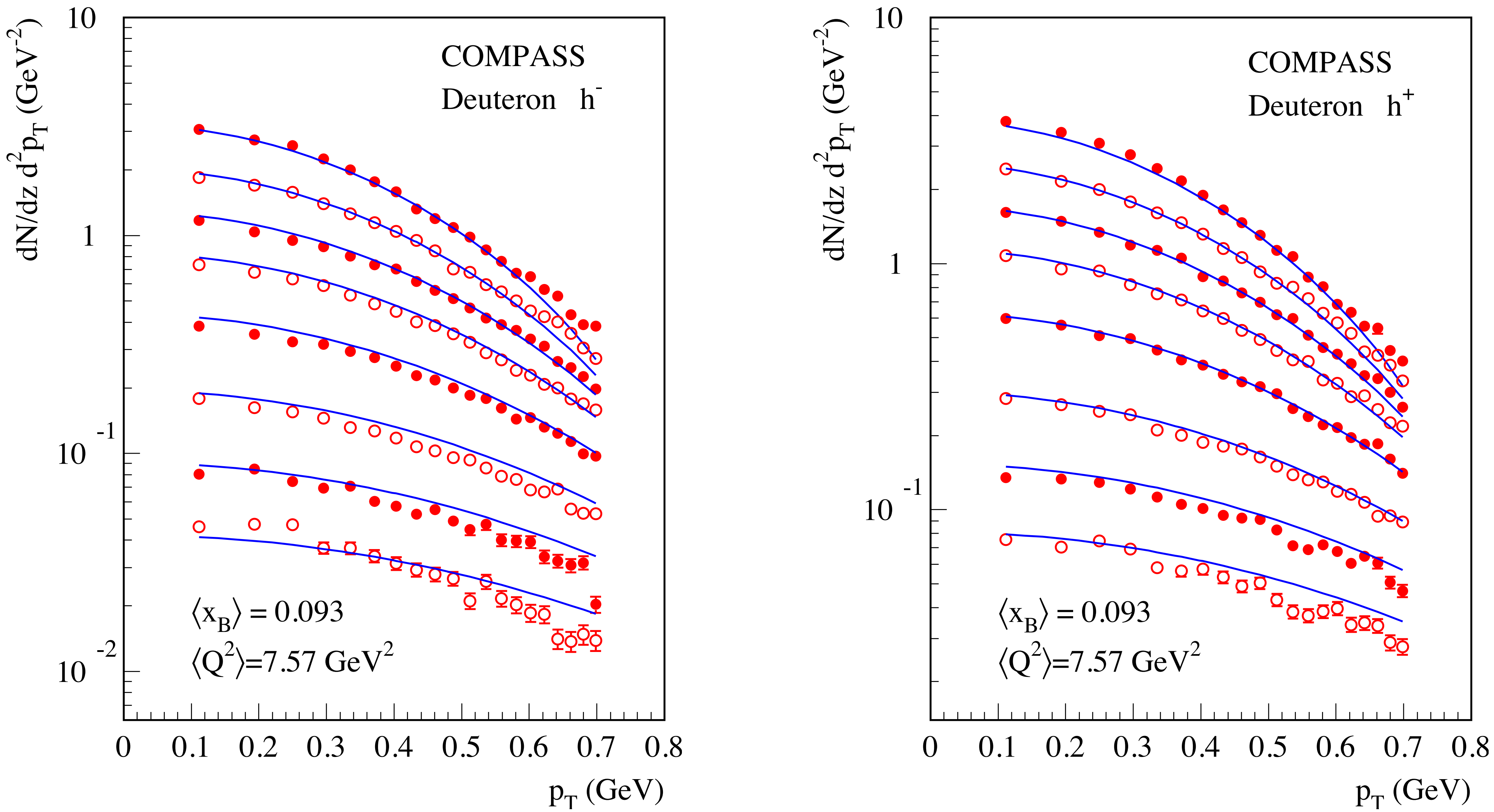


FIG. 2. The comparison with the COMPASS data (deuteron target) [7] at $\langle Q^2 \rangle = 7.57$ GeV^2 and $\langle x_B \rangle = 0.093$. The data points from top to bottom correspond to different z region: $[0.2, 0.25]$, $[0.25, 0.3]$, $[0.3, 0.35]$, $[0.35, 0.4]$, $[0.4, 0.5]$, $[0.5, 0.6]$, $[0.6, 0.7]$, and $[0.7, 0.8]$.

