Non-perturbative TMD parameters and p_T-spectra of electroweak bosons

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"Studies of 3D Structure of Nucleon"

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Motivation I

♠ TMD PDFs concept and TMD-factorization: CRUCIAL for processes sensitive to intrinsic parton transverse momentum.

♠ We have many examples in which TMD-factorization is at work at small transverse momentum: Drell-Yan (DY), (SIDIS), etc.

♠ TMD PDFs: useful tool in the construction of Monte Carlo event generators, where the details of final state kinematics are important.

♠ GPD and TMD PDFs: a road map for understanding the 3D structure of the proton

Motivation II

In this talk :

★ we illustrate how recent Drell-Yan data can be used to extract the nonperturbative component of the CSS resummed cross section and estimate its dependence on arbitrary resummation scales and other factors.

★ We examine if the ϕ_η^* DY data corroborate the universal behavior of the resummed nonperturbative terms that is expected from the TMD factorization theorem.

★ The analysis: technically challenging, it requires to examine several effects that were negligible in the previous studies of the resummed nonperturbative terms.

Recent theory developments

A lot of progress is going on TMD factorization

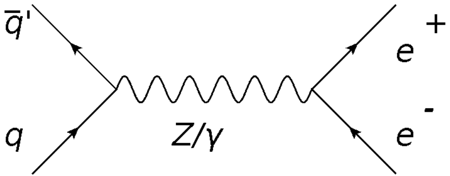
- Collins and Rogers, PRD 2013
- Aybat, Collins, Qiu and Rogers, PRD 2012
- Echevarriá, Idilbi, Schäfer, Scimemi EPJ C 2012
- Aybat and Rogers PRD 2011
- J. C. Collins and F. Hautmann, PLB 472, 129 (2000); J. C. Collins and F. Hautmann, JHEP0103, 016 (2001).
- A. A. Henneman, D. Boer and P. J. Mulders, NPB 620, 331 (2002).
- A. V. Belitsky, X. Ji and F. Yuan, NPB 656, 165 (2003).
- D. Boer, P. J. Mulders and F. Pijlman, NPB667, 201 (2003).
- J.C. Collins, Acta Phys. Polon. B 34, 3103 (2003).
- F. Hautmann and D. E. Soper, PRD 75, 074020 (2007).
- J. C. Collins, T. C. Rogers and A. M. Stasto, PRD 77, 085009 (2008).
- I. O. Cherednikov and N. G. Stefanis, Phys. Rev. D 77, 094001 (2008); I. O. Cherednikov, A. I. Karanikas and N. G. Stefanis, NPB 840, 379 (2010).

What we are going to show

- Analysis of the Z/γ^* distribution in terms of the novel variable ϕ^*_η at D0 Tevatron and LHC.
  D0 Coll. Phys.Rev.Lett. 106 (2011) 122001;
  ATLAS Coll. Phys.Lett. B705 (2011) 415-434:
- These measurements are compared to a new improved version of ResBos at approx NNLO + NNLL accuracy.
- We constrain non-perturbative effects in the small ϕ^*_η region (small Q_T) and we take into account the systematic uncertainties in the $70 \leq Q \leq 110$ GeV range.
- New simple parametrization for the NP Gaussian smearing: relevant for precise measurements of M_W.

New ϕ^*_η measurements will be available in the near future from DØ, ATLAS and CMS coll.

The Drell-Yan process



In the Drell-Yan process we consider decay angles ϕ_{CS}, θ_{CS} in the Collins-Soper frame (PRD 1977)

The CS frame is a rest frame of the vector boson in which the z axis bisects the angle formed by the momenta $\vec{p}_1$ and $-\vec{p}_2$ of the incident quark and antiquark. In the CS frame, the decay leptons escape back-to-back ($\vec{l}_1 + \vec{l}_2 = 0$).

It requires knowledge of the lepton momenta and is thus susceptible to the effects of lepton momentum resolution.

Z/γ^* boson transverse momentum distribution in ϕ^*_η

A new variable ϕ^*_η has been proposed by Banfi et al. EPJC 2011, PLB701 2011 to describe final state electron and muon angular distributions in hadronic collisions.

$$\phi^*_\eta = \tan\left(\phi_{acop}/2\right)\sin\theta^*_\eta, \qquad \cos\theta^*_\eta = \tanh\left(\frac{\eta^- - \eta^+}{2}\right), \tag{1}$$

$\phi_{acop} = \pi - \Delta\varphi$ and $\Delta\varphi$: azimuthal angle φ between the leptons; $\sin\theta^*_\eta$: scattering angle of the dileptons with respect to the beam in the dilepton rest frame. In the $Q_T \to 0$ limit one has

$$\phi^*_\eta \approx a_T/M \qquad \cos\theta^*_\eta \approx \cos\theta_{cs}, \tag{2}$$

M is the dilepton invariant mass and a_T the component of Q_T normal to an axis in the transverse plane which coincides with the lepton axis (Banfi, Dasgupta, Delgado, JHEP 2009).

- less sensitivity to experimental resolution on lepton momenta.
- ϕ^*_η is accessed by a direct experimental measurement of track directions $\to$ very precise!

ϕ_η^* and the CSS variables

In the limit $Q_T \to 0$, ϕ_η^* simplifies to

$$\phi_\eta^* \approx (Q_T/Q) \sin \varphi_{CS}, \tag{3}$$

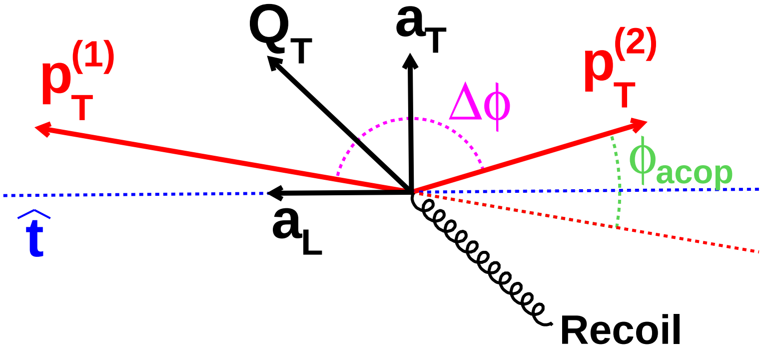
since $\tan(\phi_{acop}/2) = \sqrt{(1 + \cos \Delta\varphi) / (1 - \cos \Delta\varphi)}$, and

$$\theta_\eta^* \to \theta_{CS}, \qquad \cos \Delta\varphi \to -1 + 2 \left(\frac{Q_T}{Q} \frac{\sin \varphi_{CS}}{\sin \theta_{CS}} \right)^2. \tag{4}$$

Measurement of ϕ_η^* thus directly probes Q_T/Q at $Q_T << Q$.

