Phenomenology (partial) review

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UNIVERSITÀ DI PAVIA

TMDs with polarization

quark pol. / nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

TMDs in black survive transverse-momentum integration
TMDs in red are T-odd

see, e.g., AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

TMDs with polarization

helicity

quark pol.

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Twist-2 TMDs

transversity

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TMDs with polarization

helicity

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Twist-2 TMDs

Sivers

transversity

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TMDs with polarization

helicity

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Boer-Mulders

Sivers

Twist-2 TMDs

transversity

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see, e.g., AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

TMDs with polarization

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Twist-2 TMDs

helicity

Boer-Mulders

Sivers

transversity

pretzelosity

TMDs in black survive transverse-momentum integration
TMDs in red are T-odd

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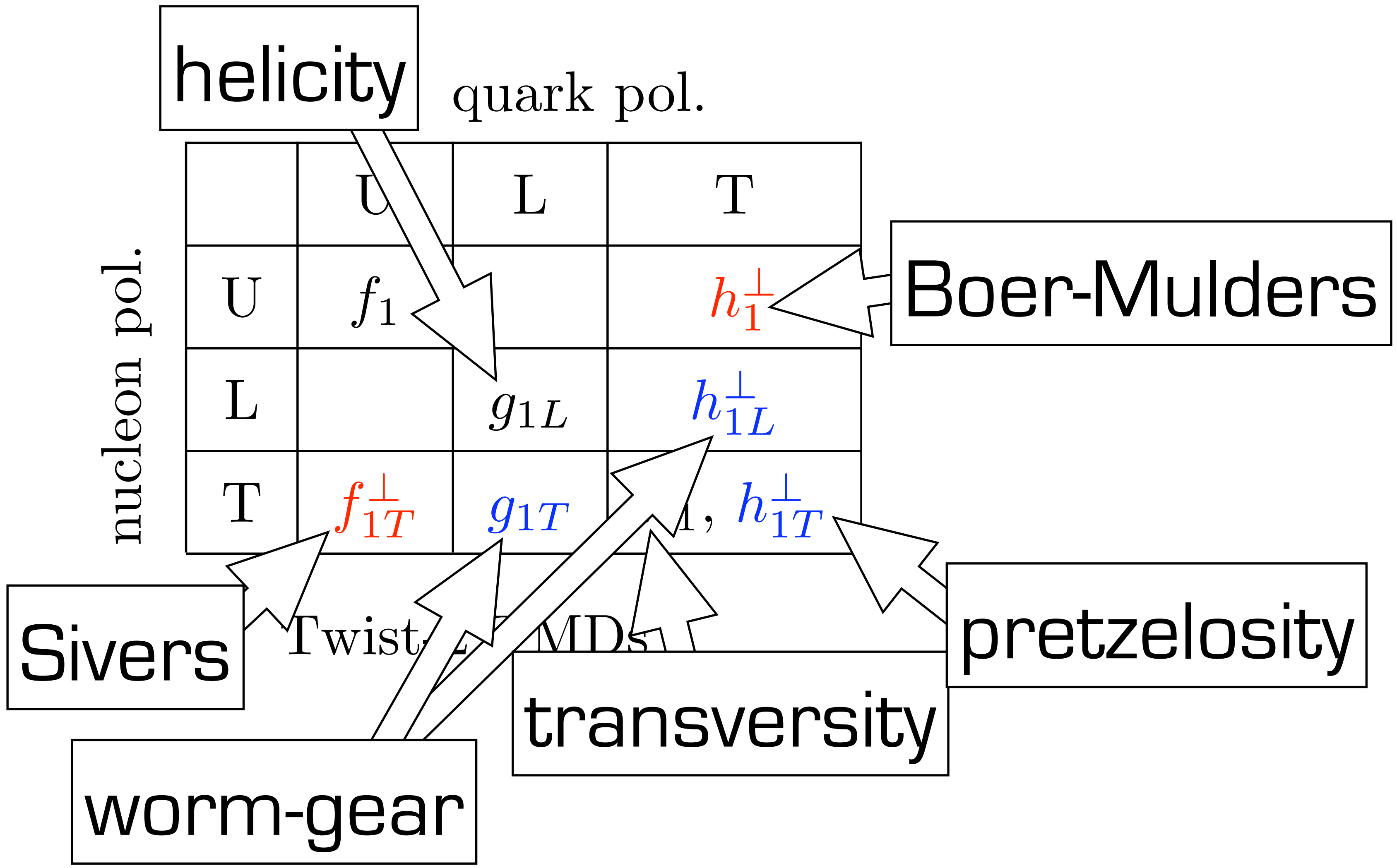
TMDs with polarization



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TMDs with polarization

helicity

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T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

Boer-Mulders

Sivers

worm-gear

transversity

"the one who shall not be named"

TMDs in black survive transverse-momentum integration
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TMDs

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
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TMD Parton Distribution Functions (TMD PDFs)

quark pol.

U	L	T
D_1		$H_1^\perp$

TMD Fragmentation Functions (TMD FFs)

TMDs

quark pol.

nucleon pol.		U	L	T
	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
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TMD Parton Distribution Functions
(TMD PDFs)

quark pol.

U	L	T
D_1		$H_1^\perp$

Collins

TMD Fragmentation Functions
(TMD FFs)

Focus on

Focus on

- Unpolarized TMD PDFs and FFs

Focus on

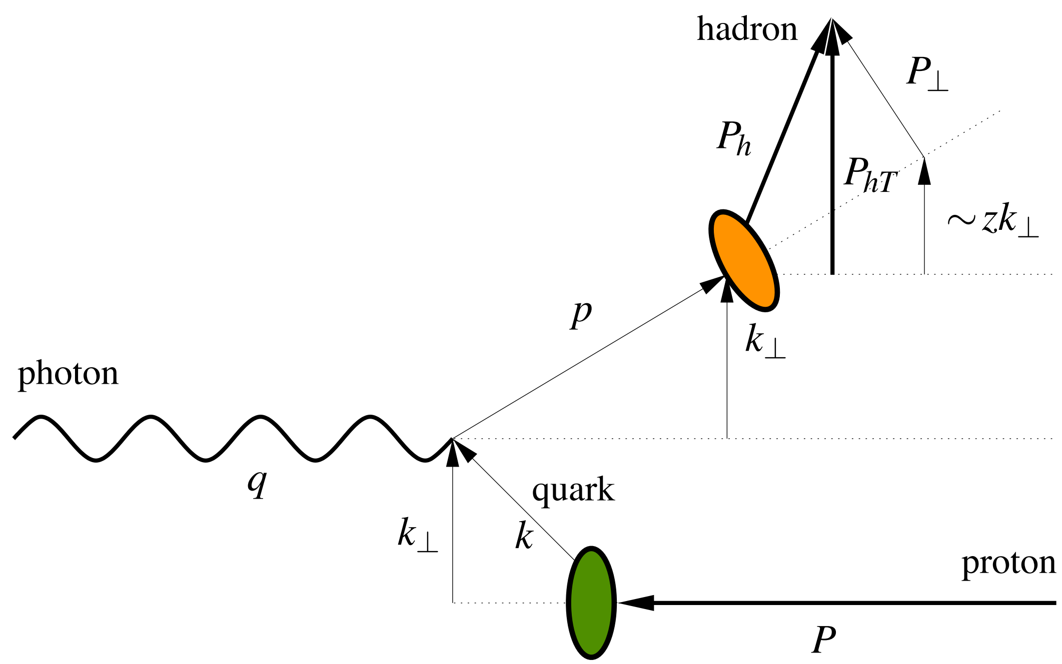
- Unpolarized TMD PDFs and FFs
- Sivers function

Focus on

- Unpolarized TMD PDFs and FFs
- Sivers function
- Collins function and transversity

Unpolarized TMDs

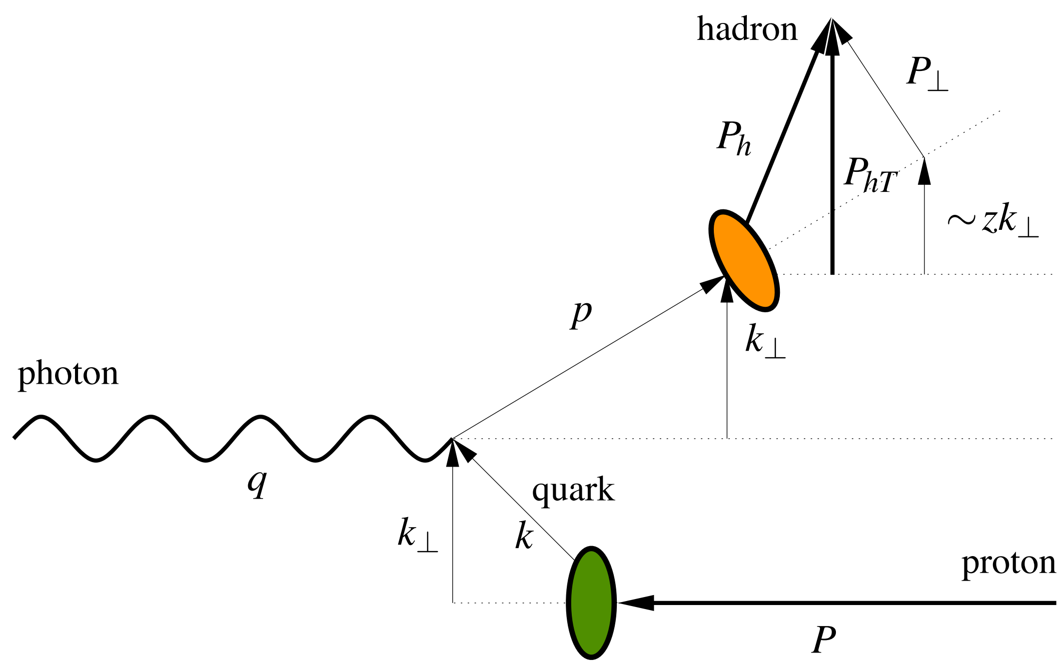
Structure functions



"Parton model"

$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2) = \sum_a \int d\boldsymbol{k}_\perp\, d\boldsymbol{P}_\perp\, f_1^a\big(x,\boldsymbol{k}_\perp^2\big)\, D_1^{a\to h}\big(z,\boldsymbol{P}_\perp^2\big)\, \delta\big(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp\big) + \mathcal{O}\big(M^2/Q^2\big)$$

Structure functions



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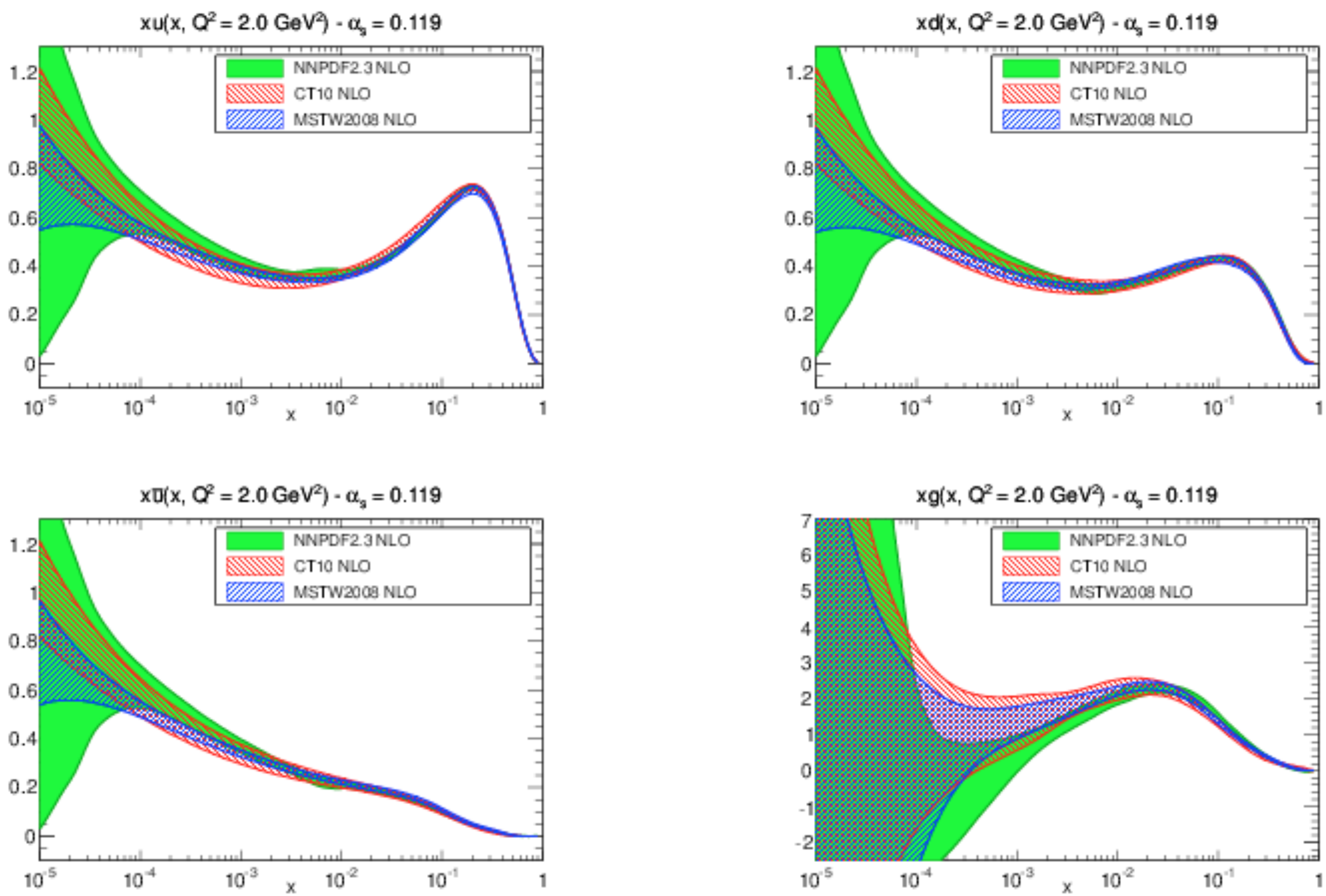
With QCD corrections

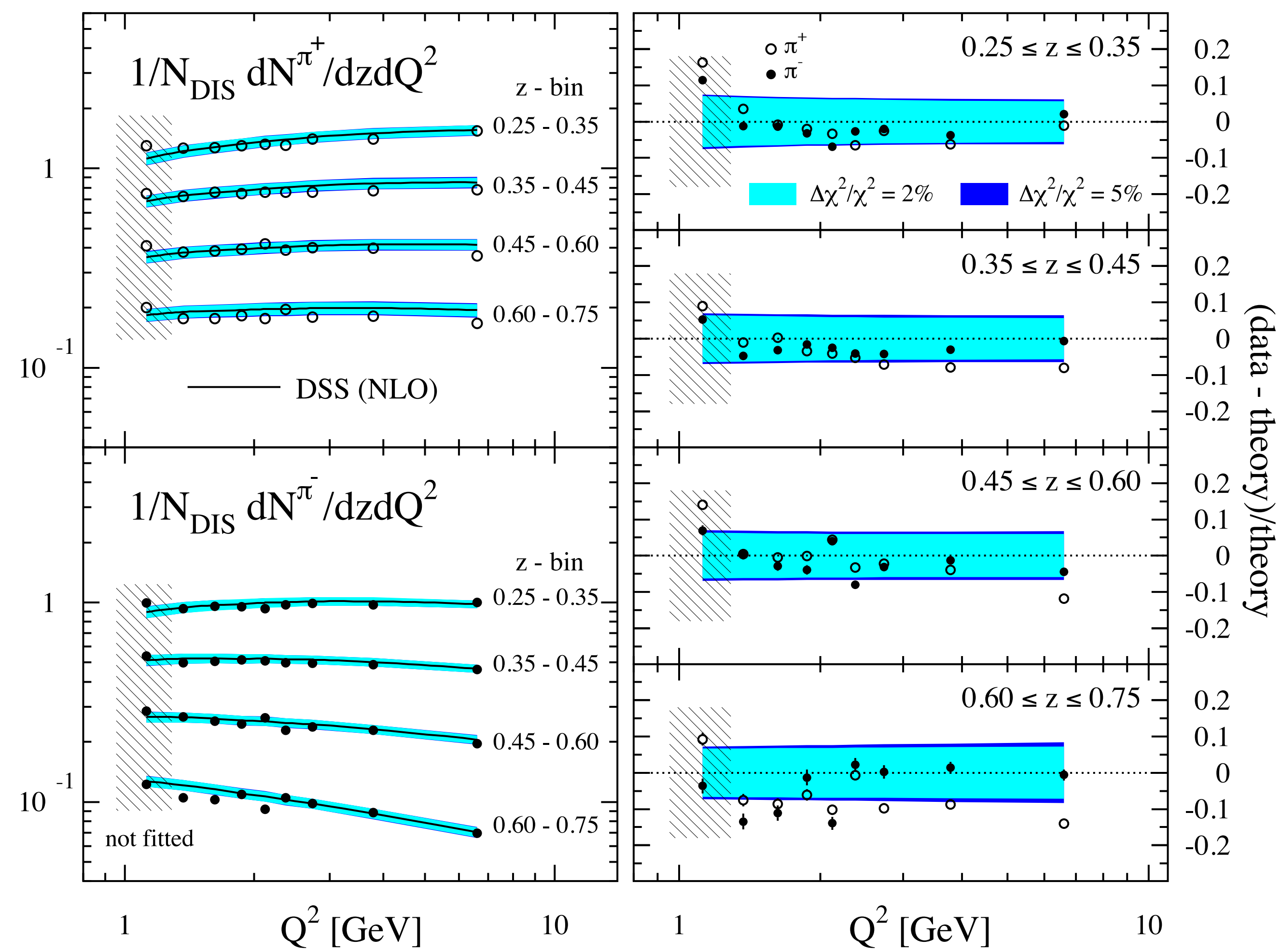
$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2) = x\sum_a \mathcal{H}^a_{UU,T}(Q^2;\mu^2) \int d\boldsymbol{k}_\perp \, d\boldsymbol{P}_\perp \, f_1^a\big(x,\boldsymbol{k}_\perp^2;\mu^2\big)\, D_1^{a\to h}\big(z,\boldsymbol{P}_\perp^2;\mu^2\big)\, \delta\big(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp\big)$$
$$+ Y_{UU,T}\big(Q^2,\boldsymbol{P}_{hT}^2\big) + \mathcal{O}\big(M^2/Q^2\big)$$

Collinear PDFs

NNPDF http://nnpdf.hepforge.org

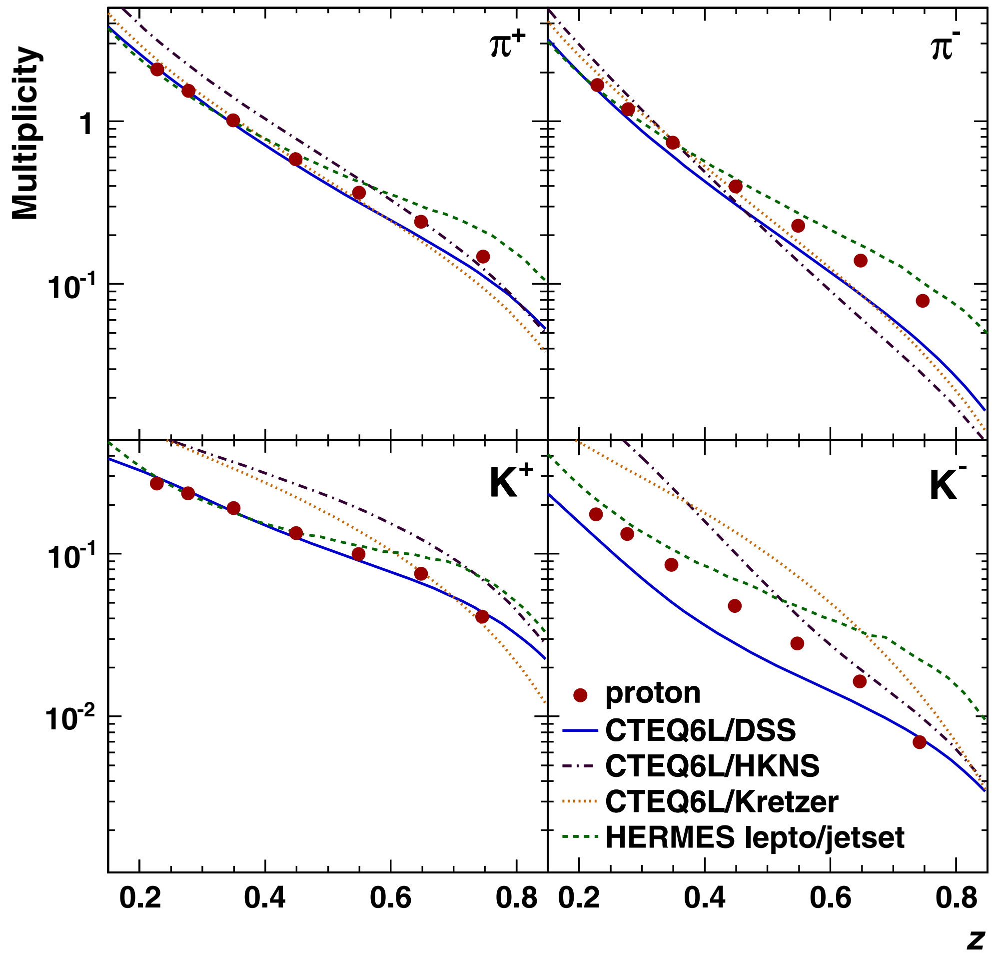


Collinear FFs

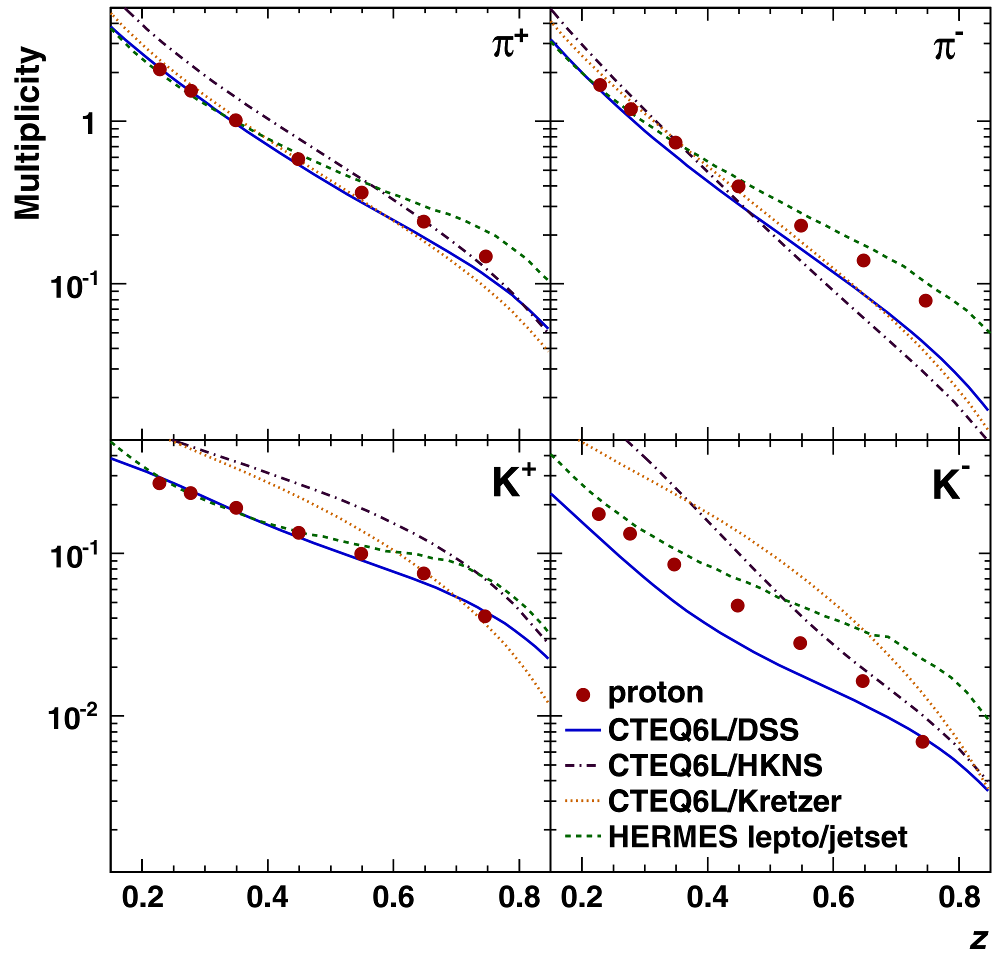


Epele, Llubarof, Sassot, Stratmann, PRD68 (12)

Is the collinear description good?



Is the collinear description good?



χ^2/d.o.f.				
	$Q^2 > 1.4\,\mathrm{GeV}^2$	$Q^2 > 1.4\,\mathrm{GeV}^2$ (no VM subtr.)	$Q^2 > 1.4\,\mathrm{GeV}^2$ (with evolution)	$Q^2 > 1.6\,\mathrm{GeV}^2$
global	2.86	3.90	3.55	2.29
$p \to K^-$	2.25	2.27	1.38	2.38
$p \to \pi^-$	3.39	6.58	5.03	2.70
$p \to \pi^+$	1.87	2.45	2.74	1.16
$p \to K^+$	0.89	0.85	1.13	0.59
$D \to K^-$	4.26	4.22	2.81	4.45
$D \to \pi^-$	5.05	8.66	7.96	3.42
$D \to \pi^+$	3.33	4.61	5.19	2.29
$D \to K^+$	1.80	1.57	2.17	1.31

With MSTW08 + DSS

table from Signori, Bacchetta, Radici, Schnell, JHEP 11 (13)

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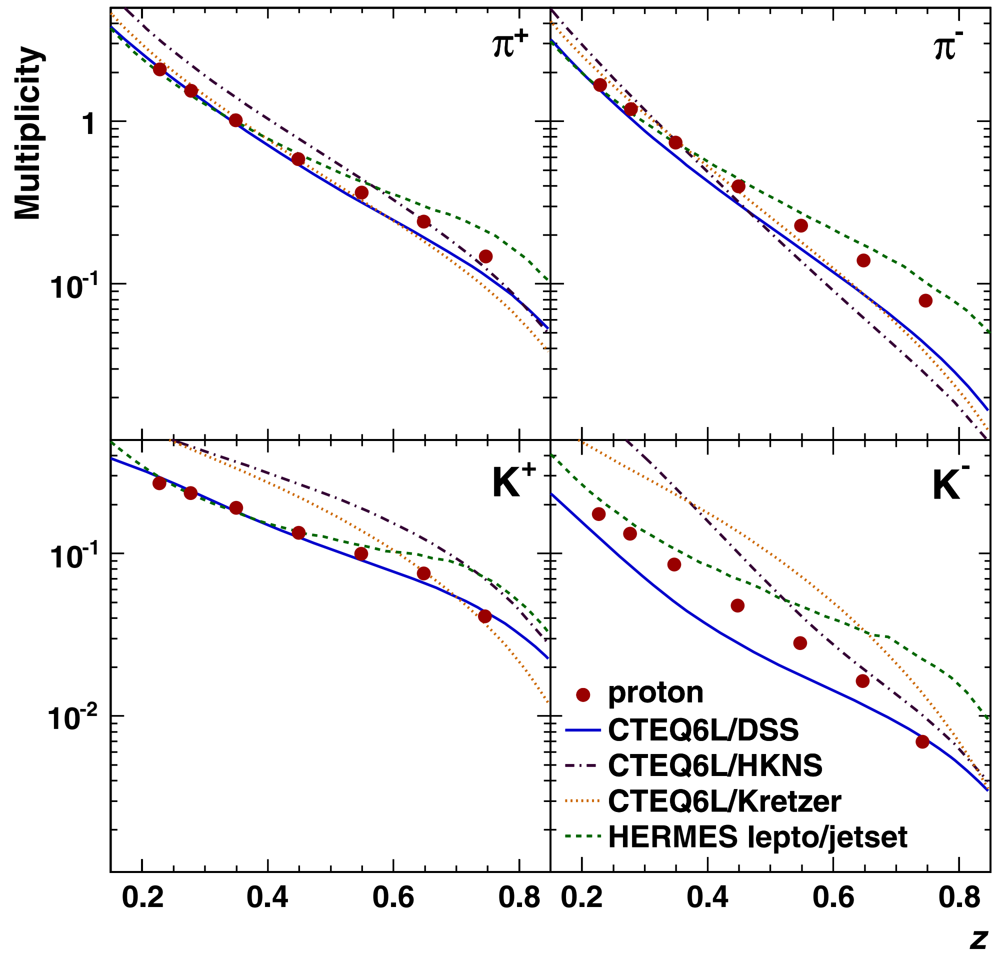

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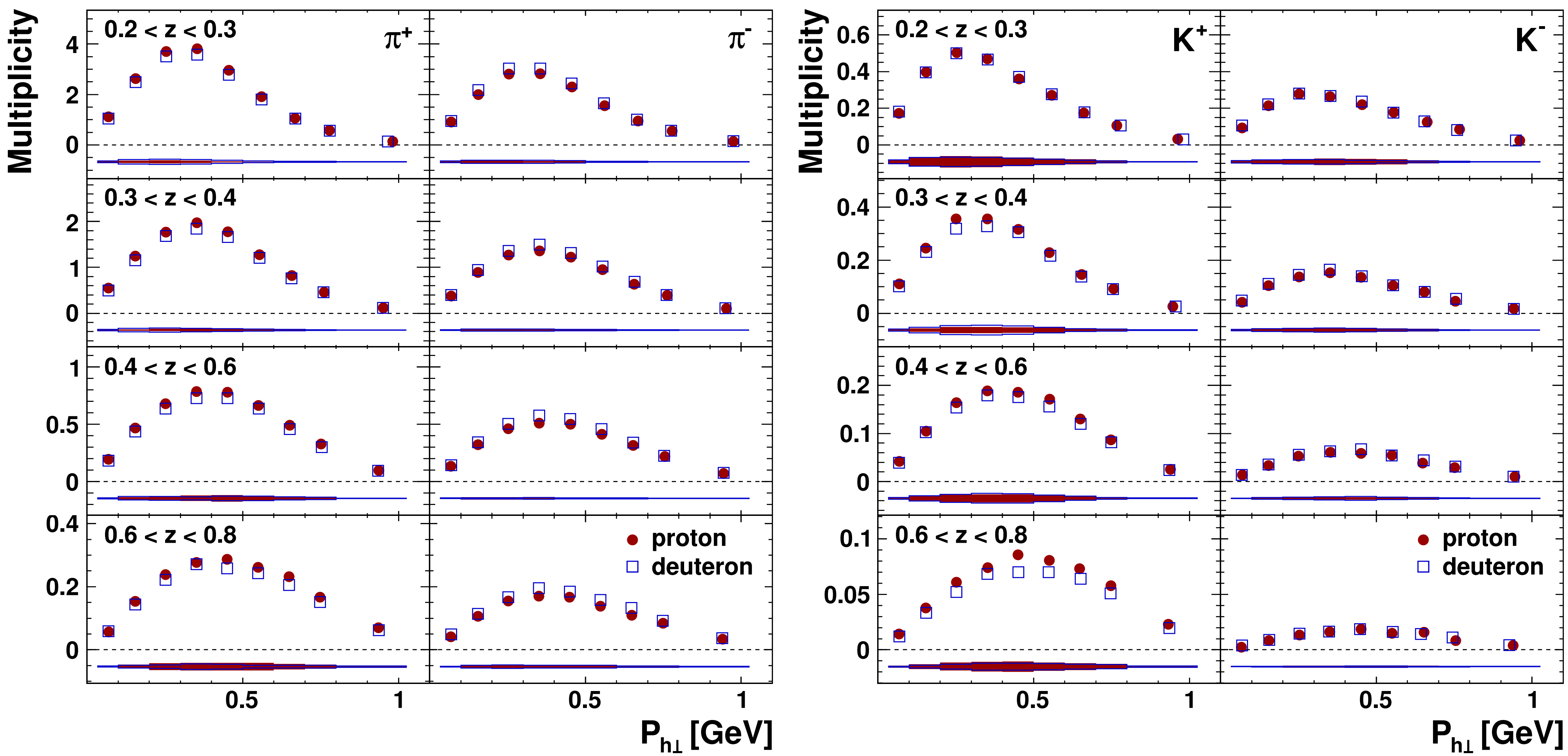
With MSTW08 + DSS

table from Signori, Bacchetta, Radici, Schnell, JHEP 11 (13)

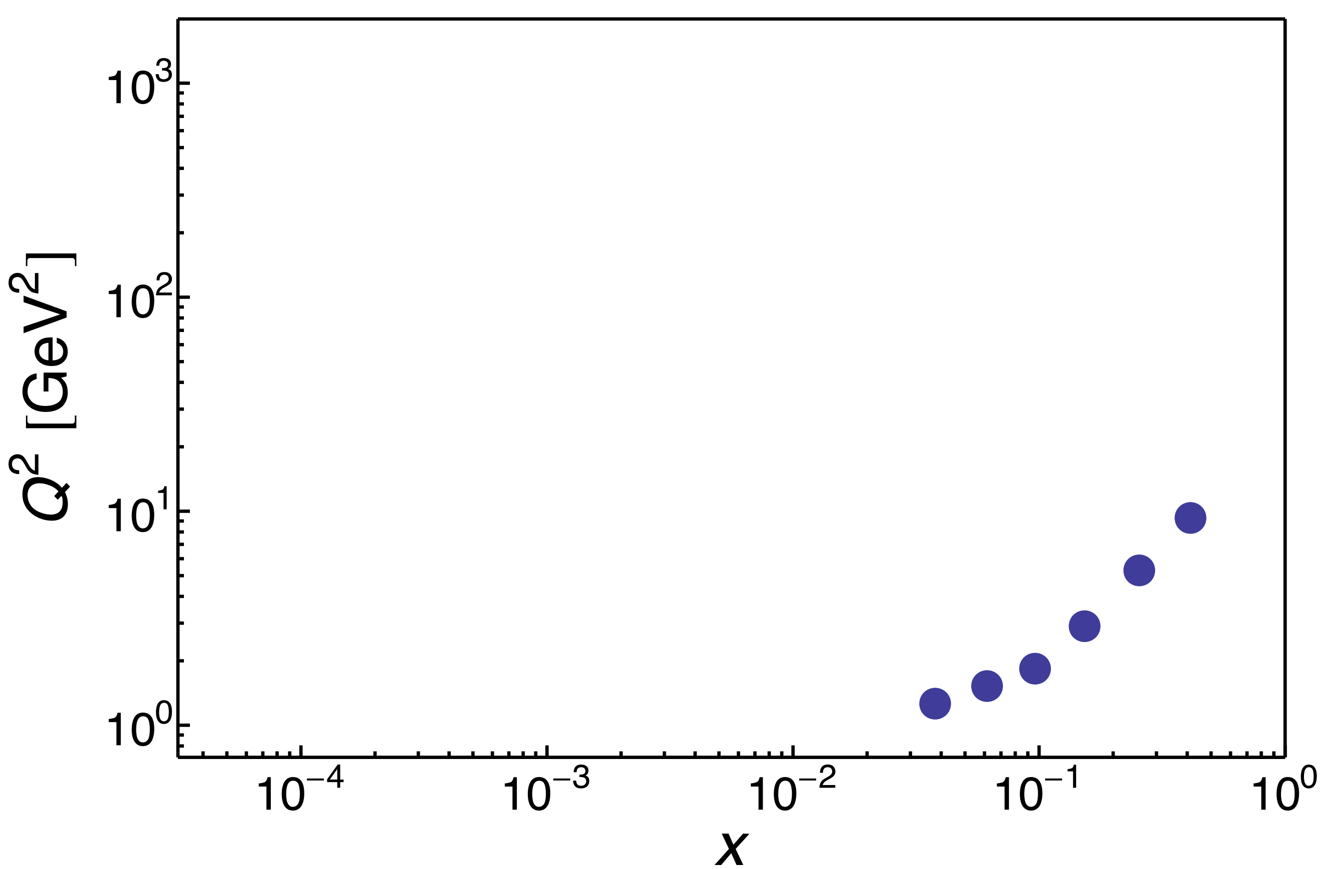
Now, let's move to the transverse-momentum dependence...

Very recent data

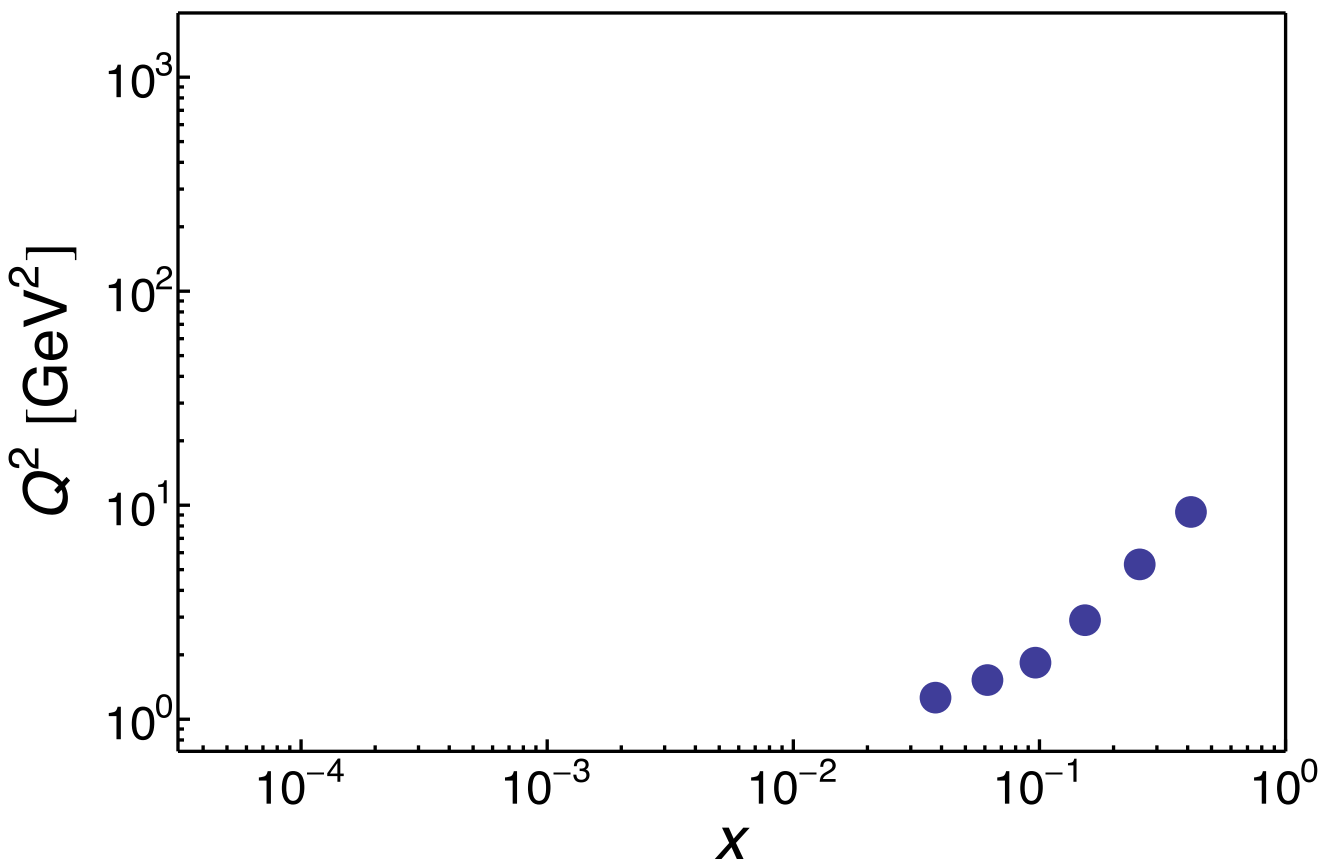


Airapetian et al., PRD87 (2013)

Limited x - Q^2 coverage



Limited x - Q^2 coverage



6 bins in x, 8 bins in z,

7 bins in P_{hT},

2 targets, 4 final-state hadrons,

= 2688 data points

The Pavia-Amsterdam-Bilbao fit

The Pavia-Amsterdam-Bilbao fit

Signori, Bacchetta, Radici, Schnell, JHEP 11 (13)

The Pavia-Amsterdam-Bilbao fit

- x dependence of distribution transverse momentum

Signori, Bacchetta, Radici, Schnell, JHEP 11 (13)

The Pavia-Amsterdam-Bilbao fit

- x dependence of distribution transverse momentum
- z dependence of fragmentation transverse momentum

Signori, Bacchetta, Radici, Schnell, JHEP 11 (13)

The Pavia-Amsterdam-Bilbao fit

- x dependence of distribution transverse momentum
- z dependence of fragmentation transverse momentum
- flavor dependence

Signori, Bacchetta, Radici, Schnell, JHEP 11 (13)

The Pavia-Amsterdam-Bilbao fit

- x dependence of distribution transverse momentum
- z dependence of fragmentation transverse momentum
- flavor dependence
- error treatment based on replica method

Signori, Bacchetta, Radici, Schnell, JHEP 11 (13)

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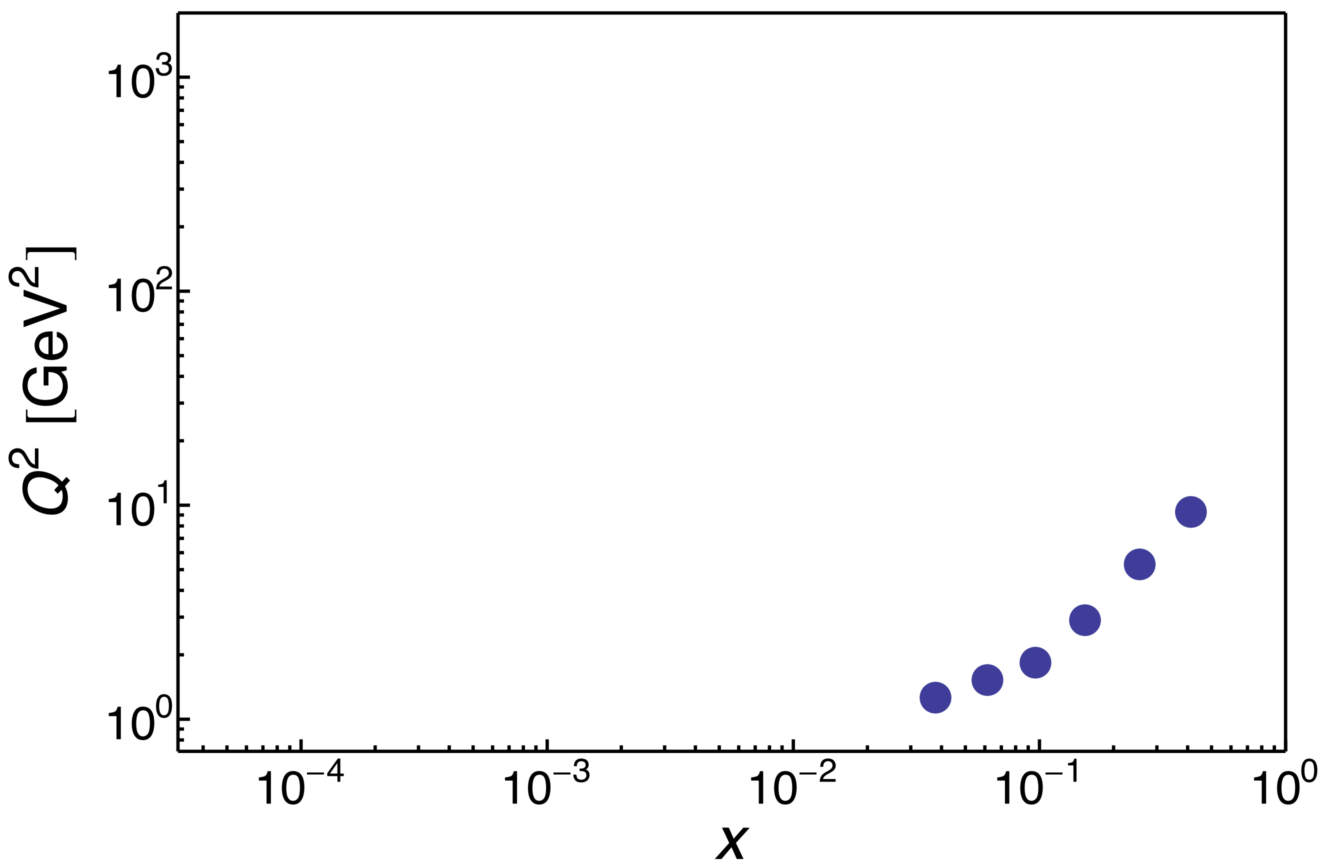
Signori, Bacchetta, Radici, Schnell, JHEP 11 (13)

The Pavia-Amsterdam-Bilbao fit

- x dependence of distribution transverse momentum
- z dependence of fragmentation transverse momentum
- flavor dependence
- error treatment based on replica method
- no evolution (not even collinear!)

Signori, Bacchetta, Radici, Schnell, JHEP 11 (13)

Pavia fit (no evo)



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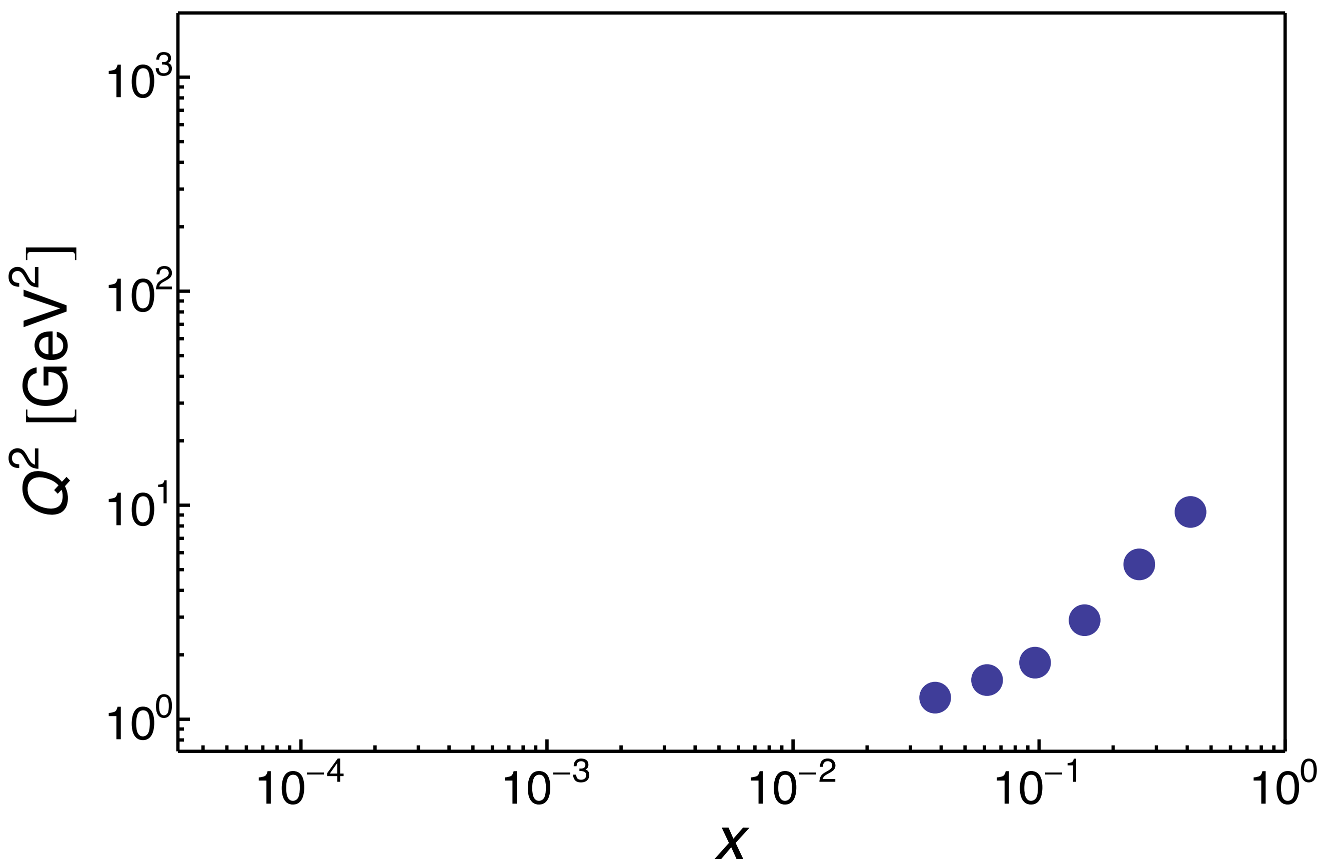

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We selected 1538 data points

$Q^2 > 1.4\ \mathrm{GeV}^2$

$z < 0.7$

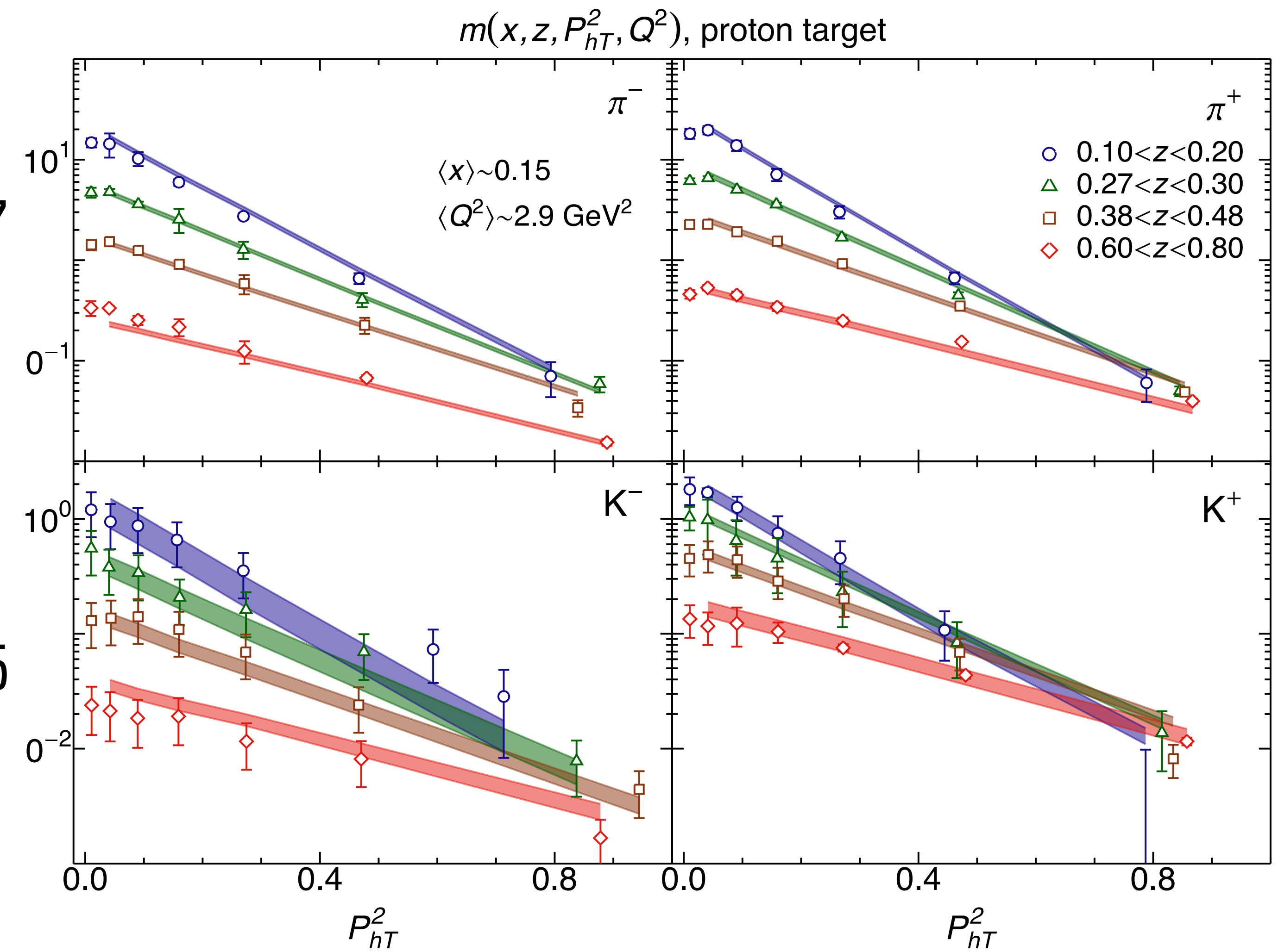
$0.15\ \mathrm{GeV}^2 < P_{hT} < Q^2/3$

Pavia fit (no evo)

Global $\chi^2/\text{dof} = 1.63 \pm 0.12$

1.80±0.27

2.64±0.21



0.78±0.15

0.46±0.07

Pavia fit (no evo)

