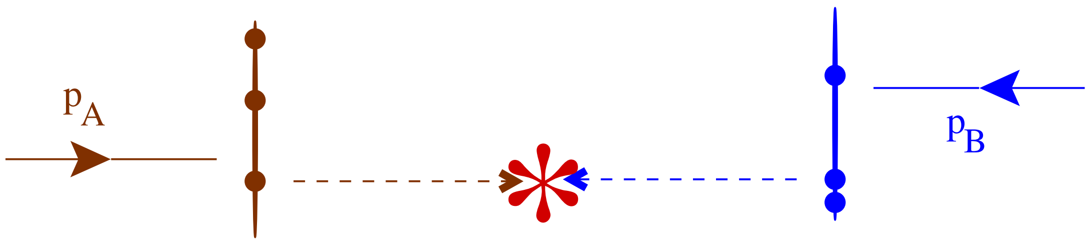
Issues in the QFT foundations of MCEG

John Collins (Penn State)

- Factorized inclusive cross sections v. MCEGs
- Whence MCEG?
- Issues/complications/. . .

Factorization v. MCEG: Factorized inclusive cross sections

E.g., Drell-Yan ($H_A + H_B \to \mu^+\mu^-$(or c.) $+ X$)



$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y} = \sum \int \mathrm{d}\xi_A\,\mathrm{d}\xi_B\, f_{i/A}(\xi_A,\mu) f_{j/B}(\xi_B,\mu) \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}Q^2\,\mathrm{d}y}$$

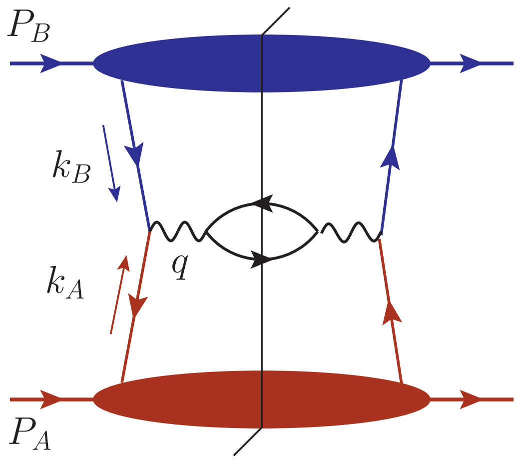
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} f_{i/H}(x,\mu) = \sum_j \int_{x-}^{1+} P_{i/j}\left(\frac{x}{\xi},\mu\right) f_{j/H}(\xi,\mu)$$

Predictive power from

- Universality of pdfs (& fragmentation fns.) — incl. non-perturbative parts;
- Expand $\mathrm{d}\hat{\sigma}$ in powers of $\alpha_s(Q)$;
- Expand DGLAP kernel in powers of $\alpha_s(\mu)$;

Same idea applies to many inclusive processes. Extends to TMD factorization, etc.

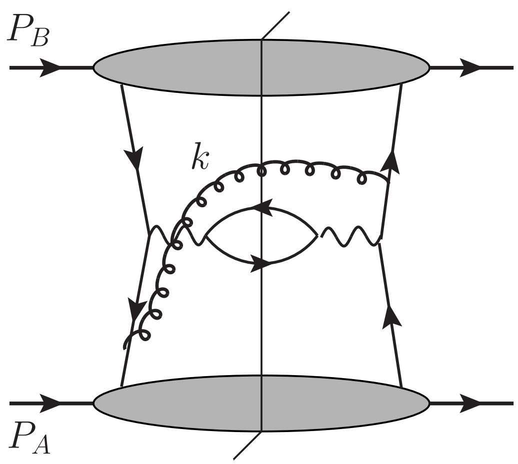
MCEGs answer: “What accompanies the DY pair (etc)?”