More details with some algebra in the backup.

A graphical illustration



taken from EPJC (2011) 71 1600; PRL 106 (2011) 122001, arXiv:1010.0262 [hep-ex]

CSS Formalism: a brief overview

It is known for a long time that to have a good perturbative description of QCD observables like transverse momentum distributions, logarithmic contributions of the type $L = \ln(Q_T^2/Q^2)$ that have a singular behavior when $Q_T \to 0$, have to be resummed.

This reorganization is achieved by the Collins, Soper and Sterman (CSS) formalism (NPB 1985), according to which the Q_T distribution of hadronically produced lepton pairs $h_1 h_2 \to V(\to l_1 l_2)X$ is described by the following combination

$$\frac{d\sigma}{dQ_T} = \frac{d\sigma}{dQ_T}\bigg|_{W-Resummed} + \frac{d\sigma}{dQ_T}\bigg|_{F.O.} - \frac{d\sigma}{dQ_T}\bigg|_{Asymptotic} \tag{5}$$

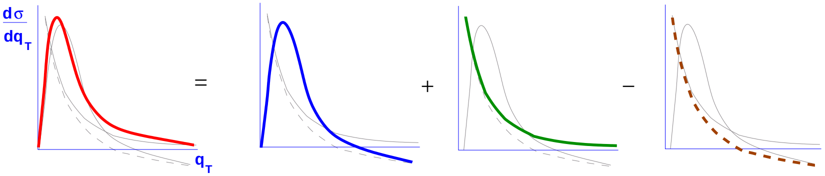
CSS Formalism: the basic structure

$$\frac{d\sigma}{dQT} = \text{RES} + \text{FO} - \text{SUBTRACTION}$$

RES: Piece containing the Sudakov exponent

FO: Calculate in perturbative QCD at order α_s^n

SUBTRACTION: Calculate as an expansion of RES up to order α_s^n



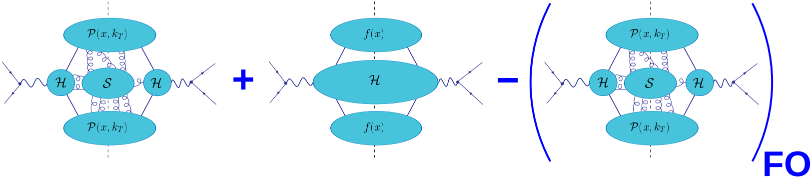
QCD factorization as a function of q_T

(according to Collins, Soper, and Sterman approach)

Small-q_T term — $\Lambda^2_{QCD} \ll q_T^2 \ll Q^2$

Large-q_T term — $\Lambda^2_{QCD} \ll q_T^2 \sim Q^2$

Overlap term



- k_T-dependent PDFs $\mathcal{P}(x, \vec{k}_T)$
- Sudakov function $\mathcal{S}(x, \vec{k}_T)$

▷ actually, their impact parameter (b) space transforms

- Collinear PDFs $f_a(x, \mu)$
- hard matrix elements $\mathcal{H}$ of order N

- Truncated perturbative expansion

$$\sum_{k=0}^{N} \alpha_s^k \sum_{m=0}^{2k-1} c_{km} \ln^m \left(\frac{q_T^2}{Q^2} \right)$$

Factorization at $Q_T \ll Q$

$$\left.\frac{d\sigma_{AB\to VX}}{dQ^2 dy dQ_T^2}\right|_{Q_T^2 \ll Q^2} = \sum_{a,b=g,\overset{(-)}{u},\overset{(-)}{d},\ldots} \int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{q}_T\cdot\vec{b}} \widetilde{W}_{ab}(b, Q, x_A, x_B)$$

$$\widetilde{W}_{ab}(b, Q, x_A, x_B) = |\mathcal{H}_{ab}|^2 \; e^{-\mathcal{S}(b,Q)} \overline{\mathcal{P}}_a(x_A, b) \overline{\mathcal{P}}_b(x_B, b)$$

$\mathcal{H}_{ab}$ is the hard vertex, $\mathcal{S}$ is the soft (Sudakov) factor, $\overline{\mathcal{P}}_a(x, b)$ is the unintegrated PDF in the gauge $\eta \cdot \mathcal{A} = 0$, $\eta^2 < 0$

$\overline{\mathcal{P}}_a(x, b) = \int d^{n-2}\vec{k}_T e^{i\vec{k}_T\cdot\vec{b}} \mathcal{P}_a(x, \vec{k}_T)$.

When $b \ll 1$ GeV^{-1}, $\mathcal{S}(b, Q)$ and $\overline{\mathcal{P}}_a(x, b)$ are calculable in perturbative QCD;

$$\overline{\mathcal{P}}_{a/A}(x, b) = \left(\mathcal{C}_{ja} \otimes f_{a/A}\right)(x, b; \mu_F) + \mathcal{O}(b^2)$$

The differential cross section

The result for the differential cross section is given by

$$\frac{d\sigma\left(h_1 h_2 \to Z(\to l_1 \bar{l}_2)X\right)}{dQ^2 dy dq_T^2 d\Omega} = \frac{1}{48\pi S}\frac{Q^2}{(Q^2 - M_Z^2)^2 + Q^4\Gamma_Z^2/M_Z^2}$$

$$\times \frac{1}{(2\pi)^2}\left\{ \int d^2 b e^{i\vec{q}_T\cdot\vec{b}} \sum_{j,k} \tilde{W}_{j\bar{k}}(b_*, Q, x_1, x_2, \Omega, C_1, C_2, C_3) * \right.$$

$$\left. \tilde{W}_{j\bar{k}}^{NP}(b, Q, x_1, x_2) + Y(q_T, Q, x_1, x_2, \Omega, C_4) \right\}, \qquad (6)$$

where (Balazs and Yuan PRD 1997)

$$\tilde{W}_{j\bar{k}}(b_*, Q, x_1, x_2, \Omega, C_1, C_2, C_3) \propto$$

$$e^{-S(b,Q,C_1,C_2)} \left(\mathcal{C}_{ja} \otimes f_{a/h_1}\right)(x_1) \left(\mathcal{C}_{\bar{k}a} \otimes f_{b/h_2}\right)(x_2). \qquad (7)$$

The parameter b_* is the separation scale at which the perturbative $\tilde{W}$ factorizes from the non-perturbative $\tilde{W}^{NP}$.