Global χ^2/dof = 1.63±0.12

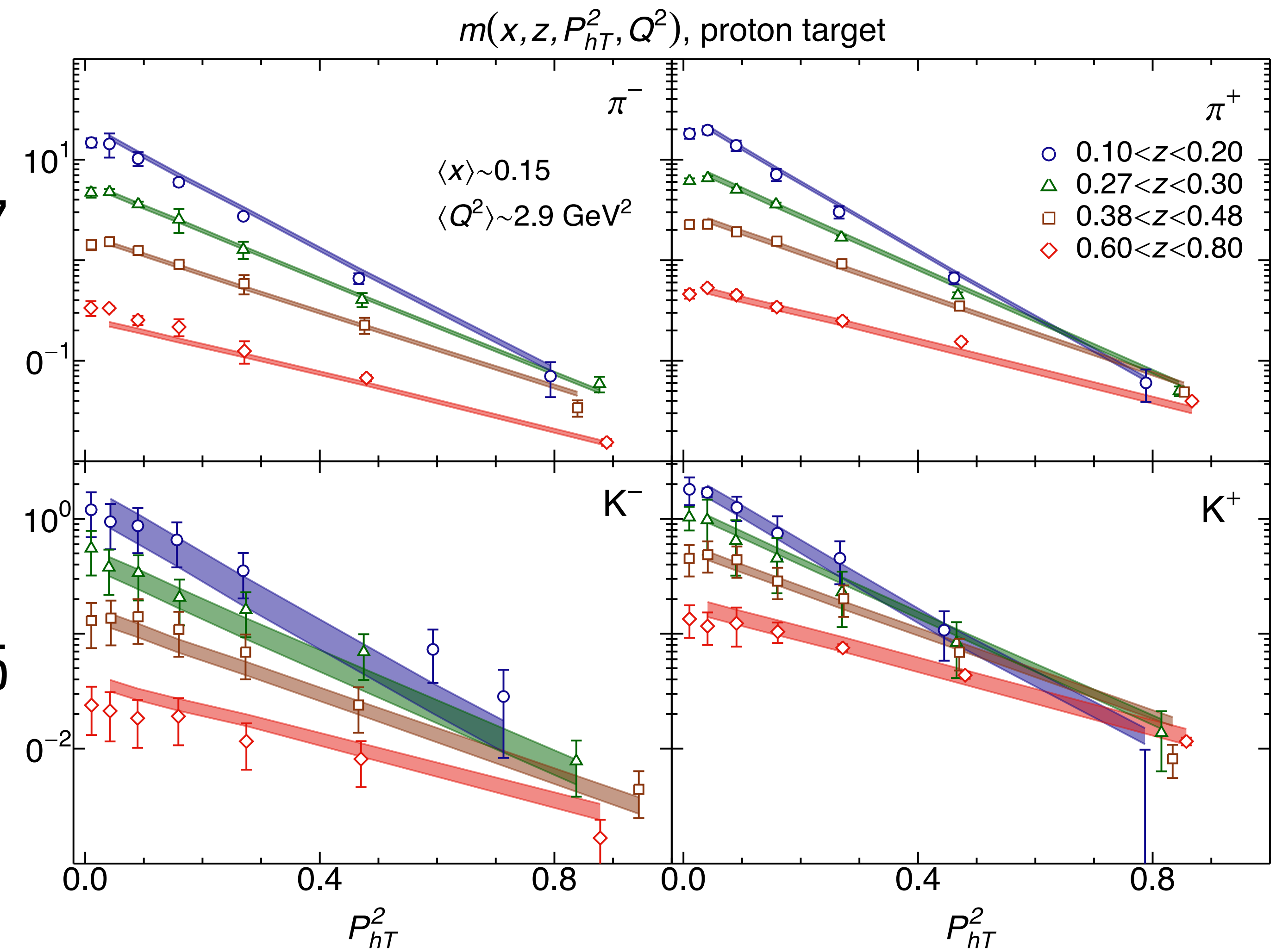
Without flavor dep.: global χ^2/dof = 1.72±0.11

1.80±0.27

1.83±0.25

2.64±0.21

2.89±0.23



0.78±0.15

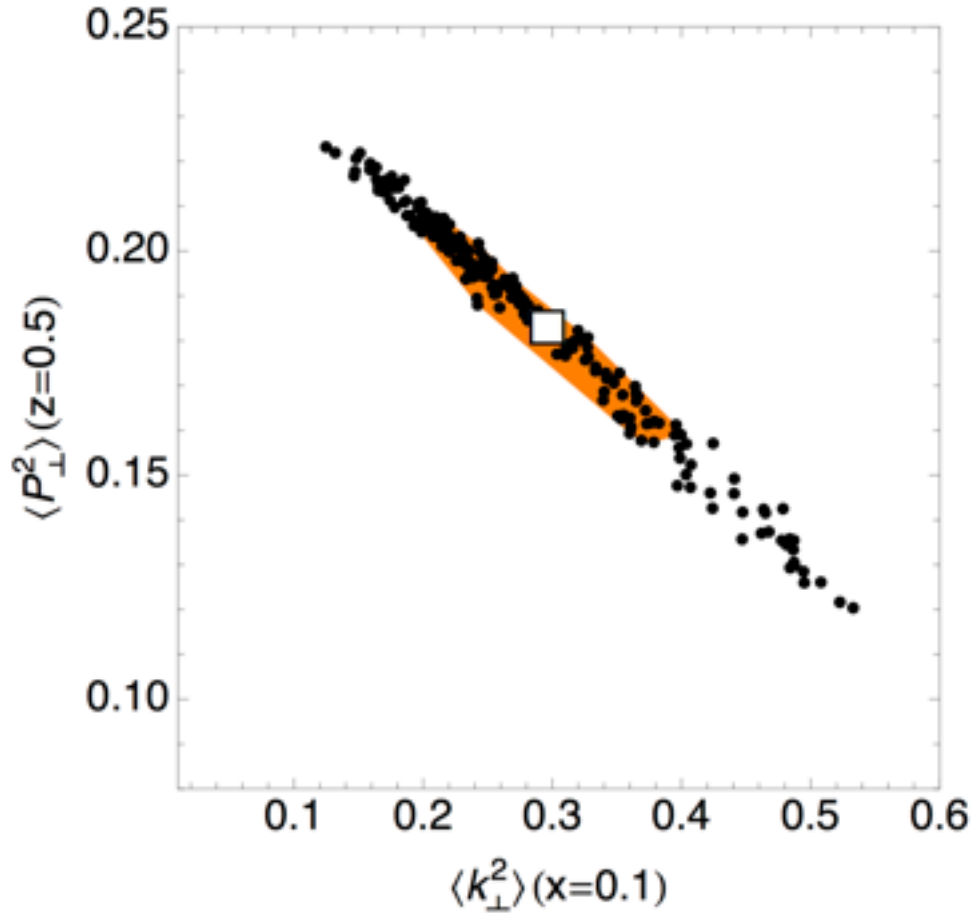
0.87±0.16

0.46±0.07

0.43±0.07

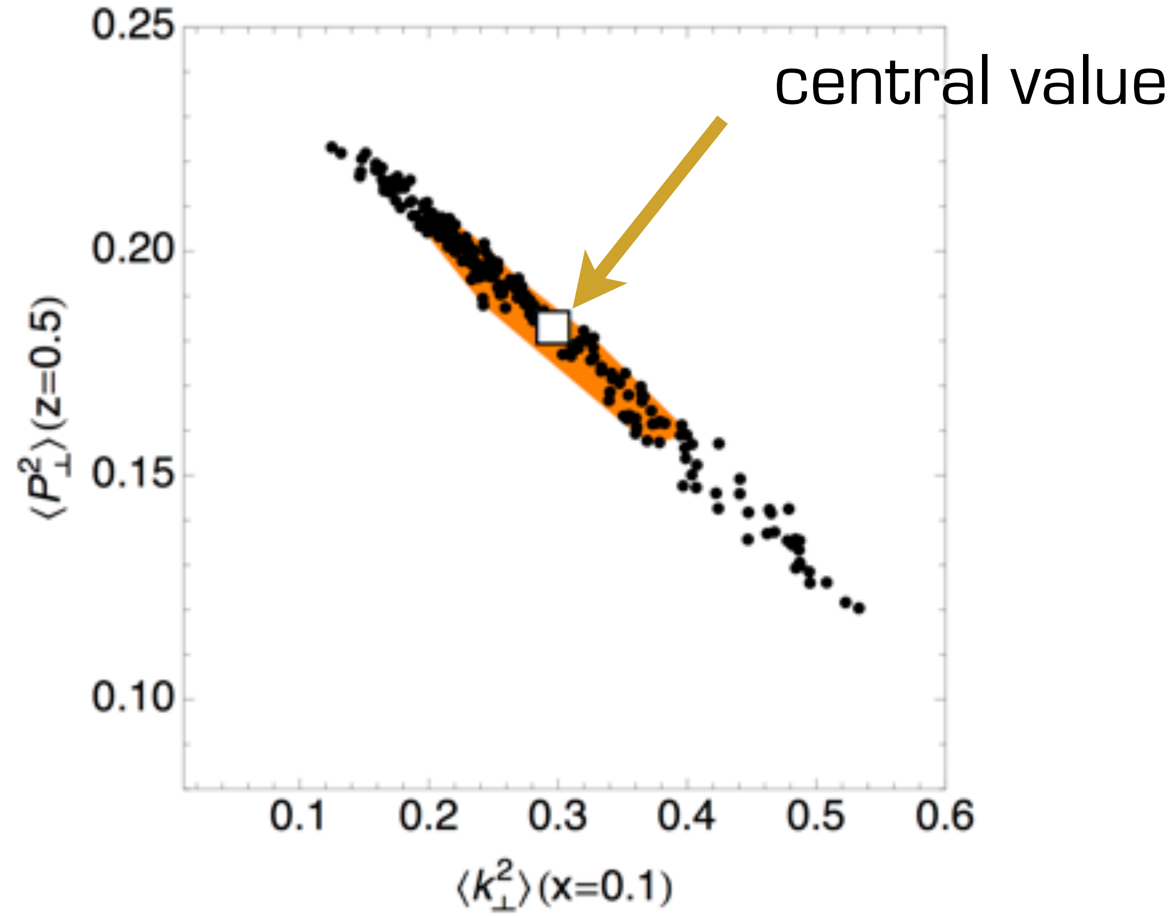
Results: flavor-independent fit

Transverse momentum
in FFs



Transverse momentum
in PDFs

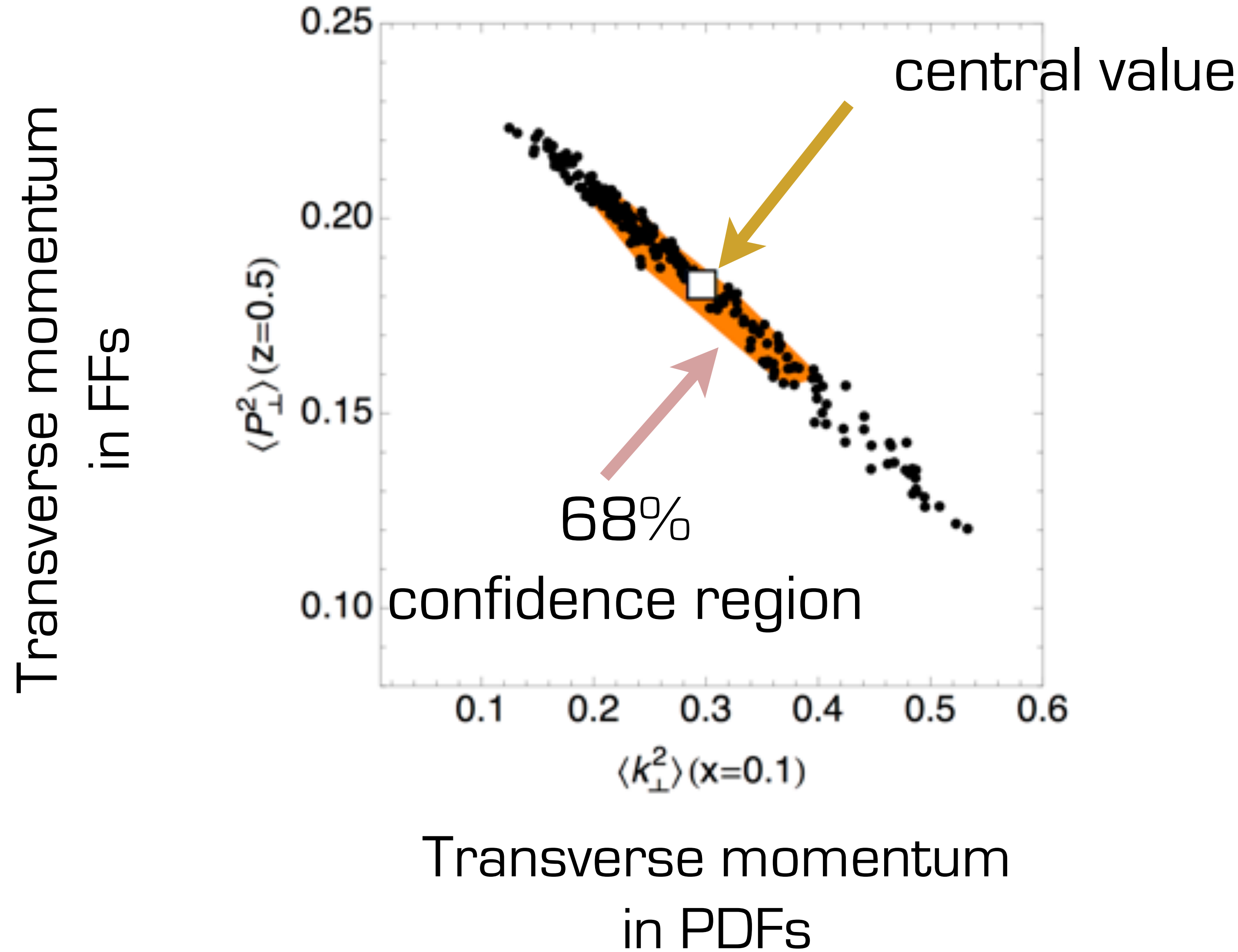
Results: flavor-independent fit



Transverse momentum in FFs

Transverse momentum in PDFs

Results: flavor-independent fit



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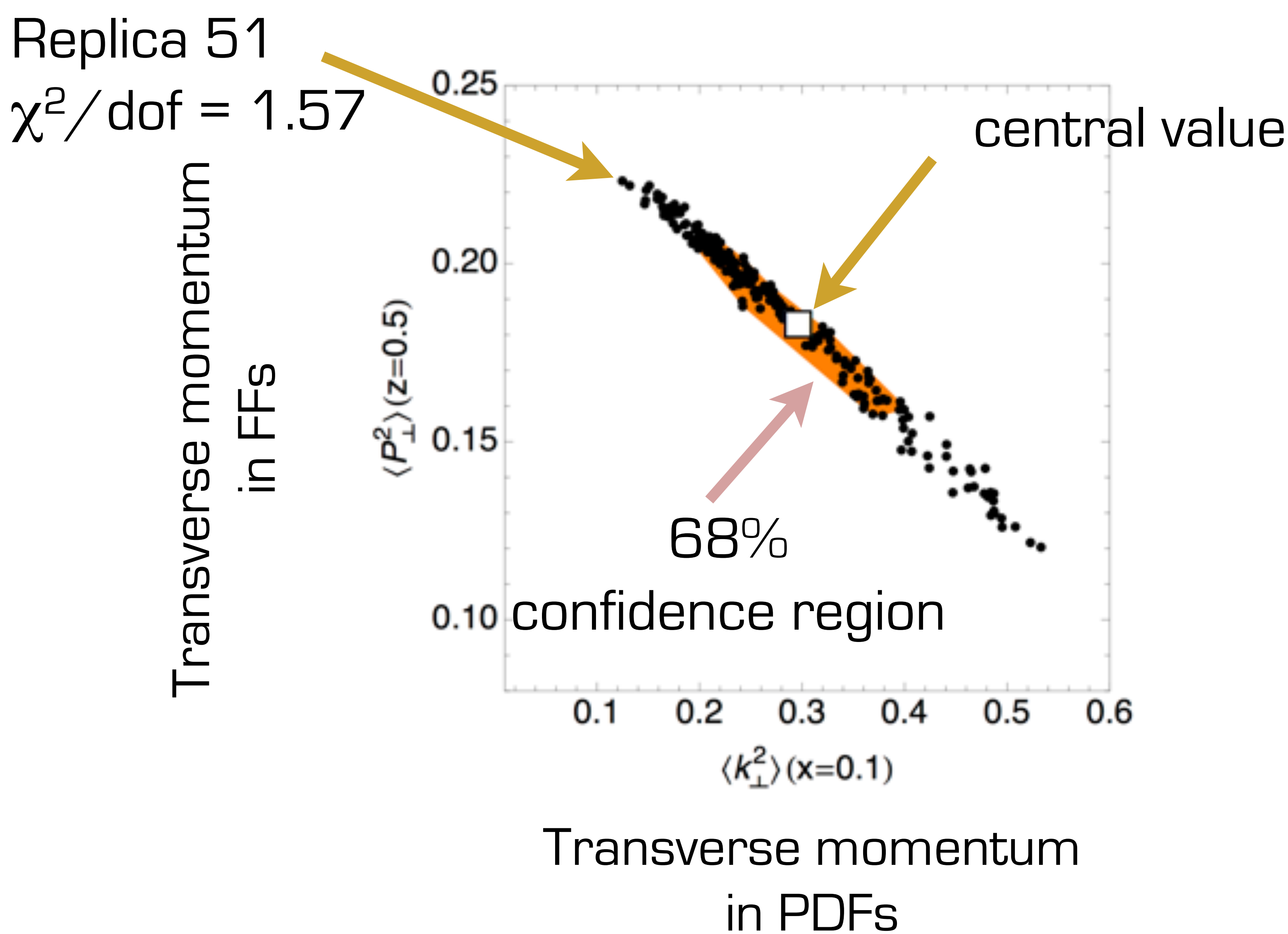

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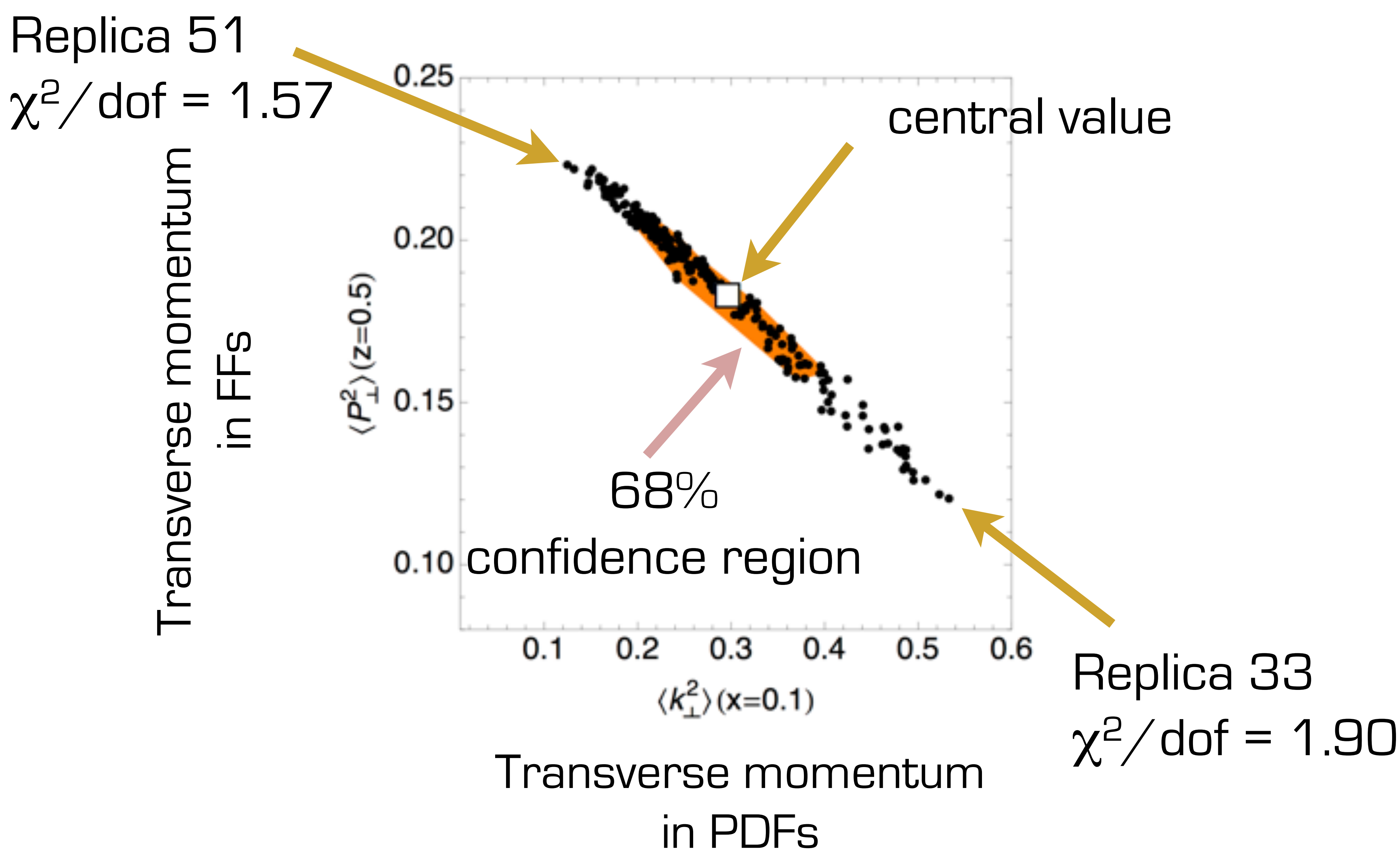

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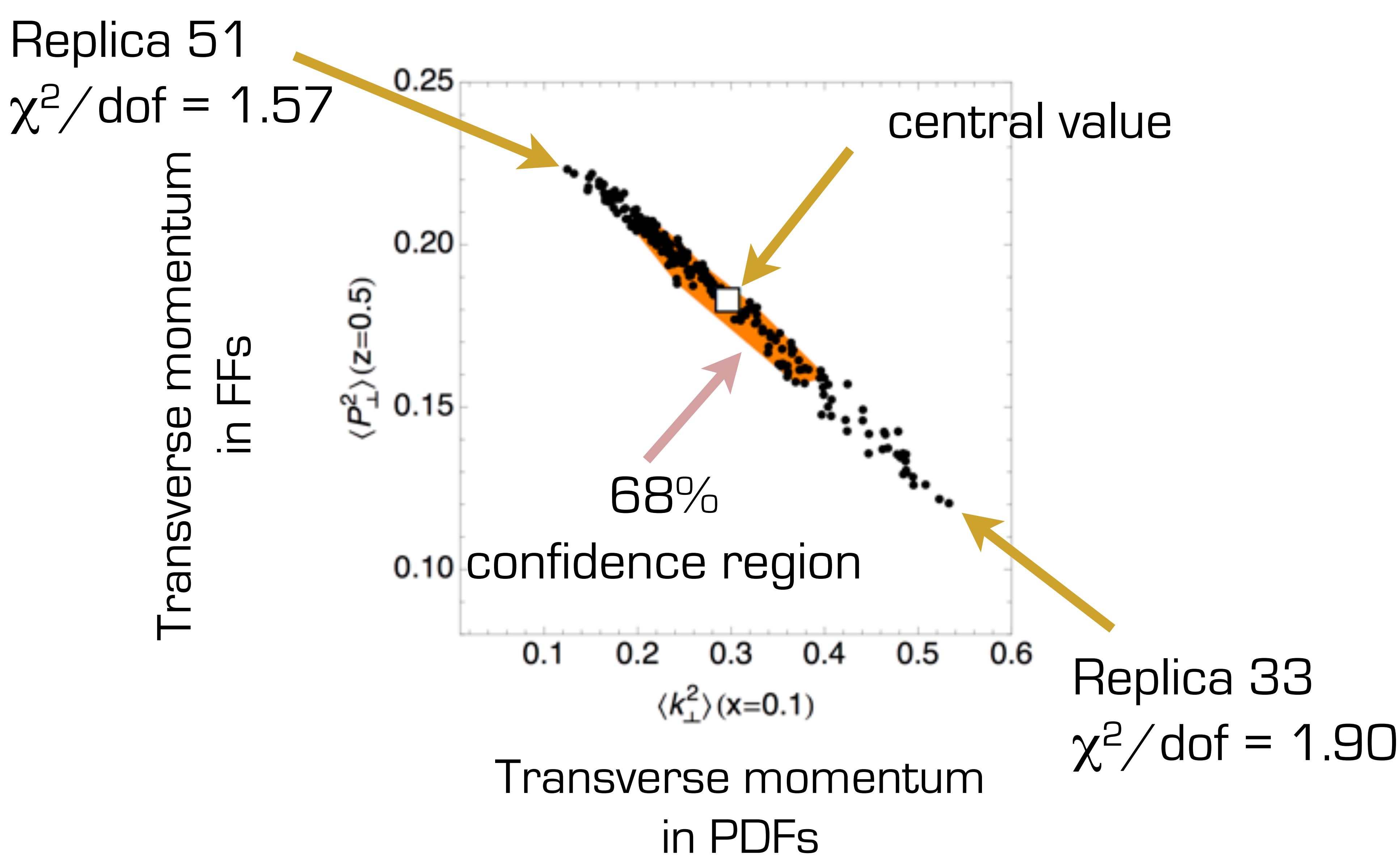
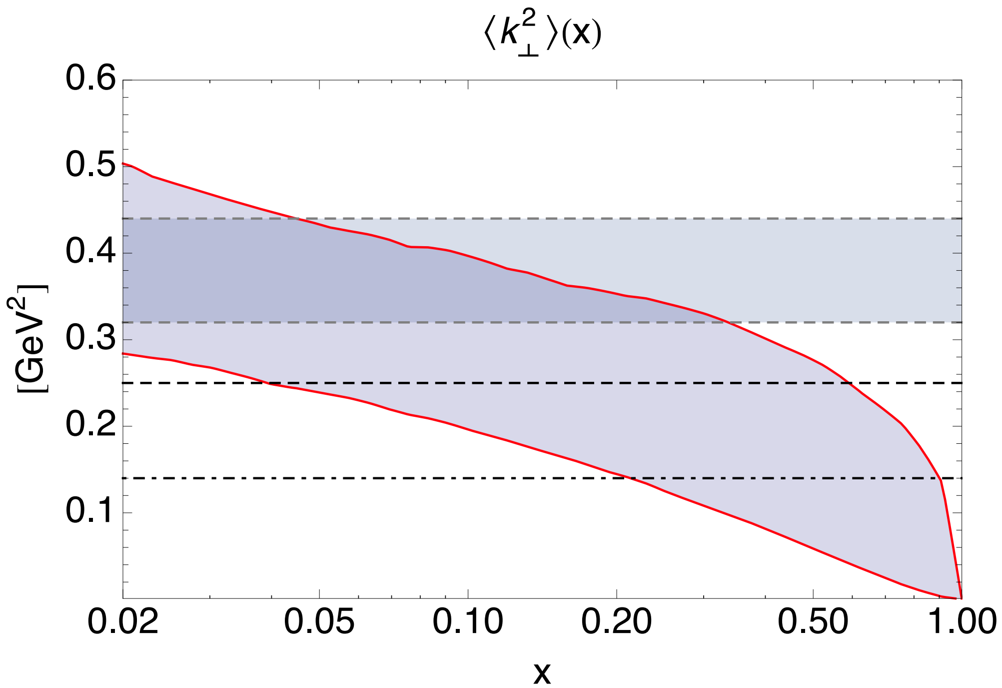
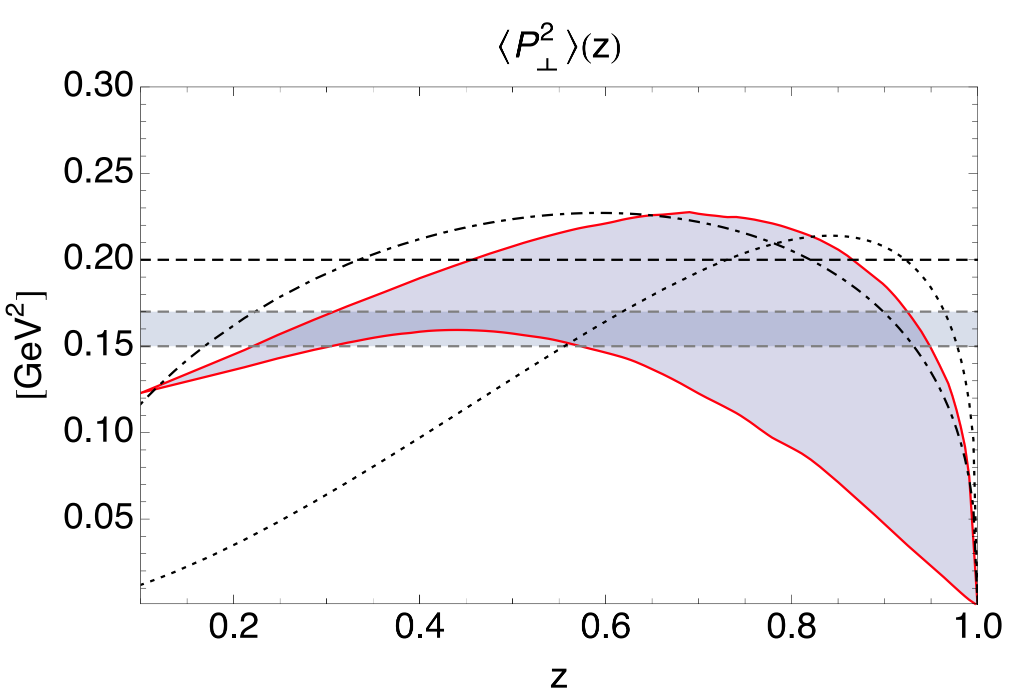


Strong anticorreleation between distribution and fragmentation

68% bands

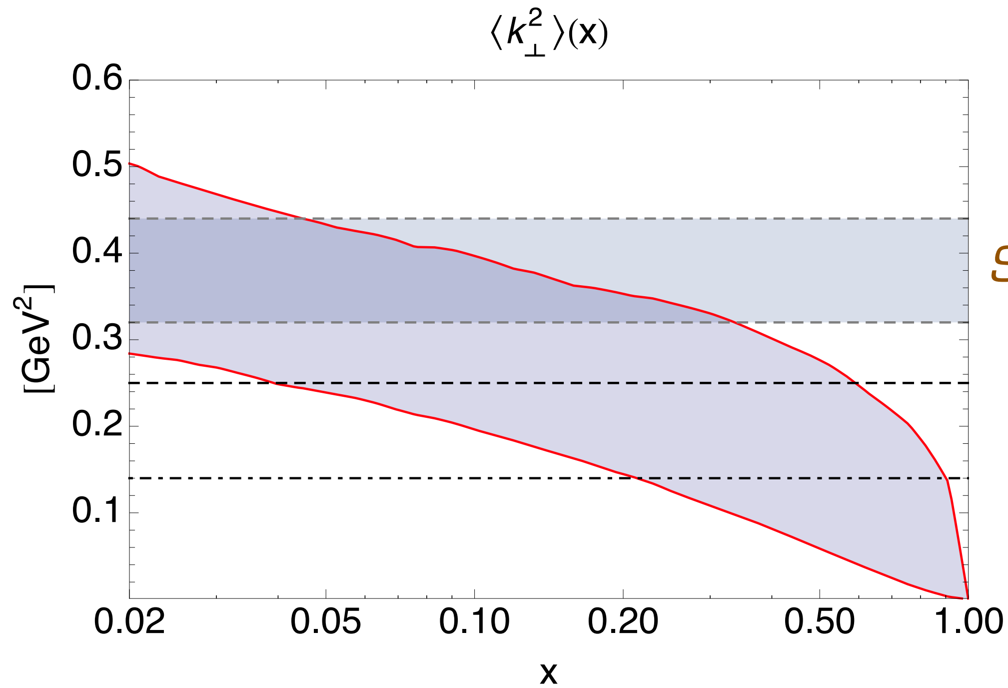


$\langle k_\perp^2 \rangle$(x)
[GeV^2]
0.6
0.5
0.4
0.3
0.2
0.1
0.02
0.05
0.10
0.20
0.50
1.00
x

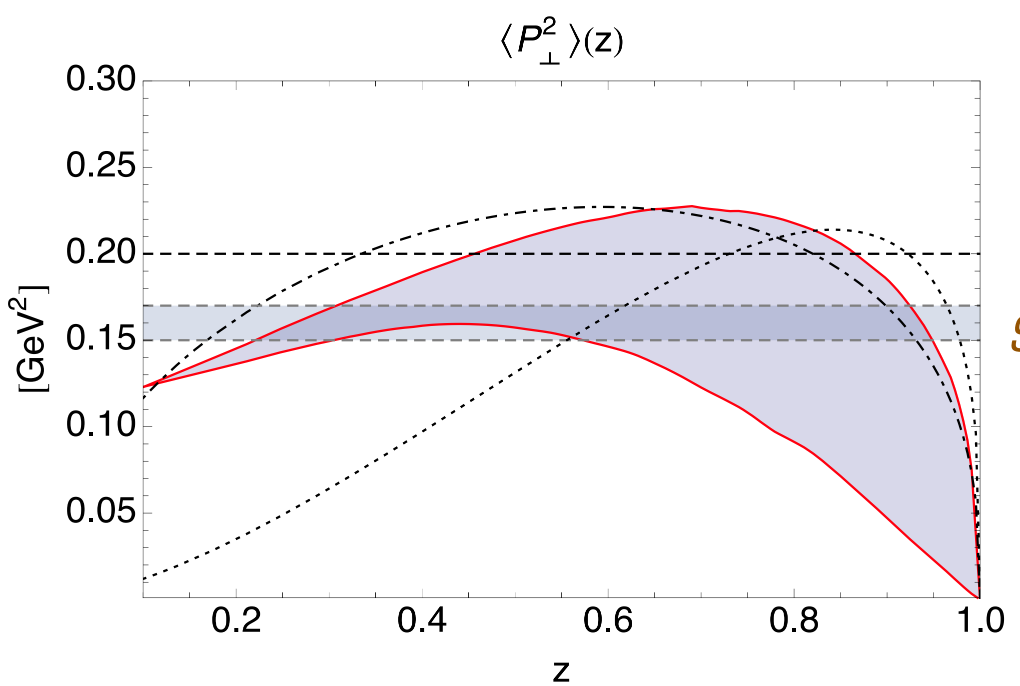


$\langle P_\perp^2 \rangle$(z)
[GeV^2]
0.30
0.25
0.20
0.15
0.10
0.05
0.2
0.4
0.6
0.8
1.0
z

68% bands



Schweitzer, Teckentrup, Metz, PRD 81 (2010)



Schweitzer, Teckentrup, Metz, PRD 81 (2010)

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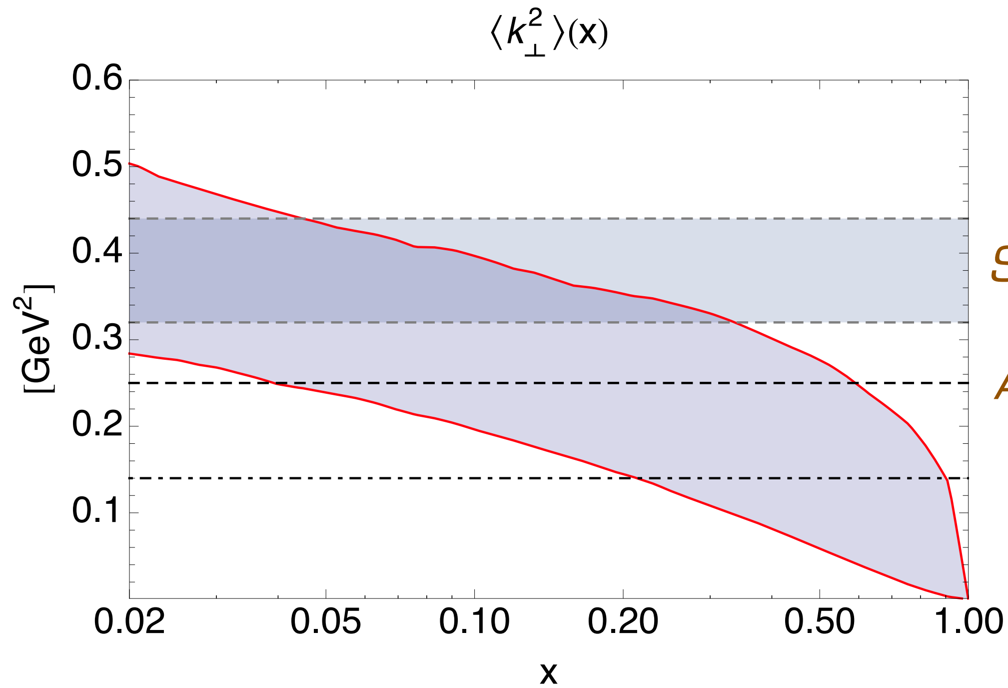
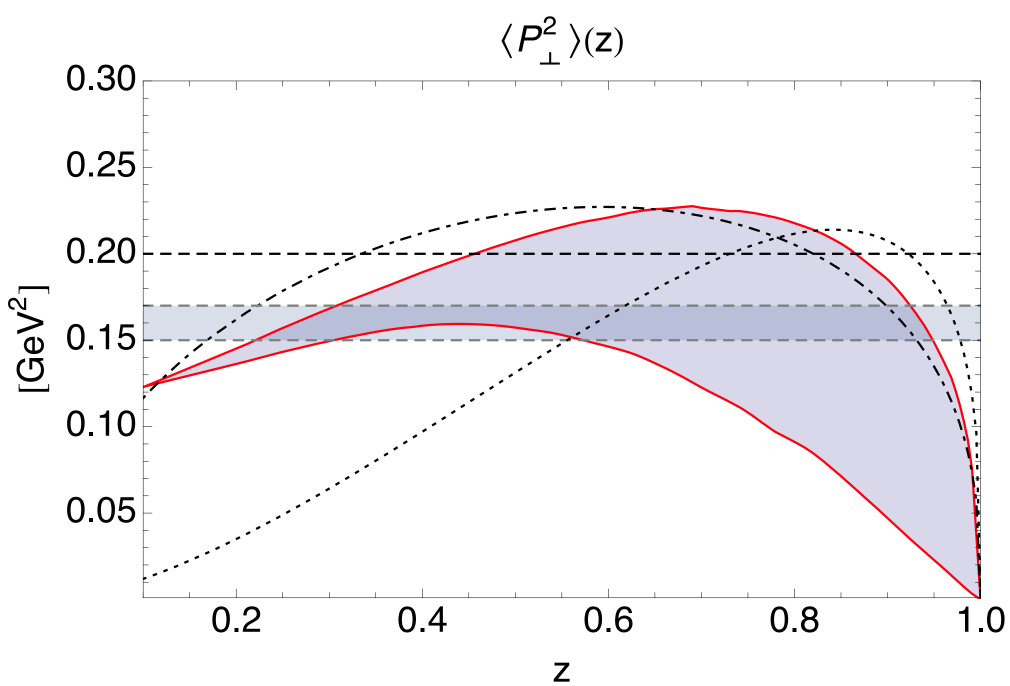


Schweitzer, Teckentrup, Metz, PRD 81 (2010)

Anselmino et al., PRD 71 (2005)



Anselmino et al., PRD 71 (2005)

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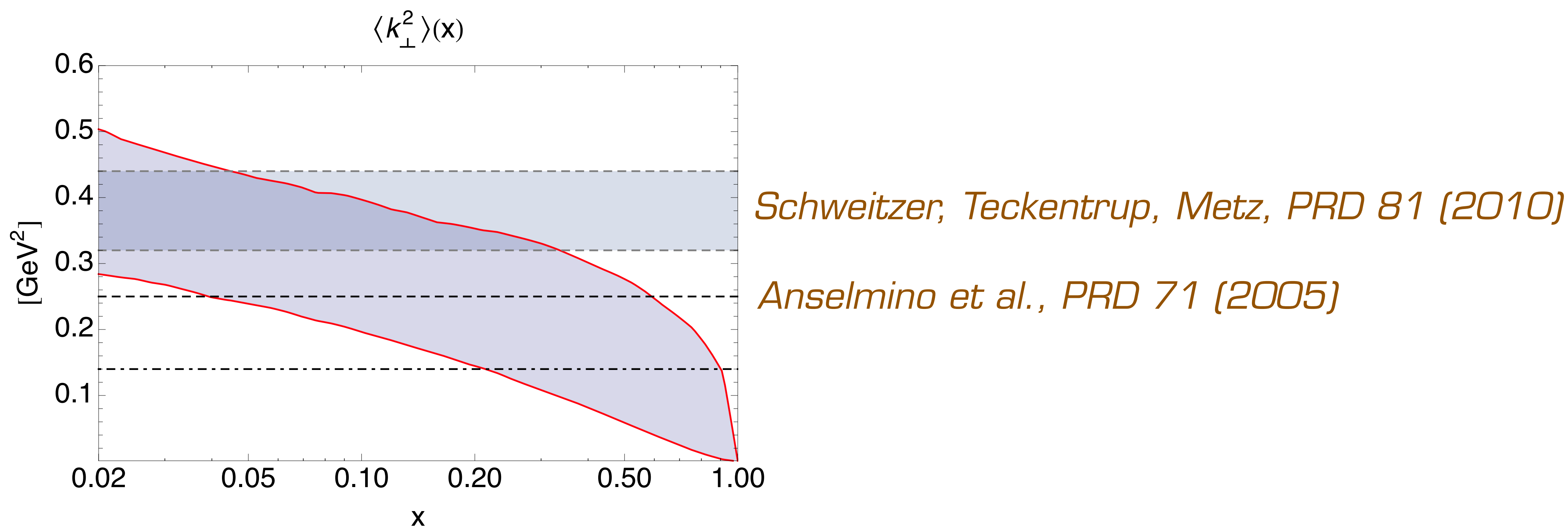

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[GeV^2]
0.6
0.5
0.4
0.3
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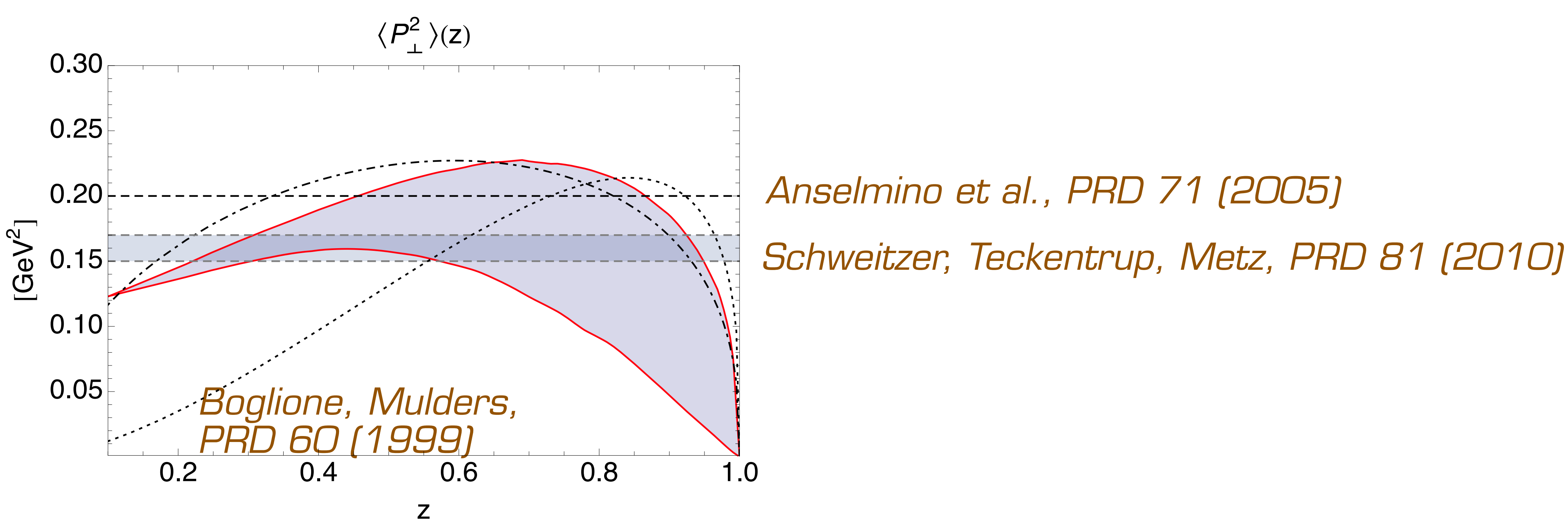
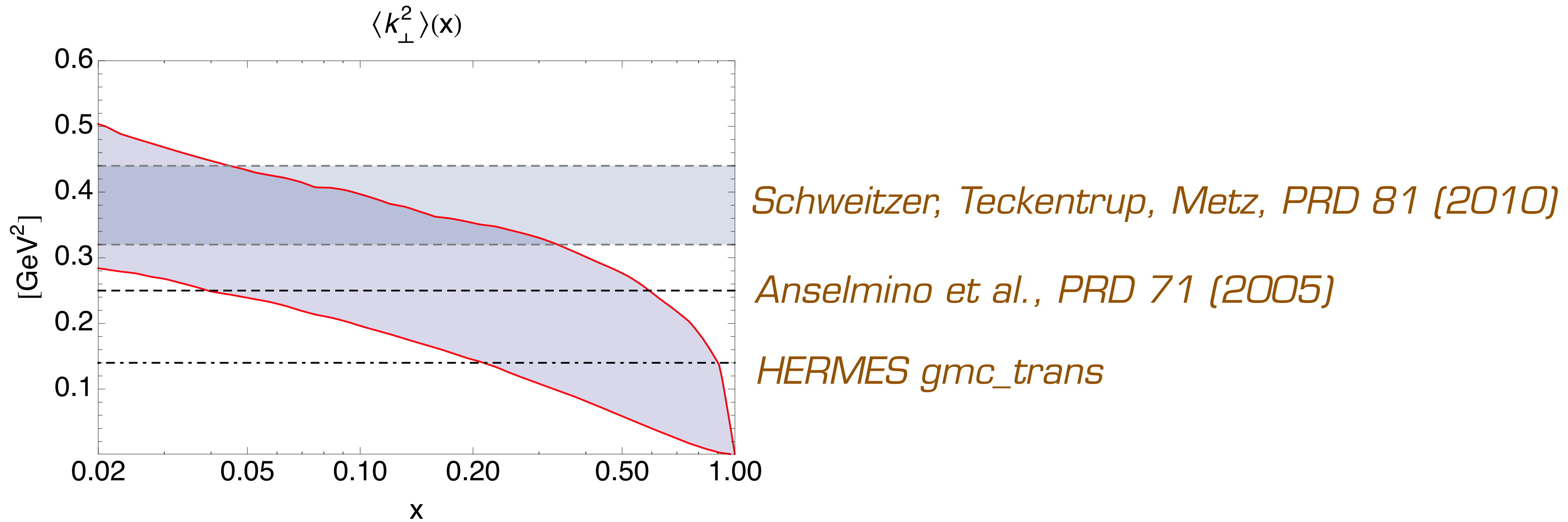
0.1
0.02
0.05
0.10
0.20
0.50
1.00
x
Schweitzer, Teckentrup, Metz, PRD 81 (2010)
Anselmino et al., PRD 71 (2005)



$\langle P_\perp^2 \rangle$(z)
[GeV^2]
0.30
0.25
0.20
0.15
0.10
0.05
0.2
0.4
0.6
0.8
1.0
z
Anselmino et al., PRD 71 (2005)
Schweitzer, Teckentrup, Metz, PRD 81 (2010)
Boglione, Mulders,
PRD 60 (1999)

68% bands



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Schweitzer, Teckentrup, Metz, PRD 81 (2010)
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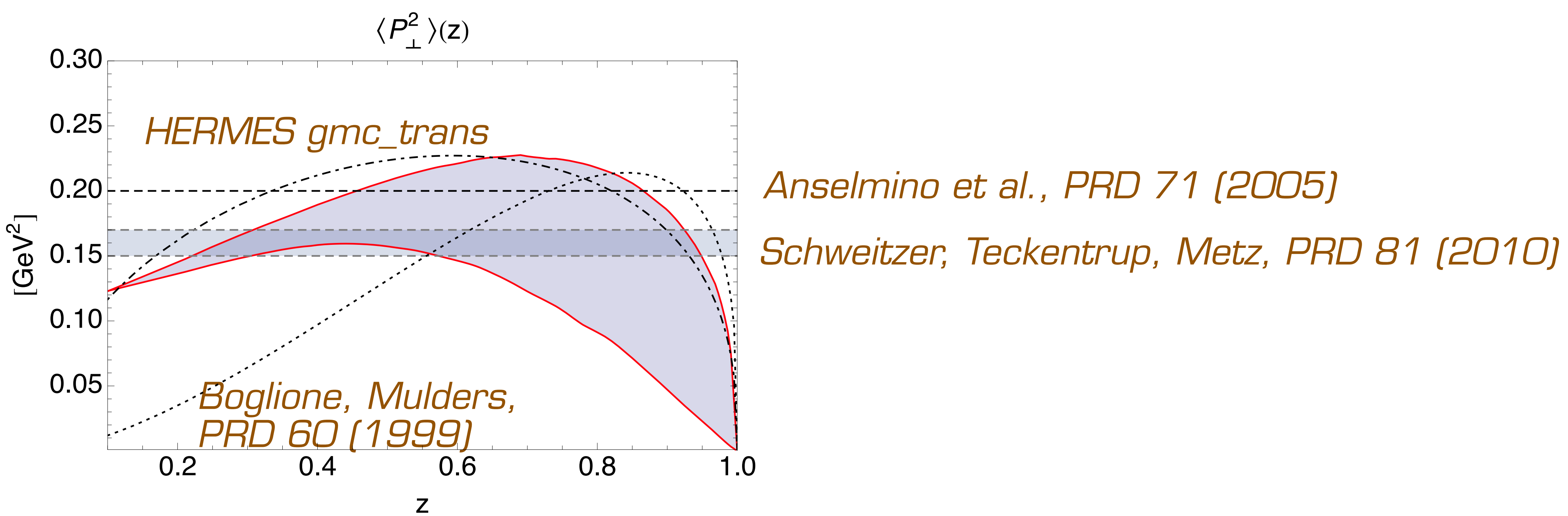
HERMES gmc_trans



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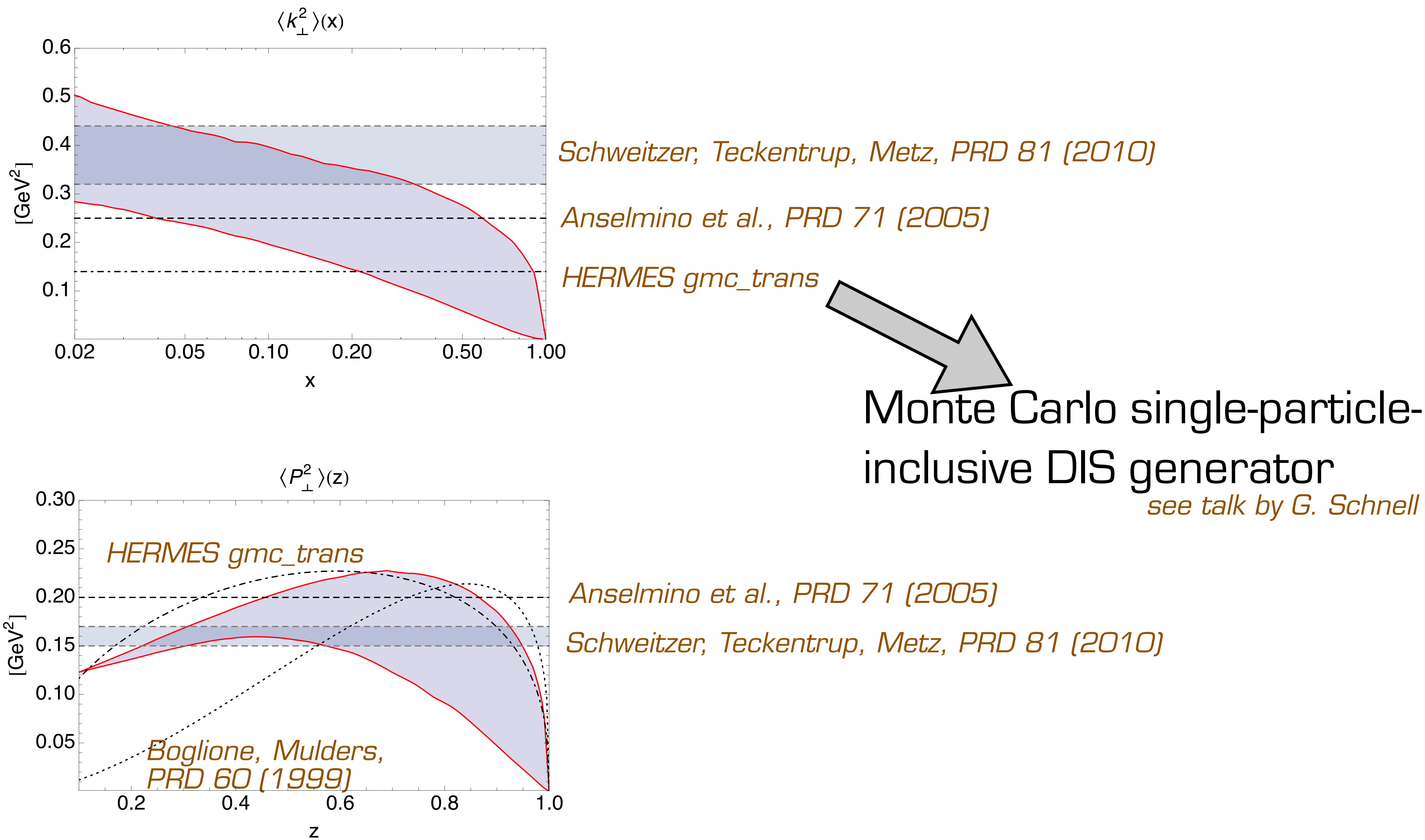

