INT Workshop INT-14-55W

"Studies of 3D Structure of Nucleon"

Limitations in the phenomenological extraction of TMDs

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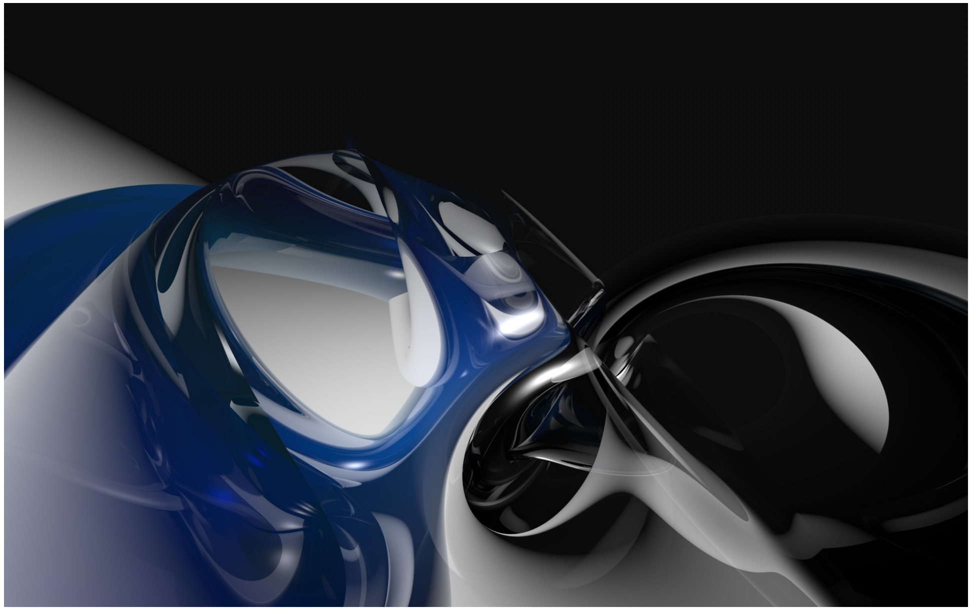
First of all ...

My deepest gratitude to
Alessandro Bacchetta
Alexei Prokudin
Zhongbo Kang
and Ted Rogers
for beautifully
explaining all
the bright sides
of ***TMDs*** *...*



... graciously
leaving me the
honour to
illustrate their
Darkest Side

Phenomenology

THEORY
- Perturbative QCD
- Factorization theorem
- ...

PHENOMENOLOGY
Mission: devise simple flexible and efficient models to link THEORY with EXPERIMENTS

EXPERIMENTS
- Drell-Yan scattering
- Di-hadron production from e+e- scattering
- DIS and SIDIS processes
- Inclusive single particle production from hadronic scattering

Limitations

Phenomenology suffers of …

- ***Experimental limitations***
  (statistics, acceptance, angular coverage, …)

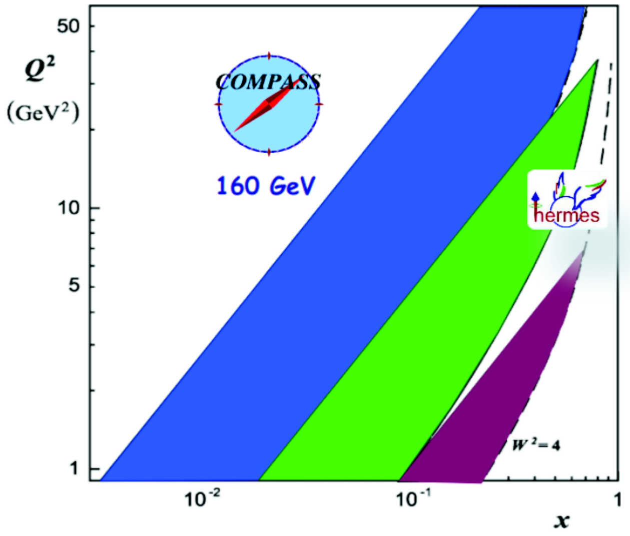
- ***Theoretical limitations***
  (theory exists …but it cannot be straightforwardly applied)

- ***Phenomenological limitations***
  (model dependence, over-simplifications, limited kinematics coverage, "matching", inconsistencies among different experimental measurements, …)

Limitations

To do phenomenology we inevitably have to deal with ...

- Different data sets from different experiments (beams, targets, final hadron production, ...)
- Different kinematics ranges (x, z, P_T, Q^2, ...)
- Different experimental analyses (choice for binning, unfolding, kinematical corrections, background subtraction, nuclear target corrections ...)
- Normalization



Limitations

Let's start with some examples ...

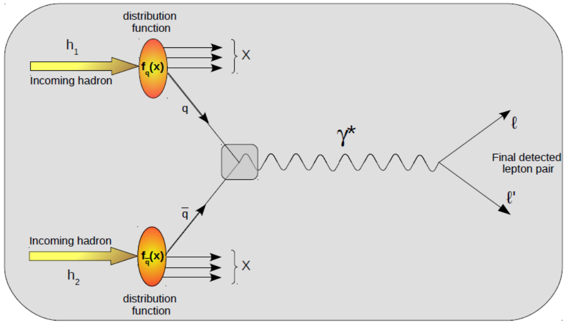
Extraction of the unpolarized distribution and fragmentation TMDs

from ...

- SIDIS
- Drell-Yan scattering
- e^+e^- scattering

Experimental Data for TMD studies

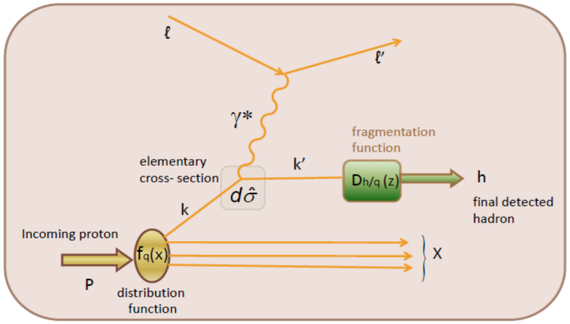
Unpolarized and Polarized Drell-Yan scattering



$$\sigma_{Drell-Yan} = f_q(x,k_\perp) \otimes f_{\bar{q}}(x,k_\perp) \otimes \hat{\sigma}^{q\bar{q}\to\ell\ell'}$$

Allows extraction of distribution functions

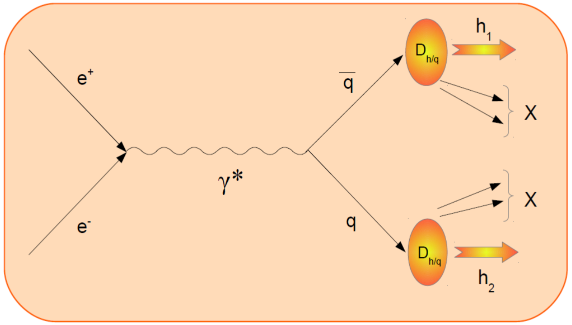
Unpolarized and Polarized SIDIS scattering



$$\sigma_{SIDIS} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

Allows extraction of distribution and fragmentation functions

$e^+ e^- \to h_1 h_2 X$



$$\sigma_{h1\,h2} \propto D(z_1) \otimes D(z_2) \otimes \hat{\sigma}$$

Allows extraction of fragmentation functions

TMD parametrizations

- TMDs are parametrized in a form in which the x and $k_\perp$ dependences are factorized, and only the collinear part evolves in Q

Unpolarized TMD PDF

$$\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x; Q) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

Collinear PDF (DGLAP evolution)
Normalized Gaussian (no evolution)

Unpolarized TMD FF

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle},$$

Collinear FF (DGLAP evolution)

Normalized Gaussian (no evolution)

Extracting the unpolarized TMD gaussian widths from SIDIS data

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, ArXiv:1312.6261

- Data: Hermes (p and d targets, p^+, p^-, K^+, K^- production)

  2660 data points in (x, z, P_T, Q^2 bins)

  Compass (d target, h^+, h^- production)

  18627 data points in (x, z, P_T, Q^2 bins)

A. Airapetian et al., Phys. Rev. D87 (2013) 074029

C. Adolph et al., Eur. Phys. J. C73, 2531 (2013)

- Parameterizations:

$$\widehat{f}_{q/p}(x,k_\perp;Q) = f_{q/p}(x;Q)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

$$D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle},$$

CTEQ6L (DGLAP evolution)

1 free parameter
(no evolution)

DSS (DGLAP evolution)

1 free parameter
(no evolution)

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities

COMPASS HERMES

$$\frac{d^2 n^h(x_B, Q^2, z_h, P_T)}{dz_h\, dP_T^2} = \frac{1}{2P_T} M_n^h(x_B, Q^2, z_h, P_T) =$$

$$= \frac{\pi \sum_q e_q^2 f_{q/p}(x_B)\, D_{h/q}(z_h)}{\sum_q e_q^2 f_{q/p}(x_B)} \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi\langle P_T^2\rangle}$$

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities

COMPASS **HERMES**

$$\frac{d^2 n^h(x_B, Q^2, z_h, P_T)}{dz_h\, dP_T^2} = \frac{1}{2 \ldots} \ldots =$$

$$= \frac{\pi \sum_q e_q^2\, f_{q/p}(x_B)\, D_{h/q}(z_h)}{\sum_q e_q^2\, f_{q/p}(x_B)} \frac{e^{-P_T^2/\langle P_T^2\rangle}}{\pi \langle P_T^2\rangle}$$

Phenomenological limitation

Model dependence

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities

In the simplest form of this model:

Flavor-independent average transverse momenta

No x-dependence

No z-dependence

Two parameters in total

→

Gaussian model:

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle.$$

$$\sigma \propto \frac{1}{\pi \langle P_T^2 \rangle} e^{-P_T^2/\langle P_T^2 \rangle}$$

