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Towards a Recursive Monte-Carlo generator of quark jets with spin

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Introduction

• A jet model which takes into account the quark spin of freedom must start with quantum amplitudes rather than probabilities.

• A « toy model » following this principle was built [X. Artru, DSPIN-09] using Pauli spinors and inspired from the *multiperipheral* model and the classical "***string + $^3P_0$*** " mechanism

• It generated *Collins and Longitudinal jet handedness* effects.

• This model was too simplified: hadrons were not on mass-shell.

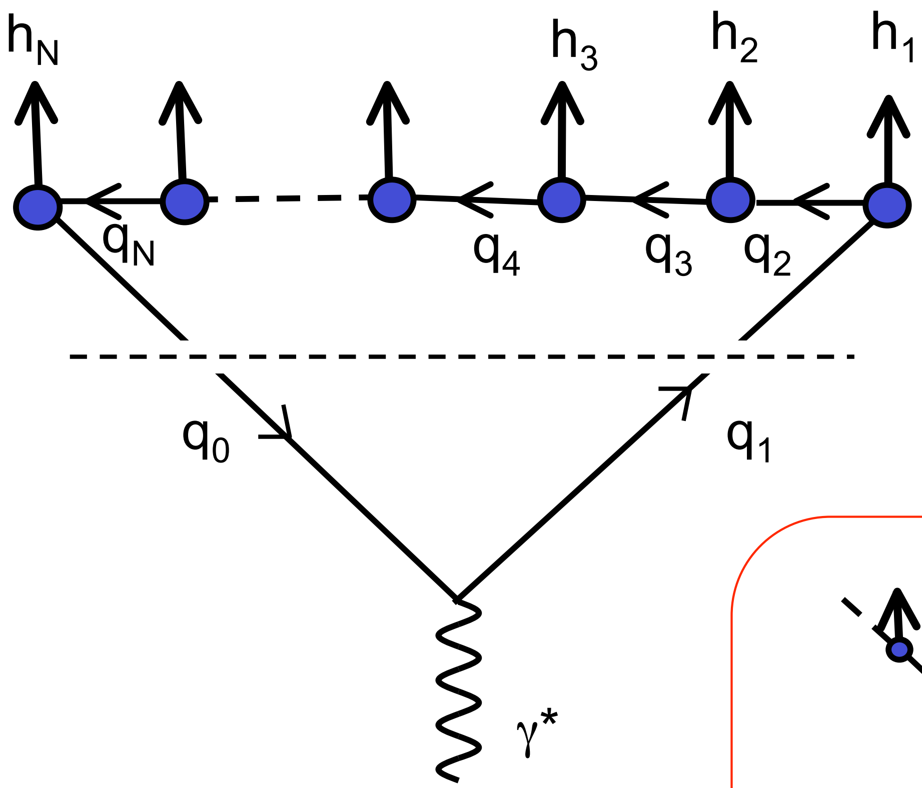
• We present an improved model with mass-shell constraints.

Outlines

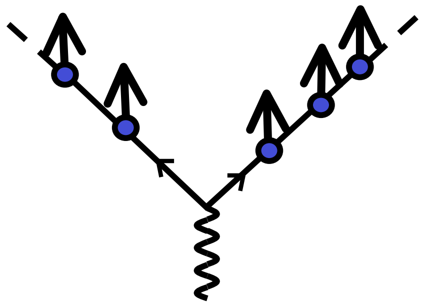
- *Quark-Multiperipheral* (Q-M) and *String Fragmentation* (SF) models.
- SF model = particular type of Q-M model
- The semi-classical "***string* + $^3P_0$** " mechanism : discussion and some predictions
- Semi-quantization of the string fragmentation model with spin
- *ab initio* splitting algorithm (for recursive Monte-Carlo code)
- "renormalized" input
- a non-recursive mechanisms : permuted string diagrams

Two models of $q+\bar{q}$ hadronization

Quark Multiperipheral (QM) model

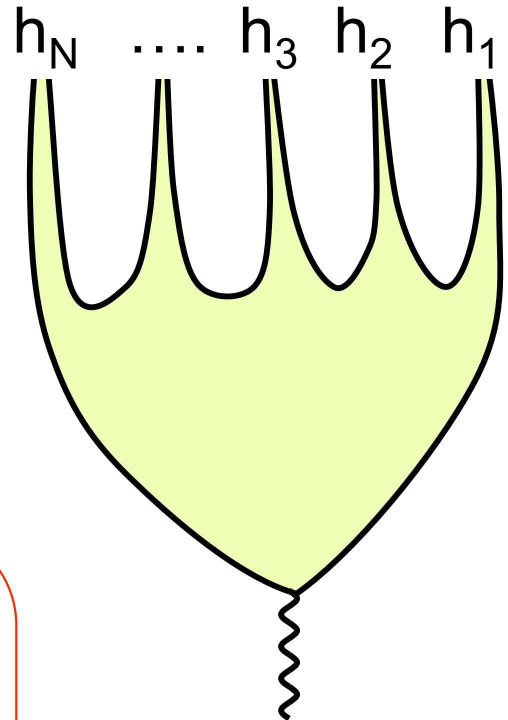


Feynman diagram
with a loop



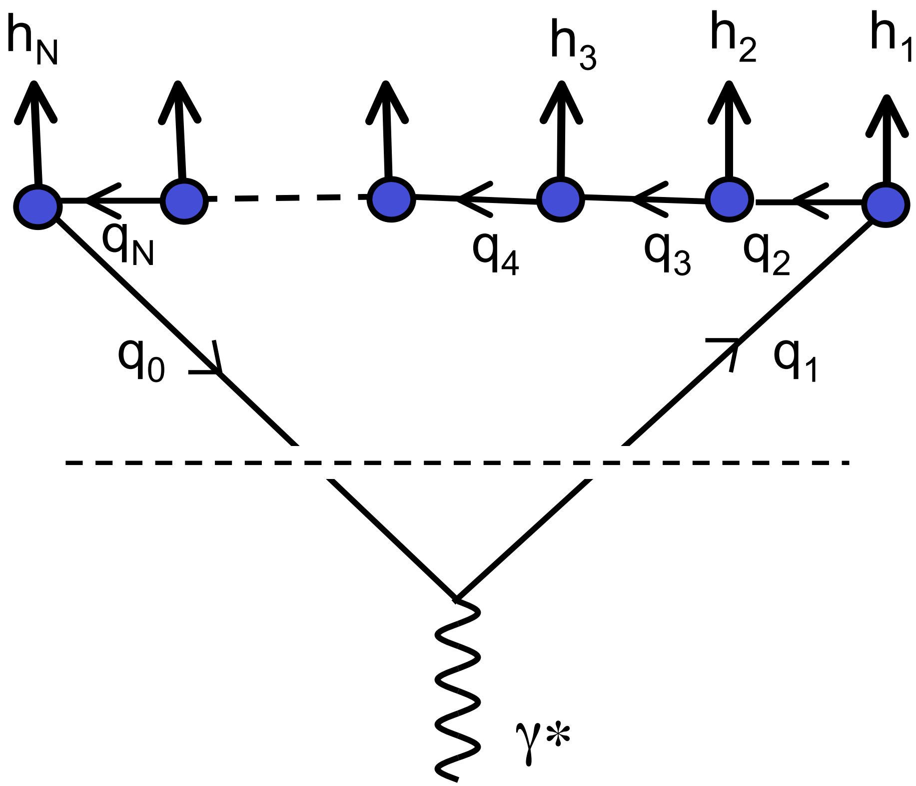
Cascade of "quark decays" = bad model : confinement violated

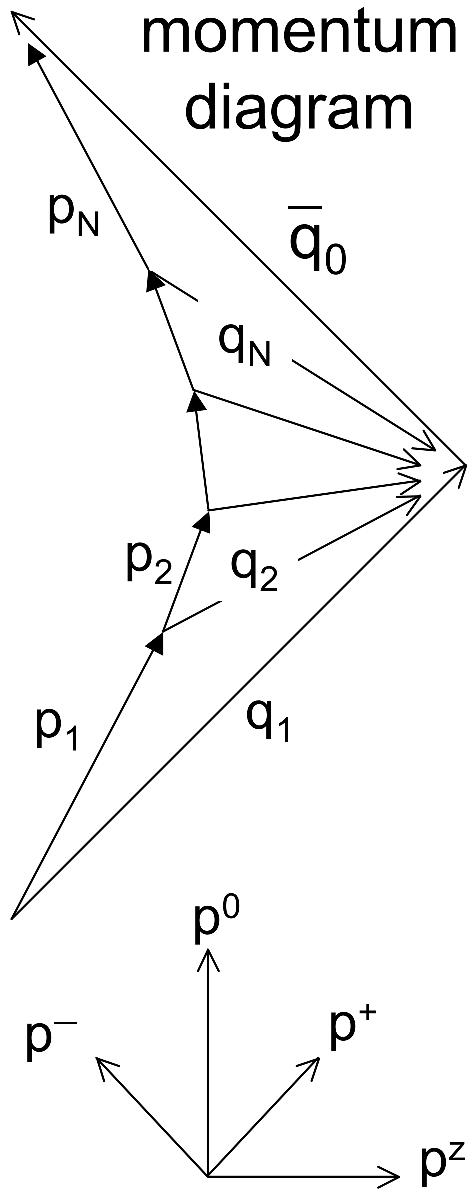
String Fragmentation (SF) model



String diagram
Confinement built-in

Quark momenta in the Quark Multiperipheral model.
1) time- and longitudinal components