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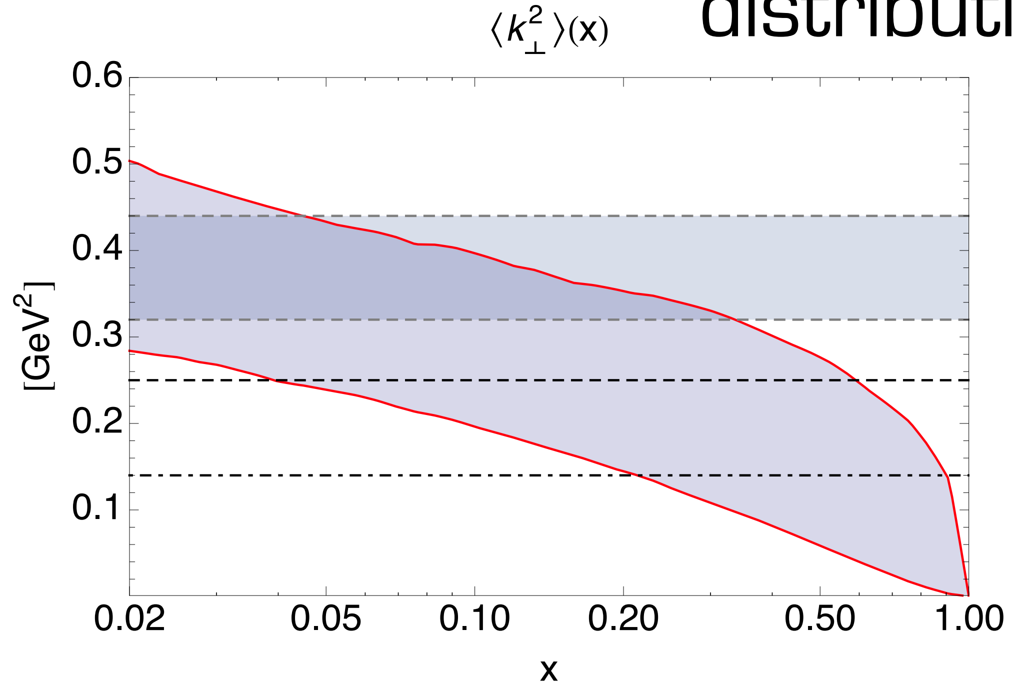
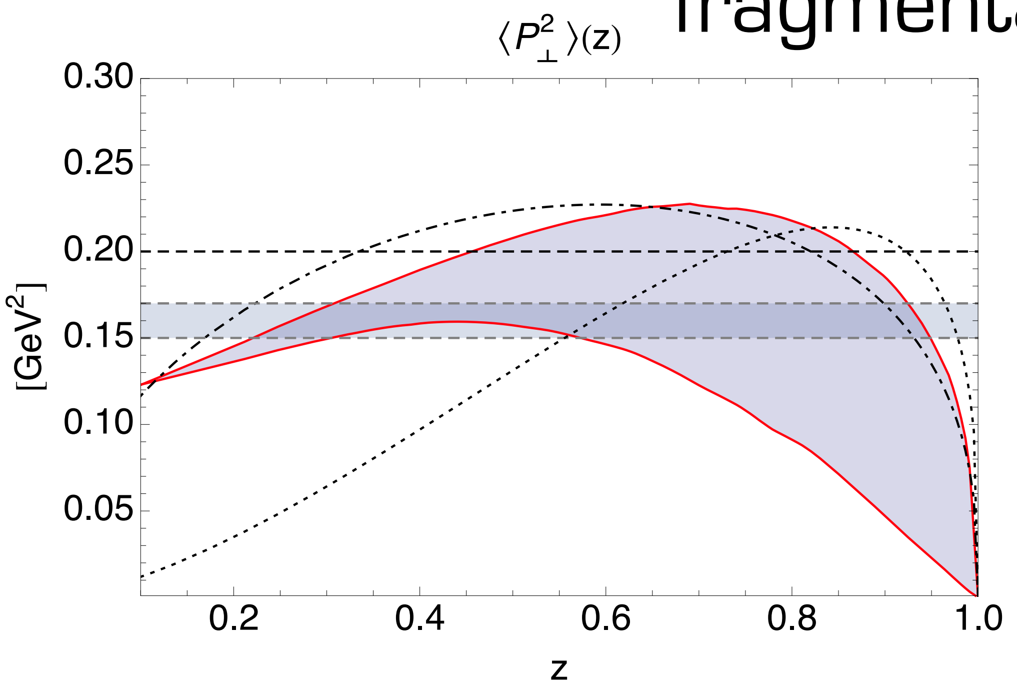
Schweitzer, Teckentrup, Metz, PRD 81 (2010)
Anselmino et al., PRD 71 (2005)
HERMES gmc_trans
Monte Carlo single-particle-inclusive DIS generator
see talk by G. Schnell
⟨$P_\perp^2$⟩(z)
[GeV^2]
0.30
0.25
0.20
0.15
0.10
0.05
0.2
0.4
0.6
0.8
1.0
z
HERMES gmc_trans
Anselmino et al., PRD 71 (2005)
Schweitzer, Teckentrup, Metz, PRD 81 (2010)
Boglione, Mulders, PRD 60 (1999)

68% bands

distribution

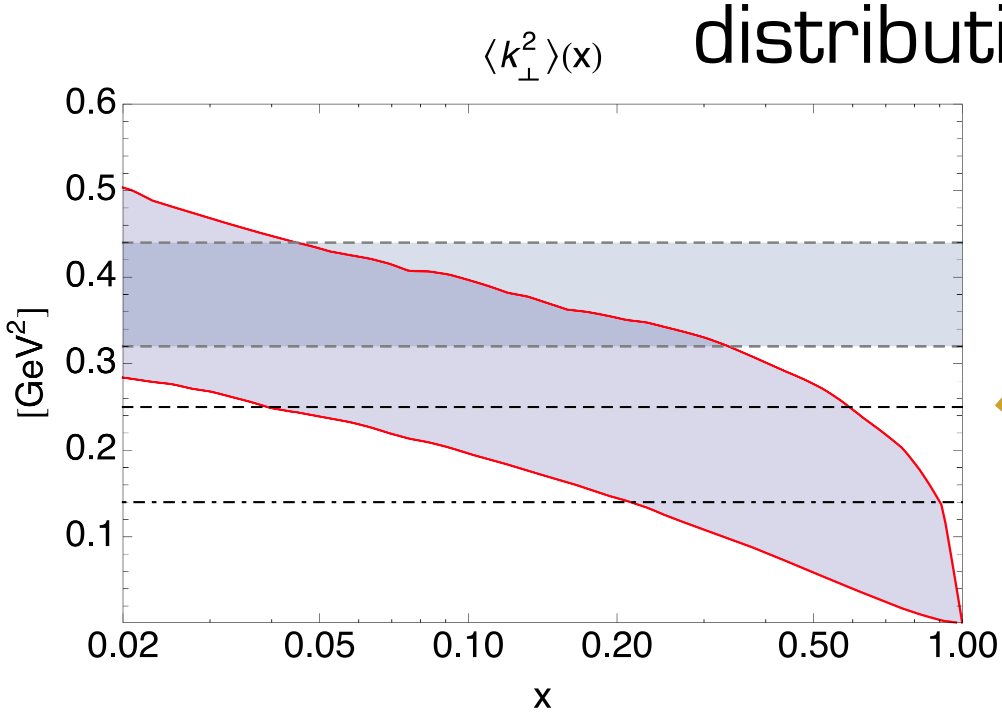


fragmentation

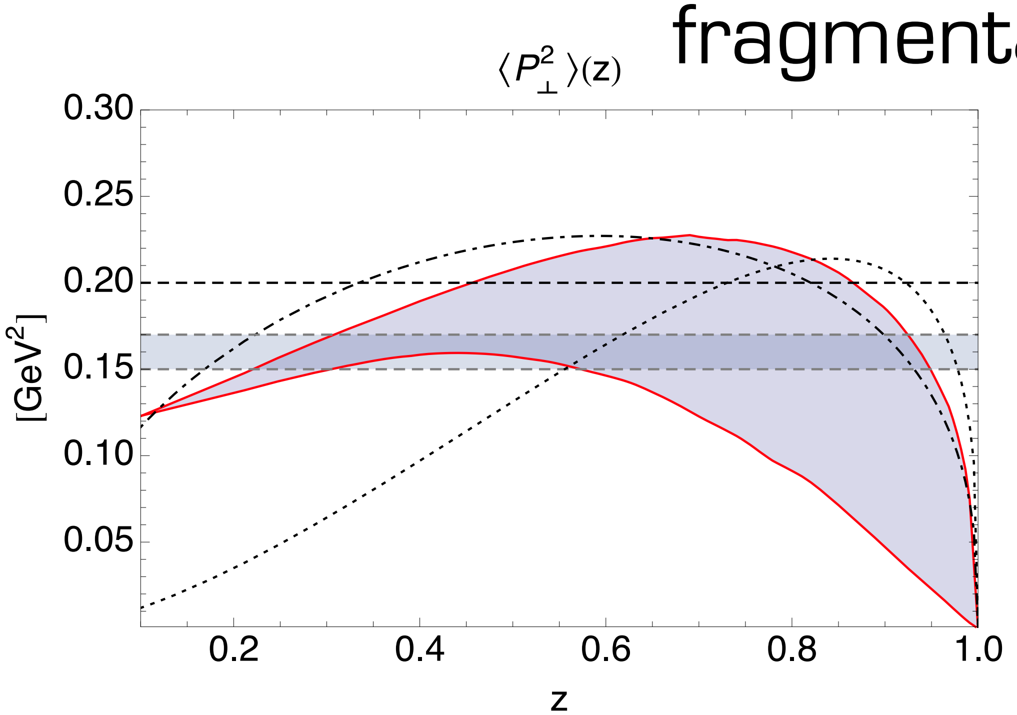


68% bands

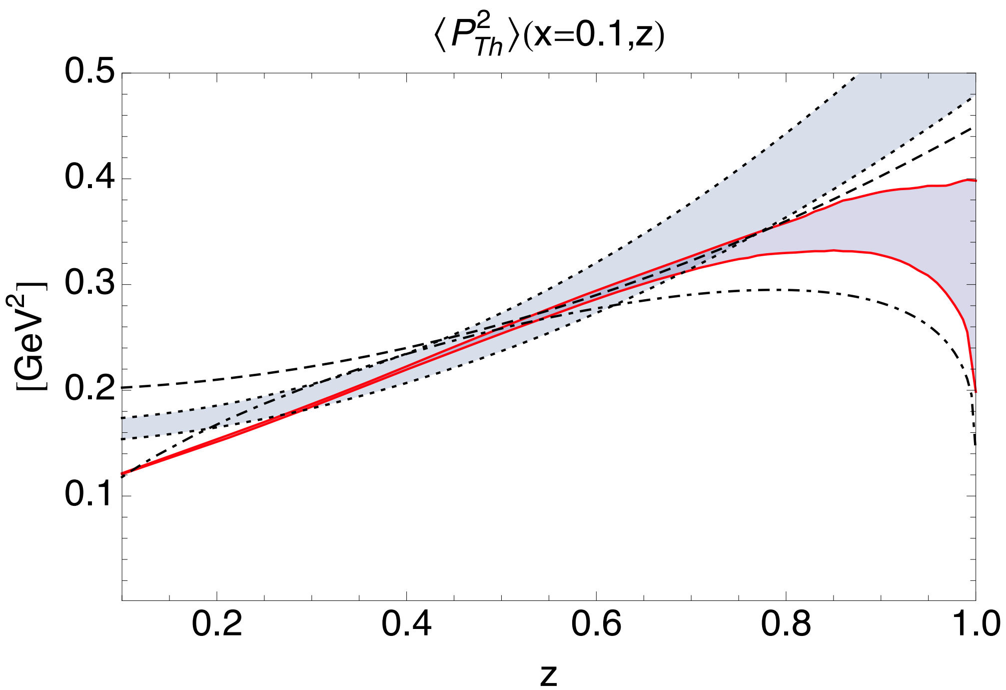
distribution



fragmentation

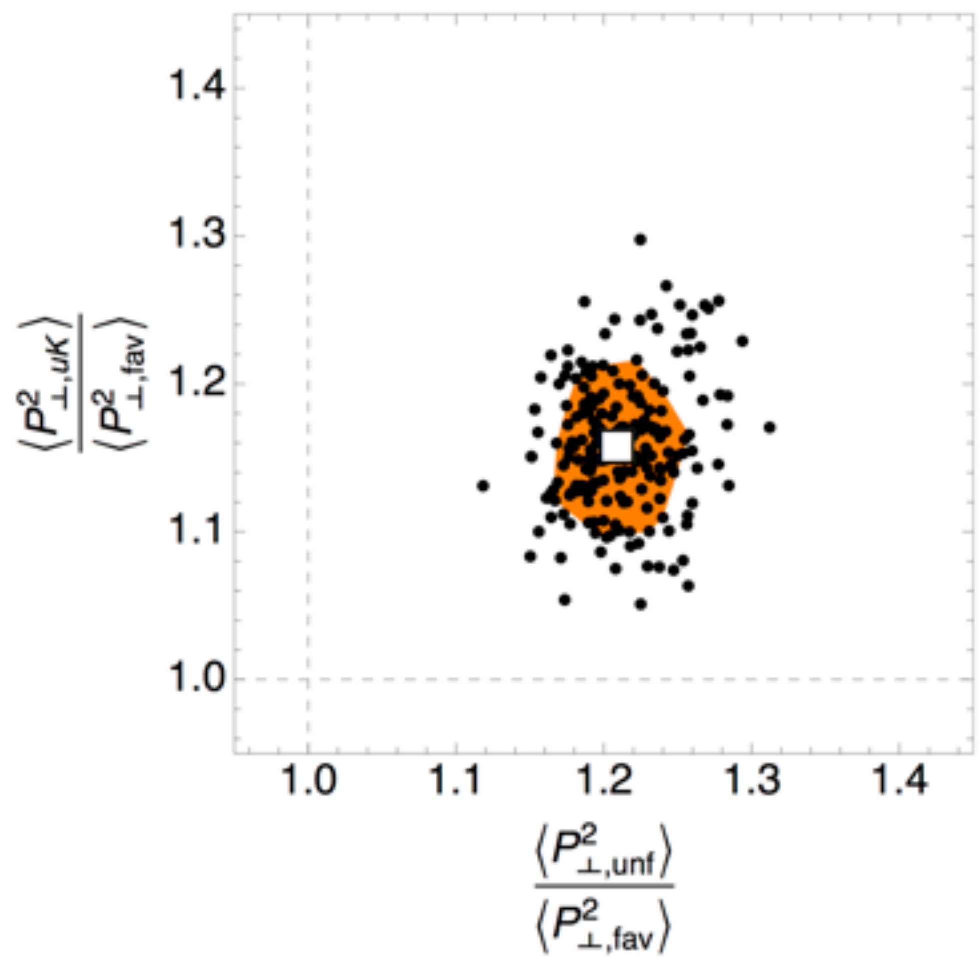


observed

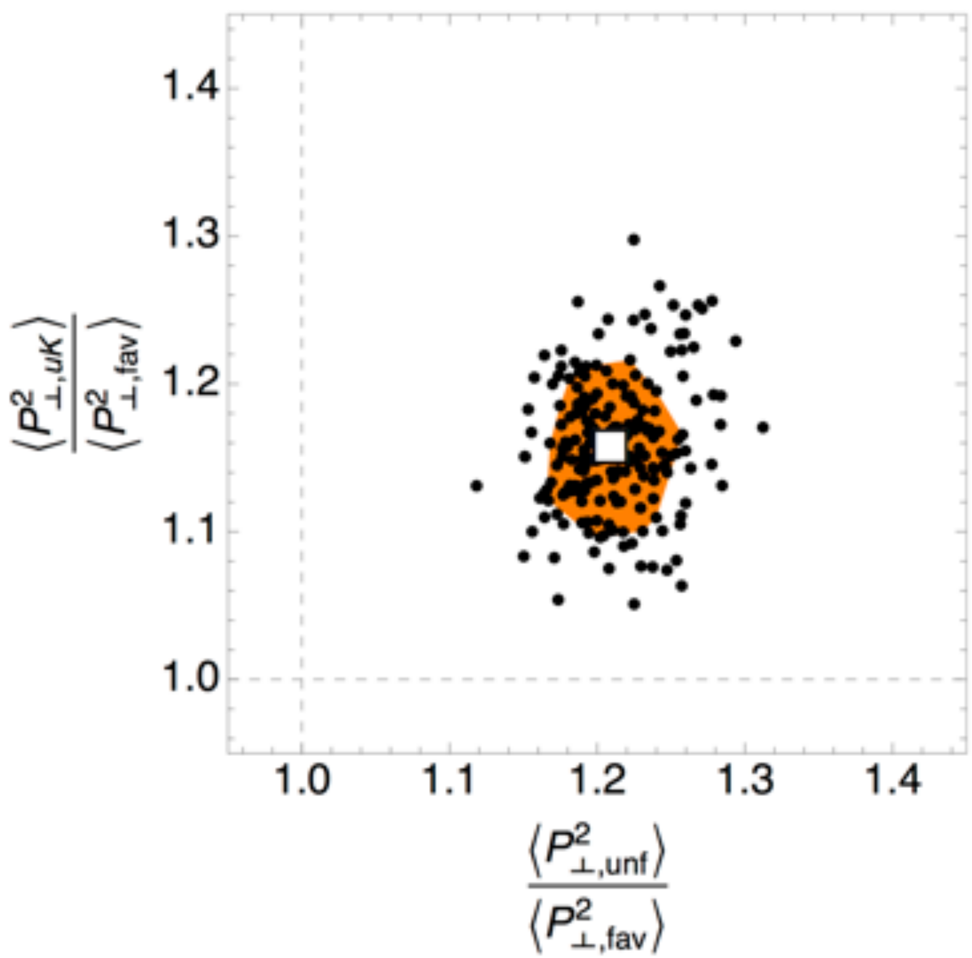


We need data from electron-positron annihilation

Flavor dependence in FFs

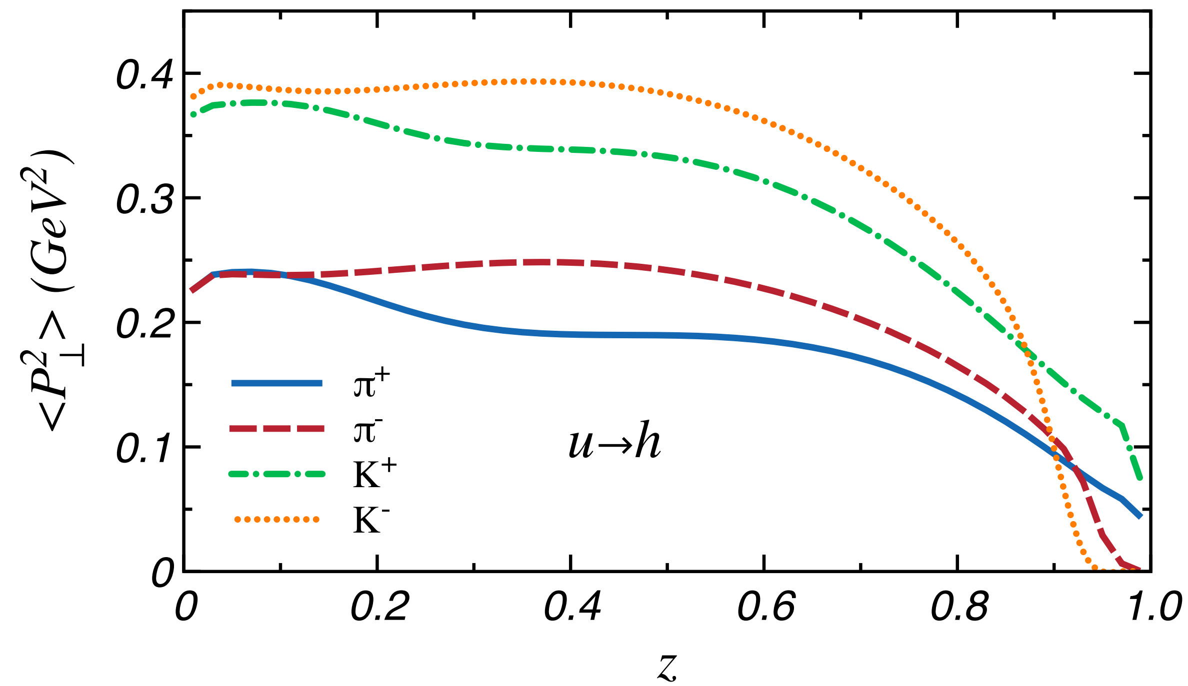


Flavor dependence in FFs



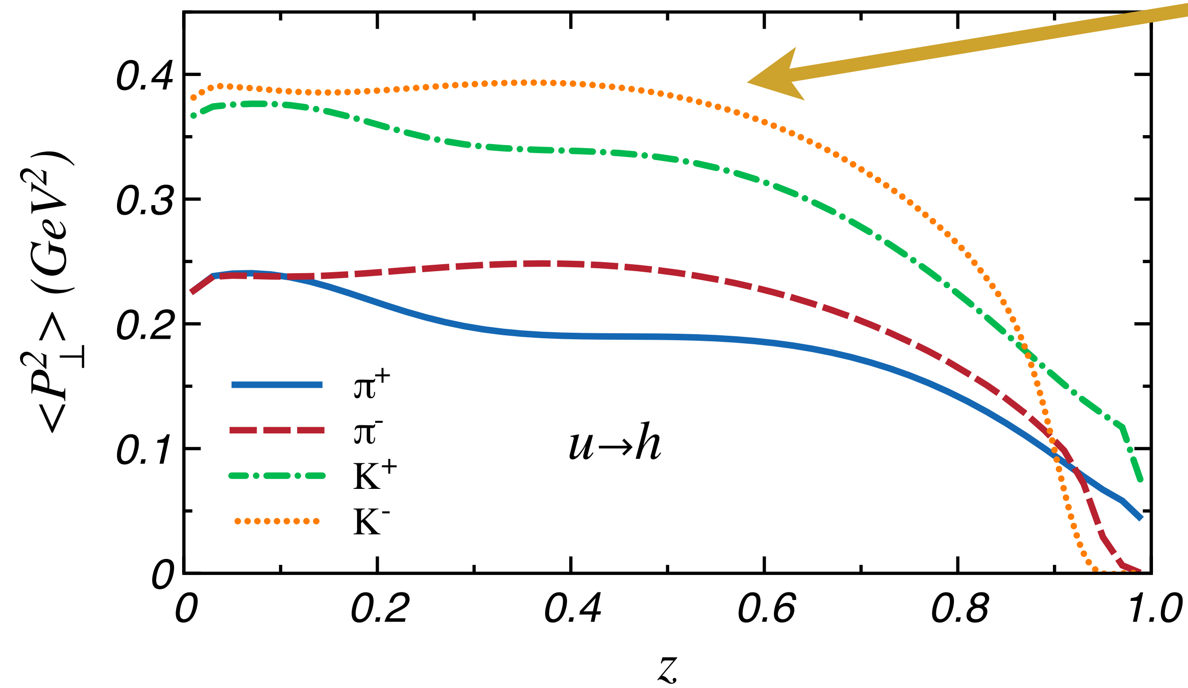
We find significant evidence that pion-unfavored and kaon fragmentation functions are wider than pion-favored

Models of fragmentation functions



Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)

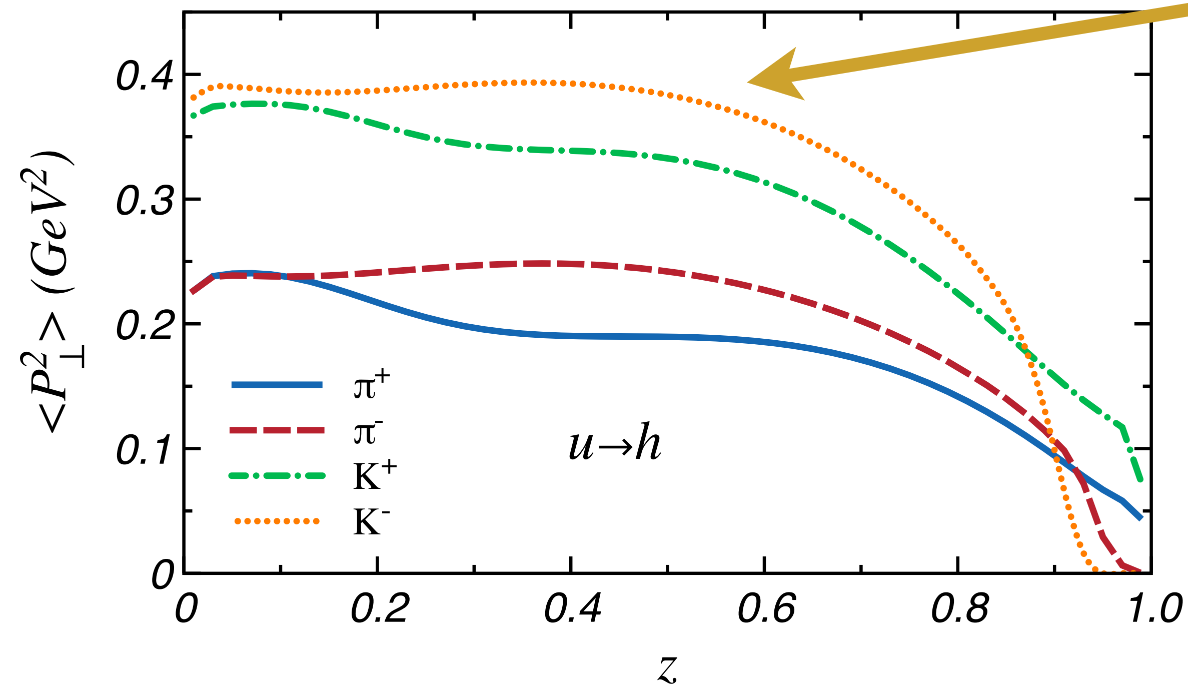
Models of fragmentation functions



higher average transverse momentum

Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)

Models of fragmentation functions

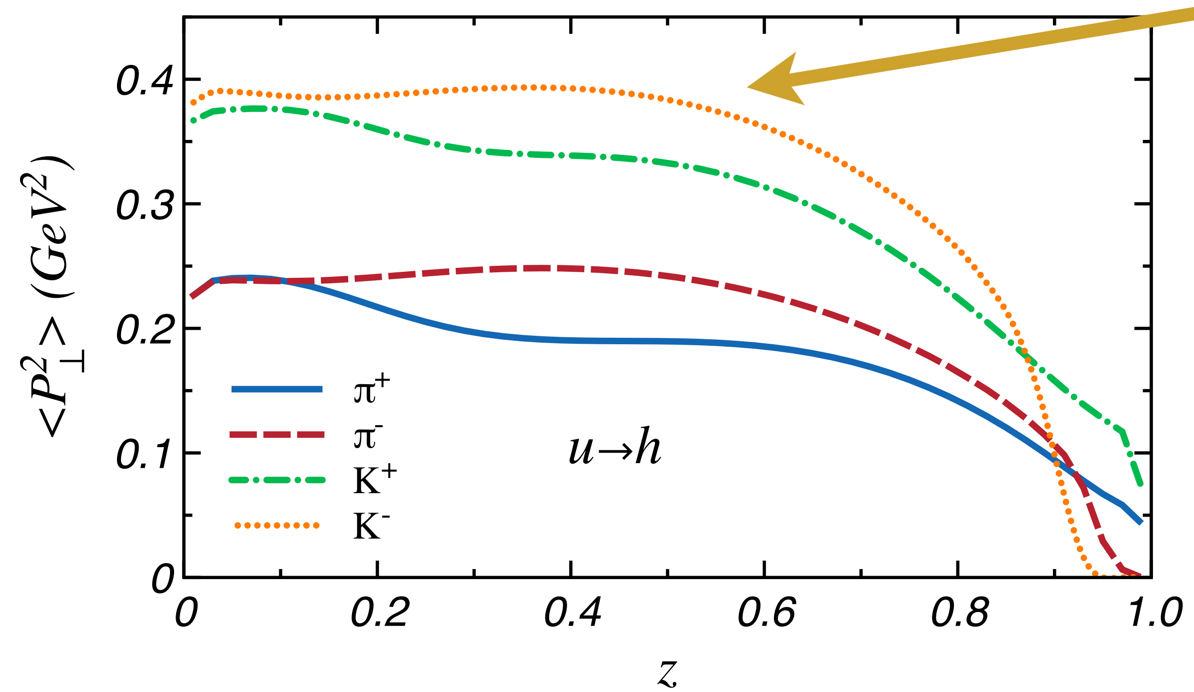


higher average transverse momentum

Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)

Unfavored pion fragmentation and kaon fragmentation are wider than favored pion fragmentation

Models of fragmentation functions



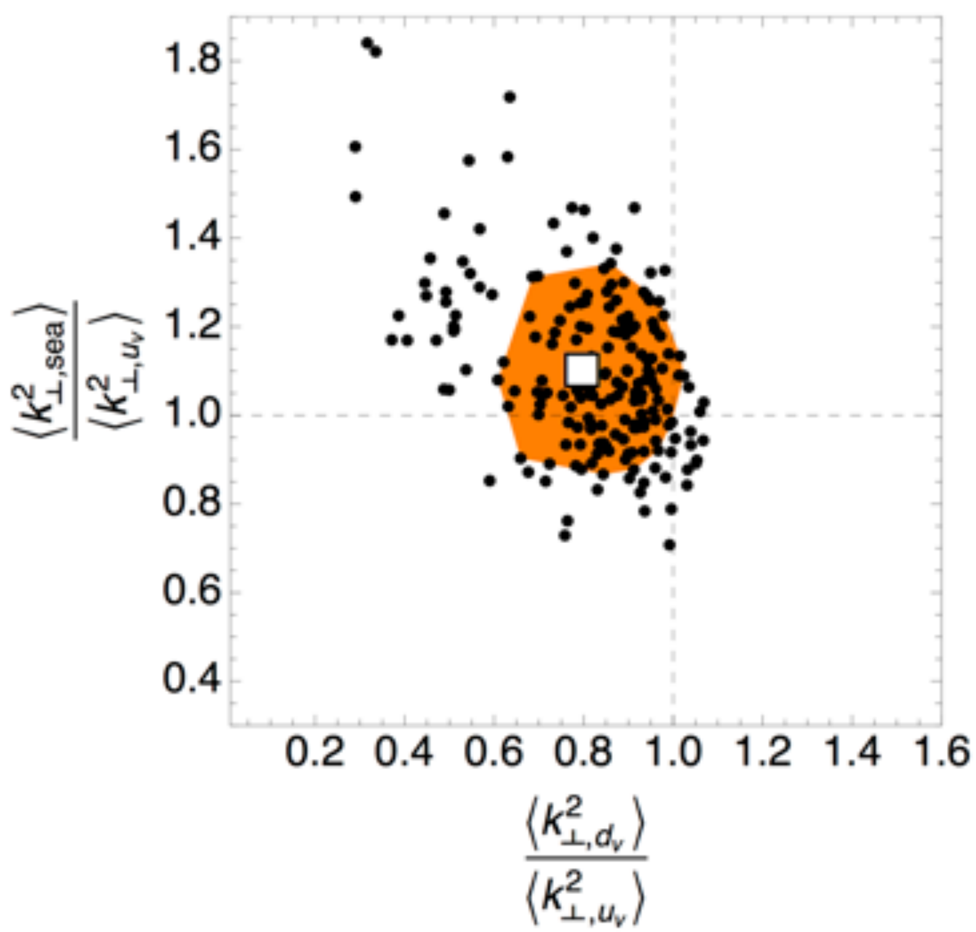
higher average transverse momentum

Matevosyan, Bentz, Cloet, Thomas, PRD 85 (2012)

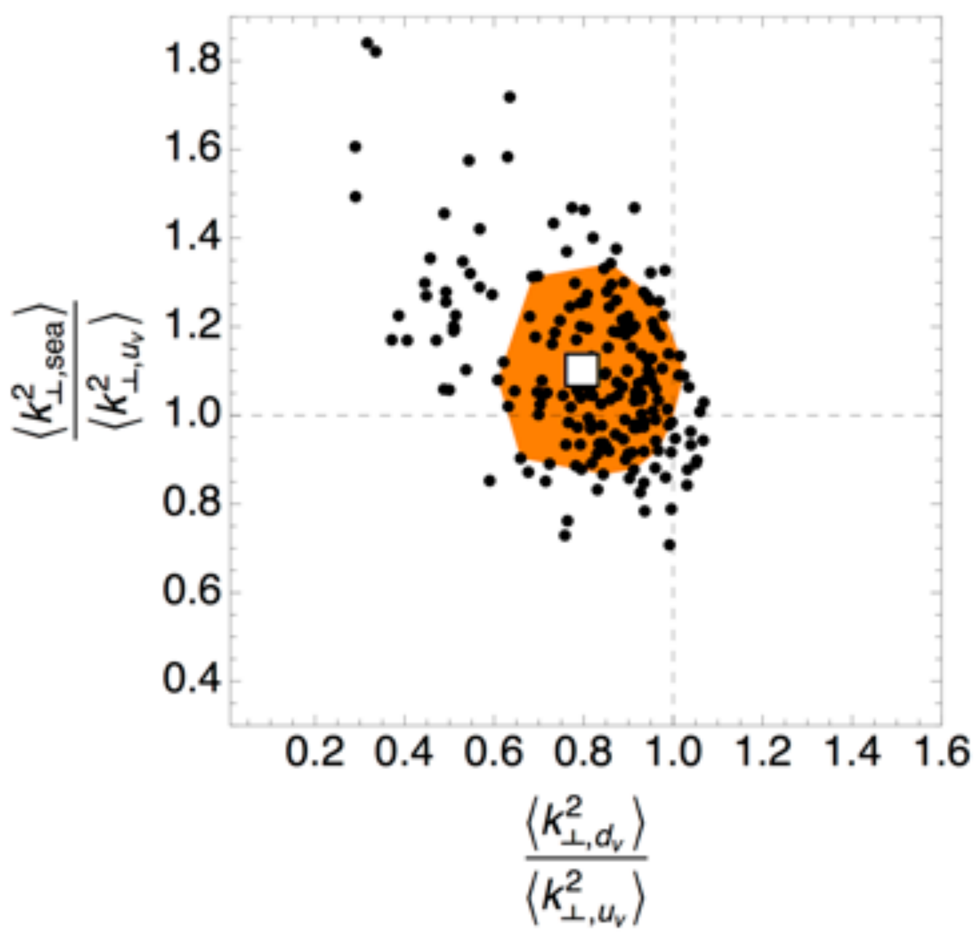
Unfavored pion fragmentation and kaon fragmentation are wider than favored pion fragmentation

see also talk by H. Matevosyan

Flavor dependence in PDFs

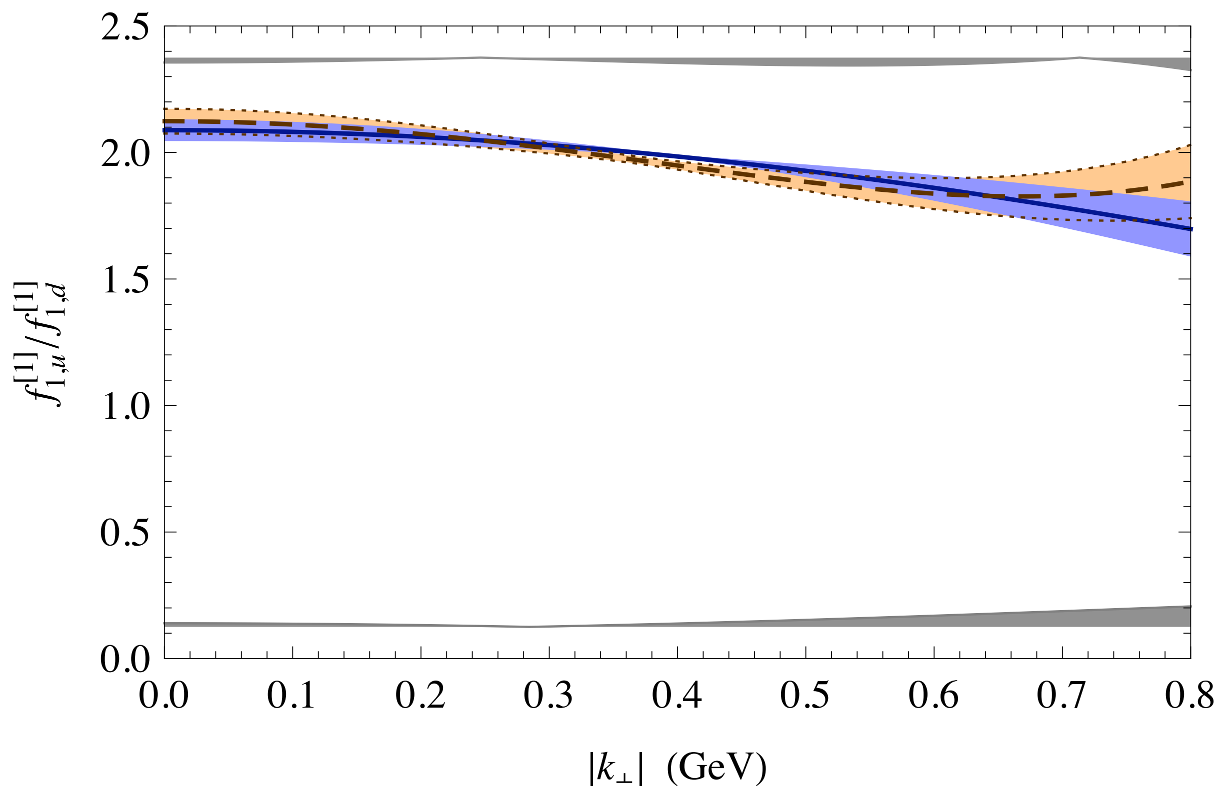


Flavor dependence in PDFs



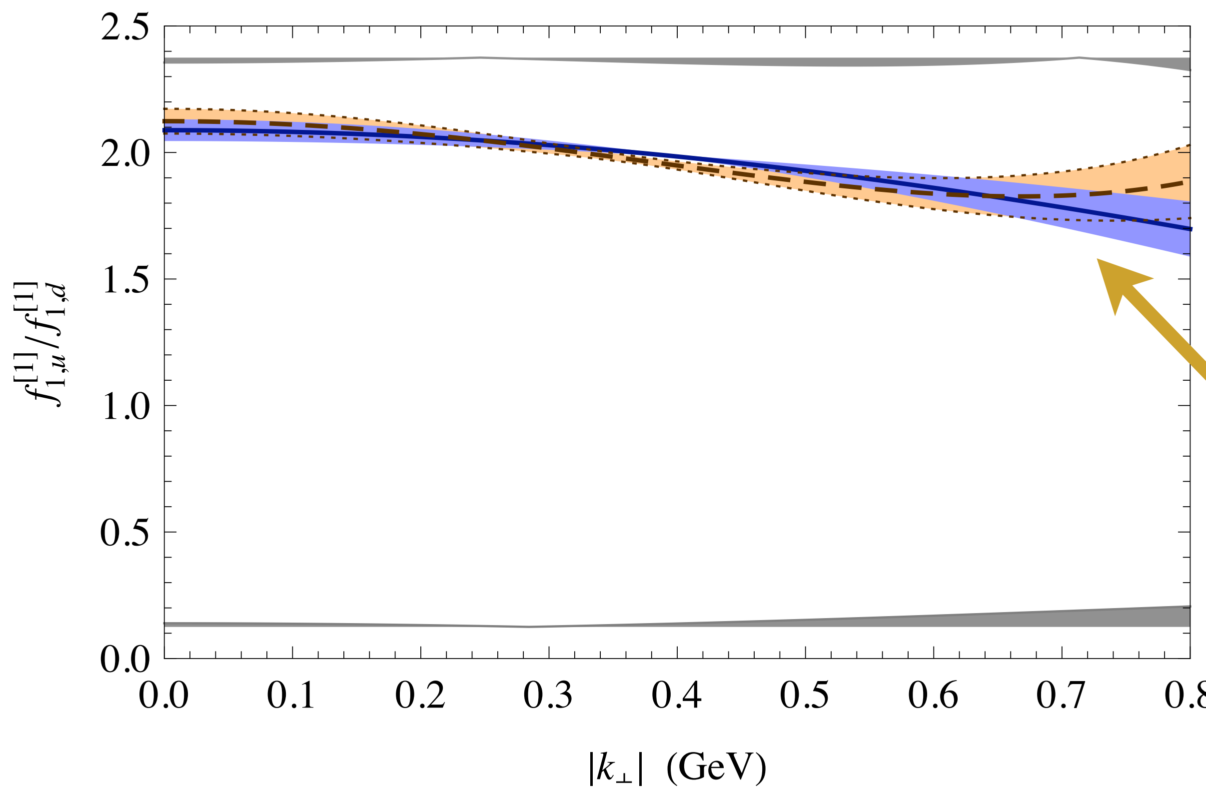
There is a lot of room for flavor dependence...

Indications from lattice QCD



Musch, Hagler, Negele, Schafer, PRD 83 (11)

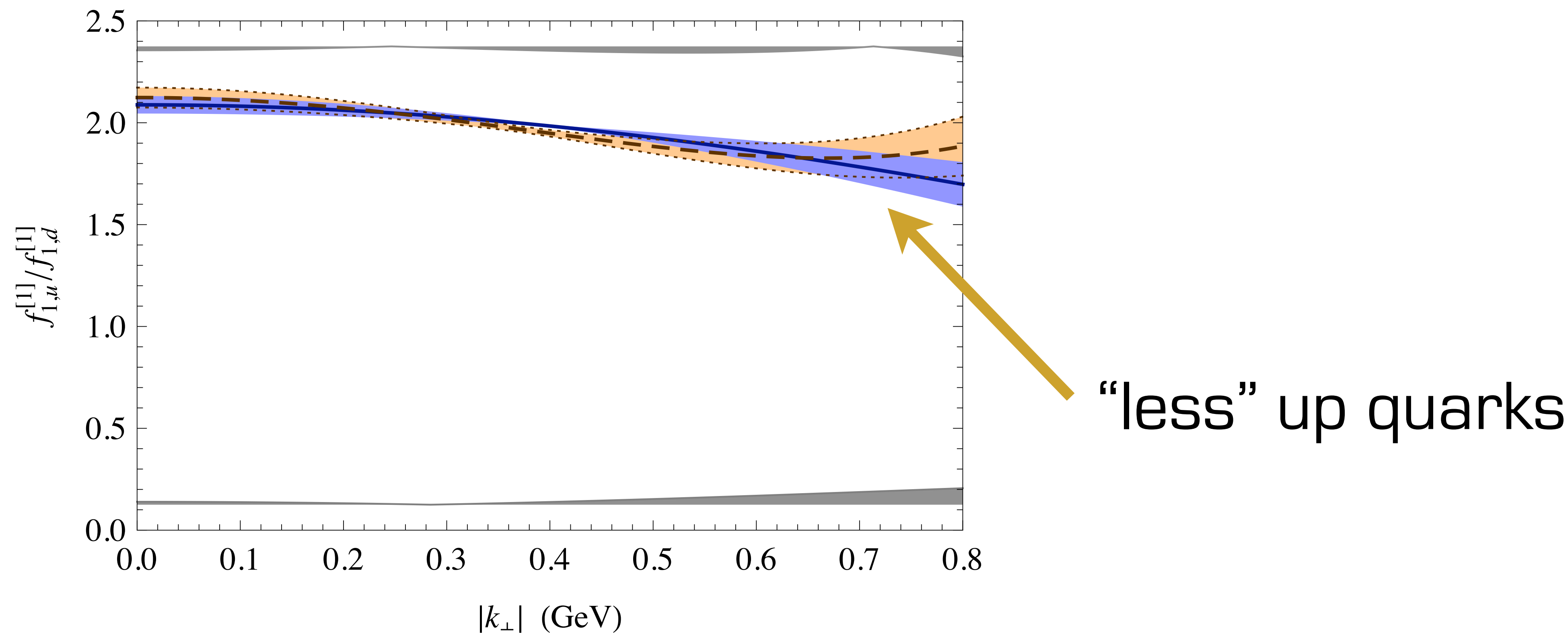
Indications from lattice QCD



"less" up quarks

Musch, Hagler, Negele, Schafer, PRD 83 (11)

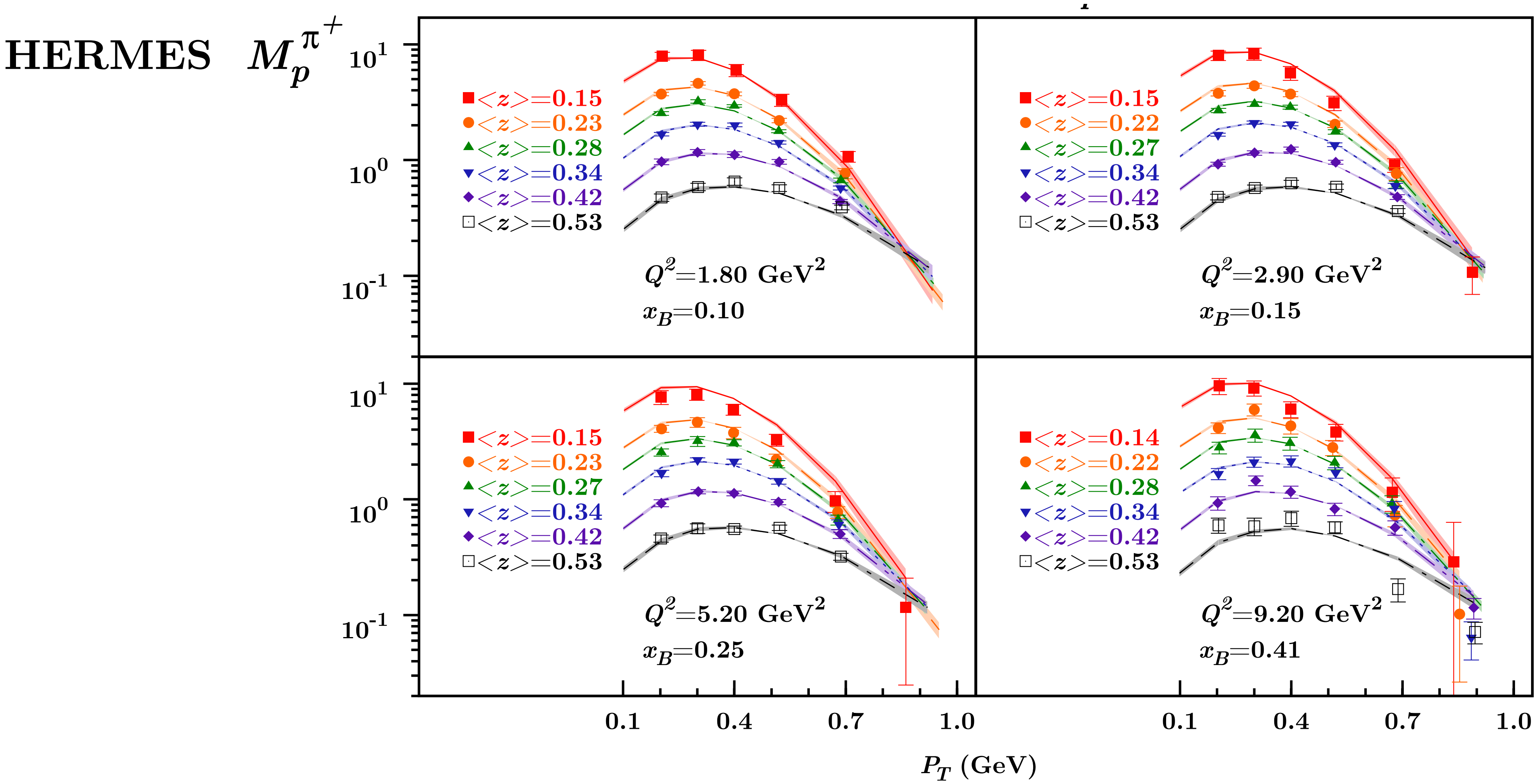
Indications from lattice QCD



Musch, Hagler, Negele, Schafer, PRD 83 (11)

Pioneering lattice-QCD studies hint at a down distribution being wider than up

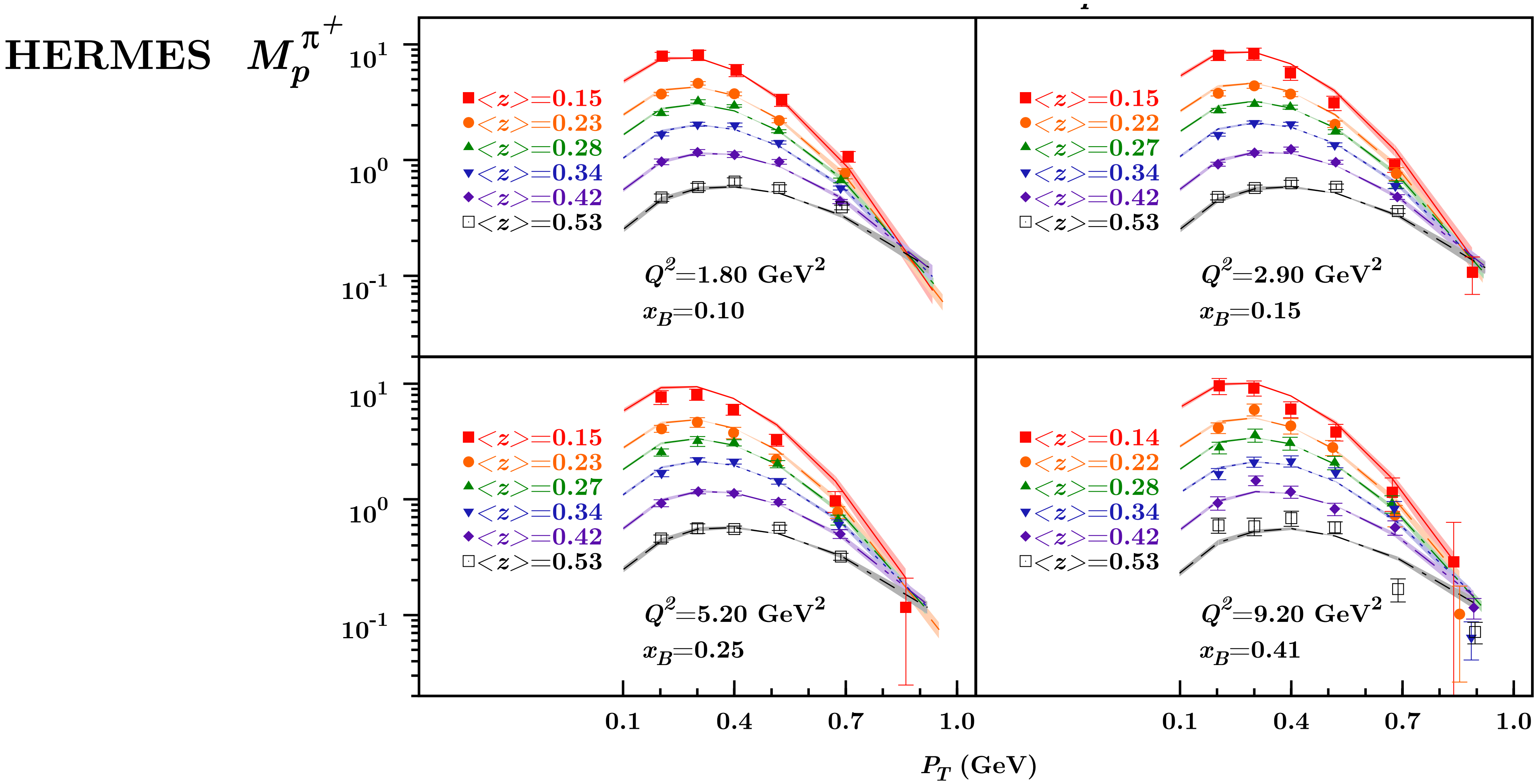
Torino fit to HERMES (no evo)



Cuts	χ^2_{dof}	n. points	$[\chi^2_{\text{point}}]^{\pi^+}$	$[\chi^2_{\text{point}}]^{\pi^-}$	Parameters
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.6$	1.69	497	1.93	1.45	$\langle k_\perp^2\rangle = 0.57 \pm 0.08$ GeV^2 $\langle p_\perp^2\rangle = 0.12 \pm 0.01$ GeV^2
$Q^2 > 1.69$ GeV^2 $0.2 < P_T < 0.9$ GeV $z < 0.7$	2.62	576	2.56	2.68	$\langle k_\perp^2\rangle = 0.46 \pm 0.09$ GeV^2 $\langle p_\perp^2\rangle = 0.13 \pm 0.01$ GeV^2

Torino fit to HERMES (no evo)