Drell-Yan as typical example

Basic parton-model set-up

Blobs: momenta collinear to parent hadron



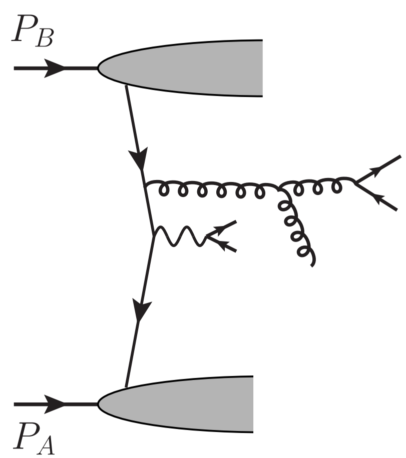
+ etc

- Extra lines $\Longrightarrow$ large effect
- Strongly ordered kinematics $\Longrightarrow$ $\int \frac{\mathrm{d}k_T^2}{k_T^2} \int \mathrm{d}y$ for one gluon

$\Longrightarrow$ $\ln^2 Q$ per α_s

MC algorithm: conditional distribution (k, gluon) given hard scattering, etc

MCEG: Now iterate



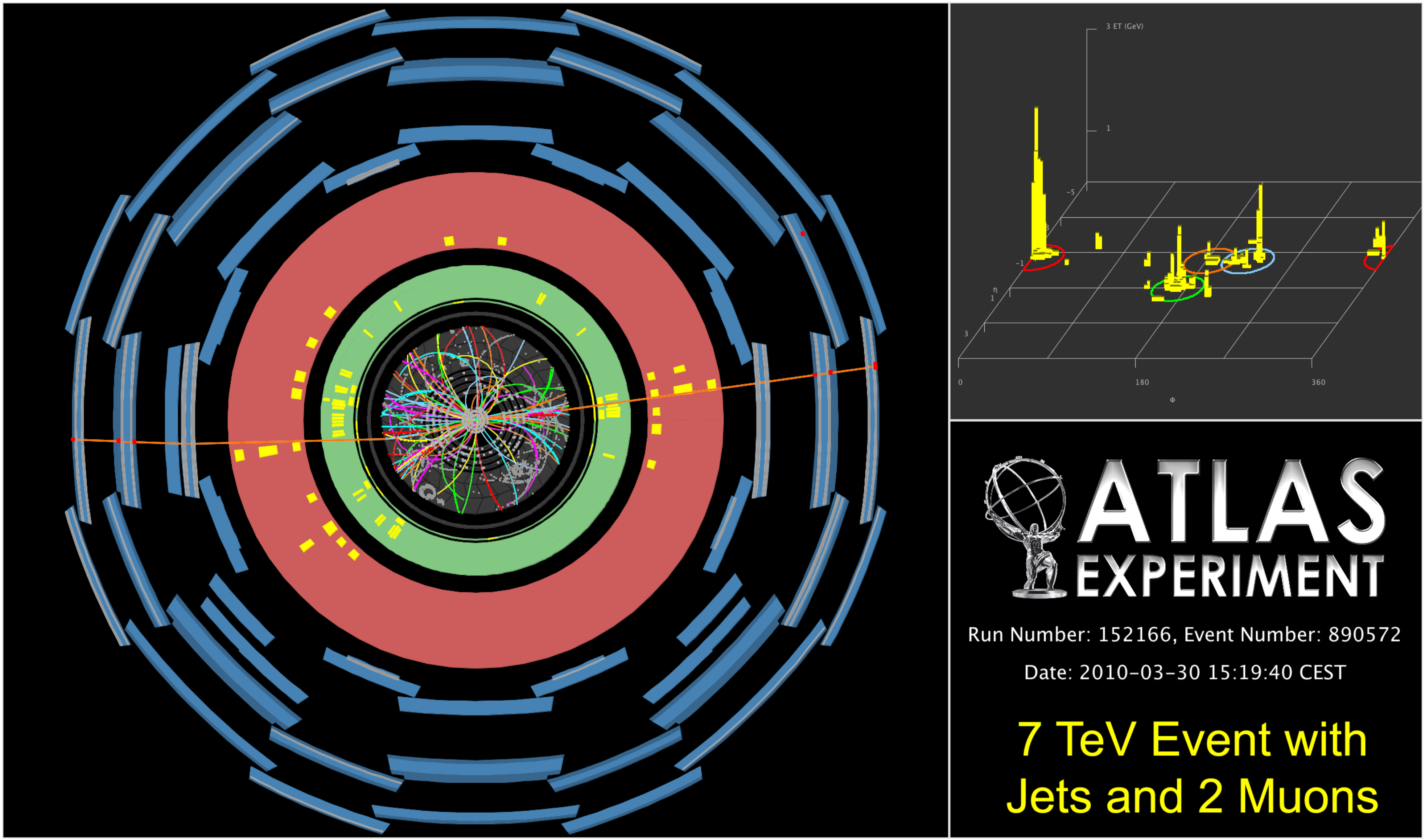
& etc

Large & variable dimension for distribution of final state particles

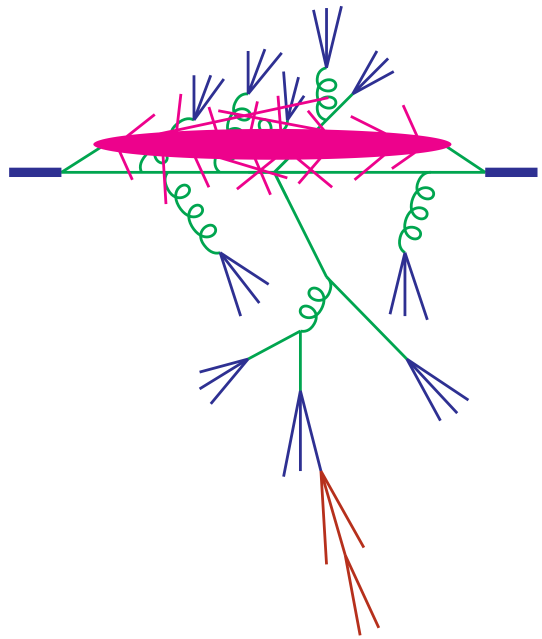
Hence MC implementation is appropriate

Add in non-perturbative model of hadronization/final-state interaction, etc

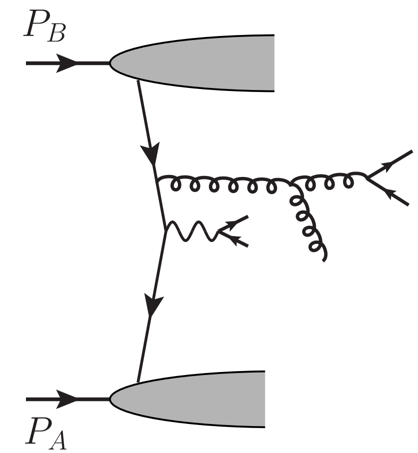
$$pp \to (Z \to \mu^+\mu^-) + \textbf{jets}$$



Importance of MCEG



Seymour & Marx, arXiv:1304.6677



