Parton distributions
concepts, processes, acronyms

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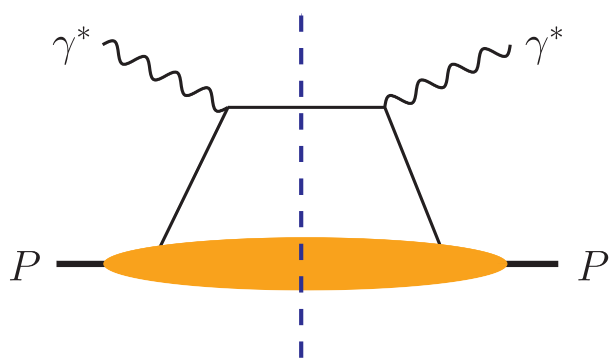
Motivation: two aspects

- parton distributions quantify hadron structure
  - ★ aim: understand dynamics of QCD in nonperturbative sector
  - ★ different types of distributions → more information
- quantitative description of scattering processes
  - ★ parton distributions as inevitable nonperturbative input, want highest possible precision on these
  - ★ try to minimize number of quantities to be fitted, focus largely on conventional unpolarized PDFs

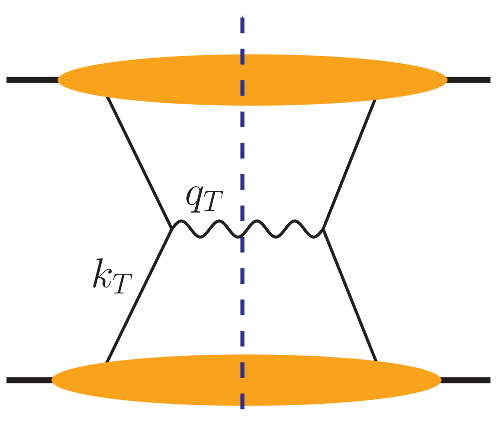
The main players

An incomplete overview

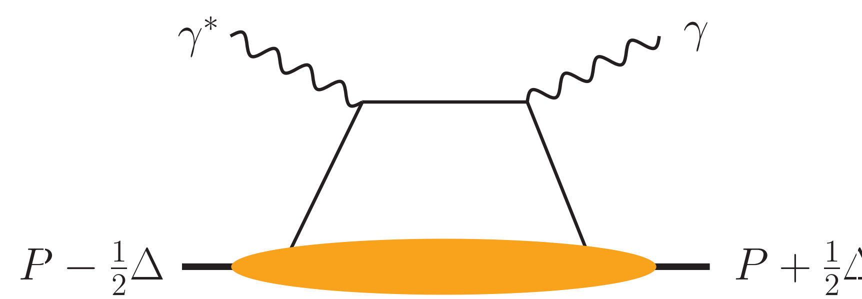
- PDF factorization
  - ★ inclusive processes
  - ★ p_T ~ hardest scale or unmeasured



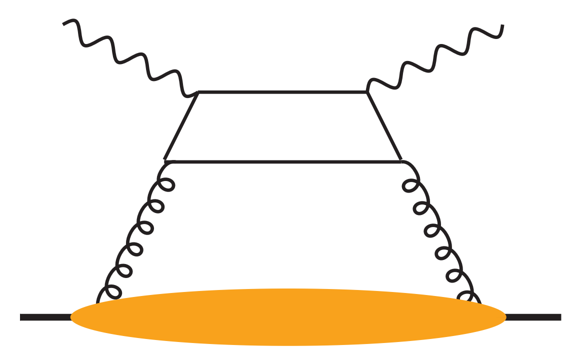
- TMD factorization
  - ★ inclusive
  - ★ p_T « hardest scale



- GPD factorization
  - ★ exclusive processes
  - ★ non-forward kinematics



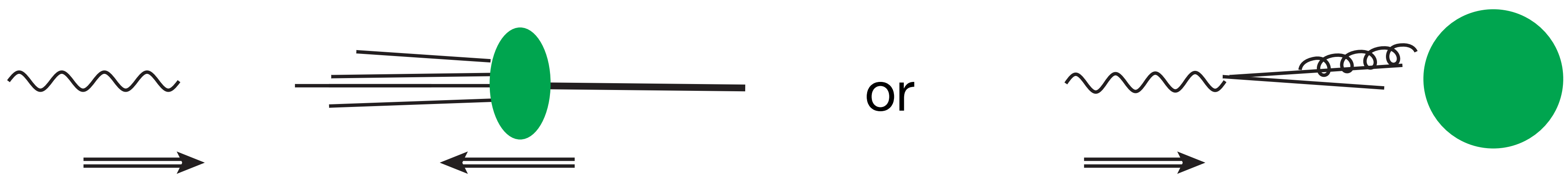
- small-x factorization
  - ★ inclusive or exclusive
  - ★ unintegrated gluon dist's



Part 1: some basics

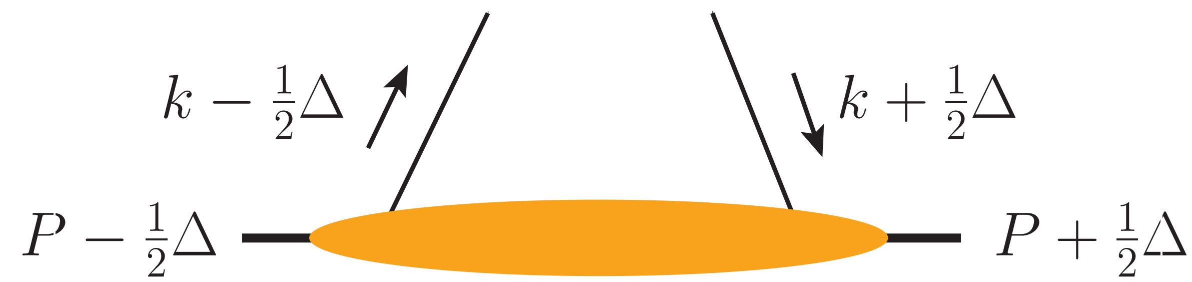
renormalization, Wilson lines, etc.
→ later