Normalization

Gaussian width

Limitations

Experimental limitations

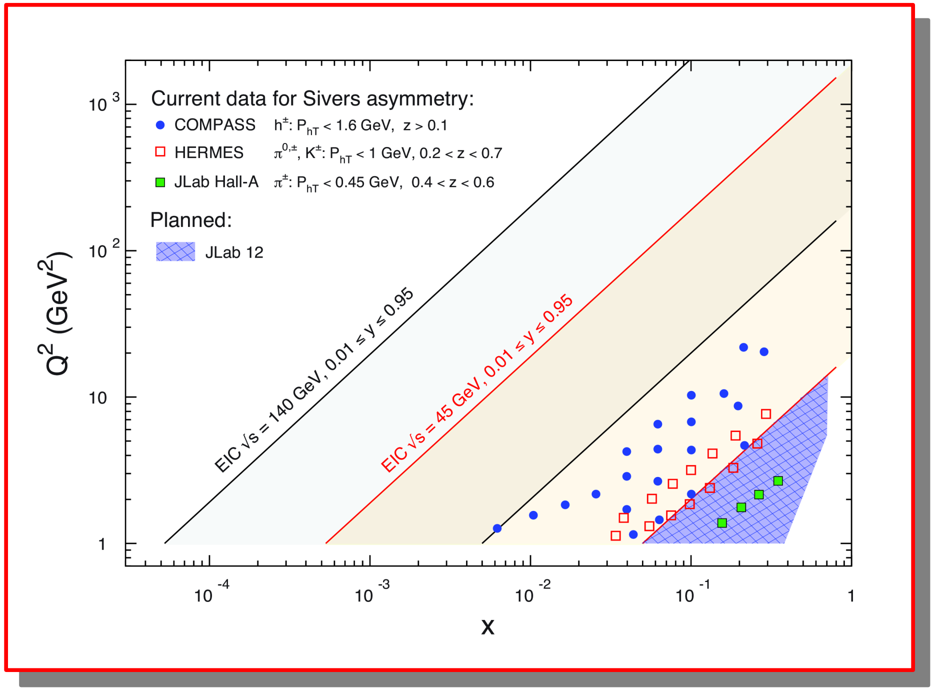
- Kinematics: HERMES and COMPASS cover similar ranges in z and Q^2, but different ranges in x and P_T
- Q^2 range is not very wide, and Q^2 values are small
- Data stretch to very large values of z, where they are affected by exclusive production and large-z resummation effects
- ...

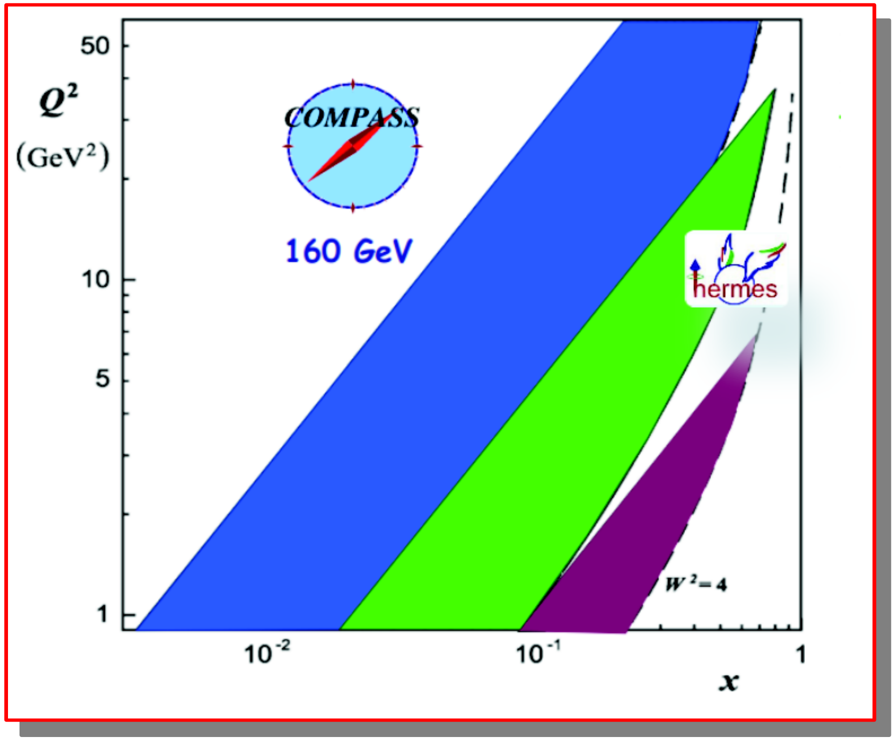
Phenomenological limitations

- Model dependence (gaussian assumption, factorization between x and $k_\perp$, ...)
- Number of free parameters (flavour dependence of the gaussian widths, x and z dependence of the gaussian widths...)
- Neural networks
- ...
- Q^2 is small: are we looking at a DIS processes ? Higher twist contributions ?
- What about scale evolution ?
- ...

Limitations

Consistency among different data sets covering different kinematical regions

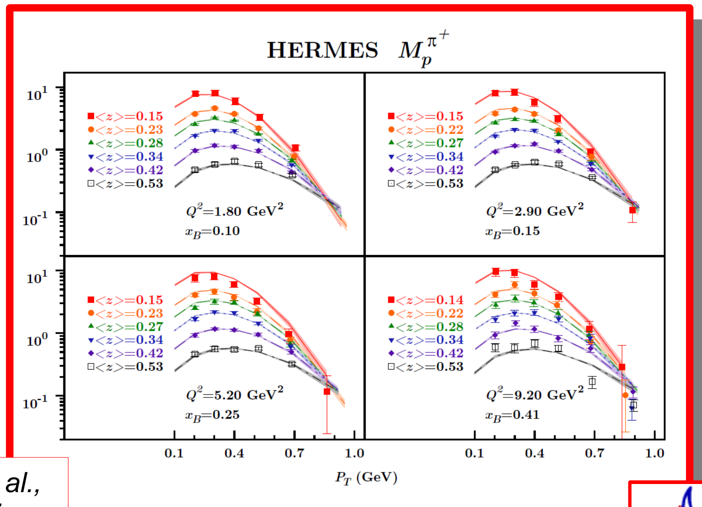


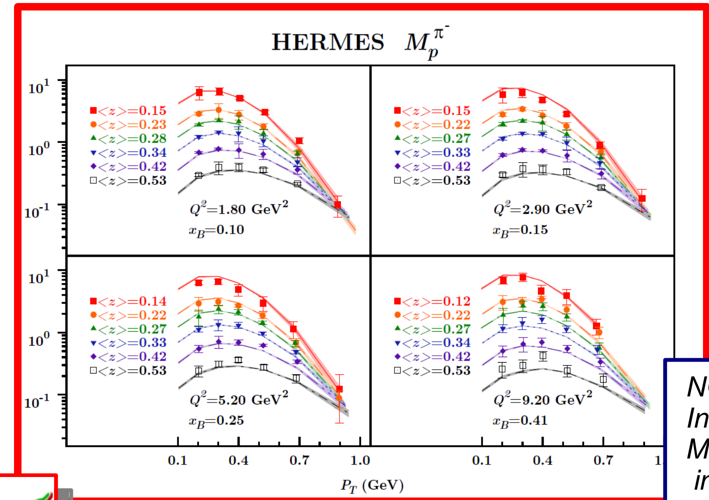


Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, ArXiv:1312.6261

$\chi^2 = 1.69$





A. Airapetian et al.,
Phys. Rev. D87
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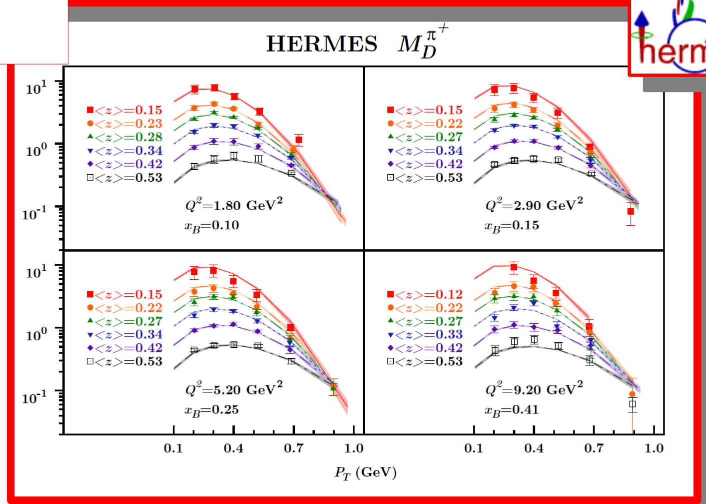
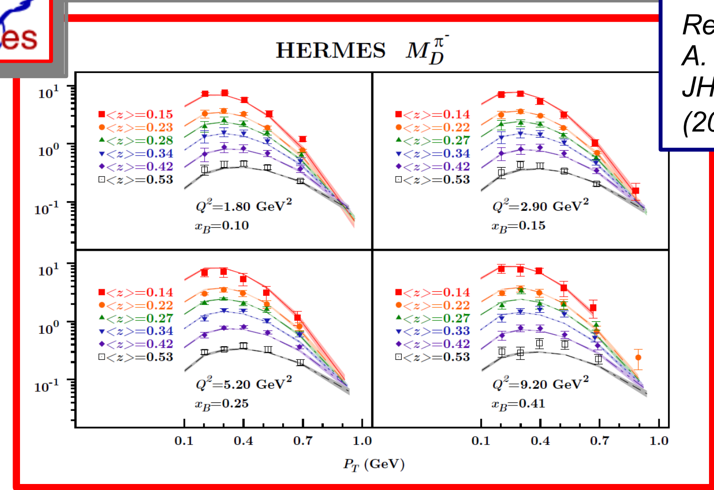