Multiplicity distribution in SIDIS 2

- Comparison with HERMES data

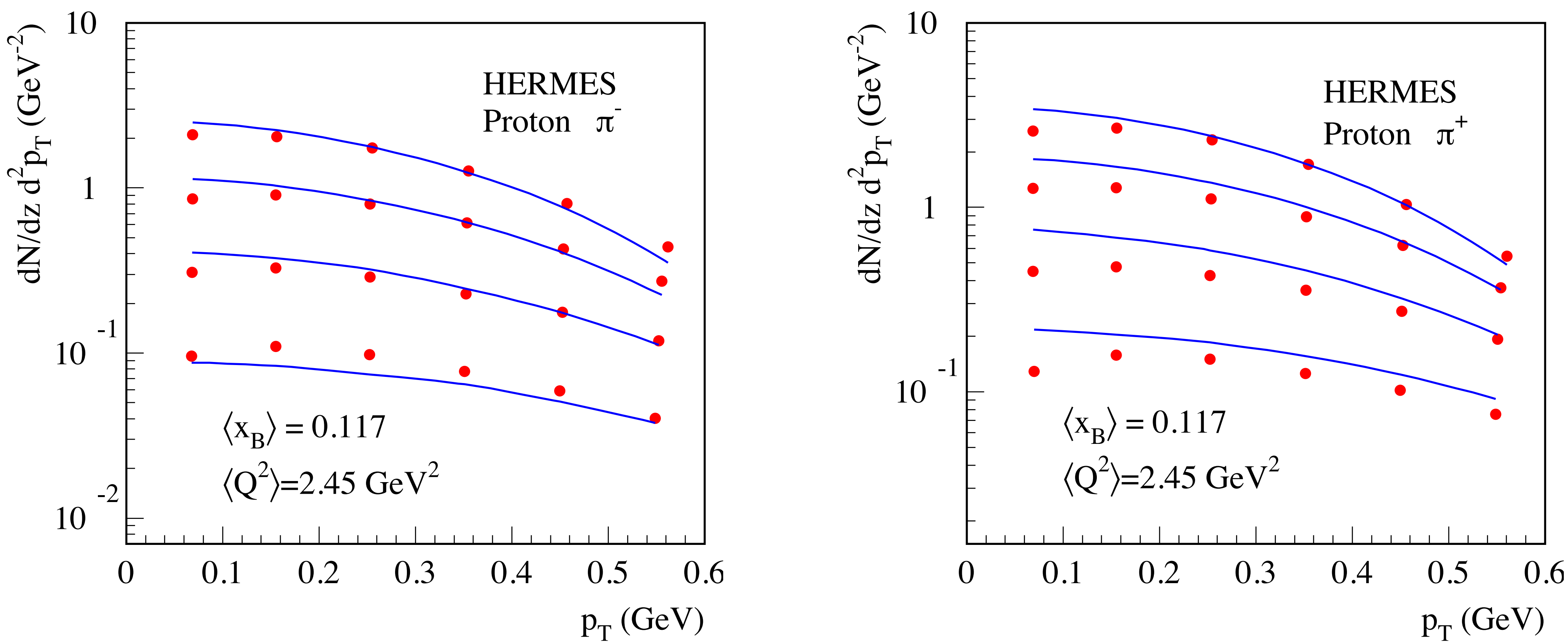


FIG. 1. The comparison with the HERMES data (proton target) [6]. The data points from top to bottom correspond to different z region: $[0.2, 0.3]$, $[0.3, 0.4]$, $[0.4, 0.6]$, and $[0.6, 0.8]$.

Sivers effect

- Now let us try to use the same formalism to describe Sivers effect

$$F(x,b;Q) = F(x,b;c/b)\exp\left\{-\int_{c/b}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\}$$

- Now F(x,b; Q) is given by

$$f_{1T}^{\perp q(\alpha)}(x,b;Q) = \frac{1}{M}\int d^2k_\perp e^{-ik_\perp\cdot b}k_\perp^\alpha f_{1T}^{\perp q}(x,k_\perp^2;Q)$$

- The perturbative expansion gives Qiu-Sterman function

$$f_{1T}^{\perp q(\alpha)}(x,b;\mu=c/b) = \left(\frac{-ib^\alpha}{2}\right)T_{q,F}(x,x,\mu=c/b)$$

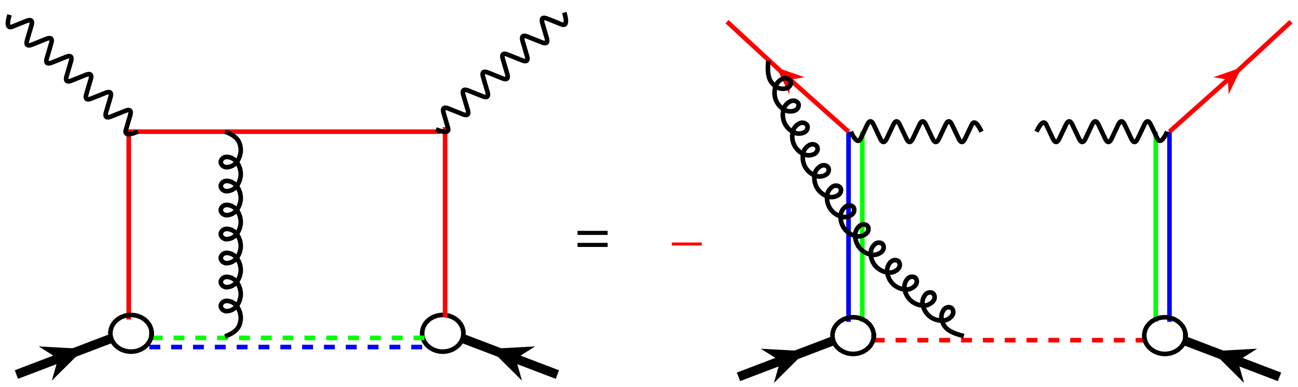
Sivers function

- Sivers function: an asymmetric parton distribution in a transversly polarized nucleon (kt correlated with the spin of the nucleon)

Spin-dependent

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/h^\uparrow}(x,k_\perp)\,\vec{S}\cdot\hat{p}\times\hat{\mathbf{k}}_\perp$$

Spin-independent



$$\Delta^N f^{\rm DIS}_{q/h^\uparrow}(x,k_\perp) = -\Delta^N f^{\rm DY}_{q/h^\uparrow}(x,k_\perp)$$

NSAC milestone: most important property of the Sivers function, need to be tested

Fitting parameters

- Similar form for non-perturbative Sudakov factor (note: g2 is spin-independent, so use the same g2)

$$S_{\rm NP}^{\rm sivers}(b,Q) = b^2\left[g_1^{sivers} + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right] \qquad g_1^{sivers} = \frac{\langle k_{s\perp}^2\rangle}{4}$$

  - Intrinsic kt-width for Sivers has to be fitted
  - x-dependence has to be fitted
- Qiu-Sterman function