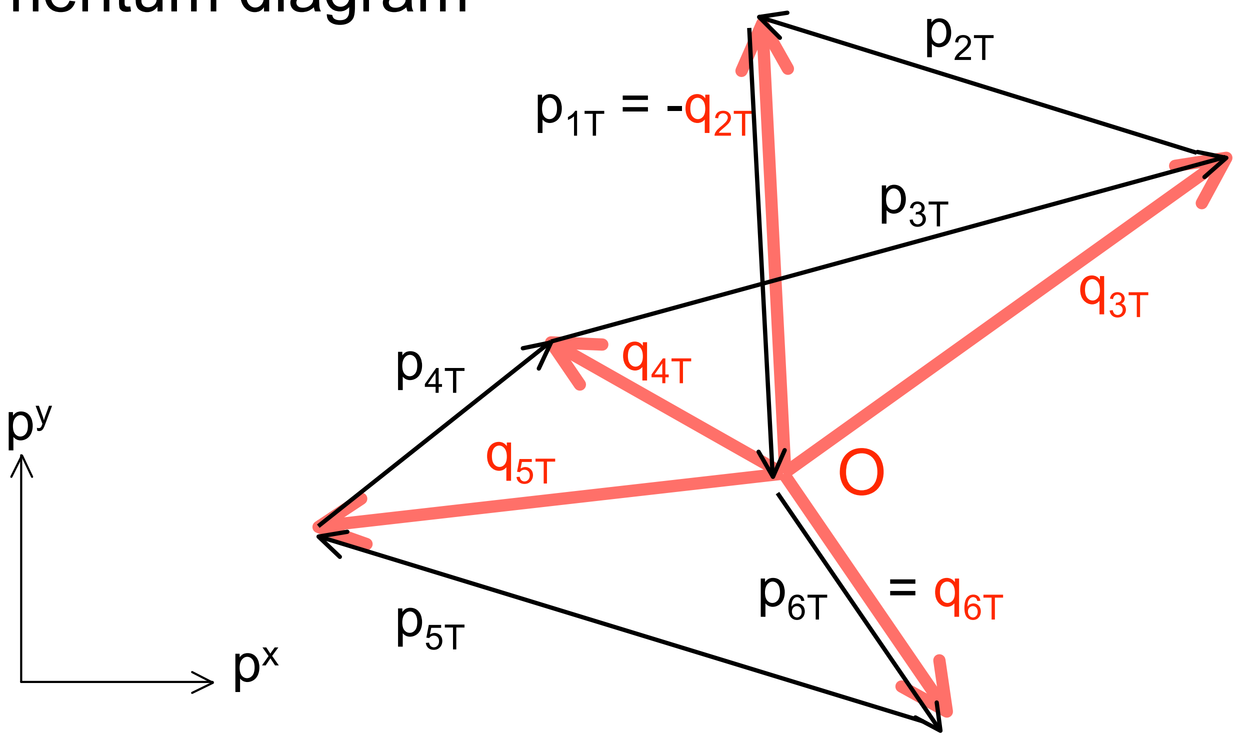


q_1 = *internal momentum*
→ no classical meaning.

Cutoff in $|q^+q^-|$ → approximate ordering of p_1, p_2, p_3,… in rapidity

Quark momenta in the Quark Multiperipheral model.
2) transverse momenta

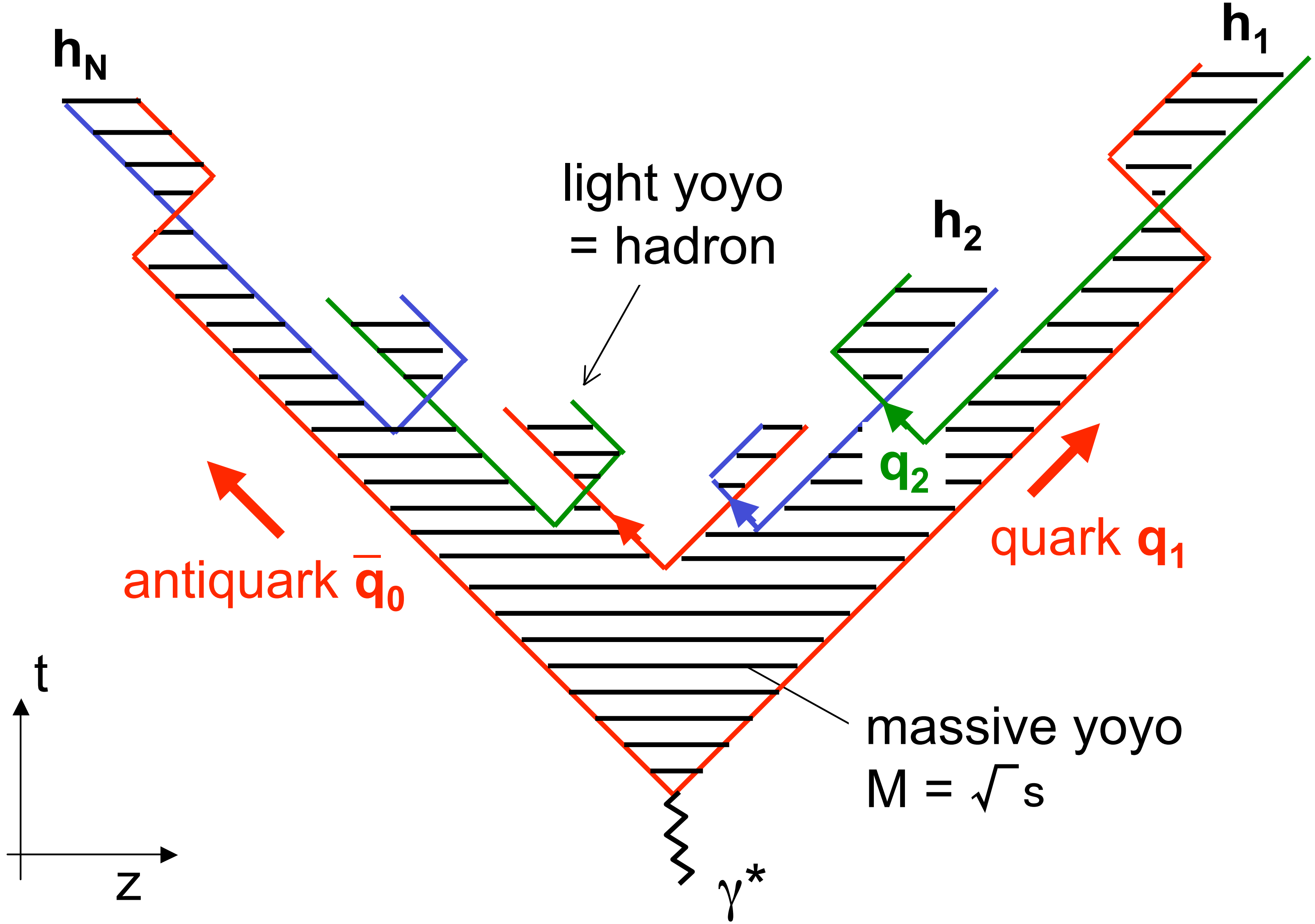
Transverse momentum diagram



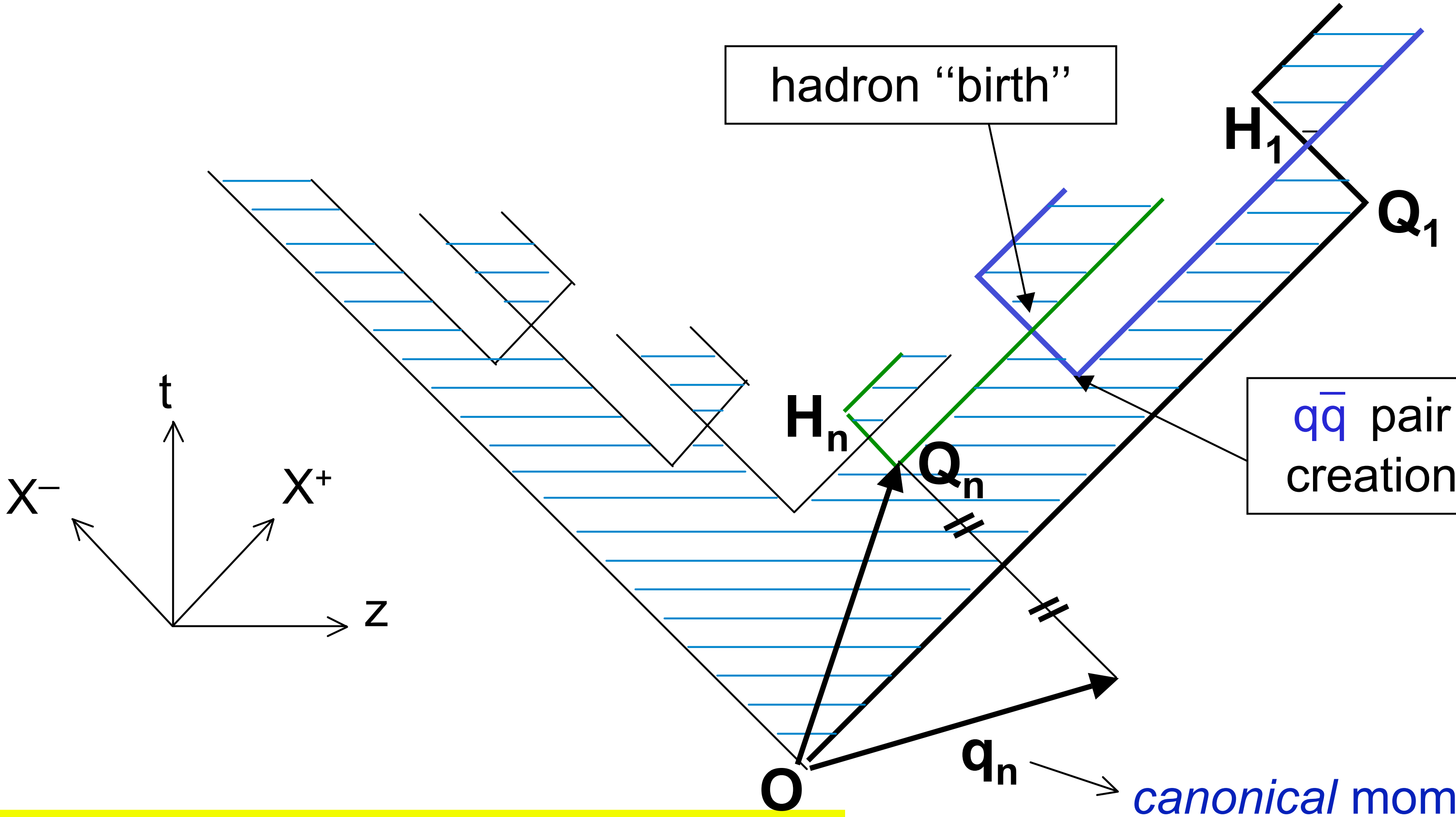
Cutoff in q_T → cutoff in p_T
+ ***L**ocal **C**ompensation of **T**ransverse **M**omenta*

- a cutoff in p_T alone **does not** give a cutoff in q_T
- what about JETSET (LUZDIS) ?