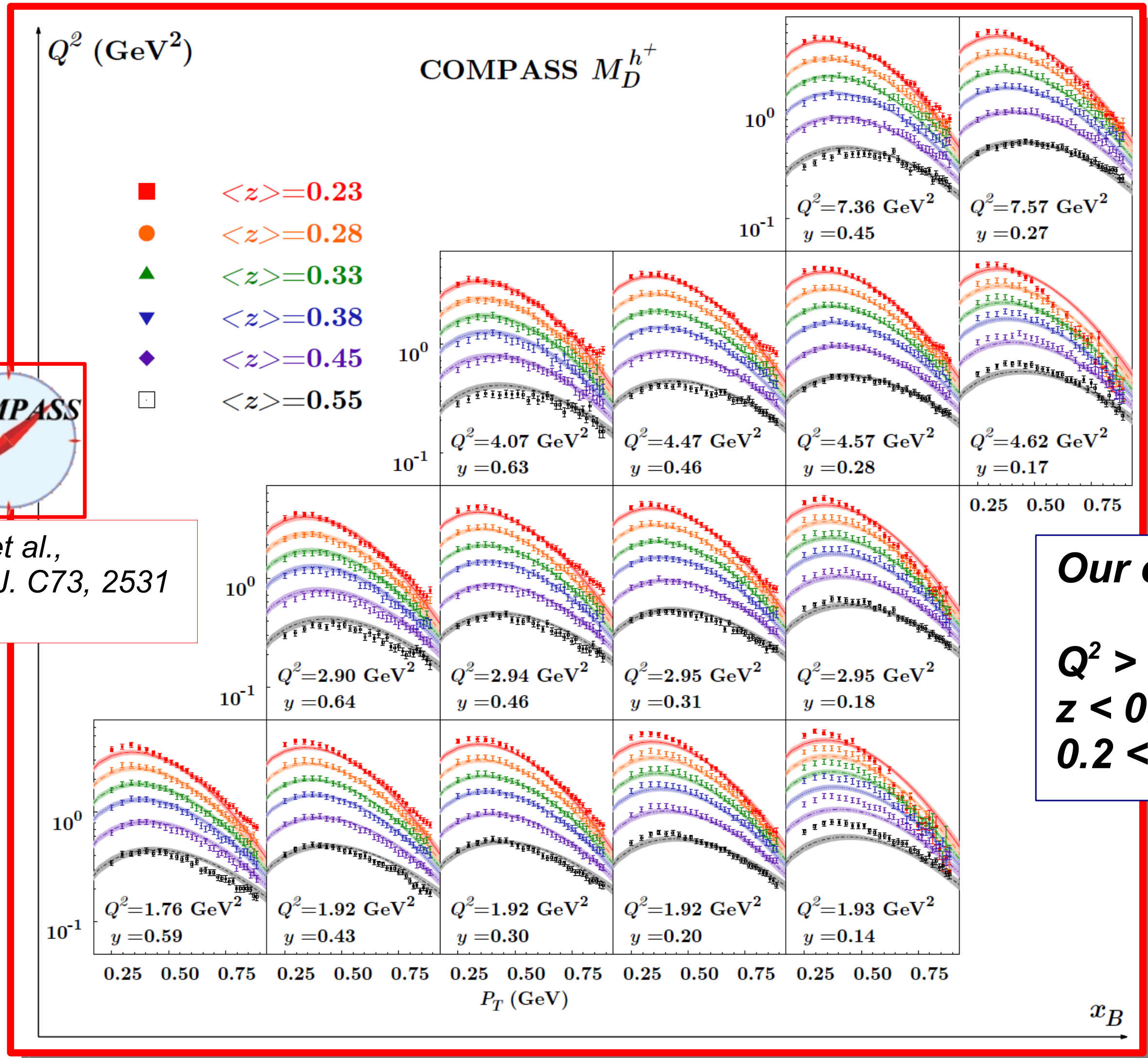
NO flavour dep. In distr. fns., MILD flavour dep. in fragm. fns,

Results agree with A. SIgnori et al., JHEP 1311 (2013) 194

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, ArXiv:1312.6261

$\chi^2 = 3.42$



C. Adolph et al., Eur. Phys. J. C73, 2531 (2013)

NO flavour dep. In distr. fns., NO flavour dep. in fragm. fns,

Our cuts:

$Q^2 > 1.6$ GeV^2
$z < 0.6$
$0.2 < P_T < 0.9$

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, ArXiv:1312.6261

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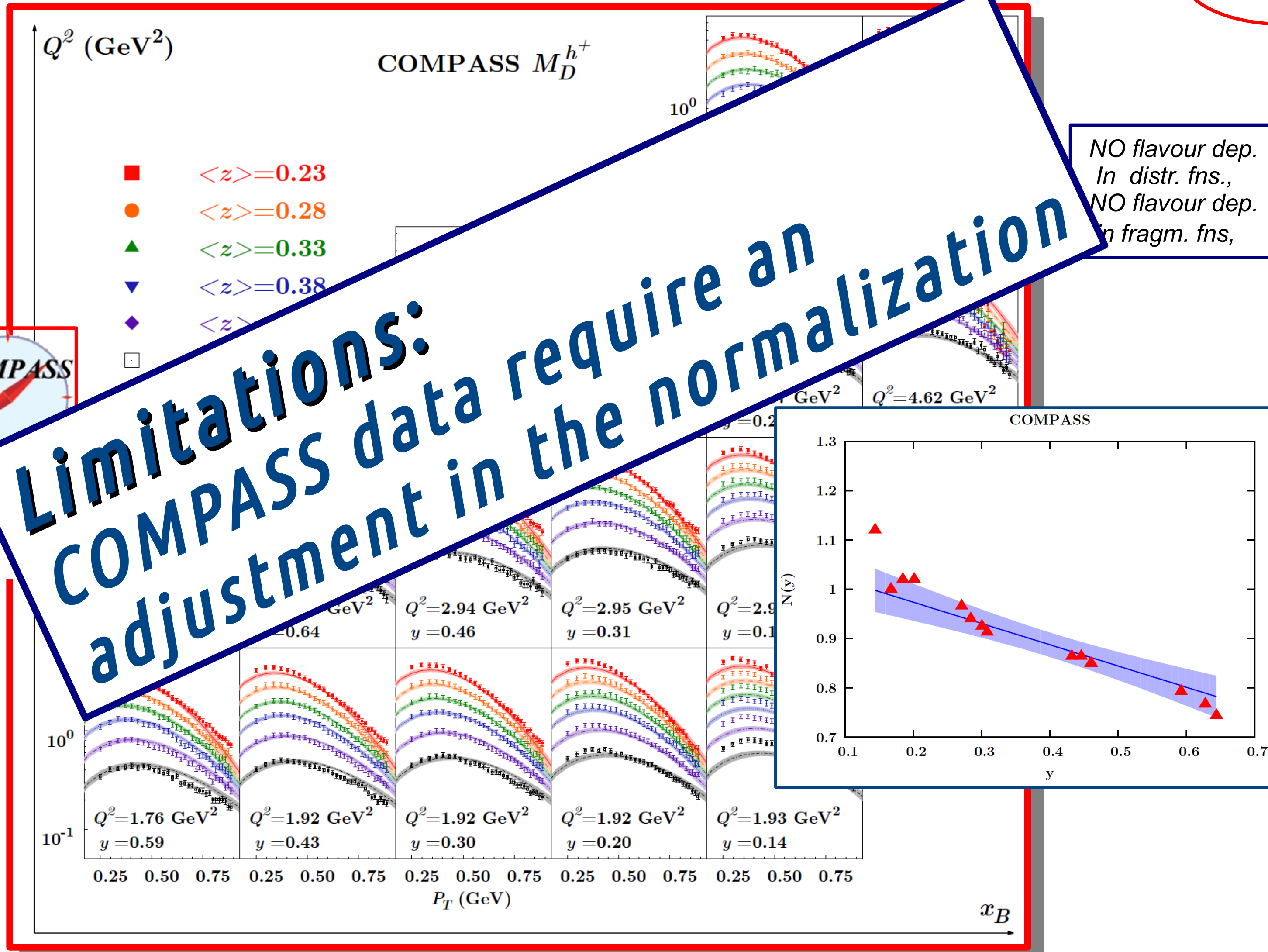

NO flavour dep. In distr. fns., NO flavour dep. in fragm. fns,

C. Adolph
Eur. Phys.
(2013)

Limitations:
COMPASS data require an adjustment in the normalization

Parameter determination

HERMES					
Cuts	χ^2_{dof}	n. points	$[\chi^2_{points}]^{\pi^+}$	$[\chi^2_{points}]^{\pi^-}$	Parameters
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.6$	1.69	497	1.93	1.45	$\langle k_\perp^2 \rangle = 0.57 \pm 0.08$ GeV^2 $\langle p_\perp^2 \rangle = 0.12 \pm 0.01$ GeV^2
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.7$	2.62	576	2.56	2.68	$\langle k_\perp^2 \rangle = 0.46 \pm 0.10$ GeV^2 $\langle p_\perp^2 \rangle = 0.13 \pm 0.01$ GeV^2

COMPASS					
Cuts	χ^2_{dof}	n. points	$[\chi^2_{dof}]^{h^+}$	$[\chi^2_{dof}]^{h^-}$	Parameters
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.6$	8.54	5385	8.94	8.15	$\langle k_\perp^2 \rangle = 0.61 \pm 0.20$ GeV^2 $\langle p_\perp^2 \rangle = 0.19 \pm 0.02$ GeV^2
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.6$ $N_y = A + B\,y$	3.42	5385	3.25	3.60	$\langle k_\perp^2 \rangle = 0.60 \pm 0.14$ GeV^2 $\langle p_\perp^2 \rangle = 0.20 \pm 0.02$ GeV^2 $A = 1.06 \pm 0.06$ $B = -0.43 \pm 0.14$

Parameter determination

HERMES					
Cuts	$\chi^2_{\rm dof}$	n. points	$[\chi^2_{\rm points}]^{\pi^+}$	$[\chi^2_{\rm points}]^{\pi^-}$	Parameters
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$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.7$	2.62	576	2.56	[illegible]	[illegible]$.46 \pm 0.10$ GeV^2 [illegible]13 ± 0.01 GeV^2