Anselmino, Boglione, Gonzalez, Melis, Prokudin, arXiv:1312.6261



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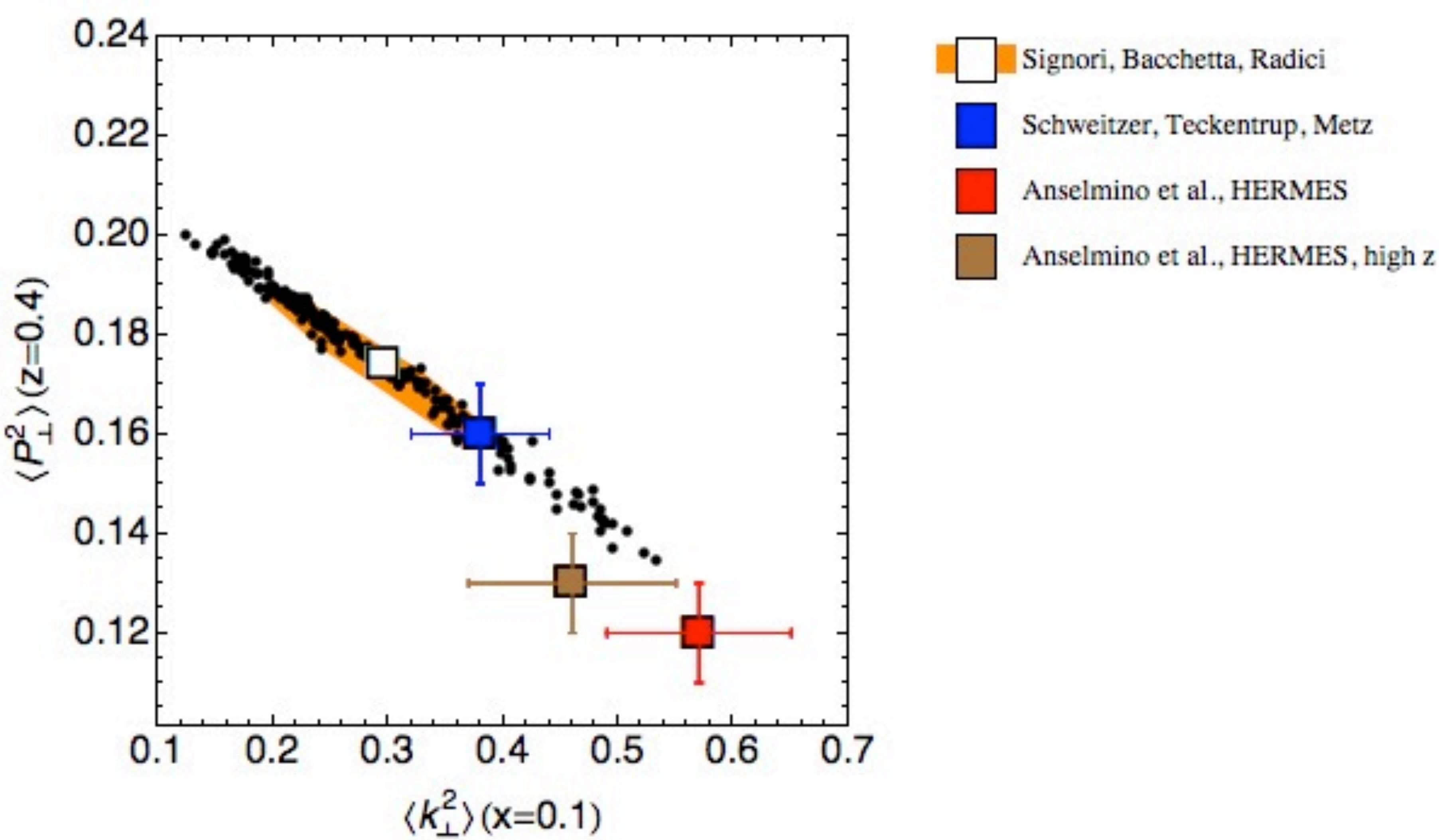
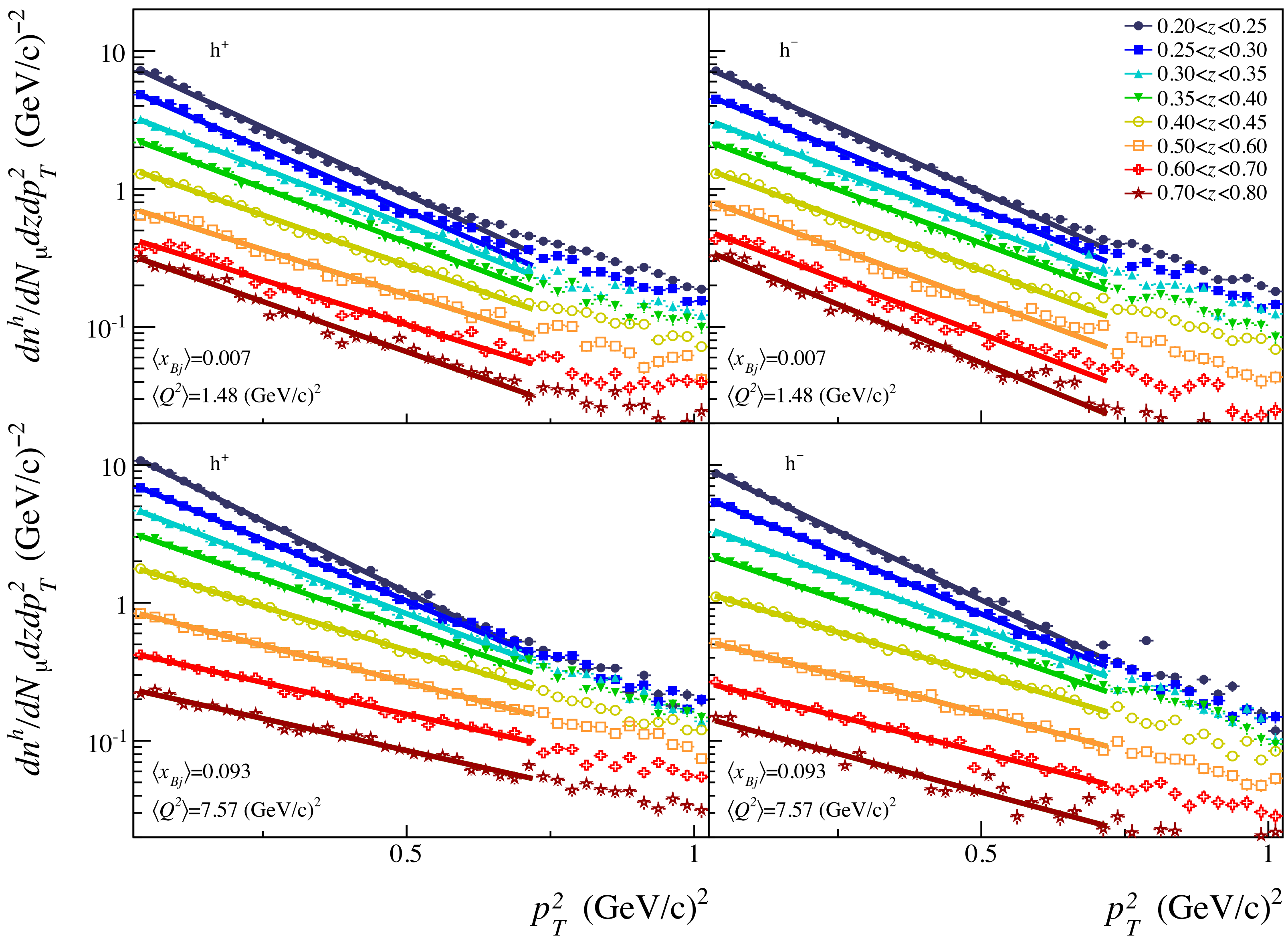
Comparison Pavia-Torino (HERMES)

Transverse momentum in FFs



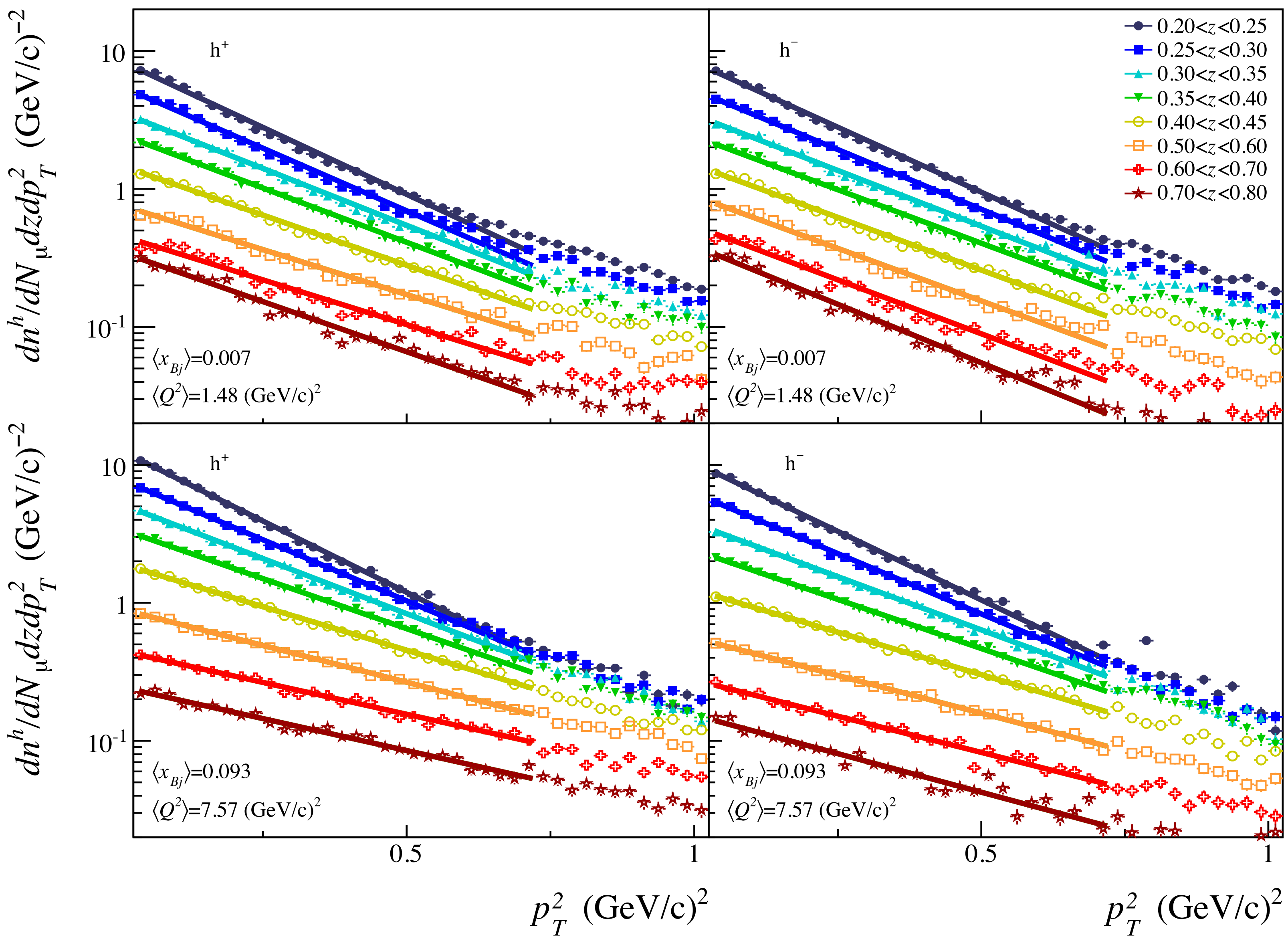
Transverse momentum in PDFs

COMPASS multiplicities



Adolph et al., EPJ C73 (13)

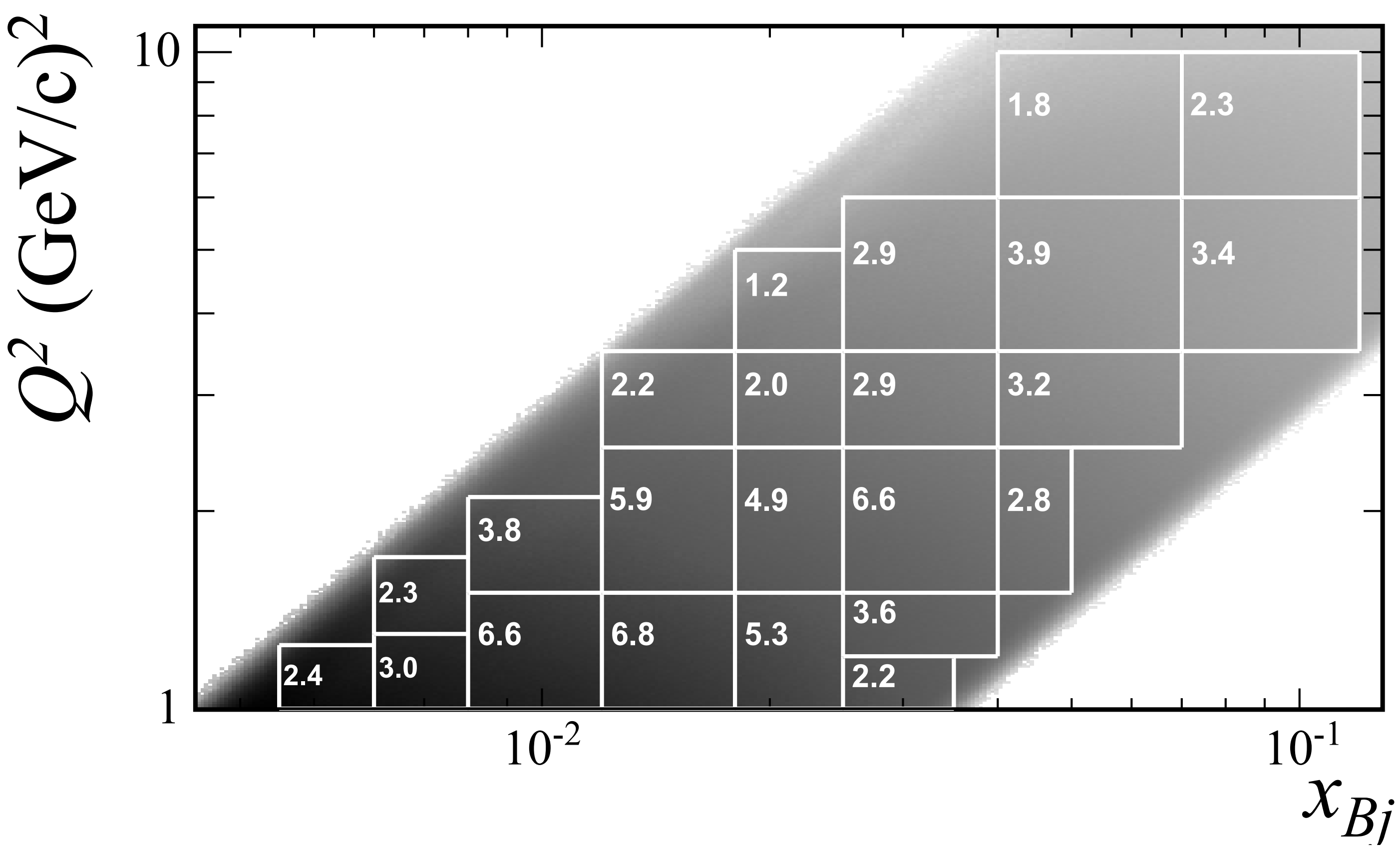
COMPASS multiplicities



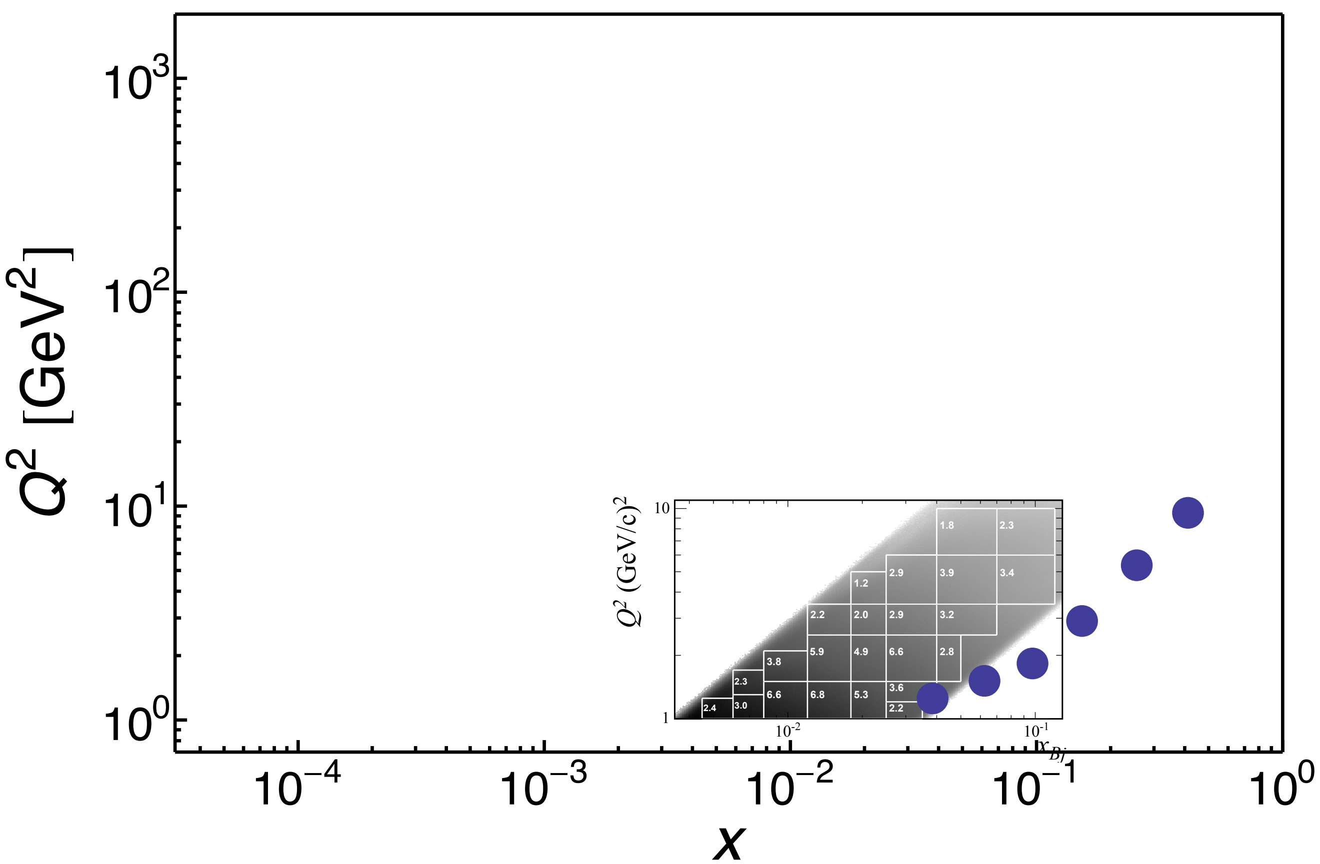
Adolph et al., EPJ C73 (13)

About 20000 data points!

Limited x - Q^2 coverage



Limited x - Q^2 coverage



Torino COMPASS

Two versions of the fits:

- without any normalization factor
- with a y dependent normalization factor

Torino COMPASS

Anselmino, Boglione, Gonzalez, Melis, Prokudin, arXiv:1312.6261

see talk by Elena Boglione

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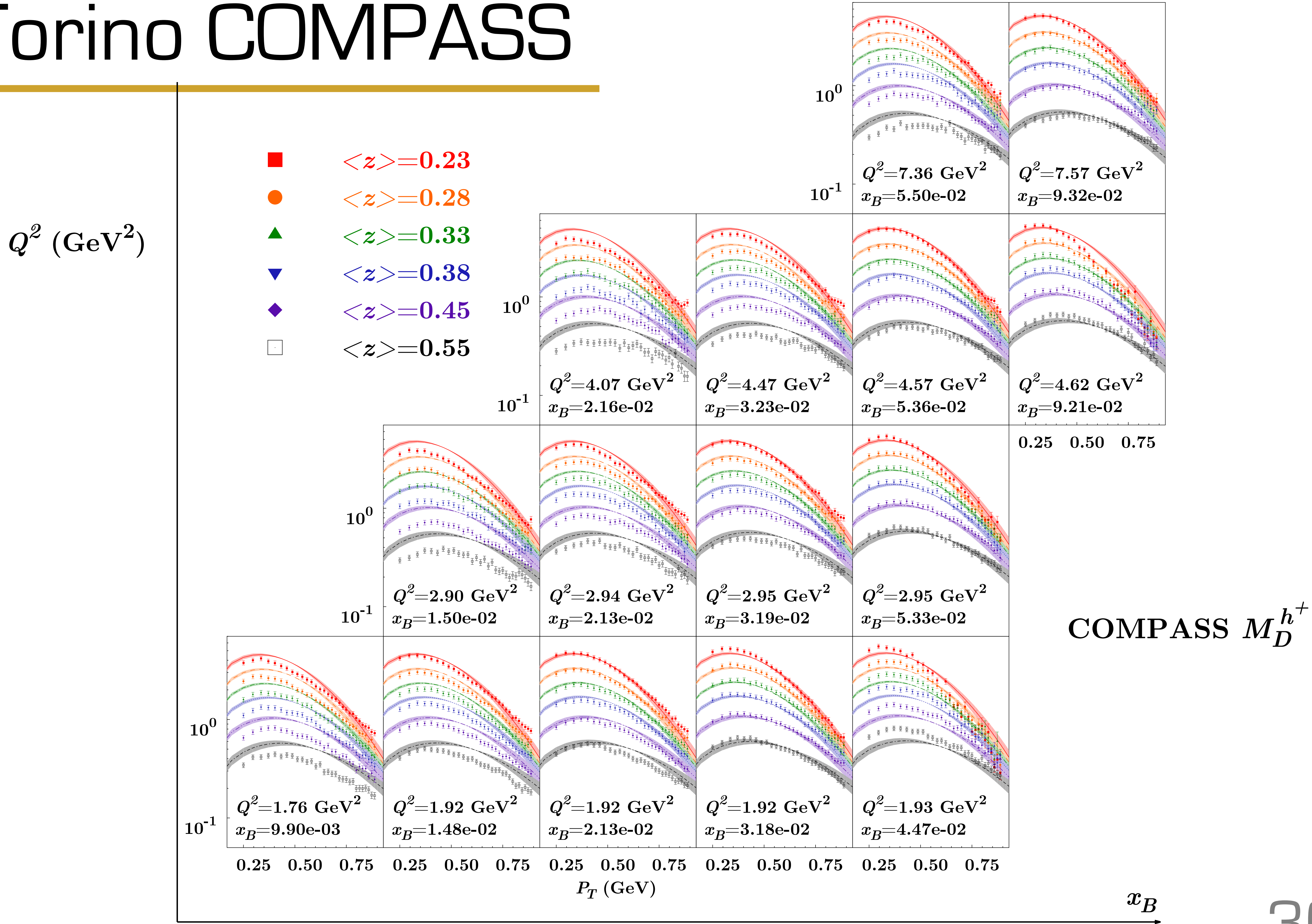
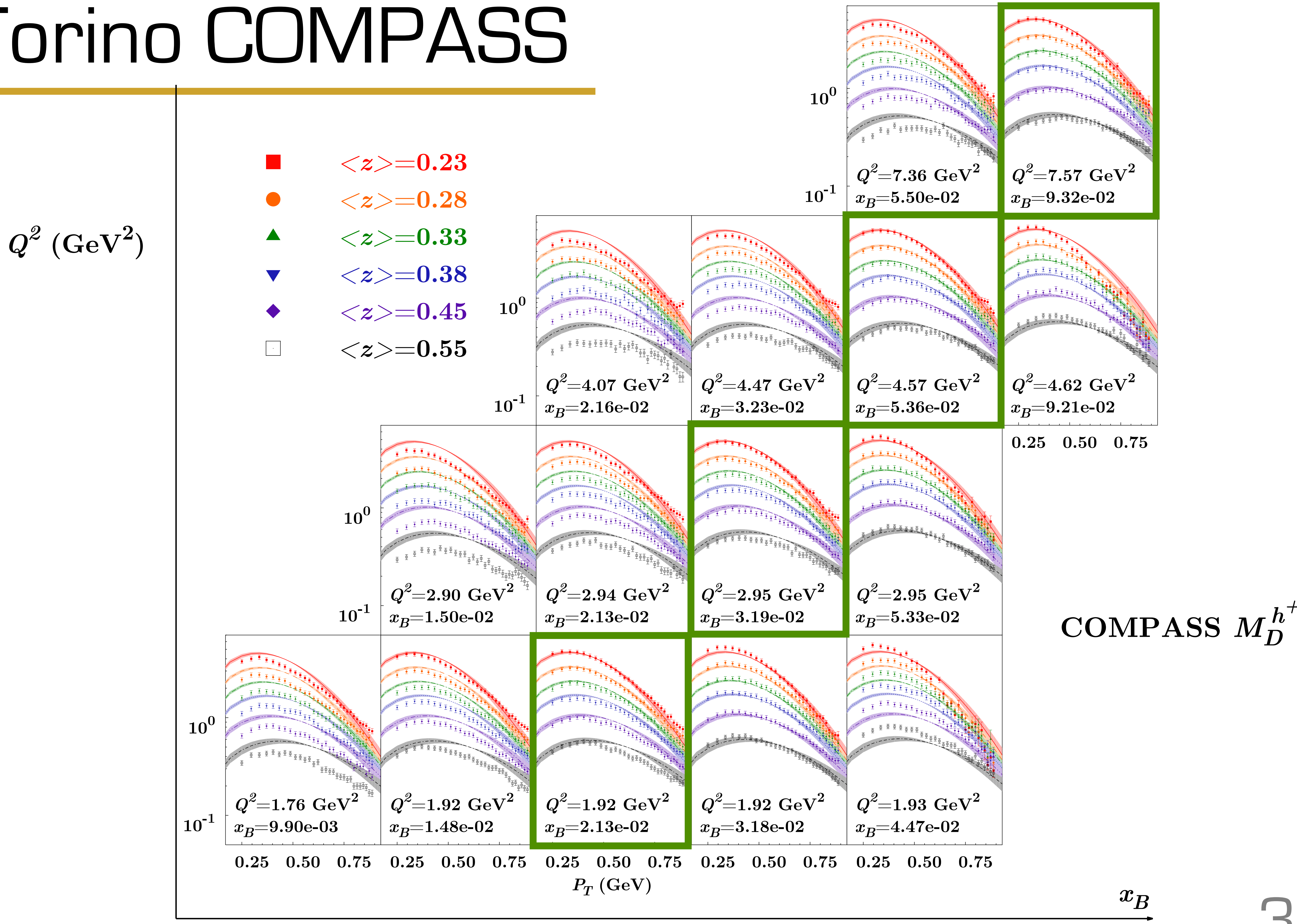


Tuesday, 25 February 14

Torino COMPASS



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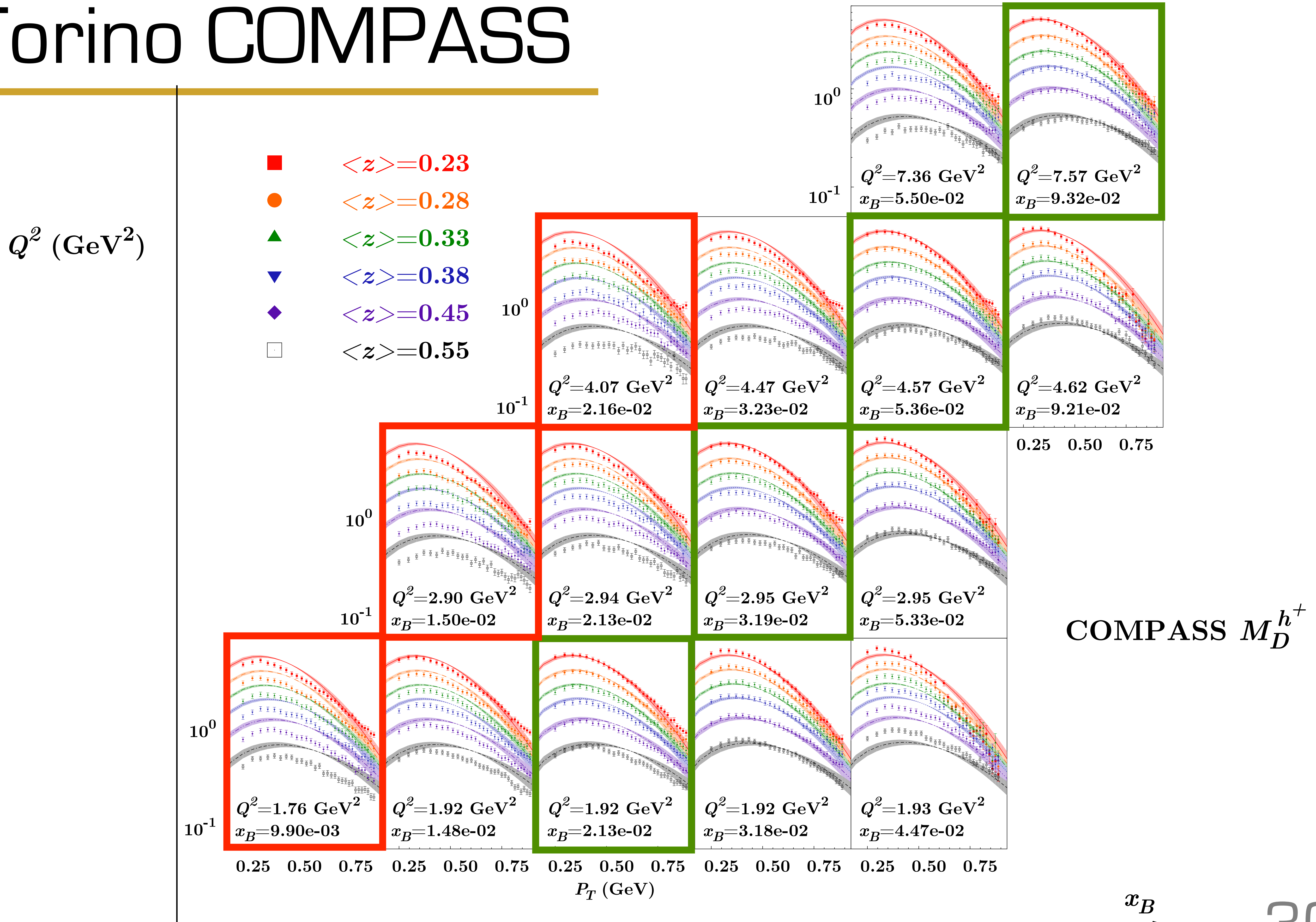
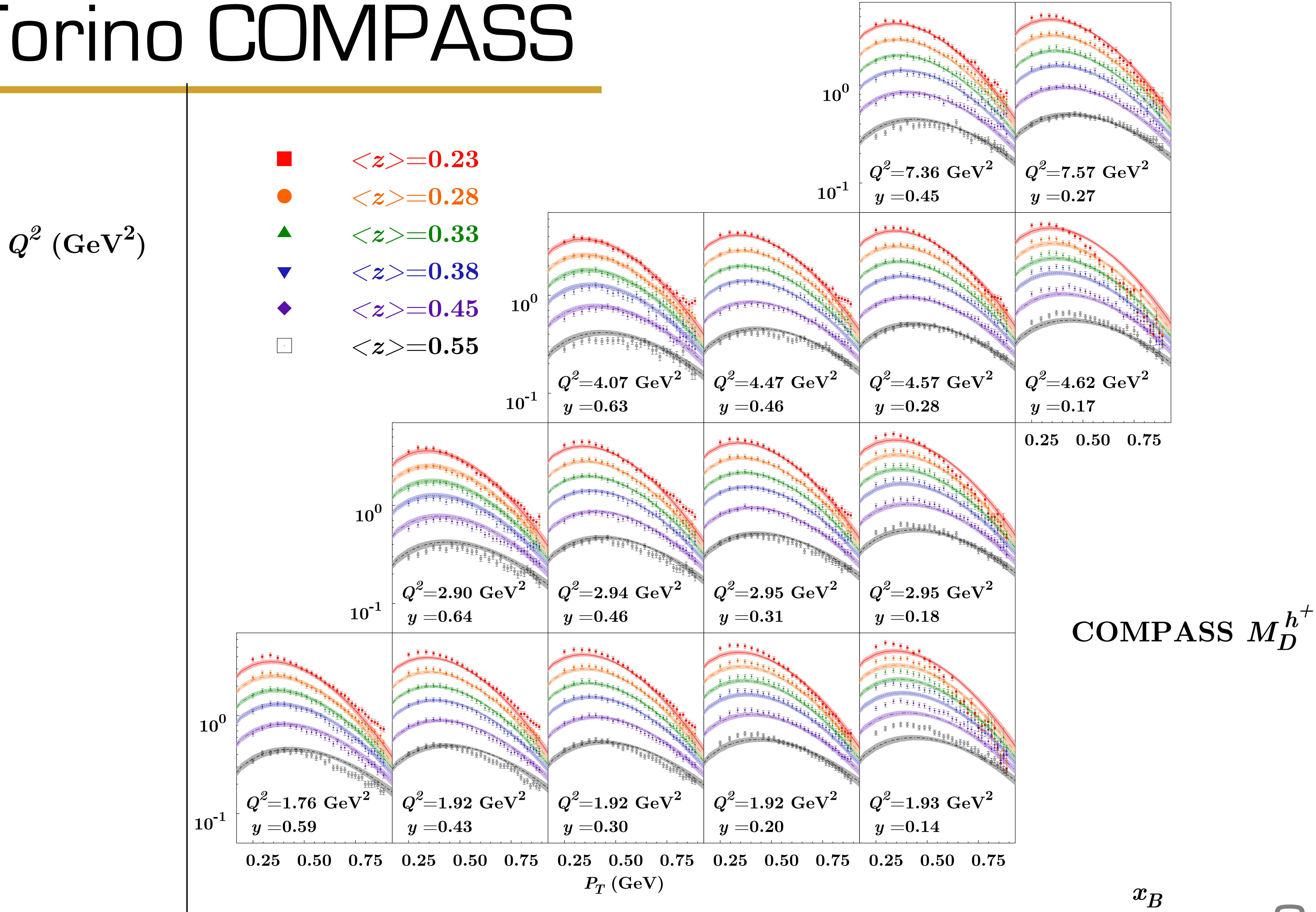
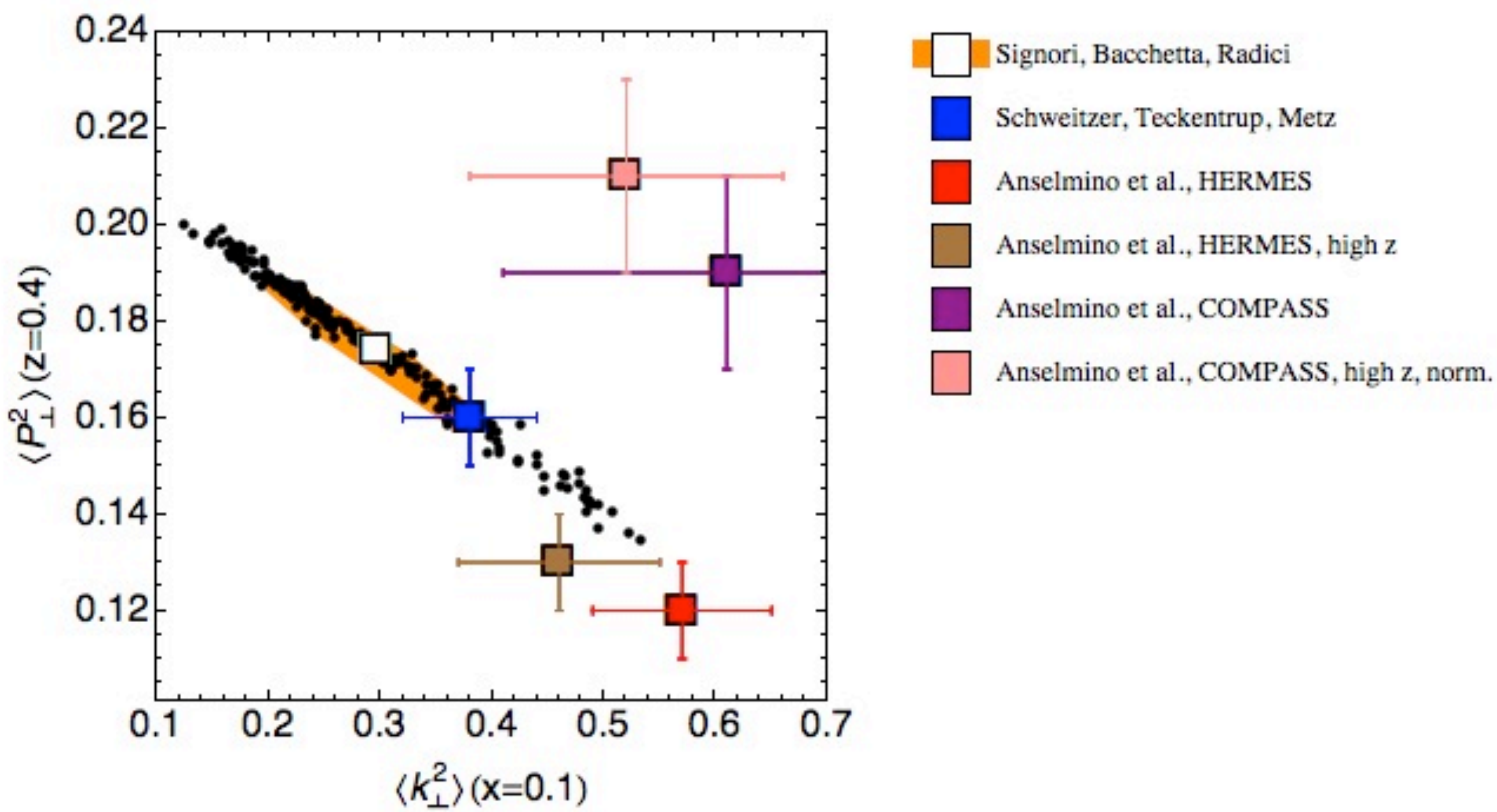


Torino COMPASS



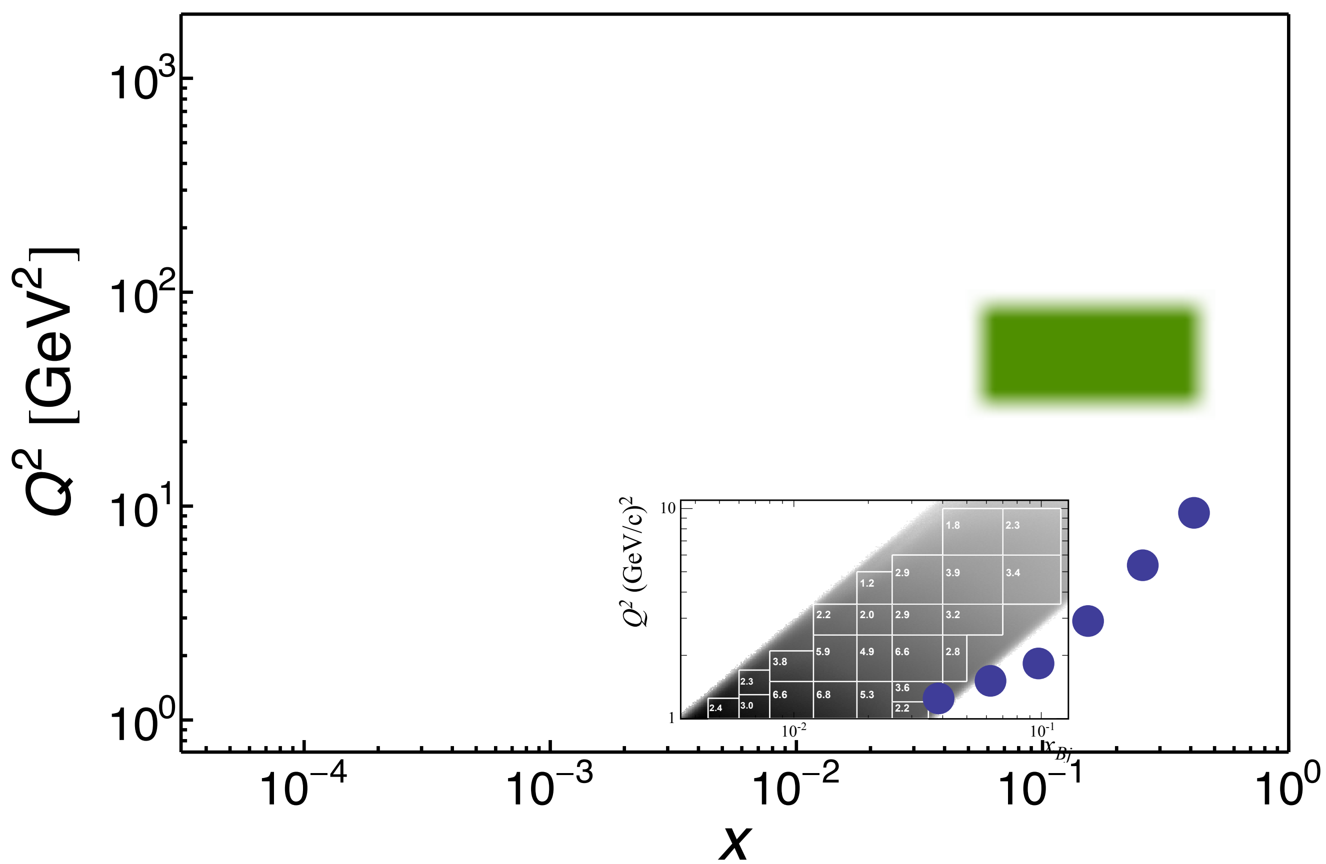
Comparison



Transverse momentum in FFs

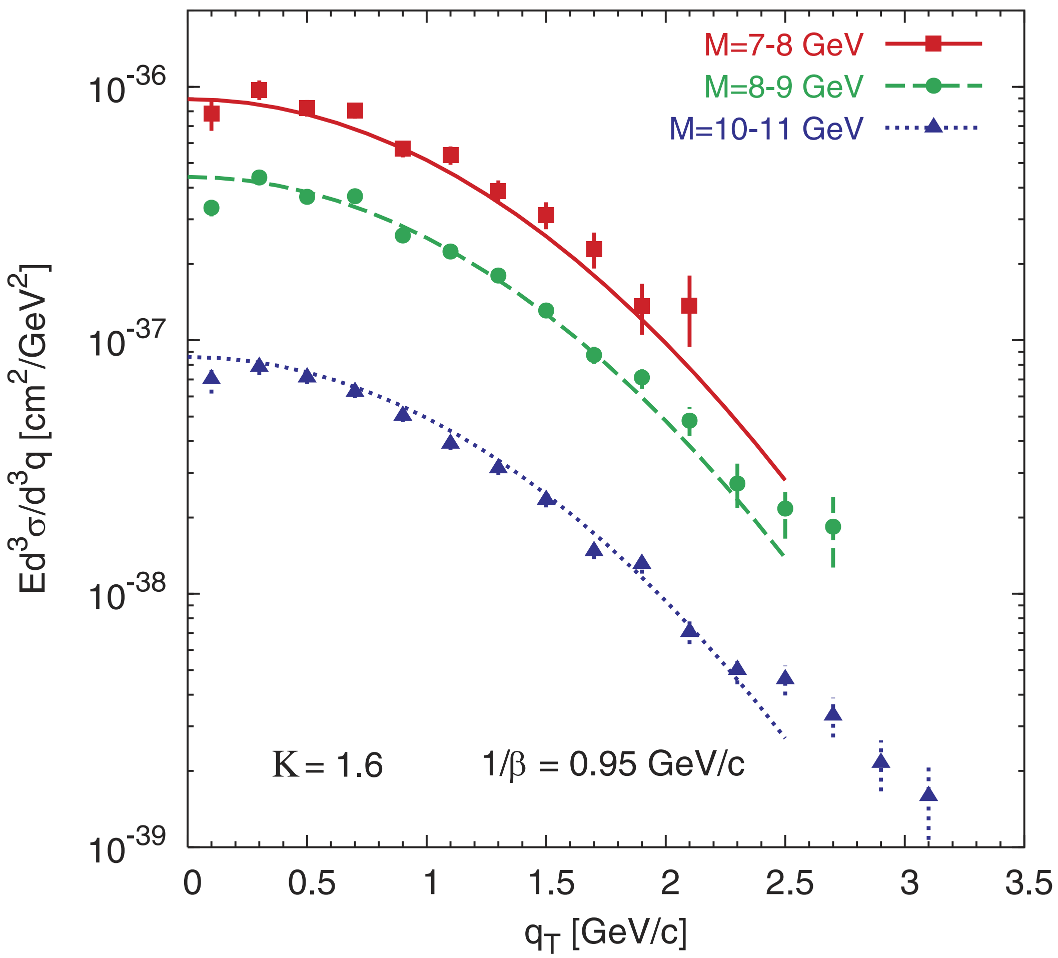
Transverse momentum in PDFs

Let us turn to Drell-Yan

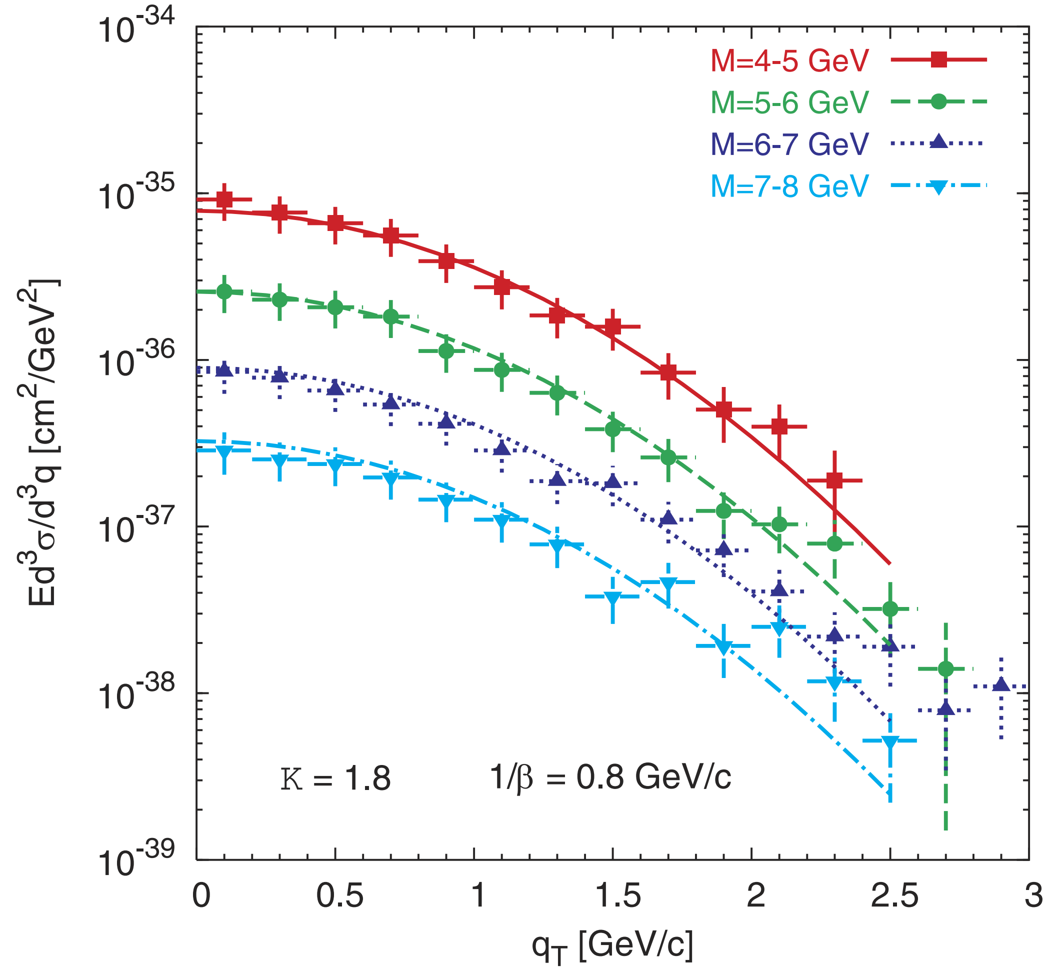


Cagliari Drell-Yan (no evo)

D'Alesio, Murgia, PRD70 (04)



E605



E288

$$\langle k_T^2 \rangle \approx 1.3 - 1.8 \text{ GeV}^2$$

Inclusion of pQCD corrections

$$F_{UU,T}(x, z, \boldsymbol{P}_{hT}^2, Q^2) = x \sum_a \mathcal{H}_{UU,T}^a(Q^2; \mu^2) \int d\boldsymbol{k}_\perp \, d\boldsymbol{P}_\perp \, f_1^a\big(x, \boldsymbol{k}_\perp^2; \mu^2\big) \, D_1^{a \to h}\big(z, \boldsymbol{P}_\perp^2; \mu^2\big) \, \delta\big(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp\big)$$
$$+ Y_{UU,T}\big(Q^2, \boldsymbol{P}_{hT}^2\big) + \mathcal{O}\big(M^2/Q^2\big)$$

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$$f_1^a(x,\boldsymbol{k}_\perp^2;\mu^2) \equiv \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{b}_T\cdot\boldsymbol{k}_\perp}\,\tilde{f}_1^a(x,b_T;\mu^2)$$

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$$F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2) = x\sum_a \mathcal{H}^a_{UU,T}(Q^2;\mu^2)\int \frac{d\boldsymbol{b}_\perp^2}{4\pi} J_0(|\boldsymbol{b}_T||\boldsymbol{P}_{h\perp}|)\tilde{f}_1^a\big(x,z^2\boldsymbol{b}_\perp^2;\mu^2\big)\,\tilde{D}_1^{a\to h}\big(z,\boldsymbol{b}_\perp^2;\mu^2\big)$$
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see talk by L. Gamberg

Evolved TMDs à la Collins

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$

Evolved TMDs à la Collins

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collinear PDF

Evolved TMDs à la Collins

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collinear PDF

pQCD

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collinear PDF

nonperturbative part of evolution

pQCD

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collinear PDF

pQCD

nonperturbative part of evolution

nonperturbative part of TMD

Evolved TMDs à la Collins

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collinear PDF

pQCD

nonperturbative part of evolution

nonperturbative part of TMD

$$b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}} \qquad \mu_b = 2e^{-\gamma_E}/b_* \equiv b_0/b_*$$

Evolved TMDs à la Collins

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^a(x, b_T)$$

collinear PDF

pQCD

nonperturbative part of evolution

nonperturbative part of TMD

$$b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$

$$\mu_b = 2e^{-\gamma_E}/b_* \equiv b_0/b_*$$

Many talks: Rogers, Vogelsang, Sun, Kang...

Evolved TMDs à la Collins

$$\widetilde{f}_1^a(x,b_T;\mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x,b_*;\mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x,b_T)$$

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nonperturbative part of evolution

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Many talks: Rogers, Vogelsang, Sun, Kang...

