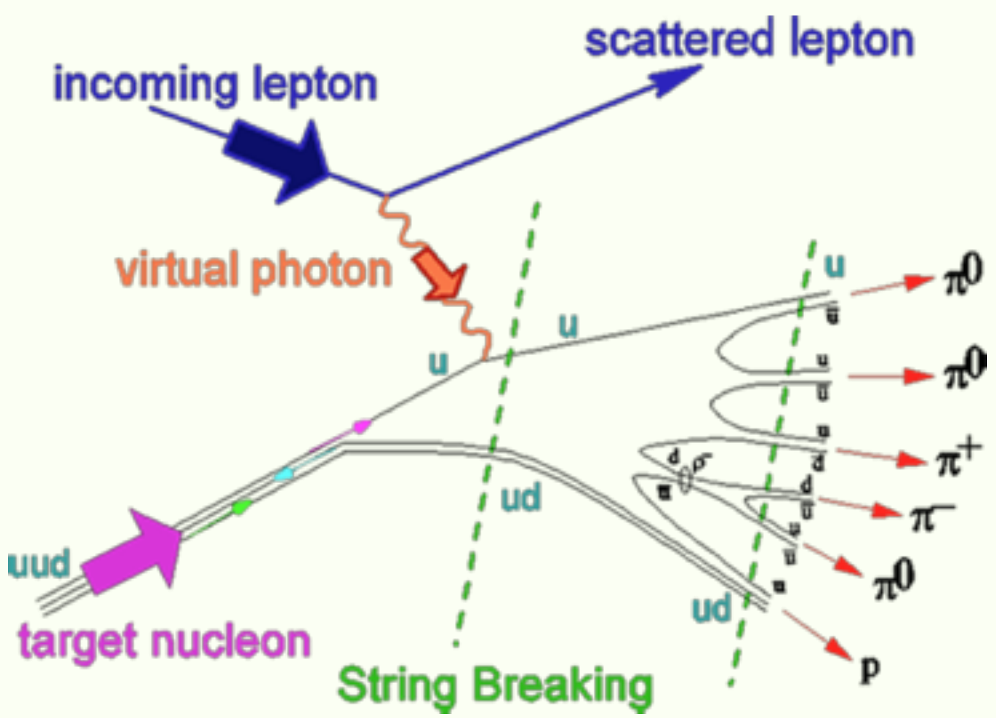
Bessel Weighted Asymmetries in SIDIS



INT Workshop INT-14-55W
Studies of 3D Structure of Nucleon
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Based on Boer, LG,Musch,Prokudin JHEP 2011 and "in progress"

Outline

- **BW and Fourier Transform of SIDIS CS & “FT” TMDs**
- Merit of Bessel Weighted Asymmetries (BWA) “S/T” pic of SIDIS
- BWA in Parton model connection w/ conventional weighting
- Impact on studying TMD evolution Sivers
- **Summary Elements Factorization-SIDIS**
- Cancellation Soft, Pert. some other univ. factors BWA-JMY
- Cancellation of Universal & flavor indep. factors in BWAs-Collins Factorization
- A study of BWA

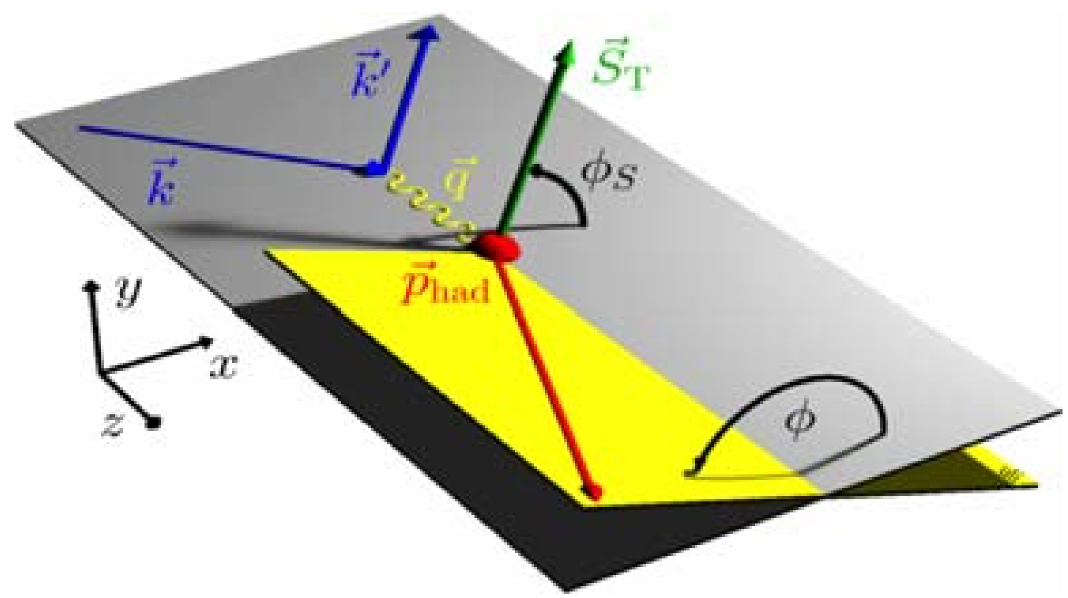
Comments

- We exhibit the SIDIS cross section in b_T space
- Demonstrate how this rep results in model indep. observables BWAs generalization of conventional WAs Kotzinian & Mulders PLB97
- Explore impact BWA in studying scale dependence in SIDIS at small to moderate P_T where TMD description framework reliable $P_T \ll Q$

SIDIS-CS expressed thru structure functions

$$\frac{d\sigma}{dxdydzd\phi_h dP_{h\perp}^2} \sim \left\{ F_{UU,T} \cdots + \ldots |S_\perp| \left(\sin(\phi_h - \phi_S) F_{UT,T}^{\sin(\phi_h - \phi_S)} + \sin(\phi_h + \phi_S) \varepsilon F_{UT}^{\sin(\phi_h + \phi_S)} \ldots \right) \ldots \right.$$

Kotzinian NPB 95,
Mulders Tangermann NPB 96,
Boer & Mulders PRD 97
Bacchetta et al JHEP 08



Structure functions & spin asymmetry projected from cross section

$$A_{XY}^{\mathcal{F}} \equiv 2 \frac{\int d\phi_h \, d\phi_S \, \mathcal{F}(\phi_h, \phi_S) \, \left(d\sigma^{\uparrow} - d\sigma^{\downarrow}\right)}{\int d\phi_h d\phi_S \, (d\sigma^{\uparrow} + d\sigma^{\downarrow})},$$

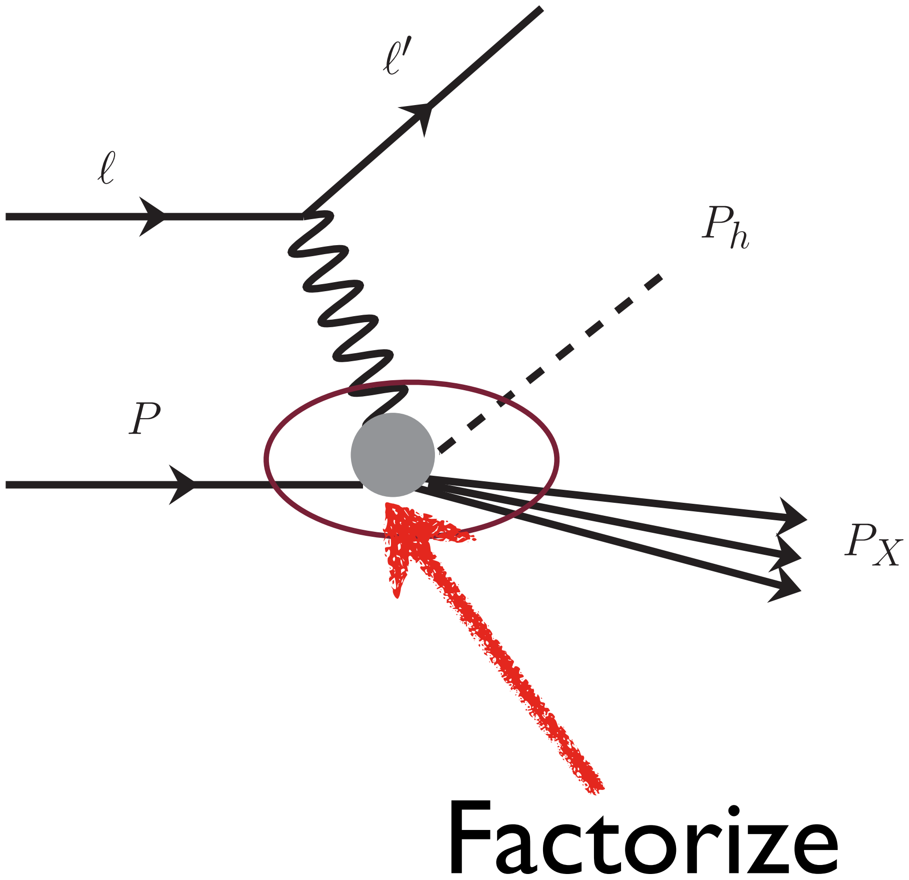
XY-polarization e.g.

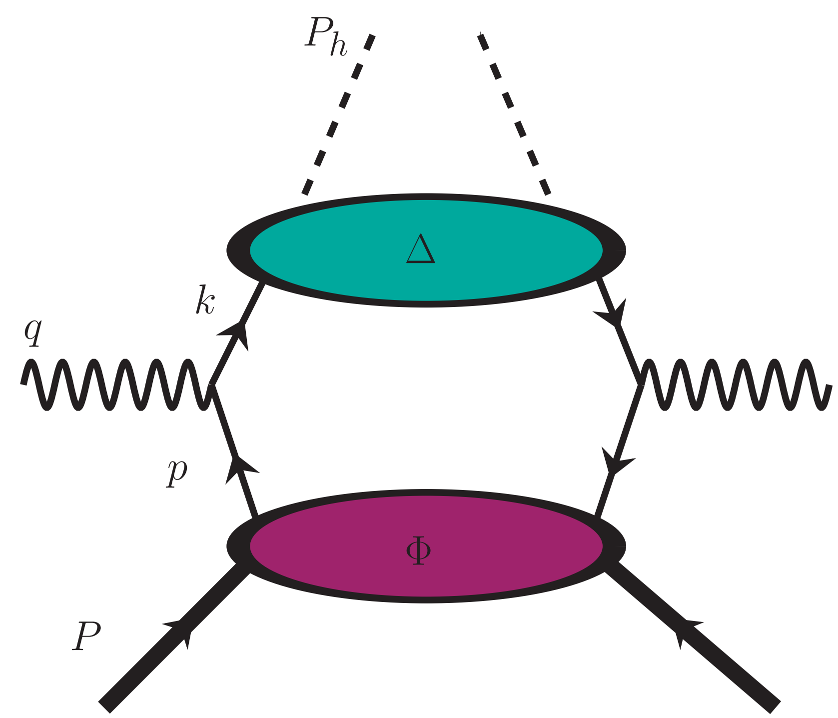
$$\mathcal{F}(\phi_h, \phi_S) = \sin(\phi_h - \phi_S).$$

$$
\begin{aligned}
\frac{d\sigma}{dx_B\,dy\,d\psi\,dz_h\,d\phi_h\,dP_{h\perp}^2} &= \frac{\alpha^2}{x_B y Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)\Bigg\{F_{UU,T}+\varepsilon\,F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&+ \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&+ S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&+ S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&+ |\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&+ \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&+ |\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},
\end{aligned}
$$