The Sudakov exponent

The Sudakov exponent is given by

$$S(b, Q, C_1, C_2) = \int_{C_1^2/b^2}^{C_2^2 Q^2} \frac{d\bar{\mu}^2}{\mu^2} \left[A(\alpha_s(\bar{\mu}), C_1) \ln \left(\frac{C_2^2 Q^2}{\bar{\mu}^2} \right) + B(\alpha_s(\bar{\mu}), C_1, C_2) \right],$$

$$A(\alpha_s(\bar{\mu}), C_1) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s(\bar{\mu})}{\pi} \right)^n A^{(n)}(C_1),$$

$$B(\alpha_s(\bar{\mu}), C_1, C_2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_s(\bar{\mu})}{\pi} \right)^n B^{(n)}(C_1, C_2), \qquad (9)$$

where coefficients $A^{(n)}(C_1)$ and $B^{(n)}(C_1, C_2)$ are known from the literature, while parton convolutions are defined as

$$\left(\mathcal{C}_{ja} \otimes f_{a/h_1} \right)(x_1) =$$

$$\int_{x_1}^{1} \frac{d\xi_1}{\xi_1} \mathcal{C}_{ja} \left(\frac{x_1}{\xi_1}, b, \mu = \frac{C_3}{b}, C_1, C_2 \right) f_{a/h_1}(\xi_1, \mu = \frac{C_3}{b}). (10)$$

The Y contribution

The Y term which is defined as the difference between the fixed order perturbative contribution and those obtained by expanding the perturbative part of $\tilde{W}_{j\bar{k}}$ is given by

$$Y(Q_T, Q, x_1, x_2, \theta, \phi, C_4) = \int_{x_1}^{1} \frac{d\xi_1}{\xi_1} \int_{x_2}^{1} \frac{d\xi_2}{\xi_2} \sum_{n=1}^{\infty} \left[\frac{\alpha_s(C_4 Q)}{\pi}\right]^n$$

$$f_{a/h_1}(\xi_1, C_4 Q) R_{ab}^{(n)} \left(Q_T, Q, \frac{x_1}{\xi_1}, \frac{x_2}{\xi_2}, \theta, \phi\right) f_{b/h_2}(\xi_2, C_4 Q), \quad (11)$$

where $R_{ab}^{(n)}$ are less singular than Q_T^{-2} or $Q_T^{-2}\left(Q_T^2/Q^2\right)$ when $Q_T \to 0$.

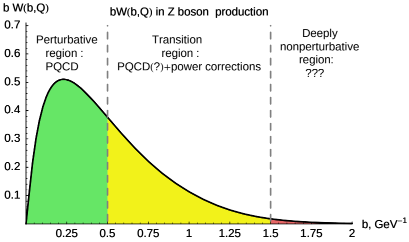
Non-perturbative function

The non-perturbative $\tilde{W}^{NP}(b, Q)$ function as originally parametrized in CSS paper (1985), is given by

$$\tilde{W}^{NP}_{j\bar{k}}(b, Q, Q_0, x_1, x_2) = \exp\left[-F_1(b)\ln\left(\frac{Q^2}{Q_0^2}\right) - F_{j/h_1}(x_1, b) - F_{\bar{k}/h_2}(x_2, b)\right], \quad (12)$$

where functions $F_1(b)$, F_{j/h_1} and $F_{\bar{k}/h_2}$ have to be determined by fits to the experimental data.

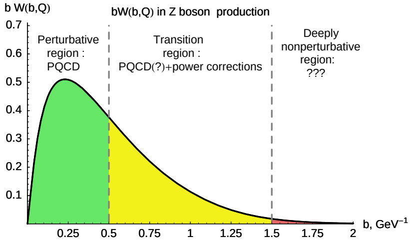
$b\widetilde{W}(b,Q)$ in Z boson production



- $b \lesssim 0.5$ GeV^{-1} $(\mu_b \sim 1/b > 2$ GeV$)$

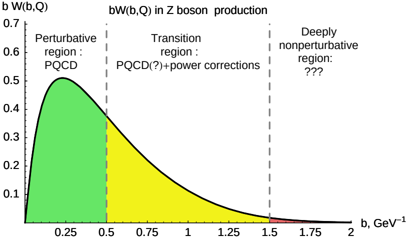
- dominated by the leading-power (logarithmic) term, calculable in PQCD:
  $\widetilde{W}(b,Q) \approx \widetilde{W}_{LP}(b,Q)$
- contributes most of the rate at large Q

$b\widetilde{W}(b,Q)$ in Z boson production



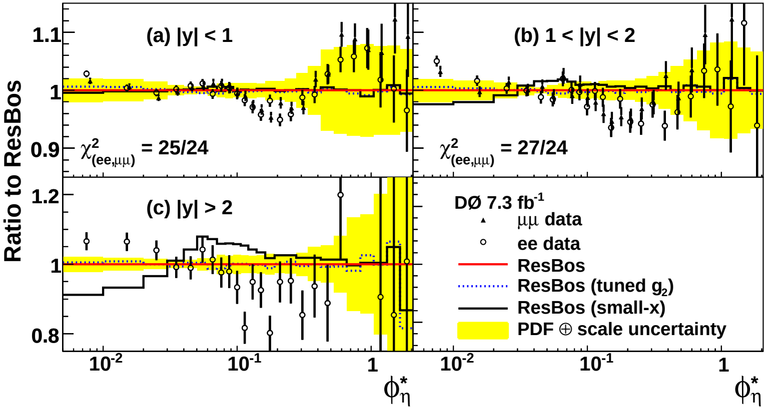
- $0.5 \lesssim b \lesssim 1.5 - 2$ GeV^{-1} $(0.5 - 0.7 \lesssim \mu_b \lesssim 2$ GeV$)$
- higher-order terms in α_s and b^p modify $d\sigma/dQ_T$ at $Q_T \lesssim 10$ GeV
- constrained within a global Q_T fit (similar to PDF's), especially by the Drell-Yan process at $Q = 3 - 10$ GeV

$b\widetilde{W}(b,Q)$ in Z boson production



- $b \gtrsim 1.5 - 2\ \text{GeV}^{-1}$

- terra incognita; tiny contributions ?
- negligible effect ?



