Polyakov loop correlators and cyclic Wilson loop from lattice QCD

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Overview

1. Motivation and overview
2. Short review: static quark potential and free energies
3. Perturbative predictions
4. Numerical results
    1. Free energies
    2. Screening functions
    3. Thermal modification of the potential
    4. Effective (running) coupling
    5. Screening masses
    6. Cyclic Wilson loops
5. Summary and outlook

Motivation

Screening of colour charges in the deconfined phase

- At finite temperature ($T > T_c$), QCD has deconfined QGP phase
- In QGP, interactions between colour charges are screened

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- At finite temperature ($T > T_c$), QCD has deconfined QGP phase
- In QGP, interactions between colour charges are screened
- Scale separation: in which distance regime is perturbation theory applicable?
- Screened charges: what is the effective running coupling constant for several scales?
- Screening masses: how to define the concept of screening?

[O. Kaczmarek, F. Zantow (2005))], [O. Kaczmarek et. al. (2004))], [O. Kaczmarek (2007))]

Simulation setup

- HotQCD configurations with 2+1 flavours of HISQ fermions
- Lines of constant physics, Goldstone pion mass of $M_\pi \sim 160\, MeV$, $m_s = 20 m_l$

NT	N_{ens}	N_{cfg}	β	T MeV
4	21	$\lesssim 6000$	$[5.900, 8.000]$	$[200, 1400]$
6	19	$\lesssim 3000$	$[6.050, 8.000]$	$[154, 947]$
8	25	$\lesssim 12000$	$[6.245, 8.400]$	$[139, 999]$

- All data shown are preliminary

Free energy of a static $Q\bar{Q}$ pair

Relation to temporal Wilson lines

- Interquark forces between static quarks from static quark 4-point correlation function

$$G_{Q\bar{Q}}(r,T) = \langle W(r) W^\dagger(0) \rangle$$

with temporal Wilson line $W(\mathbf{x}) = \mathcal{P}\exp(ig\int_0^{1/T} d\tau A_0(\tau,\mathbf{x}))$ [L. McLerran, B. Svetitsky (1981)]

- Physical quarks represented by colour-averaged Wilson lines $\mathrm{Tr}_c W/3$ (Polyakov loop)
- Colour singlet/octet components defined with colour projectors $P_{1,8}$

$$G^{1,8}_{Q\bar{Q}}(r,T) = \frac{\mathrm{Tr}_c P_{1,8} G_{Q\bar{Q}}(r,T)}{\mathrm{Tr}_c P_{1,8}} = \exp(-\frac{F_{1,8}(r,T)}{T})$$

[P. Petreczky, hep-lat/0502008]

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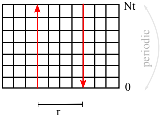

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Singlet and octet configurations not gauge-invariant: fix to Coulomb gauge

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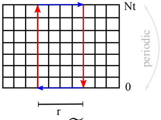

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Free energy of a static $Q\bar{Q}$ pair

Colour-averaged free energy

- Colour-averaged correlation function is gauge-invariant sum of colour components

$$G^{\mathrm{avg}}_{Q\bar{Q}}(r,T) = \frac{\mathrm{Tr}_c(P_1 + 8P_8)G_{Q\bar{Q}}(r,T)}{\mathrm{Tr}_c(P_1 + 8P_8)} = \frac{1}{9}\langle \mathrm{Tr}_c W(r) \mathrm{Tr}_c W^\dagger(0)\rangle$$

- Exponentiated colour-averaged free energy is related to singlet and octet by

$$G^{\mathrm{avg}}_{Q\bar{Q}}(r,T) = \exp(-\frac{F_{\mathrm{avg}}(r,T)}{T}) = \exp(-\frac{F_1(r,T)}{T}) + 8\exp(-\frac{F_8(r,T)}{T})$$

[N. Brambilla et. al. (2010)]

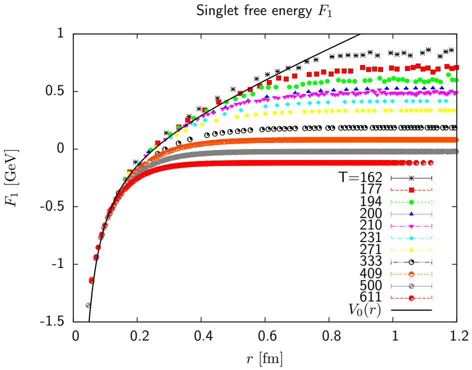
- Colour averaged free energy of single static (anti-)quark is the Polyakov loop

$$G_Q(T) = G_{\bar{Q}}(T) = \langle 1/3\,\mathrm{Tr}_c W(0)\rangle$$

Distance regimes

Very short distance regime (vacuum physics)

- Screening of charges not yet in effect, remnant of confining force seen
- Match $F_1(r, T)$ to static potential $V(r)$ at $T = 0$ for $r < r_0$; r_0 independent of T



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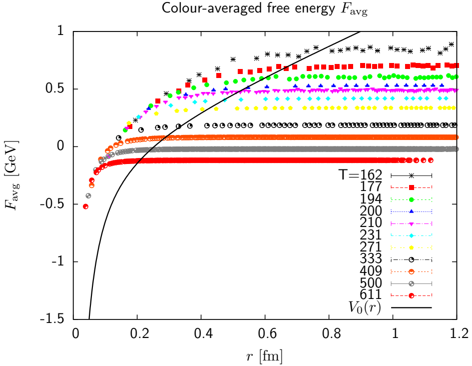
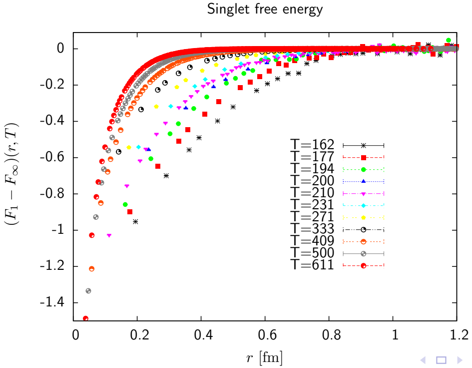


Distance regimes

Large distance regime (medium effects)

- Free energy of two charges in screened potential eventually flattens for large separation
- Contribution of two fully-screened charges $2\langle F_Q(T)\rangle$ independent of colour configuration
- Divide out asymptotic contribution from correlators and study

$$C^1_{Q\bar{Q}}(r,T) = G^1_{Q\bar{Q}}(r,T)/\big(G_Q(T)\big)^2, \quad C^{\rm avg}_{Q\bar{Q}}(r,T) = G^{\rm avg}_{Q\bar{Q}}(r,T)/\big(G_Q(T)\big)^2$$