String Fragmentation model: space-time picture



Time- and longitudinal quark momenta in the SF model

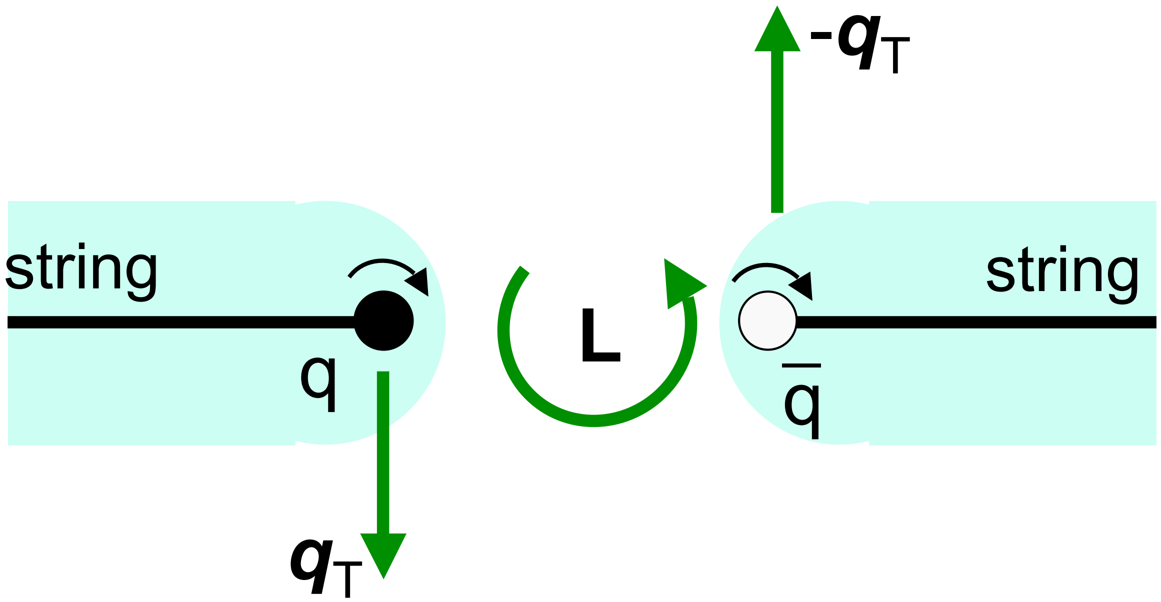


canonical momentum
= *mechanical* momentum
of the quark
+ flow of string
momentum through OQ_n

SF model ≈ QM model:
$\mathbf{q}_n$ is *symmetrical of* $\mathbf{OQ_n}$
about the X^+ *axis*
(in units κ= string tension =1)

The semi-classical « *string* + $^3P_0$ » mechanism

Hypothesis : the ($q\bar{q}$) pair is created with the vacuum quantum numbers, therefore in the $\mathbf{0^{++}} = {}^3\mathbf{P}_0$ state. [Le Yaouang *et al*]



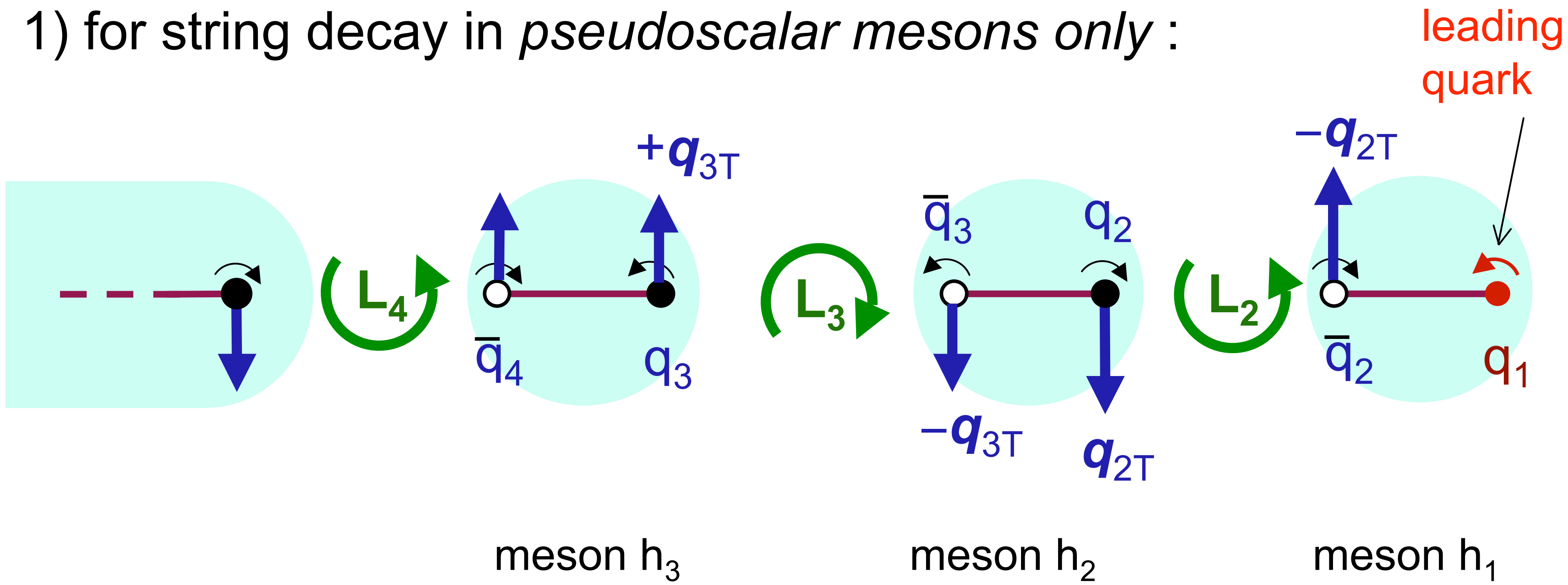
Azimutal correlation between $\boldsymbol{q}_T$ and $\mathbf{S}_{quark}$

→ transverse polarization of inclusive hyperons [Lund group]

→ mechanism of **Collins effect** [X.A, J. Czyzewski, H.Yabuki]

Predictions of *string* + $^3P_0$

1) for string decay in *pseudoscalar mesons only* :

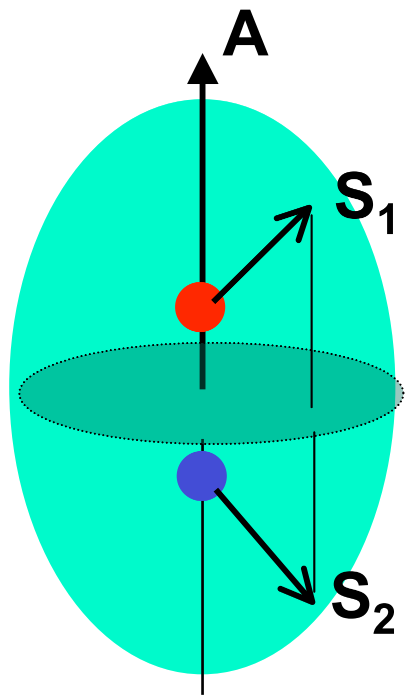


- alternate Collins effects
- large *Relative Collins Effect* (or "***I**nterference **F**ragmentation **F**unction*")
- large Collins effect for the 2nd-rank (or *unfavored*) meson

  (due to $\boldsymbol{q}_T$ - $\mathbf{S_q}$ correlations on both sides)

2) for a *leading* vector meson

The quark model says :
in a 1^- *vector meson*
of ***linear*** *polarization* $\boldsymbol{A}$*,*
the quark and antiquark
polarizations are symmetrical
about the plane $\perp$ $\boldsymbol{A}$



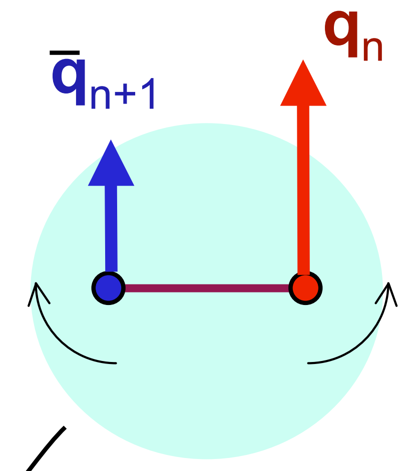
- If **A** *along Oz* the Collins effect is in azimuth $\phi(\mathbf{S_1}) + \pi/2$ (opposite to the pion one)

- If **A** $\perp$ *Oz* it is in the azimuth $2\phi(\mathbf{A}) - \phi(\mathbf{S_1}) - \pi/2$

- On the average, the Collins effect is -1/3 the pion one [J. Czyzewski 1996]

3) *Hidden spin* effects (effect without external polarization)

The $\mathbf{S}_q$ - $\mathbf{q}_T$ correlation acts upon the $\langle \mathbf{p}_T^2 \rangle$ of the *unfavored* hadrons.

