Parton Physics and Large Momentum Effective Field Theory (LaMET)

XIANGDONG JI

UNIVERSITY OF MARYLAND

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Outline

- Wilson’s unsolved problem
- Large-momentum effective field theory (LaMET)
- An example
- Practical considerations
- Closing remarks

Wilson’s unsolved problem

Parton physics

- The physics of the bound state is described by parton properties in Inifite Momentum Frame (*a la* Feynman)
  - Parton distributions
  - Parton distribution amplitudes (light-cone wave funcitons)
  - GPDs (generalized parton distributions)
  - TMDs transverse-momentum-dependent parton distributions)
  - higher twists…

Light-front formulation

- Formulating a bound state problem in the IMF directly is complicated, because of the delicate cancellation of P=∞ factors, although the physics is straight forward *a la* Feynman & Weinberg.
- The most economic formulation of parton physics is through the so-called light-front quantization (Dirac):

“simple” theoretically, but a somewhat exoteric to non-experts.

Parton distribution in LF

- Can be formulated in as the matrix elements of the boost-invariant light-front correlations .

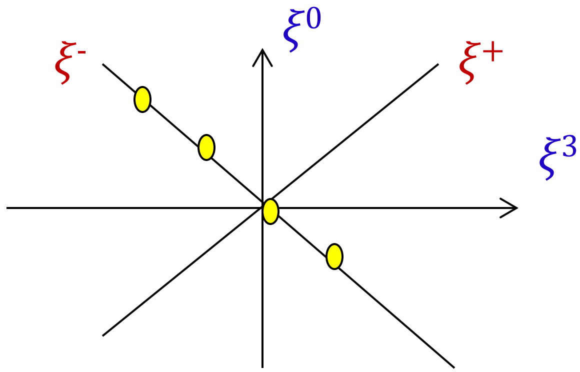
$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ \times \exp\left(-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\psi(0)|P\rangle \,,$$

where $\xi^{\pm} = (\xi^0 \pm \xi^3)/\sqrt{2}$ are light-cone variables, A^+ is the gluon potential etc.

The distribution is independent of the momentum P.

Light-front correlations

- Quark and gluon fields are distributed along the light-cone ξ^-direction



- Parton physics involves time-dependent dynamics.
- This is very general, parton physics = “light cone physics” of bound states.

Problems in calculating light-cone physics on lattice

- Much of the parton physics in theory so far is done through models!
- Lattice is the only non-perturbative ab inito approach, but cannot handle real time.
- One can form local moments to get rid of the time-dependence $\langle x^n \rangle = \int q(x) x^n dx$
  →matrix elements of local operators
  - However, one can only calculate lowest few moments in practice.
  - Many other parton properties cannot be not related to local operators, e. g. TMDs.

Rise of Light-front quantization in early 1990's

- Choose a new system of coordinates with ξ^+ as the new "time" (light-cone time) and ξ^- as the new "space" (light-cone space); Also choose the light-cone gauge $A^+ = 0$
- Light-cone correlation becomes "equal-time" correlation.
- Parton physics is manifest through light-cone quantization (LCQ)

$$\psi_+(\xi^+ = 0, \xi^-, \xi_\perp) = \int \frac{d^2k_\perp}{(2\pi)^3}\frac{dk^+}{2k^+}\sum_\lambda \left[b_\lambda(k)u(k\lambda)e^{-i(k^+\xi^- - \vec{k}_\perp\cdot\vec{\xi}_\perp)} + d^\dagger_\lambda(k)v(k\lambda)e^{i(k^+\xi^- - \vec{k}_\perp\cdot\vec{\xi}_\perp)}\right] .$$

Wilson's unsolved problem

- In early 1990's, Ken Wilson became a strong proponent for LCQ as a non-perturbative approach to solve QCD, and thus a way to calculate parton physics.
  - Wilson published 44 papers in his life time.
  - His h-index is 33, with average citation 300.
  - He published 14 paper after 1993, among which 10 was on LCQ
- However, despite many years of efforts by many, light-front quantization has not yielded a systematic approach to calculate non-perturbative QCD physics!

Large momentum effective field theory (LaMET)

What is LaMET good for?

- LaMET is a theory allowing ab initio computation of light-cone (light-front, parton) physics on a Euclidean lattice!

Step 1: Constructing lattice operators and evaluate the ME

- Construct a *frame-dependent, Euclidean* quasi-operator "O".
- In the IMF limit, O becomes a light-cone (light-front, parton) operator *o*.

$$O_1 = A^0 \;\rightarrow\; \mathrm{o} = \Lambda\, A^+$$

There are many operators leading to the same light-cone operator.

$$O_2 = A^3 \;\rightarrow\; \mathrm{o} = \Lambda$$
$$O_3 = \alpha A^0 + (1-\alpha)A^3 \;\rightarrow\; \mathrm{o} = \Lambda\, A^+$$

Step 2: lattice calculations

- Compute the matrix element of O on a lattice
- It will depend on the momentum of the hadron P, O(P,a).
- It also depends on the details of the lattice actions (UV specifics).