Can new data sets from hadron colliders set constraints on W^{NP} ?

Let's check it out!

We need tools:

★ *ResBos* by Balazs, Yuan (1997); Balazs Qiu and Yuan (1995); Brock, Landry, Nadolsky, Yuan (2002)

★ Fully differential NNLO computations

- *FEWZ* by K. Melnikov and F. Petriello PRD 74 (2006) 114017
- *DYNNLO* by G.Bozzi, S.Catani, M.Grazzini, D. De Florian, NPB 737 (2006) 73

★ Resummed NNLL/NNLO computation

- Ferrera, Grazzini et al.
- A.Banfi, M.Dasgupta, S.Marzani, L.Tomlinson JHEP 1201 (2012) 044

Current version of ResBos

(see Pavel Nadolsky's talk for a more general description of ResBos)

★ Approximated Resummed NNLL/NNLO computations

- ResBos + CANDIA ($\leftarrow$ M.G., A. Cafarella, C. Corianò 2006)

  -Approximate NNLO ($\mathcal{C}^{(2)}$ found numerically)

  - NNLL resummation $A^{(3)}, B^{(2)}$

♠ The current accuracy of *ResBos* is competitive with full NNLL/NNLO resummed computations.

♠ It's fast and includes all the dominant components of the full NNLO calculation.

Systematic uncertainties in this study

Main corrections in our computation

- EM corrections $\approx 2\%$ at small ϕ^*_η,
- NNLO corrections,
- Kinematic corrections $\rightarrow$ dependence on the matching (already discussed in Nadolsky's talk)

Main source of systematic uncertainties

- Scale dependence,
- Non-perturbative function,
- PDF uncertainty.

Scale dependence of the CSS resummed form factor

At small b, the scale-dependent expression of the CSS resummed form factor takes the form

$$\widetilde{W}_{\alpha,j}^{pert} = \sum_{j=u,d,s\ldots} \left|H_{\alpha,j}(Q,\Omega,C_2 Q)\right|^2$$

$$\times \exp\left[-\int_{C_1^2/b^2}^{C_2^2 Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} A(\bar{\mu};C_1)\ln\left(\frac{C_2^2 Q^2}{\bar{\mu}^2}\right) + B(\bar{\mu};C_1,C_2)\right]$$

$$\times \sum_{a=g,q,\bar{q}} \left[\mathcal{C}_{ja}\otimes f_{a/h_1}\right]\left(\chi_1,\frac{C_1}{C_2},\frac{C_3}{b}\right) \sum_{b=g,q,\bar{q}} \left[\mathcal{C}_{\bar{j}b}\otimes f_{b/h_2}\right]\left(\chi_2,\frac{C_1}{C_2},\frac{C_3}{b}\right),$$

$\mu_F = C_3/b$ is the factorization scale at which Wilson coefficient functions are evaluated.

In our study we have used “non-canonical” choice representations corresponding to $C_1 = C_3 = \{2b_0, b_0\}$, with $C_2 = 1/2$.

This sets the resummation scale to $M_Z/2$ and improves the agreement with the ϕ_η^* data.

Perturbative and NP form factors

Given the strong suppression of the deeply nonperturbative large-b region in Z boson production, only contributions from the transition region of b of about 1 GeV^{-1} are non-negligible compared to the perturbative contribution from $b < 1\ \mathrm{GeV}^{-1}$. In the transition region, $\widetilde{W}(b,Q)$ is approximately given by the extrapolated leading-power, or perturbative, part $\widetilde{W}^{pert}(b,Q)$, and the nonperturbative smearing factor $\widetilde{W}^{NP}(b,Q)$:

$$\widetilde{W}_{\alpha,j}(b,Q,y_Z) = \widetilde{W}^{pert}_{\alpha,j}(b_*,Q,y_Z)\widetilde{W}^{NP}(b,Q,y_Z). \tag{13}$$

When b is large, the slow b dependence in $\widetilde{W}^{pert}_{\alpha,j}(b_*,Q)$ can be neglected, compared to the rapidly changing $\widetilde{W}_{NP}(b,Q)$.

b_* prescription: one way of doing the separation

To avoid divergence due to the Landau pole in $\alpha_s(\overline{\mu})$ at $\overline{\mu} \to 0$, scales of order $1/b$ in $\widetilde{W}^{pert}(b, Q)$ are redefined according to the b_* prescription (J.Collins 1981,1984) dependent on two parameters. In the Sudakov exponential, the lower limit $(C_1/b)^2$ is replaced by $(C_1/b_*(b, b_{max}))^2$, with

$$b_*(b, b_{max}) \equiv \frac{b}{\sqrt{1 + (b/b_{max})^2}}, \qquad (14)$$

where b_{max} is set to 1.5 GeV^{-1} (Konychev, Nadolsky 2005)

More on the NP function

In a broad range of Q values in the Drell-Yan process, the behavior of experimentally observed Q_T distributions is described by

$$\widetilde{W}^{NP}(b,Q) = \exp\left[-b^2\left(a_1 + a_2 \ln\left(\frac{Q}{2\ Q_0}\right) + a_3 \ln\left(\frac{x_1^{(0)} x_2^{(0)}}{0.01}\right)\right)\right], \tag{15}$$

with $x_{1,2}^{(0)} = \frac{Q}{\sqrt{s}} e^{\pm y}$, free parameters $a_1,\ a_2,\ a_3$, and a fixed dimensional parameter $Q_0 = 1.6$ GeV.