Limitations: NO global fit

COMPASS					
C[illegible]	$\chi^2_{\rm dof}$	n. points	$[\chi^2_{\rm dof}]^{h^+}$	$[\chi^2_{\rm dof}]^{h^-}$	Parameters
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.6$	8.54	5385	8.94	8.15	$\langle k_\perp^2 \rangle = 0.61 \pm 0.20$ GeV^2 $\langle p_\perp^2 \rangle = 0.19 \pm 0.02$ GeV^2
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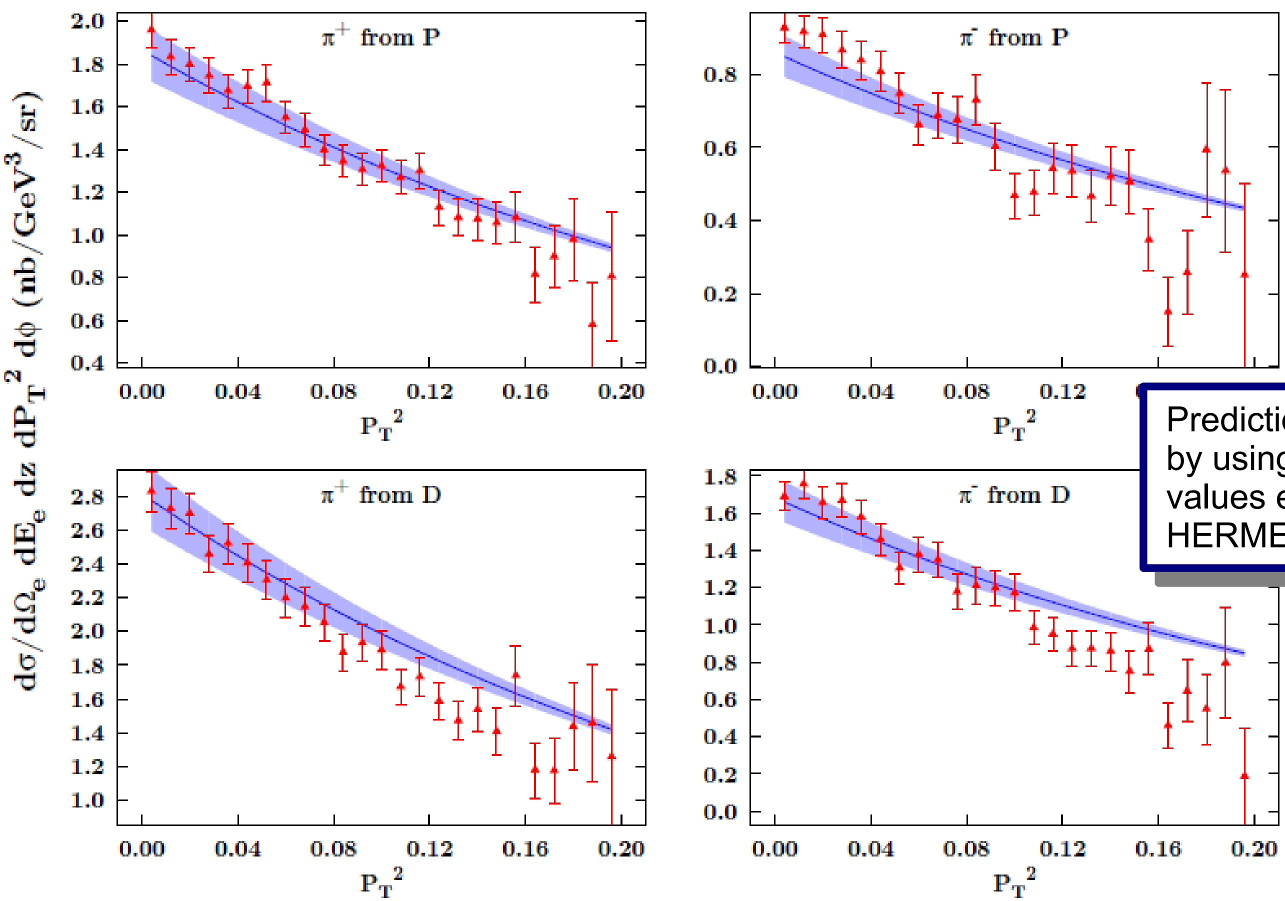
Comparison with Jlab data HALL C

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, ArXiv:1312.6261

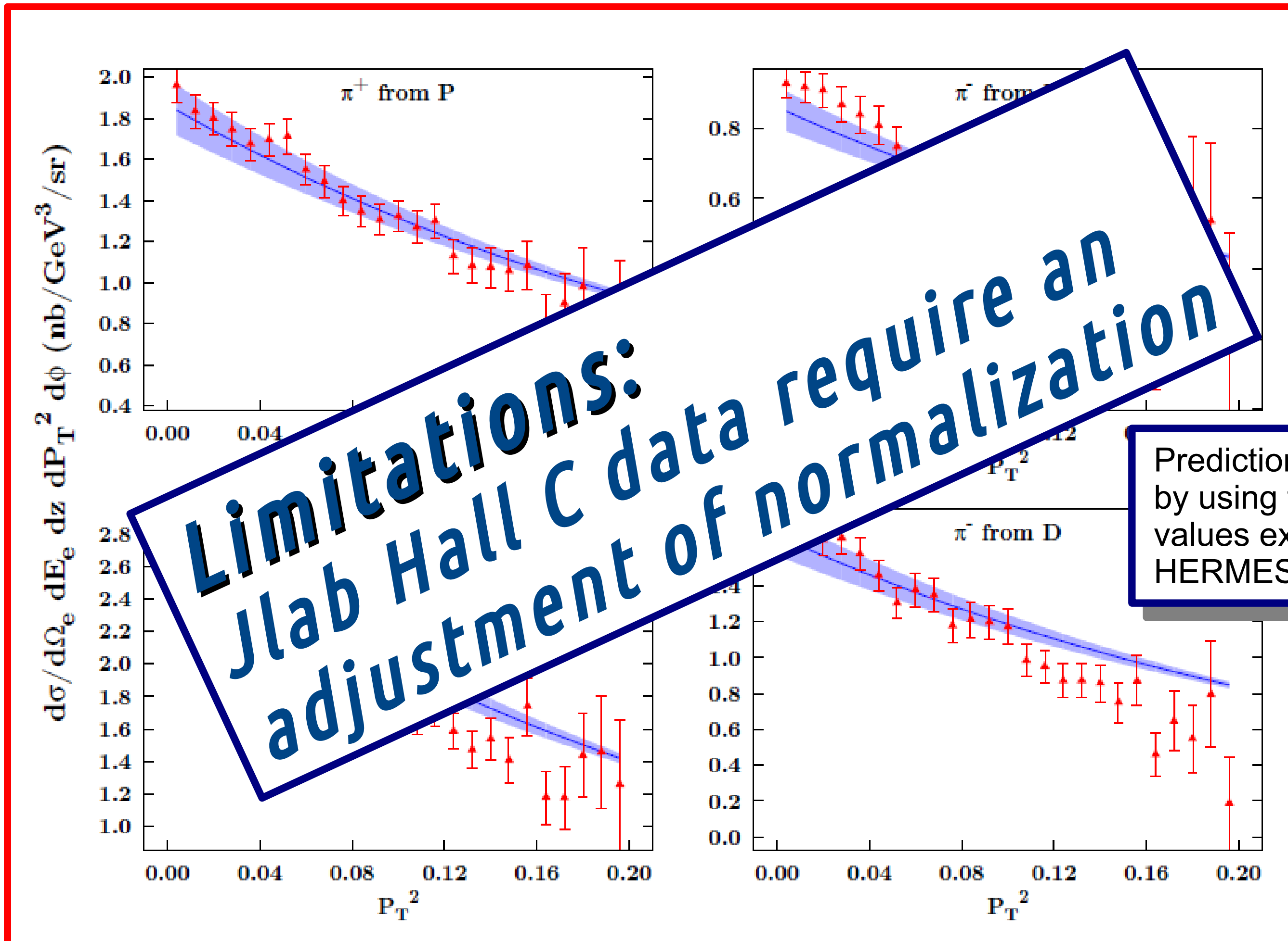


Predictions obtained by using the parameter values extracted from HERMES multiplicities