Step 3: Extracting the light-cone physics from the lattice ME

- Extract light-front physics o(μ) from O(P,a) at large P through a EFT matching condition or factorization theorem,

$$O(P,a) = Z(\frac{\mu}{P})o(\mu) + \frac{c_2}{P^2} + \frac{c_4}{P^4} + \cdots$$

  Where Z is perturbatively calculable.

- Infrared physics of O(P,a) is entirely captured by the parton physics o(μ). In particular, it contains all the collinear divergence when P gets large.

Factorization

- Z contains all the lattice artifact (scheme dependent), but only depends on the UV physics, can be calculated in perturbation theory, containing large logarithms of $\ln(P/\mu)$ when P is large.
- Summation over large momentum logs through RG equation.

Momentum dependence in RG

- Large logs can be resumed through renormalization group equation.
- Define,

$$\gamma(\alpha_s) = \frac{1}{Z}\frac{\partial Z}{\partial lnP}$$

we have,

$$\frac{\partial O(P)}{\partial \ln P} = \gamma(\alpha_s)O(P) + 0(\frac{1}{\mathrm{P}^2})$$

Why factorization exists?

- When taking $p \to \infty$ first, before a UV regularization imposed, one recovers from O, the light-cone operator. [This is done through construction.]
- The lattice matrix element is obtained at large P, with UV regularization (lattice cut-off) imposed first.

Order of limits

- Thus the difference between the matrix elements o and O is the order of limits:

  o: $P \rightarrow \infty$, followed by UV cut-off

  O: UV cut-off imposed first, followed by $P \rightarrow \infty$

- This is the starndard set-up for effective field theory, such as HQET. The generic argument for factorization follow through. Hence we have large-momentum effective field theory: LaMET.
- Perturbative proof case by case.

Comparison with high-energy expts

- This extraction of light-cone observable is similar to factorization of an experimental cross section in high-energy scattering.

	"Observables"	Scale separation	Effective quantities
High-Energy Scattering, Large momentum transfer Q	Cross sections (Q)	Factorization theorems, Soft-Collinear ET (SCET)	Parton physics including distributions etc.
Lattice QCD calculation, Large hadron momentum P	Quasi-observables (P)	LaMET, matching	Parton physics, all kinds of parton properties

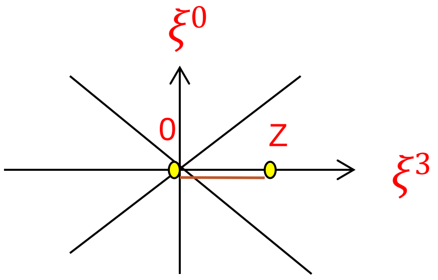
Universality class

- Just like the same parton distribution can be extracted from different hard scattering processes, the same light-cone physics can be extracted from different lattice operators.
- All operators that yield the same light-cone physics form a universality class.
- Universality class allows one exploring different operator O so that a result at finite P can be as close to that at large P as possible.

An example

A Euclidean quasi-distribution

- Consider space correlation in a large momentum P in the z-direction.



$$\tilde{q}(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \times \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle$$

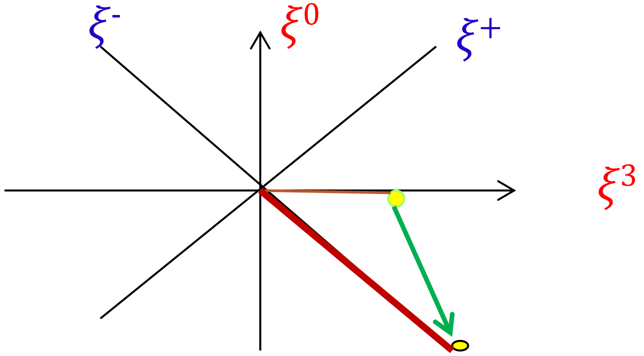
- Quark fields separated along the z-direction
- The gauge-link along the z-direction
- The matrix element depends on the momentum P.

Taking the limit P-> ∞ first

- After renormalizing all the UV divergences, one has the standard quark distribution!
  - One can prove this using the standard OPE
  - One can also see this by writing

    |P> = U(Λ(p)) |p=0>

  and applying the boost operator on the gauge link.



Finite but large P

- The distribution at a finite but large P is the most interesting because it is potentially calculable in lattice QCD.
- Since it differs from the standard PDF by simply an infinite P limit, it shall have the same infrared (collinear) physics.
- It shall be related to the standard PDF by a matching factor $Z(\frac{\mu}{P})$ which is perturbatively calculable.

One-loop matching

FIG. 1: One loop corrections to quasi quark distribution.