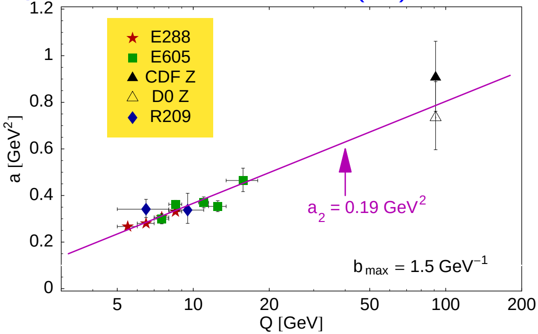
In the vicinity of Q around M_Z, Eq. (15) reduces to

$$\widetilde{W}^{NP}(b, Q \approx M_Z) = \exp\left[-b^2 a_Z\right] \tag{16}$$

with

$$a_Z = a_1 + a_2 \ln\left(\frac{M_Z}{2\ Q_0}\right) + a_3 \ln\left(\frac{M_Z^2}{0.01\ s}\right). \tag{17}$$

Independent scans of $a(Q)$ in 5 experiments



$\mathcal{F}_{NP}(b, Q) \approx a(Q) b^{2-\beta}$

- All experiments prefer $\beta \approx 0$
- $a(Q) \approx a_1 + a_2 \ln(Q/3.2)$
- $a_2 \sim 0.18$ GeV^2 agrees well with the IR renormalon + lattice QCD estimate, $(a_2)_{IR} = 0.19^{+0.12}_{-0.09}$ GeV^2
- Improve the large-Q constraints using ϕ^*_η data.
- Include scale dependence and other factors.

Gaussian smearing from previous global p_T fits

$a_{1,2,3}$ found from the fit are correlated with the assumed form of $\widetilde{W}_{NP}$ (value of b_{max})

Landry, Brock, Nadolsky, Yuan, 2002 ($b_{max} = 0.5$ GeV^{-1}):

$$a(Q) = \underbrace{0.21}_{a_1} + \underbrace{0.68}_{a_2} \ln \frac{Q}{3.2} - \underbrace{0.13}_{a_3} \ln(100 x_A x_B)$$

- a_3 is comparable to a_1, a_2
- For $\sqrt{s} = 1.96$ TeV, $a(M_Z) \approx 2.7$ GeV^2 (surprisingly large)

Konychev-Nadolsky, 2006 ($b_{max} = 1.5$ GeV^{-1}):

$$a(Q) = 0.20 + 0.19 \ln \frac{Q}{3.2} - 0.03 \ln(100 x_A x_B)$$

- $a_2 \sim 0.19$ GeV^2
- $a_3 \ll a_1, a_2$; in Z production, $a(M_Z) \approx 0.9$ GeV^2
- reduced $\chi^2/d.o.f.$ in the fit

It is worthy to do a new investigation on the b_{max} dependence by using ϕ_η^* data from Tevatron and LHC. We want to know where transitions happen.

Important message

The modifications on the Q_T (ϕ^*_η) spectrum due to variations of the scale dependence (scale parameters C_1, C_2, C_3, C_4 in CSS)

are DIFFERENT

from the modifications due to variations of the NP function!

The small-Q_T/ϕ^*_η spectrum cannot be fully described by employing perturbative scale variations only. The constraining power of ϕ^*_η differential distribution data allows us to estimate the size of these nonperturbative effects.

Results of the analysis by which we determine the NP parameter a_Z from very precise data on the Z ϕ^*_η distribution at DØ Tevatron.

Z and W production: a_Z from fit to the data.

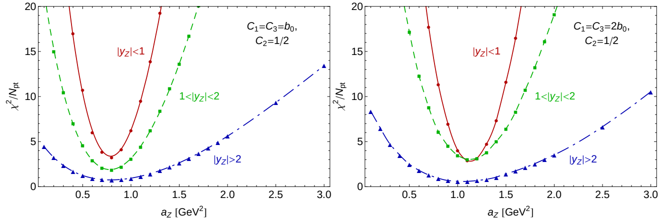
We determine the value of a_Z from a fit to the

D0 ϕ^*_η distribution data using two methods.

★ Method I: we compute the χ^2 without including the shifts due to variations of C_1, C_2, C_3.

★ Method II: we compute the χ^2 by including the covariance matrix due to these shifts.

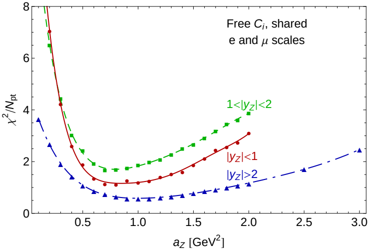
Method I: χ^2/N_{pt} as a function of a_Z with fixed $C_{1,2,3}$.



Fit results for $\phi^*_\eta \leq 0.1$

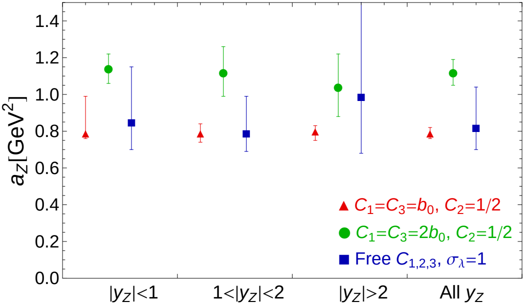
	N_{pt}	χ^2_{min}/N_{pt}	$\bar{a}_Z \pm \delta a_Z$ (GeV^2)
$\lvert y_Z \rvert \leq 1, \quad e+\mu$	24	3.24	$0.79^{+0.2}_{-0.03}$
		2.83	1.14 ± 0.08
$1 \leq \lvert y_Z \rvert \leq 2, \quad e+\mu$	24	1.87	0.79 ± 0.05
		3.03	$1.12^{+0.14}_{-0.13}$
$\lvert y_Z \rvert \geq 2, \quad e$	12	0.74	$0.8^{+0.03}_{-0.05}$
		0.58	$1.04^{+0.18}_{-0.16}$
All y_Z bins, weighted average	60	2.19	0.79 ± 0.03
		2.46	1.12 ± 0.07

Method II: we allow for shifts in $C_{1,2,3}$.