R. Asaturyan et al., Phys. Rev. C85, 015202 (2012)

Comparison with Jlab data HALL C

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, ArXiv:1312.6261

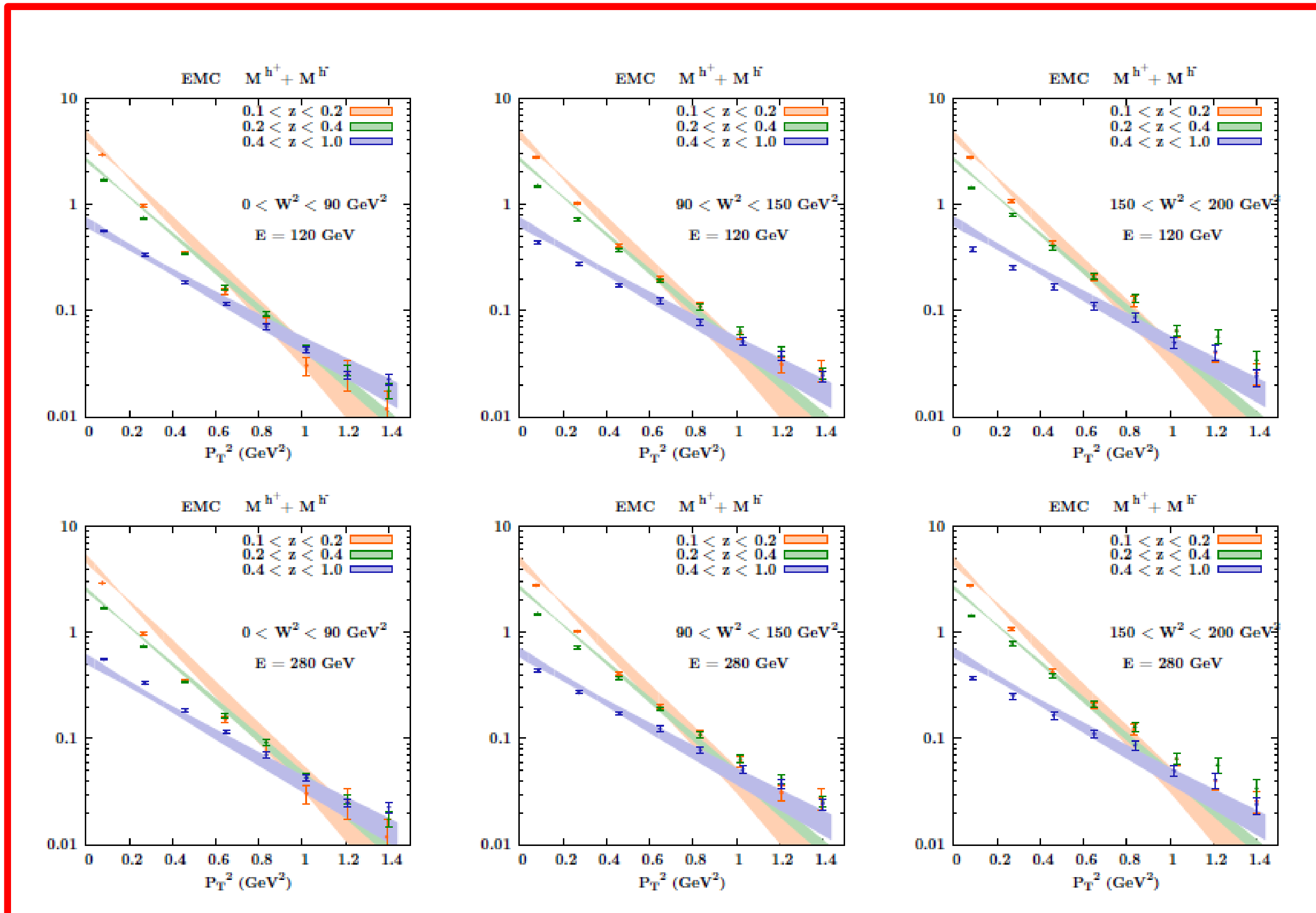


Predictions obtained by using the parameter values extracted from HERMES multiplicities

R. Asaturyan et al., Phys. Rev. C85, 015202 (2012)

Comparison with EMC data

M. Ashman et al., Z. Phys. C52, 361 (1991)



Predictions obtained by using the parameter values extracted from COMPASS multiplicities

5 energies, 3 targets, positively and negatively charged particles, averaged in 1 data set

What about scale evolution ?

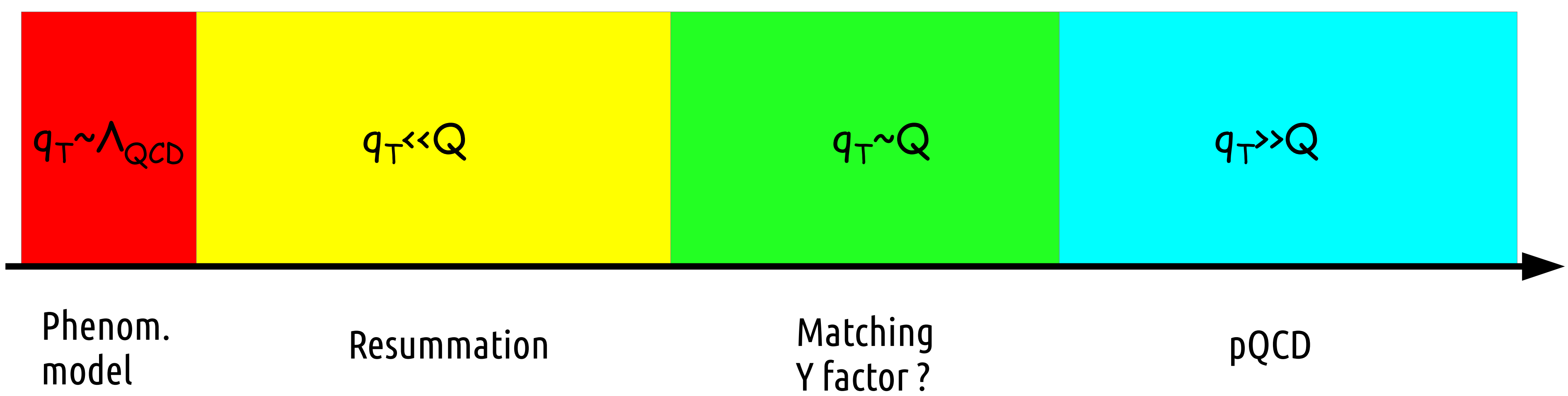
Limitations ...

- To study TMD evolution, from a phenomenological point of view, we need high precision experimental data, covering sufficiently **wide Q^2 ranges**
- Evolution studies have traditionally been performed on Drell-Yan cross sections, which cover wide Q^2 ranges at rather large q_T.
- HERMES and COMPASS SIDIS data cover very limited ranges of (rather small) Q^2
- SIDIS data alone are not sufficient to fix all the non perturbative behaviour of TMDs
- Need Drell-Yan (moderate Q^2, moderate energy) and e+e- data for **global analyses**
- But ... global analyses need to rely on consistent data sets !

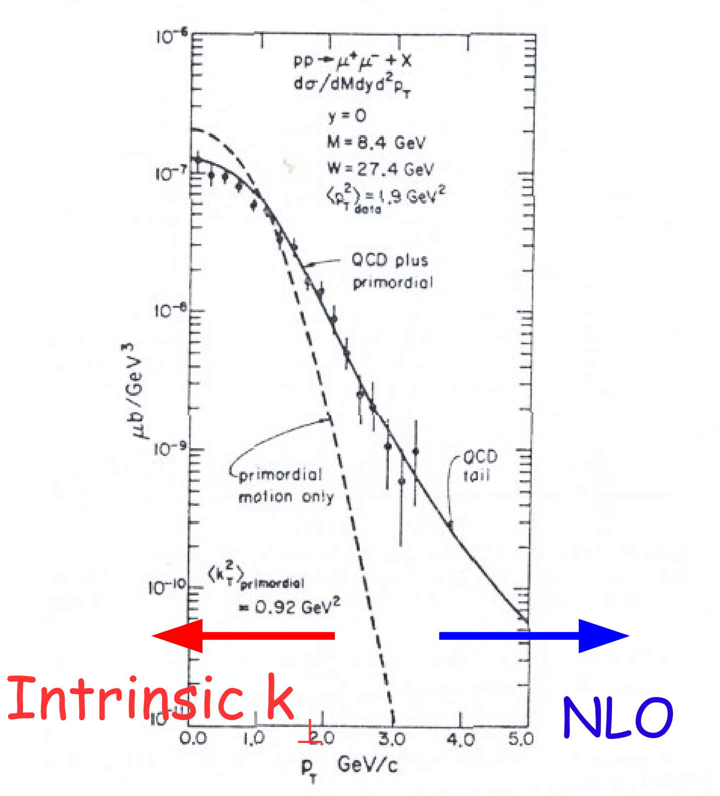
From non-perturbative to perturbative QCD

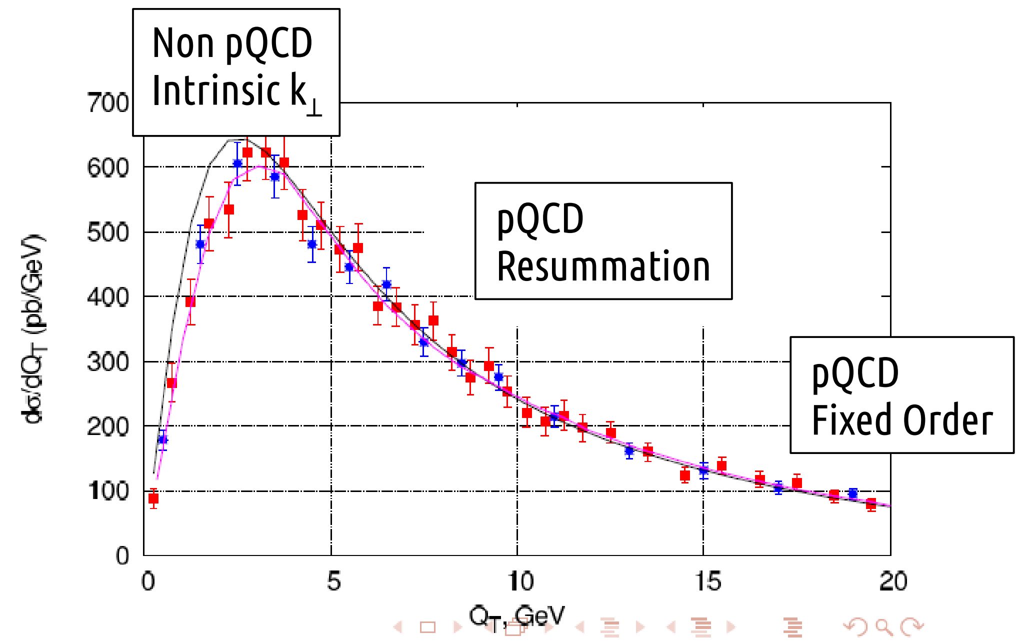
Non perturbative region
q_T "intrinsic"

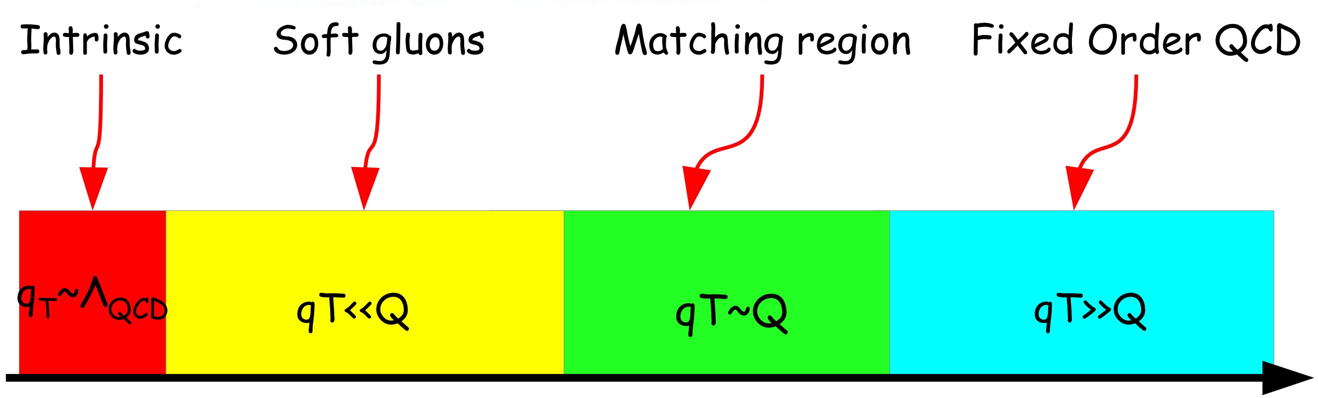
Perturbative region
q_T is generated by gluon radiation