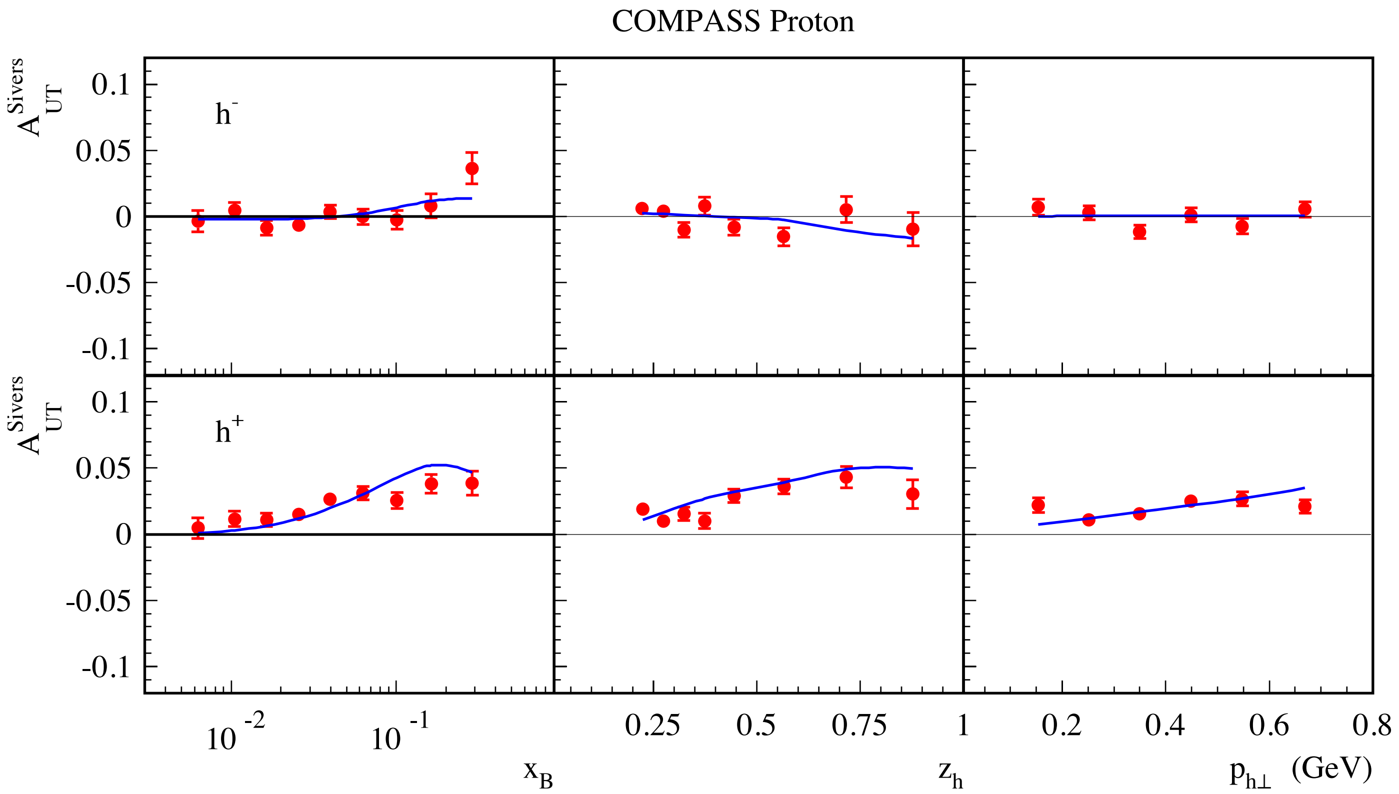
$$T_{q,F}(x,x,\mu) = N_q\frac{(\alpha_q+\beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q}\beta_q^{\beta_q}}x^{\alpha_q}(1-x)^{\beta_q}f_{q/A}(x,\mu)$$

- Total parameters (11):

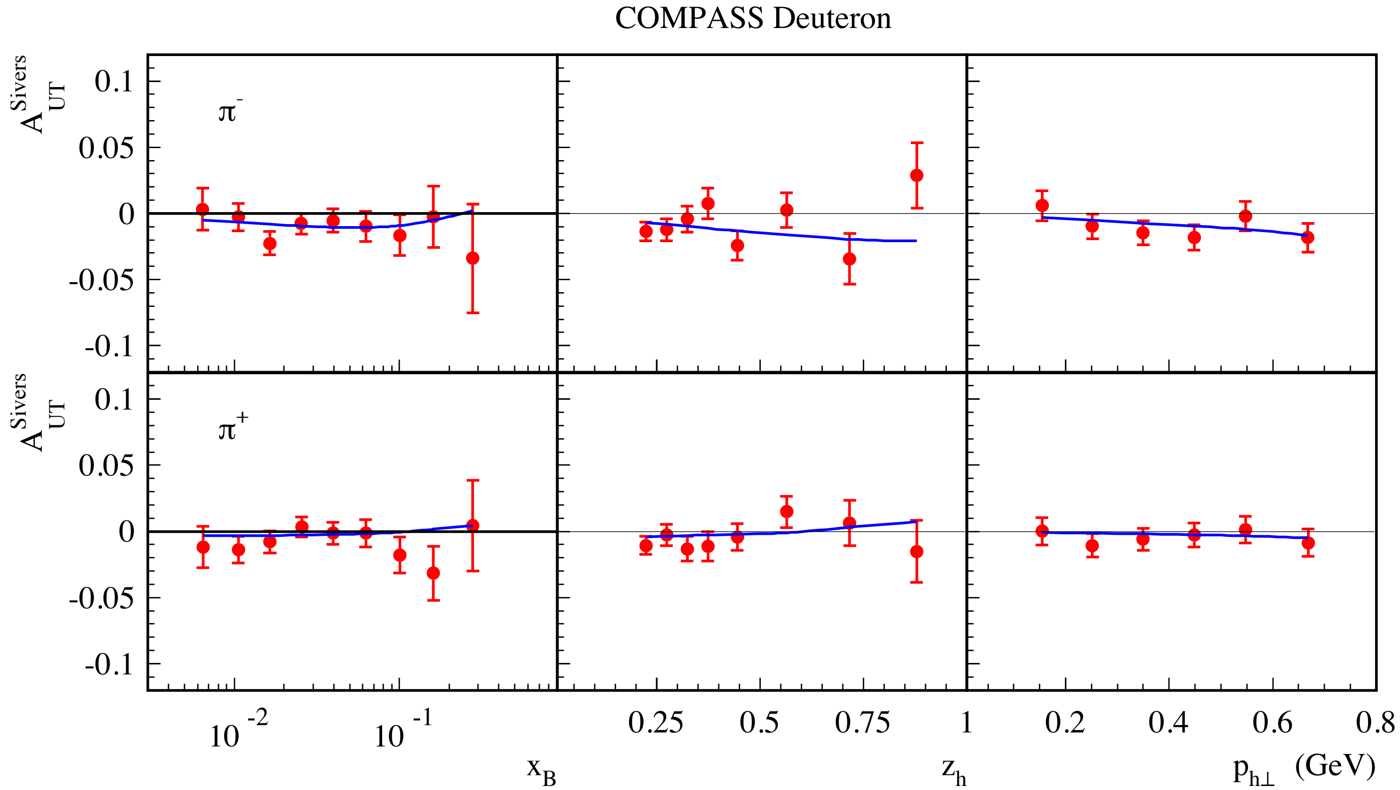
$$\langle k_{s\perp}^2\rangle, N_u, N_d, N_{\bar{u}}, N_{\bar{d}}, N_s, N_{\bar{s}}, \alpha_u, \alpha_d, \alpha_{\rm sea}, \beta$$