$$\tilde{q}(x,\mu^z,P^z) = (1+\tilde{Z}_F^{(1)}+\dots)\delta(x-1)+\tilde{q}^{(1)}(x)+\dots \qquad ($$

with

$$\tilde{q}^{(1)}(x) = \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{1+x^2}{1-x}\ln\frac{x(\Lambda(x)-xP^z)}{(x-1)(\Lambda(1-x)+P^z(1-x))}+1-\frac{xP^z}{\Lambda(x)}+\frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2P^z}\,, & x>1\,, \\ \frac{1+x^2}{1-x}\ln\frac{(P^z)^2}{m^2}+\frac{1+x^2}{1-x}\ln\frac{4x(\Lambda(x)-xP^z)}{(1-x)(\Lambda(1-x)+(1-x)P^z)}-\frac{4x}{1-x}+1-\frac{xP^z}{\Lambda(x)} \\ +\frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2P^z}\,, & 0<x<1 \\ \frac{1+x^2}{1-x}\ln\frac{(x-1)(\Lambda(x)-xP^z)}{x(\Lambda(1-x)+(1-x)P^z)}-1-\frac{xP^z}{\Lambda(x)}+\frac{x\Lambda(1-x)+(1-x)\Lambda(x)}{(1-x)^2P^z}\,, & x<0 \end{cases} \qquad ($$

$$\Lambda(x) = \sqrt{\mu^2+x^2(P^z)^2}$$

Properties

- It was done in cut-off regulator so that the result will be similar for lattice perturbation theory.
- It does not vanish outside $0<x<1$, because there are backward moving particles.
- All soft divergences cancels. All collinear divergences appear in $0<x<1$, exactly same as the light-cone distribution.
- As $p \to \infty$ first, one recover the standard LC distribution.

Factorization or matching at one-loop

- Since the IR behavior of the quasi-distribution is the same as the LC one, one can write down easily a factorization to one-loop order.

$$\tilde{q}(x,\mu^2,P^z) = \int_{-1}^{1}\frac{dy}{|y|}Z\left(\frac{x}{y},\frac{\mu}{P^z}\right)q(y,\mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right)$$

$$Z(x,\mu/P^z) = \delta(x-1) + \frac{\alpha_s}{2\pi}Z^{(1)}(x,\mu/P^z) + \ldots$$

- Where the matching factor is perturbative.

bution to the light-cone quark distribution. For $\xi > 1$, one has

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi}{\xi-1} + 1 + \frac{1}{(1-\xi)^2}\frac{\mu}{P^z}\,,$$

whereas for $0 < \xi < 1$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{(P^z)^2}{\mu^2} + \left(\frac{1+\xi^2}{1-\xi}\right)\ln\left[4\xi(1-\xi)\right] - \frac{2\xi}{1-\xi} + 1 + \frac{\mu}{(1-\xi)^2P^z}$$

and for $\xi < 0$

$$Z^{(1)}(\xi)/C_F = \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi-1}{\xi} - 1 + \frac{\mu}{(1-\xi)^2P^z}\,.$$

All order argument

- Consider $A^3 = 0$ gauge (no subtle spurious singularity in this case).
- There are no soft-singularities in the quasi-distribution.
- All collinear singularities are in ladder diagrams, happen when P large and gluons are collinear with the P. This singularity is the same as that of the light-distribution.

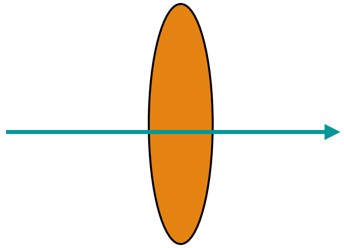
Comments

- One can make similar factorization for other observables.
- One must calculate the matching factor with lattice action, but the leading log shall be the same in any scheme.

Practical considerations

Practical considerations

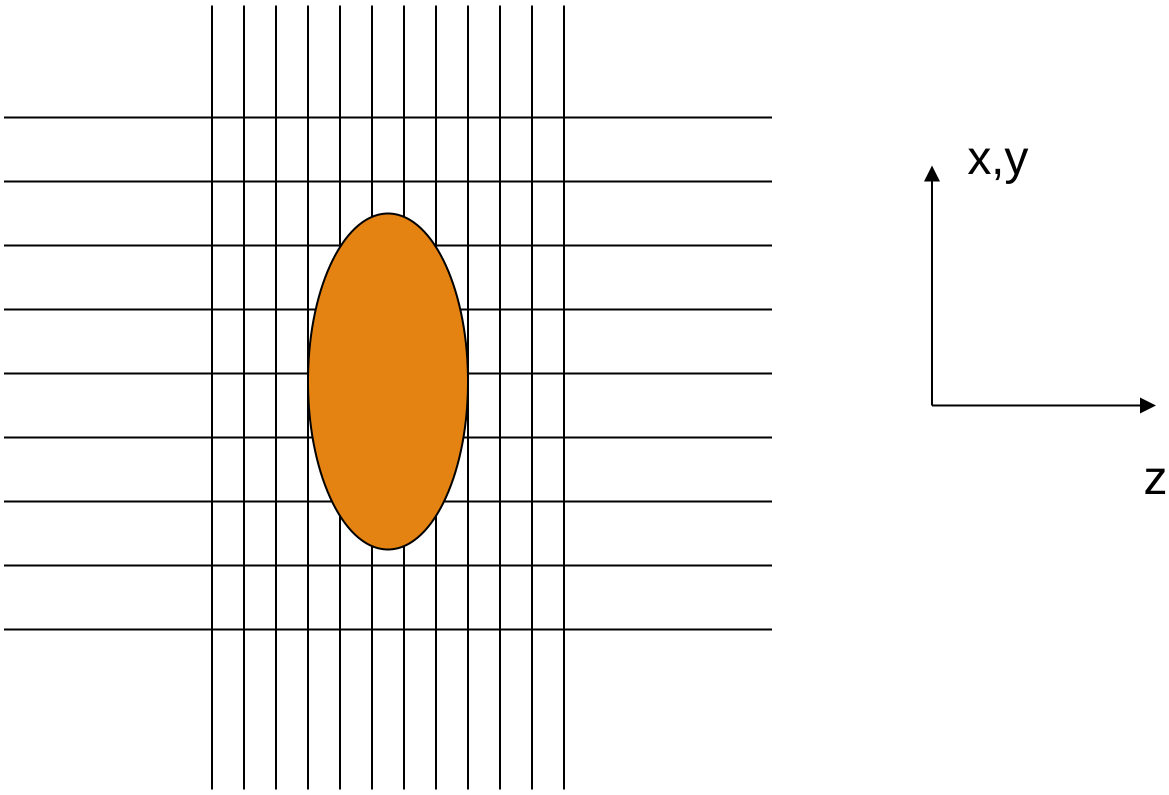
- For a fixed x, large P_z means large k_z, thus, as P_z gets larger, the valence quark distribution in the z-direction get Lorentz contracted, $z \sim 1/k_z$.