Remark: MC generators with parton shower should partially reproduce the effect of evolution

Fits by Nadolsky et al. (CSS formalism)

$$\widetilde{f}_1^f(x, b_T; \mu^2) = \sum_i (\tilde{C}_{f/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\mathrm{NP}}^q(x, b_T)$$

Brock, Landry, Nadolsky, Yuan, PRD67 (03)

Fits by Nadolsky et al. (CSS formalism)

$$\widetilde{f}_1^f(x,b_T;\mu^2)=\sum_i\big(\tilde{C}_{f/i}\otimes f_1^i\big)(x,b_*;\mu_b)e^{\tilde{S}(b_*;\mu_b,\mu)}e^{g_K(b_T)\ln\frac{\mu}{\mu_0}}\underbrace{\hat{f}^q_{\mathrm{NP}}(x,b_T)}_{e^{-b_T^2/\langle b_T^2\rangle}}$$

Brock, Landry, Nadolsky, Yuan, PRD67 (03)

Fits by Nadolsky et al. (CSS formalism)

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$$e^{-b_T^2/\langle b_T^2 \rangle}$$

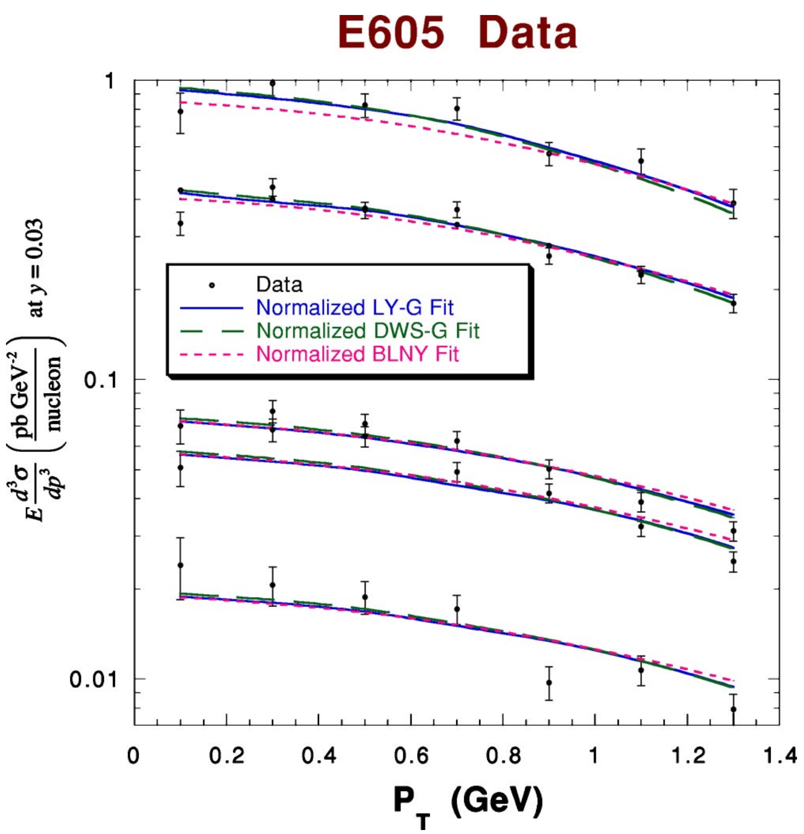
$$\frac{1}{\langle b_T^2 \rangle} = \frac{1}{2}\left(g_1 + g_2 \log\left(\frac{Q}{2Q_0}\right) + g_1 g_3 \, \log(10x) \right)$$

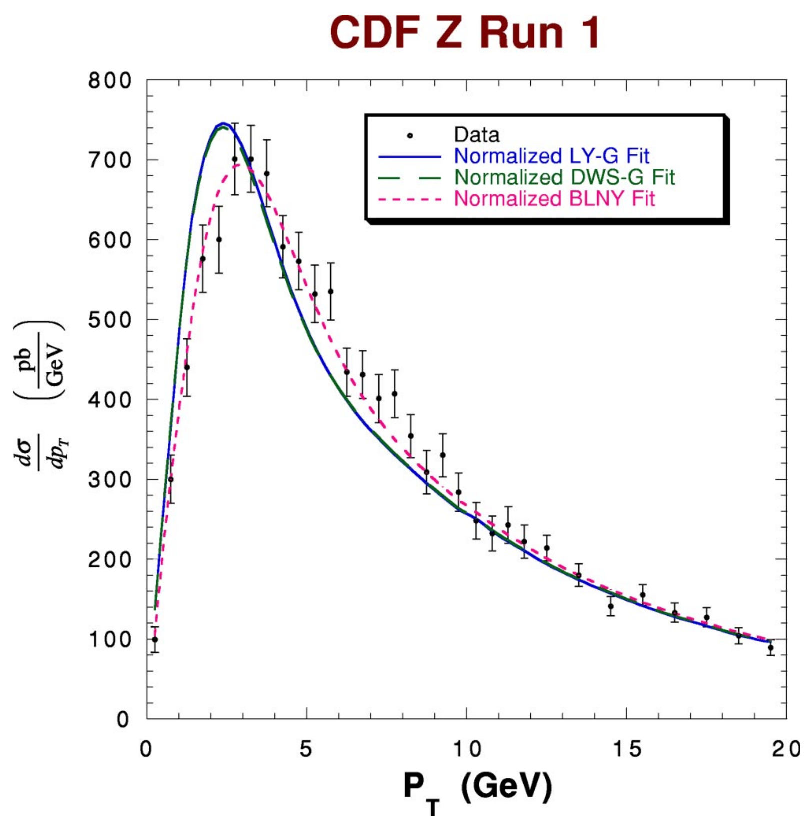
$b_{\rm max}$

Brock, Landry, Nadolsky, Yuan, PRD67 (03)

Nadolsky et al. fits (CSS formalism)

Nadolsky et al. fits (CSS formalism)



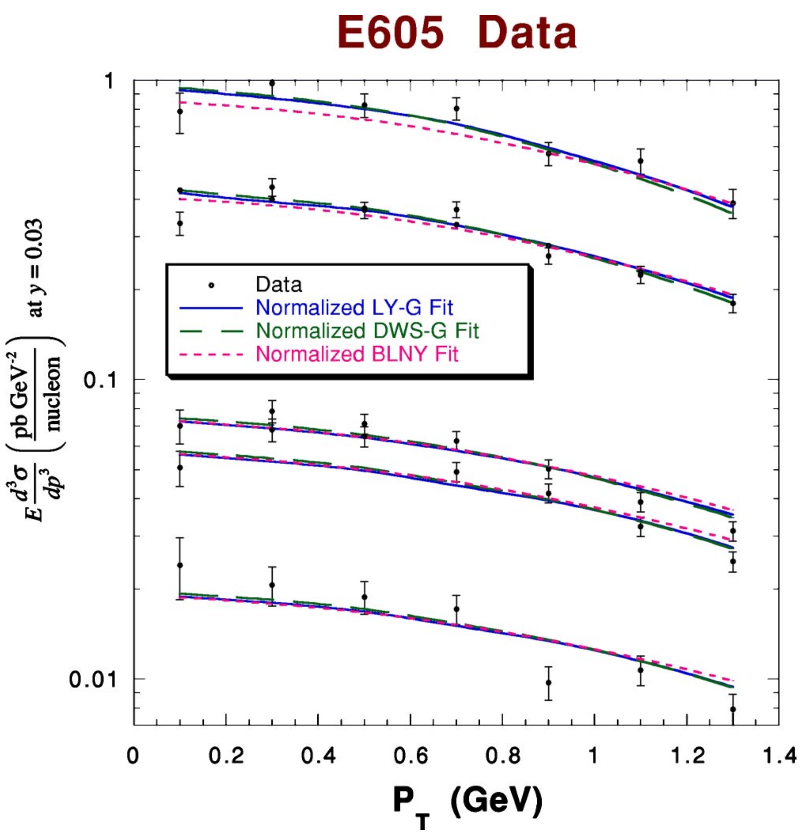


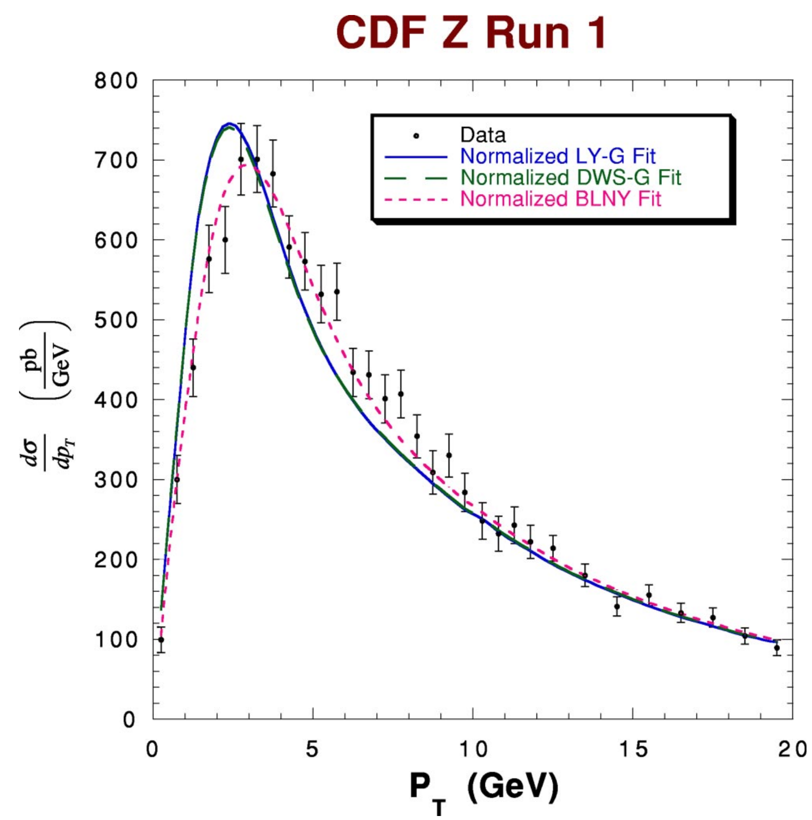
111 data points

Drell-Yan

Q^2>5 GeV

Nadolsky et al. fits (CSS formalism)





111 data points

Drell-Yan

$Q^2>5$ GeV

$$\frac{1}{\langle b_T^2\rangle} = \frac{1}{2}\left(0.21 + 0.68\,\log\left(\frac{Q}{2Q_0}\right) - 0.25\,\log\left(10x\right)\right)$$

$$b_{\max} = 0.5\ \mathrm{GeV}^{-1}$$

Brock, Landry, Nadolsky, Yuan, PRD67 (03)

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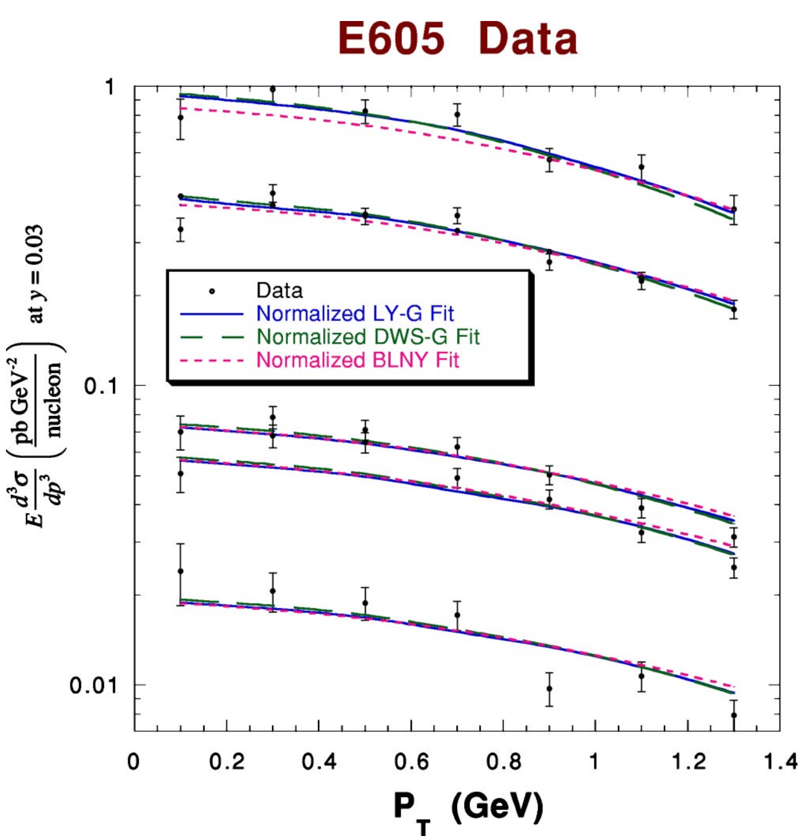
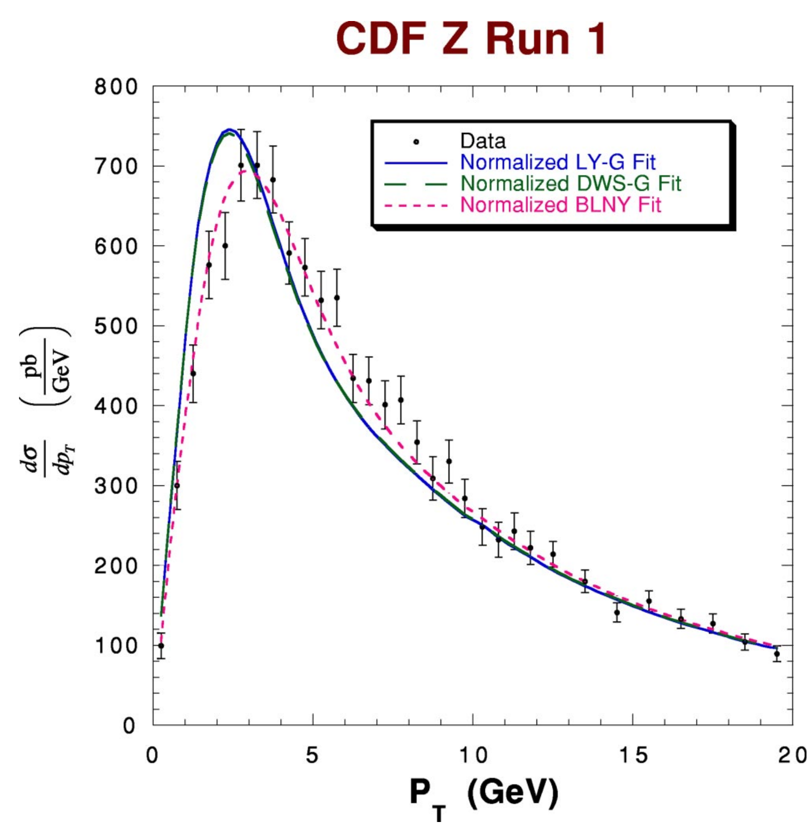




111 data points
Drell-Yan
Q^2>5 GeV

$$\frac{1}{\langle b_T^2\rangle} = \frac{1}{2}\left(0.21 + 0.68\,\log\left(\frac{Q}{2Q_0}\right) - 0.25\,\log\left(10x\right)\right)$$

$$b_{\max} = 0.5\ \mathrm{GeV}^{-1}$$

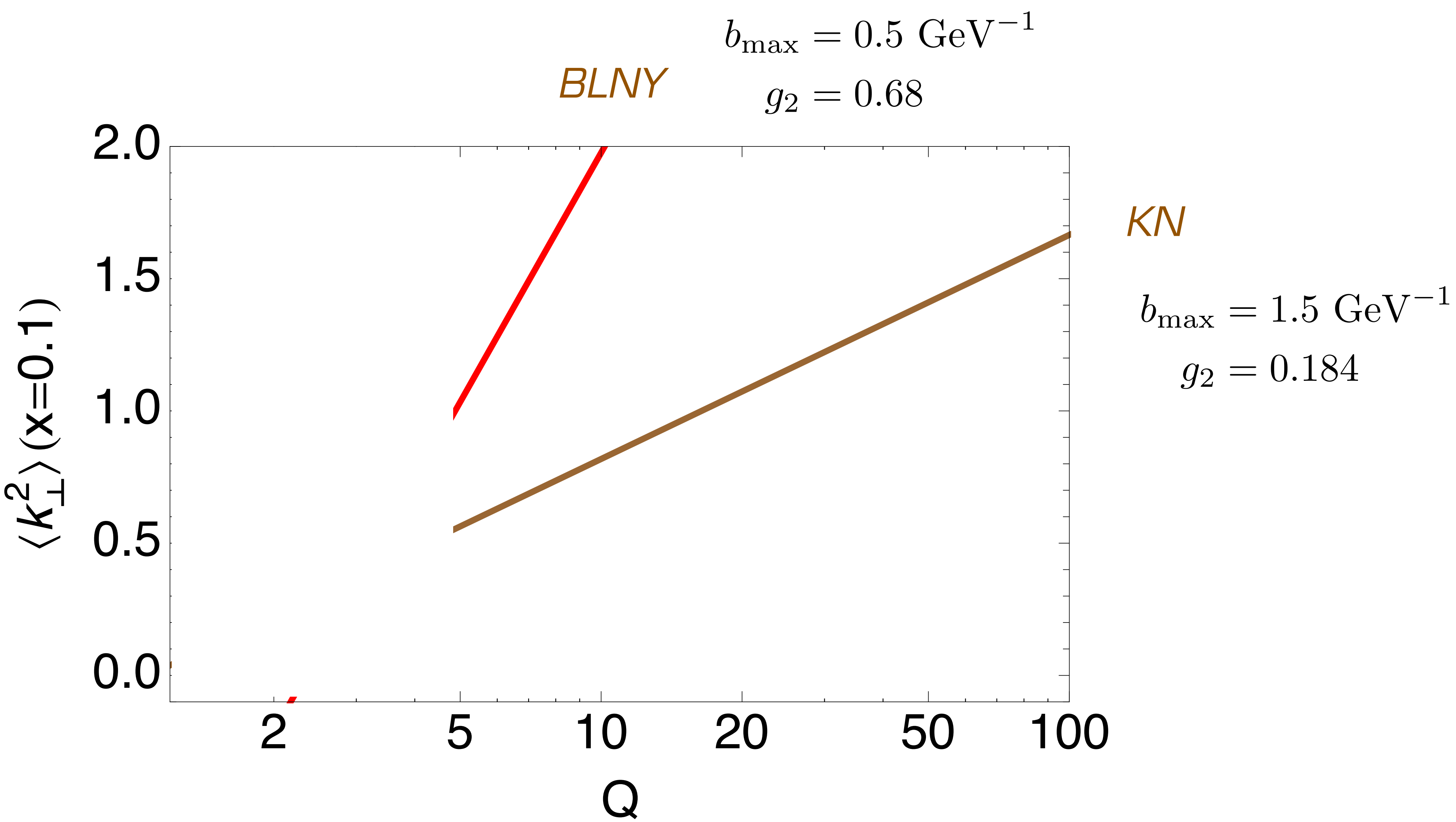
Brock, Landry, Nadolsky, Yuan, PRD67 (03)

$$\frac{1}{\langle b_T^2\rangle} = \frac{1}{2}\left(0.20 + 0.184\,\log\left(\frac{Q}{2Q_0}\right) - 0.026\,\log\left(10x\right)\right)$$

$$b_{\max} = 1.5\ \mathrm{GeV}^{-1}$$

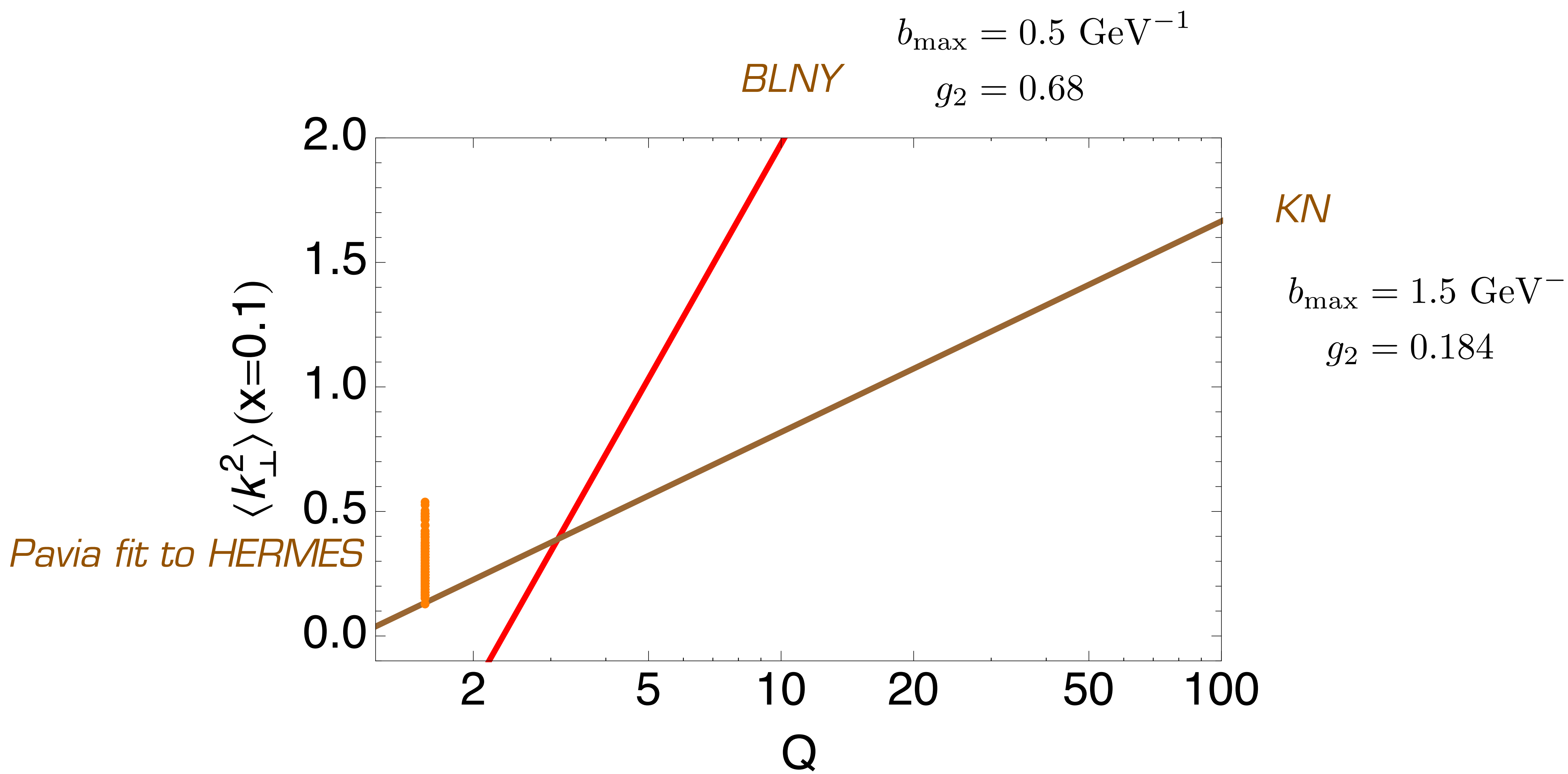
Konychev, Nadolsky, PLB633 (06)
see talk by M. Guzzi

Dependence of Q



Sun, Yuan, PRD88 (13)

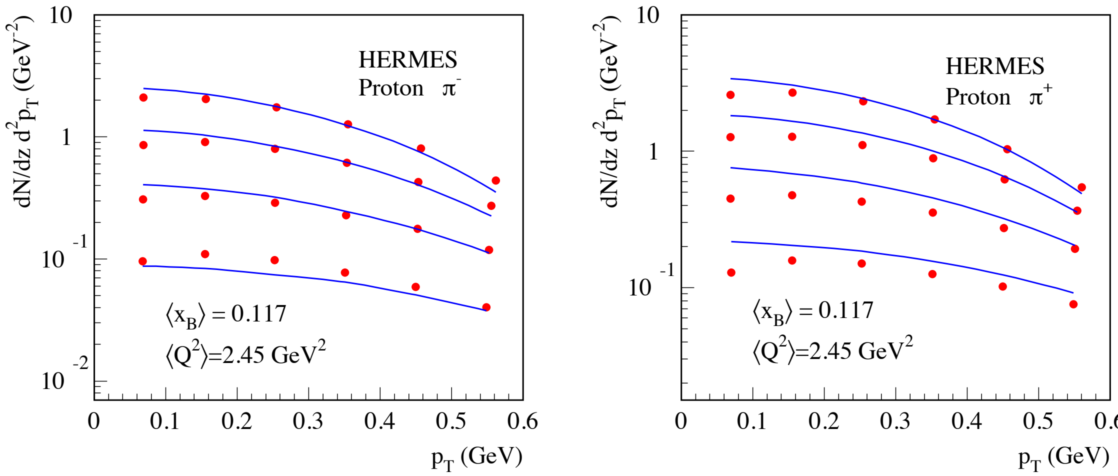
Dependence of Q



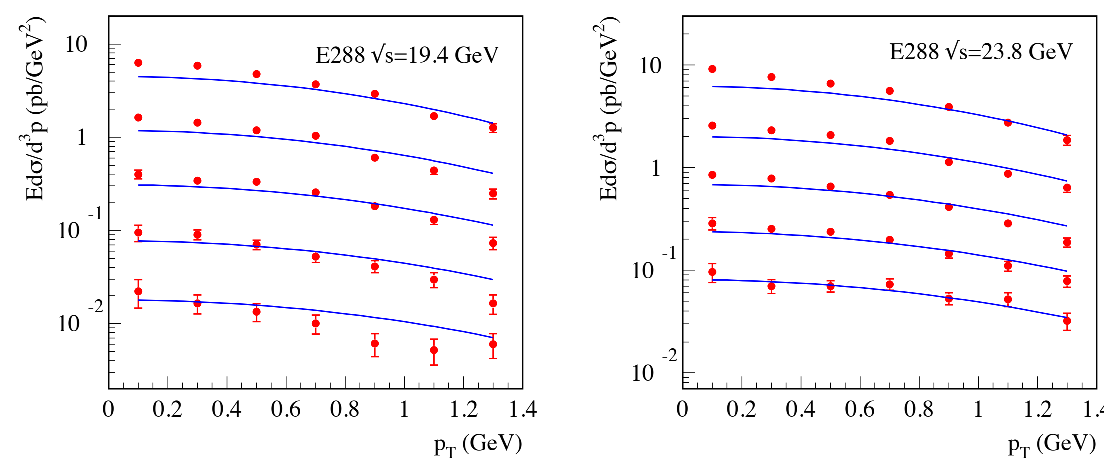
Sun, Yuan, PRD88 (13)

Echevarria, Idilbi, Kang, Vitev

SIDIS



DRELL-YAN



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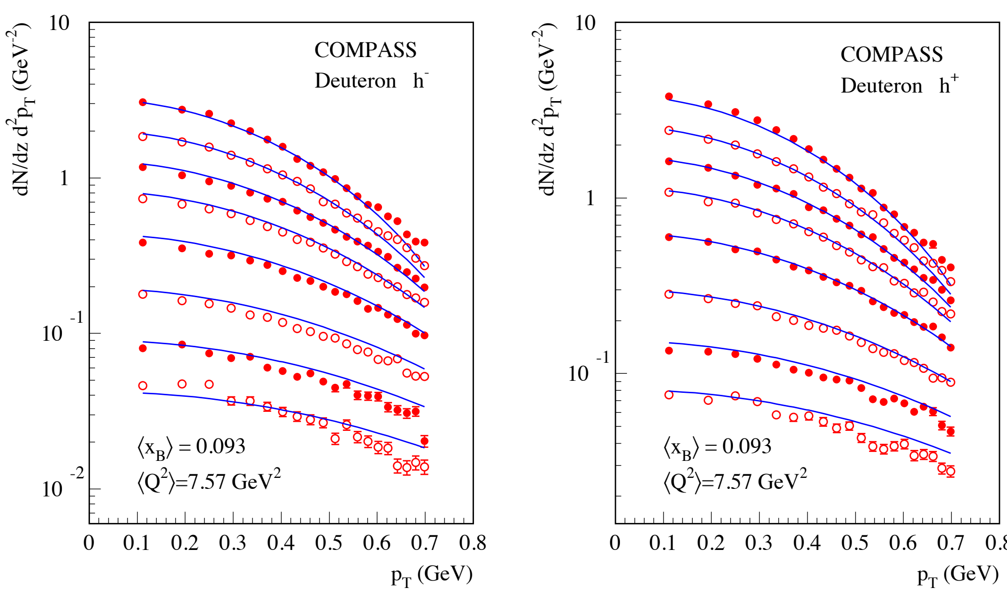
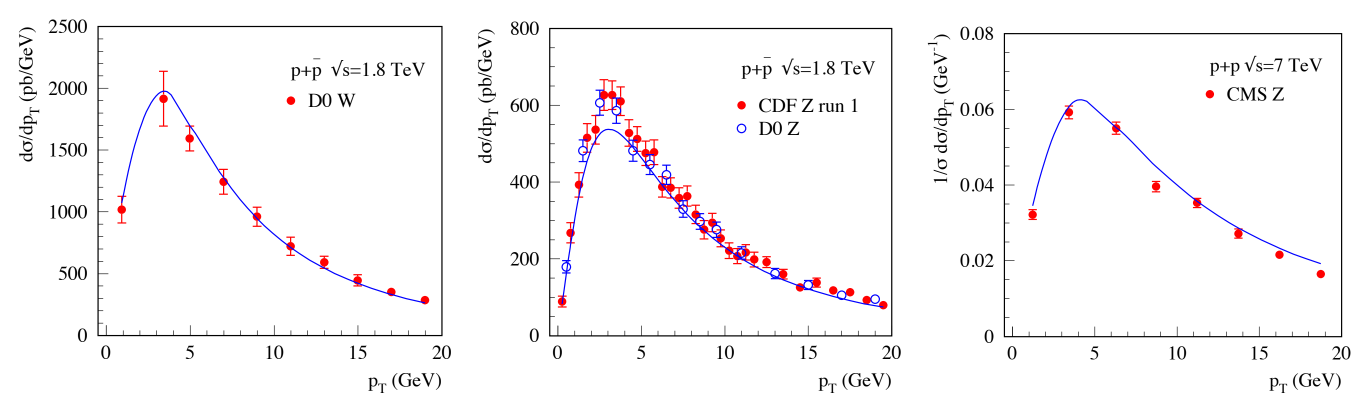


W AND Z PRODUCTION

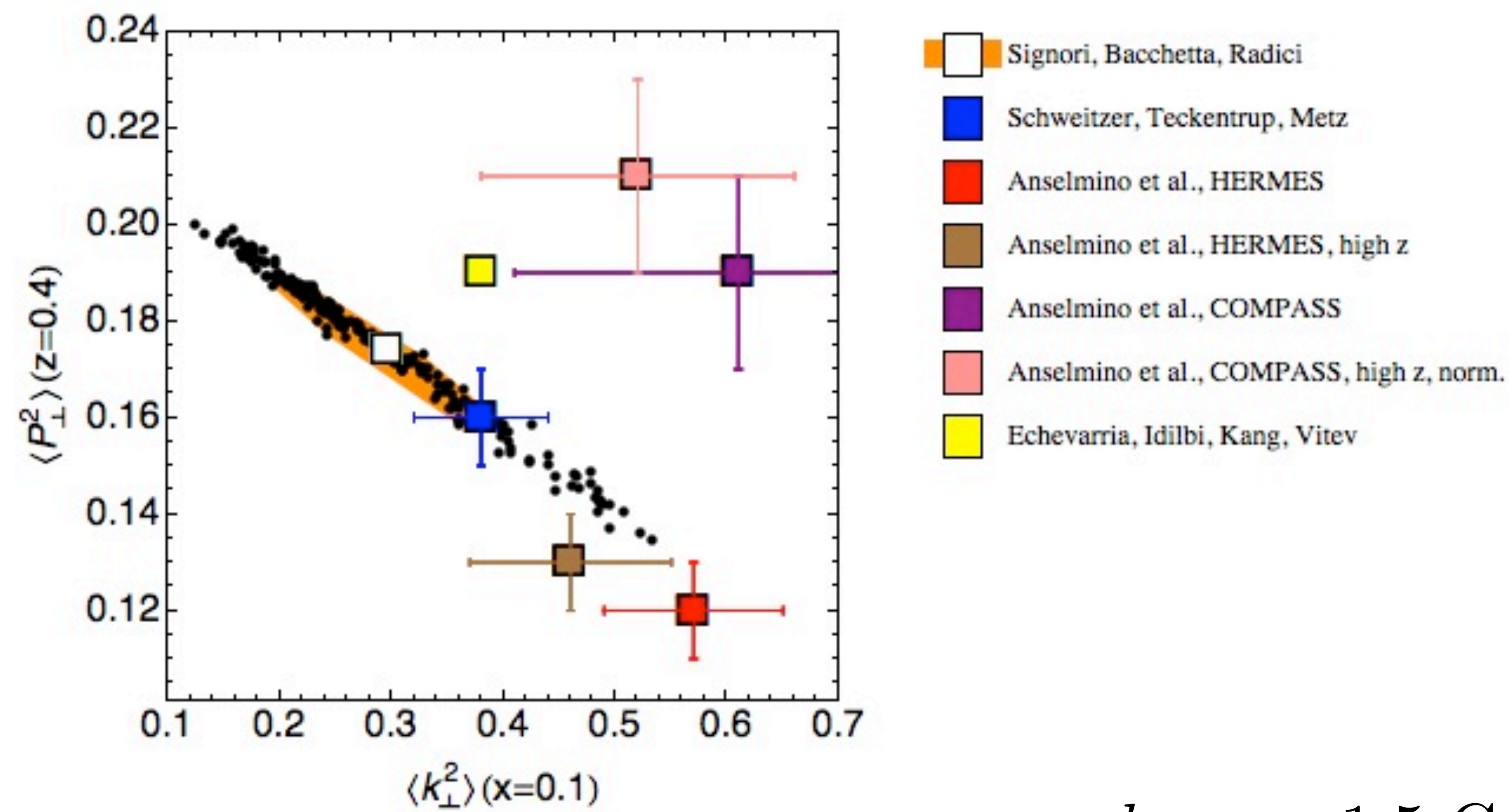


$$b_{\max} = 1.5\ \text{GeV}^{-1}$$

$$g_2 = 0.16$$

Comparison

Transverse momentum
in FFs



$$b_{\max} = 1.5 \text{ GeV}^{-1}$$

$$g_2 = 0.16$$

Transverse momentum
in PDFs

Sun-Yuan

Sun, Yuan, PRD88 (13)

$$\widetilde{f}_1^f(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{f/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^q(x, b_T)$$

“standard” CSS $\quad \exp\left\{-2C_F \int_{\mu_b = b_0/b_\star}^{Q} \frac{d\mu'}{\mu'} \frac{\alpha_s(\mu')}{\pi}\left[\ln\left(\frac{Q^2}{\mu'^2}\right) - \frac{3}{2}\right] + g_2 b_T^2 \ln\left(\frac{Q}{Q_0}\right)\right\}$

Sun-Yuan

Sun, Yuan, PRD88 (13)

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Sun-Yuan

Sun, Yuan, PRD88 (13)

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Sun-Yuan

$$\exp\left\{-2C_F \int_{Q_0}^{Q} \frac{d\mu'}{\mu'} \frac{\alpha_s(\mu')}{\pi}\left[\ln\left(\frac{Q^2}{\mu'^2}\right) + \ln\left(\frac{Q_0^2 b_T^2}{C_1^2}\right) - \frac{3}{2}\right]\right\}$$

see talk by Peng Sun

Sun-Yuan

Sun, Yuan, PRD88 (13)

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Sun-Yuan

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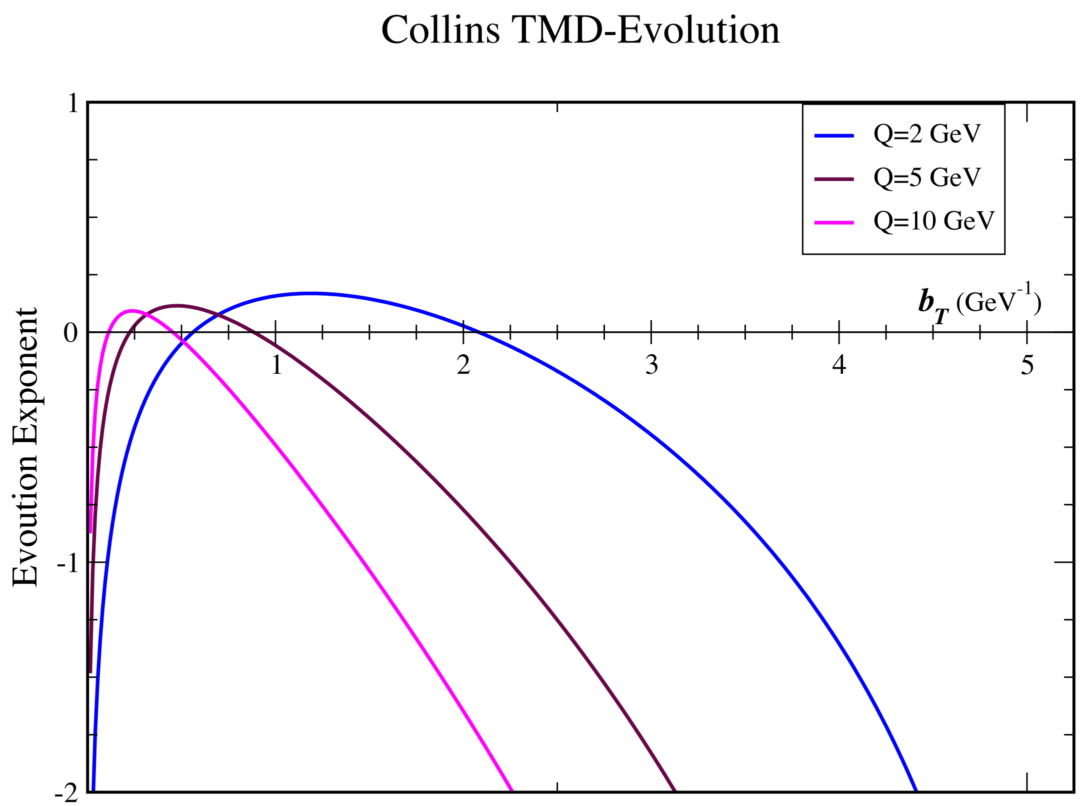
see talk by Peng Sun

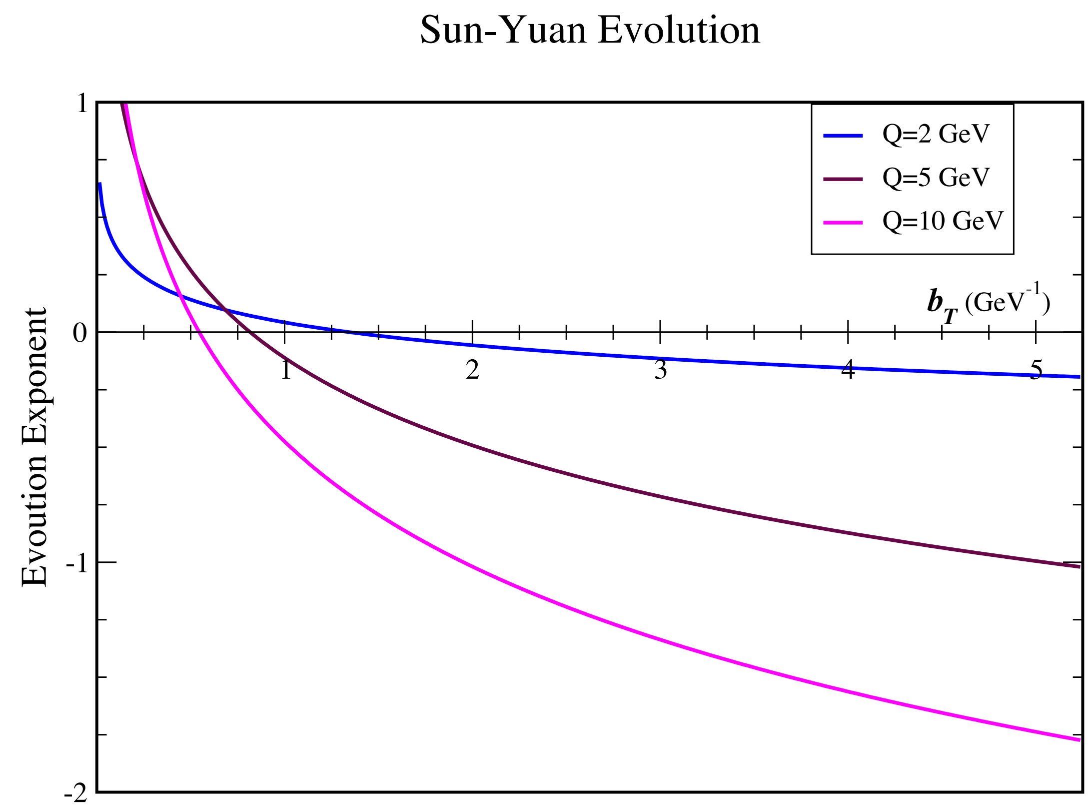
Other prescriptions are possible! E.g., complex b prescription

see talk by W. Vogelsang

Comparison Collins/Sun-Yuan

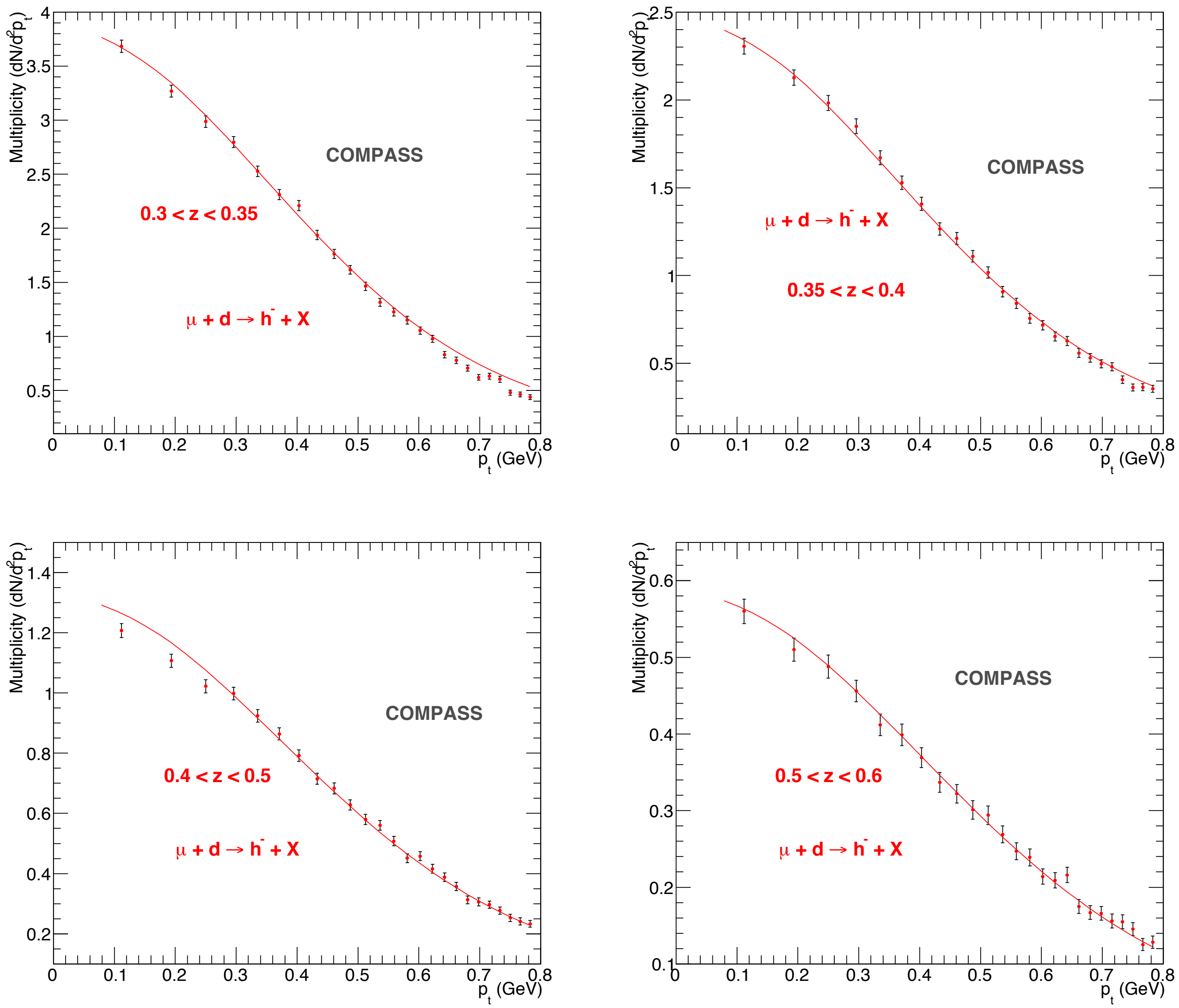
Aidala et al.: arXiv:1401.2654

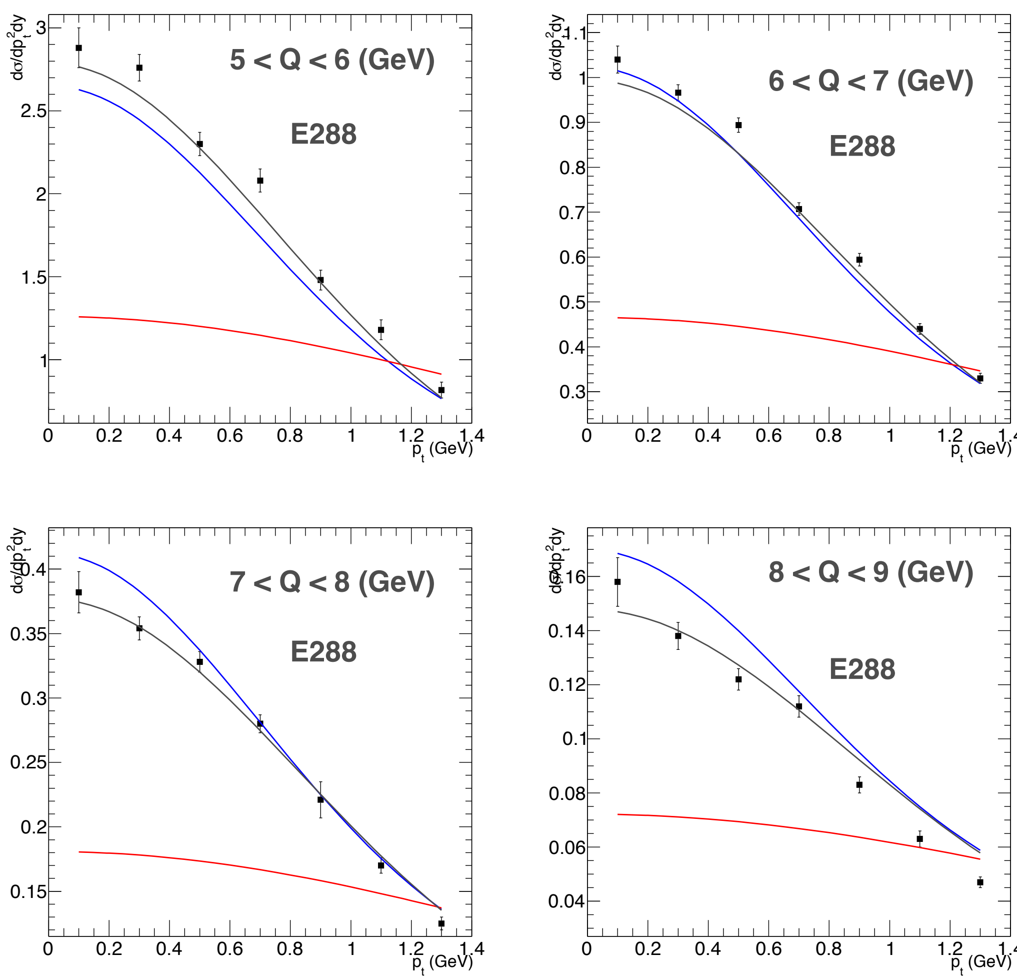




Sun-Yuan

Sun, Yuan, PRD88 (13)

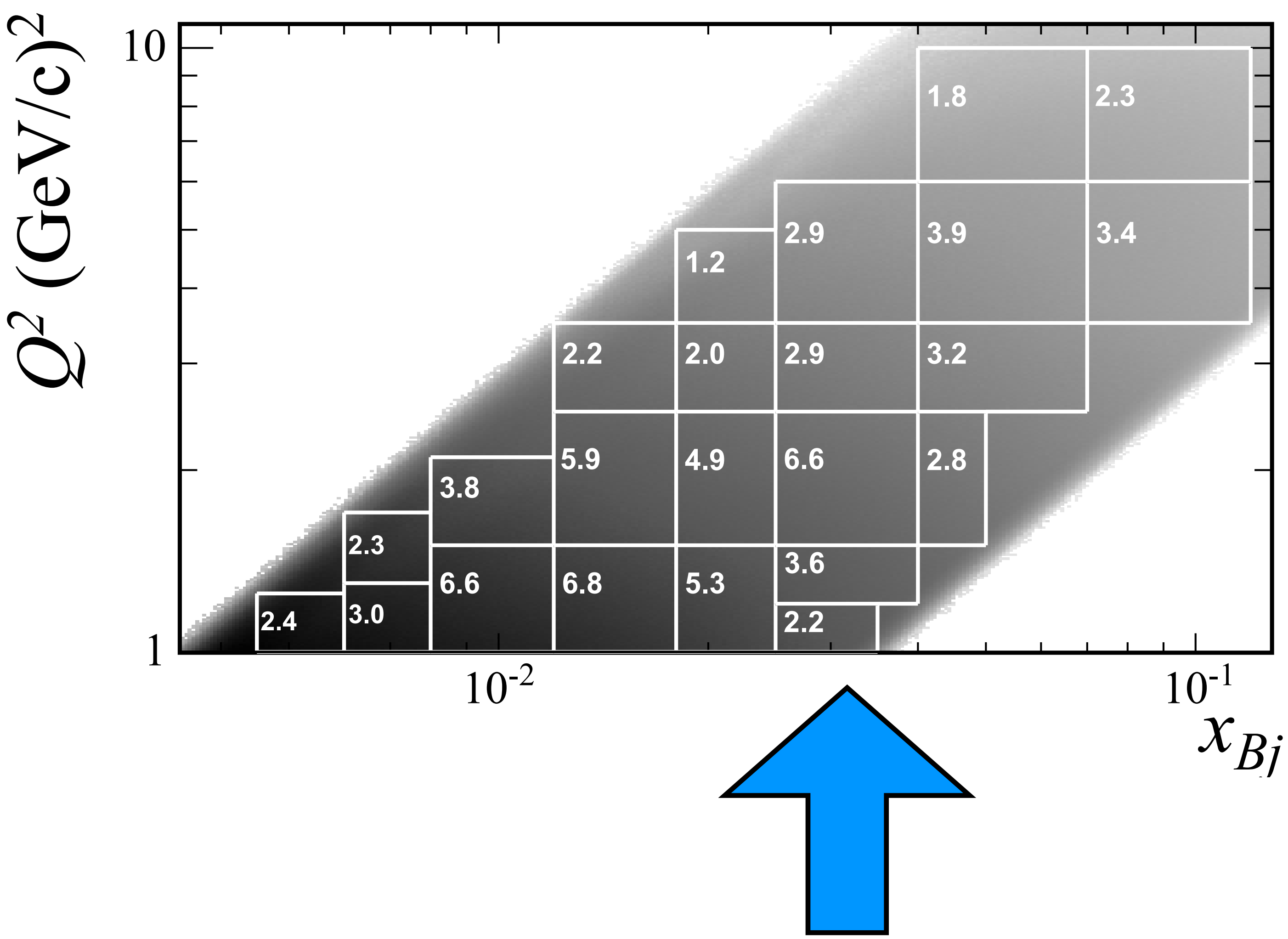




The prescription seems to be working phenomenologically

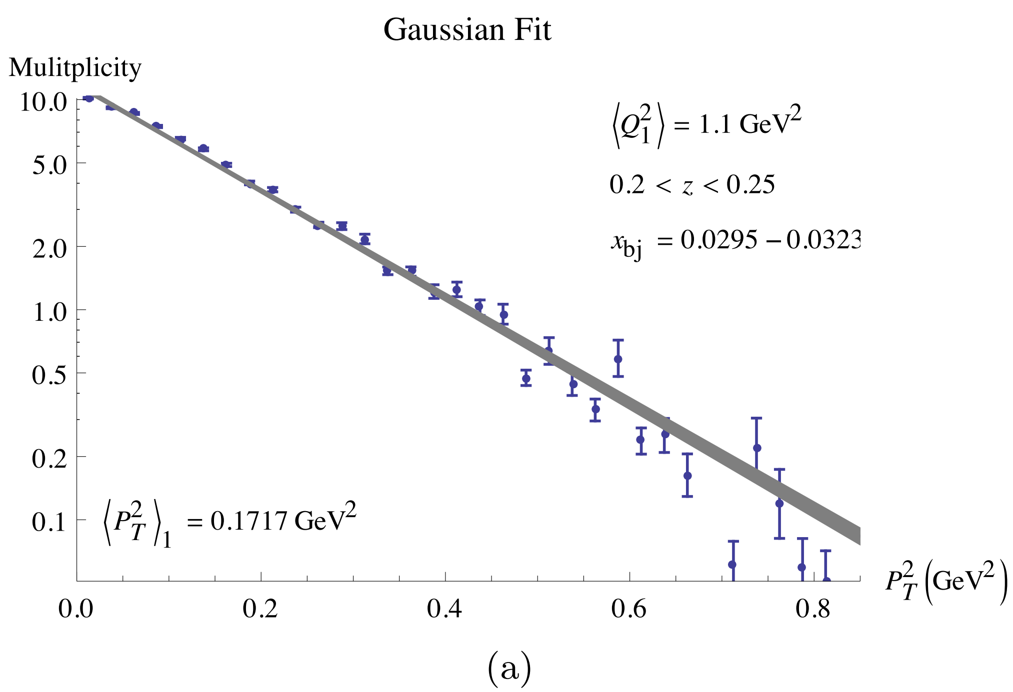
Aidala, Field, Gamberg, Rogers

Aidala et al.: arXiv:1401.2654

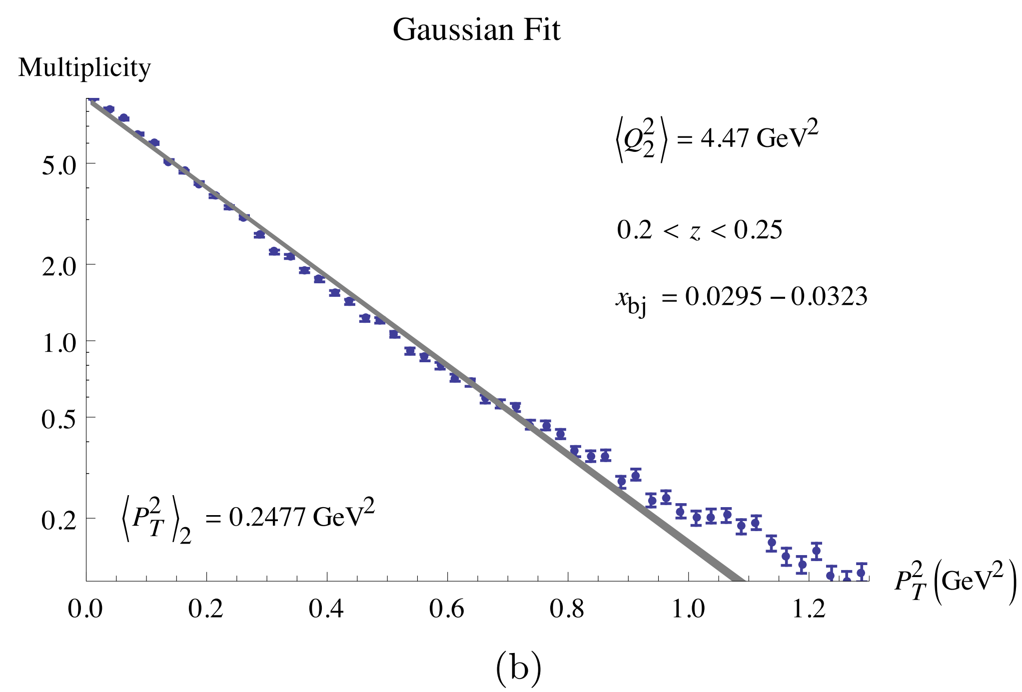


Aidala, Field, Gamberg, Rogers

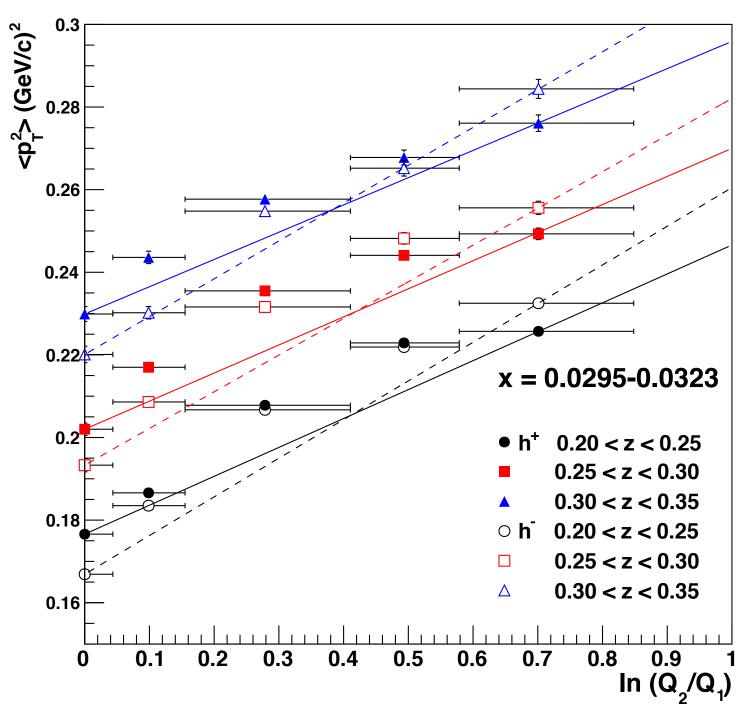
Aidala et al.: arXiv:1401.2654



(a)

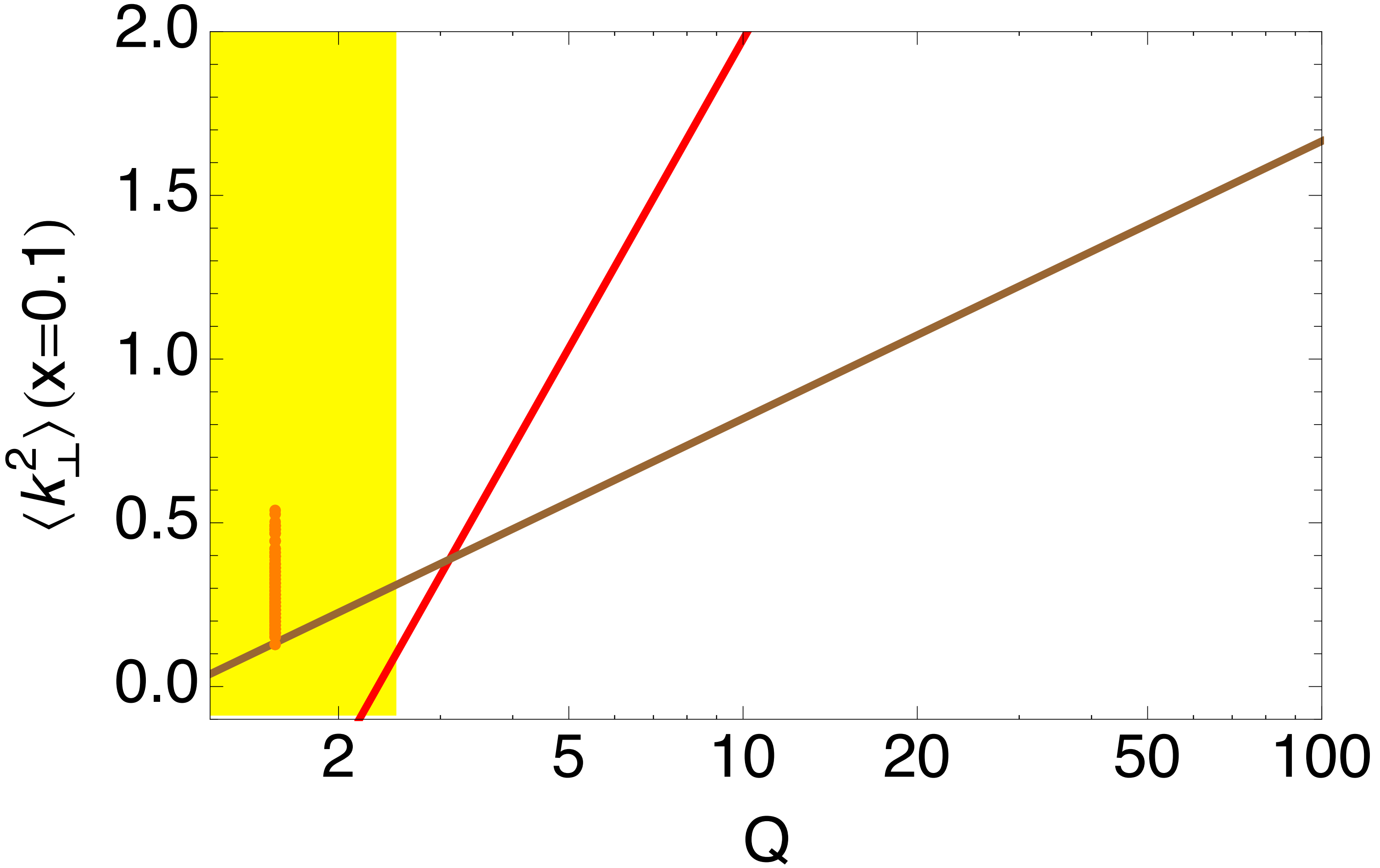


(b)