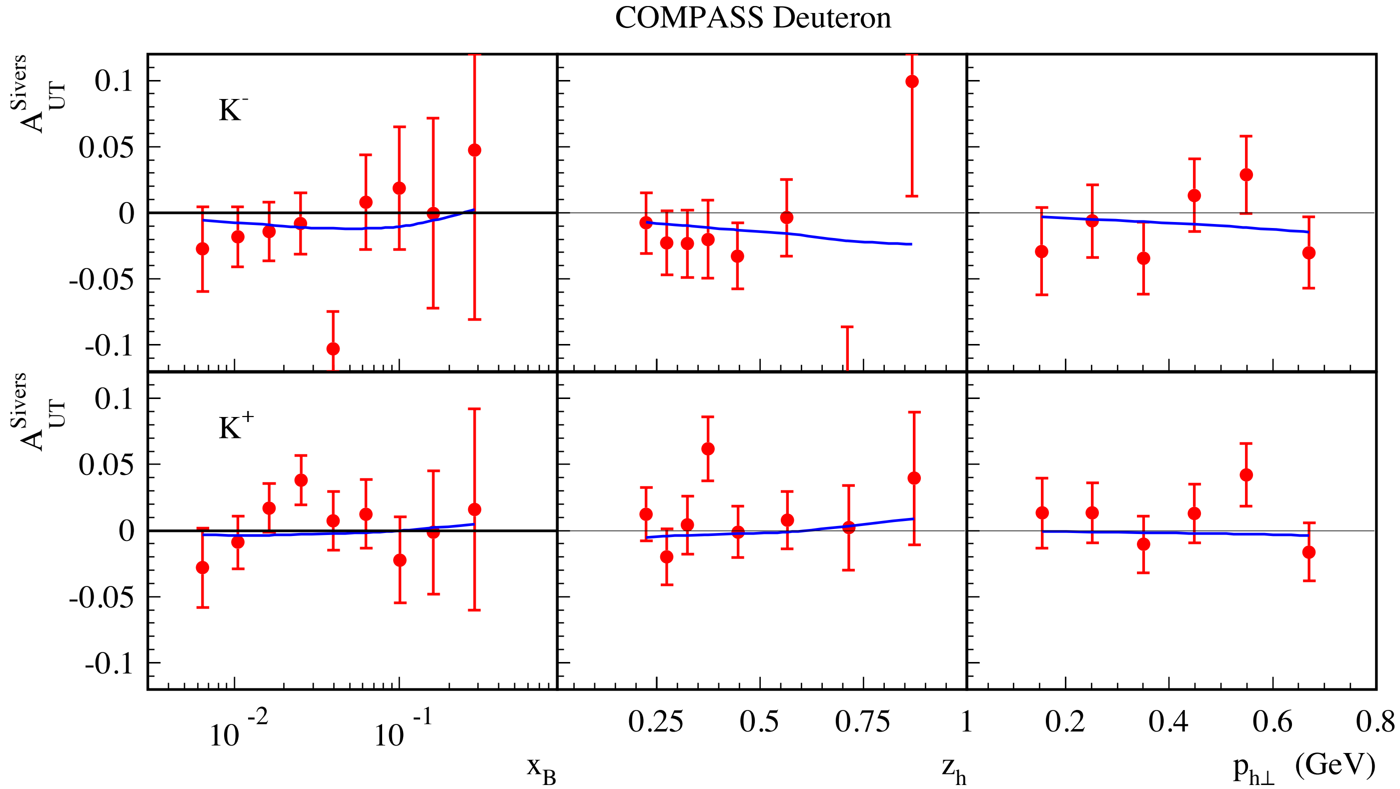
Fitted results

- COMPASS proton

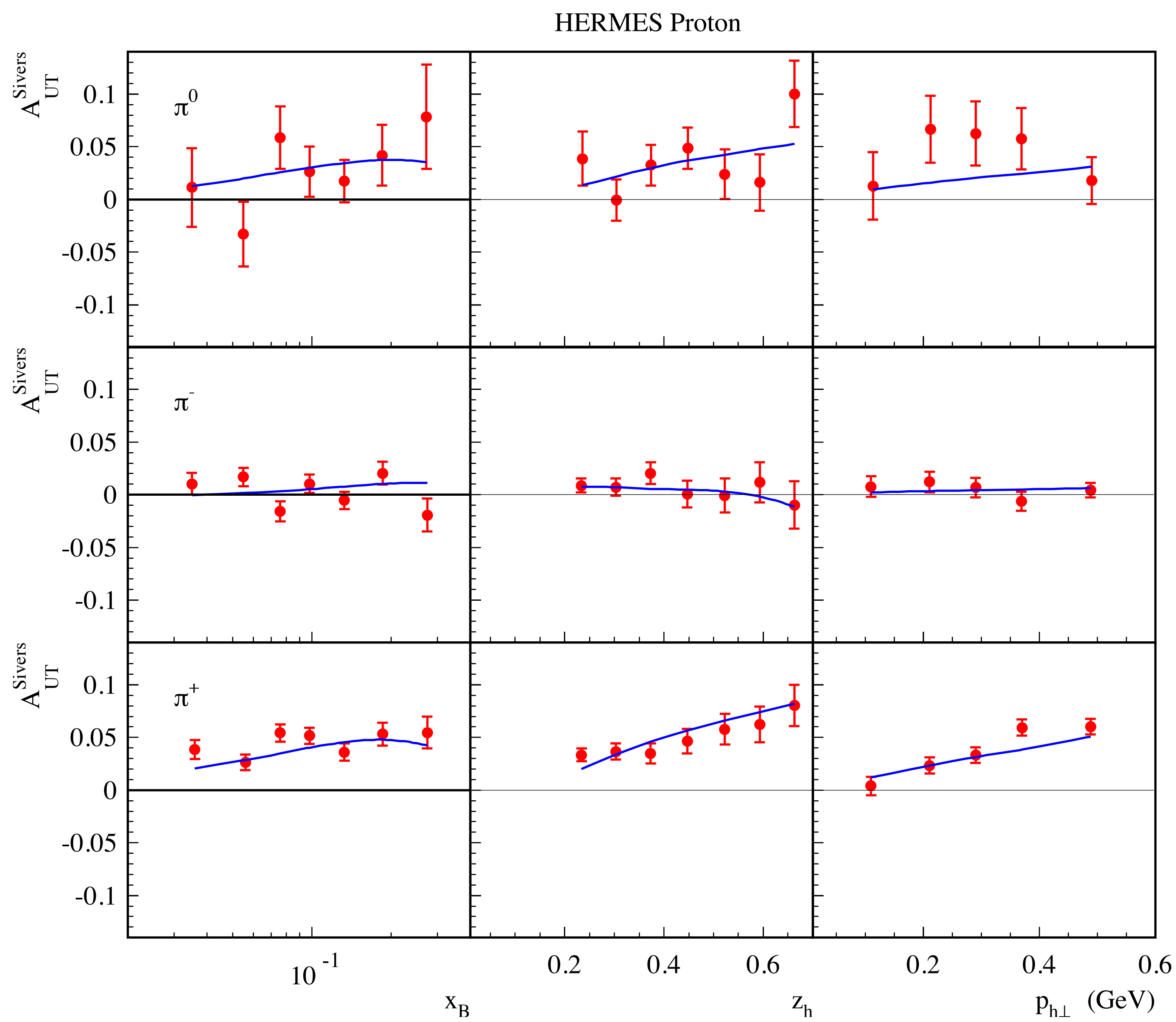


COMPASS Deuteron target



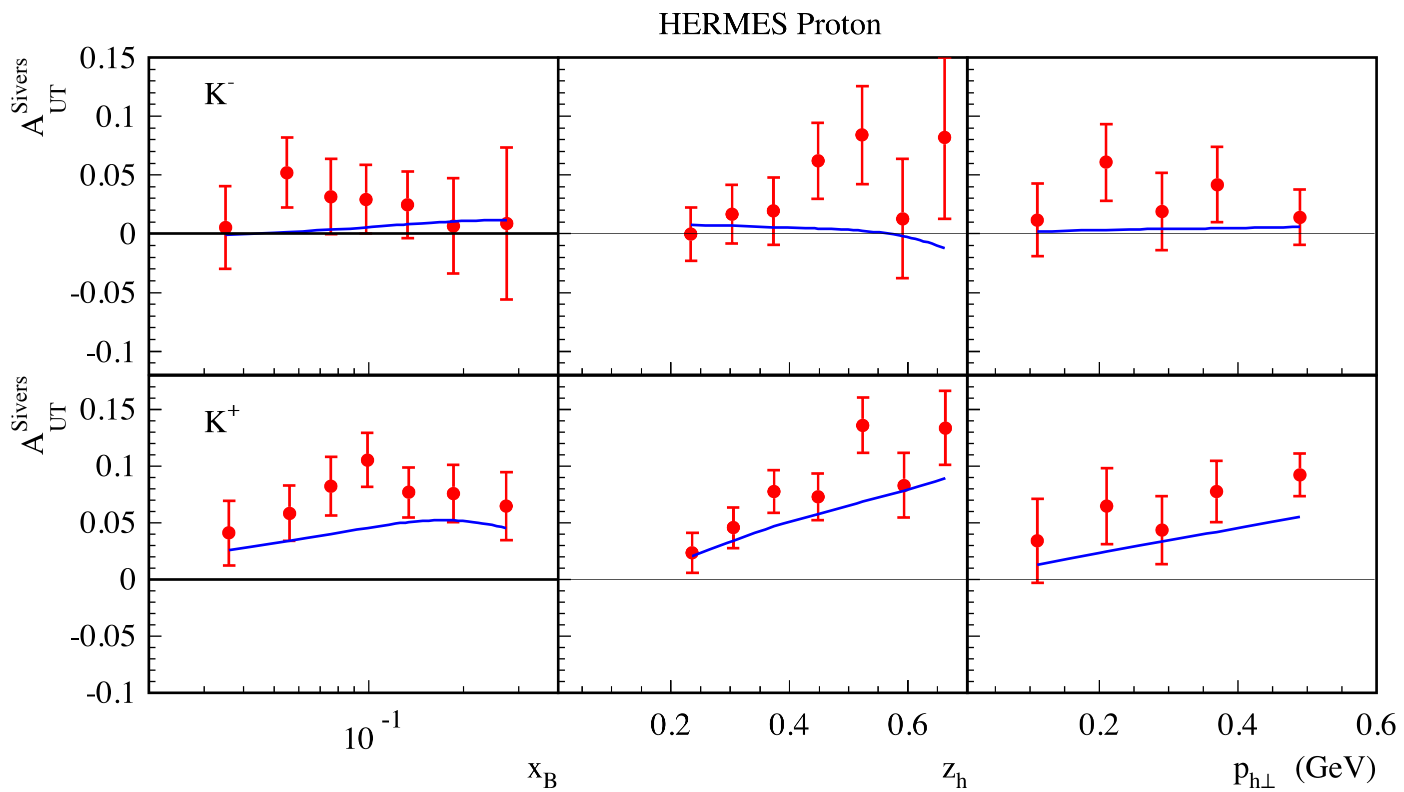