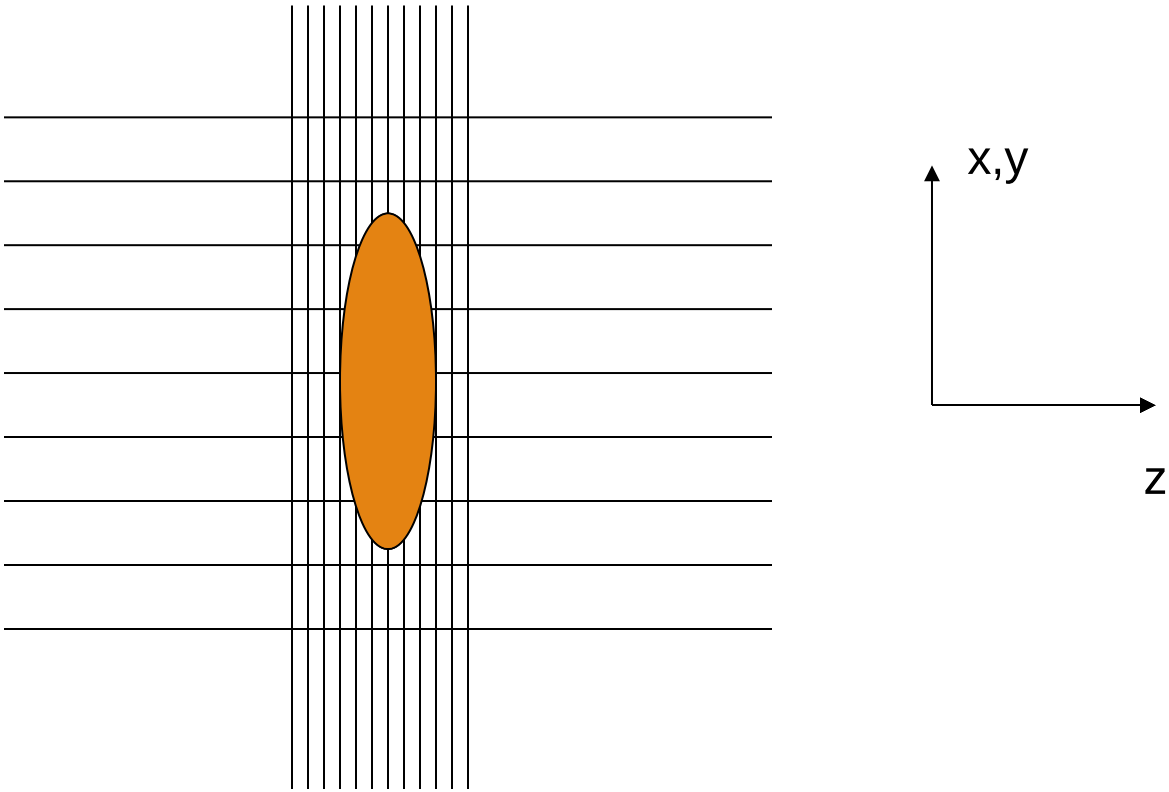
- Thus one needs increasing resolution in the z-direction for a large-momentum nucleon. Roughly speaking: $a_L/a_T \sim \gamma$

One needs special kinds of lattices

γ=2



γ=4



Small x partons

- The smallest x partons that one accesses for a nucleon momentum P is roughly,

$$x_{min} = \Lambda_{QCD}/P \sim 1/3\gamma$$

small x physics needs large γ as well.

- Consider x ~ 0.01, one needs a γ factor about 10~30. This means 100 lattice points along the z-direction.
- A large momentum nucleon costs considerable resources!