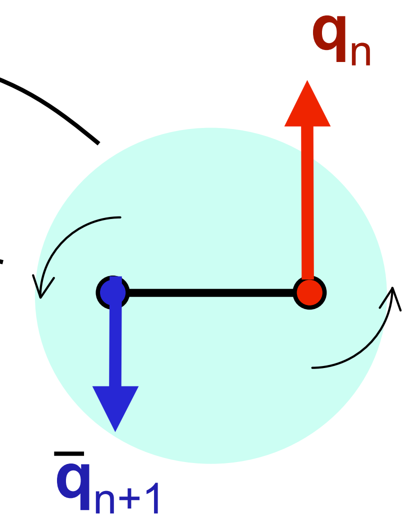


$$\langle \mathbf{p}_T^2 \rangle_{\mathbf{pion}} > 2 \langle \mathbf{q}_T^2 \rangle_{\mathbf{quark}}$$

For a *vector meson,* of linear polarization

- *along Oz* : $\langle \mathbf{p}_T^2 \rangle_{\mathbf{v.m.}} < 2 \langle \mathbf{q}_T^2 \rangle_{\mathbf{quark}}$

- *along Ox* :



$$\langle \mathbf{p}_{\mathbf{x}}^2 \rangle_{\mathbf{v.m.}} < \langle \mathbf{q}_T^2 \rangle_{\mathbf{quark}} < \langle \mathbf{p}_{\mathbf{y}}^2 \rangle_{\mathbf{v.m.}}$$

On the average, $\langle \mathbf{p}_T^2 \rangle_{\mathbf{v.m.}} < \langle \mathbf{p}_T^2 \rangle_{\mathbf{pion}}$

Semi-quantized hadronization model

Spin has non-classical properties (positivity constraints, entanglement).

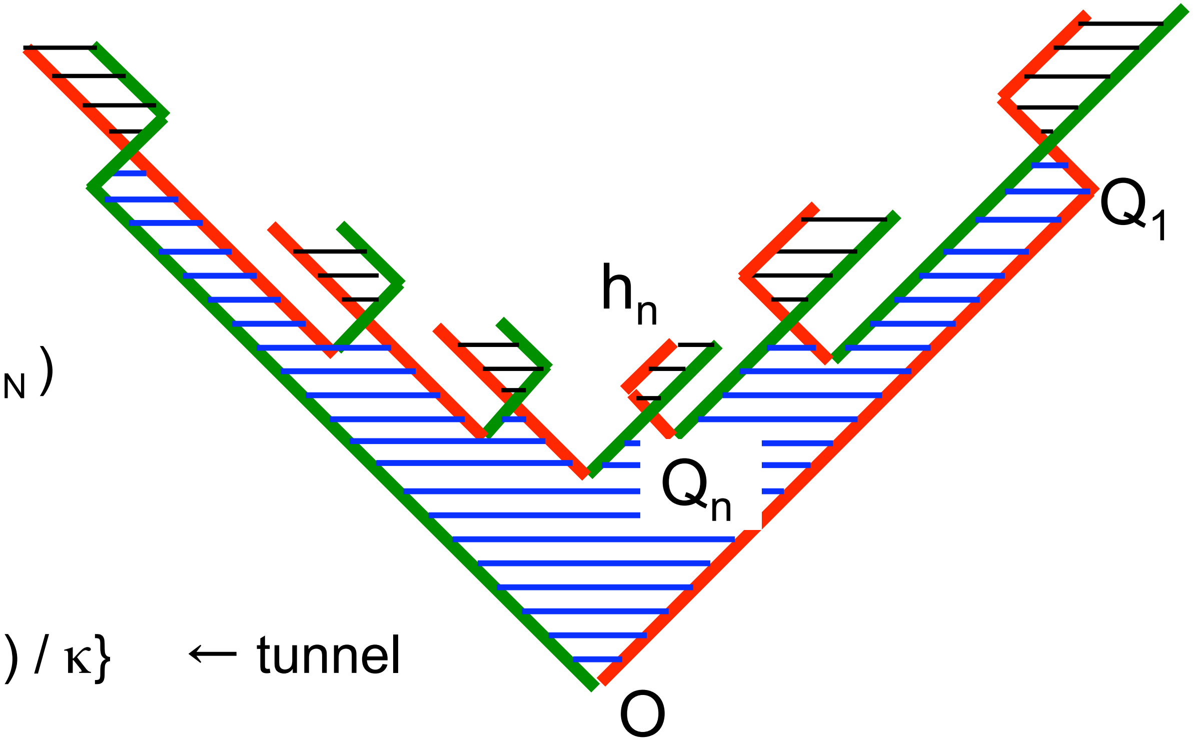
One needs a (semi-) quantum model of hadronization.

We propose a model which borrows:

- The spin structure of the Quark Multiperipheral model (but using *Pauli* spinors)

- The dynamics of the String Fragmentation model

Non-quantized, spin-blind, string model : the *Lund-symmetric model*



Exclusive distribution :

Probability($q\bar{q} \rightarrow h_1+h_2 \ldots +h_N$)

$\propto$ exp{-2b × (blue area) }

× *contour terms*

× $\prod_{n=2}^{n=N}$ exp{$-\pi$ ($m_{qn}^2+q_{nT}^2$) / κ}   ← tunnel

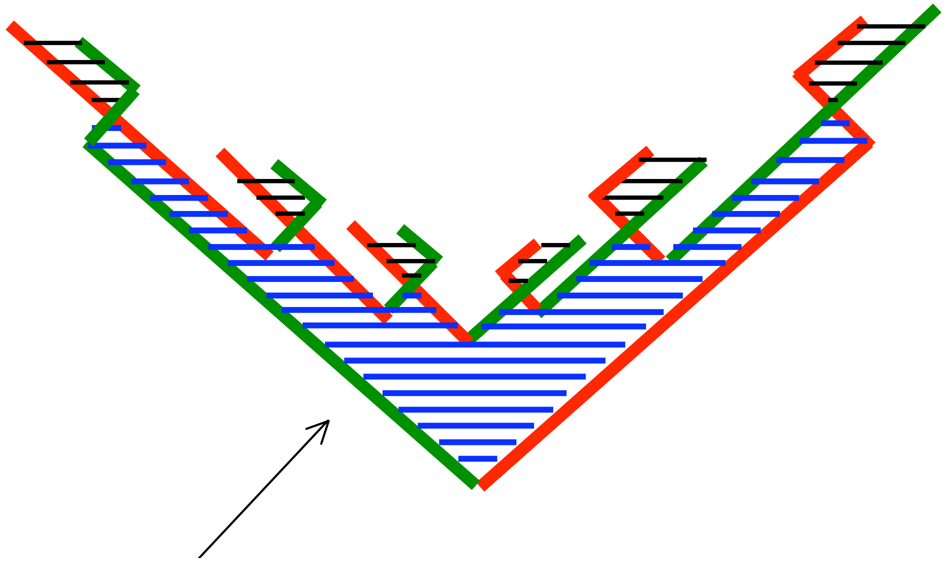
κ = string tension; 2b = *string fragility*

Splitting function :

$f(q \rightarrow h+q') \propto \exp\{-b\,(m_h^2+p_T^2)/z\} \times (1-z)^a \times \exp\{-\pi\,(m'^2_q+\boldsymbol{q}'^2_T)/\kappa\}$

↑ area   ↑ contour   ↑ tunnel

Semi-quantization of the SF model, *incuding spin*



Following Feynman, to each *classical history* we associate a *quantum amplitude*

$M_N(q_0, q_N, \ldots q_2, q_1)$ = exp{ i × (string action) }

× quark propagators

× vertex matrices

The amplitude M_N is invariant under

- rotation about jet axis
- boost along jet axis

} Lorentz subgroup

- parity
- quark chain reversal (basis of the *symmetric Lund model*)

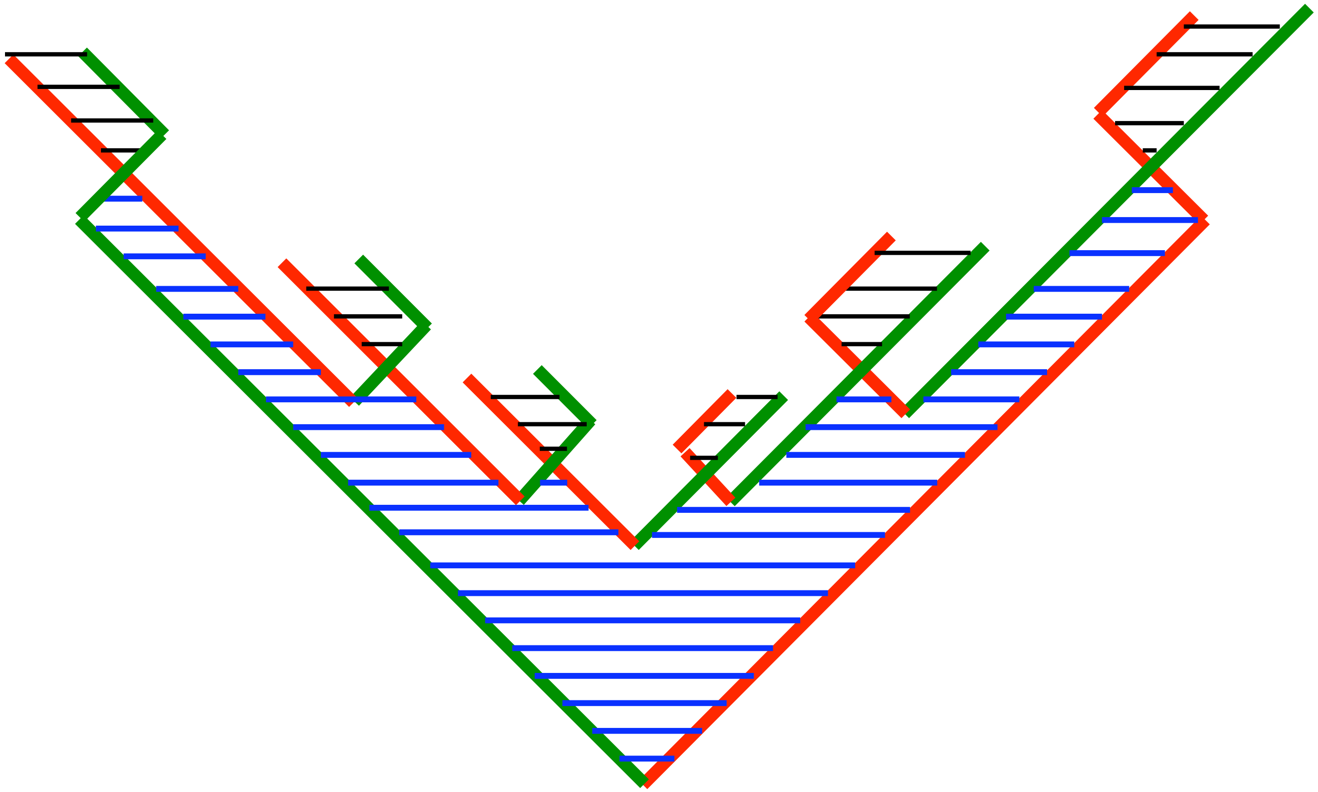
But :

- No full Lorentz invariance

  (not required, once the jet axis is given).