- Produce N particles $\Longrightarrow$ need roughly $N!$ graphs. (N.B. + loops, + non-pert.)
- MCEG reduces $O(>N!)$ computation to approximation with $O(N)$ computation
- Why use MCEG:
  - Full final state,
  - Use with data analysis,
  - Estimate complicated observables

Some examples of mismatches and conceptual complications

Generally: Mismatches between verbal summary of physics and actual implementation and reality.

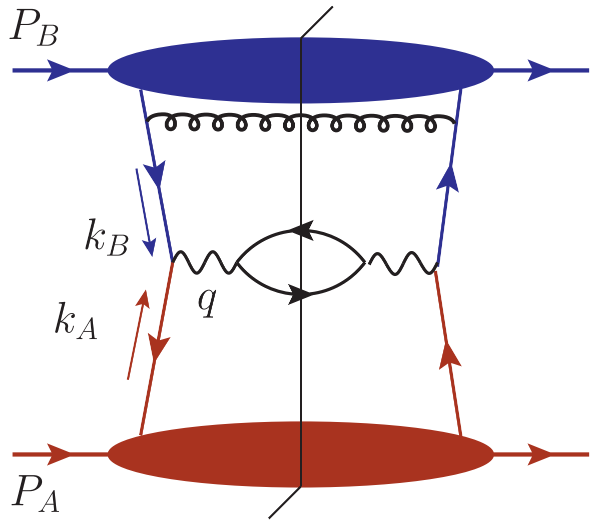
- Momentum is not conserved (in standard approximations)
- Feynman graphs don't match exactly entities in MCEG
- Momentum-space calculations v. coordinate-space understanding
- Multiple regions for single graphs
- Feynman-graph-like physics beyond literal perturbation theory
- Spin v. classical-like simulation in MCEG
- Entanglement of states in different parts of process
- Intuition of local evolution v. Schrödinger and Heisenberg picture formalisms.