Light-cone coordinates



- hard processes single out (at least one) spatial direction
  - ★ longitud. and transv. directions play different roles
    loss of manifest 3d rotation invariance
  - ★ light-cone coordinates $v^{\pm} = (v^0 \pm v^3)/\sqrt{2}$
    and 2d transverse component

Two-parton correlation functions



$$H(k^\mu, \Delta^\mu, P^\mu) = (2\pi)^{-4} \int d^4 z \; e^{izk} \left\langle P + \tfrac{1}{2}\Delta \right| \bar{q}(-\tfrac{1}{2}z)\, \Gamma q(\tfrac{1}{2}z) \left| P - \tfrac{1}{2}\Delta \right\rangle$$

$$f(k^\mu, P^\mu) = (2\pi)^{-4} \int d^4 z \; e^{izk} \left\langle P \right| \bar{q}(-\tfrac{1}{2}z)\, \Gamma q(\tfrac{1}{2}z) \left| P \right\rangle$$

- independent variables:
  - ★ parton: k^+, k_T and k^- (or virtuality k^2)
  - ★ protons: P^+, P_T and Δ^+, Δ_T
    minus components fixed by mass shell conditions
    typically chose frame with $P_T = 0$
- Dirac matrix $\Gamma \leftrightarrow$ quark polarization and twist of dist'n

Longitudinal momentum and position

$$f(k^\mu, P^\mu) = (2\pi)^{-4} \int d^4z \; e^{izk} \, \langle P \big| \, \bar{q}(-\tfrac{1}{2}z) \, \Gamma q(\tfrac{1}{2}z) \, \big| P \rangle$$

$$\int dk^- \; f = (2\pi)^{-3} \int dz^- \, d^2z_T \; e^{izk} \, \langle P \big| \, \bar{q}(-\tfrac{1}{2}z) \, \Gamma q(\tfrac{1}{2}z) \, \big| P \rangle \Big|_{z^+=0}$$

- $\int dk^-$ sets field arguments $z^+ = 0$ (light front)
  - ★ in light front quantization expand fields at $z^+ = 0$ into creation/annihilation operators
    - ➡ interpret as “free” partons like in parton model

$$\int dk^- \, d^2k_T \; f = (2\pi)^{-1} \int dz^- \; e^{izk} \, \langle P \big| \, \bar{q}(-\tfrac{1}{2}z) \, \Gamma q(\tfrac{1}{2}z) \, \big| P \rangle \Big|_{z^+=0, z_T=0}$$

- $\int dk^- \, d^2k_T$ puts field separation on light cone $z^2 = 0$

Transverse momentum and position

- variables related by Fourier transform, e.g.
  - ★ quark field $\tilde{q}(k_T, z^-) = \int d^2 z_T \; e^{i z_T k_T} \; q(z_T, z^-)$
  - ★ proton state $|p^+, b_T\rangle = \int d^2 p_T \; e^{-i b_T p_T} \; |p^+, p_T\rangle$

$$\bar{\tilde{q}}(k_T)\tilde{q}(l_T) = \int d^2 y_T \, d^2 z_T \; e^{-i(y_T k_T - z_T l_T)} \, \bar{q}(y_T) \, q(z_T)$$

$$y_T k_T - z_T l_T = \tfrac{1}{2}(y_T + z_T)(k_T - l_T) + \tfrac{1}{2}(y_T - z_T)(k_T + l_T)$$

  - ★ ‘average’ momentum ↔ position difference
  - ★ ‘average’ position ↔ momentum transfer

- Wigner distributions depend on ‘average’ momentum and position
  - ➡ no probability interpretation

Parton correlation functions and their descendants

parton correlation function $H(k^\mu, \Delta^\mu, P^\mu)$

$\downarrow \int dk^-$

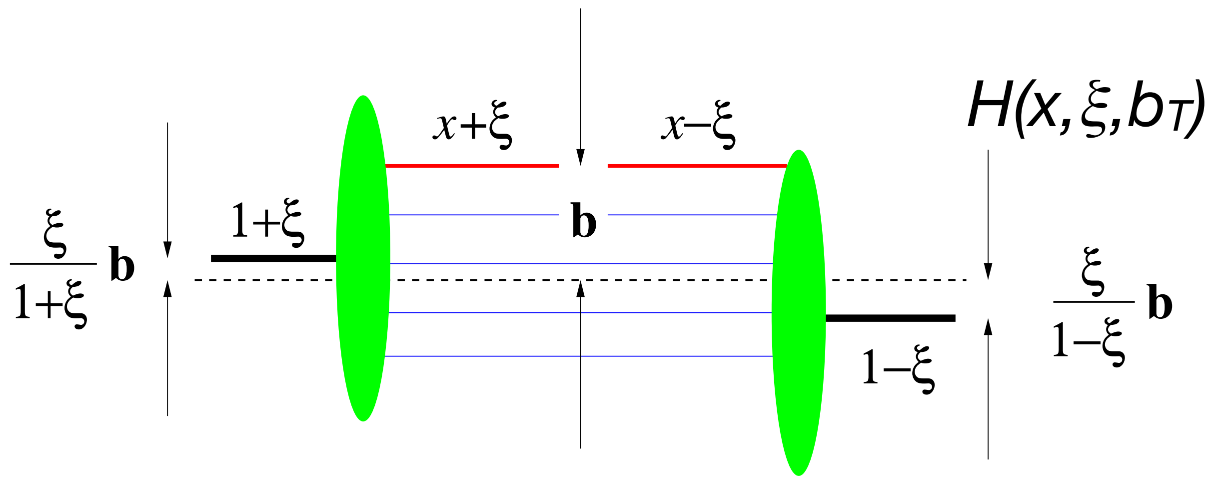
$x = k^+/P^+$

$\xi = -2\Delta^+/P^+$

$H(x, k_T, \xi, b_T) \overset{FT}{\longleftrightarrow} H(x, k_T, \xi, \Delta_T)$ **GTMD**