HERMES pion

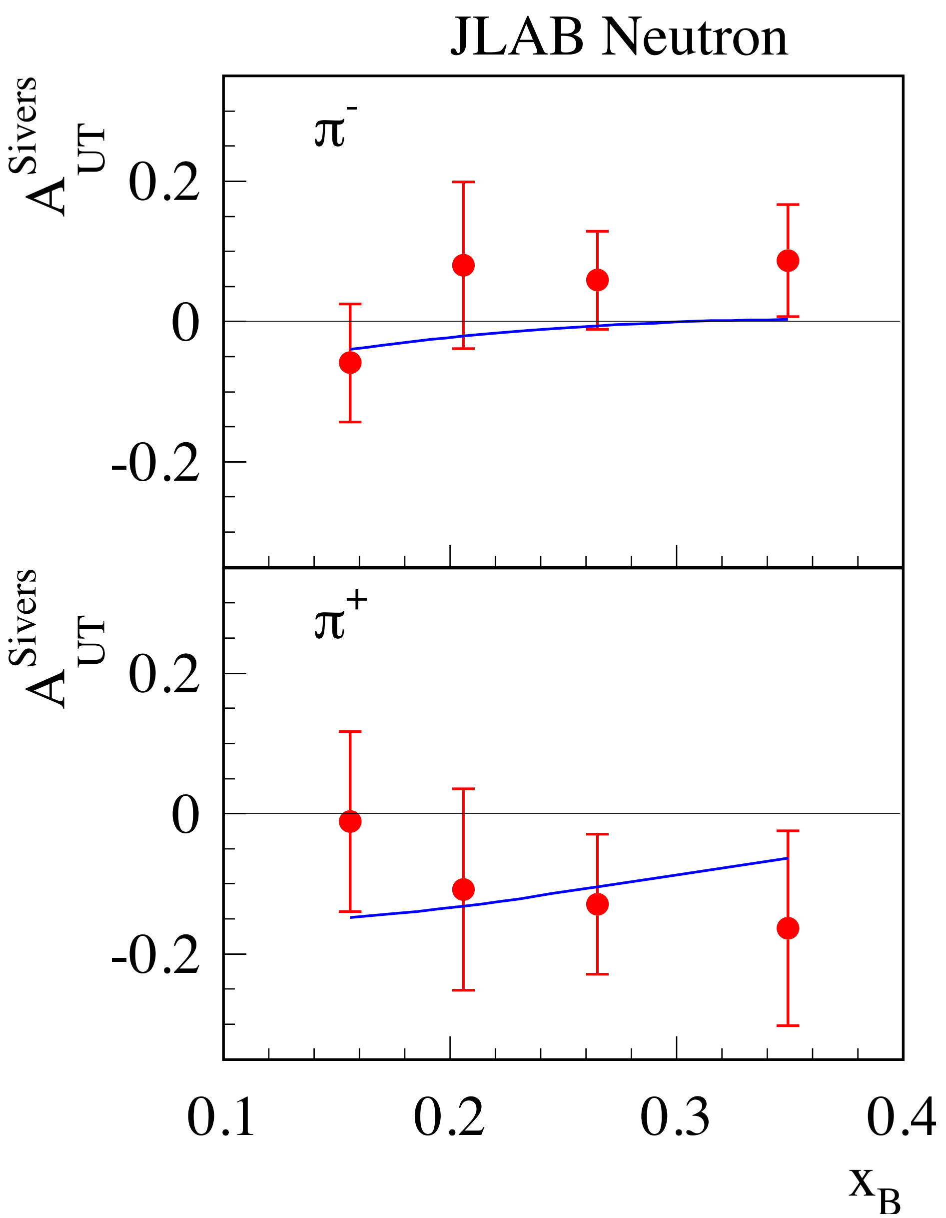


HERMES Proton
A_{UT}^{Sivers}
π^0
π^-
π^+
0.1
0.05
0
-0.05
-0.1
10^{-1}
x_B
0.2
0.4
0.6
z_h
$p_{h\perp}$ (GeV)