From non-perturbative to perturbative QCD

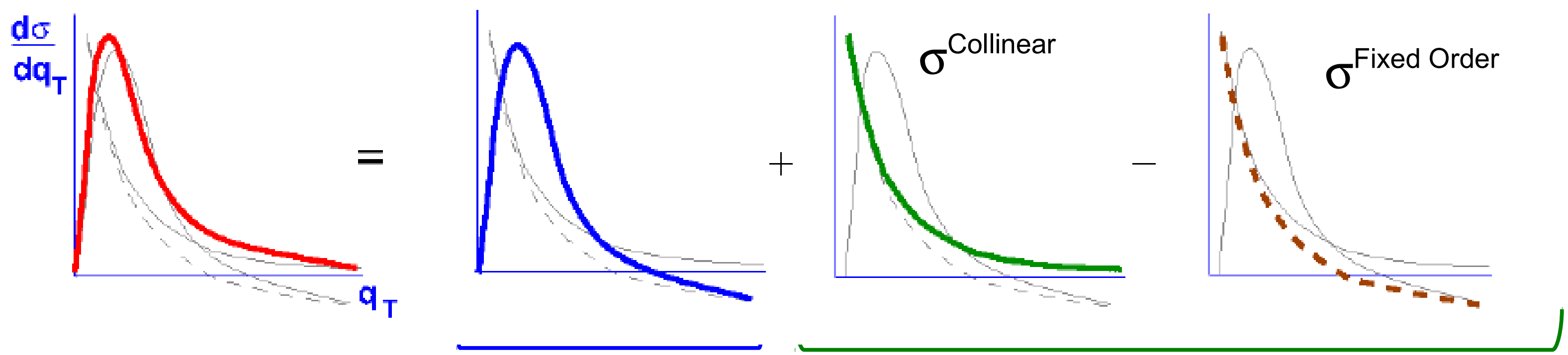






In Drell-Yan processes these regions are rather well defined

From non-perturbative to perturbative QCD



$W = \sigma^{\text{Resummed}}$ $\qquad$ $Y = \sigma^{\text{Collinear}} - \sigma^{\text{Fixed Order}}$

See talks by Nadolsky and Guzzi

- Y factor under controll in Drell-Yan
- Y factor problematic in SIDIS

See talk by Peng Sun

From non-perturbative to perturbative QCD

Limitations:
In SIDIS (HERMES and COMPASS – low Q^2)
The matching region sets in at low q_T

See ta
Nadols ... zzi

$W = \sigma^{\text{Resummed}}$ $Y = \sigma^{\text{Collinear}} - \sigma^{\text{Fixed Order}}$

- Y factor under controll in Drell-Yan
- Y factor problematic in SIDIS

See talk by Peng Sun

CSS formalism

Collins, Soper, Sterman, Nucl. Phys. B250, 199 (1985)

$$\frac{1}{\sigma_0}\frac{d\sigma}{dQ^2dydq_T^2} = \int\frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}\sum_j e_j^2 W_j(x_1,x_2,b_T,Q) + Y(x_1,x_2,q_T,Q)$$

Resummed part

Regular part

$$W_j(x_1,x_2,b_T,Q) = \exp\left[S_j(b_T,Q)\right]\sum_{i,k} C_{ji}\otimes f_i(x_1,C_1^2/b_T^2)\; C_{\bar{j}k}\otimes f_k(x_2,C_1^2/b_T^2)$$

Sudakov factor

$$S_j(b_T,Q) = \int_{C_1^2/b_T^2}^{Q^2}\frac{d\kappa^2}{\kappa^2}\left[A_j(\alpha_s(\kappa))\ln\left(\frac{Q^2}{\kappa^2}\right) + B_j(\alpha_s(\kappa))\right]$$

$$A_j(\alpha(\mu)) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n A_j^{(n)}$$

$$B_j(\alpha(\mu)) = \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n B_j^{(n)}$$

Leading Log (LL)	:	$A^{(1)}$;
Next to LL (NLL)	:	$A^{(2)}, B^{(1)}, C^{(1)}$;
Next to NLL (NNLL)	:	$A^{(3)}, B^{(2)}, C^{(2)}$;
Fixed order $\alpha_s(FXO)$	:	$A^{(1)}, B^{(1)}, C^{(1)}$;

TMD evolution

Collins, Foundations of perturbative QCD, Cambridge University Press (2011); Rogers and Aybat, Phys. Rev. D83, 114042

$$\zeta_F = Q^2 \qquad b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{max}^2}} \qquad \mu_b = C_1/b_*$$

$$\tilde{F}(x, b_T, Q, \zeta_F \equiv Q^2) = \sum_j \tilde{C}_{f/j}(x/y, b_*, \mu_b, \mu_b^2) \otimes f_j(y, \mu_b) \exp[S_{RAC}(b_*, Q^2)] F_{NP}(x, b_T, Q)$$

Non perturbative function
To be determined
phenomenologically

It can be show easily that at first order in the strong coupling constant:

$$S_{RAC}(b_T, Q^2) = C_F \int_{\mu_b}^{Q} \frac{d\kappa}{\kappa} \frac{\alpha_s(\kappa)}{\pi} \left[\frac{3}{2} - \ln\left(\frac{Q^2}{\kappa^2}\right) \right] \equiv \frac{1}{2} S_{CSS}(b_T, Q^2)$$

TMD evolution

At LO the evolution equation can be summarized by the following expression:

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{-g_K(b_T) \ln \frac{Q}{Q_0}\right\}$$

Output function at the scale **Q** in the impact parameter space

Input function at the scale $\mathbf{Q_0}$ in the impact parameter space

Evolution kernel

This approach maximizes the non perturbative content of the evolution

TMD evolution

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\ \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

$$g_K(b_T) = \frac{1}{2} g_2 b_T^2 \qquad \text{with} \qquad g_2 = 0.68$$

$$\widetilde{f}_{q/p}(x, b_T; Q_0) = f_{q/p}(x, Q_0) \exp\left\{ -\alpha^2 b_T^2 \right\}$$

$$f_{q/p}(x, k_\perp; Q_0) = f_{q/p}(x, Q_0)\, \frac{1}{\pi \langle k_\perp^2 \rangle}\, e^{-k_\perp^2/\langle k_\perp^2 \rangle}$$

$$\alpha^2 = \langle k_\perp^2 \rangle / 4$$

Scale Evolution of unpolarized SIDIS

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, ArXiv:1312.6261

HERMES and COMPASS multiplicities cover the same range in Q^2 ...

$$\langle k_\perp^2 \rangle = g_1 + g_2 \ln(Q^2/Q_0^2) + g_3 \ln(10\, e\, x)$$
$$\langle p_\perp^2 \rangle = g_1' + z^2 g_2' \ln(Q^2/Q_0^2)$$

$$\langle P_T^2 \rangle = g_1' + z^2 [g_1 + g_2 \ln(Q^2/Q_0^2) + g_3 \ln(10\, e\, x)]$$

- HERMES multiplicities show no sensitivity to these parameters
- COMPASS fitting is much more involved.

After correcting for normalization, we find that the total χ^2 goes down to 2.69.

Scale Evolution of unpolarized SIDIS

M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, ArXiv:1312.6261

HERMES and COMPASS multiplicities cover the same range in

$$\langle k_\perp^2\rangle = g_1 + g_2 \ln(Q^2/Q_0^2) + g_3 \ln(1\ldots$$
$$\langle p_\perp^2\rangle = g_1' + z^2 g_2' \ln(Q^2/Q^2\ldots$$

$$\langle P_T^2\rangle = g' \ldots g_3 \ln(10\, e\, x)]$$

Limitations:
Q^2 range is not wide enough
Little sensitivity to scale evolution
COMPASS data need to be better understood

- HERMES ... ensitivity to these parameters
- COMPASS fi... ...ch more involved.

After correcting for normalization, we find that the total χ^2 goes down to 2.69.

TMD evolution of the Sivers function

TMD evolution of the Sivers function

$$\widetilde{F}(x,\boldsymbol{b}_T;Q) = \widetilde{F}(x,\boldsymbol{b}_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\;\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}$$

$$g_K(b_T) = \frac{1}{2}\,g_2\,b_T^2 \qquad \text{with} \qquad g_2 = 0.68$$

$$\widetilde{f}_{q/p}(x,b_T;Q_0) = f_{q/p}(x,Q_0)\exp\left\{-\alpha^2 b_T^2\right\}$$

$$f_{q/p}(x,k_\perp;Q_0) = f_{q/p}(x,Q_0)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$\alpha^2 = \langle k_\perp^2\rangle/4$$

Sivers function from HERMES and COMPASS SIDIS data

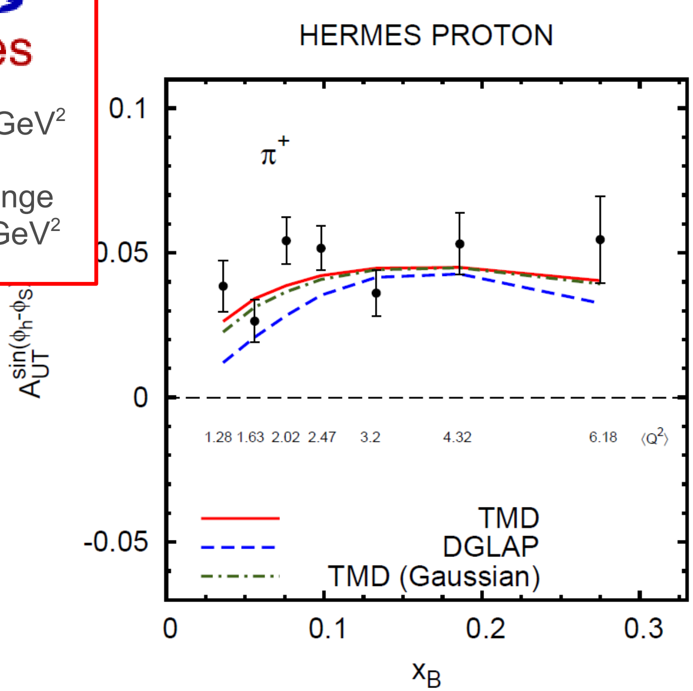
- **2 different fits:**
- TMD-fit (computing TMD evolution equations numerically)
- DGLAP evolution equation for the collinear part of the TMD)