Bacchetta et al JHEP 08

Factorization Parton Model





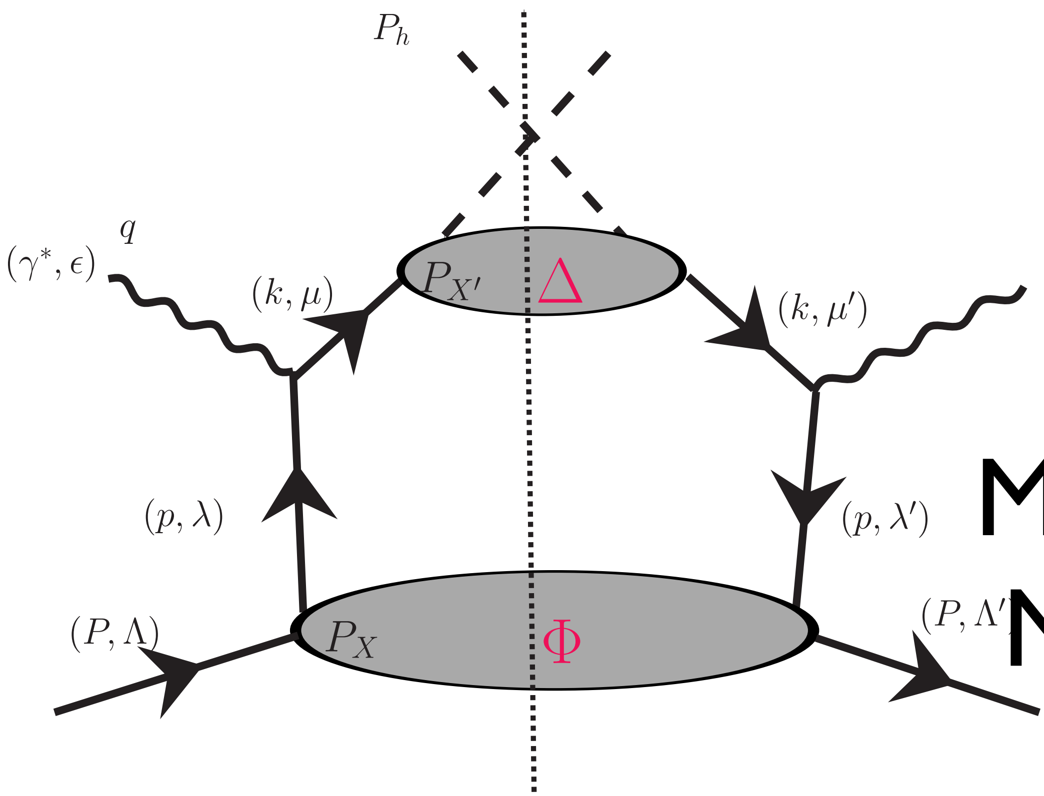
$$\frac{d\sigma}{dx_B\, dy\, d\psi\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}|\, d|\boldsymbol{P}_{h\perp}|} = \frac{\alpha^2}{x_B y Q^2}\, \frac{y^2}{(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x_B}\right) L_{\mu\nu} W^{\mu\nu},$$

Factorization P_T of hadron small sensitive to intrinsic transv. momentum of partons

$$W^{\mu\nu}(q,P,S,P_h) = \int \frac{d^2\mathbf{p}_T}{(2\pi)^2} \int \frac{d^2\mathbf{k}_T}{(2\pi)^2} \delta^2\left(\mathbf{p}_T - \frac{\mathbf{P}_{h\perp}}{z_h} - \mathbf{k}_T\right) \mathrm{Tr}\left[\Phi(x,\mathbf{p}_T)\gamma^\mu \Delta(z,\mathbf{k}_T)\gamma^\nu\right]$$

$$\Phi(x,\mathbf{p}_T) = \int dp^- \Phi(p,P,S)|_{p^+=x_B P^+}\,, \qquad \Delta(z,\mathbf{k}_T) = \int dk^- \Delta(k,P_h)|_{k^-=\frac{P^-}{z_h}}$$

Small transverse momentum



Purely Kinematic-integrate over small momentum component

Must also respect gauge invariance

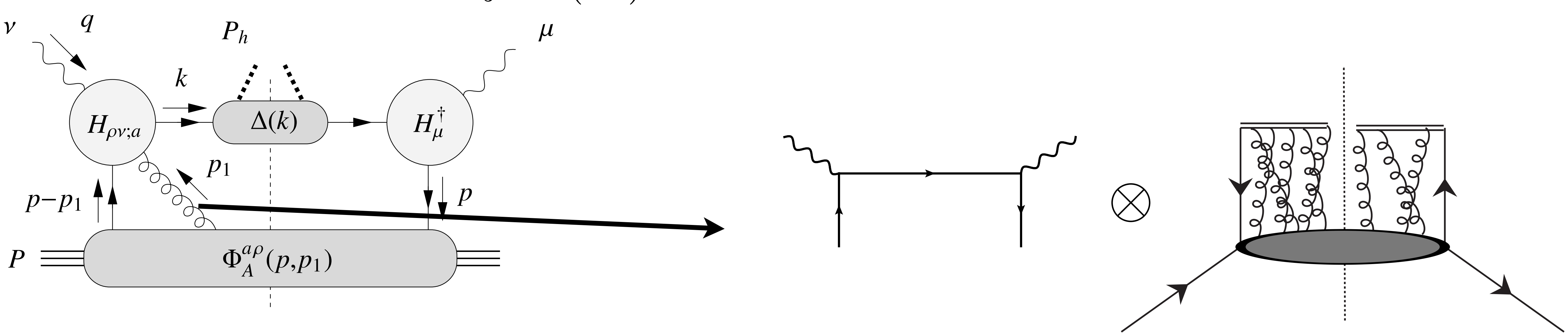
Minimal requirement satisfy color gauge invariance

Factorization Parton Model-predicts existence of T-odd PDFs and TSSAs--Boer-Mulders PRD 1998

Minimal Requirement for PARTON MDL Factorization

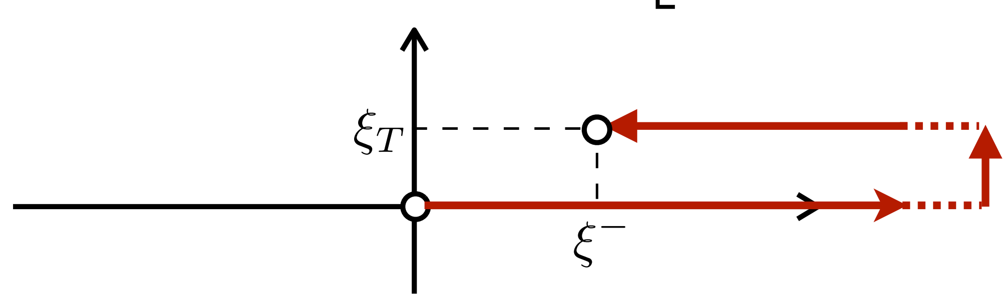
Gauge link determined re-summing leading gluon interactions btwn soft and hard
Efremov,Radyushkin Theo. Math. Phys. 1981, Collins, Soper NPB 1981, 1982,Collins PLB 2002, Belitsky, Ji, Yuan NPB 2003, Boer, Bomhof, Mulders Pijlman, et al. 2003 - 2008- NPB, PLB, PRD, Collins 2011

$$\Phi^{[\mathcal{U}[\mathcal{C}]]}(x,p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip\cdot\xi} \langle P|\overline{\psi}(0)\mathcal{U}^{[C]}_{[0,\xi]}\psi(\xi^-,\xi_T)|P\rangle|_{\xi^+=0}$$



- **The path $[C]$ is fixed by hard subprocess within hadronic process.**

$$W_{\mu\nu}(q,P,S,P_h) = \int d^4p d^4k \delta^4(p+q-k) \mathrm{Tr}\left[\Phi^{\mathcal{U}^{[C]}_{[\infty;\xi]}}(p) H^\dagger_\mu(p,k)\Delta(k)H_\nu(p,k)\right]$$



$U_{[+]}$

Partonic picture Structure Functions momentum CONVOLUTION

$$\mathcal{C}\big[w\,f\,D\big] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big],$$

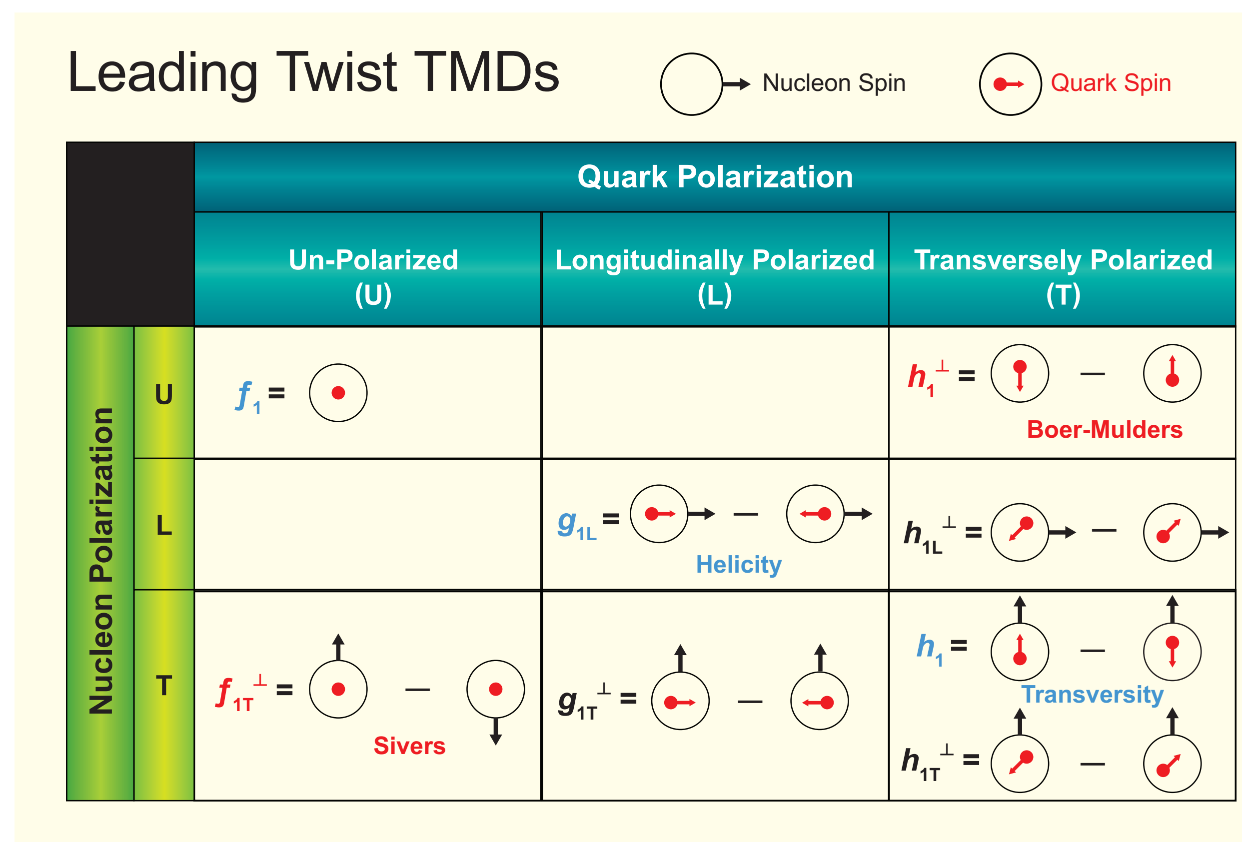
$$F_{LL} = \mathcal{C}\big[g_{1L} D_1\big],$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1L}^{\perp} H_1^{\perp}\right],$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_1^{\perp} H_1^{\perp}\right],$$