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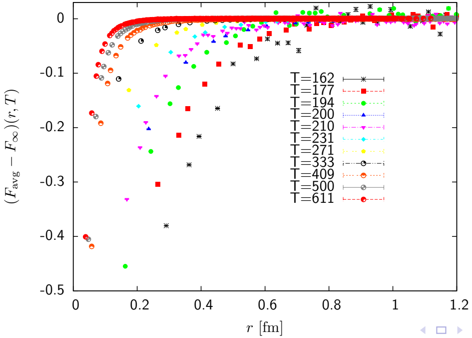
Colour-averaged free energy



Perturbative predictions

Leading contribution to quark anti-quark scattering [P. Petreczky, hep-lat/0502008]

- Quark anti-quark potential perturbatively related to scattering amplitude
- Leading contribution: one-gluon exchange in non-relativistic limit

$$V^{ab}(r) = \langle T^a T^b \rangle g^2 \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} D_{00}(k)$$

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Group structure

- Colour factors $\langle T^a T^b \rangle$ evaluate to $-4/3$ for singlet and $+1/6$ for octet contribution
- Colour-averaged term is subject to cancellations between singlet and octet

Perturbative predictions

Cutoff effects at short distance in $D_{00}(k)$

[S. Necco, R. Sommer (2002))]

[O. Kaczmarek, F. Karsch, P. Petreczky, F. Zantow (2004))]

- $D_{00}(k)$ in Coulomb gauge for Symanzik-improved action sensitive to violation of rotational symmetry at short distance (free theory expression)

$$D_{00}^{-1}(k) = 4a^2 \sum_{i=1}^{3} \Big(\sin^2(\frac{ak_i}{2}) + \frac{1}{3}\sin^4(\frac{ak_i}{2})\Big)$$

- Use of improved radii r_{I} to smooth out lattice artefacts for short distances

$$r_{\mathrm{I}}^{-1} = 4\pi \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} D_{00}(k)$$

- Numerical data shown always uses r_{I} instead of r in the following

Perturbative predictions

Debye mass pole of $D_{00}(k)$ at finite temperature

[E. Braaten, A. Nieto (1995))]

- Gluon propagator has non-trivial infrared limit of $\Pi_{00}(\mathbf{k})$ for $T \neq 0$

$$\Pi_{00}(k \to 0) = m_D^2 = (gT)^2 (N_c/3 + N_f/6) + \mathcal{O}(g^4)$$

- Extract effective coupling $\tilde{\alpha} = \sqrt{(m_D/T)^2/(4\pi)}$ from Debye mass

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Screening for large distances $r \gg 1/T$

[P. Petreczky, hep-lat/0502008]

- For large distances, Debye mass pole dominates $V^{ab}(r) = \langle T^a T^b \rangle \frac{g^2}{4\pi r} e^{(-m_D r)}$

$$F_{1,8}(r,T) = \left(-\frac{4}{3}, +\frac{1}{6}\right) \frac{g^2}{4\pi r} \exp\left(-m_D r\right)$$

- Cancellations in colour-averaged term predict for $r \gg 1/T$

$$F_{\mathrm{avg}}(r) = -\frac{1}{9T} \left(\frac{g^2}{4\pi r}\right)^2 \exp\left(-2 m_D r\right)$$

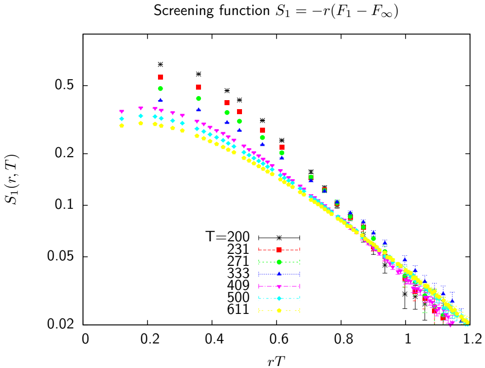
Screening functions

Colour singlet screening function $S_1(r, T)$

- Effective coupling defined by colour singlet screening function $S_1(r, T)$

$$S_1(r,T) = -r(F_1(r,T) - F_\infty(T)) = -r \log C^1_{Q\bar{Q}}(r,T)$$

- Rise for $rT \lesssim 0.5$, weak temperature depence for $rT \gtrsim 0.8$



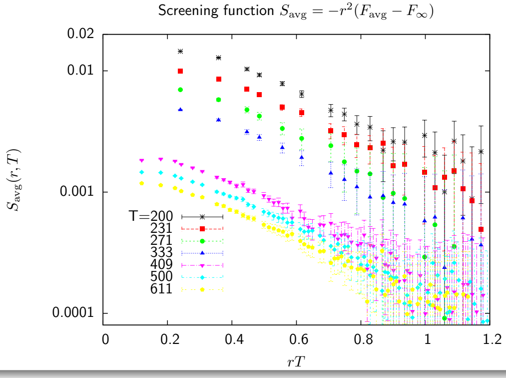
Screening functions

Colour-averaged screening function $S_{\text{avg}}(r, T)$

- Effective coupling defined by colour-averaged screening function $S_{\text{avg}}(r, T)$

$$S_{\text{avg}}(r, T)) = -r^2(F_{\text{avg}}(r, T) - F_{\infty}(T)) = -r^2 \log C^{\text{avg}}_{Q\bar{Q}}(r, T)$$

- Order of magnitude suppression, screening mass weakly temperature dependent



Thermal modification of the potential

- Singlet free energy $F_1(r, T)$ approximates zero temperature vacuum potential $V_0(r)$
- Steep rise of thermal corrections within interval of ≈ 0.1 fm
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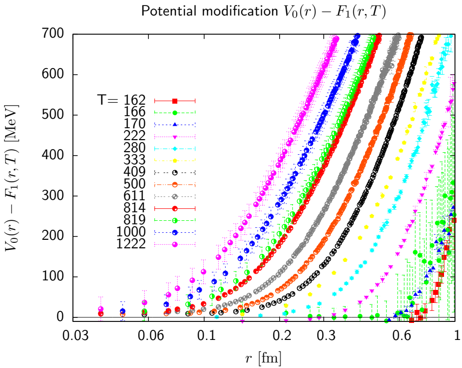

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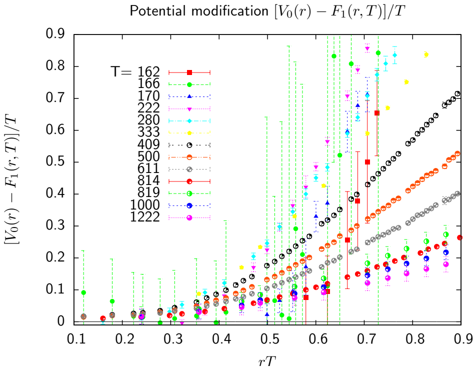