- Pauli spinors are sufficient.

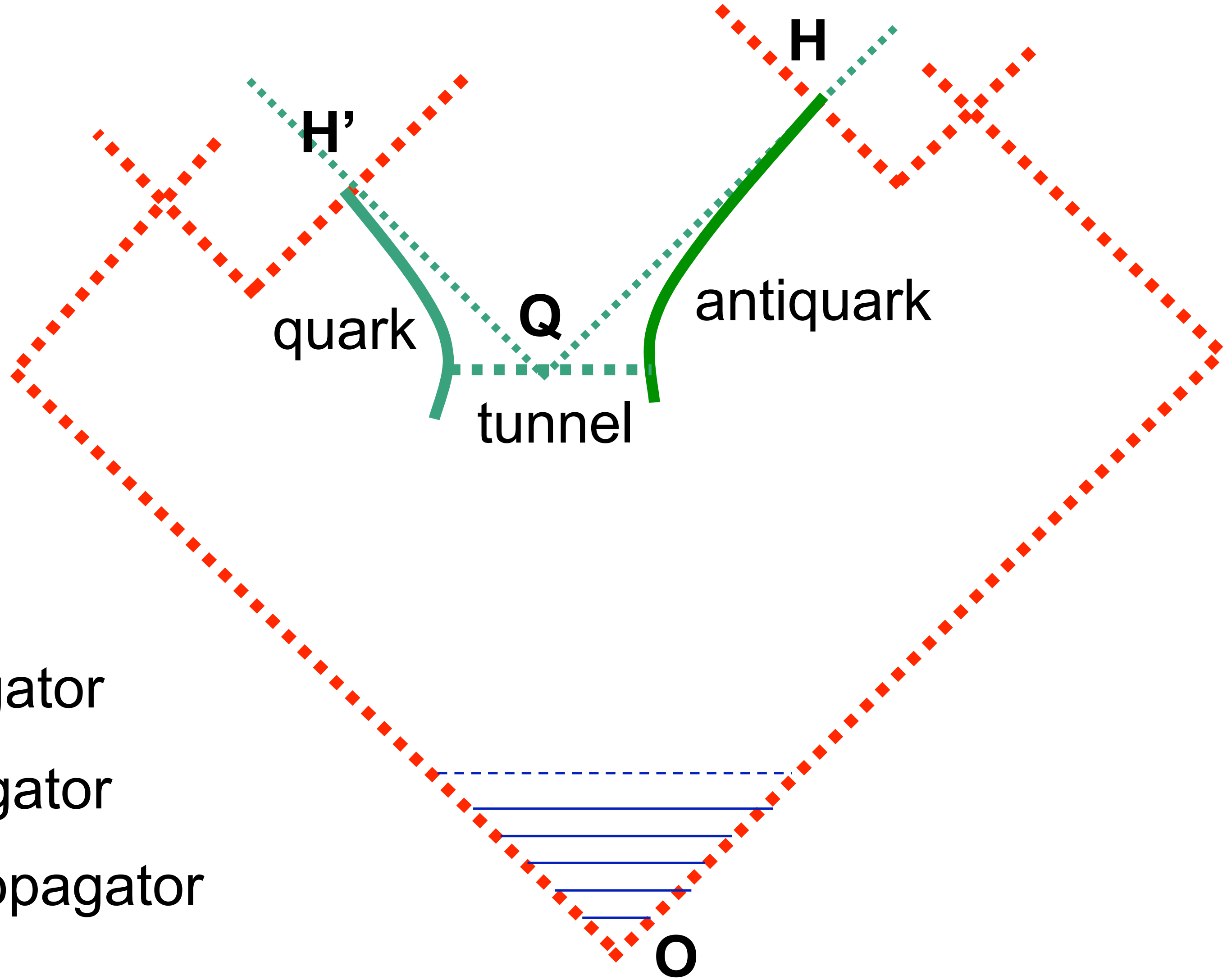
1) action of the massive string



$A_{string} = -\,(\kappa - i\,b) \times$ {hatched bleu area}

$\kappa - i\,b$ is analoguous to $m - i\,\gamma/2$

Quark propagator (1/2)



H — H' propagator

= quark propagator

× antiquark propagator

× tunnel factor

× spin-dependent "Feynman numerator"

Quark propagator (2/2)

quark propagator : $(p'^{-})^{\alpha}$

antiquark propagator : $(p^{+})^{\alpha}$

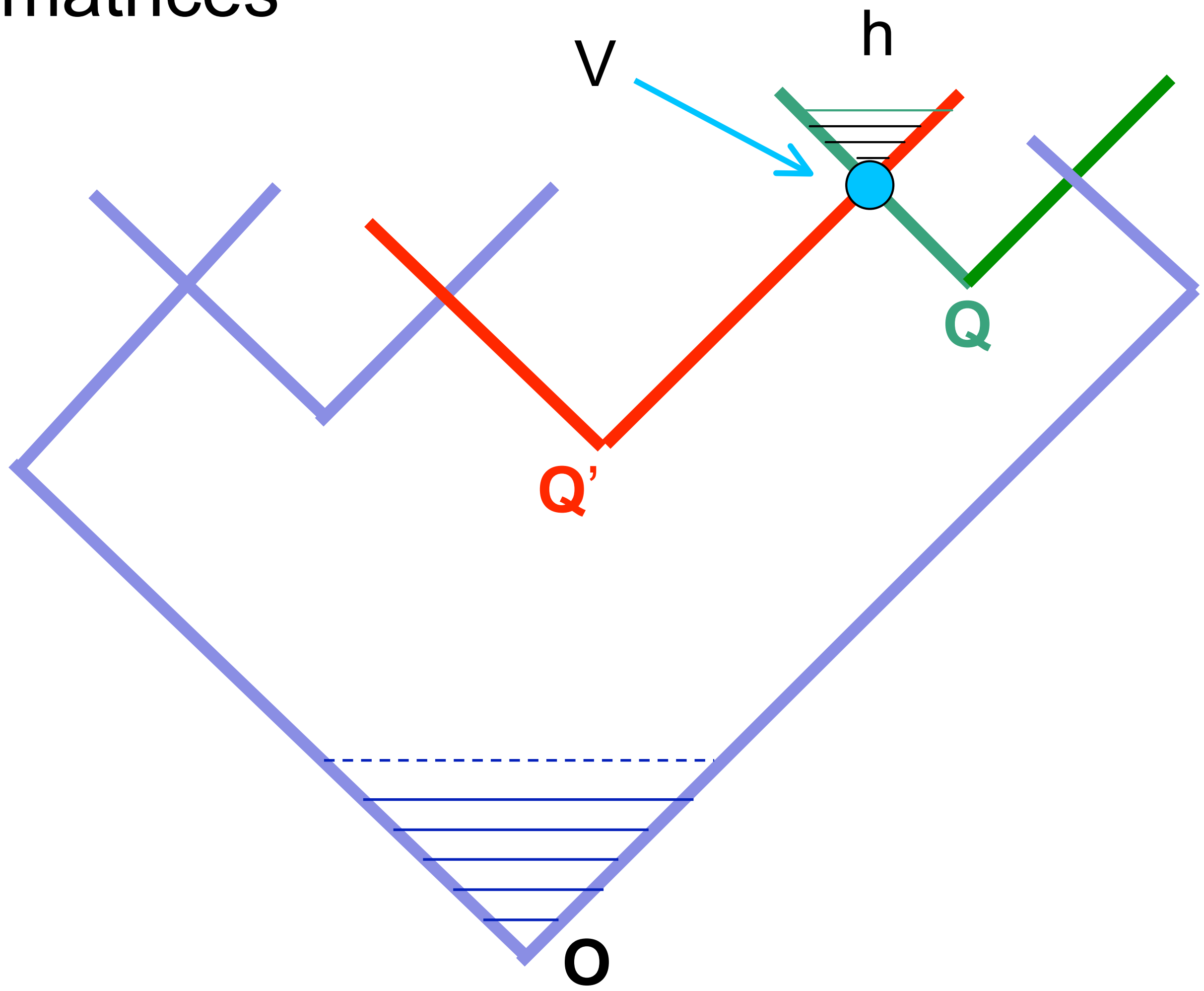
tunnelling factor $\exp\{-\pi(m_q^2+q_T^2)/2\kappa\}$

"spin numerator" : $\mu - \sigma_z\, \sigma.\boldsymbol{q}_T$ (analogue of $m+\gamma\cdot p$)

Total : $(p^{+}p'^{-})^{\alpha}\ \exp\{-\pi(m_q^2+q_T^2)/2\kappa\}\ (\mu - \sigma_z\, \sigma.\boldsymbol{q}_T)$

- μ is a *complex* parameter $\neq m_q$
- $(p^{+}p'^{-})^{\alpha}$ gives a Regge-like rapidity gap distribution
- Schwinger mechanism : $\alpha = (m_q^2+q_T^2)(b - i\kappa)/2$

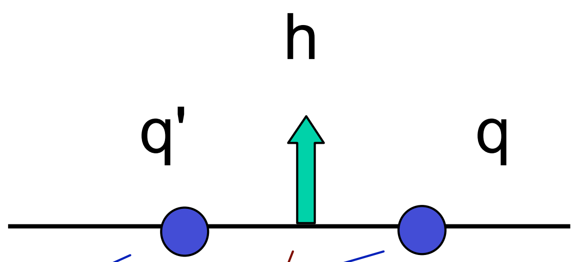
Vertex spin matrices



Pseudo-scalar meson (π, K, η^0) : $V = \sigma_z$ (analogue of γ_5)

Vector meson : $V = G_L A_z + G_T\, \sigma.\mathbf{A}_T\, \sigma_z$

Translation into propagator and vertex function for the QM picture



propagators :

$$\Delta(q) = \exp\{ (i\kappa - b)\,(q^+ q^-)/2 \}\,(q^+ q^- - i0)^{\alpha}\,(\mu - \sigma_z\, \sigma . \boldsymbol{q}_T)$$

vertex function (for pseudoscalars) :

$$\Gamma(q', h, q) = \exp\{ (b - i\kappa) q^+ q'^- /2 \}\,(-q^+ q'^-)^{-\alpha}\ \sigma_z$$

What *splitting function* does it give for a recursive Monte-Carlo code ?