Anselmino, Boglione, Melis, Phys. Rev. D86 (2012) 014028

hermes

$\langle Q^2 \rangle$ = 2.4 GeV^2

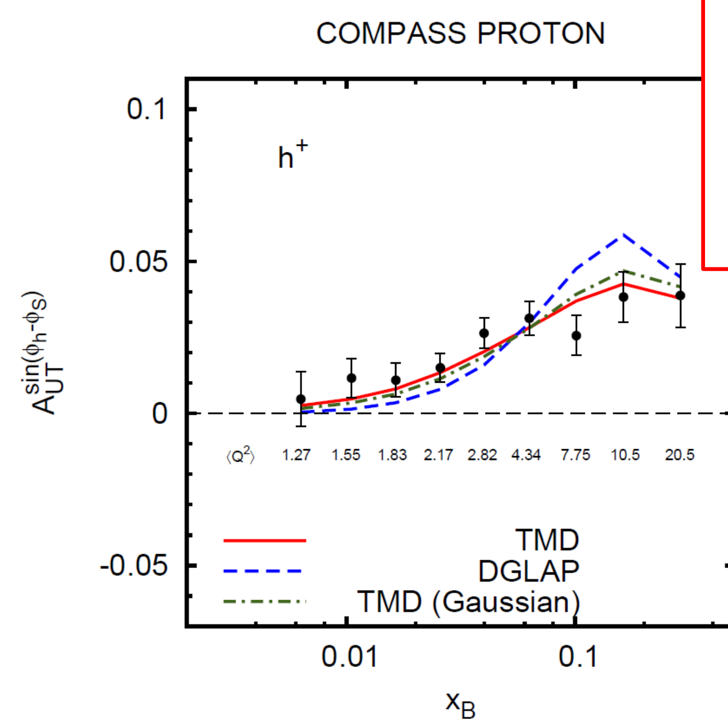
Q^2 in the range [1.3 – 6.2] GeV^2



COMPASS

$\langle Q^2 \rangle$ = 3.8 GeV^2

Q^2 in the range [1.3 – 20.5] GeV^2



A. Airapetian et al., Phys. Rev. Lett. 103, (2009) 152002

C. Adolph et al., Phys. Lett. B717 (2012) 383

g_2 alert !

- g_2 controls the b_T gaussian width and its spreading as b_T varies.

$$g_K(b_T) = \frac{1}{2}\, g_2\, b_T^2 \qquad \text{with} \qquad g_2 = 0.68$$

$$b_{\max} = 0.5\ \mathrm{GeV}^{-1}$$

- ***We did not extract the value of g_2 from our fit***

- *We used a fixed value previously, determined in a fit of D-Y data. Landry, Brock, Nadolsky, Yuan, Phys. Rev. D67(2003) 073016 We could have done it, and probably got a smaller value, but it is important to remember that* ***SIDIS data are very little sensitive to the precise value of g_2.***

- *D-Y data, instead, are extremely sensitive to it: this requires a new, careful, global analysis on all SIDIS and D-Y, re-starting from unpolarized cross sections.*

SIDIS vs Drell-Yan

- Numerator of the asymmetry in analytical approximation for a DY process

$$N_{DY} \propto \Delta^N f(x_1,Q_0) f(x_2,Q_0) \sqrt{2e}\,\frac{P_T}{M_1}\,\frac{\langle k_\perp^2\rangle_{Siv}^2}{\langle k_{\perp 1}^2\rangle \langle P_T^2\rangle_{Siv}^2}\, e^{-P_T^2/\langle P_T^2\rangle_{Siv}}$$

$$\langle P_T^2\rangle_{Siv}^{DY} = \omega_{Siv}^2 + \omega_2^2$$

$$w_S^2(Q,Q_0) = \langle k_\perp^2\rangle_S + 2 g_2 \ln\frac{Q}{Q_0}$$

$$w^2(Q,Q_0) = \langle k_\perp^2\rangle + 2\, g_2 \ln\frac{Q}{Q_0}$$

- Here it is squared, strongly suppresses the asymmetry as it becomes larger and larger

- g_2 is more crucial for DY processes than for the present SIDIS data because of the larger range spanned by Q

SIDIS vs Drell-Yan

- Numerator of the asymmetry in analytical approximatio… …ess

$$N_{DY} \propto \Delta^N f(x_1,Q_0) f(x_2,Q_0)\sqrt{2e} \ldots$$

$$\langle P_T^2 \rangle_{Siv}^{DY} = \ldots$$

$$w^2 \ldots \ln \frac{Q}{Q_0}$$

…ere it is squared, strongly suppresses the asymmetry as it becomes larger and larger

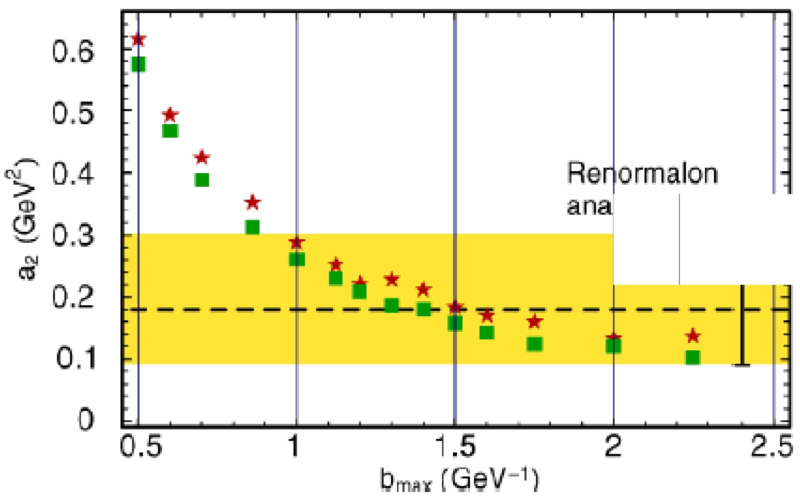
Limitations:
Need global fits
Need new high precision Drell-Yan data at moderate energies and low q_T

- g_2 is more crucial for DY processes than for the present SIDIS data because of the larger range spanned by Q

SIDIS vs Drell-Yan

Konychev, Nadolsky, Phys. Lett. B633 (2006) 710

- g_2 depends on the prescription for the separation of the perturbative region from the non -perturbative one. Depends also on the "order" at which you stop in the perturbative expansion.



$a_2=g_2$, stars correspond to the choice $C1=2\exp(-\gamma_e)$, squares to $C1=4\exp(-\gamma_e)$

Low-Q Drell-Yan experiments (E288,E605 and R209) show a preference for b_{max} larger than 0.5 GeV^{-1} (around 1.5), while higher Q data are not very sensitive to this value.

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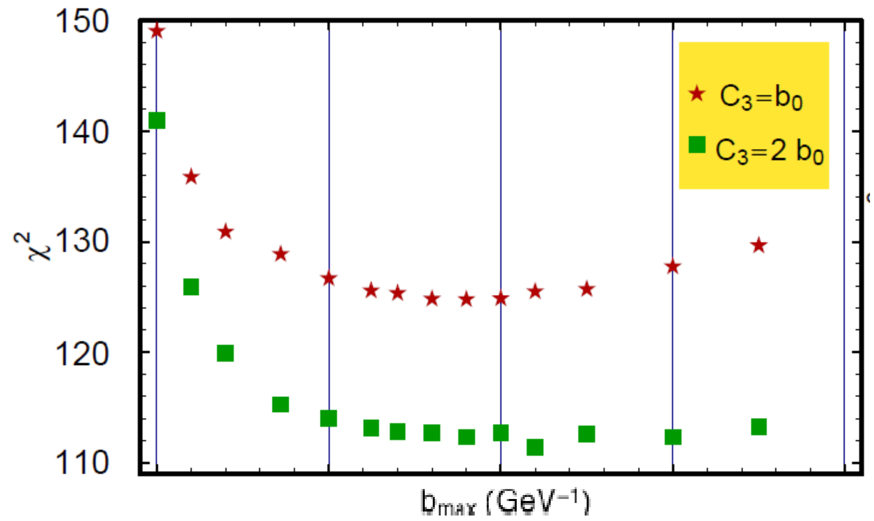

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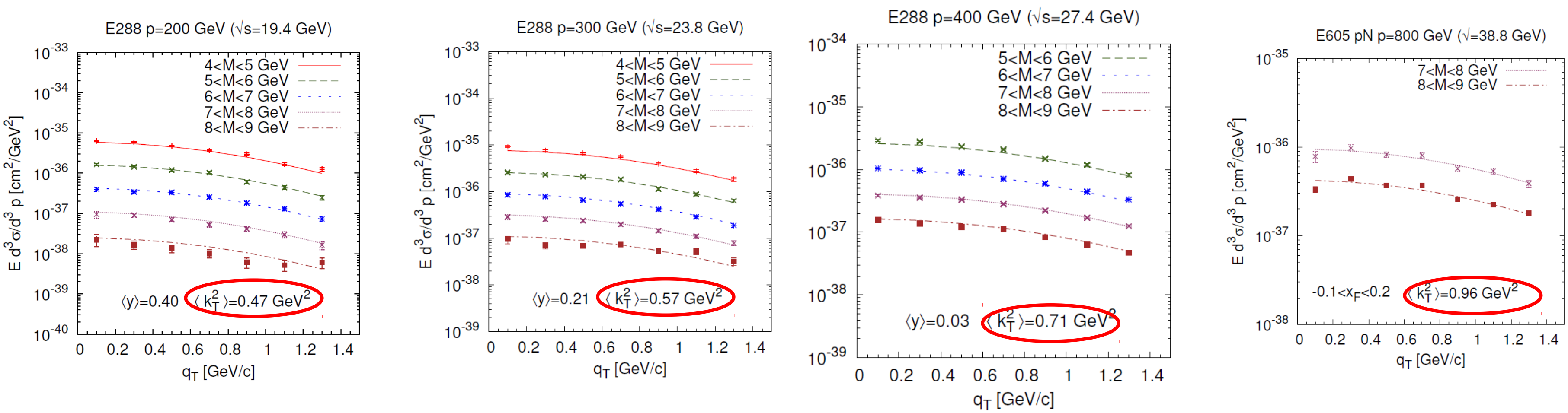
Drell-Yan phenomenology

Stefano Melis preliminary studies



The fit on E288 and E605 Drell-Yan data is performed by assuming a gaussian $k_\perp$ dependence with a DGLAP evolution of the factorized PDFs.

$$\widehat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x; Q)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}$$

The gaussian width is fitted independently for each different energy data set.