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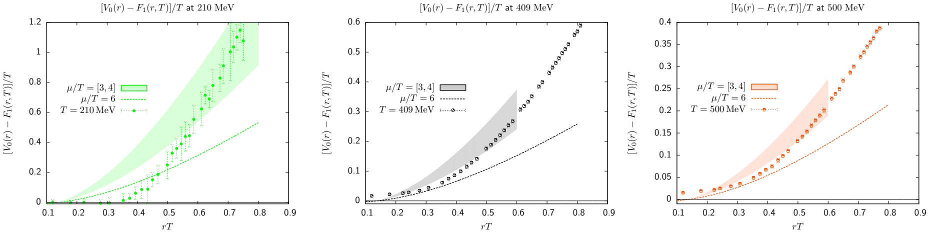
only 1-loop runnning coupling



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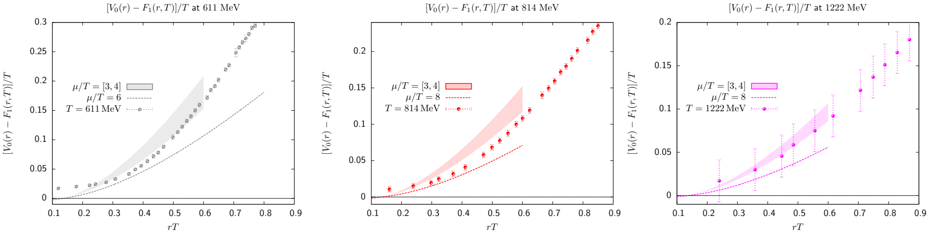
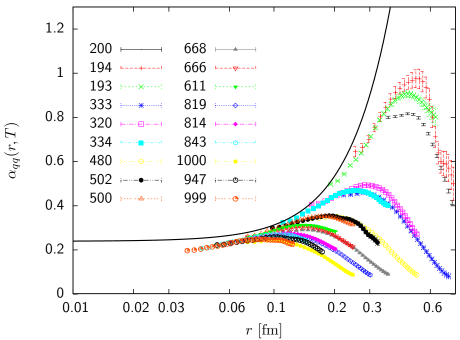


Remnant of confining force

Effective coupling $\alpha_{\mathrm{qq}}(r, T)$

- Interpolate $F_1(r, T)$ in radius r at short distance, compute force as derivative
- Study effective coupling $\alpha_{\mathrm{qq}}(r, T) \equiv \frac{3}{4} r^2 \frac{\partial F_1(r,T)}{\partial r}$ obtained from force $-\frac{\partial F_1(r,T)}{\partial r}$
-

Effective coupling $\alpha_{qq}(r,T)$ at short distance vs radius



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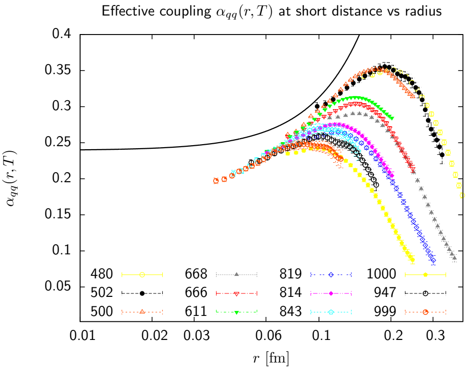

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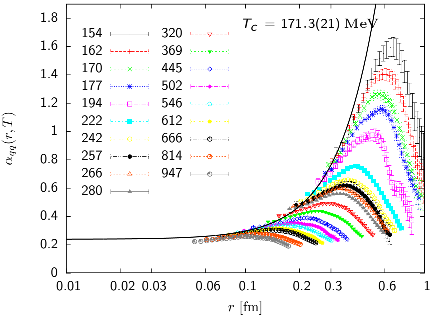
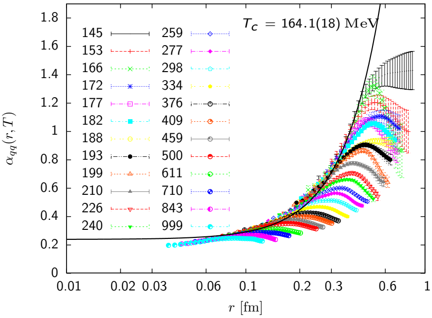
- Quadratic rise of $\alpha_{Q\bar{Q}}(r, T)$, smooth passage through transition temperature T_c

Effective coupling $\alpha_{qq}(r,T)$ vs radius at $(N_t = 6)$



Effective coupling $\alpha_{qq}(r,T)$ vs radius at $(N_t = 8)$

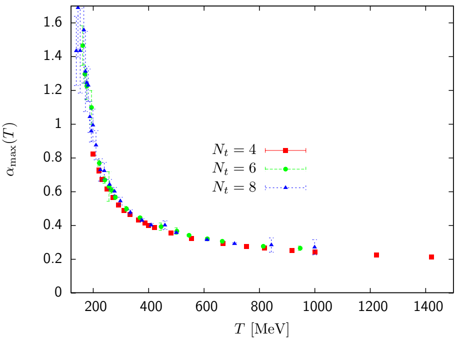


Effective coupling

Onset of screening at radius $r_{\max}(T)$

- Effective coupling exhibits maximum $\alpha_{\max}(T) = \alpha_{qq}(r_{\max}, T)$
- $\alpha_{\max}(T)$ is smooth function of T

Coupling $\alpha_{\max}(T)$ vs temperature



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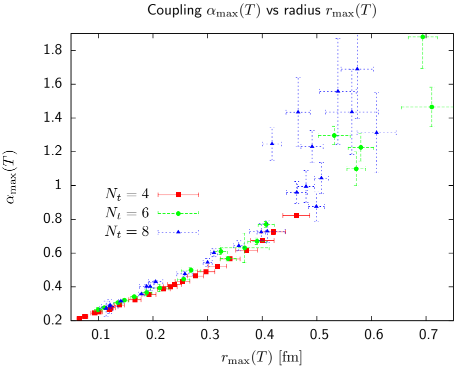

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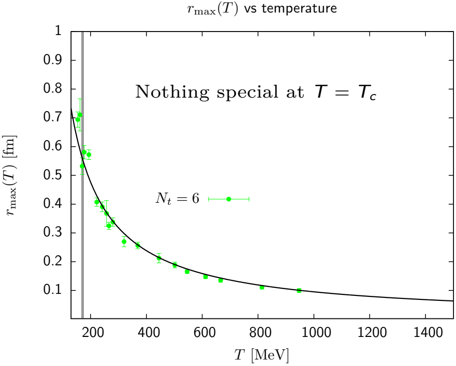