Comments on Weighting

- Transverse momentum weighting with an appropriate power of $P_{h\perp}$ possible to convert the convolutions in the cross section into simple products

- Transverse momentum weighted asymmetries provide model independent observables which are generalizations of conventional WA
Kotzinian, Mulders PLB 97, Boer, Mulders PRD 98

- Explore impact these BWA have on studying the scale dependence of the SIDIS cross section at small to moderate transverse momentum where the TMD framework is expected to give a good description of the cross section
Boer, LG,Musch,Prokudin JHEP 2011

Weighted asymmetries proposed as *model independent deconvoltuion* of CS in terms of moments of TMDs

Kotzinian, Mulders PLB 97, Boer, Mulders PRD 98

$$A_{UT,T}^{w_1 \sin(\phi_h - \phi_S)} = 2\,\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}| d\phi_h\, d\phi_S\, w_1(|\boldsymbol{P}_{h\perp}|)\ \sin(\phi_h - \phi_S)\ \{d\sigma(\phi_h, \phi_S) - d\sigma(\phi_h, \phi_S + \pi)\}}{\int d|\boldsymbol{P}_{h\perp}|\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d\phi_S\, w_0(|\boldsymbol{P}_{h\perp}|)\ \{d\sigma(\phi_h, \phi_S) + d\sigma(\phi_h, \phi_S + \pi)\}},$$

e.g. $$\mathcal{W}_{\text{Sivers}} = \frac{|\boldsymbol{P}_{h\perp}|}{zM} \sin(\phi_h - \phi_S)$$

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M} \sin(\phi_h - \phi_s)} = -2\,\frac{\sum_a e_a^2\ f_{1T}^{\perp(1)}(x)\ D_1^{a(0)}(z)}{\sum_a e_a^2\ f_1^{a(0)}(x)\ D_1^{a(0)}(z)}$$

Undefined w/o subtractions perscription regularization to subtract infinite contribution at large transverse momentum

Problems

$$f_{1T}^{\perp(1)}(x) = \int d^2k_T \frac{k_T^2}{2M} f_{1T}^{\perp}(x, k_T)$$

- Sivers tail $\qquad f_{1T}^{\perp}(x, k_T) \sim \frac{M^2}{\left(k_T^2 + M^2\right)^2}$
- moment diverges

Bacchetta et al. JHEP 08, Aybat Collins Rogers Qiu PRD 2012,

Comments

- Propose generalize Bessel Weights-"BW"
- BW procedure has advantages
  - ★ Structure functions become simple product $\mathcal{P}[\quad]$ rather than convolution $\mathcal{C}[\quad]$
  - ★ CS has simple S/T interpretation as a multipole expansion in terms of $\boldsymbol{P}_{h\perp}$ conjugate to $b_T\,[\mathrm{GeV}^{-1}]$

- ★ The usefulness of Fourier-Bessel transforms in studying the factorization as well as the scale dependence of transverse momentum dependent cross section has been known for over 30 years.
- ★ Is the natural language for TMD Evolution
- ★ Collins Soper (81), Collins, Soper, Sterman (85), Boer (01) (09) (13), Ji,Ma,Yuan (04), Collins-Cambridge University Press (11), Aybat Rogers PRD (11), Abyat, Collins, Qiu, Rogers (11), Aybat, Prokudin, Rogers (11), Bacchetta, Prokudin (13), Sun, Yuan (13), Aidala, Field, Gamberg, Rogers (14)

Further Comments

- Provides a regularization of infinite contributions at lg transverse momentum when $\mathcal{B}_T^2$ is non-zero for moments
- Study scale changes in TMD picture, soft factor eliminated from Sivers and ....weighted asymmetries
  Boer, LG,Musch,Prokudin JHEP 2011
- Cancellation of perturbative Sudakov Broadening
  mentioned by D. Boer NPB 1999, 2007
- Cancellation hard cross section
  Boer, LG,Musch,Prokudin JHEP 2011
- Some asymmetry e.g. Sivers TSSA less sensitive scale changes- observable for different scales.... could be useful for EIC

Advantages of Bessel Weighting

“Deconvolution”-CS-struct fncts simple product “$\mathcal{P}$“

$$W^{\mu\nu}(\boldsymbol{P}_{h\perp}) \equiv \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2}\, e^{-i\boldsymbol{b}_T\cdot\boldsymbol{P}_{h\perp}}\; \tilde{W}^{\mu\nu}(\boldsymbol{b}_T)\,,$$

$$\tilde{\Phi}_{ij}(x, z\boldsymbol{b}_T) \equiv \int d^2\boldsymbol{p}_T\, e^{iz\boldsymbol{b}_T\cdot\boldsymbol{p}_T}\; \Phi_{ij}(x,\boldsymbol{p}_T)$$

$$\tilde{\Delta}_{ij}(z,\boldsymbol{b}_T) \equiv \int d^2\boldsymbol{K}_T\, e^{i\boldsymbol{b}_T\cdot\boldsymbol{K}_T}\; \Delta_{ij}(z,\boldsymbol{K}_T)$$

$$\frac{d\sigma}{dx_B\, dy\, d\psi\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d|\boldsymbol{P}_{h\perp}|} = \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{-i\boldsymbol{b}_T\cdot\boldsymbol{P}_{h\perp}} \left\{ \frac{\alpha^2}{x_B y Q^2}\frac{y^2}{(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x_B}\right) L_{\mu\nu}\tilde{W}^{\mu\nu} \right\}.$$

$$2M\tilde{W}^{\mu\nu} = \sum_a e_a^2\, \mathrm{Tr}\left(\tilde{\Phi}(x, z\boldsymbol{b}_T)\gamma^\mu\tilde{\Delta}(z,\boldsymbol{b}_T)\gamma^\nu\right)\,.$$

Example Sivers Function

"Deconvolution"-Structure function simple product " $\mathcal{P}$ "

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

"dipole structure"

$$\mathcal{C}\left[w\, f\, D\right] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\left(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\right) w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

★ $$F_{UT,T}^{\sin(\phi_h-\phi_S)} = -x_B \sum_a e_a^2 \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T|^2 J_1(|\boldsymbol{b}_T|\,|\boldsymbol{P}_{h\perp}|) M z\; \tilde{f}_{1T}^{\perp a(1)}(x, z^2\boldsymbol{b}_T^2)\, \tilde{D}_1^a(z, \boldsymbol{b}_T^2).$$

$\tilde{f}_1$, $\tilde{f}_{1T}^{\perp(1)}$, and $\tilde{D}_1$ are Fourier Transf. of TMDs/FFs and finite

Boer, LG, Musch, Prokudin JHEP 2011

- Transversity and Collins

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)\left(\boldsymbol{p}_T\cdot\boldsymbol{k}_T\right)+\boldsymbol{p}_T^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)-4\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)}{2M^2M_h}\,h_{1T}^{\perp}H_1^{\perp}\right]$$

Write out in cylindrical polar-traceless irreducible tensor no mixture of Bessel "J_3"

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = x_B\sum_a e_a^2\int\frac{d|\boldsymbol{b}_T|}{(2\pi)}|\boldsymbol{b}_T|^4 J_3(|\boldsymbol{b}_T|\,|\boldsymbol{P}_{h\perp}|)\frac{M^2M_hz^3}{4}\,\tilde{h}_{1T}^{\perp a(2)}(x,z^2\boldsymbol{b}_T^2)\,\tilde{H}_1^{\perp a(1)}(z,\boldsymbol{b}_T^2)\,.$$

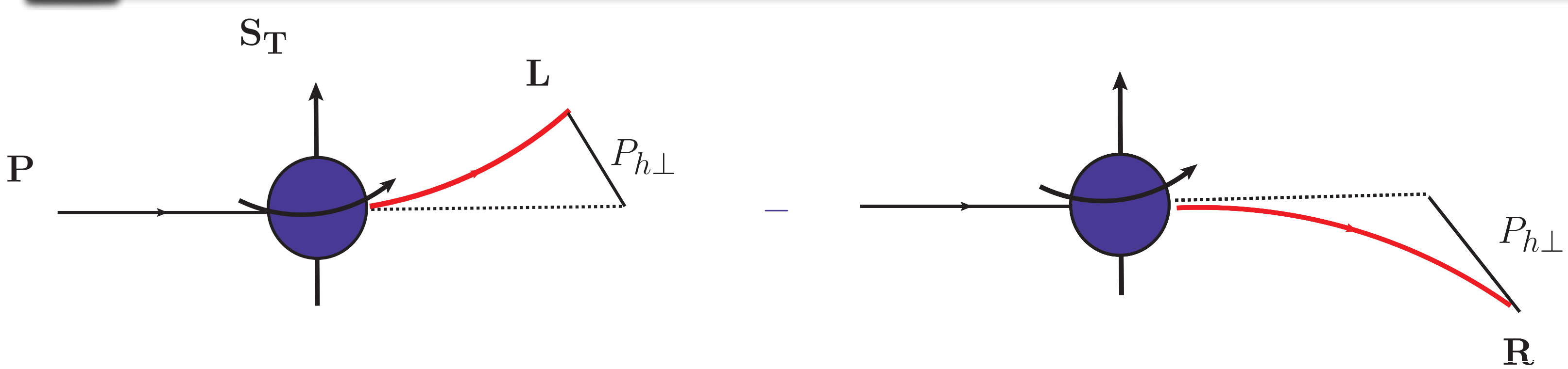
Simple product " $\mathcal{P}$ "