$\downarrow \int d^2k_T$

$H(x, \xi, b_T) \overset{FT}{\longleftrightarrow} H(x, \xi, t)$ **GPD**



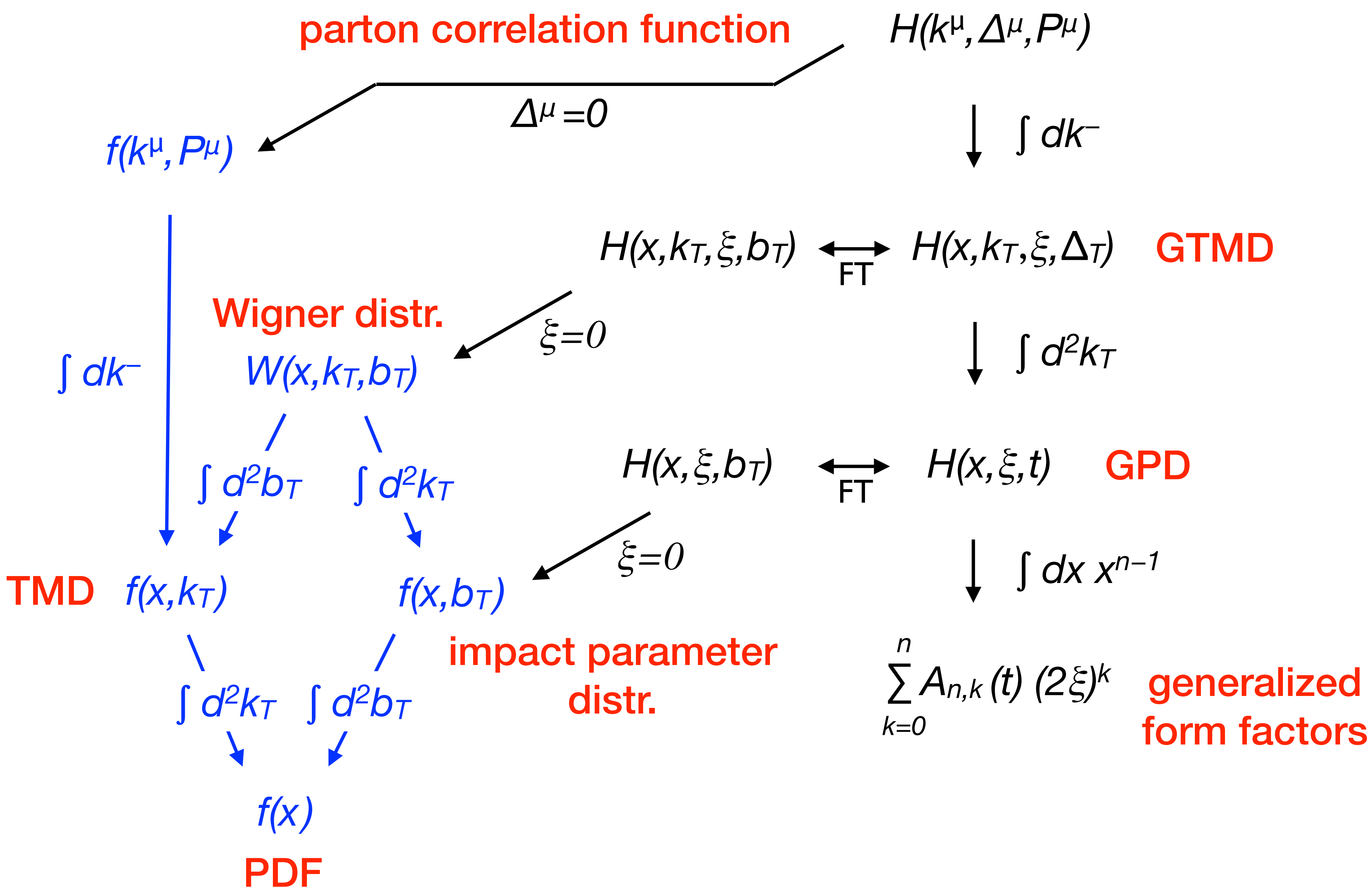
$\downarrow \int dx\, x^{n-1}$

$\sum_{k=0}^{n} A_{n,k}(t)\,(2\xi)^k$ **generalized form factors**

FT = Fourier transform $b_T \longleftrightarrow k_T$

$t = \Delta^2$ = function of ξ and Δ_T

Parton correlation functions and their descendants



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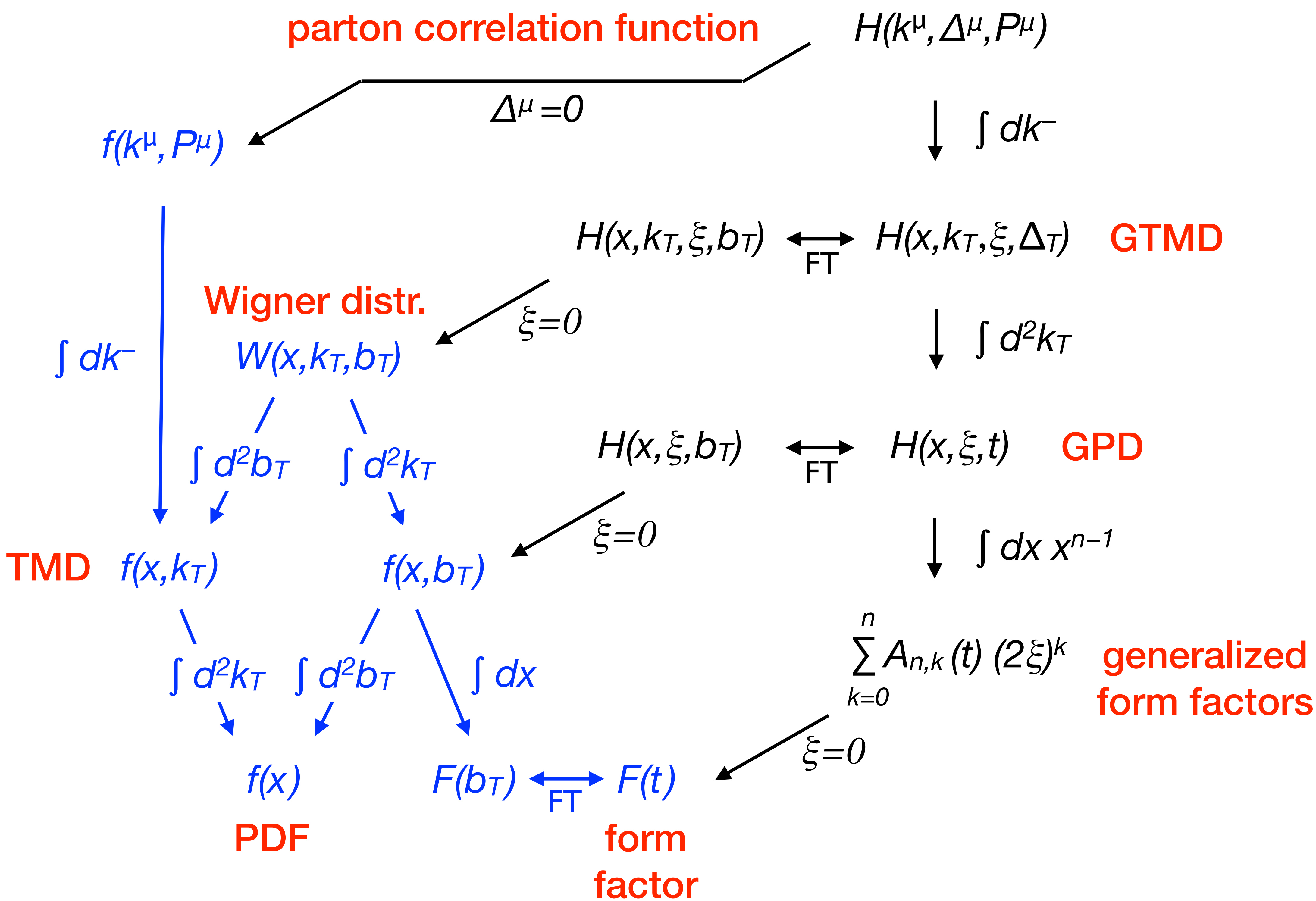


Longitudinal position

- 3d Fourier transform of $H(x, k_T, \xi, \Delta_T)$ — Belitsky, Ji, Yuan '03

  w.r.t. momentum transfer in Breit frame ($\Delta^0 = 0$)

  ➡ Wigner function with 3d position of proton

  ★ interpretation problematic for positions ~ $1/m_p$
  situation known from Sachs form factors

- 3d FT of Compton amplitude $A(\xi, \Delta_T)$ — Brodsky et al '06, '07

  ➡ diffraction patterns in longitudinal position

- FT of collinear distributions (PDFs, GPDs) w.r.t. x — Balitsky, Braun; Müller et al; Radyushkin