C_1, C_2, C_3 are independent in each y_Z bin				
	N_{pt}	χ^2_{min}/N_{pt}	$\bar{a}_Z \pm \delta a_Z$ (GeV^2)	Best-fit $C_{1,2,3}$
$\|y_Z\| \leq 1, \quad e+\mu$	24	1.0 1.16	$0.56^{+0.95}_{-0.02}$ $0.85^{+0.3}_{-0.15}$	0.21, 0.18, 7.56 1.47, 0.3, 1.46
$1 \leq \|y_Z\| \leq 2, \quad e+\mu$	24	1.48 1.70	$1.22^{+0.27}_{-0.36}$ $0.79^{+0.2}_{-0.1}$	18, 0.58,0.1 1.69, 0.37, 0.77
$\|y_Z\| \geq 2, \quad e$	12	- 0.59	- $0.99^{+0.99}_{-0.31}$	- 1.74, 0.48, 2.12
Weighted average of all bins	60		0.97 ± 0.25 0.82 ± 0.12	

Recapitulation: main findings of this analysis

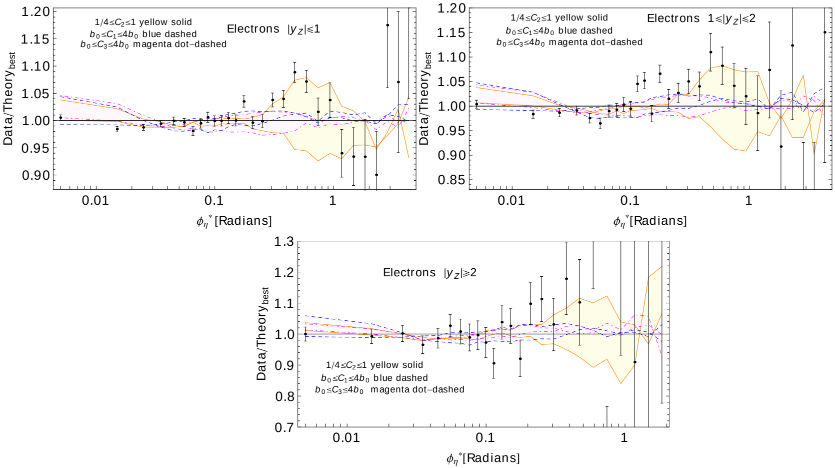


68% C.L. ranges for a_Z in individual y_Z bins and in all bins.

All fits consistently yield a_Z values that are at least 5σ from zero.

No pronounced rapidity dependence in contrast to Berge, Nadolsky, Olness, Yuan

$C_{1,2,3}$ scale dependence for the electron channel



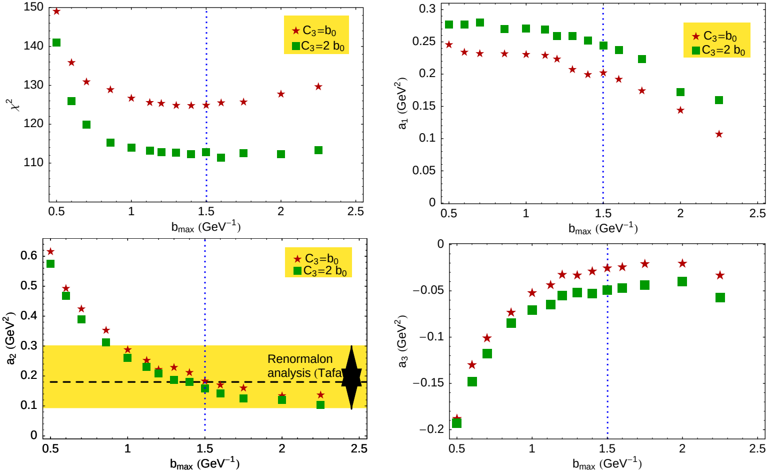
Muons in the backup

Conclusions

- We have shown that a significant nonperturbative Gaussian smearing is necessary to describe features of the low ϕ_η^* spectrum. A non-zero NP function is present even if all the perturbative scale parameters of the CSS formalism are varied.
- Values of a_Z smaller than 0.5 GeV^2 are disfavoured by the fit to the recent DØ data.
- The constraining power of ϕ_η^* differential distribution data allows us to estimate the size of these nonperturbative effects.
- ResBos is a valuable tool for investigations at low transverse momentum regions at colliders. It will be
- Precise measurements of hadronic cross sections at small Q_T will verify the TMD formalism for QCD factorization and shed light on the nonperturbative QCD dynamics.

BACKUP

Scan over b_{max}



Best fit: $b_{max} \approx 1.5\ \mathrm{GeV}^{-1}$, $\beta = 0^{+0.3}_{-0.4}$ (set to 0), $a_1 \approx 0.23$, $a_2 \approx 0.18$, $a_3 \approx -0.05$

The 2005 p_T analysis

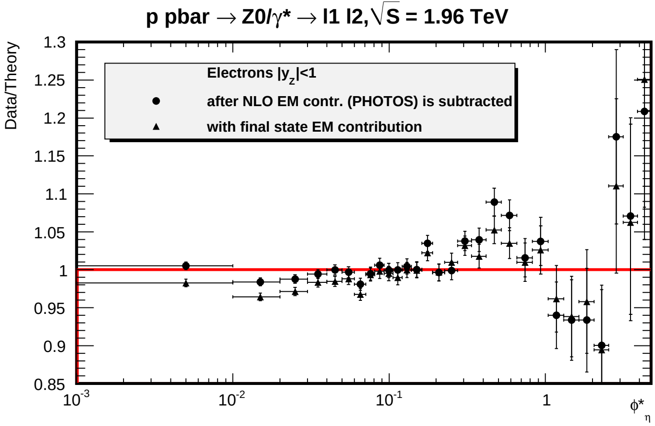
- leads to a consistent picture of the power-suppressed term
- supports dominance of soft contributions in $\mathcal{F}_{NP}(b, Q)$
- suggests
  - Gaussian
    $\mathcal{F}_{NP}(b, Q) = b^2 \left[0.20 + 0.19 \ln(Q/3.2) - 0.026 \ln(100 x_A x_B)\right]$
  - linear $\ln Q$ dependence (consistent with SIDIS)
  - small $\sqrt{s}$ dependence
  - no tangible flavor dependence
  - uncertainty translates into $\delta M_W \approx 15$ MeV
- **applies to light-flavor (u, d, s) scattering at $x \gtrsim 10^{-2}$**

$O(\alpha_s^2)$ corrections in ResBos at approx: current setup

- the *FO* contribution, is computed up to $O(\alpha_s^2)$ for the leading structure functions.
- The $Y = Y_{NLO} K_{NNLO}$ piece is computed up to $O(\alpha_s^2)$ by using the K-factors as in Arnold and Reno Nucl.Phys. B319 (1989); Arnold and Kauffman Nucl.Phys. B349 (1991), for the dominant F_{-1} only.
- The W piece is computed up to NNLL approximation in the Sudakov exponent, while the finite part of the coefficient functions $\mathcal{C}^{(n)}(\xi, b, \mu, C_1, C_2)$ is computed up to $O(\alpha_s^2)$ by using K-factors obtained by *CANDIA* (Cafarella, Corianò, M.G., JHEP 0708 (2007); CPC 179 (2008)).
- In $\mathcal{C}^{(2)}(\xi, b, \mu, C_1, C_2)$ included is also the logarithmic dependence on the coefficients C_1, C_2 up to $O(\alpha_s^2)$.