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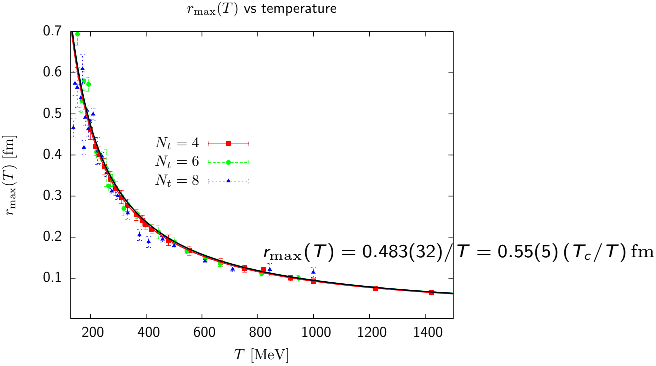

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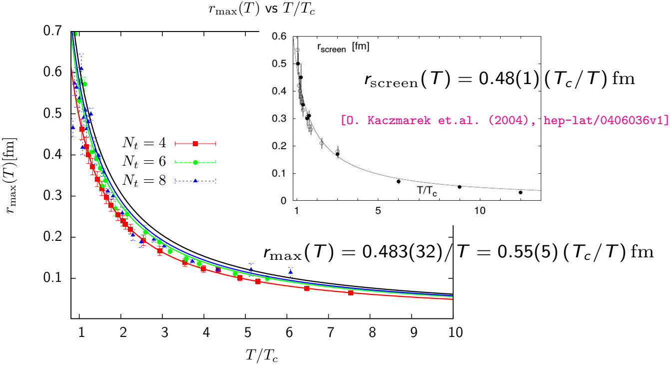
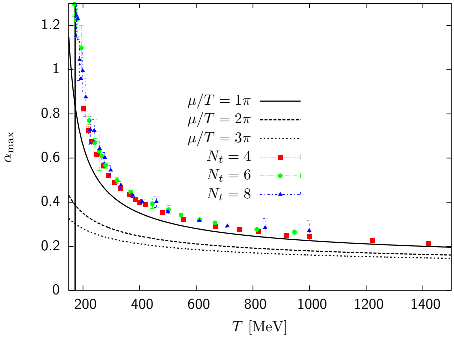


Regime for perturbation theory?

- Coupling $\alpha_{\max}(T) \lesssim 0.5$ for $T > 300$ MeV – start of perturbative regime?

Comparison of $\alpha_{\max}$ to perturbation theory (NLO)



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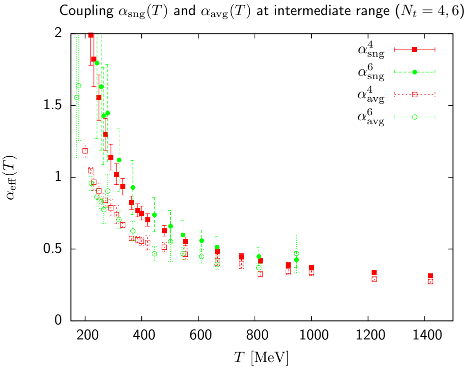

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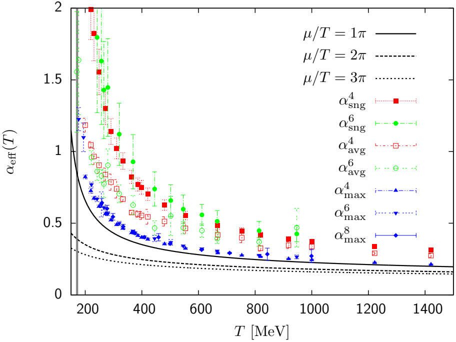
Coupling $\alpha_{\rm sng}(T)$ and $\alpha_{\rm avg}(T)$ vs T (NLO, $N_t = 4, 6$)



Comparison to perturbation theory

Screening masses

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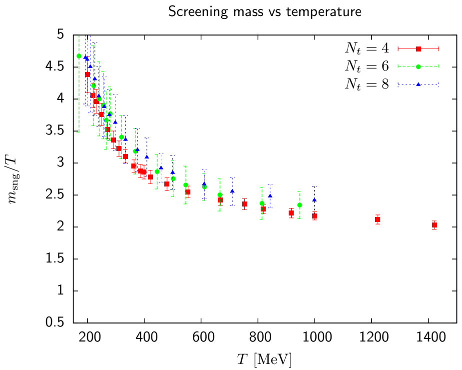
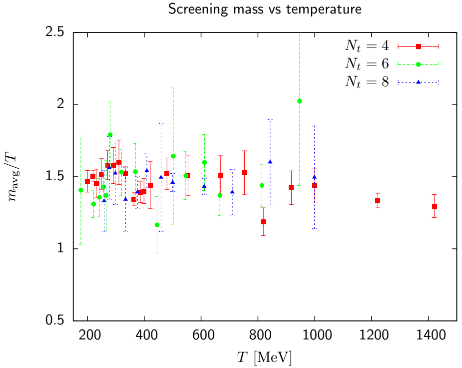


Comparison to perturbation theory

Screening masses

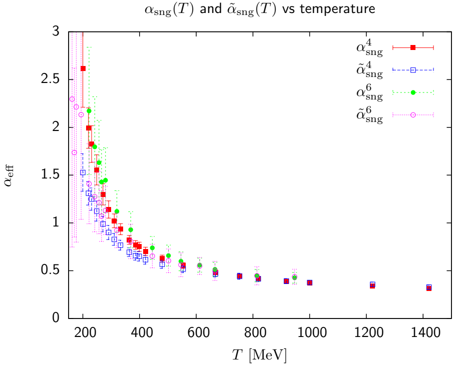
- Determine screening masses m_{sng} and m_{avg} for $r_{\max}(T) \lesssim r \lesssim 1/T$
- No logarithmic rise of screening mass m_{avg} for low temperatures



Comparison to perturbation theory

Relation between running coupling and screening mass

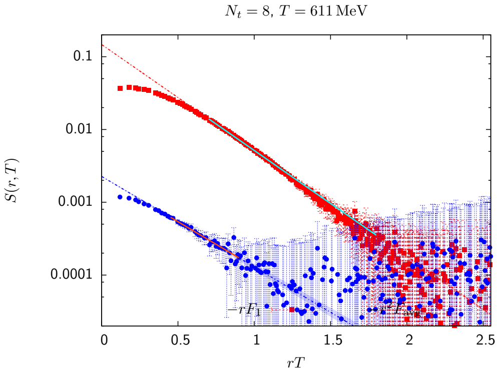
- $\widetilde{\alpha}_{\rm sng}(T) = \sqrt{(m_{\rm sng}/T)^2/(4\pi))}$ and $\alpha_{\rm sng}$ consistent for $T \gtrsim 500\,\mathrm{MeV}$
- Effective coupling indicates that $T < 300\,\mathrm{MeV}$ is strongly coupled QGP