  ➡ depend on field separation z^- along light-cone

  ★ common representation of scale evolution for
  PDFs, GPDs, distribution amplitudes

Part 2: some more detail

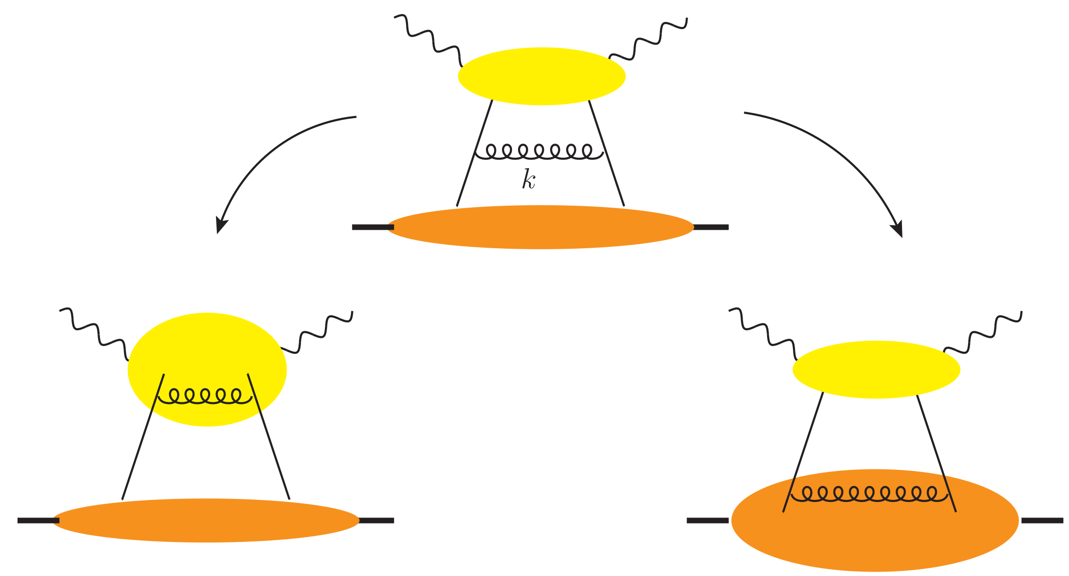
still an overview
many issues will be picked up in talks this week

apologies: citations will be
sparse and incomplete

Collinear factorization and PDFs

$$f(x) = (2\pi)^{-1} \int dz^- \; e^{ixP^+z^-} \, \langle P | \, \bar{q}(-\tfrac{1}{2}z) \, \Gamma q(\tfrac{1}{2}z) \, | P \rangle \Big|_{z^+=0, z_T=0}$$

- "naive operator definition" insufficient, must
  - ★ renormalize UV divergences → scale μ, DGLAP evolution lose literal interpretation as densities
  - ★ include Wilson line for gauge invariance
- both steps also required in factorization formulae for scattering processes
  - ★ prevent double counting
  - ★ proper treatment of A^- gluon polarization



- different renormalization schemes
  - ★ $\overline{\text{MS}}$ vs DIS; more schemes for polarized PDFs (axial anomaly)
  - ➡ separation of "hadron structure" and "probe" not unique