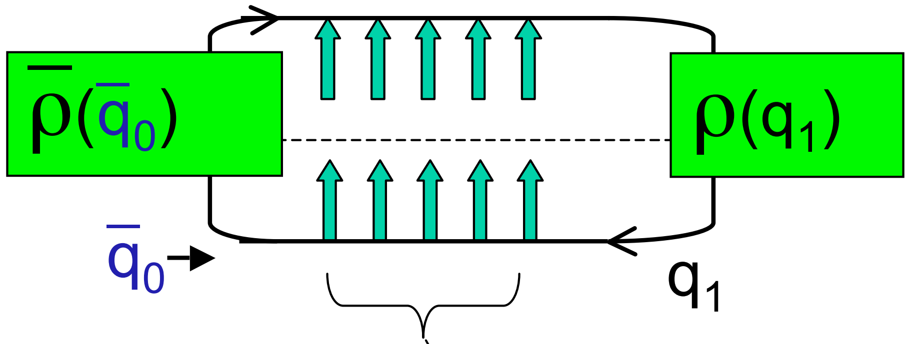
- the *ab initio* method has been presented at DSPIN-11 [X.A. & Zouina Belghobsi]. The "*initio*" or *input* consists in the quark propagator and q-h-q vertices
- it requires a preliminary task : solving a *matrix-valued integral equation*
- using a "renormalized input", we can replace the integral equation by an ordinary integration (much easier)

Next slides:

- review the *ab initio* method
- present the "renormalized input" method

The ab initio method

Ladder unitarity diagram
(multiperipheral picture)

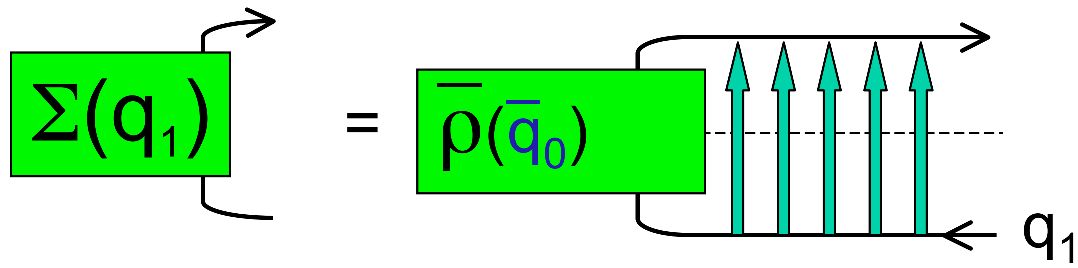


hadronization "cross section"

$\sigma\,(q_1 + \bar{q}_0 \rightarrow X) = \Sigma_N \int dp_1 \ldots dp_N \,\mathrm{Tr}\{\, M_N^{\dagger}\, \rho(\bar{q}_0)\, M_N\, \rho(q_1) \,\}$

$= \mathrm{Tr}\{\, \boldsymbol{\Sigma}(q_1)\, \rho(q_1) \,\}$ (implicit q_0-dependence)

Cross section matrix,
or ***acceptance*** *matrix* for q_1



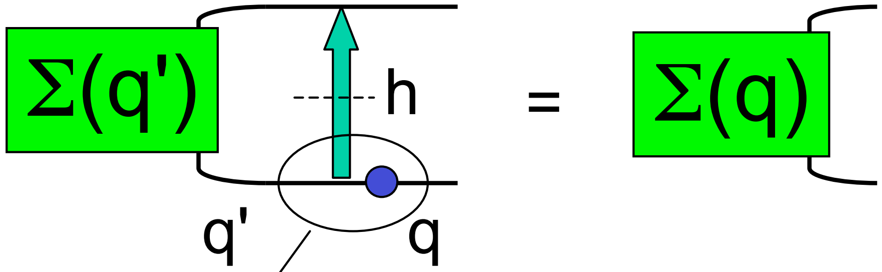
$\boldsymbol{\Sigma}(q) = [\ \beta(\boldsymbol{q}_T^{\,2}) + \gamma(\boldsymbol{q}_T^{\,2})\ \boldsymbol{\sigma} \cdot (z \times \boldsymbol{q}_T)\ ]\ |q_0^- q^+|^{\alpha(\mathrm{ladder})}$ ("output" Regge)

Contains the single-spin asymmetry $A_N = |\boldsymbol{q}_T|\ \gamma / \beta$

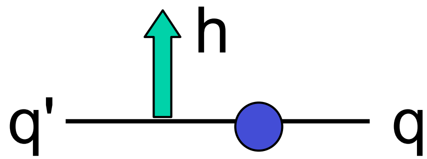
Integral equation for the acceptance matrix Σ(q)

In the *ladder approximation,*

$$\int d^3\mathbf{p}\ \ \mathbf{T}^\dagger(q',h,q)\ \ \Sigma(q')\ \ \mathbf{T}(q',h,q)\ \ =\ \ \Sigma(q)$$



$\mathbf{T} = \Gamma\ \Delta$ =

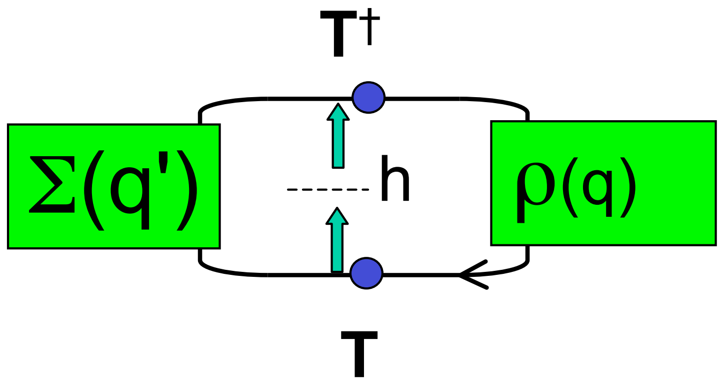


Δ and Γ are the inputs

Monte-Carlo algorithm for the splitting $q \rightarrow h+p'$

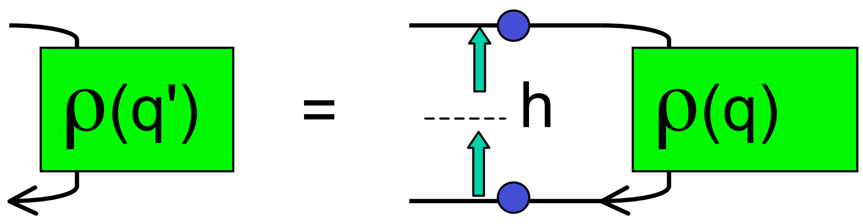
- step 1 : draw the momentum ***p*** with the probabilty

$$F(q \rightarrow h+X) \propto d^3\boldsymbol{p}\ \mathrm{Trace}\{\, \Sigma(q')\, \mathbf{T}\, \rho(q)\, \mathbf{T}^\dagger \}$$



- step 2 : calculate the density matrix of the new quark

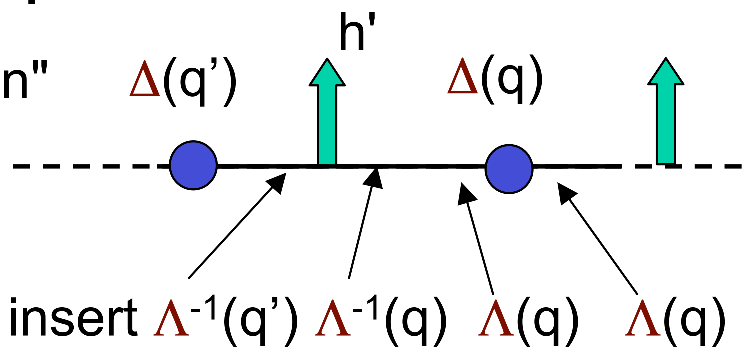
$$\rho(q') = \mathbf{T}\, \rho(q)\, \mathbf{T}^\dagger$$



- generate the jet by iterating step 1 and step 2

Renormalized input

The physic is invariant under the "renormalization"



$\Delta(q) \quad \rightarrow |q^{+}q^{-}|^{\lambda}\ \Lambda(q)\, \Delta(q)\ \Lambda(q)$

$\Gamma(q',h,q) \rightarrow |q'^{+}q^{-}|^{\lambda}\ \Lambda^{-1}(q')\ \Gamma(q',h,q)\ \Lambda^{-1}(q)$

wherefrom $\Sigma(q) \rightarrow |\, q_0^{-}\, q^{+}|^{2\lambda}\ \Lambda^{\dagger}(q)\ \Sigma(q)\ \Lambda(q)$

One can choose λ and Λ such that $\Sigma(q) \rightarrow \mathbf{1}$ (unit matrix).