Size of electromagnetic corrections

Final state EM radiation is accounted for by using PHOTOS (Barberio and Was CPC79 1994). Here Theory is ResBos at approx NNLO



These are around 2% at small ϕ^*_η

ϕ^*_η and a_T

$$\sin\theta^*_\eta = \frac{2l_T}{M} \tag{18}$$

since

$$\tan(\phi_{acop}/2) = \sqrt{(1+\cos\Delta\varphi)\,/\,(1-\cos\Delta\varphi)} \tag{19}$$

$Q_T^2 = l_{T1}^2 + l_{T2}^2 + 2l_{T1}^2 l_{T2}^2 \cos\Delta\varphi$
In the soft limit $Q_T \to 0$ we have $l_{T1} \approx l_{T2}$.
In this limit the leptons are nearly back-to-back in the transverse plane which results in $l_{T1}^2 - l_{T2}^2 = Q_T \cos\alpha$ where α is the angle made by the Q_T vector with the lepton axis in the transverse plane.

$$Q_T^2 \sin^2\alpha \approx 2l_T^2(1+\cos\Delta\varphi)$$

$$\tan(\phi_{acop}/2) = \sqrt{(1+\cos\Delta\varphi)\,/\,(1-\cos\Delta\varphi)} = \frac{Q_T\sin\alpha}{2l_T} = a_T/M$$

ϕ^*_η and the CSS variables: general expression

One can write $\cos\theta^*_\eta$ as a function of the lepton momenta in the lab frame as

$$\cos\theta^*_\eta = \tanh\left(\frac{\eta_1-\eta_2}{2}\right) = \frac{\sqrt{l_1^+ l_2^-} - \sqrt{l_1^- l_2^+}}{\sqrt{l_1^+ l_2^-} + \sqrt{l_1^- l_2^+}} = \frac{f\left(\cos\theta_{CS}\right) - f\left(-\cos\theta_{CS}\right)}{f\left(\cos\theta_{CS}\right) + f\left(-\cos\theta_{CS}\right)}, \tag{20}$$

where $l^{\pm}_{1,2} = (l^0_{1,2} \pm l^z_{1,2})/\sqrt{2}$,

$$f(\cos\theta_{CS}) \equiv \sqrt{M_T^2 + 2M_T Q\cos\theta_{CS} + Q^2\cos^2\theta_{CS} - Q_T^2\sin^2\theta_{CS}\cos^2\varphi_{CS}}, \tag{21}$$

and $M_T^2 = Q^2 + Q_T^2$. We also write $\cos\Delta\varphi$ as

$$\cos\Delta\varphi = (Q_T^2 - Q^2\sin^2\theta_{CS} - Q_T^2\sin^2\theta_{CS}\cos^2\varphi_{CS}) \times[(Q^2\sin^2\theta_{CS} + Q_T^2\sin^2\theta_{CS}\cos^2\varphi_{CS} + Q_T^2)^2 - 4M_T^2Q_T^2\sin^2\theta_{CS}\cos^2\varphi_{CS}]^{-\frac{1}{2}}. \tag{22}$$

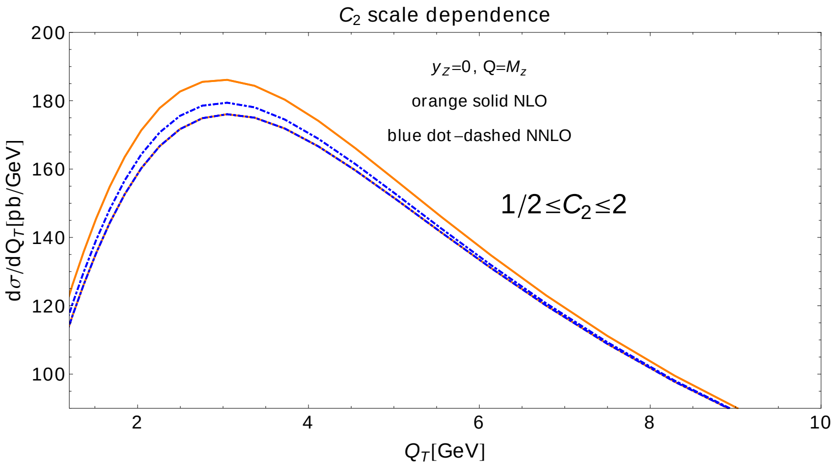
Approximated $\mathcal{C}^{(2)}$ Wilson Coeff. function

$$\begin{aligned}\mathcal{C}_{ja}^{(2)}\left(\xi, \frac{C_1}{C_2}, C_3\right) &= \mathcal{C}_{ja}^{(2,c)}(\xi) + \delta_{ja}\delta(1-\xi)L^{(2)}(C_1, C_2) \\ &+ \left\{\frac{\beta_0}{2}\mathcal{C}_{jb}^{(1,c)}(\xi) - [\mathcal{C}_{jb}^{(1,c)} \otimes P_{ba}^{(1)}](\xi) - P_{ja}^{(2)}(\xi)\right\} \ln\frac{\mu_F b}{b_0} \\ &+ \frac{1}{2}[P_{jb}^{(1)} \otimes P_{ba}^{(1)}](\xi)\ \ln^2\frac{\mu_F b}{b_0}. \qquad (23)\end{aligned}$$

$$\mathcal{C}_{ja}^{(2)}(\xi, C_1/C_2, C_3) \approx \left\{\langle\delta\mathcal{C}^{(2,c)}\rangle + L^{(2)}(C_1, C_2)\right\}\delta(1-\xi)\,\delta_{ja}, \qquad (24)$$