HERMES Kaons

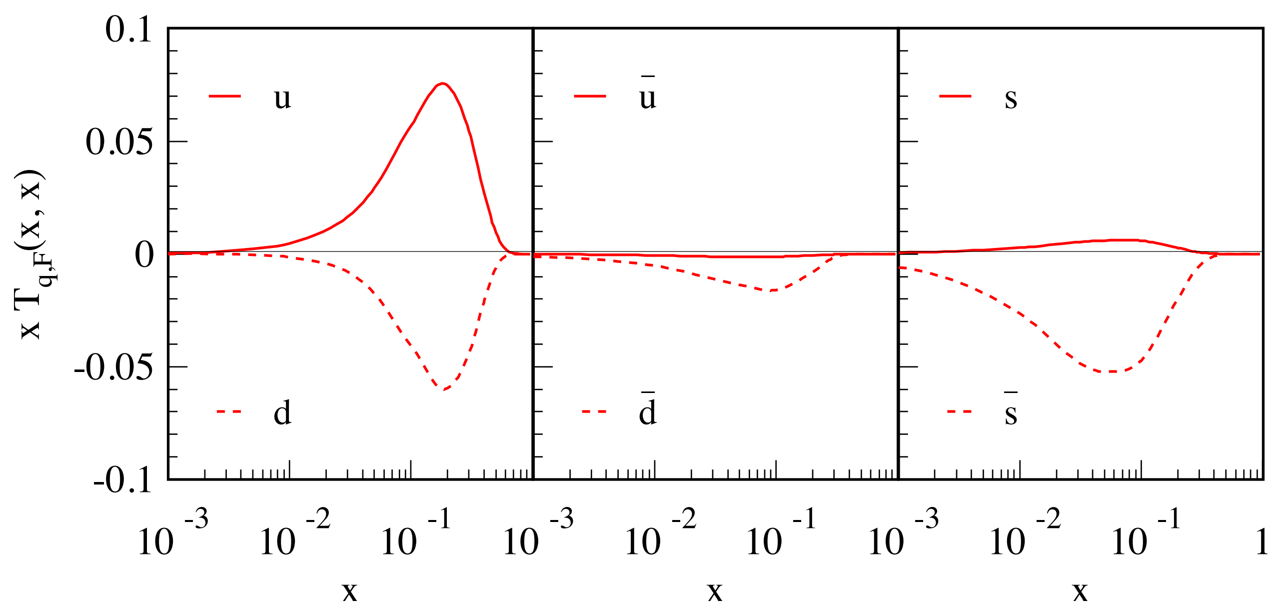


JLab neutron target



Fitted Qiu-Sterman function

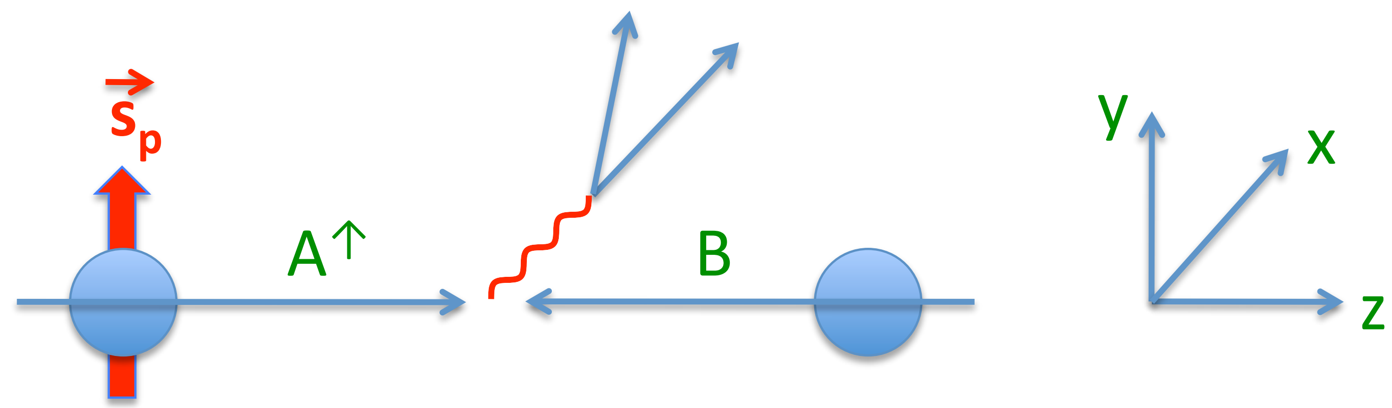
- chi2/d.o.f = 1.3: slightly larger than the usual Gaussian fit. Feel more confident when extrapolated to the whole Q range

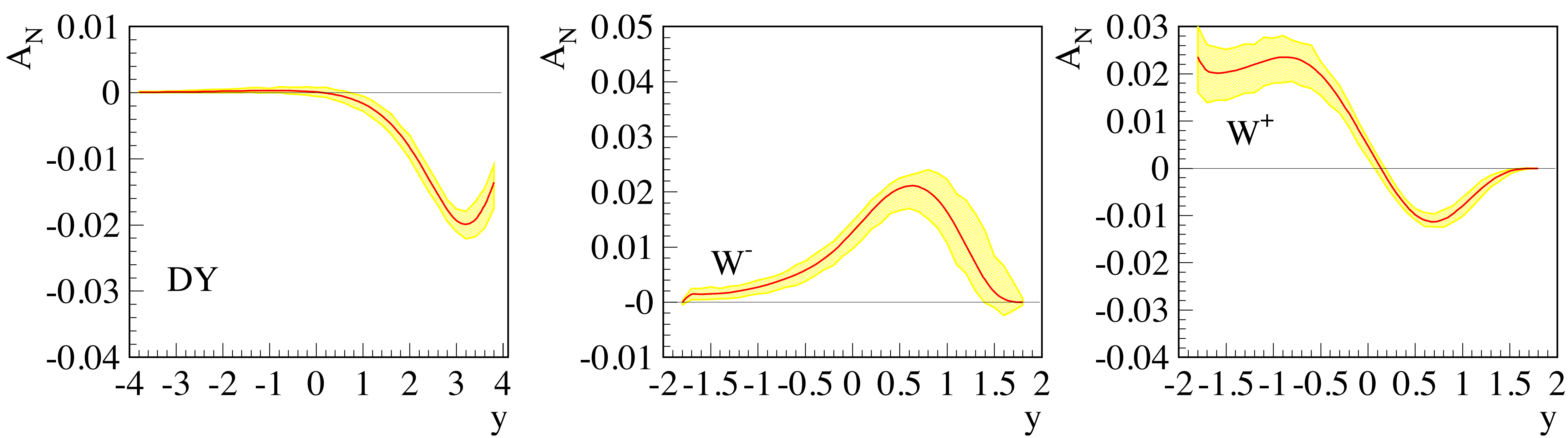