Collinear factorization: higher twist

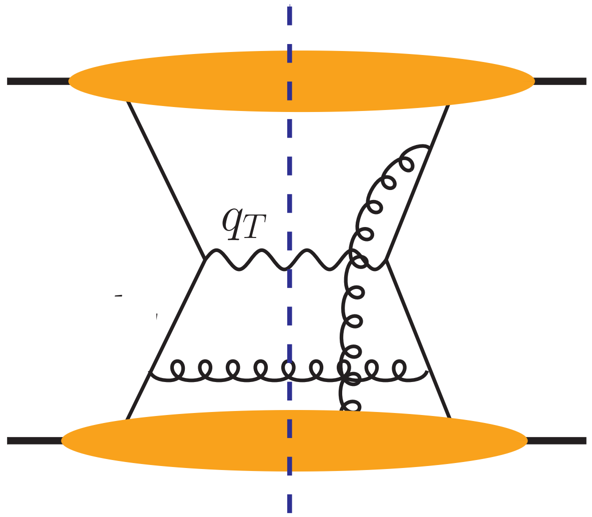
- three or more fields separated along light cone
  e.g. $\bar{q}(z_1)\, A_T(z_2)\, \Gamma q(z_3)$ joined by Wilson lines

  ★ UV renormalization as for PDFs
  evolution equations more involved

- appear in hard processes with suppression factor
  (Λ/hard scale)^n

- twist 3 distributions prominent in spin asymmetries

- interpretation: parton correlations
  (not densities like PDFs)



TMD factorization

$$f(x, z_T) = (2\pi)^{-1} \int dz^- \; e^{ixP^+z^-} \, \langle P | \, \bar{q}(-\tfrac{1}{2}z) \, \Gamma q(\tfrac{1}{2}z) \, | P \rangle \Big|_{z^+=0} = \text{FT of } f(x, k_T)$$

- must subtract divergences
  - ★ field renormalization (scale μ, usual RGE)
  - ★ rapidity divergences (scale ζ , Collins-Soper evolution, Sudakov logarithms) → Wilson lines
    - ✦ technically convenient in z_T space (often called b space)
- factorization: soft gluon exchange → Wilson lines
  - ★ process dependent paths → reduced universality
    textbook example: Sivers distribution (unpol. quarks in transv. pol. proton)
- different technical implementations/schemes Collins; Ji, Ma, Yuan; ...
  - ★ also in SCET, often called "beam functions"
    Mantry, Petriello; Chiu et al.; Becher, Neubert; Echeverria et al, ...

TMD factorization

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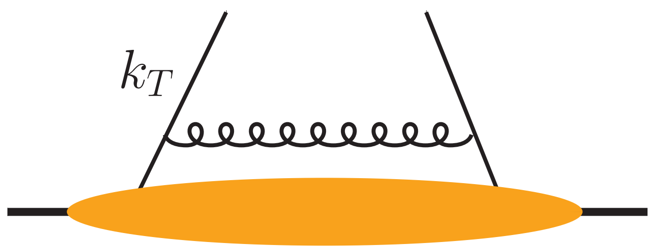
- factorization: soft gluon exchange → Wilson lines
  - ★ process dependent paths → reduced universality
  - ★ TMD factorization established for limited class of processes SIDIS, Drell-Yan, some more candidates otherwise expect factorization breaking
  - ★ probably connected with results found in perturbative calculations Forshaw, Seymour; Catani, de Florian, Rodrigo

Connection between TMDs and PDFs

- for $k_T \gg \Lambda$ can compute $f(x,k_T)$ as PDF $\otimes$ splitting kernel
  - ➡ increased predictive power
  - ★ more complicated for polarized PDFs → later

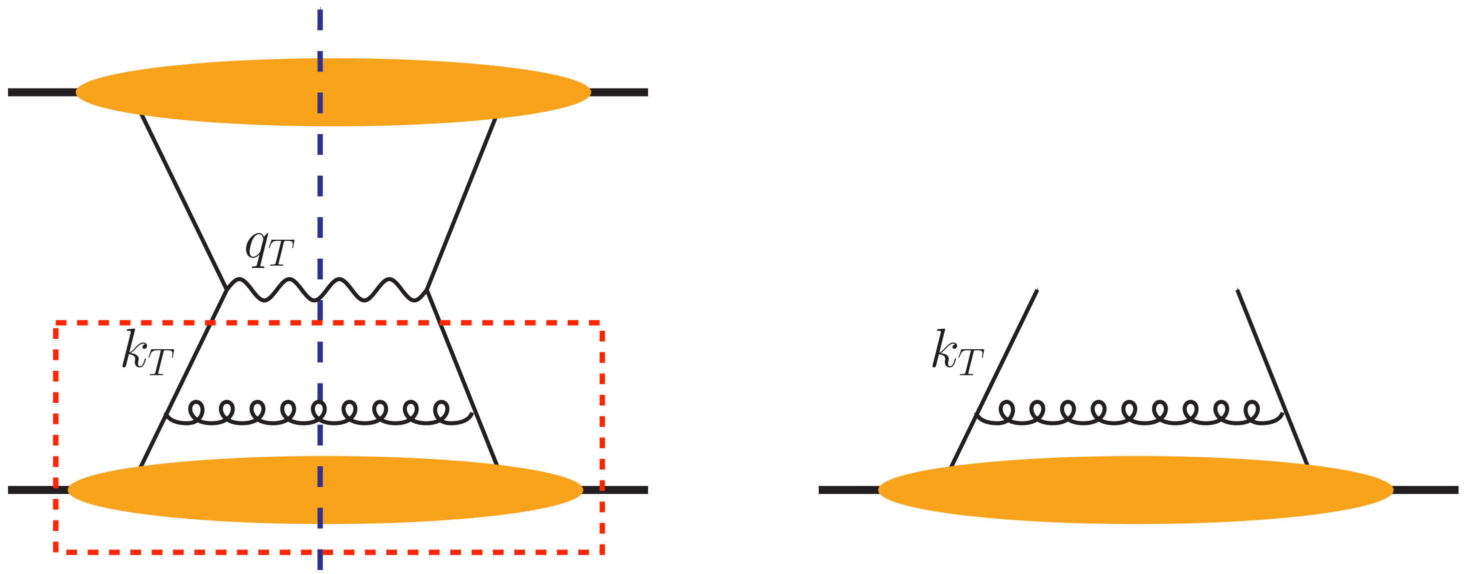


  - ★ in b space for small b:
    - ✦ $f(x,b,\zeta,\mu) = f(x,\mu) + f \otimes$ splitting kernel

      natural scales $\zeta \sim \mu \sim 1/b$
    - ✦ recent calculation of two-loop splitting kernels
      [Gehrmann, Lübbert, Yang ’12-’14]
      using rapidity regulator of Becher/Neubert