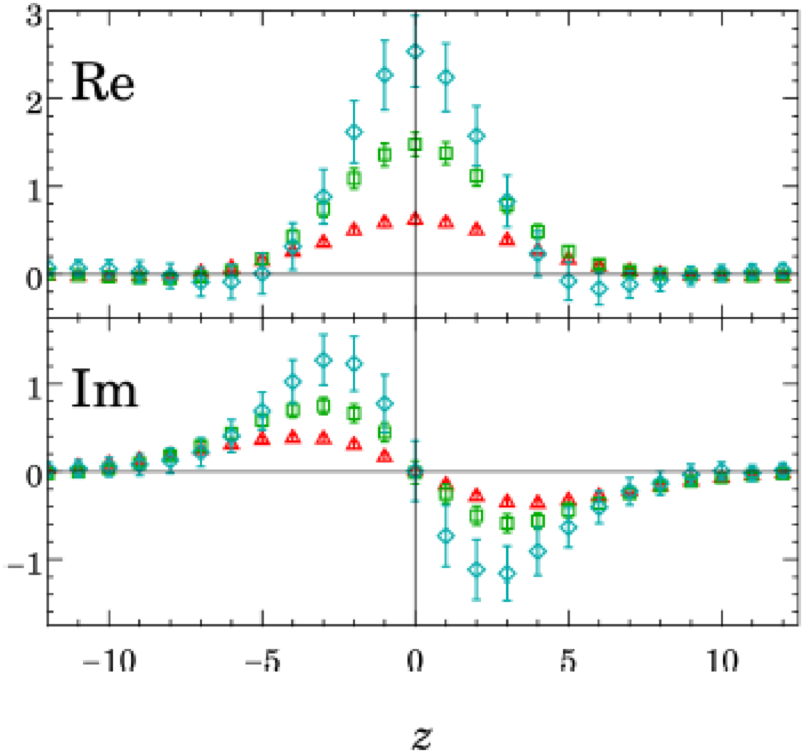
Additional remarks

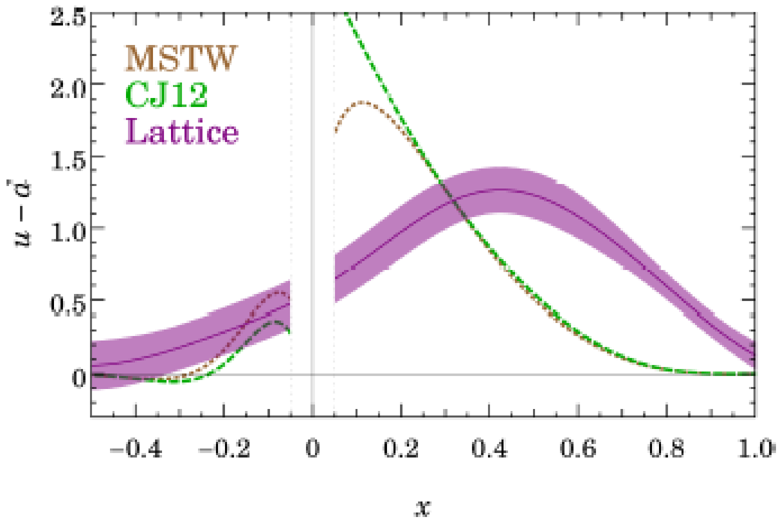
- One has learnt how to calculate in lattice QCD with large momentum scales in
  - Heavy-quark physics
  - Elastic form factors
  - Excited baryons
- For real applications, P=(2-3) GeV might be good enough for valence quarks and at not so small-x.
- Small time-resolution in necessary, thus lattice size shall be

$$(\gamma L)^2 \, \mathrm{L}^2$$

A lattice QCD calculation (H. W. Lin et al)

- $m_\pi = 310\ MeV; a = 0.12\ fm; u - d\ distribution$





Parton physics on lattice

- Gluon helicity distribution and total gluon spin
- GPDs

$$F(x,\xi,t) = \int \frac{dz}{2\pi} e^{-izk^z} \langle P'|\overline{\psi}(-z/2)\gamma^z \times L(-z/2,z/2)\psi(z/2)|P\rangle$$

- TMDs

$$q(x,k_\perp,P_z\mu^2) = \int \frac{dz}{4\pi} d^2\vec{r}_\perp e^{i(zk^z+\vec{k}_\perp\vec{r}_\perp)} \langle P|\overline{\psi}(\vec{r}_\perp,z) \times L^\dagger(\pm\infty;(\vec{r}_\perp,z))\,\gamma^z L(\pm\infty;0)\psi(0)|P\rangle$$

- Light-cone wave functions
- Higher twist observables….

Some recent work

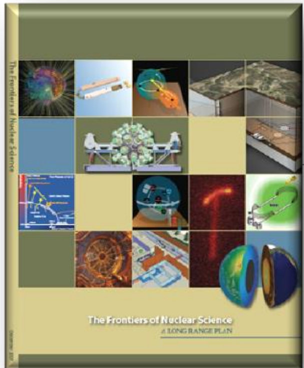
- LP^3: lattice parton physics project
- Gluon helicity distribution (Y. Zhao's talk)
- Universality class: there are infinite number of quasi-observables yielding the same physics
- Quark and gluon parton orbital angular momentum
- GPD and distribution amplitude
- …
- A workshop at Maryland, March 30, 2014