Large distance regime

Weakly coupled QGP

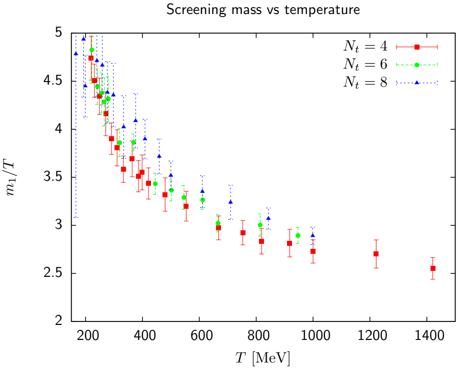
- Long distance regime $r \ll 1/T$ is weakly coupled QGP
- Fit screening functions with perturbative ansatz $S_1(r,T) = \frac{4}{3}\alpha_1 \exp(-m_1 r)$
- General underestimation of Debye mass taken care of as systematic error



Debye mass

Cutoff effects and continuum limit

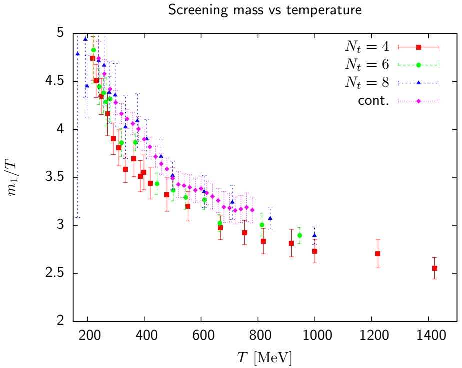
- Conservative estimate of systematic error and evidence of cutoff effects
-



Debye mass

Cutoff effects and continuum limit

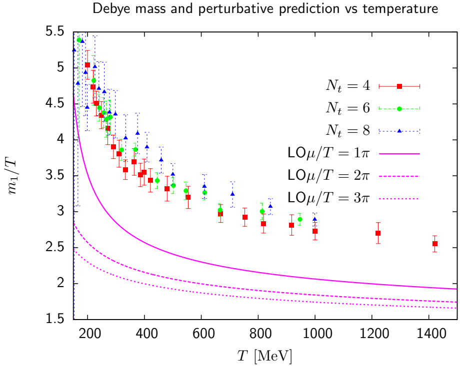
- Conservative estimate of systematic error and evidence of cutoff effects
- Continuum limit agrees with $N_t = 8$ within errors



Debye mass

Comparison of m_D/T with LO and NLO prediction [E. Braaten, A. Nieto (1995))]

- Leading order prediction below data



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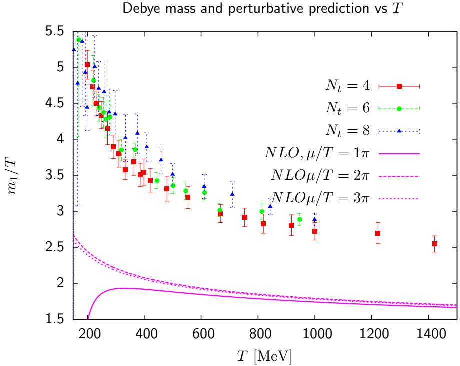
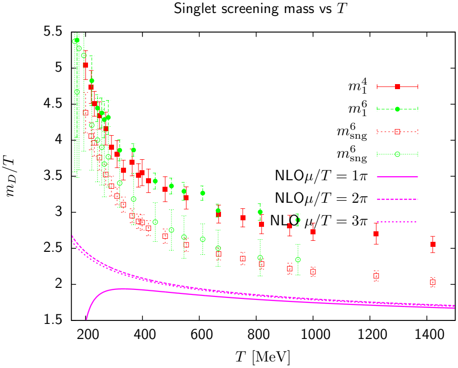


Debye mass

Comparison of m_D/T with LO and NLO prediction [E. Braaten, A. Nieto (1995))]

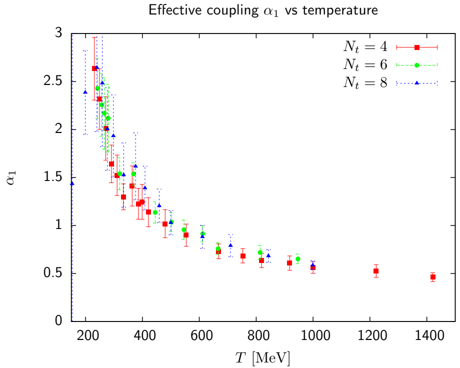
- Leading order prediction below data, next-to-leading order even lower
- Screening masses at intermediate distance lie closer to perturbative prediction



Effective coupling

Effective coupling α_1

- Perturbative prediction $\alpha_1 \to \alpha_s$



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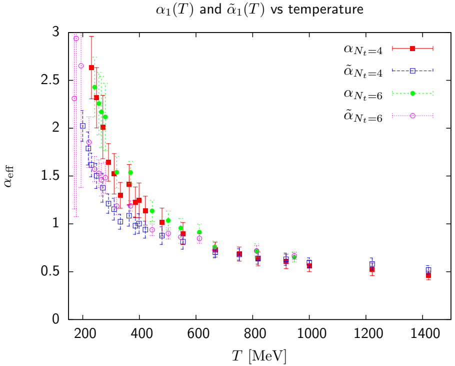

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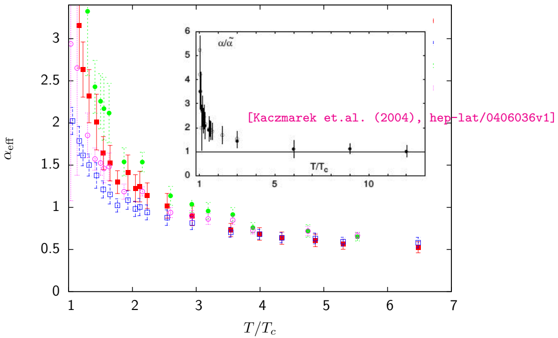
$\alpha_1(T)$ and $\tilde{\alpha}_1(T)$ vs T/T_c