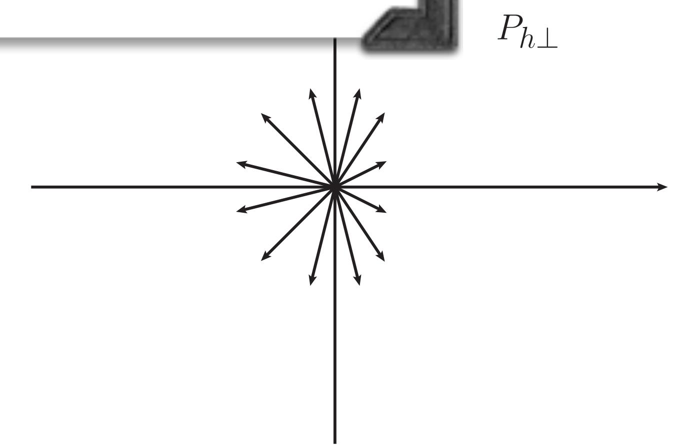
Structure Functions deconvolute

$$\begin{aligned}
\mathcal{F}_{UU,T} &= \mathcal{P}[\tilde{f}_1^{(0)}\ \tilde{D}_1^{(0)}]\,,\\
\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} &= -\mathcal{P}[\tilde{f}_{1T}^{\perp(1)}\ \tilde{D}_1^{(0)}]\,,\\
\mathcal{F}_{LL} &= \mathcal{P}[\tilde{g}_{1L}^{(0)}\ \tilde{D}_1^{(0)}]\,,\\
\mathcal{F}_{LT}^{\cos(\phi_h-\phi_s)} &= \mathcal{P}[\tilde{g}_{1T}^{(1)}\ \tilde{D}_1^{(0)}]\,,\\
\mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} &= \mathcal{P}[\tilde{h}_1^{(0)}\ \tilde{H}_1^{\perp(1)}]\,,\\
\mathcal{F}_{UU}^{\cos(2\phi_h)} &= \mathcal{P}[\tilde{h}_1^{\perp(1)}\ \tilde{H}_1^{\perp(1)}]\,,\\
\mathcal{F}_{UL}^{\sin(2\phi_h)} &= \mathcal{P}[\tilde{h}_{1L}^{\perp(1)}\ \tilde{H}_1^{\perp(1)}]\,,\\
\mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} &= \frac{1}{4}\mathcal{P}[\tilde{h}_{1T}^{\perp(2)}\ \tilde{H}_1^{\perp(1)}].
\end{aligned}$$

$$\mathcal{P}[\tilde{f}^{(n)}\,\tilde{D}^{(m)}] \equiv x_B \sum e_a^2\,(zM|\boldsymbol{b}_T|)^n\,(zM_h|\boldsymbol{b}_T|)^m\,\tilde{f}^{a(n)}(x, z^2\boldsymbol{b}_T^2)\,\tilde{D}^{a(m)}(z, \boldsymbol{b}_T^2)$$

What is transverse single spin asymmetry TSSAs





$\sigma^{\downarrow}(x, P_\perp) = \sigma^{\uparrow}(x, -P_\perp)$ Rotational Invariance *"Left-Right" Asymmetry*

$$A_N = \frac{\sigma^{\uparrow}(x, P_\perp) - \sigma^{\uparrow}(x, -P_\perp)}{\sigma^{\uparrow}(x, P_\perp) + \sigma^{\uparrow}(x, -P_\perp)} \equiv \Delta\sigma$$

QCD is Parity Conserving TSSAs Scattering plane transverse to spin

Naively "T-odd"

$$\Delta\sigma \sim iS_T \cdot (\mathbf{P} \times P_\perp) \otimes (\text{"}T - odd\text{" QCD} - \text{phases})$$

Spin orbit

Correlator w/ explicit *spin orbit* correlations

$$\tilde{\Phi}^{[\gamma^+]}(x, \boldsymbol{b}_T) = \tilde{f}_1(x, \boldsymbol{b}_T^2) - i\,\epsilon_T^{\rho\sigma} b_{T\rho} S_{T\sigma}\, M\, \tilde{f}_{1T}^{\perp(1)}(x, \boldsymbol{b}_T^2)\,,$$

$$\tilde{\Phi}^{[\gamma^+\gamma^5]}(x, \boldsymbol{b}_T) = S_L\, \tilde{g}_{1L}(x, \boldsymbol{b}_T^2) + i\, \boldsymbol{b}_T \cdot \boldsymbol{S}_T M\, \tilde{g}_{1T}^{(1)}(x, \boldsymbol{b}_T^2)\,,$$

$$\tilde{\Phi}^{[i\sigma^{\alpha+}\gamma^5]}(x, \boldsymbol{b}_T) = S_T^\alpha\, \tilde{h}_1(x, \boldsymbol{b}_T^2) + i\, S_L\, b_T^\alpha M\, \tilde{h}_{1L}^{\perp(1)}(x, \boldsymbol{b}_T^2) + \frac{1}{2}\left(b_T^\alpha b_T^\rho + \frac{1}{2}\, \boldsymbol{b}_T^2\, g_T^{\alpha\rho}\right) M^2\, S_{T\rho} \tilde{h}_{1T}^{\perp(2)}(x, \boldsymbol{b}_T^2) - i\, \epsilon_T^{\alpha\rho} b_{T\rho} M \tilde{h}_1^{\perp(1)}(x, \boldsymbol{b}_T^2)\,,$$

N.B. Transverse sep. of quarks in correlator

★ CS has simpler *S/T* interpretation--multipole expansion in terms of $b_T\,[\mathrm{GeV}^{-1}]$ conjugate to $\boldsymbol{P}_{h\perp}$

$$
\begin{aligned}
&\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz_h\, d\phi_h\, |\boldsymbol{P}_{h\perp}| d|\boldsymbol{P}_{h\perp}|} = \\
&\frac{\alpha^2}{x_B y Q^2} \frac{y^2}{(1-\varepsilon)} \left(1+\frac{\gamma^2}{2x_B}\right) \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T| \Big\{ J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UU,T} + \varepsilon\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UU,L} \\
&+ \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UU}^{\cos(2\phi_h)} \\
&+ \lambda_e \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LU}^{\sin\phi_h} \\
&+ S_\parallel \Big[\sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UL}^{\sin 2\phi_h} \Big] \\
&+ S_\parallel \lambda_e \Big[\sqrt{1-\varepsilon^2}\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LL}^{\cos\phi_h} \Big] \\
&+ |\boldsymbol{S}_\perp| \Big[\sin(\phi_h-\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|) \left(\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, \mathcal{F}_{UT,L}^{\sin(\phi_h-\phi_S)} \right) \\
&\quad + \varepsilon \sin(\phi_h+\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UT}^{\sin(\phi_h+\phi_S)} \\
&\quad + \varepsilon \sin(3\phi_h-\phi_S)\, J_3(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UT}^{\sin\phi_S} \\
&\quad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h-\phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{UT}^{\sin(2\phi_h-\phi_S)} \Big] \\
&+ |\boldsymbol{S}_\perp| \lambda_e \Big[\sqrt{1-\varepsilon^2}\, \cos(\phi_h-\phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LT}^{\cos(\phi_h-\phi_S)} \\
&\quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LT}^{\cos\phi_S} \\
&\quad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h-\phi_S)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{F}_{LT}^{\cos(2\phi_h-\phi_S)} \Big] \Big\}
\end{aligned}
$$

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Sivers

- Taking the asymptotic form of the Bessel function the conventional weights appear as the leading term of the Taylor expansion of the of Bessel Weight

$$|\boldsymbol{P}_{h\perp}|^n \to J_n(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)\, n! \left(\frac{2}{\mathcal{B}_T}\right)^n$$

“Parton Model”

Bessel weighting-projecting out Sivers

orthogonality of Bessel Fncts.

$$\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM} = \frac{2\,J_1(|\boldsymbol{P}_{hT}|\mathcal{B}_T)}{zM\mathcal{B}_T}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)}(\mathcal{B}_T) =$$

$$2\,\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)\,\left(d\sigma^\uparrow - d\sigma^\downarrow\right)}{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\;\mathcal{J}_0^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)\;\left(d\sigma^\uparrow + d\sigma^\downarrow\right)}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) = -2\,\frac{\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}{\sum_a e_a^2\;\tilde{f}_1^a(x,z^2\mathcal{B}_T^2)\,\tilde{D}_1^a(z,\mathcal{B}_T^2)}$$

Traditional weighted asymmetry recovered but UV divergent

$$\lim_{\mathcal{B}_T \to 0} w_1 = 2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)/zM\mathcal{B}_T \longrightarrow |\boldsymbol{P}_{h\perp}|/zM$$

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M}\sin(\phi_h - \phi_s)} = -2\frac{\sum_a e_a^2\, f_{1T}^{\perp(1)}(x)\; D_1^{a(0)}(z)}{\sum_a e_a^2\, f_1^{a(0)}(x)\; D_1^{a(0)}(z)}$$

Bacchetta et al. JHEP 08

undefined w/o regularization

TMDs in “config” space--Bessel MOMENTS

a) F.T. SIDIS cross section w/ following Bessel moments

$$\tilde{f}(x,\boldsymbol{b}_T^2) \equiv \int d^2\boldsymbol{p}_T \, e^{i\boldsymbol{b}_T\cdot\boldsymbol{p}_T} \, f(x,\boldsymbol{p}_T^2)$$

$$= 2\pi \int d|\boldsymbol{p}_T||\boldsymbol{p}_T| \, J_0(|\boldsymbol{b}_T||\boldsymbol{p}_T|) \, f^a(x,\boldsymbol{p}_T^2) \ ,$$

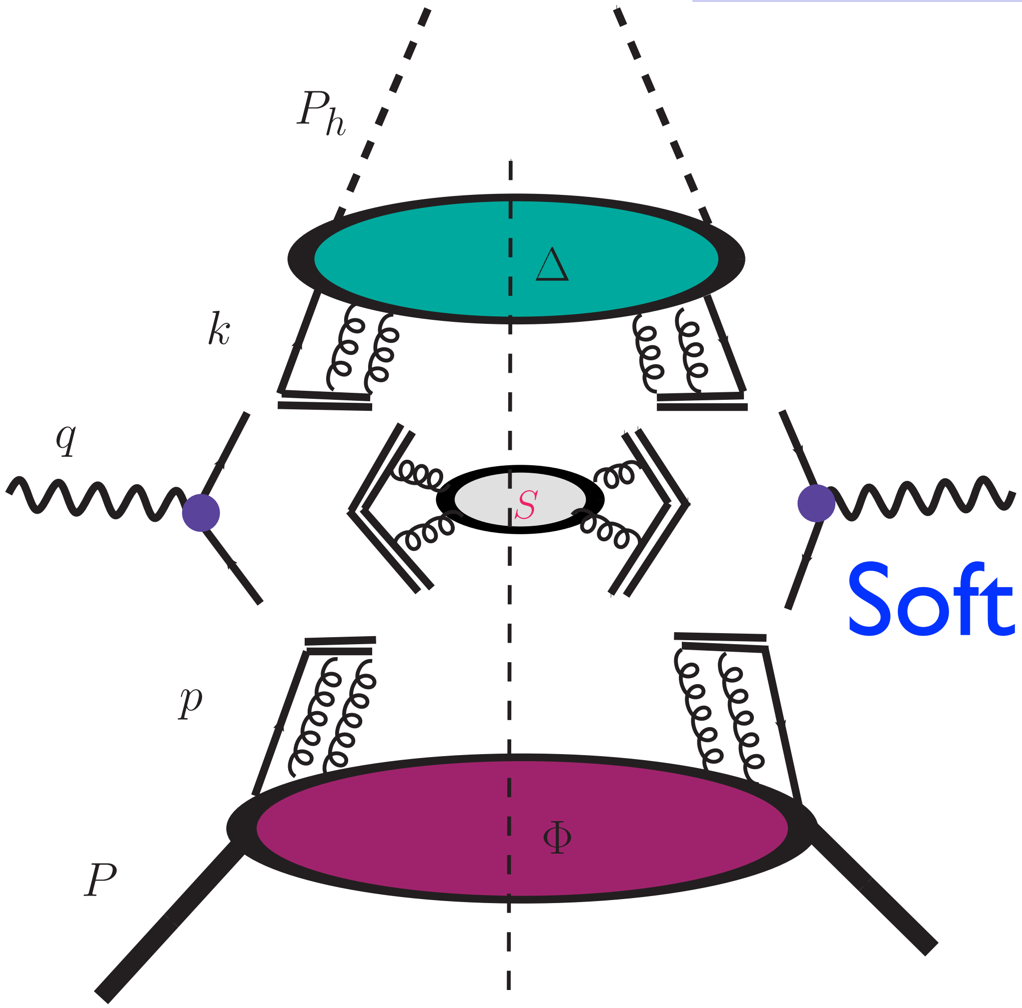
$$\tilde{f}^{(n)}(x,\boldsymbol{b}_T^2) \equiv n!\left(-\frac{2}{M^2}\partial_{\boldsymbol{b}_T^2}\right)^n \tilde{f}(x,\boldsymbol{b}_T^2)$$

$$= \frac{2\pi \ n!}{(M^2)^n}\int d|\boldsymbol{p}_T||\boldsymbol{p}_T| \left(\frac{|\boldsymbol{p}_T|}{|\boldsymbol{b}_T|}\right)^n J_n(|\boldsymbol{b}_T||\boldsymbol{p}_T|) \, f(x,\boldsymbol{p}_T^2) \ ,$$

b) n.b. connection to $\boldsymbol{p}_T$ moments

$$\tilde{f}^{(n)}(x,0) = \int d^2\boldsymbol{p}_T \left(\frac{\boldsymbol{p}_T^2}{2M^2}\right)^n f(x,\boldsymbol{p}_T^2) \equiv f^{(n)}(x)$$