- Only u and d quark Sivers functions are constrained by SIDIS data, all the sea quark Sivers functions are not constrained
  - If set all sea quark Sivers functions vanishing, one still obtains the almost the same chi2/d.o.f.

Some predictions for asymmetries of DY and W

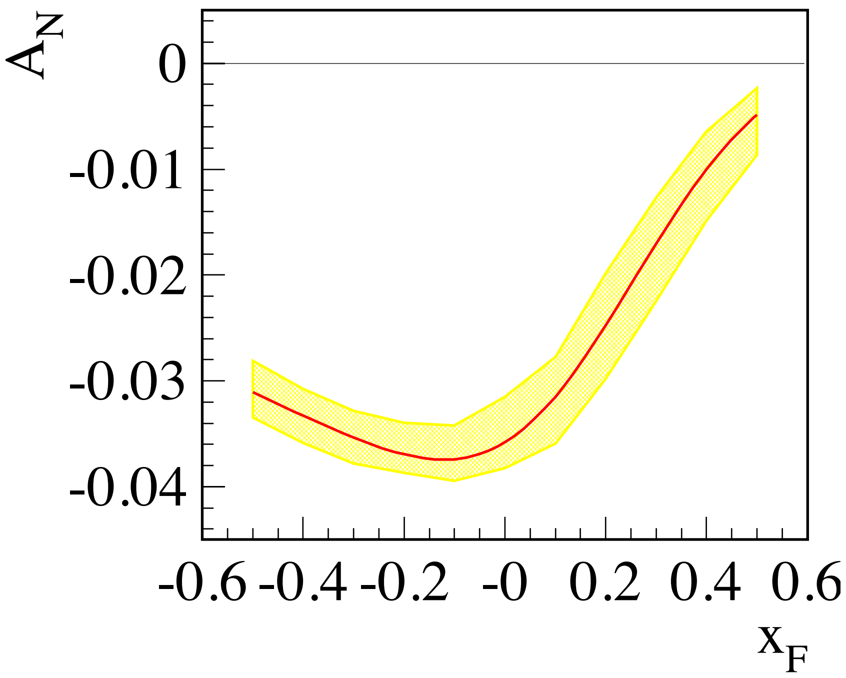
- At 510 GeV RHIC energy (DY: pt [0,1], Q [4,9] W: pt [0,3] GeV)