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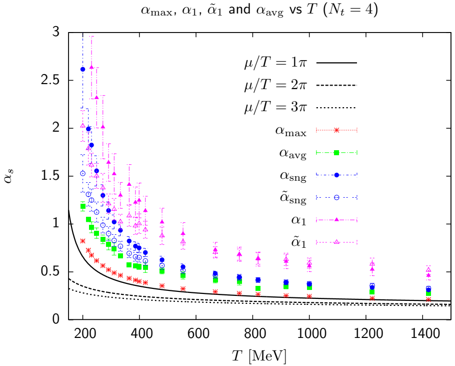
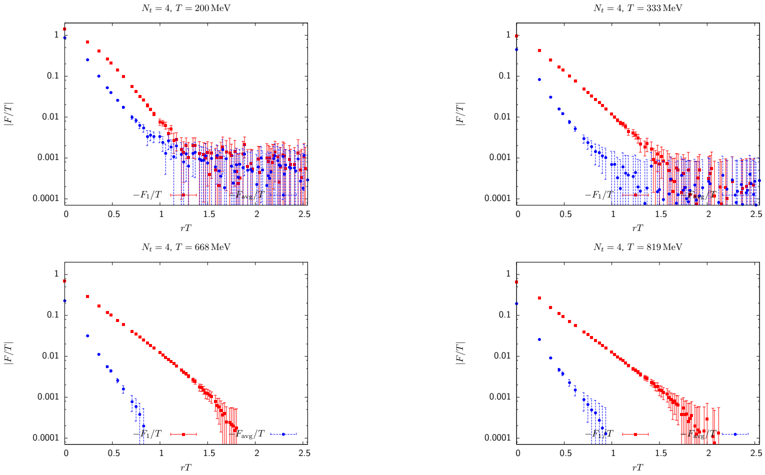


Summary & Outlook

Summary & Outlook

- Study of colour-singlet and colour-averaged free energies is work in progress
- Short distance effects in medium: thermal modification of the potential
- Intermediate distance: onset of screening
- Large distance: Debye screening, weakly coupled, but non-perturbative.
- $T < 300$ MeV: strongly coupled QGP. $T > 500$ Mev: weakly coupled QGP.
- Correlation of running coupling and screening mass in weakly coupled QGP.
- Need more statistics for study of colour averaged free energy
- Utilise cyclic Wilson loops for study of singlet free energy
- Still far away from perturbative regime

Free energies $F_1/T = -\log C_1(r,T)$ and $F_{\rm avg}/T = -\log C_{\rm avg}(r,T)$



$F_{\rm avg}(r,T)$ decays faster than $F_1(r,T)$ in high temperature region ($T > 2T_c \simeq 310\,$MeV)

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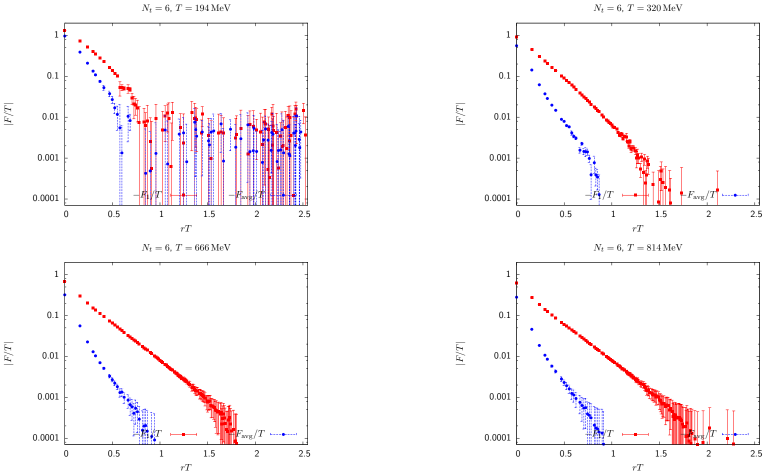
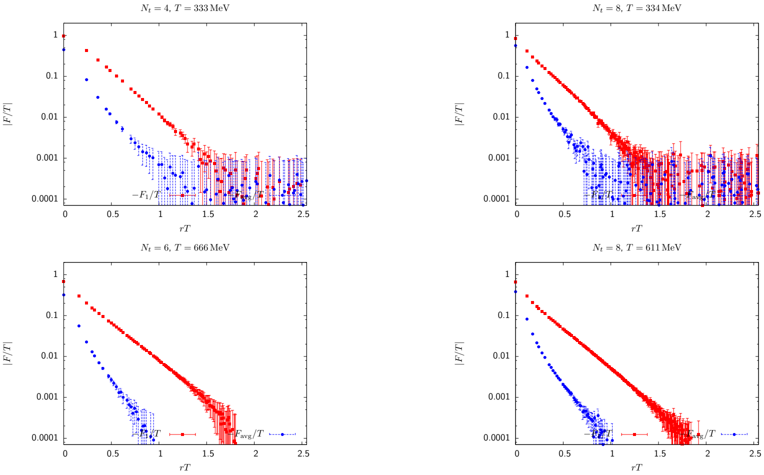


Quantitatively good agreement between $NT = 4$ and $NT = 6$

Free energies $F_1/T = -\log C_1(r,T)$ and $F_{\mathrm{avg}}/T = -\log C_{\mathrm{avg}}(r,T)$



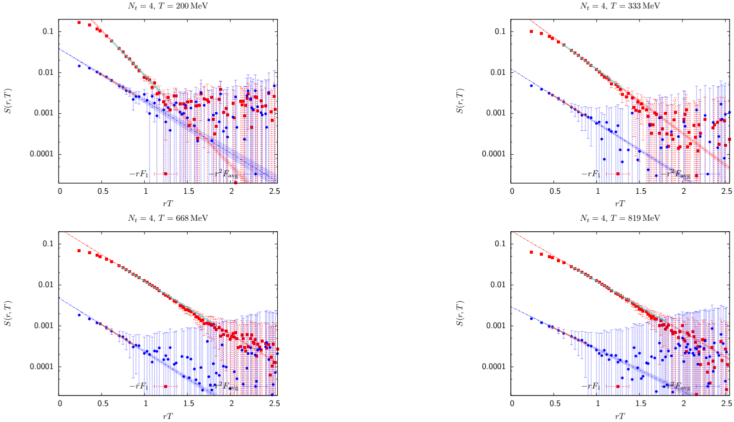
Signal-to-noise ratio worse. Much more statistics required.