Further Beyond "tree level" factorization



Soft factor

CS NPB 81,CSS NPB 1985 Collins, Hautman PLB 00,
Boer NPB 2001, Collins Metz PRL 2004
Idilbi, Ji, Ma, Yuan PRD 05,
Boer NPB 2009
Cherednikov, Karanikas, Stefanis NPB 10,
Collins Oxford Press 2011,
Abyat, Rogers PRD 2011,
Abyat, Collins, Qiu, Rogers PRD 2012 ...
Echevarria,Idilbi, Scimemi JHEP 2012

- Extra divergences at one loop and higher
- Extra variables needed to regulate
  light-cone, soft & collinear divergences
- Modifies convolution integral introduction of soft factor
- Effects cancel in Bessel weighted asymmetries

Collins Soper NPB 1981, Collins Metz PRL 2004, Ji, Ma, Yuan PRD 2004, Collins 2011, Collins Rogers 2012

Comments on Soft factor

- Collective effect soft gluons not associated with distribution frag function-factorizes into a matrix of Wilson lines in QCD vacuum

- Subtracts rapidity divergences from TMD pdf and FF

- Considered to be universal in hard processes
(Collins Soper 81, ... , Collins & Metz PRL 04, Ji, Ma, Yuan PRD 05)

- At tree level (zeroth order α_s) unity-parton model

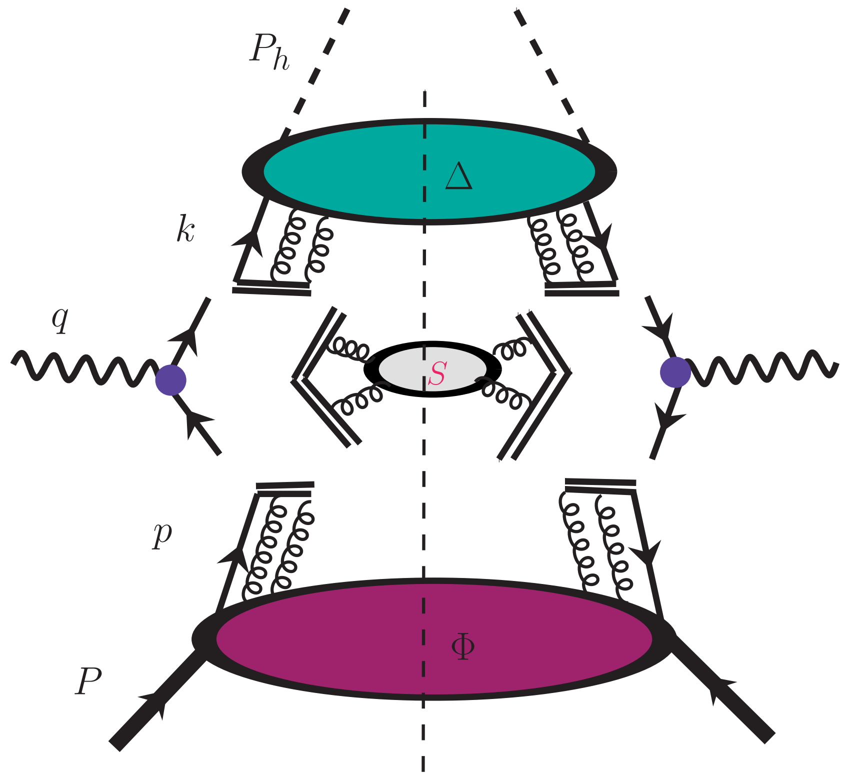
- Absent tree level pheno analyses of experimental data
(e.g. Anselmino et al PRD 05 & 07, Efremov et al PRD 07)

- Potentially, results of analyses can be difficult to compare at different energies **issue for EIC**

- Correct description of energy scale dependence of cross section and asymmetries in TMD picture, soft factor must be included
(Ji, Ma, Yuan 2004, Collins Camb. Univ. Press 2011, Abyat, Collins, Rogers PRD 2011)

- However, possible to consider observables where its affects cancels e.g. weighted asymmetries Boer, LG, Musch, Prokudin JHEP 2011

Momentum space convolution



CS 81, Idilbi, Ji, Ma, Yuan PRD 05 ....

Hard

$$\mathcal{C}\left[H; wfSD\right] \equiv x_B H(Q^2, \mu^2, \rho) \sum_a e_a^2 \int d^2p_T\, d^2K_T\, d^2\ell_T\, \delta^{(2)}(zp_T + K_T + \ell_T - P_{h\perp})\, w\left(p_T, -\frac{K_T}{z}\right)$$
$$\times f^a(x, p_T^2, \mu^2, x\zeta, \rho)\, S(\ell_T^2, \mu^2, \rho)\, D^a(z, K_T^2, \mu^2, \hat{\zeta}/z, \rho)$$

TMD

Soft

FF

Soft factor deconvoluted in Fourier Bessel rep cross sec.

$\mathcal{P}$ versus $\mathcal{C}$

$$\frac{d\sigma}{dx_B\, dy\, d\phi_S\, dz_h\, d\phi_h\, d|\boldsymbol{P}_{h\perp}|^2} \propto \frac{\alpha^2}{x_B Q^2} \int \frac{d|\boldsymbol{b}_T|}{(2\pi)} |\boldsymbol{b}_T|\, \tilde{S}(\boldsymbol{b}_T^2) \Big\{ \quad \ldots$$

$$+ J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{f}_1 \ \tilde{D}_1]$$

$$+ |\boldsymbol{S}_\perp| \sin(\phi_h - \phi_S)\, J_1(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{f}_{1T}^{\perp(1)} \ \tilde{D}_1]$$

$$+ \varepsilon \cos(2\phi_h)\, J_2(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\, \mathcal{P}[\tilde{h}_1^{\perp(1)} \ \tilde{H}_1^{\perp(1)}]$$

$$+ \quad \ldots 15 \quad \text{more} \quad \text{structure} \quad \text{functions}$$

Soft factor is

- spin blind
- flavor blind
- factors in $\mathcal{P}$
- Universal

Idilbi,Ji,Ma,Yuan PRD 05

Products in terms of " $\boldsymbol{b}_T$ moments "

$$\mathcal{F}_{UT,T}^{\sin(\phi_h-\phi_S)} = H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\ \tilde{S}^{(+)}(\boldsymbol{b}_T^2,\mu^2,\rho)\ \mathcal{P}[\tilde{f}_{1T}^{(1)}\,\tilde{D}_1^{(0)}] + \tilde{Y}_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\boldsymbol{b}_T^2)\ .$$

Comment

- *Y* term corrects the structure functions at $P_T \sim Q$, where the factorized structure fnct. does a good job in the $P_T << Q$
- *We will focus the kinematic regime $P_T << Q$ where TMD factorization is appropriate*

See talks of Pavel Nadowsky, Ted Rogers, Marco Guzzi

Bessel Weighting & cancellation of soft factor

Bessel weighting-projecting out Sivers using orthogonality of Bessel Fncts.

$$\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM} = \frac{2\,J_1(|\boldsymbol{P}_{hT}|\mathcal{B}_T)}{zM\mathcal{B}_T}$$

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)}(\mathcal{B}_T) =$$

$$2\,\frac{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\,\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_S)\left(d\sigma^{\uparrow}-d\sigma^{\downarrow}\right)}{\int d|\boldsymbol{P}_{h\perp}|\,|\boldsymbol{P}_{h\perp}|\,d\phi_h\,d\phi_S\;\mathcal{J}_0^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)\;\left(d\sigma^{\uparrow}+d\sigma^{\downarrow}\right)}$$

Sivers asymmetry with full dependences

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) \quad =$$

$$-2\frac{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UU,T}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_1^a(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}$$

Circumvents the problem of ill-defined $\boldsymbol{p}_T$ moments

$$A_{UT}^{\frac{\mathcal{J}_1^{\mathcal{B}_T}(|\boldsymbol{P}_{hT}|)}{zM}\sin(\phi_h-\phi_s)}(\mathcal{B}_T) \quad =$$

$$-2\frac{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UT,T}^{\sin(\phi_h-\phi_S)}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_{1T}^{\perp(1)a}(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}{\tilde{S}(\mathcal{B}_T^2,\mu^2,\rho^2)H_{UU,T}(Q^2,\mu^2,\rho)\sum_a e_a^2\,\tilde{f}_1^a(x,z^2\mathcal{B}_T^2;\mu^2,\zeta,\rho)\,\tilde{D}_1^a(z,\mathcal{B}_T^2;\mu^2,\hat{\zeta},\rho)}$$

Traditional weighted asymmetry recovered but UV divergent

$$\lim_{\mathcal{B}_T\to 0} w_1 = 2J_1(|\boldsymbol{P}_{h\perp}|\mathcal{B}_T)/zM\mathcal{B}_T \longrightarrow |\boldsymbol{P}_{h\perp}|/zM$$

$$A_{UT}^{\frac{|\boldsymbol{P}_{h\perp}|}{z_h M}\sin(\phi_h-\phi_s)} = -2\frac{\sum_a e_a^2\, f_{1T}^{\perp(1)}(x)\ D_1^{a(0)}(z)}{\sum_a e_a^2\, f_1^{a(0)}(x)\ D_1^{a(0)}(z)}$$