Deep conceptual issues are encountered just under the surface

Momentum is not conserved in standard approximation for factorization, I

LO for hard scattering for DY $\mathrm{d}\sigma \,/\, \mathrm{d}Q^2\, \mathrm{d}y$ arises from

$$K \int \mathrm{d}^2\mathbf{q}_T\, \mathrm{d}^4k_A\, \mathrm{d}^4k_B\, \delta^{(4)}(q - k_A - k_B)$$



(Extra gluon is example of possible structure inside collinear factors.)

- Use light-front coordinates $(+, -, T)$:

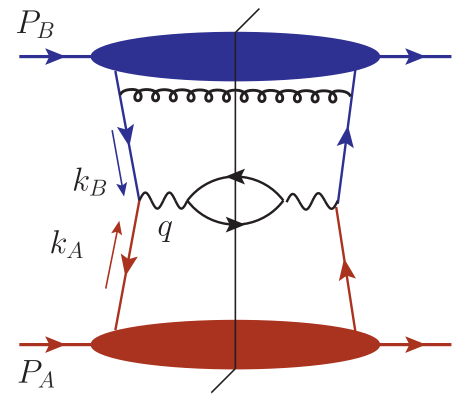
$$p_A \simeq \left(\sqrt{s/2},\ m^2/2p_A^+,\ \mathbf{0}_T\right), \quad p_B \simeq \left(m^2/2p_B^-,\ \sqrt{s/2},\ \mathbf{0}_T\right),$$

$$q = \left(e^y\sqrt{(Q^2+q_T^2)/2},\ e^{-y}\sqrt{(Q^2+q_T^2)/2},\ \mathbf{q}_T\right)$$

- . . .

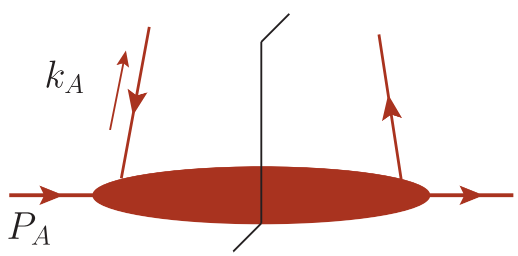
Momentum is not conserved in standard approximation II

$$K \int \mathrm{d}^2\mathbf{q}_T \, \mathrm{d}^4k_A \, \mathrm{d}^4k_B \, \delta^{(4)}(q - k_A - k_B)$$

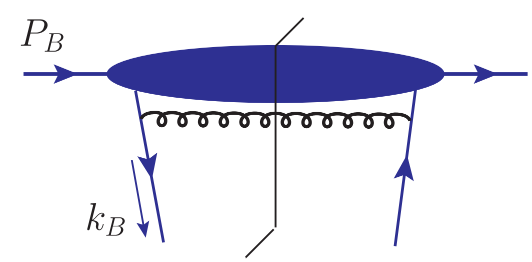


- In $\delta^{(4)}(q - k_A - k_B)$, replace $k_A \mapsto (k_A^+, 0, \mathbf{0}_T) \quad k_B \mapsto (0, k_B^-, \mathbf{0}_T)$.
  I.e., retain only large components of parton momenta.
- Good to leading power in k_T/Q (with similar change in Dirac numerators)
- Get factorization with standard parton densities:

$$\int \mathrm{d}k_A^- \, \mathrm{d}^2\mathbf{k}_{A,T}$$



$$\times \int \mathrm{d}k_B^+ \, \mathrm{d}^2\mathbf{k}_{B,T}$$



- Change of momentum values ⇔ Non-conservation of momentum

Why doesn't momentum-non-conservation matter for *inclusive* cross sections?

- Inclusive cross-section doesn't involve whole final state

- Approximation is valid to leading power in $k_{A,T}/Q$, $k_{B,T}/Q$, m/Q.
- When $k_{A,T}, k_{B,T}$ are order $\Lambda_{\rm QCD}$, that's fine (if rest of final-state is *not* used).
- *But* when $k_{A,T}, k_{B,T}$ get closer to Q, there are larger errors.
- These are corrected with NLO, NNLO, etc, hard scattering and evolution, with subtractions to prevent double counting.

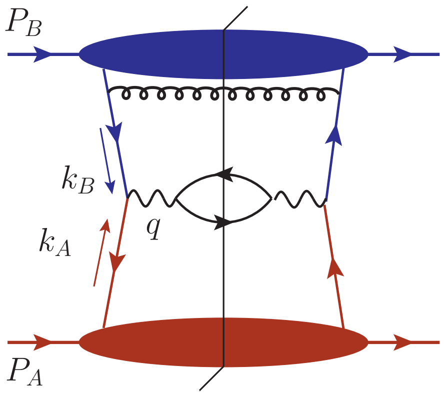
- But it's *not* obvious exactly what is the final state!!