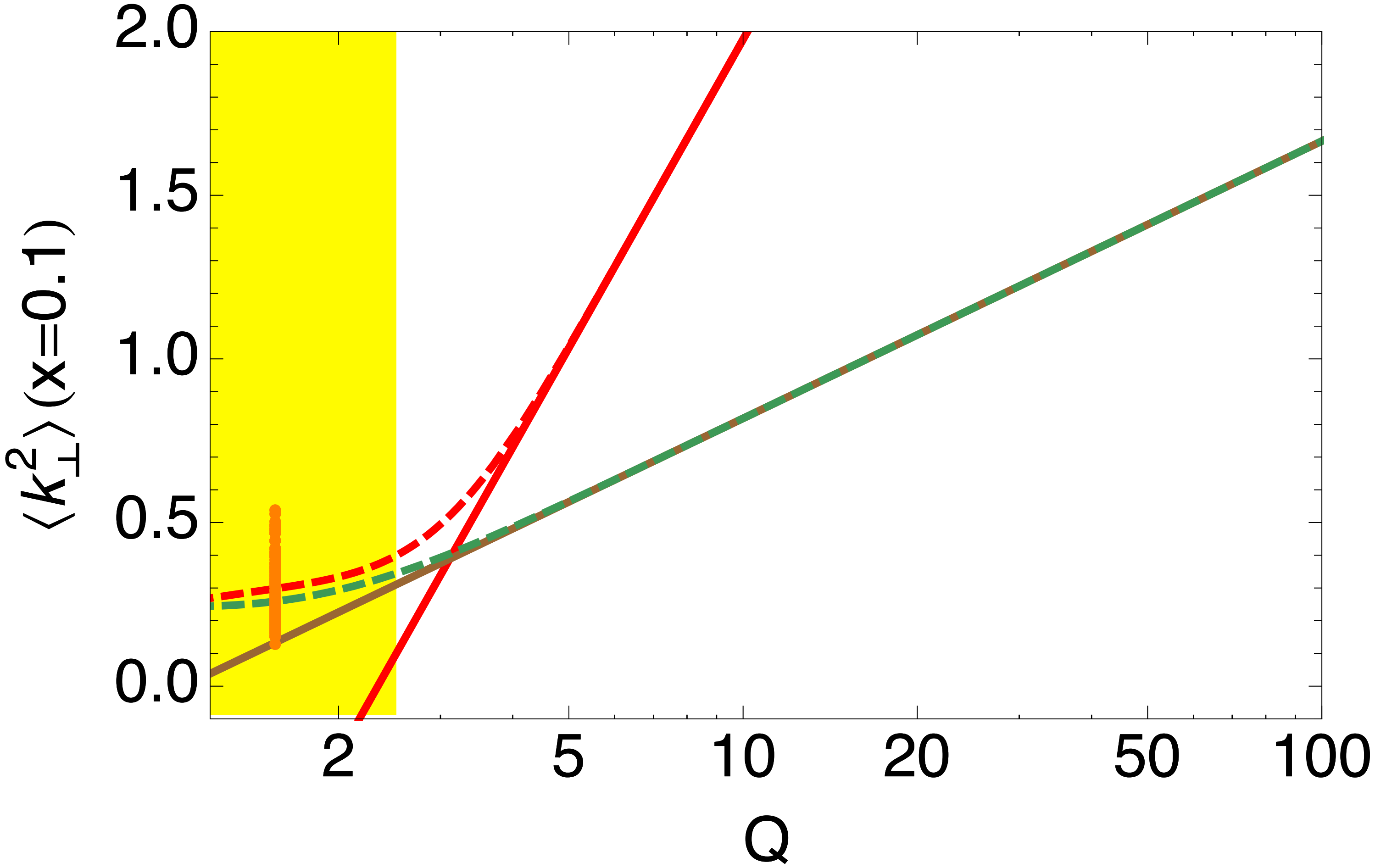


Conclusion: g_2 cannot be large in this region

Dependence of Q

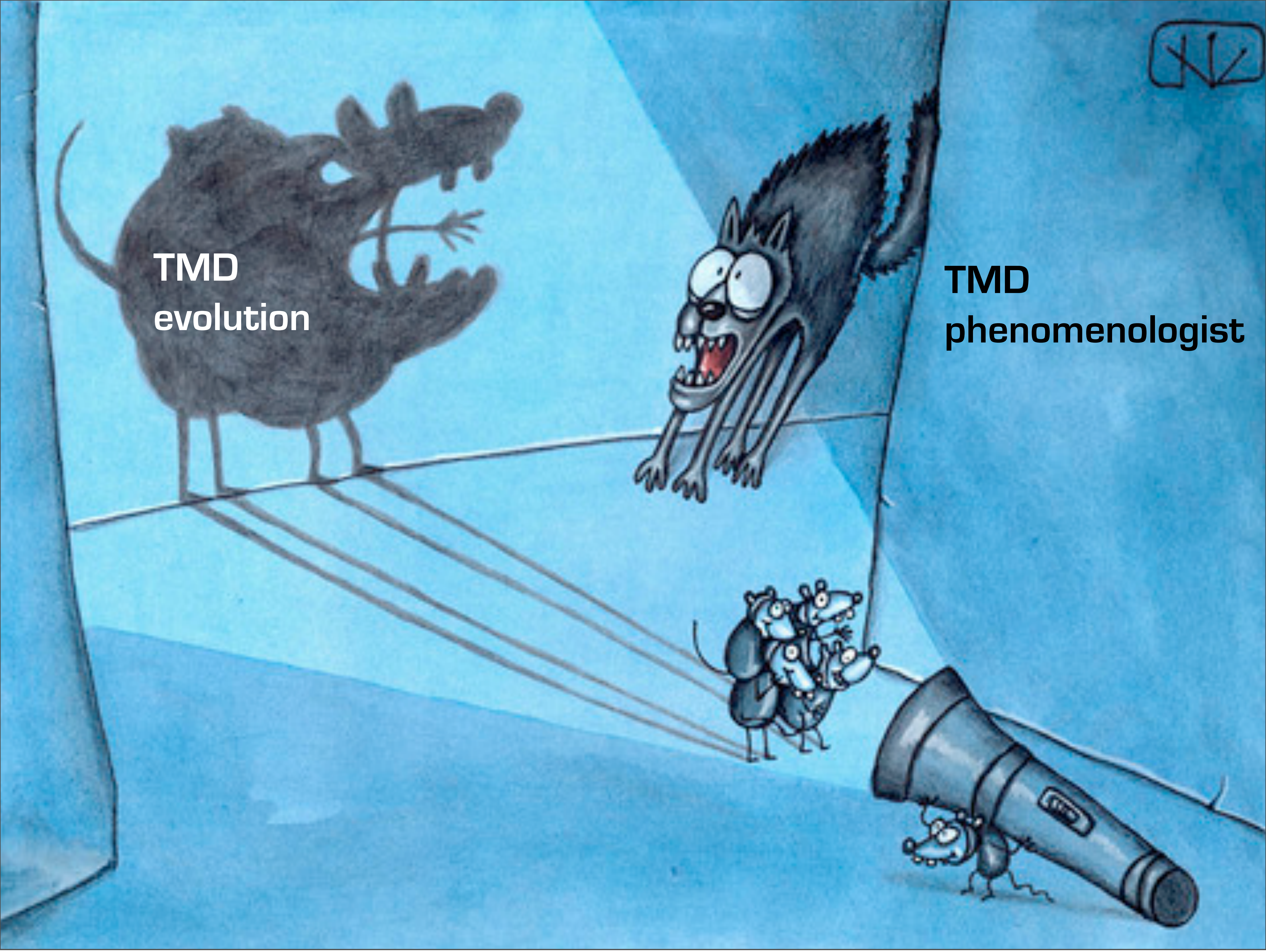


Dependence of Q

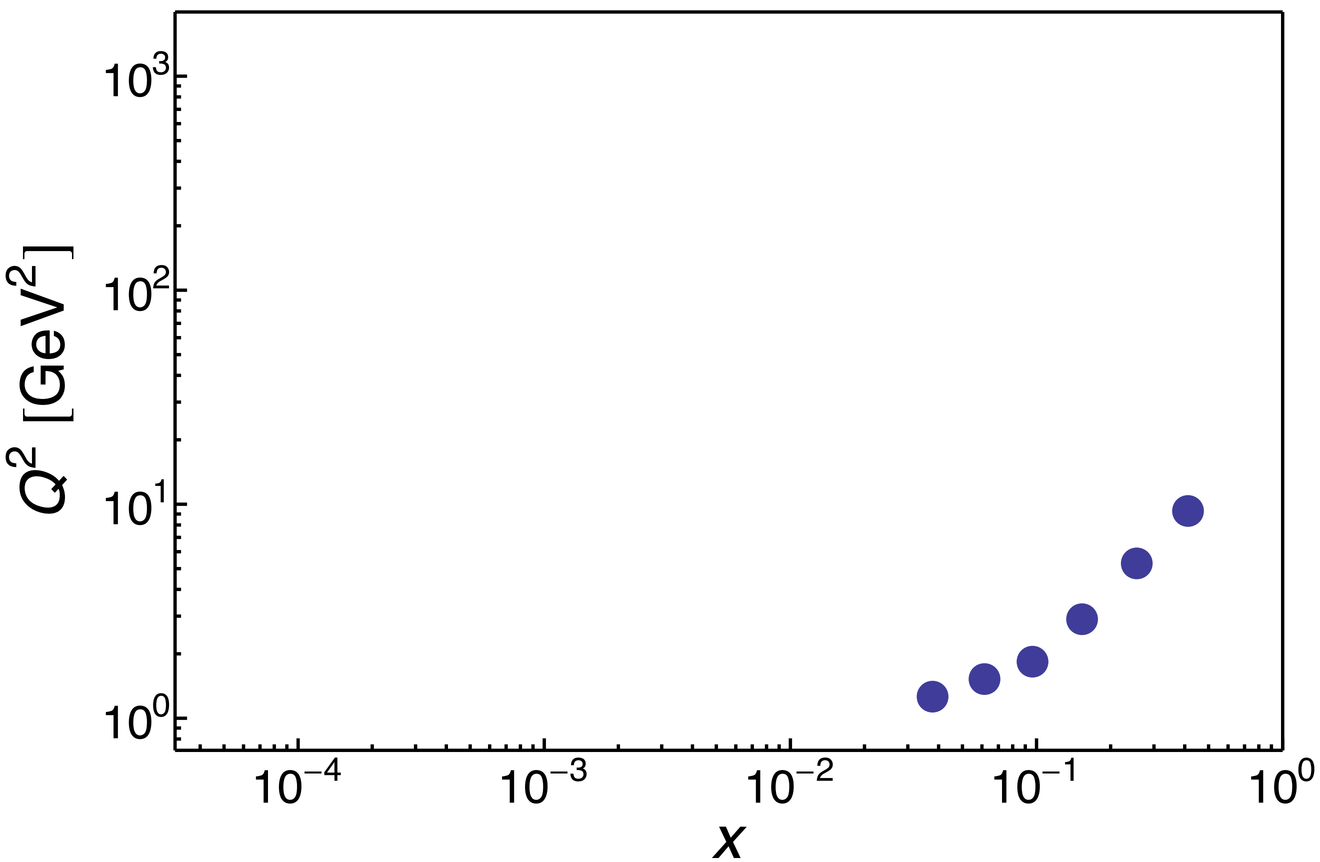


The effect of evolution on unpolarized TMDs below 10 GeV^2 is small

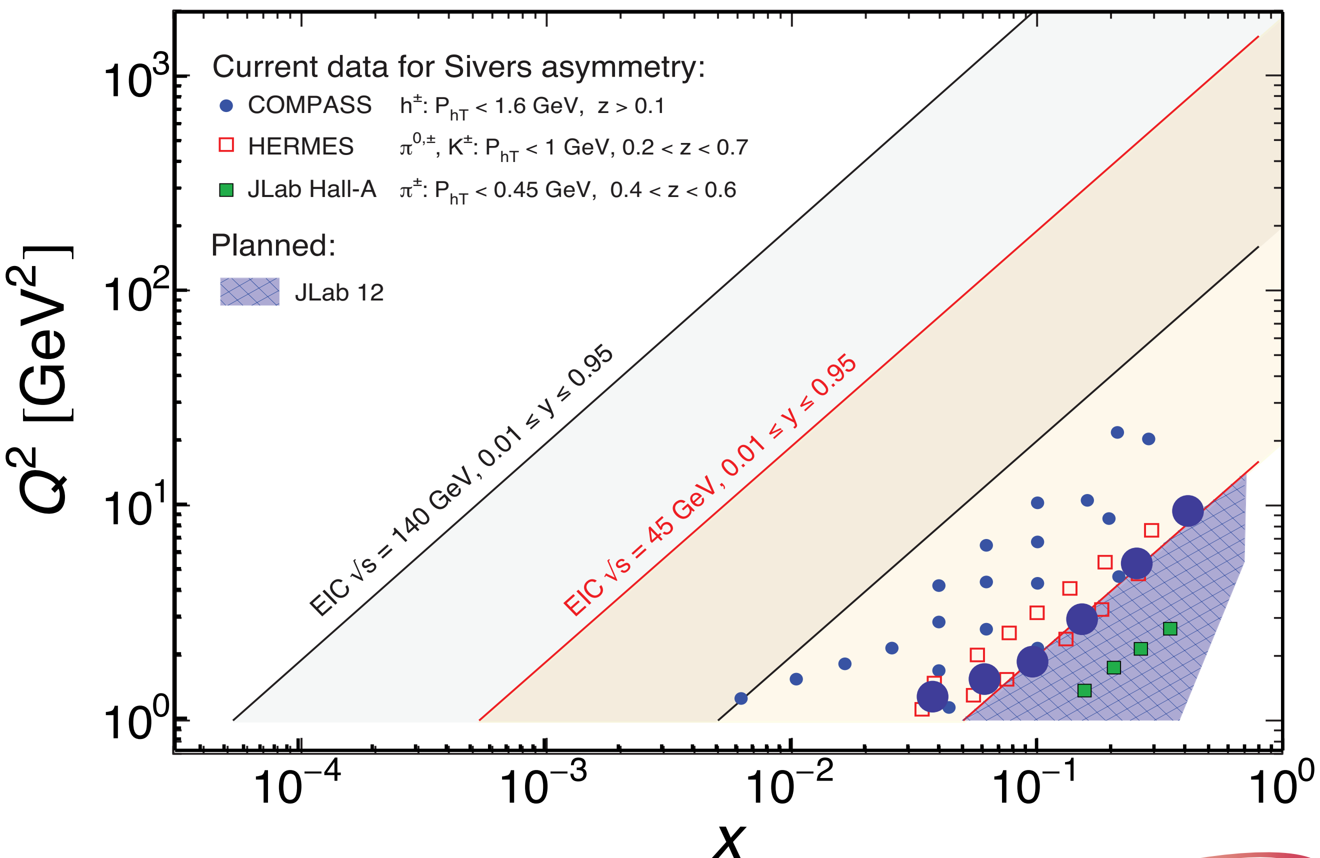

TMD
evolution
TMD
phenomenologist


TMD
evolution
TMD
phenomenologist

Fun in the future...



Fun in the future...



EIC

Jefferson Lab

Unpolarized gluon TMD/uPDF?

Parametrize unintegrated gluon distribution at a starting scale

Unpolarized gluon TMD/uPDF?

Parametrize unintegrated gluon distribution at a starting scale

$$xA_0(x, k_t) = Nx^{-B}(1-x)^C(1-Dx)e^{-(k_t-\mu)^2/\sigma^2}$$

Unpolarized gluon TMD/uPDF?

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Run a Monte Carlo that implements gluon radiation (according to CCFM formalism)

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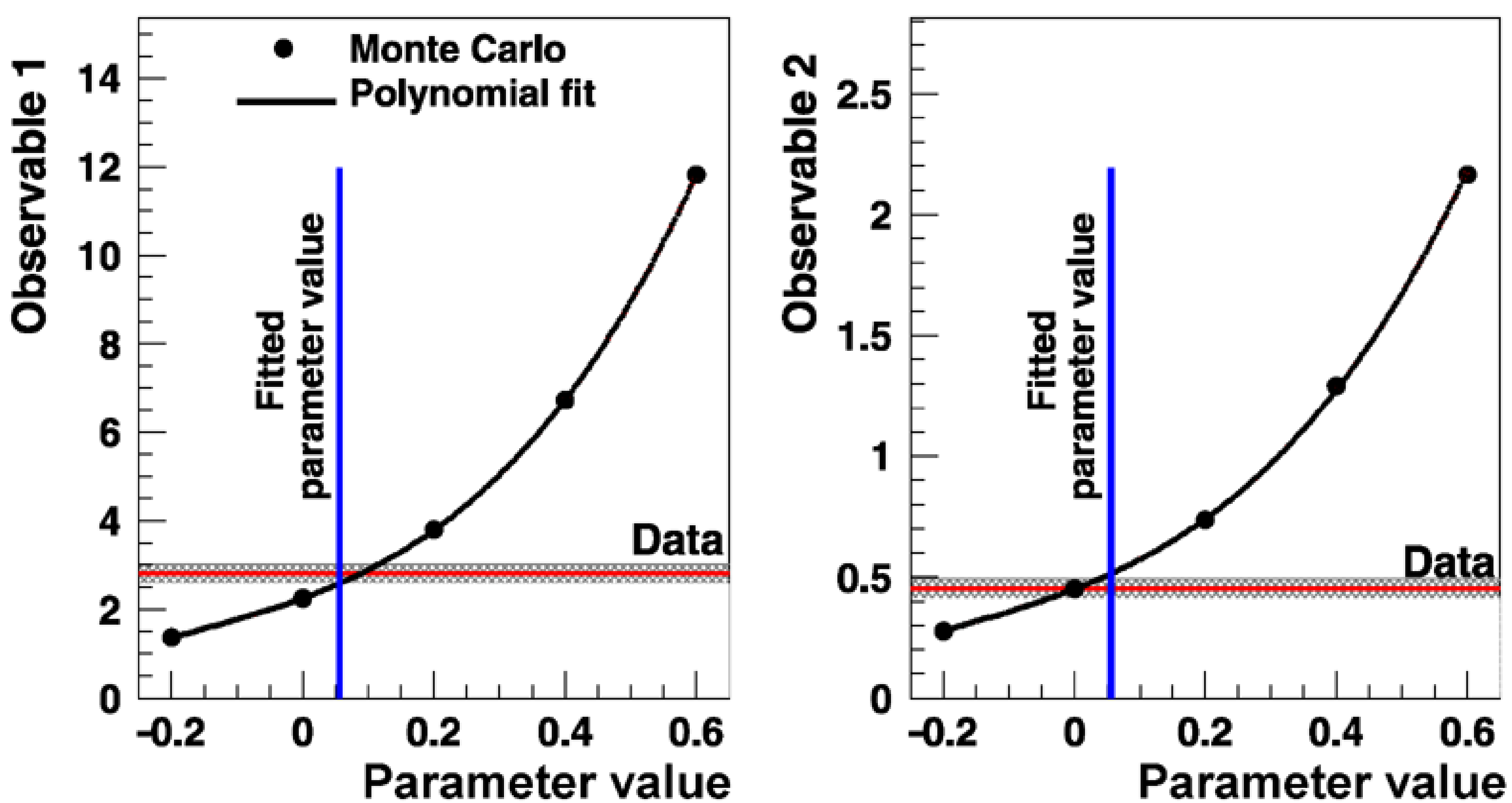
Run a Monte Carlo that implements gluon radiation
(according to CCFM formalism)

Predict an observable

Tune the above parameters

Event generator tuning



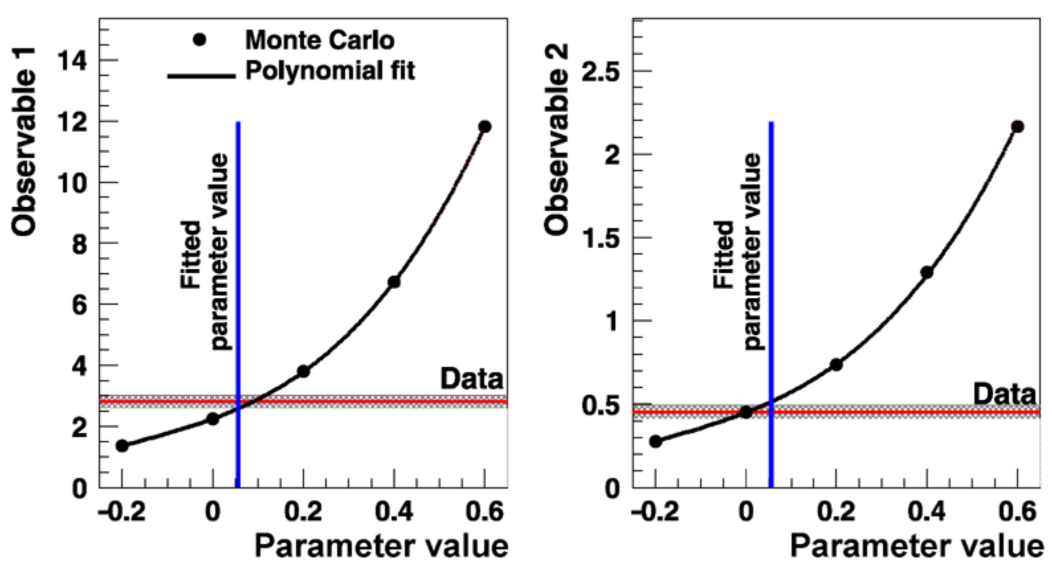
Bacchetta, Jung, Knutsson, Kutak, Samson-Himmelstjerna, EPJC70 (10)

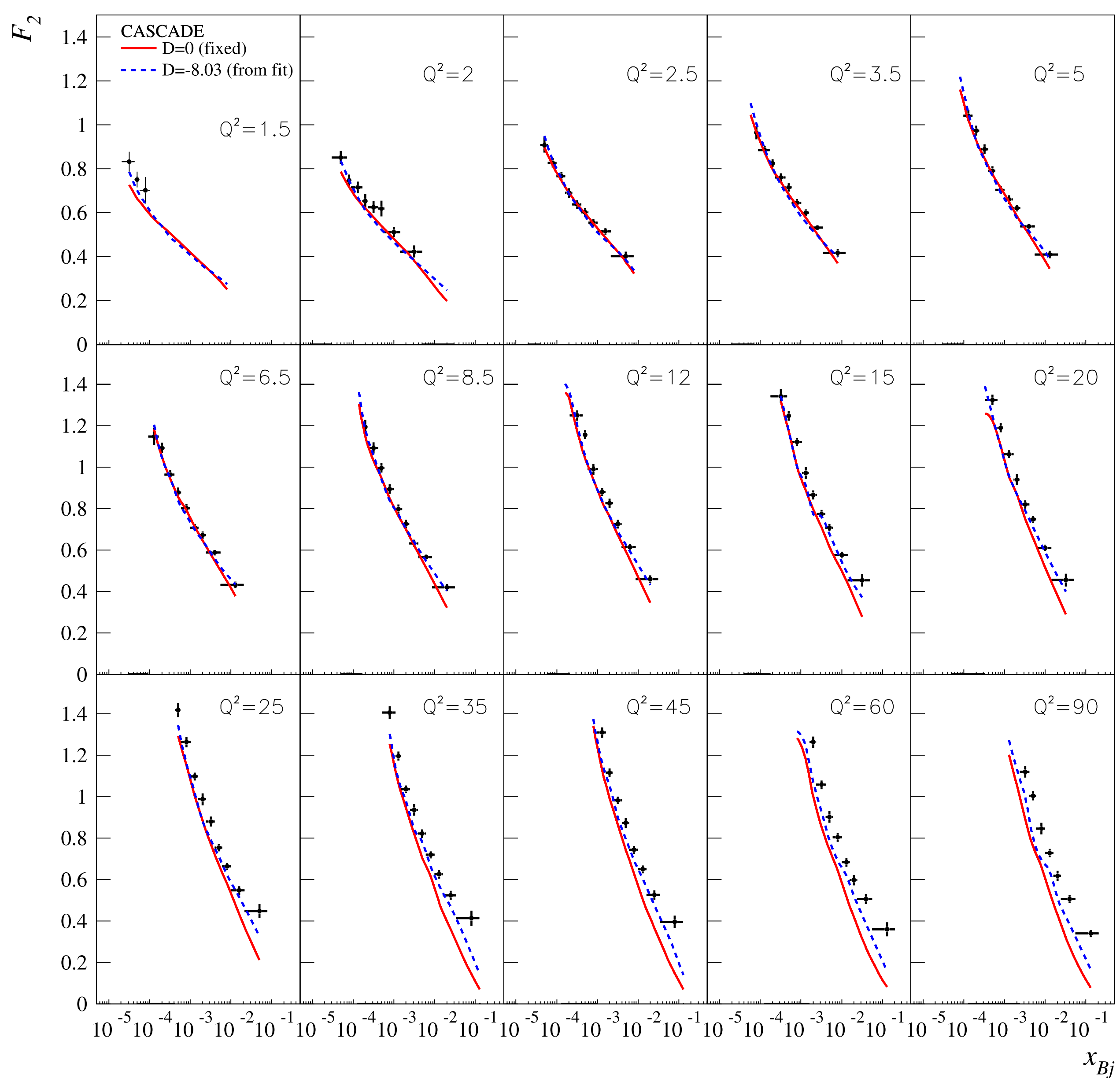
Unpolarized gluon TMD/uPDF?

Input gluon TMD/uPDF

$$xA_0(x,k_t) = N x^{-B}(1-x)^C(1-Dx)e^{-(k_t-\mu)^2/\sigma^2}$$

Monte Carlo tuning





Results: large negative D required

Bacchetta, Jung, Knutsson, Kutak, Samson-Himmelstjerna , EPJC70 (10)

Unpolarized gluon TMD/updf

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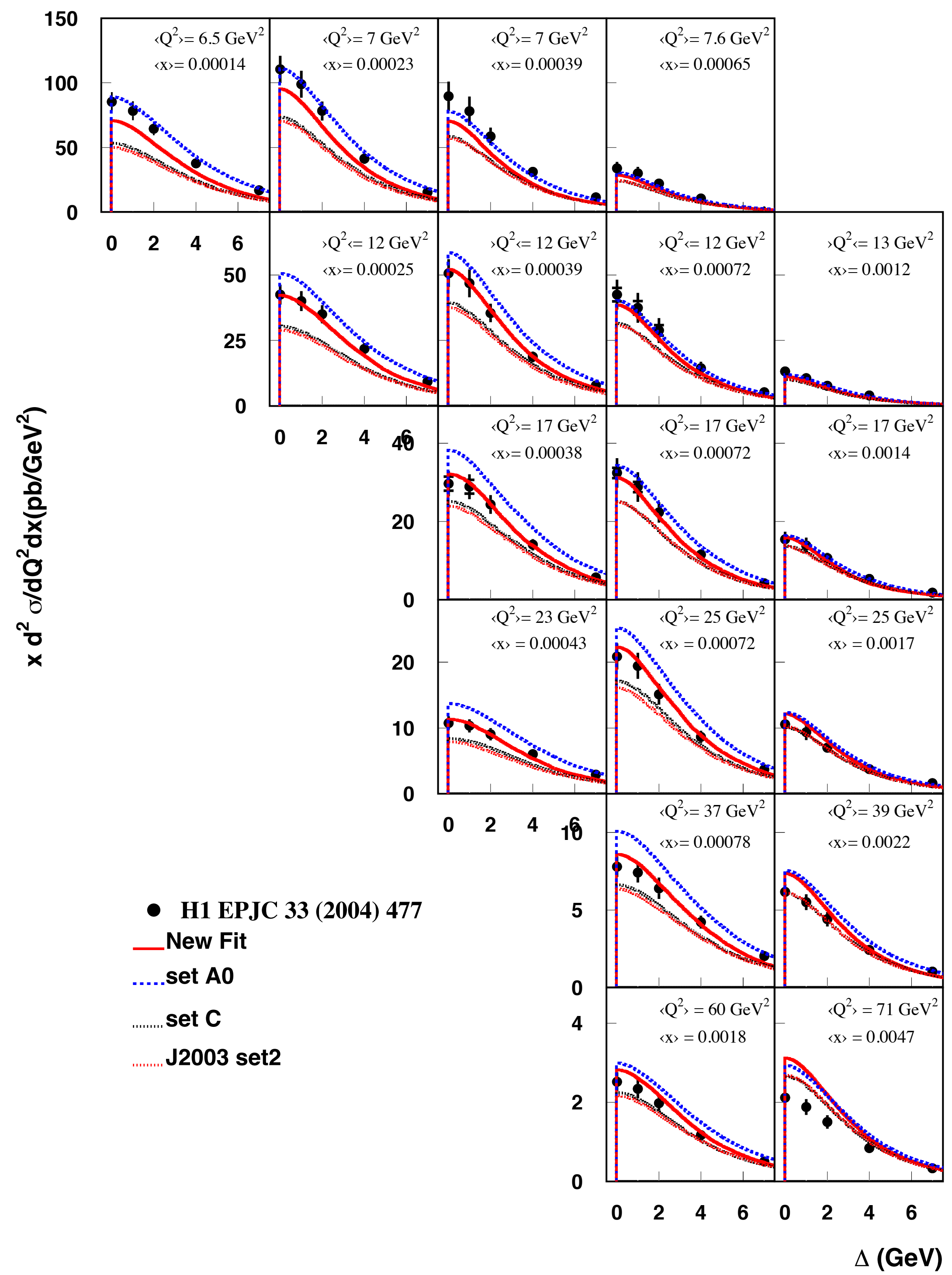


Results:

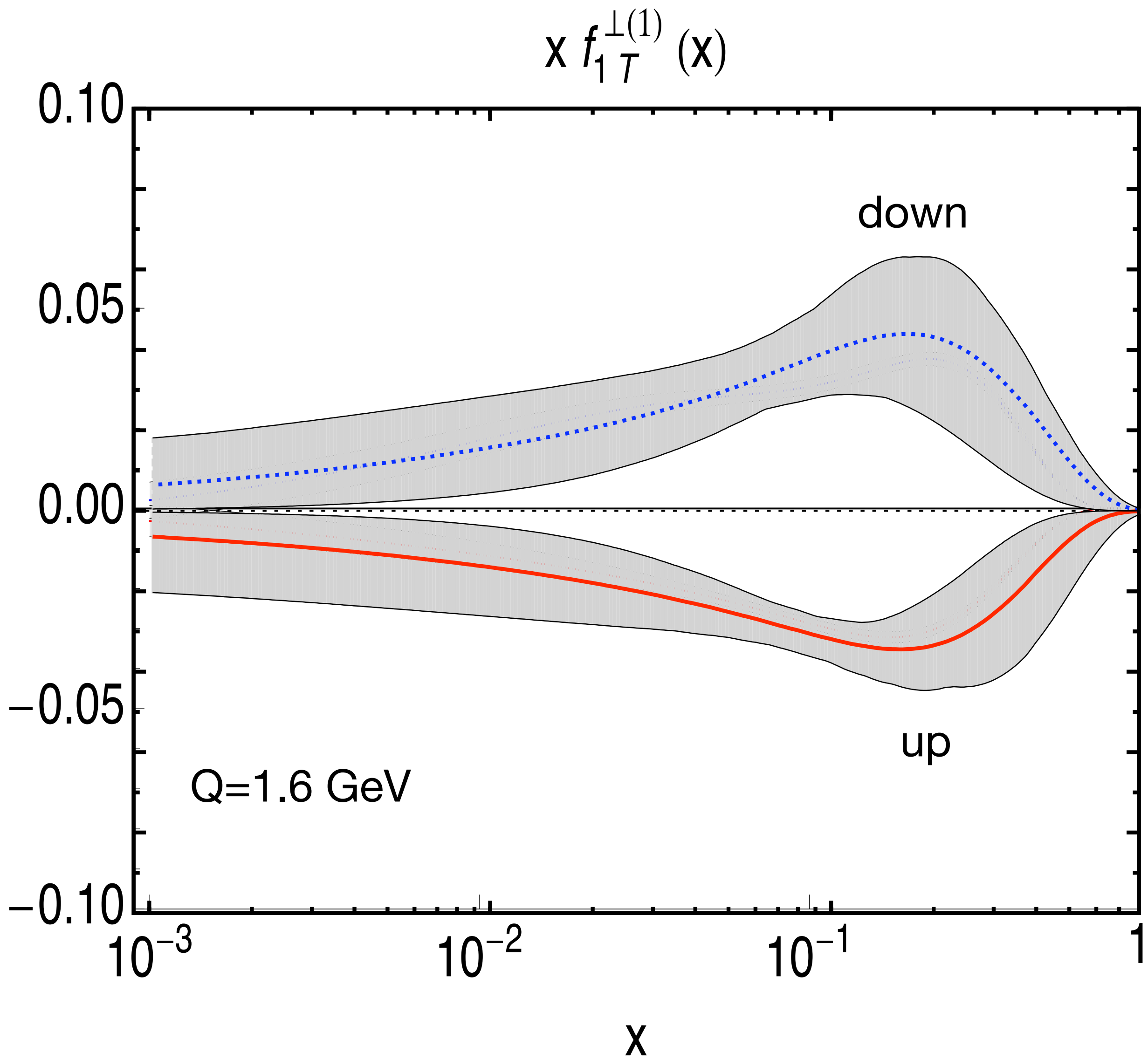
large μ required

Bacchetta, Jung, Knutsson, Kutak, arXiv:0808.0847

Sivers function

Torino fit (no evo)

Anselmino et al., arXiv:1107.4446



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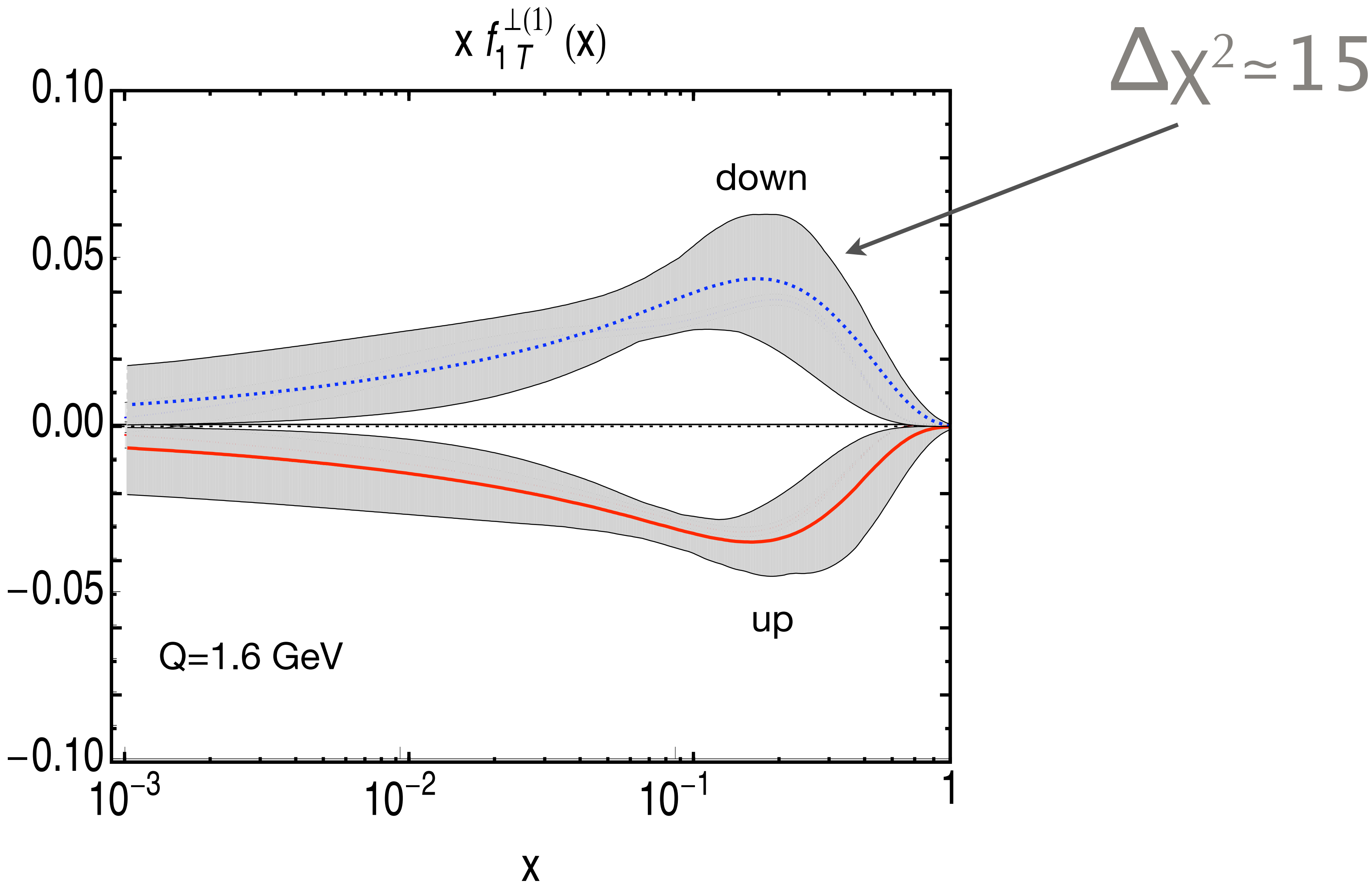

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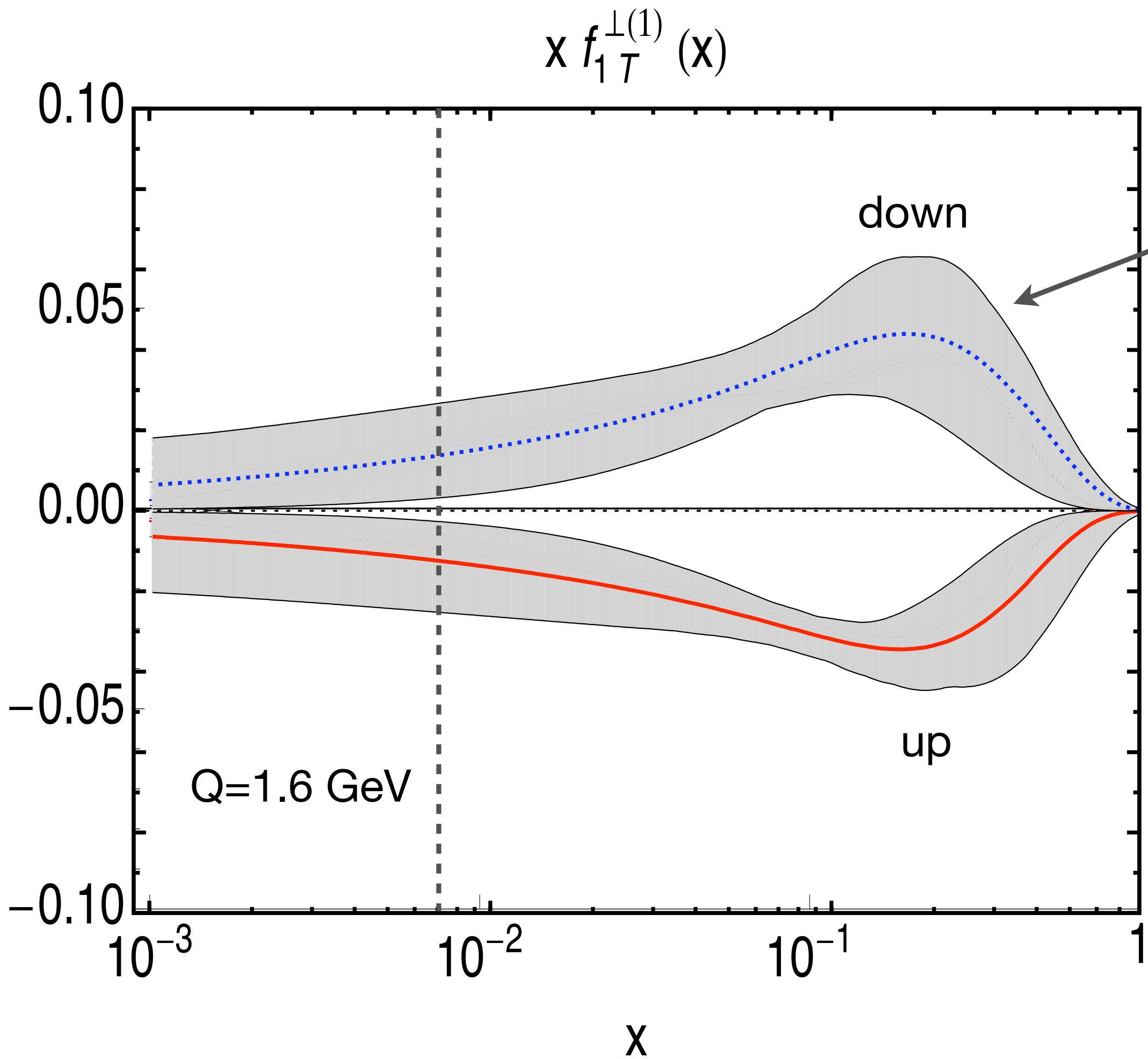
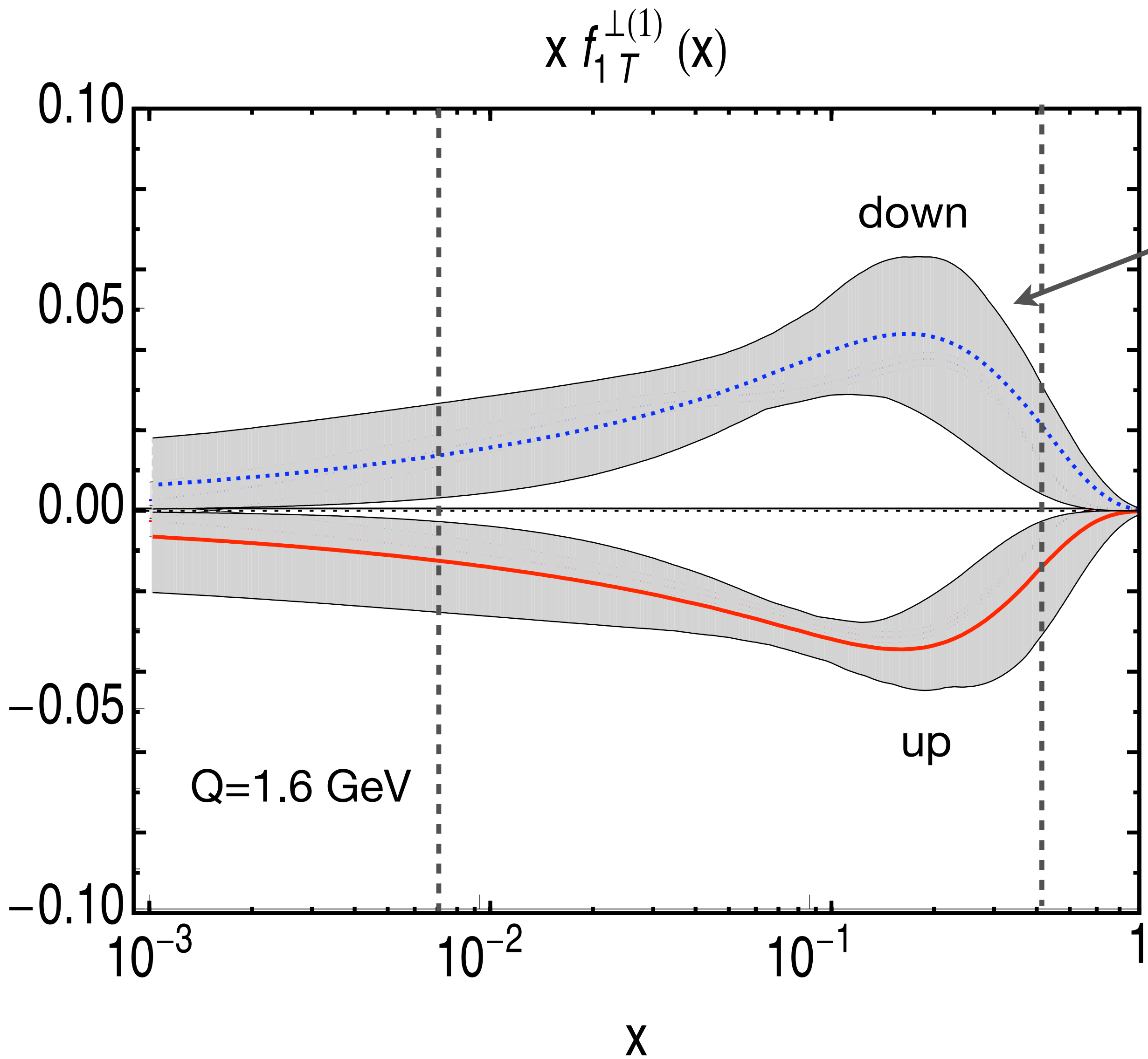


$\Delta\chi^2 \simeq 15$

Torino fit (no evo)

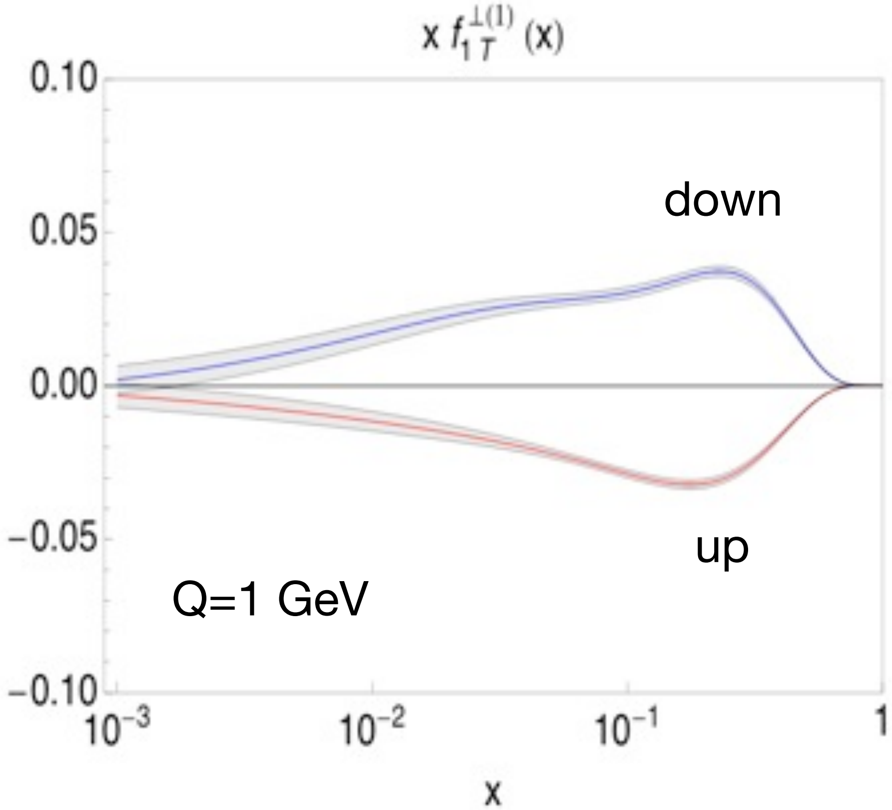
Anselmino et al., arXiv:1107.4446

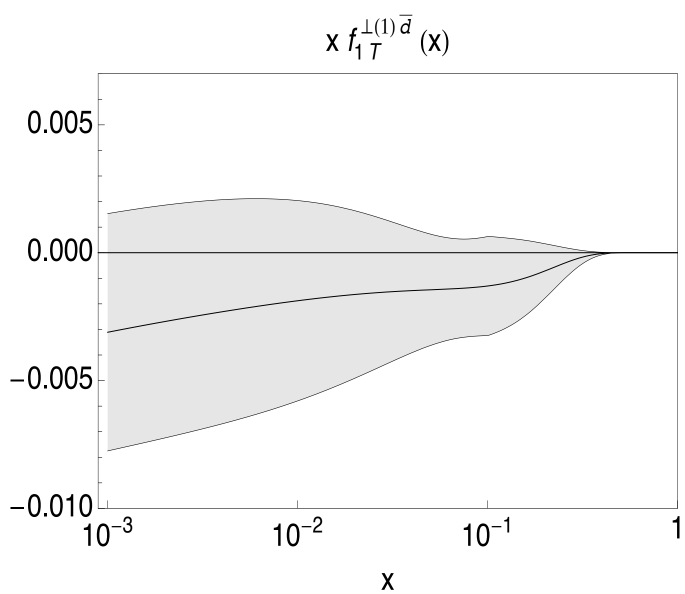


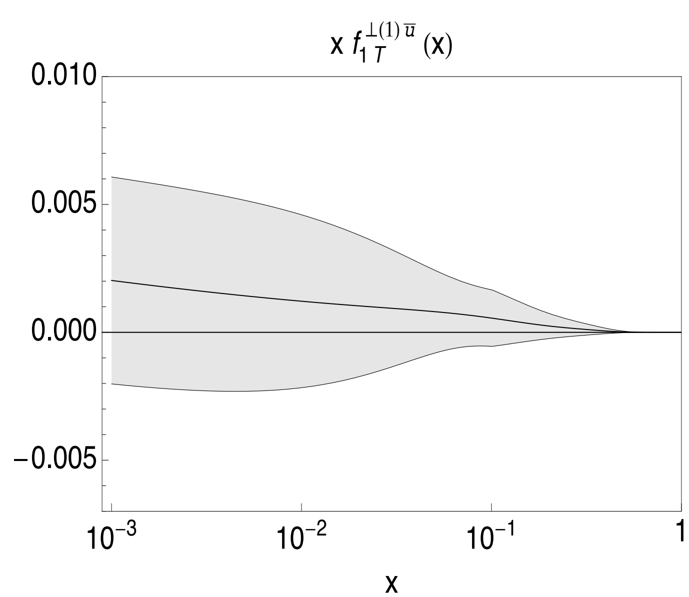
$\Delta\chi^2 \simeq 15$

Pavia fit (no evo)

Bacchetta, Radici, PRL 107 (2011)



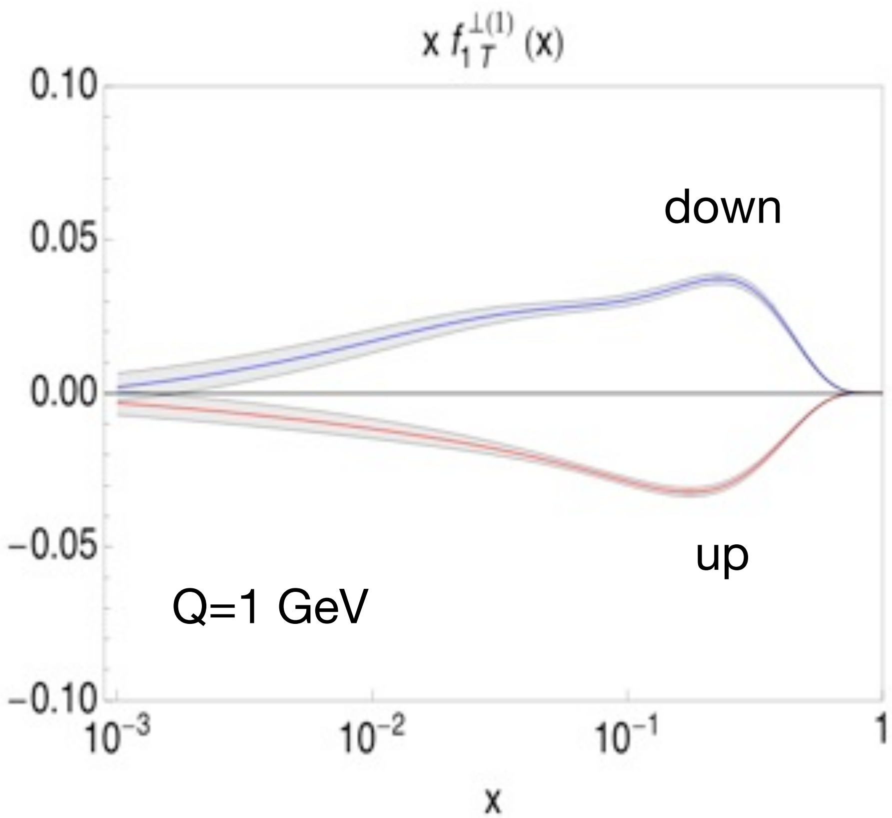


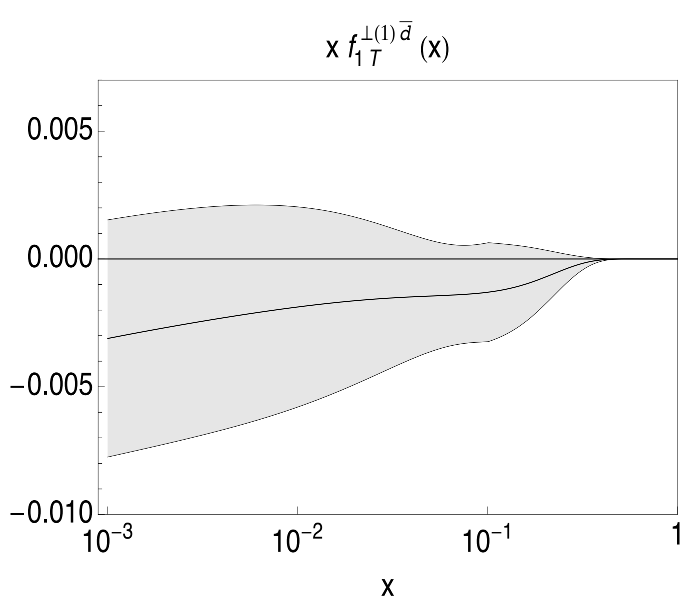


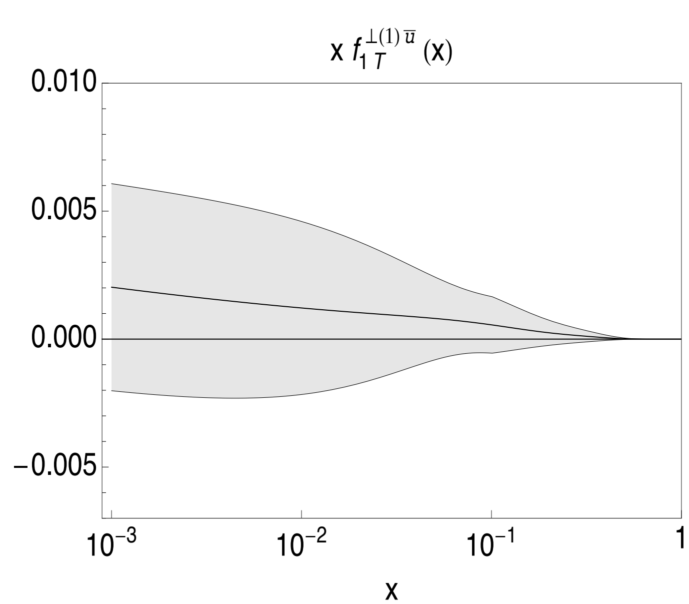
Torino

Pavia fit (no evo)