Bacchetta et al. JHEP 08

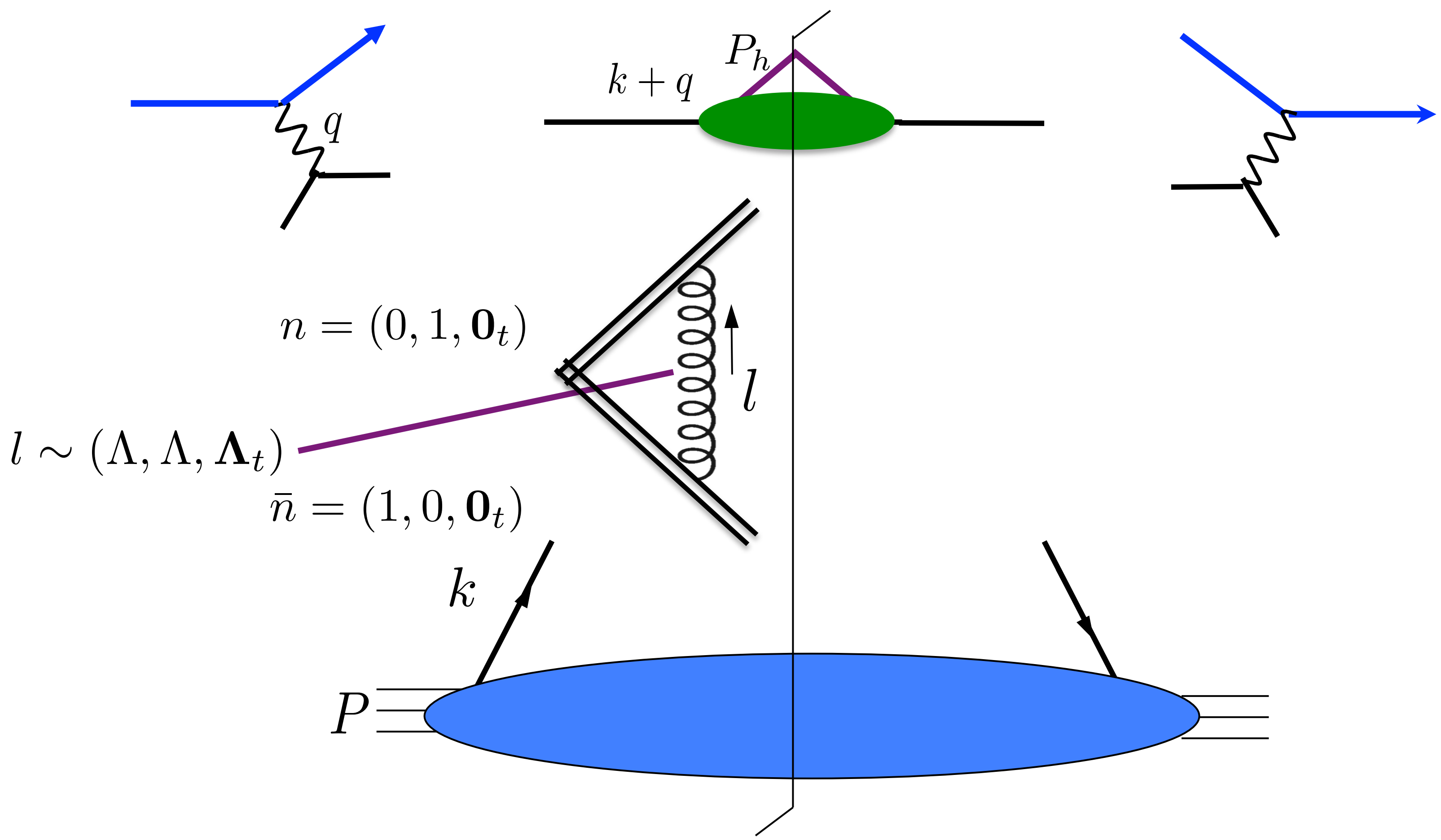
undefined w/o regularization

How does this emerge in CSS + JCC 2011 Factorization formulation

- Here we see the cancellation of spin independent & “Universal” parts of the evolution kernal

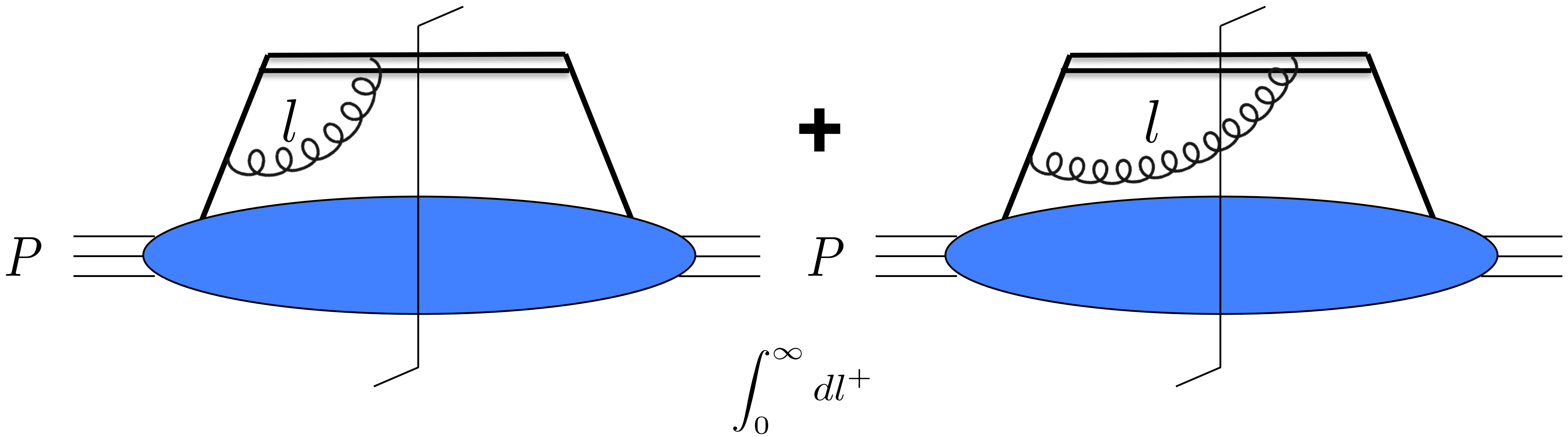
TMD Factorization & treatment of LC/Rapidity divergences
Collins 2011, Aybat & Rogers 2011

Soft Gluons



Power counting "region analysis" soft and collinear factors

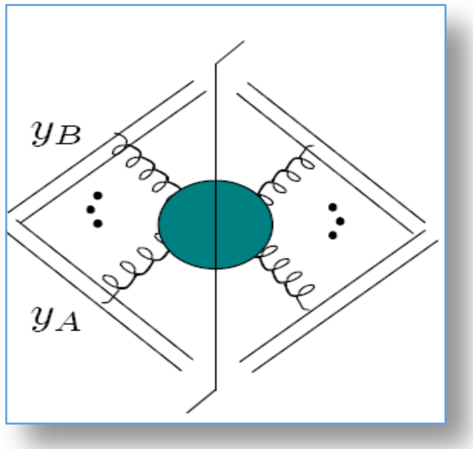
Gauge links have light-cone divergences they must cancel or you must regulate



- Divergent contribution at $l^+ = 0$.
- Cancelation in the integral over all l_t.
- What if we don't integrate? ←

Again .... Emergence of Soft Factor in CS

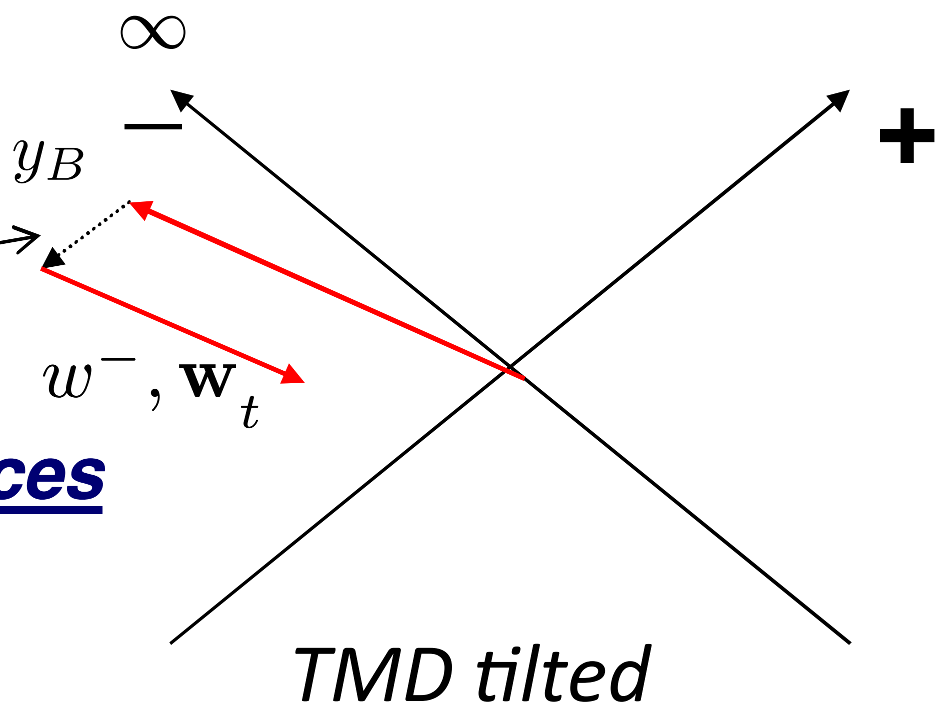
- Lightlike Wilson lines in TMDs
  - Infinite rapidity QCD radiation in the wrong direction.
  - In soft factor/fragmentation function too.



- Finite rapidity Wilson lines
  - Regulate rapidity of extra gluons.

Introduces rapidity scale parameter

$n_{\rm B} = (-e^{2y_{\rm B}}, 1, \mathbf{0}_t)$.



Tilt to regulate rapidity divergences

$$\zeta_F = 2M_p^2 x^2 e^{2(y_P - y_s)} \quad \Longleftrightarrow \quad y$$

Emergence of Soft Factor in Cross section

$$d\sigma = |\mathcal{H}|^2 \, \frac{\tilde{F}_1^{\text{unsub}}(y_1 - (-\infty)) \times \tilde{F}_2^{\text{unsub}}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}$$

TMDs are still "entangled" not yet full factorization

Collins 2011 Cam. Univ. Press

Emergence of Soft Factor in TMDs

Start with only the hard part factorized:

$$d\sigma = |\mathcal{H}|^2 \frac{\tilde{F}_1^{\text{unsub}}(y_1 - (-\infty)) \times \tilde{F}_2^{\text{unsub}}(+\infty - y_2)}{\tilde{S}(+\infty, -\infty)}$$

Soft factor repartitioned
This is done to both

1) cancel LC divergences and
2) separate "right & left" movers i.e. factorize

$$d\sigma = |\mathcal{H}|^2 \left\{ F_1^{\text{unsub}}(y_1 - (-\infty)) \sqrt{\frac{\tilde{S}(+\infty, y_s)}{\tilde{S}(+\infty, -\infty)\tilde{S}(y_s, -\infty)}} \right\} \times \left\{ \tilde{F}_2^{\text{unsub}}(+\infty - y_2) \sqrt{\frac{\tilde{S}(y_s, -\infty)}{\tilde{S}(+\infty, -\infty)\tilde{S}(+\infty, y_s)}} \right\}$$

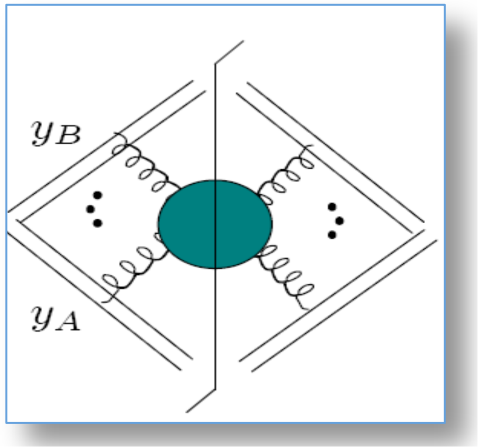
Separately Well-defined

TMD Evolution...CSS + JCC 2011

Evolution follows from their operator definition

Collins-Soper Equation:

$$\frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$



Now effects of Soft factor soft gluon radiation in evolution kernal

$$\tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{\tilde{S}(b_T; y_n, -\infty)}{\tilde{S}(b_T; +\infty, y_n)}$$