Connection between TMDs and PDFs

- for $k_T \gg \Lambda$ can compute $f(x,k_T)$ as PDF ⊗ splitting kernel



- two equivalent descriptions for Drell-Yan with $Q \gg k_T \gg \Lambda$
  - ★ TMD factorization with TMD = PDF ⊗ hard kernel
  - ★ fixed order calculation with PDF
  - ➡ “nested” perturbative description for scales Q and k_T
    use Collins-Soper evolution to resum large logarithms

Collins, Soper, Sterman (CSS) ’84

Connection between TMDs and PDFs

TMD $f(x,k_T)$

$\int d^2k_T$ ↓

$f(x)$

PDF

cannot hold literally since at high k_T

$$f(x, k_T) \sim 1/k_T^2$$

→ must regulate k_T integral

- at leading order may use cutoff

$$\pi \int_0^{\mu^2} dk_T^2\, f(x, k_T) = f(x, \mu)$$

  ★ used in small x phenomenology Martin, Ryskin; ...

- *b* space regulator:

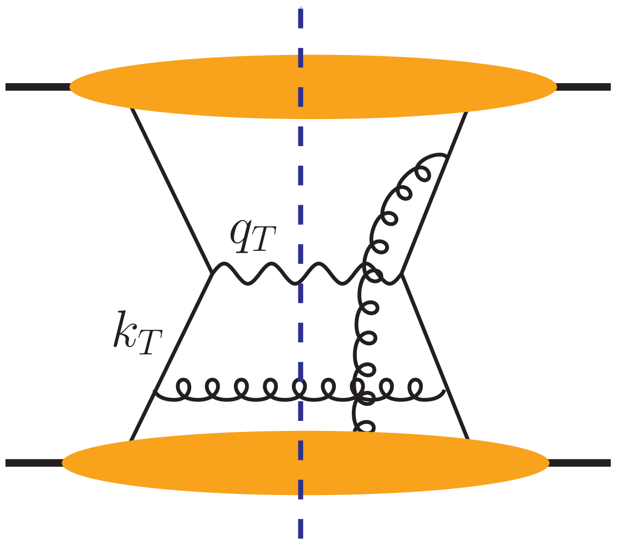
$$f(x, b) = \pi \int dk_T^2\, J_0(b\, k_T) f(x, k_T) = f(x, \mu \sim 1/b) + \mathcal{O}(\alpha_s)$$

indeed

$$\int dk_T^2\, J_0(b\, k_T) f(x, k_T) = \int_0^{\mu^2} dk_T^2\, f(x, k_T) + \mathcal{O}(b\Lambda) \quad \text{for } \mu = \#/b$$

TMDs with spin dependence

- depending on spin structure express TMDs at high k_T in terms of PDF or higher-twist collinear function
- TMD and collinear description equivalent for some spin asymmetries, but not for others Bacchetta et al '08
  - ★ explicit matching shown e.g. for Drell-Yan Sivers asymmetry Ji et al; Eguchi et al; Vogelsang et al; ...



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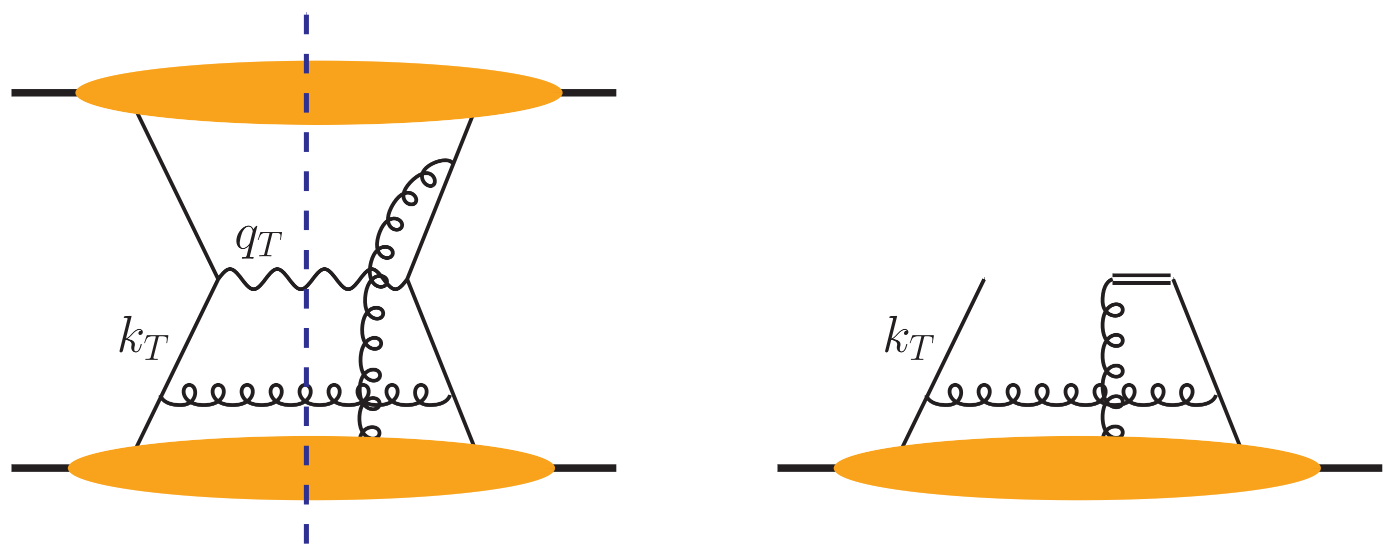


- as in unpol. case, need UV regulator for integral relations like

$$\int d^2k_T\, k_T^2 \times \text{Sivers fct} = \text{number} \times \text{Qiu-Sterman fct}$$

Parton correlation functions

- not integrated over parton k^- → depend on parton virtuality
- also doubly/fully unintegrated parton distributions
  or "beam functions" (in SCET community)
- advocated in different contexts:
  - ★ small x, computed in terms of PDFs (valid for large k^2)
    Martin, Ryskin, Watt '03
  - ★ control of final-state kinematics, esp. for MC generators
    Collins, Jung '05; Collins, Rogers, Staśto '08
- also considered: correlation fcts integrated over k_T but not k^-
  - ★ resummation for observables sensitive to beam jets in SCET
    computed in terms of PDFs at two loops
    Stewart, Tackmann , Waalewijn '09, Gaunt, Stahlhofen, Tackmann '14
- process dependence? universality? factorization breaking?