The new $\Gamma(q',h,q)$ is taken as *renormalized input*.

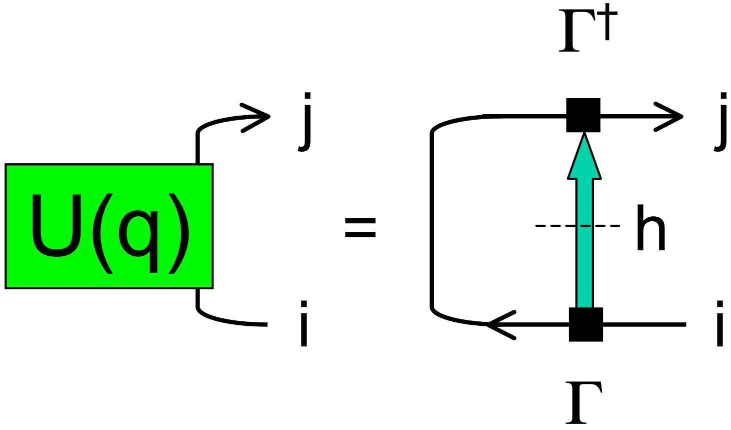
The new $\Delta(q)$ is deduced from

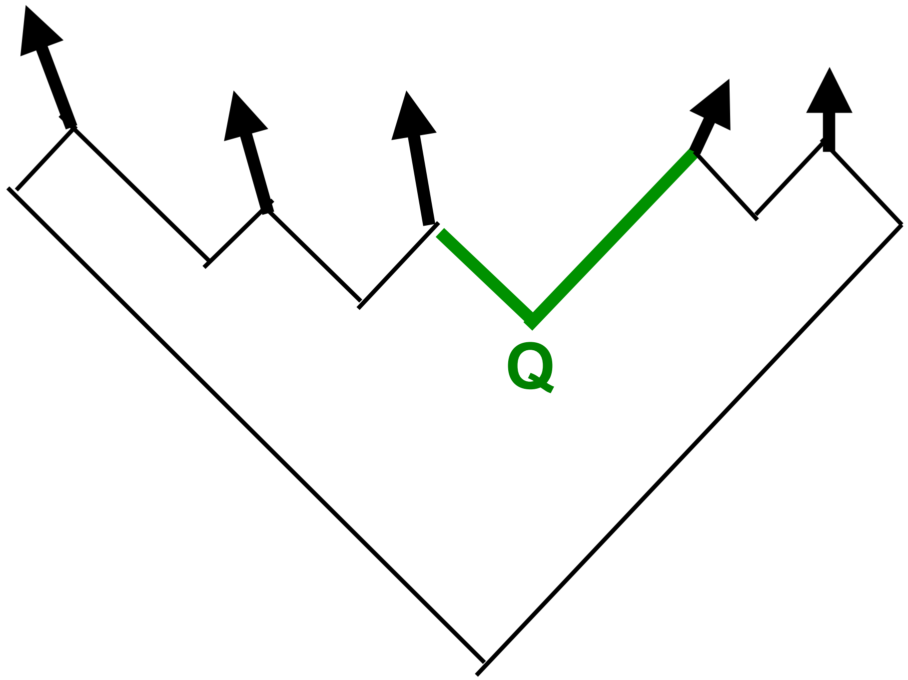
$[\ \Delta^{\dagger}(q)\ \Delta(q)\ ]^{-1} \equiv U(q) = \int d^3\mathbf{p}\ \ \Gamma^{\dagger}(q',h,q)\ \Gamma(q',h,q)$

This equation replaces the integral equation $\int \mathbf{T}^{\dagger}\, \Sigma\, \mathbf{T} = \Sigma$

Steps 1 and 2 proceeds as before with $\Sigma(q') = \mathbf{1}$

Physical meaning of U(q) in the string model





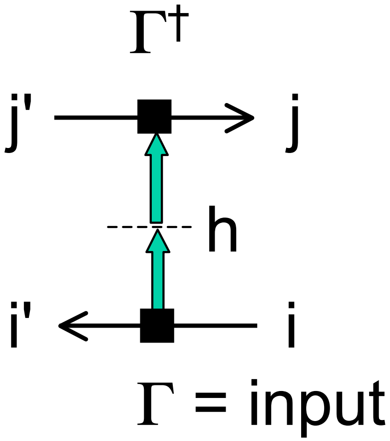
U(q) is the *density of propagators* in the $M^4(q)$ space.

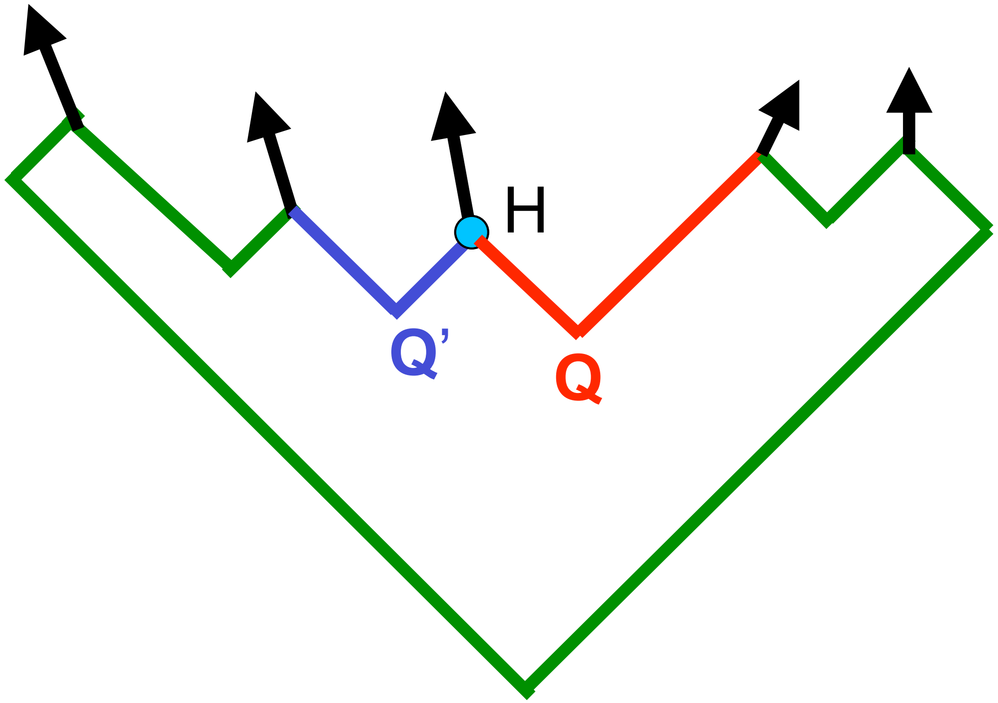
For the spin variable, it is a *density matrix*

$$U(q) = \exp(-b\, Q^+Q^-) \times (Q^+Q^-)^a \times [\, \beta(\boldsymbol{q}_T^2) + \gamma(\boldsymbol{q}_T^2)\, \sigma \cdot (z \times \boldsymbol{q}_T)\,]$$

Symmetric Lund model *spin dependence*

Physical meaning of renormalized $\Gamma(q',h,q)$





$W(q',h,q) = \Gamma^{\dagger}(q',h,q)\, \Gamma(q',h,q)$
is the *density of vertices* in the $M^4(q') \times M^4(q)$ space

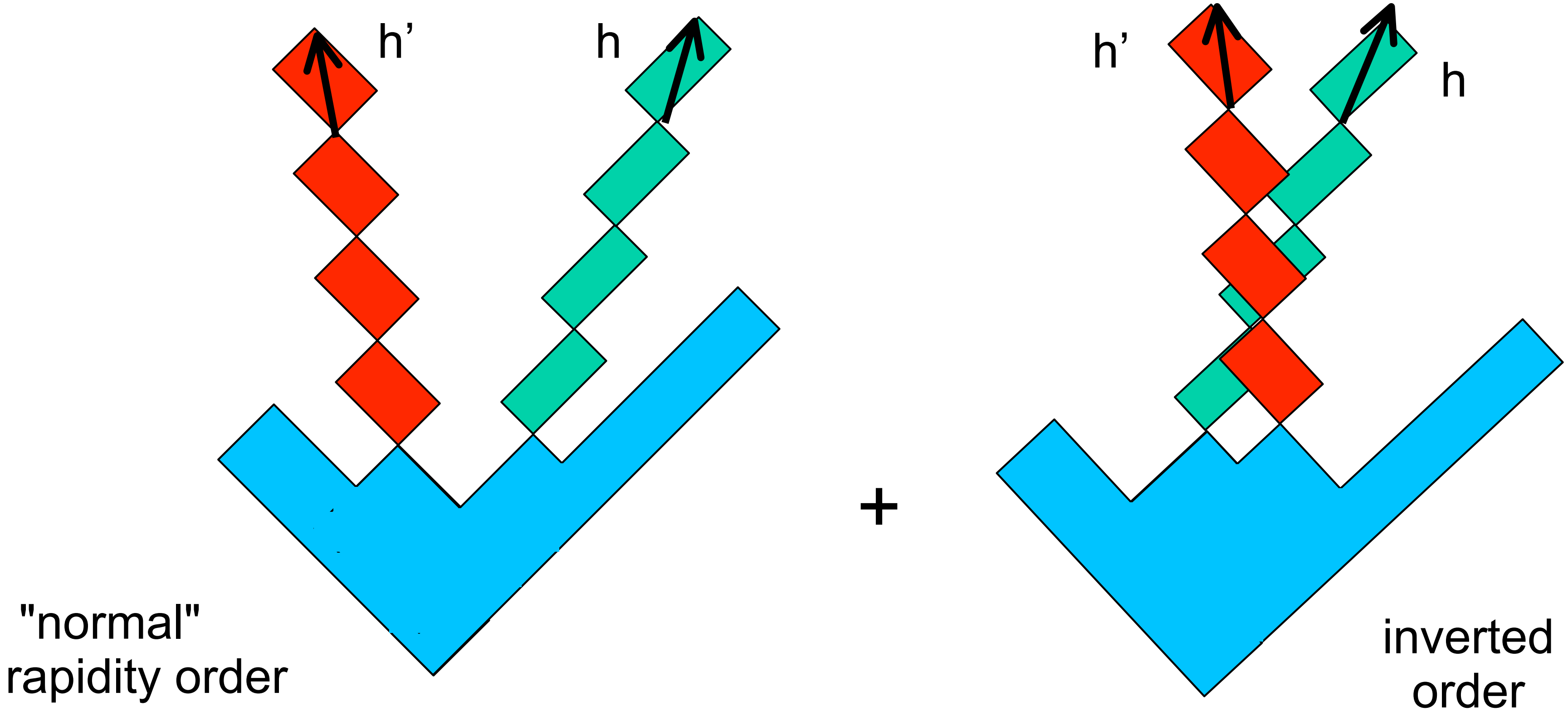
In spin space, it is a positive (4×4) matrix (after a partial transposition).