Along with .... RGE

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d\ln\tilde{F}(x, b_T; \mu, \zeta)}{d\ln\mu} = -\gamma_F(g(\mu); \zeta/\mu^2)$$

.... and RGE

Solve CS eq. & RGE equation to obtain Evolution kernal

One TMD factorization entire range of P_T or b_T

Collins Soper Sterman NPB 85

- TMD formalism of Collins 2011 interpolates/ matches the "TMD" and collinear picture
- Maximizes the perturbative content while providing a TMD formalism that is applicable over the entire range of P_T

$$\mathbf{b}_* = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}, \qquad \mu_b = \frac{C_1}{b_*}.$$

Partition the perturbative and nonperturbative parts of evolution Kernal $\tilde{K}(b_T, \mu)$

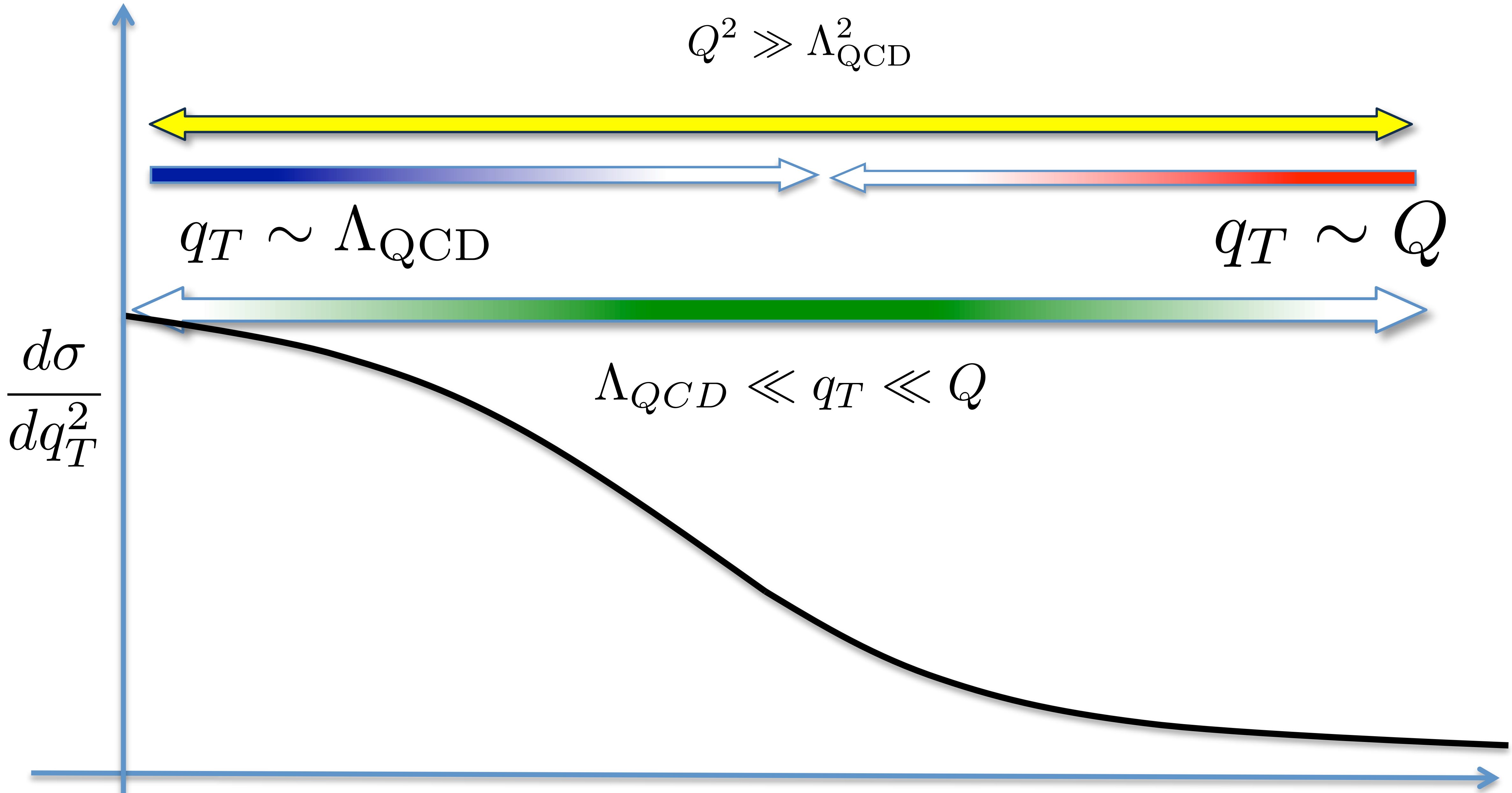
Collins Soper Sterman NPB 85

$$\mathbf{b}_* = \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\max}^2}}, \qquad \mu_b = \frac{C_1}{b_*}.$$

$$\tilde{K}(b_T;\mu) = \tilde{K}(b_*;\mu_b) - \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \gamma_K(g(\mu')) - g_K(b_T)$$

$b_{\max}$ chosen so that $\mathbf{b}_*$ doesnt go too far beyond the pertb. region maximize perturbative content

Transverse Momentum matching extension of Parton Model picture



TMD formalism of Collins 2011 interpolates/matches between "TMD" and so called collinear picture seems to be able to combine pictures

- ★ Collins-Cambridge Univ (11), Aybat Rogers PRD (11), Abyat, Collins, Qiu, Rogers (11), Aybat, Prokudin, Rogers (11), Bacchetta, Prokudin (13)

Structure Function *beyond* Parton Model

$$\mathcal{F}_{UU}(x,z,b,Q^2)=\sum_a \tilde{F}^a_{H1}(x,b_T,\mu,\zeta_F)\tilde{D}^a_{H2}(z_h,b_T,\mu,\zeta_D)H_{UU}(Q^2,\mu^2)$$

$$\tilde{F}_{H_1}(x,b_T;Q,Q^2)=\tilde{F}_{H_1}(x,b_*;\mu_b,\mu_b^2)\exp\left\{\underline{-g_1(x,b_T;b_{\max})}-\underline{g_K(b_T;b_{\max})}\ln\left(\frac{Q}{Q_0}\right)\right.$$
$$\left.+\ln\left(\frac{Q}{\mu_b}\right)\tilde{K}(b_*;\mu_b)+\int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_{\rm PDF}(\alpha_s(\mu');1)-\ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right]\right\}$$

$$\tilde{D}_{H_2}(z,b_T;Q,Q^2)=\tilde{D}_{H_2}(z,b_*;\mu_b,\mu_b^2)\exp\left\{\underline{-g_2(z,b_T;b_{\max})}-\underline{g_K(b_T;b_{\max})}\ln\left(\frac{Q}{Q_0}\right)\right.$$
$$\left.+\ln\left(\frac{Q}{\mu_b}\right)\tilde{K}(b_*;\mu_b)+\int_{\mu_b}^{Q}\frac{d\mu'}{\mu'}\left[\gamma_{\rm FF}(\alpha_s(\mu');1)-\ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right]\right\}$$

One TMD formalism for entire range of PT

$$\frac{d\sigma}{dP_T^2} \propto \mathcal{H}(\alpha_s(Q)) \int d^2 b_T e^{i b_T \cdot P_T} \, \tilde{F}_{H_1}(x, b_T; Q, Q^2) \, \tilde{D}_{H_2}(z, b_T; Q, Q^2) \;+\; Y_{\rm SIDIS}$$

$$\frac{d\sigma}{dP_T^2} \propto \text{ F.T.} \exp\left\{ -g_{\rm PDF}(x, b_T; b_{\max}) - g_{\rm FF}(z, b_T; b_{\max}) - 2 g_K(b_T; b_{\max}) \ln\left(\frac{Q}{Q_0}\right) + \right.$$

$$\left. + \, 2 \ln\left(\frac{Q}{\mu_b}\right) \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_{\rm PDF}(\alpha_s(\mu'); 1) + \gamma_{\rm FF}(\alpha_s(\mu'); 1) - 2 \ln\left(\frac{Q}{\mu'}\right) \gamma_K(\alpha_s(\mu')) \right] \right\}$$

$$+ \, Y_{\rm SIDIS}$$

Sivers Structure Function

$$F_{UT}(x,z,b,Q) = (\tilde{C}_{f/i} \otimes f^{(1)}_{1T\,i/P})(x,b_\star;\mu_b)(\tilde{C}_{j/H} \otimes d_{H/j})(z,b_\star;\mu_b)e^{-S^{pert}(b_*,Q)}e^{-S^{NP}_{UT}(b,Q,x,z)}$$

★ Abyat, Collins, Qiu, Rogers PRD (11), $b_* = \frac{b}{\sqrt{1+(b/b_{max})^2}}$

$$e^{-S^{NP}_{UT}}(b,Q,x,z) = \exp\left\{-\left[g_1(x,b_T;b_{\max}) + g_2(z,b_T;b_{\max}) + 2g_k(b_T)\ln\left(\frac{Q}{Q_0}\right)\right]\right\}_{UT}$$

Non perturbative factor contribution must be fit

$\left\{g_i\right\} \to 0 \quad \text{as} \quad \text{b} \to 0 \quad \text{perturbative}$ CSS NPB 85

Unpolarized and Sivers evolve in same way !!!

Recall correlator in *b*-space From Bessel Transform

$$\tilde{\Phi}^{[\gamma^+]}(x,\boldsymbol{b}_T) \;=\; \tilde{f}_1(x,\boldsymbol{b}_T^2) - i\,\epsilon_T^{\rho\sigma}\,b_{T\rho}S_{T\sigma}\,M\,\tilde{f}_{1T}^{\perp(1)}(x,\boldsymbol{b}_T^2)$$

$$\frac{\partial\,\tilde{\phi}^{i}_{f/P}(x,\mathbf{b}_{\mathrm{T}};\mu,\zeta_F)\epsilon_{ij}S_T^j}{\partial\ln\sqrt{\zeta_F}} = \tilde{K}(b_T;\mu)\,\tilde{\phi}^{i}_{f/P}(x,\mathbf{b}_{\mathrm{T}};\mu,\zeta_F)\epsilon_{ij}S_T^j.$$

Sivers BWA: Cancellation of Universal *NP* and flavor blind hard contributions

When $\Lambda^2_{QCD} \ll P_h^2 \ll Q^2$

$$\mathcal{A}_{UT}(x,z,b,Q^2) = \frac{\tilde{f}_{1T}^{\perp(1)}(x,z^2\boldsymbol{b}^2,\mu_0^2,Q_0)\tilde{D}_1(z_h,\boldsymbol{b}^2,\mu_0^2,Q_0)\tilde{H}_{UT}(\mu_0^2,Q_0)e^{-S^{\rm pert}(b_*,Q)}e^{-2g_k(b_T)\ln\left(\frac{Q}{Q_0}\right)}}{\tilde{f}_1(x,z^2\boldsymbol{b}^2,\mu_0^2,Q_0)\tilde{D}_1(z_h,\boldsymbol{b}^2,\mu_0^2,Q_0)\tilde{H}_{UU}(\mu_0^2,Q_0)e^{-S^{\rm pert}(b_*,Q)}e^{-2g_k(b_T)\ln\left(\frac{Q}{Q_0}\right)}}$$

BWA less sensitivity to Evolution

In prep. Boer, LG, B. Musch, A. Prokudin....