Notice that $\langle k_\perp^2 \rangle$ grows as energy grows

Schweitzer, Teckentrup, Metz, Phys.Rev. D81 (2010) 094019
D'Alesio, Murgia, Phys. Rev. D70 (2004) 074009

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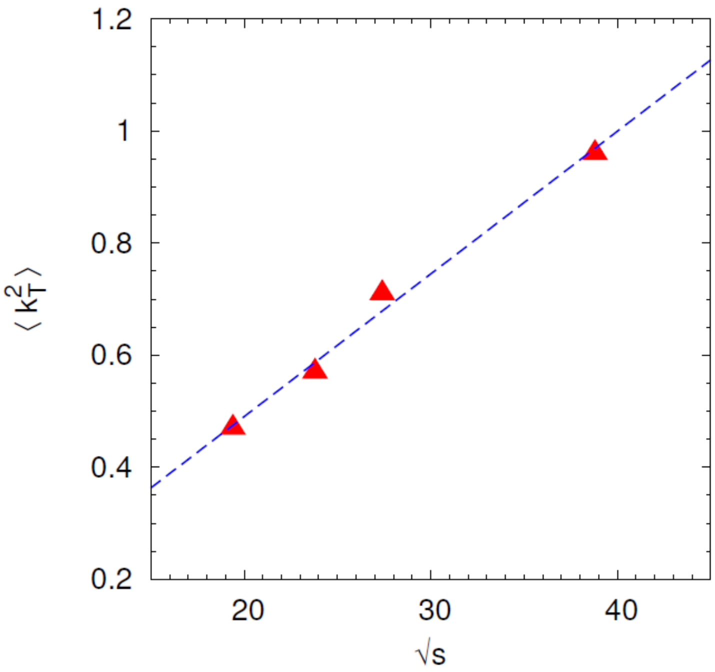


The dependence of $\langle k_\perp^2 \rangle$ on the energy is roughly linear

Loads of room for phenomenology !

See talk by Z. Kang

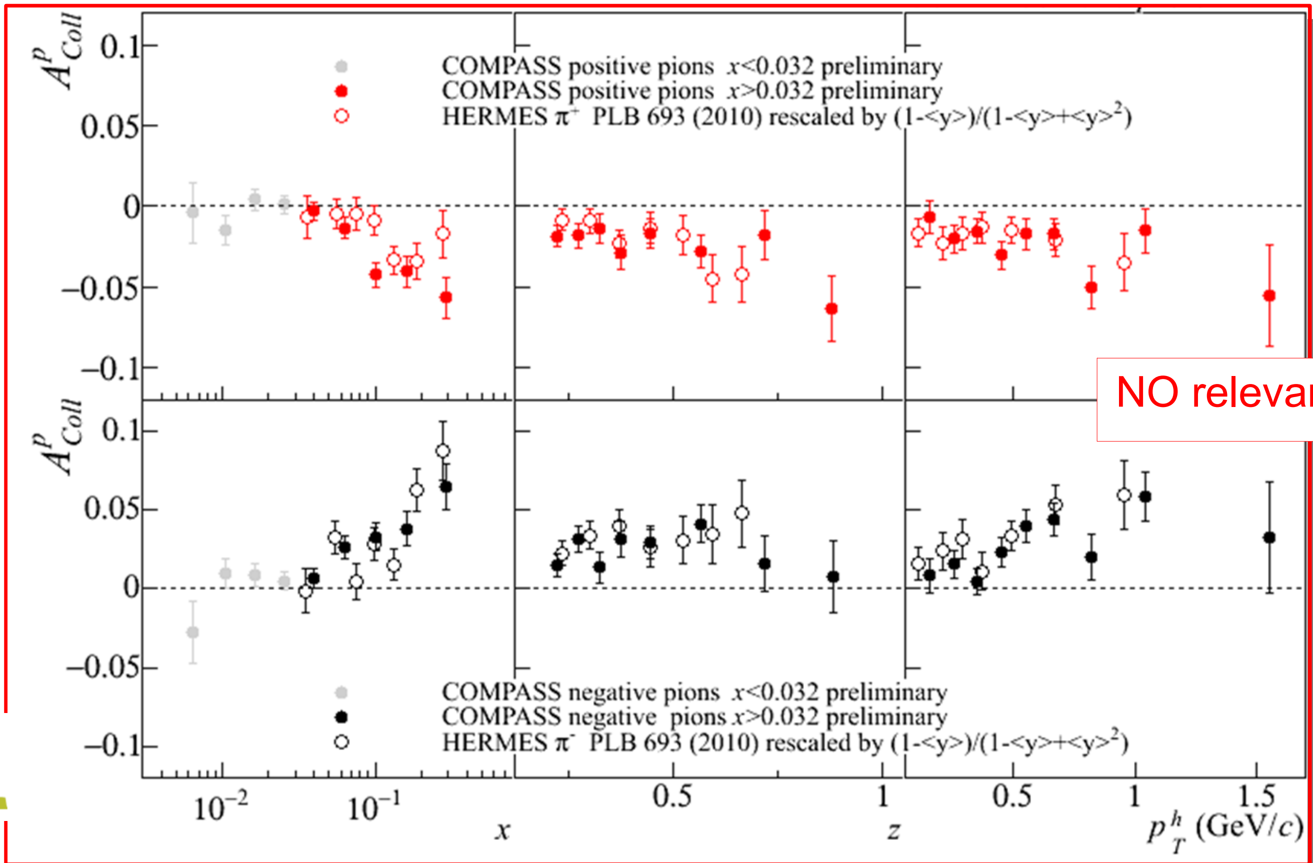
TMD evolution of the Collins function

Does most recent SIDIS data suggest TMD evolution for the Collins function ?

Collins asymmetry on **proton** (x > 0.032)

Charged pions (and kaons), 2010 data

Comparison with HERMES results



NO relevant hints of TMD evolution

π+

● COMPASS $\langle Q^2 \rangle$ = 3.2 GeV²

○ hermes $\langle Q^2 \rangle$ = 2.4 GeV²

π-

● COMPASS $\langle Q^2 \rangle$ = 3.2 GeV²

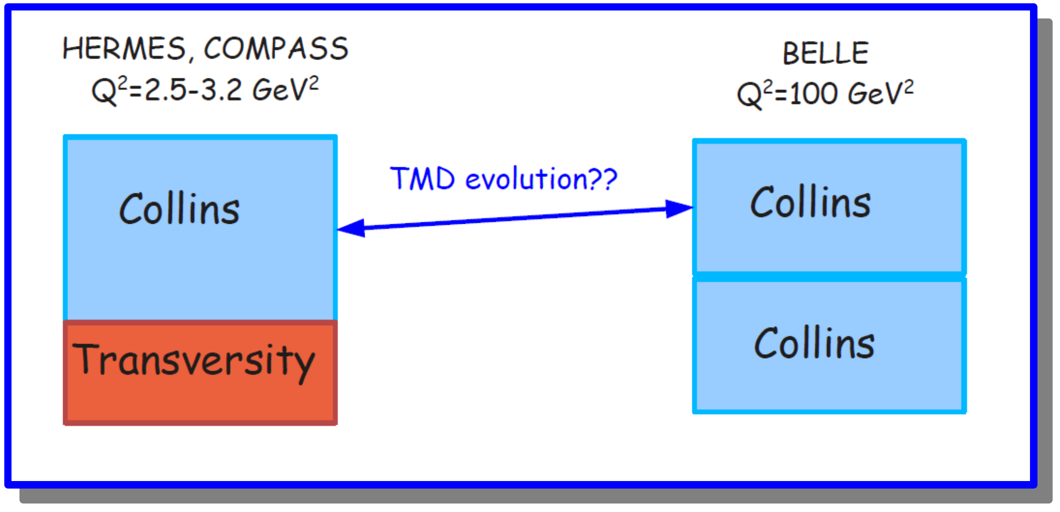
○ hermes $\langle Q^2 \rangle$ = 2.4 GeV²

Anna Martin

DIS 2013

TMD Evolution of the Collins function

- TMD evolution tends to reduce the size of distribution and
- fragmentation functions as Q^2 grows.
  D. Boer, Nucl. Phys. B603 (2001); Nucl. Phys. B803 (2009),
  BUT the azimuthal moments involved in the Collins
- asymmetries are probably not smaller !
  Need a more detailed investigation ...



Courtesy of Stefano Melis

Outlook and conclusions

- New experimental data on SIDIS multiplicities should allow to perform a global analysis of Drell-Yan as well as SIDIS unpolarized cross sections, to determine the basic parameters needed for the implementation of the TMD evolution schemes.
- Afterwards, we will perform the same analysis for the Sivers, transversity and Collins TMD functions.
- As far as TMD evolution is concerned we have recently come a long way.
- We now have evolution schemes and some first attempts to the phenomenological study of the unpolarized distribution and fragmentation TMDs, of the TMD transversity and of the Sivers functions.
- Preliminary studies are now being refined, especially as far as the parametrization of unknown phenomenological quantities are concerned.
- From the experimental side, we need more SIDIS (polarized and unpolarized) data at larger values of x (Jlab 12) and spanning a larger Q^2 range (EIC) as well as more (and more precise) Drell-Yan data, for which new experiments are being planned (COMPASS, RHIC, Fermilab, NICA, JPARK).

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