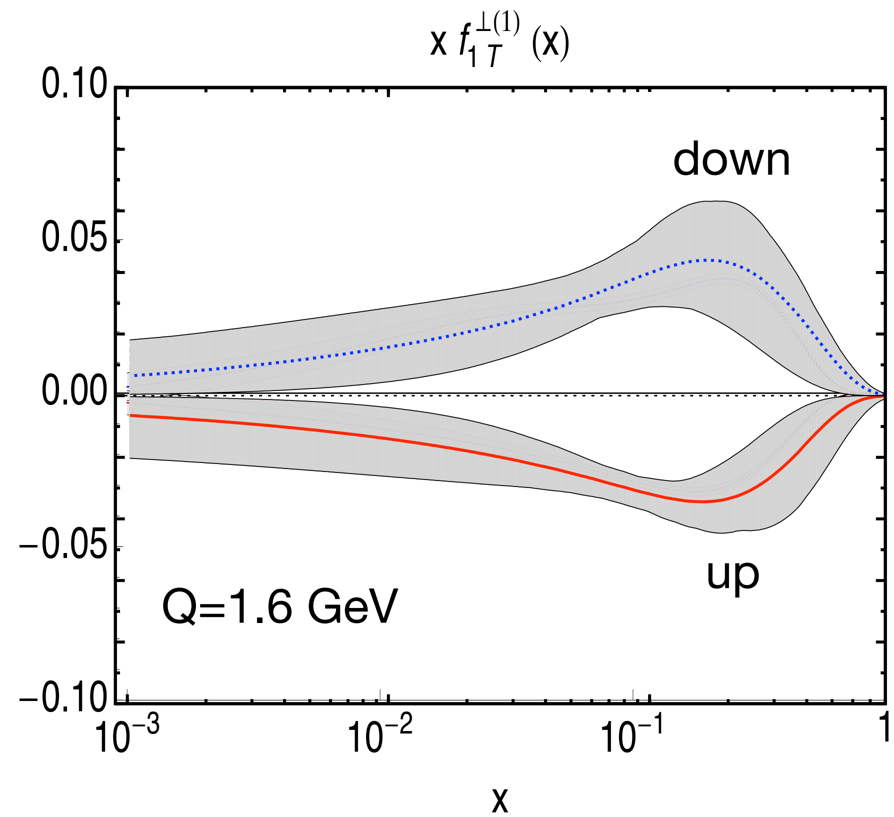
Bacchetta, Radici, PRL 107 (2011)







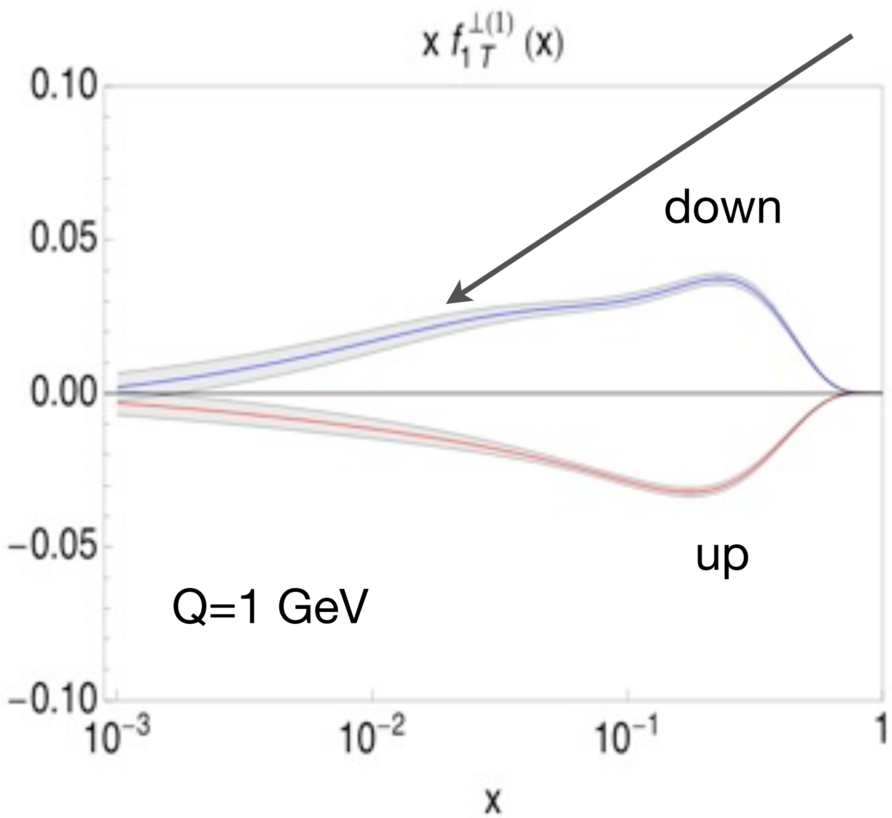
Torino

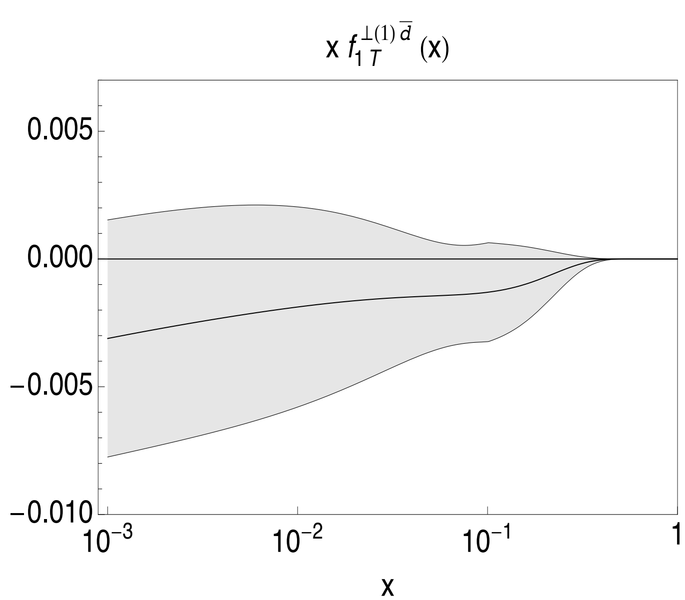


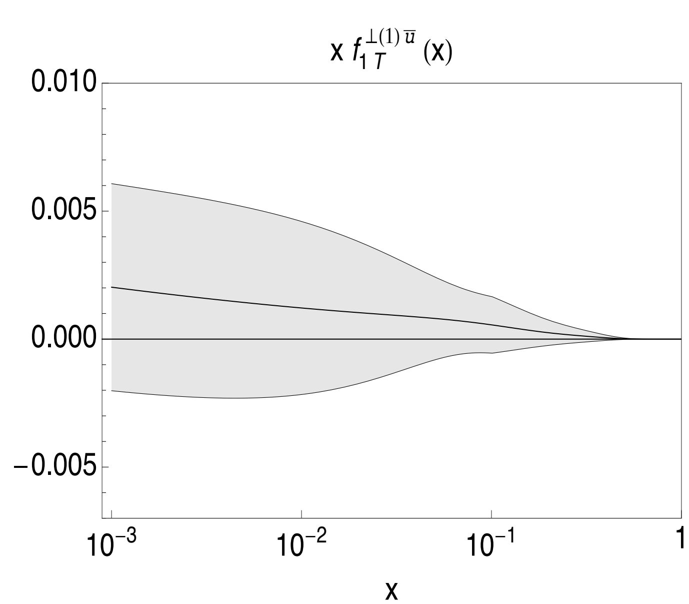
Pavia fit (no evo)

Bacchetta, Radici, PRL 107 (2011)

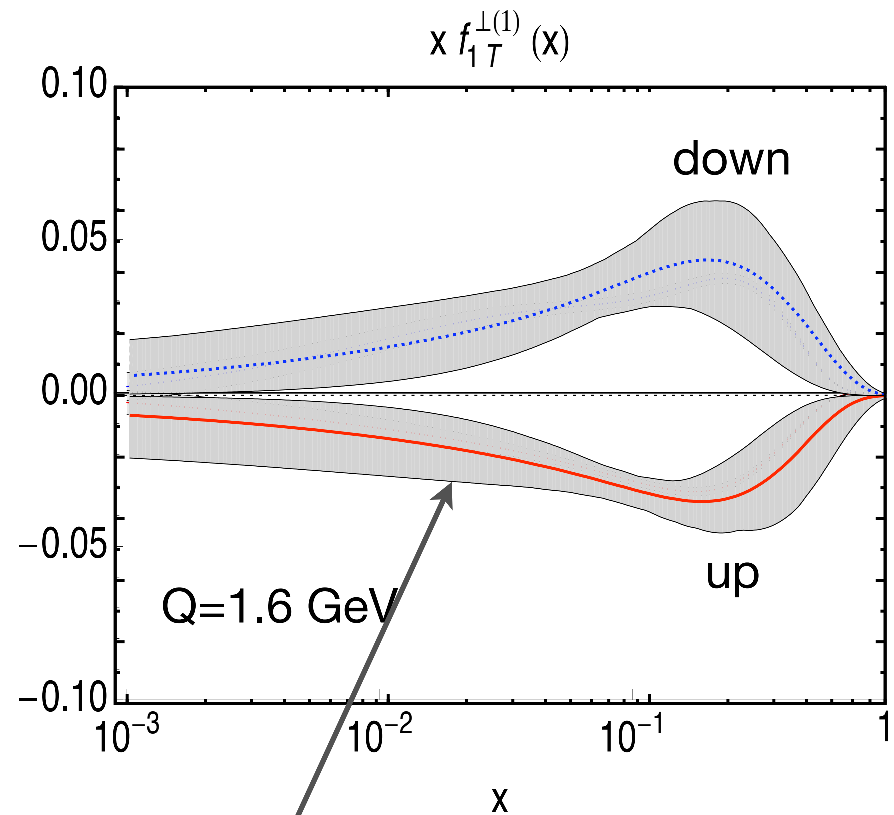
$\Delta\chi^2 = 1$







Torino



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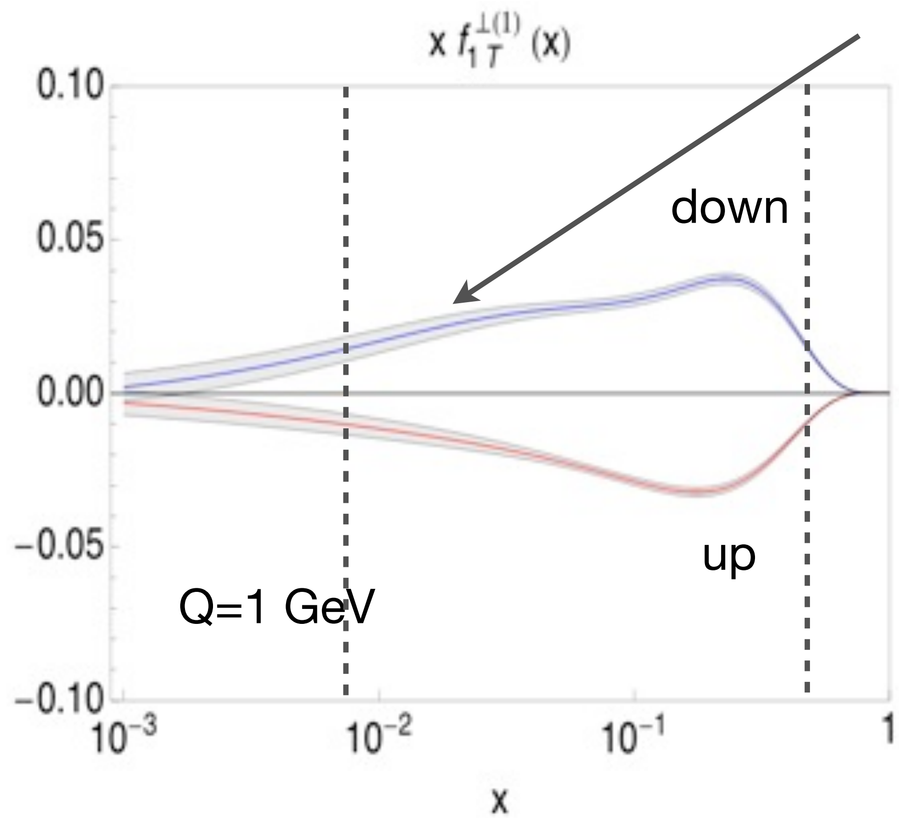
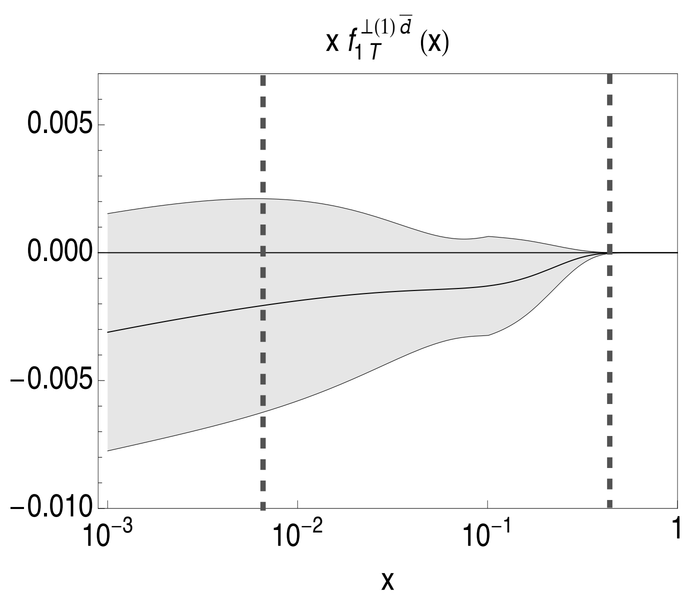
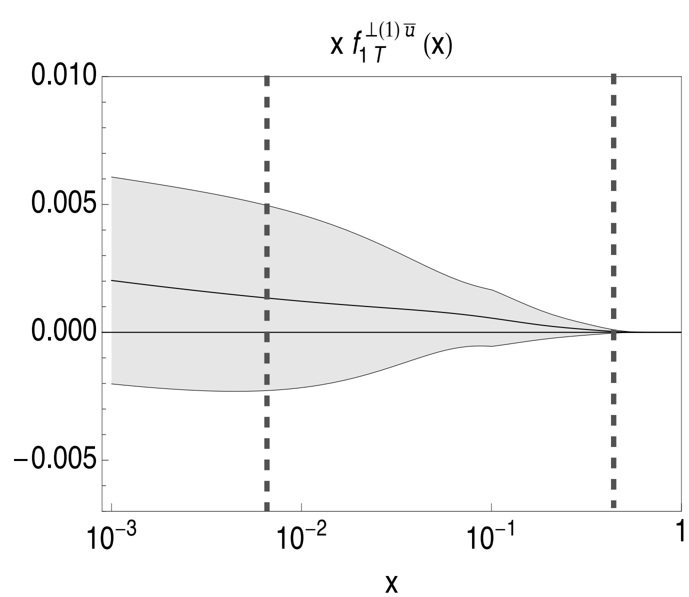
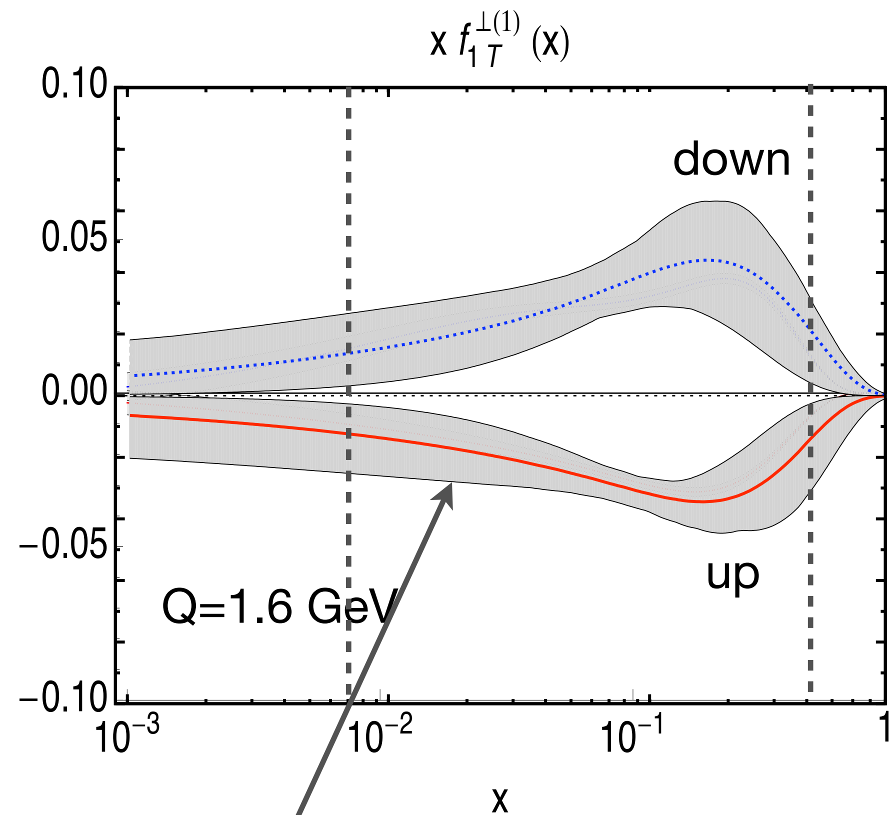






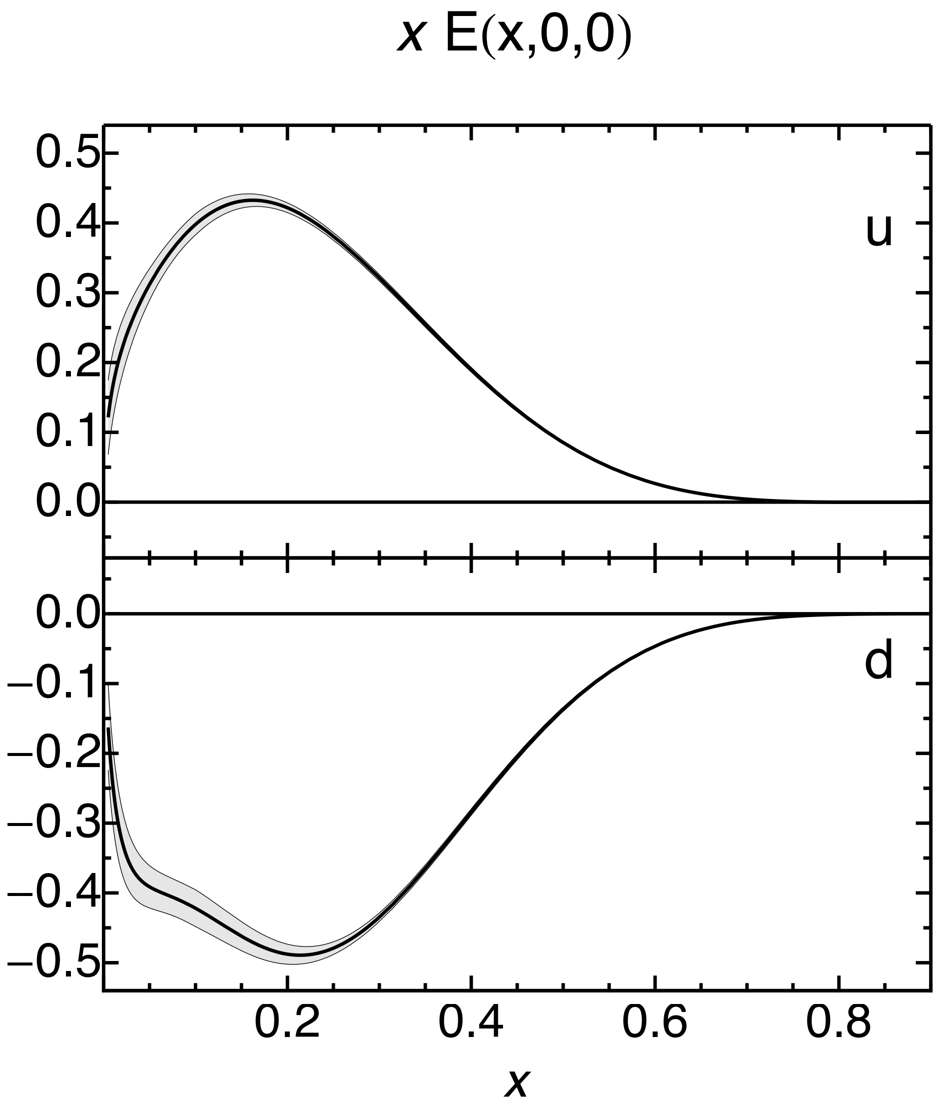
Torino

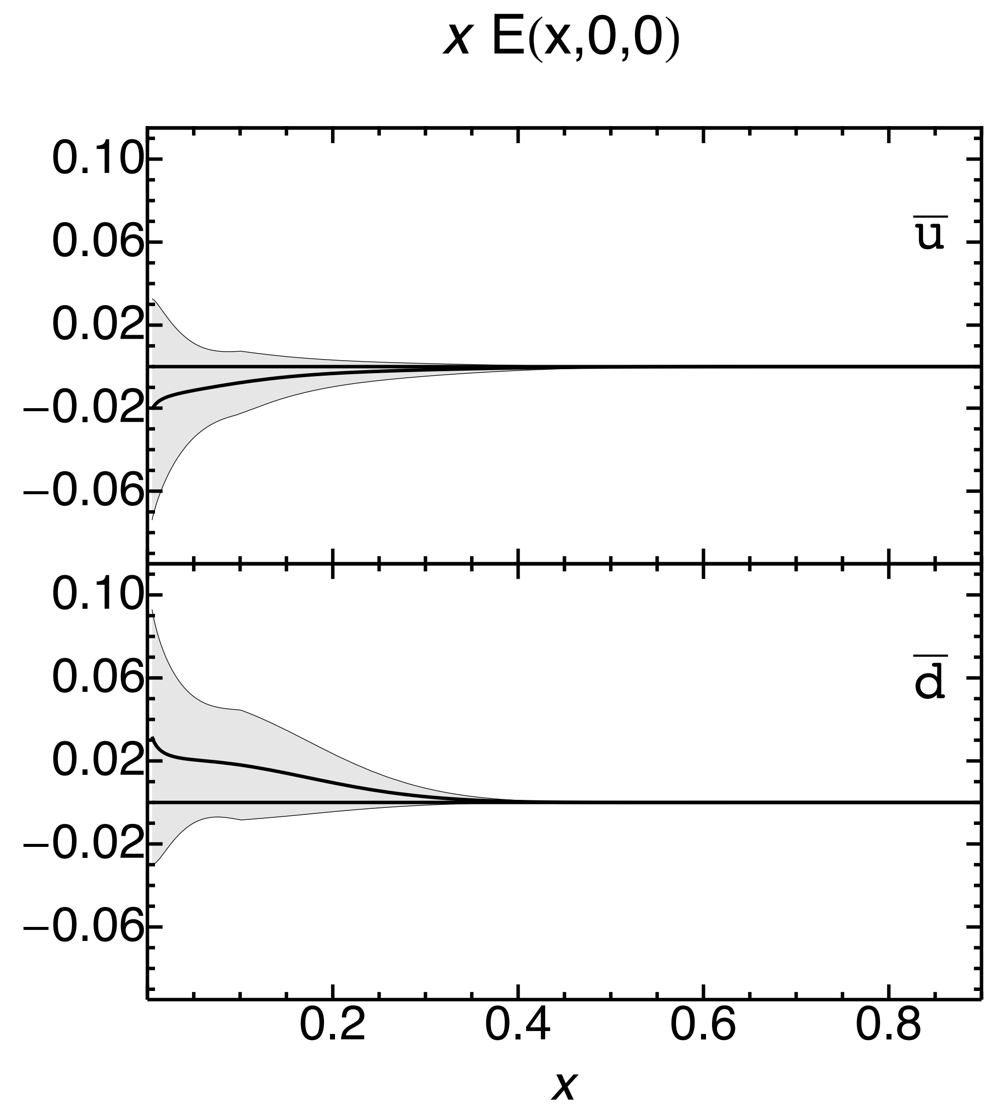


$\Delta\chi^2 \simeq 15$

Connection with GPDs

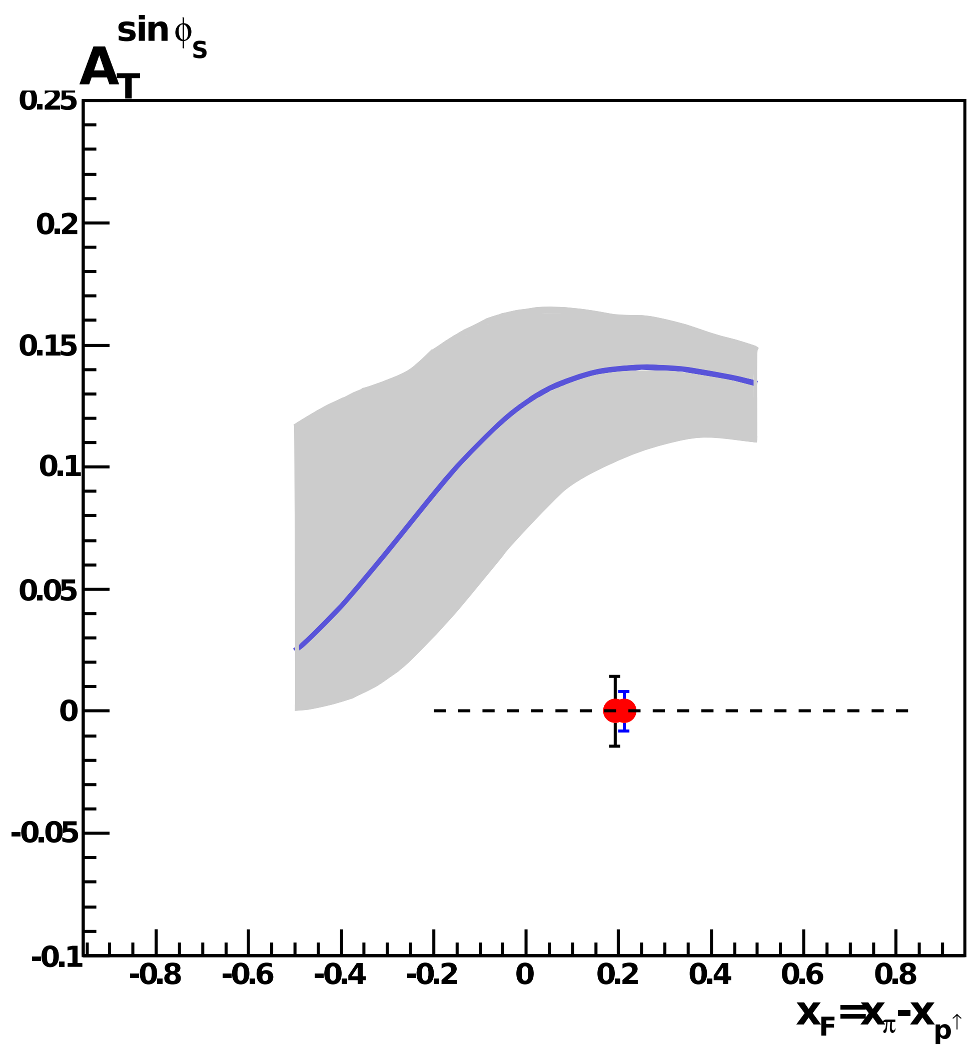
$$E^q(x,0,0;Q_L^2) \propto -\frac{C^q}{K}\,(1-x/\alpha^q)\,(1-x)^{1+\eta} f_1^q(x:Q_L^2)$$





Predictions for Drell-Yan

Anselmino et al., PRD 79 (09)



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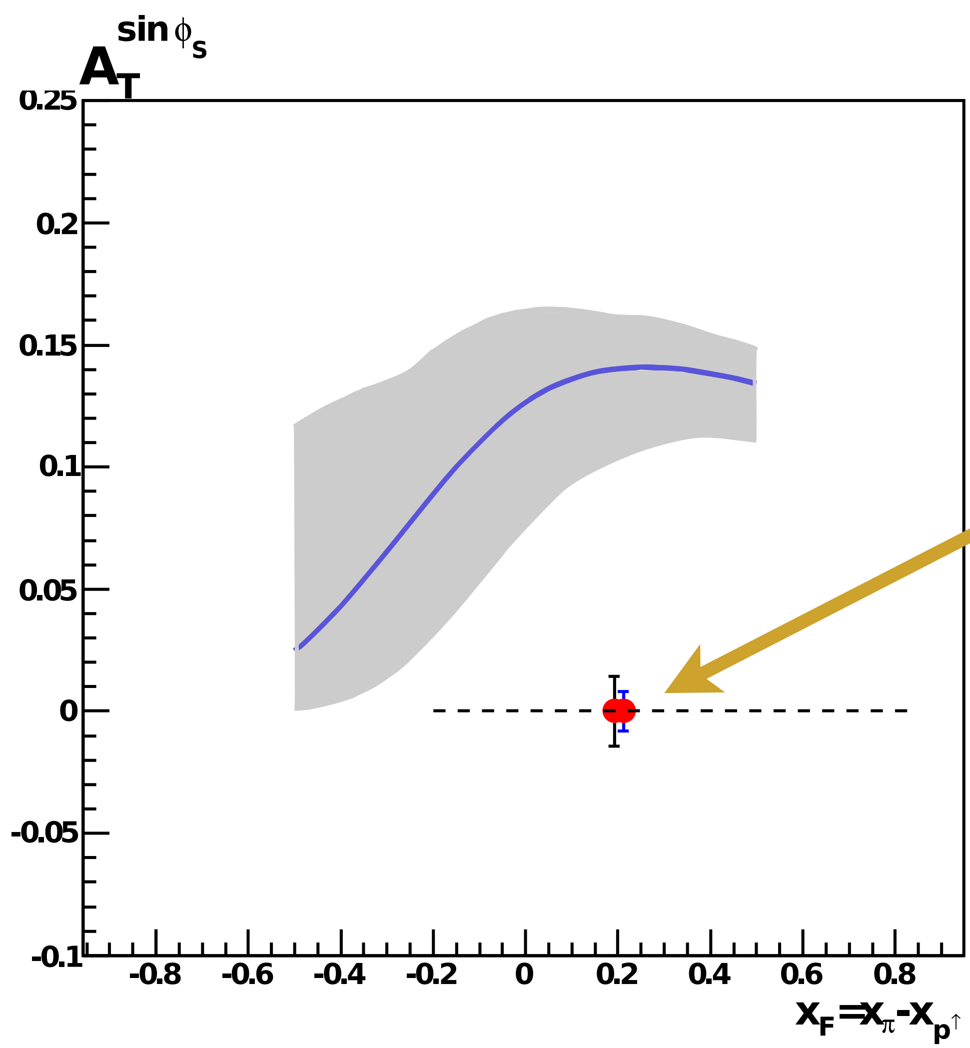
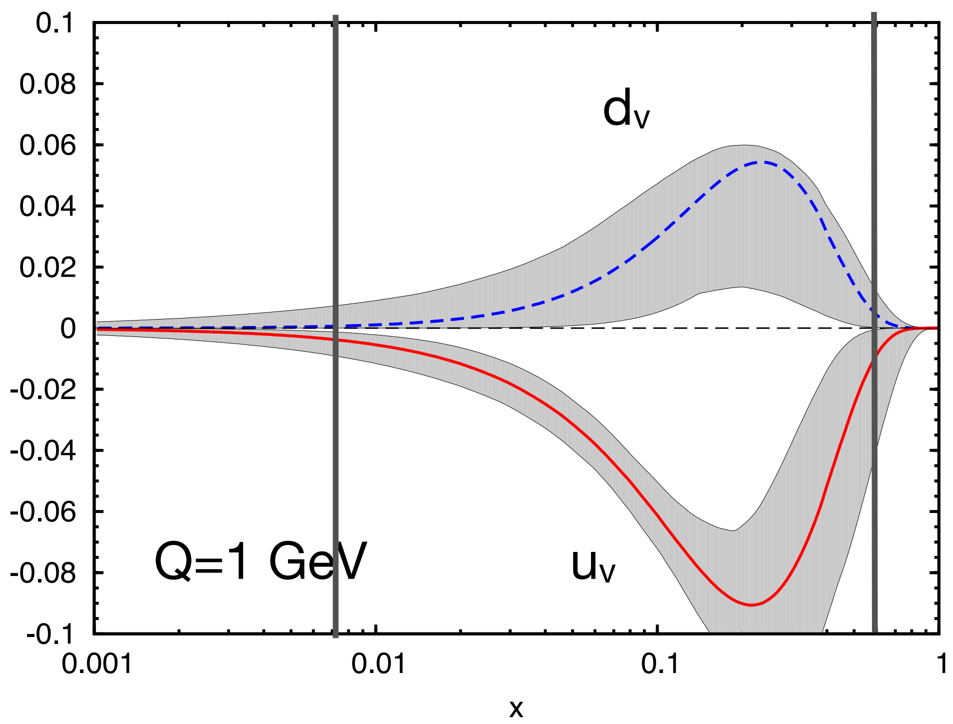


COMPASS estimated error

Torino fit with evolution

Theory: Aybat, Rogers, PRD85 (2012)
First application: Aybat, Prokudin, Rogers, PRL108 (2012)

First fit: Anselmino, Boglione, Melis, PRD86 (2012)



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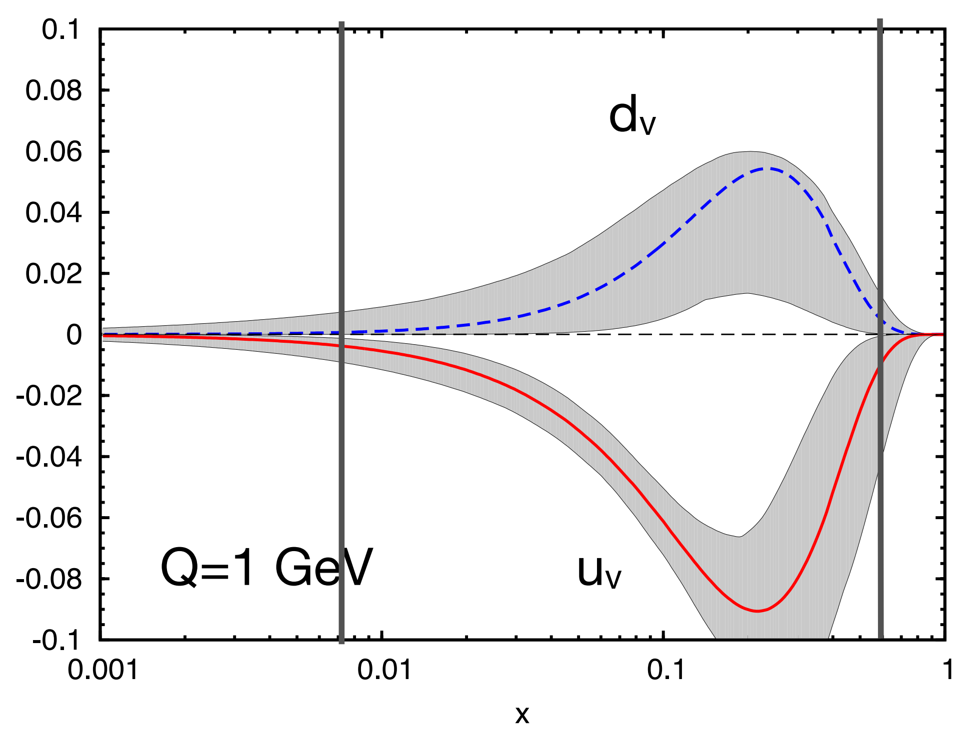

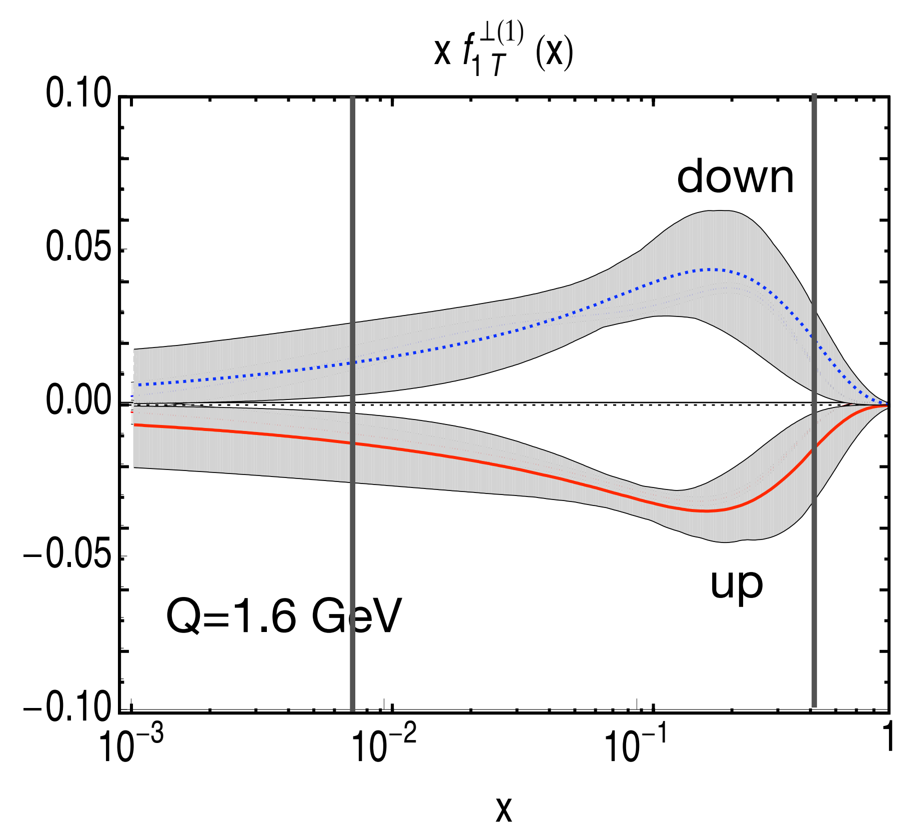

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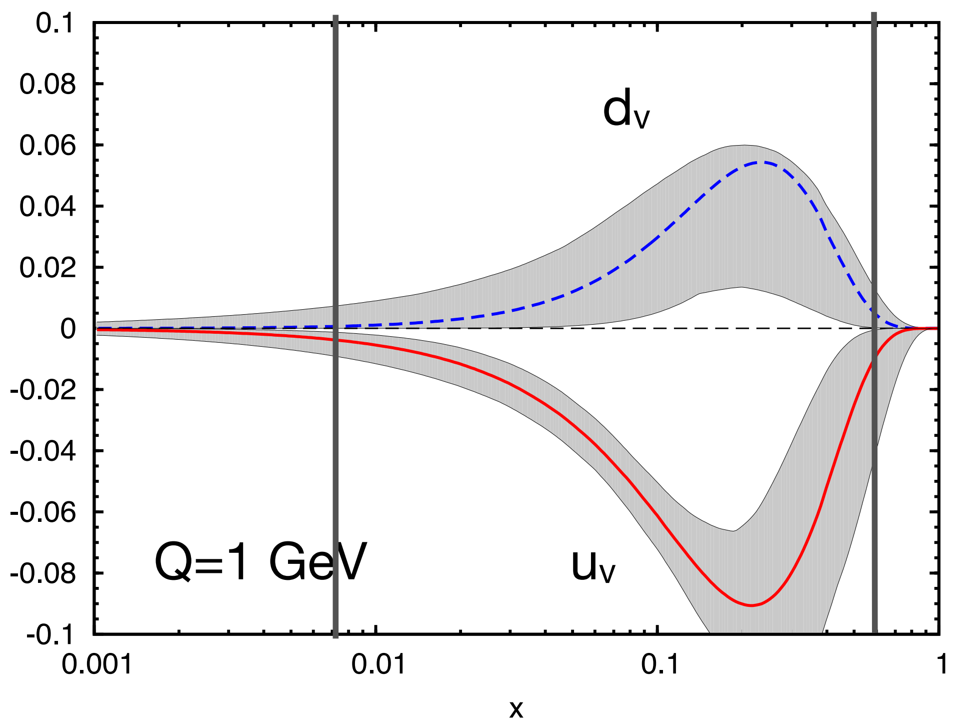

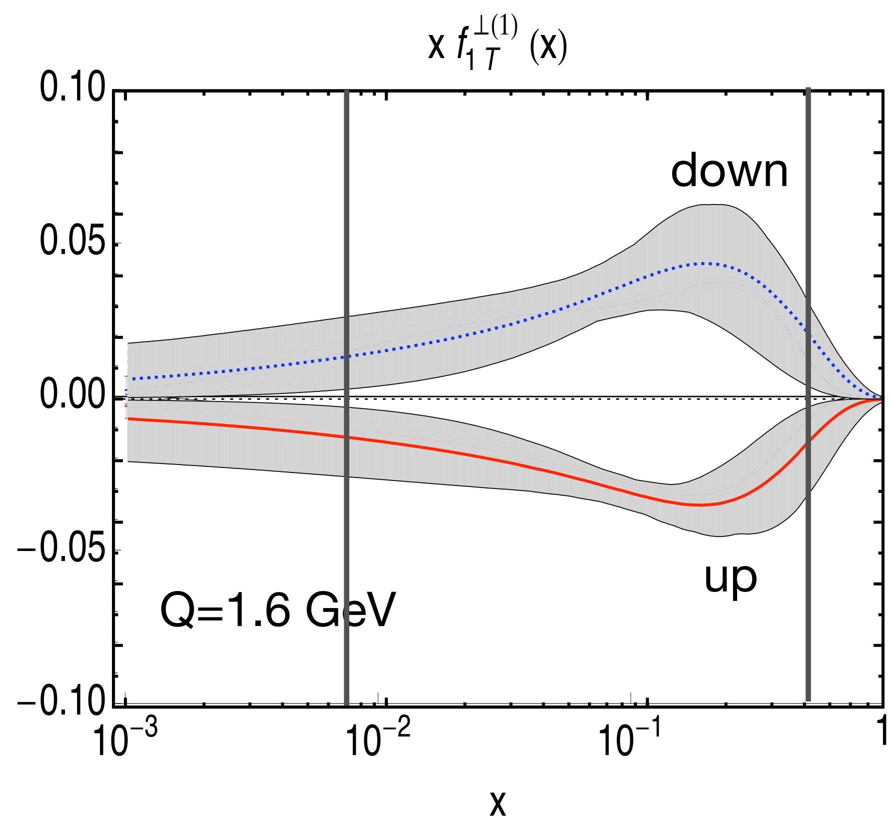

Strong suppression with evolution was predicted.
An increase in the Sivers function was needed to describe data.

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First application: Aybat, Prokudin, Rogers, PRL108 (2012)

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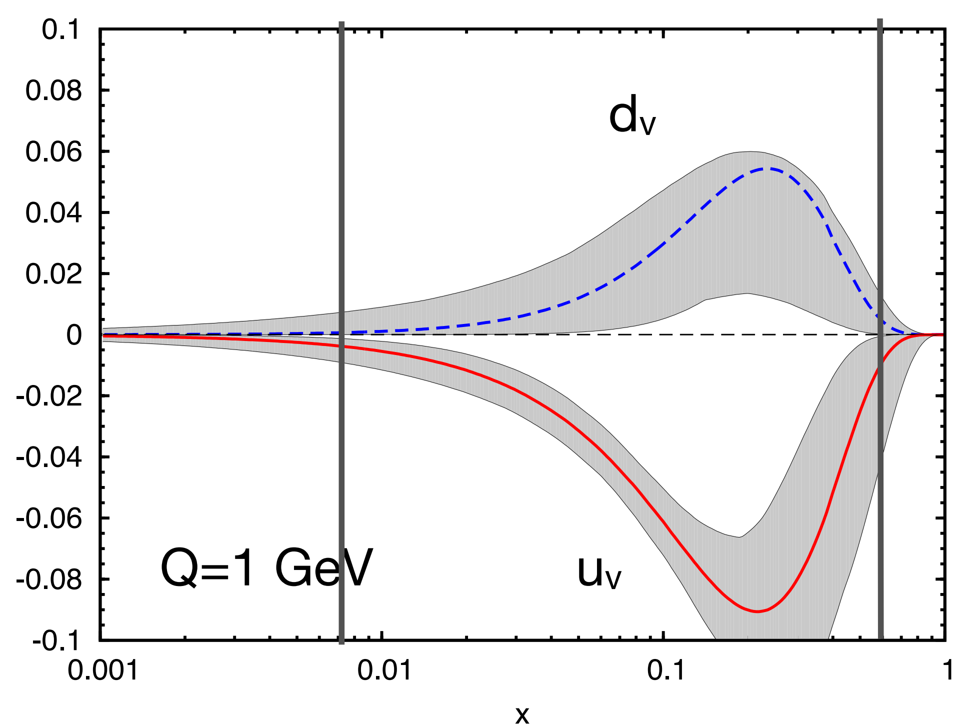
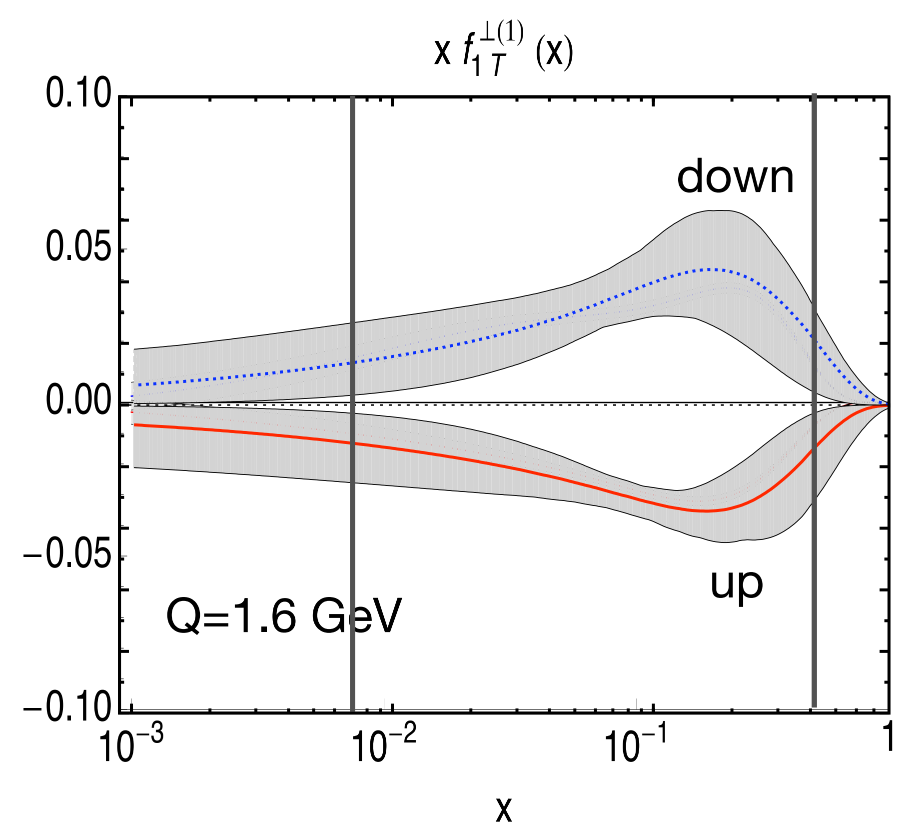
Using BLNY parameters g_2 and b_{max}





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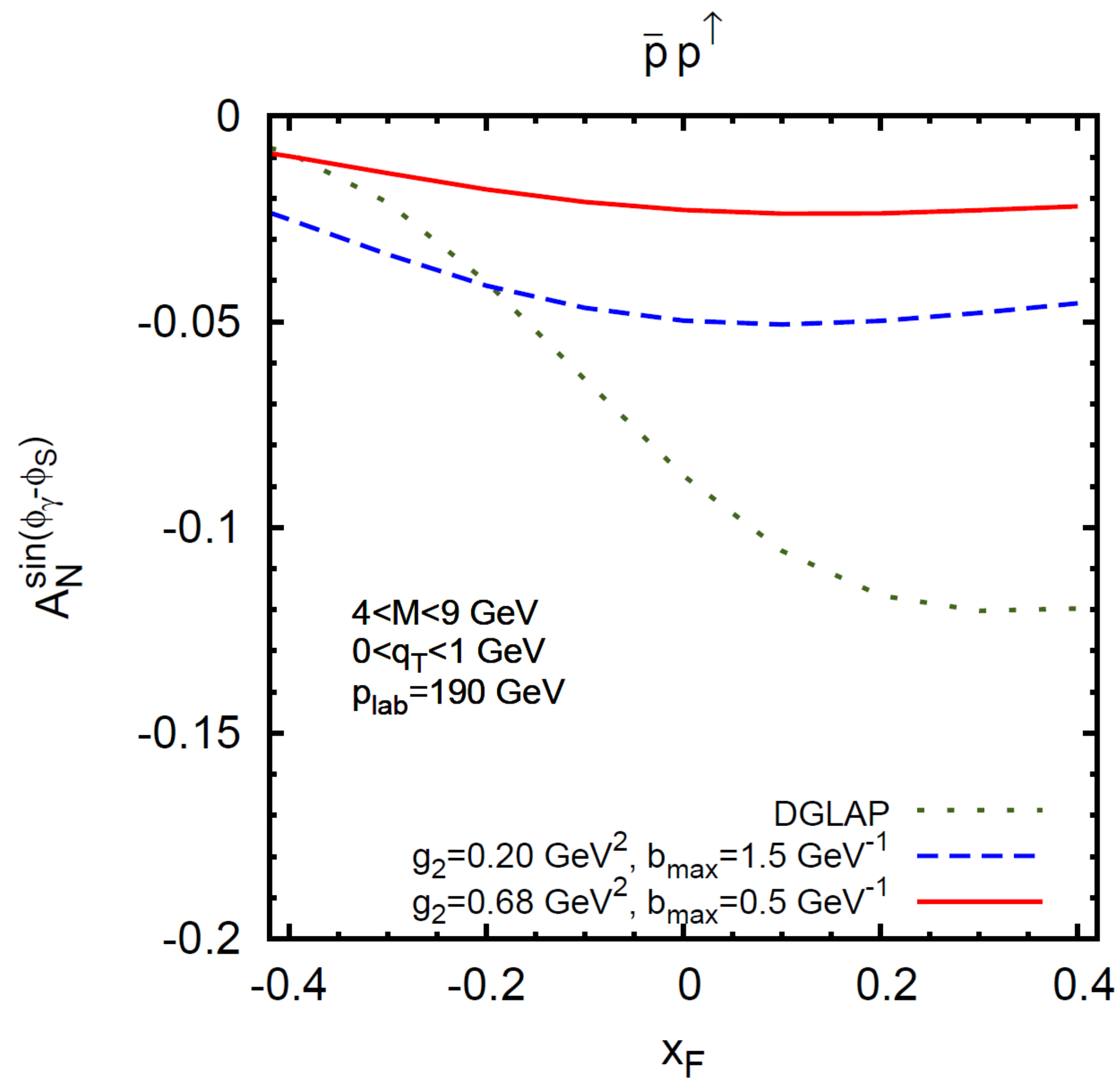
Consequence on Drell-Yan predictions



The Drell-Yan signal is reduced by a factor 1/4

Sun-Yuan fit with evo

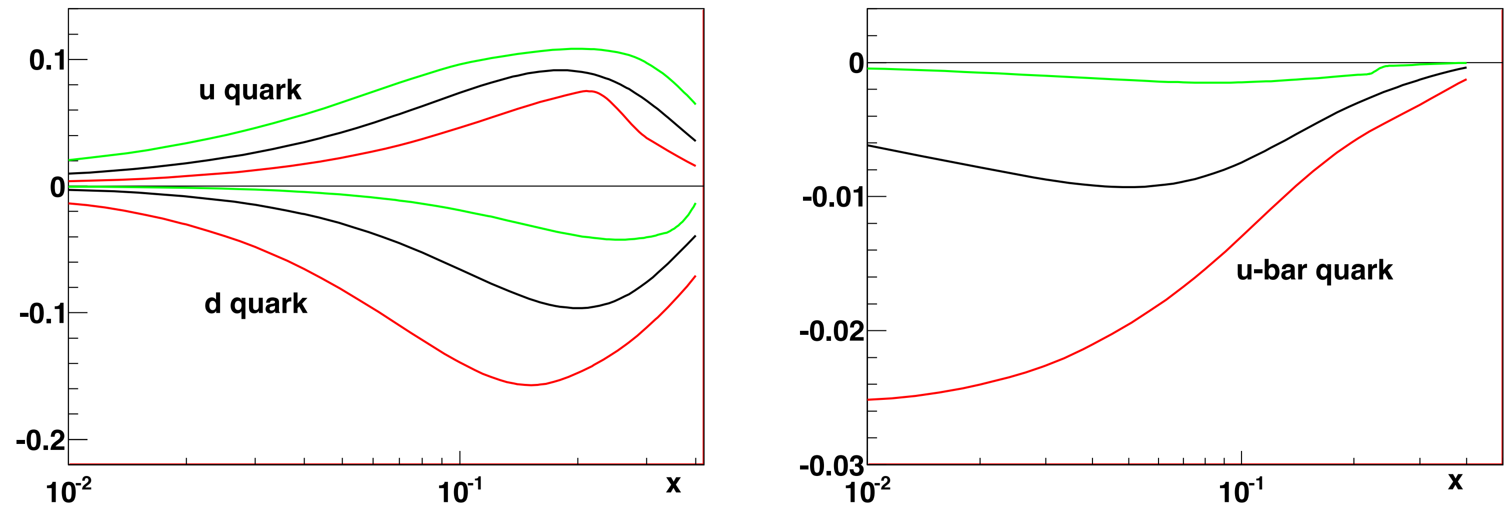


FIG. 9: Moments of the quark Sivers functions $\Delta f_q = T_F(x,x)/M$ fitted to HERMES and COMPASS data: up and down quark (left) and anti-up quark (right). Upper and lower curves for the uncertainties.