First Attempts

PROCEEDINGS OF SCIENCE

Studies of TMDs with CLAS

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Studies of single and double-spin asymmetries in pion electro-production in semi-inclusive deep-inelastic scattering of 5.8 GeV polarized electrons from unpolarized and longitudinally polarized targets at the Thomas Jefferson National Accelerator Facility using CLAS discussed. We present a Bessel-weighting strategy to extract transverse-momentum-dependent parton distribution functions.

XXI International Workshop on Deep-Inelastic Scattering and Related Subject -DIS2013,
22-26 April 2013
Marseilles,France

Use of Bessel Weighting

- Study by Mher Aghasyan using MC see talk of yesterday

Project Upol. and Doubly polarized Structure Function

$$\tilde{\sigma}(B_T) = 2\pi \int dP_{h\perp} P_{h\perp} J_0(B_T P_{h\perp}) \frac{d\sigma}{dx\, dy\, d\psi\, dz\, d\phi_h\, dP_{h\perp}\, P_{h\perp}}$$

$$= 2\pi \int dP_{h\perp} P_{h\perp} J_0(B_T P_{h\perp}) \int \frac{db_T\, b_T}{2\pi} J_0(b_T\, P_{h\perp}) \times K(x,y) \left(\frac{\mathcal{F}_{UU,T}(b_T)}{x} + S_{||}\lambda_e \sqrt{1-\varepsilon^2} \frac{\mathcal{F}_{LL}(b_T)}{x} \right)$$

$$= K(x,y) \left(\frac{\mathcal{F}_{UU,T}(B_T)}{x} + S_{||}\lambda_e \sqrt{1-\varepsilon^2} \frac{\mathcal{F}_{LL}(B_T)}{x} \right)$$

$$S_{||}\lambda_e = \pm 1$$

$$\tilde{\sigma}^{\pm}(b_T) = K(x,y)\left(\frac{\mathcal{F}_{UU,T}(b_T)}{x} \pm \sqrt{1-\varepsilon^2}\frac{\mathcal{F}_{LL}(b_T)}{x}\right)$$

$$A_{LL}^{J_0(b_T P_{hT})}(b_T) = \frac{\tilde{\sigma}^{+}(b_T) - \tilde{\sigma}^{-}(b_T)}{\tilde{\sigma}^{+}(b_T) + \tilde{\sigma}^{-}(b_T)} = \frac{\tilde{\sigma}_{LL}(b_T)}{\tilde{\sigma}_{UU}(b_T)} = \sqrt{1-\varepsilon^2}\frac{\sum_q e_a^2 \tilde{g}_1^q(x, z^2\boldsymbol{b}_{\boldsymbol{T}}^2)\tilde{D}_1^q(z, \boldsymbol{b}_{\boldsymbol{T}}^2)}{\sum_q e_a^2 \tilde{f}_1^q(x, z^2\boldsymbol{b}_{\boldsymbol{T}}^2)\tilde{D}_1^q(z, \boldsymbol{b}_{\boldsymbol{T}}^2)},$$

Discretize

$$K(x,y)\sqrt{1-\varepsilon^2}\frac{\mathcal{F}_{LL}(b_T)}{x} = \pi\int dP_{h\perp}P_{h\perp}J_0(B_T P_{h\perp})\left(\frac{d\sigma^+}{dx\,dy\,dz\,dP_{h\perp}P_{h\perp}} - \frac{d\sigma^-}{dx\,dy\,dz\,dP_{h\perp}P_{h\perp}}\right)$$

$$d\Phi \equiv dx\,dy\,d\psi\,dz\,dP_{h\perp}P_{h\perp}$$

$$\Delta\Phi \equiv \Delta x\,\Delta y\,\Delta z\,\Delta P_{h\perp}P_{h\perp}$$

$$\int dP_{h\perp}J_0(B_T P_{h\perp})\frac{dn}{dx\,dy\,dz\,d\phi_h\,dP_{h\perp}} = \sum_{i\,\epsilon\,\mathrm{bin}[x_i,y_i,z_i]} J_0(B_T P_{h\perp\,i})\frac{1}{\Delta x\,\Delta y\,\Delta z}$$

$$K(x,y)\,\sqrt{1-\varepsilon^2}\,\frac{\mathcal{F}_{LL}(B_T)}{x} =$$

$$\Rightarrow \frac{1}{2}\left\{\frac{2\pi}{N_0^+}\sum_{i\,\epsilon\,\mathrm{bin}[x_i,y_i,z_i]} J_0(B_T P_{h\perp\,i})\frac{n_i^+}{\Delta x_i\,\Delta y_i\,\Delta z_i} \;-\; \frac{2\pi}{N_0^-}\sum_{i\,\epsilon\,\mathrm{bin}[x_i,y_i,z_i]} J_0(B_T P_{h\perp\,i})\frac{n_i^-}{\Delta x_i\,\Delta y_i\,\Delta z_i}\right\}$$

Sum over events in bin to sum over events

$$K(x,y)\,\sqrt{1-\varepsilon^2}\,\frac{\mathcal{F}_{LL}(B_T)}{x} =$$

$$\Rightarrow \left\{\sum_{j\ \mathrm{events}}^{N^+} J_0(B_T P_{h\perp\,j}) \;-\; \sum_{j\ \mathrm{events}}^{N^-} J_0(B_T P_{h\perp\,j})\right\}$$

$$\tilde{\sigma}^{\pm}(b_T) = S^{\pm} \equiv \sum_{i=1}^{N^{\pm}} J_0(b_T P_{hT\,i})$$

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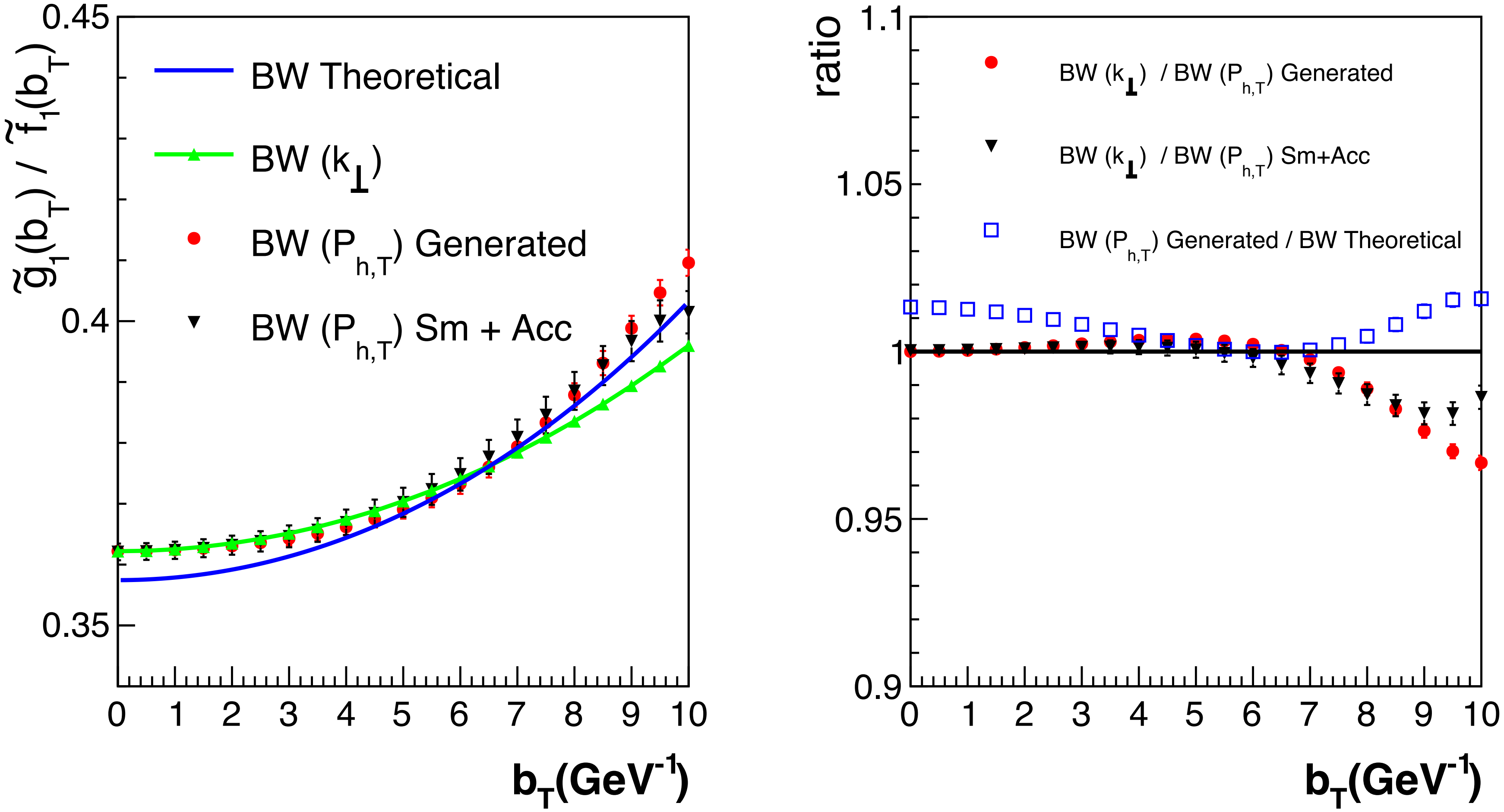

$$A_{LL}^{J_0(b_T P_{hT})}(b_T) = \frac{\tilde{\sigma}^+(b_T) - \tilde{\sigma}^-(b_T)}{\tilde{\sigma}^+(b_T) + \tilde{\sigma}^-(b_T)} = \frac{\tilde{\sigma}_{LL}(b_T)}{\tilde{\sigma}_{UU}(b_T)} = \sqrt{1-\varepsilon^2}\frac{\sum_q e_a^2 \tilde{g}_1^q(x, z^2\boldsymbol{b}_T^2)\tilde{D}_1^q(z, \boldsymbol{b}_T^2)}{\sum_q e_a^2 \tilde{f}_1^q(x, z^2\boldsymbol{b}_T^2)\tilde{D}_1^q(z, \boldsymbol{b}_T^2)}$$

Conclusions-II

- Propose generalized Bessel Weights
- Theoretical weighting procedure-advantages
- Introduces a free parameter $\mathcal{B}_T\,[\mathrm{GeV}^{-1}]$ that is Fourier conjugate to $\boldsymbol{P}_{h\perp}$
- Provides a regularization of infinite contributions at lg. transverse momentum when $\mathcal{B}_T^2$ is non-zero
- Soft, Hard CS, eliminated from weighted asymmetries, Sudakov dpnds coupling of *b* & *Q*
- Possible to compare observables at different scales.... could be useful for an EIC

Cancellation of Soft Factor on level of the Matrix elements ***(summarize)***

- So far we get ratios of moments of TMDs and FFs that are free/insensitive to soft gluon radiation
- It was not necessary to specify explicit def. of TMDs and FFs
- We also analyze ratio of moments of TMDs directly on level of matrix elements of TMDs & FFs
- Again we find cancellation of soft factors in ratio
- Impact for Lattice calculation of moments of TMDS, Musch, Ph. Hagler, M. Engelhardt, J.W. Negele, A. Schafer arXiv 2011