Closing remarks

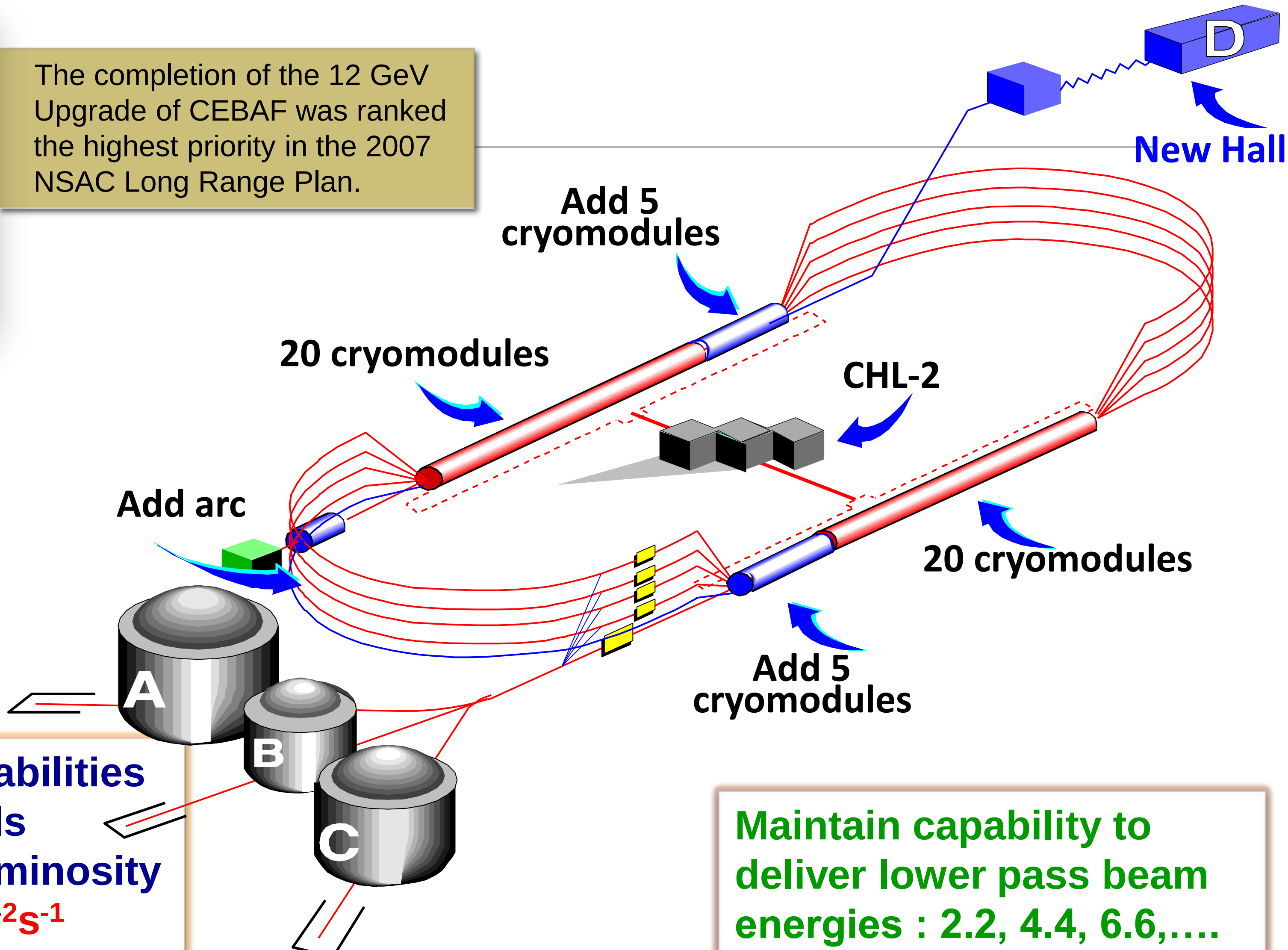
There are still LARGE GAPS in our understanding that how the protons and neutrons are made of the underlying fundamental degrees of freedom: quarks and gluons

- Where is the proton spin come from?
- How important is the quark orbital motion?
- What is the role of the gluons?
- What does the nucleon look like in a 3D tomography?

12 GeV Upgrade



The completion of the 12 GeV Upgrade of CEBAF was ranked the highest priority in the 2007 NSAC Long Range Plan.