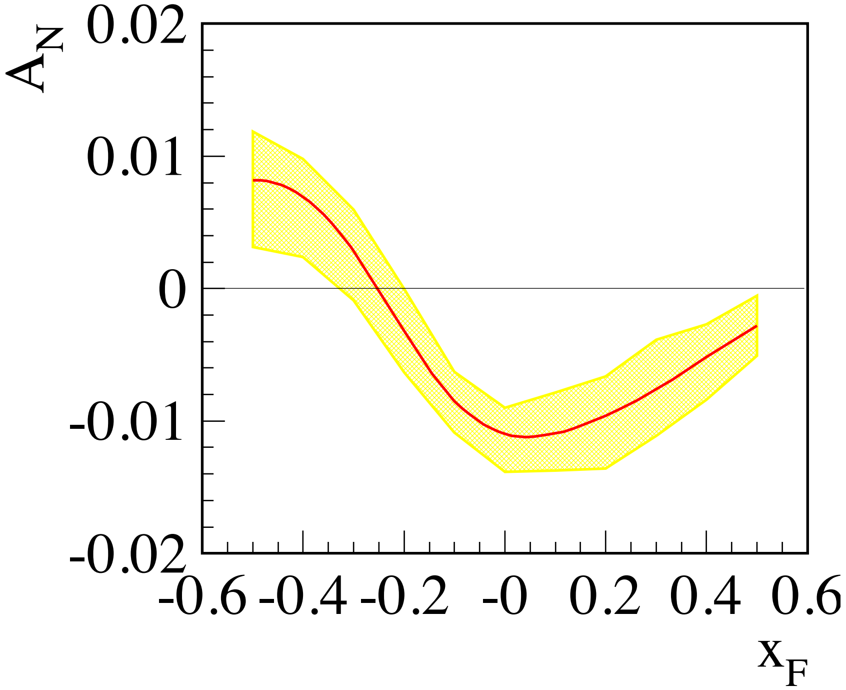


Predictions for other experiments

- DY at COMPASS: 190 GeV pi- beam

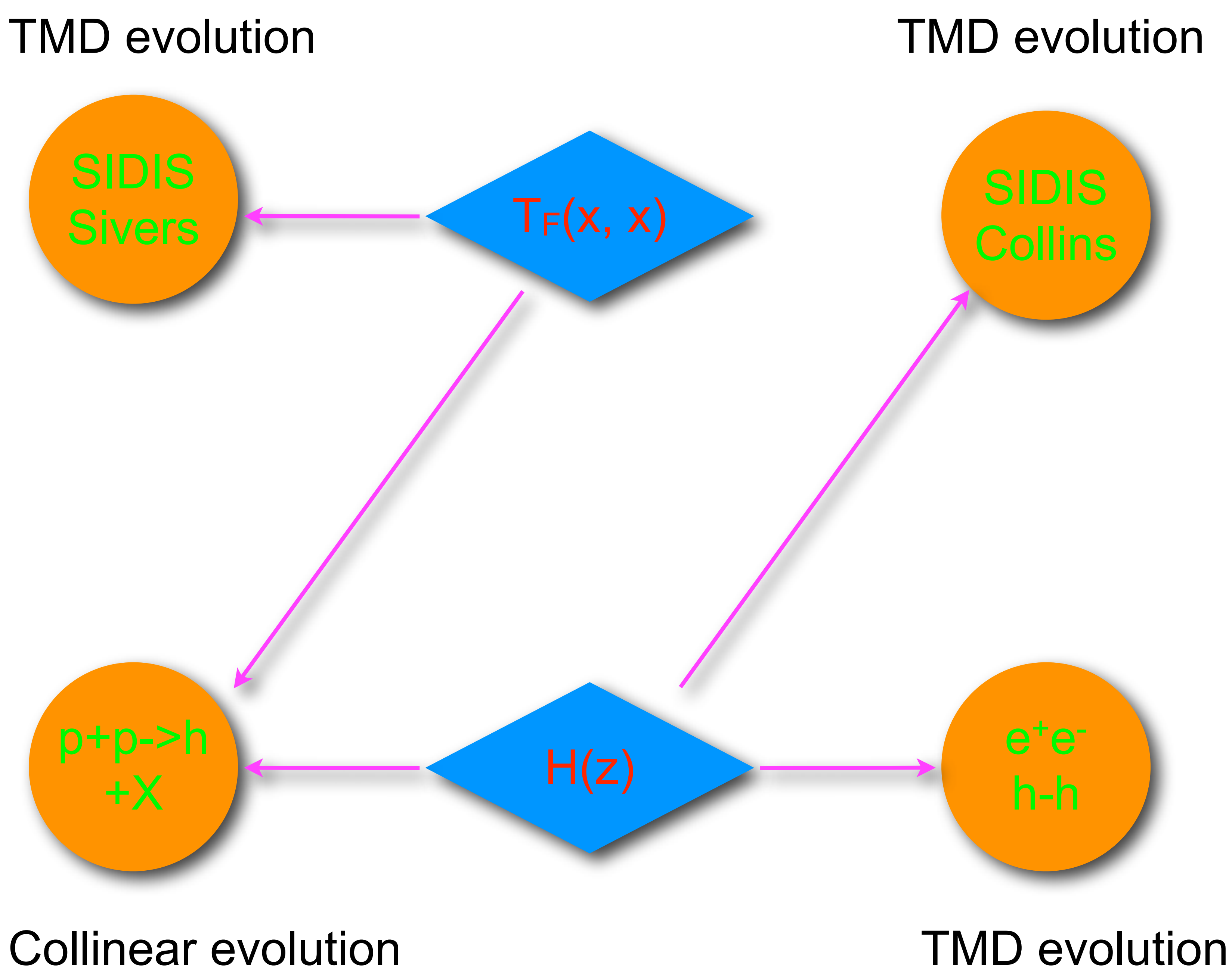


- DY at Fermilab: xf>0 polarized beam; xf<0 polarized target



Roadmap for global analysis of spin asymmetries

Gamberg-Kang-Prokudin, in preparation



Summary

- Perturbatively, the QCD evolution kernel for TMDs are the same in all existing approaches
- The difficult on the QCD evolution comes from pinning down the non-perturbative part, which has to be fitted from experimental data
- We find some simple non-perturbative form, which can describe all the data on SIDIS, DY, W/Z production
- Use the same non-perturbative form, we extract the Sivers function and predict the asymmetry for DY and W production at RHIC energy

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Thank you