Generalized parton distributions

- GPDs measured in exclusive processes
  - ★ at LO sensitive to $H(\xi,\xi,t)$
  - ★ access to $x \neq \xi$ via evolution and NLO effects
- cannot compute $H(x,\xi,t=0)$ from $f(x)$ using 1st principles
  - ★ connection via Shuvaev transform [Martin, Ryskin et al] is a model
- generalized form factors computed in lattice QCD (including Ji's sum rule)

GTMD $H(x,k_T,\xi,\Delta_T)$

$$\downarrow \int d^2k_T$$

GPD $H(x,\xi,t)$

$$\downarrow \int dx\, x^{n-1}$$

$$\sum_{k=0}^{n} A_{n,k}(t)\,(2\xi)^k$$ generalized form factors

Generalized parton distributions

- access to GTMDs?

  spin decomposition:
  Meissner et al

- similar case:
  distribution amplitudes (DAs) vs.
  light-cone wave fcts (LCWFs)

  ★ LCWFs used in $\gamma^* \gamma \to \pi$
  and meson production

  Li, Sterman; ...
  Goloskokov, Kroll

  ✦ non-pert. k_T behavior → power corrections

- GTMDs used in Guichon, Guidal, Vanderhaeghen ’99

- very complicated theory and phenomenology

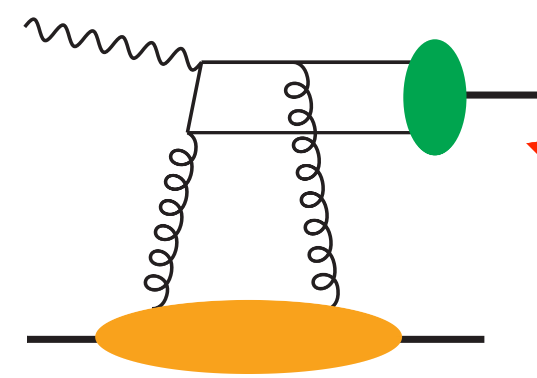
GTMD $H(x,k_T,\xi,\Delta_T)$ **LCWF** $\Psi(z,k_T)$

$\downarrow \int d^2k_T$ $\downarrow \int d^2k_T$

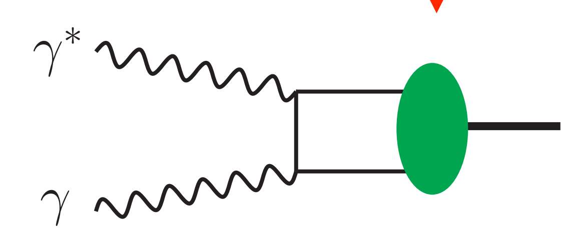
GPD $H(x,\xi,t)$ **DA** $\varphi(z)$

$\downarrow \int dx\, x^{n-1}$

$$\sum_{k=0}^{n} A_{n,k}(t)\,(2\xi)^k$$



DA or LCWF



Small-x factorization

high-energy/low-x factorization		hard-scattering factorization (collinear or TMD)
	separate dynamics according to	
rapidity		virtuality/transverse mom.
	expand in	
log(1/x)		1/(hard scale)

small-x formalism(s):

- evolution equations in log(1/x) ~ rapidity
  - ★ BFKL, CCFM
- gluon saturation → nonlinear evolution: BK, JIMWLK
- primary quantities are not parton distributions, but
  - ★ impact factors, BFKL kernel, dipole scattering amplitude and generalizations (formulated in terms of Wilson lines)

Small-x factorization

- however, unintegrated (k_T dependent) gluon dist'n emerges in suitable processes and kinematics (non-saturating, hard scale)
- work in color glass condensate formalism:
  two gluon distributions, with different Wilson lines
  - ★ Weizsäcker-Williams (future pointing WLs)
    has density interpretation
  - ★ Dipole gluon distribution (past and future point WLs)
    $\propto$ FT of dipole scattering amplitude

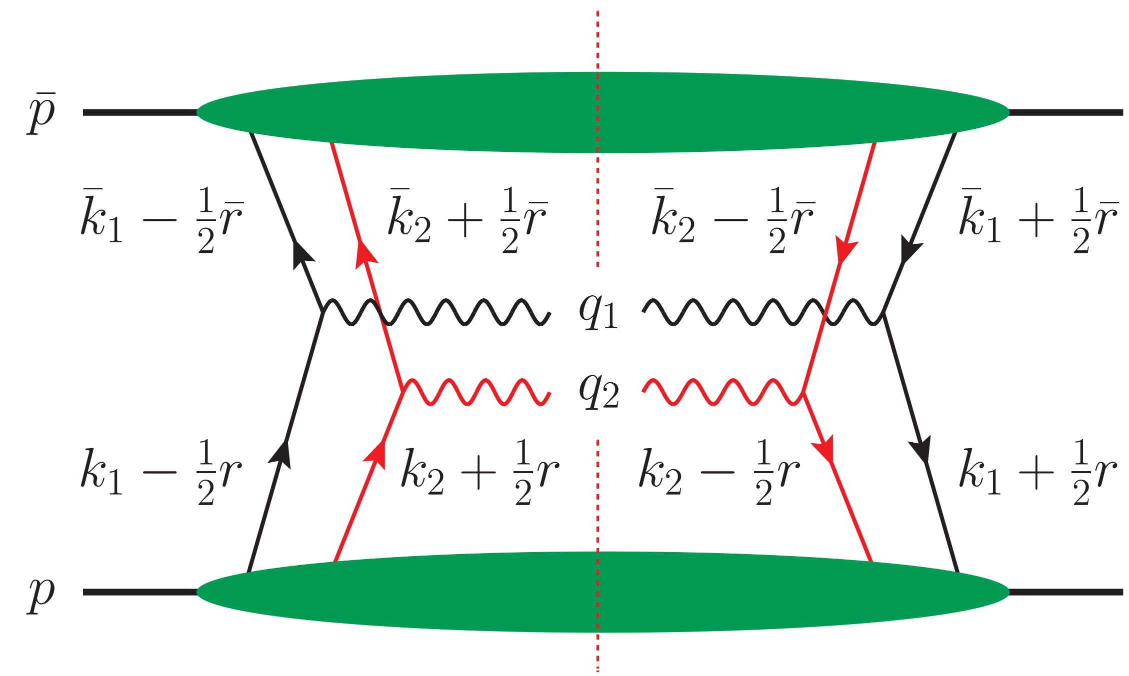
	DIS and DY	SIDIS	hadron in pA	photon-jet in pA	Dijet in DIS	Dijet in pA
$G^{(1)}$ (WW)	$\times$	$\times$	$\times$	$\times$	$\surd$	$\surd$
$G^{(2)}$ (dipole)	$\surd$	$\surd$	$\surd$	$\surd$	$\times$	$\surd$