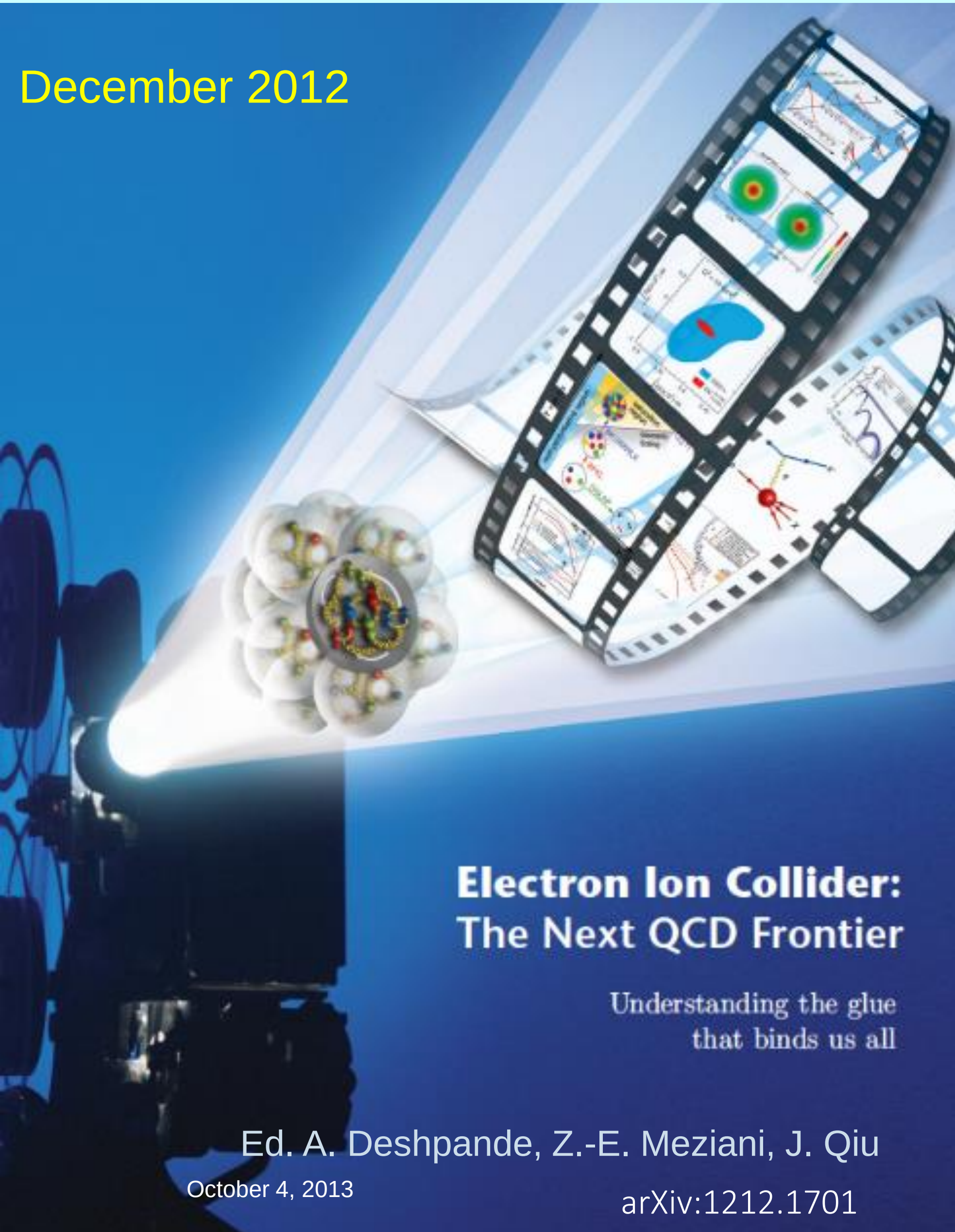


- **Enhanced capabilities in existing Halls**
- **Increase of Luminosity**
  10^{35} - ~10^{39} cm^{-2}s^{-1}

Maintain capability to deliver lower pass beam energies : 2.2, 4.4, 6.6,….

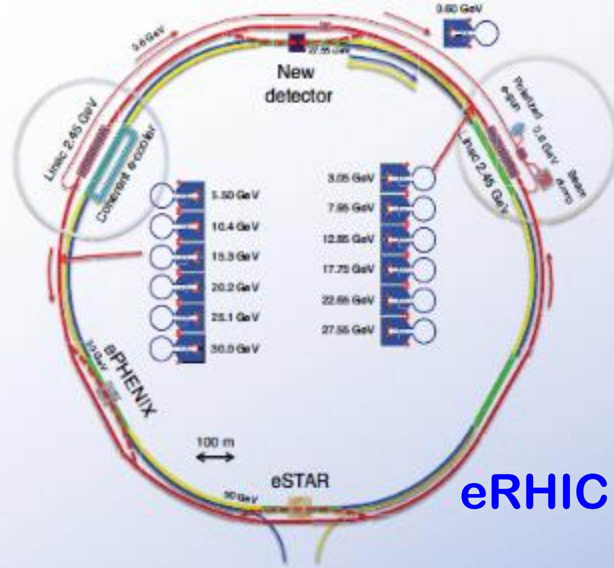
White Paper for the Electron-Ion Collider