For h = pseudoscalar meson,

$\langle\, i'\, j \,|\; W(q',h,q) \;|\, j'\, i \,\rangle$

$= \exp(-b\, Q'^{+} Q^{-})\; (Q'^{+} Q^{-})^{a}\;\; \delta(p^2 - m_h^2)\; \langle\, j \,|\, \sigma_z \,|\, j' \,\rangle\; \langle\, i' \,|\, \sigma_z \,|\, i \,\rangle$

Another mechanism of Collins effect: Interference between permuted *diagrams*



The amplitude has a factor exp {– i(κ–i b) × bleu area}. The difference in blue areas leads to a phase difference between the two amplitudes. For identical h and h', it gives a Bose-Einstein correlation [Anderson & Hoffman]. It also give a relative Collins effect.

CONCLUSION

- We have built a recursive quark fragmentation model with quark spin.
- Important constraints : quark line reversal, cutoff in $\mathbf{q}_T$ (not only in $\mathbf{p}_T$)
- With complex parameter μ it reproduces the Collins effects of the classical "***string* + $^3P_0$** " mecanism and gives *jet handedness* in addition.
- It can serve as a guide to find efficient *estimators* in quark polarimetry.
- even in unpolarized experiments, hidden spin effects should be taken into account. It could lead to a better fit and a better understanding of hadronization.
- The "renormalized" input is a symmetric vertex amplitude $\Gamma(q',h,q)$. For instance,

$$\Gamma(q',h,q) = \exp\{- 0.5 \, (b \, |q'^{+}q^{-}| + B \, \mathbf{q'}_T^{2} + B \, \mathbf{q}_T^{2}) \, (\sigma_z + \mu \, \sigma_z \, \sigma . \, \mathbf{p}_T)$$

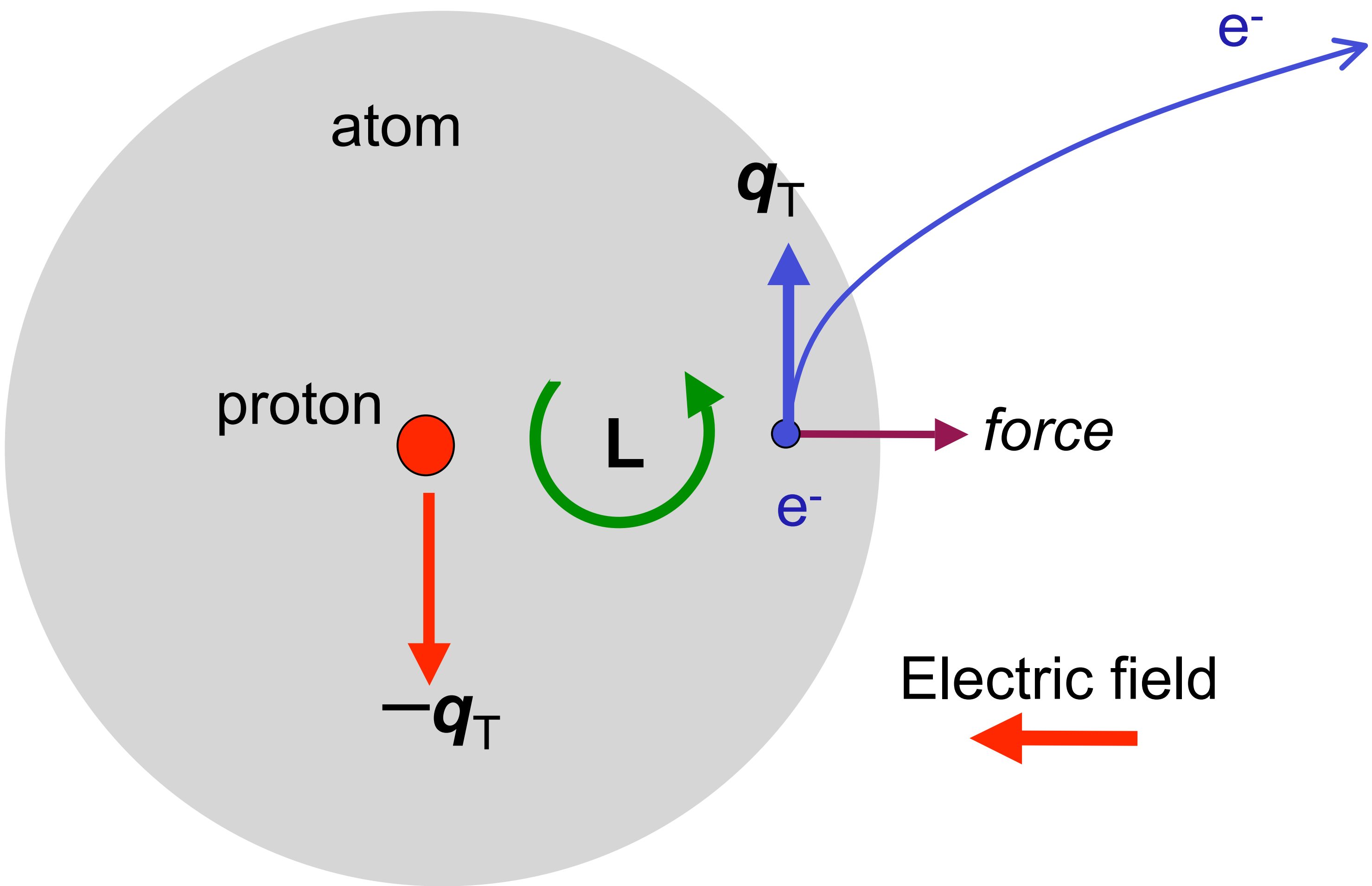
- A preliminary task is to calculate the propagator with an ordinary integration.
- The model differs from the Lund Symmetric model by the inclusion of spin matrices and a two-step algorithm for the splitting $q \rightarrow h + q'$

Thank you for attention !

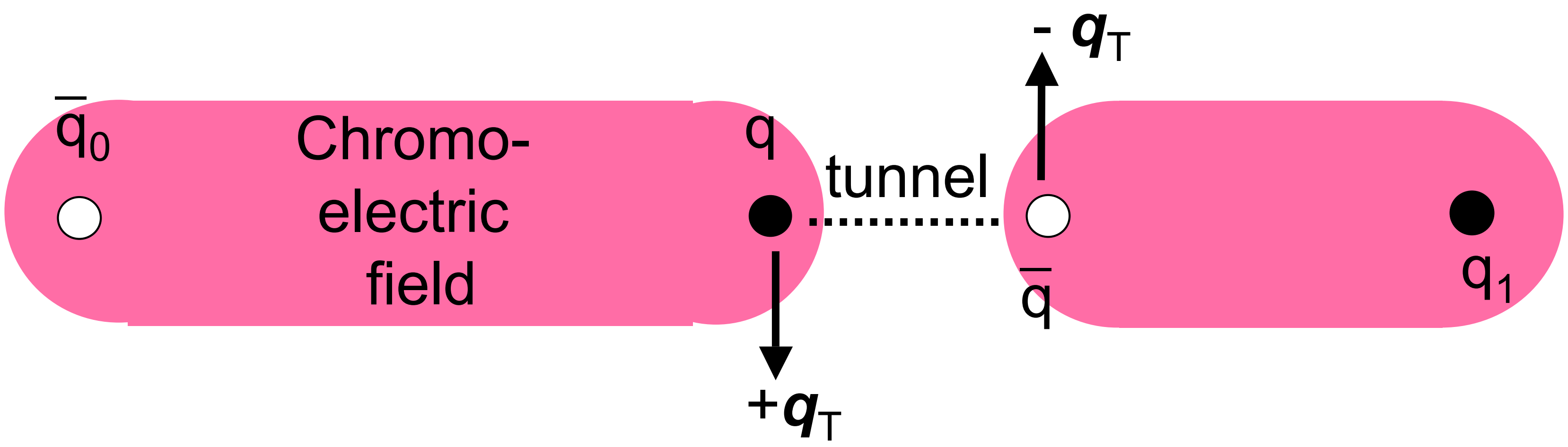
(Complement)

Analogue in atomic physics :

Ionisation of a hydrogen by tunnel effect in a static electric field [Essma Redouane-Salah, X.A.]



(Complement) The Schwinger mechanism



Explains the suppression of heavy quark, by the tunnel factor

$$\exp\{-\pi\,(m_q^{\,2} + \boldsymbol{q}_T^{\,2})\,/\kappa\}$$

… but predicts no q_T - S_q correlation [X.A., J. Czyzewski]

(contrary to the *string* + $^3P_0$ model).
The disagreement between the Schwinger and *"string+$^3P_0$"* mechanisms deserves attention.

(complement)
Unitarity diagram - bis