from: Dominguez et al '11

- relation with TMD formalism not fully worked out
  recent work on Sudakov resummation in small-x formalism

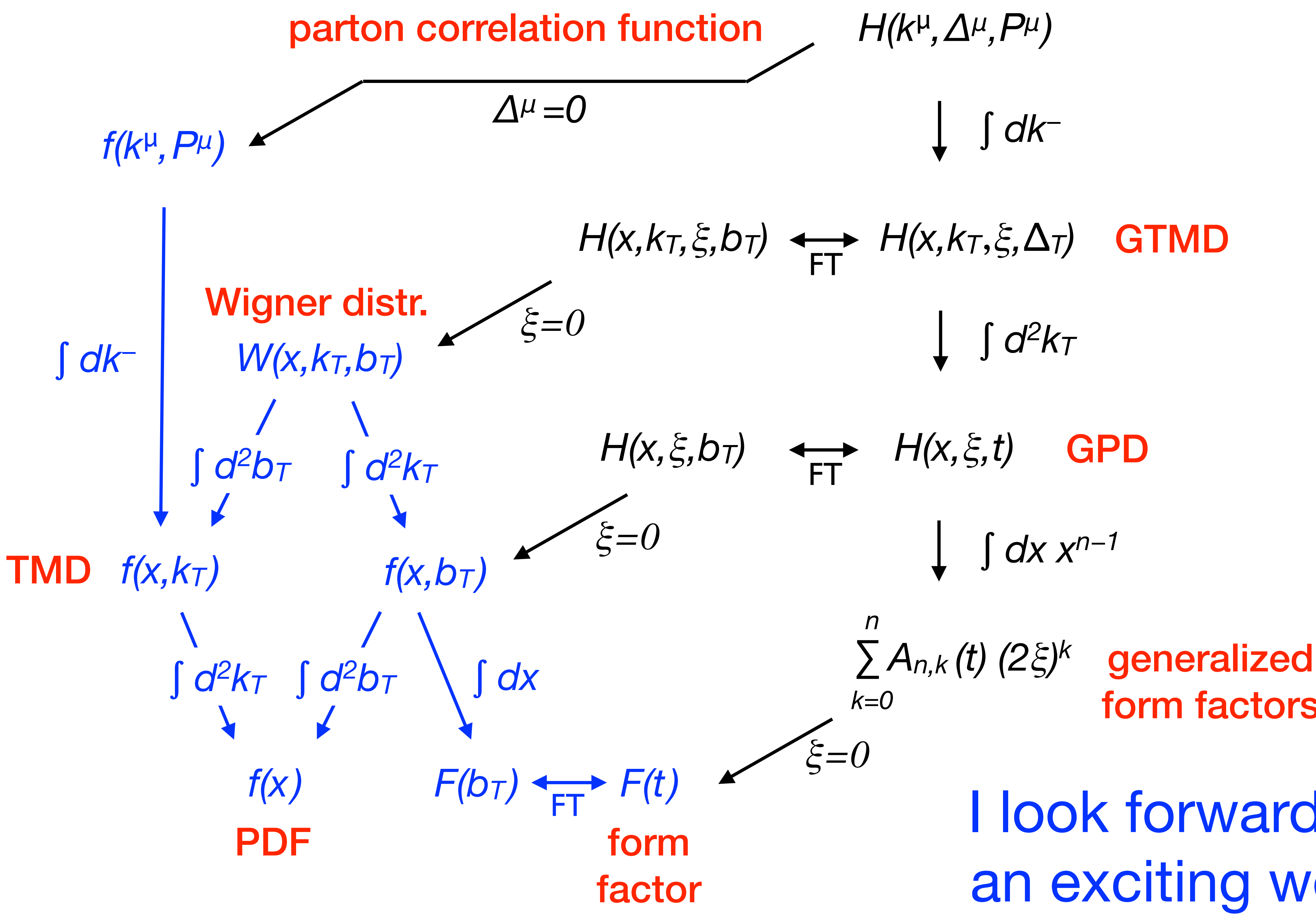
Mueller, Xiao, Yuan '13

Bonus slide: double parton distributions



- two hard interactions in same proton collision
  - ★ e.g. $pp \to WW + X$ or $pp \to W + \text{jets} + X$
- competes with single hard scattering
  - ★ power suppressed if integrate over final state k_T
  - ★ enhanced at small x due to steep rise of parton densities
- double parton distributions in collinear or TMD formalism $F(x_1, x_2, y_T)$ or $F(x_1, x_2, k_{T1}, k_{T2}, y_T)$
  - ★ y_T = distance between two partons
    Fourier conjugate to momentum mismatch r_T
- sensitive to correlations between partons
  - ★ in momentum, position, polarization, color, ...

Instead of a summary



I look forward to
an exciting week