- For MCEG, momentum non-conservation does matter, especially when $k_{A,T}, k_{B,T}$ get near Q . . .

Momentum-non-conservation and MCEGs

- For MCEG, momentum non-conservation does matter, especially when $k_{A,T}, k_{B,T}$ get near Q. . . .
- E.g., std. approx. for $\delta^{(4)}(q - k_A - k_B)$ in

$$K \int \mathrm{d}^2\mathbf{q}_T \, \mathrm{d}^4k_A \, \mathrm{d}^4k_B \, \delta^{(4)}(q - k_A - k_B)$$

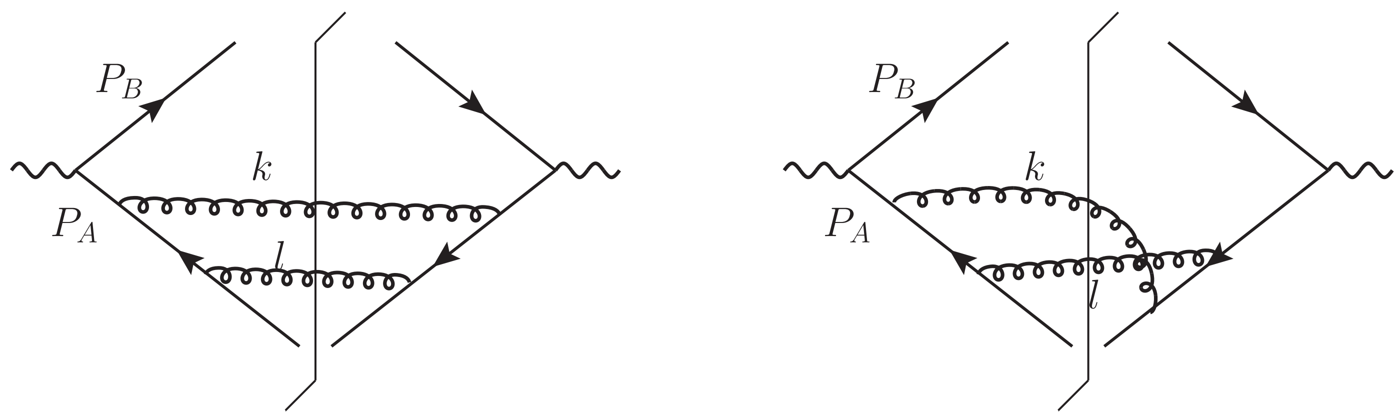


replaces k_A and k_B by large components: $k_A \mapsto (k_A^+, 0, \mathbf{0}_T)$ $\quad k_B \mapsto (0, k_B^-, \mathbf{0}_T)$.

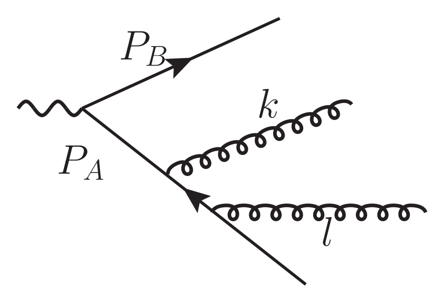
- MCEG *must* correct the kinematics by some prescription.
  See documentation *and* code!
- This messes up simple minded construction of NLO corrections, because it messes up the double-counting subtractions.
- [Hence: Entanglement of the QM states for the beam remnants.]

Subgraphs v. basic entities in MCEG algorithms

E.g., $e^+e^- \to$ hadrons



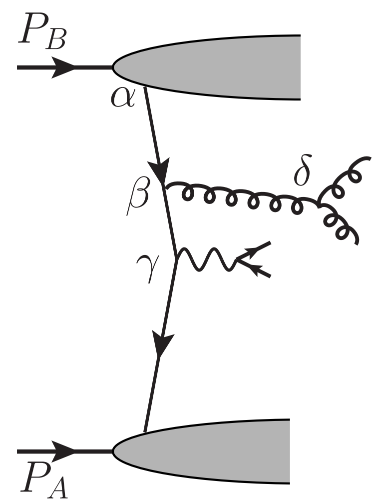
- In strongly-ordered region $\theta_{l,p_A} \ll \theta_{k,p_A}$, MCEG algorithm uses tree structure:

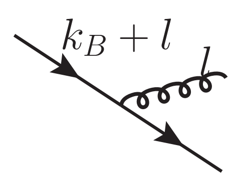


and so makes k given $\{e^+e^- \to q\bar{q}\}$, and then l given $\{e^+e^- \to q\bar{q}$ & gluon $k\}$

- But *both* graphs (etc) are needed (and others).
- So correct MCEG object $\neq$ obvious (approximated) subgraph.