Screening functions $S_1 = -rF_1(r,T)$ and $S_{avg} = -r^2 F_{avg}(r,T)$

$NT = 4$

[P. Petreczky, hep-lat/0502008]

- Fit as $F_1(r,T) = -\frac{4}{3}\alpha_1 \frac{\exp(-m_1 r)}{r}$ and $F_{avg}(r,T) = -\frac{1}{9}\alpha_{avg}^2 \frac{\exp(-2m_{avg} r)}{r^2}$



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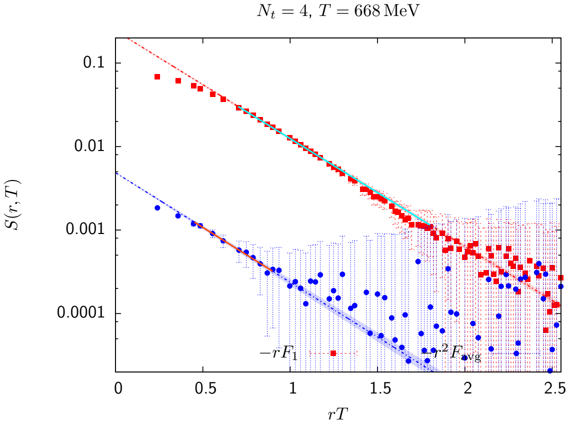

Screening functions $S_1 = -rF_1(r,T)$ and $S_{\rm avg} = -r^2 F_{\rm avg}(r,T)$

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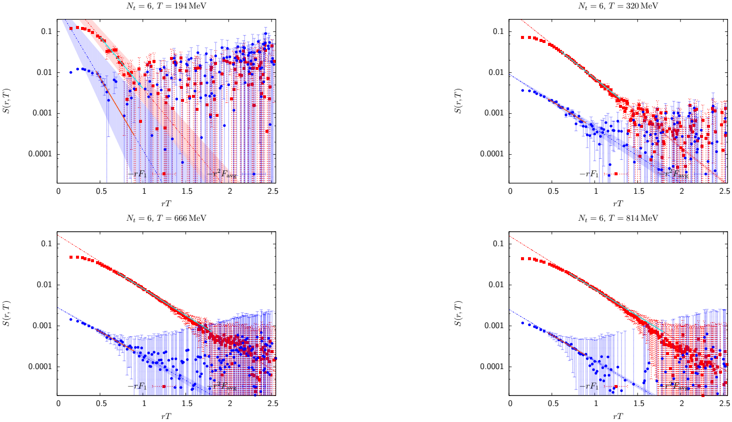

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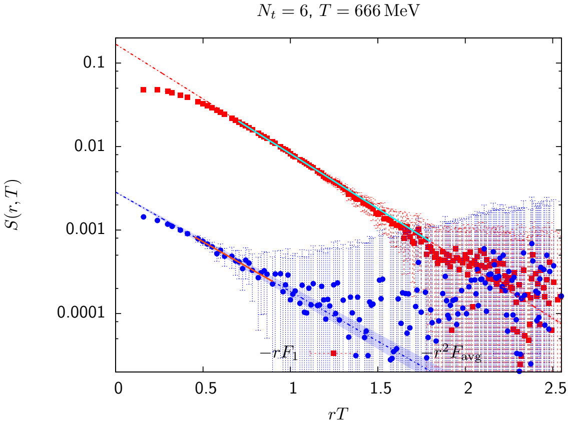
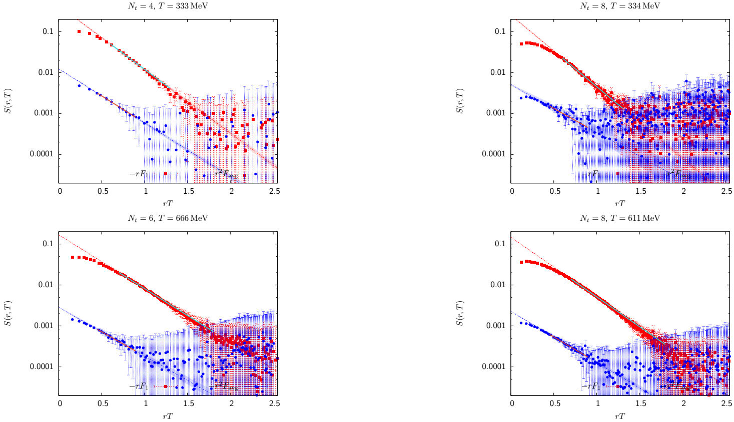


Screening functions $S_1 = -rF_1(r,T)$ and $S_{\rm avg} = -r^2 F_{\rm avg}(r,T)$

$NT = 8$

- Screening masses m_1 and $2m_{\rm avg}$ not related by factor two



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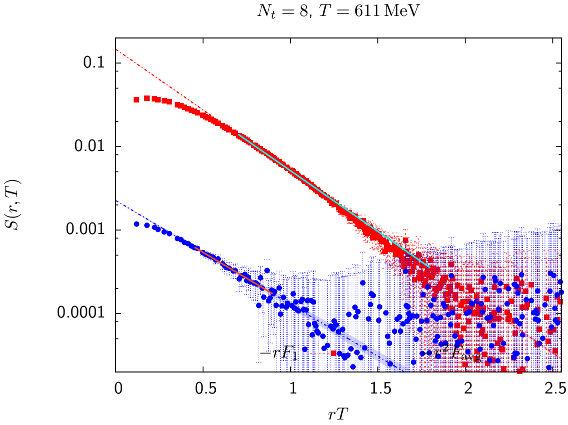
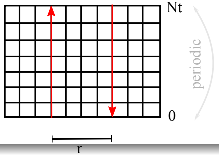


Cyclic Wilson loops

Relation to singlet free energy

- Cyclic Wilson loop $W_c(r, T) = \frac{1}{3}\mathrm{Tr}_c\langle S(r;0)W(r)S^\dagger(r;N_t)W^\dagger(0)\rangle$
- Cyclic Wilson loop and potential loop both observables for $\exp(-F_1(r, T)/T)$

$\frac{1}{3}\mathrm{Tr}_c\langle W(r)W^\dagger(0)\rangle$



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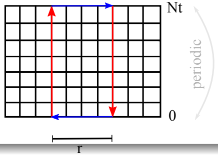

Cyclic Wilson loops

Why potential loops are easier

- Zero temperature Wilson loop: *cusp* divergences at corners
- Cyclic Wilson loop: additional *intersection* divergences [M. Berwein, N. Brambilla, A. Vairo (2014)]
- Divergences not present in Coulomb gauge or for link-smeared Wilson loops

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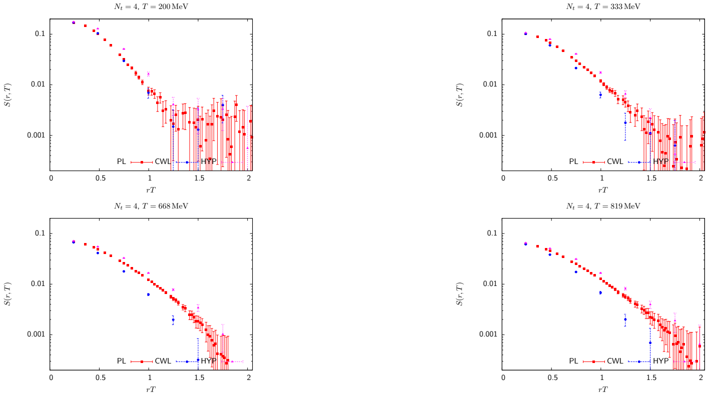
Why cyclic Wilson loops are still very interesting