Sun-Yuan fit with evo

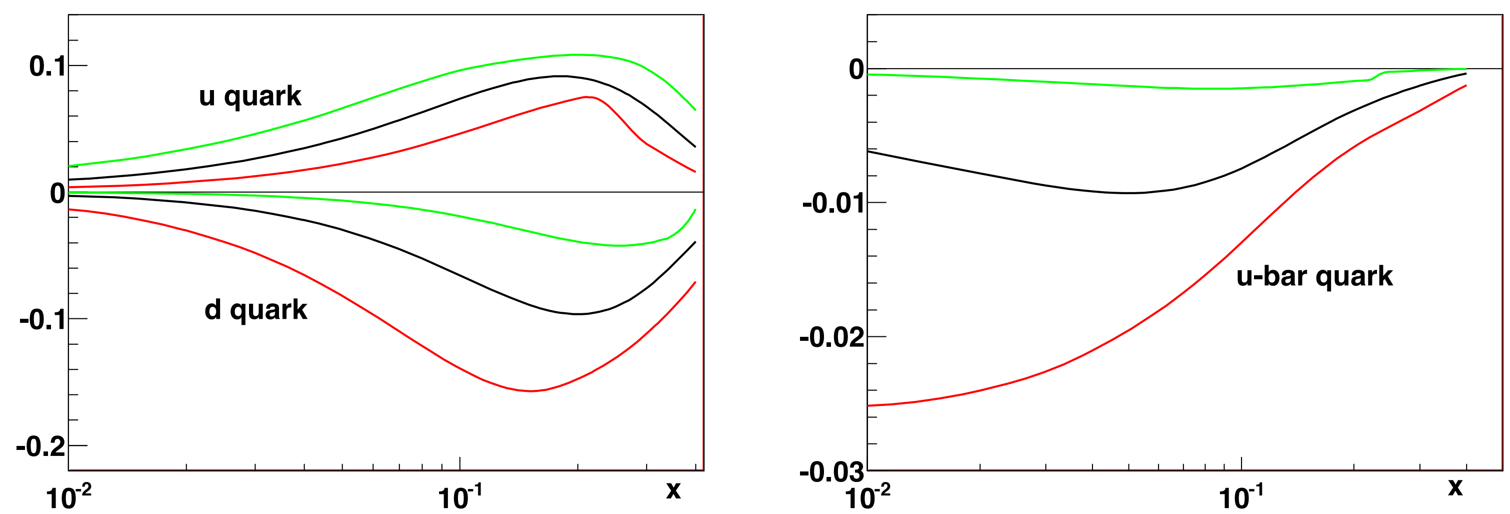
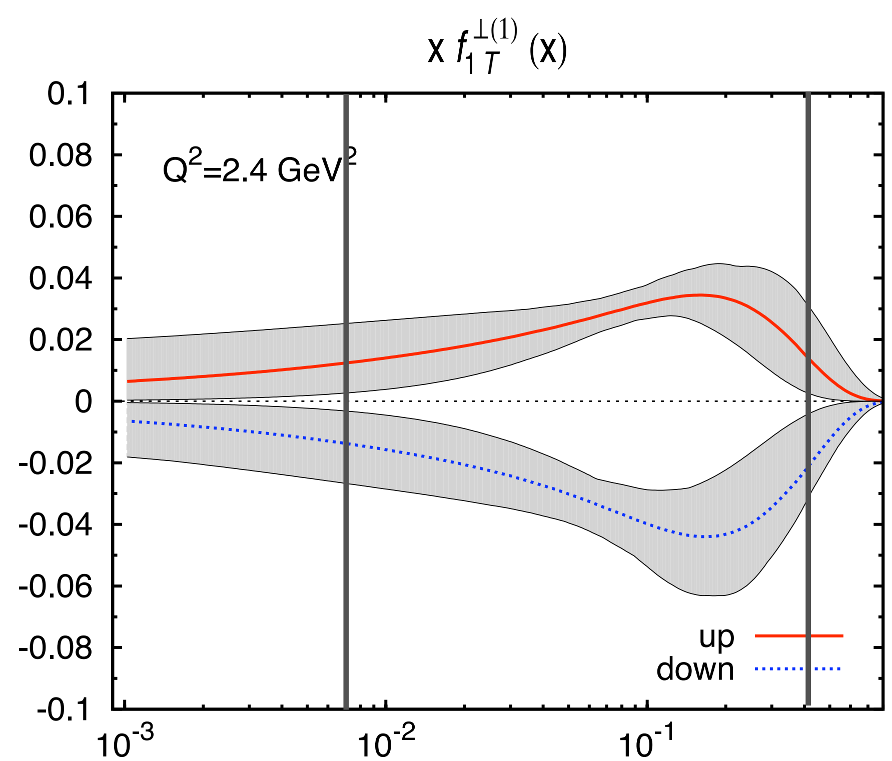


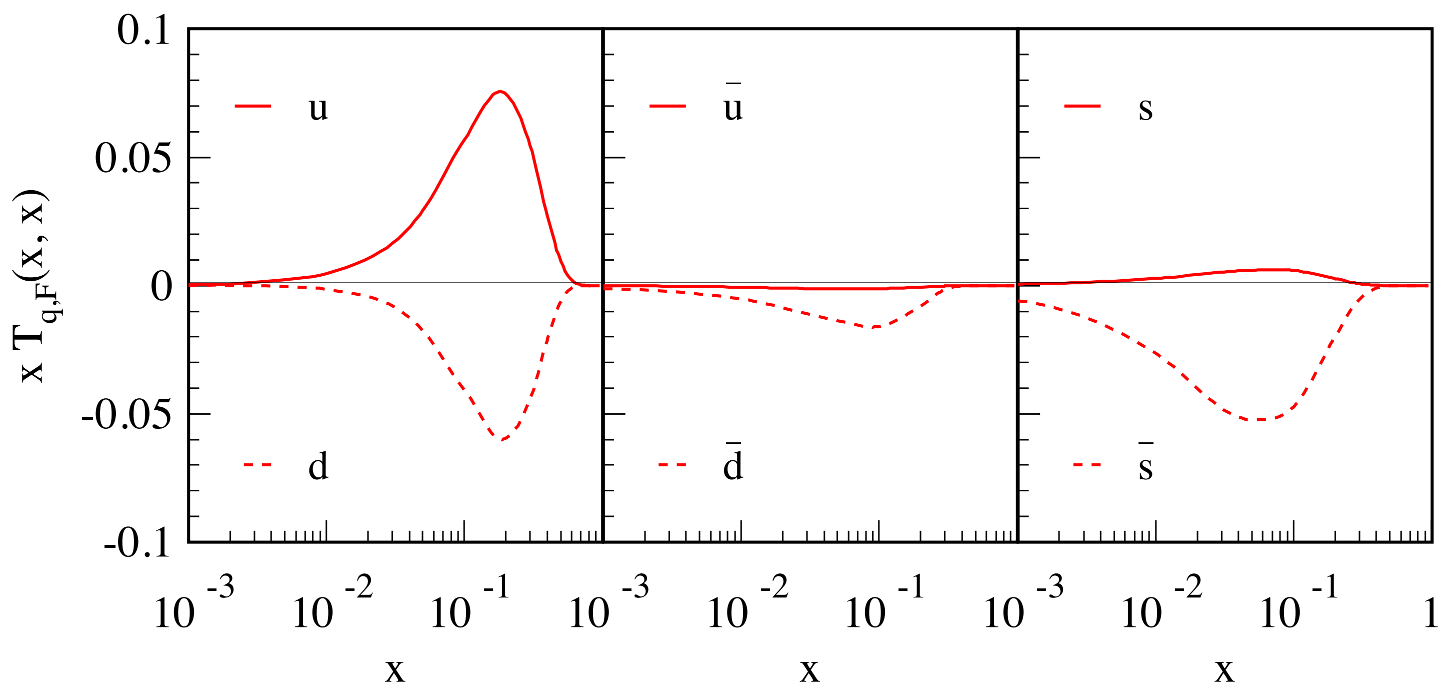
FIG. 9: Moments of the quark Sivers functions $\Delta f_q = T_F(x,x)/M$ fitted to HERMES and COMPASS data: up and down quark (left) and anti-up quark (right). Upper and lower curves for the uncertainties.

Old fit with no evo



$$f_{1T}^{\perp(1)} = \frac{T_F(x,x)}{2M}$$

Echevarria, Idilbi, Kang, Vitev with evo



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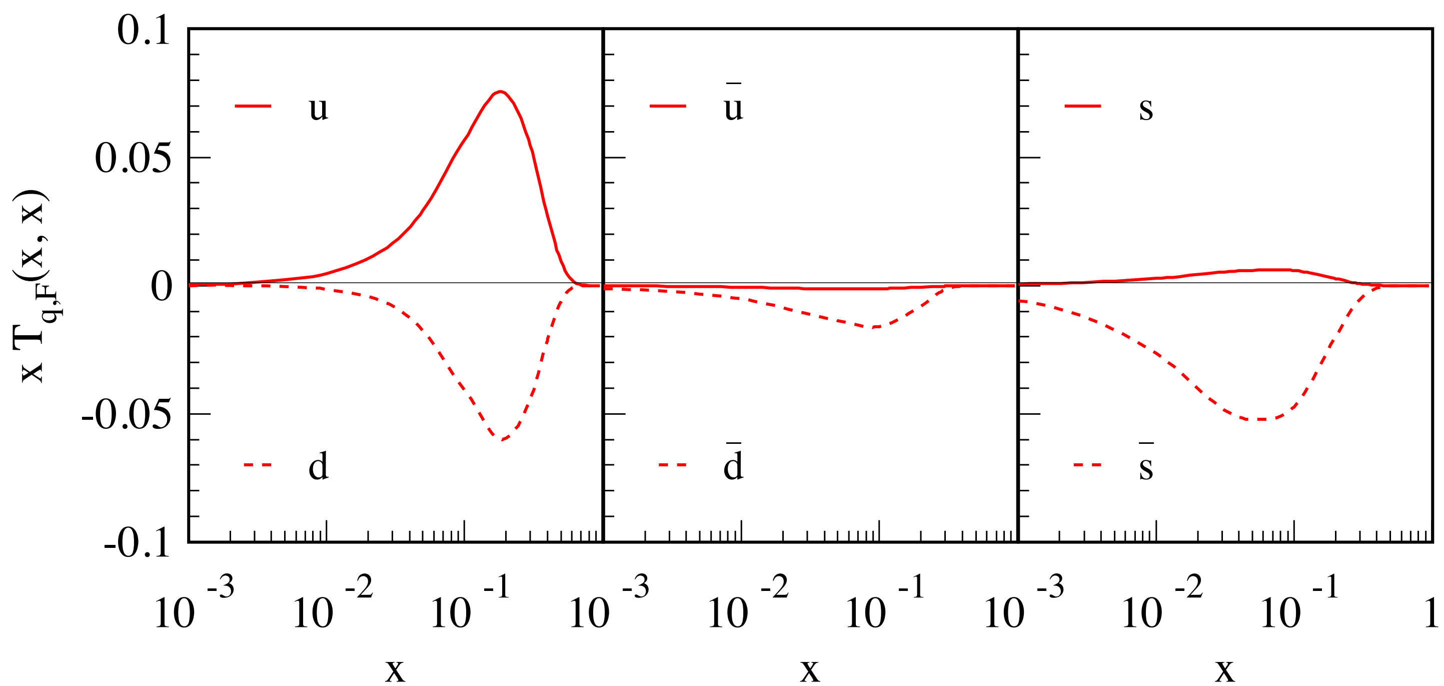
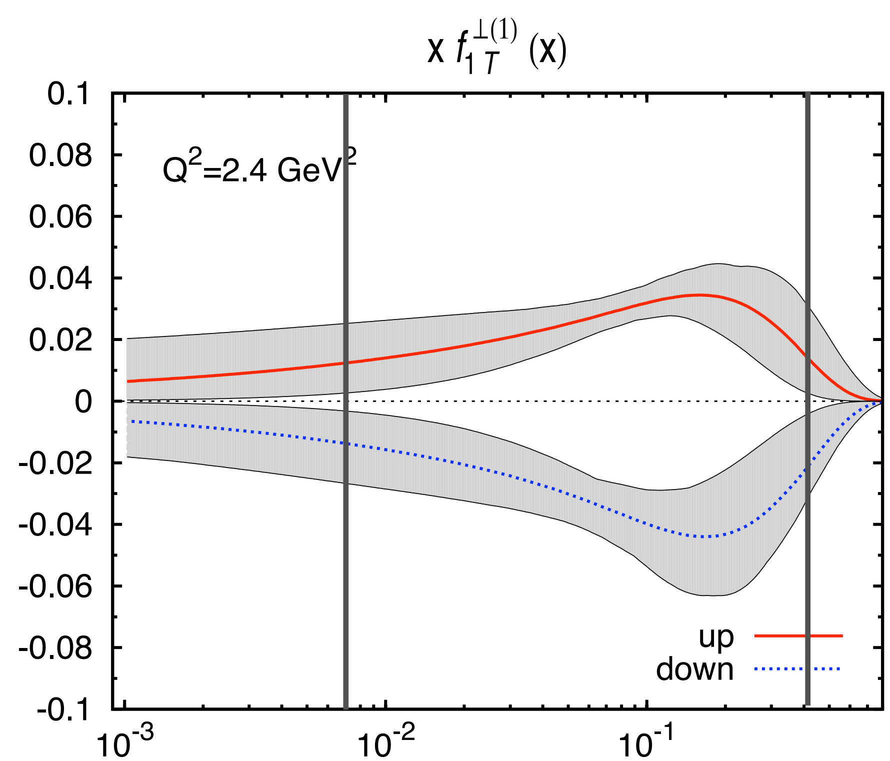


Old fit with no evo

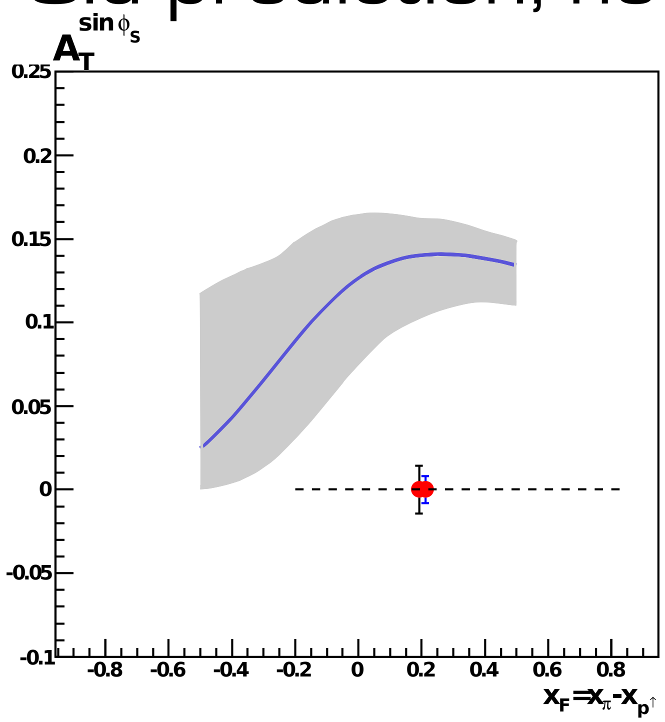


$$f_{1T}^{\perp(1)} = \frac{T_F(x,x)}{2M}$$

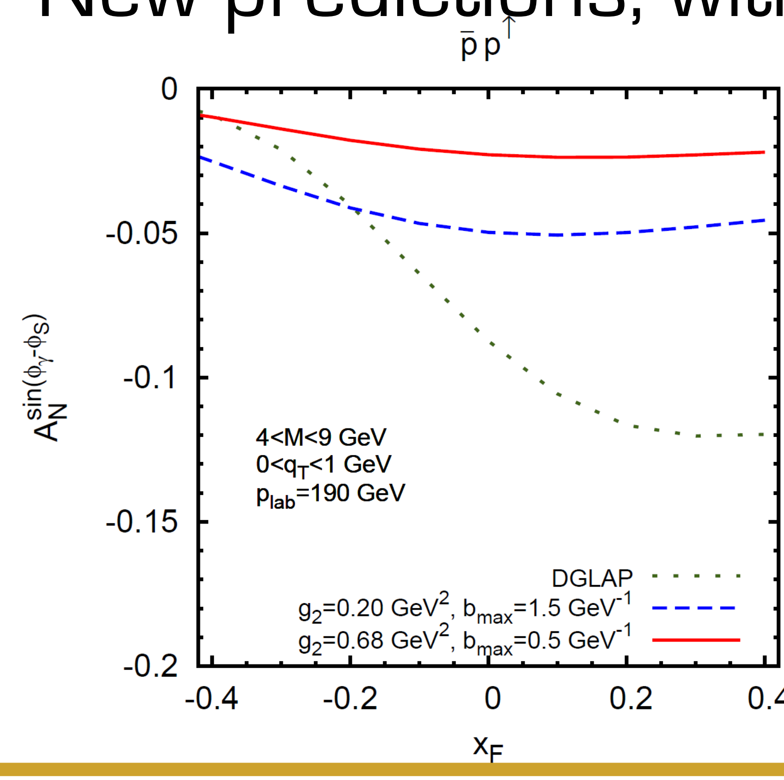
The effect of evolution on Sivers TMDs below 10 GeV^2 is small

Consequences on Drell-Yan predictions

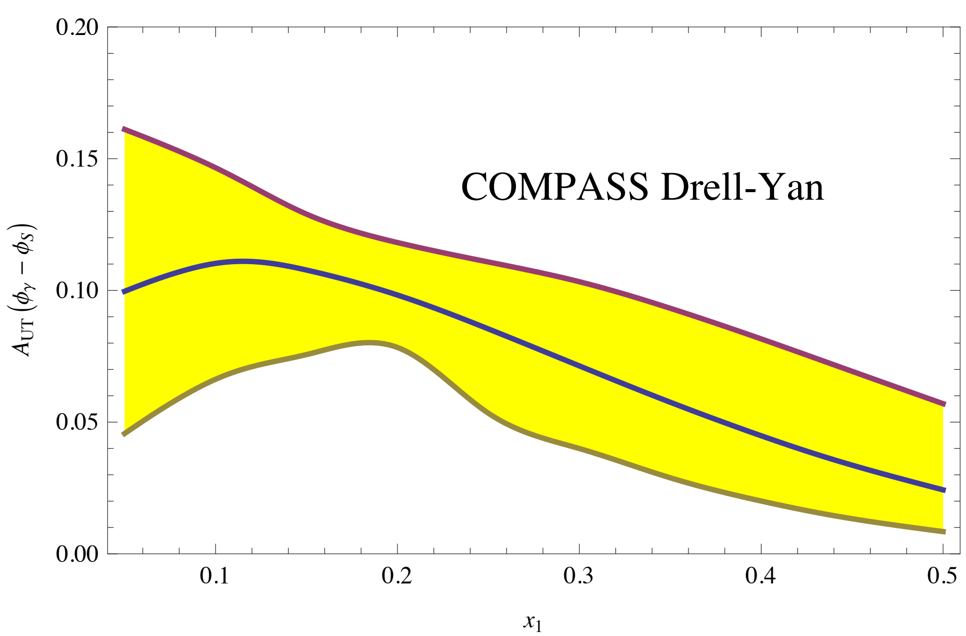
Old prediction, no evo



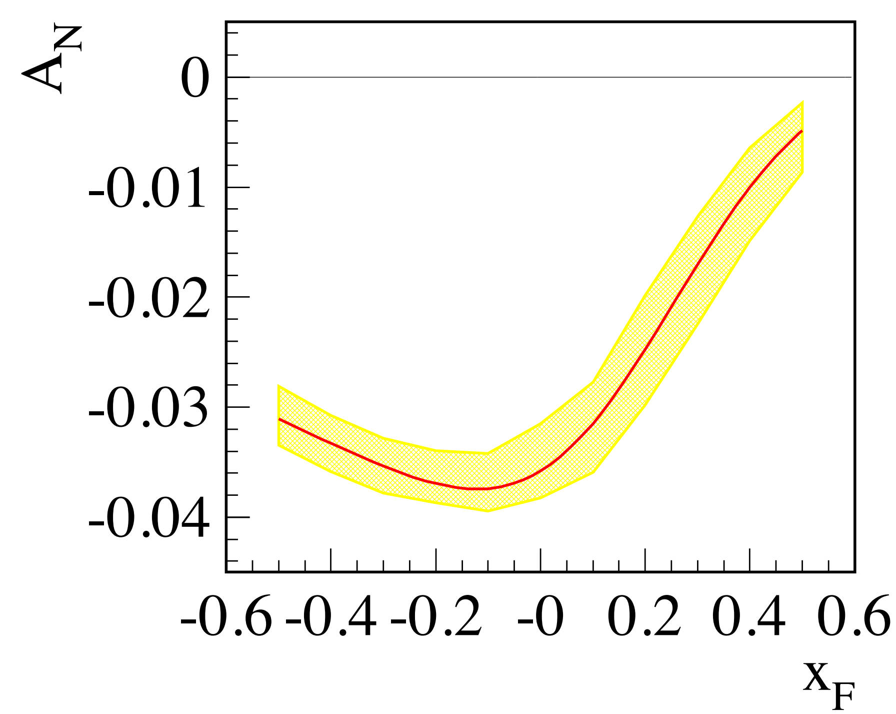
New predictions, with evo



Sun-Yuan, with evo



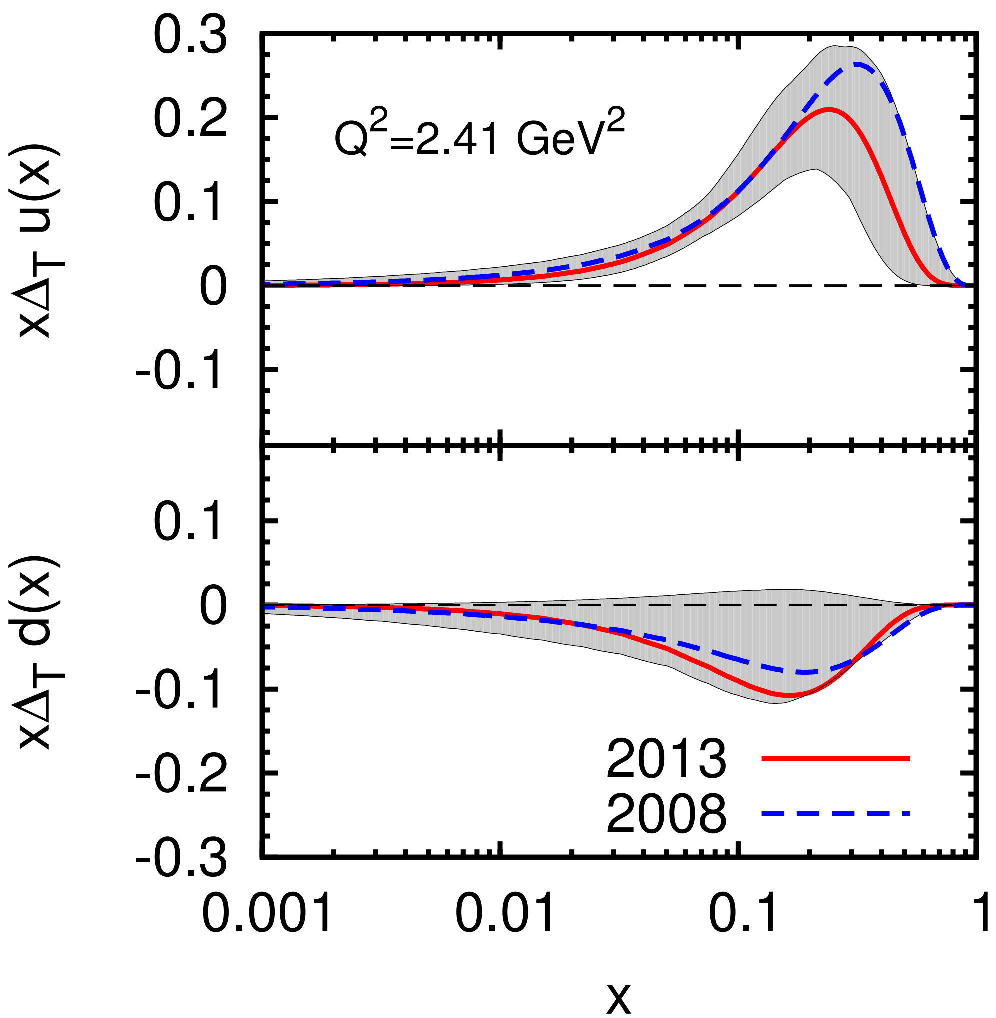
Echevarria et al., with evo



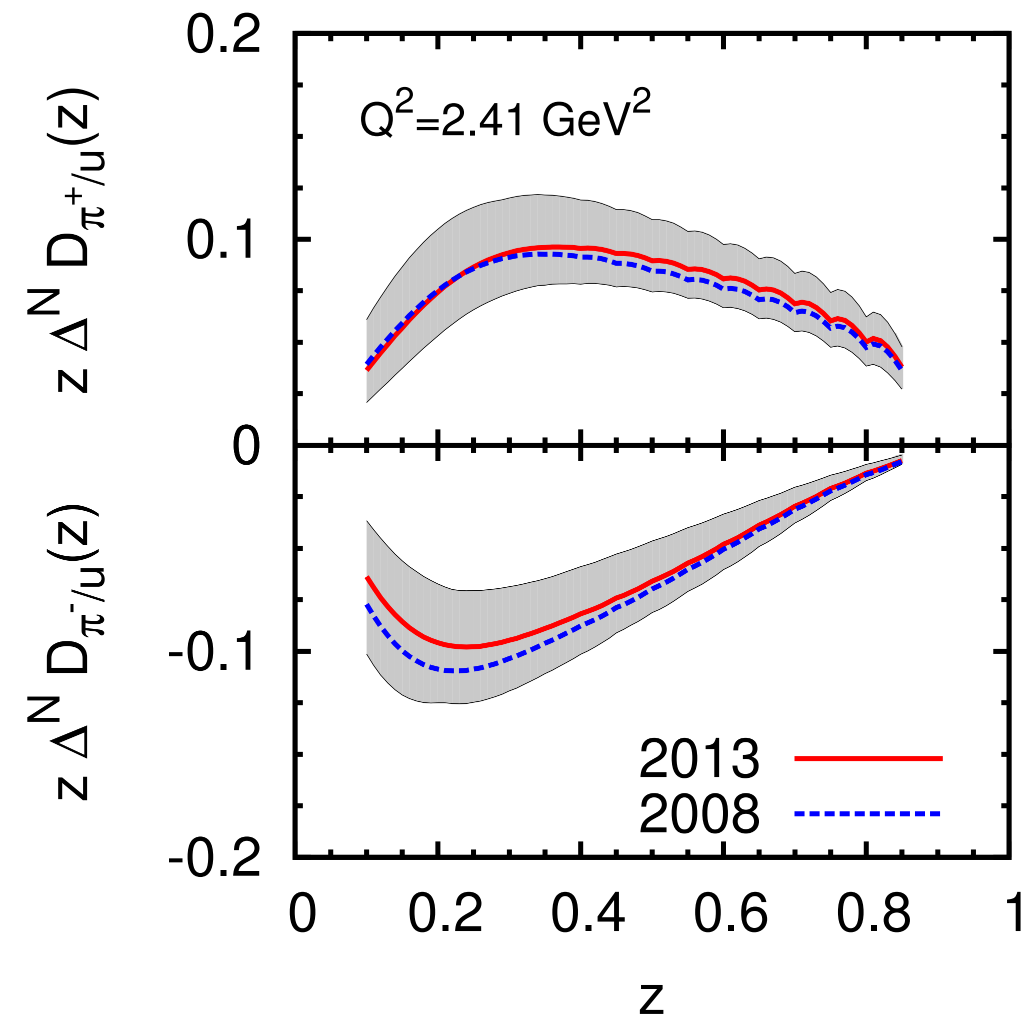
Collins function and transversity

Torino fit 2013, no evo

Anselmino et al., PRD87 (2013)



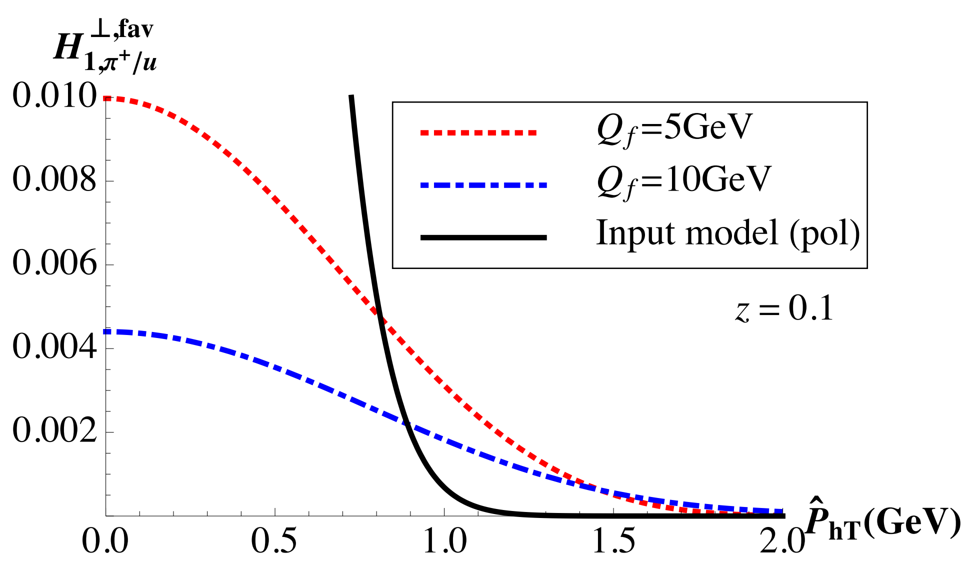
Transversity



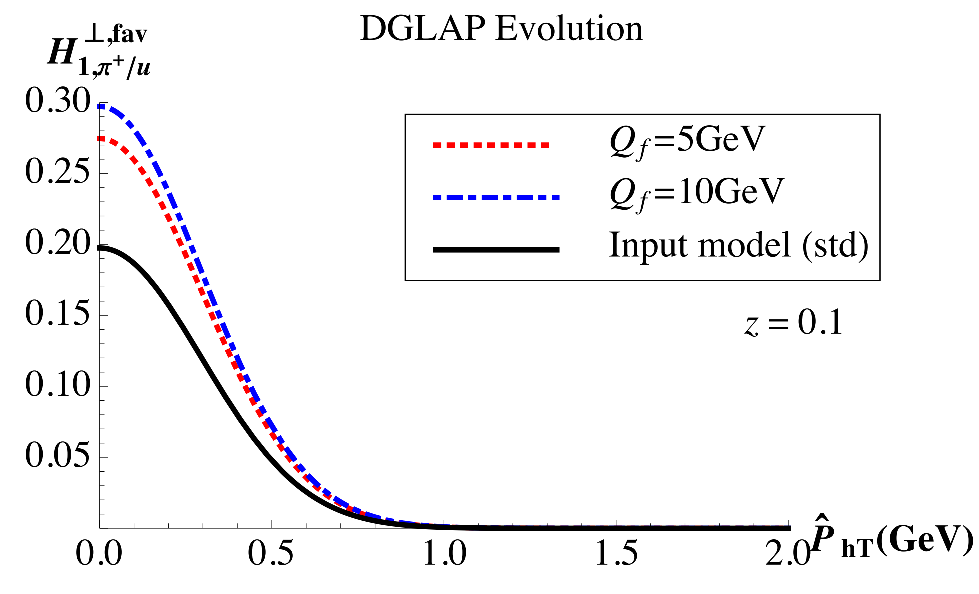
Collins function

TMD evolution of the Collins function

Echevarria, Idilbi, Scimemi, arXiv:1402.0869



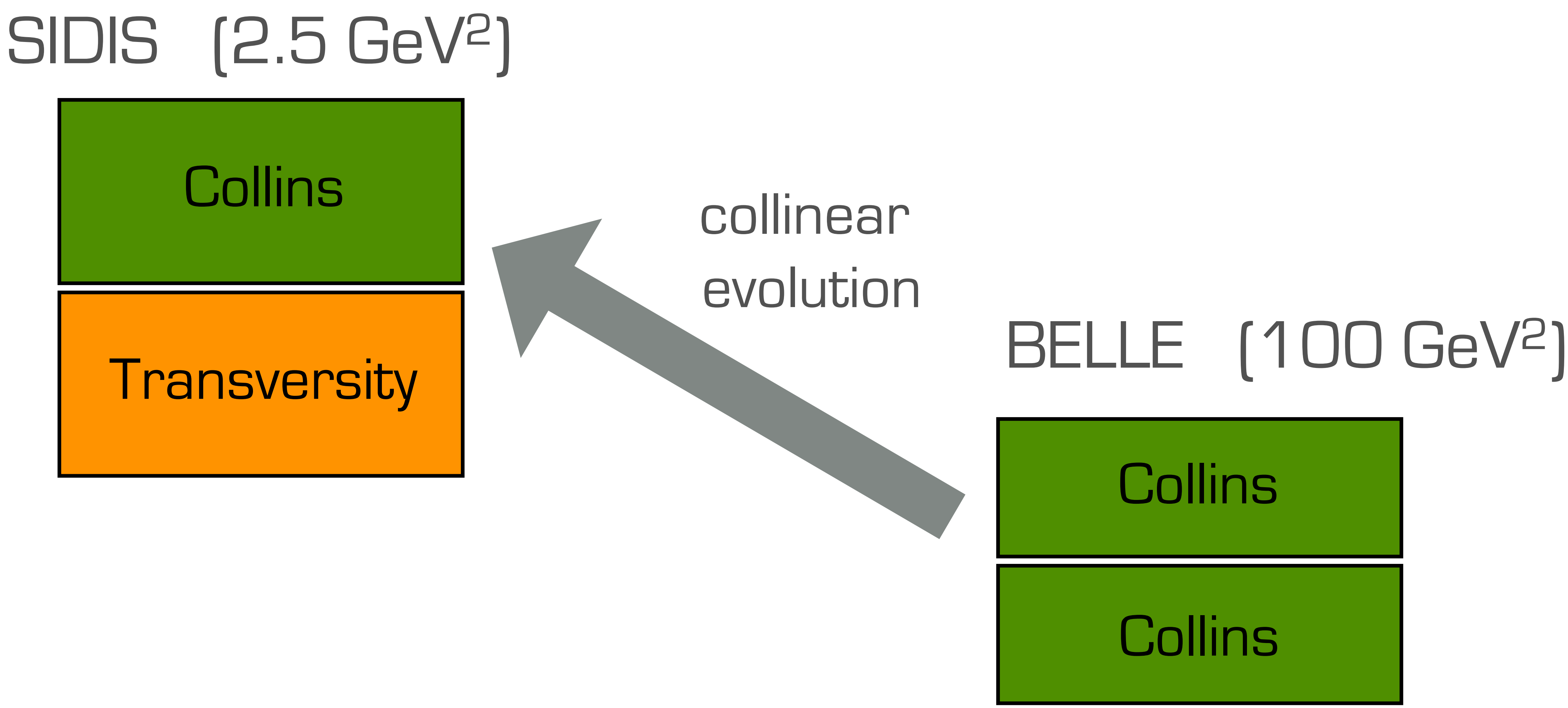
(a)



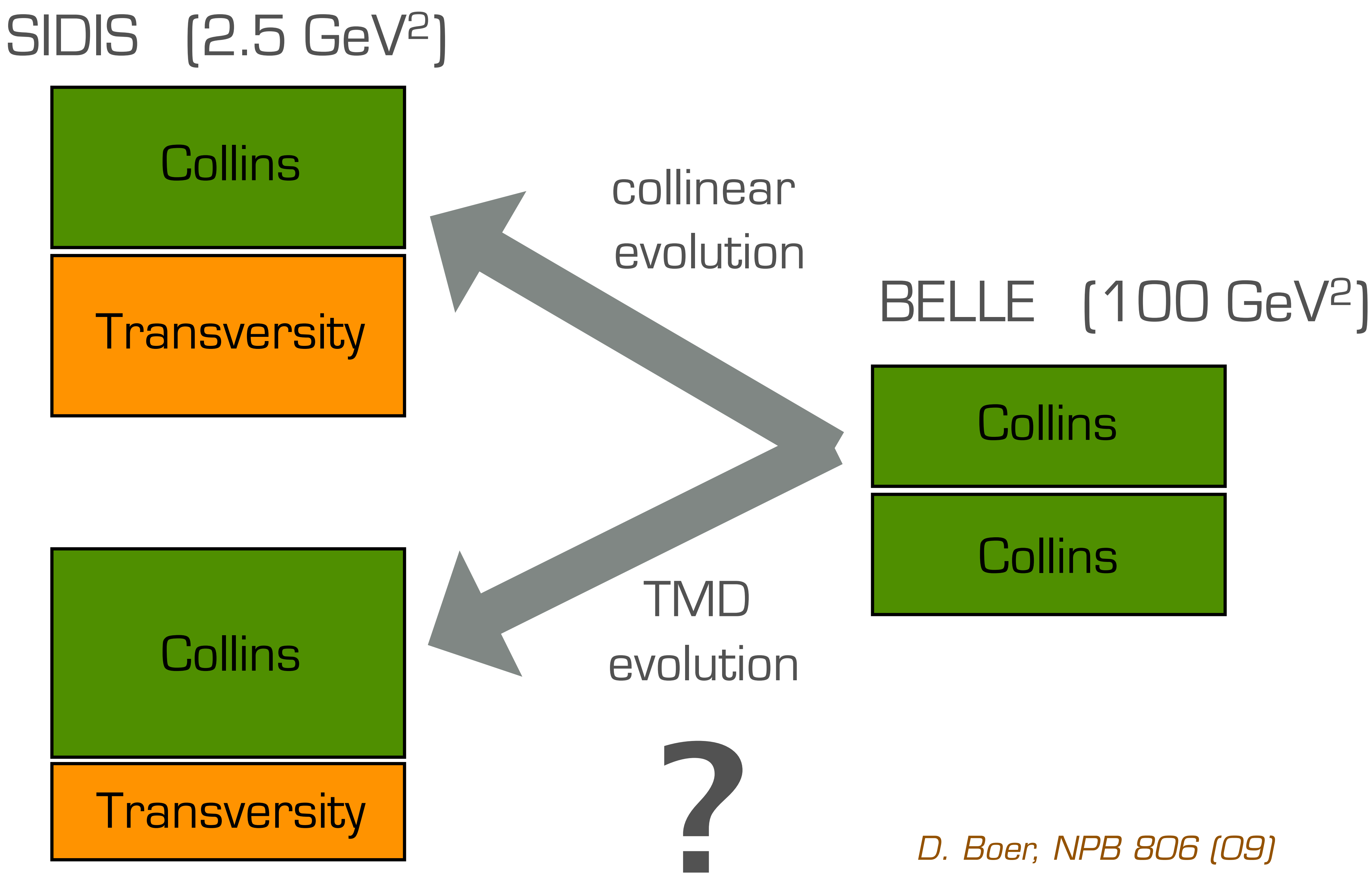
(b)

Very large reduction factors!

Collins function evolution



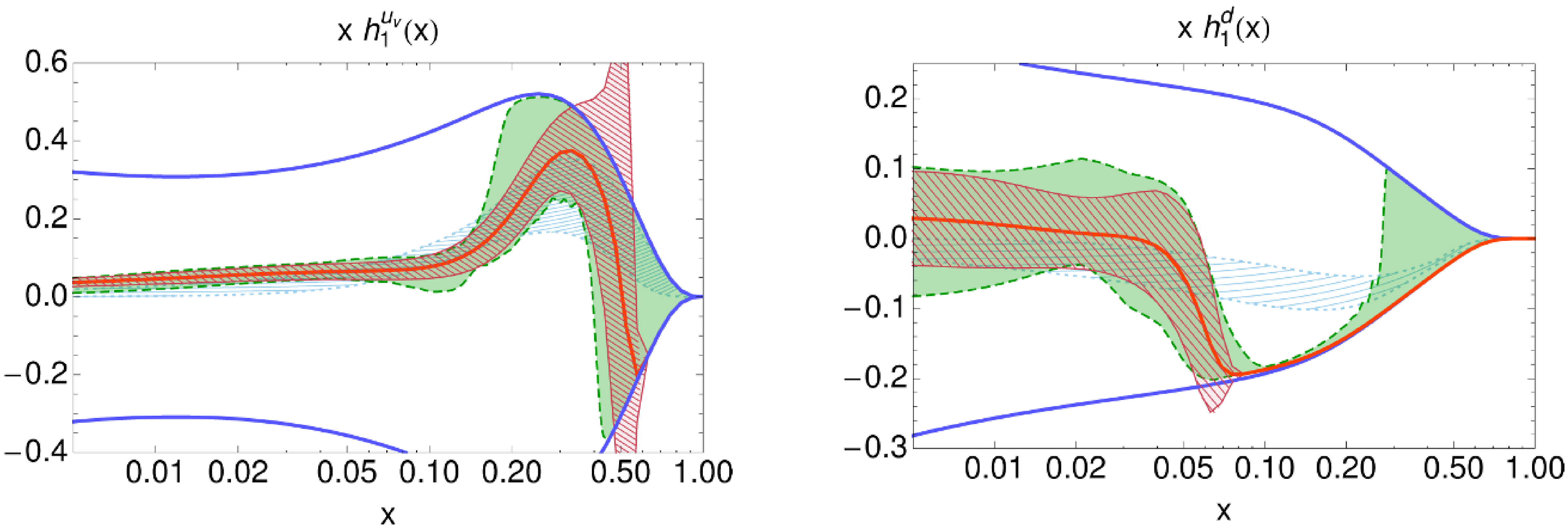
Collins function evolution



D. Boer, NPB 806 (09)

Transversity from dihadron FFs

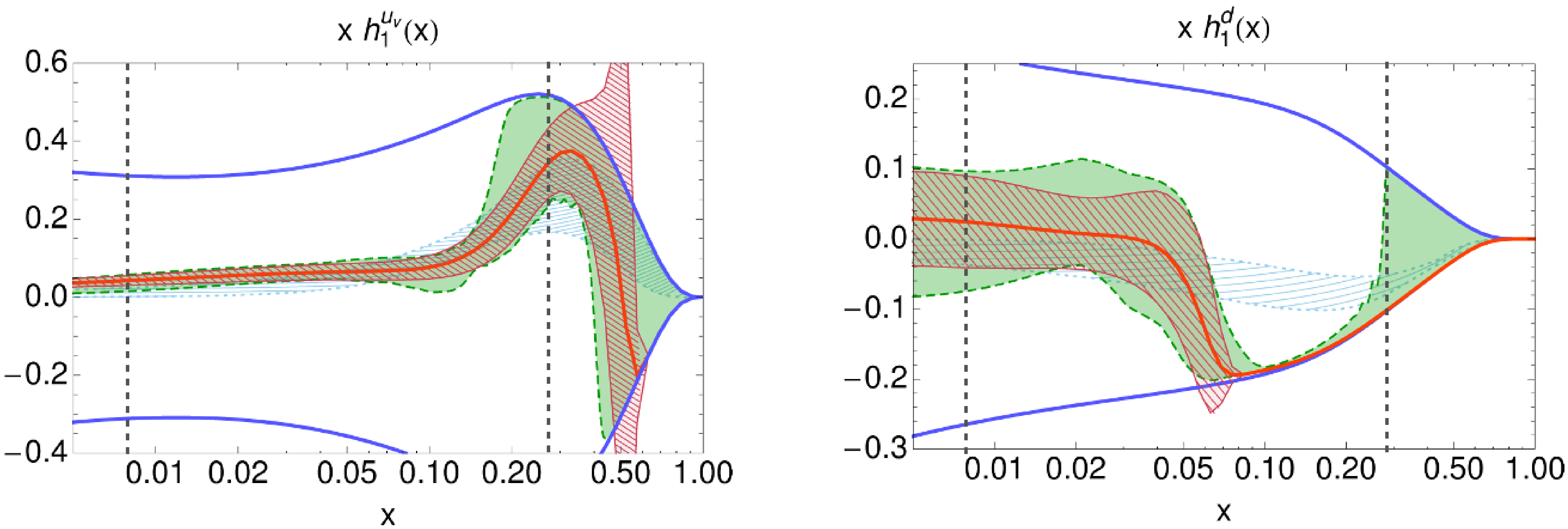
Bacchetta, Courtoy, Radici, JHEP 1303 (2013)



Based on collinear factorization

Transversity from dihadron FFs

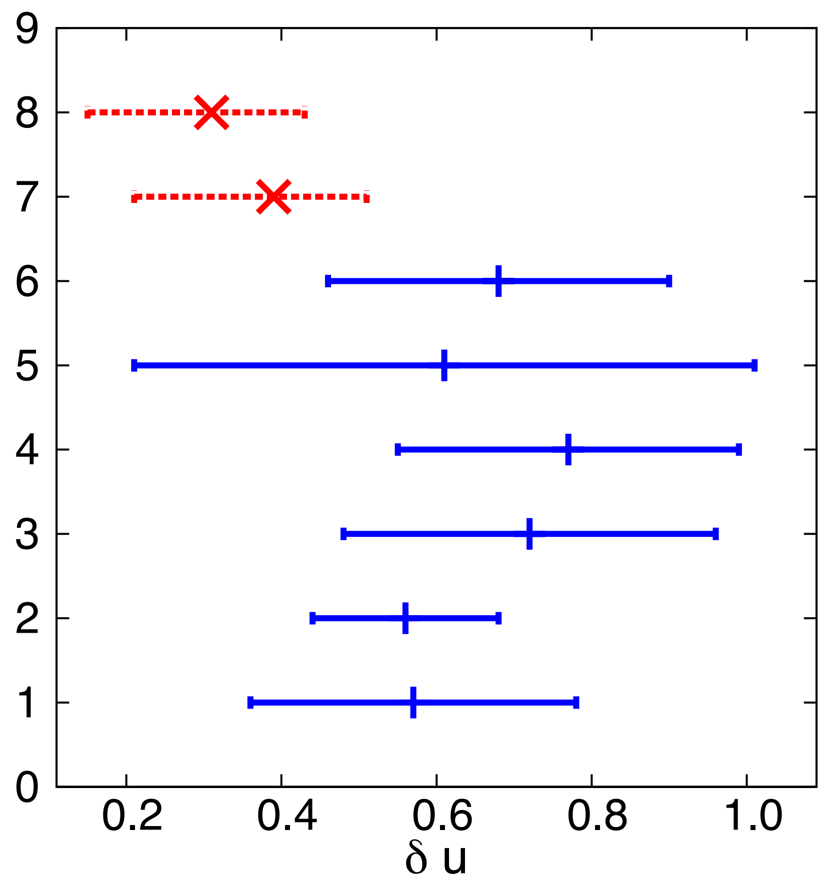
Bacchetta, Courtoy, Radici, JHEP 1303 (2013)



Based on collinear factorization

Tensor charge

$$\delta q = \int_{\sim 0}^{1} dx\, h_1^{q_v}(x)$$



8. fit of A_0

7. fit of A_{12}

6. MC extra flexible

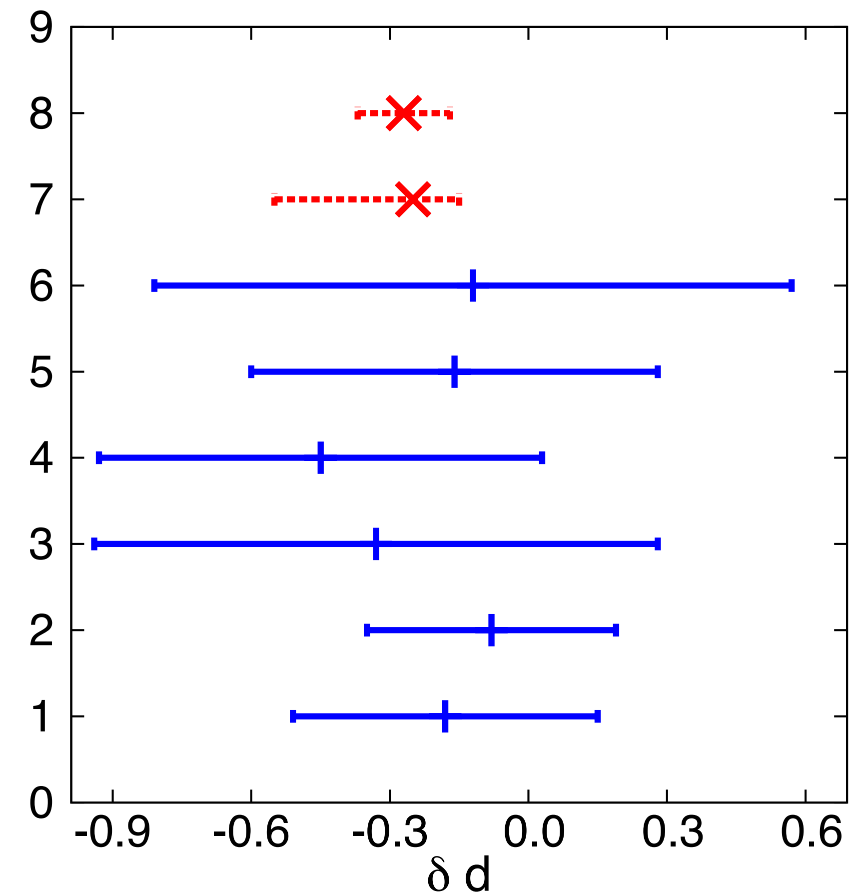
5. standard extra flexible

4. MC flexible

3. standard flexible

2. MC rigid

1. standard rigid



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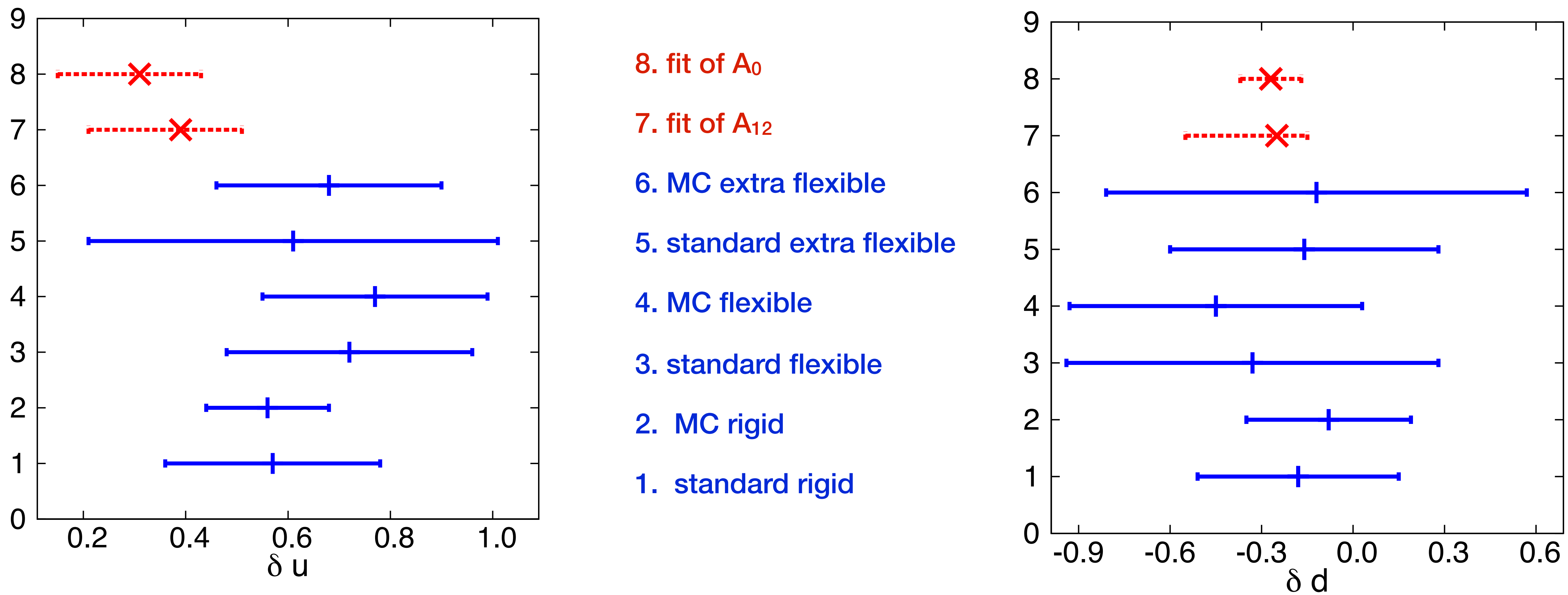

From lattice QCD:

LHPC $\delta u - \delta d = 1.038(20)$

MILC $\delta u - \delta d = 1.083(48)$

see talk by Huey-Wen Lin

Conclusions

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- The Sivers function at low scales is under control
- The Collins function and transversity are not yet under control

TMD “evolution”

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