December 2012



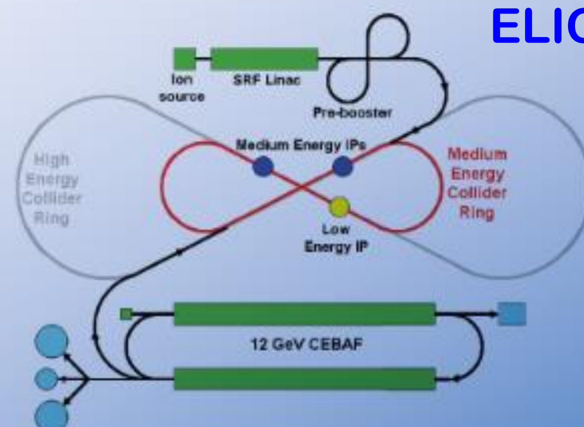
Ed. A. Deshpande, Z.-E. Meziani, J. Qiu

arXiv:1212.1701



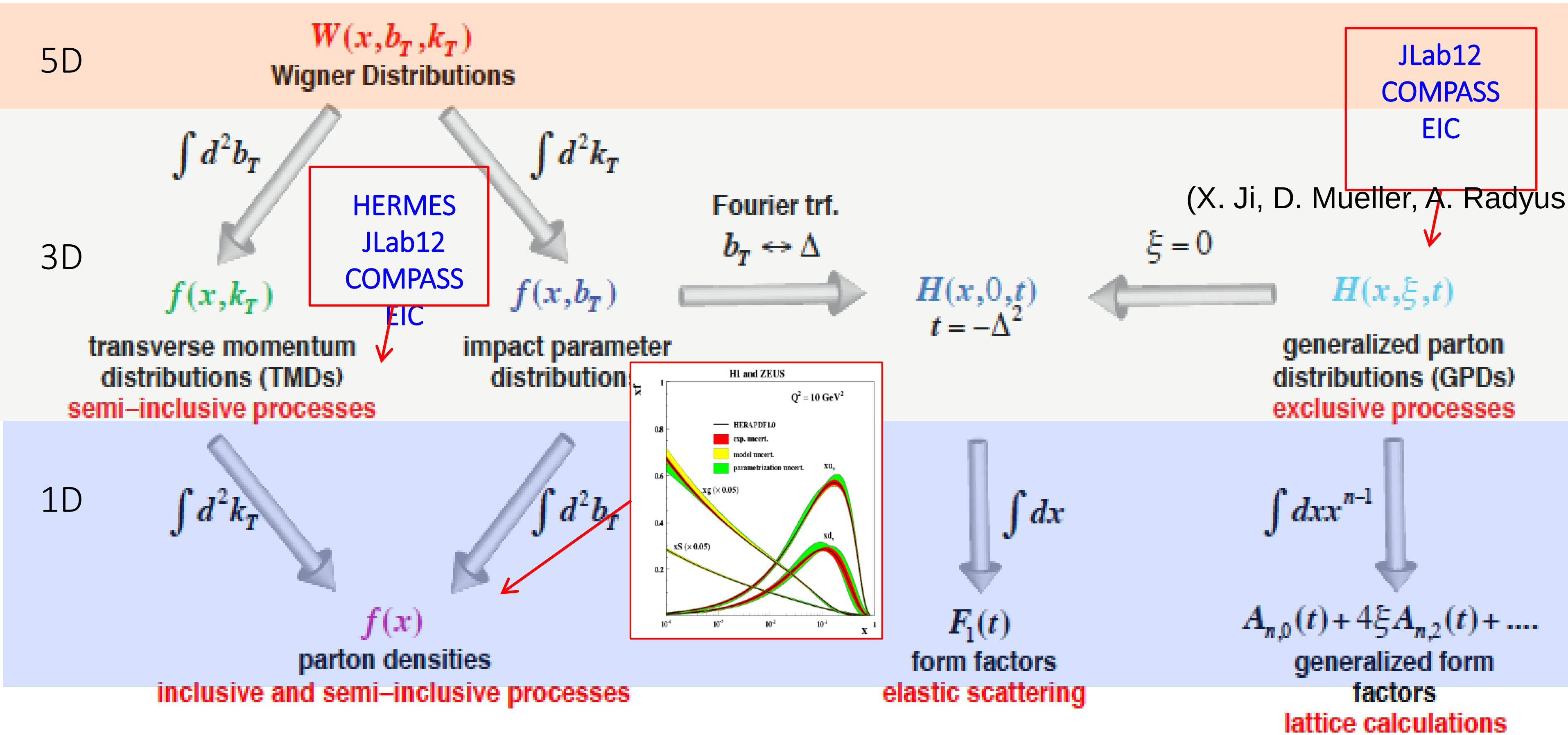
eRHIC (BNL)

ELIC (JLab)



Unified view of the Nucleon Parton Structure

- Wigner distributions (Belitsky, Ji, Yuan)



Summary

- Parton physics demands novel technique to solve non-perturbative QCD
- New progress recently allows one calculate parton physics using Euclidean lattice. Thus interesting properties like GPDs and TMDs can now be calculated.
- With 12 GeV upgrade at Jlab and future EIC, there is an exciting era of precision comparison between experimental data and theoretical calculations.

Papers

- X. Ji, Parton Physics on a Euclidean Lattice, Phys. Rev. Lett. 110, 262002 (2013).
- X. Ji, J. Zhang, and Y. Zhao, Physics of Gluon Helicity Contribution to Proton Spin, Phys. Rev. Lett. 111, 112002 (2013).
- Y. Hatta, X. Ji, and Y. Zhao, Gluon Helicity ΔG from a Universality Class of Operators on a Lattice, arXiv:1310.4263v1
- H. W. Lin, Chen, Cohen and Ji, arXiv:1402.1462
- …

Collaborators

- Yong Zhao, Ph.D. Student at U. Maryland
- Xiaonu Xiong, Ph. Student at Peking U.
- Jianhui Zhang, Postdoc at Shanghai Jiao Tong U
- Huey-wen Lin, Research Assistant Prof. At U. Washington
- Jiunn-Wei Chen, Prof. at National Taiwan U.
- Feng Yuan, Senior research scientist at LBL
- Y. Hatta, Prof. at Kyoto University