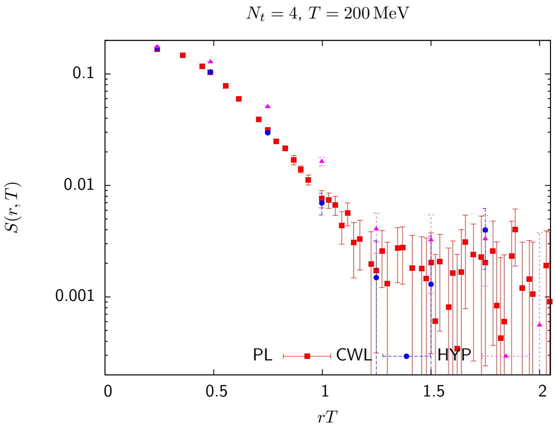
- Gauge-invariant difference $\left[W_c - P_{\text{avg}}\right](r,T)$ projected to colour octet with Polyakov loop correlator $P_{\text{avg}} = \frac{1}{9}\langle \text{Tr}_c W(r) \text{Tr}_c W^\dagger(0)\rangle$
- Renormalisation as $[W_c - P_{\text{avg}}]^{\text{ren}}(r,T) = \exp[-2\Lambda_F/T - \Lambda_A r]\, Z_{int}\, [W_c - P_{\text{avg}}](r,T)$ with Λ_F, Λ_A lineraly divergent constants [M. Berwein, N. Brambilla, A. Vairo (2014)]
- Divergence-free ratio $\frac{[W_c - P_{\text{avg}}](r,T)[W_c - P_{\text{avg}}](2r_0 - r,T)}{([W_c - P_{\text{avg}}](r_0,T))^2}$ useful for colour octet study? [M. Berwein (2014)]

Cyclic Wilson loops



- CWL computed without gauge fixing

Cyclic Wilson loops



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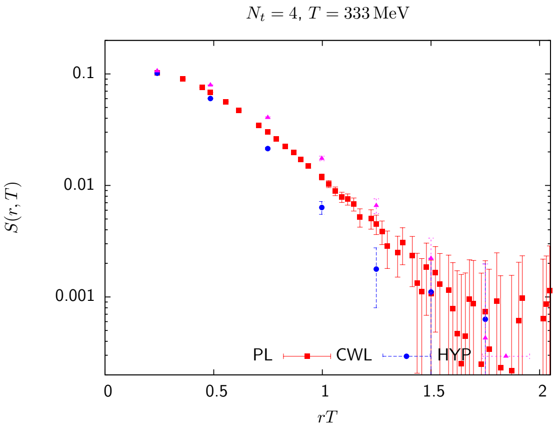

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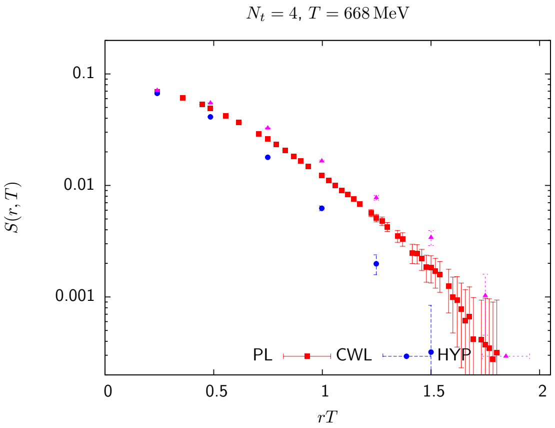

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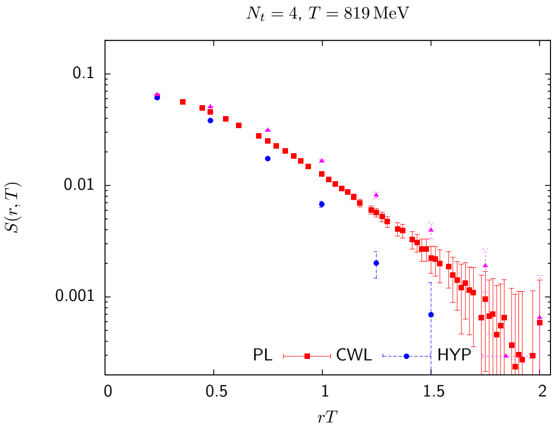

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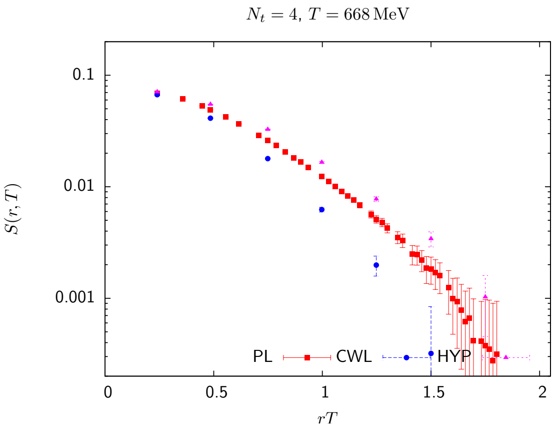
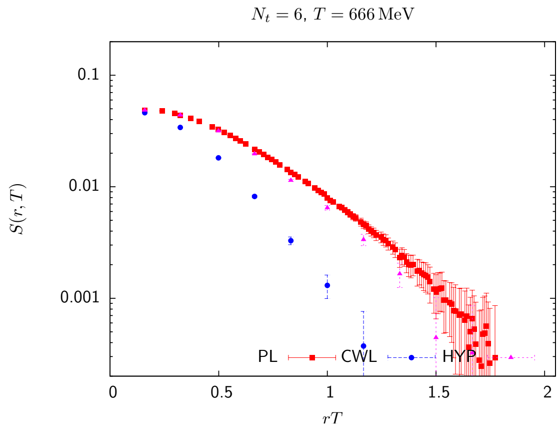


- Finite size effects in CWL
- Comparable in smeared and unsmeared CWL

Cyclic Wilson loops



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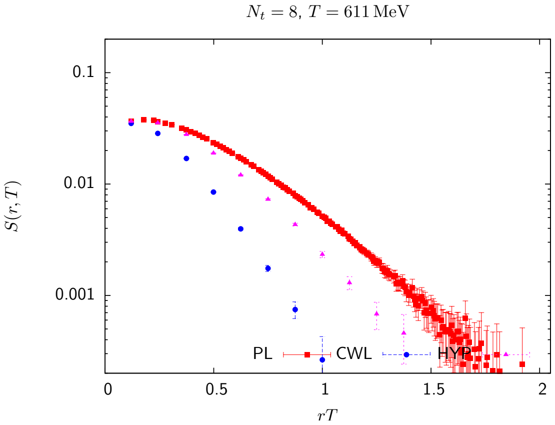

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