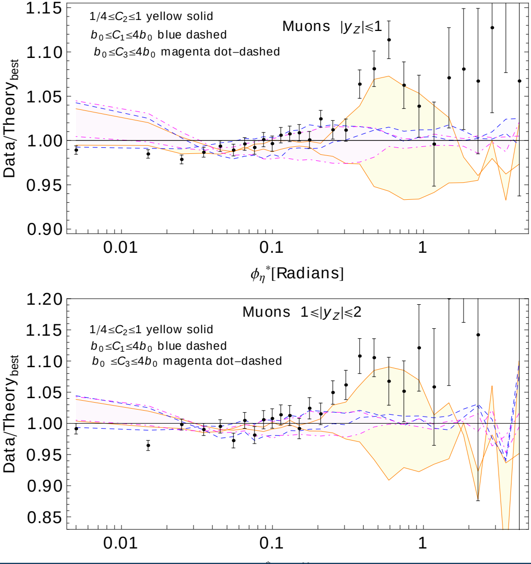
where $\langle\delta\mathcal{C}^{(2,c)}\rangle$ denotes the average value of the Wilson coefficient in Z production for the canonical scale combination and $L^{(2)}(C_1, C_2)$ contains the scale dep. when C_1, C_2, C_3 is "non-canonical".

Reduction on C_2-sensitivity



Renormalization constant C_2 dependence: approx NNLO vs NLO.

$C_{1,2,3}$ scale dependence Muons



χ^2 definition for Method I

In method I a_Z is determined from the DØ data by minimization of a function

$$\chi^2(a_Z) = \sum_{i=1}^{N_{pt}} \left(\frac{D_i - \bar{T}_i(a_Z)}{s_i} \right)^2, \qquad (25)$$

where D_i are the data points; $\bar{T}_i(a_Z)$ are the theoretical predictions for fixed scale parameters $\{\bar{C}_1, \bar{C}_2, \bar{C}_3\}$; s_i are the uncorrelated experimental uncertainties; and N_{pt} is the number of points.

χ^2 definition for Method II

We introduce a linearized approximation for the covariance matrix. For each scale parameter C_α, $\alpha = 1, 2, 3$, we define a nuisance parameter $\lambda_\alpha \equiv \log_2(C_\alpha/\bar{C}_\alpha)$ and compute the finite-difference derivatives of theory cross sections

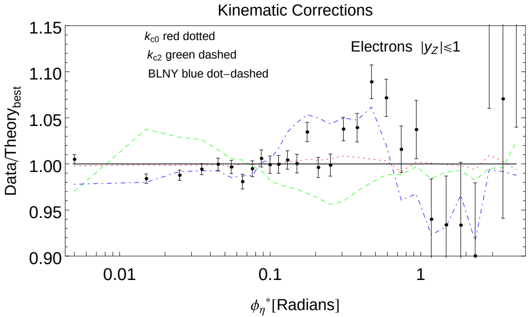
$$\beta_{i\alpha} \equiv \frac{T_i(a_Z, \lambda_\alpha = +1) - T_i(a_Z, \lambda_\alpha = -1)}{2}, \qquad \alpha = 1, 2, 3; \quad i = 1, \dots, N_{pt}$$

over the interval $\lambda_\alpha = \pm 1$ corresponding to $\bar{C}_\alpha/2 \leq C_\alpha \leq 2\bar{C}_\alpha$. Variations of λ_α introduce correlated shifts in theory cross sections $T_i(a_Z, C_{1,2,3})$ with respect to the fixed-scale theory cross sections $T_i(a_Z, \bar{C}_{1,2,3}) \equiv \bar{T}_i(a_Z)$. We can reasonably assume that the probability distribution over each λ_α is similar to a Gaussian one with a central value of 0 and half-width σ_λ, taken to be the same for all λ_α. The goodness-of-fit function is then defined as

$$\chi^2(a_Z, \lambda_{1,2,3}) = \sum_{i=1}^{N_{pt}} \left(\frac{D_i - \bar{T}_i(a_Z) - \sum_{\alpha=1}^{3} \beta_{\alpha i} \lambda_\alpha}{s_i} \right)^2 + \sum_{\alpha=1}^{3} \frac{\lambda_\alpha^2}{\sigma_\lambda^2}. \quad (26)$$

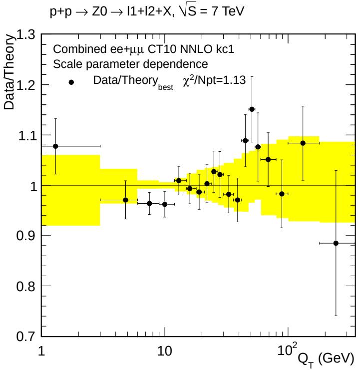
The minimum with respect to λ_α can be found algebraically for every a_Z

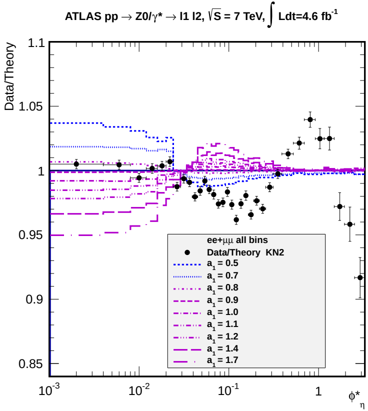
Comparison to other predictions



The ratios to the central theoretical prediction of the DØ electron data at $|y_Z| \leq 1$ and alternative theoretical predictions. The central prediction is computed assuming $C_1 = C_3 = 2b_0$, $C_2 = 1/2$, $a_Z = 1.1$ GeV^2, and kinematical correction 1. Theory predictions based on alternative kinematical corrections (0 and 2) and BLNY nonperturbative parametrization are also shown.

ϕ_η^* at ATLAS





Data vs. theory ratios for the Q_T distribution by ATLAS 7 TeV, $35-40$ pb^{-1} (ATLAS coll. 2011) and ϕ_η^* distribution ATLAS 7 TeV, 4.6 fb^{-1} (ATLAS coll. 2012)