Feynman graph in momentum space v. coordinate-space ideas

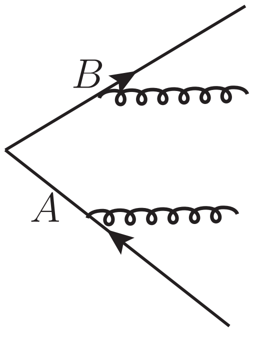


- Graphs are calculated in momentum space. (Plane waves are uniform in space.)
- But coordinate-space description is: Quark is emitted at α, *then* it emits gluon at α, *then* it gets to hard scattering at γ. Gluon splits later at δ.
- But α, β, γ space-like; time-ordering is frame dependent.
- Calculation of splitting 

has $k_B + l$ and l on-shell, as appropriate *approximation*; they are off-shell in reality.
- But they aren't on-shell, and they always hadronize, and color gets neutralized.

Spin

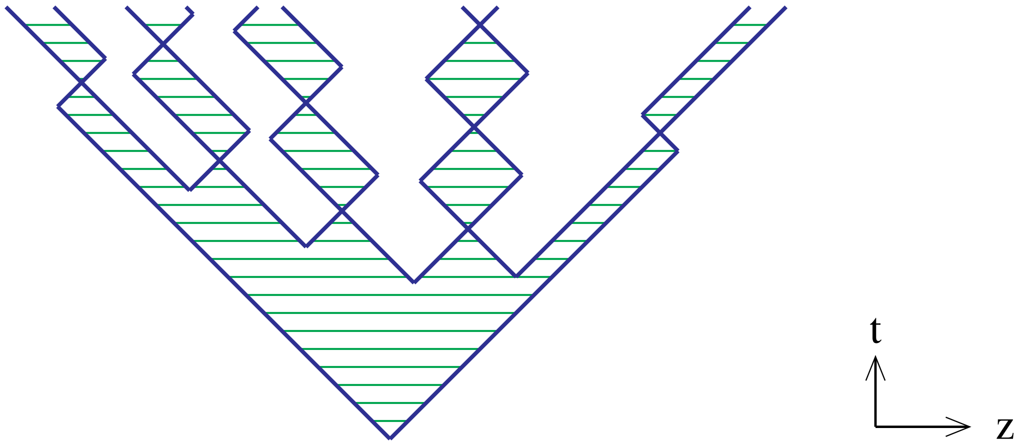
- Suppose intermediate state has N partons. The density matrix is $2^N \times 2^N$.
- But there's an $O(N)$ algorithm (JCC, NPB 304, 794 (1988)), used in HERWIG.
- E.g., in



  - Single quarks unpolarized
  - Split A, unpolarized
  - Azimuth $\Longrightarrow$ measurement $\Longrightarrow$ density-like matrix for A
  - A and B's spin state entangled
  - Deduce conditional density matrix of B
  - Generate azimuthal dependence of its decay.
- Result: Correct correlations to appropriate accuracy, $O(N)$ computation, but anticausal algorithm.

Feynman graphs v. hadronization & color neutralization

Lund string (t–z) in e^+e^- annihilation:



Lund: Production of $q\bar{q}$ pairs uniform in space-time volume in flux tube.

N.B. Time-dilation of hadronization time near q and $\bar{q}$.

Intuition (approximate): Local evolution in space-time.

To be contrasted with Heisenberg and Schrödinger pictures.

Conclusions

- MCEG: $O(N)$ approximation to $O(>N!)$ computational problem.
- Biggest practical issues:
  - What's the nature of the approximation?
  - How to improve it *systematically*?
- Issues about spin etc. Anti-intuitive $O(N)$ algorithm.
- Mismatches of words and deeds
- Link to fundamental issues in QFT and QM